
ELECTRICAL SENSORS AND TRANSDUCERS

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MAGNETIC SENSORS

CHAPTER OBJECTIVES

1. To understand the theory and operation of the following magnetic field-sensing devices:
 - a. The magnetic reed switch
 - b. The Hall effect transistor
 - c. The inductive pickup coil
 - d. The vibrating wire transducer

4-1 INTRODUCTION

In this chapter we study those sensors that either use or interact with a magnetic field to cause the sensor to operate. This subject covers quite a wide range of sensing devices, so we will study only the more commonly used ones frequently encountered in industry.

The magnetic field referred to above is one that is usually produced by a permanent magnet. This magnet is often mounted externally to the transducer body and associated somehow with the measurand. A typical example is given below. We discuss the various configurations in which magnetic sensors are found; additionally, we will see how and why these configurations all work.

4-2 COMMONLY SENSED MEASURANDS

The measurands most often sensed using magnetic sensors are (1) position, (2) motion, and (3) flow. In all three cases the sensing is contactless, which is very desirable. However, as we will find out later in our discussions, there is a drawback to this kind of sensing produced by what is known as *viscous damping*. Viscous damping is an interaction between a magnetic field and a moving member reacting to that field and causing the member to have its motion impeded, so to speak, due to the attractive forces involved.

As we discuss the various types of magnetic sensor applications we will see how each of the three measurands—position, motion, and flow—is detected.

4-3 MAGNETIC REED SENSOR

The magnetic reed sensor is perhaps the simplest of all magnetic sensors in its construction. It came into existence around the mid-1960s and has been manufactured in a variety of forms. The thing to bear in mind about this type of sensor is the fact that it is a digital device. In other words, it is either in one state or another (i.e., either electrically on or off, as in a relay contact). Its theory of operation and various physical configurations are discussed below.

4-3.1 Theory of Operation

Figure 4-1 illustrates a typical magnetic reed sensor. Usually, the sensor is constructed inside an evacuated glass bulb or one that contains an inert gas such as nitrogen. Inside the envelope is a set of electrical contacts in which one side of the contacts, which is movable, is made from a ferrous metal. The other contact half, or stationary side, is made of a nonferrous material. The movable contact or reed is “oil-canned”; that is, it is constructed so there is a built-in snap action that takes place when the contact is moved slightly.

In the example shown in Figure 4-1 there are actually two sets of contacts

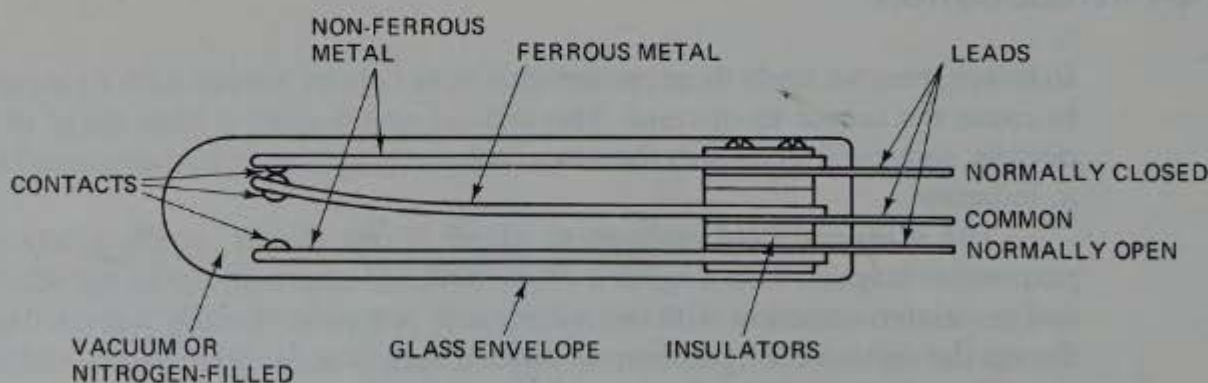


Figure 4-1 Magnetic reed sensor.

shown. The outer two contacts are nonferrous. Only the middle “common” contact is ferrous. In this design, then, one set of contacts is normally “closed” in the nonenergized state, whereas the other set is normally “open” in the nonenergized state. When a small magnet is brought near the glass envelope of the reed switch, the common contact is attracted toward the magnet, causing it to snap into its new position against the other contact. When the magnet’s magnetic field is removed, the middle reed snaps back to its original position against the other contact. The reed switch in this example is being used as a proximity switch.

4-3.2 Operating Characteristics

The triggering distance between the switch and the magnet will vary according to the strength of the magnet’s magnetic field. Typically, this distance is between 5 and 25 mm.

The contacts of the reed switch are not made to handle very large currents or voltages. Maximum voltages are usually no greater than 60 to 70 V, although some manufacturers have rated their contacts for 125 V ac. Such high voltages for the small contacts being used are usually not recommended, however.

Current ratings are normally no greater than 0.5 A at 50 V dc (i.e., 25 W). Any larger amounts of current and voltage will cause excessive arcing and pitting of the contacts.

As long as the power ratings of the contacts are not exceeded, the life span of the reed switch is extremely long. Typically, a reed switch can withstand at least 10 million switch closures before failing. A hundred million cycles is not uncommon with these devices.

As for specifying magnetic field intensities needed to cause switch closures with the reed switch, these data are not often published by the manufacturer. In reality, it is generally assumed that the user will, with some experimentation, determine the size of magnet needed and the spacing to cause the switch to trip.

4-3.3 Physical Configurations

Figure 4-2 shows a typical example of a manufactured reed switch. The size and shape can easily be configured by the manufacturer for many types of specialized applications.

4-3.4 Practical Applications

As already mentioned in Section 4-3.2, one of the primary functions of the reed switch is to act as a proximity switch, making simple contact openings and closures. The one popular aspect of using reed switches for this kind of application is that they are so easy to install onto existing equipment or structures. Let’s look at some examples.

Figure 4-3 shows another possibility for installation, in which it is desired to

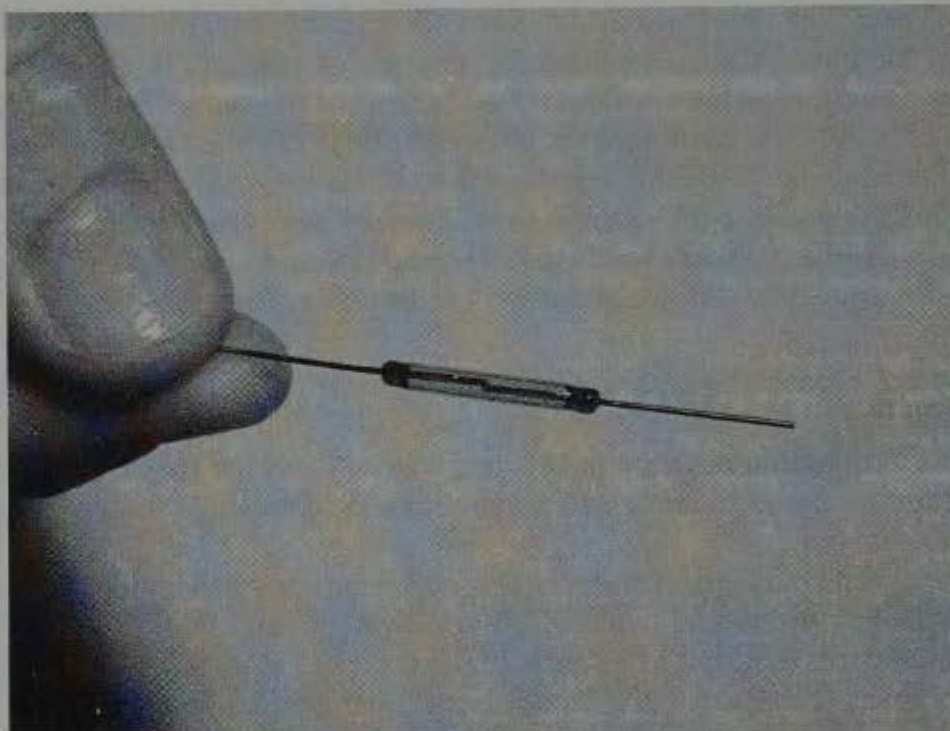


Figure 4-2 Typical reed switches.

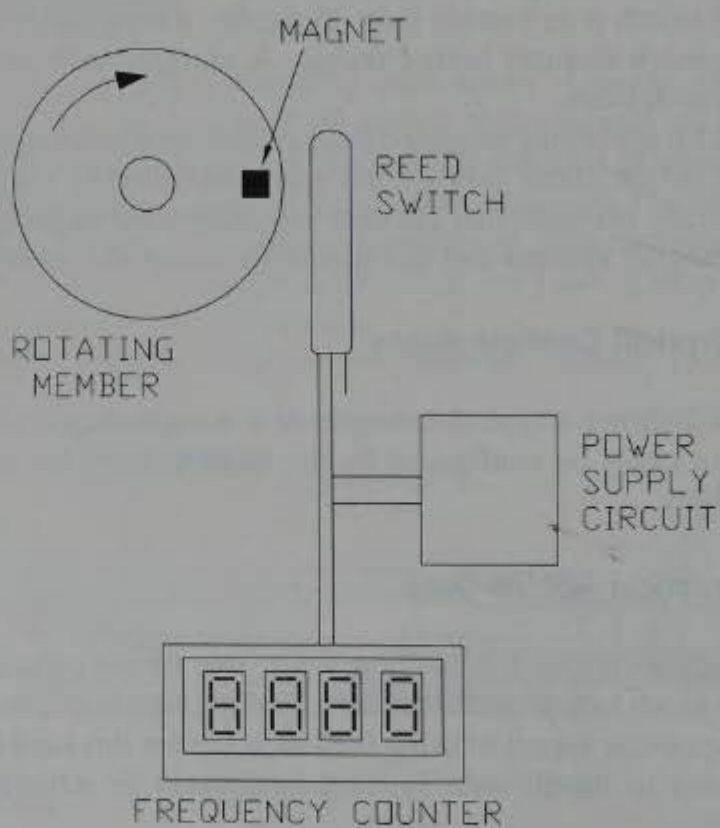


Figure 4-3 Monitoring a rotating machine member using a reed switch.

know the rpm of a rotating machine member. A magnet is mounted (glued) onto the rotating member and a reed switch mounted in close proximity to the magnet on a stationary portion of the machine. Each passage of the magnet would then trip the switch's contacts, thereby indicating one complete revolution.

4-4 HALL EFFECT TRANSISTOR

The *Hall effect* is a phenomenon that was predicted for the transistor soon after its development in the late 1950s. The discovery that the current-carrying behavior of a conductor can be altered when it is immersed in a magnetic field became evident many years before. The Hall effect was named in honor of the American physicist Edwin Hall (1855–1938). We explore the reason for the existence of this effect in our next discussion.

4-4.1 Theory of Operation

In Figure 4-4 we see an illustration depicting an electrical conductor subjected to a magnetic field. The magnetic field is at right angles to a current flow that has already been established within the conductor. One electron, which is a member of this current, is shown. Its velocity of travel is v . Note that the direction of travel is opposite the current direction following the convention that current flow is always

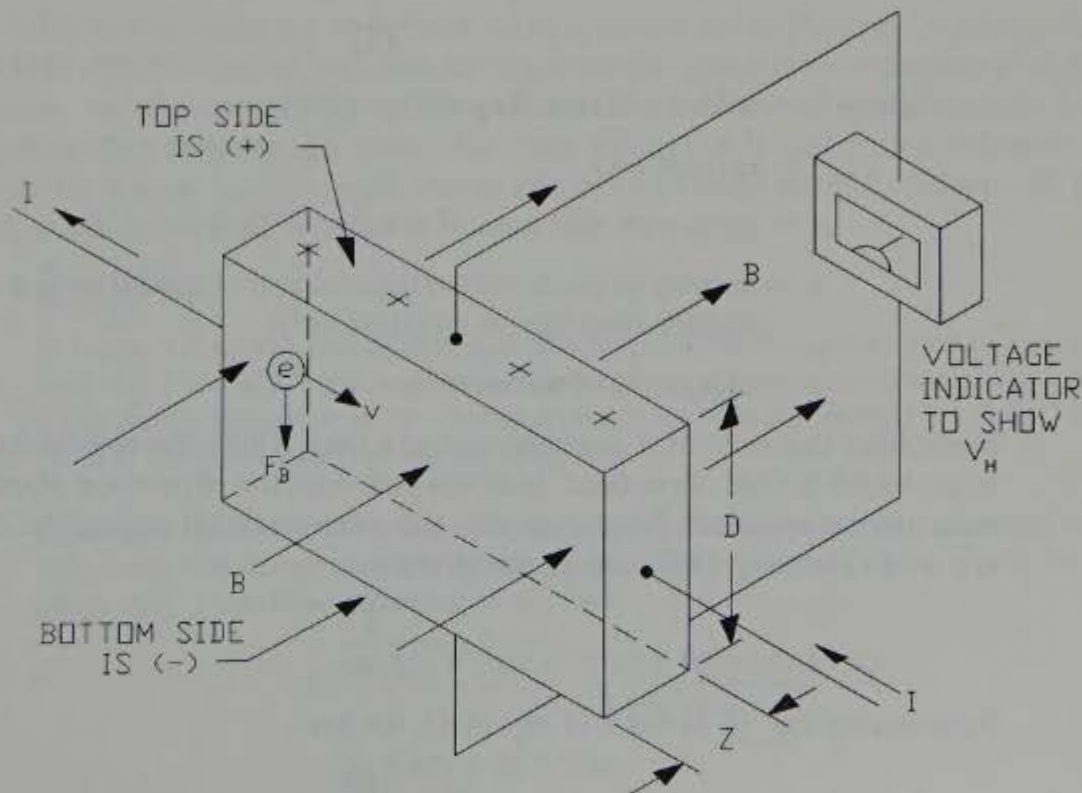


Figure 4-4 How the Hall effect takes place.

in the opposite direction of electron flow. According to the right-hand rule for determining the direction of force experienced by an electron traveling through a magnetic field (i.e., the extended fingers of the hand point in the direction of the magnetic flux, the thumb points in the opposite direction of electron travel, and the palm of the hand faces the direction of the exerted force), a force F_L (called the Lorentz force) will be generated on the electron as shown. This means that all the electrons traveling as shown in the slab of conductor will congregate toward the bottom of the conductor. As a result, the bottom edge of the conductor will become negatively charged while the upper edge becomes positively charged. In other words, an EMF will develop across the width of the slab that is in proportion to the amount of flux, current, and resultant charge separation, d .

The amount of generated voltage due to the Hall effect, V_H , can be calculated using the relationship

$$V_H = avd \quad (4-1)$$

where V_H = Hall effect voltage (V)

a = flux density of magnetic field [Wb/m or tesla (T)]

v = velocity of traveling charge (m/s)

d = height or width of conductor (m)

To find the velocity, v , of the electrons we have to go to physics, which states that

$$v = \frac{I}{Ane} \quad (4-2)$$

where v = velocity of traveling charge (m/s)

I = current (A)

A = cross-sectional area of conductor (m²)

n = number of electrons per unit volume of conductor [i.e., the electron density (number of electrons/m³)]

e = charge per electron (1.6×10^{-19} C)

Remember that current, I , is charge per unit time. This is the reason that the velocity term, which is time dependent, is developed from the expression above where time units are not apparent. Note, too, that the cross-sectional area of the conductor, A , is $z \times d$. Then eq. (4-2) can be rewritten as

$$v = \frac{I}{z dne} \quad (4-3)$$

Substituting Eq. (4-3) for v in eq. (4-1), we get

$$V_H = \frac{\beta I}{zne} \quad (4-4)$$

where V_H = Hall effect voltage (V)

β = flux density of magnetic field (Wb/m or T)

I = current flowing through conductor (A)

z = thickness of conductor (m)

n = number of electrons per unit volume (number of electrons/m³)

e = electron charge (1.6×10^{-19} C)

Finally, often what is done in Eq. (4-4) is to assign a constant, the Hall effect constant, K_H , to the variables $1/n$ and $1/e$ so that eq. (4-4) becomes

$$V_H = \frac{K_H \beta I}{z} \quad (4-5)$$

where V_H = Hall effect voltage (V)

K_H = Hall effect constant (m³/number of electrons-C)

β = flux density of magnetic field (Wb/m or T)

I = current flowing through conductor (A)

z = thickness of conductor (m)

Up to this point we have been using a conductor to illustrate the behavior of the Hall effect. Actually, semiconducting material is used to manufacture Hall effect devices, but the explanation of how electrons are deflected at right angles to the magnetic flux remains the same. The only difference is that with semiconducting materials you are working with charge carriers and holes instead of electrons. Let's look at an example.

Example 4-1

In Figure 4-5 we see a piece of semiconducting material being subjected to a magnetic field of 1.8 Wb/m. The dimensions of the piece are given. A supply current of 350 mA is circulating through it in the direction shown and the K_H value is 1.6×10^{-4} m³/C. What is the generated Hall effect voltage?

Solution: Because of the flux and current directions shown, and according to the right-hand rule stated earlier, the charge electrons would drift to the top of the Hall effect slab. Therefore, according to eq. (4-5),

$$\begin{aligned} V_H &= \frac{(1.6 \times 10^{-4} \text{ m}^3/\text{C})(1.8 \text{ Wb/m})(0.35 \text{ A})}{2.5 \times 10^{-3} \text{ m}} \\ &= 0.403 \times 10^{-1} \text{ Wb/s} \\ &= 40.3 \text{ mV} \end{aligned}$$

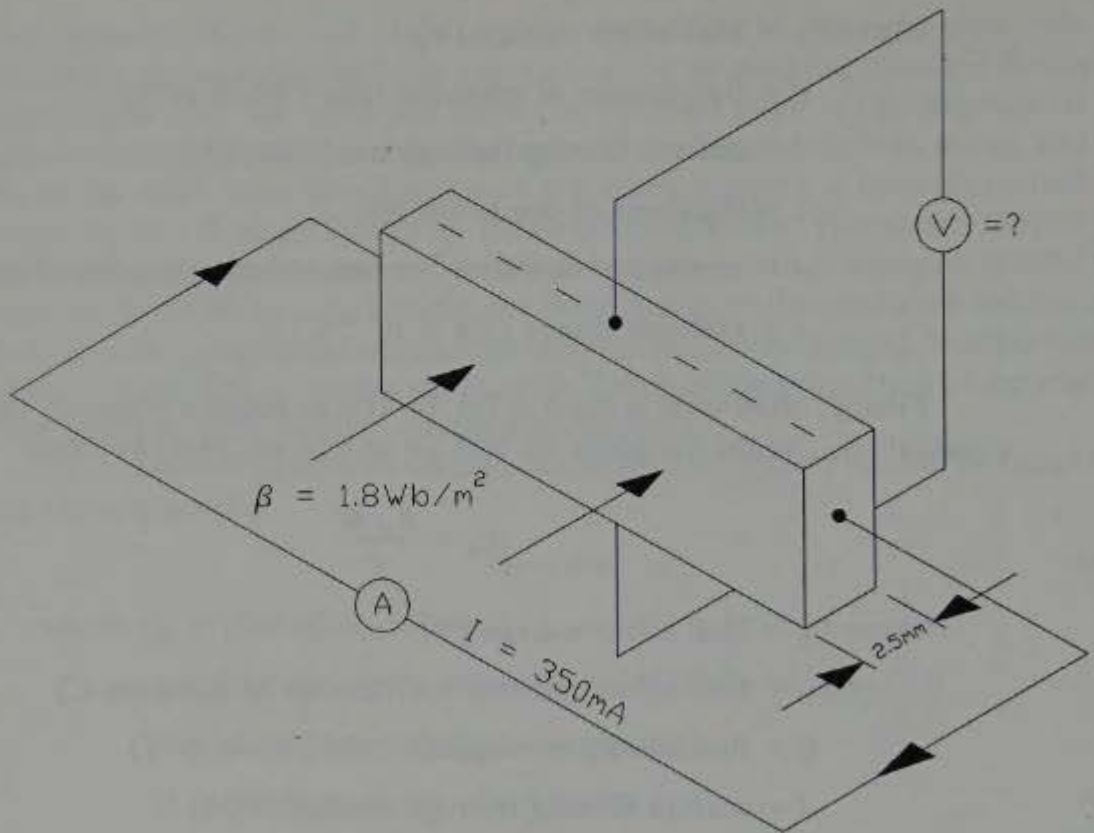


Figure 4-5 Illustration for Example 4-1.

4-4.2 Operating Characteristics

Hall effect devices may be manufactured from either p-type or n-type semiconducting materials. The only difference between the way these two materials behave is in the internal flow direction of electrons. Figure 4-6 shows that when using n-type material, you are dealing with a flow of electrons whereas when working with p-type material, you are working with a flow of hole carriers. For all practical purposes these are the same as positively charged particles, or positrons, that flow in the opposite direction from the n-type electrons. Consequently, the outward results of using these two types of material are identical; only the polarities of the biasing currents are reversed. The Hall effect sensor can be used as either a proportional sensing device or as a proximity sensor.

4-4.3 Typical Construction

Hall devices can be manufactured to fit into a variety of packages. Figure 4-7 shows some of the packages commonly used. These are housings normally associated with transistors and other solid-state devices. In many cases it is very difficult to single out the Hall device within a transducer system because of the integral design packaging that is often used.

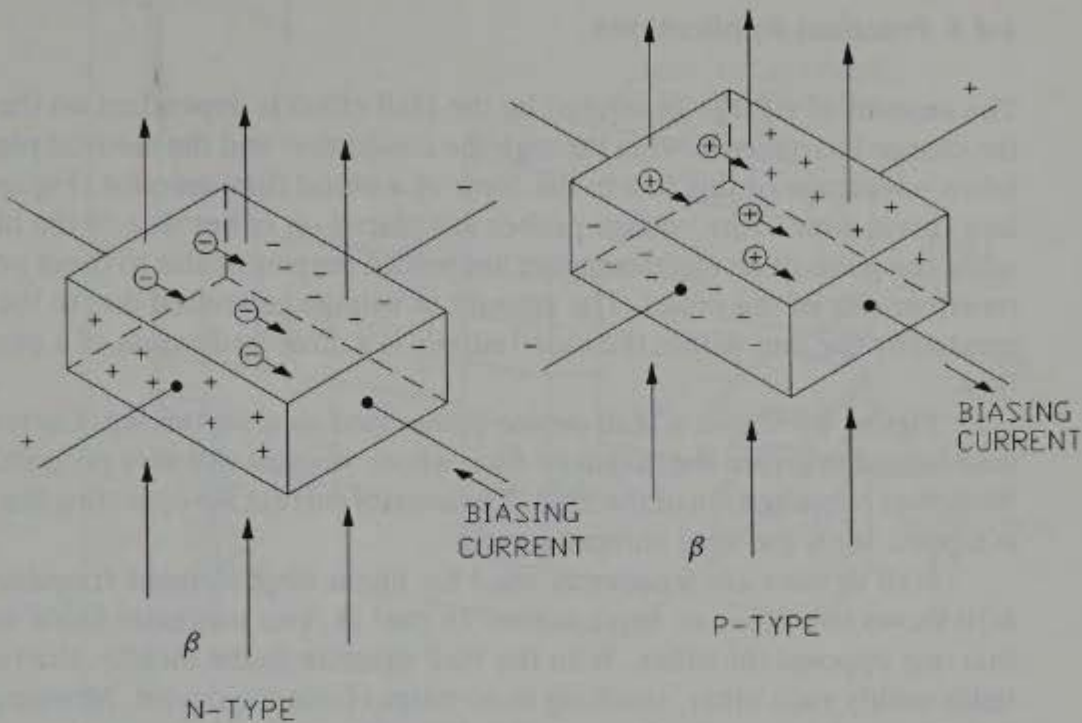


Figure 4-6 How p-type and n-type materials react to form Hall effect sensors.

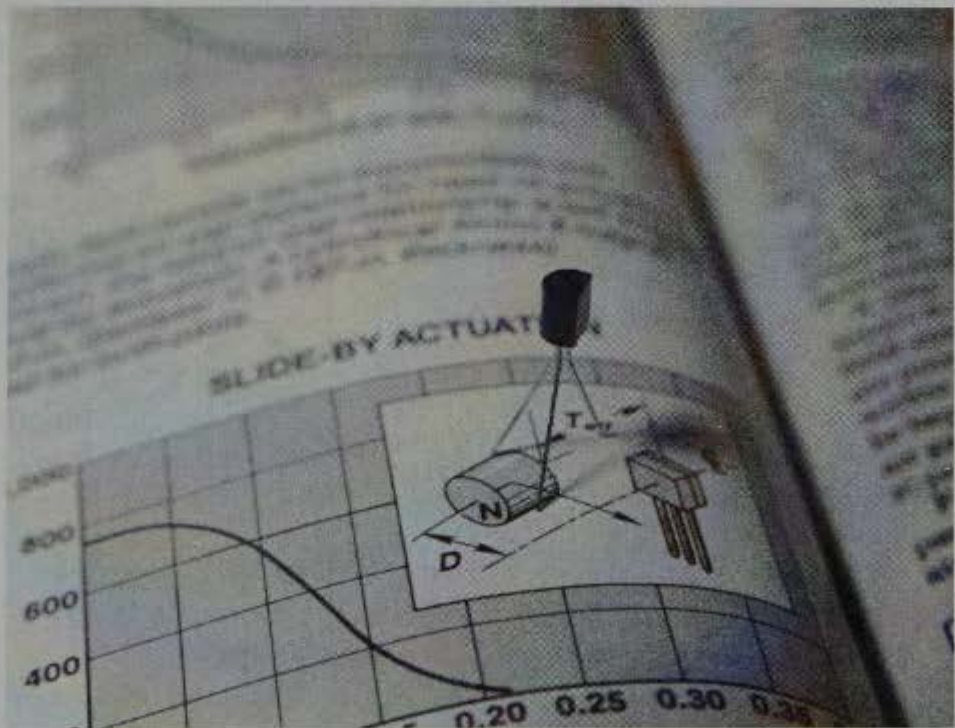


Figure 4-7 Various packaging of Hall effect devices.

4-4.4 Practical Applications

The amount of voltage generated by the Hall effect is dependent on the velocity of the charged particles flowing through the conductor, and the medical profession has taken advantage of this fact in the form of a blood flow monitor. Figure 4-8 shows how this is done. Tiny voltage probes are placed on either side of the blood vessel, while the poles of an electromagnet are placed perpendicular to these probes, again on either side of the vessel. The amount of voltage generated due to the Hall effect created by the ions within the blood supply is a direct indication of a person's blood flow.

Figure 4-9 shows a Hall device being used as a wattmeter. Current from the load is used to create the magnetic flux, whose amount will vary proportionally with the power consumption of the load. The control current for operating the Hall device is tapped from the load current supply.

Hall devices are frequently used for linear displacement transducers. Figure 4-10 shows two types of applications. In part A, two magnetic fields are set up so that one opposes the other. With the Hall detector in the middle, the two magnetic fields nullify each other, resulting in no output from the device. Movement to either side of center causes a voltage to develop. The polarity of this voltage depends on which side of center the device is located.

Figure 4-11 shows a Hall device being used as a tachometer. A rotating gear causes a nearby magnetic field to become redirected toward one of its teeth as it rotates past the Hall detector. The redirection, in turn, causes the field to sweep

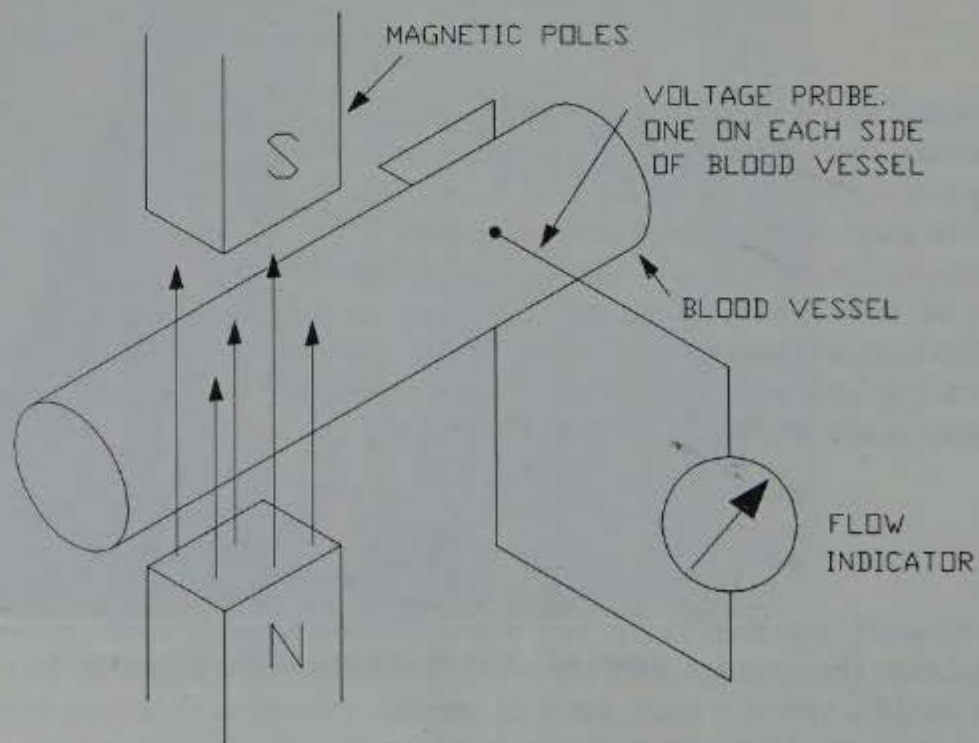


Figure 4-8 Utilizing the Hall effect in measuring blood flow.

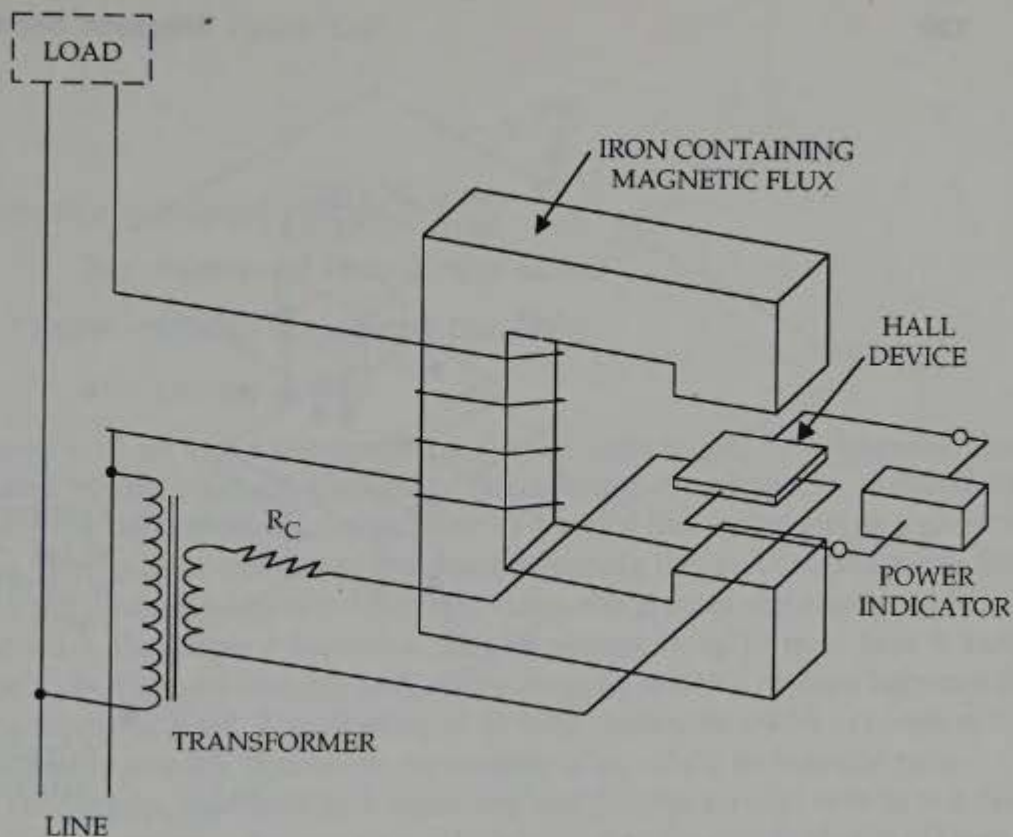


Figure 4-9 Using a Hall effect device in a wattmeter application.

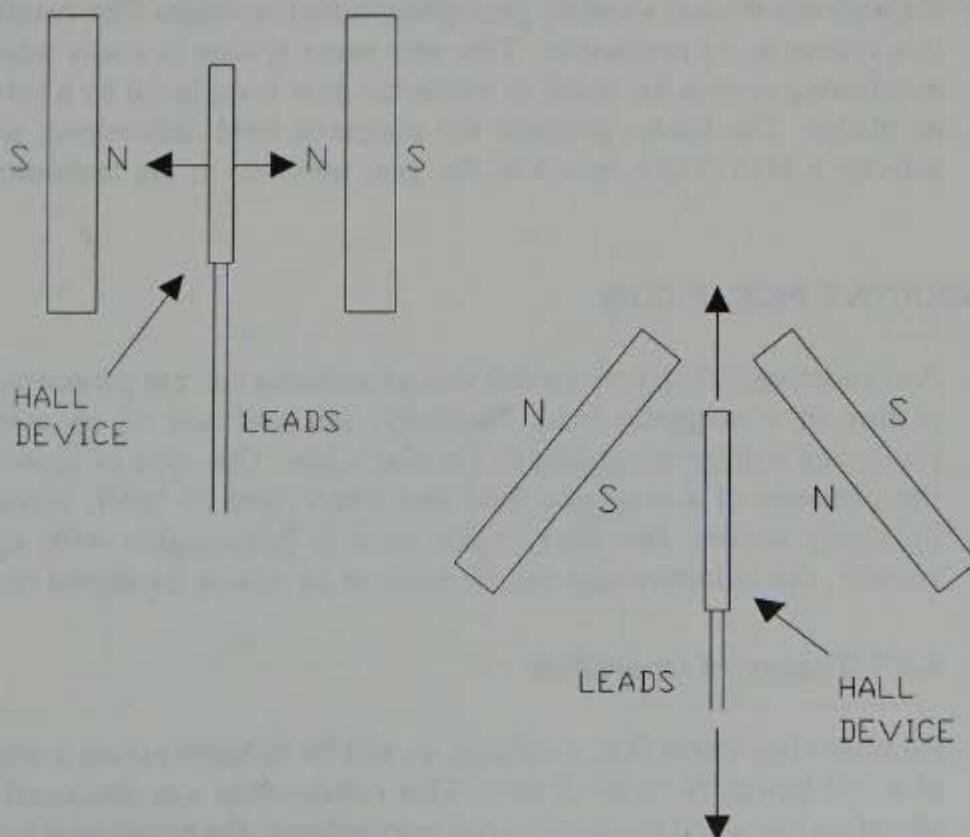


Figure 4-10 Using Hall effect sensors for linear displacement measurements.

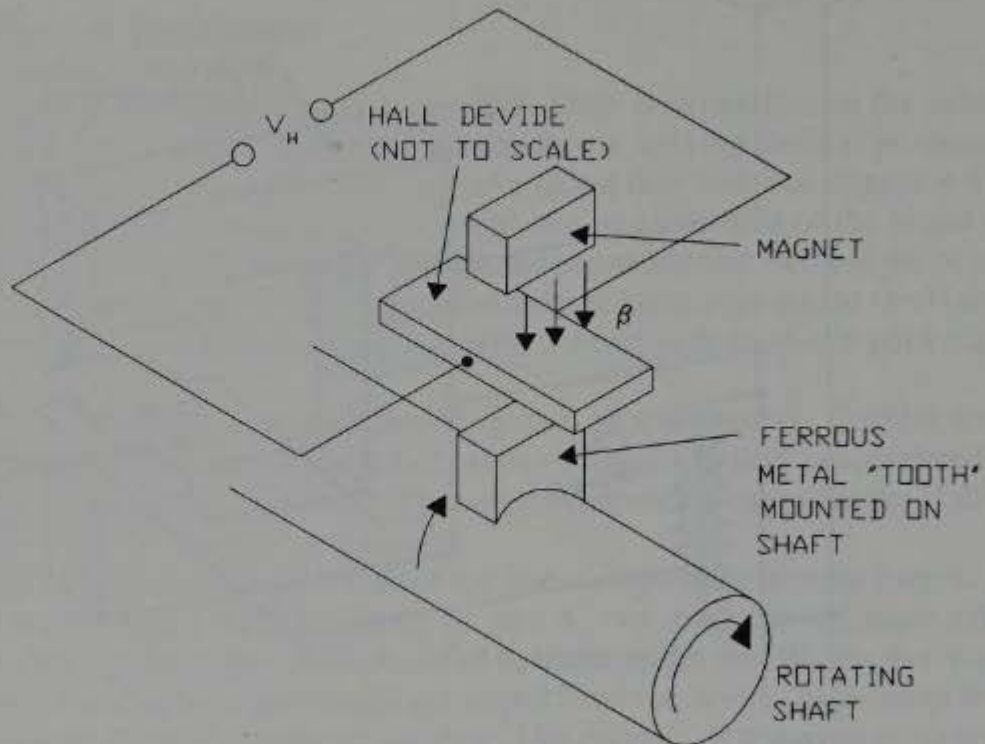


Figure 4-11 Hall effect device being used as a digital tachometer.

through the device, thereby generating a Hall voltage. The rotational accuracy of this system is $\pm \frac{1}{2}$ revolution. This very same system is easily adaptable to a flow-monitoring system for fluids in which the gear is replaced by a rotating turbine and its blades. The blades produce the magnetic field deflections, which periodically activate a Hall device, much as the gear teeth do in the tachometer.

4-5 INDUCTIVE PICKUP COIL

An inductive pickup coil is a coil that generates a current proportional to the change in flux of a magnetic field. Basically, this coil acts as an electrical generator, producing voltage according to Faraday's law. This type of sensor is used to sense the presence of a magnetic field and when used by itself, is used primarily as a proximity sensor. However, when used in conjunction with signal-conditioning circuits, the inductive coil can be used as an analog or digital device.

4-5.1 Theory of Operation

Faraday's law states that a voltage, e , will be induced across a conductor in a form of a coil having N turns of wire. This relationship was discussed in Section 3-6.1, where we discussed the moving-coil microphone; the equation is being repeated here for clarity. Faraday's law states the following relationship:

$$e = N \frac{d\Phi}{dt} \quad (4-6)$$

where e = induced voltage (V)

N = number of turns of wire in coil

$d\Phi$ = change in magnetic flux (Wb)

dt = change in time

In Figure 4-12 we see a conductor moving at right angles to a magnetic flux field produced by the poles of a magnet. The amount of voltage, e , induced into the conductor is dependent on the number of lines of flux being cut in a given instant of time. The greater number of conductors passing through this magnetic field, the greater the amount of induced voltage. Also, the greater the change in flux of this coil ($d\Phi/dt$), the larger e becomes. The important thing to note here is that for a voltage to be induced into the coil, there must be relative motion between the coil and the magnetic field. The absence of motion causes the $d\Phi/dt$ expression to equal zero, thereby causing the entire expression in eq. (4-6) to become zero.

The pickup head used in a recording and playback audio system is a magnetic induction coil whose construction is similar to that of the design shown in Figure 4-13. Note the added core in the coil's construction. The purpose of the core is to concentrate or localize the magnetic flux "sensitivity" of the coil. In this particular application the pickup head must be able to respond to very small areas of magnetism

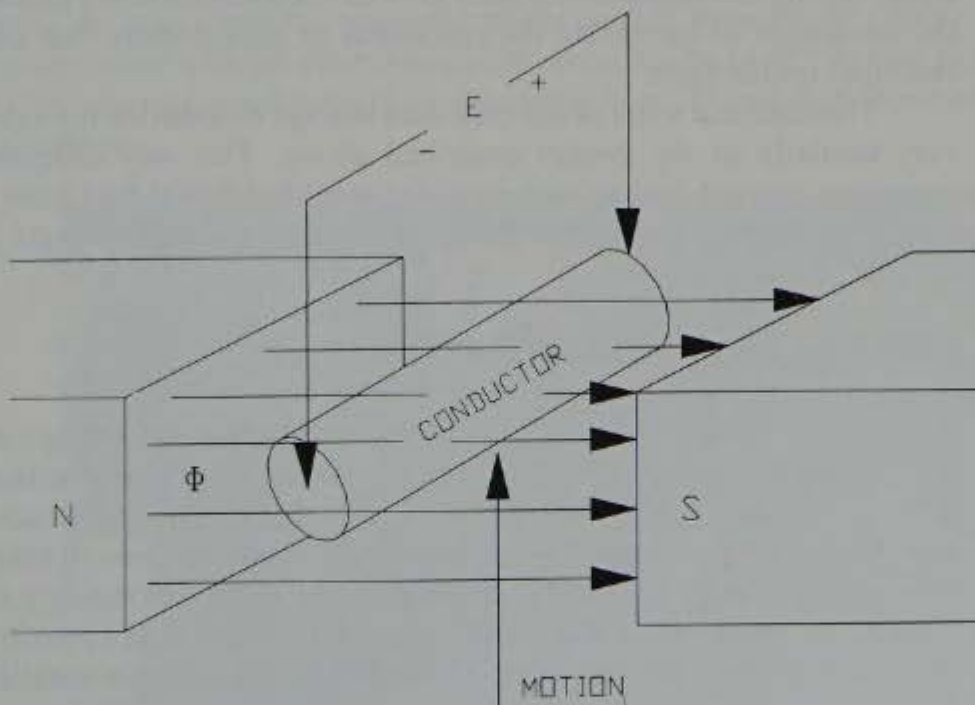


Figure 4-12 How current is induced into a conductor by a moving conductor in a magnetic field.

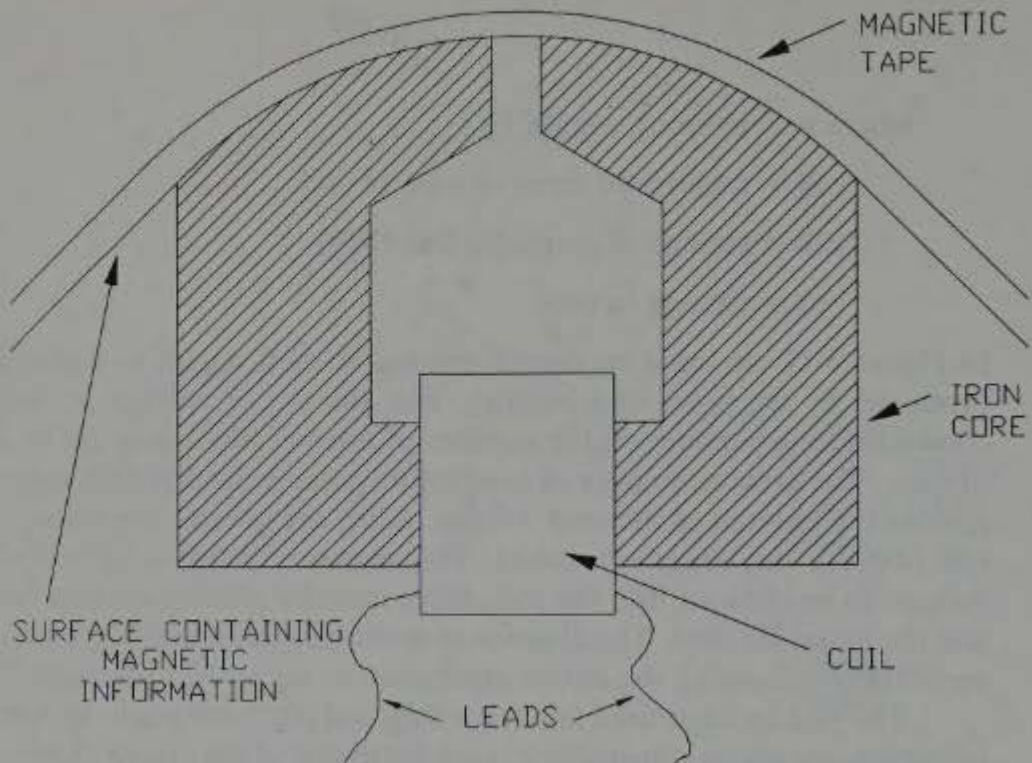


Figure 4-13 Using an induction coil in a magnetic tape audio system.

on a moving tape. Since it is too difficult to wind very small coils for this application, a core having low reluctance is used instead to concentrate the pickup area. This has the advantage of increasing the resolution or data density that can be previously recorded on the tape.

The read and write heads on a data storage disk device for a computer operate very similarly to the system described above. The only difference is that in a computer, you are dealing with receiving recorded digital data from the media in the form of magnetic pulses rather than in the form of variable-strength analog magnetic signals.

4-5.2 Operating Characteristics

Because the inductive coil functions only when there is a change in magnetic flux, it is relatively easy to use this type of device for a proximity sensor. However, for it to function in this role, the magnetic flux and coil must experience relative motion at right angles to each other for optimum voltage generation to take place. Relative motion at any angle other than 90° will produce correspondingly smaller values of e according to the sine of that angle. A typical installation as a proximity switch would be virtually identical to that of the Hall effect detector shown installed in Figure 4-10.

Two very important quantities associated with magnetic induction coils are their sensitivity to magnetic fields and their frequency response. Sensitivity has to do with the coil's ability to detect weak magnetic fields and is defined as the ratio

of the change in the coil's output signal to a change in the magnetic field at the coil's input producing the change in the output. The units associated with this sensitivity value are $V\text{-m/Wb}$ or V/T .

Frequency response for a magnetic induction coil refers to the coil's ability to respond to a particular range of magnetic signal changes per second. The unit of measure is the hertz but is rapidly being replaced by the unit *bits per second* (bps) in digital applications, where the concern is with *data rate* capability rather than frequency response.

One of the limiting factors that determine the frequency response of a coil is its *hysteresis*, a retention property in which the coil's magnetic core retains magnetism from a previous magnetic signal. This, in effect, desensitizes or biases the next-received magnetic signal, resulting in distortion or a false signal.

4-5.3 Practical Applications

The most frequent application for the magnetic induction coil is in the manufacture of the reading and writing heads used in data handling. These heads detect the very small magnetic pulses or variable magnetic areas implanted on magnetic media during a recording session. ("Magnetic media" refers to the material used in the manufacturing of recording tape, cassettes, floppy disks, and hard drives on computers, videocassettes, and so on. This material is comprised of iron oxide or other easily magnetized particles suspended on a plastic tape. Refer to Figure 4-16.) Figure 4-14 is a photograph of a typical recording head used in the common cassette player. Figure 4-15 is a photograph of a "read/write" head on a floppy disk drive in a computer. In both cases the magnetic media passing beneath the pickup head has been magnetized with an electromagnetic recording head or "writing head," very similar in construction to the pickup or reading head. Figure 4-16 illustrates how all of this is done.



Figure 4-14 Pickup head on a cassette player system.

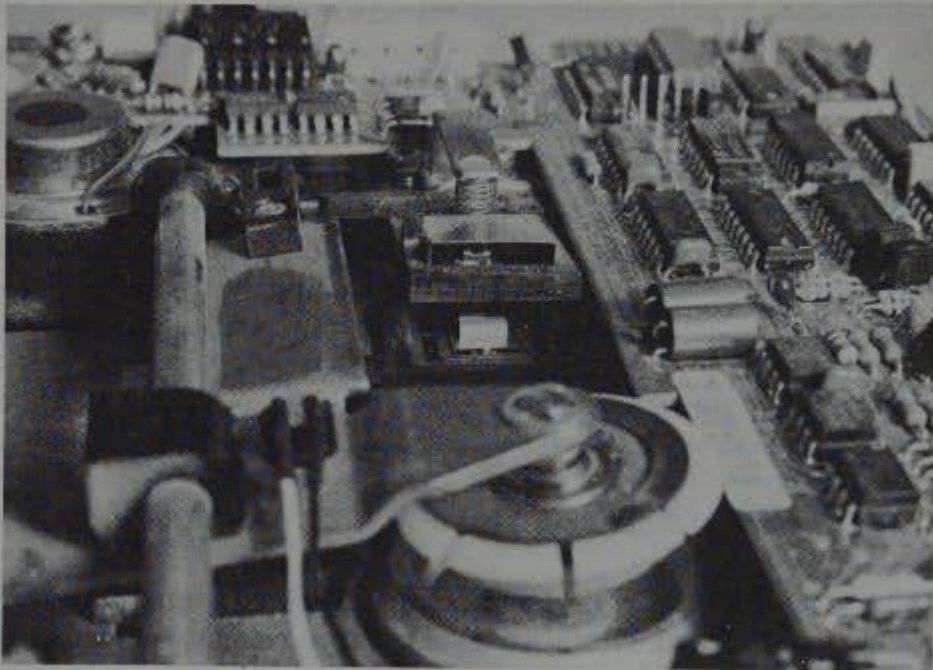


Figure 4-15 Read/write head on a floppy drive in a PC. Heads are visible in center of photo.

4-6 VIBRATING-WIRE TRANSDUCER

Various transducers have been designed around the mechanical vibration properties of stretched members inside an unknown magnetic field. The following transducer description is such a device. It is used for monitoring magnetic fields in general where it is desirable to know if any fluctuations are taking place within that field.

4-6.1 Theory of Operation

In Figure 4-17 we see a stretched wire between the poles of a permanent magnet. Its ends are fastened between two stationary posts, *a* and *b*. Because of the wire's

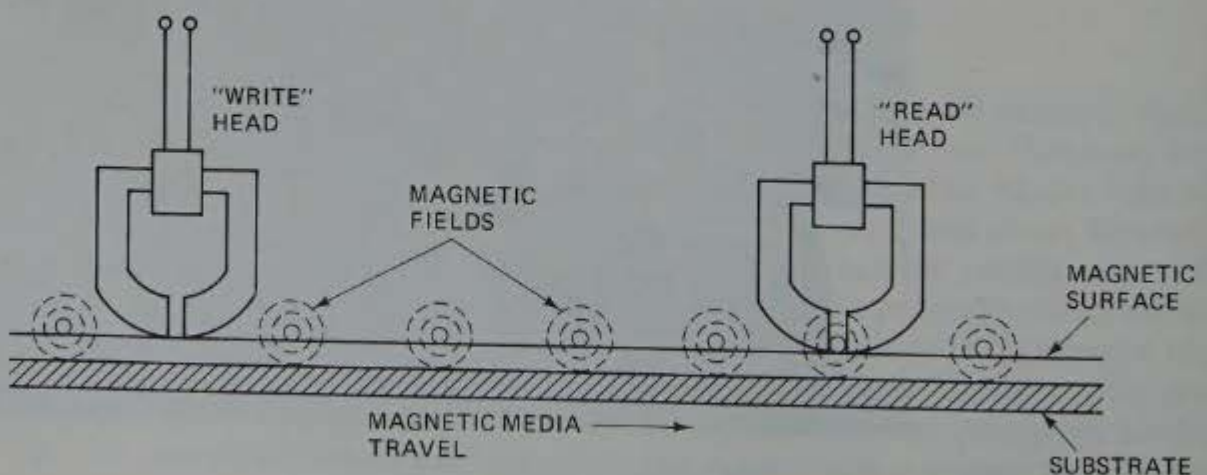


Figure 4-16 How charges are stored and read on magnetic media.

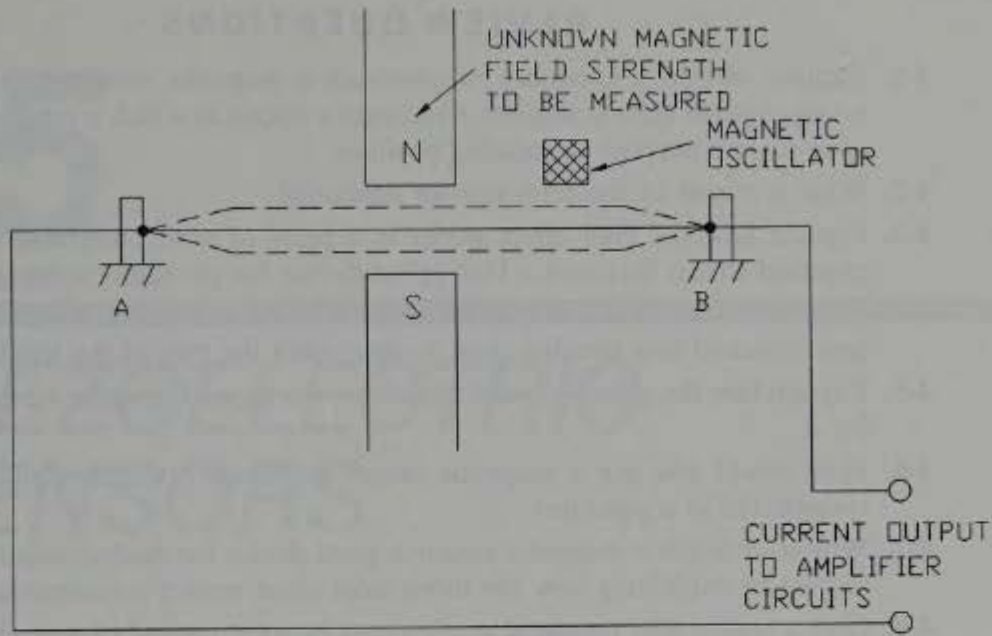


Figure 4-17 How the vibrating wire transducer works.

length, it will have a natural resonant frequency, f_r . This value is determined by the equation

$$f_r = \frac{1}{2L} \sqrt{\frac{T}{m}} \quad (4-7)$$

where f_r = natural resonant frequency (Hz)

L = length of wire (m)

T = tension force on wire (N)

m = mass of wire per unit length (kg/m)

Initially, the wire is made to oscillate by a smaller oscillating magnetic field located near the wire. A small ferromagnetic sleeve or other attachment is installed on the wire so that it will respond to the field. The frequency of oscillation coincides with the natural resonant frequency of the wire. The induced current in the wire will then be in proportion to the larger magnetic field being monitored since the wire's frequency of vibration is held constant.

4-6.2 Practical Applications

This type of transducer can be used in applications where constantly maintained magnetic fields are needed, such as in certain research applications. Using a vibrating wire system for these measurements represents an independent source of measurement, rather than depending on related parameters associated with generating and maintaining the magnetic field.

REVIEW QUESTIONS

- 4-1. Explain why it is necessary to construct a magnetic reed switch inside a sealed, nitrogen-filled glass container. Construct a circuit in which a magnetic reed switch is used for the purpose of locating position.
- 4-2. What is meant by the term *viscous damping*?
- 4-3. Explain how the Hall effect works in a piece of semiconducting material. Show a practical circuit that uses a Hall effect device for proximity sensing.
- 4-4. Explain how an inductive pickup device works in conjunction with a gear tooth on a gear attached to a rotating shaft to determine the rpm of the shaft.
- 4-5. Explain how the vibrating-wire transducer works, and describe a practical application for it.
- 4-6. How would you use a magnetic sensor to measure the flow rate of water being transported in a pipeline?
- 4-7. Why is or is not a magnetic sensor a good device for analog sensing? Support your answer by explaining how the three most often sensed measurands are sensed.
- 4-8. Give a reason why magnetic sensing may be used instead of a mechanical proximity switch for certain applications.
- 4-9. Give two factors that limit the life or usefulness of a magnetic reed switch.
- 4-10. What advantage is there in using a Hall effect device for proximity sensing rather than a magnetic reed switch? Can you think of a disadvantage?

PROBLEMS

- 4-11. Calculate the Hall effect voltage generated by a charge traveling at 2.8×10^8 m/s in a conductor having a width of 0.5 cm which is exposed to a magnetic field of 1.93×10^{-7} Wb/m.
- 4-12. A vibrating wire transducer is made from a piece of No. 26 AWG steel wire of length 13 cm. The tension used to stretch the wire is 84 N, and its mass per length is 0.00127 kg/m. Determine the wire's resonant frequency.
- 4-13. Determine the tension needed for an 18-cm length of wire, with a mass per length of 0.0025 kg/m, to vibrate in a vibrating wire transducer at a frequency of 315 Hz.
- 4-14. Using the voltage calculated in Problem 4-1, calculate the Hall effect constant, K_H , associated with the system described in that problem. Assume a current flow of 13.4 μ A.

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9

VARIABLE-INDUCTANCE SENSORS

CHAPTER OBJECTIVES

1. To review the concept of inductance.
2. To study the linear and rotary motion inductor.
3. To understand the advantages and disadvantages of the inductive sensor.

9-1 INTRODUCTION

Sensors that vary the inductance of a looped or coiled conductor to perform their measurand sensing are called variable-inductance sensors. In the section that follows, we first discuss the theory of inductance and then look at the various forms of variable-inductance sensors that fall under this category. Unfortunately, from an intuitive approach, inductance is not as easily understood as capacitance. Hopefully, though, the following review will help to clarify this situation.

9-2 REVIEW OF INDUCTANCE

An inductor is a coiled loop made of a conducting material whose purpose is either to “choke” back alternating current to prevent it from flowing in a circuit, or to encourage oscillations at a particular frequency when used either in series or in parallel with a capacitor. More exactly, inductance is an indication of the ability of

a coil to *oppose any change in current flowing through that coil*. Sometimes the term *self-inductance* is used to describe that ability, but often the term *self* is dropped. It should be pointed out here that inductance must not be confused with inductive reactance even though the two terms are related. The amount of inductance for a given coil may be calculated using the following equation:

$$L = \frac{\mu N^2 A}{l} \quad (9-1)$$

where L = inductance [henry (H)]

μ = permeability "constant" [see discussion below; units are webers/ampere turns-meter (Wb/At-m)]

N = number of loops or turns

A = area of the inside core of loop (m^2)

l = length of core (m)

When we talk about inductance, we also have to talk about permeability. *Permeability* refers to a material's ability to support the existence of magnetic lines of flux that pass through the core of an inductor whenever the inductor becomes energized with a current flow. Strictly speaking, the permeability constant, μ , referred to in eq. (9-1) is not really a constant since its value depends on the flux density, B , and the magnetizing force, H . Both are present inside the coil's core when current is flowing through the coiled conductor. Because $\mu = B/H$, it can be seen that μ is dependent on the amount of current flowing through the coil at the time. Also, eq. (9-1) is valid only for a single-layered coil winding. Multiple-layered coils require a considerably modified equation. Note, too, that the unit of inductance in eq. (9-1) is the henry. Equation (9-1) is considered a "basic" equation that is derived from the basic definition of induction. The use of this equation is demonstrated in the following example.

Example 9-1

Calculate the inductance of the coil in Figure 9-1. Assume air as its core material. The permeability of air is 12.57×10^{-7} Wb/At-m.

Solution: We first calculate the area of the core having a diameter of 2.54 cm according to Figure 9-1. Since

$$A = \frac{\pi d^2}{4}$$

then

$$\begin{aligned} A &= \frac{\pi(0.0254 \text{ m})^2}{4} \\ &= 5.067 \times 10^{-4} \text{ m}^2 \end{aligned}$$

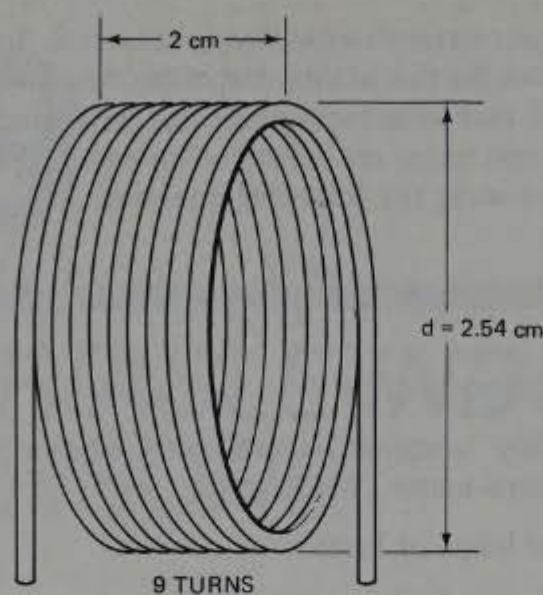


Figure 9-1 Illustration for Example 9-1.

Note that in the calculations above, we converted all lengths to meters, the basic unit of length in the SI system. This is because we are working with an equation containing no conversion factors. We now use eq. (9-1) to get

$$\begin{aligned} L &= \frac{\mu N^2 A}{l} \\ &= \frac{(12.57 \times 10^{-7} \text{ Wb/At-m})(9 \text{ turns})^2(5.067 \times 10^{-4} \text{ m}^2)}{0.02 \text{ m}} \\ &= 2.58 \times 10^{-6} \text{ H} \quad \text{or} \quad 2.58 \mu\text{H} \end{aligned}$$

Table 9-1 lists the *relative permeability* of several common materials. (The relative permeability is merely a relative comparison or ratio of the permeability of a substance to that of a vacuum, the permeability of a vacuum being 12.57×10^{-7} Wb/At-m.) In reality, all nonferromagnetic materials have a relative permeability approximating 1.0.

TABLE 9-1 APPROXIMATE RELATIVE PERMEABILITIES OF
SOME COMMON MATERIALS

Nickel	50
Cast iron	90
Machine steel	450
Transformer iron	5500
Air	1.0006
Aluminum	1.00000
Silver	0.99999
Wood	0.99999

Source: Adapted from W. H. Buchsbaum, *Practical Electronic Reference Data*, 2nd ed. (Englewood Cliffs, NJ: Prentice Hall, 1980).

Notice in Example 9-1 that no core was involved in construction of the coil. The coil, by itself, possessed an amount of inductance simply because air was, in effect, the core. If we had inserted a core of some different material, say brass or copper, we would have introduced a different variable permeability factor that would have depended on how far the core was inserted into the coil. In other words, we would have had a permeable material comprised of a mixture of brass or copper, and air. The problem's solution would have been somewhat complicated. Note the following problem and its solution.

Example 9-2

Using the coil in Example 9-1, we now insert an iron core one-third of the way into it. Determine its total inductance.

Solution: We break this problem into two solutions, each solution resulting from treating that portion of the coil having its own particular core material as a separate coil. The final answer will be the sum of these two solutions. The first solution results from using a coil having $\frac{1}{3}$ of 9 turns, or $N = 3$; $\frac{1}{3}$ of l , or 0.667 cm; and using iron as the core material. The second solution results from using the same coil but with using $\frac{2}{3}$ of 9 turns, or $N = 6$; $\frac{2}{3}$ of l , or 1.33 cm; and using air as the core material.

Solution for coil portion having iron core: From Table 9-1 we see that iron as a relative permeability of 5500. Therefore,

$$\begin{aligned} L &= \frac{(12.57 \times 10^{-7} \text{ Wb/At-m})(5500)(3 \text{ turns})^2(5.067 \times 10^{-4} \text{ m}^2)}{0.00667 \text{ m}} \\ &= 0.00473 \text{ H} \\ &= 4.73 \text{ mH} \end{aligned}$$

Solution for coil portion having air core:

$$\begin{aligned} L &= \frac{(12.57 \times 10^{-7} \text{ Wb/At-m})(6 \text{ turns})^2(5.067 \times 10^{-4} \text{ m}^2)}{0.0133 \text{ m}} \\ &= 0.00000172 \text{ H} \\ &= 1.72 \text{ } \mu\text{H} \end{aligned}$$

We can see that the air-core portion of the coil contributes an insignificant amount of inductance to that of the iron core; consequently, we can ignore the 1.72 μH value and state that the total inductance of the coil is 4.73 mH.

9-3 COMMONLY SENSED MEASURANDS

The inductive sensor must be designed in such a manner that the measurand alters the self-inductance of the inductive sensor's coil to produce either a change in current through the coil or a change in the voltage across the coil. One of the most common ways of producing this situation is to design a movable core that moves back and forth inside a coil (Figure 9-2) so that as the core is displaced, a variable permeability is

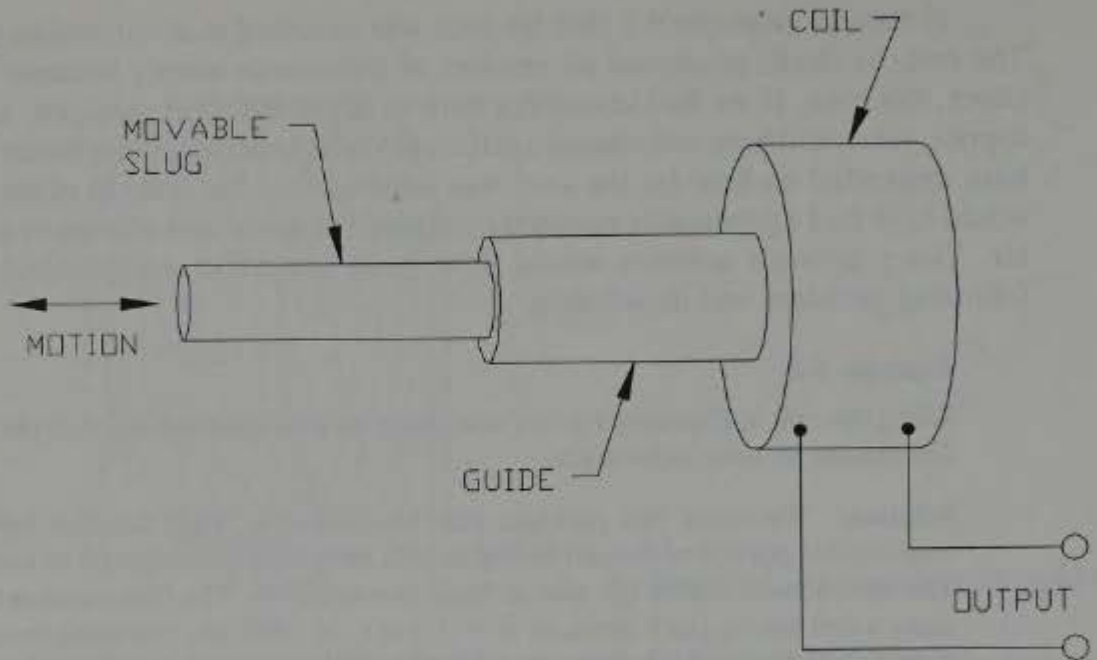


Figure 9-2 Coil with movable slug for varying its inductance.

created within the coil. The core is usually made of a fairly highly permeable material such as ferrite or iron.

Using this scheme it is possible to sense a wide range of measurands. As a matter of fact, the measurands that are sensed in this manner are those that were mentioned in Section 8-3 for the capacitor. In that section it is noted that for the capacitor, a common variation of displacement sensing is the detection of force or pressure. In the case of the induction sensor, either can cause a core to change its position inside the sensor's coil.

Another frequent measurement is liquid-level sensing. However, virtually any other measurand can be detected also, provided that it can be transformed into motion.

9-4 TYPES OF INDUCTIVE SENSING

Both linear motion and rotary motion detectors are used in industry. There is also a third category of inductive sensing, which falls under the heading of proximity sensing. This is the sensing of unknown materials that are generally hidden from view. We cover the theories of operation of each of these categories in the next three sections.

9-4.1 Linear Motion Inductor

The linear motion inductor is an inductor having a movable slug that slides into and out of the coil's central core. As in the case of the linear capacitor, the term *linear* refers to the straight-line mechanical motion of the movable slug and not necessarily to the output characteristic of the inductor's signal.

The construction of a typical linear motion inductor is illustrated in Figure 9-3, a cutaway view of the inductor's interior. The core is usually made from a powdered ferrite material that has been pressed into a core's desired shape. Often, elements such as nickel, zinc, and cobalt are added to the ferrite to obtain certain desirable characteristics. Ferrite itself is an iron material mixed with certain binders and other elements to create a material having very high permeability characteristics for certain current and frequency amounts.

Attached to the core's one end is a guide shaft that maintains alignment between the core and the coil's central core area. The shaft rides inside a bearing surface, usually a sleeved bearing made from a lubricant-impregnated brass- or bronze-like material, to permit smooth, relatively friction-free motion.

One of the characteristics of the linear motion inductance sensor is its relatively linear output. As the inductor's core is moved into and out of its coil, the value of μ is varied in linear fashion. As the inductance, L , is varied linearly, the reactance for any given frequency of current passing through the coil is also produced in a linear fashion:

$$X_L = 2\pi fL \quad (9-2)$$

where X_L = inductive reactance (Ω)

f = frequency (Hz)

L = inductance (H)

Since inductive reactance by itself is an imaginary mathematical expression denoted either by the imaginary symbol $+jX_L$ or its polar equivalent, $+X_L/90^\circ$, the impedance created by the inductor's coil is equal to the vector sum of its dc resistance, R , and its imaginary component. In other words,

$$Z_L = \sqrt{R^2 + jX_L^2} \quad (9-3)$$

where Z_L = impedance across inductor (Ω)

R = coil's dc resistance (Ω)

jX_L = inductive reactance (Ω)

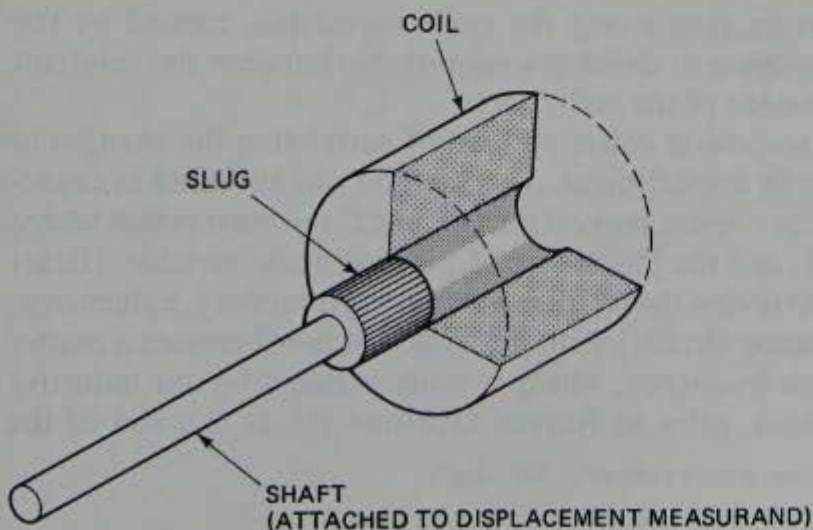


Figure 9-3 Internal construction of a linear-motion inductor.

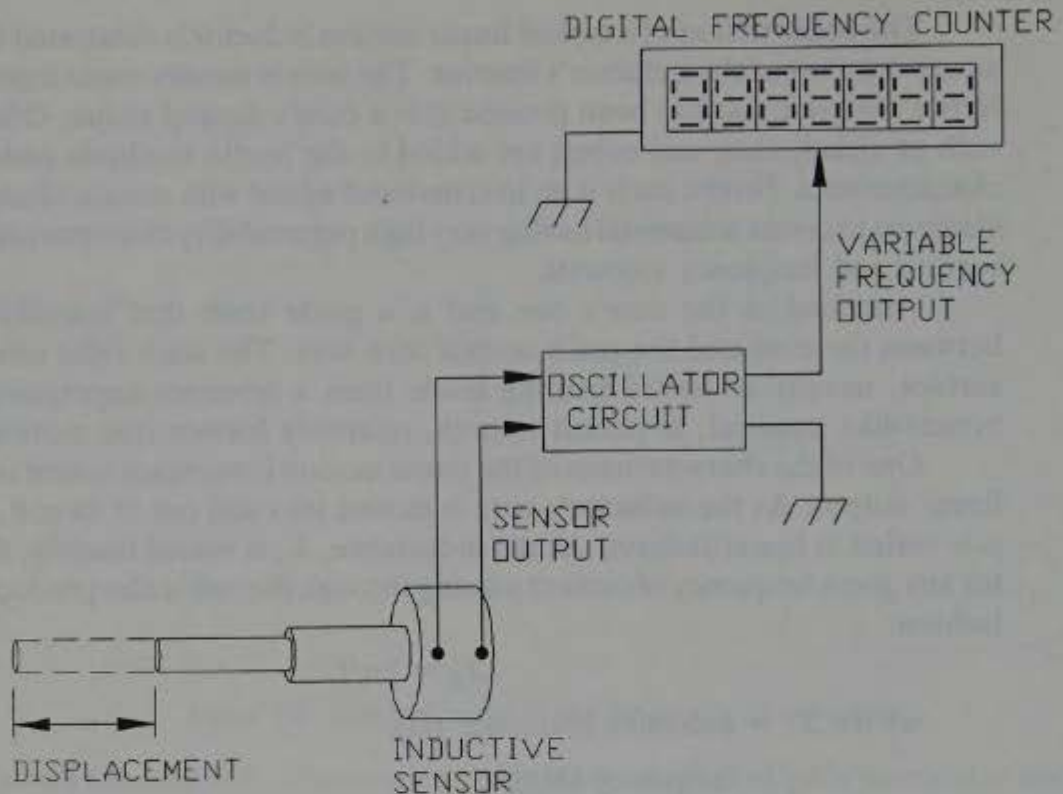


Figure 9-4 Circuit for translating displacement into a digital readout.

Then the voltage drop across the inductor will vary because

$$V_L = I_L Z_L \quad (9-4)$$

where V_L = voltage drop across coil (V)

I_L = current through coil (A)

Z_L = coil's impedance (Ω)

Equation (9-4) tells us that by monitoring the coil's impedance created by the changing inductance, it is possible to develop a relationship between the resultant impedance and the displacement of the coil's core.

There is, however, a somewhat easier method of correlating the changes in inductance with the changes in displacement. This method was also used in capacitive sensing. For our inductor, we can make it part of an *LC* resonant circuit where the capacitance, C , is fixed, and the inductance, L , is now made variable. [Refer to eq. (8-2) in Section 8-4.1 to review the relationships among frequency, inductance, and capacitance in an oscillating circuit.] With this system it now becomes a matter of monitoring the oscillation frequency, which is made variable by our inductive sensor. (See Figure 9-4. Also, refer to Review Question 9-6 at the end of the chapter.)

9-4.2 Rotary Motion Inductor

A linear motion inductive sensor can be modified to sense rotary motion. All that one has to do is to modify the linear shaft's sliding bearing surface by including machined threads so that the surface becomes a screw-type mechanism. By rotating the shaft, the core is now made to move in and out of the coil according to the direction of shaft's rotation. A variation on this concept is to install a ball-screw mechanism, which may help reduce the rotational friction common to threaded surfaces. An advantage of the threaded or ball-screw system is that the pitch of thread used will determine the rotational resolution of the system. That is, it may be possible to have several 360° turns of the shaft to cause a desired incremental change in core displacement as opposed to having only one shaft revolution to produce the same incremental change.

Another rotary motion type of construction is illustrated in Figure 9-5. This particular design requires that the inductive-sensing coil is built surrounding a circular shaft containing the high-permeability core. There are two obvious disadvantages to this system: (1) the coil must be wound on a form having the same radius of curvature as the rotating circular shaft, and (2) there is no possibility of increasing the rotational resolution of the system as there was with the threaded or ball-screw systems described earlier.

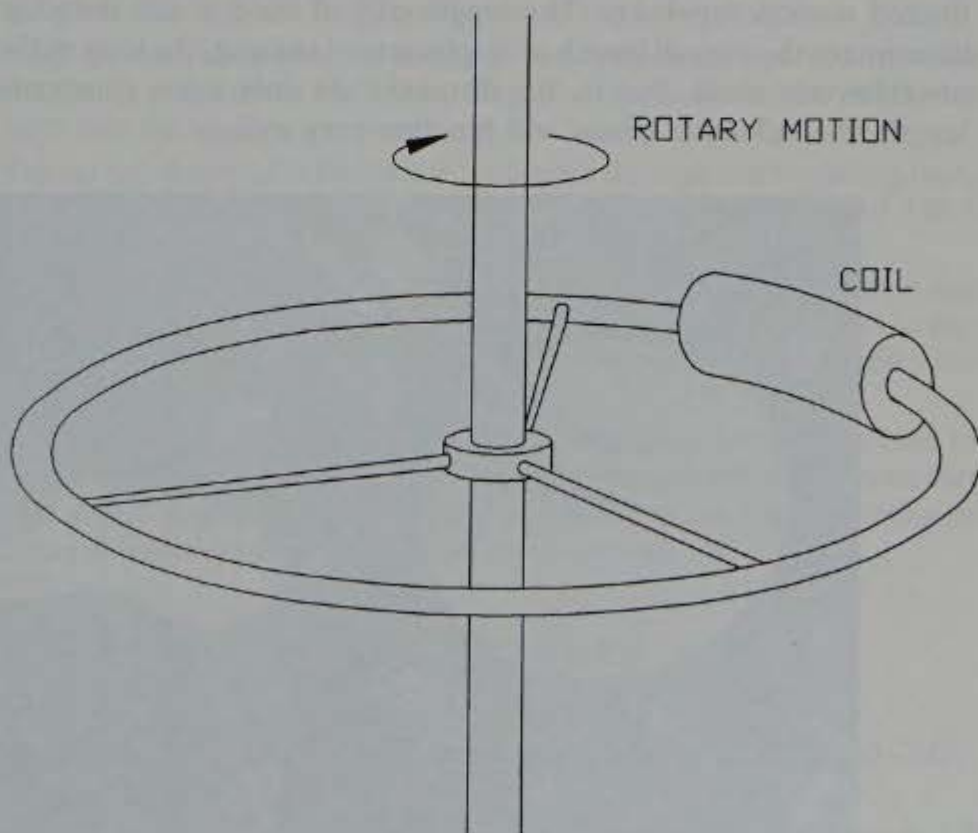


Figure 9-5 Another rotary-motion inductor design.

9-4.3 Inductive Proximity Sensing

Inductive proximity sensing is used primarily for the detection or location of metals. This particular activity has become rather popular in recent years among hobbyists trying to locate buried minerals or metallic treasures. In industry this type of sensing is used for the detection of minerals in the earth.

The inductive proximity sensor is usually comprised of a large sensing coil which is part of an *LC* RF circuit (Figure 9-6). The circuit's frequency of oscillation is typically in the hundreds or thousands of kilohertz. When a concentration of metal is brought near the sensing coil the frequency of oscillation is changed because of the altering of the oscillator circuit's inductive value. This shift in frequency causes a voltage imbalance to take place in an otherwise balanced voltage differential circuit. This imbalance either sounds an alarm or varies an indicating circuit.

9-5 ADVANTAGES AND DISADVANTAGES OF USING THE INDUCTIVE SENSOR

The inductive sensor is a relatively inexpensive sensor, which explains its popularity with users. Also, it is considered to be capable of producing fairly linear outputs. However, the inductive sensor has one major disadvantage. This has to do with its limited motion capability. The length of coil used in the inductor's construction determines the overall length of displacement sensing. As long as the displacements are relatively small, that is, the distances are only a few centimeters or inches in length, the inductive sensor will function very well.



Figure 9-6 Pickup head in a metal detector unit for detecting the location of metallic substances.

REVIEW QUESTIONS

- 9-1. Why is inductive sensing considered to be such a versatile method of sensing compared with, say, a piezoelectric sensor?
- 9-2. Explain how you would set up an inductive sensor for the purpose of separating copper metal from its ore rock.
- 9-3. How would you design a mechanism for the purpose of measuring hydraulic pressure in a pressure line using a variable inductive circuit?
- 9-4. How would you characterize the input impedances of most inductive sensors? Would you say the impedance would probably be very high or rather low? Justify your answer with an explanation.
- 9-5. Explain the difference between the terms *self-inductance* and *inductive reactance*.
- 9-6. In Section 9-4.1 the statement was made that a resonant circuit made from a fixed capacitor and a variable inductor could be used to monitor a measurand by monitoring this circuit's output frequency. Assuming a linear relationship between the coil's inductance and the measurand, what would be a major disadvantage of this system? Support your answer mathematically.

PROBLEMS

- 9-7. Calculate the inductance for a coil to be used in a displacement transducer that is to be made from 250 turns of wire in a single-layered winding wound on a 10-mm-diameter form. The coil will be 70 mm in length and have an air core.
- 9-8. Find the inductance of a coil having 85 turns of a single-layered wire winding that is 0.45 in. in diameter and is 2.33 in. long. The core is made from steel that is inserted 60% into the winding's center.
- 9-9. For the coil described in Problem 9-7, calculate the impedance resulting from supplying the coil with a 230-kHz signal at 6.3 V ac. The coil's dc resistance is 3.9 Ω . How much current does the coil require?
- 9-10. How many windings are necessary to create a 30-mm-long coil 6 mm in diameter that will use a movable iron core coupled to the measurand? The coil will be supplied with a 3.5-V ac 3.25-MHz signal. Assume no dc resistance in this coil and a current draw of 4 mA.
- 9-11. Make a sketch showing how a variable-inductance device may be used to sense air pressure. Show how this device may be part of a remote telemetry system that transmits the sensed pressure values by radio signals. Use block diagramming rather than discrete components to show the circuitry involved.

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