RADIO ENGINEERS' HANDBOOK

BY

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RADIO ENGINEERS' HANDBOOK

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PREFACE

The purpose of the "Radio Engineers' Handbook" is to provide a reference book summarizing the body of engineering knowledge that is the basis of radio and electronics.

Particular effort has been made to achieve thoroughness. Some two thousand technical articles were reviewed during the preparation of the manuscript, and the book in its final form presents in an organized way the more important contributions made to the art by this vast body of technical literature. In the selection of material an attempt has been made to avoid the common failing of most handbooks, which instead of summarizing *all* of the important facts about a topic, and bringing together formulas and procedures useful in actual design, too often devote most of the available space to introductory material, such as found in good textbooks, and to descriptions of particular equipments.

The text is well documented, there being about fifteen hundred footnote references to published articles. Practically all the pertinent engineering literature in English that bears on any particular subject is referred to in connection with the text on that subject. The reader who wishes to make an intensive study of a particular topic is thus in most cases spared the necessity of engaging in a time-consuming search through many indexes to find out what has been published. At the same time, the essential information, formulas, etc., are insofar as possible included in this book. The reader, therefore, does not need to have an entire library within arm's reach in order to obtain the answer to most problems, and yet knows just where to go for further information.

Attention is called to certain sections of the book which present in one place material previously widely scattered. Thus the formulas and curves on skin effect, inductance, mutual inductance, and capacity in Section 2 represent the most complete collections ever presented at one place. The same is true of the transmission-line equations in Section 3, the formulas for field patterns and radiation resistance of antennas in Section 11, the treatment of ground-wave and ultra-high-frequency propagation in Section 10, etc. Similarly, the important subject of network theory is covered in a straightforward and comprehensive way that, it is hoped, will widen the understanding of such important matters as Foster's reactance theorem, attenuation and phase equalizers, lattice and ladder filters of various types, impedance matching and insertion loss, the relation between attenuation and phase shift, etc. The treatment on

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electron optics is noteworthy in that it is the first summary of the subject that has appeared in handbook form, and also because it presents the most complete collection of data on electron lenses that has ever been published.

The "Radio Engineers' Handbook" is unique among handbooks in that it is essentially a one-man job. This has made it possible to coordinate different sections of the book very closely. Thus, related subjects in different parts of the book are treated from the same viewpoint and are fully cross-referenced, duplication is avoided, and there is no trouble from gaps such as inevitably appear when the writer of one section is not familiar with the details of all of the other sections. At the same time, the one-man approach has necessarily limited the range of topics covered to the basic engineering principles which the radio man thinks of as constituting radio engineering. Related communication fields, such as wire communication, as well as the physics aspects of radio and electronics, including particularly the physics literature and special applications to problems in physics, are covered only very incompletely. This is not because these things are not important, but rather because what one man can do in one book is inevitably limited. Within the scope of the book, the coverage will be found to be almost complete, however.

There is one omission that the inquiring reader will wonder about, namely, the lack of a section on television. This was to be Section 14, but as the author became deeply involved in war work early in 1942, it became impossible to follow the original plans without delaying publication indefinitely. Television material does appear in sections dealing with amplifiers, antennas, etc., but the coordinated presentation of television systems is regrettably missing.

In selecting the material to go into "Radio Engineers' Handbook" it has of course been necessary to avoid any material that even hints at the new developments that have evolved from the needs of war. Certain topics were in fact deliberately deemphasized to avoid indicating the directions in which the new developments are being made. Accordingly, even when readers are disappointed in not finding descriptions and explanations of the new techniques evolved from the war effort, they should not feel that the author is oblivious to them. Quite the contrary is the case.

Many people and organizations have helped in one way and another to bringing the "Radio Engineers' Handbook" into being. The most important contribution was made by Dr. Karl Spangenberg, the author's associate at Stanford University, who wrote Section 4 on Vacuum Tubes and Electronics, and who was largely responsible for the material on wave guides and cavity resonators in Section 3 and the collection of formulas giving inductance, mutual inductance, and capacity in Section 2.

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Acknowledgment is also due Dr. W. Y. Pan and Donald MacQuivey, who prepared most of the illustrations, and to W. S-Y Yeh, T. H. Pi, C. K. Chang, Robert Soderman, O. G. Villard, Jr., Dr. E. A. Yunker, Clark Cahill, Robert Barnes, and others who assisted in various ways at one time or another. Thanks are also due the International Standard Electric Corporation for making available the data on electron lenses that appears in Section 4, and to the General Radio Company, Driver Harris Co., RCA Manufacturing Co., Inc., Ohmite Manufacturing Company, and Phelps Dodge Copper Products Corporation for permission to reproduce various charts and tables.

In a work of this type some mistakes in equations, curves, etc., will inevitably occur no matter how much care is taken. Where these are found, the author would appreciate their being called to his attention so that subsequent printings may be corrected.

FREDERICK EMMONS TERMAN.

CAMBRIDGE, MASSACHUSETTS,
May, 1943.

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SECTION 1

TABLES, MATHEMATICAL RELATIONS, AND UNITS

TABLES

1. Decibel Table

Db	Current and voltage ratio		Power ratio		Db	Current and voltage ratio		Power ratio	
	Gain	Loss	Gain	Loss		Gain	Loss	Gain	Loss
0.1	1.01	0.989	1.02	0.977	8.0	2.51	0.398	6.31	0.158
0.2	1.02	0.977	1.05	0.955	8.5	2.66	0.376	7.08	0.141
0.3	1.03	0.966	1.07	0.933	9.0	2.82	0.355	7.94	0.126
0.4	1.05	0.955	1.10	0.912	9.5	2.98	0.335	8.91	0.112
0.5	1.06	0.944	1.12	0.891	10.0	3.16	0.316	10.00	0.100
0.6	1.07	0.933	1.15	0.871	11.0	3.55	0.282	12.6	0.079
0.7	1.08	0.923	1.17	0.851	12.0	3.98	0.251	15.8	0.063
0.8	1.10	0.912	1.20	0.832	13.0	4.47	0.224	19.9	0.050
0.9	1.11	0.902	1.23	0.813	14.0	5.01	0.199	25.1	0.040
1.0	1.12	0.891	1.26	0.794	15.0	5.62	0.178	31.6	0.032
1.1	1.13	0.881	1.29	0.776	16.0	6.31	0.158	39.8	0.025
1.2	1.15	0.871	1.32	0.759	17.0	7.08	0.141	50.1	0.020
1.3	1.16	0.861	1.35	0.741	18.0	7.94	0.126	63.1	0.016
1.4	1.17	0.851	1.38	0.724	19.0	8.91	0.112	79.4	0.013
1.5	1.19	0.841	1.41	0.708	20.0	10.00	0.100	100.0	0.010
1.6	1.20	0.832	1.44	0.692	25.0	17.8	0.056	3.16×10^{2}	3.16×10^{-3}
1.7	1.22	0.822	1.48	0.676	30.0	31.6	0.032	10^{3}	10^{-3}
1.8	1.23	0.813	1.51	0.661	35.0	56.2	0.018	3.16×10^{3}	3.16×10^{-4}
1.9	1.24	0.803	1.55	0.646	40.0	100.0	0.010	10^{4}	10^{-4}
2.0	1.26	0.794	1.58	0.631	45.0	177.8	0.006	3.16×10^{4}	3.16×10^{-5}
2.2	1.29	0.776	1.66	0.603	50.0	316	0.003	10^{5}	10^{-5}
2.4	1.32	0.759	1.74	0.575	55.0	562	0.002	3.16×10^{5}	3.16×10^{-6}
2.6	1.35	0.741	1.82	0.550	60.0	1,000	0.001	10^{6}	10^{-6}
2.8	1.38	0.724	1.90	0.525	65.0	1,770	0.0006	3.16×10^{6}	3.16×10^{-7}
3.0	1.41	0.708	1.99	0.501	70.0	3,160	0.0003	10^{7}	10^{-7}
3.2	1.44	0.692	2.09	0.479	75.0	5,620	0.0002	3.16×10^{7}	3.16×10^{-8}
3.4	1.48	0.676	2.19	0.457	80.0	10,000	0.0001	10^{8}	10^{-8}
3.6	1.51	0.661	2.29	0.436	85.0	17,800	0.00006	3.16×10^{8}	3.16×10^{-9}
3.8	1.55	0.646	2.40	0.417	90.0	31,600	0.00003	10^{9}	10^{-9}
4.0	1.58	0.631	2.51	0.398	95.0	56,200	0.00002	3.16×10^{9}	3.16×10^{-10}
4.2	1.62	0.617	2.63	0.380	100.0	100,000	0.00001	10^{10}	10^{-10}
4.4	1.66	0.603	2.75	0.363	105.0	178,000	0.000006	3.16×10^{10}	3.16×10^{-11}
4.6	1.70	0.589	2.88	0.347	110.0	316,000	0.000003	10^{11}	10^{-11}
4.8	1.74	0.575	3.02	0.331	115.0	562,000	0.000002	3.16×10^{11}	3.16×10^{-12}
5.0	1.78	0.562	3.16	0.316	120.0	1,000,000	0.000001	10^{12}	10^{-12}
5.5	1.88	0.531	3.55	0.282	130.0	3.16×10^{6}	3.16×10^{-7}	10^{13}	10^{-13}
6.0	1.99	0.501	3.98	0.251	140.0	10^{7}	10^{-7}	10^{14}	10^{-14}
6.5	2.11	0.473	4.47	0.224	150.0	3.16×10^{7}	3.16×10^{-8}	10^{15}	10^{-15}
7.0	2.24	0.447	5.01	0.199	160.0	10^{8}	10^{-8}	10^{16}	10^{-16}
7.5	2.37	0.422	5.62	0.178	170.0	3.16×10^{8}	3.16×10^{-9}	10^{17}	10^{-17}

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2. Natural Sines and Cosines*

NOTE.—For cosines use right-hand column of degrees and lower line of tenths.

Deg	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
0°	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	89
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	88
2	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506	87
3	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680	86
4	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854	85
5	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028	84
6	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201	83
7	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374	82
8	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547	81
9	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719	80°
10°	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891	79
11	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062	78
12	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232	77
13	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402	76
14	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571	75
15	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740	74
16	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907	73
17	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074	72
18	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239	71
19	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404	70°
20°	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567	69
21	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730	68
22	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891	67
23	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051	66
24	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210	65
25	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368	64
26	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524	63
27	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679	62
28	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833	61
29	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985	60°
30°	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5900	0.5105	0.5120	0.5135	59
31	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284	58
32	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432	57
33	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577	56
34	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721	55
35	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864	54
36	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004	53
37	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143	52
38	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280	51
39	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414	50°
40°	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547	49
41	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678	48
42	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807	47
43	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934	46
44	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059	45
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg

* From "Standard Handbook for Electrical Engineers," 7th ed.

2. Natural Sines and Cosines.*—(*Concluded*)

Deg	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
45	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181	44
46	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302	43
47	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420	42
48	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536	41
49	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649	40°
50°	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760	39
51	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869	38
52	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976	37
53	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080	36
54	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181	35
55	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281	34
56	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377	33
57	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471	32
58	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563	31
59	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652	30°
60°	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738	29
61	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821	28
62	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902	27
63	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980	26
64	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056	25
65	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128	24
66	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198	23
67	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265	22
68	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330	21
69	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391	20°
70°	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449	19
71	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505	18
72	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558	17
73	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608	16
74	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655	15
75	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699	14
76	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740	13
77	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778	12
78	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813	11
79	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845	10°
80°	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874	9
81	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900	8
82	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923	7
83	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943	6
84	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960	5
85	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974	4
86	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985	3
87	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993	2
88	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998	1
89	0.9998	0.9999	0.9999	0.9999	0.9999	1.000	1.000	1.000	1.000	1.000	0°
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg

* From "Standard Handbook for Electrical Engineers," 7th ed.

3. Natural Tangents and Cotangents*

NOTE.—For cotangents use right-hand column of degrees and lower line of tenths.

Deg	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
0°	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	89
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	88
2	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507	87
3	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682	86
4	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857	85
5	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033	84
6	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210	83
7	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388	82
8	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566	81
9	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745	80°
10°	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926	79
11	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107	78
12	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290	77
13	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475	76
14	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2643	0.2661	75
15	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849	74
16	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038	73
17	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230	72
18	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424	71
19	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620	70°
20°	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819	69
21	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020	68
22	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224	67
23	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431	66
24	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642	65
25	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856	64
26	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073	63
27	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295	62
28	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520	61
29	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750	60°
30°	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985	59
31	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224	58
32	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469	57
33	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720	56
34	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976	55
35	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239	54
36	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508	53
37	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785	52
38	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069	51
39	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361	50°
40°	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662	49
41	0.8693	0.8724	0.8754	0.8785	0.8816	0.8347	0.8878	0.8910	0.8941	0.8972	48
42	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293	47
43	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623	46
44	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965	45
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg

* From "Standard Handbook for Electrical Engineers," 7th ed.

Ex. 1044-0014

3. Natural Tangents and Cotangents.*—(*Concluded*)

Deg	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
45	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319	44
46	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686	43
47	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067	42
48	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463	41
49	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875	40°
50°	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305	39
51	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753	38
52	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222	37
53	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713	36
54	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229	35
55	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770	34
56	1.4326	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340	33
57	1 5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5318	1.5880	1.5941	32
58	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577	31
59	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251	30°
60°	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966	29
61	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728	28
62	1.8307	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542	27
63	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413	26
64	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348	25
65	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355	24
66	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445	23
67	2.3599	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627	22
68	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916	21
69	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326	20°
70°	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878	19
71	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595	18
72	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	2.2506	17
73	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646	16
74	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062	15
75	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812	14
76	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972	13
77	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646	12
78	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970	11
79	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140	10°
80°	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2433	9
81	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264	8
82	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285	7
83	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572	6
84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20	5
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95	4
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46	3
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27	2
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08	1
89	57.29	63.66	71.62	81.85	94.49	114.6	143.2	191.0	286.5	573.0	0°
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg

* From "Standard Handbook for Electrical Engineers," 7th ed.

4. Common Logarithms of Numbers*

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

* From "Standard Handbook for Electrical Engineers," 7th ed.

Ex. 1044-0016

4. Common Logarithms of Numbers.*—(*Concluded*)

N	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8850
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

* From "Standard Handbook for Electrical Engineers," 7th ed.

5. Natural, Napierian, or Hyperbolic Logarithms*

N	0	1	2	3	4	5	6	7	8	9
0	$-\infty$	0.0000	0.6931	1.0986	1.3863	1.6094	1.7918	1.9459	2.0794	2.1972
10	2.3026	2.3979	2.4849	2.5649	2.6391	2.7081	2.7726	2.8332	2.8904	2.9444
20	2.9957	3.0445	3.0910	3.1355	3.1781	3.2189	3.2581	3.2958	3.3322	3.3673
30	3.4012	3.4340	3.4657	3.4965	3.5264	3.5553	3.5835	3.6109	3.6376	3.6636
40	3.6889	3.7136	3.7377	3.7612	3.7842	3.8067	3.8286	3.8501	3.8712	3.8918
50	3.9120	3.9318	3.9512	3.9703	3.9890	4.0073	4.0254	4.0431	4.0604	4.0775
60	4.0943	4.1109	4.1271	4.1431	4.1589	4.1744	4.1897	4.2047	4.2195	4.2341
70	4.2485	4.2627	4.2767	4.2905	4.3041	4.3175	4.3307	4.3438	4.3567	4.3694
80	4.3820	4.3944	4.4067	4.4188	4.4308	4.4427	4.4543	4.4659	4.4773	4.4886
90	4.4998	4.5109	4.5218	4.5326	4.5433	4.5539	4.5643	4.5747	4.5850	4.5951
100	4.6052	4.6151	4.6250	4.6347	4.6444	4.6540	4.6634	4.6728	4.6821	4.6913
110	4.7005	4.7095	4.7185	4.7274	4.7362	4.7449	4.7536	4.7622	4.7707	4.7791
120	4.7875	4.7958	4.8040	4.8122	4.8203	4.8283	4.8363	4.8442	4.8520	4.8598
130	4.8675	4.8752	4.8828	4.8903	4.8978	4.9053	4.9127	4.9200	4.9273	4.9345
140	4.9416	4.9488	4.9558	4.9628	4.9698	4.9767	4.9836	4.9904	4.9972	5.0039
150	5.0106	5.0173	5.0239	5.0304	5.0370	5.0434	5.0499	5.0562	5.0626	5.0689
160	5.0752	5.0814	5.0876	5.0938	5.0999	5.1059	5.1120	5.1180	5.1240	5.1299
170	5.1358	5.1417	5.1475	5.1533	5.1591	5.1648	5.1705	5.1761	5.1818	5.1874
180	5.1930	5.1985	5.2040	5.2095	5.2149	5.2204	5.2257	5.2311	5.2364	5.2417
190	5.2470	5.2523	5.2575	5.2627	5.2679	5.2730	5.2781	5.2832	5.2883	5.2933
200	5.2983	5.3033	5.3083	5.3132	5.3181	5.3230	5.3279	5.3327	5.3375	5.3423
210	5.3471	5.3519	5.3566	5.3613	5.3660	5.3706	5.3753	5.3799	5.3845	5.3891
220	5.3936	5.3982	5.4027	5.4072	5.4116	5.4161	5.4205	5.4250	5.4293	5.4337
230	5.4381	5.4424	5.4467	5.4510	5.4553	5.4596	5.4638	5.4681	5.4723	5.4765
240	5.4806	5.4848	5.4889	5.4931	5.4972	5.5013	5.5053	5.5094	5.5134	5.5175
250	5.5215	5.5255	5.5294	5.5334	5.5373	5.5413	5.5452	5.5491	5.5530	5.5568
260	5.5607	5.5645	5.5683	5.5722	5.5759	5.5797	5.5835	5.5872	5.5910	5.5947
270	5.5984	5.6021	5.6058	5.6095	5.6131	5.6168	5.6204	5.6240	5.6276	5.6312
280	5.6348	5.6384	5.6419	5.6454	5.6490	5.6525	5.6560	5.6595	5.6630	5.6664
290	5.6699	5.6733	5.6768	5.6802	5.6836	5.6870	5.6904	5.6937	5.6971	5.7004
300	5.7038	5.7071	5.7104	5.7137	5.7170	5.7203	5.7236	5.7268	5.7301	5.7333
310	5.7366	5.7398	5.7430	5.7462	5.7494	5.7526	5.7557	5.7589	5.7621	5.7652
320	5.7683	5.7714	5.7746	5.7777	5.7807	5.7838	5.7869	5.7900	5.7930	5.7961
330	5.7991	5.8021	5.8051	5.8081	5.8111	5.8141	5.8171	5.8201	5.8230	5.8260
340	5.8289	5.8319	5.8348	5.8377	5.8406	5.8435	5.8464	5.8493	5.8522	5.8551
350	5.8579	5.8608	5.8636	5.8665	5.8693	5.8721	5.8749	5.8777	5.8805	5.8833
360	5.8861	5.8889	5.8916	5.8944	5.8972	5.8999	5.9026	5.9054	5.9081	5.9108
370	5.9135	5.9162	5.9189	5.9216	5.9243	5.9269	5.9296	5.9322	5.9349	5.9375
380	5.9402	5.9428	5.9454	5.9480	5.9506	5.9532	5.9558	5.9584	5.9610	5.9636
390	5.9661	5.9687	5.9713	5.9738	5.9764	5.9789	5.9814	5.9839	5.9865	5.9890
400	5.9915	5.9940	5.9965	5.9989	6.0014	6.0039	6.0064	6.0088	6.0113	6.0137
410	6.0162	6.0186	6.0210	6.0234	6.0259	6.0283	6.0307	6.0331	6.0355	6.0379
420	6.0403	6.0426	6.0450	6.0474	6.0497	6.0521	6.0544	6.0568	6.0591	6.0615
430	6.0638	6.0661	6.0684	6.0707	6.0730	6.0753	6.0776	6.0799	6.0822	6.0845
440	6.0868	6.0890	6.0913	6.0936	6.0958	0.0981	6.1003	6.1026	6.1048	6.1070
450	6.1092	6.1115	6.1137	6.1159	6.1181	6.1203	6.1225	6.1247	6.1269	6.1291
460	6.1312	6.1334	6.1356	6.1377	6.1399	6.1420	6.1442	6.1463	6.1485	6.1506
470	6.1527	6.1549	6.1570	6.1591	6.1612	6.1633	6.1654	6.1675	6.1696	6.1717
480	6.1738	6.1759	6.1779	6.1800	6.1821	6.1841	6.1862	6.1883	6.1903	6.1921
490	6.1944	6.1964	6.1985	6.2005	6.2025	6.2046	6.2066	6.2086	6.2106	6.2126

* From "Standard Handbook for Electrical Engineers," 7th ed.

NOTE 1: Moving the decimal point n places to the right (or left) in the number is equivalent to adding (or subtracting) n times 2.3026.

NOTE 2:

$$\log_e x = 2.3026 \log_{10} x$$
$$\log_{10} x = 0.4343 \log_e x$$
$$\log_e 10 = 2.3026$$
$$\log_{10} e = 0.4343$$

n	$n \times 2.3026$
1	2.3026 = 0.6974–3
2	4.6052 = 0.3948–5
3	6.9078 = 0.0922–7
4	9.2103 = 0.7897–10
5	11.5129 = 0.4871–12
6	13.8155 = 0.1845–14
7	16.1181 = 0.8819–17
8	18.4207 = 0.5793–19
9	20.7233 = 0.2767–21

Ex. 1044-0018

5. Natural, Napierian, or Hyperbolic Logarithms.*—(*Concluded*)

N	0	1	2	3	4	5	6	7	8	9
500	6.2146	6.2166	6.2186	6.2206	6.2226	6.2246	6.2265	6.2285	6.2305	6.2324
510	6.2344	6.2364	6.2383	6.2403	6.2422	6.2442	6.2461	6.2480	6.2500	6.2519
520	6.2538	6.2558	6.2577	6.2596	6.2615	6.2634	6.2653	6.2672	6.2691	6.2710
530	6.2729	6.2748	6.2766	6.2785	6.2804	6.2823	6.2841	6.2860	6.2879	6.2897
540	6.2916	6.2934	6.2953	6.2971	6.2989	6.3008	6.3026	6.3044	6.3063	6.3081
550	6.3099	6.3117	6.3135	6.3154	6.3172	6.3190	6.3208	6.3226	6.3244	6.3261
560	6.3279	6.3297	6.3315	6.3333	6.3351	6.3368	6.3386	6.3404	6.3421	6.3439
570	6.3456	6.3474	6.3491	6.3509	6.3256	6.3544	6.3561	6.3578	6.3596	6.3613
580	6.3630	6.3648	6.3665	6.3682	6.3699	6.3716	6.3733	6.3750	6.3767	6.3784
590	6.3801	6.3818	6.3835	6.3852	6.3869	6.3886	6.3902	6.3919	6.3936	6.3953
600	6.3969	6.3986	6.4003	6.4019	6.4036	6.4052	6.4069	6.4085	6.4102	6.4118
610	6.4135	6.4151	6.4167	6.4184	6.4200	6.4216	6.4232	6.4249	6.4265	6.4281
620	6.4297	6.4313	6.4329	6.4345	6.4362	6.4378	6.4394	6.4409	6.4425	6.4441
630	6.4457	6.4473	6.4489	6.4505	6.4520	6.4536	6.4552	6.4568	6.4583	6.4599
640	6.4615	6.4630	6.4646	6.4661	6.4677	6.4693	6.4708	6.4723	6.4739	6.4754
650	6.4770	6.4785	6.4800	6.4816	6.4831	6.4846	6.4862	6.4877	6.4892	6.4907
660	6.4922	6.4938	6.4953	6.4968	6.4983	6.4998	6.5013	6.5028	6.5043	6.5058
670	6.5073	6.5088	6.5103	6.5117	6.5132	6.5147	6.5162	6.5177	6.5191	6.5206
680	6.5221	6.5236	6.5250	6.5265	6.5280	6.5294	6.5309	6.5323	6.5338	6.5352
690	6.5367	6.5381	6.5396	6.5410	6.5425	6.5439	6.5453	6.5468	6.5482	6.5497
700	6.5511	6.5525	6.5539	6.5554	6.5568	6.5582	6.5596	6.5610	6.5624	6.5639
710	6.5653	6.5667	6.5681	6.5695	6.5709	6.5723	6.5737	6.5751	6.5765	6.5779
720	6.5793	6.5806	6.5820	6.5834	6.5848	6.5862	6.5876	6.5889	6.5903	6.5917
730	6.5930	6.5944	6.5958	6.5971	6.5985	6.5999	6.6012	6.6026	6.6039	6.6053
740	6.6067	6.6080	6.6093	6.6107	6.6120	6.6134	6.6147	6.6161	6.6174	6.6187
750	6.6201	6.6214	6.6227	6.6241	6.6254	6.6267	6.6280	6.6294	6.6307	6.6320
760	6.6333	6.6346	6.6359	6.6373	6.6386	6.6399	6.6412	6.6425	6.6438	6.6451
770	6.6464	6.6477	6.6490	6.6503	6.6516	6.6529	6.6542	6.6554	6.6567	6.6580
780	6.6593	6.6606	6.6619	6.6631	6.6644	6.6657	6.6670	6.6682	6.6695	6.6708
790	6.6720	6.6733	6.6746	6.6758	6.6771	6.6783	6.6796	6.6809	6.6821	6.6834
800	6.6846	6.6859	6.6871	6.6884	6.6896	6.6908	6.6921	6.6933	6.6946	6.6958
810	6.6970	6.6983	6.6995	6.7007	6.7020	6.7032	6.7044	6.7056	6.7069	6.7081
820	6.7093	6.7105	6.7117	6.7130	6.7142	6.7154	6.7166	6.7178	6.7190	6.7202
830	6.7214	6.7226	6.7238	6.7250	6.7262	6.7274	6.7286	6.7298	6.7310	6.7322
840	6.7334	6.7346	6.7358	6.7370	6.7382	6.7393	6.7405	6.7417	6.6429	6.7441
850	6.7452	6.7464	6.7476	6.7488	6.7499	6.7511	6.7523	6.7534	6.7546	6.7558
860	6.7569	6.7581	6.7593	6.7604	6.7616	6.7627	6.7639	6.7650	6.7662	6.7673
870	6.7685	6.7696	6.7708	6.7719	6.7731	6.7742	6.7754	6.7765	6.7776	6.7788
880	6.7799	6.7811	6.7822	6.7833	6.7845	6.7856	6.7867	5.7878	6.7890	6.7901
890	6.7912	6.7923	6.7935	6.7946	6.7957	6.7968	6.7979	6.7991	6.8002	6.8013
900	6.8024	6.8035	6.8046	6.8057	6.8068	6.8079	6.8090	6.8101	6.8112	6.8123
910	6.8134	6.8145	6.8156	6.8167	6.8178	6.8189	6.8200	6.8211	6.8222	6.8233
920	6.8244	6.8255	6.8265	6.8276	6.8287	6.8298	6.8309	6.8320	6.8330	6.8341
930	6.8352	6.8363	6.8373	6.8384	6.8395	6.8405	6.8416	6.8427	6.8437	6.8448
940	6.8459	6.8469	6.8480	6.8491	6.8501	6.8512	6.8522	6.8533	6.8544	6.8554
950	6.8565	6.8575	6.8586	6.8596	6.8607	6.8617	6.8628	6.8638	6.8648	6.8659
960	6.8669	6.8680	6.8690	6.8701	6.8711	6.8721	6.8732	6.8742	6.8752	6.8763
970	6.8773	6.8783	6.8794	6.8804	6.8814	6.8824	6.8835	6.8845	6.8855	6.8865
980	6.8876	6.8886	6.8896	6.8906	6.8916	6.8926	6.8937	6.8947	6.8957	6.8967
990	6.8977	6.8987	6.8997	6.9007	6.9017	6.9027	6.9037	6.9047	6.9057	6.9068

* From "Standard Handbook for Electrical Engineers," 7th ed.

Ex. 1044-0019

6. Hyperbolic Sines* $\sinh x = (e^x - e^{-x})/2$

x	0	1	2	3	4	5	6	7	8	9	Av. diff.
0.0	0.0000	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0701	0.0801	0.0901	100
0.1	0.1002	0.1102	0.1203	0.1304	0.1405	0.1506	0.1607	0.1708	0.1810	0.1911	101
0.2	0.2013	0.2115	0.2218	0.2320	0.2423	0.2526	0.2629	0.2733	0.2837	0.2941	103
0.3	0.3045	0.3150	0.3255	0.3360	0.3466	0.3572	0.3678	0.3785	0.3892	0.4000	106
0.4	0.4108	0.4216	0.4325	0.4434	0.4543	0.4653	0.4764	0.4875	0.4986	0.5098	110
0.5	0.5211	0.5324	0.5438	0.5552	0.5666	0.5782	0.5897	0.6014	0.6131	0.6248	116
0.6	0.6367	0.6485	0.6605	0.6725	0.6846	0.6967	0.7090	0.7213	0.7336	0.7461	122
0.7	0.7586	0.7712	0.7838	0.7966	0.8094	0.8223	0.8353	0.8484	0.8615	0.8748	130
0.8	0.8881	0.9015	0.9150	0.9286	0.9423	0.9561	0.9700	0.9840	0.9981	1.012	138
0.9	1.027	1.041	1.055	1.070	1.085	1.099	1.114	1.129	1.145	1.160	15
1.0	1.175	1.191	1.206	1.222	1.238	1.254	1.270	1.286	1.303	1.319	16
1.1	1.336	1.352	1.369	1.386	1.403	1.421	1.438	1.456	1.474	1.491	17
1.2	1.509	1.528	1.546	1.564	1.583	1.602	1.621	1.640	1.659	1.679	19
1.3	1.698	1.718	1.738	1.758	1.779	1.799	1.820	1.841	1.862	1.883	21
1.4	1.904	1.926	1.948	1.970	1.992	2.014	2.037	2.060	2.083	2.106	22
1.5	2.129	2.153	2.177	2.201	2.225	2.250	2.274	2.299	2.324	2.350	25
1.6	2.376	2.401	2.428	2.454	2.481	2.507	2.535	2.562	2.590	2.617	27
1.7	2.646	2.674	2.703	2.732	2.761	2.790	2.820	2.850	2.881	2.911	30
1.8	2.942	9.973	3.005	3.037	3.069	3.101	3.134	3.167	3.200	3.234	33
1.9	3.268	3.303	3.337	3.372	3.408	3.443	3.479	3.516	3.552	3.589	36
2.0	3.627	3.665	3.703	3.741	3.780	3.820	3.859	3.899	3.940	3.981	39
2.1	4.022	4.064	4.106	4.148	4.191	4.234	4.278	4.322	4.367	4.412	44
2.2	4.457	4.503	4.549	4.596	4.643	4.691	4.739	4.788	4.837	4.887	48
2.3	4.937	4.988	5.039	5.090	5.142	5.195	5.248	5.302	5.356	5.411	53
2.4	5.466	5.522	5.578	5.635	5.693	5.751	5.810	5.869	5.929	5.989	58
2.5	6.050	6.112	6.174	6.237	6.300	6.365	6.429	6.495	6.561	6.627	64
2.6	6.695	6.763	6.831	6.901	6.971	7.042	7.113	7.185	7.258	7.332	71
2.7	7.406	7.481	7.557	7.634	7.711	7.789	7.868	7.948	8.028	8.110	79
2.8	8.192	8.275	8.359	8.443	8.529	8.615	8.702	8.790	8.879	8.969	87
2.9	9.060	9.151	9.244	9.337	9.431	9.527	9.623	9.720	9.819	9.918	96
3.0	10.02	10.12	10.22	10.32	10.43	10.53	10.64	10.75	10.86	10.97	11
3.1	11.08	11.19	11.30	11.42	11.53	11.65	11.76	11.88	12.00	12.12	12
3.2	12.25	12.37	12.49	12.62	12.75	12.88	13.01	13.14	13.27	13.40	13
3.3	13.54	13.67	13.81	13.95	14.09	14.23	14.38	14.52	14.67	14.82	14
3.4	14.97	15.12	15.27	15.42	15.58	15.73	15.89	16.05	16.21	16.38	16
3.5	16.54	16.71	16.88	17.05	17.22	17.39	17.57	17.74	17.92	18.10	17
3.6	18.29	18.47	18.66	18.84	19.03	19.22	19.42	19.61	19.81	20.01	19
3.7	20.21	20.41	20.62	20.83	21.04	21.25	21.46	21.68	21.90	22.12	21
3.8	22.34	22.56	22.79	23.02	23.25	23.49	23.72	23.96	24.20	24.45	24
3.9	24.69	24.94	25.19	25.44	25.70	25.96	26.22	26.48	26.75	27.02	26
4.0	27.29	27.56	27.84	28.12	28.40	28.69	28.98	29.27	29.56	29.86	29
4.1	30.16	30.47	30.77	31.08	31.39	31.71	32.03	32.35	32.68	33.00	32
4.2	33.34	33.67	34.01	34.35	34.70	35.05	35.40	35.75	36.11	36.48	35
4.3	36.84	37.21	37.59	37.97	38.35	38.73	39.12	39.52	39.91	40.31	39
4.4	40.72	41.13	41.54	41.96	42.38	42.81	43.24	43.67	44.11	44.56	43
4.5	45.00	45.46	45.91	46.37	46.84	47.31	47.79	48.27	48.75	49.24	47
4.6	49.74	50.24	50.74	51.25	51.77	52.29	52.81	53.34	53.88	54.42	52
4.7	54.97	55.52	56.08	56.64	57.21	57.79	58.37	58.96	59.55	60.15	58
4.8	60.75	61.36	61.98	62.60	63.23	63.87	64.51	65.16	65.81	67.47	64
4.9	67.14	67.82	68.50	69.19	69.88	70.58	71.29	72.01	72.73	73.46	71
5.0	74.20										

If $x > 5$, $\sinh x = e^x/2$ correct to four significant figures.

* From Lionel S. Marks, "Mechanical Engineers' Handbook."

Ex. 1044-0020

7. Hyperbolic Cosines* $\cosh x = (e^x + e^{-x})/2$

x	0	1	2	3	4	5	6	7	8	9	Av. diff.
0.0	1.000	1.000	1.000	1.000	1.001	1.001	1.002	1.002	1.003	1.004	1
0.1	1.005	1.006	1.007	1.008	1.010	1.011	1.013	1.014	1.016	1.018	2
0.2	1.020	1.022	1.024	1.027	1.029	1.031	1.034	1.037	1.039	1.042	3
0.3	1.045	1.048	1.052	1.055	1.058	1.062	1.066	1.069	1.073	1.077	4
0.4	1.081	1.085	1.090	1.094	1.098	1.103	1.108	1.112	1.117	1.122	5
0.5	1.128	1.133	1.138	1.144	1.149	1.155	1.161	1.167	1.173	1.179	6
0.6	1.185	1.192	1.198	1.205	1.212	1.219	1.226	1.233	1.240	1.248	7
0.7	1.255	1.263	1.271	1.278	1.287	1.295	1.303	1.311	1.320	1.329	8
0.8	1.337	1.346	1.355	1.365	1.374	1.384	1.393	1.403	1.413	1.423	10
0.9	1.433	1.443	1.454	1.465	1.475	1.486	1.497	1.509	1.520	1.531	11
1.0	1.543	1.555	1.567	1.579	1.591	1.604	1.616	1.629	1.642	1.655	13
1.1	1.669	1.682	1.696	1.709	1.723	1.737	1.752	1.766	1.781	1.796	14
1.2	1.811	1.826	1.841	1.857	1.872	1.888	1.905	1.921	1.937	1.954	16
1.3	1.971	1.988	2.005	2.023	2.040	2.058	2.076	2.095	2.113	2.132	18
1.4	2.151	2.170	2.189	2.209	2.229	2.249	2.269	2.290	2.310	2.331	20
1.5	2.352	2.374	2.395	2.417	2.439	2.462	2.484	2.507	2.530	2.554	23
1.6	2.577	2.601	2.625	2.650	2.675	2.700	2.725	2.750	2.776	2.802	25
1.7	2.828	2.855	2.882	2.909	2.936	2.964	2.992	3.021	3.049	3.078	28
1.8	3.107	3.137	3.167	3.197	3.228	3.259	3.290	3.321	3.353	3.385	31
1.9	3.418	3.451	3.484	3.517	3.551	3.585	3.620	3.655	3.690	3.726	34
2.0	3.762	3.799	3.835	3.873	3.910	3.948	3.987	4.026	4.065	4.104	38
2.1	4.144	4.185	4.226	4.267	4.309	4.351	4.393	4.436	4.480	4.524	42
2.2	4.568	4.613	4.658	4.704	4.750	4.797	4.844	4.891	4.939	4.988	47
2.3	5.037	5.087	5.137	5.188	5.239	5.290	5.343	5.395	5.449	5.503	52
2.4	5.557	5.612	5.667	5.723	5.780	5.837	5.895	5.954	6.013	6.072	58
2.5	6.132	6.193	6.255	6.317	6.379	6.443	6.507	6.571	6.636	6.702	64
2.6	6.769	6.836	6.904	6.973	7.042	7.112	7.183	7.255	7.327	7.400	70
2.7	7.473	7.548	7.623	7.699	7.776	7.853	7.932	8.011	8.091	8.171	78
2.8	8.253	8.335	8.418	8.502	8.587	8.673	8.759	8.847	8.935	9.024	86
2.9	9.115	9.206	9.298	9.391	9.484	9.579	9.675	9.772	9.869	9.968	95
3.0	10.07	10.17	10.27	10.37	10.48	10.58	10.69	10.79	10.90	11.01	11
3.1	11.12	11.23	11.35	11.46	11.57	11.69	11.81	11.92	12.04	12.16	12
3.2	12.29	12.41	12.53	12.66	12.79	12.91	13.04	13.17	13.31	13.44	13
3.3	13.57	13.71	13.85	13.99	14.13	14.27	14.41	14.56	14.70	14.85	14
3.4	15.00	15.15	15.30	15.45	15.61	15.77	15.92	16.08	16.25	16.41	16
3.5	16.57	16.74	16.91	17.08	17.25	17.42	17.60	17.77	17.95	18.13	17
3.6	18.31	18.50	18.68	18.87	19.06	19.25	19.44	19.64	19.84	20.03	19
3.7	20.24	20.44	20.64	20.85	21.06	21.27	21.49	21.70	21.92	22.14	21
3.8	22.36	22.59	22.81	23.04	23.27	23.51	23.74	23.98	24.22	24.47	23
3.9	24.71	24.96	25.21	25.46	25.72	25.98	26.24	26.50	26.77	27.04	26
4.0	27.31	27.58	27.86	28.14	28.42	28.71	29.00	29.29	29.58	29.88	29
4.1	30.18	30.48	30.79	31.10	31.41	31.72	32.04	32.37	32.69	33.02	32
4.2	33.35	33.69	34.02	34.37	34.71	35.06	35.41	35.77	36.13	36.49	35
4.3	36.86	37.23	37.60	37.98	38.36	38.75	39.13	39.53	39.93	40.33	39
4.4	40.73	41.14	41.55	41.97	42.39	42.82	43.25	43.68	44.12	44.57	43
4.5	45.01	45.47	45.92	46.38	46.85	47.32	47.80	48.28	48.76	49.25	47
4.6	49.75	50.25	50.75	51.26	51.78	52.30	52.82	57.35	57.89	54.43	52
4.7	54.98	55.53	56.09	56.65	57.22	57.80	58.38	58.96	59.56	60.15	58
4.8	60.76	61.37	61.99	62.61	63.24	63.87	64.52	65.16	65.82	66.48	64
4.9	67.15	67.82	68.50	69.19	69.89	70.59	71.30	72.02	72.74	73.47	71
5.0	74.21										

If $x > 5$, $\cosh x = e^x/2$ correct to four significant figures.

* From Lionel S. Marks, "Mechanical Engineers' Handbook."

8. Hyperbolic Tangents*

$\tanh x = (e^x - e^{-x})/(e^x + e^{-x}) = \sinh x/\cosh x$

x	0	1	2	3	4	5	6	7	8	9	Av. diff.
0.0	0.0000	0.0100	0.0200	0.0300	0.0400	0.0500	0.0599	0.0699	0.0798	0.0898	100
0.1	0.0997	0.1096	0.1194	0.1293	0.1391	0.1489	0.1587	0.1684	0.1781	0.1878	98
0.2	0.1974	0.2070	0.2165	0.2260	0.2355	0.2449	0.2543	0.2636	0.2729	0.2821	94
0.3	0.2913	0.3004	0.3095	0.3185	0.3275	0.3364	0.3452	0.3540	0.3627	0.3714	89
0.4	0.3800	0.3885	0.3969	0.4053	0.4137	0.4219	0.4301	0.4382	0.4462	0.4542	82
0.5	0.4621	0.4700	0.4777	0.4854	0.4930	0.5005	0.5080	0.5154	0.5227	0.5299	75
0.6	0.5370	0.5441	0.5511	0.5581	0.5649	0.5717	0.5784	0.5850	0.5915	0.5980	67
0.7	0.6044	0.6107	0.6169	0.6231	0.6291	0.6352	0.6411	0.6469	0.6527	0.6584	60
0.8	0.6640	0.6696	0.6751	0.6805	0.6858	0.6911	0.6963	0.7014	0.7064	0.7114	52
0.9	0.7163	0.7211	0.7259	0.7306	0.7352	0.7398	0.7443	0.7487	0.7531	0.7574	45
1.0	0.7616	0.7658	0.7699	0.7739	0.7779	0.7818	0.7857	0.7895	0.7932	0.7969	39
1.1	0.8005	0.8041	0.8076	0.8110	0.8144	0.8178	0.8210	0.8243	0.8275	0.8306	33
1.2	0.8337	0.8367	0.8397	0.8426	0.8455	0.8483	0.8511	0.8538	0.8565	0.8591	28
1.3	0.8617	0.8643	0.8668	0.8693	0.8717	0.8741	0.8764	0.8787	0.8810	0.8832	24
1.4	0.8854	0.8875	0.8896	0.8917	0.8937	0.8957	0.8977	0.8996	0.9015	0.9033	20
1.5	0.9052	0.9069	0.9087	0.9104	0.9121	0.9138	0.9154	0.9170	0.9186	0.9202	17
1.6	0.9217	0.9232	0.9246	0.9261	0.9275	0.9289	0.9302	0.9316	0.9329	0.9342	14
1.7	0.9354	0.9367	0.9379	0.9391	0.9402	0.9414	0.9425	0.9436	0.9447	0.9458	11
1.8	0.9468	0.9478	0.9488	0.9498	0.9508	0.9518	0.9527	0.9536	0.9545	0.9554	9
1.9	0.9562	0.9571	0.9579	0.9587	0.9595	0.9603	0.9611	0.9619	0.9626	0.9633	8
2.0	0.9640	0.9647	0.9654	0.9661	0.9668	0.9674	0.9680	0.9687	0.9693	0.9699	6
2.1	0.9705	0.9710	0.9716	0.9722	0.9727	0.9732	0.9738	0.9743	0.9748	0.9753	5
2.2	0.9757	0.9762	0.9767	0.9771	0.9776	0.9780	0.9785	0.9789	0.9793	0.9797	4
2.3	0.9801	0.9805	0.9809	0.9812	0.9816	0.9820	0.9823	0.9827	0.9830	0.9834	4
2.4	0.9837	0.9840	0.9843	0.9846	0.9849	0.9852	0.9855	0.9858	0.9861	0.9863	3
2.5	0.9866	0.9869	0.9871	0.9874	0.9876	0.9879	0.9881	0.9884	0.9886	0.9888	2
2.6	0.9890	0.9892	0.9895	0.9897	0.9899	0.9901	0.9903	0.9905	0.9906	0.9908	2
2.7	0.9910	0.9912	0.9914	0.9915	0.9917	0.9919	0.9920	0.9922	0.9923	0.9925	2
2.8	0.9926	0.9928	0.9929	0.9931	0.9932	0.9933	0.9935	0.9936	0.9937	0.9938	1
2.9	0.9940	0.9941	0.9942	0.9943	0.9944	0.9945	0.9946	0.9947	0.9949	0.9950	1
3.0	0.9951	0.9959	0.9967	0.9973	0.9978	0.9982	0.9985	0.9988	0.9990	0.9992	4
4.0	0.9993	0.9995	0.9996	0.9996	0.9997	0.9998	0.9998	0.9998	0.9999	0.9999	1
5.0	0.9999	If $x > 5$, $\tanh x = 1.0000$ to four decimal places.									

* From Lionel S. Marks, "Mechanical Engineers' Handbook."

Ex. 1044-0022

9. Degrees and Minutes Expressed in Radians*

Degrees						Hundredths				Minutes	
1°	0.0175	61°	1.0647	121°	2.1118	0°.01	0.0002	0°.51	0.0089	1′	0.0003
2	0.0349	62	1.0821	122	2.1293	0 .02	0.0003	0 .52	0.0091	2′	0.0006
3	0.0524	63	1.0996	123	2.1468	0 .03	0.0005	0 .53	0.0093	3′	0.0009
4	0.0698	64	1.1170	124	2.1642	0 .04	0.0007	0 .54	0.0094	4′	0.0012
5°	0.0873	65°	1.1345	125°	2.1817	0 .05	0.0009	0 .55	0.0096	5′	0.0015
6	0.1047	66	1.1519	126	2.1991	0 .06	0.0010	0 .56	0.0098	6′	0.0017
7	0.1222	67	1.1694	127	2.2166	0 .07	0.0012	0 .57	0.0099	7′	0.0020
8	0.1396	68	1.1868	128	2.2340	0 .08	0.0014	0 .58	0.0101	8′	0.0023
9	0.1571	69	1.2043	129	2.2515	0 .09	0.0016	0 .59	0.0103	9′	0.0026
10°	0.1745	70°	1.2217	130°	2.2689	0°.10	0.0017	0°.60	0.0105	10′	0.0029
11	0.1920	71	1.2392	131	2.2864	0 .11	0.0019	0 .61	0.0106	11′	0.0032
12	0.2094	72	1.2566	132	2.3038	0 .12	0.0021	0 .62	0.0108	12′	0.0035
13	0.2269	73	1.2741	133	2.3213	0 .13	0.0023	0 .63	0.0110	13′	0.0038
14	0.2443	74	1.2915	134	2.3387	0 .14	0.0024	0 .64	0.0112	14′	0.0041
15°	0.2618	75°	1.3090	135°	2.3562	0 .15	0.0026	0 .65	0.0113	15′	0.0044
16	0.2793	76	1.3265	136	2.3736	0 .16	0.0028	0 .66	0.0115	16′	0.0047
17	0.2967	77	1.3439	137	2.3911	0 .17	0.0030	0 .67	0.0117	17′	0.0049
18	0.3142	78	1.3614	138	2.4086	0 .18	0.0031	0 .68	0.0119	18′	0.0052
19	0.3316	79	1.3788	139	2.4260	0 .19	0.0033	0 .69	0.0120	19′	0.0055
20°	0.3491	80°	1.3963	140°	2.4435	0°.20	0.0035	0°.70	0.0122	20′	0.0058
21	0.3665	81	1.4137	141	2.4609	0 .21	0.0037	0 .71	0.0124	21′	0.0061
22	0.3840	82	1.4312	142	2.4784	0 .22	0.0038	0 .72	0.0126	22′	0.0064
23	0.4014	83	1.4486	143	2.4958	0 .23	0.0040	0 .73	0.0127	23′	0.0067
24	0.4189	84	1.4661	144	2.5133	0 .24	0.0042	0 .74	0.0129	24′	0.0070
25°	0.4363	85°	1.4835	145°	2.5307	0 .25	0.0044	0 .75	0.0131	25′	0.0073
26	0.4538	86	1.5010	146	2.5482	0 .26	0.0045	0 .76	0.0133	26′	0.0076
27	0.4712	87	1.5184	147	2.5656	0 .27	0.0047	0 .77	0.0134	27′	0.0079
28	0.4887	88	1.5359	148	2.5831	0 .28	0.0049	0 .78	0.0136	28′	0.0081
29	0.5061	89	1.5533	149	2.6005	0 .29	0.0051	0 .79	0.0138	29′	0.0084
30°	0.5236	90°	1.5708	150°	2.6180	0°.30	0.0052	0°.80	0.0140	30′	0.0087
31	0.5411	91	1.5882	151	2.6354	0 .31	0.0054	0 .81	0.0141	31′	0.0090
32	0.5585	92	1.6057	152	2.6529	0 .32	0.0056	0 .82	0.0143	32′	0.0093
33	0.5760	93	1.6232	153	2.6704	0 .33	0.0058	0 .83	0.0145	33′	0.0096
34	0.5934	94	1.6406	154	2.6878	0 .34	0.0059	0 .84	0.0147	34′	0.0099
35°	0.6109	95°	1.6581	155°	2.7053	0 .35	0.0061	0 .85	0.0148	35′	0.0102
36	0.6283	96	1.6755	156	2.7227	0 .36	0.0063	0 .86	0.0150	36′	0.0105
37	0.6458	97	1.6930	157	2.7402	0 .37	0.0065	0 .87	0.0152	37′	0.0108
38	0.6632	98	1.7104	158	2.7576	0 .38	0.0066	0 .88	0.0154	38′	0.0111
39	0.6807	99	1.7279	159	2.7751	0 .39	0.0068	0 .89	0.0155	39′	0.0113
40°	0.6981	100°	1.7453	160°	2.7925	0°.40	0.0070	0°.90	0.0157	40′	0.0116
41	0.7156	101	1.7628	161	2.8100	0 .41	0.0072	0 .91	0.0159	41′	0.0119
42	0.7330	102	1.7802	162	2.8274	0 .42	0.0073	0 .92	0.0161	42′	0.0122
43	0.7505	103	1.7977	163	2.8449	0 .43	0.0075	0 .93	0.0162	43′	0.0125
44	0.7679	104	1.8151	164	2.8623	0 .44	0.0077	0 .94	0.0164	44′	0.0128
45°	0.7854	105°	1.8326	165°	2.8798	0 .45	0.0079	0 .95	0.0166	45′	0.0131
46	0.8029	106	1.8500	166	2.8972	0 .46	0.0080	0 .96	0.0168	46′	0.0134
47	0.8203	107	1.8675	167	2.9147	0 .47	0.0082	0 .97	0.0169	47′	0.0137
48	0.8378	108	1.8850	168	2.9322	0 .48	0.0084	0 .98	0.0171	48′	0.0140
49	0.8552	109	1.9024	169	2.9496	0 .49	0.0086	0 .99	0.0173	49′	0.0143
50°	0.8727	110°	1.9199	170°	2.9671	0°.50	0.0087	1°.00	0.0175	50′	0.0145
51	0.8901	111	1.9373	171	2.9845	....	.....	....	.....	51′	0.0148
52	0.9076	112	1.9548	172	3.0020	....	.....	....	.....	52′	0.0151
53	0.9250	113	1.9722	173	3.0194	....	.....	....	.....	53′	0.0154
54	0.9425	114	1.9897	174	3.0369	....	.....	....	.....	54′	0.0157
55°	0.9599	115°	2.0071	175°	3.0543	....	.....	....	.....	55′	0.0160
56	0.9774	116	2.0246	176	3.0718	....	.....	....	.....	56′	0.0163
57	0.9948	117	2.0420	177	3.0892	....	.....	....	.....	57′	0.0166
58	1.0123	118	2.0595	178	3.1067	....	.....	....	.....	58′	0.0169
59	1.0297	119	2.0769	179	3.1241	....	.....	....	.....	59′	0.0172
60°	1.0472	120°	2.0944	180°	3.1416	....	.....	....	.....	60′	0.0175

Arc 1° = 0.01745. Arc 1′ = 0.0002909. Arc 1″ = 0.000004848.
1 radian = 57°.296 = 57° 17′.75 = 57° 17′ 44″.81.
* From Lionel S. Marks, "Mechanical Engineers' Handbook."

10. Radians Expressed in Degrees*

										Interpolation	
0.01	0°.57	0.64	36°.67	1.27	72°.77	1.90	108°.86	2.53	144°.96		
0.02	1°.15	0.65	37°.24	1.28	73°.34	1.91	109°.43	2.54	145°.53	0.0002	0°.01
0.03	1°.72	0.66	37°.82	1.29	73°.91	1.92	110°.01	2.55	146°.10	0.0004	0°.02
0.04	2°.29	0.67	38°.39	1.30	74°.48	1.93	110°.58	2.56	146°.68	0.0006	0°.03
0.05	2°.86	0.68	38°.96	1.31	75°.06	1.94	111°.15	2.57	147°.25	0.0008	0°.05
0.06	3°.44	0.69	39°.53	1.32	75°.63	1.95	111°.73	2.58	147°.82	0.0010	0°.06
0.07	4°.01	0.70	40°.11	1.33	76°.20	1.96	112°.30	2.59	148°.40	0.0012	0°.07
0.08	4°.58	0.71	40°.68	1.34	76°.78	1.97	112°.87	2.60	148°.97	0.0014	0°.08
0.09	5°.16	0.72	41°.25	1.35	77°.35	1.98	113°.45	2.61	149°.54	0.0016	0°.09
0.10	5°.73	0.73	41°.83	1.36	77°.92	1.99	114°.02	2.62	150°.11	0.0018	0°.10
0.11	6°.30	0.74	42°.40	1.37	78°.50	2.00	114°.59	2.63	150°.69	0.0020	0°.11
0.12	6°.88	0.75	42°.97	1.38	79°.07	2.01	115°.16	2.64	151°.26	0.0022	0°.13
0.13	7°.45	0.76	43°.54	1.39	79°.64	2.02	115°.74	2.65	151°.83	0.0024	0°.14
0.14	8°.02	0.77	44°.12	1.40	80°.21	2.03	116°.31	2.66	152°.41	0.0026	0°.15
0.15	8°.59	0.78	44°.69	1.41	80°.79	2.04	116°.88	2.67	152°.98	0.0028	0°.16
0.16	9°.17	0.79	45°.26	1.42	81°.36	2.05	117°.46	2.68	153°.55	0.0030	0°.17
0.17	9°.74	0.80	45°.84	1.43	81°.93	2.06	118°.03	2.69	154°.13	0.0032	0°.18
0.18	10°.31	0.81	46°.41	1.44	82°.51	2.07	118°.60	2.70	154°.70	0.0034	0°.19
0.19	10°.89	0.82	46°.98	1.45	83°.08	2.08	119°.18	2.71	155°.27	0.0036	0°.21
0.20	11°.46	0.83	47°.56	1.46	83°.65	2.09	119°.75	2.72	155°.84	0.0038	0°.22
0.21	12°.03	0.84	48°.13	1.47	84°.22	2.10	120°.32	2.73	156°.42	0.0040	0°.23
0.22	12°.61	0.85	48°.70	1.48	84°.80	2.11	120°.89	2.74	156°.99	0.0042	0°.24
0.23	13°.18	0.86	49°.27	1.49	85°.37	2.12	121°.47	2.75	157°.56	0.0044	0°.25
0.24	13°.75	0.87	49°.85	1.50	85°.94	2.13	122°.04	2.76	158°.14	0.0046	0°.26
0.25	14°.32	0.88	50°.42	1.51	86°.52	2.14	122°.61	2.77	158°.71	0.0048	0°.28
0.26	14°.90	0.89	50°.99	1.52	87°.09	2.15	123°.19	2.78	159°.28	0.0050	0°.29
0.27	15°.47	0.90	51°.27	1.53	87°.66	2.16	123°.76	2.79	159°.86	0.0052	0°.30
0.28	16°.04	0.91	52°.14	1.54	88°.24	2.17	124°.33	2.80	160°.43	0.0054	0°.31
0.29	16°.62	0.92	52°.71	1.55	88°.81	2.18	124°.90	2.81	161°.00	0.0056	0°.32
0.30	17°.19	0.93	53°.20	1.56	89°.38	2.19	125°.48	2.82	161°.57	0.0058	0°.33
0.31	17°.76	0.94	53°.86	1.57	89°.95	2.20	126°.05	2.83	162°.15	0.0060	0°.34
0.32	18°.33	0.95	54°.43	1.58	90°.53	2.21	126°.62	2.84	162°.72	0.0062	0°.36
0.33	18°.91	0.96	55°.00	1.59	91°.10	2.22	127°.20	2.85	163°.29	0.0064	0°.37
0.34	19°.48	0.97	55°.58	1.60	91°.67	2.23	127°.77	2.86	163°.87	0.0066	0°.38
0.35	20°.05	0.98	56°.15	1.61	92°.25	2.24	128°.34	2.87	164°.44	0.0068	0°.39
0.36	20°.63	0.99	56°.72	1.62	92°.82	2.25	128°.92	2.88	165°.01	0.0070	0°.40
0.37	21°.20	1.00	57°.30	1.63	93°.39	2.26	129°.49	2.89	165°.58	0.0072	0°.41
0.38	21°.77	1.01	57°.87	1.64	93°.97	2.27	130°.06	2.90	166°.16	0.0074	0°.42
0.39	22°.35	1.02	58°.44	1.65	94°.54	2.28	130°.63	2.91	166°.73	0.0076	0°.44
0.40	22°.92	1.03	59°.01	1.66	95°.11	2.29	131°.21	2.92	167°.30	0.0078	0°.45
0.41	23°.49	1.04	59°.59	1.67	95°.68	2.30	131°.78	2.93	167°.88	0.0080	0°.46
0.42	24°.06	1.05	60°.16	1.68	96°.26	2.31	132°.35	2.94	168°.45	0.0082	0°.47
0.43	24°.64	1.06	60°.73	1.69	96°.83	2.32	132°.93	2.95	169°.02	0.0084	0°.48
0.44	25°.21	1.07	61°.31	1.70	97°.40	2.33	133°.50	2.96	169°.60	0.0086	0°.49
0.45	25°.78	1.08	61°.88	1.71	97°.98	2.34	134°.07	2.97	170°.17	0.0088	0°.50
0.46	26°.36	1.09	62°.45	1.72	98°.55	2.35	134°.65	2.98	170°.74	0.0090	0°.52
0.47	26°.93	1.10	63°.03	1.73	99°.12	2.36	135°.22	2.99	171°.31	0.0092	0°.53
0.48	27°.50	1.11	63°.60	1.74	99°.69	2.37	135°.79	3.00	171°.89	0.0094	0°.54
0.49	28°.07	1.12	64°.17	1.75	100°.27	2.38	136°.36	3.01	172°.46	0.0096	0°.55
0.50	28°.65	1.13	64°.74	1.76	100°.84	2.39	136°.94	3.02	173°.03	0.0098	0°.56
0.51	29°.22	1.14	65°.32	1.77	101°.41	2.40	137°.51	3.03	173°.61		
0.52	29°.79	1.15	65°.89	1.78	101°.99	2.41	138°.08	3.04	174°.18	Multiples of π	
0.53	30°.37	1.16	66°.46	1.79	102°.56	2.42	138°.66	3.05	174°.75		
0.54	30°.94	1.17	67°.04	1.80	103°.13	2.43	139°.23	3.06	175° 33		
0.55	31°.51	1.18	67°.61	1.81	103°.71	2.44	139°.80	3.07	175°.90	1 3.1416	180°
0.56	32°.09	1.19	68°.18	1.82	104°.28	2.45	140°.37	3.08	176°.47	2 6.2832	360°
0.57	32°.66	1.20	68°.75	1.83	104°.85	2.46	140°.95	3.09	177°.04	3 9.4248	540°
0.58	33°.23	1.21	69°.33	1.84	105°.42	2.47	141°.52	3.10	177°.62	4 12.5664	720°
0.59	33°.80	1.22	69°.90	1.85	106°.00	2.48	142°.09	3.11	178°.19	5 15.7080	900°
										6 18.8496	1080°
0.60	34°.38	1.23	70°.47	1.86	106°.57	2.49	142°.67	3.12	178°.76	7 21.9911	1260°
0.61	34°.95	1.24	71°.05	1.87	107°.14	2.50	143°.24	3.13	179°.34	8 25.1327	1440°
0.62	35°.52	1.25	71°.62	1.88	107°.72	2.51	143°.81	3.14	179°.91	9 28.2743	1620°
0.63	36°.10	1.26	72°.19	1.89	108°.29	2.52	144°.39	3.15	180°.48	10 31.4159	1800°

* From Lionel S. Marks, "Mechanical Engineers' Handbook."

Ex. 1044-0024

11. Exponentials*
(e^n and e^{-n})

n	e^n	Diff.	n	e^n	Diff.	n	e^n	n	e^{-n}	Diff.	n	e^{-n}	n	e^{-n}
0.00	1.000	10	0.50	1.649	16	1.0	2.718†	0.00	1.000	−10	0.50	0.607	1.0	0.368
0.01	1.010	10	0.51	1.665	17	1.1	3.004	0.01	0.990	−10	0.51	0.600	1.1	0.333
0.02	1.020	10	0.52	1.682	17	1.2	3.320	0.02	0.980	−10	0.52	0.595	1.2	0.301
0.03	1.030	11	0.53	1.699	17	1.3	3.669	0.03	0.970	−9	0.53	0.589	1.3	0.273
0.04	1.041	10	0.54	1.716	17	1.4	4.055	0.04	0.961	−10	0.54	0.583	1.4	0.247
0.05	1.051	11	0.55	1.733	18	1.5	4.482	0.05	0.951	−9	0.55	0.577	1.5	0.223
0.06	1.062	11	0.56	1.751	17	1.6	4.953	0.06	0.942	−10	0.56	0.571	1.6	0.202
0.07	1.073	10	0.57	1.768	18	1.7	5.474	0.07	0.932	−9	0.57	0.566	1.7	0.183
0.08	1.083	11	0.58	1.786	18	1.8	6.050	0.08	0.923	−9	0.58	0.560	1.8	0.165
0.09	1.094	11	0.59	1.804	18	1.9	6.686	0.09	0.914	−9	0.59	0.554	1.9	0.150
0.10	1.105	11	0.60	1.822	18	2.0	7.389	0.10	0.905	−9	0.60	0.549	2.0	0.135
0.11	1.116	11	0.61	1.840	19	2.1	8.166	0.11	0.896	−9	0.61	0.543	2.1	0.122
0.12	1.127	12	0.62	1.859	19	2.2	9.025	0.12	0.887	−9	0.62	0.538	2.2	0.111
0.13	1.139	11	0.63	1.878	18	2.3	9.974	0.13	0.878	−9	0.63	0.533	2.3	0.100
0.14	1.150	12	0.64	1.896	20	2.4	11.02	0.14	0.869	−8	0.64	0.527	2.4	0.0907
0.15	1.162	12	0.65	1.916	19	2.5	12.18	0.15	0.861	−9	0.65	0.522	2.5	0.0821
0.16	1.174	11	0.66	1.935	19	2.6	13.46	0.16	0.852	−8	0.66	0.517	2.6	0.0743
0.17	1.185	12	0.67	1.954	20	2.7	14.88	0.17	0.844	−9	0.67	0.512	2.7	0.0672
0.18	1.197	12	0.68	1.974	20	2.8	16.44	0.18	0.835	−8	0.68	0.507	2.8	0.0608
0.19	1.209	12	0.69	1.994	20	2.9	18.17	0.19	0.827	−8	0.69	0.502	2.9	0.0550
0.20	1.221	13	0.70	2.014	20	3.0	20.09	0.20	0.819	−8	0.70	0.497	3.0	0.0498
0.21	1.234	12	0.71	2.034	20	3.1	22.20	0.21	0.811	−8	0.71	0.492	3.1	0.0450
0.22	1.246	13	0.72	2.054	21	3.2	24.53	0.22	0.803	−8	0.72	0.487	3.2	0.0408
0.23	1.259	12	0.73	2.075	21	3.3	27.11	0.23	0.795	−8	0.73	0.482	3.3	0.0369
0.24	1.271	13	0.74	2.096	21	3.4	29.96	0.24	0.787	−8	0.74	0.477	3.4	0.0334
0.25	1.284	13	0.75	2.117	21	3.5	33.12	0.25	0.779	−8	0.75	0.472	3.5	0.0302
0.26	1.297	13	0.76	2.138	22	3.6	36.60	0.26	0.771	−8	0.76	0.468	3.6	0.0273
0.27	1.310	13	0.77	2.160	21	3.7	40.45	0.27	0.763	−7	0.77	0.463	3.7	0.0247
0.28	1.323	13	0.78	2.181	22	3.8	44.70	0.28	0.756	−8	0.78	0.458	3.8	0.0224
0.29	1.336	14	0.79	2.203	23	3.9	49.40	0.29	0.748	−7	0.79	0.454	3.9	0.0202
0.30	1.350	13	0.80	2.226	22	4.0	54.60	0.30	0.741	−8	0.80	0.449	4.0	0.0183
0.31	1.363	14	0.81	2.248	22	4.1	60.34	0.31	0.733	−7	0.81	0.445	4.1	0.0166
0.32	1.377	14	0.82	2.270	23	4.2	66.69	0.32	0.726	−7	0.82	0.440	4.2	0.0150
0.33	1.391	14	0.83	2.293	23	4.3	73.70	0.33	0.719	−7	0.83	0.436	4.3	0.0136
0.34	1.405	14	0.84	2.316	24	4.4	81.45	0.34	0.712	−7	0.84	0.432	4.4	0.0123
0.35	1.419	14	0.85	2.340	23	4.5	90.02	0.35	0.705	−7	0.85	0.427	4.5	0.0111
0.36	1.433	15	0.86	2.363	24			0.36	0.698	−7	0.86	0.423		
0.37	1.448	14	0.87	2.387	24	5.0	148.4	0.37	0.691	−7	0.87	0.419	5.0	0.00674
0.38	1.462	15	0.88	2.411	24	6.0	403.4	0.38	0.684	−7	0.88	0.415	6.0	0.00248
0.39	1.477	15	0.89	2.435	25	7.0	1097.	0.39	0.677	−7	0.89	0.411	7.0	0.000912
0.40	1.492	15	0.90	2.460	24	8.0	2981.	0.40	0.670	−6	0.90	0.407	8.0	0.000335
0.41	1.507	15	0.91	2.484	25	9.0	8103.	0.41	0.664	−7	0.91	0.403	9.0	0.000123
0.42	1.522	15	0.92	2.509	26	10.0	22026.	0.42	0.657	−6	0.92	0.399	10.0	0.000045
0.43	1.537	16	0.93	2.535	25	$\pi/2$	4.810	0.43	0.651	−7	0.93	0.395		
0.44	1.553	15	0.94	2.560	26	$2\pi/2$	23.14	0.44	0.644	−6	0.94	0.391		
0.45	1.568	16	0.95	2.586	26	$3\pi/2$	111.3	0.45	0.638	−7	0.95	0.387	$\pi/2$	0.208
0.46	1.584	16	0.96	2.612	26	$4\pi/2$	535.5	0.46	0.631	−6	0.96	0.383	$2\pi/2$	0.0432
0.47	1.600	16	0.97	2.638	26	$5\pi/2$	2576	0.47	0.625	−6	0.97	0.379	$3\pi/2$	0.00898
0.48	1.616	16	0.98	2.664	27	$6\pi/2$	12392	0.48	0.619	−6	0.98	0.375	$4\pi/2$	0.00187
0.49	1.632	17	0.99	2.691	27	$7\pi/2$	59610	0.49	0.613	−6	0.99	0.372	$5\pi/2$	0.000388
0.50	1.649		1.00	2.718		$8\pi/2$	286751	0.50	0.607		1.00	0.368	$6\pi/2$	0.000081

$e = 2.71828$. $1/e = 0.367879$. $\log_{10} e = 0.4343$. $1/(0.4343) = 2.3026$.

* From Lionel S. Marks, "Mechanical Engineers' Handbook."

† NOTE: Do not interpolate in this column.

Ex. 1044-0025

12. Sine Integral Si(x)*

$$Si(x) = \int_0^x \frac{\sin u}{u}\, du$$

x	$Si(x)$	x	$Si(x)$	x	$Si(x)$	x	$Si(x)$	x	$Si(x)$	x	$Si(x)$
0.0	0.00000	5.0	1.54993	10.0	1.65835	15.0	1.61819	20.0	1.54824	25.0	1.53148
0.1	0.09994	5.1	1.53125	10.1	1.65253	15.1	1.62226	20.1	1.55289	50.0	1.55162
0.2	0.19956	5.2	1.51367	10.2	1.64600	15.2	1.62575	20.2	1.55767		
0.3	0.29850	5.3	1.49732	10.3	1.63883	15.3	1.62865	20.3	1.56253		
0.4	0.39646	5.4	1.48230	10.4	1.63112	15.4	1.63093	20.4	1.56743		
0.5	0.49311	5.5	1.46872	10.5	1.62294	15.5	1.63258	20.5	1.57232		
0.6	0.58813	5.6	1.45667	10.6	1.61439	15.6	1.63359	20.6	1.57714		
0.7	0.68122	5.7	1.44620	10.7	1.60556	15.7	1.63396	20.7	1.58186		
0.8	0.77210	5.8	1.43736	10.8	1.59654	15.8	1.63370	20.8	1.58641		
0.9	0.86047	5.9	1.43018	10.9	1.58743	15.9	1.63280	20.9	1.59077		
1.0	0.94608	6.0	1.42469	11.0	1.57831	16.0	1.63130	21.0	1.59489		
1.1	1.02869	6.1	1.42087	11.1	1.56927	16.1	1.62921	21.1	1.59873		
1.2	1.10805	6.2	1.41871	11.2	1.56042	16.2	1.62657	21.2	1.60225		
1.3	1.18396	6.3	1.41817	11.3	1.55182	16.3	1.62339	21.3	1.60543		
1.4	1.25623	6.4	1.41922	11.4	1.54356	16.4	1.61973	21.4	1.60823		
1.5	1.32468	6.5	1.42179	11.5	1.53571	16.5	1.61563	21.5	1.61063		
1.6	1.38918	6.6	1.42582	11.6	1.52835	16.6	1.61112	21.6	1.61261		
1.7	1.44959	6.7	1.43121	11.7	1.52155	16.7	1.60627	21.7	1.61415		
1.8	1.50582	6.8	1.43787	11.8	1.51535	16.8	1.60111	21.8	1.61525		
1.9	1.55778	6.9	1.44570	11.9	1.50981	16.9	1.59572	21.9	1.61590		
2.0	1.60541	7.0	1.45460	12.0	1.50497	17.0	1.59014	22.0	1.61608		
2.1	1.64870	7.1	1.46443	12.1	1.50088	17.1	1.58443	22.1	1.61582		
2.2	1.68763	7.2	1.47509	12.2	1.49755	17.2	1.57865	22.2	1.61510		
2.3	1.72221	7.3	1.48644	12.3	1.49501	17.3	1.57285	22.3	1.61395		
2.4	1.75249	7.4	1.49834	12.4	1.49327	17.4	1.56711	22.4	1.61238		
2.5	1.77852	7.5	1.51068	12.5	1.49234	17.5	1.56146	22.5	1.61041		
2.6	1.80039	7.6	1.52331	12.6	1.49221	17.6	1.55598	22.6	1.60806		
2.7	1.81821	7.7	1.53611	12.7	1.49287	17.7	1.55070	22.7	1.60536		
2.8	1.83210	7.8	1.54894	12.8	1.49430	17.8	1.54568	22.8	1.60234		
2.9	1.84219	7.9	1.56167	12.9	1.49647	17.9	1.54097	22.9	1.59902		
3.0	1.84865	8.0	1.57419	13.0	1.49936	18.0	1.53661	23.0	1.59546		
3.1	1.85166	8.1	1.58637	13.1	1.50292	18.1	1.53264	23.1	1.59168		
3.2	1.85140	8.2	1.59810	13.2	1.50711	18.2	1.52909	23.2	1.58772		
3.3	1.84808	8.3	1.60928	13.3	1.51188	18.3	1.52600	23.3	1.58363		
3.4	1.84191	8.4	1.61981	13.4	1.51716	18.4	1.52339	23.4	1.57945		
3.5	1.83313	8.5	1.62960	13.5	1.52291	18.5	1.52128	23.5	1.57521		
3.6	1.82195	8.6	1.63857	13.6	1.52905	18.6	1.51969	23.6	1.57097		
3.7	1.80862	8.7	1.64665	13.7	1.53352	18.7	1.51863	23.7	1.56676		
3.8	1.79333	8.8	1.65379	13.8	1.54225	18.8	1.51810	23.8	1.56262		
3.9	1.77650	8.9	1.65993	13.9	1.54917	18.9	1.51810	23.9	1.55860		
4.0	1.75820	9.0	1.66504	14.0	1.55621	19.0	1.51863	24.0	1.55474		
4.1	1.73874	9.1	1.66908	14.1	1.56330	19.1	1.51967	24.1	1.55107		
4.2	1.71837	9.2	1.67205	14.2	1.57036	19.2	1.52122	24.2	1.54762		
4.3	1.69732	9.3	1.67393	14.3	1.57733	19.3	1.52324	24.3	1.54444		
4.4	1.67583	9.4	1.67473	14.4	1.58414	19.4	1.52572	24.4	1.54154		
4.5	1.65414	9.5	1.67446	14.5	1.59072	19.5	1.52863	24.5	1.53897		
4.6	1.63246	9.6	1.67316	14.6	1.59702	19.6	1.53192	24.6	1.53672		
4.7	1.61101	9.7	1.67084	14.7	1.60296	19.7	1.53357	24.7	1.53484		
4.8	1.58998	9.8	1.66757	14.8	1.60851	19.8	1.53954	24.8	1.53333		
4.9	1.56956	9.9	1.66338	14.9	1.61360	19.9	1.54378	24.9	1.53221		

* From P. O. Pedersen, "Radiation from a Vertical Antenna over Flat Perfectly Conducting Earth," G. E. C. Gad, Copenhagen.

Ex. 1044-0026

13. $S_1(x)$ and Cosine Integral $Ci(x)$*

$$S_1(x) = \log_e x + 0.5772 - Ci(x) = \int_0^x \frac{1 - \cos x}{x}\, dx$$

$$Ci(x) = -\int_x^\infty \frac{\cos u}{u}\, du = \log_e x + 0.5772 - Si(x)$$

x	$S_1(x)$	x	$S_1(x)$	x	$S_1(x)$	x	$S_1(x)$	x	$S_1(x)$	x	$S_1(x)$
0.0	0.00000	5.0	2.37669	10.0	2.92527	15.0	3.23899	20.0	3.52853	25.0	3.80295
0.1	0.00249	5.1	2.38994	10.1	2.94327	15.1	3.25090	20.1	3.53173	50.0	4.49486
0.2	0.00998	5.2	2.40113	10.2	2.96050	15.2	3.26308	20.2	3.53535		
0.3	0.02241	5.3	2.41044	10.3	2.97688	15.3	3.27552	20.3	3.53946		
0.4	0.03973	5.4	2.41801	10.4	2.99234	15.4	3.28814	20.4	3.54402		
0.5	0.06185	5.5	2.42402	10.5	3.00688	15.5	3.30087	20.5	3.54905		
0.6	0.08866	5.6	2.42866	10.6	3.02045	15.6	3.31363	20.6	3.55456		
0.7	0.12002	5.7	2.43210	10.7	3.03300	15.7	3.32641	20.7	3.56049		
0.8	0.15579	5.8	2.43452	10.8	3.04457	15.8	3.33911	20.8	3.56687		
0.9	0.19578	5.9	2.43610	10.9	3.05514	15.9	3.35167	20.9	3.57368		
1.0	0.23981	6.0	2.43704	11.0	3.06467	16.0	3.36401	21.0	3.58085		
1.1	0.28766	6.1	2.43749	11.1	3.07323	16.1	3.37612	21.1	3.58840		
1.2	0.33908	6.2	2.43764	11.2	3.08083	16.2	3.38790	21.2	3.59629		
1.3	0.39384	6.3	2.43766	11.3	3.08749	16.3	3.39932	21.3	3.60446		
1.4	0.45168	6.4	2.43770	11.4	3.09322	16.4	3.41032	21.4	3.61288		
1.5	0.51233	6.5	2.43792	11.5	3.09814	16.5	3.42088	21.5	3.62155		
1.6	0.57549	6.6	2.43847	11.6	3.10225	16.6	3.43096	21.6	3.63037		
1.7	0.64088	6.7	2.43947	11.7	3.10561	16.7	3.44050	21.7	3.63935		
1.8	0.70820	6.8	2.44106	11.8	3.10828	16.8	3.44947	21.8	3.64842		
1.9	0.77713	6.9	2.44335	11.9	3.11038	16.9	3.45788	21.9	3.65751		
2.0	0.84739	7.0	2.44643	12.0	3.11190	17.0	3.46568	22.0	3.66662		
2.1	0.91865	7.1	2.45040	12.1	3.11301	17.1	3.47288	22.1	3.67568		
2.2	0.99060	7.2	2.45534	12.2	3.11370	17.2	3.47945	22.2	3.68465		
2.3	1.06295	7.3	2.46130	12.3	3.11412	17.3	3.48543	22.3	3.69348		
2.4	1.13540	7.4	2.46834	12.4	3.11429	17.4	3.49077	22.4	3.70216		
2.5	1.20764	7.5	2.47649	12.5	3.11436	17.5	3.49553	22.5	3.71059		
2.6	1.27939	7.6	2.48577	12.6	3.11437	17.6	3.49969	22.6	3.71879		
2.7	1.35038	7.7	2.49619	12.7	3.11438	17.7	3.50330	22.7	3.72670		
2.8	1.42035	7.8	2.50775	12.8	3.11453	17.8	3.50639	22.8	3.73427		
2.9	1.48903	7.9	2.52044	12.9	3.11484	17.9	3.50895	22.9	3.74153		
3.0	1.55620	8.0	2.53423	13.0	3.11540	18.0	3.51107	23.0	3.74838		
3.1	1.62163	8.1	2.54906	13.1	3.11628	18.1	3.51276	23.1	3.75483		
3.2	1.68511	8.2	2.56491	13.2	3.11754	18.2	3.51404	23.2	3.76089		
3.3	1.74646	8.3	2.56171	13.3	3.11924	18.3	3.51500	23.3	3.76651		
3 4	1.80552	8.4	2.59938	13.4	3.12142	18.4	3.51568	23.4	3.77170		
3.5	1.86211	8.5	2.61786	13.5	3.12414	18.5	3.51610	23.5	3.77644		
3.6	1.91613	8.6	2.63704	13.6	3.12745	18.6	3.51633	23.6	3.78072		
3.7	1.96745	8.7	2.65686	13.7	3.13134	18.7	3.51645	23.7	3.78459		
3.8	2.01600	8.8	2.67721	13.8	3.13587	18.8	3.51648	23.8	3.78801		
3.9	2.06170	8.9	2.69799	13.9	3.14104	18.9	3.51648	23.9	3.79101		
4.0	2.10449	9.0	2.71909	14.0	3.14688	19.0	3.51660	24.0	3.79360		
4.1	2.14438	9.1	2.74042	14.1	3.15338	19.1	3.51661	24.1	3.79582		
4.2	2.18131	9.2	2.76186	14.2	3.16054	19.2	3.51685	24.2	3.79767		
4.3	2.21535	9.3	2.78332	14.3	3.16835	19.3	3.51727	24.3	3.79917		
4.4	2.24648	9.4	2.80468	14.4	3.17677	19.4	3.51790	24.4	3.80036		
4.5	2.27479	9.5	2.82583	14.5	3.18583	19.5	3.51879	24.5	3.80129		
4.6	2.30033	9.6	2.84669	14.6	3.19545	19.6	3.52002	24.6	3.80197		
4.7	2.32317	9.7	2.86713	14.7	3.20564	19.7	3.52156	24.7	3.80243		
4.8	2.34344	9.8	2.88712	14.8	3.21630	19.8	3.52348	24.8	3.80271		
4.9	2.36124	9.9	2.90651	14.9	3.22746	19.9	3.52578	24.9	3.80288		

* From P. O. Pedersen, "Radiation from a Vertical Antenna over Flat Perfectly Conducting Earth." C. E. C. Gad, Copenhagen.

Ex. 1044-0027

MATHEMATICAL RELATIONS

14. Trigonometric Relations

$$\sin x = \frac{A}{B}$$

$$\cos x = \frac{C}{B}$$

$$\tan x = \frac{A}{C} = \frac{\sin x}{\cos x}$$

$$\sec x = \frac{B}{C} = \frac{1}{\cos x}$$

$$\operatorname{cosec} x = \frac{B}{A} = \frac{1}{\sin x}$$

$$\cot x = \frac{C}{A} = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

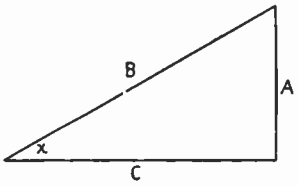


FIG. 1.

$$\sin(-x) = -\sin x$$
$$\cos(-x) = \cos x$$
$$\tan(-x) = -\tan x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \sin\left(\frac{\pi}{2} + x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x \qquad \cos\left(\frac{\pi}{2} + x\right) = -\sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \qquad \tan\left(\frac{\pi}{2} + x\right) = -\cot x$$

$$\sin(\pi - x) = \sin x \qquad \sin(\pi + x) = -\sin x$$
$$\cos(\pi - x) = -\cos x \qquad \cos(\pi + x) = -\cos x$$
$$\tan(\pi - x) = -\tan x \qquad \tan(\pi + x) = \tan x$$

$$\left.\begin{aligned}\sin(x + 2\pi n) &= \sin x\\ \cos(x + 2\pi n) &= \cos x\\ \tan(x + 2\pi n) &= \tan x\end{aligned}\right\} (n \text{ a positive or negative integer})$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\sin 2x = 2 \sin x \cos x \qquad \cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} \qquad \cot 2x = \frac{(\cot^2 x - 1)}{2 \cot x}$$

$$\sin 3x = 3 \sin x - 4 \sin^3 x \qquad \cos 3x = 4\cos^3 x - 3 \cos x$$

$$\sin\frac{1}{2}x = \left[\frac{1}{2}(1 - \cos x)\right]^{\frac{1}{2}} = \frac{1}{2}(1 + \sin x)^{\frac{1}{2}} - \frac{1}{2}(1 - \sin x)^{\frac{1}{2}}$$

$$\cos\frac{1}{2}x = \left[\frac{1}{2}(1 + \cos x)\right]^{\frac{1}{2}} = \frac{1}{2}(1 + \sin x)^{\frac{1}{2}} + \frac{1}{2}(1 - \sin x)^{\frac{1}{2}}$$

$$\tan\frac{1}{2}x = \frac{(1 - \cos x)^{\frac{1}{2}}}{(1 + \cos x)^{\frac{1}{2}}} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^3 x = \frac{1}{4}(3 \sin x - \sin 3x) \qquad \cos^3 x = \frac{1}{4}(\cos 3x + 3 \cos x)$$

Ex. 1044-0028

$$\sin x \sin y = \frac{1}{2}\cos (x - y) - \frac{1}{2}\cos (x + y)$$

$$\cos x \cos y = \frac{1}{2}\cos (x - y) + \frac{1}{2}\cos (x + y)$$

$$\sin x \cos y = \frac{1}{2}\sin (x - y) + \frac{1}{2}\sin (x + y)$$

$$\sin x + \sin y = 2 \sin \frac{1}{2}(x + y) \cos \frac{1}{2}(x - y)$$

$$\sin x - \sin y = 2 \cos \frac{1}{2}(x + y) \sin \frac{1}{2}(x - y)$$

$$\cos x + \cos y = 2 \cos \frac{1}{2}(x + y) \cos \frac{1}{2}(x - y)$$

$$\cos x - \cos y = -2 \sin \frac{1}{2}(x + y) \sin \frac{1}{2}(x - y)$$

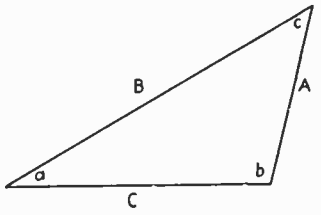


FIG. 2.

$$\tan x + \tan y = \frac{\sin (x + y)}{\cos x \cos y} \qquad \tan x - \tan y = \frac{\sin (x - y)}{\cos x \cos y}$$

$$\sin^2 x - \sin^2 y = \sin (x + y) \sin (x - y)$$
$$\cos^2 x - \cos^2 y = -\sin (x + y) \sin (x - y)$$
$$\cos^2 x - \sin^2 y = \cos (x + y) \cos (x - y)$$

$$\sin^2 x + \cos^2 x = 1 \qquad \sec^2 x - \tan^2 x = 1$$

In any triangle (Fig. 2):

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c} \qquad \text{(law of sines)}$$

$$A^2 = B^2 + C^2 - 2BC \cos a \qquad \text{(law of cosines)}$$

$$\frac{A + B}{A - B} = \frac{\tan \frac{1}{2}(a + b)}{\tan \frac{1}{2}(a - b)} = \frac{\sin a + \sin b}{\sin a - \sin b} \qquad \text{(law of tangents)}$$

$$a + b + c = 180^\circ \qquad A = B \cos c + C \cos b$$
$$B = C \cos a + A \cos c \qquad C = A \cos b + B \cos a$$

15. Properties of Hyperbolic Functions

$$\sinh x = \frac{\epsilon^x - \epsilon^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\cosh x = \frac{\epsilon^x + \epsilon^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\sinh (-x) = -\sinh x$$
$$\cosh (-x) = \cosh x$$
$$\cosh^2 x = 1 + \sinh^2 x$$

$$\frac{d (\sinh x)}{dx} = \cosh x$$

$$\frac{d (\cosh x)}{dx} = \sinh x$$

$$\int \sinh x \cdot dx = \cosh x$$
$$\int \cosh x \cdot dx = \sinh x$$
$$\sinh (x \pm j\pi) = -\sinh x$$
$$\cosh (x \pm j\pi) = -\cosh x$$
$$\sinh (x \pm j2\pi) = \sinh x$$
$$\cosh (x \pm j2\pi) = \cosh x$$

Sawtooth wave (Fig. 3*c*):

$$y = \frac{2}{\pi} E \left(\sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x - \frac{1}{4} \sin 4x + \cdots \right)$$

Short square pulse (Fig. 3*d*):

$$y = E \left\{ k + \frac{2}{\pi} \left(\sin k\pi \cos x + \frac{1}{2} \sin 2k\pi \cos 2x + \frac{1}{3} \sin 3k\pi \cos 3x \cdots + \frac{1}{n} \sin nk\pi \cos nx \cdots \right) \right\}$$

The relative values of the coefficients of cos nx are plotted in Fig. 4 as a function of nk for the case $k << 1$, *i.e.*, for a short pulse.. Under these conditions the coefficients of cos nx are proportional to $(\sin nk\pi)/nk\pi$.

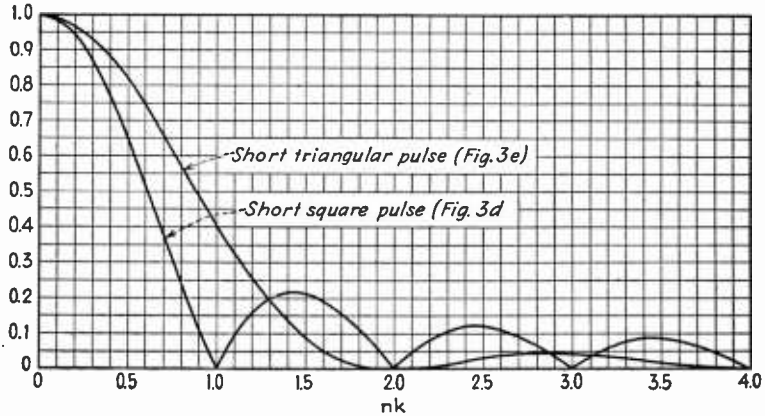


FIG. 4.—Relative amplitude of harmonics for waves of Fig. 3*d* and *e* for $k << 1$.

Short triangular pulse (Fig. 3*e*):

$$y = E \left[\frac{k}{2} + \sum_{n=1} \left\{ \frac{2}{n\pi} \sin n\pi k - \frac{2}{n^2\pi^2 k} \left(n\pi k \sin n\pi k - 2 \sin^2 \frac{n\pi k}{2} \right) \right\} \cos nx \right]$$

The relative values of the coefficients of cos nx are plotted in Fig. 4 as a function of nk, for $k << 1$, *i.e.*, for a short pulse.

Short pulse that is a section of a sine wave (Fig. 3*f*):

$$y = \frac{E}{\pi \left(1 - \cos \frac{\theta}{2} \right)} \left[\left(\sin \frac{\theta}{2} - \frac{\theta}{2} \cos \frac{\theta}{2} \right) + \left(\frac{\theta}{2} - \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right) \cos x + \cdots \left(\frac{\sin (n+1) \frac{\theta}{2}}{n+1} + \frac{\sin (n-1) \frac{\theta}{2}}{n-1} - \frac{2 \sin n \frac{\theta}{2} \cos \frac{\theta}{2}}{n} \right) \cos nx + \cdots \right]$$

Half-wave rectifier output (Fig. 3*g*):

$$y = \frac{1}{\pi} E \left(1 + \frac{\pi}{2} \cos x + \frac{2}{3} \cos 2x - \frac{2}{15} \cos 4x + \frac{2}{35} \cos 6x \cdots (-1)^{\frac{n}{2}+1} \frac{2}{n^2 - 1} \cos nx \cdots \right) \qquad (n \text{ even})$$

Ex. 1044-0030

Full-wave rectifier output (Fig. 3*h*):

$$y = \frac{2}{\pi} E \left(1 + \frac{2}{3}\cos 2x - \frac{2}{15}\cos 4x + \frac{2}{35}\cos 6x \cdots (-1)^{\frac{n}{2}+1} \frac{2}{n^2 - 1}\cos nx \cdots\right) \qquad (n \text{ even})$$

UNITS

18. Units of Length

1 kilometer (km) = 1,000 meters
1 decimeter (dm) = 0.1 meter
1 centimeter (cm) = 0.01 meter
1 millimeter (mm) = 0.001 meter
1 micron (μ) = 0.000,001 meter = 0.001 millimeter
1 millimicron (mμ) = 0.000,000,001 meter = 0.001 micron
1 angstrom (A) = 0.000,000,1 millimeter
= 0.0001 micron
= 0.1 millimicron
1 statute mile = 1,760 yards = 5,280 feet
1 foot = $\frac{1}{3}$ yard = 12 inches
1 inch = $\frac{1}{36}$ yard = $\frac{1}{12}$ foot
1 mil = 1/1,000 inch
1 U.S. nautical mile, 1 sea mile, 1 geographical mile = 1,853.248 meters = 6,080.20 feet
1 international nautical mile = 1,852 meters
= 6,076.10 feet
1 fathom = 6 feet

19. Conversion Table for Units of Length

Centimeters	Meters	Kilometers	Inches	Feet	Miles
1	0.01	10^{-5}	0.3937	0.03281	0.6214×10^{-5}
2.540	0.02540	2.54×10^{-5}	1	0.0833	0.1578×10^{-4}
100	1	0.001	39.37	3.281	0.0006214
100,000	1,000	1	39,370	3,281	0.6214
30.48	0.3048	30.48×10^{-5}	12	1	0.0001894
160,935	1,609	1.609	63,360	5,280	1

NOTE: All numbers in the same horizontal column represent the same length.

20. Systems of Electrical Units

Electromagnetic Cgs Units.—The electromagnetic system of cgs units (often abbreviated emu) results if one uses centimeters, grams, and seconds, and then arbitrarily assumes that the magnetic permeance μ of a centimeter cube in a vacuum is unity. These units are sometimes designated by the prefix *ab*, such as abohm, abvolt, etc.

Electrostatic Cgs Units.—The electrostatic system of cgs units (sometimes abbreviated esu) results if one uses centimeters, grams, and seconds, and then arbitrarily assumes that the capacity across the faces of a centimeter cube is unity. These units are sometimes designated by the prefix *stat*, such as statohm, statvolt, etc.

"Practical" Electrical Units.—This term refers to the ordinary system of volts, amperes, etc., used in practice. These units have more convenient values than the electromagnetic and electrostatic systems.

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Relationship between Systems of Units.—The fundamental relation between the cgs electromagnetic units and the cgs electrostatic units, as determined experimentally, is

$$1 \text{ abfarad} = 8.9878 \times 10^{20} \text{ statfarads}$$
$$\cong 9 \times 10^{20} \text{ statfarads}$$

The fundamental relation between the cgs electromagnetic system of units and the practical system is

$$1 \text{ abcoulomb} = 10 \text{ coulombs}$$
$$1 \text{ erg} = 10^{-7} \text{ watt-seconds (or joules)}$$

The erg is the unit of energy in the cgs electromagnetic system and the watt-second (or joule) that of the practical system.

It is convenient to note that when an equation is expressed in electromagnetic units one can convert to practical units by substituting 30 ohms for the velocity of light (which will appear either directly or indirectly in the relation and if it does not appear directly can be inserted by multiplying by $\frac{c}{3 \times 10^{10}} = 1$).

21. Relations between the Practical and Corresponding Cgs Units

Quantity	Symbol	Practical unit	Cgs electromagnetic unit	Cgs electrostatic unit
Emf	E	Volt $= 10^8$ abvolts Volt $= 3.\dot{3} \times 10^{-3}$ statvolt	Abvolt $= 10^{-8}$ volt Abvolt $= 3.\dot{3} \times 10^{-11}$ statvolt	Statvolt $= 300$ volts very nearly Statvolt $= 3 \times 10^{10}$ abvolts
Resistance	R	Ohm $= 10^9$ abohms Ohm $= 1.1 \times 10^{-12}$ statohm	Abohm $= 10^{-9}$ ohm Abohm $= 1.\dot{1} \times 10^{-21}$ statohm	Statohm $= 9 \times 10^{11}$ ohms Statohm $= 9 \times 10^{20}$ abohms
Current	I	Ampere $= 10^{-1}$ abampere Ampere $= 3 \times 10^9$ statamperes	Abampere $= 10$ amperes Abampere $= 3 \times 10^{10}$ statamperes	Statampere $= 3.\dot{3} \times 10^{-10}$ ampere Statampere $= 3.\dot{3} \times 10^{-11}$ abampere
Charge	Q	Coulomb $= 10^{-1}$ abcoulomb Coulomb $= 3 \times 10^9$ statcoulombs	Abcoulomb $= 10$ coulombs Abcoulomb $= 3 \times 10^{10}$ statcoulombs	Statcoulomb $= 3.\dot{3} \times 10^{-10}$ coulomb Statcoulomb $= 3.\dot{3} \times 10^{-11}$ abcoulomb
Capacity	C	Farad $= 10^{-9}$ abfarad Farad $= 9 \times 10^{11}$ statfarads	Abfarad $= 10^9$ farads Abfarad $= 9 \times 10^{20}$ statfarads	Statfarad $= 1.\dot{1} \times 10^{-12}$ farad $= 1.\dot{1}\,\mu\mu f$ Statfarad $= 1.\dot{1} \times 10^{-21}$ abfarad
Inductance	L	Henry $= 10^9$ abhenrys Henry $= 1.\dot{1} \times 10^{-12}$ stathenry	Abhenry $= 10^{-9}$ henry Abhenry $= 1.\dot{1} \times 10^{-21}$ stathenry	Stathenry $= 9 \times 10^{11}$ henrys Stathenry $= 9 \times 10^{20}$ abhenrys
Energy	W	Joule $= 10^7$ ergs	Erg $= 10^{-7}$ joule	Erg $= 10^{-7}$ joule
Power	P	Watt $= 10^7 \frac{\text{ergs}}{\text{sec}}$	$\frac{\text{Erg}}{\text{Sec}} = 10^{-7}$ watt	$\frac{\text{Erg}}{\text{Sec}} = 10^{-7}$ watt

NOTE: A dot over a number indicates a recurring decimal.

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SECTION 2

CIRCUIT ELEMENTS

RESISTANCE AND RESISTORS

1. Specific Resistance and Temperature Coefficient of Resistance.—The resistance R of a conductor of length l and cross-sectional area A is

$$R = \rho \frac{l}{A} \qquad (1)$$

where ρ is termed the *specific resistance*, or *resistivity*, and is a property of the conductor material and the temperature.

The specific resistance ρ can be expressed in terms of the resistance of a wire one mil in diameter and one foot long (ohms per circular mil foot), in which case the length in Eq. (1) is in feet and the area A in circular mils.[1] The specific resistance is also often expressed in terms of the resistance existing between opposite faces of a cube one centimeter on a side (ohms per cm cube), in which case it is termed the *volume resistivity*, and corresponds to the length l in Eq. (1), expressed in centimeters, and the area A, in square centimeters. Values of specific resistance for a number of common materials are given in Table 1.

The resistance of a conductor depends upon the temperature as well as the material and form. To a first approximation, the change in resistance produced by a variation in temperature is proportional to the temperature increment, provided this increment is not too large. Hence the relation between the resistances R_{T_1} and R_{T_2} of a conductor at two temperatures T_1 and T_2, respectively, is

$$R_{T_2} = R_{T_1}[1 + \alpha(T_2 - T_1)] \qquad (2)$$

where α is the fractional change in resistance per degree of temperature rise, and is termed the *temperature coefficient of resistance*. Values of temperature coefficient for a number of common materials are given in Table 1.

2. Resistance Wire.—Although the usefulness of copper is due largely to its low specific resistance, other materials, particularly certain alloys, are important because of their high specific resistance. The most important of these resistance alloys for radio purposes are listed in Table 1, together with their principal properties. A wire table giving further properties of some of these resistance alloys also appears below.

The nichrome group of resistance alloys listed in Table 1 is capable of operating at high temperatures, and is the wire generally employed for rheostats, heating elements, enamel-coated resistance tubes, etc.

Advance and the related resistance alloys are characterized by zero temperature coefficient of resistance and a high thermoelectric voltage against copper. This type of wire is used for precision purposes and also for thermocouples. Its maximum working temperature is appreciably lower than that of nichrome, but considerably higher than manganin.

[1] A circular mil is the area a circle one mil (0.001 inch) in diameter. The area of a wire in circular mils is hence the square of the diameter when the diameter is expresesd in mils.

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TABLE 1

Material	Resistivity		Temperature coefficient of resistance per °C at 20°C	Remarks
	Ohms per cir mil ft at 20°C	Microhms per cm cube at 20°C		
Aluminum	17	2.828	0.0049	
Brass	45	7.5	0.002–0.007	
Chromium	16	2.7		
Carbon (graphite)	400–1,100	33–185	−0.0006 to −0.0012	
Copper	10.37	1.724	0.00393	
Iron	59	9.8	0.006	
Mercury	575	95.8	0.00089	
Molybdenum	34	5.7	0.0033	
Nickel	60	10	0.005	Max. working temp. = 500°C
Phosphor bronze	70	11.5	0.004	
Silver	9.8	1.629	0.00381	
Tantalum	93	15.5	0.0031	
Tin	69	11.5	0.0042	
Tungsten	33	5.51	0.0045	
Zinc	36	5.9	0.0035	
Resistance alloys:				
Nichrome, Nichrome I–V, Chromel A-C, etc.	650	108	0.0002	Max. working temp. = 1100°C. Thermal emf against copper = 22 μv per °C
Advance, Constantan, Copel, Ideal, $I_a - I_a$, etc.	295	49	0	Max. working temp. = 500°C. Thermal emf against copper = 43 μv per °C
Manganin	290	48	±0.00001	Thermal emf against copper = 2 μv per °C
Ohmax	1,000	167	−0.00035	Max. working temp. = 500°C
Radiohm	800	133	0.0007	
Monel	255	43	0.0019	Max. working temp. = 425°C
German silver	185	31	0.00027	
Midohm	180	30	0.00018	

NOTE: The foregoing are typical values. Actual values in the case of elements will depend on purity, heat treatment, etc., and also upon exact composition in the case of alloys.

Manganin is used primarily for precision resistors. It has negligible temperature coefficient of resistance at ordinary room temperatures, and also an extremely small thermoelectric coefficient against copper. The resistance of manganin is affected appreciably by mechanical strains such as are introduced by winding, but when these are relieved by baking the completed resistance units for 24 hours at a temperature of 120°C, the resistance is extremely stable in its characteristics, provided that the resistance is not subsequently overheated.

Ohmax is an aluminum-bearing alloy having a high specific resistance, but is available only in the smaller sizes (10 mils and less). It is used where high resistance is required in a small space, as in radio rheostats. It has a negative temperature coefficient of resistance and is sometimes employed in combination with nichrome to

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give zero over-all temperature coefficient in precision resistors requiring high resistance in such a small space that manganin is not suitable.

Radiohm is similar to ohmax, but has a slightly lower specific resistance, is available in a greater range of sizes, and has a positive temperature coefficient of resistivity.

Midohm is a copper-nickel alloy having a lower temperature coefficient of resistivity than other alloys of similar specific resistance.

3. Wire Tables.

TABLE 2.—COPPER-WIRE TABLE, STANDARD ANNEALED COPPER
American Wire Gage (B & S)

Gage No.	Diameter, mils at 20°C	Cross section at 20°C		Ohms per 1,000 ft at 20°C (= 68°F)	Lb. per 1,000 ft	Ft per lb	Ft per ohm at 20°C (= 68°F)	Ohms per lb at 20°C (= 68°F)	Gage No.
		Circular mils	Sq in						
0000	460.0	211,600.0	0.1662	0.04901	640.5	1.561	20,400.0	0.00007652	0000
000	409.6	167,800.0	0.1318	0.06180	507.9	1.968	16,180.0	0.0001217	000
00	364.8	133,100.0	0.1045	0.07793	402.3	2.482	12,830.0	0.0001935	00
0	324.9	105,500.0	0.08289	0.09827	319.5	3.130	10,180.0	0.0003076	0
1	289.3	83,690.0	0.06573	0.1239	253.3	3.947	8,070.0	0.0004891	1
2	257.6	66,370.0	0.05213	0.1563	200.9	4.977	6,400.0	0.0007778	2
3	229.4	52,640.0	0.04134	0.1970	159.3	6.276	5,075.0	0.001237	3
4	204.3	41,740.0	0.03278	0.2485	126.4	7.914	4,025.0	0.001966	4
5	181.9	33,100.0	0.02600	0.3133	100.2	9.980	3,192.0	0.003127	5
6	162.0	26,250.0	0.02062	0.3951	79.46	12.58	2,531.0	0.004672	6
7	144.3	20,820.0	0.01635	0.4282	63.02	15.87	2,007.0	0.007905	7
8	128.5	16,510.0	0.01297	0.6282	49.98	20.01	1,592.0	0.01257	8
9	114.4	13,090.0	0.01028	0.7921	39.63	25.23	1,262.0	0.01999	9
10	101.9	10,380.0	0.008155	0.9983	31.43	31.82	1,001.0	0.03178	10
11	90.74	8,234.0	0.006467	1.260	24.92	40.12	794.0	0.05053	11
12	80.81	6,530.0	0.005129	1.588	19.77	50.59	629.6	0.08035	12
13	71.96	5,178.0	0.004067	2.003	15.68	63.80	499.3	0.1278	13
14	64.08	4,107.0	0.003225	2.525	12.43	80.44	396.0	0.2032	14
15	57.07	3,257.0	0.002558	3.184	9.858	101.4	314.0	0.3230	15
16	50.82	2,583.0	0.002028	4.016	7.818	127.9	249.0	0.5136	16
17	45.26	2,048.0	0.001609	5.064	6.200	161.3	197.5	0.8167	17
18	40.30	1,624.0	0.001276	6.385	4.917	203.4	156.6	1.299	18
19	35.89	1,288.0	0.001012	8.051	3.899	256.5	124.2	2.065	19
20	31.96	1,022.0	0.0008023	10.15	3.092	323.4	98.50	3.283	20
21	28.46	810.1	0.0006363	12.80	2.452	407.8	78.11	5.221	21
22	25.35	642.4	0.0005046	16.14	1.945	514.2	61.95	8.301	22
23	22.57	509.5	0.0004002	20.36	1.542	648.4	49.13	13.20	23
24	20.10	404.0	0.0003173	25.67	1.223	817.7	38.96	20.99	24
25	17.90	320.4	0.0002517	32.37	0.9699	1,031.0	30.90	33.37	25
26	15.94	254.1	0.0001996	40.81	0.7692	1,300.0	24.50	53.06	26
27	14.20	201.5	0.0001583	51.47	0.6100	1,639.0	19.43	84.37	27
28	12.64	159.8	0.0001255	64.90	0.4837	2,067.0	15.41	134.2	28
29	11.26	126.7	0.00009953	81.83	0.3836	2,607.0	12.22	213.3	29
30	10.03	100.5	0.00007894	103.2	0.3042	3,287.0	9.691	339.2	30
31	8.928	79.70	0.00006260	130.1	0.2413	4,145.0	7.685	539.3	31
32	7.950	63.21	0.00004964	164.1	0.1913	5,227.0	6.095	857.6	32
33	7.080	50.13	0.00003937	206.9	0.1517	6,591.0	4.833	1,364.0	33
34	6.305	39.75	0.00003122	260.9	0.1203	8,310.0	3.833	2,168.0	34
35	5.615	31.52	0.00002476	329.0	0.09542	10,480.0	3.040	3,448.0	35
36	5.000	25.00	0.00001964	414.8	0.07568	13,210.0	2.411	5,482.0	36
37	4.453	19.83	0.00001557	523.1	0.06001	16,660.0	1.912	8,717.0	37
38	3.965	15.72	0.00001235	659.6	0.04759	21,010.0	1.516	13,860.0	38
39	3.531	12.47	0.000009793	831.8	0.03774	26,500.0	1.202	22,040.0	39
40	3.145	9.888	0.000007766	1,049.0	0.02993	33,410.0	0.9534	35,040.0	40

4. Resistance at Radio Frequencies—Skin Effect.—In circuits carrying alternating currents, particularly high-frequency currents, the power loss is often greater than where a direct current of the same value is being carried. This is because of such

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TABLE 3.—RESISTANCE WIRE TABLE

B & S No.	Diam., in.	Nichrome		Advance		Manganin		B & S No.
		Ohms per ft at 68°F (20°C)	Weight per 1,000 ft bare wire, lb	Ohms per ft at 68°F (20°C)	Weight per 1,000 ft bare wire, lb	Ohms per ft at 68°F (20°C)	Weight per 1,000 ft bare wire, lb	
10	0.102	0.06248	29.60	0.02826	31.35	.....	.....	10
11	0.091	0.07849	23.65	0.03550	25.06	.....	.....	11
12	0.081	0.09907	18.67	0.04481	19.77	.....	.....	12
13	0.072	0.1255	14.75	0.05672	15.62	.....	.....	13
14	0.064	0.1588	11.65	0.07178	12.35	.....	.....	14
15	0.057	0.2000	9.240	0.09049	9.788	0.0893	9.054	15
16	0.051	0.2499	7.400	0.1130	7.836	0.1115	7.249	16
17	0.045	0.3209	5.760	0.1452	6.100	0.1432	5.642	17
18	0.040	0.4062	4.550	0.1837	4.822	0.1813	4.460	18
19	0.036	0.5015	3.690	0.2270	3.906	0.2238	3.613	19
20	0.032	0.6347	2.915	0.2871	3.085	0.2832	2.854	20
21	0.0285	0.8002	2.291	0.3619	2.430	0.3570	2.248	21
22	0.0253	1.017	1.807	0.4557	1.946	0.4531	1.771	22
23	0.0226	1.272	1.453	0.5756	1.539	0.5678	1.424	23
24	0.0201	1.609	1.139	0.7277	1.218	0.7178	1.116	24
25	0.0179	2.029	0.9110	0.9176	0.9655	0.9051	0.8931	25
26	0.0159	2.571	0.7190	1.163	0.7619	1.147	0.7047	26
27	0.0142	3.228	0.5740	1.458	0.6076	1.438	0.5620	27
28	0.0126	4.090	0.4540	1.852	0.4786	1.826	0.4427	28
29	0.0113	5.090	0.3570	2.302	0.3785	2.271	0.3501	29
30	0.0100	6.500	0.2845	2.940	0.3014	2.900	0.2788	30
31	0.0089	8.206	0.2253	3.722	0.2387	3.662	0.2208	31
32	0.0080	10.16	0.1821	4.594	0.1929	4.531	0.1784	32
33	0.0071	12.90	0.1434	5.833	0.1519	5.754	0.1405	33
34	0.0063	16.37	0.1129	7.408	0.1196	7.305	0.1106	34
35	0.0056	20.72	0.08922	9.375	0.09451	9.247	0.08742	35
36	0.0050	26.00	0.07113	11.76	0.07534	11.60	0.06969	36
37	0.0045	32.09	0.05758	14.52	0.06100	14.32	0.05642	37
38	0.0040	40.62	0.04552	18.37	0.04822	18.13	0.04460	38
39	0.0035	53.06	0.03482	24.00	0.03689	23.67	0.03412	39
40	0.0031	67.63	0.02731	30.60	0.02893	30.18	0.02676	40
	0.00275	85.98	0.02128	38.88	0.02254	38.36	0.02085	
	0.00250	104.00	0.01775	47.04	0.01880	46.40	0.01739	
	0.00225	128.5	0.01423	58.07	0.01507	57.31	0.01394	
	0.00200	162.5	0.01138	73.50	0.01205	72.50	0.01115	
	0.00175	212.4	0.008706	96.00	0.009222			
	0.00150	288.9	0.006396	130.70	0.006774			
	0.0014	331.6	0.005577					
	0.0013	384.6	0.004809					
	0.0012	451.4	0.004097					
	0.0011	537.2	0.003443					
	0.001	650.0	0.002845					

* This table is reproduced by permission from *Booklet R-36* of the Driver-Harris Company.

factors as dielectric hysteresis, eddy currents, skin effect, etc. As a result, it is customary, in dealing with alternating-current circuits, to consider that the effective or equivalent circuit resistance is that quantity which when multiplied by the square of the current equals the power dissipated in the circuit.

Skin Effect.—At high frequencies the current carried by a conductor is not uniformly distributed over the conductor cross section, as is the case with direct currents, but rather tends to be concentrated near the surface. This action, termed *skin effect*, is a result of magnetic flux lines that circle part but not all of the conductor. Those parts of the cross section which are circled by the largest number of flux lines have higher inductance than other parts of the conductor, and hence a greater reactance. The result is a redistribution of current over the cross section in such a way as to

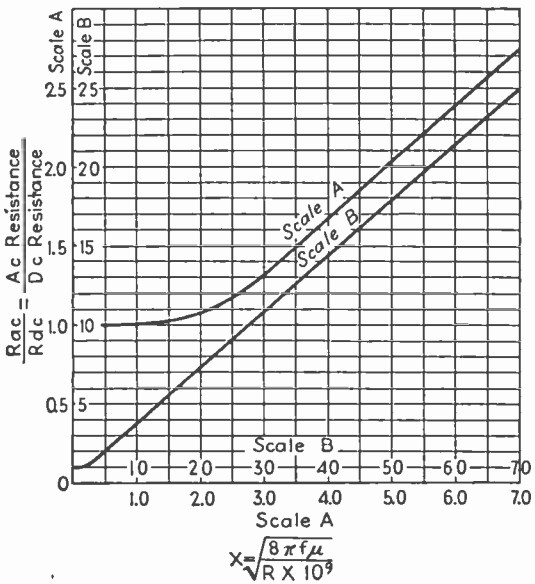


FIG. 1.—Resistance ratio in isolated round wires as a function of parameter x defined by Eq. (3).

cause those parts of the conductor having the highest reactance, *i.e.*, those parts nearer the center, to carry the least current. With a round wire this causes the current density to be maximum at the surface and least at the center. With a square bar the greatest concentration of current is at the corners, with the flat sides coming next and the center carrying the least current. With a flat strip the current density is greatest at the edges, considerable at the flat surfaces, and again least in the center. In every case it will be noticed that the current is so distributed as to cause those parts of the cross section that are inclosed by the greatest number of flux lines to carry the least current.

The redistribution of current over a conductor cross section that is associated with skin effect causes the ratio R_{ac}/R_{dc} of effective alternating-current resistance to direct-current resistance to be greater than unity, because in the presence of skin effect, portions of the conductor are not fully effective in carrying current. Along with this increase in resistance, there is also a decrease in the inductance of the conductor because of the fact that the redistribution of current is always of such a character

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as to make the flux linkages, and hence the inductance, less than with a uniform current distribution. The magnitude of these effects on inductance and a-c resistance increases with frequency, conductivity, magnetic permeability, and size of the the conductor. This is because a high frequency increases the difference in reactance resulting from the different inductance of various current paths, whereas greater conductivity makes the same difference in reactance more effective in modifying current distribution, and a greater magnetic permeability increases the flux.

In dealing with skin-effect problems, it is often convenient to make use of the principle of similitude, which states that in the case of nonmagnetic materials, if a conductor or combination of conductors has a given ratio R_{ac}/R_{dc} at a certain value of f/R_{dc}, then, if the relative shapes are not altered, changes in the frequency, conductor, resistivity, and size that do not alter f/R_{dc} will not alter the resistance ratio R_{ac}/R_{dc}.[1]

Resistance Ratio of Isolated Conductors.—The ratio R_{dc}/R_{ac} of alternating-current to direct-current resistance for straight cylindrical wires is given in Fig. 1 and

TABLE 4.—RATIO OF ALTERNATING-CURRENT RESISTANCE TO DIRECT-CURRENT RESISTANCE FOR A SOLID ROUND WIRE

x	$\frac{R_{ac}}{R_{dc}}$	x	$\frac{R_{ac}}{R_{dc}}$	x	$\frac{R_{ac}}{R_{dc}}$
0	1.0000	5.2	2.114	14.0	5.209
0.5	1.0003	5.4	2.184	14.5	5.386
0.6	1.0007	5.6	2.254	15.0	5.562
0.7	1.0012	5.8	2.324	16.0	5.915
0.8	1.0021	6.0	2.394	17.0	6.268
0.9	1.0034	6.2	2.463	18.0	6.621
1.0	1.005	6.4	2.533	19.0	6.974
1.1	1.008	6.6	2.603	20.0	7.328
1.2	1.011	6.8	2.673	21.0	7.681
1.3	1.015	7.0	2.743	22.0	8.034
1.4	1.020	7.2	2.813	23.0	8.387
1.5	1.026	7.4	2.884	24.0	8.741
1.6	1.033	7.6	2.954	25.0	9.094
1.7	1.042	7.8	3.024	26.0	9.447
1.8	1.052	8.0	3.094	28.0	10.15
1.9	1.064	8.2	3.165	30.0	10.86
2.0	1.078	8.4	3.235	32.0	11.57
2.2	1.111	8.6	3.306	34.0	12.27
2.4	1.152	8.8	3.376	36.0	12.98
2.6	1.201	9.0	3.446	38.0	13.69
2.8	1.256	9.2	3.517	40.0	14.40
3.0	1.318	9.4	3.587	42.0	15.10
3.2	1.385	9.6	3.658	44.0	15.81
3.4	1.456	9.8	3.728	46.0	16.52
3.6	1.529	10.0	3.799	48.0	17.22
3.8	1.603	10.5	3.975	50.0	17.93
4.0	1.678	11.0	4.151	60.0	21.47
4.2	1.752	11.5	4.327	70.0	25.00
4.4	1.826	12.0	4.504	80.0	28.54
4.6	1.899	12.5	4.680	90.0	32.07
4.8	1.971	13.0	4.856	100.0	35.61
5.0	2.043	13.5	5.033	∞	∞

[1] See H. B. Dwight, Skin Effect on Tubular and Flat Conductors, *Trans. A.I.E.E.*, Vol. 37, p. 1379, 1918.

Ex. 1044-0039

Table 4[1] in terms of a parameter x defined by the equation

$$\left.\begin{aligned} x &= \pi d \sqrt{\frac{2\mu f}{\rho \times 10^9}} = \sqrt{\frac{8\pi\mu f A}{\rho \times 10^9}} \\ &= \sqrt{\frac{8\pi\mu f}{R \times 10^9}} = 0.1585 \sqrt{\frac{\mu f_{mc}}{R}} \end{aligned}\right\} \tag{3}$$

where A = wire area, sq cm.
d = wire diameter, cm.
μ = permeability of conductor ($\mu = 1$ for air).
ρ = specific resistivity in ohms per cm cube.
f and f_{mc} = frequency in cycles and megacycles, respectively.
R = resistance to direct current for 1 cm of conductor, ohms.

In the case of copper wire, $\mu = 1$, and $\rho = 1.724 \times 10^{-6}$, so that Eq. (3) becomes

$$x \text{ (for copper)} = 0.271 d_m \sqrt{f_{mc}} \tag{4}$$

where d_m is the diameter in mils. The largest diameter that is permissible at various frequencies for a resistance ratio not exceeding 1.01 is tabulated in Table 5 for several common materials.

TABLE 5.—LARGEST PERMISSIBLE WIRE DIAMETER IN MILS FOR SKIN-EFFECT RATIO OF 1.01 OR LESS

Frequency, kc	Nichrome	Advance and manganin	Copper
100	104.5	70.2	14.0
200	74.0	49.6	9.9
500	46.8	31.4	6.3
1,000	33.0	22.2	4.4
2,000	23.4	15.7	3.1
5,000	14.7	9.9	2.0
10,000	10.4	7.0	1.4
20,000	7.4	5.0	1.0
50,000	4.7	3.1	0.6

For a resistance ratio of 1.1, multiply the preceding diameters by 1.78. For a resistance ratio of 1.001, multiply the preceding diameters by 0.55.

In the case of isolated tubular conductors, the resistance ratio is always closer to unity than for a solid conductor of the same outside diameter. This is because the center of a solid wire does not do its full share in the carrying of current; so if the center is removed to form a tube the resistance ratio will be improved. However, removing the center to form a tube increases the d-c resistance sufficiently so that the a-c resistance of the tube is greater than for the corresponding solid wire, even though the resistance ratio is less. Values of resistance ratio for isolated nonmagnetic tubular conductors are given in Fig. 2.[2]

A conductor consisting of a flat rectangular strip (ribbon) will have a lower resistance ratio than a solid round wire of the same cross section when the frequency is

[1] From *Bur. Standards Circ.* 74.
For the derivation of the resistance ratio of a round wire, see A. E. Kennelly, F. A. Laws, and P. H. Pierce, Experimental Researches on Skin Effect in Conductors, *Trans. A.I.E.E.*, Vol. 34, p. 1953, 1915; or see L. F. Woodruff, "Electric Power Transmission and Distribution," 2d ed., p. 54, Wiley, New York, 1938.

[2] The analysis of the tubular conductor case is given by Woodruff, *op. cit.*, and also by H. B. Dwight, A Precise Method of Calculating Skin Effect in Isolated Tubes, *A.I.E.E. Jour.*, Vol. 42, p. 827, August, 1923.

Ex. 1044-0040

high enough to make the resistance ratio appreciable, but has a higher resistance ratio than would be obtained by forming the ribbon into a tube of the same cross-sectional area and the same wall thickness. This is because the current in a ribbon tends to

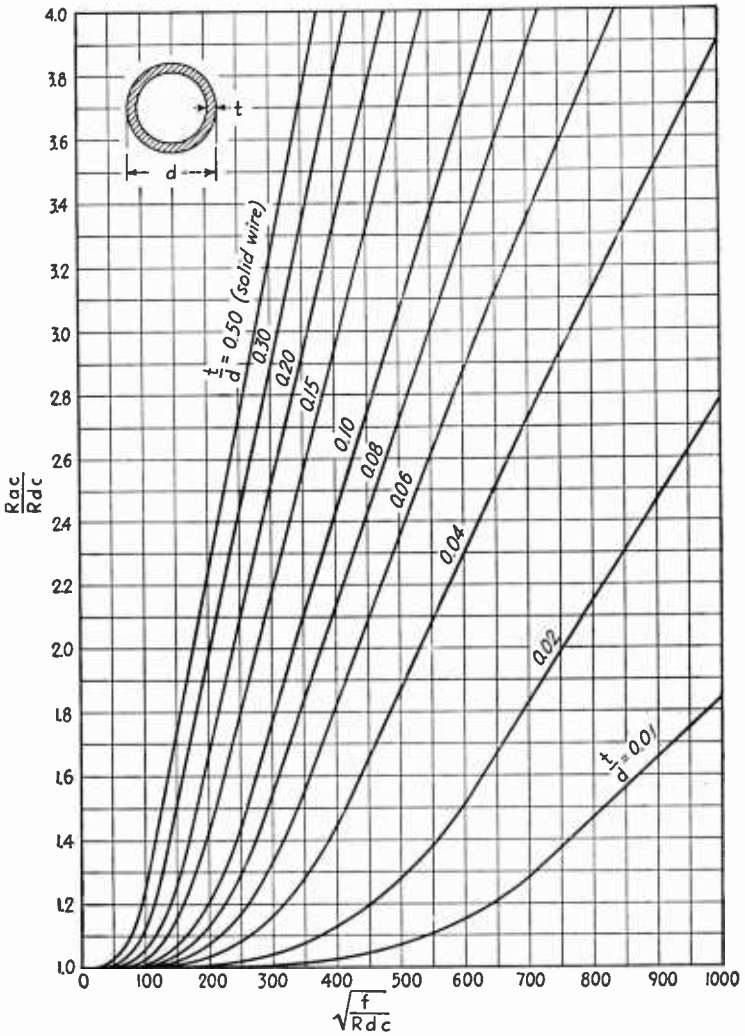


FIG. 2.—Resistance ratio of isolated nonmagnetic tubular conductors in terms of parameter $\sqrt{f/R_{dc}}$, where f is the frequency in cycles, and R_{dc} the direct-current resistance per thousand feet.

concentrate more at the edges than at the sides, since the edges are circled by the fewest flux lines. Information on the resistance ratio of rectangular conductors of various shapes is given in Fig. 3.[1] It is seen that for shape ratios exceeding about

[1] Most of the results in Fig. 3 are based on experimental data given by S. J. Haefner, Alternating Current Resistance of Rectangular Conductors, *Proc. I.R.E.*, Vol. 25, p. 434, April, 1937.

Ex. 1044-0041

2:1, the ribbon has lower a-c resistance than a solid round wire of the same cross-section except when the resistance ratio approaches unity.

Skin Depth and Resistance at Very High Frequencies.—At frequencies so great that the resistance ratio is large, substantially all of the current carried by the conductor is concentrated very close to the surface. Under these conditions, the a-c resistance offered by the conductor is approximately the same as the d-c resistance of a hollow

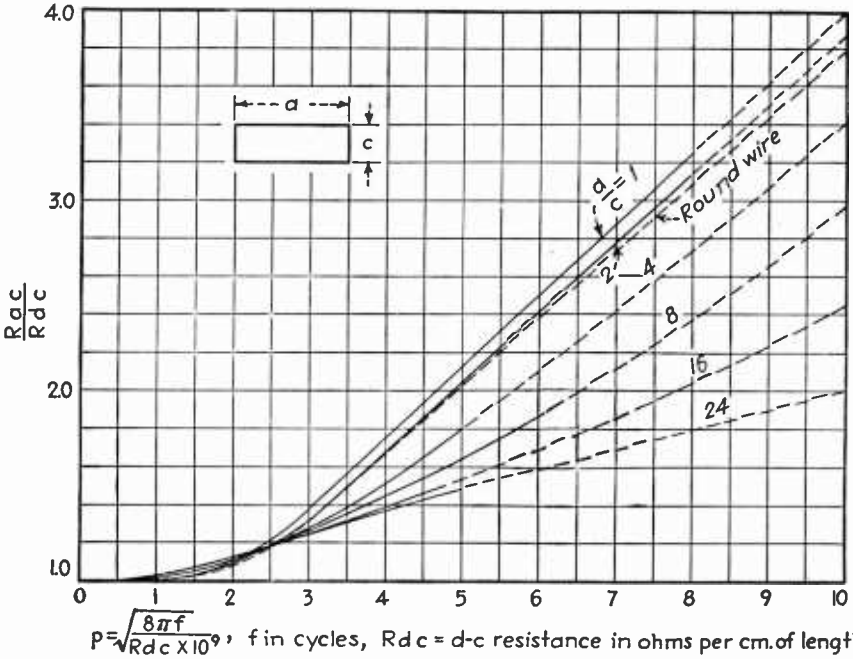


FIG. 3.—Resistance ratio of rectangular conductors in terms of parameter p, involving the frequency f in cycles, and R_{dc} the resistance in ohms per centimeter of length. These curves are a combination of experimental and calculated results, with the dotted portions representing low-frequency results extrapolated to join on with the theoretical results for high frequencies.

conductor having the same external shape as the actual conductor but having a thickness equal to

$$\text{Skin depth, cm} = \epsilon = \frac{1}{2\pi}\sqrt{\frac{\rho \times 10^9}{\mu f}} = 5{,}033\sqrt{\frac{\rho}{\mu f}} \qquad (5)$$

where ρ is in ohms per cm cube and f is in cycles. This thickness, termed the *skin depth*, is a rough measure of the current penetration since the current density drops off to a very small value at a distance corresponding to several skin depths. For copper at 20°C, the skin depth is $6.62/\sqrt{f}$ cm, or $2.61/\sqrt{f}$ inches, where f is in cycles. The skin depth of copper is given graphically in Fig. 4.

It will be noted that where the skin depth controls the a-c resistance, a-c resistance is directly proportional to the square root of frequency and inversely proportional to the conductor size (or perimeter).

The skin-depth concept can be used to calculate the radio-frequency resistance of conductors of any shape *provided the conductor thickness is everywhere considerably greater than the skin depth and provided the radius of curvature of the conductor surface*

Ex. 1044-0042

is likewise everywhere appreciably greater than the skin depth and does not vary too rapidly around the periphery. Under these conditions, the high-frequency resistance of a copper conductor is

$$\text{High-frequency resistance, ohms per centimeter} = \frac{261\sqrt{f} \times 10^{-9}}{P} \qquad (6)$$

where P is the perimeter in centimeters and f is in cycles. This equation is also plotted in Fig. 4. Equation (6) can be expected to hold reasonably well for resistance ratios exceeding 2, and quite closely when the ratio is of the order of 5 or more. If the radius of curvature varies too rapidly around the periphery, the a-c resistance is greater than given by Eq. (6) by a factor that depends on the shape. It will be noted

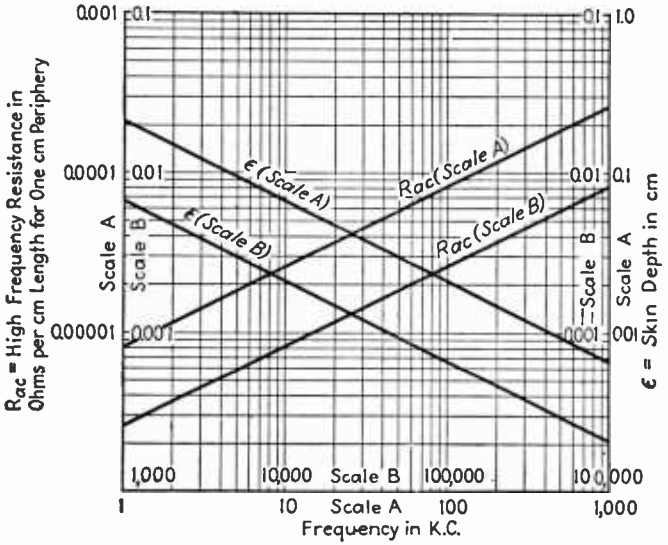


FIG. 4.—Skin depth and high-frequency resistance of copper at frequencies so high that substantially the entire current is concentrated near the conductor surface.

that when Eq. (6) holds, the alternating-current resistance is directly proportional to the square root of the frequency, and inversely proportional to the conductor size (or perimeter).

In the special case where the conductor is a round wire or tube, one has $P = \pi d$, where d is the outside diameter in centimeters, and Eq. (6) becomes

$$\left.\begin{array}{l}\text{High-frequency resistance of wire or tube}\\ \text{at high frequencies, ohms per cm}\end{array}\right\} = \frac{83.2\sqrt{f}}{d} \times 10^{-9} \qquad (7)$$

In the case of a rectangular conductor, the radius of curvature varies so rapidly at the corners that Eq. (6) must be modified as follows:

$$\text{High-frequency resistance of ribbon, ohms per centimeter} = K\frac{261\sqrt{f}}{2(a+c)} \times 10^{-9} \qquad (8)$$

where a and c are the width and thickness, respectively, in centimeters, f is in cycles, and K is a constant determined by the ratio a/c, and given by Fig. 5.[1] Equation (8)

[1] This figure is calculated from formulas derived by J. D. Cockroft, Skin Effect in Rectangular Conductors at High Frequency, *Proc. Roy. Soc.* (*London*), Vol. 122, No. A790, p. 533, Feb. 4, 1929.

will give reasonably accurate results when the thickness is appreciably greater than twice the skin depth.

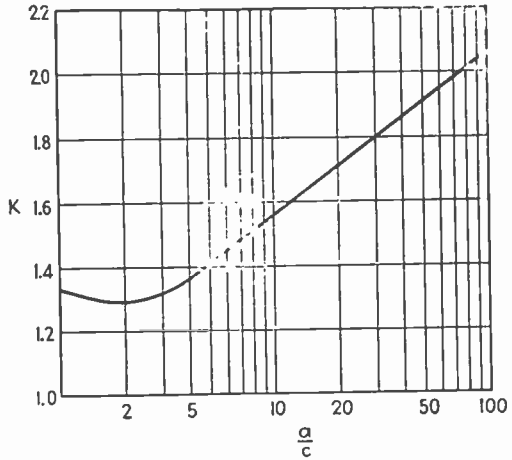


FIG. 5.—Factor K for Eq. (8).

Proximity Effect.—When two or more adjacent conductors are carrying current, the current distribution in one conductor is affected by the magnetic flux produced by the adjacent conductors, as well as the magnetic flux produced by the current in the conductor itself. This effect is termed *proximity effect*, and ordinarily causes the resistance ratio to be greater than in the case of simple skin effect. Proximity effect is very important in radio-frequency inductance coils, and is sometimes a factor in other cases.

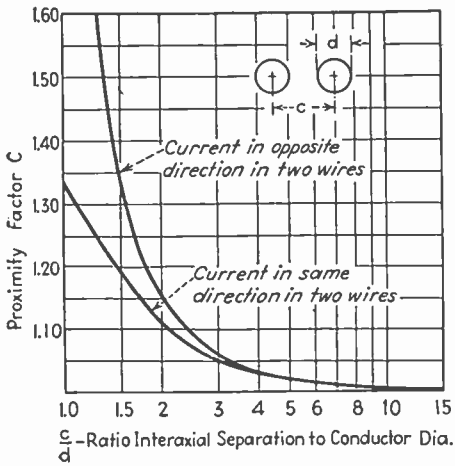


FIG. 6.—Proximity factor in parallel cylinders or tubes for very high frequencies.

Formulas for accurately calculating the proximity effect have been worked out for only very simple cases. For a two-wire line employing round or tubular wires and operating at frequencies so high that the skin depth is small, the factor C by which proximity effect increases the resistance of the conductor is given in Fig. 6.[1] When the frequency is low enough so that the skin depth is not negligible compared with other dimensions involved, the proximity effect will be less than that given by Fig. 6 and can be calculated according to formulas available in the literature.[2]

[1] The curve for currents in the opposite direction is calculated by formulas given by Sallie Pero Mead, Wave Propagation over Parallel Tubular Conductors: The Alternating Current Resistance, *Bell System Tech. Jour.*, Vol. 4, p. 327, April, 1925. The curve applying to currents flowing in the same direction is from S. Butterworth, On the Alternating Current Resistance of Solenoidal Coils, *Proc. Roy. Soc.* (*London*), Vol. 107*A*, p. 693, 1925.

[2] See Mead, *op. cit.*; also, H. B. Dwight, Proximity Effect in Wires and Thin Tubes, *Trans. A.I.E.E.*, Vol. 42, p. 850, 1923.

Ex. 1044-0044

The case of two flat wound ribbons with parallel flat sides adjacent to each other and very close together has not been worked out, but experiments indicate that the proximity effect in this case, assuming the ribbons carry equal currents flowing in the same direction, is very small and sometimes even negative (*i.e.*, may actually reduce the total resistance to alternating currents).

Litz Wire.—The resistance ratio of a conductor can be made to approach unity very closely, at least at the lower radio frequencies, by use of a conductor consisting of a large number of strands of fine wire that are insulated from each other except at the ends where the various wires are connected in parallel. Such a conductor is termed a *litz* or Litzendraht conductor.

In order that a litz conductor may be effective, the strands must be woven or transposed in such a way that each strand occupies all possible positions in the cable to approximately the same extent. In this way the total flux linkages surrounding the various individual strands of the cable, when totaled over an appreciable length, will be substantially the same for all strands even though at any one spot the strands near the center are circled by more flux than the strands that are near the surface at that particular point. The result of this equalizing the flux linkages, and hence reactances, of the individual strands is to cause the current to divide uniformly between strands. The resistance ratio then tends to approximate unity.

The ratio of a-c to d-c resistance of an isolated litz wire is[1]

$$\frac{\text{Resistance to alternating currents}}{\text{Resistance to direct currents}} = H + k\left(\frac{nd_s}{d_0}\right)^2 G \qquad (8a)$$

where H = resistance ratio of individual strand when isolated, as obtained from Table 18 (or Table 4), with x evaluated for the *diameter of the individual strand.*

G = constant taking into account the proximity of neighboring wires, and given by Table 18.

n = number of strands in cable.

d_s = diameter of individual strand.

d_0 = diameter of cable.

k = constant depending on n, given in the following table.

n =	3	9	27	Infinity
k =	1.55	1.84	1.92	2.0

Practical litz conductors are very effective at frequencies below 500 kc, but as the frequency becomes higher the benefits obtained are less. This is because irregularities in stranding and capacity between the individual strands result in failure to realize the ideal conditions. As a consequence, litz wire becomes progressively less effective as the frequency is increased and seldom is useful at frequencies greater than about 2 mc.

5. Types of Resistors Most Frequently Used in Communication.—Resistors used in communication work are commonly divided into wire-wound and composition types according to whether the resistance element is metallic, or is a conducting compound, respectively.

Fixed Wire-wound Resistors.—Fixed wire-wound resistors consist of resistance wire, normally nichrome or an equivalent type, wound on an insulating form. Where

[1] S. Butterworth, Effective Resistance of Inductance Coils at Radio Frequency, *Exp. Wireless and Wireless Eng.*, Vol. 3, p. 483, August, 1926.

Ex. 1044-0045

appreciable power is to be dissipated, the form is either a ceramic tube, or a metal sheet covered with asbestos. In the former case the resistance wire is protected against mechanical injury and corrosion by a cement or vitreous enamel coating. Power resistors of these types are available in a large variety of sizes and mounting arrangements, and are also available with intermediate taps. The power ratings

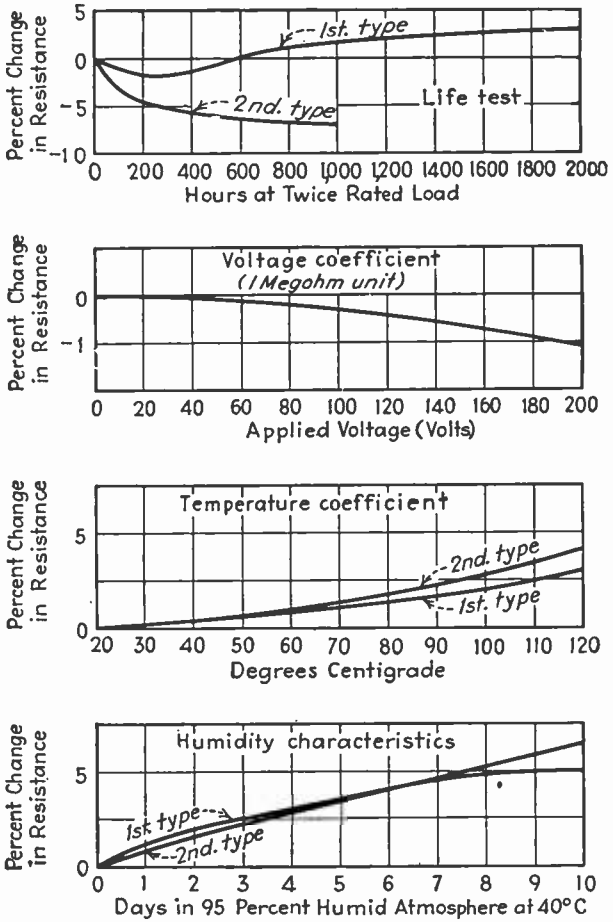


FIG. 7.—Various characteristics of typical carbon resistors. The behavior will vary greatly with ohmage, wattage rating, manufacturer, etc.

depend upon the size, method of construction, and freedom with which air is allowed to circulate around the resistor.

Resistors that are called upon to dissipate only a few watts commonly employ insulated wire wound on a bakelite strip, or on a molded or ceramic spool. The spool-type resistor is widely used for meter multipliers and similar applications, while the strip type is employed for the center-tapped resistance across filaments, etc.

Composition Resistors.—The commonest type of composition resistor is the "carbon" resistor, widely used in radio work. This obtains its resistance by combining powdered carbon or graphite with a synthetic resin bond and an inert material such as

Ex. 1044-0046

talc, in the proper proportions. Another type of composition resistor makes use of a conducting filament that is painted or deposited on a glass rod or other insulator. The "metalized" resistors are of this type.

Composition resistors are made in ratings from ¼ watt to several watts dissipation, and can be obtained with a variety of terminal arrangements. The conducting filament (or metalized) type and also many of the carbon types are inclosed in an insulating tube. With carbon resistors, the resistance material is, however, often simply formed in the shape of a stick, provided with terminals, and then painted.

Composition resistors differ from wire-wound resistors in that the resistance is less stable with time, drops somewhat when large voltages are applied, and has a large negative temperature coefficient of resistance.

Typical performance curves of composition resistors are shown in Fig. 7. Standard procedures for making these and other tests on composition resistors have been worked out by the Radio Manufacturers Association.

Color Code.—Composition resistors normally have their resistance value indicated by a color code established by the Radio Manufacturers Association. In this code, the color of the body of the resistor designates the first digit. The color at the end of the resistor designates the second digit, and the color of the dot (or band) in the center of the resistor indicates the number of ciphers after the first two digits. The colors used to designate different numbers are as follows:

0	Black	5	Green
1	Brown	6	Blue
2	Red	7	Violet
3	Orange	8	Gray
4	Yellow	9	White

Thus a resistor with a black body, red end, and brown band would be 20 ohms, whereas one with a green body, black end, and yellow band would be 500,000 ohms.

When direct current is passed through a composition resistor, there are minute irregularities in the voltages appearing across the resistor. These are the result of random variations in the resistance due to intermittent contacts between conducting granules, and cause small audio-frequency (and also radio-frequency) voltages to appear across the resistor terminals.[1] As a result of this "noise," it is not permissible to use most types of composition resistors as the coupling resistance in the first stages of high gain amplifiers.

Variable Resistors Used in Communication Work–Volume Controls.—When a resistor is to be used as a rheostat, potentiometer, or volume control, an adjustable connection to the resistance element is required. The construction depends upon the power that must be dissipated. An arrangement very widely used in laboratory work to handle powers in excess of 25 watts makes use of resistance wire wound on a ceramic metal tube provided with a sliding contact. The wire is ordinarily bare and is oxidized to provide insulation between turns. In many cases all the wire except that over which the slider operates is coated with vitreous enamel to provide mechanical protection and additional insulation. Another arrangement employs a rotating contact arm, with the resistance wire wound on a circular ceramic core or on an asbestos-covered metal strip.

Where the power to be dissipated is only a few watts, the resistance element is generally wound on a fiber strip that is bent into a circular arc and mounted so that a slider operates over an exposed edge. The wire is usually enameled, with the

[1] C. J. Christensen and G. L. Pearson, Spontaneous Resistance Fluctuations in Carbon Microphones and Other Granular Resistances, *Bell System Tech. Jour.*, Vol. 15, p. 181, April, 1936.

Ex. 1044-0047

enamel being cleaned off where the slider touches. Wire-wound variable resistors of these types are available up to about 100,000 ohms, and are capable of dissipating up to about 15 watts, according to size.

When a variable resistance of very low ohmage is required, it is customary to use a slide wire. In its simplest form this consists of a disk that has a single resistance wire mounted on its periphery and is rotated in contact with a slider.

Resistors used for volume control and similar applications need dissipate only negligible power, but often must have very high resistance. They must also generally be tapered, *i.e.*, the variation of resistance with rotation must be nonlinear. Such resistors may be either of the wire-wound or composition types. The wire-wound type is suitable for total resistances up to about 50,000 ohms, and, although more expensive than the composition type, has greater durability, is able to dissipate more power, and is less likely to become noisy. Wire-wound resistors can be tapered by varying the shape of the form upon which the wire is wound to give a turn length that varies for different parts of the resistor, by changing the winding pitch, and by using a different size of wire in different parts of the resistor.

Composition-type volume controls in a typical case utilize a resistance element consisting of either an absorbent paper upon which a conducting solution is painted or a bakelite form upon which the conducting compound is deposited. The desired taper is obtained by the shape of the pattern formed by the conducting paint, the number of coats of paint, and the thickness and composition of this paint. Contact with the rotating element can be made without introducing excessive wear by means of a roller, a floating disk, or very fine sliding wires. Composition-type volume controls are inexpensive, provide high resistances, and make possible almost any conceivable taper. Their chief disadvantage is lack of durability and low dissipation.

6. Inductance and Capacity Effects Associated with Resistors and Resistor Behavior at High Frequencies.—Every resistor has a certain amount of inductance and capacity associated with it. The inductance arises because whenever current flows through a conductor, magnetic fields are produced. Capacity is inevitably present because of the capacity between terminals, between various parts of the resistor and terminals, and between parts of the resistor. As a result, a resistor can be represented by the equivalent circuit shown in Fig. 8, in which L is an equivalent inductance that takes into account the magnetic fields produced by the current flowing through the resistor and C is the equivalent lumped capacity which when connected in shunt with the resistance terminals will act in the same way as the actual distributed capacities associated with the resistor. This is a low Q parallel resonant circuit, the properties of which are discussed in Sec. 3, Par. 2.

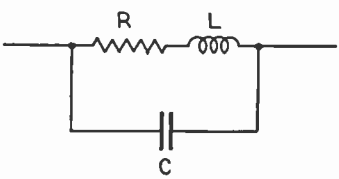


FIG. 8.—Equivalent circuit of resistor, showing inductive and capacitive effects associated with a resistor.

When effects from the inductance are to be minimized, it is necessary that $\omega L << R$ and that $1/\omega C >> R$. Under these conditions, the resistor can be thought of as consisting of a resistance R_{eq} in series with an equivalent inductance L_{eq}, for which

$$\text{Equivalent resistance} = R_{eq} = R[1 + \omega^2 C(2L - R^2 C)] \quad (9)$$

$$\text{Equivalent inductance} = L_{eq} = L - R^2 C \quad (10)$$

These equations show that if $2L = R^2C$, then the equivalent resistance R_{eq} equals the actual resistance, and no error arises in the equivalent series resistance as a result of the associated inductance and capacity (provided $R >> \omega L$ and $R << 1/\omega C$) The resistance will, however, under these conditions, act as though it has an equivalent

Ex. 1044-0048

series inductance of $-L$, *i.e.*, will have a capacitive phase angle. When, on the other hand, $L = R^2C$, then the equivalent reactance associated with the resistance becomes zero, but the resistance between terminals drops as the frequency increases according to the equation $R_{eq} = R[1 + \omega^2 LC]$. When $L > R^2C$, as tends to be the case with low resistances, an inductive impedance is obtained, whereas if $L < R^2C$, capacitive effects predominate.

The amount by which the power-factor angle of a resistor departs from unity is termed the *phase angle*. When the phase angle is small, its value is to a good approximation given by the equation

$$\text{Phase angle, radians} = \frac{\omega(L - R^2C)}{R} \tag{11}$$

The merit of a resistance unit from the point of view of freedom from reactance effects is also sometimes expressed in terms of the *time constant* ($= L_{eq}/R_{eq}$), which may be as low as 10^{-8} when care is taken to minimize capacitive and inductive effects.

Phase-angle Characteristics of Commercial Wire-wound Resistors.—The ordinary wire-wound resistors, rheostats, and potentiometers used in commercial equipment normally have reasonably low phase angles at audio frequencies, even though designed on the basis of current-carrying capacity. Some typical values are given in Table 6. Although the phase angle varies widely according to the method of construction, it is so small at audio frequencies that the reactances associated with the resistance have negligible effect upon the impedance. These commercial resistance devices are satisfactory for such applications as voltmeter multipliers, attenuators, and voltage dividers intended for audio-frequency service. It will be noted that when the resistance is low, the impedance tends to be inductive, whereas high-resistance units have a capacitive phase angle.

Characteristics of Composition Resistors at High Frequencies.—Composition resistors, because of their small physical size and short current paths in proportion to resistance, have negligible series inductance. The principal reactive effect is accordingly

TABLE 6.—PHASE ANGLES OF TYPICAL COMMERCIAL WIRE-WOUND RESISTORS

Description	Resistance, ohms	Phase angle at 1,000 cycles
Wire-wound potentiometer	200	6′ lag
Wire-wound potentiometer	360	8′ lag
Wire-wound potentiometer	27	9′ lag
Wire-wound potentiometer	50,000	24′ lead
50-watt vitreous enamel tube	100	14′ lag
50-watt vitreous enamel tube	10	26′ lag
200-watt vitreous enamel tube	100,000	2°54′ lead
1-watt wire wound	18,000	6′ lead
1-watt wire wound	3,500	1′ lag
Service test box	50,000	1°40′ lead

produced by the electrostatic field existing between the terminal electrodes and between electrodes and the resistance element itself. Approximate analysis indicates that the effects produced by these electrostatic fields associated with the resistor are equivalent to a lumped shunting capacity that increases with the length and diameter of the resistance unit.[1]

[1] G. W. O. Howe, The Behavior of High Resistances at High Frequencies, *Wireless Eng.*, Vol. 12, p 291, June, 1935.

Ex. 1044-0049

The effective resistance of a composition resistor, considered independently of the equivalent shunting capacity, is influenced by two factors. Skin effect tends to make the resistance increase with frequency, whereas the capacity between conducting granules separated by the inert material in the mix tends to cause the resistance to drop with increasing frequency.[1] With carbon resistors, the second effect is far greater than the first, and the effective resistance falls off very greatly at high frequencies. The extent of this falling off depends upon the composition of the resistor, and in general tends to be greatest with units having high resistance. The behavior of a particular carbon resistor at high frequencies is shown in Fig. 9.

Composition resistors of the filament or metalized type normally have a resistance that is much less affected by frequency than carbon resistors. This is because, first, the conductor is in the form of a thin tubular film so that skin effect is small, and,

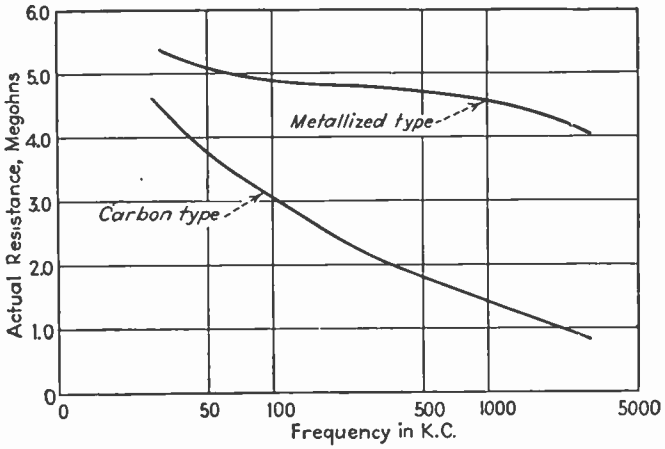


FIG. 9.—Effect of frequency on the resistance of typical metalized and carbon resistors, showing the superior performance of the former type and the very pronounced falling off in resistance with increase in frequency in the case of the carbon resistor.

second, the small conductor cross section permits the use of higher conductivity mix containing proportionately less inert material than carbon resistors. In a typical metalized resistor, the two opposing influences on the resistance are of the same order of magnitude, with one or the other predominating according to the frequency or con-

TABLE 7.—PERCENTAGE CHANGE IN EFFECTIVE RESISTANCE FROM DIRECT-CURRENT VALUES

I.R.C. Ceramic-type Resistors (1-watt Size)

Nominal value, ohms	30 mc	50 mc	100 mc	200 mc	250 mc
10,000	0	0	− 4	− 2	0
20,000	0	....	− 9	−10	−7
50,000	− 7	−11	−16	−10	−6
100,000	− 9	−12	−18	−11	−3
200,000	−17	−17	−23	−17	−4

[1] O. S. Puckle, The Behavior of High Resistances at High Frequencies, *Wireless Eng.*, Vol. 12, p. 303, June, 1935.

Ex. 1044-0050

struction. Some experimental data on the high-frequency behavior of metalized resistors are shown in Table 7.[1] The excellent behavior at extremely high frequencies is apparent.

Nonreactive Wire-wound Resistors.—Reactance effects associated with wire-wound resistors can be minimized by special winding arrangements.

The inductance of a resistor is determined primarily by the number of turns of wire and the area inclosed by the individual turns. To keep the inductance low, each turn should inclose the minimum possible area, and the wire should have as many ohms per foot of length as possible so that the length required to obtain the desired resistance will be small. In addition, it is desirable that adjacent turns carry current in opposite directions so that the residual inductance of an individual turn is neutralized by the effect of adjacent turns. A low capacitive reactance associated with a

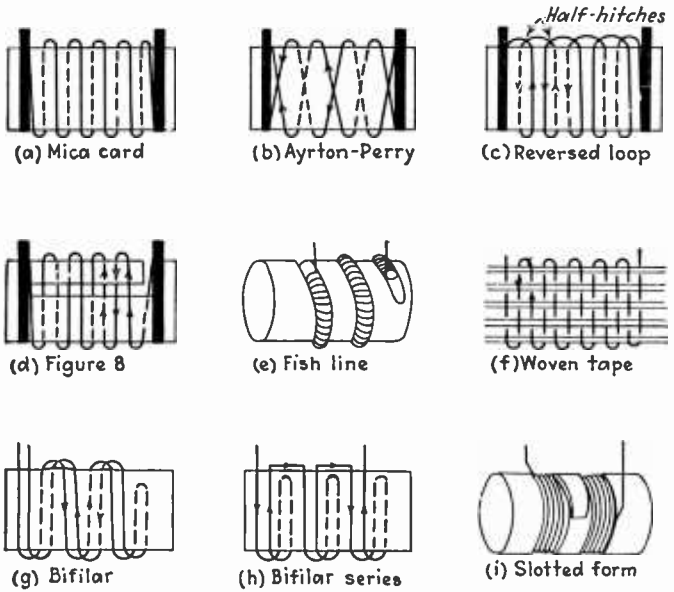


FIG. 10.—Types of resistor windings that minimize reactive effects.

resistor is obtained by arranging the winding in such a way that adjacent turns of wire have a low potential difference between them and are as far apart as possible.

Methods that can be used to minimize the reactive effects associated with a resistor are shown in Fig. 10. The mica-card type of resistor uses a single-layer winding on a thin mica form provided with copper end strips that serve as terminals and reinforcing. A low inductance can be obtained by making the card very thin and using small wire to give a high resistance per turn.[2] The Ayrton-Perry type of resistor is constructed by winding a spaced layer of insulated wire on a thin strip, after which a second wire is wound in the opposite direction between turns of the first winding. The two windings are connected in parallel and thereby produce practically zero resultant magnetic effect. The distributed capacity is low, because adjacent turns have very little potential difference between them. The reversed-loop winding obtains low inductive effects by making a half hitch at the end of each turn and thus reversing the direction of current in adjacent turns. The winding of Fig. 10*d* accom-

[1] Data by J. M. Miller and B. Salzberg, Measurement of Admittances at Ultra-high Frequencies, *RCA Rev.*, Vol. 3, p. 486, April, 1939.

[2] The inductance of windings on cards can be calculated by Eq. (41) of this section.

plishes substantially the same result in a different way. It should be noted that in this winding the wire is passed through the slot only on the *alternate* times it comes to the slot. The fish-line type of resistor consists of a fine resistance wire wound over a silk cord that serves as a core, and the resulting "fish line" is then space-wound on a cylindrical form. The tape resistor is made by weaving the resistance wire into a fabric in which the wire serves as the weft and silk thread functions as the warp.[1] The bifilar winding has negligible inductance, but the capacity is relatively large, because the beginning and end of the resistance are close together. This capacity effect can be minimized to some extent by subdividing the total resistance into several bifilar sections, as shown in Fig. 10*h*. The slotted-form winding will have moderately low capacity because of the subdivision of the winding, and the inductance can be kept moderately low by reversing the direction of the winding in adjacent slots and by using small wire to keep down the required size and the number of turns.

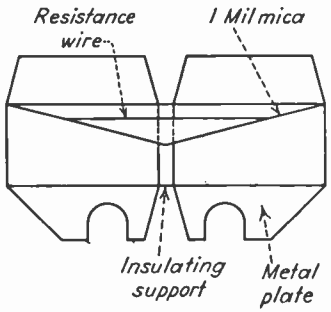


FIG. 11.—Fixed resistor having low reactive effects at very high frequencies.

The mica-card, reversed-loop, and figure-eight types of resistors can be made to have very low phase angles and are the types used in radio-frequency attenuators. The mica-card, fish-line, and woven-tape types of construction are commonly used in decade resistance boxes designed to have a low phase angle at radio frequencies, particularly for the high-resistance units. The Ayrton-Perry winding is also suitable for use at radio frequencies, particularly for resistances up to several thousand ohms. The simple bifilar winding is suitable at radio frequencies only for resistances so low that capacity effects are of no importance. The slotted type of construction is used in very high resistance units where only moderately low phase angle is essential, as in the case of voltmeter multipliers.

A radically different arrangement that is suitable for providing a fixed resistance having low reactance effects at very high frequencies is illustrated in Fig. 11. Here the resistance element is a very short, straight length of fine wire in which the inductance is minimized by mounting this wire against a metal plate with only a thin mica sheet for insulation. The shielding effect of the metal at high frequencies is such as to prevent the penetration of magnetic flux into the metal, with a consequent virtual elimination of the magnetic field around the wire. Capacity is still present, but can be minimized by using a wire of such small diameter and high resistivity that only a short length is required. This type of construction is particularly suitable for resistances of the order of five to two hundred ohms, and has good resistance and reactance characteristics to frequencies exceeding 10 mc. The power-dissipating ability is reasonably large in spite of the small dimensions because of the proximity of the metal plates.[2]

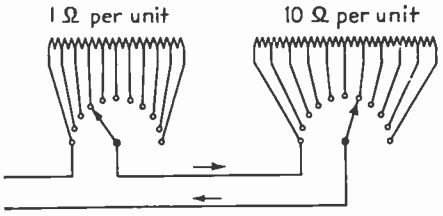


FIG. 12.—Circuit arrangement of decade resistance box.

[1] For a discussion of various types of weaves that may be used, see L. Behr and R. E. Tarpley, Design of Resistors for Precise High-frequency Measurements, *Proc. I.R.E.* Vol. 20, p. 1101, July, 1932.

[2] Further information on this type of resistance is given by D. B. Sinclair, Type 663 Resistor—A Standard for Use at High Frequencies, *Gen. Rad. Exp.*, Vol. 13, p. 6, January, 1939.

Ex. 1044-0052

Decade Resistance Boxes.—A decade resistance box consists of a number of individual resistance units associated with switches so that the total resistance is adjustable in decade increments. Such boxes are used primarily in laboratory work and in test instruments. The usual circuit arrangement of a decade resistance box is shown in Fig. 12, where ten resistance units are connected in series to form a decade controlled by an eleven-point switch. The use of ten instead of nine resistance units provides an overlap between decades and is extremely desirable.

The phase angle of a resistance box depends upon the characteristics of the individual resistance units and upon the wiring, the shielding, and the capacities introduced by the switches. When it is desired to keep the phase angle small, the resistance units must employ one of the nonreactive types of windings illustrated in Fig. 10, the switch and resistance units should be as compact as possible, and the wiring must be carefully arranged. In particular, the outgoing and return wires should be placed reasonably close together, as illustrated in Fig. 12 in order to minimize inductance effects. Even with the utmost precautions it is always found, however, that the inductance introduced by the connecting wires, and the extra shunting capacity produced by the leads and the switch, cause the phase angle of the resistance box to be much larger than that of the individual resistance units. The characteristics of the individual decade resistance assemblies of a particular type available commercially are given in Table 8.[1]

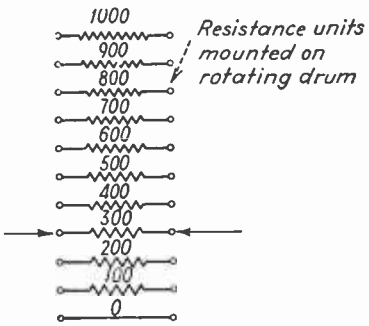


FIG. 13.—Schematic diagram showing decade resistance box which is adjusted by rotating a drum to connect the appropriate resistance unit.

The effect of the leads and switch can be minimized by rotating the resistance elements instead of the switch in the manner indicated schematically in Fig. 13. In this way, it is possible to keep the leads as short as possible, and also to reduce the capacity effects, because the unused resistance units are disconnected. Furthermore, by proper design, the inductance and capacity effects can be made independent of the value of resistance, which is of advantage in some cases.

TABLE 8.—CHARACTERISTICS OF COMMERCIAL DECADE UNITS FOR MAXIMUM SETTING OF DECADE

Decade, ohms per step	Accuracy of adjustment, per cent	Type of winding	Percentage error in resistance			Percentage error in impedance		
			Frequency, kc			Frequency, kc		
			100	1,000	5,000	100	1,000	5,000
0.1	1.0	Bifilar	0.1%	5 %	....	0.7%		
1.0	0.25	Ayrton-Perry	0	1	....	0.2		
10	0.1	Ayrton-Perry	0	0.5	11%	0	2 %	
100	0.1	Ayrton-Perry	0	0.3	4	0	0.3	5%
1,000	0.1	Unifilar on mica	− 0.1	−11	....	− 0.1	−6	
10,000	0.1	Unifilar on mica	−12	.....	....	−10		

The best resistance boxes have inclosed switches to protect the contacts from dust and are mounted in electrostatically shielding containers. In this way the resistance

[1] Further and more detailed information on the behavior of decade resistance boxes is given by D. B. Sinclair, Radio-frequency Characteristics of Decade Resistors, *Gen. Rad. Exp.*, Vol. 15, p. 1, December, 1940.

Ex. 1044-0053

and switch assembly are protected from electrostatic couplings with neighboring objects. The shield may be connected to either terminal of the resistance box or may be left floating. The phase angle will vary depending upon the way in which the shield is handled.[1]

Inductance-compensated Resistors.—In certain types of measurements, particularly in the determination of small inductances at audio frequencies, it is necessary that the equivalent series inductance of a resistance unit be independent of the resistance setting, although at the same time the inductance need not be zero. This result can be obtained by compensating for the inductance of the resistances switched in and out by switching an equivalent inductance in and out of the circuit. An arrangement of this type is illustrated in Fig. 14*a*, where the opposite ends of the switch control, respectively, resistance and inductance windings. The arrangement is so designed that as the switch is rotated the total inductance stays absolutely constant, and the resistance is increased or decreased by the desired increments.

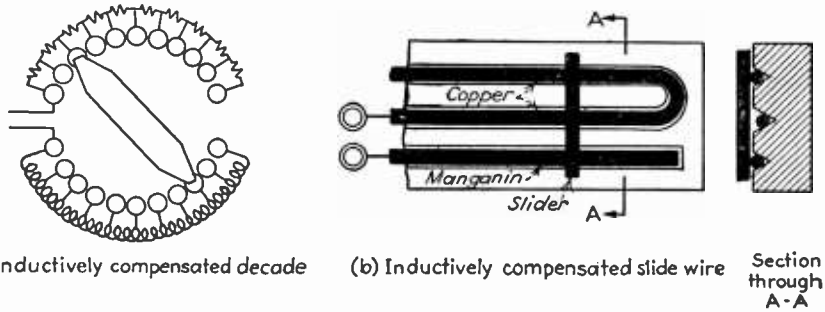


FIG. 14.—Inductively compensated adjustable resistors.

Inductance compensation can also be used with slide wires, a typical arrangement being shown in Fig. 14*b*. Here the moving slider causes a length of manganin wire to be replaced by a length of copper in an equivalent position, so that the resulting inductance is constant at all times.

Load Resistors for Absorbing Radio-frequency Power.—Resistors used as radio-frequency loads, *i.e.*, as dummy antennas, present a particularly difficult problem, since here one desires a nonreactive unit capable of dissipating appreciable wattage. When the power is of the order of fifty watts or less, several satisfactory arrangements are available. One consists of a bifilar resistance element supported on mica and mounted in a glass bulb filled with inert gas, preferably hydrogen.[2] Another arrangement consists of a mica-card type of unit mounted between two large aluminum castings that are for the purpose of conducting away the heat. Both these arrangements give excellent phase-angle characteristics.

Where larger amounts of power are to be handled, various expedients are used. In most of these the reactance is eliminated by tuning, and the dissipated power is evaluated by a calorimetric or photometric method, or by measuring the equivalent circuit resistance at the frequency involved. Another possibility is to use a metalized

[1] For a detailed discussion of the problem of shielding resistance units, see J. G. Ferguson, Shielding in High Frequency Measurements, *Trans. A.I.E.E.*, Vol. 48, p. 1286, October, 1929.

[2] The use of hydrogen as a cooling medium is very effective because of the high heat conductivity of this gas. Thin wires in hydrogen will dissipate as much as 1,000 times as much heat at a given temperature as in a vacuum. Furthermore, with small wires, the heat dissipated at a given temperature does not change greatly with wire diameter, thus permitting a wide change in resistance without changing dissipative ability and wire length. Detailed information on the theory and design of resistors of this type is given by E. G. Linder, The Use of Gas-filled Lamps as High Dissipation, High-frequency Resistors Especially for Power Measurements, *RCA Rev.*, Vol. 4, p. 83, July, 1939.

Ex. 1044-0054

type of resistor immersed in cooling water.[1] In this way the rating can be increased to 50 to 80 times that for air, and is of the order of 100 watts per square inch of surface. The power being dissipated can be determined from the rate of flow and temperature rise of the cooling water. By making the resistor the central conductor of a concentric line shorted at the receiving end, the reactance at the input terminals can by suitable design proportions be made zero.

INDUCTANCE AND MUTUAL INDUCTANCE

7. Self-inductance.—Self-inductance is that property of electrical circuits which causes them to oppose a change in the current flowing through them. The opposition to the change in current manifests itself in the form of a back emf that is developed when the current is changed. A circuit is said to have an inductance of one henry if it develops a back emf of one volt when the current through it changes at the rate of one ampere per second.

By virtue of the way in which the back emf in a circuit is related to the time rate of change of the magnetic field, inductance is related to the number of flux linkages per unit current in the circuit. A flux linkage represents one flux line encircling the circuit current once. Flux linkages may be fractional as well as integral. The relation between inductance, total flux linkages, and current is given by

$$\text{Inductance } L, \text{ henrys} = \frac{\text{flux linkages}}{\left\{\begin{matrix}\text{current producing}\\ \text{the flux, amperes}\end{matrix}\right\}} \times 10^{-8} \tag{12}$$

Inductance is also related to the energy stored in the magnetic field by the relation

$$L = 2\frac{W}{I^2} \text{ henrys} \tag{13}$$

in which W is the energy stored in the field in watt-seconds.

Units and Dimensions.—The practical unit of inductance is the *henry*. This is such a large unit of inductance that it is often more convenient to express inductance in terms of millihenrys (10^{-3} henrys) or microhenrys (10^{-6} henrys).

Inductance has the dimensions of length. It will be found that in all inductance formulas the inductance is proportional to a linear dimension and to the square of the number of turns when the shape of the inductance is kept constant.

Inductance Formulas.—For various geometrical configurations it is possible to calculate the number of flux lines set up or the energy stored in the magnetic field for an assumed current. The inductance formulas that follow have been calculated from Eqs. (12) or (13) in this manner. The collection of inductance formulas given here will be found complete enough to satisfy the ordinary needs of engineers and physicists. In general the formulas are accurate to better than 0.5 per cent. For more extensive collections, the reader is referred to treatises on the subject.[2]

Because of the way in which the current distribution within a wire changes with frequency, the inductance of a circuit changes somewhat with frequency. Of most interest are the values that the inductance assumes at very low frequencies when the current distribution through the wire cross section is uniform, and the limiting value that the inductance assumes as the frequency approaches infinity. This latter inductance is always less than the low-frequency inductance, since the redistribution

[1] See G. H. Brown and J. W. Conklin, Water-cooled Resistors for Ultrahigh Frequencies, *Electronics*, Vol. 14, p. 24, April, 1941.

[2] E. B. Rosa and F. W. Grover, Formulas and Tables for the Calculation of Mutual and Self Inductance, *Bur. Standards Bull.*, Vol. 8, No. 1, pp. 1–237, Jan. 1, 1912.
Radio Instruments and Measurements, *Bur. Standards Circ.* 74, pp. 242–296.
J. Hak, "Eisenlose Drosselspulen [Air-core Inductances]," Verlag K. F. Koehler, Leipzig, 1938.

Ex. 1044-0055

of current at high frequencies is always of such a character as to reduce the flux linkages. The difference between the low-frequency and high-frequency inductance is usually not great, however.

8. Self-inductance of Straight Conductors.—The self-inductance of an unclosed circuit is here taken as the inductance of the unclosed circuit considered as a part of a closed circuit. The total self-inductance of the closed circuit is equal to the sum of the self-inductances of all of its parts plus the sum of the mutual inductances of each one of the component parts on every other part.

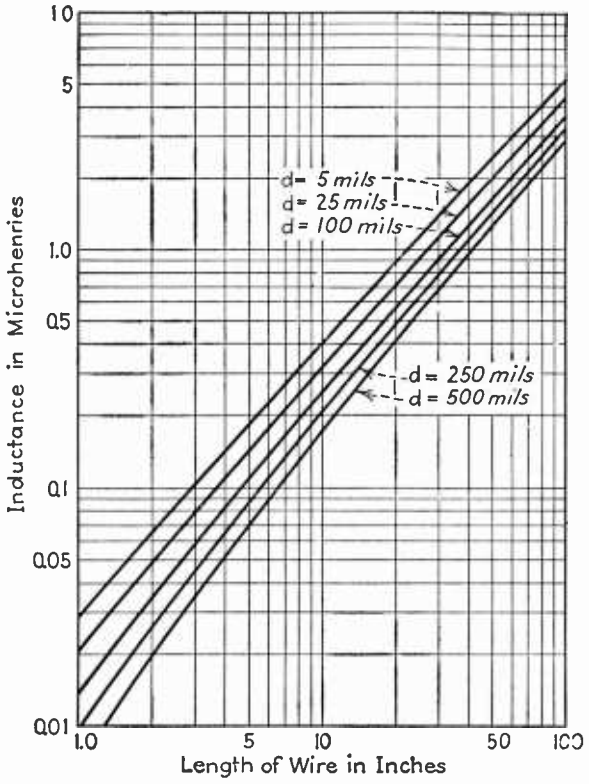


FIG. 15.—Self-inductance of a straight round wire at high frequencies.

Straight Round Wires.—The self-inductance of a round straight wire of length l, diameter d, and permeability μ is

$$L_0 = 0.00508l\left(2.303 \log_{10}\frac{4l}{d} - 1 + \frac{\mu}{4}\right) \text{ microhenrys} \tag{14}$$

where L_0 is the low-frequency inductance and the dimensions are in inches. For wires of nonmagnetic materials such as copper, this becomes

$$L_0 = 0.00508l\left(2.303 \log_{10}\frac{4l}{d} - 0.75\right) \text{ microhenrys} \tag{15}$$

For such very short lengths of wire that $l < 100d$, a term $d/2l$ should be inserted within the parentheses above.

Ex. 1044-0056

As frequency increases, the inductance of a straight round wire assumes a limiting value of

$$L = 0.00508l \left(2.303 \log_{10} \frac{4l}{d} - 1\right) \text{ microhenrys} \tag{16}$$

Limiting values of inductance for various wire lengths and diameters are shown in

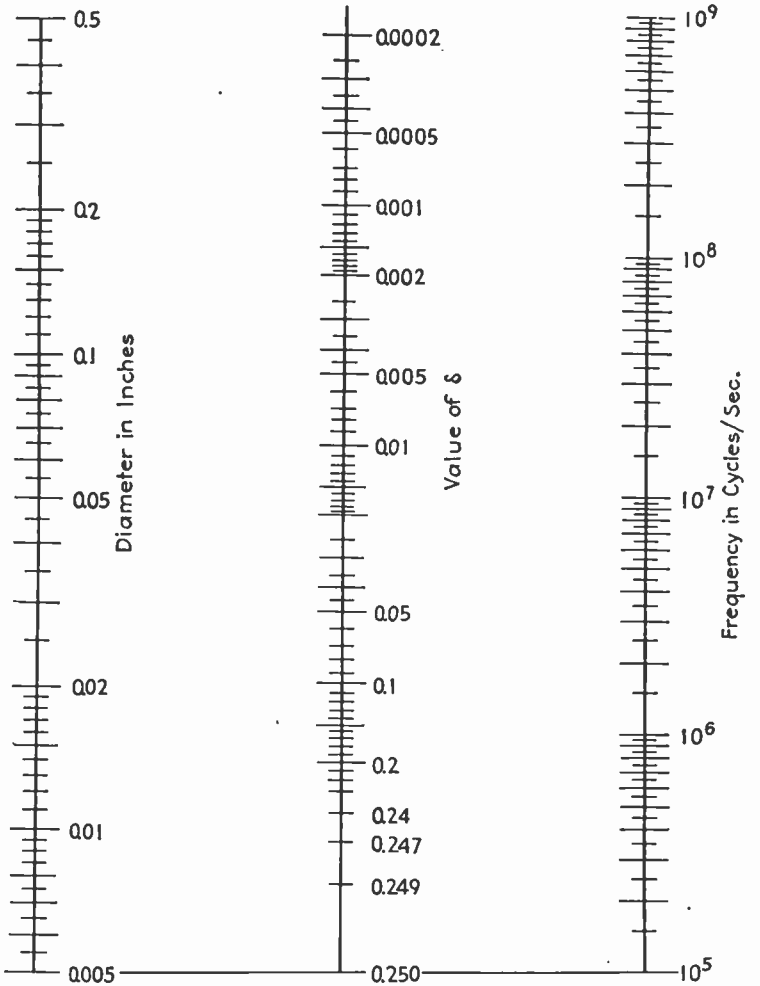


FIG. 16.—Skin-effect correction factor δ as a function of wire diameter and frequency.

Fig. 15. For intermediate frequencies, the inductance is

$$L = 0.00508l \left(2.303 \log_{10} \frac{4l}{d} - 1 + \mu\delta\right) \text{ microhenrys} \tag{17}$$

where δ is a skin-effect factor that depends upon the same quantity x defined in Eq. (3)

Ex. 1044-0057

$$x = \left(d\sqrt{\frac{\mu f}{\rho}}\right) \times \text{constant} \tag{18}$$

where f is the frequency, ρ is the volume resistivity, and μ the magnetic permeability. Values of δ for copper wire, as determined by d and f, may be obtained from the nomogram of Fig. 16. To obtain from this chart the value of δ for materials other than copper, an equivalent value of frequency, $f' = f\rho_c/\rho$ should be used instead of the actual value of frequency f, where ρ_c is the resistivity of copper. The factor δ approaches the value of 0.25 for low frequencies. For high frequencies, δ is inversely proportional to the square root of frequency, and approaches the value zero as the frequency becomes very large.

The changes of inductance with frequency are comparatively small. Skin effect at very high frequencies decreases inductance from about 6 per cent for short wires to about 2 per cent for long wires.

Return Circuit of Two Parallel Round Wires.—The self-inductance of a pair of parallel round wires carrying current in opposite directions, each having a length l, a diameter d, and spaced a distance D from one another (dimensions in inches), is

$$L = 0.01016l\left(2.303\log_{10}\frac{2D}{d} - \frac{D}{l} + \mu\delta\right) \text{ microhenrys} \tag{19}$$

This neglects the inductance of the connecting link between the ends of the wires. As before, δ is a skin-effect factor that may be determined from Fig. 16.

Grounded Horizontal Round Wire.—The self-inductance of a wire of diameter d, length l, height above ground h, placed horizontally, with the earth used as a return circuit, is

$$\begin{aligned} L = {} & 0.001170l\left[\log_{10}\left(\frac{l + \sqrt{l^2 + d^2/4}}{l + \sqrt{l^2 + 4h^2}}\right) + \log_{10}\frac{4h}{d}\right] \\ & + 0.00508\left[\sqrt{l^2 + 4h^2} - \sqrt{l^2 + d^2/4} + \mu l\delta - 2h + \frac{d}{2}\right] \text{ microhenrys} \end{aligned} \tag{20}$$

The dimensions are in inches, and δ has the same meaning as above. In most applications the diameter is very small compared with the length of the wire, and the preceding expression may be simplified to the following forms:

For $2h/l \leqq 1$:

$$L = 0.00508l\left(2.303\log_{10}\frac{4h}{d} - P + \mu\delta\right) \text{ microhenrys} \tag{21}$$

For $l/2h \leqq 1$:

$$L = 0.00508l\left(2.303\log_{10}\frac{4l}{d} - Q + \mu\delta\right) \text{ microhenrys} \tag{22}$$

Here P and Q are functions of $2h/l$ and $l/2h$ respectively, which may be determined with sufficient accuracy from the values in Table 9.

Further simplification is possible when the length is much greater than the height. The inductance per *foot* for this case is

$$L = 0.1404\log_{10}\frac{4h}{d} + 0.061\,\mu\delta \text{ microhenrys per foot} \tag{22a}$$

Grounded Round Wires in Parallel.—The formulas given here are for n parallel wires evenly spaced in a plane. The resultant expression depends upon the self-inductance of each wire and the mutual inductances between the wires. Exact expressions for this case are quite complicated. The following are approximate expressions accurate to about 1 per cent.

$$L = l\left[\frac{L_1 + (n-1)M_1}{n} - 0.00254k\right] \text{ microhenrys} \tag{23}$$

where l is the length in inches, k is a function of the number of wires n, as tabulated in Table 10, L_1 is the inductance per inch of a single grounded wire, as determined from the Eqs. (21) and (22) in the previous section, and M_1 is the mutual inductance per unit length between two adjacent wires, as determined from the following:

For $2h/l \leqq 1$:

$$M_1 = \frac{M}{l} = 0.00508\left(2.303 \log_{10}\frac{2h}{D} - P + \frac{D}{l}\right) \tag{24}$$

For $1/2h \leqq 1$:

$$M_1 = \frac{M}{l} = 0.00508\left(2.303 \log_{10}\frac{2l}{D} - Q + \frac{D}{l}\right) \tag{25}$$

in which P and Q may be found from the values in Table 9.

TABLE 9.—VALUES OF P AND Q IN EQS. (21), (22), (24), AND (25)

$2h/l$	P	$l/2h$	Q
0.0	0.0000	0.0	1.0000
0.1	0.0975	0.1	1.0499
0.2	0.1900	0.2	1.0997
0.3	0.2778	0.3	1.1489
0.4	0.3608	0.4	1.1975
0.5	0.4393	0.5	1.2452
0.6	0.5136	0.6	1.2918
0.7	0.5840	0.7	1.3373
0.8	0.6507	0.8	1.3819
0.9	0.7139	0.9	1.4251
1.0	0.7740	1.0	1.4672

Straight Rectangular Bar.—The inductance at low frequencies of a straight rectangular bar of length l, width b, and thickness c inches is

$$L_0 = 0.00508l\left(2.303 \log_{10}\frac{2l}{b+c} + 0.5 + 0.2235\frac{b+c}{l}\right) \tag{26}$$

The last term may be neglected for $l > 50(b + c)$.

TABLE 10.—VALUES OF k IN EQ. (23)

n	k
2	0
3	0.308
4	0.621
5	0.906
6	1.18
7	1.43
8	1.66
9	1.86
10	2.05
11	2.22
12	2.37
13	2.51
14	2.63
15	2.74
16	2.85
17	2.95
18	3.04
19	3.14
20	3.24

Inductance of Concentric Cable.—The low-frequency inductance is

$$L_0 = 0.140 \log_{10} \frac{r_2}{r_1} + 0.015 \text{ microhenrys per foot} \tag{27}$$

where r_1 is the radius of the outside of the inner conductor and r_2 is the radius of the inner side of the outer conductor.

Return Circuit of Two Parallel Rectangular Bars.—The low-frequency inductance of two parallel rectangular bars of length l, width b, breadth c, and center spacing D inches is

$$L_0 = 0.01016l \left(2.303 \log_{10} \frac{D}{b+c} + 1.5 - \frac{D}{l} + 0.2235 \frac{b+c}{l}\right) \text{ microhenrys} \tag{28}$$

The accuracy can be increased slightly by taking D as the geometrical mean distance between the wires.[1]

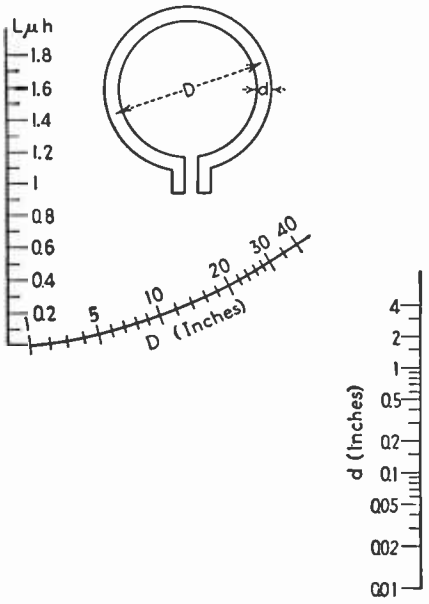


FIG. 17.—High-frequency inductance of a circular loop.

9. Self-inductance of Single-turn Loops. *Circular Ring of Circular Section.* The inductance of a circular loop of diameter D of wire of diameter d inches is

$$L = 0.01595D \left(2.303 \log_{10} \frac{8D}{d} - 2 + \mu\delta\right) \text{ microhenrys} \tag{29}$$

In Fig. 17 is given a nomogram from which the limiting value of inductance at high frequencies may be obtained for any loop and wire diameter. The nomogram may be extended by making use of the fact that inductance is proportional to linear dimension.

Circular Ring of Tubular Section.—The low-frequency inductance of a tube of inner diameter d_1 and outer diameter d_2, bent into a circle of mean diameter D inches, is

$$L_0 = 0.01595D \left[2.303 \log_{10} \frac{8D}{d_2} - 1.75 - \frac{d_1^2}{2(d_2^2 - d_1^2)} + 2.303 \frac{d_1^4}{(d_2^2 - d_1^2)^2} \log_{10} \frac{d_2}{d_1}\right] \text{ microhenrys} \tag{30}$$

For infinite frequencies this assumes the value

$$L_\infty = 0.01595D \left(2.303 \log_{10} \frac{8D}{d_2} - 2\right) \text{ microhenrys} \tag{31}$$

Square of Round Wire.—The inductance of a square of side s of wire of diameter d inches is

$$L = 0.02032s \left(2.303 \log_{10} \frac{2s}{d} + \frac{d}{2s} - 0.774 + \mu\delta\right) \text{ microhenrys} \tag{32}$$

[1] For a discussion of geometrical mean distance, see E. B. Rosa and F. W. Grover, *Bur. Standards Bull.*, Vol. 8, p. 166, 1912. For parallel rectangular bars, the geometrical mean distance is slightly greater than the center distance. The difference is inappreciable when the cross-sectional dimensions are small compared to the center spacing.

Ex. 1044-0060

Rectangle of Round Wire.—The inductance of a rectangle of sides s_1 and s_2 and diagonal g of wire of diameter d inches is

$$\left.\begin{aligned} L = 0.02339 \Big[(s_1 + s_2) \log_{10} \frac{4s_1s_2}{d} - s_1 \log_{10} (s_1 + g) \\ - s_2 \log_{10} (s_2 + g) \Big] + 0.01016 \Big[\mu\delta(s_1 + s_2) \\ + 2\left(g + \frac{d}{2}\right) - 2(s_1 + s_2) \Big] \text{ microhenrys} \end{aligned}\right\} \quad (33)$$

Rectangle of Rectangular Wire.—The low-frequency inductance of a rectangle of sides s_1 and s_2 and diagonal g of rectangular wire of thickness b and width c, as shown in Fig. 18, with dimensions in inches, is

$$L_0 = 0.02339 \left[(s_1 + s_2) \log_{10} \frac{2s_1s_2}{b + c} - s_1 \log_{10} (s_1 + g) - s_2 \log_{10} (s_2 + g) \right] + 0.01016 \left[2g - \frac{s_1 + s_2}{2} + 0.447(b + c) \right] \text{ microhenrys} \quad (34)$$

Simplified Loop Formula for Regular Figures.—Examination of the foregoing loop-inductance equations shows them to be of the same general form regardless of the shape. If l is the total perimeter of the figure, then the inductance at high frequencies can be written as

$$L_\infty = 0.00508l \left(2.303 \log_{10} \frac{4l}{d} - \theta \right) \text{ microhenrys} \quad (35)$$

The dimensions are in inches. The quantity θ is a constant that depends upon the shape of the loop.[1] Values of θ for the commonest regular figures are given in Table 11.

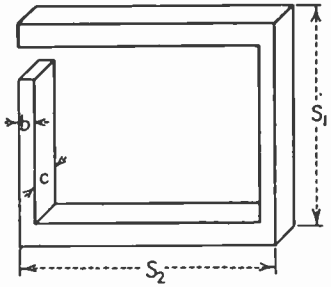


FIG. 18.—Rectangular loop of rectangular cross section.

The above formula is fairly accurate when the perimeter of the figure is large compared to the wire diameter. For ordinary loop shapes it is correct to within 0.5 per cent.

TABLE 11.—VALUES OF θ FOR USE IN EQ. (35)

Circle	$\theta = 2.451$
Regular octagon	$\theta = 2.561$
Regular hexagon	$\theta = 2.636$
Regular pentagon	$\theta = 2.712$
Square	$\theta = 2.853$
Equilateral triangle	$\theta = 3.197$
Isosceles right-angled triangle	$\theta = 3.332$

10. Inductance of Single-layer Coils. *Single-layer Solenoid of Round Wire.*—The low-frequency inductance of a single-layer solenoid is given by[2]

$$L_0 = Fn^2d \text{ microhenrys} \quad (36)$$

[1] For the development of the preceding expression, see V. I. Bashenoff, Inductance of Oddly Shaped Loops, *Proc. I.R.E.*, Vol. 15, p. 1013, December, 1927. For an extension of Bashenoff's work, see P. L. Kalantaroff and V. I. Worobieff, Self Inductance of Polygonal Circuits, *Proc. I.R.E.*, Vol. 24, p. 1585, December, 1936, which tabulates constants for 51 polygonal shapes, including many with reentrant angles.

[2] This is a rearrangement of the well-known Nagaoka formula, which is frequently written in the form $L_0 = \dfrac{2.54 \times 0.03948 \left(\frac{d}{2}\right)^2 n^2}{l} K$, where K is a constant depending upon the ratio of diameter d to length l. Factor F in Eq. (36) is $\left(2.54 \times 0.03948 \times \frac{d}{4l} K\right)$, which is a function only of d/l. See E. B. Rose and F. W. Grover, *Bur. Standards Bull.*, Vol. 8, p. 119, 1912.

Ex. 1044-0061

TABLE 12.—VALUES OF F FOR USE IN EQ. (36)

Diameter / Length	F	Difference	Diameter / Length	F	Difference
0.00	0.0000000	+0.0004972	3.00	0.03228	+0.00049
0.02	0.0004972	+0.0004888	3.10	0.03277	+0.00048
0.04	0.000986	+0.000481	3.20	0.03325	+0.00047
0.06	0.001467	+0.000472	3.30	0.03372	+0.00045
0.08	0.001939	+0.000465	3.40	0.03417	+0.00044
0.10	0.002404	+0.000457	3.50	0.03461	+0.00042
0.12	0.002861	+0.000449	3.60	0.03503	+0.00042
0.14	0.003310	+0.000442	3.70	0.03545	+0.00041
0.16	0.003752	+0.000434	3.80	0.03586	+0.00040
0.18	0.004186	+0.000428	3.90	0.03626	+0.00039
0.20	0.004614	+0.000420	4.00	0.03665	+0.00038
0.22	0.005034	+0.000413	4.10	0.03703	+0.00037
0.24	0.005447	+0.000407	4.20	0.03740	+0.00036
0.26	0.005854	+0.000400	4.30	0.03776	+0.00035
0.28	0.006254	+0.000393	4.40	0.03811	+0.00035
0.30	0.006647	+0.000387	4.50	0.03846	+0.00034
0.32	0.007034	+0.000381	4.60	0.03880	+0.00033
0.34	0.007415	+0.000375	4.70	0.03913	+0.00033
0.36	0.007790	+0.000369	4.80	0.03946	+0.00032
0.38	0.008159	+0.000363	4.90	0.03978	+0.00031
0.40	0.008522	+0.000358	5.00	0.04009	+0.00061
0.42	0.008883	+0.000352	5.20	0.04070	+0.00058
0.44	0.009232	+0.000346	5.40	0.04128	+0.00057
0.46	0.009578	+0.000341	5.60	0.04185	+0.00055
0.48	0.009919	+0.000336	5.80	0.04240	+0.00053
0.50	0.01026	+0.00081	6.00	0.04293	+0.00051
0.55	0.01107	+0.00079	6.20	0.04344	+0.00050
0.60	0.01186	+0.00076	6.40	0.04394	+0.00049
0.65	0.01262	+0.00073	6.60	0.04443	+0.00046
0.70	0.01335	+0.00071	6.80	0.04489	+0.00045
0.75	0.01406	+0.00068	7.00	0.04534	+0.00044
0.80	0.01474	+0.00066	7.20	0.04578	+0.00043
0.85	0.01540	+0.00064	7.40	0.04621	+0.00043
0.90	0.01604	+0.00062	7.60	0.04664	+0.00041
0.95	0.01666	+0.00060	7.80	0.04705	+0.00040
1.00	0.01726	+0.00058	8.00	0.04745	+0.00095
1.05	0.01784	+0.00056	8.50	0.04840	+0.00091
1.10	0.01840	+0.00055	9.00	0.04931	+0.00085
1.15	0.01895	+0.00053	9.50	0.05016	+0.00081
1.20	0.01948	+0.00052	10.00	0.05097	
1.25	0.02000	+0.00050	10.0	0.05097	+0.00151
1.30	0.02050	+0.00049	11.0	0.05248	+0.00138
1.35	0.02099	+0.00048	12.0	0.05386	+0.00128
1.40	0.02147	+0.00046	13.0	0.05514	+0.00119
1.45	0.02193	+0.00045	14.0	0.05633	+0.00110
1.50	0.02238	+0.00044	15.0	0.05743	+0.00102
1.55	0.02282	+0.00043	16.0	0.05845	+0.0095
1.60	0.02325	+0.00042	17.0	0.05940	+0.0090
1.65	0.02367	+0.00041	18.0	0.06030	+0.0086
1.70	0.02408	+0.00040	19.0	0.06116	+0.0081
1.75	0.02448	+0.00039	20.0	0.06197	+0.00151
1.80	0.02487	+0.00038	22.0	0.06348	+0.00138
1.85	0.02525	+0.00037	24.0	0.06486	+0.00127
1.90	0.02562	+0.00037	26.0	0.06613	+0.00119
1.95	0.02599	+0.00036	28.0	0.06732	+0.00112
2.00	0.02635	+0.00069	30.0	0.0684	+0.0024
2.10	0.02704	+0.00067	35.0	0.0708	+0.0022
2.20	0.02771	+0.00065	40.0	0.0730	+0.0020
2.30	0.02836	+0.00062	45.0	0.0750	+0.0016
2.40	0.02898	+0.00059	50.0	0.0766	+0.0028
2.50	0.02957	+0.00058	60.0	0.0794	+0.0025
2.60	0.03015	+0.00056	70.0	0.0819	+0.0022
2.70	0.03071	+0.00054	80.0	0.0841	+0.0019
2.80	0.03125	+0.00052	90.0	0.0850	+0.0017
2.90	0.03177	+0.00051	100.0	0.0877	

Ex. 1044-0062

where n is the number of turns, F a quantity that depends upon the ratio of the diameter to the length of the coil, and d is the diameter of the coil in inches. The quantity F is tabulated for a wide range of values of the ratio of (diameter/length) in Table 12 and plotted in Fig. 19.

Where great accuracy is required, a correction factor may be applied to Eq. (36) to take account of the fact that the coil is wound of spaced round wires rather than with a uniform current sheet.[1] This correction rarely exceeds 0.5 per cent, is greatest for widely spaced turns, and increases with the number of turns.

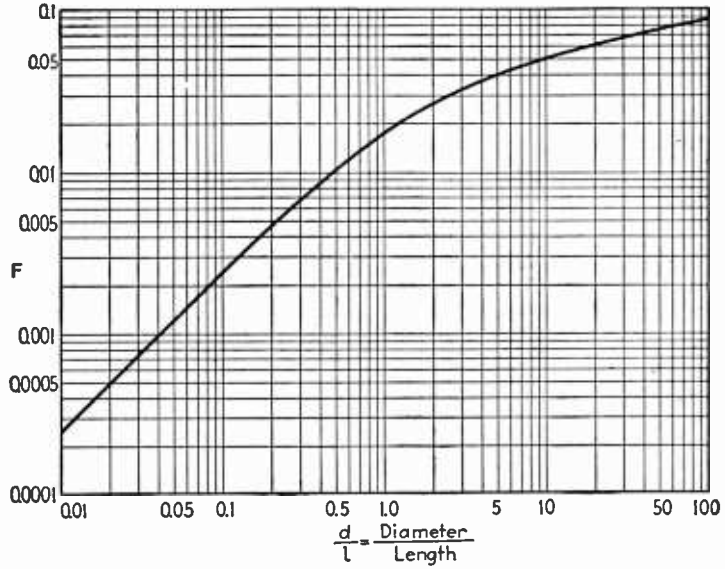


FIG. 19.—Factor F for single-layer solenoid in Eq. (36).

A simple *approximate* formula for the low-frequency inductance of a single-layer solenoid is[2]

$$L_0 = \frac{r^2n^2}{9r + 10l} \text{ microhenrys} \tag{37}$$

where r is the radius of the coil and l its length in inches. This formula is accurate to within one per cent for $l > 0.8r$, *i.e.*, if the coil is not too short.

Single-layer Solenoid of Wire of Rectangular Cross Section.—The low-frequency inductance is most conveniently given by

$$L_0 = Fn^2d - \frac{0.01596n^2dc}{l}(0.693 + B_s) \tag{38}$$

where n = number of turns of the winding.
d = diameter of the coil to center of winding, in.
l = length of the coil, in.
c = radial thickness of the wire strip, in.

F = function of $\frac{\text{diameter}}{\text{length}}$ given in Table 12 and Fig. 19.

B_s = function of $\frac{\text{length}}{\text{radial thickness } c \text{ of wire}}$ given in Table 13.

[1] Radio Instruments and Measurements. *Bur. Standards Circ.* 74, p. 253.
[2] H. A. Wheeler, Simple Inductance Formulas for Radio Coils, *Proc. I.R.E.*, Vol. 16, p. 1398, October, 1928.

Ex. 1044-0063

It will be recognized that the first term gives the inductance of the equivalent solenoid of round wire and that the second is a correction for the fact that the wires are not round.

TABLE 13.—VALUES OF B_s FOR USE IN EQ. (38)

l/c	B_s	l/c	B_s	l/c	B_s
1	0.0000	11	0.2844	21	0.3116
2	0.1202	12	0.2888	22	0.3131
3	0.1753	13	0.2927	23	0.3145
4	0.2076	14	0.2961	24	0.3157
5	0.2292	15	0.2991	25	0.3169
6	0.2446	16	0.3017	26	0.3180
7	0.2563	17	0.3041	27	0.3190
8	0.2656	18	0.3062	28	0.3200
9	0.2730	19	0.3082	29	0.3209
10	0.2792	20	0.3099	30	0.3218

Single-layer Polygonal Coil.—The inductance of a single-layer polygonal coil can be calculated from the formula for a single-layer solenoid, Eq. (36), by assuming that the polygonal coil is equivalent to a single-layer solenoid whose diameter is equal to the mean of the radii of the circumscribed and inscribed circles of the polygon. Thus if a solenoid of N sides has a circumscribed diameter d, then the equivalent solenoid diameter is

$$d_0 = d \cos^2 \frac{\pi}{2N} \tag{39}$$

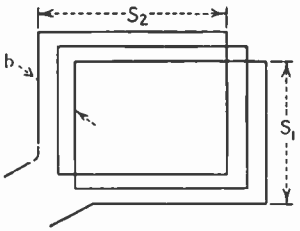


FIG. 20.—Single-layer rectangular coil.

The use of this equivalent diameter generally gives results accurate to within one per cent. The results become more accurate as the number of sides of the polygon increases.

Single-layer Rectangular Coil.—The low-frequency inductance is

$$L_0 = pn^2(G + H) \text{ microhenrys} \tag{40}$$

where p = perimeter, $2(s_1 + s_2)$.
s_1 = length of short side, in.
s_2 = length of long side, in.
b = axial length of the coil, in.
n = number of turns.
G = factor determined by s_1/s_2 and b/s_2 given in Fig. 21.
H = factor determined by n and $\dfrac{\text{diameter of wire}}{\left\{\begin{matrix}\text{spacing between} \\ \text{adjacent turns}\end{matrix}\right\}}$ and given in Fig. 21.

The foregoing expression is accurate only when the distance between turns is small compared to the short side of the coil.

Single-layer Square Coil.—This is conveniently obtained by setting $s_1 = s_2$ in the formula, Eq. (40), of the previous section.

Winding Very Long Compared with Smallest Cross-sectional Dimension.—The low-frequency inductance of such a winding, irrespective of the shape of the cross section, is

$$L_0 = 0.0319 \frac{An^2}{l} \text{ microhenrys} \tag{41}$$

Ex. 1044-0064

where A is the area of the cross section in square inches, l is the length of the coil in inches, and n is the number of turns.

Toroidal Coil of Circular Cross Section.—The low-frequency inductance of a toroidal coil of n turns with a winding of circular cross section of diameter d and a

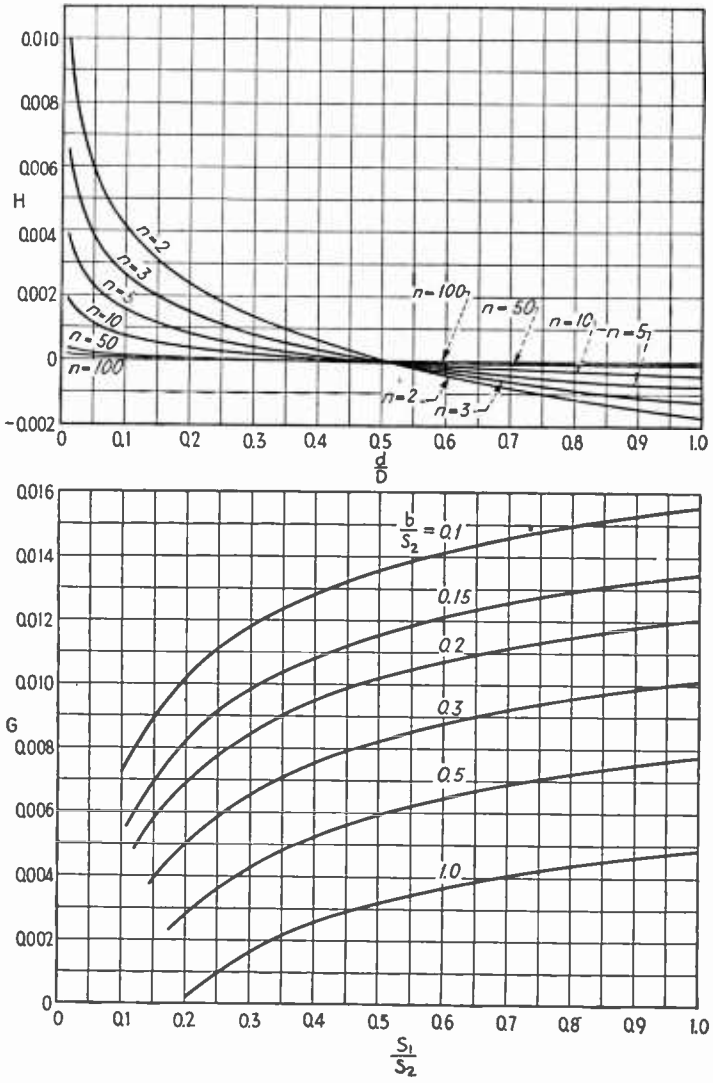


FIG. 21.—Factors G and H for single-layer rectangular coil, used in Eq. (40).

radius of revolution $D/2$ inches, as shown in Fig. 22, is

$$L_0 = 0.01595n^2[D - (D^2 - d^2)^{1/2}] \text{ microhenrys} \tag{42}$$

For toroidal coils in which the turn diameter is much less than the toroid diameter (*i.e.*, $d < 0.1D$), the inductance is *approximately*

Ex. 1044-0065

$$L_0 = 0.007975 \frac{d^2n^2}{D} \text{ microhenrys} \quad (43)$$

Toroidal Coil of Rectangular Cross Section.—When the axial depth of the winding is h and the inner and outer diameters d_1 and d_2 inches, respectively, as shown in Fig. 23, the low-frequency inductance is

$$L_0 = 0.01170n^2h \log_{10} \frac{d_2}{d_1} \text{ microhenrys} \quad (44)$$

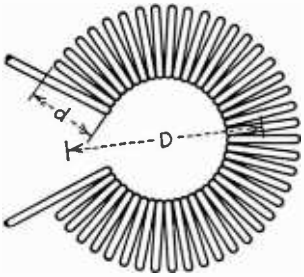


FIG. 22.—Toroidal coil of circular cross section showing notation of Eq. (42).

Flat Spiral.—The low-frequency inductance of a flat spiral of *round wire* of mean radius a and radial depth of winding c inches, as shown in Fig. 24, is conveniently expressed by

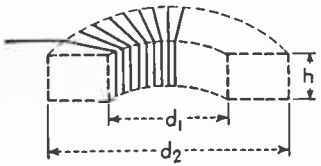


FIG. 23.—Toroidal coil of rectangular cross section illustrating notation of Eq. (44).

$$L_0 = an^2K \text{ microhenrys} \quad (45)$$

in which n is the number of turns and K is a quantity depending upon a/c as given in Fig. 24.

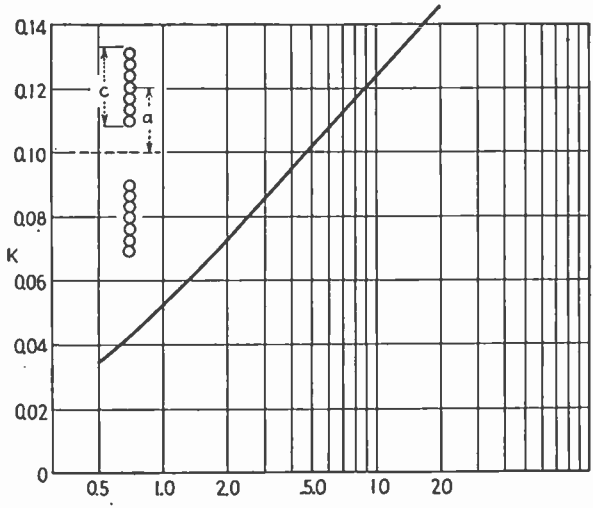


FIG. 24.—Factor K for flat spiral coil used in Eq. (45).

An *approximate* formula for this type of inductance is[1]

$$L_0 = \frac{a^2n^2}{8a + 11c} \text{ microhenrys} \quad (46)$$

This formula is accurate to within 5 per cent for $c > 0.2a$.

[1] See Wheeler, *loc. cit.*

The inductance of a flat spiral of *flat ribbon* is most readily found by using Eq. (51) for a short solenoidal multilayer coil. This gives results that are, in general, small by less than one per cent.

TABLE 14.—VALUES OF CORRECTION TERM A IN EQ. (48)

d/D	A	Difference	d/D	A	Difference
1.00	0.557	−0.051	0.20	−1.053	−0.051
0.95	0.506	−0.054	0.19	−1.104	−0.054
0.90	9.452	−0.057	0.18	−1.158	−0.057
0.85	0.394	−0.061	0.17	−1.215	−0.061
0.80	0.334	−0.065	0.16	−1.276	−0.064
0.75	0.269	−0.069	0.15	−1.340	−0.069
0.70	0.200	−0.074	0.14	−1.409	−0.074
0.65	0.126	−0.080	0.13	−1.483	−0.080
0.60	0.046	−0.087	0.12	−1.563	−0.087
0.55	−0.041	−0.095	0.11	−1.650	−0.096
0.50	−0.136	−0.041	0.10	−1.746	−0.105
0.48	−0.177	−0.043	0.09	−1.851	−0.118
0.46	−0.220	−0.044	0.08	−1.969	−0.133
0.44	−0.264	−0.047	0.07	−2.102	−0.154
0.42	−0.311	−0.048	0.06	−2.256	−0.173
0.40	−0.359	−0.052	0.05	−2.439	−0.223
0.38	−0.411	−0.054	0.04	−2.662	−0.288
0.36	−0.465	−0.057	0.03	−2.950	−0.405
0.34	−0.522	−0.061	0.02	−3.355	−0.693
0.32	−0.583	−0.064	0.01	−4.048	
0.30	−0.647	−0.069			
0.28	−0.716	−0.074			
0.26	−0.790	−0.080			
0.24	−0.870	−0.087			
0.22	−0.957	−0.096			

Flat Rectangular Coil.—The low-frequency inductance of a flat rectangular coil whose average dimensions are s_1 and s_2 with an average diagonal $g = \sqrt{s_1^2 + s_2^2}$ wound of n complete turns of wire of diameter d with a pitch of winding D, as shown in Fig. 25, is

$$L_0 = 0.02339n^2\left[(s_1 + s_2)\log_{10}\frac{2s_1s_2}{nD} - s_1\log_{10}(s_1 + g) - s_2\log_{10}(s_2 + g)\right]$$

$$\therefore$$

$$+ 0.01016n^2\left(2g - \frac{s_1 + s_2}{2} + 0.447nD\right) \qquad (47)$$

$$- 0.01016n(s_1 + s_2)(A + B) \text{ microhenrys}$$

The dimensions are in inches. In the preceding, A and B are constants depending upon the wire spacing and number of turns, respectively. Values of A corresponding to various values of d/D are given in Table 14. Values of B corresponding to various values of n are given in Table 15.

Flat Square Coil.—If $s = s_1 = s_2$ are set in the previous section, the expression for inductance reduces to

$$L_0 = 0.02032n^2s\left[2.303\log_{10}\frac{s}{nD} + 0.2235\frac{nD}{s} + 0.726\right] \qquad (48)$$

$$- 0.02032ns(A + B) \text{ microhenrys}$$

TABLE 15.—VALUES OF CORRECTION TERM B IN EQ. (48)

Number of turns, n	B	Number of turns, n	B	Number of turns, n	B
1	0.000	15	0.286	80	0.326
2	0.114	20	0.297	90	0.327
3	0.166	25	0.304	100	0.328
4	0.197	30	0.308	150	0.331
5	0.218	35	0.312	200	0.333
6	0.233	40	0.315	300	0.334
7	0.244	45	0.317	400	0.335
8	0.253	50	0.319	500	0.336
9	0.260	60	0.322	700	0.336
10	0.266	70	0.324	1,000	0.336

where s is the average length of side, n is the number of turns, d is the diameter of the wire, D is the distance between turns, and A and B are constants that may be determined from Tables 14 and 15.

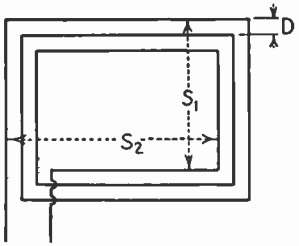


FIG. 25.—Flat rectangular coil.

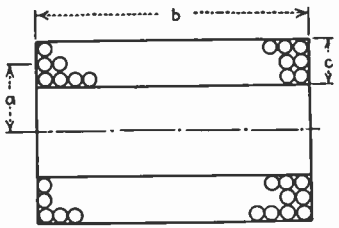


FIG. 26.—Long multiple-layer coil of solenoidal form.

11. Inductance of Multiple-layer Coils. *Long Solenoidal Form.*—The low-frequency inductance of a long multiple-layer coil of solenoidal form, as shown in Fig. 26, where b and c are the axial length and radial thickness of the winding, both measured to the outside of the wires, and a is the mean radius, is given by

$$L_0 = Fn^2d - \frac{0.03193n^2ac}{b}(0.693 + B_s) \text{ microhenrys} \quad (49)$$

The dimensions are in inches. It will be recognized that the first term of this expression is the equation for the inductance of a single-layer solenoid given in Eq. (36), F being the factor given in Table 12. The second term is a correction for the thickness of the winding with the quantity B_s the same quantity given in Table 13, except that b/c is substituted for l/c in the table. The foregoing expression is generally accurate to within 0.5 per cent.

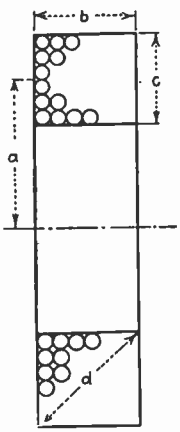


FIG. 27.—Short multiple-layer coil of solenoidal form.

If the insulation thickness is an appreciable fraction of the wire thickness, the following correction should be added:

$$\Delta L = 0.03193an\left(2.303 \log_{10}\frac{D}{d} + 0.155\right) \text{ microhenrys} \quad (50)$$

where D is the distance between wire centers and d is the diameter of the bare wire in inches. This incremental correction increases as the insulation thickness increases.

Ex. 1044-0068

Short Solenoidal Form.—The inductance of the multiple layer coil of the form shown in Fig. 27, in which b and c are the axial length and radial thickness of the winding measured to the outside of the winding, d the diagonal of the winding cross section, and a the mean radius, may be expressed in several ways. If n is the number

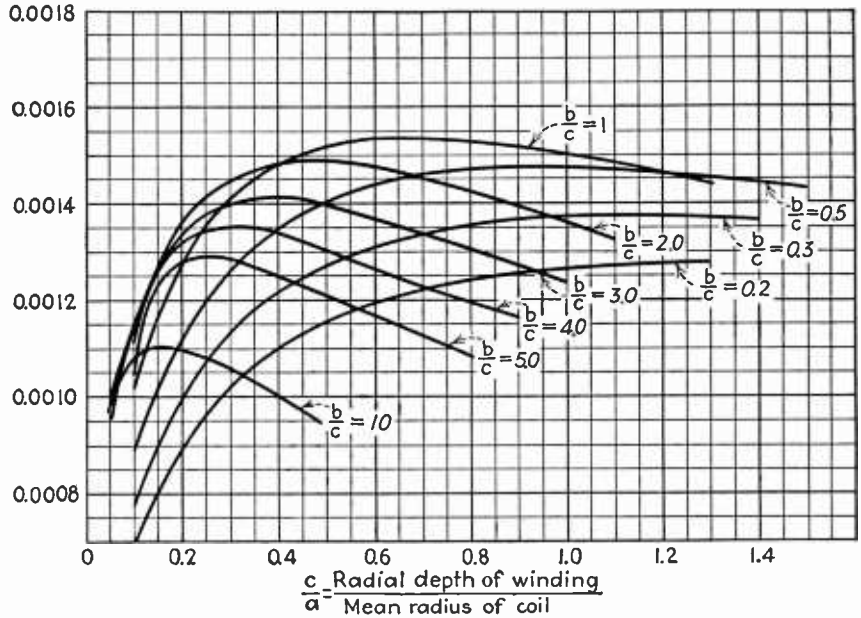


FIG. 28.—Factor I for use in Eq. (51) applying to multiple-layer coil of solenoidal form.

of turns, l the total length of the wire, and D the distance between centers of adjacent wires, then the low-frequency inductance may be expressed by (dimensions in inches)

$$L_0 = \frac{l^{5/3}}{D^{2/3}} I \text{ microhenrys} \tag{51}$$

or

$$L_0 = an^2 J \text{ microhenrys} \tag{52}$$

where I and J are the factors given in Figs. 28 and 29, respectively, in terms of c/a and b/c. It will be observed that the maximum inductance is obtained from a given length of wire when the cross section of the winding is square and the side of the cross section is 0.662 times the mean coil radius.

Where greater accuracy is desired than may be obtained from the curves of Figs. 28 and 29, the following formulas may be used:

For $b > c$:

$$L_0 = 0.03193an^2\left[2.303\left(1 + \frac{b^2}{32a^2} + \frac{d^2}{96a^2}\right)\log_{10}\frac{8a}{d} - y_1 + \frac{b^2}{16a^2}y_2\right] \tag{53}$$

where y_1 and y_2 are constants depending upon b/c and given in Table 16 below.

Ex. 1044-0069

For $b < c$:

$$L_0 = 0.03193an^2\left[2.303\left(1+\frac{b^2}{32a^2}+\frac{c^2}{96a^2}\right)\log_{10}\frac{8a}{d} - y_1 + \frac{c^2}{16a^2}y_2\right] \text{ microhenrys} \quad (54)$$

where y_1 and y_2 are constants depending upon b/c, as given in Table 16 below. The foregoing expressions are accurate to within 0.1 per cent. When the insulation

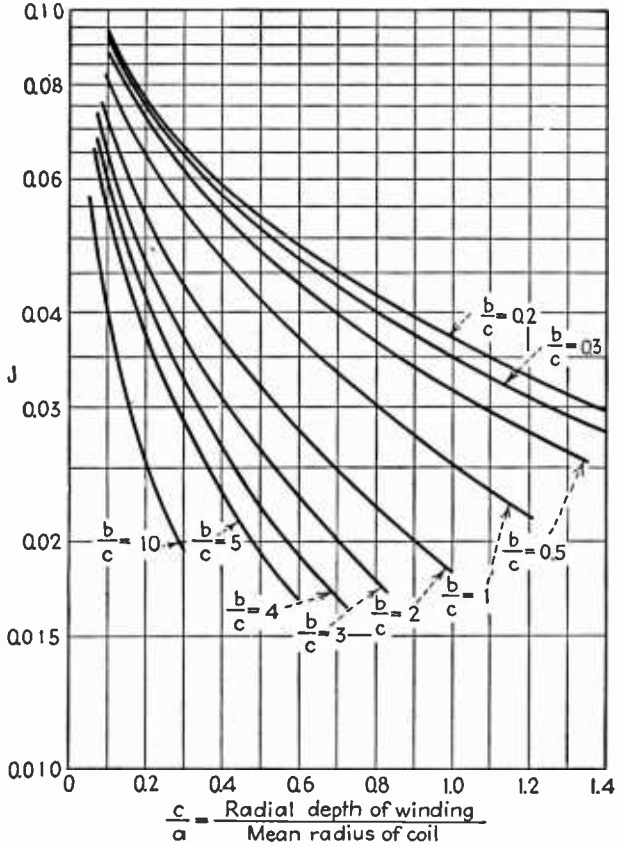


FIG. 29.—Factor J for use in Eq. (52) applying to multiple-layer coil of solenoidal form.

thickness of the wire is an appreciable fraction of the wire diameter, the following correction should be added:

$$\Delta L = +0.03193an\left(2.303\log\frac{D}{d} + 0.155\right) \text{ microhenrys} \quad (55)$$

where D is the distance between adjacent wire centers and d is the diameter of the bare wire in inches.

A simple *approximate* formula for a short multiple-layer inductance of solenoidal form[1] is

$$L_0 = \frac{0.8a^2n^2}{6a + 9b + 10c} \text{ microhenrys} \quad (56)$$

[1] See Wheeler, *loc. cit.*

Ex. 1044-0070

where the symbols have the significance indicated in Fig. 27 and the dimensions are in inches. This formula is accurate to within one per cent when the terms of the denominator are of about the same size, *i.e.*, for a coil of about the shape shown in Fig. 27.

Circular Coil of Circular Cross Section.—The low-frequency inductance of such a coil is

$$L_0 = 0.01595Dn^2\left(2.303 \log \frac{8D}{d} - 1.75\right) \text{ microhenrys} \qquad (57)$$

TABLE 16.—VALUES OF y_1, y_2, AND y_3 FOR USE IN EQS. (53) AND (54)

b/c or c/b	y_1	Difference	c/b	y_2	Difference	b/c	y_3	Difference
0	0.5000	0.0253	0	0.125	0.002	0	0.597	0.002
0.025	0.5253	0.0237						
0.05	0.5490	0.0434	0.05	0.127	0.005	0.05	0.599	0.003
0.10	0.5924	0.0386	0.10	0.132	0 010	0.10	0.602	0.006
0.15	0.6310	0.0342	0.15	0.142	0.013	0.15	0.608	0.007
0.20	0.6652	0.0301	0.20	0.155	0.016	0.20	0.615	0.009
0.25	0.6953	0.0266	0.25	0.171	0.020	0.25	0.624	0.009
0.30	0.7217	0.0230	0.30	0.192	0.023	0.30	0.633	0.010
0.35	0.7447	0.0198	0.35	0.215	0.027	0.35	0.643	0.011
0.40	0.7645	0.0171	0.40	0.242	0.031	0.40	0.654	0.011
0.45	0.7816	0.0144	0.45	0.273	0.034	0.45	0.665	0.012
0.50	0.7960	0.0121	0.50	0.307	0.037	0.50	0.667	0.013
0.55	0.8081	0.0101	0.55	0.344	0.040	0.55	0.690	0.012
0.60	0.8182	0.0083	0.60	0.384	0.043	0.60	0.702	0.013
0.65	0.8265	0.0066	0.65	0.427	0.047	0.65	0.715	0.014
0.70	0.8331	0.0052	0.70	0.474	0.049	0.70	0.729	0.013
0.75	0.8383	0.0039	0.75	0.523	0.053	0.75	0.742	0.014
0.80	0.8422	0.0029	0.80	0.576	0.056	0.80	0.756	0.015
0.85	0.8451	0.0019	0.85	0.632	0.059	0.85	0.771	0.015
0.90	0.8470	0.0010	0.90	0.690	0.062	0.90	0.786	0.015
0.95	0.8480	0.0003	0.95	0.752	0.064	0.95	0.801	0.015
1.00	0.8483	.....	1.00	0.816	.....	1.00	0.816	

where D is the mean diameter of the turns and d is the diameter of the circular winding cross section in inches. This will be recognized as being simply n^2 times the expression for a single turn of wire of diameter d. The expression neglects the space occupied by the insulation between wires.

Rectangular Coil of Rectangular Cross Section.—The low frequency inductance may be represented by

$$L_0 = pn^2G \text{ microhenrys} \qquad (58)$$

where p is to be defined as being the perimeter of the rectangle in inches, n is the number of turns, and G is a quantity depending upon the ratio s_1/s_2 of the length of the short side to the length of the long side, as given in Fig. 21. The quantity b/s_2 appearing in this figure is taken to be the circumference of the winding cross section divided by twice the length of the longer side (*i.e.*, for b use b = circumference/$2S_2$).

Where greater accuracy is desired than may be obtained from the preceding formula, the following equation should be used. As before, let s_1 and s_2 be the short and long sides of the rectangle, respectively. Let t and w be the thickness and width of the cross section of the winding and g. the diagonal of the coil in inches. Let n represent the number of turns. Then

Ex. 1044-0071

$$L_0 = 0.02339(s_1 + s_2)n^2\left[\log_{10}\frac{2s_1s_2}{t+w} - \frac{s_1}{s_1+s_2}\log_{10}(s_1+g) - \frac{s_2}{s_1+s_2}\log_{10}(s_2+g)\right] + 0.01016(s_1+s_2)n^2\left[2\left(\frac{g}{s_1+s_2}\right) - \frac{1}{2} + 0.447\frac{(t+w)}{(s_1+s_2)}\right] \text{ microhenrys} \quad (59)$$

If the insulation thickness is an appreciable fraction of the wire diameter, the correction of Eq. (55) should be added.

Square Coil of Rectangular Cross Section.—The inductance of a square coil may be obtained by setting $s_1 = s_2 = s$ in Eqs. (58) and (59). With this substitution, Eq. (59) simplifies to

$$L_0 = 0.04678sn^2\left[\log_{10}\frac{2s^2}{t+w} - \log_{10}2.414s\right] + 0.02032sn^2\left[0.914 + 0.2235\frac{t+w}{s}\right] \text{ microhenrys} \quad (60)$$

12. Mutual Inductance and Coefficient of Coupling.—When the relative position of two inductances is such that lines of flux from one inductance link with turns of the other, the two inductances are said to be inductively coupled, and mutual inductance exists between them. Mutual inductance may be defined in terms of the number of flux linkages in the second coil per unit current in the first coil, or vice versa. In practical units, the relation is

$$M = \frac{\left\{\begin{matrix}\text{flux linkages in second coil} \\ \text{produced by current in first coil}\end{matrix}\right\}}{\text{current in first coil}} 10^{-8} \quad (61a)$$

$$= \frac{\left\{\begin{matrix}\text{flux linkages in first coil} \\ \text{produced by current in second coil}\end{matrix}\right\}}{\text{current in second coil}} 10^{-8} \quad (61b)$$

where M is the mutual inductance in henrys.

Mutual inductance may also be defined as the voltage induced in the second circuit when the current in the first circuit is changing at a unit rate. If the current flowing in the first circuit is sinusoidal, then the voltage induced in the second circuit is

$$E_2 = -j\omega M I_1. \quad (62)$$

The negative of the ratio of the voltage induced in the second circuit to the current in the first circuit is called the *mutual impedance*. That is

$$Z_m = j\omega M = \frac{-E_2}{I_1} \quad (63)$$

where Z_m is the mutual impedance and the quantity $-j$ indicates that the induced voltage lags the current by 90°. The concept of mutual impedance can be generalized to give the mutual impedance between any two circuits coupled in any fashion no matter how complex. The generalized mutual impedance is the vector ratio $-E_2/I_1$, where E_2 is the voltage appearing across the opened terminals of the second circuit and I_1 is the current flowing in the first.

Coefficient of Coupling.—The maximum value of mutual inductance that can exist between two coils of inductance L_1 and L_2 is $\sqrt{L_1L_2}$, which occurs when all the flux of one coil links with all the turns of the other. The ratio of the mutual inductance actually present to the maximum possible value that can occur is called the *coefficient of coupling* and is written as

$$k = \frac{M}{\sqrt{L_1L_2}} \quad (64)$$

The coefficient of coupling k is a dimensionless quantity having a maximum value of 1.

Ex. 1044-0072

Closely coupled coils usually have a value of k of 0.5 or greater. Coils with a coefficient of 0.01 or less are said to be loosely coupled.

Circuits may be coupled capacitively or resistively as well as inductively. In the general case the coefficient of coupling is the ratio of the mutual impedance, as defined

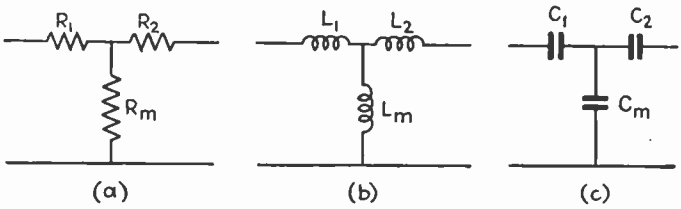


FIG. 30.—Directly coupled circuits.

above, to the square root of the product of the total self-impedances in the coupled circuits of the same nature as the coupling element. Thus, in general

$$k = \frac{Z'_m}{(Z'_1 Z'_2)^{1/2}} \tag{65}$$

where the primes indicate that all impedances are of the same kind. For circuits coupled resistively, as in Fig. 30*a*, the coefficient of coupling is

$$k = \frac{R_m}{\sqrt{(R_1 + R_m)(R_2 + R_m)}} \tag{66}$$

Similarly, for circuits directly coupled by a common inductance, as shown in Fig. 30*b*, the coefficient of coupling is

$$k = \frac{L_m}{\sqrt{(L_1 + L_m)(L_2 + L_m)}} \tag{67}$$

For circuits that are directly coupled by a common capacity, as shown in Fig. 30*c*, the coefficient of coupling is

$$k = \frac{\sqrt{C_1 C_2}}{\sqrt{(C_1 + C_m)(C_2 + C_m)}} \tag{68}$$

Combinations of Inductances Involving Mutual Inductances.—The total self-inductance of combinations of inductances depends upon the self- and mutual inductances involved. The total inductance resulting from two inductances in *series* is

$$L = L_1 + L_2 \pm 2M \tag{69}$$

where L_1 and L_2 are the self-inductances of the two coils and M is the mutual inductance between them. In general the total self-inductance of a number of inductances in series is the sum of the self-inductances of all the components plus the algebraic sum of the mutual inductances of each one of the component parts to all the other parts.

The total inductance resulting when two inductances are in *parallel* is

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 \mp 2M} \tag{70}$$

13. Mutual Inductance between Straight Conductors. *Two Parallel Wires Side by Side.*—The mutual inductance between two straight parallel wires of the same length, as shown in Fig. 31, is given by

$$M = 0.00508l \left(2.303 \log_{10} \frac{2l}{D} - 1 + \frac{D}{l}\right) \text{ microhenrys} \tag{71}$$

Ex. 1044-0073

where l is the length of the wire in inches and D is the distance between the wires. The formula assumes that the length of the wires is much greater than their spacing and that the effect of the cross section is unappreciable.

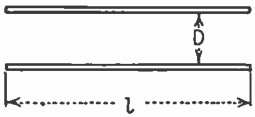


FIG. 31.—Two parallel wires side by side.



FIG. 32.—Two wire segments in same line, end-to-end.

Two Wire Segments in the Same Line End to End.—If the wire lengths are l_1 and l_2 in inches, as shown in Fig. 32, the mutual inductance is

$$M = 0.005850 \left(l_1 \log_{10} \frac{l_1 + l_2}{l_1} + l_2 \log_{10} \frac{l_1 + l_2}{l_2} \right) \text{ microhenrys} \tag{72}$$



FIG. 33.—Two wire segments in same line, ends separated.

Two Wire Segments in the Same Line, Ends Separated.—If the wire lengths are l_1 and l_2 in inches and the ends are separated a distance D, as in Fig. 33, mutual inductance is

$$\begin{aligned} M = 0.005850[&(l_1 + l_2 + D) \log_{10} (l_1 + l_2 + D) + D \log_{10} D \\ &- (l_1 + D) \log_{10} (l_1 + D) - (l_2 + D) \log_{10} (l_2 + D)] \text{ microhenrys} \end{aligned} \tag{73}$$

Two Parallel Segments.—The mutual inductance for this case is

$$\begin{aligned} M = 0.00294 &\left[l_1 \log_{10} \left(\frac{R_{1.46}}{R_{1.35}} \right) + l_2 \log_{10} \left(\frac{R_{1.46}}{R_{2.35}} \right) + e \log_{10} \left(\frac{R_{1.46}}{R_{1.36}} \times \frac{R_{2.35}}{R_{2.46}} \right) \right] \\ &- 0.00254(r_{14} - r_{13} + r_{23} - r_{24}) \text{ microhenrys} \end{aligned} \tag{74}$$

where

$$R_{k.nm} = \frac{r_{kn} + r_{km}}{r_{kn} - r_{km}}$$

where

$$r_{14} = \sqrt{a^2 + (l_1 + l_2 + e)^2}$$
$$r_{16} = l_1 - l_2 + e, \text{ etc.}$$

and the other symbols are the lengths and distances indicated in Fig. 34 in inches.

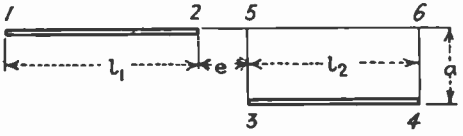


FIG. 34.—Two parallel wire segments.

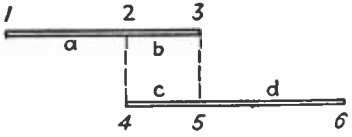


FIG. 35.—Two overlapping parallel wire segments.

The formula of Eq. (74) holds for $e = 0$, but not when one wire overlaps the other. When the wires overlap, as in Fig. 35

$$M = M_{a.cd} + M_{bc} + M_{bd} \text{ microhenrys} \tag{75}$$

where $M_{a.cd}$ is calculated by Eq. (74), using $e = 0$, using the length 1–2 for l_1, and using the length 4–6 for l_2. Similarly, for M_{bd}, use $e = 0$, length 2–3 for l_1, length 5–6 for l_2. M_{bc} is calculated from Eq. (71).

Two Parallel Symmetrically Placed Wires.—For the configuration of Fig. 36, the mutual inductance is

$$M = 0.00508 \left[4.606\, l_1 \log_{10} \left(\frac{l_1 + l_2 + \sqrt{(l_1 + l_2)^2 + D^2}}{D} \right) + 2.303(l_1 + l_2) \log_{10} \left(\frac{l_1 + l_2 + \sqrt{(l_1 + l_2)^2 + D^2}}{l_2 - l_1 + \sqrt{(l_2 - l_1)^2 + D^2}} \right) + \sqrt{(l_1 - l_2)^2 + D^2} - \sqrt{(l_1 + l_2)^2 + D^2} \right] \text{ microhenrys} \quad (76)$$

where the dimensions are in inches.[1]

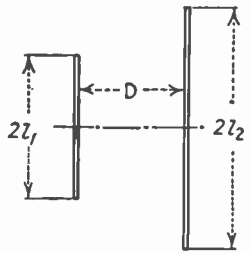


FIG. 36.—Two parallel symmetrically placed wires.

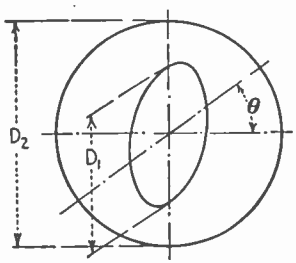


FIG. 38.—Two concentric circles.

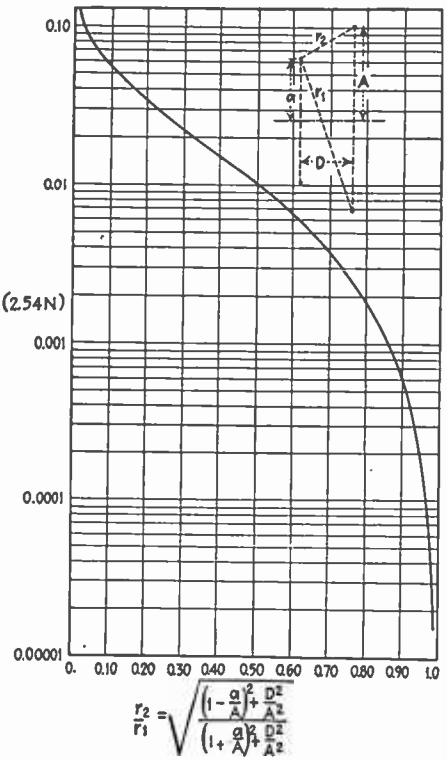


FIG. 37.—Factor $2.54N$ for use in Eq. (77) applying to two parallel coaxial circles.

14. Mutual Inductance between Single-turn Coils. *Two Parallel Coaxial Circles.* The mutual inductance between two parallel coaxial circles having the configuration shown in Fig. 36 is given by

$$M = 2.54\, N \sqrt{Aa} \text{ microhenrys} \quad (77)$$

where A is the radius of the large circle in inches, a is the radius of the small circle, and N depends upon r_1/r_2, as given in Fig. 37 and Table 17. The quantities r_1 and r_2 are, respectively, the longest and the shortest distances between points on the circum-

[1] For formulas for wire segments in positions other than given here, see Hak, *op. cit.*, p. 81, 1938; V. I. Bashenoff, Mutual Inductance of Circuits of Arbitrary Form, *Elektrotech. u. Machinenbau*, p. 1027, 1929.

Ex. 1044-0075

ferences of the two circles. Their ratio in terms of the circle radii and spacing is

$$\frac{r_2}{r_1} = \sqrt{\frac{\left(1 - \frac{a}{A}\right)^2 + \frac{D^2}{A^2}}{\left(1 + \frac{a}{A}\right)^2 + \frac{D^2}{A^2}}} \tag{78}$$

Two Concentric Circles.—The mutual inductance of any two concentric circles of diameters D_2 and D_1 turned so that the angle between their planes is θ as in Fig. 38 may be expressed by

$$M = 0.00254 D_2 \phi \text{ microhenrys} \tag{79}$$

where D_2 is the diameter of the larger circle in inches and ϕ depends upon the ratio of diameters D_2/D_1, and the angle θ, as given by the curves of Fig. 39. For a ratio

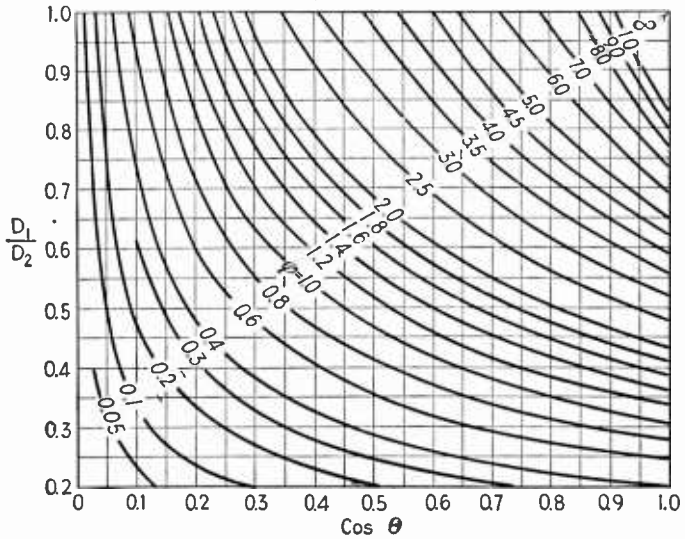


FIG. 39.—Factor ϕ for use in Eq. (79) applying to two concentric circles.

of diameters of 0.55, the mutual inductance is almost exactly proportional to the angle between the planes of the circles.[1]

Formulas are also available for the mutual inductance between two circles of any size in any arbitrary relative position.[2] These formulas tend to be quite complex, being usually given as a series expansion of spherical harmonics.

Two Equal Parallel Coaxial Rectangles.—Two equal parallel coaxial rectangles having the configuration shown in Fig. 40, with sides of length l_1 and l_2 in inches and spaced a distance D, have a mutual inductance given by

$$\begin{aligned} M = 0.02339 \Bigg[& l_1 \left(\log_{10} \frac{l_1 + \sqrt{l_1^2 + D^2}}{l_1 + \sqrt{l_1^2 + l_2^2 + D^2}} \times \frac{\sqrt{l_1^2 + D^2}}{D} \right. \\ & \left. + l_2 \left(\log_{10} \frac{l_2 + \sqrt{l_2^2 + D}}{l_2 + \sqrt{l_1^2 + l_2^2 + D^2}} \times \frac{\sqrt{l_1^2 + D^2}}{D} \right) \right] \\ & + 0.02032 \left(\sqrt{l_1^2 + l_2^2 + D^2} - \sqrt{l_1^2 + D^2} - \sqrt{l_2^2 + D_2^2} + D \right) \text{ microhenrys} \end{aligned} \tag{80}$$

[1] Hak, *op. cit.*, p. 78, 1938.

[2] Hak, *op. cit.*, p. 77; Chester Snow, Mutual Inductance of Any Two Circles, *Bur. Standards Jour. Research*, Vol. 1, p. 531, October, 1928.

Ex. 1044-0076

TABLE 17.—VALUES OF N FOR USE IN EQ. (77)

r_2/r_1	N	Difference	r_2/r_1	N	Difference
.....	.....	.....	0.30	0.008844	−0.000341
0.000	∞	.....	0.31	0.008503	−0.000328
0.010	0.05016	−0.00120	0.32	0.008175	−0.000314
0.011	0.04897	−0.00109	0.33	0.007861	−0.000302
0.012	0.04787	−0.00100	0.34	0.007559	−0.000290
0.013	0.04687	−0.00093	0.35	0.007269	−0.000280
0.014	0.04594	−0.00087	0.36	0.006989	−0.000270
0.015	0.04507	−0.00081	0.37	0.006720	−0.000260
0.016	0.04426	−0.00148	0.38	0.006460	−0.000249
0.018	0.04278	−0.00132	0.39	0.006211	−0.000241
0.020	0.04146	−0.00119	0.40	0.005970	−0.000232
0.022	0.04027	−0.00109	0.41	0.005738	−0.000225
0.024	0.03918	−0.00100	0.42	0.005514	−0.000217
0.026	0.03818	−0.00093	0.43	0.005297	−0.000210
0.028	0.03725	−0.00086	0.44	0.005087	−0.000202
0.030	0.03639	−0.00081	0.45	0.004885	−0.000195
0.032	0.03558	−0.00076	0.46	0.004690	−0.000189
0.034	0.03482	−0.00071	0.47	0.004501	−0.000183
0.036	0.03411	−0.00068	0.48	0.004318	−0.000178
0.038	0.03343	−0.00064	0.49	0.004140	−0.000171
0.040	0.03279	−0.00061	0.50	0.003969	−0.000166
0.042	0.03218	−0.00058	0.51	0.003803	−0.000160
0.044	0.03160	−0.00055	0.52	0.003643	−0.000156
0.046	0.03105	−0.00053	0.53	0.003487	−0.000150
0.048	0.03052	−0.00051	0.54	0.003337	−0.000146
0.050	0.03001	−0.00226	0.55	0.003191	−0.000141
0.060	0.02775	−0.00191	0.56	0.003050	−0.000137
0.070	0.02584	−0.00164	0.57	0.002913	−0.000133
0.080	0.02420	−0.00144	0.58	0.002780	−0.000128
0.090	0.02276	−0.00128	0.59	0.002652	−0.000125
0.100	0.02148	−0.00116	0.60	0.002527	−0.000120
0.11	0.02032	−0.00104	0.61	0.002407	−0.000117
0.12	0.01928	−0.00096	0.62	0.002290	−0.000113
0.13	0.01832	−0.00089	0.63	0.002177	−0.000109
0.14	0.01743	−0.00082	0.64	0.002068	−0.000106
0.15	0.01661	−0.00075	0.65	0.001962	−0.000103
0.16	0.01586	−0.00071	0.66	0.001859	−0.000099
0.17	0.01515	−0.00066	0.67	0.001760	−0.000096
0.18	0.01449	−0.00062	0.68	0.001664	−0.000093
0.19	0.01387	−0.00059	0.69	0.001571	−0.000090
0.20	0.01328	−0.00055	0.70	0.001481	−0.000087
0.21	0.01273	−0.00052	0.71	0.001394	−0.000084
0.22	0.01221	−0.00050	0.72	0.001310	−0.000081
0.23	0.01171	−0.00047	0.73	0.001228	−0.000078
0.24	0.01124	−0.00045	0.74	0.001150	−0.000076
0.25	0.010792	−0.000425	0.75	0.0010741	−0.0000731
0.26	0.010366	−0.000408	0.76	0.0010010	−0.0000704
0.27	0.009958	−0.000388	0.77	0.0000931	−0.0000680
0.28	0.009570	−0.000371	0.78	0.0000863	−0.0000643
0.29	0.009199	−0.000355	0.79	0.0000797	−0.0000628

TABLE 17.—VALUES OF N FOR USE IN EQ. (77).—(*Continued*)

r_2/r_1	N	Difference	r_2/r_1	N	Difference
0.80	0.0007345	−0.0000604	0.950	0.00008107	−0.00000494
0.81	0.0006741	−0.0000579	0.952	0.00007613	−0.00000482
0.82	0.0006162	−0.0000555	0.954	0.00007131	−0.00000470
0.83	0.0005607	−0.0000531	0.956	0.00006661	−0.00000458
0.84	0.0005076	−0.0000507	0.958	0.00006202	−0.00000446
0.85	0.0004569	−0.0000484	0.960	0.00005756	−0.00000436
0.86	0.0004085	−0.0000460	0.962	0.00005320	−0.00000421
0.87	0.0003625	−0.0000437	0.964	0.00004899	−0.00000409
0.88	0.0003188	−0.0000413	0.966	0.00004490	−0.00000397
0.89	0.0002775	−0.0000389	0.968	0.00004093	−0.00000383
0.90	0.0002386	−0.0000365	0.970	0.00003710	−0.00000370
0.91	0.0002021	−0.0000341	0.972	0.00003340	−0.00000356
0.92	0.0001680	−0.0000316	0.974	0.00002984	−0.00000341
0.93	0.0001364	−0.0000290	0.976	0.00002643	−0.00000327
0.94	0.0001047	−0.0000263	0.978	0.00002316	−0.00000312
0.95	0.00008107	−0.00002351	0.980	0.00002004	−0.00000296
0.96	0.00005756	−0.00002046	0.982	0.00001708	−0.00000278
0.97	0.00003710	−0.00001706	0.984	0.00001430	−0.00000262
0.98	0.00002004	−0.00001301	0.986	0.00001168	−0.00000242
0.99	0.00000703	−0.00000703	0.988	0.00000926	−0.00000223
1.00	0.00000		0.990	0.00000703	−0.00000201
			0.992	0.00000502	−0.00000177
			0.994	0.00000326	−0.00000148
			0.996	0.00000177	−0.00000115
			0.998	0.00000062	−0.00000062

Two Equal Parallel Squares.—If l is the side of each of two squares spaced a distance D inches, then Eq. (80) becomes

$$M = 0.04678 \left(l \log_{10} \frac{l + \sqrt{l^2 + D^2}}{l + \sqrt{2l^2 + D^2}} \times \frac{\sqrt{l^2 + D^2}}{D} \right) + 0.02032 \left(\sqrt{2a^2 + D^2} - 2\sqrt{a^2 + D^2} + D \right) \text{ microhenrys} \quad (81)$$

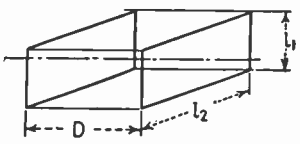


FIG. 40.—Two equal parallel coaxial rectangles.

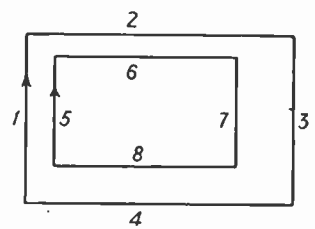


FIG. 41.—Coplanar rectangles with parallel sides.

Coplanar Rectangles with Parallel Sides.—If the two rectangles have their sides numbered as in Fig. 41, then

$$M = M_{15} + M_{26} + M_{37} + M_{48} - M_{17} - M_{28} - M_{35} - M_{46} \text{ microhenrys} \quad (82)$$

If the rectangles are concentric

$$M_{15} = M_{37}, \qquad M_{26} = M_{48}, \qquad M_{17} = M_{35}, \qquad M_{28} = M_{46},$$

Ex. 1044-0078

so that

$$M = 2(M_{15} + M_{26}) - 2(M_{17} + M_{28}) \text{ microhenrys} \tag{83}$$

If the figures are concentric squares

$$M = 4(M_{15} - M_{17}) \tag{84}$$

The separate terms are calculated by Eq. (76) if the sides are symmetrically placed. If they are not symmetrically placed, the terms are calculated by Eqs. (76) and (74).

Coaxial Solenoids not Concentric

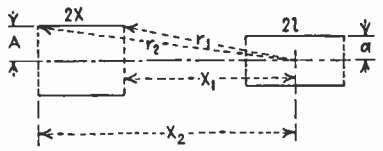


FIG. 42.—Coaxial solenoids not concentric.

Coaxial Concentric Solenoids-Outer Coil the Longer

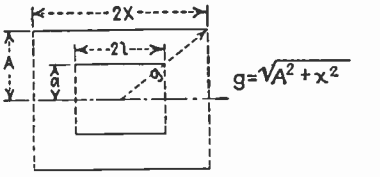


FIG. 43.—Coaxial concentric solenoids, outer coil the longer.

15. Mutual Inductance between Single-layer Solenoids. *Coaxial Coils Not Concentric.*—When the configuration is as shown in Fig. 42, the mutual inductance is given by

$$M = 0.02505 \frac{a^2 A^2 n_1 n_2}{4lx} (K_1 k_1 + K_3 k_3 + K_5 k_5) \text{ microhenrys} \tag{85}$$

where a = smaller radius, measured from the axis of the coil to the center of the wire, in.

A = larger radius, measured in the same way, in.

$2l$ = length of coil of smaller radius = number of turns times pitch of winding, in.

$2x$ = length of coil of larger radius, measured in the same way, in.

n_1 and n_2 = total number of turns on the two coils.

$$K_1 = \frac{2}{A^2}\left(\frac{x_2}{r_2} - \frac{x_1}{r_1}\right), \qquad k_1 = 2l$$

$$x_1 = D - x, \qquad r_1 = \sqrt{x_1^2 + A^2}$$

$$x_2 = D + x, \qquad r_2 = \sqrt{x_2^2 + A^2}$$

$$K_3 = \frac{1}{2}\left(\frac{x_1}{r_1^5} - \frac{x_2}{r_2^5}\right), \qquad k_3 = a^2 l\left(3 - \frac{4l^2}{a^2}\right)$$

$$K_5 = -\frac{A^2}{8}\left[\frac{x_1}{r_1^9}\left(3 - \frac{4x_1^2}{A^2}\right) - \frac{x_2}{r_2^9}\left(3 - \frac{4x_2^2}{A^2}\right)\right]$$

$$k_5 = a^4 l\left(\frac{5}{2} - 10\frac{l^2}{a^2} + 4\frac{l^2}{a^4}\right)$$

D = axial distance between centers of coils, in.

Coaxial Concentric Coils—Outer Coil the Longer.—For coils having the configuration shown in Fig. 43 and for symbols having the significance shown (measured in inches), the mutual inductance is given by

$$M = 0.0501 \frac{a^2 n_1 n_2}{g}\left[1 + \frac{A^2 a^2}{8g^4}\left(3 - 4\frac{l^2}{a^2}\right)\right] \text{ microhenrys} \tag{86}$$

A simple *approximate* expression for the coefficient of coupling for coils having the configuration of Fig. 43 is given by

$$k = \frac{a^2 l}{A^2 x} \tag{87}$$

Coaxial Concentric Solenoids—Outer Coil the Shorter.—When the coils have the configuration shown in Fig. 44, mutual inductance is again given by Eq. (86), provided g is defined as indicated in Fig. 44 and all dimensions are in inches.

Discussion of Formulas.—In the preceding formulas for the mutual inductance between solenoids, the expressions involve differences between quantities of nearly the same size. To obtain a high degree of accuracy it is hence necessary to carry out the calculations with a considerable number of significant figures. Other formulas are also available for calculating the mutual inductance of solenoids in which the calculations are not so exacting but involve a greater number of terms.[1] Where great accuracy is not demanded and many calculations are to be made, time can be saved by using a combined graphical and computational method. Formulas are also available for other cases of mutual inductance between solenoids than those given here.[2] These are invariably quite complex, involving elliptic integrals or series expansions with spherical harmonics.

16. Mutual Inductance between Multilayer Coils. *Two Coaxial Circular Coils of Small Rectangular Cross Section.*—For coils having the approximate configuration shown in Fig. 45, the mutual inductance is given by

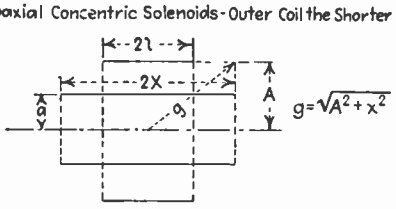


FIG. 44.—Coaxial concentric solenoids, outer coil the shorter.

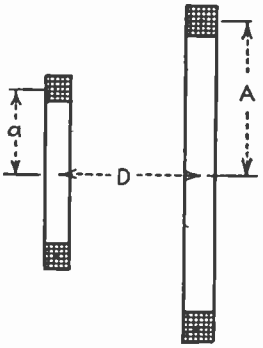


FIG. 45.—Two coaxial circular coils of small rectangular cross section.

$$M = n_1 n_2 M_0 \text{ microhenrys} \tag{88}$$

where n_1 and n_2 are the number of turns of the two coils and M_0 is the mutual inductance of the coaxial circles located at the centers of the cross sections of the two windings. Thus if a is the mean radius in inches of the small coil and A is the mean radius of the large coil and D the distance between their planes, the value of M_0 is that computed by Eq. (77) and Table 17 for the two coaxial circles thus defined. This formula is best suited for coils with square cross section and gives results in error by less than 0.5 per cent, even with relatively large cross-sectional dimensions, except when the coils are close together.

Two Coaxial Circular Coils of Rectangular Cross Section.—For coils whose cross-sectional dimensions are relatively large, compared to the coil diameters, as shown in Fig. 46, the mutual inductance is given by

$$M = \frac{n_1 n_2}{6}\Big[(M_{12'} + M_{13'} + M_{14'} + M_{15'}) + (M_{1'2} + M_{1'3} + M_{1'4} + M_{1'5}) - 2M_{11'}\Big] \text{ microhenrys} \tag{89}$$

[1] H. B. Dwight and F. W. Grover, Some Series Formulas for Mutual Inductance of Solenoids, *Elec. Eng.*, Vol. 56, p. 347, March, 1937; F. W. Grover, Tables for the Calculation of the Mutual Inductance of Any Two Coaxial Single-layer Coils. *Proc. I.R.E.*, Vol. 21, p. 1039, July, 1933.

[2] F. W. Grover, Mutual Inductance of Coils not Coaxial, *Communications*, Vol. 18, p. 10, August, 1938. See also Chester Snow, Mutual Inductance of Concentric Solenoids, *Bur. Standards Jour. Research*, Vol. 1, p. 531, 1928

Ex. 1044-0080

where the various terms represent the mutual inductance between the coaxial circles corresponding to the subscripts as indicated in Fig. 46. The separate terms are calculated by Eq. (77) and Table 17.[1]

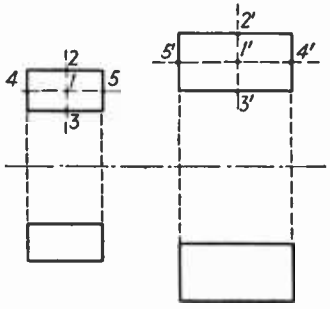


FIG. 46.—Two coaxial circular coils of rectangular cross section.

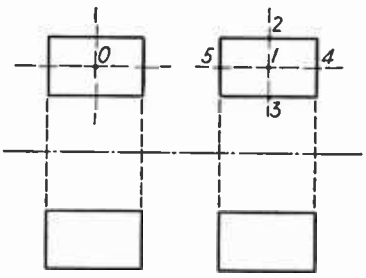


FIG. 47.—Two identical coaxial coils.

Two Identical Coaxial Coils.—When the coils are identical and of rectangular cross section, as shown in Fig. 47, the mutual inductance is given by

$$M = \frac{n^2}{3}(M_{02} + M_{03} + M_{04} + M_{05} - M_{01}) \text{ microhenrys} \qquad (90)$$

where n is the number of turns per coil and the separate terms are the mutual inductances between the coaxial circles corresponding to the subscripts, as indicated in the figure. As before, the separate terms are calculated by Eq. (77) and Table 17.

Two Coils of Equal Diameter.—When the coils are of equal diameter and radial depth but of different lengths, as shown in Fig. 48, the mutual inductance is given by

$$M = \frac{n_1 n_2}{6}(2M_{11'} + M_{14'} + M_{15'} + M_{1'4} + M_{1'5}) \text{ microhenrys} \qquad (91)$$

where n_1 and n_2 are the number of turns of the two coils and the M terms are the mutual inductances between the coaxial circles corresponding to the subscripts as indicated in the figure as calculated by Eq. (77) and Table 17.

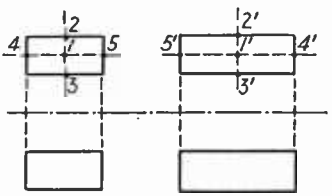


FIG. 48.—Two coaxial coils of equal diameter.

COILS WITH AIR CORES

17. Types of Windings Used in Air-cored Coils.—Most air-cored coils are single or multilayer arrangements wound on cylindrical forms. Other types occasionally used are toroids, flat spirals (or disks), and single or multilayer windings on square, hexagonal, etc., forms.

Single-layer coils are used to provide inductances for resonant circuits at broadcast and higher frequencies and for radio-frequency choke coils at very high frequencies. Multilayer coils are used when a large inductance is to be obtained in a small space. Such coils are employed in resonant circuits at broadcast and lower frequencies, as radio-frequency choke coils and as inductance standards for audio frequencies.

[1] Hak, *op. cit.*, p. 61, 1938.

Ex. 1044-0081

18. Losses in Air-cored Coils at Radio Frequencies. *Method of Expressing Losses.*—Coil losses can be expressed in terms of the equivalent series resistance R, in terms of power factor of the coil, or in terms of the ratio $\omega L/R = Q$. The coil power factor and Q are related by the equation

$$\text{Power factor} = \frac{1}{\sqrt{Q^2 + 1}} \tag{92}$$

When $Q > 5$, the power factor can be taken as $1/Q$.

Factors Contributing to Coil Losses.—The equivalent series resistance of a coil is greater than the d-c resistance at all but the very lowest frequencies, as a result of:[1] (1) skin effect in the conductor that modifies the current distribution over the cross section of the individual conductor; (2) proximity effect with respect to adjacent conductors in the coil, which produces an additional modification in the current distribution; (3) losses in dielectrics in the vicinity of the coil, particularly in the coil form; (4) eddy-current losses in neighboring metal objects. These effects tend to increase with frequency, with the result that the ratio $\omega L/R = Q$ varies only slowly with variations in frequency.

Q of Single-layer Coils.—The Q of a single-layer coil depends upon the frequency, size, and shape of the coil, and such constructional details as the number of turns, size, and character of the wire, etc. With a given coil, the Q, as observed experimentally, varies somewhat with frequency and goes through a broad maximum at a frequency that is usually either in or near the working frequency range of the coil. With a given inductance and coil diameter, the Q is maximum when the ratio (length)/(diameter) is of the order of ½:1, although the relation is not at all critical. With everything fixed except the size of wire, it is usually found that maximum Q is obtained with wire somewhat smaller than the largest that could be used in the available space. When the size of a coil is increased while maintaining the inductance and shape constant, the Q will be roughly proportional to square root of the coil diameter, provided the wire diameter is always maintained at the optimum value.

Edgewise-wound strip and ribbon conductors are sometimes used in large single-layer and pancake coils, respectively. Such conductors can be expected to give lower Q than solid wire of the same surface area, but if the dimensions are large and the strip or ribbon of ample cross section, the losses will still be low.

Litz wire gives a higher Q than solid wire at the lower radio frequencies, while at very high frequencies solid wire is as good as or better than litz. The frequency at which litz loses its advantage depends upon circumstances, but is normally of the order of 1,000 to 2,000 kc. The proper size of litz wire to use depends upon the frequency and coil construction, and, in general, does not have the same cross section as does the corresponding solid wire of optimum proportions. The effect of broken or insulated strands in a coil wound with litz wire is far less than the loss in d-c conductivity.

The form, binder, etc., used with single-layer coils have relatively little effect upon the losses, except possibly at very high frequencies, provided the dielectrics involved do not absorb moisture. Thus bakelite, polystyrene, isolantite, etc., are quite satisfactory, whereas untreated paper or cardboard is not. Experiments indicate that there is no unusual advantage, even at very high frequencies, of using types of construction such as "basket-weave" coils, in which the amount of dielectric associated with the coil is reduced to the barest minimum.

[1] Considerable attention is sometimes paid to coil proportions that have the lowest possible d-c resistance. Such considerations are, however, of little significance at radio frequencies because of skin and proximity effect. Their only usefulness is in connection with inductances for low frequencies, such as standard inductances for audio-frequency bridge work.

Ex. 1044-0082

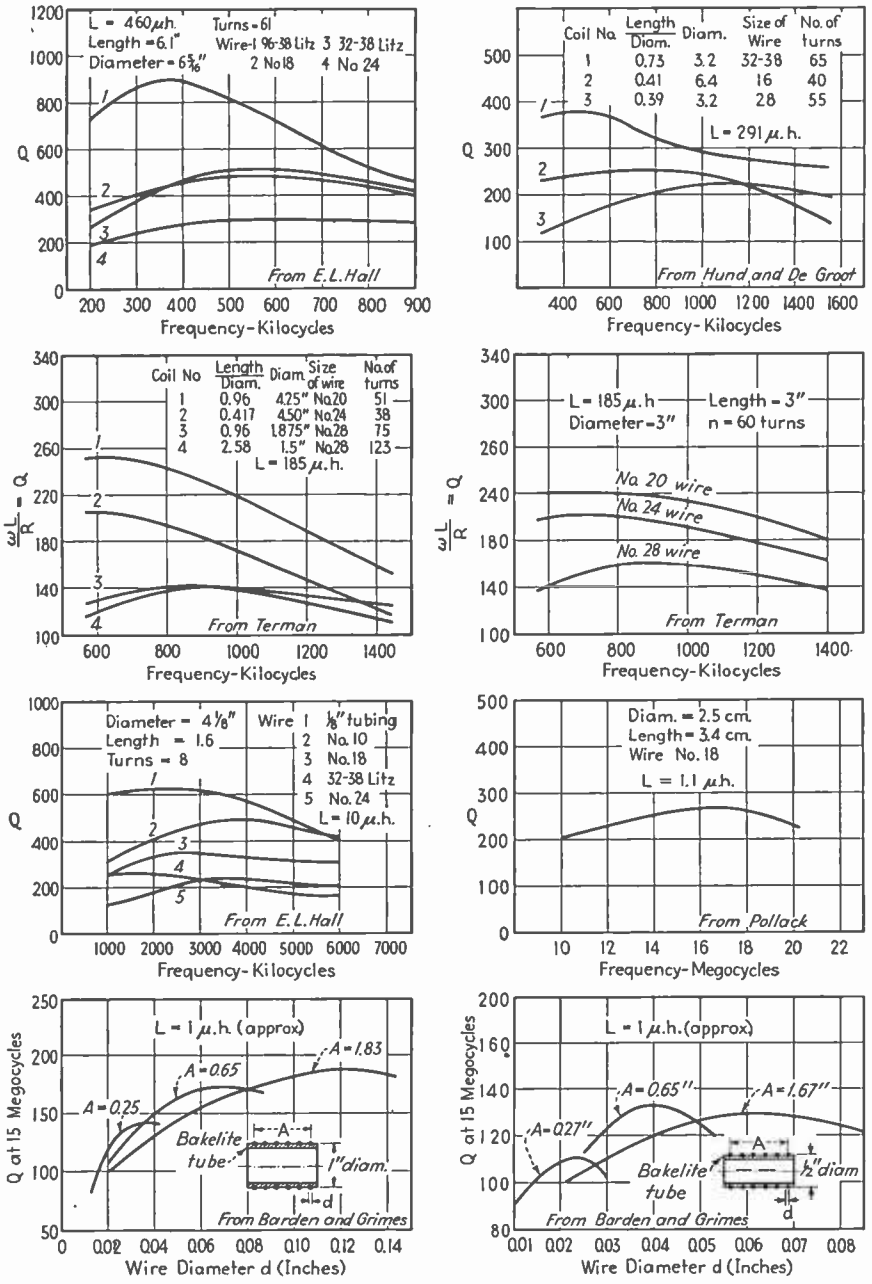


FIG. 49.—Values of circuit Q for single-layer coils designed for different frequency ranges, as reported by various investigators. All of these data were measured on the assumption that the tuning condenser had zero losses (*i.e.*, the condenser losses are charged against the coil).

Ex. 1044-0083

Experimental data giving Q as a function of frequency for a variety of single-layer coils are given in Fig. 49. These data have been obtained from various sources, and are representative of what can be expected.[1,2]

Q of Multilayer Coils.—The losses in a multilayer coil depend upon the frequency, size, and shape of the coil, the arrangement of the winding, the size and kind of wire, etc., just as in the single-layer coil. With a given coil, the Q varies only slowly with frequency and goes through a broad maximum at a frequency that is usually in or near the working range of the coil. The Q increases with the size of the coil providing the optimum wire size and arrangement is maintained, but the effect of size is not as

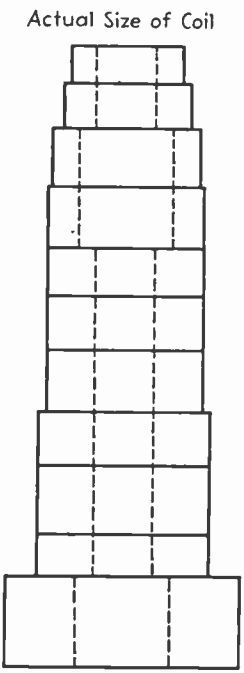


FIG. 50.—Characteristics of typical multilayer coils such as used at intermediate frequencies.

great as might be expected, and is frequently overshadowed by other factors. The exact shape and arrangement of the winding space is not critical, and satisfactory multilayer coils can be built in such widely different arrangements as a thin winding having two or three layers of many turns, and a disk or pancake winding having many layers, each of only a few turns. Good efficiency will be obtained provided the proportions do not depart too greatly from the relation $3t + 2b = D$, where t is the radial thickness of the winding, b the axial length, and D the outer diameter.

[1] Values given by the curves involve in nearly every case the assumption that the condenser used to tune the coil to resonance when the measurements were being made had zero losses. As a result, the true coil losses are probably somewhat less than those presented in the figures, particularly toward the high-frequency end of the useful range of the coil.

[2] D. Pollack, The Design of Inductances for Frequencies between 4 and 25 Megacycles, *Elec. Eng.*, Vol. 56, p. 1169, September, 1937; D. Grimes and W. Barden, Coil Design for Short Wave Receivers, *Electronics*, Vol. 7, p. 174, June, 1934; F. E. Terman, "Radio Engineering," 2d ed., p. 38; A. J. Palermo and F. W. Grover, Study of the High-frequency Resistance of Single Layer Coils, *Proc. I.R.E.*, Vol. 18, p. 2041, December, 1930; E. L. Hall, Resistance of Conductors of Various Types and Sizes as Windings of Single-layer Coils at 150 to 6,000 Kilocycles, *Bur. Standards Tech. Paper* 330, October, 1926; August Hund and H. B. DeGroot, Radio-frequency Resistance and Inductance of Coil used in Radio Reception, *Bur. Standards Tech. Paper* 298, October, 1925.

Ex. 1044-0084

The optimum size of wire ordinarily has a wire diameter that gives a cross section of copper considerably less than the cross section of the winding space, corresponding to a not too closely packed winding. Litz gives a higher Q than solid conductors at broadcast and lower frequencies, which are the frequencies where multilayer coils find their principal use. The size and number of the strands in litz for best results depends upon the frequency, physical size of the coil, etc., and ranges from three strands of No. 40 or five strands of No. 41 wire for very compact coils intended for broadcast and intermediate frequencies, to proportionately larger and more numerous strands as the physical size of the coil is increased.

Data on typical multilayer coils of the type used in radio receivers are shown in Fig. 50, and indicate the type of behavior that can be expected.[1]

Q of Toroidal Coils.—The Q's obtained with toroidal coils are always appreciably lower than for ordinary single-layer or multilayer coils of corresponding physical dimensions. In fact, an ordinary radio-frequency coil that is well designed and enclosed in a shield will have at least as high a Q and no more external magnetic field than a toroid of corresponding dimensions, and will also have zero external electrostatic field.

Maximum Q with given outer diameter is obtained in single-layer toroids of circular cross section wound with the best size of wire when the diameter of the winding section is approximately 0.35 times the over-all diameter. With single-layer toroids of rectangular cross section wound with the optimum size of wire, the optimum radial depth of winding is approximately 0.3 times the outside diameter, and the optimum height is approximately 0.4 times the outside diameter.[2]

The losses of toroidal coils can be reduced by using a single-layer winding and a relatively flat conductor so curved that the flat side follows exactly the surface of the toroid, as in Fig. 52.[3] The maximum Q of such a toroid with circular cross section occurs at $r/R = 0.71$ for fixed outer diameter. For rectangular cross section, the optimum condition for given height h and outer diameter are $r_1/r_2 = 0.279$. The Q of such coils is theoretically proportional to the size and to the square root of the frequency. With large dimensions and very high frequencies, Q's of the order of 1,000 to 10,000 are theoretically realizable. There are practical difficulties of construction, however.

19. Calculation of Copper Loss in Coils.—The most important loss in coils is the copper loss as influenced by skin and proximity effects. The problem of deriving formulas for calculating coil losses when skin and proximity effects are involved has been attacked by several investigators. The most extensive work of this character is that of Butterworth[4], who considered a coil to be composed of a number of short cylindrical parallel sections of conductor, and then determined the losses in each of these cylinders as a result of the magnetic fields produced by the current in each cylinder and in the neighboring cylinders. The results of these calculations are summarized below.

Copper Losses in Single-layer Solenoids.—For not too closely wound single-layer solenoids employing solid wire, one has

$$\frac{\text{A-c resistance}}{\text{D-c resistance}} = H + u\left(\frac{d_0}{c}\right)^2 G \tag{93}$$

[1] D. Grimes and W. S. Barden, A Study of Litz Wire Coils, *Electronics*, Vol. 6, p. 303, November; p. 342, December, 1933.

[2] S. Butterworth, The High-frequency Resistance of Toroidal Coils, *Exp. Wireless and Wireless Eng.*, Vol. 6, p. 13, January, 1929.

[3] F. E. Terman, Some Possibilities for Low Loss Coils, *Proc. I.R.E.*, Vol. 23, p. 1069, September, 1935.

[4] S. Butterworth, Effective Resistance of Inductance Coils at Radio Frequencies, *Exp. Wireless and Wireless Eng.*, Vol. 3, p. 203, April, 1926; p. 392, May, 1926; p. 417, July, 1926; p. 483, August, 1926; On the Alternating Current Resistance of Solenoidal Coils, *Proc. Roy. Soc. (London)*, Vol. 107, p. 693, 1925. A summary of Butterworth's work is given by B. B. Austin, Effective Resistance of Inductance Coils at Radio Frequency, *Wireless Eng. and Exp. Wireless*, Vol. 11, p. 12, January, 1934.

Ex. 1044-0085

TABLE 18.—VALUES OF THE FUNCTIONS H AND G

d = diameter of wire, cm; ρ = resistivity, cgs units; f = frequency, cycles per sec; $x = \pi d\sqrt{2f/\rho}$. For copper of resistivity 1,700 cgs units $x = 0.1078\, d\sqrt{f}$

x	H	G	x	H	G	x	H	G	x	H	G
0.0	1.000		2.5	1.175	0.2949	5.0	2.043	0.755	10.0	3.799	1.641
0.1	1.000	$x^4/64$	2.6	1.201	0.3184	5.2	2.114	0.790	11.0	4.151	1.818
0.2	1.000		2.7	1.228	0.3412	5.4	2.184	0.826	12.0	4.504	1.995
0.3	1.000		2.8	1.256	0.3632	5.6	2.254	0.861	13.0	4.856	2.171
0.4	1.000		2.9	1.286	0.3844	5.8	2.324	0.896	14.0	5.209	2.348
0.5	1.000	0.00097	3.0	1.318	0.4049	6.0	2.394	0.932	15.0	5.562	2.525
0.6	1.001	0.00202	3.1	1.351	0.4247	6.2	2.463	0.967	16.0	5.915	2.702
0.7	1.001	0.00373	3.2	1.385	0.4439	6.4	2.533	1.003	17.0	6.268	2.879
0.8	1.002	0.00632	3.3	1.420	0.4626	6.6	2.603	1.038	18.0	6.621	3.056
0.9	1.003	0.01006	3.4	1.456	0.4807	6.8	2.673	1.073	19.0	6.974	3.233
1.0	1.005	0.01519	3.5	1.492	0.4987	7.0	2.743	1.109	20.0	7.328	3.409
1.1	1.008	0.02196	3.6	1.529	0.5160	7.2	2.813	1.144	21.0	7.681	3.586
1.2	1.011	0.03059	3.7	1.566	0.5333	7.4	2.884	1.180	22.0	8.034	3.763
1.3	1.015	0.04127	3.8	1.603	0.5503	7.6	2.954	1.216	23.0	8.388	3.940
1.4	1.020	0.0541	3.9	1.640	0.5673	7.8	3.024	1.251	24.0	8.741	4.117
1.5	1.026	0.0691	4.0	1.678	0.5842	8.0	3.094	1.287	25.0	9.094	4.294
1.6	1.033	0.0863	4.1	1.715	0.601	8.2	3.165	1.322	30.0	10.86	5.177
1.7	1.042	0.1055	4.2	1.752	0.618	8.4	3.235	1.357	40.0	14.40	6.946
1.8	1.052	0.1265	4.3	1.789	0.635	8.6	3.306	1.393	50.0	17.93	8.713
1.9	1.064	0.1489	4.4	1.826	0.652	8.8	3.376	1.428	60.0	21.46	10.48
2.0	1.078	0.1724	4.5	1.863	0.669	9.0	3.446	1.464	70.0	25.00	12.25
2.1	1.094	0.1967	4.6	1.899	0.686	9.2	3.517	1.499	80.0	28.54	14.02
2.2	1.111	0.2214	4.7	1.935	0.703	9.4	3.587	1.534	90.0	32.07	15.78
2.3	1.131	0.2462	4.8	1.971	0.720	9.6	3.657	1.570	100.0	35.61	17.55
2.4	1.152	0.2708	4.9	2.007	0.738	9.8	3.728	1.605			
2.5	1.175	0.2949	5.0	2.043	0.755	10.0	3.799	1.641	Large	$\frac{\sqrt{2}x+1}{4}$	$\frac{\sqrt{2}x-1}{8}$

TABLE 19.—VALUES OF u FOR SINGLE-LAYER SOLENOIDS OF MANY TURNS (SPACED)

D = diameter of coil
b = length of coil
u_1 = contribution of radial component of field
u_2 = contribution of axial component of field

b/D	u_1	u_2	$u = u_1 + u_2$
0.0	3.29	0.00	3.29
0.2	3.13	0.50	3.63
0.4	2.83	1.23	4.06
0.6	2.51	1.99	4.50
0.8	2.22	2.71	4.93
1.0	1.94	3.35	5.29
2	1.11	5.47	6.58
4	0.51	7.23	7.74
6	0.31	8.07	8.38
8	0.21	8.52	8.73
10	0.17	8.73	8.90
∞	0.00	9.87	9.87

Ex. 1044-0086

where H = resistance ratio of wire when isolated, as given by Table 4 (or Table 18).
G = proximity effect factor, given by Table 18.
u = a constant given by Table 19 or 20.

$$d_0/c = \frac{\text{wire diameter}}{\text{spacing between centers of adjacent turns}}.$$

This equation holds for single-layer solenoids having many turns provided u is obtained from Table 19 and $d_0/c \lesssim 0.6$ (*i.e.*, turns not too closely spaced[1]). It also applies to coils having a few spaced turns provided the length is small compared with the diameter and u is obtained from Table 20.

TABLE 20.—VALUES OF u FOR SINGLE-LAYER SOLENOIDS OF FEW TURNS (SPACED)

No. of turns	2	4	6	8	10	12	16	24	32	∞
u	1.00	1.80	2.16	2.37	2.51	2.61	2.74	2.91	3.00	3.29

TABLE 21.—VALUES OF α, β, γ IN EQ. (94)*

d_0/c	$x = 1$			$x = 2$			$x = 3$		
	α	β	γ	α	β	γ	α	β	γ
1.0	1.01	1.02	0.96	1.09	1.34	0.67	1.31	2.29	0.49
0.9	1.00	1.02	0.97	1.06	1.29	0.72	1.20	1.99	0.55
0.8	....	1.02	0.98	1.04	1.23	0.78	1.13	1.73	0.62
0.7	....	1.02	0.98	1.02	1.18	0.83	1.08	1.52	0.68
0.6	....	1.01	0.99	1.00	1.13	0.87	1.04	1.36	0.75
0.5	....	1.01	0.99	....	1.09	0.91	1.02	1.24	0.82
0.4	....	1.01	0.99	....	1.06	0.94	1.01	1.14	0.88
0.3	....	1.00	1.00	....	1.04	0.97	1.00	1.06	0.93
0.2	....	....	....	....	1.01	0.99	....	1.03	0.97
0.1	....	....	....	....	1.00	1.00	....	1.01	0.99

d_0/c	$x = 4$			$x = 5$			$x = \infty$		
	α	β	γ	α	β	γ	α	β	γ
1.0	1.43	3.61	0.43	1.50	4.91	0.41	1.71	inf.	0.35
0.9	1.30	2.75	0.49	1.37	3.39	0.46	1.55	12.45	0.39
0.8	1.21	2.12	0.55	1.25	2.48	0.53	1.41	4.83	0.44
0.7	1.12	1.71	0.62	1.15	1.94	0.60	1.27	2.87	0.52
0.6	1.07	1.51	0.70	1.09	1.60	0.68	1.16	2.03	0.60
0.5	1.03	1.32	0.78	1.04	1.37	0.76	1.08	1.59	0.69
0.4	1.02	1.19	0.85	1.02	1.22	0.84	1.03	1.33	0.78
0.3	1.00	1.10	0.91	1.00	1.11	0.90	1.01	1.17	0.87
0.2	....	1.04	0.96	....	1.05	0.96	1.00	1.07	0.94
0.1	....	1.01	0.99	....	1.01	0.99	....	1.02	0.98

* x has the same meaning as in Tables 4 and 18.

[1] At frequencies such that H does not differ greatly from unity, the formula holds even when the spacing is less.

With closely wound coils having very many turns of solid wire, the resistance ratio for any spacing between turns, even very close spacing, is

$$\frac{\text{A-c resistance}}{\text{D-c resistance}} = \alpha H + (\beta u_1 + \gamma u_2) G \left(\frac{d_0}{c}\right)^2 \tag{94}$$

where α, β, and γ depend on (d_0/c) and are given by Table 21, u_1 and u_2 depend upon the ratio of length to diameter of the coil, as in Table 19, and the remaining notation is as in Eq. (93).

At very high frequencies, Eqs. (93) and (94) reduce to the following:

For Eq. (93), assuming copper wire:

$$\frac{\text{A-c resistance}}{\text{D-c resistance}} = 38 \sqrt{f_{mc}} d_0 \left[1 + \frac{1}{2} u \left(\frac{d_0}{c}\right)^2\right] \tag{93a}$$

For Eq. (94), assuming copper wire:

$$\frac{\text{A-c resistance}}{\text{D-c resistance}} = H\phi \tag{94a}$$

Here d_0 is the wire diameter in centimeters. The factor ϕ is given in Table 22, and depends upon the ratio of coil length to diameter and upon the closeness of the winding.

When litz wire is used, the resistance ratio of single-layer coils of many turns not too closely spaced is

$$\frac{\text{A-c resistance}}{\text{D-c resistance}} = H + \left[k + u \left(\frac{d_0}{c}\right)^2\right] \left(\frac{d_s}{d_0}\right)^2 n^2 G \tag{95}$$

where d_0 = diameter of cable.
c = spacing between centers of adjacent turns.
d_s = diameter of individual strand of cable.
n = number of strands in cable.
k = constant depending on n, given in Table 23.
G and H are the same as above, except that x in Table 18 is evaluated for the *diameter of the individual strand.*

TABLE 22.—VALUES OF FACTOR ϕ IN EQ. (94a)

$\frac{d_0}{c}$	$\frac{b}{D} = \frac{\text{coil length}}{\text{coil diameter}}$											
	0	0.2	0.4	0.6	0.8	1.0	2	4	6	8	10	∞
1.0	∞	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	3.41
0.9	18.2	17.5	16.1	14.6	13.2	11.9	8.02	5.27	4.39	3.96	3.78	3.11
0.8	6.49	6.32	5.96	5.57	5.23	4.89	3.91	3.20	3.04	2.97	2.92	2.82
0.7	3.59	3.53	3.43	3.29	3.17	3.07	2.74	2.61	2.51	2.51	2.50	2.52
0.6	2.36	2.35	2.32	2.29	2.26	2.23	2.16	2.15	2.14	2.16	2.16	2.22
0.5	1.73	1.74	1.75	1.75	1.75	1.76	1.77	1.85	1.85	1.86	1.86	1.93
0.4	1.38	1.39	1.41	1.42	1.44	1.45	1.49	1.56	1.57	1.59	1.60	1.65
0.3	1.16	1.19	1.21	1.22	1.22	1.24	1.28	1.34	1.34	1.35	1.36	1.39
0.2	1.07	1.08	1.08	1.09	1.10	1.10	1.13	1.16	1.16	1.17	1.17	1.19
0.1	1.02	1.02	1.03	1.03	1.03	1.03	1.04	1.04	1.04	1.04	1.04	1.05

TABLE 23

No. of strands, n	3	9	27	∞
k	1.55	1.84	1.92	2

Ex. 1044-0088

Calculation of Copper Losses in Multilayer Coils.—Butterworth has also derived a formula for the a-c/d-c resistance ratio of multilayer coils having many turns not too compactly wound. For solid wire one has

$$\frac{\text{A-c resistance}}{\text{D-c resistance}} = H + \frac{1}{4}\left(\frac{Kbm}{D}\right)^2\left(\frac{d_0}{c}\right)^2 G \tag{96}$$

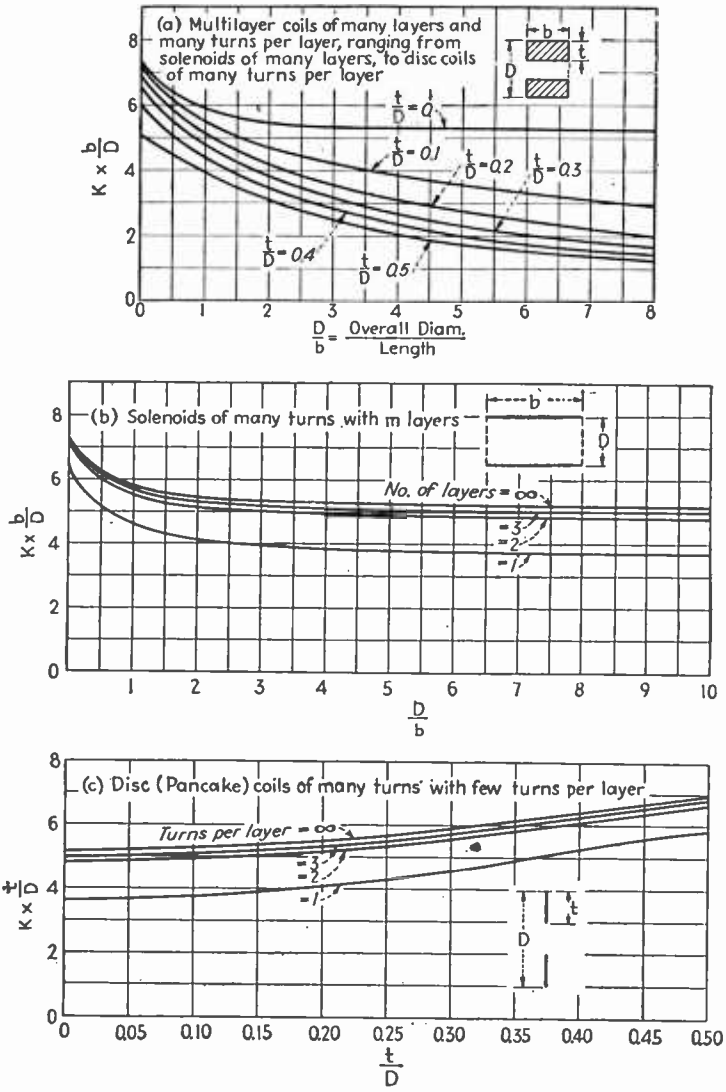


FIG. 51.—Values of constant K in Eqs. (96) and (97) for multilayer coils.

where K = a constant depending upon the type of winding, obtainable from Fig. 51.
b = axial length of winding.
D = over-all diameter.
m = number of layers.

Ex. 1044-0089

$$d_0/c = \frac{\text{diameter of wire}}{\text{spacing between centers of adjacent turns in same layer}}$$

H and G = constants given by Table 18.

For litz, one has

$$\frac{\text{A-c resistance}}{\text{D-c resistance}} = H + \left[k + \frac{1}{4}\left(\frac{Kbm}{D}\right)^2 \left(\frac{d_0}{c}\right)^2\right]\left(\frac{d_s}{d_0}\right)^2 n^2 G \qquad (97)$$

The notation in Eq. (97) is the same as in Eqs. (95) and (96).

Equations (96) and (97) apply to all shapes of coils, ranging from solenoids in one extreme ($t/D \rightarrow 0$) to disk or pancake coils in the other extreme ($D/b \rightarrow \infty$). The only restriction is that there must be many turns and these must be reasonably well spaced (*i.e.*, not too compactly wound). In the particular case of a single-layer solenoid, the value of K is such that Eqs. (96) and (97) reduce to Eqs. (93) and (95).

Calculation of Copper Losses in Toroidal Coils.—The resistance ratio of toroidal coils is given by Eq. (96) when solid round wire is used, and by Eq. (97) when litz wire is employed, provided that the following modified definitions are used.[1]

D = over-all diameter of toroid.

$b = \pi D$ = outer periphery of winding section.

c = spacing between centers of adjacent turns of same layer at outer periphery.

K = a constant given, depending upon the type of toroid and the proportions, as given in Table 24.

TABLE 24.—VALUES OF K FOR SINGLE-LAYER TOROIDS

Circular cross section		Rectangular cross section				
			K			
Diam. of winding / Over-all diam.	K	Radial depth / Over-all diam.	Axial length / Over-all diam.			
			0.125	0.250	0.375	0.500
0.05	2.110					
0.10	2.243					
0.15	2.409	0.1	2.25	2.25	2.26	2.26
0.20	2.602	0.2	2.65	2.67	2.69	2.70
0.25	2.913	0.3	3.37	3.47	3.54	3.58
0.30	3.327	0.4	5.25	5.68	5.96	6.14
0.35	3.978					
0.40	5.180					
0.45	8.341					

In the case of single-layer toroids wound with conductor of the type illustrated in Fig. 52, the Q obtained with optimum proportions for Fig. 52*a* is[2]

$$Q \text{ for optimum } \frac{r_2}{r_1} = \frac{84.3\sqrt{f}\, r_2}{\left(1 + 0.556\frac{r_2}{h}\right)} \qquad (98)$$

[1] S. Butterworth, The High Frequency Resistance of Toroidal Coils, *Exp. Wireless and Wireless Eng.*, Vol. 6, p. 13 January, 1929.

[2] Terman, *loc. cit.* See also G. Reber, Optimum Design of Toroidal Inductances, *Proc. I.R.E.*, Vol. 23, p. 1056, September, 1935.

Ex. 1044-0090

For a toroid of circular cross section (Fig. 52*b*),

$$Q \text{ for optimum } \frac{r_2}{r_1} = 52.1b\sqrt{f} \tag{99}$$

The dimensions in these equations are in centimeters, and the frequency f is in megacycles.

Practical Usefulness of Formulas for Calculating Copper Losses.—Equations (93) to (99) take into account only the copper losses resulting from skin and proximity effects. At frequencies somewhat less than that giving maximum Q, these losses represent nearly the entire loss of energy in the coil. At higher frequencies, dielectric losses become important and lower the Q.[1]

The chief value of the formulas for calculating losses is in indicating the relative effect that such factors as size, shape, wire size, and other proportions have upon

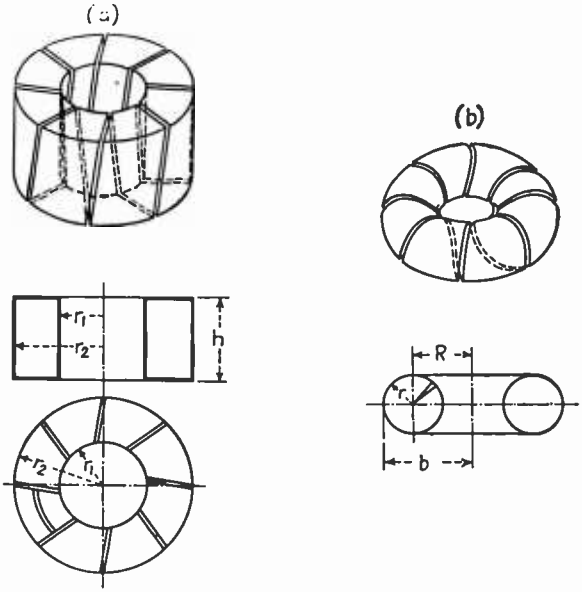


FIG. 52.—Toroidal coils wound with strip conductor that is curved to follow the surface of revolution formed by the toroid.

the coil Q. These formulas will satisfactorily compare the merit of different proposed designs, and indicate which are most promising, even under conditions where the absolute accuracy of the formulas is not particularly high.

The formulas are especially useful in indicating the size of wire for winding a given form. In the case of a single-layer solenoid in which Eq. (93*a*) applies, the wire diameter giving maximum Q when the length, diameter, and number of turns is fixed is

$$\frac{\text{Optimum diameter}}{\left\{\begin{matrix}\text{Spacing between centers}\\ \text{of adjacent turns}\end{matrix}\right\}} = \sqrt{\frac{2}{\mu}} \tag{100}$$

where μ is given by Table 19.

[1] A comparison of calculated and observed coil losses is given by W. Jackson, Measurements of the High-frequency Resistance of Single-layer Solenoids, *Jour. I.E.E.*, Vol. 80, p. 844, 1937; see also Wireless Section, *I.E.E.*, Vol. 12, p. 133, June, 1937.

Ex. 1044-0091

20. Distributed Capacity of Coils.—The voltage difference that exists between different parts of a coil produces an electrostatic field in the air and in the dielectric near the coil. The effect of the resulting storage of electrostatic energy upon the coil behavior is to a good approximation equivalent to the effect produced by a small capacity shunted across the terminals of the coil. Such a hypothetical shunting capacity is termed the *distributed capacity* of the coil. Under practical conditions, there are also capacities between the coil terminals, and between lead wires, which increase the total effective shunting capacity.[1]

The distributed capacity of an isolated single-layer solenoid is approximately proportional to the diameter of the coil and decreases slowly with increasing length. The distributed capacity is largely independent of the number of turns if there are a considerable number of turns and these are not too closely spaced. Very close spacing increases the capacity somewhat, however, particularly if there are only a few turns.

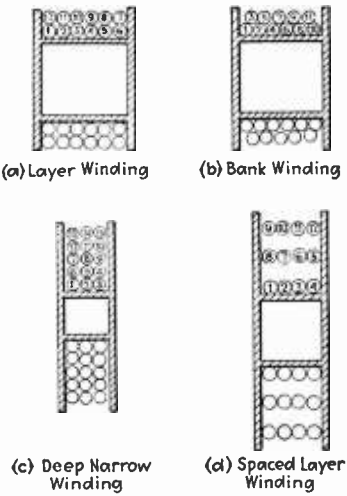


FIG. 53.—Several types of multilayer windings. The numbers in the drawing indicate the order of the turns.

Multilayer coils tend to have higher distributed capacity than do single-layer coils. In order to avoid excessive distributed capacity, the winding of a multilayer coil must be arranged so that the difference of potential between adjacent turns is only a small fraction of the total voltage applied to the coil. Thus a simple two-layer winding, as shown in Fig. 53*a*, will have very high distributed capacity, whereas with a winding in which the turns are wound as indicated in Fig. 53*b* (bank winding), the distributed capacity will be only moderate. Other means of minimizing distributed capacity consist of employing a narrow winding of many layers with few turns per layer, as shown in Fig. 53*c*, or the use of spaced layers, as in Fig. 53*d*. The common universal or honeycomb-type multilayer coil has a reasonably low distributed capacity, since it is equivalent to a coil of many layers with few turns per layer.

Dielectric in the field of a coil, such as the form upon which the coil is wound, the insulation of the wire, etc., increase the distributed capacity. Metal objects such as a shield, metal panel, etc., near a coil increase the distributed capacity, particularly if the distance from the coil is of the same order of magnitude as the coil dimensions or is less.

Effects of Distributed Capacity.—The presence of distributed capacity introduces losses, because some of the electrostatic energy storage involves solid dielectrics, which are never perfect. Dielectric losses have an effect equivalent to adding a resistance in series with the inductance coil according to the equation

$$\left.\begin{array}{l}\text{Equivalent series}\\ \text{resistance representing}\\ \text{dielectric losses}\end{array}\right\} = \tau\omega^3 L^2 C \qquad (101)$$

where $\omega = 2\pi \times$ frequency.

L = true coil inductance.

[1] The effective distributed capacity will also depend to some extent upon the current distribution in the coil, and will in general be larger when the coil is shunted with a large external tuning capacity than when the coil is resonated with its self-capacity.

Ex. 1044-0092

C = distributed capacity.

τ = power factor of distributed capacity.

The resistance given by Eq. (101) is a part of the coil resistance R in Fig. 54*a*. Experiments indicate that the dielectric losses are not over 20 per cent of the total loss in the working range of the coil, unless the form or wire insulation has absorbed moisture.[1] When hygroscopic forms are used, and when the wire has cotton insulation, it is advisable to impregnate the coil with wax, shellac, or lacquer to prevent the absorption of moisture.

The presence of the distributed capacity causes a partial resonance that modifies the apparent resistance and reactance of the coil as viewed from the terminals. The apparent series resistance L_{eq} and R_{eq} that appear to be present when the coil is

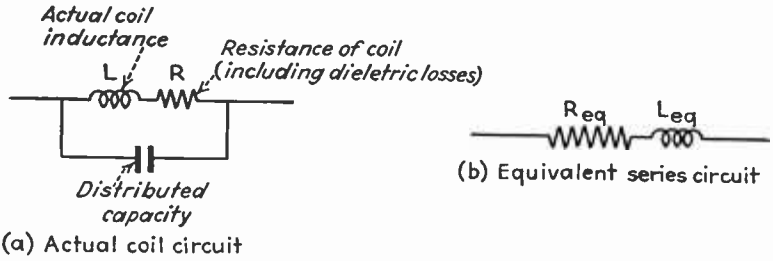


FIG. 54.—Actual circuit of coil with distributed capacity, together with equivalent series circuit.

viewed from its terminals (see Fig. 54*b*) are related to the true resistance and inductance according to the equations

$$\left.\begin{matrix}\text{Apparent inductance } L_{eq} \text{ of} \\ \text{coil with distributed capacity}\end{matrix}\right\} = \frac{L}{1-\gamma^2} \tag{102}$$

$$\left.\begin{matrix}\text{Apparent resistance } R_{eq} \text{ of} \\ \text{coil with distributed capacity}\end{matrix}\right\} = \frac{R}{(1-\gamma^2)^2} \tag{103}$$

where L and R are the true coil inductance and resistance, respectively, and γ is the ratio of actual frequency to the frequency at which the coil inductance would be resonant with the distributed capacity.

21. Temperature Coefficient of Inductance.[2]—The inductance of a coil varies with temperature as a result of (1) changes in dimensions with temperature and (2) changes in current distribution in the wire as a result of the change of wire resistivity with temperature modifying the skin and proximity effects.

The relation between the dimensions of a coil and the temperature depend upon several factors. In the ideal case of a freely suspended coil expanding according to the coefficient of linear expansion of the metal, the temperature coefficient of inductance will be the same as the temperature coefficient of expansion of the metal. Actually, however, the mechanical strains existing in the conductor ordinarily cause added changes in dimensions, with the result that the temperature coefficient of such a coil is

[1] W. Jackson, Dielectric Losses in Single-layer Coils at Radio Frequencies, *Exp. Wireless and Wireless Eng.*, Vol. 5, p. 255, May, 1928.

[2] H. A. Thomas, The Stability of Inductance Coils for Radio Frequencies, *Jour. I.E.E.*, Vol. 77, p. 702, 1935; also Wireless Section, *Jour. I.E.E.*, Vol. 11, p. 44, March, 1936; D. A. Bell, Temperature Coefficient of Inductance, *Wireless Eng.*, Vol. 16, p. 240, May, 1939; H. A. Thomas, The Dependence on Frequency of the Temperature-coefficient of Inductance of Coils, *Jour. I.E.E.*, Vol. 84, p. 101, 1939; also, Wireless Section, *Jour. I.E.E.*, Vol. 14, p. 19, March, 1939; E. B. Moullin, The Temperature Coefficient of Inductances for Use in a Valve Generator, *Proc. I.R.E.*, Vol. 26, p. 1385, November, 1938; Janusz Groszkowski, The Temperature Coefficient of Inductance, *Proc. I.R.E.*, Vol. 25, p. 448, April, 1937; Janusz Groszkowski, Temperature Coefficient of Inductance, *Wireless Eng.*, Vol. 12, p. 650, December, 1935.

Ex. 1044-0093

usually a number of times the coefficient of linear expansion. When the coil is wound on a form instead of being freely suspended, the changes of dimension tend to be controlled by the form, but even here it is found that the temperature coefficient of inductance will normally be considerably greater than the expansion coefficient of either the form or the wire.

The change in wire resistivity with temperature modifies the skin and proximity effect and so causes the inductance to vary. Inasmuch as skin and proximity effects depend upon the frequency, this causes the temperature coefficient of inductance to depend likewise upon the frequency. The contribution to the temperature coefficient resulting from resistance changes is low at low frequencies when the skin and proximity effect are small, and is also low at very high frequencies, where these effects are very large. The change in inductance is maximum at a frequency between the extremes, where the skin and proximity effects are only moderate and the current distribution is particularly sensitive to resistance changes.

It is frequently found that the variation of coil inductance with temperature is noncyclical; *i.e.*, if the coil undergoes temperature cycles the inductance will not follow the temperature exactly, but, rather, will vary irregularly and will not return to its initial value when the temperature returns to normal. This is apparently the result of mechanical changes such as result from slippage of the wire over the form, and permanent changes in physical dimensions occasioned by the relieving of initial stress.

Practical Inductance Coils Having Low Temperature Coefficients.—Several methods have been suggested for designing coils to have a low and stable temperature coefficient. Probably the best arrangement is a single-layer solenoid with flat conductor wound directly on an insulating form having a low coefficient of expansion. The conductor can be a flat strip, but is preferably a conducting film deposited directly on the surface. If the radial depth of the flat conductor is very small and if the turns are well spaced axially, then the temperature coefficient of inductance will be of the same order of magnitude as that of the form, will be independent of frequency, and will be stable over repeated temperature cycles. Temperature coefficients of inductance of 5 to 15 parts per million per°C can be obtained in this way.

Another possible method of construction is to anneal the turns to relieve initial stress, and then mount them so that the coefficient of expansion in an axial direction is controlled by material having a relatively large temperature coefficient, while the coefficient of expansion in a radial direction is fixed by material having a low expansion coefficient. By suitable choice of the relative coefficients, it is then possible to obtain either zero or any desired temperature coefficient. Such coils must, however, be very carefully constructed or their behavior will not repeat on successive temperature cycles.[1]

Small inductances having low temperature coefficient can be constructed by employing a single-layer coil in which the conductor is a copper- or silver-plated invar (or nilvar) wire wound on a ceramic form having a low temperature coefficient of expansion. It is possible in this way to obtain an inductance coefficient of about one part in a million per°C.[2]

22. Practical Air-cored Coils. *Coils for Resonant Circuits in Radio Receivers.*—Coils for resonant circuits of radio receivers must be compact, and at the same time have a moderate to high Q. At frequencies in excess of 1,500 kc, the coils are practically always single-layer solenoids using spaced turns of solid wire wound on insu-

[1] For further information on such coils, see W. H. F. Griffiths, Recent Improvements in Air-cored Inductances, *Wireless Eng.*, Vol. 19, p. 8, January, 1942.

[2] S. W. Seeley and E. I. Anderson, UHF Oscillator Frequency-stability Considerations, *RCA Rev.*, Vol. 5, p. 77, July, 1940.

Ex. 1044-0094

lating tubes. Coils for broadcast frequencies (550 to 1,500 kc) are usually also single-layer solenoids, although bank and universal windings are sometimes employed. At frequencies below 550 kc, the coils for resonant circuits are nearly always multilayer, usually of the universal or similar type.[1] Litz wire is generally preferred for these frequencies, although solid wire is often used because of lower cost.

Disk, toroidal, and other types of coils are practically never used for resonant circuits in receivers. At broadcast and lower frequencies, coils with finely subdivided iron cores are sometimes used (see Par. 27).

Coils for Resonant Circuits of Radio Transmitters.—Coils for transmitters are subjected to high voltage stresses and must handle large amounts of reactive circulating energy. This requires relatively large spacings in order to obtain the required voltage insulation and heat-dissipating ability. The size of coils for transmitters tends to be directly proportional to the power and inversely proportional to frequency.

In short-wave transmitters, it is customary to use coils wound with solid wire on ceramic forms when the power is low, and to use coils of copper tubing supported from the terminals for high power. In high-power broadcast transmitters, coils may be

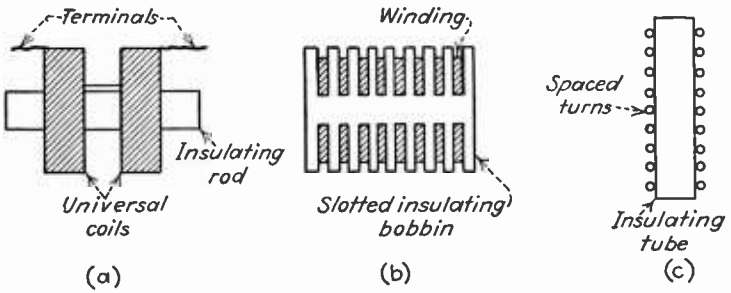


FIG. 55.—Typical examples of radio-frequency choke coils.

either large single-layer solenoids wound with copper tubing or edgewise wound strip. Single-layer disk or pancake coils using ribbon or solid conductors are also sometimes employed. When large powers are to be handled at still lower frequencies, it is customary to use a very large solenoid, either single or multilayer, with conductor consisting of large copper tubing, or litz, of large cross section.

Radio-frequency Choke Coils.—A radio-frequency choke coil is an inductance designed to offer a high impedance to alternating currents over the frequency range for which coil is to be used. This result is obtained by making the inductance high, and the distributed capacity low. Radio-frequency choke coils are commonly used at frequencies in the neighborhood of the natural resonant frequency of the coil. Thus the coils will normally offer a capacitive impedance to an applied voltage over at least a part of their working range.

The usual radio-frequency choke consists of two or more universal wound coils mounted some distance apart on an insulating rod, as shown in Fig. 55*a*. Another method of construction is to use a series of "pies" wound in deep narrow slots (Fig. 55*b*). A long single-layer solenoid, as in Fig. 56*c*, is sometimes used at very high frequencies.

The important characteristic of a radio-frequency choke coil is the variation of impedance with frequency. This depends upon the inductance and capacity of the

[1] The design of such multilayer windings is discussed by A. A. Joyner and V. D. Landon, Theory and Design of Progressive Universal Coils, *Communications*, Vol. 18, p. 5, September, 1938; A. W. Simon, Winding the Universal Coil, *Electronics*, Vol. 9, p. 22, October, 1936; L. M. Hershey, The Design of the Universal Winding, *Proc. I.R.E.*, Vol. 29, p. 442, August, 1941.

Ex. 1044-0095

sections, the mutual inductance and mutual capacities between sections, the capacity to ground, and the lead arrangements. The behavior in a simple idealized case is illustrated in Fig. 56, in which the choke is considered as consisting of two inductances each with distributed capacities, but with mutual inductance and mutual capacities

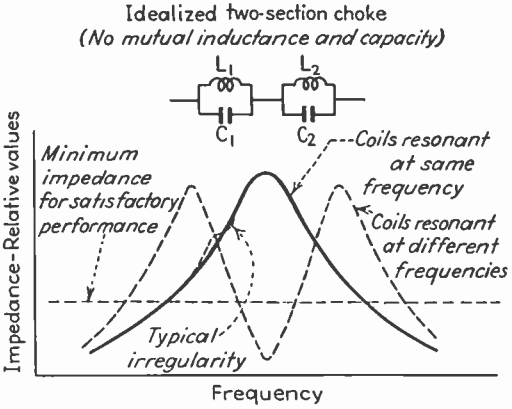


FIG. 56.—Impedance characteristics of idealized radio-frequency choke coils.

ignored. If the resonant frequencies of the two sections are the same, *i.e.*, if $L_1C_1 = L_2C_2$ in Fig. 56, then the choke acts as a single resonant circuit and has an impedance characteristic as shown by the solid line of Fig. 56. On the other hand, if the two sections have different resonant frequencies, the impedance will vary as a function of frequency in the manner shown by the dotted line. At a frequency some-

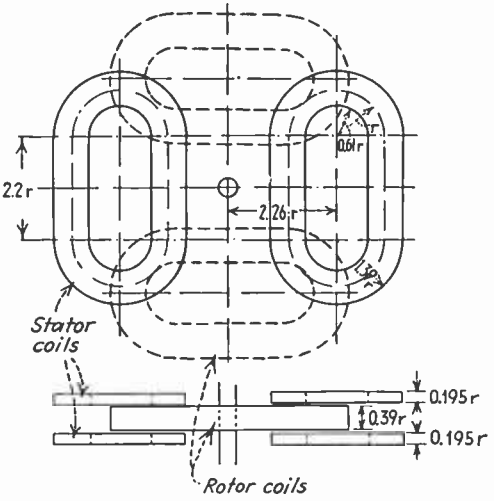


FIG. 57.—Brooks inductometer.

where between the two resonant frequencies, the choke has negligible impedance as a result of one section offering an inductive reactance of the correct magnitude to be in series resonance with the capacitive reactance of the second section. The actual situation existing in a practical radio-frequency choke coil is more complicated than

Ex. 1044-0096

that of Fig. 56, because of mutual inductance and capacity effects and capacity to ground. The behavior is, however, of the same character, though more complex. By careful balancing of inductances, capacities, and couplings, it is possible to obtain performance approximating that corresponding to a single resonant frequency.[1]

Practical radio-frequency choke coils are designed either to have a single resonant frequency or to be proportioned so that the "holes" in the impedance characteristic occur at frequencies other than those for which the coil is to be used. Where the choke coil is to be shunted across the resonant circuit of a receiver that is tuned over a frequency band, it is necessary that the choke be very carefully proportioned to avoid even minor irregularities in the impedance curve, as shown by the dashed line in Fig. 56.

Variable Inductances.—The most common type of variable inductance is the ordinary variometer or inductometer. This consists of two systems of coils that are connected in series and rotated with respect to each other in order to vary the mutual inductance. The total inductance is $L_1 + L_2 \pm 2M$, and if the coefficient of coupling is large, the ratio of maximum to minimum inductance obtainable will be of the order of 10:1.

Two methods of constructing variable inductances are shown in Figs. 57 and 58. In the variometer, the rotating coil is wound on a spherical section, and mounted inside a corresponding fixed coil. In the Brooks inductometer, the same result is achieved in a slightly different way. The Brooks inductometer has the advantage that the calibration is not appreciably affected by axial displacements of the rotating coil and, further, that by proportioning the coils as shown in Fig. 57, the inductance varies linearly with the angle of rotation.[2]

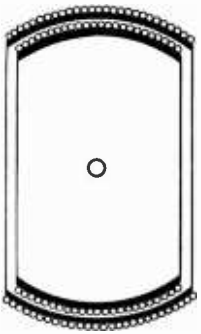


FIG. 58.—Variometer type of variable inductance.

The inductance of single-layer coils wound with relatively large wire can be varied by providing a wheel or trolley assembly that rides on the wire and provides a sliding contact that is adjusted by rotating the coil to vary the position of contact.[3] With proper mechanical design, such an arrangement provides a satisfactory and very fine control of the inductance.

Small variations in the inductance of coils used at radio frequencies can be obtained by rotating a copper disk or ring in the magnetic field of the coil. Such a copper object reduces the inductance in proportion to the magnetic flux lines that it intercepts, and so gives maximum inductance when the plane of the ring or disk is at right angles to the magnetic flux. The extra losses resulting from this method of controlling inductance are small, provided that the range of control is not excessive.

Fixed Inductances for Audio Frequencies.—Fixed inductances for such purposes as standards at audio frequencies are made of several types, as illustrated in Fig. 59. The conventional multilayer coil shown at Fig. 59*a* gives the largest inductance in proportion to the direct-current resistance, but it has the disadvantage of a large external field.[4] Inductances of this type will have minimum loss when proportioned so that $b = c$, and $c/a = 0.66$. The toroid has no external field but requires a very

[1] The design of radio-frequency choke coils for optimum performance is discussed by Harold A. Wheeler, The Design of Radio-frequency Choke Coils, *Proc. I.R.E.*, Vol. 24, p. 850, June, 1936; Herman P. Miller, Jr., Multi-band R-f Choke Coil Design, *Electronics*, Vol. 8, p. 254, August, 1935.

[2] For detailed design information on the Brooks inductometer see H. B. Brooks and F. C. Weaver, A Variable Self and Mutual Inductor, *Bur. Standards, Sci. Paper* 290.

[3] Paul Ware, A New System of Inductive Tuning, *Proc. I.R.E.*, Vol. 26, p. 308, March, 1939; L. E. Herborn, A Precise High-frequency Inductometer, *Bell Lab. Rec.*, Vol. 17, p. 351, July, 1939.

[4] The design of standard inductances of this type is thoroughly discussed by H. B. Brooks, Design of Standards of Inductance, and the Proposed Use of Model Reactors in the Design of Air-core and Iron-core Reactors, *Bur. Standards Jour. Research*, Vol. 7, p. 289, August, 1931.

Ex. 1044-0097

large amount of wire in proportion to the inductance obtained, and so has high direct-current resistance. The double-*D* winding is intermediate in its characteristics between the toroid and the multilayer coil with respect to both external field and direct-current resistance. This type of coil has the advantage that the individual coils can be wound with a suitable number of turns, impregnated, taped, and mounted, all before final adjustment. The adjustment to give the exact value of inductance desired is then made by shifting the relative position of the two coils slightly with respect to each other before clamping them into final position.

Fixed inductances for use at radio frequencies should have low losses, good mechanical stability, and a minimum of dielectric loss. Litz wire is desirable for frequencies up to and including broadcast frequencies. The construction must be substantial, the form should be of good dielectric material, and the coil should be protected from moisture by suitable "dope." The distributed capacity should be low in order that the frequency at which the inductance is resonant with the distributed capacity will be as high as possible. In this way, the working range of frequencies over which the

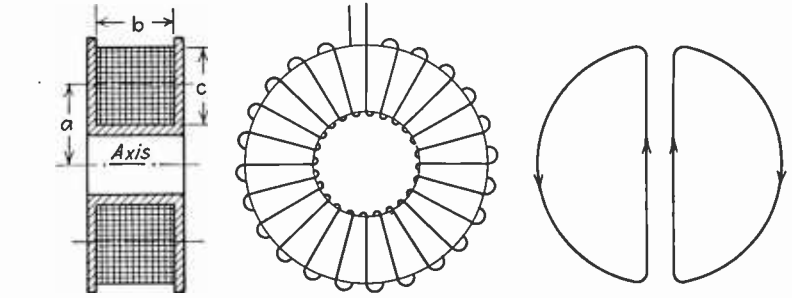


FIG. 59.—Types of fixed inductances used in audio-frequency work.

apparent and the true inductance are approximately the same will be as great as possible.

The temperature coefficient of inductance standards should be low. However, it is impractical to achieve a particularly low coefficient in multilayer and toroidal coils, as there is no material for the coil form that has the requisite low temperature coefficient combined with other desirable characteristics, such as stability, machinability, etc. Wood impregnated with beeswax and certain grades of porcelain are to be preferred, while bakelite and hard rubber are not recommended because of their high coefficients of expansion.

COILS WITH MAGNETIC CORES

23. Fundamental Properties of Magnetic Substances.—The relationship between flux density and magnetizing force in the core of a coil when the core is of magnetic material that has been previously demagnetized is shown in Fig. 60*a*, and is termed the saturation or magnetization curve. When such a core is carried through a complete cycle of magnetization, as when the magnetizing force is supplied by an alternating current, the relationship between flux density and magnetizing force follows a loop, as illustrated in Fig. 60*b*. This is termed the hysteresis loop. The residual flux B_r remaining after a positive magnetizing force has been removed is termed the *retentivity* of the material. The negative magnetizing force H_c required to reduce the flux to zero is termed the *coercive force.*

Ex. 1044-0098

Permeability and Incremental Permeability.—The permeability of a magnetic material is defined as the ratio B/H, where B is the flux density in lines per square centimeter (gauss) and H the magnetizing force in oersteds (1 ampere turn per centimeter = 1.256 oersteds). With these units the permeability of air is unity. The

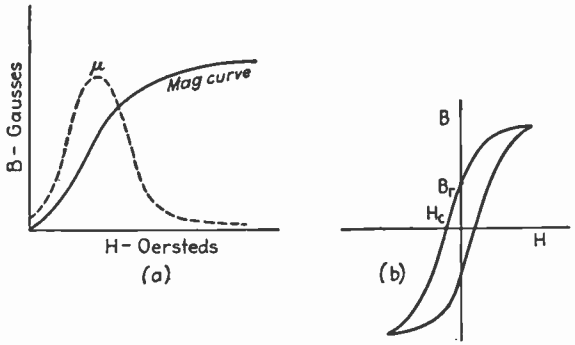


FIG. 60.—Typical magnetization curve, permeability curve, and hysteresis loop of a magnetic material.

permeability μ depends upon the flux density, and is in general small at low flux densities, maximum at moderate values of B, and quite small when the core is saturated, as shown in Fig. 60*a*.

The permeability at very low magnetizing forces is termed the *initial permeability*, and is of considerable importance in communication equipment. With ordinary magnetic materials, the initial permeability is very small.

The permeability offered to an alternating magnetizing force superimposed upon a d-c magnetizing force is termed *incremental permeability*, apparent permeability, or a-c permeability. Under such conditions, the cycle of magnetization follows a small displaced hysteresis such as is illustrated in Fig. 61; and the permeability to the alternating magnetization is determined by the slope of the line joining the tips of this displaced hysteresis loop.[1] The incremental permeability increases with increasing alternating flux densities (up to the point at which the core is saturated) and decreases with increasing d-c magnetization. The behavior in a typical case is illustrated in Fig. 62.

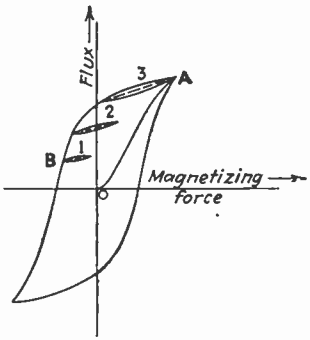


FIG. 61.—Typical hysteresis loop showing displaced minor hysteresis loops 1, 2, and 3, obtained when a small alternating current is superimposed on a direct-current magnetization. The effective permeability to the superimposed alternating current is proportional to the slope of the line joining the tips of the displaced hysteresis loop.

When both alternating and direct magnetizing forces are acting simultaneously, the permeability offered to the d-c magnetization in the low or moderate flux-density range is increased by a small amount of superimposed a-c excitation, and decreased by a large amount.[2] With large d-c magnetizations, the permeability to d-c is reduced by a superimposed alternating magnetization.

[1] Thomas Spooner. Permeability, *Trans. A.I.E.E.*, Vol. 43, p. 340, 1923.
[2] R. F. Edgar, Silicon Steel with A-C and D-C Excitation, *Trans. A.I.E.E.*, Vol. 53, p. 318, February, 1934.

Ex. 1044-0099

Core Loss.—When a core is subjected to an alternating magnetization, there is a dissipation of energy in the core as a result of hysteresis and eddy currents. The total power loss in core materials used in transformers is often expressed in curves,

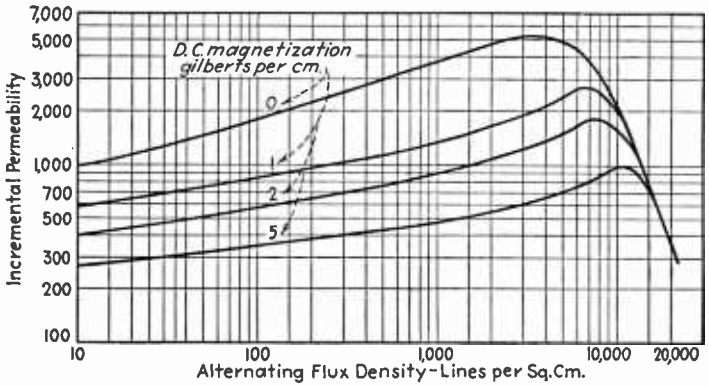


FIG. 62.—Incremental permeability of a typical sample of silicon steel as a function of alternating-current magnetization, for several values of superimposed direct current.

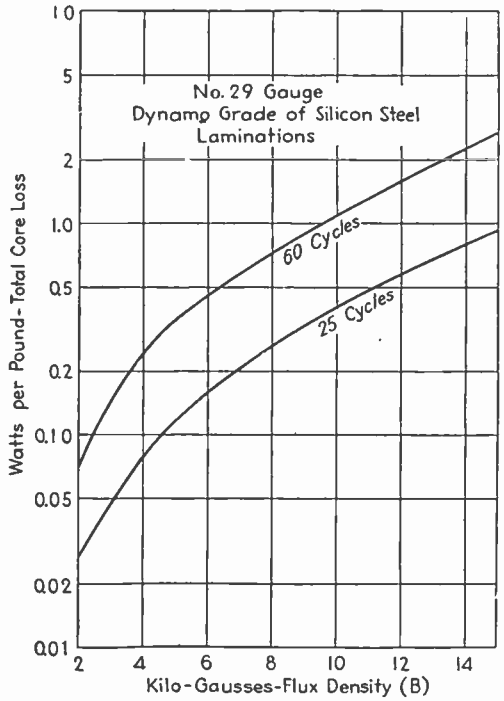


FIG. 63.—Core-loss curves of typical silicon steel sample.

such as those of Fig. 63, that give the watt loss per pound of material as a function of flux density for a given frequency. Such curves are of convenience in the design of power and audio transformers.

Ex. 1044-0100

A coil having hysteresis and eddy-current losses can be represented by postulating series resistances R_h and R_e, respectively, in series with an inductance having no losses, together with a resistance R_c representing copper losses, as shown in Fig. 64*b*. In many cases, it is found convenient to represent the eddy-current losses as a resistance R_{sh} shunted across the inductance as in Fig. 64*c*, instead of as a series resistance. When the eddy-current losses are small, this shunting resistance is independent of frequency, whereas the series resistance R_e is proportional to the square of the frequency.

The eddy-current loss in a core is the result of currents induced in the core by the varying magnetic flux. The magnitude of the power loss per unit volume is proportional to the square of the frequency and to the square of the flux density, inversely proportional to the resistivity of the core material, and directly proportional to the square of the thickness (or diameter) of the individual core laminations (or particles). The eddy-current loss is not affected by d-c saturation of the core.

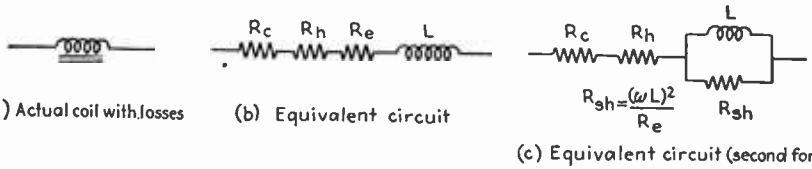


FIG. 64.—Equivalent circuits showing methods of representing hysteresis and eddy current losses in a coil having a magnetic core.

The equivalent series resistance R_e in Fig. 64*b* that can be considered as representing the eddy-current loss in a laminated core is[1]

$$R_e = 0.00658 \frac{t^2}{\rho_1} \mu_m f X_L \tag{104}$$

where t = thickness of the laminations, cm.
ρ_1 = resistivity of core material, microhm-cm.
μ_m = permeability of core corresponding to the maximum flux density produced by the magnetization ($\mu_m = 1$ for air).
X_L = inductive reactance of the coil with its magnetic core.

The value of the equivalent series resistance R_e for the case of a powdered core, on the assumption that the individual magnetic particles are of spherical shape is

$$R_e = \frac{0.00197 t^2}{\rho_1 k^{1/3}} \mu_m f X_L \tag{105}$$

where μ_m, f, ρ_1, and X_L have the same meaning as before, while t^2 is now defined as the mean square diameter of the spherical magnetic particles in cm and k is the fraction of the core volume that is occupied by the magnetic material. Equations (104) and (105) assume that the core is sufficiently subdivided so that magnetic skin effect is negligible.

Hysteresis is a form of magnetic friction, and for a given maximum flux density gives rise to a power loss per unit volume that is proportional to the frequency and to the area under the hysteresis loop. At flux densities sufficiently low that the permeability at the maximum flux density is not over 10 to 20 per cent greater than the initial permeability of the material, the hysteresis power loss per unit volume of most core materials is proportional to the fractional increase of the permeability over the initial value, and is approximately[2]

[1] V. E. Legg, Magnetic Measurements at Low Flux Densities using the Alternating Current Bridge, *Bell System Tech. Jour.*, Vol. 15, p. 39, January, 1936.

[2] This is a consequence of the Rayleigh hysteresis loop. See Legg, *loc. cit.*

Ex. 1044-0101

$$R_h = 0.425 \left(1 - \frac{\mu_0}{\mu_m}\right) X_L \tag{106}$$

where μ_0 and μ_m are the initial permeability and the permeability corresponding to the maximum flux density, respectively, and X_L is the reactance of the coil with its iron core. With low magnetizing forces, the value of $\left(1 - \frac{\mu_0}{\mu_m}\right)$ is proportional to the magnetizing force, and hence to the peak flux density B_m. Under these conditions the equivalent resistance R_h representing hysteresis loss is proportional to B_m, and the power loss due to hysteresis[1] is proportional to B_m^3. When the flux density is moderately high, the hysteresis loss increases less rapidly with flux density. Thus, for silicon steel at flux densities such as used in ordinary transformers, the hysteresis power loss per volume is approximately proportional to $B_m^{1.6}$, corresponding to an equivalent resistance R_h that decreases slowly with increase in B_m.

Hysteresis loss is affected by d-c saturation and also by the presence of currents of other frequencies. The relations are relatively complicated but can, in general, be summarized as follows: With alternating flux densities below a critical value (13,000 to 14,000 gausses for silicon steel), the hysteresis loss rises with increasing d-c magnetization, when constant alternating flux density is assumed. With alternating flux densities greater than this critical value the hysteresis loss with constant alternating flux density decreases with superimposed d-c excitation.[2] When high- and low-frequency magnetizations are superimposed, the hysteresis loss to the high frequency is greater than it would be in the absence of the low-frequency magnetization, and the hysteresis loss of the latter is reduced. This action, termed *flutter effect*,[3] is of practical importance in loaded telephone lines carrying superimposed telegraph currents. The magnitude of the flutter effect is proportional to the area of the normal hysteresis loop and to the frequency of the high-frequency current.

Harmonics and Cross-modulation Caused by Magnetic Cores.[4]—The nonlinear relation existing between flux and magnetizing force in magnetic cores results in the production of harmonics when a sine-wave magnetizing force is applied to the coil, and also to cross modulation when two or more frequencies are applied.

With sine-wave excitation, the distortion components are in the form of odd harmonics, of which the third is the most important. With high maximum flux densities, the principal cause of such harmonic generation is the variation of permeability with flux density. With very low flux densities, hysteresis is the principal factor accounting for harmonic generation.

When the magnetizing force consists of a sine wave upon which a d-c current is superimposed, there results a second-harmonic distortion component having an amplitude depending upon the core material and upon the direct and alternating magnetizing forces involved.

When two alternating magnetizations are superimposed upon a core, the result is to produce combination frequencies (cross-modulation), the magnitude of which is dependent upon the amplitude of the magnetizing forces involved and upon the magnetic material. It has been determined experimentally that the third harmonic produced with a sinusoidal magnetizing force is an index of the tendency toward cross-modulation, which, accordingly, depends primarily upon hysteresis effects at low flux densities and upon the variation of permeability at high flux densities.

[1] There is also a component of hysteresis loss at low flux densities that is proportional to B_m^2. This is termed a "residual loss" and is of importance only at very low frequencies. For further discussion, see Legg, *loc. cit.*

[2] See Edgar, *loc. cit.*

[3] W. Fondiller and W. H. Martin, Hysteresis Effects with Varying Superimposed Magnetizing Forces, *Trans. A.I.E.E.*, Vol. 40, p. 553, 1921.

[4] Eugene Peterson, Harmonic Production in Ferromagnetic Materials at Low Frequencies and Low Flux Densities, *Bell System Tech. Jour.*, Vol. 7, p. 762, October, 1928.

Ex. 1044-0102

Permalloy is many times better than silicon steel in freedom from harmonic generation and cross-modulation, while perminvar is many times better than permalloy.

24. Magnetic Materials Used in Communication.[1] *Silicon Steel.*[2]—The most widely used magnetic material in communication work is silicon steel. Silicon steel is available in various grades and types, according to the particular properties to be

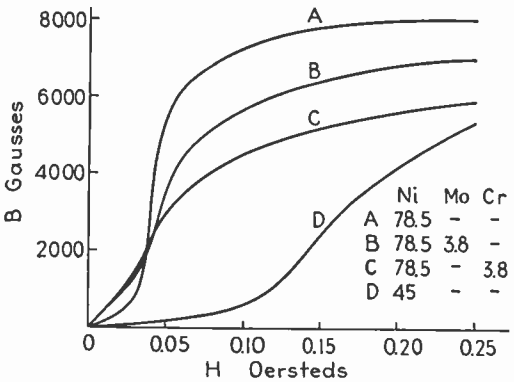


FIG. 65.—Typical magnetization curves of various nickel-iron alloys.

emphasized. It is used for power transformers, filter chokes, and as a core material for most audio-frequency, interstage, and output transformers.

Magnetic Alloys of Iron, Nickel, and Cobalt (Permalloys, Perminvar, and Permendur).[3]—Alloys of iron, nickel, and cobalt have some remarkable properties that are very useful in specialized applications. The characteristics of these alloys depend upon their composition and heat treatment, and a wide variety of properties are obtainable.

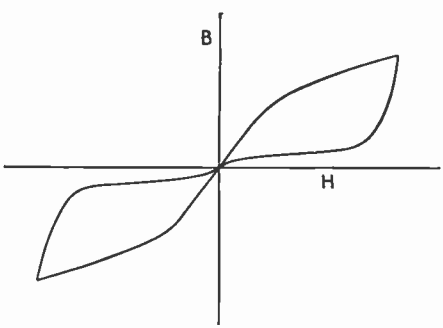


FIG. 66.—Hysteresis loop of perminvar at moderate flux density, showing the negligible coercive force and residual magnetization.

An important class of alloys of this type are those consisting of nickel and iron in the range 45 to 80 per cent nickel, with small amounts of chromium and molybdenum added in some cases. Such alloys go by such names as *Permalloy, Hipernik, Electric metal, Mumetal,* etc., according to the exact composition and the manufacturer. When properly heat-treated, they are all characterized by high maximum permeability and high initial permeability. The flux density at which saturation occurs is, however, the same as or even slightly less than for silicon steel. Typical magnetization curves are shown in Fig. 65. The addition of molybdenum to permalloy increases the resistivity, and hence reduces the eddy-current losses, without serious sacrifice in the low hysteresis loss which permalloy has. Molybdenum per-

[1] V. E. Legg, Survey of Magnetic Materials and Applications in the Telephone Systems, *Bell System Tech. Jour.*, Vol. 18, p. 438, July, 1939.

[2] C. H. Crawford and E. J. Thomas, Silicon Steel in Communication Equipment, *Elec. Eng.*, Vol. 54, p. 1348, December, 1935.

[3] G. W. Elmen, Magnetic Alloys of Iron, Nickel, and Cobalt, *Elec. Eng.*, Vol. 54, p. 1292 December, 1935.

Ex. 1044-0103

malloy accordingly gives low losses at relatively high frequencies provided the flux densities are not too great.

Alloys of nickel, iron, and cobalt are termed perminvars, and when suitably heat-treated are characterized by substantially constant permeability and extremely low hysteresis loss at low flux densities. At medium flux densities, the hysteresis loop is severely constricted in the middle, as shown in Fig. 66, so that although the area of the hysteresis loop is not zero, the coercive force and retentivity are very small. At high flux densities, this constriction disappears, and the hysteresis loop has normal shape. Typical characteristics of perminvar are shown in Fig. 67.

Iron and cobalt in equal parts yield a useful magnetic alloy termed *permendur*. Its outstanding property is high permeability at high flux densities, corresponding to a saturation flux density higher than that of most materials. Permendur also has higher incremental permeability than most magnetic materials at relatively high d-c magnetizing forces. A magnetization curve of permendur is shown in Fig. 67.

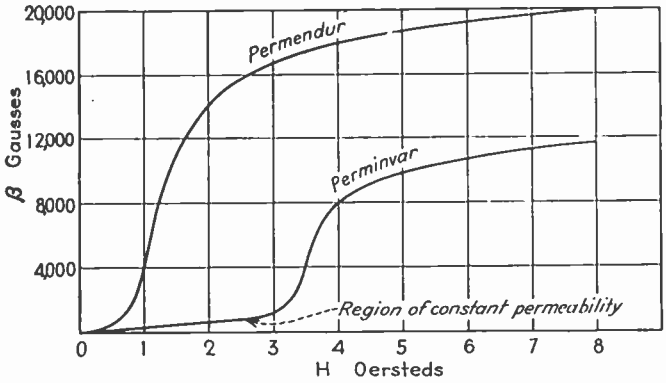


FIG. 67.—Typical magnetization curves of permendur and perminvar, showing the high flux density at saturation of the former and the constant permeability of low flux densities of the latter.

Permanent Magnets.[1]—The merit of a magnetic substance for use as a permanent magnet can be expressed in terms of the coercive force H_c the retentivity B_r, and the maximum value of the energy product BH of the demagnetization curve. The value of the energy product BH gives the relative energy available to sustain flux in the external magnetic circuit per unit volume of magnetic material. For most efficient utilization of the magnetic material, it is accordingly necessary to make the cross section of the magnet such that the desired flux will be obtained in the external circuit with a flux density in the permanent magnet corresponding to the flux giving maximum energy product. The demagnetizing force corresponding to the flux density existing in the magnet represents the magnetizing force per unit length of magnet available to force this flux through the reluctance of the magnetic circuit external to the permanent magnet. As a result of these factors, the total cross section of the permanent magnet is determined by the total flux required in the external circuit, and the length of the magnet is determined by the reluctance of the circuit.

The most desirable materials for permanent magnets are listed in Table 25, and have characteristics such as shown in Fig. 68 in the demagnetization portion of their

[1] C. S. Williams, Permanent Magnet Materials, *Elec. Eng.*, Vol. 55, p. 19, January, 1936; K. L. Scott, Magnet Steels and Permanent Magnets—Relationship among Their Magnetic Properties, *Bell System Tech. Jour.*, Vol. 11, p. 383, July 1932.

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hysteresis curves.[1] Alnico and the related Alnic and Alnico-cu, are the best compromises between high coercive force and high retentivity. These alloys are inexpensive, but have the disadvantage that they must be cast to shape and finished by grinding, since they are so brittle that drilling and machining are impossible. The oxide material has the highest coercive force. It is also the least expensive and has an

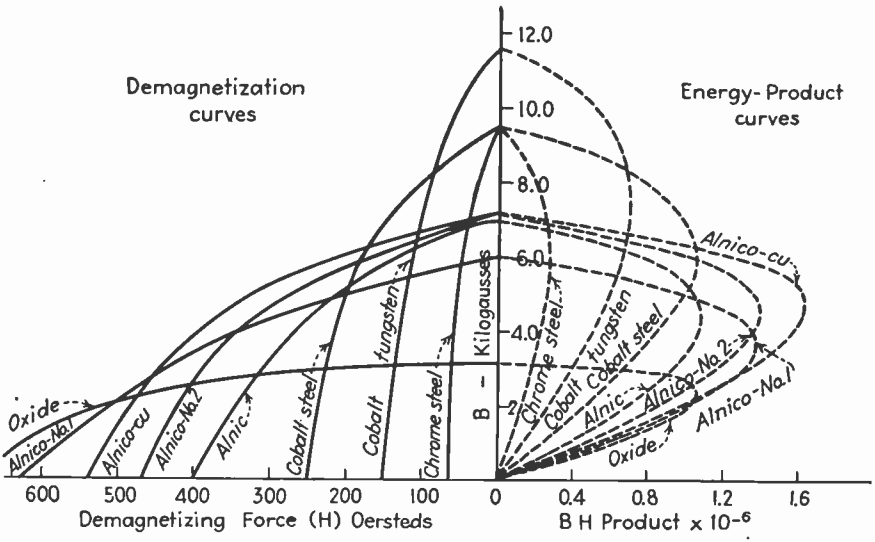


FIG. 68.—Demagnetization curves for permanent magnet materials.

unusually low specific gravity, but is even weaker mechanically than Alnico, since it consists of a sintered mass of oxides. Cobalt steel has a very high retentivity but low coercive force, is machinable, but is relatively expensive.

TABLE 25

Name	Component elements*	Coercive force, oersteds	Retentivity, gausses
Alnic	Al-Ni-Fe	100–600	8 000–4,000
Alnico	Al-Ni-Co-Fe	450–650	7,000–6,000
Alnico-cu	Al-Ni-Co-Cu-Fe	550	7,000
Cobalt steel	Co-Fe	150–250	10,000
Cobalt-tungsten	Co-W-Fe	30–150	13,500–11,000
Chrome	Cr-Fe	60	10,000
Oxide	CoO, Fe_2O_3, Fe_3O_4	300–1,100	6,000–1,500

* Ni = nickel; Al = aluminum; Fe = iron; Co = cobalt; W = tungsten

25. Calculation of Properties of Iron-cored Coils. *Inductance.*—The inductance of an iron-cored coil subjected to an alternating magnetization corresponding to a maximum a-c flux density B_{max} in the iron is as follows:

[1] Another permanent magnet material called Vicalloy has recently been announced, and promises to be of commercial importance. It is composed of vanadium, iron, and cobalt, and is described in *Bell Lab. Rec.*, Vol. 19, p. 36, September, 1940.

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Dimensions in centimeters:

$$\text{Inductance in henrys} = L = \frac{1.256N^2}{\left(\frac{l_i}{\mu a_i} + \frac{l_a}{a_a}\right)} \times 10^{-8} \qquad (107a)$$

Dimensions in inches:

$$\text{Inductance in henrys} = L = \frac{3.19N^2 \times 10^{-8}}{\left(\frac{l_i}{\mu a_i} + \frac{l_a}{a_a}\right)} \qquad (107b)$$

where N = number of turns.
l_i = length of magnetic circuit in iron.
l_a = length of magnetic circuit in air.
μ = permeability of magnetic material evaluated at the appropriate maximum alternating flux density B_{max}.
a_i = cross section of iron (not including the area of insulation between laminations).
a_a = effective cross section of air gap.

The effective cross section a_a of the air gap will in general be somewhat greater than the cross section a_i of the iron because of fringing flux, and because the insulation between laminations makes the effective cross section of iron less than the cross section calculated from the external dimensions of the core.

The relationship between the maximum alternating flux density in the core and the rms value of a sinusoidal voltage is

$$\text{Voltage (rms)} = 4.44fNAB_{max} \times 10^{-8} \qquad (108)$$

where f = frequency in cycles per sec.
N = number of turns.
A = net area of core.
B_{max} = maximum flux density in core in lines per unit area.

Incremental Inductance.—The incremental inductance, sometimes called also the apparent, or a-c, inductance, is also given by Eq. (107) for any given alternating flux density B_{max} provided the permeability μ appearing in the equation is interpreted to mean the incremental permeability evaluated at the appropriate a-c and d-c flux densities from data such as are presented in Fig. 62.

The value of d-c flux density B_0 existing in an iron core having a winding of N turns carrying a d-c current I_0 can be determined from the fact that it is necessary to satisfy the relation.

$$1.256NI_0 = H_i l_i + B_0 \left(\frac{a_i}{a_a}\right) l_a \qquad (109)$$

The notation is the same as above, with the addition that H_i represents the magnetizing force in oersteds per centimeter required to support a d-c flux density of B_0 lines per square centimeter in the iron. The value of d-c flux density B_0 that satisfies Eq. (109) can be obtained by the cut-and-try process of calculating the value of the right-hand side of Eq. (109) for different assumed flux densities B_0 until a value is obtained that satisfies the equation. An alternative method is to employ the graphical solution shown in Fig. 69. Here B_0 is found as the intersection of the BH magnetization curve of the magnetic material, with a straight line crossing the horizontal axis at

$$H = 1.256NI/l_i$$

and the vertical axis at $B = 1.256NI/[l_a(a_i/a_a)]$.

Leakage Inductance.—The leakage inductance existing between two coils wound on the same core depends upon the geometry of the windings. The leakage induc-

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tance is independent of the character of the core material or of the d-c magnetization, since the paths of the leakage flux are largely in air, and the core contributes negligible reluctance to them.

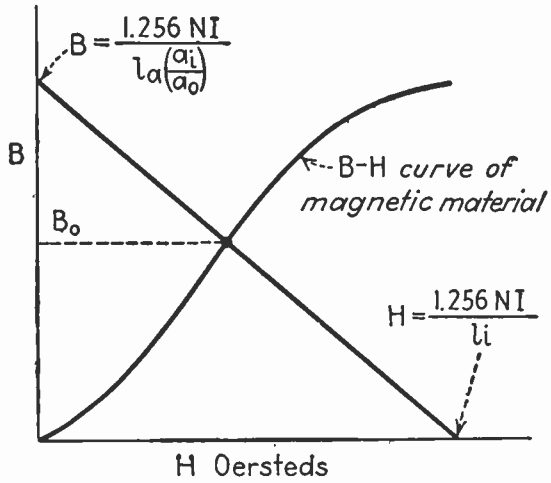


FIG. 69.—Graphical method of determining the flux density B_0 existing in a magnetic core having a series air gap and subjected to a known magnetizing force.

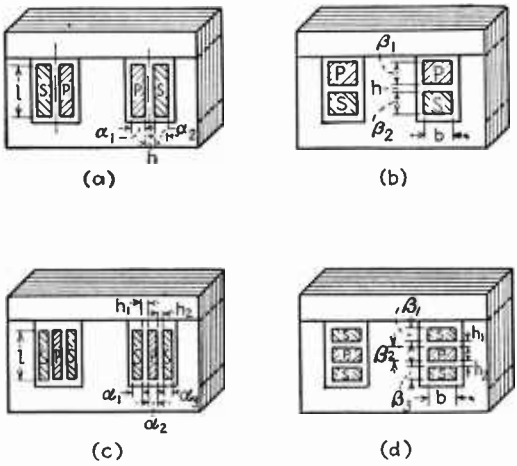


FIG. 70.—Typical coil arrangements in audio transformers.

When the secondary is wound on the outside of the primary, as shown in Fig. 70*a*, the total leakage inductance of the windings, referred to the primary side, is[1]

$$L_p' = \frac{2.9CN_p^2}{l}\left(h + \frac{\alpha_1 + \alpha_2}{3}\right) \times 10^{-8} \text{ henry} \qquad (110)$$

where C is the length of the mean turn in inches, N_p the number of primary turns, and

[1] The total leakage inductance referred to the secondary side is obtained by replacing N_p in Eq. (110) with the number of secondary turns N_s. The total leakage inductance L_p' referred to the primary

the remaining symbols have the meaning indicated in Fig. 70*a* (all measured in inches). This equation applies to coils wound on both circular and rectangular forms.

If the primary and secondary windings are arranged side by side as in Fig. 70*b*, the leakage inductance referred to the primary side is

$$L_p' = \frac{3.2CN_p^2}{b}\left(h + \frac{\beta_1 + \beta_2}{3}\right) \times 10^{-8} \text{ henry} \qquad (111)$$

where the notation is as in Fig. 70*b*, or is as above.

The leakage inductance can be reduced by dividing the primary and secondary coils into sections and interspersing these. Simple arrangements of this type are shown in Figs. 70*c* and 70*d*, and have leakage inductances referred to the primary that are

For Fig. 70*c*:

$$L_p' = \frac{0.72CN_p^2}{l}\left(h_1 + h_2 + \frac{\alpha_1 + \alpha_2 + \alpha_3}{3}\right) \times 10^{-8} \text{ henry} \qquad (112)$$

For Fig. 70*d*:

$$L_p' = \frac{0.8CN_p^2}{b}\left(h_1 + h_2 + \frac{\beta_1 + \beta_2 + \beta_3}{3}\right) \times 10^{-8} \text{ henry} \qquad (113)$$

Sectionalization of windings, although greatly reducing the leakage inductance, simultaneously increases the winding capacities. The reduction achieved in leakage inductance is accordingly not all net gain.

In push-pull output transformers, the primary is frequently sectionalized to maintain symmetry. When this is done, the leakage reactance between each half of the primary and the secondary can be kept at a minimum by dividing the secondary into two identical parts, as in Fig. 71*a*, and connecting these in *parallel*.

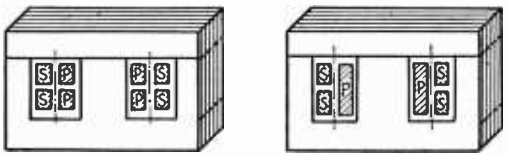


FIG. 71.—Transformers with divided windings.

Distributed and Other Capacities of Iron-cored Coils.—The distributed capacity of multilayer windings such as commonly wound on iron cores is made up primarily of the layer-to-layer capacities.[1] The turn-to-turn capacities can be neglected unless there are only a few layers. When the coil is wound in the normal manner so that the voltage between one end of the adjacent layers is twice the voltage per layer and the voltage between layers at the other end of the same layers is zero, the equivalent distributed capacity C_s that can be considered as shunted across the terminals of the

side is $L_p'' + L_s''/n^2$, where n is the step-up ratio of the transformer and L_p'' and L_s'' are the leakage inductances of the individual primary and secondary windings, respectively.

In the textbooks where the leakage inductance for Fig. 70*a* is calculated for an idealized case, the numerical constant in the equation is found to be 3.2 instead of 2.9. This latter value is obtained by introducing an empirical modifying factor of 0.9 to give more accurate agreement with observed results, as recommended by J. G. Story, Design of Audio-frequency Input and Intervalve Transformers, *Wireless Eng.*, Vol. 15, p. 69, February, 1938.

[1] Glenn Koehler, The Design of Transformers for Audio-frequency Amplifiers with Preassigned Characteristics, *Proc. I.R.E.*, Vol. 16, p. 1742, December, 1928.

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winding is

$$C_s = \frac{0.30PlK}{dT} \mu\mu f \tag{114}$$

where P = mean circumference of coil turns, in.
l = axial length of coil, in.
d = distance between copper of adjacent layers, in.
T = number of layers.
K = average dielectric constant of insulation between layers, in. For paper-insulated wax-impregnated wire, K is approximately 3.

It is to be noted that Eq. (114) gives the distributed capacity of the coil referred to its own terminals. The equivalent value of the same distributed capacity referred to

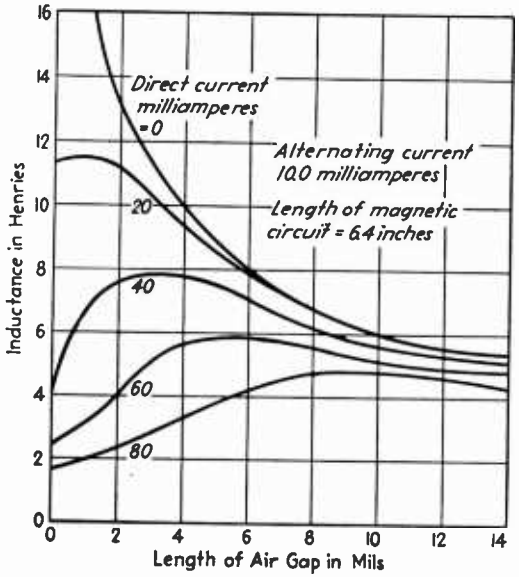


FIG. 72.—Example of incremental inductance as a function of air-gap length with various d-c magnetizing currents.

another winding on the core is obtained by converting in accordance with the inverse of the square of the turn ratio.

The coil capacity may be reduced by splitting the coil into two or more sections as in Fig. 71, each coil having the same number of layers as the single coil, but fewer turns per layer. In such an arrangement the capacity of each section can be determined by Eq. (114), and the capacities of the various sections then considered as acting in series. Thus subdivision into two equal sections quarters the distributed capacity. Subdivision will, however, increase the capacity of the winding to ground, and so may not improve the performance of the coil.

26. Design of Coils and Transformers with Magnetic Cores.—The design of coils with magnetic cores is normally carried out by a cut-and-try process. A typical procedure is to start by assuming a core cross section on the basis of previous experience. The space available for winding is then allotted to the various coils to be wound on the core, and a tentative selection is made of wire size and winding details in accordance with the insulation required, current to be carried, flux densities desired,

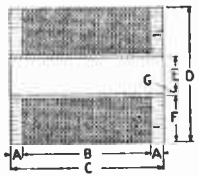
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the space available, etc. The performance of the resulting reactor or transformer is then calculated, and the tentative design modified as required.

Winding Data.—The coils of chokes, transformers, etc., using magnetic cores are normally layer-wound by machine. The insulation employed depends upon the size of wire and the voltage between turns and between layers. In the case of small coils using relatively large wire, cotton-covered insulation is often sufficient. With very small wire, such as is used in audio-frequency transformers, the bulk of cotton or silk insulation is proportionately excessive, and the coils usually use enameled wire with paper insulation between layers in many cases.

TABLE 26*a*.—TURNS POSSIBLE PER SQUARE INCH OF WINDING SPACE, USING ONE LAYER OF PAPER OF PROPER THICKNESS AS SHOWN
Table Based on Average Commercial Practice

B & S gage	Plain enamel	Layer insulation	S.C.E.	D.C.C.	S.C.C.	S.S.E.	S.S.C.	D.S.C.
10	70	.010 in. Kraft	56	56	64			
11	86	.010 in. Kraft	72	64	72			
12	105	.010 in. Kraft	90	90	90			
13	126	.010 in. Kraft	111	111	120			
14	162	.010 in. Kraft	143	125	143			
15	191	.010 in. Kraft	180	143	168	Sq in. of winding space = $B \times F$		
16	238	.010 in. Kraft	208	180	208			
17	310	.007 in. Kraft	255	238	270			
18	396	.007 in. Kraft	304	288	340			
19	473	.007 in. Kraft	378	340	396			
20	621	.005 in. Kraft	460	418	504			
21	750	.005 in. Kraft	572	504	621			
22	952	.005 in. Kraft	696	621	750			
23	1,140	.005 in. Kraft	832	725	980			
24	1,596	.0022 in. Glassine	1,120	928	1,258			
25	1,974	.0022 in. Glassine	1,330	1,054	1,517	1,596	1,800	1,517
26	2,392	.0022 in. Glassine	1,596	1,258	1,800	1,974	2,200	1,800
27	3,009	.0022 in. Glassine	1,886	1,440	2,156	2,392	2,640	2,150
28	3,894	.0015 in. Glassine	2,397	1,720	2,646	2,964	3,355	2,538
29	4,745	.001b in. Glassine	2,750	1,978	3,074	3,591	4,020	3,074
30	5,904	.0015 in. Glasine	3,300	2,254	3,648	4,340	4,884	3,648
31	7,189	.0015 in. Glassine	3,712	2,548	4,216	5,025	5,757	4,216
32	8,888	.0013 in. Glassine	4,414	2,860	4,891	6,068	6,804	4,891
33	11,172	.0013 in. Glassine	5,100	3,132	5,688	7,280	8,245	5,688
34	14,336	.001 in. Glassine	6,150	3,658	6,715	8,910	10,085	6,794
35	17,324	.001 in. Galssine	6,880	3,965	7,728	10,272	12,051	7,728
36	21,330	.001 in. Glassine	7,840	4,352	8,722	12,168	13,860	8,722
37	25,404	.001 in. Glassine				14,112	15,182	10,379
38	31,878	.001 in. Glassine				16,184	18,270	11,845
39	42,020	.0007 in. Glassine				19,140	22,594	14,022
40	51,168	.0007 in. Glassine				21,855	25,992	15,600
41	62,746	.0007 in. Glassine						
42	75,392	.0005 in. Glassine						
43	91,800	.0005 in. Glassine						
44	105,534	.0005 in. Glassine						



A—margins*
B—winding length
C—over-all length
D—outside diameter
E—inside diameter
F—build-up
G—tube

* Margins are determined by wire sizes. The following are the minimum allowances for commercial windings:

Wire gage:
18–29 1/16 in. each end
30–37 3/32 in. each end
38–44 1/16 in. each end

Data useful in the winding of coils are given in Tables 26*a* to 26*d*.[1]

Some Factors Involved in the Design of Inductances Carrying Direct Current.—There is a particular air gap for which the incremental or a-c inductance is maximum when the core material and dimensions, the number of turns, and the d-c magnetizing current are fixed. As the d-c magnetizing current is increased, the optimum air gap becomes longer and the corresponding incremental (a-c) inductance less. The behavior in a typical case is shown in Fig. 72.[2]

This best length of air gap can be obtained by plotting curves of LI^2/V as a function of NI/l for various percentage air-gap lengths, as shown in Fig. 73, where L is the

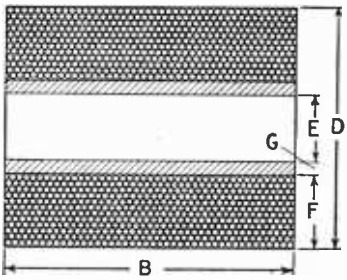
[1] Data from *Inca Bull.* 3, Phelps Dodge Copper Products Corporation, Inca Mfg. Division.
[2] H. M. Turner, Inductance as Effected by the Initial Magnetic State. Air-gap, and Superimposed Currents, *Proc. I.R.E.*, Vol. 17, p. 1822, October, 1929.

Ex. 1044-0110

TABLE 26b.—RANDOM WOUND COILS SUCH AS DYNAMIC SPEAKER FIELDS USING ENAMELED WIRE

Table Based on Average Commercial Practice

B & S gage	Turns possible per linear in. of winding space*	Turns possible per sq in. of cross section	Ohms possible per cu in. of winding space
10	9	85	0.00716
11	10	106	0.0112
12	11	131	0.01733
13	12	160	0.02673
14	14	209	0.0440
15	15	251	0.0666
16	17 (90 %)	319	0.1068
17	19	99	0.168
18	22	519	0.275
19	24	631	0.424
20	27	794	0.672
21	30	987	1.053
22	34	1,255	1.689
23	31	1,525	2.59
24	35	1,933	4.14
25	39	2,428	6.55
26	44	3,080	10.48
27	49	3,891	17.6
28	56	5,001	27.3
29	62	6,200	42.4
30	69	7,797	67.1
31	77 (75 %)	9,800	106.2
32	86	12,210	166.9
33	98	15,740	271.5
34	109	19,520	425
35	121	24,680	677
36	136	31,280	1,082
37	150	38,500	1,681
38	170	49,350	2,720
39	192	63,400	4,370
40	243	67,500	5,910
41	274	85,800	9,370
42	304 (85 %)	104,900	14,410
43	340	130,900	23,300
44	369	153,850	33,300



Linear in. of winding space = B

Sq in. of winding space = $B \times F$

Cu in. of winding space = $[(D^2 \times 0.7854) - (E + 2G)^2 \times 0.7854]B$

No insulation used between layers on random wound coils.

* Percentages give normal space factor.

incremental (a-c) inductance, I the direct current in the winding, V the volume of the core, N the number of turns, and l the length of the magnetic circuit.[1] The envelope of these curves, shown dotted in Fig. 73, gives the maximum incremental inductance per unit volume obtainable for a particular current I, with any given d-c magnetizing force per unit length NI/l, and corresponds to a percentage air gap given

[1] C. R. Hanna, Design of Reactances and Transformers which Carry Direct Current, *Trans. A.I.E.E.*, Vol. 46, p. 155, 1927. An alternative method of design is given by John Minton and Ioury G. Maloff, Design Methods for Soft Magnetic Materials in Radio, *Proc. I.R.E.*, Vol. 17, p. 1021, June, 1929.

Ex. 1044-0111

by the particular curve tangent to the envelope at the point in question. Information such as shown in Fig. 73 is particularly useful where a large number of cores are to be designed employing the same material, since then it is worth while to calculate or measure the special curves required for the particular material involved. Where only a single core is to be designed, it is simpler to determine the optimum air gap by assuming various values and calculating the behavior of each case.

TABLE 26c.—OHMS POSSIBLE PER CUBIC INCH IN LAYER INSULATED COILS*
Table Based on Average Commercial Practice

B & S gage	Plain enamel	S.C.E.	D.C.C.	S.C.C.	S.S.E.	S.S.C.	D.S.C.
10	0.00584	0.00466	0.00466	0.00534			
11	0.00904	0.00756	0.00672	0.00756			
12	0.0139	0.0119	0.0119	0.0119			
13	0.0210	0.01852	0.01852	0.02003			
14	0.0341	0.0301	0.0263	0.0301			
15	0.05075	0.04775	0.03895	0.0446			
16	0.0797	0 0696	0.06025	0.0696			
17	0.1305	0.1074	0.1002	0.1136			
18	0.21	0.1614	0.153	0.1805			
19	0.318	0.254	0.2285	0.266			
20	0.526	0.3895	0.354	0.426			
21	0.800	0.610	0.5375	0.6625			
22	1.275	0.936	0.835	1.09			
23	1.935	1.412	1.23	1.613			
24	3.41	2.395	1.983	2.685			
25	5.325	3.59	2.84	4.09	4.31	4.86	4.09
26	8.140	5.43	4.28	6.125	6.725	7.48	6.125
27	12.79	8.10	6.18	9.25	10.82	11.35	9.25
28	21.05	12.95	9.30	14.33	16.05	18.15	13.71
29	32.81	18.80	13.50	21.00	24.50	27.43	20.99
30	50.90	28.40	19.40	31.42	37.35	42.05	31.40
31	77.95	40.25	27.65	45.70	54.50	62.50	45.70
32	121.4	60.40	39.10	66.80	82.95	93.75	66.80
33	192.5	87.95	54.15	98.05	125.60	142.20	98.05
34	312.3	133.75	79.50	146.25	194.25	219.00	147.75
35	475	188.65	108.85	212.00	280.65	330.15	212.00
36	729	271.50	150.50	301.50	421.00	479.00	301.50
37	1,110	.....	.....	.....	616	662	452
38	1,750	.....	.....	.....	882	1,002	649
39	2,920	.....	.....	.....	1,330	1,568	974
40	4,480	.....	.....	.....	1,913	2,269	1,365
41	6,860						
42	10,390						
43	16,380						
44	22,850						

* This table based on using one layer of paper insulation of proper thickness.

In the design of filter reactors, the allowable d-c voltage drop in the winding is frequently the limiting factor rather than the heating of the winding. Under these conditions, a larger core and winding will be required to obtain a desired inductance than would otherwise be the case.

Factors Involved in the Design of Small Transformers.—In the design of transformers, the first step is to determine the volt-ampere ratings of the windings. In the case of

TABLE 26*d*.—TURNS POSSIBLE PER LINEAR INCH OF WINDING SPACE IN LAYER INSULATED COILS*

Table Based on Average Commercial Practice

B & S gage	Plain enamel†	S.C.E.†	D.C.C.†	S.C.C.†	S.S.E.†	S.S.C.†	D.S.C.†
10	9	8	8	8			
11	10	9	8	9			
12	11	10	10	10			
13	12	11	11	12			
14	14	13	12	13			
15	15	14	13	14			
16	17	16	15	16			
17	19	17	17	18			
18	22 } 90 %	19 } 90 %	18 } 90 %	20 } 90 %			
19	24	21	20	22			
20	27	23	22	24			
21	30	26	24	27			
22	34	29	27	30			
23	38	32	29	35			
24	42	35	32	37			
25	47	38	34	41	42	45	41
26	52	42	37	45	47	50	45
27	59	46	40	49	52	55	49
28	66 } 89 %	51 } 89 %	43 } 89 %	54 } 89 %	57 } 89 %	61 } 89 %	54 } 89 %
29	73	55	46	58	63	67	58
30	82	60	49	64	70	74	64
31	91	64	52	68	75	81	68
32	101	69	55	73	82	88	73
33	114 } 88 %	75 } 88 %	58 } 88 %	79 } 88 %	91 } 88 %	97 } 88 %	79 } 88 %
34	128	82	62	85	99	106	86
35	142	86	65	92	107	117	92
36	158 } 87 %	92	68	98	117	125	98
37	174				126	134	107
38	198				136 } 87 %	145 } 87 %	115 } 87 %
39	220 } 86 %				145	158	123
40	246				155	171	130
41	274						
42	304						
43	340 } 85 %						
44	369						

* These figures are for paper layer insulated coils.

† The percentages represent the space factor under normal conditions.

small transformers used for heating filaments, etc., this is done on the assumption of 80 to 90 per cent efficiency and 90 per cent power factor in the primary and unity power factor on the secondaries. In transformers used with rectifiers, it is commonly necessary to increase the volt-ampere ratings, particularly of the secondary, because the utilization factor is appreciably less than unity. The next step in the design is to select a suitable core area. Experience with small transformers shows that, with ordinary silicon steel, the net area of iron in the core, in square inches, should be at least[1] $0.16\sqrt{\text{primary volt-amperes}}$. Where a conservative design is to be used, a somewhat larger core area than this minimum is recommended.

[1] The actual cross section of the core will be somewhat larger because of the insulation between laminations. The ratio of the area of iron to the gross area of the core, *i.e.*, the stacking factor, is commonly of the order of 0.9.

Ex. 1044-0113

The third step in the design is to select a suitable maximum flux density, and then to calculate the turns per volt from the equation

$$\text{Turns per volt} = \frac{10^8}{4.44 f B_{max} A} \tag{115}$$

The notation is the same as in Eq. (108). The maximum flux density B_{max} should normally not exceed 75,000 lines per square inch for 60-cycle operation and 90,000 lines per square inch for 25-cycle operation. Lower flux densities will reduce the core losses and also the exciting current, but require more copper and iron.

The next step is the determination of the wire size and winding arrangements. It is customary to allow from 500 to 1,000 circular mils per ampere for transformers under 50 watts, and up to 1,500 to 1,750 circular mils per ampere for larger trans-

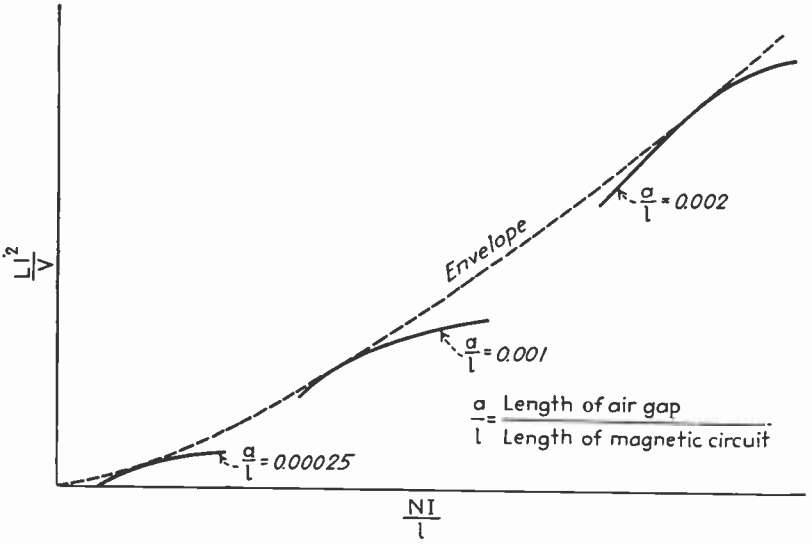


FIG. 73.—Graphical method of determining best air-gap length.

formers. The insulation between turns and layers, and between the winding and core are arranged in accordance with the voltages involved.

A core is now selected that will have sufficient window space to accommodate the winding to be used. The core is customarily composed of laminations of standard *E-I* punchings. This gives a shell-type core that is assembled in interleaved fashion to eliminate butt joints and minimize the air gap. Standard laminations of this type are available in a wide variety of shapes and in sizes large enough for power transformers of moderate power transmitters. With 60-cycle operation, the laminations are normally 29 gage (0.014 inches) silicon steel.

This completes the formal design of the transformer. The copper losses of the various windings can now be calculated from the currents and resistances of the windings, and the core loss can be evaluated from the volume of the core, the frequency, and the flux density, with the use of curves such as those of Fig. 63. The efficiency and the power that must be dissipated in the transformer can then be readily calculated. Voltage regulation can be determined from the resistances of the windings, and the leakage inductance as calculated by Eqs. (110) to (113). After the design has been completed and the important features of the performance cal-

Ex. 1044-0114

culated, it is always desirable to examine the design as a whole. This may indicate that modifications will give an improved performance or a satisfactory performance at a lower cost.

27. Magnetic Cores in Audio- and Radio-frequency Coils.—The permeability of iron (and other magnetic materials) drops off slowly with increasing frequency, but the reduction is not great at broadcast and even short-wave frequencies. Thus tests with iron and nickel have indicated that the permeability does not drop to unity until the frequency is in the range 10^9 to 10^{11} cycles.[1] Magnetic cores can accordingly be employed in high-frequency coils. It is merely necessary that the core be subdivided sufficiently that the ratio R_e/X_L in Eqs. (104) and (105) are not too great.

If the core is not sufficiently subdivided (*i.e.*, if t^2f in Eqs. (104) and (105) is large), not only will the eddy-current losses be excessive, but also the magnetic flux produced by the eddy currents will be sufficient to modify the flux distribution over the cross section of the magnetic material. The net result is then to reduce flux density at the center of the laminations or particles, producing a form of *magnetic skin effect* or shielding that reduces the effective cross section of the core and lowers the inductance.[2]

Hysteresis does not introduce any special problems at high frequencies, since the ratio of hysteresis loss to inductive reactance is independent of frequency, *i.e.*, the power loss is proportional to the number of cycles per second.

Use of Laminated Cores at Audio and Radio Frequencies.—The silicon steel laminations of No. 29 gage (0.014 inches thick) used in 60-cycle transformers, also have reasonably low losses at the lower audio frequencies. Such laminations are also used in filter reactors for smoothing rectified 60-cycle currents, for the cores of audio-frequency input, interstage, and output transformers, and in some cases in wave filters having pass bands below 1,000 to 1,500 cycles. At higher frequencies it is necessary to use laminations thinner than 0.014 inches if excessive eddy current losses are to be avoided. Thicknesses as small as 2 mils have been used, and in this way magnetic cores can be obtained that have fairly low loss even at moderately high radio frequencies.[3] Thin laminations of this character were at one time used to a considerable extent in communication work, but have now been largely displaced by cores of compressed powdered magnetic material.

Coils Using Cores of Compressed Powdered Magnetic Material.[4]—The most effective way to employ magnetic material at the higher audio frequencies and at radio frequencies is to mix the magnetic material in the form of a very fine powder or dust

[1] For a discussion of the effect of frequency on permeability see G. W. O. Howe, Permeability at Very High Frequencies, *Wireless Eng.*, Vol. 16, p. 541, November, 1939.

[2] Analyses are available for calculating the extent of the magnetic shielding in the case of laminations of rectangular cross section. See K. L. Scott, Variation of Inductance of Coils Due to the Magnetic Shielding Effect of Eddy Currents in the Cores, *Proc. I.R.E.*, Vol. 18, p. 1750, October, 1930.

In practice there is a thin surface layer, normally about 1 mil thick, having lower permeability than the main body of the laminations. This surface film is apparently the result of the rolling operation used in producing the lamination, and is not affected by annealing. Its effect is important at high frequencies where the penetration of the flux is small, and at these frequencies the simple theory will accordingly fail to give correct results. Where the layer is important, it can be removed by etching.

For further discussion of magnetic skin effect, with particular reference to the oxide film layer, see E. Peterson and L. R. Wrathall, Eddy Currents in Composite Laminations, *Proc. I.R.E.*, Vol. 24, p. 275, February, 1936; M. Reed, An Experimental Investigation of the Theory of Eddy Currents in Laminated Cores of Rectangular Section, *Jour. I.E.E.*, Vol. 80, p. 567, 1937; also Wireless Section, *I.E.E.*, Vol. 12, p. 167, June, 1937; M. Reed, Losses in Ferromagnetic Laminae at Radio Frequencies, *Wireless Eng.*, Vol. 15, p. 263, May, 1938.

[3] Information on losses of thin silicon steel laminations at radio frequencies is given by Thomas Spooner, High Frequency Iron Losses, *A.I.E.E. Jour.*, Vol. 39, p. 809, September, 1920; Leon T. Wilson, The Behavior of Iron in Alternating Fields of between 100,000 and 1,500,000 cycles, *Proc. I.R.E.*, Vol. 9, p. 56, February, 1921.

[4] W. J. Polydoroff, Ferro-inductors and Permeability Tuning, *Proc. I.R.E.*, Vol. 21, p. 690, May, 1933; V. E. Legg and F. J. Given, Compressed Powdered Molybdenum Permalloy for High-quality Inductance Coils, *Trans. A.I.E.E.*, Vol. 59, p. 1940; W. J. Shackelton and I. G. Barber, Compressed Powdered Permalloy, *Trans. A.I.E.E.*, Vol. 47, p. 429, April, 1928; Buckner Speed and G. W. Elmen, Magnetic Properties of Compressed Powdered Iron, *Trans. A.I.E.E.*, Vol. 40, p. 596, 1921.

Ex. 1044-0115

with an insulating binder, and then compress the combination into cores of the appropriate form. In such cores the insulating binder surrounds the magnetic particles, thereby giving a very complete subdivision of the core, and low eddy-current losses.

The size of the magnetic particles and the amount of magnetic material in proportion to insulating binder control the effective permeability of the resulting core and also the frequency at which the coil Q is highest. The smaller the particles the less the effective permeability, and also the higher the optimum frequency. The effective permeabilities used in practice range from 75 to 150 for coils designed to be optimum at

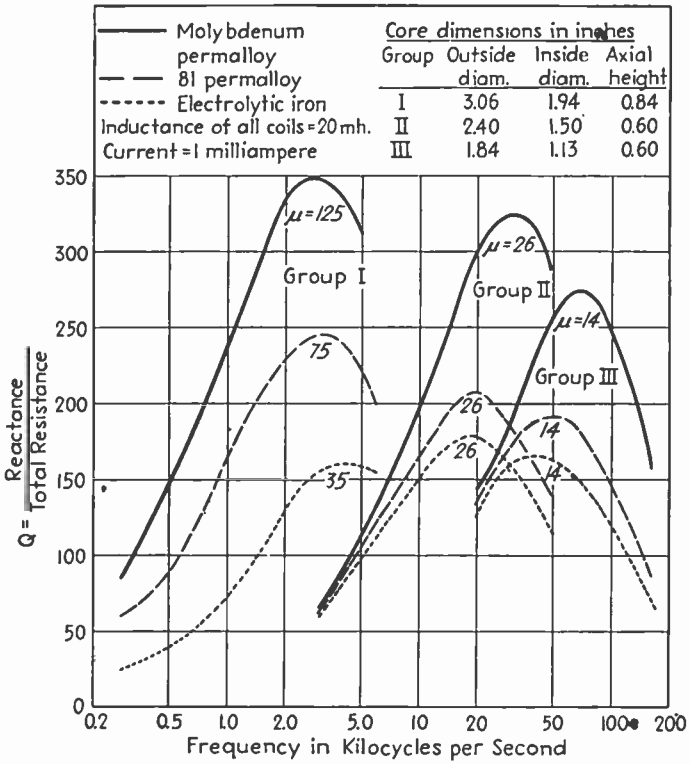


FIG. 74.—Variation of coil Q with frequency for several coils using toroidal dust cores.

audio frequencies, to values of 4 to 6 for coils intended for operation at frequencies of 500 to 1,500 kc.

The materials most frequently used in powdered cores are iron, permalloy, and molybdenum permalloy. Molybdenum permalloy has the lowest losses and hence gives the highest Q, while iron has the highest losses. The total core loss in any case will be quite small, however, provided that the core is used in its proper frequency range.

Coils employing powdered iron cores are often used in the intermediate-frequency and radio-frequency tuned circuits of radio receivers, up to and beyond the broadcast range.[1] For such radio-frequency application, the cores are commonly in the form of

[1] An excellent discussion of the properties of such cores and of the measurement of these properties is given by D. E. Foster and A. E. Newlon, Measurement of Iron Cores at Radio Frequencies, *Proc. I.R.E.*, Vol. 29, p. 266, May, 1941.

Ex. 1044-0116

cylindrical slugs upon which the coil is wound. At intermediate (175 to 460 kc) and lower frequencies, the use of iron-dust cores makes it possible to realize a decidedly higher Q in a very small coil than is obtainable by the use of an air-cored coil of comparable size. In the case of intermediate-frequency transformers, magnetic cores also have the advantage that the resonant frequency can be controlled by adjusting the position of the core, thereby eliminating the trimmer condenser otherwise needed.

Toroidal coils using cores of compressed powdered molybdenum permalloy, permalloy, and iron-dust rings are widely used in telephone work for loading coils, in wave filters, etc. The losses of such coils are quite small, as shown by Fig. 74, with molybdenum permalloy the most desirable core material and permalloy next. Not only are the losses lower than with iron but also the inductance is less affected by the intensity of the magnetizing force, and the harmonics and cross-modulation are also lower than with iron. Another advantage is that a large d-c magnetization has less permanent effect upon the inductance.

CAPACITY AND CAPACITORS

28. Definitions of Capacity.—Any pair of electrical conductors that will store electrical charge when a difference in potential is applied to them constitutes a condenser. The defining relation for capacity may be written

$$C = \frac{Q}{E} \tag{116}$$

where C is the capacity in farads, Q is the charge in coulombs, and E is the potential in volts. Capacity may also be defined in terms of the energy stored in the electrostatic field about the condenser plates; thus

$$W = \tfrac{1}{2}CE^2 \tag{117}$$

where W is the energy stored in watt-seconds (joules), C is the capacity in farads, and E is the potential in volts. Since current is the time rate of change of charge, the current flowing into any condenser is proportional to the time rate of change of the potential across it. This relation is given by

$$I = C\frac{dE}{dt} \tag{118}$$

where I is the current in amperes, t is the time in seconds, and the other symbols have their previous significance.

Capacity has the dimensions of length when expressed in electrostatic units. As a result the capacities of similar condensers are proportional to the first power of their linear dimensions. Doubling all dimensions of a condenser doubles its capacity.

29. Condenser Losses and Their Representation.—A perfect condenser when discharged gives up all the electrical energy that was supplied to it in charging. Actual condensers never realize this ideal perfectly but, rather, dissipate some of the energy delivered to them. Most of the loss in ordinary condensers occurs in the dielectric. Other ways by which energy can be lost in a condenser are from the resistance of the leads and metal plates, from leakage resistance between plates, and as a result of corona.

The merit of a condenser from the point of view of freedom from losses is usually expressed in terms of the power factor or phase angle.[1] The power factor represents the fraction of the input volt-amperes that is dissipated in the condenser, while the phase angle is the angle by which the current flowing into the condenser fails to be 90°

[1] The merit of a condenser is occasionally expressed in terms of the condenser Q, which represents the ratio of the capacitive reactance of the condenser to the equivalent series resistance. For all practical purposes the condenser Q is the reciprocal for the power factor.

Ex. 1044-0117

out of phase with the applied voltage. When the losses are low, as is practically always the case, the phase angle expressed in radians is equal to the power factor. The power factor (or phase angle) is a ratio representing the fraction of the input volt-amperes that is dissipated in the condenser, and for a given type of condenser tends to be independent of the applied voltage, of the condenser size, etc., and with ordinary dielectrics is substantially constant over wide frequency ranges.

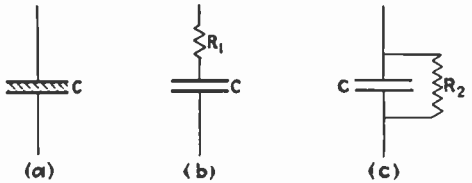


FIG. 75.—Representation of imperfect condenser by a perfect condenser of same capacity with series resistance, and by a perfect condenser with shunt resistance.

The effect of the condenser losses on the circuit in which the condenser is connected can be obtained by replacing the actual condenser by the combination of a perfect condenser of the same capacity with a resistance in series, as shown in Fig. 75*b* (or with a resistance in parallel, as in Fig. 75*c*). The value of the series (or shunt) resistance is so selected that the power factor of the combination of perfect condenser with its associated resistance is the same as the power factor of the actual condenser. The value of the resistances are

$$\text{Series resistance} = R_1 = \frac{\text{power factor}}{2\pi fC} \tag{119}$$

$$\text{Shunt resistance} = R_2 = \frac{1}{(2\pi fC)(\text{power factor})} \tag{120}$$

30. Dielectrics.—The insulating medium that separates the plates of a condenser is known as the *dielectric*, and plays an important role in determining the characteristics of the condenser. The presence of a dielectric other than a vacuum raises the capacity of the condenser in comparison to its capacity in the absence of the dielectric by a factor known as the *dielectric constant* (also sometimes called *specific inductive capacity*). The dielectric constants of the commonest insulating materials are given in Table 27, which also gives other properties of dielectrics.

Two very important classes of dielectrics listed in Table 27 are plastic and ceramic types. Some of the plastic dielectrics as, for example, bakelite, cellulose acetate, vinylite, etc., have very poor power factors. Others, notably polystyrene, have an unusually low power factor, which in some cases approaches that of quartz.

The more important ceramic dielectrics include steatite, cordierite, and titanium dioxide ceramics, and mycalex.[1] The term *steatite* refers to ceramic products that contain hydrous magnesium silicate (talc) as a predominant constituent of the fired body. Steatite materials combine low dielectric losses, high dielectric strength, and high insulation resistance. They are particularly suitable for general insulation purposes at high frequencies and high temperatures. It is possible to employ dry pressing in forming the product, so that parts can be manufactured in automatic presses to good accuracy and with economy. Cordierite is a complex magnesium aluminum silicate. Ceramics based upon it can be made that have an unusually low coefficient of thermal expansion, so that this material is particularly suitable for coil forms. The titanium dioxide ceramics are characterized by a high dielectric constant and low dielectric losses. This material finds its chief use as the dielectric of condensers and gives high mechanical stability and a high capacity in a relatively small volume. The temperature coefficient of capacity of such condensers depends upon the

[1] For further information on ceramic dielectrics, see Hans Thurnauer, Ceramic Insulating Materials, *Elec. Eng.*, Vol. 59, p. 451, November, 1940; M. D. Rigterink, Improved Ceramic Dielectric Materials, *Rev. Sci. Instruments*, Vol. 12, p. 527, November, 1941; W. G. Robinson, Ceramic Insulations for High-frequency Work, *Jour. I.E.E.*, Vol. 87, p. 570, 1940.

Ex. 1044-0118

TABLE 27.—PROPERTIES OF DIELECTRICS*

	Dielectric constant	Power factor, per cent			Specific gravity	Tensile strength, lb. per sq in (multiply by 10^3)	Compressive strength, lb per sq in (multiply by 10^3)	Softens at °C	Stable at °C	Specific heat	Coefficient of linear expansion, parts in 10^6 per °C	Heat conductivity cgs	Machine ability	Water absorption, per cent in 24 hr	Remarks
		60 cycles	1 kc	1 mc											
Amber	2.9	....	.....	0.2	1.1	.....	.....	250	180	.....	44	.....	Very good	0	Natural petrified resin
Casein—moulded	6.4	....	.....	6	1.33	7	.....	177	165	.....	80	.....	Very good	4–9	
Cellulose acetate	6–8	7	.....	3–6	1.3	3	4	70	65	0.5	150	0.0005	Very good	4	Tenite, safety film—burns very slowly
Cellulose nitrate	4–7	5–9	5	5	1.5	3–6	.....	85	85	0.36	140	0.0003	Very good	2–3	Celluloid, pyralin, pyroxylin—burns rapidly
Cordierite ceramics	5–5.5	....	0.6–1	0.4–0.7	2.1	2–5	40–60	1,405	.....	.....	1–2	0.003	.....	0–1	Temp. coefficient of cap. = $+5 \times 10^{-4}$ per°C; used for coil forms
Fiber	4–5	6–9	5	5	1.3	10	25	130	95	.....	25	0.0011	Very good	30	
Glass—crown	6.2	....	1	.....	2.48	2–5	10–30	1,100	.....	0.161	8.9	0.0025	No	0	Window glass
Glass—flint	7	....	0.45	0.4	3.7	3–6	6–10	.....	.....	0.117	7.9	0.002	No	0	
Glass—pyrex	4.5	....	0.5	0.2	2.25	.....	40	600	520	0.2	3.2	0.0027	Very poor	0	
Methacrylic resin	2.8	3	2	2	1.19	8–9	12	135	90	0.45	70	0.00055	Very good	0.3	Lucite, plexiglass—slow burning
Mica—clear India	7–7.3	0.03	0 02	0.02	2.8	.....	.....	1,200	600	2.06	3–7	0.0018			
Mycalex	6–8	....	0.6	0.3	3.5	6–8	25–40	.....	350	0.22	8–9	0.0014	Poor	0.035	Mica and lead borate
Marble—white	7–9	....	.....	4	2.7	2	8–15	.....	.....	0.21	8–12	0.0015	Fair	Very high	
Phenol—pure	5	2	.....	1	1.3	5–11	15–30	.....	120	0.3	28	0.0004	Very good	0.15	Catalin, bakelite—burns very slowly
Phenol—yellow	5.3	2.5	1.4	0.7	1.9	5.5	.....	.....	130	.....	.....	.....	Poor	0.2	Low-loss bakelite—nearly non-burning
Phenol—black molded	5.5	8	6	3.5	1.35	7.5	30	.....	140	0.35	40	0.0005	Fair	0.3	Nearly nonburning
Phenol—paper base	5.5	6	5	3.5	1.35	10–15	30	.....	125	0.3	30	0.00065	Good	0.2–1	Nearly nonburning
Phenol—cloth base	5.6	5	5	5	1.38	11	35	.....	115	0.35	20	0.0005	Good	0.7	Nearly nonburning
Porcelain—wet process	6.5–7	2	1	0.6	2.4	3–6	30–50	1,610	1,050	0.25	4–5	0.0025	No	Low	
Porcelain—dry process	6.2–7.5	2	1	0.7	2.3	2–3	30–50	.....	1,050	0.26	3–4	0.0025	No	0.1–1	
Quarts—fused	4.2	0.03	0.03	0.03	2.21	7–10	200	1,430	1,150	0.18	0.45	0.0024	Very poor	0	SiO_2 conducts at 800°C
Rubber—hard	2–3	1	1	0.5–0.9	1.15	4–7	7	70	65	0.33	70–80	0.0004	Fair	0.02	Burns slowly
Slate	6–8	....	0.9	.....	2.8	5	15	.....	.....	0.22	10	0.005	Fair	High	
Steatite	6.1	1	0.4	0.3	2.5	8–10	50–100	1,500	1,000	.....	6–8	.....	No	0.02	Magnesium silicate—isolantite, lava—temp. coefficient of cap. = $+1.5 \times 10^{-4}$ per°C
Styrene (polymerised)	2.4–2.9	0.02	0.02	0.03	1.05	6–9	14	90	75	0.324	70	0.0004	Good	0.01	Victron, trolitul, polystyrene, amphenol—very slow burning
Sulphur	3–3.8	....	.....	.....	2.05	.....	.....	113	95	0.17	64	0.0006	.....	.....	Burns rapidly
Shellac	2.5–4	2.5	.....	0.9	1.1	0.9	7	85	75	.....	.....	0.0006	.....	0.1	Burns readily
Titanium dioxide	90–170	....	0.1	0.06	4–5	4	60	1,600	.....	.....	7–8	.....	No	0	Rutile—temp. coeff. of cap. = 6.5×10^{-4} per°C
Titanium dioxide magnesium titanate	12–18	....	0.1	0.02–0.1	3.1	8–12	70–85	1,400	.....	.....	6–10	0.010	.....	0	Coefficient of cap. = $\pm 0.5 \times 10^{-4}$
Titanium dioxide titanium—zirconium dioxides	40–60	....	0.1–0.3	0.05–0.10	3.6	4–7	40–80	1,500	.....	.....	6–7	.....	.....	0	Temp. coeff. of cap. = $\pm 0.5 \times 10^{-4}$
Urea—formaldehyde compounds	6–7	5	3.8	3	1.48	6–9	25–30	200	80	.....	.....	0.00017	Fair	0.4	Beetle, plaskon
Vinyl resins—unfilled	4	....	1.4	1.7	1.35	8–10	.....	.....	50	0.244	70	0.0005	Very good	0.15	Vinylite—nonburning

* Most of the data in this table are from the *Gen. Rad. Exp.* for June, 1939.

Ex. 1044-0119

mixture, and values can be obtained that range from substantially zero to relatively large negative values. This gives the possibility of providing a condenser that will correct for the positive temperature coefficient of the remaining coils and condensers in a circuit.

Mycalex is composed of ground mica and lead borate glass mixed and fired. Metal inserts can be molded into mycalex pieces because of the relatively low firing temperature and the fact that there is no shrinkage. Mycalex can be machined, although with difficulty. It is used in radio transmitters for coil supports, control shafts, antenna insulators, and in other places where an insulator must be employed that is capable of withstanding high voltage.

Dielectric Losses—Loss Factor.—The merit of a dielectric from the point of view of losses can be expressed by the equivalent power factor or phase angle. When these terms are applied to a dielectric (as contrasted with a condenser), they are defined as the power factor (or phase angle) that would be obtained with a condenser having no losses other than those in the dielectric. Power factors of common dielectrics are given in Table 27.

The power factor (or phase angle) is independent of the size and shape of the dielectric or the applied voltage. It also tends to be independent of frequency, since the fraction of the energy lost during each cycle of charge and discharge is substantially independent of the number of cycles per second, over wide frequency ranges. The power factor of a dielectric depends not only upon the kind of dielectric involved but also upon the conditions of temperature and moisture under which it is used. In general, moisture and high temperature increase the power factor.

The power loss (and hence heating) per unit volume of dielectric is[1]

$$\left.\begin{matrix}\text{Power loss in}\\ \text{watts per cu-}\\ \text{bic inch}\end{matrix}\right\} = 2\pi f G^2 K\tau \times 0.2244 \times 10^{-12} \tag{121}$$

where f = frequency in cycles.
G = voltage gradient in dielectric, rms volts per in.
K = dielectric constant.
τ = power factor of dielectric.

The rate at which heat is generated in a dielectric is proportional to $K\tau$, which is termed the *loss factor* of the dielectric. The loss factor is the best single criterion of the ability of a solid insulating material to withstand high radio-frequency voltages.

31. Calculation of Condenser Capacity. *Parallel Plate Condenser.*—The capacity of a parallel plate condenser is

$$C = 0.2244K \frac{A}{d} \mu\mu f \tag{122}$$

where K = dielectric constant of the dielectric between the plates.
A = area of one plate, sq in.
d = spacing of plates, in.

If the dimensions are in centimeters

$$C = 0.08842K \frac{A}{d} \mu\mu f \tag{122a}$$

These expressions neglect the fringing flux lines at the edges of the plates and so give a result slightly less than the actual capacity.

[1] E. T. Hoch, Power Losses in Insulating Materials, *Bell System Tech. Jour.*, Vol. 1, p. 110, October, 1922.

Ex. 1044-0120

Long Parallel Strips in Air.—The capacity between two parallel strips of great length compared with the width or spacing is given by the following approximate formulas:[1]

For $R = \dfrac{\text{width of strips}}{\text{separation of strips}} < 1$:

$$C = \frac{0.3063}{\log_{10}\left(\frac{4}{R}\right)} \mu\mu\text{f per inch} \tag{123}$$

This expression becomes more accurate as the value of R becomes smaller. For R less than 0.5, the formula gives results less than 2 per cent high, and for $R = 1$, the results are about 10 per cent high.

For $R > 1$:[2]

$$C = 0.2244R\left(1 + \frac{1}{\pi R}(1 + 2.303 \log_{10} 2\pi R)\right) \mu\mu\text{f per inch} \tag{124}$$

This expression gives values that are lower than the exact value, and whose accuracy increases as the value of R increases. The error is 4 per cent for $R = 2$ and less than 10 per cent for $R = 1$.

When the plates of the condenser are immersed in a dielectric of constant K (as if the dielectric were a liquid), the results obtained from Eqs. (123) and (124) must be multiplied by K. With solid dielectric between the plates but not projecting beyond the edges, these equations still hold, but with less accuracy. When the solid dielectric projects considerably beyond the edges of the plates, fair results can be obtained by multiplying values from Eqs. (123) and (124) by the dielectric constant.

Concentric Spheres.

$$C = 1.412K \frac{d_1 d_2}{d_2 - d_1} \mu\mu\text{f} \tag{125}$$

where d_1 = outer diameter of inside sphere, in.
d_2 = inner diameter of outside sphere, in.
K = dielectric constant of material between spheres.

Isolated Sphere.

$$C = 1.412d \ \mu\mu\text{f} \tag{126}$$

where d is the diameter of the sphere in inches and the sphere is assumed to be in free space (vacuum). This will be seen to correspond to the capacity of a set of concentric spheres with d_2 = infinity.

Isolated Disk.

$$C = 0.8992d \ \mu\mu\text{f} \tag{127}$$

where d is the diameter of the disk in inches. The disk is presumed to be of negligible thickness, and is assumed located in free space (vacuum).

Single Wire Parallel to Ground.—When the wire can be considered as infinitely long, then

$$C = \frac{7.354}{\log_{10}\left\{\frac{2h}{d}\left(1 + \sqrt{1 - \frac{1}{(2h/d)^2}}\right)\right\}} \mu\mu\text{f per foot} \tag{128}$$

where h = height of wire center above earth.
d = diameter of wire.

[1] The exact expression for this case involves elliptic integrals and can be solved only by trial and error. See H. B. Palmer, Capacitance of a Parallel Plate Capacitor by the Schwartz-Christoffel Transformation, *Trans. A.I.E.E.*, Vol. 56, No. 3, p. 363, March, 1937.

[2] A. E. H. Love, Some Electrostatic Distributions in Two Dimensions, *Proc. London Math. Soc.*, Vol. 22, p. 337, Mar. 8, 1923.

Ex. 1044-0121

In the usual case, h is considerably greater than d, and Eq. (128) can be written as

$$C = \frac{7.354}{\log_{10}\frac{4h}{d}} \mu\mu f \text{ per foot} \tag{129}$$

Curves of capacity per inch as calculated by Eq. (129) for various wire sizes and heights above ground are given in Fig. 76. These curves are useful for determining wiring capacity in radio receivers.

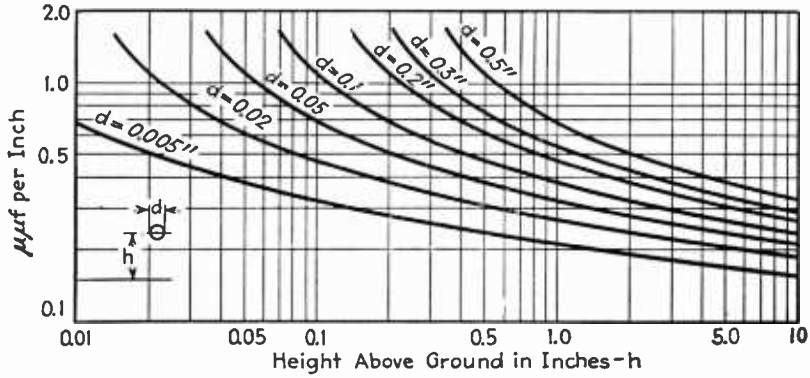


FIG. 76.—Capacity per inch of single-wire conductor of various diameters as a function of height above ground.

When the wire cannot be considered infinitely long, but still has a small diameter compared with the height, then[1]

$$C = \frac{7.354l}{\log_{10}\left(\frac{4h}{d}\right) - S} \mu\mu f \tag{130}$$

where l is the length in feet, h and d are the height and wire diameter, respectively, in the same units, and S is a constant given in Table 28.

TABLE 28.—VALUES OF THE CONSTANT S*

$2h/l$	S	$l/2h$	S	$l/2h$	S
0	0	1.00	0.336	0.50	0.541
0.1	0.042	0.95	0.350	0.45	0.576
0.2	0.082	0.90	0.364	0.40	0.617
0.3	0.121	0.85	0.379	0.35	0.664
0.4	0.157	0.80	0.396	0.30	0.721
0.5	0.191	0.75	0.414	0.25	0.790
0.6	0.223	0.70	0.435	0.20	0.874
0.7	0.254	0.65	0.457	0.15	0.990
0.8	0.283	0.60	0.482	0.10	1.155
0.9	0.310	0.55	0.510	0.05	1.445
1.0	0.336	0.50	0.541		

* The argument to be used is either $2h/l$ or $l/2h$, according to which is less than unity.

[1] Frederick W. Grover, Methods, Formulas, and Tables for the Calculation of Antenna Capacity, *Bur. Standards Sci. Paper* 568.

Ex. 1044-0122

Capacity to Ground of Parallel Wires of the Same Height Joined Together.[1]—On the assumption that the wire diameter is small compared with the spacing between wires, then

$$C = \frac{7.36l}{F}\ \mu\mu f \tag{131}$$

where $F = \dfrac{P + (n-1)Q}{n} - S_n$.

$P = \log_{10}\dfrac{4h}{d} - S$.

$Q = \log_{10}\dfrac{2h}{D} - S$.

l = length of wire, ft (assumed same for all wires).
d = wire diameter, ft.
n = number of wires.
D = spacing between adjacent wires, ft (assumed same for all adjacent pairs).
h = height above the surface of the earth, ft.
S = constant given by Table 28.
S_n = constant given by Table 29.

This equation assumes that $(n-1)D \gtrsim l/4$.

TABLE 29.—VALUES OF THE CONSTANT S_n

n	S_n	n	S_n	n	S_n	n	S_n
2	0	8	0.347	14	0.550	20	0.688
3	0.067	9	0.388	15	0.576	30	0.847
4	0.135	10	0.425	16	0.601	40	0.970
5	0.197	11	0.460	17	0.625	50	1.063
6	0.252	12	0.492	18	0.647	100	1.357
7	0.302	13	0.522	19	0.668		

For the case of two wires, Eq. (131) reduces to

$$C = \frac{14.73l}{\log_{10}\dfrac{4h}{d} + \log_{10}\dfrac{2h}{D} - 2S} \tag{132}$$

Capacity to Ground of a Horizontal Cage.[1]—When the wires are closely spaced around the circumference of the cage, the capacity to ground can be obtained very approximately by using Eq. (130), if the cage is considered to be a wire having a diameter equal to the diameter of the cage. The capacity obtained in this way is large. Accurate results are given by

$$C = \frac{7.36ln}{\log_{10}\dfrac{4h}{d} + \displaystyle\sum_{r=1}^{r=n-1}\left(\log_{10}\frac{2h}{D_r} + 0.434\,\frac{D_r}{l}\right) - nS}\ \mu\mu f \tag{133}$$

where l = length of wires in cage, ft.
n = number of wires in cage.
d = diameter of wires in cage, ft.
h = height of cage center above ground, ft.
S = constant from Table 28.

[1] Grover, *loc. cit.*

Ex. 1044-0123

D_r = distance between any given wire and another wire, ft. = $\delta \sin r\frac{\pi}{n}$, where δ is the diameter of the cage, ft.

Capacity to Ground of Vertical Wire and Cage Antennas.[1]—For a single vertical wire

$$C = \frac{7.36m}{\log_{10}\frac{2m}{d} - k} \mu\mu f \tag{134}$$

where m = length of vertical wire, ft.
h' = height of lower end of wire above ground, ft.
d = diameter of wire, ft.
k = constant given by Table 30 and depending on h'/m.

For accurate results, h'/m must be small.

The capacity to ground of a vertical cage of n wires of diameter d, length m, arranged as elements of a cylinder of diameter δ whose axis is vertical and whose lower end is at a height h' above ground is (dimensions in feet):

$$C = \frac{7.36nm}{\log_{10}\frac{2m}{d} + \sum_{r=1}^{r=n-1}\left(\log_{10}\frac{m}{D_r} + 0.434\frac{D_r}{m}\right) - nk} \mu\mu f \tag{135}$$

where k is obtained from Table 30 and

$$D_r = \delta \sin r\frac{\pi}{n} \tag{136}$$

TABLE 30.—VALUES OF THE CONSTANT k*

h'/m	k	h'/m	k	m/h'	k
....	.....	0.3	0.280	1.0	0.207
0.02	0.403	0.4	0.261	0.9	0.202
0.04	0.384	0.5	0.247	0.8	0.196
0.06	0.369	0.6	0.236	0.7	0.190
0.08	0.356	0.7	0.227	0.6	0.184
0.10	0.345	0.8	0.219	0.5	0.177
0.15	0.323	0.9	0.2125	0.4	0.170
0.20	0.305	1.0	0.207	0.3	0.162
0.25	0.291	...	.....	0.2	0.153
0.30	0.280	...	.....	0.1	0.144
....	.....	...	.....	0	0.133

* The argument is h'/m or m/h' according to which is less than unity.

Capacity to Ground of Single-wire T and Inverted L Antennas.[1]—The capacity in these cases can be determined approximately by adding the capacities calculated separately for the vertical and horizontal components. This gives a result somewhat high, however, because of mutual effects between the vertical and horizontal portions. More accurate results are obtainable by the formulas given below.

For a single-wire T having a horizontal length l at a height h above ground and a vertical wire of length m attached to the center of l, then, if the dimensions are in feet

$$C = \frac{7.36(l + m)}{U} \tag{137}$$

[1] Grover, *loc. cit.*

Ex. 1044-0124

where

$$U = \frac{l}{l+m}\left(\log_{10}\frac{4h}{d} - S\right) + \frac{m}{l+m}\left(\log_{10}\frac{2m}{d} - k\right) + \frac{l+2m}{l+m}X \qquad (138)$$

The constants S and k are given by Tables 28 and 30; while X is given by Fig. 77. In using these tables, it is to be noted that the height h' of the lower end of the vertical above ground is $(h - m)$.

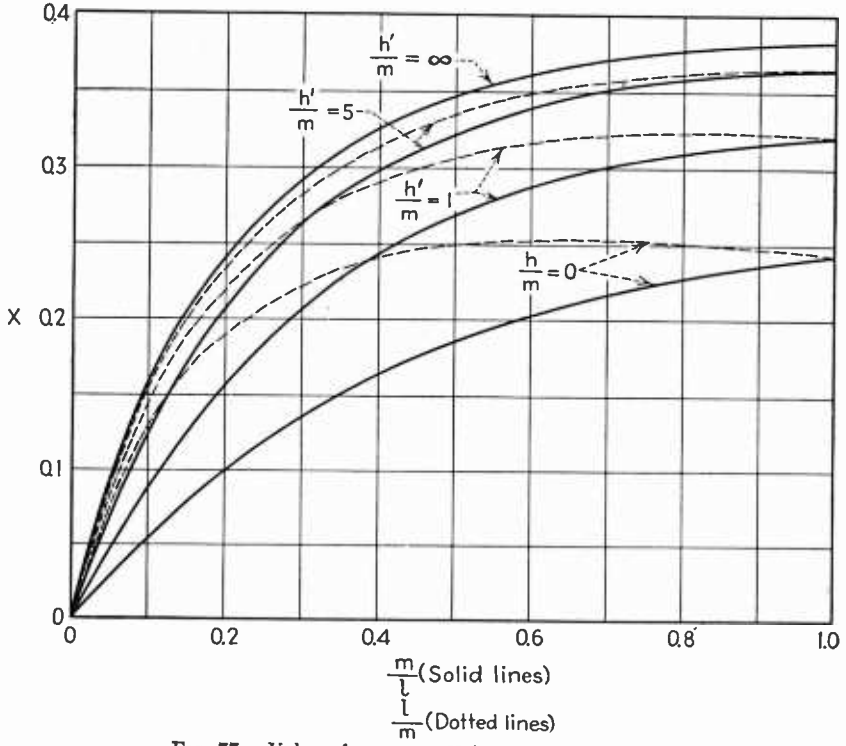


FIG. 77.—Value of constant X for use in Eq. (138).

The capacity to ground for the case of a single-wire inverted L antenna (*i.e.*, where the down-lead is attached to the end instead of being center of the horizontal portion) is also given by Eq. (138), provided U is defined as

$$U = \frac{l}{l+m}\left(\log_{10}\frac{4h}{d} - S\right) + \frac{m}{l+m}\left(\log_{10}\frac{2m}{d} - k\right) + X \qquad (139)$$

Capacity to Ground of Inclined Wire, Vertical Fan, etc.—The capacity to ground of a considerable variety of arrangements such as vertical cone, vertical fan, inclined wires, horizontal V, etc., are given in convenient form by F. W. Grover in the reference cited above.

Capacity between Two Horizontal Parallel Wires Remote from the Ground.—When the wires are infinitely long, the capacity per unit length is half that given by Eqs. (128) and (129), calculated by replacing h in these equations by $D/2$, where D is the spacing between wire centers. This gives

Ex. 1044-0125

$$C = \frac{3.677}{\log_{10}\left\{\frac{D}{d}\left(1 + \sqrt{1 - \frac{1}{(D/d)^2}}\right)\right\}} \ \mu\mu\text{f per foot} \tag{140}$$

For the common case of d/D small, this becomes

$$C = \frac{3.677}{\log_{10}\frac{2D}{d}} \ \mu\mu\text{f per foot} \tag{141}$$

$$C = \frac{0.01941}{\log_{10}\left(\frac{2D}{d}\right)} \ \mu\text{f per mile} \tag{142}$$

The curves of Fig. 78 give the capacities for various wire sizes and spacings.

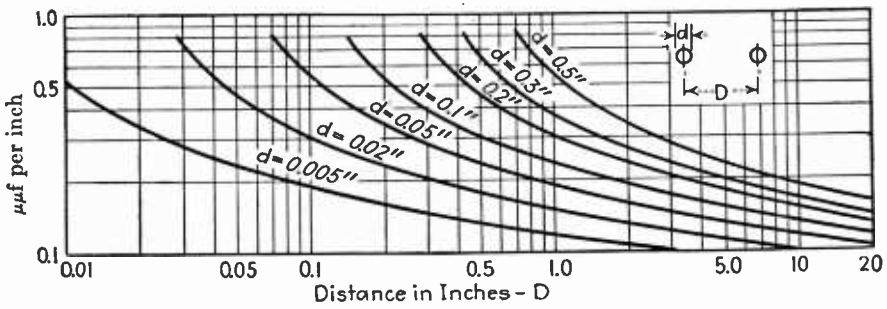


FIG. 78.—Capacity per inch of two-wire line with various wire diameters as a function of spacing.

The case for finite length, corresponding to Eq. (130), is

$$C = \frac{3.68l}{\log_{10}\frac{4h}{d} - S} \ \mu\mu\text{f} \tag{143}$$

where the notation has the same meaning as in Eq. (130).

Capacity between Two Parallel Wires in Presence of Ground.—In the usual case where d/l and d/D are very small, one has

$$C = \frac{3.677}{\log_{10}\frac{2D}{d} \frac{1}{\sqrt{1 + \left(\frac{D}{2h}\right)^2}}} \ \mu\mu\text{f per foot} \tag{144}$$

The presence of the ground increases the capacity for this case by the same amount that would result if the spacing were assumed to have an effective value of $D/\sqrt{1 + (D/2h)^2}$.

Capacity between Two Horizontal Parallel Wires One above the Other.—The capacity for this case may be determined approximately from Eq. (144) by taking for the value of h the average of the heights of the wires $\frac{h_1 + h_2}{2}$.

Capacity of Concentric Cable.—Let

D = inside diameter of the outside cylinder.

d = outside diameter of the inner conductor.

K = dielectric constant of material between conductors.

Ex. 1044-0126

Then

$$C = \frac{7.354K}{\log_{10}\left(\frac{D}{d}\right)} \ \mu\mu\text{f per foot} \tag{145}$$

The capacities of concentric cables with various inner and outer diameters are given in Fig. 79.

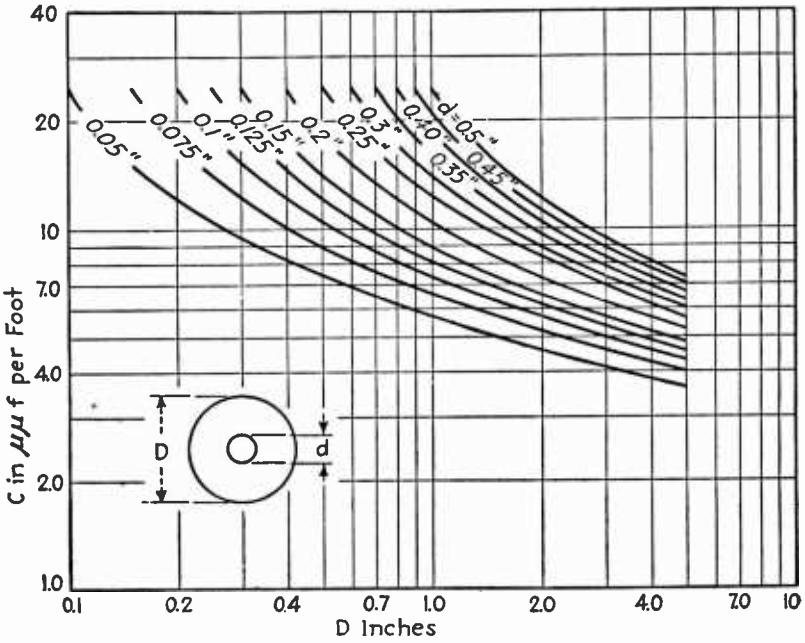


FIG. 79.—Capacity per foot of concentric lines with air dielectric.

32. Condensers with Air Dielectric.—An air-dielectric condenser consists of two assemblies of spaced plates held in fixed relationship to each other by insulating members so that the plates mesh but do not touch. Such condensers are normally

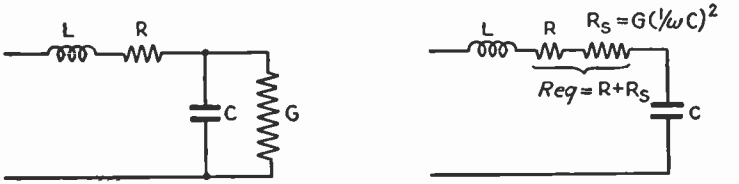


FIG. 80.—Equivalent electrical networks of a variable air condenser.

made so that one set of plates can be rotated to adjust the capacity. Variable air-dielectric condensers are characterized by very low losses, and by having reproducible capacity settings. They are used for adjusting the resonant frequency of tuned circuits in both transmitters and receivers.

Resistance and Inductance of Air-dielectric Condensers.—Even though air is a perfect dielectric with zero power factor, air condensers have losses because of the resistance of the plates, rods, washers, etc., and because of the losses in the dielectric used to mount the two sets of plates with respect to each other. Air condensers also have a small inductance as a consequence of the magnetic flux produced by the currents flowing through the condenser and its connections.

The actual condenser can be represented by the electrical network of Fig. 80*a* (or by the series equivalent of this, as given in Fig. 80*b*), where C is the capacity of the condenser.[1]

The condenser inductance L in Fig. 80 takes into account the magnetic effect of the currents in the condenser, and is to a first approximation independent of the capacity setting or the frequency. It depends upon the physical size of the condenser and the method of making connections to the two sets of condenser plates. This inductance can be reduced by making the condenser of small physical size and by bringing the connections to the centers of the plate assemblies, rather than to the ends as is usually done. The effect of the inductance is to cause the apparent capacity of the condenser, as observed at the terminals, to be greater than the actual capacity according to the equation

$$\text{Apparent capacity} = \frac{C}{1 - \omega^2 LC} \tag{146}$$

where C and L are the capacity and inductance in farads and henrys, respectively, and ω is 2π times the frequency in cycles. The difference between the apparent and actual capacities is negligible except at high frequencies, but then increases very rapidly. The order of magnitude that can be expected for L is indicated by a value of 0.06 μh for a physically large standard variable condenser having a maximum capacity of 1,000 $\mu\mu f$ and 0.0055 for a physically small (145 $\mu\mu f$ maximum) variable condenser having center-fed plates, and otherwise designed for low inductance.

The conductance G in Fig. 80*a* represents the losses in the solid dielectrics used for support, and is independent of the capacity setting but is directly proportional to the frequency. The conductance is determined by the type and amount of dielectric and the method of using it. The merit of a condenser with respect to freedom from dielectric losses can be expressed in terms of the quantity $R_s\omega C^2$, where R_s is the series resistance corresponding to G (see Fig. 80). For any given condenser this figure of merit is independent of frequency and capacity.

The resistance R represents the series resistance of the leads, washers, connecting rods, etc. It is substantially independent of the capacity setting, but increases with frequency as the result of skin effect, being proportional to the square root of the frequency at higher frequencies.

The ratio $R_{ac}/(1/\omega C)$ in Fig. 80 is termed the condenser power factor (the reciprocal of the condenser power factor is the condenser Q). The value of the power factor is determined primarily by the dielectric losses G at low frequencies and by the series resistance R at high frequencies. For a given variable condenser, the power factor is maximum at low capacity settings for low frequencies, whereas at high frequencies the power factor commonly goes through a minimum at something less than the maximum capacity. The behavior in some typical cases is shown in Fig. 81.[2] It will be noted that condenser power factors, although small, are by no means zero, particularly at small capacity settings and also at very high frequencies.

[1] R. F. Field and D. B. Sinclair, A Method for Determining the Residual Inductance and Resistance of a Variable Air Condenser at Radio Frequencies, *Proc. I.R.E.*, Vol. 24, p. 255, February, 1936.

[2] See Field and Sinclair, *loc. cit.*; also, W. Jackson, The Analysis of Air Condenser Loss Resistance, *Proc. I.R.E.*, Vol. 22, p. 957, August, 1934.

Ex. 1044-0128

Law of Capacity Variation.—The plates of a variable air condenser can be shaped to give any desired law of capacity variation as a function of angle of rotation.[1] The types of condensers of greatest interest are condensers in which the capacity is a linear function of angle of rotation (termed straight-line capacity and abbreviated SLC); condensers in which the plates are shaped so that, when used to tune a coil to resonance, the wave length at resonance is a linear function of the angle of rotation (straight-line wave length, abbreviated SLW); condensers that, when used to tune an inductance, give a resonant frequency that is a linear function of the angle of rotation (straight-line frequency, abbreviated SLF); and, finally, condensers in which either the logarithm of capacity or the logarithm of frequency of resonance with a coil is a straight-line function of the angle of rotation. Practical condensers frequently use intermediate characteristics or combinations of these basic types. Thus the tuning

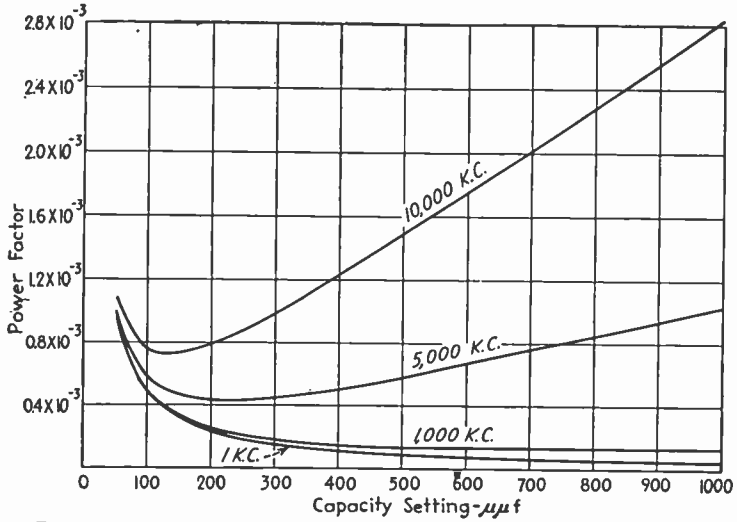


FIG. 81.—Power factor at various frequencies of a laboratory-type variable condenser as a function of capacity setting.

condensers used in a broadcast receiver usually approximate a straight-line wave-length law at low capacity settings and a straight-line frequency law at large capacity settings. Still other shapes are used to meet special requirements, as, for example, in beat-frequency oscillators.[2]

Straight-line capacity characteristics can be obtained by using semicircular rotating and stationary plates, as illustrated in Fig. 82. The straight-line wave-length characteristic is obtained by modifying the shape of the rotating plates according to the following equation:

$$r = \sqrt{\frac{5dK(K\theta + \lambda_{min})}{NL} + r_1^2} \text{ inches} \qquad (147)$$

[1] Condensers have also been designed with wedge-shaped plates so arranged that the thickness of the air gap varies with the position of the rotating plates, thereby introducing an additional factor modifying the law of capacity variation. Such condensers are used only to a limited extent, however, because the plates cannot be made of standard sheet material.

[2] Thus see E. R. Meissner, Logarithmic Scale for Beat-frequency Oscillator, *Proc. I.R.E.*, Vol. 17, p. 879, May, 1929; W. H. F. Griffiths, Notes on the Laws of Variable Air Condensers, *Exp. Wireless and Wireless Eng.*, Vol. 3, p. 14, January, and p. 743, December, 1926.

Ex. 1044-0129

where d = separation between plates, in.

$K = \frac{\lambda_{max} - \lambda_{min}}{\pi}$.

N = number of air spaces in condenser.

r_1 = radius of cutout on stationary plates, in.

L = tuned inductance, microhenrys.

θ = angle of rotation, radians

λ = wave length, meters.

Similarly, a straight-line frequency characteristic is obtained with rotating plates cut so that the radius of the plates as a function of the angular position is given by[1]

$$r = \sqrt{\frac{4D^2}{nkK^2\left(\frac{D}{K\sqrt{C_0}} - \theta\right)^3} + r_1^2} \text{ cm} \qquad (148)$$

where $D = 1/2\pi\sqrt{L}$, a constant (L, henrys).

$K = \frac{f_{max} - f_{min}}{\pi}$, cycles per radian on dial.

n = number of air spaces in condenser.

$k = 10^{-11}/36\pi d$, a condenser constant.

C_0 = total minimum capacity when $\theta = 0$, farads.

θ = angle of rotation of rotor plates in radians.

r_1 = radius of cutout on stationary plates, cm.

d = air gap between plates, cm.

The approximate plate shape and the way in which the capacity varies with the angle of rotation in a typical case are shown in Fig. 82.

Temperature Coefficient of Capacity.—The principal factors that may cause the capacity of an air-dielectric condenser to change with temperature are (1) differential expansion of different portions of the condenser, particularly effects causing bending of the plates, (2) linear expansion that causes changes in surface areas, etc., (3) changes in dimensions and dielectric constant of the insulation used for mounting, (4) changes in residual stress with temperature that give rise to deformations.

The temperature coefficient of capacity of ordinary air-dielectric condensers may vary over a wide range. Thus values from +150 to −65 parts per million per °C have been observed for different designs. The behavior of ordinary condensers with respect to changes in temperature is also frequently noncyclic, *i.e.*, the capacity does not return to its original value when the temperature comes back to normal.

Air condensers with small and adjustable temperature coefficients, having cyclic behavior can be realized by proper design.[2] Such a result is facilitated by making the fixed and rotating plate assemblies of the same material throughout and annealing to relieve residual stress. The air gap should be large so that the capacity will not be critical with small deformations of the plates. The insulating supports should have low temperature coefficients of expansion and of dielectric constant, and possess cyclic behavior. Certain of the ceramics and plastics are the best. The exact temperature coefficient of such a condenser can be adjusted as desired by controlling the axial position of the rotating plates with respect to the fixed plates by means of metal rods of a second material having a different coefficient of expansion. In this way it is possi-

[1] H. C. Forbes, The Straight-line Frequency Variable Condenser, *Proc. I.R.E.*, Vol. 13, p. 507, August, 1925. See also O. C. Roos, Simplified SLF and SLW Design, *Proc. I.R.E.*, Vol. 14, p. 773, December, 1926; Griffiths, *loc. cit.*

[2] H. A. Thomas, The Development of a Small Variable Air Condenser Compensated for Rapid Changes of Temperature, *Jour. I.E.E.*, Vo.. 84, p. 495, 1939; also, Wireless Section, *I.E.E.*, Vol. 14, p. 157, June, 1939; The Electrical Stability of Condensers, *Jour. I.E.E.*, Vol. 79, p. 297, 1936; also, Wireless Section, *I.E.E.*, Vol. 11, p. 202, September, 1936.

Ex. 1044-0130

ble to obtain zero temperature coefficient of capacity by starting with a proper initial displacement of the rotating plates away from the mid-position between fixed plates, or one can obtain either a positive or negative temperature coefficient as desired by using a greater or less displacement than that giving zero coefficient.[1]

Transmitting Condensers with Air Dielectric.—In transmitting condensers, the air gap between the plates must be in proportion to the voltage for which the condenser is rated, and care must be used in the design so that there will be no point at which electrostatic flux will concentrate unduly and cause even the slightest corona at rated

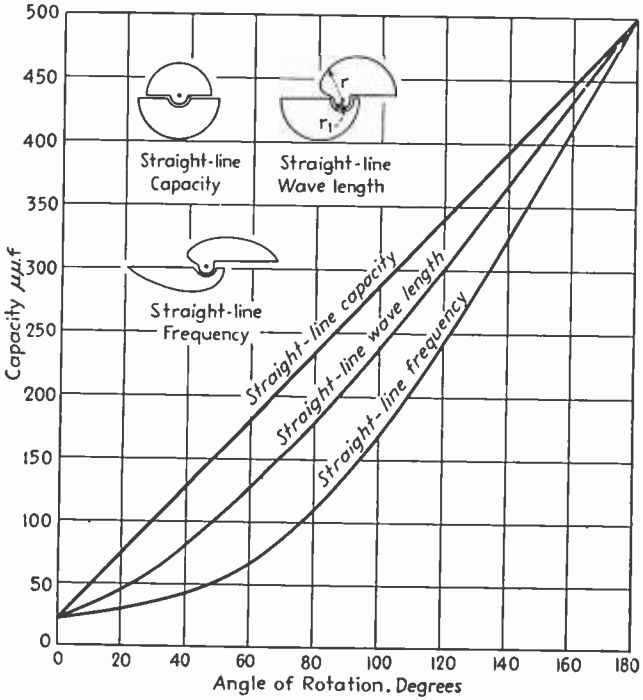


FIG. 82.—Characteristics of straight-line capacity, straight-line wavelength, and straight-line frequency condensers, showing capacity as a function of angle of rotation in typical cases, together with approximate shape of plates.

voltage. Furthermore, the solid insulation must have a low loss factor and be arranged so that the voltage gradient in it is low.

The dielectric strength of air is somewhat less at radio frequencies than at 60 cycles. Furthermore, a slight trace of corona such as would be of negligible consequence at 60 cycles, will cause intense ionization, accompanied by high power loss and possible breakdown when the frequency is high.[2]

The effective utilization of air as a dielectric in high-voltage condensers requires large spacings between plates and careful rounding of all sharp edges to avoid flux

[1] Another method of obtaining zero coefficient in a variable condenser involves a condenser in which all parts determining length are aluminum, and as are also all the stator plates, while half of the rotor plates are invar and half aluminum. See T. Slonczewski, High Accuracy Heterodyne Oscillators, *Bell System Tech. Jour.*, Vol. 19, p. 407, July, 1940.

[2] P. A. Ekstrand, Radio Frequency Spark-over in Air, *Proc. I.R.E.*, Vol. 28, p. 262; A. Alford and S. Pickles, Radio-frequency High Voltage Phenomena, *Trans. A.I.E.E.*, Vol. 59, p. 129, March, 1940; E. W. Seward, The Electric Strength of Air at High Frequencies, *Jour. I.E.E.*, Vol. 84, p. 288, 1939; also, Wireless Section, *I.E.E.*, Vol. 14, p. 31, March, 1939.

Ex. 1044-0131

concentrations. In particular, the plates should be polished, have rounded edges, and have a thickness from one-third to one-fourth of the air gap.

Pressure and Vacuum Condensers.—The voltage rating of an air-dielectric condenser can be increased by increasing the air pressure, since the relationship between breakdown voltage and pressure at radio frequencies has the character shown in Fig. 83. Variable and fixed condensers based on this principle are commercially available[1] and are used in high-power radio transmitters at broadcast and higher frequencies for tuning tank circuits, as neutralizing condensers, and in antenna coupling networks. Practical compressed-gas condensers employ dry nitrogen from a tank to maintain the pressure, and operate at approximately 20 atm. This gives a voltage rating of four to five times that obtainable at atmospheric pressure.

Another means of increasing the voltage rating of an air-dielectric condenser is to place it in a vacuum comparable with that used in vacuum tubes. Under these condi-

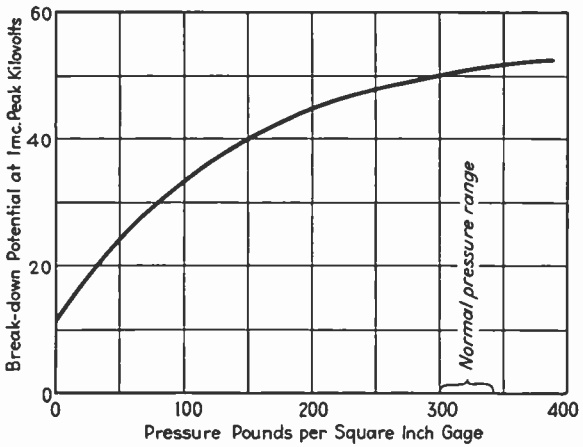


FIG. 83.—Breakdown voltage as a function of pressure at a frequency of one megacycle.

tions, there is so little gas present that ionization is not possible in appreciable amounts, and the dielectric strength is very high. Condensers of this type can be either fixed or variable. In the case of variable condensers, the adjustment is made from outside the vacuum by means of a copper bellows or flexible diaphragm.

33. Condensers with Solid Dielectric. *Types of Dielectrics Used.*—Solid dielectric is used in most fixed and in some adjustable condensers. The dielectrics most commonly employed are mica, paper, ceramics, and electrolytic films.

Mica is characterized by an unusually low power factor and high voltage strength. Mica condensers are commonly constructed by piling alternate sheets of mica and copper foil to form a stack that is held under pressure and impregnated. Such condensers are used for by-pass and blocking purposes and also whenever a fixed capacity of very low losses and appreciable capacity is required.[2]

Paper condensers normally consist of two strips of tin or aluminum foil insulated by paper and rolled into a bundle. This is vacuum-impregnated with oil or wax, and sealed against moisture. Paper condensers are characterized by relatively low cost, high capacity in proportion to voltage rating and size, and a power factor that is

[1] Pressure Capacitors, *Electronics*, Vol. 12, p. 16, April, 1939.

[2] The capacity of small mica condensers is frequently indicated by a series of three colored dots, which indicate the capacity in $\mu\mu f$ by the same color code used with resistors, described on page 39. The sequence in which the dots should be read is indicated by an arrow.

Ex. 1044-0132

reasonably high (commonly of the order of 0.5 per cent). Such condensers are used primarily for by-pass and blocking purposes and for filter condensers in power-supply systems. The inexpensive type of paper condensers sealed in waxed cardboard containers deteriorate with time as a result of gradual penetration of moisture, and so have a limited life. This life is also dependent upon the d-c voltage applied to the condenser.[1]

Ceramic dielectrics are used to obtain controlled temperature coefficients. The ceramics used frequently contain titanium oxide (see Table 27), to give a very high dielectric constant. The power factor of such condensers can be quite low, and is frequently less than 0.001.

Electrolytic condensers are considered in Par. 34.

Power Factor, Resistance, and Reactance of Condensers.—Condensers with solid dielectric can be represented by the same equivalent electrical circuit shown in Fig. 80 for air condensers. The capacity C in this equivalent circuit is determined by the area, thickness, and dielectric constant of the dielectric. The inductance arises from flux linkages associated mainly with the leads. The loss in the condenser at low and moderate frequencies arises primarily from the conductance G, so the condenser power factor at such frequencies is equal to the power factor of the dielectric, and is independent of frequency.[2] However, as the frequency is increased the series resistance R begins to be of importance, and the power factor decreases. This series resistance tends to be proportional to the square root of frequency, making the power factor proportional to $f^{3/2}$ when the frequency f is so high that the series resistance is the principal source of loss.

Temperature Coefficient of Capacity.—Change in temperature can cause the capacity of condensers having solid dielectric to vary as a result of the effect of temperature on the dimensions, dielectric constant, and mechanical stress (and hence pressure) in the condenser. In ordinary mica and impregnated paper condensers, the effect of the temperature upon the pressures in the condenser assembly is the most important factor causing the capacity to change, and results in temperature coefficients of capacity that are large and noncyclic.

A low and very stable coefficient of temperature can be obtained by depositing the condenser electrodes either directly upon the opposite sides of a mica sheet or upon the inside and outside of a suitable ceramic tube. The metalized mica type of condenser gives a temperature coefficient of capacity of approximately +20 parts in a million per°C. The coefficient of the ceramic type of condenser can be controlled by the ceramic material employed, with values ranging all the way from about +200 to −800 parts in a million per°C. being available in commercial condensers.

Voltage Rating.—The voltage rating of a condenser at low frequencies is the voltage at which the dielectric will spark through. As the frequency is increased the voltage rating becomes less as a result of energy dissipated in the condenser. When the power factor of the dielectric accounts for most of the loss, as is the case at moderate radio frequencies, the voltage rating will be inversely proportional to the square root of the frequency. At higher frequencies, where the series resistance is also a factor, the permissible voltage decreases still more rapidly with frequency.

The voltage rating of a typical mica condenser at various frequencies is given in Table 31. These ratings assume that the voltage is applied continuously for a con-

[1] It has been stated that the life of an ordinary paper condenser such as is used in radio receivers is proportional to the fifth power of the applied voltage. This fact is sometimes used for accelerated life tests by operating the condenser at excess voltage and then using this rule to determine the significance of the results. Such tests neglect the time factor involved in moisture penetration, however, and so are only approximate.

[2] At quite low frequencies, there is usually a slight rise in the power factor as a result of leakage and absorption.

Ex. 1044-0133

siderable length of time. When the voltage acts for a brief period in such a way that the condenser does not have time to exceed its allowable operating temperature, correspondingly higher voltages may be applied at the higher frequencies.

TABLE 31.—VOLTAGE RATING OF A PARTICULAR 0.001 μf MICA CONDENSER*

Frequency, kc	Rated effective voltage, volts	Rated current, amp.	Rated energy, kva
Direct current	10,000		
1	10,000	0.063	0.63
100	3,000	1.9	5.7
300	3,000	5.7	17.0
1,000	1,780	11.2	20.0
3,000	605	11.4	8.9
10,000	178	11.2	2.0

* I. G. Maloff, Mica Condensers in High Frequency Circuits, *Proc. I.R.E.*, Vol. 20, p. 647, April, 1932.

34. Electrolytic Condensers.[1]—An electrolytic condenser consists of positive and negative electrodes, usually of aluminum, placed in a suitable conducting liquid. The dielectric of such a condenser is provided by a thin insulating film formed directly on the positive electrode by suitable treatment.

Electrolytic condensers are characterized by a low cost per microfarad, a very large capacity in proportion to volume, a high power factor, and appreciable leakage. The characteristics such as capacity, leakage resistance, and power factor depend greatly upon the applied voltage, temperature, previous history of the condenser, etc. The ordinary electrolytic condensers used in radio work are polarized, so that the voltage on the anode plate must never be allowed to become negative even instantaneously. This limits the use of electrolytic condensers to circumstances where the alternating voltage present is less than the superimposed d-c voltage.

Electrolytic condensers are used as by-pass and filter condensers in radio receivers in parts of the circuit where a d-c potential is present and where a moderate leakage current can be tolerated.

Electrolytic Condensers.—Electrolytic condensers used in radio work are nearly always of the "dry" type. In such condensers, the positive and negative electrodes consist of aluminum foil separated by paper or gauze that is saturated with an electrolyte that is either a highly viscous liquid or a fudgelike solid. The entire assembly is wound in a roll and mounted in a waxed cardboard tube or box.[2] The anode is usually chemically etched to increase its effective area. The insulating film on the anode is formed chemically before assembly by passing the foil through suitable solutions and applying a proper sequence of voltages between the foil and the solution. After the condenser has been assembled any damage that may have occurred to the

[1] For further information, particularly on constructional details, see P. M. Deeley, "Electrolytic Capacitors," Cornell Dubilier Electric Corp., 1938; P. R. Coursey, "Electrolytic Condensers," 2d ed., Chapman and Hall, 1939.

[2] In the "wet" type of condenser, the anode consists of folded aluminum foil; the cathode electrode is usually an aluminum container lined inside with a perforated insulating material to prevent contact between cathode and anode. The electrolyte is a liquid, frequently moderately viscous.

Compared with the dry electrolytic condenser, the wet type has the disadvantages of higher power factor, narrower temperature range over which satisfactory operation can be obtained, and greater deterioration while lying idle. Furthermore, wet condensers must be provided with a vent to prevent excessive internal pressures from building up and must be mounted in a particular position to avoid leakage. The wet type is capable of giving voltage limiting action, since it can be designed to draw an excessively large leakage current when the voltage on the circuit becomes larger than normal. In such service, the wet type has both greater reliability and life and also possesses a more rapid rise of leakage current with voltage than the dry type.

Ex. 1044-0134

anode film through handling, etc., is repaired by a reforming process that consists of applying voltage to the condenser under suitably controlled conditions.

Characteristics of Electrolytic Condensers.[1]—The capacity of an electrolytic condenser is influenced by the applied voltage, length and continuity of service, voltage at which the insulating film was formed on the anode, the temperature, and the frequency. In general the capacity decreases with age, increased forming voltage, increased temperature, and increased frequency. In condensers that must carry appreciable alternating currents, as, for example, the first condenser in a condenser-input rectifier filter system, there is a tendency for an insulating film to form on the cathode during use, with consequent excessive loss of capacity.

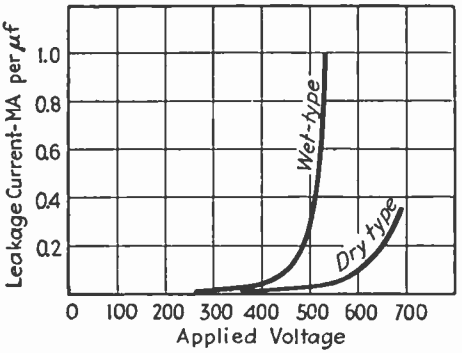


FIG. 84.—Leakage current in typical electrolytic condensers.

The leakage resistance of an electrolytic condenser depends upon the ratio of voltage applied to the voltage used in forming the anode film, and is affected by temperature, amount and conditions of use, etc. The leakage current varies with applied voltage in the manner shown in Fig. 84. It is quite small with applied voltages appreciably less than the forming voltage, but increasing rapidly in the region of the forming voltage, the rise being particularly great in the wet type. When voltage is applied to a condenser after a period of idleness, the leakage current is initially excessively large, but quickly drops to a normal value, as shown in Fig. 85. The leakage current for any particular operating voltage increases as the temperature becomes greater.

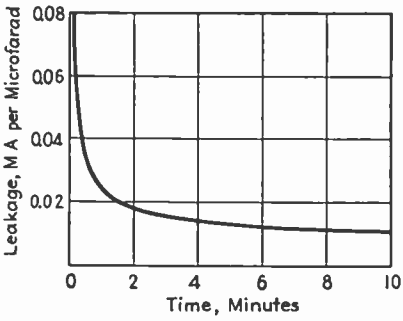


FIG. 85.—Curve illustrating typical manner in which leakage current varies with time when voltage is first applied to an electrolytic condenser that has been idle for some time.

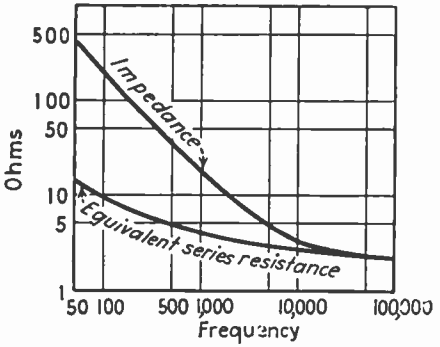


FIG. 86.—Impedance and equivalent series resistance of a typical electrolytic condenser having a normal capacity of 8 μf.

The power factor of electrolytic condensers is high, and increases with frequency and increasing applied voltage. The power factor may increase or decrease with

[1] A good summary of properties of wet and dry electrolytic condensers is given by P. R. Coursey and S. N. Ray, Electrolytic Condensers, *Jour. I.E.E.*, Vol. 85, p. 107, 1939; also, Wireless Section, *I.E.E.*, Vol. 14, p. 203, September, 1939.

Ex. 1044-0135

temperature, depending upon the construction. At high frequencies the power factor becomes so high that the condenser acts essentially as a series resistance rather than as a capacity (see Fig. 86).

The maximum voltage that can be safely applied to an electrolytic condenser depends upon the voltage at which the insulating film was formed, with the rated voltage being a little less than the forming voltage. Rated voltages ranging all the way from approximately 25 to 550 volts peak are obtainable. When the applied voltage exceeds the rated value, the leakage current becomes excessive, as shown in Fig. 84, and finally scintillations are produced as a result of breaking down of the dielectric film.

The heating produced by the d-c leakage current and the alternating currents passing through an electrolytic condenser must not produce excessive temperature rise. This means in general that the compact types of condensers, such as those using etched foil, and particularly the dry type, are at a disadvantage as compared with bulkier constructions for certain types of service, as, for example, when a large alternating current is present.

The life of an electrolytic condenser depends upon the purity of the materials used in the condenser and upon the conditions of use, particularly the applied voltage and the temperature. Leakage is an indication of life, since leakage causes the electrolyte to be used up, particularly in the wet type, with a corresponding increase in condenser power factor. The life is accordingly increased by anything that reduces the leakage current, such as operating at appreciably below the forming voltage, and by avoiding high temperatures.

SHIELDING OF MAGNETIC AND ELECTROSTATIC FIELDS

35. Conducting Shields for Magnetic Fields.—Magnetic and electrostatic fields can be confined to restricted spaces, or can be prevented from entering a particular space, by the use of suitable shields. The most effective means of controlling magnetic fields at radio frequencies is by the use of shields consisting of a good conductor, such as copper or aluminum. Magnetic flux penetrates such a shield only with great difficulty, because as the flux cuts into the conducting material it produces eddy currents that oppose the penetration. To be most effective, such shields must completely inclose the space to be protected, and all joints should be well lapped to minimize the resistance offered to the eddy currents. Where the shielding must be extremely effective, as in signal generators, joints should be soldered wherever possible, and in other cases should be sufficiently good mechanically to be literally watertight. The thickness should be a number of times the skin depth of current penetration as calculated by Eq. (5), and so has a minimum value determined by the conductivity and frequency.

Shielded Coils.[1]—One of the most important examples of a conducting shield is the copper or aluminum can frequently placed around air-cored coils. Such shields serve to confine practically all of the magnetic field to the space within the can. They also reduce the effective inductance of the coil by restricting the cross-sectional area through which the flux can pass, and increase the losses as a result of the energy dissipated in the shield by the eddy currents. However, if the clearance between the coil and its shield is everywhere at least equal to the radius of the coil, the reduction in inductance and the decrease in coil Q produced by a copper or aluminum shield will not be serious.

The loss of inductance resulting from a cylindrical shield that is long compared with the length of the coil and the shield diameter is given approximately by the curves

[1] An excellent theoretical discussion of the effect of shielding on coil resistance and inductance is given by A. G. Bogle, The Effective Resistance and Inductance of Screened Coils, *Jour. I.E.E.*, Vol. 87, p. 299, 1940; also, Wireless Section, *I.E.E.*, Vol. 15, p. 221, September, 1940.

Ex. 1044-0136

of Fig. 87.[1] The results also apply with satisfactory accuracy for finite lengths if the clearance between the ends of the shield and the coil is at least the coil radius, and the shield length exceeds the shield diameter. As these limiting conditions are approached, the actual inductance is slightly less than that calculated from Fig. 87.

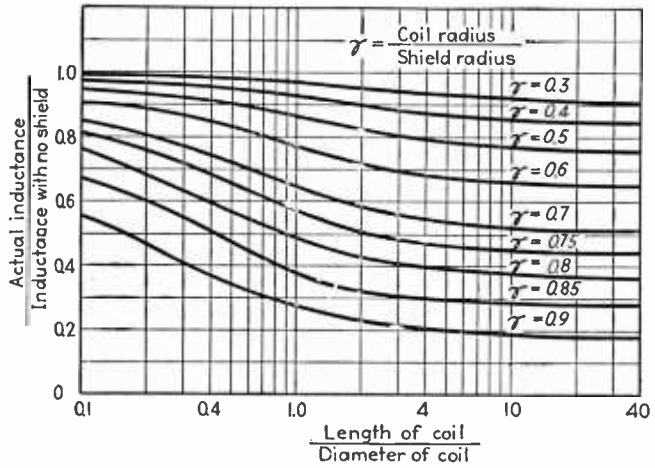


FIG. 87.—Curves giving effect of shield on the inductance on an air-cored coil.

Where Fig. 87 can be expected to be in error, the effect of the shield, including the end effects, can be more accurately taken into account by the equation[2]

$$\frac{\text{Actual inductance}}{\text{Inductance in absence of shield}} = \left[1 - \left(\frac{r_c}{r_s}\right)^2 \frac{l_c}{l_s}\frac{1}{K}\right] \tag{149}$$

where $\frac{l_c}{l_s} = \frac{\text{length of coil}}{\text{length of shield}}$.

$\frac{r_c}{r_s} = \frac{\text{radius of coil}}{\text{radius of shield}}$.

K = constant given by Fig. 88.

The resistance R_s that must be considered as added in series with the shielded coil in order to take into account the shield losses is given approximately by the equation

$$R_s = \frac{9.37 \times 10^{-4} N^2 r_c^4 \sqrt{f\rho}}{r^4} \tag{150}$$

where N = number of turns in coil.

r_c = radius of coil.

$r = \sqrt[3]{(\text{shield diameter})^2(\text{shield length})/8}$.

f = frequency in cycles.

ρ = resistivity of shield in ohms per cm cube ($= 1.724 \times 10^{-6}$ for copper).

[1] Graphic Determination of the Decrease in Inductance Produced by a Coil Shield, *R.C.A. Application Note* 48, June 12, 1935.

These curves can also be applied to the case of a square shielding can that is long compared with the coil length by assuming that the square can is equivalent to a cylindrical shield having a radius that is 0.6 times the length of one side of the can.

[2] Equations (149)–(151) are rearrangements of results given by G. W. O. Howe, The Effect of Screening Cans on the Effective Inductance and Resistance of Coils. *Wireless Eng.*, Vol. 11, p. 155, March, 1934. These equations are derived by replacing the actual shield by a sphere of diameter $2r$ that is the geometric mean of the three coordinate dimensions of the actual shield can. If these three dimensions do not differ too greatly, and if the shield clears the coil by a reasonable amount, then the results calculated for the hypothetical spherical shield approximate very closely those obtained with the actual shield.

Ex. 1044-0137

This equation assumes that the thickness of the shield is appreciably greater than the skin depth as obtained from Eq. (5) or Fig. 4. When the thickness t in cm is not small compared with the skin depth, then

$$R_s = \frac{4.7N^2\rho}{t}\left(\frac{r_c}{r}\right)^4 \qquad (150a)$$

The effectiveness with which the shield surrounding a coil reduces the external field depends primarily upon the thickness t of the shield in relation to the skin depth ϵ.

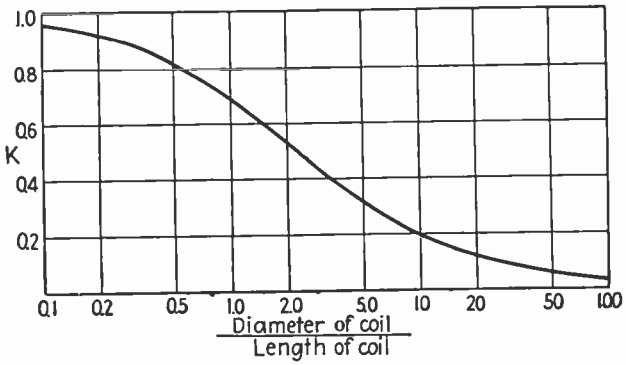


FIG. 88.—Values of constant K for use in Eq. (149).

Where $t/\epsilon > 2$, and where at the same time the shield is thin compared with its length or diameter, one has

$$\frac{\text{Magnetic field in absence of shielding}}{\text{Actual magnetic field}} = 0.24\frac{r}{\epsilon}e^{\frac{t}{\epsilon}} \qquad (151)$$

where r is as defined in Eq. (150), ϵ is the skin depth as calculated by Eq. (5), or obtained from Fig. 4, t is the thickness of the shield, and $e = 2.718$.

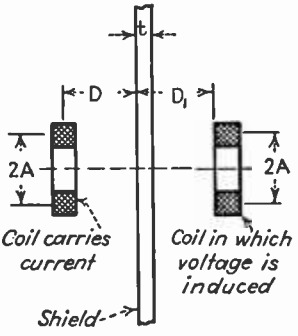


FIG. 89.—Diagram illustrating nomenclature in Eq. (152).

Planar Shield.—An important case of shielding exists where a flat sheet of conducting material is used to reduce the voltage induced in a second coil by current passing through a first coil. For the case illustrated in Fig. 89, where the two coils are coaxial and similar and the shield is a sheet placed at right angles to this axis, one has approximately[1]

$$\frac{\text{Induced voltage in absence of shield}}{\text{Induced voltage with shield}} = 0.0127At\gamma f \qquad (152)$$

where γ = conductivity of shield on basis that conductivity of standard annealed copper is 100.
A = mean radius of coil, cm.
t = thickness of shield, cm.
f = frequency in cycles.

Strictly speaking, Eq. (152) applies only where the shielding is appreciable and the shield is infinite in extent, but practically the results will not be greatly influenced by

[1] Samuel Levy, Electromagnetic Shielding Effect of an Infinite Plane Conducting Sheet Placed between Circular Coaxial Coils, *Proc. I.R.E.*, Vol. 24, p. 923, June, 1936.

Ex. 1044-0138

finite dimensions, provided the length and width of the shield are considerably greater than $2A$ or $D + D_1$ in Fig. 89, whichever is larger.

Shields Immersed in Uniform Magnetic Field.—Shields are sometimes used to protect a small space from the effects of an external field. The effectiveness of spheres and of long cylinders of conducting material in providing an interior field-free space when the shield is placed in a uniform external field is given by[1]

$$\frac{\text{Field intensity in absence of shield}}{\text{Field in interior of shield}} = 0.707S\frac{r}{\epsilon}e^{\frac{t}{\epsilon}} \qquad (153)$$

where S = a constant that is one-third for spheres and one-half for cylinders.
ϵ = skin depth as given by Eq. (5) or Fig. 4.
t = thickness of shield.
r = radius of sphere or cylinder.
e = 2.718, the base of the Napierian logarithms.

In Eq. (153) it is assumed that $t/\epsilon > 2$. In the case of cylinders, external field is assumed to be parallel to the axis, and the cylinder is considered to be of infinite length. However, if the length is a number of times the diameter, the field at the midpoint of the cylinder will be substantially as calculated, even when the ends are open. The results calculated for the sphere can be used to give an approximate indication of the shielding of cubes and similar shapes by using an equivalent sphere inclosing the same volume as the actual shield.

36. Magnetic Shields.—Shields for unidirectional and low-frequency magnetic fields are made of magnetic material, preferably having a high initial permeability. Such shields act as low reluctance paths for the flux, thereby diverting the flux away from the space to be shielded.

The most important practical use of magnetic shields in communication is to minimize voltages induced in audio transformers by magnetic fields of power frequencies. This is accomplished by placing the transformer in a magnetic shield normally consisting of a rectangular box or a short cylinder with closed ends. The degree of protection obtained in this way depends upon the size and thickness of the shield, and upon whether the shield consists of a single layer of magnetic material or several concentric layers separated by air spaces. In the case of a single-layer shield, the effectiveness is approximately[2]

$$\frac{\text{Magnetic field in absence of shielding}}{\text{Magnetic field with shielding}} = 0.22\mu\left[1 - \left(1 - \frac{t}{r_0}\right)^3\right] \qquad (154)$$

where μ = initial permeability of shield.
r_0 = radius of sphere enclosing the same volume as the outer surface of the shield.
t = thickness of shield.

Equation (154) shows that the maximum possible shielding factor obtainable is 0.22μ and that approximately 50 per cent of this possibility is realized when the thickness of the shield is one-fifth of the radius of the equivalent sphere. Relatively thick single-layer shields are accordingly uneconomic.

When the degree of shielding required is greater than that obtainable from a single shield, two or three concentric magnetic shielding shells are employed, as shown in Fig. 90. When the total thickness of such a shield is greater than one-third the radius of the equivalent sphere, this subdivision of the magnetic material into concentric layers separated by air spaces gives more effective shielding than if the air spaces were filled solidly with magnetic material (see Fig. 90).

[1] Walter Lyons, Experiments on Electromagnetic Shielding at Frequencies between One and Thirty Kilocycles, *Proc. I.R.E.*, Vol. 21, p. 574, April, 1933.
[2] W. B. Ellwood, Magnetic Shields, *Bell Lab. Rec.*, Vol. 17, p. 93, November, 1938.

Ex. 1044-0139

The effectiveness of concentric magnetic shields to alternating currents can be greatly increased by placing copper shields in the air spaces between the magnetic shields.[1] The behavior in a typical case with and without copper shields is shown in Fig. 91 as a function of frequency over the audio range. With unidirectional (d-c) fields, the copper has no effect upon shielding.

The effectiveness of magnetic shields depends primarily upon the initial permeability and is roughly proportional to μ^n where n is the number of concentric layers. Permalloy and similar magnetic materials having high initial permeability are,

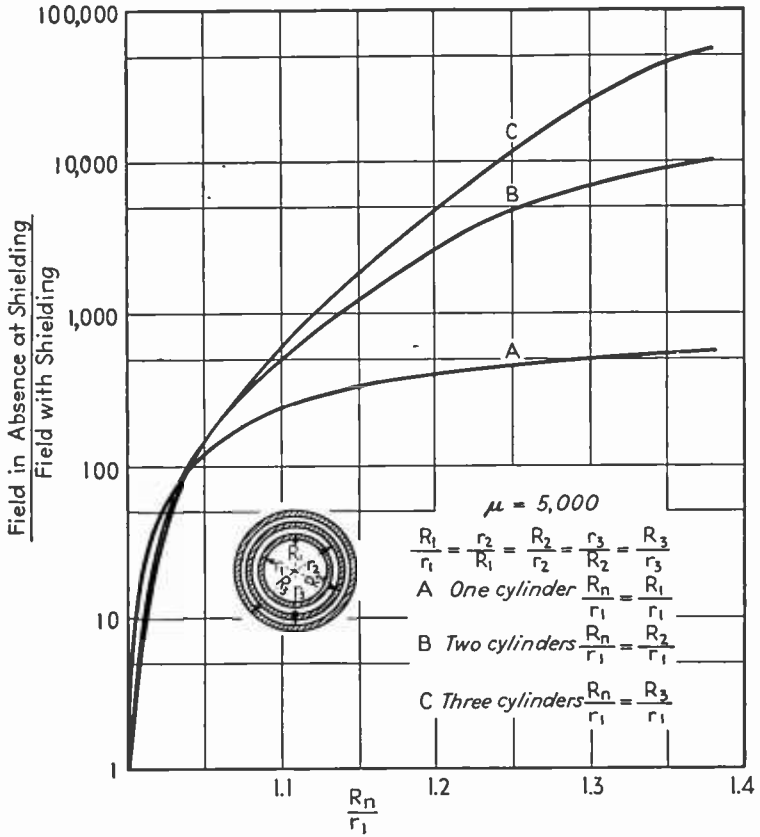


FIG. 90.—Performance of spherical magnetic shields in a typical case.

accordingly, vastly superior to ordinary cast iron or silicon steel, particularly in the case of multilayer shields.

A magnetic shield inclosing an air-cored coil causes the inductance to increase as a result of lowered reluctance offered to the flux paths by the high permeability material. The effective resistance of the shielded coil is also increased because of eddy-current and hysteresis losses in the iron.

37. Electrostatic Shields.—Any conductor can be used as an electrostatic shield. There are no particular requirements as to thickness or conductivity; it is necessary

[1] W. G. Gustafson, Magnetic Shielding of Transformers at Audio Frequencies, *Bell System Tech. Journ.*, Vol. 17, p. 416, July, 1938.

Ex. 1044-0140

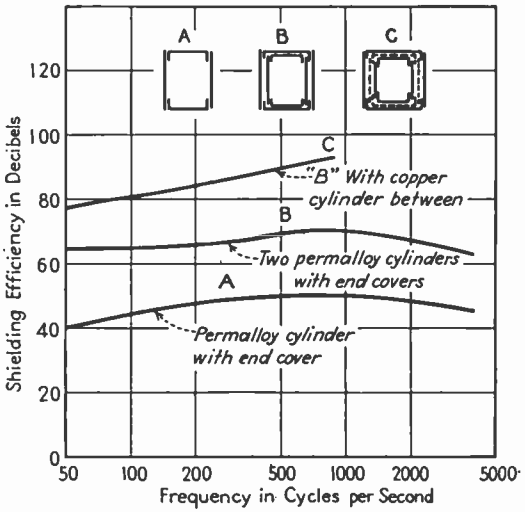


FIG. 91.—Curves showing effectiveness of various typical shielding arrangements for low-frequency fields.

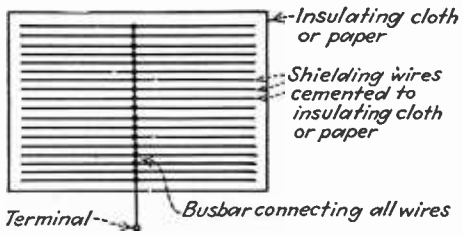


FIG. 92.—Electrostatic shield that does not affect magnetic flux.

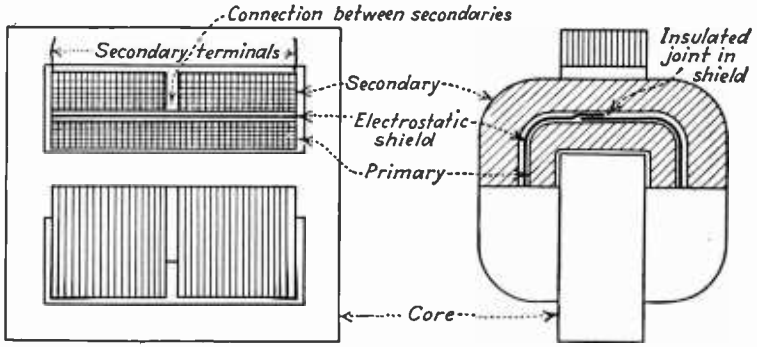


FIG. 93.—Transformer having a balanced secondary winding and an electrostatic shield between primary and secondary.

merely that the electrostatic fields terminate on the shield instead of passing through to the space being protected. To be most effective, electrostatic shields should be continuous closed surface, but even a screen will give fair shielding. The conducting and magnetic shields for magnetic flux discussed above therefore simultaneously provide electrostatic shielding along with the magnetic shielding.

Electrostatic Shielding without Magnetic Shielding.—Electrostatic shielding can be obtained without affecting magnetic flux by surrounding the space to be protected with a conducting nonmagnetic shield arranged to provide termination for electrostatic flux lines without providing closed loops of low resistance around which eddy currents can circulate. Such a shield is illustrated in Fig. 92.

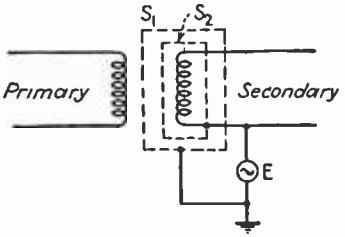


FIG. 94.—Schematic diagram of a double electrostatic shield.

Electrostatic shielding is frequently used in transformers to prevent electrostatic coupling between the various windings. Such a shield consists of a metal foil wrapped around the windings so as to inclose them completely but with an insulated lapped joint, as shown in Fig. 93, so that the foil does not act as a short-circuited turn. Such a shield replaces direct capacity coupling between coils by capacity from the individual coil to the shield (*i.e.*, to the ground).

Double electrostatic shields are sometimes required in transformers used in special applications. Thus in Fig. 94, if the secondary is at an a-c potential E above ground and it is absolutely necessary that this potential produce no voltage difference between the secondary terminals of the transformer, it is then necessary to employ a double shield around the secondary as shown. The outer shield S_1 is grounded, while the inner shield is insulated from S_1 and connected to one terminal of the secondary. In this way the capacity current flowing to ground as a result of the voltage E will not go through any part of either the secondary or primary winding, and so will not produce any voltage difference between secondary terminals.

Ex. 1044-0142

SECTION 3

CIRCUIT THEORY

SIMPLE RESONANT CIRCUITS

1. Series Resonance.—A circuit consisting of a resistance, inductance, and capacity connected in series with a voltage applied as shown in Fig. 1 is termed a *series resonant circuit.* When the circuit resistance is low, as is normally the case, the current depends upon the frequency of the applied voltage in accordance with the manner shown in Fig. 2*a*. The current is a maximum and in phase with the applied voltage at the frequency, termed the resonant frequency, at which the inductive reactance and capacitive reactances are equal. At lower frequencies the current falls off and leads, while at higher frequencies it drops and lags.

The shape of the series resonance curve of current is determined by the ratio of the inductive reactance ωL to the circuit resistance R, *i.e.*, upon $\omega L/R$. This ratio is customarily represented by the symbol Q, and is called the *circuit Q.* Its numerical value commonly approximates the Q of the coil used in the resonant circuit because most of the loss in resonant circuits is usually in the coil, but will always be at least slightly less then the coil Q because of the condenser losses. The effect upon the resonance curve of changing the circuit resistance, and hence the circuit Q, is shown in Fig. 2*a*. Increasing the circuit resistance reduces the response in the vicinity of resonance, although producing virtually no effect at frequencies differing appreciably from the resonant frequency.

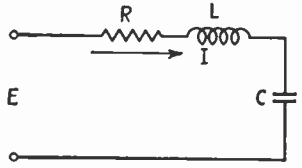


FIG. 1.—Series resonant circuit.

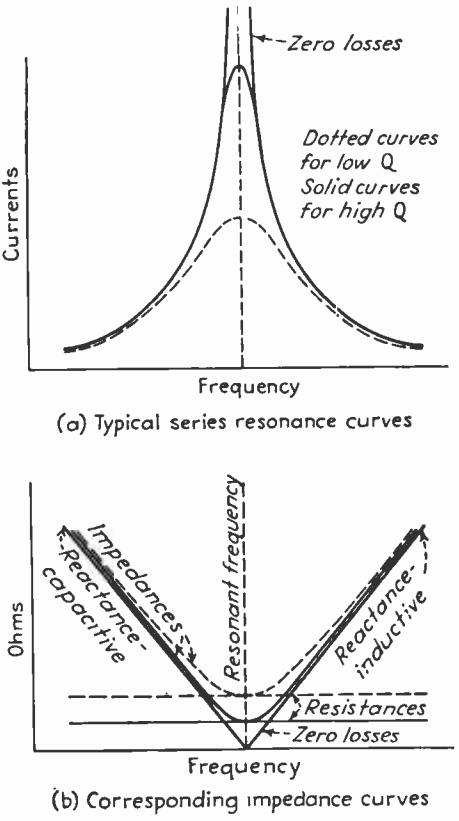


FIG. 2.—Current and impedance of a series resonant circuit as a function of frequency.

Ex. 1044-0143

The resistance and reactance components of the impedance of the series circuit vary with frequency as shown in Fig. 2*b*. In the small frequency range represented by the region around resonance, the circuit resistance is substantially constant. The reactance, on the other hand, varies substantially linearly from a relatively high capacitive value below resonance to a relatively high inductive value above resonance, and passes through zero at the resonant frequency.

General Equations of Series Resonance.—The equations given below use the following symbols:

E = voltage applied to circuit.
I = current flowing in circuit.
f = frequency in cycles.
$\omega = 2\pi f$.
$Q = \omega L/R$.
R = effective series resistance of tuned circuit.
L = inductance, henrys.
C = capacity, farads.
Z = impedance of series circuit.
θ = phase angle of impedance.
$\gamma = \dfrac{\text{actual frequency}}{\text{resonant frequency}}$.
Subscript $_0$ denotes values at resonant frequency.

$$\text{Resonant frequency} = \frac{1}{2\pi\sqrt{LC}} \tag{1}$$

$$\text{Circuit impedance} = Z = R + j\left(\omega L - \frac{1}{\omega C}\right) \tag{2a}$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{2b}$$

$$\tan\theta = \frac{\omega L - \frac{1}{\omega C}}{R} = Q\left(1 - \frac{1}{\gamma^2}\right) \tag{2c}$$

$$I = \frac{E}{Z} = \frac{E}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \tag{3}$$

$$\text{Voltage across } L = j\omega LI \tag{4a}$$

$$\text{Voltage across } C = \frac{I}{j\omega C} \tag{4b}$$

$$\text{Current at resonance} = I_0 = \frac{E}{R} \tag{5}$$

$$\text{Voltage across } L \text{ or } C \text{ at resonance} = EQ \tag{6}$$

Universal Resonance Curves.—Equation (3), when rearranged to express the current actually flowing in the circuit at resonance in terms of the circuit Q and the fractional detuning, gives the *universal resonance* curves shown in Fig. 3,[1] from which the exact resonance curve of any series circuit may be obtained without calculation when Q is known. These curves are extremely useful, because they are independent of the resonant frequency of the circuit and of the ratio of inductance to capacity.

The use of the universal resonance curves in practical calculations can be illustrated by two examples.

Example 1.—It is desired to know how many cycles one must be off resonance to reduce the current to one-half the value at resonance when the circuit has a Q of 125 and is resonant at 1,000 kc. Refer-

[1] For the derivation of these curves, see F. E. Terman, "Radio Engineering," 2d ed., p. 54, McGraw-Hill, New York, 1937.

Ex. 1044-0144

ence to Fig. 3 shows the response is reduced to 0.5 when $a = 0.86$. Hence

$$\text{Cycles off resonance} = \frac{0.86 \times 1{,}000}{125} = 6.88 \text{ kc}$$

The phase angle of the current as obtained from the curve is 60°.

Example 2.—With the same circuit as in the preceding example, it is desired to know what the response will be at a frequency 10,000 cycles below resonance. To solve this problem it is first necessary to determine a.

$$a = \left(\frac{10}{1{,}000}\right) \times 125 = 1.25$$

Reference to Fig. 3 shows that for $a = 1.25$, the response is reduced by a factor 0.365 and that the phase of the current is 68° leading.

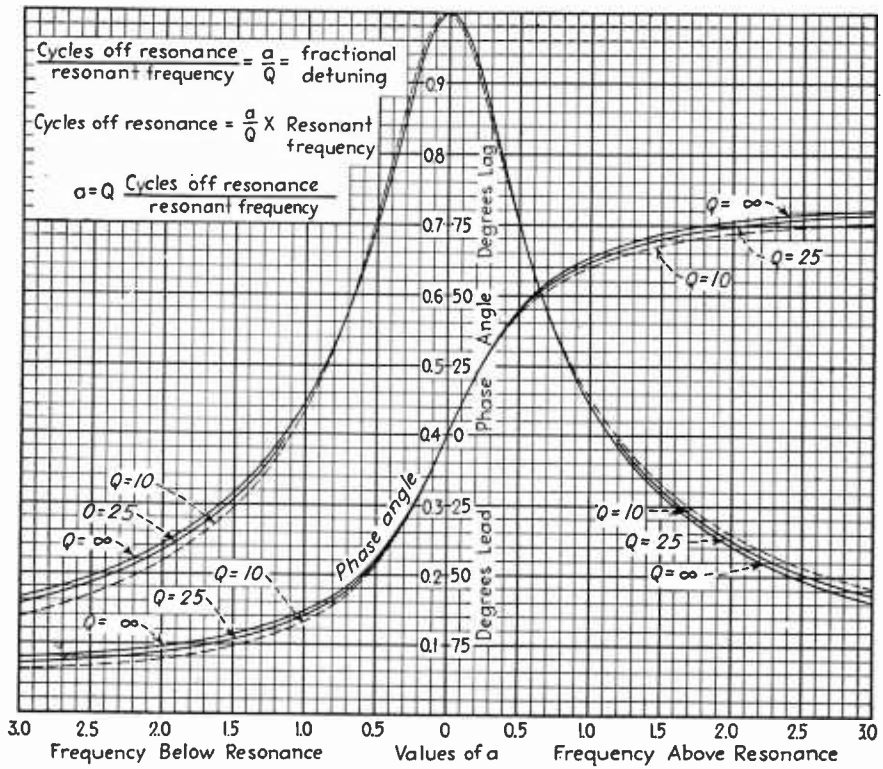


FIG. 3.—Universal resonance curve for series resonant circuit. This curve can also be applied to the parallel resonant circuit by considering the vertical scale to represent the ratio of actual parallel impedance to the parallel impedance at resonance. When applied to parallel circuits the angles shown in the figure as leading are lagging, and *vice versa*.

The only assumption involved in the universal resonance curves is that Q is the same at the frequency being considered as at the resonant frequency. *When this is true, the universal resonance curves involve no approximations whatsoever.* Over the limited range of frequencies near resonance represented in Fig. 3, the variations in Q are so small as to introduce negligible (*i.e.*, less than 1 per cent) errors from the use of the curves, provided the value of Q existing at resonance is used in determining the fractional detuning.

The universal resonance curve also gives the resonance curve of voltage developed across the inductance or capacity with an accuracy sufficient for all ordinary purposes.

Ex. 1044-0145

This is because under ordinary conditions the frequency range covered by the curves extends only a few per cent on either side of resonance. For this limited range of frequencies, the inductive (and capacitive) reactances are so nearly constant that the product of current and reactance gives a curve having essentially the same shape as the current curve.

Working Rules for Dealing with Series Resonant Circuits.—In making calculations of series resonant circuits or in estimating their behavior, the following rules and principles will frequently be found of value:

The resonance curve can for all practical purposes be considered as symmetrical about the resonant frequency. This is apparent from an examination of the universal resonance curves.

The current at resonance is always E/R, while the voltage developed across the capacitive (or inductive) reactance at resonance is Q times the applied voltage and is 90° out of phase with this voltage.

When the frequency of the applied voltage deviates from the resonant frequency by an amount that is $1/2Q$ of the resonant frequency, the current that flows is reduced to 70.7 per cent of the resonant current, and the current is 45° out of phase with the applied voltage. Correspondingly, when the fractional detuning is $1/Q$, the current is approximately 45 per cent of the current at resonance, and is 63.5° out of phase with the voltage.[1]

At frequencies that differ from the resonant frequency by amounts that exceed $3/Q$ of the resonant frequency, one can neglect the circuit resistance in determining the magnitude of the circuit impedance. The magnitude of the current is then

$$\frac{\text{Actual current when far off resonance}}{\text{Current at resonance}} = \frac{1}{Q\gamma\left(1 - \frac{1}{\gamma^2}\right)} \tag{7}$$

where $\gamma = \dfrac{\text{actual frequency}}{\text{resonant frequency}}$.

The phase angle of the current can be calculated without error by the use of Eq. (2c).

$$\tan\theta = Q\left(1 - \frac{1}{\gamma^2}\right)$$

[1] Other convenient cases are as follows:

TABLE 1

Fractional detuning $\left(= \frac{\text{cycles off resonance}}{\text{resonant frequency}}\right)$	$\frac{\text{Actual current}}{\text{Current at resonance}}$	Phase of current, °
$\frac{1}{6Q} = \frac{1}{3} \times \frac{1}{2Q}$	0.95	18½
$\frac{1}{4Q} = \frac{1}{2} \times \frac{1}{2Q}$	0.90	26½
$\frac{1}{2Q} = 1 \times \frac{1}{2Q}$	0.707	45
$\frac{1}{Q} = 2 \times \frac{1}{2Q}$	0.447	63½
$\frac{2}{Q} = 4 \times \frac{1}{2Q}$	0.242	76
$\frac{4}{Q} = 8 \times \frac{1}{2Q}$	0.124	83

Ex. 1044-0146

The voltage across the inductance and also the voltage across the condenser vary with frequency in the immediate vicinity of resonance in a way that is for all practical purposes the same as the current resonance curve. At frequencies differing somewhat from the resonant frequency, there is a slight but ordinarily unimportant difference in shape between the voltage and current curves. At frequencies that deviate from resonance by at least $3/Q$ times the resonant frequency, it is permissible to neglect the resistance in making calculations. This leads to the following formulas:

For voltage across the inductance:

$$\frac{\text{Actual voltage when far off resonance}}{\text{Voltage at resonance}} = \frac{1}{Q\left(1 - \frac{1}{\gamma^2}\right)} \tag{8a}$$

For voltage across the condenser:

$$\frac{\text{Actual voltage when far off resonance}}{\text{Voltage at resonance}} = \frac{1}{Q(\gamma^2 - 1)} \tag{8b}$$

Practical Calculation of Series Resonance Curves.—Series resonance curves can of course be calculated with the aid of Eqs. (1) to (3). Practically, however, this is not a very satisfactory way to obtain them, since in order to obtain reasonable accuracy in the region near resonance, it is necessary to use logarithms in forming the difference $\omega L - 1/\omega C$ in cases where the Q is at all appreciable.

A preferable procedure is first to calculate the resonant frequency and the response at resonance. The former can be conveniently done with the aid of the reactance charts given on the inside back cover of this volume. The working rules given above can then be applied to obtain the response at frequencies $1/2Q$, $1/Q$, etc., on either side of resonance. This gives a picture of the resonance curve that is sufficient for most practical purposes. Additional points in the vicinity of resonance can be calculated when needed with the aid of the universal resonance curve of Fig. 3. Calculations for frequencies too far from resonance to be included on the universal resonance curve can be made by the use of Eq. (8*a*) (or 8*b*) and Eq. (2*c*).

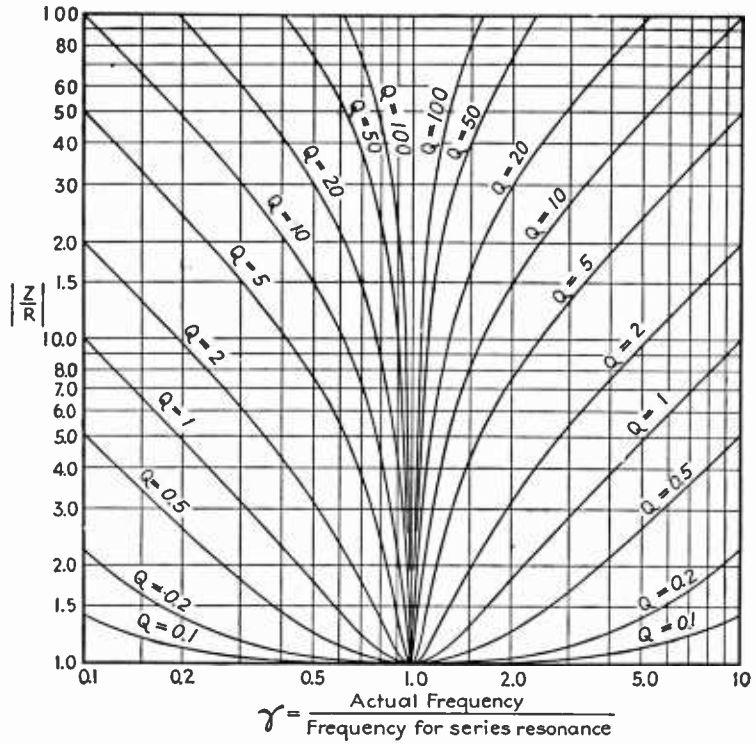
Logarithmic Decrement.—The sharpness of the series resonance curve is normally expressed in terms of the Q of the circuit. In the early days of radio, when the oscillating spark discharge represented the chief source of radio-frequency energy, the sharpness of resonance was expressed in terms of the *logarithmic decrement*, defined by the equation

$$\text{Logarithmic decrement} = \frac{R}{2f_0L} \tag{9}$$

where R and L represent circuit resistance and inductance, respectively, and f_0, the resonant frequency. It will be noted that the logarithmic decrement is equal to π/Q.

Series Resonance by Varying Circuit Reactance.—The preceding discussion of series resonance has been for the case where the circuit resistance, inductance, and capacity were constant and the frequency was varied. Under many circumstances, however, the frequency is kept constant and the circuit is adjusted to series resonance by varying either the capacity or the inductance. When the circuit reactance is varied in this way, the curve of current in the circuit as a function of the capacitive reactance (or inductive reactance) has essentially the same shape as the series resonance curves already considered. In particular, maximum current and unity power factor still occur when the capacitive and inductive reactances are equal, provided the circuit resistance does not change with the change in reactance. The curve of voltage developed across the inductance (or capacity) will, however, not be quite the same when the reactance is varied as in the case of frequency variation, since the maximum will occur at a slightly different frequency.

Ex. 1044-0147



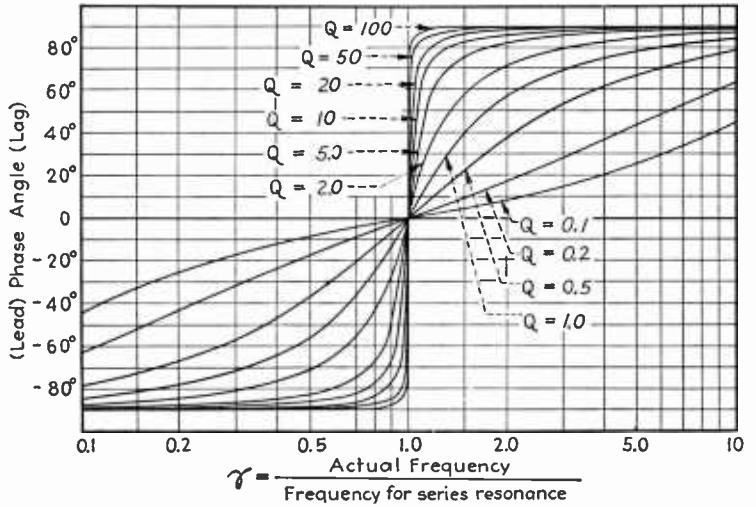


FIG. 4.—Universal curves giving the impedance as a function of frequency for a series resonant circuit having constant resistance.

Ex. 1044-0148

Series Resonant Circuits with Constant Resistance.—Series resonant circuits having a substantially constant resistance independent of frequency, particularly circuits with a relatively low Q, are often used in equalizers and in other applications. The general expression for the impedance of a series resonant circuit as given in Eq. (2*a*) can for the case of constant circuit resistance be conveniently rewritten as follows:

$$\frac{Z}{R} = 1 + jQ_0\left(\frac{\gamma^2 - 1}{\gamma}\right) \tag{10}$$

where Q_0 = circuit Q at resonance.

$\gamma = \dfrac{\text{actual frequency}}{\text{resonant frequency}}$.

Z = series impedance.

R = circuit resistance (assumed independent of frequency).

Results given by Eq. (10) are expressed graphically in Fig. 4, which not only is useful in showing at a glance the manner in which the magnitude and phase angle of the impedance will vary under different conditions but also can be used in the design of series resonant circuits to have desired characteristics.

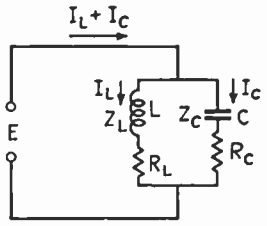


FIG. 5.—Parallel resonant circuit.

2. Parallel Resonant Circuits.—A parallel resonant circuit is obtained when the generator voltage is applied to an inductance and a capacity connected in parallel, as in Fig. 5. It will be noted that the only difference between the series and parallel circuit is in the manner of connection. In the analyses given below of the parallel-resonant circuit of Fig. 5, the following notation will be used:

E = voltage applied to circuit.

$Z_c = R_c - j/\omega C$ = impedance of capacitive branch.

$Z_L = R_L + j\omega L$ = impedance of inductive branch.

$Z_s = Z_c + Z_L$ = series impedance of circuit.

Z = parallel impedance of circuit.

$R_s = R_c + R_L$ = total series resistance of circuit.

$\omega = 2\pi$ times frequency.

$\omega_0 = 2\pi$ times frequency at which $\omega L = 1/\omega C$.

$Q = \omega L/R_s$ = circuit Q.

Q_0 = value of Q at $\omega_0/2\pi$ cycles.

The quantities L, C, R_L, and R_c refer to the inductance, capacity, and resistance components of the circuit, as indicated in Fig. 5.

The impedance offered to the generator by a parallel resonant circuit is the product of the two branch impedances divided by their sum; *i.e.*

$$\text{Parallel impedance} = Z = \frac{Z_c Z_L}{Z_c + Z_L} = \frac{Z_c Z_L}{Z_s} \tag{11}$$

$$= \frac{\left(R_c + \dfrac{1}{j\omega C}\right)(R_L + j\omega L)}{(R_c + R_L) + j\left(\omega L - \dfrac{1}{\omega C}\right)} \tag{11a}$$

The magnitude and phase of the impedance of a parallel circuit can be conveniently obtained from the universal curves of Fig. 6, which give the parallel impedance in terms of the impedance of one branch and the vector ratio of the branch impedances.[1]

[1] Somewhat similar curves are given by D. G. Fink, Parallel Impedance Chart, *Electronics*, Vol. 11, p. 31, May, 1938.

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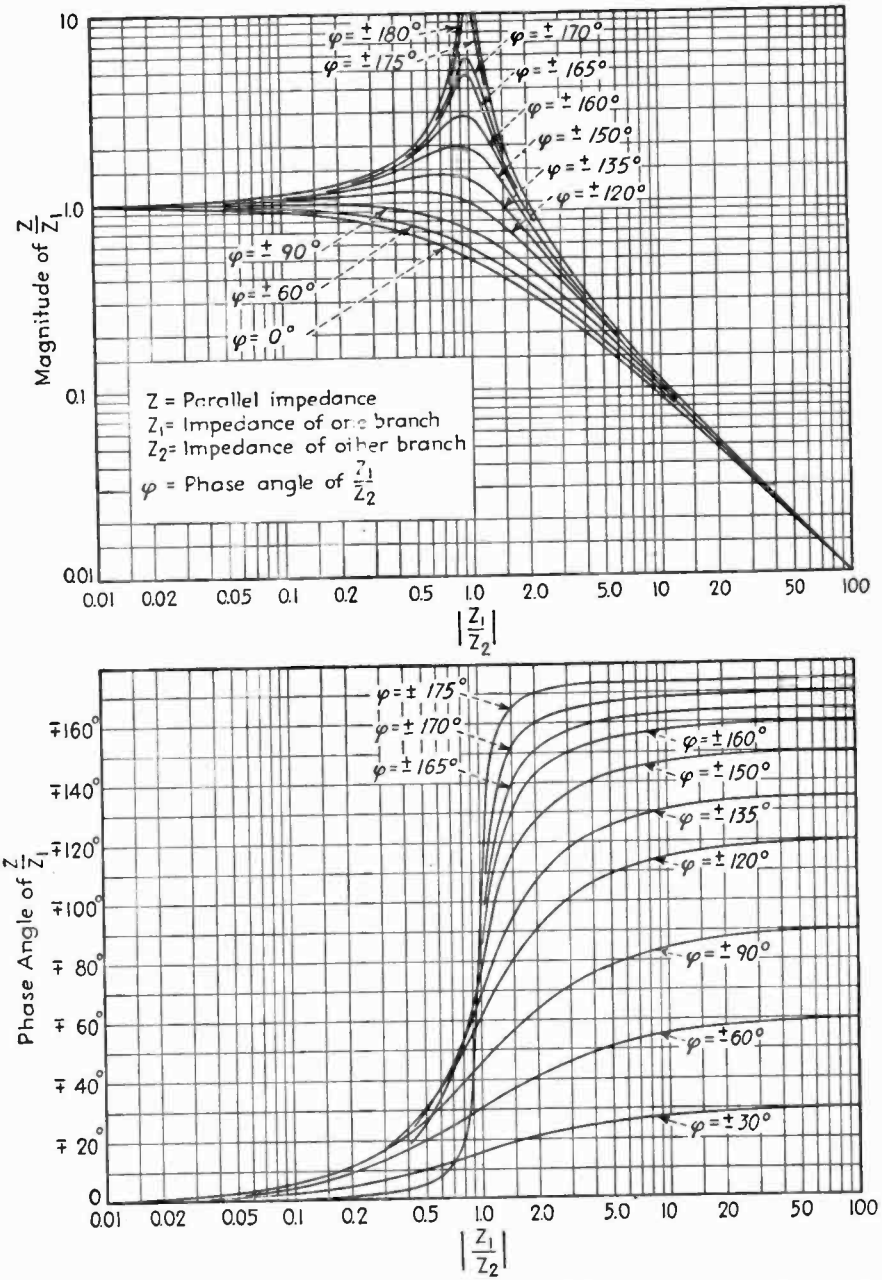


FIG. 6.—Universal curves giving the impedance of a parallel circuit in terms of the magnitudes and phases of the branch impedances.

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The current that the generator delivers to the parallel circuit can be termed the *line current*, and is the applied voltage E divided by the impedance or

$$\text{Line current} = \frac{E}{Z} \tag{12}$$

The currents in the branches are equal to the applied voltage divided by the respective branch impedances, and so are given by the equations

$$\text{Current in capacitive branch} = I_c = \frac{E}{Z_c} \tag{13a}$$

$$\text{Current in inductive branch} = I_L = \frac{E}{Z_L} \tag{13b}$$

Equations (11), (12), and (13) are the fundamental equations of the parallel circuit, and apply for all conditions, irrespective of circuit Q, frequency, or division of resistance between branches.

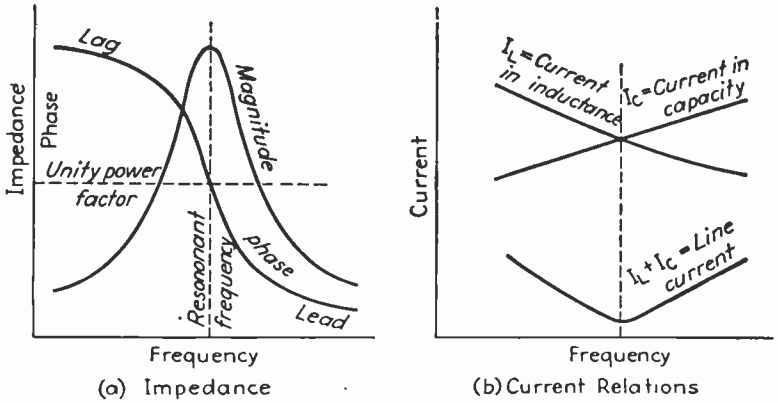


FIG. 7.—Variation of impedance and current with frequency in a parallel resonant circuit.

When the Q of a parallel circuit is not too low, the impedance that is offered to an applied voltage varies with frequency in the manner illustrated in Fig. 7*a*. At low frequencies, the inductive branch draws a large lagging current while the leading current of the capacitive branch is small, resulting in a large lagging line current and a low lagging circuit impedance. At high frequencies, the capacitive branch has a low reactance compared with the inductive branch, resulting in a large leading line current and a corresponding low circuit impedance that is leading. Between these extremes, there is a frequency, commonly termed the resonant frequency, at which the lagging current taken by the inductive branch and the leading current in the capacitive branch are equal, and since these currents are nearly 180° out of phase, they neutralize to leave only a small resultant in-phase line current. The impedance of the parallel circuit will then be a high resistance, as is apparent in Fig. 7*a*.

As a result of the fact that the parallel circuit has a high impedance at resonance and relatively low impedance at other frequencies, the line current varies with frequency as shown in Fig. 7*b* and becomes very small at resonance. The currents in the individual branches vary much more slowly with frequency, and are almost but not quite constant in the vicinity of resonance.

The resonant frequency of a parallel circuit can be defined in several ways as follows: (1) the frequency at which $\omega L = 1/\omega C$, *i.e.*, the resonant frequency of the same

circuit when operating in series resonance; (2) the frequency at which the parallel impedance of the circuit is maximum; (3) the frequency at which the parallel impedance of the circuit has unity power factor. These three definitions lead to frequencies that differ by an amount depending upon the circuit Q and the division of the resistance between the branches. A discussion of the practical importance of these various cases is given below.

Analysis of Parallel Resonant Circuits Having High Q.—The most important practical case of parallel resonance exists when the circuit Q is at least moderately high (*i.e.*, $Q \geq 10$). Under these conditions, the exact way in which the resistance is divided between the two branches is of little consequence, and it is permissible to assume that the absolute magnitude of the impedance of the individual branches is not affected by the resistance in these branches.

To the accuracy allowed by these approximations, the three possible resonant frequencies are virtually identical, so that *for all practical purposes the resonant frequency of a high Q parallel resonant circuit can be taken as the resonant frequency of the same circuit when connected in series.*

The shape of the impedance curve of a high Q parallel resonant circuit is also the same as the current curve of the same circuit acting as a series resonant circuit. The only difference between series and parallel resonance under these conditions is that the phase angles are reversed, *i.e.*, the phase angle of parallel impedance has the opposite sign from the phase angle of the current in the series circuit.

In terms of the notation given above, the equations giving the behavior of parallel resonant circuits with moderate or high Q's are

$$\text{Resonant frequency} = f_0 = \frac{1}{2\pi\sqrt{LC}} \tag{14}$$

$$\text{Parallel impedance} = \frac{(\omega_0 L)^2}{Z_s} = \frac{\left(\frac{1}{\omega_0 C}\right)^2}{Z_s} = \frac{\left(\frac{L}{C}\right)}{Z_s} \tag{15}$$

At resonance

$$\left.\begin{matrix}\text{Parallel impedance} \\ \text{at resonance}\end{matrix}\right\} = \frac{(\omega_0 L)^2}{R_s} = Q_0\omega_0 L = R_s Q_0^2 = \frac{L}{RC} \tag{16}$$

At frequencies differing from resonance by at least $3/Q$ times the resonant frequency, it is permissible to neglect the effect of the circuit resistance R in determining the magnitude of the parallel impedance. This gives

$$\frac{\text{Impedance at frequencies far from resonance}}{\text{Impedance at resonance}} = \frac{1}{Q\gamma\left(1 - \frac{1}{\gamma^2}\right)} \tag{17a}$$

where γ is the ratio of the actual to the resonant frequency. The phase angle at all frequencies is very nearly the negative of the angle of the current for series operation, and from Eq. (2c) is found to be

$$\tan\theta = Q\left(\frac{1}{\gamma^2} - 1\right) \tag{17b}$$

The recommended procedure for calculating parallel resonance in the case of moderate or high Q circuits is as follows: First, obtain the resonant frequency and the impedance at resonance, using Eqs. (14) and (16). Second, calculate the frequencies at which the impedance is 70.7 per cent and 44.7 per cent of the value at resonance with the aid of the same working rules as for series resonance, and then where other points are needed in the vicinity of resonance use either the universal resonance curve or Table 1. Finally, for frequencies too far off resonance to be within the range

Ex. 1044-0152

of the universal resonance curve, the magnitude and phase of the impedance are obtained by Eqs. (17*a*) and (17*b*). This procedure gives an accuracy far better than obtainable by slide-rule calculations using Eq. (11), and also involves considerably less work.

The impedance offered by a parallel resonant circuit can be expressed in form $R_{eq} + jX_{eq}$. A universal curve giving the variation of R_{eq}/Z_0 and X_{eq}/Z_0 as a function of frequency, where Z_0 is the impedance at resonance, is shown in Fig. 8 for the case of a high Q circuit. It will be noted that the curve of resistance is much like a resonance curve, but differs in that it has steeper sides. In particular, the resistance drops to 50 per cent of the resonant impedance when the fractional detuning is $1/2Q$

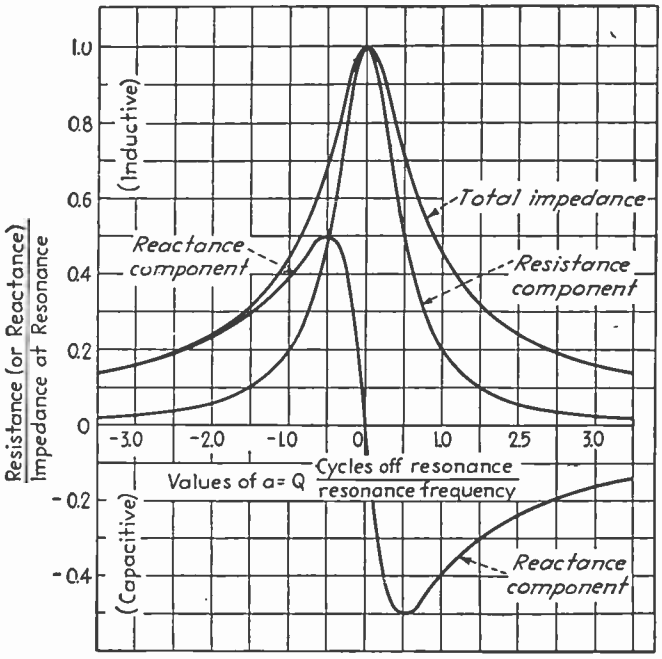


FIG. 8.—Universal curve giving the resistance and reactance components of the impedance of a high-Q parallel resonant circuit as a function of frequency.

instead of 70.7 per cent, as in the case of a resonance curve. The reactance curves of Fig. 8 are characterized by maximum inductive and capacitive reactances at frequencies that are likewise $1/2Q$ below and above resonance, respectively. These reactance maxima have an amplitude exactly 50 per cent of the impedance at resonance.

General Case of Parallel Resonance with Particular Reference to Low Q Circuits.—In the general case where the circuit Q is low, the curve of circuit impedance still tends to have the shape of a resonance curve unless the circuit Q approaches (or is less than) unity, but the peak of the curve does not necessarily occur at the frequency of series resonance, and the condition for unity power factor does not necessarily occur either at this frequency or when the impedance is a maximum. Furthermore, the details of the behavior of a low Q parallel circuit also depend upon the division of resistance between the inductive and capacitive branches, upon the way in which the resistance varies with frequency, and upon whether the adjustment to resonance is made by

Ex. 1044-0153

varying the frequency, inductance, or capacity. Some of the effects that can be obtained under different conditions are illustrated in Fig. 9.

The behavior of the impedance for any desired condition can be calculated with the aid of Eq. (11). Special relations can also be derived for particular cases, but there are so many of these special cases, and each is of such limited application that the equations are not particularly useful.[1] The results in a number of limiting cases

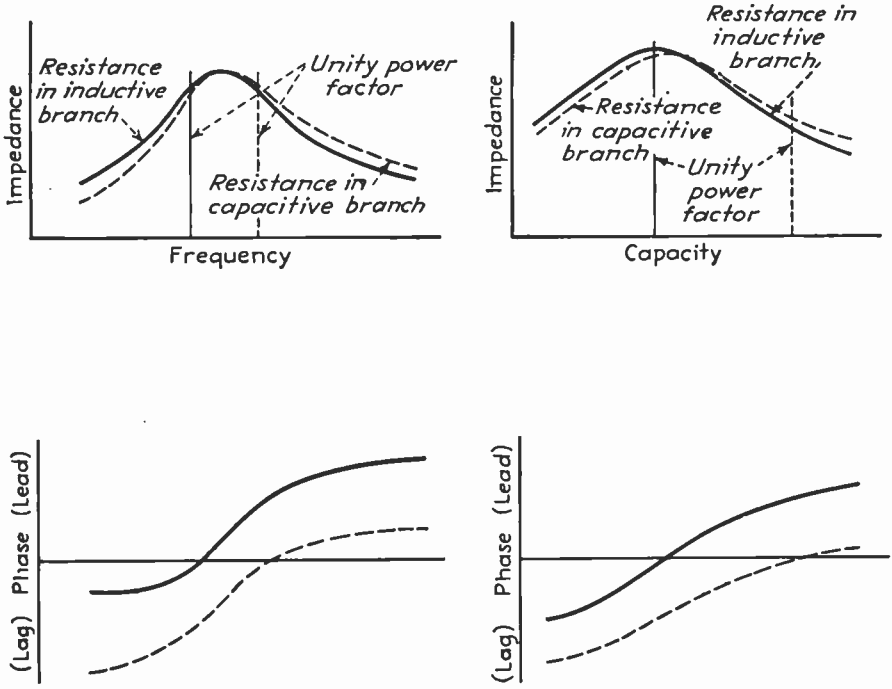


FIG. 9.—Typical characteristics of parallel resonant circuits having low Q.

are as follows: *First*, when the R, L, and C of the circuit are constant, and all of the resistance is in one branch, the frequency of unity power factor is related to the frequency of series resonance by the following equations:

For resistance in inductive branch:

$$\omega = \omega_0 \sqrt{1 - \frac{1}{Q_0^2}} \tag{18}$$

For resistance in capacitive branch:

$$\omega = \omega_0 \frac{1}{\sqrt{1 - \frac{1}{Q_0^2}}} \tag{19}$$

The notation is the same as previously used, *i.e.*, Q_0 is the circuit Q at the series resonant frequency, and $\omega_0/2\pi$ corresponds to the series resonant frequency. *Second*, when the frequency is varied, the impedance curve is not greatly affected by the

[1] A good discussion of some of these special cases is given in pp. 35–45 of R. S. Glasgow, "Principles of Radio Engineering," McGraw-Hill, New York, 1936.

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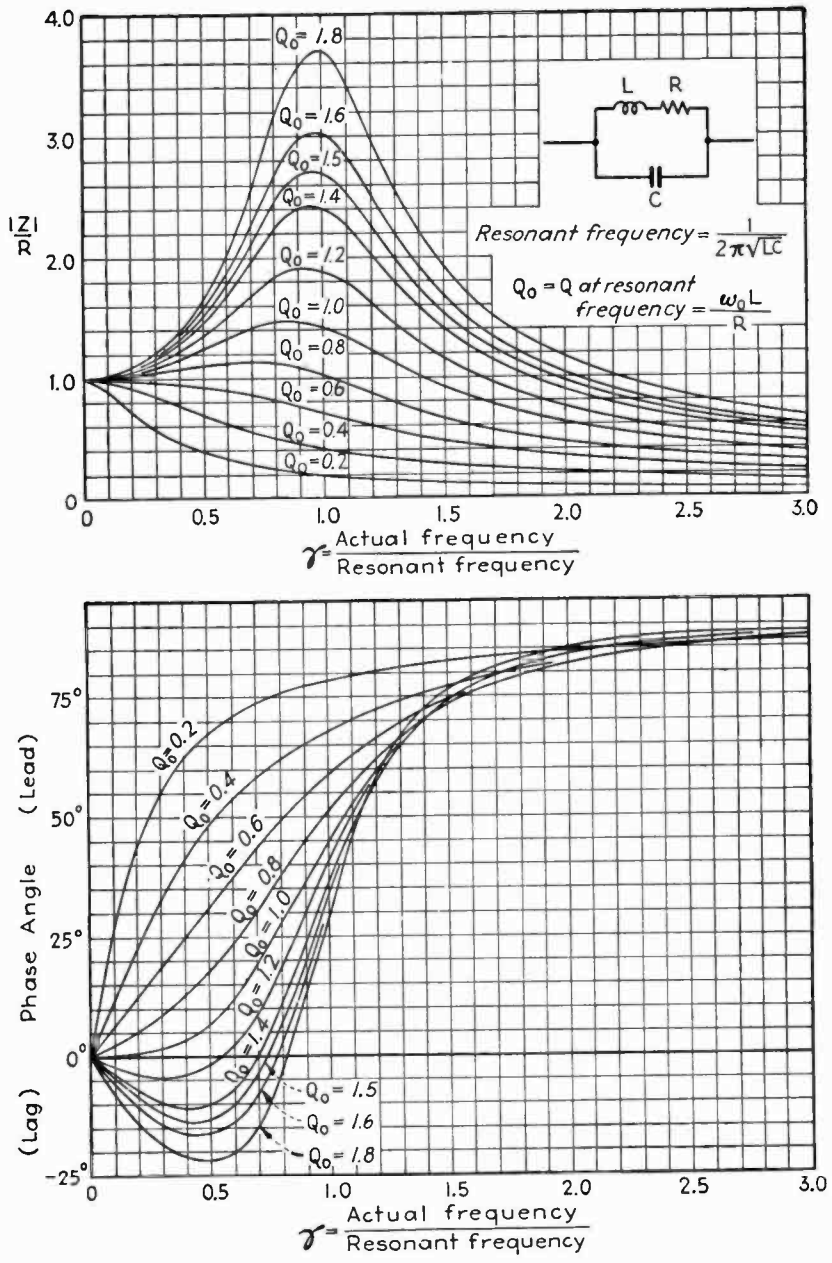


FIG. 10.—Universal curves giving magnitude and phase angle of parallel impedance as a function of frequency in parallel resonant circuits having low Q and constant circuit resistance.

Ex. 1044-0155

division of resistance between the branches, and the maximum impedance occurs very near, but not necessarily exactly at, the frequency of series resonance. *Third*, when all the resistance is in the inductive branch and the circuit capacity is varied while keeping resistance, inductance, and frequency constant, the capacity giving maximum impedance also corresponds to unity power factor. *Fourth*, when all the resistance is in the capacitive branch and the inductance is varied while keeping resistance, capacity, and frequency constant, the inductance giving maximum impedance also corresponds to unity power factor. *Fifth*, tuning by adjusting circuit by any other manner than the preceding cases, as, for example, by varying the inductance when the resistance is in the inductance branch, causes the condition for unity power factor to differ from the condition for maximum impedance.

Special Cases of Very Low Q.—Inductance and capacity are sometimes associated with a constant resistance in such a way as to form a parallel resonant circuit in which the resistance is constant and in the inductive branch, and the Q is extremely low, even less than unity. Typical examples are a resistance unit designed for low reactance effects (see Fig. 8, Sec. 2), and the coupling impedance to give shunt peaking in a video amplifier (see Fig. 51*a*, Sec. 5). Universal curves giving the magnitude and phase angle of impedance for this particular case are given in Fig. 10.

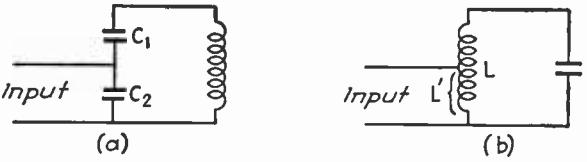


FIG. 11.—Tapped parallel resonant circuits.

Another important case of a very low Q parallel circuit is where the resistances in the two branches are the same, and are each equal to $\sqrt{L/C}$ ohms. Under this condition, the parallel impedance is a resistance at all frequencies, and has a value of $\sqrt{L/C}$ ohms.

Parallel Resonance with Tapped Circuits.—Parallel resonance circuits are frequently operated with the input connected across only a portion of the total circuit, as illustrated in Fig. 11. At frequencies near resonance, the effect of this method of operation is to make the impedance offered to the input terminals less in magnitude than the parallel impedance of the entire circuit without changing the character of the impedance curve as far as shape or equivalent Q is concerned. Tapping a parallel resonance circuit accordingly offers a means of adjusting the magnitude of impedance obtained without changing the circuit itself.

With the circuit of Fig. 11*a*, one has near the resonant frequency

$$\frac{\text{Impedance offered to input}}{\text{Parallel impedance of circuit}} = \left(\frac{C_1}{C_1 + C_2}\right)^2 \tag{20}$$

Similarly, for Fig. 11*b*

$$\frac{\text{Impedance offered to input}}{\text{Parallel impedance of circuit}} = \left(\frac{M_{eq}}{L}\right)^2 \tag{21}$$

where M_{eq} is the total equivalent mutual inductance between L′ and the entire coil L, including both common and inductive coupling.

3. Inductively Coupled Circuits.—When mutual inductance exists between coils that are in separate circuits, these circuits are said to be inductively coupled. The effect of the mutual inductance is to make possible the transfer of energy from one circuit to the other by transformer action; *i.e.*, the alternating current flowing in one

Ex. 1044-0156

circuit as a result of a voltage applied to that circuit produces magnetic flux that induces a voltage in the coupled circuit, resulting in induced currents and a transfer of energy from the first or primary circuit to the coupled or secondary circuit.

Analysis of Inductively Coupled Circuits.—The behavior of inductively coupled circuits can be determined by making use of the following principles:[1] *First*, as far as the primary circuit is concerned, the effect of the presence of the coupled secondary circuit is exactly as though an impedance $(\omega M)^2/Z_s$ had been added in series with the primary, where

M = mutual inductance.
$\omega = 2\pi f$.
Z_s = series impedance of secondary circuit when considered by itself.

Second, the voltage induced in the secondary circuit by a primary current of I_p has a

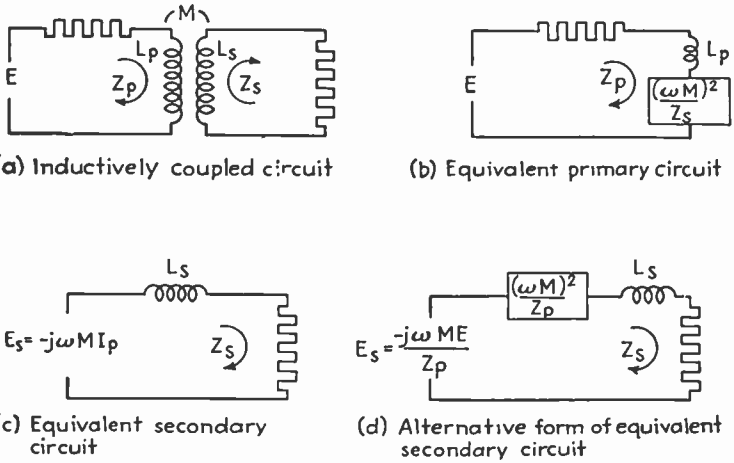


FIG. 12.—Inductively coupled circuit and also equivalent primary and secondary circuits.

magnitude of $\omega M I_p$ and lags the current that produces it by 90°. *Third*, the secondary current is exactly the same current that would flow if the induced voltage were applied in series with the secondary and if the primary were absent.

The preceding analysis shows that the actions taking place in the primary and secondary circuit are as though one had the equivalent circuits shown in Figs. 12*b* and 12*c*. Here the primary circuit consists of the actual primary impedance Z_p in series with an additional impedance $(\omega M)^2/Z_s$ that takes into account the effect the secondary produces on the primary current. This extra impedance $(\omega M)^2/Z_s$ is termed the *coupled impedance*. The equivalent secondary circuit of Fig. 12*c* consists of the voltage $-j\omega M I_p$ induced in the secondary by the primary current, in series with the secondary impedance Z_s as shown.

An alternative form of equivalent secondary circuit is given in Fig. 12*d*. This is obtained by applying Thévenin's theorem and noting that when the secondary is opened, the open circuit voltage that appears is $-j\omega M I_p$, while when the applied voltage is short-circuited, the equivalent impedance as viewed from the opened secondary terminals is $Z_s + (\omega M)^2/Z_p$. This second form of the equivalent secondary

[1] These rules for handling coupled circuits are direct consequences of the mesh equations, which for this case can be written as

$$E = I_p Z_p + j\omega M I_s \quad (22)$$

$$\text{Induced voltage} = -j\omega M I_p = I_s Z_s \quad (23)$$

Ex. 1044-0157

circuit leads to the same results as the first form, since the extra voltage that is considered as acting in the second case is just compensated for by the voltage drop in the extra impedance considered as existing in the secondary circuit.

The resistance and reactance components of the coupled impedance are

$$\text{Resistance component} = \frac{(\omega M)^2}{R_s^2 + X_s^2} R_s \tag{24}$$

$$\text{Reactance component} = -j \frac{(\omega M)^2}{R_s^2 + X_s^2} X_s \tag{25}$$

where R_s and X_s are, respectively, the resistance and reactance components of the secondary impedance Z_s. The effect that the secondary has on the primary circuit is exactly as though the resistance and reactance given by Eqs. (24) and (25) had been inserted in series with the primary circuit. The energy and reactive volt-amperes consumed by the primary current flowing through this hypothetical resistance and reactance represent the energy and reactive volt-amperes that are transferred to the secondary circuit.

Conditions for Maximum Secondary Current.[1]—In dealing with coupled circuits, it is desirable to keep in mind the adjustments giving maximum secondary current. The behavior in the more important cases is summarized below for the case of two circuits inductively coupled, with resistance and reactance in each circuit.

Case 1. Maximum Secondary Current with Variation of Secondary Reactance.—When frequency, mutual inductance, primary impedance, and secondary resistance are constant, the secondary current is maximum when

$$X_s = \frac{(\omega M)^2}{R_p^2 + X_p^2} X_p \tag{26}$$

where X_p and X_s = reactance of primary and secondary circuits, respectively.
R_p and R_s = resistance of primary and secondary circuits, respectively.
ω = 2π times frequency.
M = mutual inductance.

Case 2. Maximum Secondary Current with Variation of Primary Reactance.—If the primary rather than secondary reactance is varied, maximum secondary current is obtained when

$$X_p = \frac{(\omega M)^2}{R_s^2 + X_s^2} X_s \tag{27}$$

Case 3. Maximum Possible Secondary Current.—The secondary current has its maximum possible value when the adjustments are such that the resistance and reactance coupled into the primary circuit by the secondary are equal to the resistance of the primary circuit and the negative of the primary reactance, respectively; *i.e.*, when

$$\begin{aligned} \frac{(\omega M)^2}{R_s^2 + X_s^2} R_s &= R_p \\ -\frac{(\omega M)^2}{R_s^2 + X_s^2} X_s &= -X_p \end{aligned} \tag{28}$$

It will be noted that in order to obtain maximum possible secondary current, it is necessary that two independent variables be adjusted. One of these is commonly the mutual inductance; the other, the reactance of one or both circuits. Unless the mutual inductance satisfies the relation $\omega M \geq \sqrt{R_p R_s}$, it is impossible to satisfy the condition for maximum possible secondary current.

[1] For a more detailed discussion, see G. W. Pierce, "Electric Oscillations and Electric Waves," Chap. XI, McGraw-Hill, New York, 1920.

Ex. 1044-0158

Inductive Coupling Considered as a Transformer.—Two coils inductively coupled represent the general case of a transformer, and can be reduced to the equivalent transformer circuit of Fig. 13. Here the primary and secondary inductances are divided into leakage and closely coupled components. The leakage inductances correspond to magnetic flux lines that produce no linkages to any secondary circuit, while the coupled inductances are taken as having unity coefficient of coupling.

In a conventional transformer with iron core, the coefficient of coupling between primary and secondary is so large that the leakage inductances represent only a very small fraction of the total circuit inductances. Under these conditions it is convenient to analyze the behavior in terms of the turn-ratio, leakage inductance, etc., as is commonly done with 60-cycle power transformers. However, in the more general case where the coefficient of coupling may be small, most of the inductance is leakage, and turn ratio has relatively little significance. It is then preferable to use the general method of analysis given above.

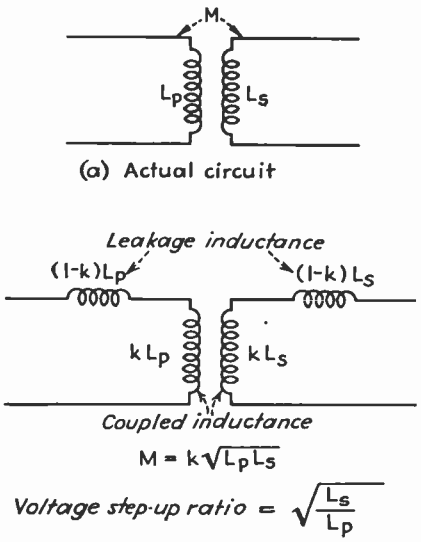


FIG. 13.—Equivalent circuit of a transformer expressed in terms of primary and secondary inductances and coefficient of coupling.

4. Analysis of Some Simple Coupled Circuits. *Untuned Secondary Consisting of a Pure Reactance.*—This case corresponds to a coil serving as the primary, with a metal mass of low resistivity serving as the secondary. In this case, which is illustrated in Fig. 14*a*, one has

$$\text{Coupled reactance} = -\frac{(\omega M)^2}{\omega L_s} = -k^2\omega L_p \tag{29}$$

where k is the coefficient of coupling between the coil and the secondary. The presence of the secondary reduces the inductance as viewed from the coil terminals to an equivalent value $(1 - k^2)L_p$.

It will be noted that the amount of reduction in inductance is determined solely by the coefficient of coupling and not by the number of primary turns, etc. It will be also observed that the effect of the secondary decreases rapidly as the coefficient of coupling becomes small.

Untuned Secondary Having Both Resistance and Reactance.—This corresponds to the practical case of a coil in a shield, or near a metal panel or other mass, with the resistance of the metal secondary taken into account. For such a secondary, the coupled impedance as given by Eqs. (24) and (25) consists of a resistance and a negative reactance. The effect of the secondary upon the primary is hence to lower the equivalent inductance and increase the equivalent resistance of the primary coil as viewed from its terminals.

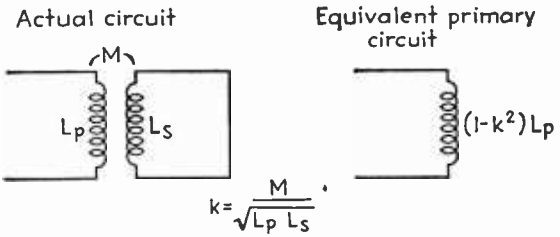
Under practical conditions, a secondary consisting of a shield or other metal object is usually of copper or aluminum, so that the secondary reactance is considerably greater than the secondary resistance. Under these conditions, the equivalent primary circuit has the values and form given in Fig. 14*b*.

Tuned Secondary, Untuned Primary.—This circuit, illustrated in Fig. 15*a*, is of importance because it represents the equivalent circuit of the transformer-coupled tuned radio-frequency amplifier, with the primary resistance R_p representing the plate

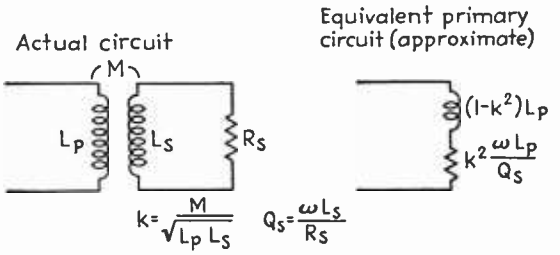
Ex. 1044-0159

resistance of the tube. When the secondary is a resonant circuit, the curve of coupled impedance $(\omega M)^2/Z_s$ with variation in frequency has substantially the same shape and characteristics as the parallel impedance curve of the secondary, but the absolute magnitude is determined by the mutual inductance.

The important characteristic of this circuit is the curve of voltage developed across the secondary condenser as a function of frequency under conditions where $R_p >> \omega L_p$. Such a characteristic is shown in Fig. 15*b*, and has substantially the same shape as a resonance curve with the peak at the resonant frequency of the secondary and an effective Q somewhat lower than the actual Q of the secondary circuit.



(a) Untuned reactive secondary with zero losses



(b) Untuned secondary with small losses

FIG. 14.—Inductively coupled circuits with untuned secondaries.

On the assumption that $R_p >> \omega L_p$, the response at resonance is

$$\frac{\left\{\begin{matrix}\text{Voltage across secondary} \\ \text{condenser at resonance}\end{matrix}\right\}}{\text{Voltage applied to primary}} = \frac{\omega M}{R_p R_s + (\omega M)^2} \times \omega L_s \tag{30}$$

With given primary and secondary circuits, the response at resonance is maximum when the coupled resistance equals the primary resistance, *i.e.*, when

$$\omega M = \sqrt{R_p R_s} \tag{31}$$

With greater or less coupling, the response is less.

The relationship between the effective Q of the response curve and the actual Q of the secondary circuit is, still assuming that $R_p >> \omega L_p$

$$\frac{\text{Effective } Q \text{ of amplification curve}}{\text{Actual } Q \text{ of tuned circuit}} = \frac{1}{1 + \dfrac{(\omega M)^2/R_s}{R_p}} \tag{32}$$

The effective Q of the response curve approaches the actual Q of the secondary circuit when the coupling is very small, and is exactly one-half of the actual Q when the coupling corresponds to the value giving maximum response.

Ex. 1044-0160

When ωL_p is not negligible compared with the primary resistance, the response curve still has the shape of a resonance curve, but the peak is shifted to a slightly higher frequency. This shift in location of the peak is due to the fact, expressed by Eq. (26), that maximum secondary current occurs when secondary is sufficiently

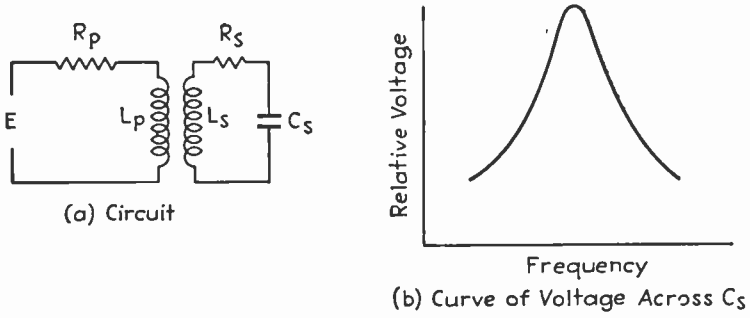


FIG. 15.—Inductively coupled circuit with tuned secondary and untuned primary.

detuned that its coupled reactance neutralizes the reactance of the primary. When the primary reactance is not negligible, this detuning is appreciable. The coupling required for maximum secondary current is also greater as the ratio $\omega L_p/R_p$ is increased.

Analysis of the output of coupled systems with untuned primary and tuned secondary can be most conveniently carried out by using the equivalent secondary circuit of Fig. 12*d*. Such equivalent circuits are shown in Fig. 16. These show how the asso-

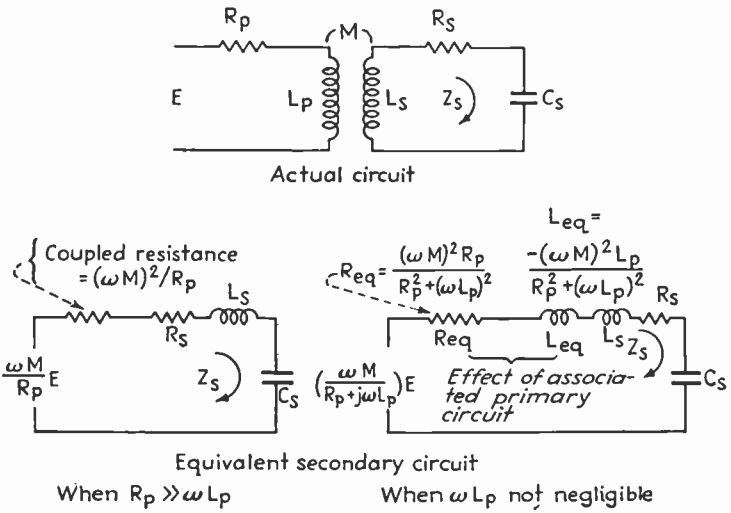


FIG. 16.—Equivalent secondary circuit for case of inductive coupling with tuned secondary and untuned primary.

ciation of the primary with the resonant secondary has the effect of increasing the effective secondary resistance and hence of lowering the effective Q, and also that an inductive primary neutralizes some of the secondary inductance and so raises the resonant frequency.

Ex. 1044-0161

5. Two Resonant Circuits Tuned to the Same Frequency and Coupled Together.[1] When two circuits resonant at the same frequency are coupled together, the resulting behavior depends very largely upon the coupling, as shown in Fig. 17. When the coefficient of coupling is small, the curve of primary current as a function of frequency approximates closely the series resonance curve of the primary circuit considered alone. The secondary current at the same time is small and varies with frequency according to a curve having a shape approximating the product of the resonance curves of

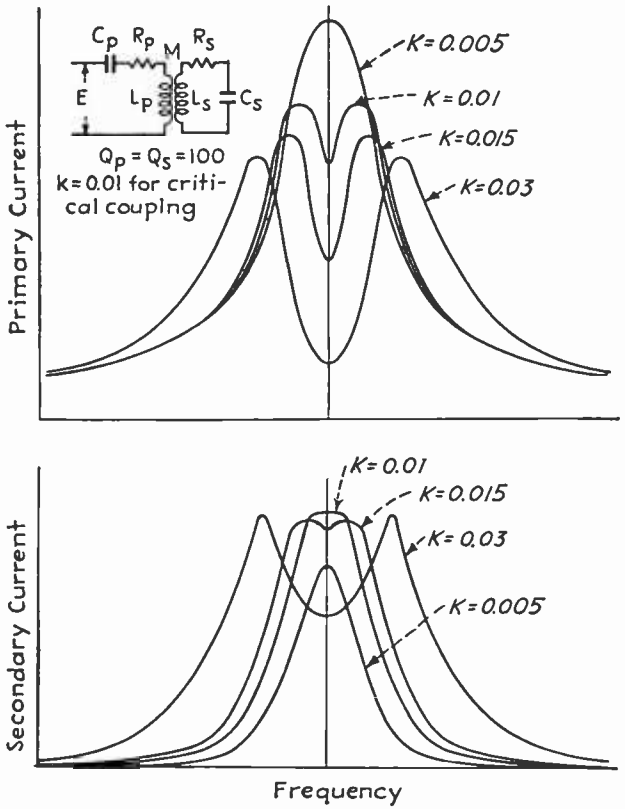


FIG. 17.—Curves showing variation of primary and secondary current with frequency for different coefficients of coupling when the primary and secondary are separately tuned to the same frequency.

primary and secondary circuits taken alone. As the coupling is increased, the curve of primary current becomes broader and the peak value of primary current is reduced. At the same time, the secondary current becomes larger and the sharpness of the secondary current curve is reduced. These trends continue until the coupling is such that the resistance that the secondary circuit couples into the primary at resonance is equal to the primary resistance. This coupling is called the *critical coupling*, and causes the secondary current to have the maximum possible value that it can attain.

[1] A very complete discussion of this subject is given by C. B. Aiken, Two-mesh Tuned Coupled Circuit Filters, *Proc. I.R.E.*, Vol. 25, p. 230, February; p. 672, June, 1937. See also E. S. Purington, Single and Coupled-circuit Systems, *Proc. I.R.E.*, Vol. 18, p. 983, June, 1930.

Ex. 1044-0162

The curve of secondary current is then somewhat broader in the immediate vicinity of the resonant frequency than the resonance curve of the secondary circuit, and the primary curve shows double peaks. With still greater coupling, the double humps in the primary current curve become more prominent, and the peaks spread farther apart. At the same time the secondary current curve begins to show double humps, with the peaks becoming progressively more pronounced and farther apart as the coefficient of coupling is increased.[1]

Basic Equations.—In practical work, the most important property of the coupled system under consideration is the ratio of voltage developed across the condenser in the secondary circuit to the voltage applied in series with the primary circuit. A useful expression for this important ratio is obtained by rearranging Eqs. (22) and (23) to give the relation[2]

$$\frac{\text{Voltage across secondary condenser}}{\text{Voltage applied in series with primary}} = \frac{E_c}{E}$$

$$= \frac{-1}{\gamma^2}\sqrt{\frac{L_s}{L_p}}\,\frac{k}{\left[k^2 + \frac{1}{Q_pQ_s} - \left(1 - \frac{1}{\gamma^2}\right)^2 + j\left(1 - \frac{1}{\gamma^2}\right)\left(\frac{1}{Q_p} + \frac{1}{Q_s}\right)\right]} \tag{33}$$

where ω = 2π times frequency.

ω_0 = 2π times resonant frequency.

$\gamma = \dfrac{\omega}{\omega_0} = \dfrac{\text{actual frequency}}{\text{resonant frequency}}$.

E = voltage applied in series with primary.

Q_p = $\omega L_p/R_p$ for primary circuit.

Q_s = $\omega L_s/R_s$ for secondary circuit.

k = $M/\sqrt{L_pL_s}$ = coefficient of coupling.

L_p = total inductance of primary circuit.

L_s = total inductance of secondary circuit.

This equation involves no approximations, other than assuming that the circuits are resonant at the same frequency.

In the special case where the circuit losses are zero, or where the frequency differs sufficiently from the resonance frequency to allow the losses to be neglected, Eq. (33) takes the form

$$\frac{E_c}{E} \text{ for zero losses} = \frac{-1}{\gamma^2}\sqrt{\frac{L_s}{L_p}}\,\frac{k}{\left[k^2 - \left(1 - \frac{1}{\gamma^2}\right)^2\right]} \tag{34}$$

Effect of Coupling on Response at Resonance—Critical Coupling.—The response at resonance is obtained by substituting $\gamma = 1$ in Eq. (33), which gives

$$\frac{E_c}{E} \text{ at resonance} = -\sqrt{\frac{L_s}{L_p}}\,\frac{k}{\left[k^2 + \frac{1}{Q_pQ_s}\right]} \tag{35}$$

This expression is maximum when the coefficient of coupling has the value

$$k_c = \frac{1}{\sqrt{Q_pQ_s}} \tag{36}$$

[1] When the circuit Q's are not equal, double humps do not appear in the curve of secondary current until the coupling is somewhat greater than the critical value. This is illustrated in the universal curves of Fig. 22.

[2] For derivation, see Terman, *op. cit.*, p. 81.

Ex. 1044-0163

This coupling k_c is often termed the *critical coupling.* The corresponding mutual inductance is

$$\omega M = \sqrt{R_p R_s} \tag{37}$$

With the mutual inductance for critical coupling, the resistance coupled into the primary circuit at resonance by the presence of the secondary equals the primary resistance. This gives the maximum possible secondary current. The resulting maximum possible voltage across the secondary condenser is

$$\text{Maximum possible } \frac{E_c}{E} = \frac{1}{2k_c}\sqrt{\frac{L_s}{L_p}} = \frac{\sqrt{Q_p Q_s}}{2}\sqrt{\frac{L_s}{L_p}} \tag{38}$$

where k_c is the critical coupling as given by Eq. (36). The voltage developed across the condenser at resonance with other couplings is less than value given by Eq. (38) by an amount depending upon the ratio of actual to critical coupling, as shown in Fig. 18.

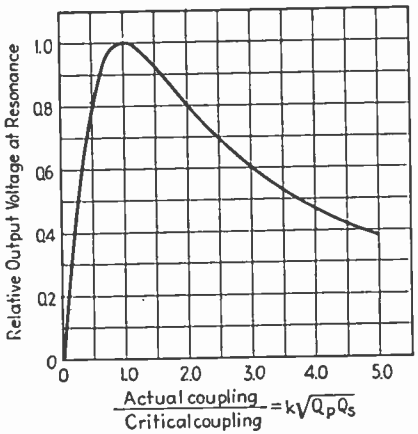


FIG. 18.—Relative output voltage obtained at resonance as a function of coefficient of coupling in the case of two circuits tuned to the same frequency and coupled together.

Conditions for Existence of Double Peak —Height, Location, and Width of Peaks.— The curve of response in the secondary circuit exhibits double peaks whenever the coefficient of coupling is sufficient to cause the right-hand side of Eq. (33) to show two maxima. This occurs when[1]

$$\frac{\text{Actual coefficient of coupling}}{\text{Critical coefficient of coupling}} \geqslant \left[\frac{1}{2}\left(\frac{Q_p}{Q_s} + \frac{Q_s}{Q_p}\right)\right]^{1/2} \tag{39}$$

It will be noted that when $Q_p = Q_s$, double peaks occur whenever the coupling exceeds the critical value, but if the Q's are different, double peaks do not occur until the coupling exceeds the critical value by an amount determined by the Q ratio.

When double humps occur, the location of the peaks in relation to the resonant frequency depends upon the actual coupling, the critical coupling, and the Q ratio. Analysis based upon Eq. (33) shows that to an accuracy sufficient for all ordinary purposes the location of the peaks is given by[2]

$$\frac{\text{Frequency at peak of secondary voltage}}{\text{Resonant frequency of tuned circuits}} = \gamma_p = \frac{1}{\sqrt{1 \pm k\left[1 - \frac{k_c^2}{2k^2}\left(\frac{Q_p}{Q_s} + \frac{Q_s}{Q_p}\right)\right]^{1/2}}} \tag{40}$$

where k_c is the critical coefficient of coupling as given by Eq. (36) and the remaining notation is the same as above. The ratio represented by the spacing between coupling peaks to the resonant frequency, as a function of the coefficient of coupling,

[1] Equation (39) assumes that the γ^2 multiplying the denominator of Eq. (33) is constant. The approximation that results is so small as to be of negligible consequence, but results in enormous simplification of the equations.

[2] Eq. (40) involves the same approximation as does Eq. (39), but again the error introduced is negligible and the simplification considerable.

Ex. 1044-0164

is given in Fig. 19 for various conditions, in terms of a parameter g given by the lower part of Fig. 19.

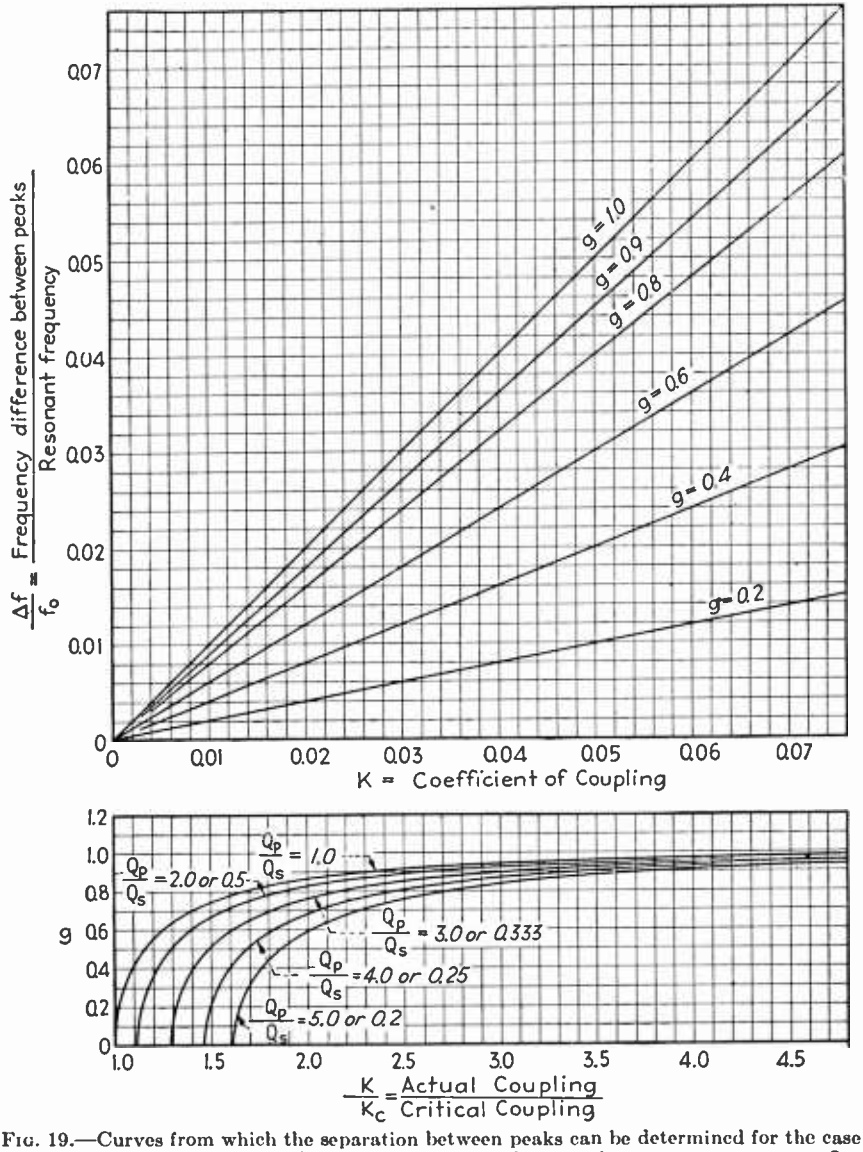


FIG. 19.—Curves from which the separation between peaks can be determined for the case of two coupled circuits resonant at the same frequency.

When the circuit Q's are not too unequal, and the coefficient of coupling greatly exceeds the critical value, Eq. (40) reduces to

$$\frac{\text{Frequency at peak of secondary voltage}}{\text{Resonant frequency of tuned circuits}} = \frac{1}{\sqrt{1 \pm k}} \qquad (41a)$$

Furthermore, in the very common case when the coefficient of coupling is small, Eq. (41a) becomes

$$\frac{\text{Frequency at peak of secondary voltage}}{\text{Resonant frequency of tuned circuits}} = 1 \pm \frac{k}{2} \quad (41b)$$

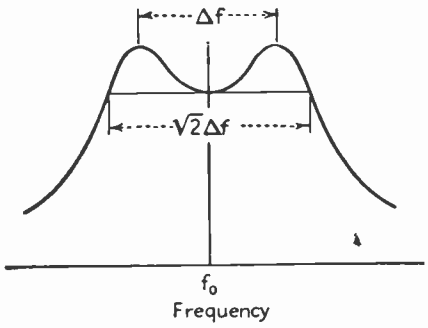


FIG. 20.—Relationship between band width, and width between secondary peaks existing when two circuits resonant at the same frequency are coupled together.

In practical work, one is commonly interested in the frequency band over which the response in the secondary circuit equals or exceeds the response at resonance. This band is illustrated schematically in Fig. 20, and can be shown to be equal to $\sqrt{2}$ times the width of the frequency band between coupling peaks.[1]

The height of the peaks of response in the secondary circuit can be obtained by substituting the value of γ_p from Eq. (40) into Eq. (33). This gives

$$\text{Value of } \frac{E_c}{E} \text{ at peaks} = \frac{1}{\gamma_p^2}\sqrt{\frac{L_s}{L_p}}\,\frac{\sqrt{Q_pQ_s}}{\left[\sqrt{\frac{Q_p}{Q_s}}+\sqrt{\frac{Q_s}{Q_p}}\right]\left[1-\frac{k_c^2}{4k^2}\left(\sqrt{\frac{Q_s}{Q_p}}-\sqrt{\frac{Q_p}{Q_s}}\right)^2\right]^{1/2}} \quad (42)$$

It will be noted that the lower frequency peak is the highest peak. However, since

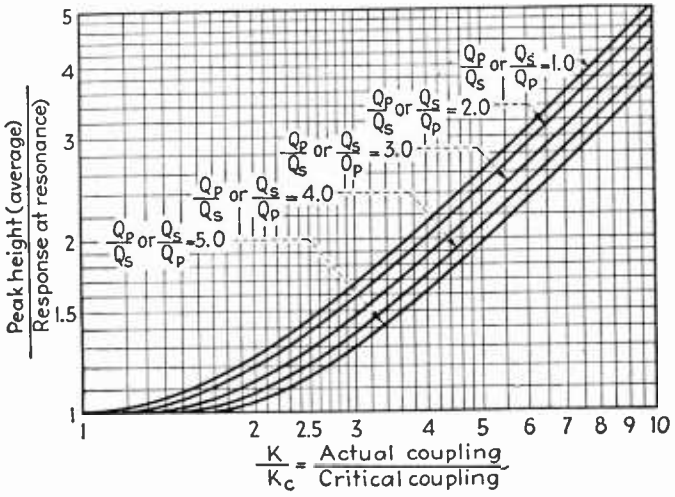


FIG. 21.—Universal curves giving the ratio of actual peak height to the response at resonance as a function of the coefficient of coupling for two coupled circuits resonant at the same frequency.

γ_p differs only slightly from unity in ordinary cases, it is customary to consider that the peaks are identical.

[1] See Aiken, *loc. cit.*

Ex. 1044-0166

When $Q_p = Q_s$, the value of E_c/E approximates the maximum possible response that can be obtained, as given by Eq. (38). On the other hand, when $Q_p \neq Q_s$, the peaks always have a height less than the maximum possible response in the secondary.

The ratio of the mean of the responses at the two peaks to the response at resonance is plotted in Fig. 21 for a number of Q ratios.

Universal Response Curves.—Equations (33) and (38) can be combined to give

$$\frac{\left\{\begin{array}{l}\text{Actual voltage across}\\ \text{secondary condenser}\end{array}\right\}}{\left\{\begin{array}{l}\text{Maximum possible voltage across secondary condenser with optimum coupling}\end{array}\right\}} = \frac{2n}{\left(1+\frac{pk_c}{2}\right)^2\left[(n^2+1)-(mp)^2+j(mp)\left(\sqrt{\frac{Q_p}{Q_s}}+\sqrt{\frac{Q_s}{Q_p}}\right)\right]} \tag{42a}$$

where $n = \dfrac{k}{k_c} = \dfrac{\text{actual coefficient of coupling}}{\text{critical coefficient of coupling}}$.

$$p = \frac{\text{cycles off resonance}}{\left(\frac{k_c}{2}\right) \times \text{resonant frequency}}.$$

$$m = \frac{1+\frac{pk_c}{4}}{\left(1+\frac{pk_c}{2}\right)^2}.$$

Universal curves based on Eq. (42*a*) are given in Figs. 22*a* and 22*b* for $k_c = 0$, and $k_c = 0.05$. These curves, especially when considered in conjunction with Figs. 18, 19, and 21, provide a complete and accurate picture of the response that can be expected when two resonance circuits are tuned to the same frequency and coupled together.

Figure 22 shows that when both primary and secondary circuits have the same Q, the response curve separates into double peaks whenever the coefficient of coupling exceeds the critical value, and these peaks maintain a substantially constant height irrespective of their location. On the other hand, when the two circuits have different Q's, then if the coupling exceeds the critical value by only a small amount, the response curve still has a single peak, but with reduced response at resonance. Double humps do not occur until the coupling exceeds the critical value by an amount that depends upon the ratio Q_p/Q_s, and the peaks become slightly lower as the coupling increases.

Practical Calculation of Coupled Systems Involving Two Resonant Circuits Tuned to the Same Frequency.—When it is desired to obtain the response curve of a given system, the relative shape of the curve of voltage developed across the secondary condenser can be determined with an accuracy sufficient for all ordinary purposes by the use of the universal curves of Fig. 22, interpolating between these in accordance with the Q ratio involved. The absolute magnitude of the curve can then be obtained by calculating the response at the resonant frequency with the aid of Eq. (35).

An alternative procedure for the case where the curve shows two peaks is to calculate the response at resonance by Eq. (35), determine the location and height of the peaks by Figs. 19 and 21, or Eqs. (40) and (42), and then to note that the response beyond the peaks falls to the response at resonance when the frequency is $\sqrt{2}$ times as far from resonance as the coupling peaks. This information is sufficient to define the shape of the double peaked curve in the vicinity of resonance. If quantitative information is desired at frequencies differing from resonance by at least twice as

Ex. 1044-0167

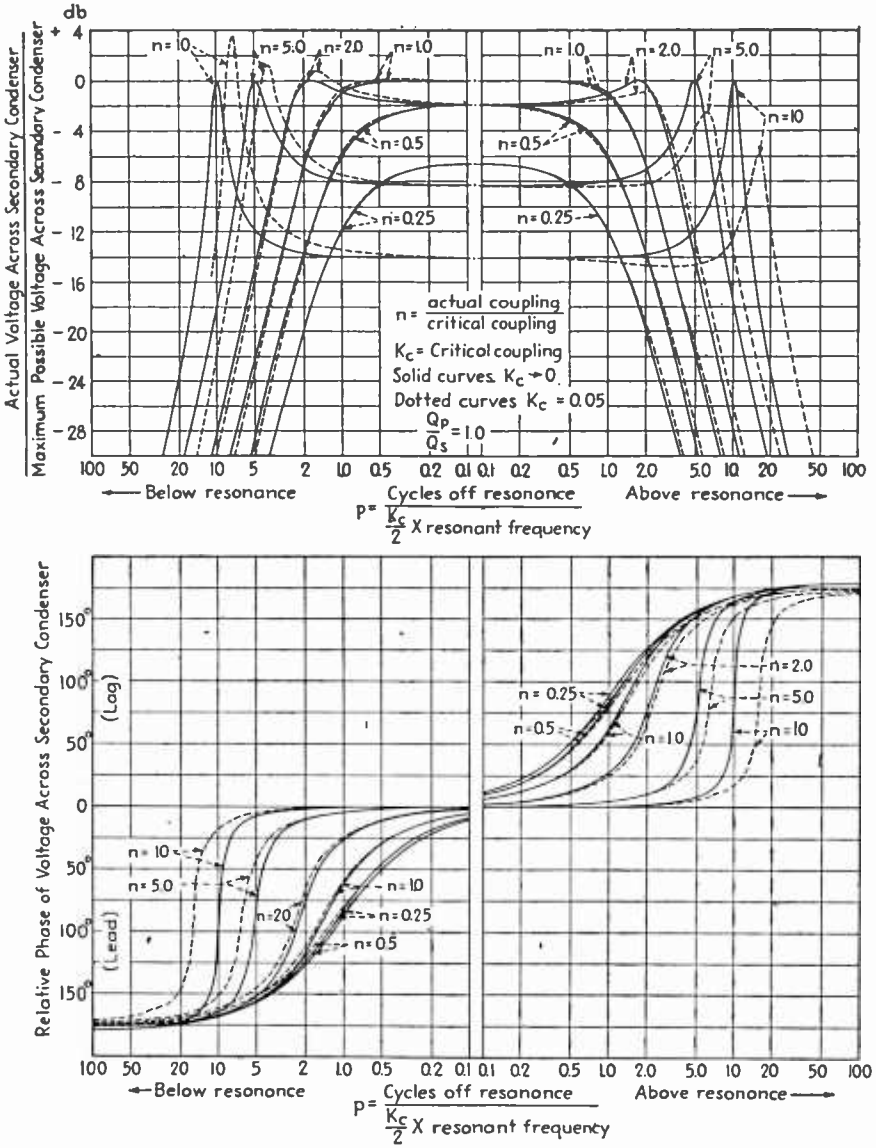


FIG. 22a.—Universal curves giving the phase and relative magnitude of the voltage across the secondary condenser for the case of two coupled circuits resonant at the same frequency and having a Q ratio of unity.

many cycles as do the peaks, this can be readily obtained by neglecting the circuit resistance and using Eq. (34).

Ex. 1044-0168

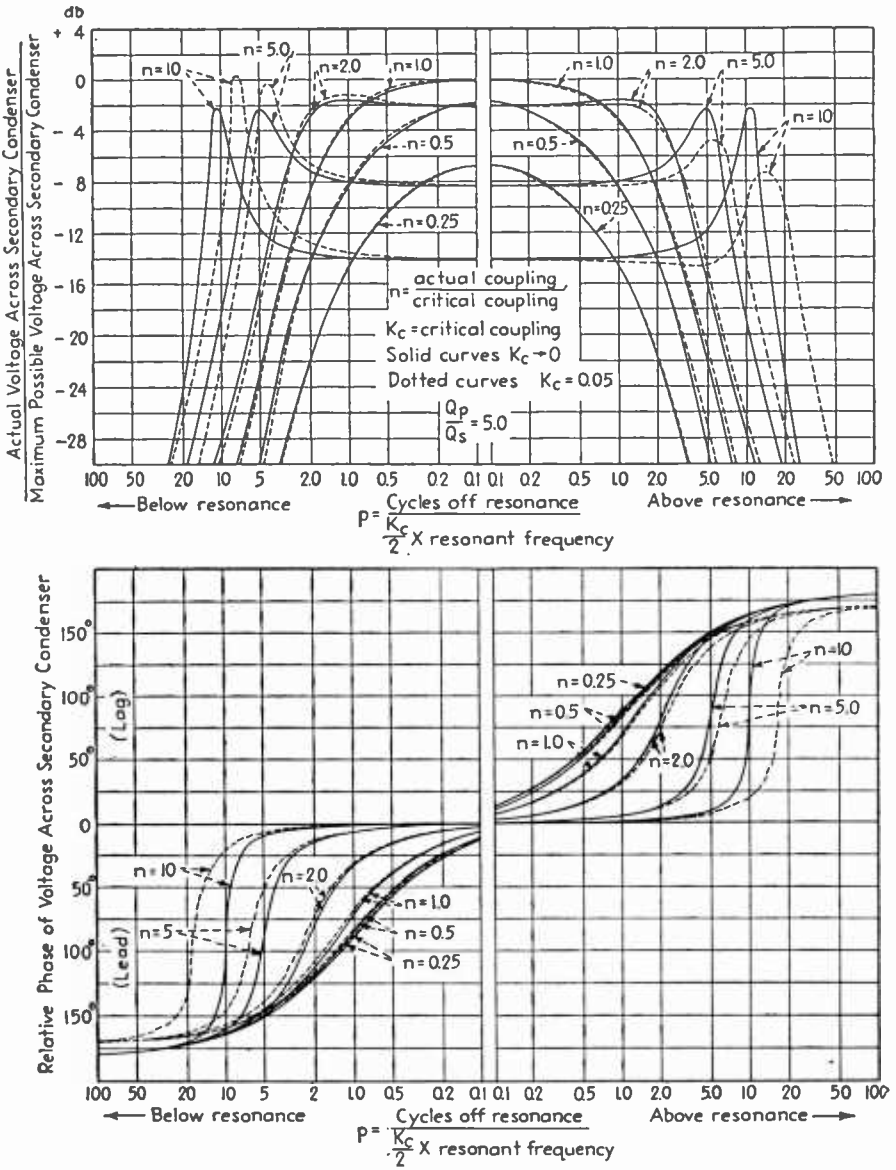


FIG. 22b.—Universal curves giving the phase and relative magnitude of the voltage across the secondary condenser for the case of two coupled circuits resonant at the same frequency and having a Q ratio of 5.

Parallel Feed in the Primary Circuit.—The analysis and discussion given above have all been for the case where the voltage was applied in series with the primary

Ex. 1044-0169

circuit, as shown in Fig. 17. In many cases where coupled resonant circuits are employed, the excitation of the primary circuit is obtained by applying the voltage in parallel with the primary circuit through a resistance, as shown in Fig. 23*a*. This arrangement can be termed *parallel excitation*, and corresponds to the case when the coupled circuits are excited from a vacuum tube.

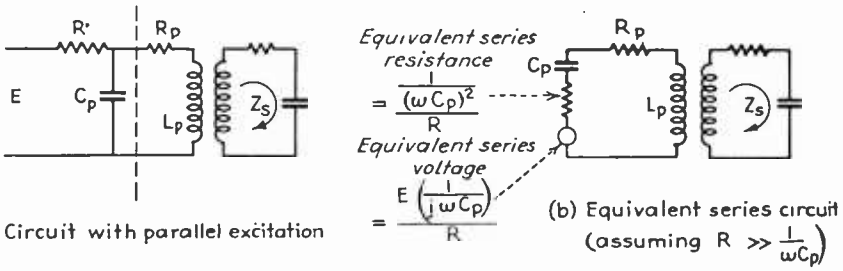
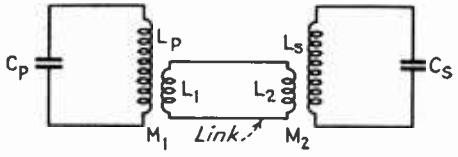


FIG. 23.—Actual and equivalent circuits for the case of parallel excitation of two coupled circuits.

The parallel excited circuit of Fig. 23*a* can be reduced by Thévenin's theorem to the equivalent arrangement shown in Fig. 23*b*. This is accomplished by considering that the portion of the circuit to the left of the dotted line is the equivalent generator circuit supplying the remainder of the circuit. The principal effect of parallel excitation is to introduce an added resistance in series with the primary circuit that becomes greater the lower the series resistance R used in the parallel feed. The equivalent



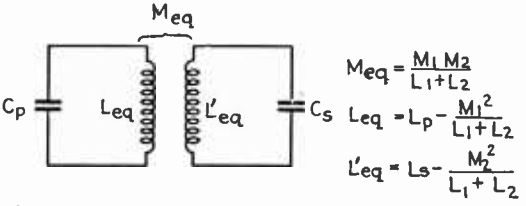


FIG. 24.—Actual and equivalent circuits for link coupling.

series voltage also varies somewhat with frequency, but in a limited range, such as the region in which the resonance phenomena occur, can be considered as being substantially constant.

Link-coupled Circuits.—Two resonant circuits are often coupled together by a link circuit, as shown in Fig. 24*a*. When this arrangement is reduced to an inductively coupled circuit by the method of Par. 8, the result for negligible losses in the link is

Ex. 1044-0170

as shown in Fig. 24*b*. It will be noted that the use of link coupling has the effect of reducing the equivalent primary and secondary inductance, and hence of raising the resonant frequencies. In other respects, the use of link coupling gives the same behavior as obtainable from ordinary inductive coupling.

6. Coupled Circuits with Resonant Primary and Secondary Tuned to Slightly Different Frequencies.—When two circuits resonant at slightly different frequencies are coupled together, the behavior depends upon the coefficient of coupling and the relative and absolute circuit Q's. When $Q_p = Q_s$, the response curve of secondary current (or voltage) has a shape of the same type as would be obtained if the two circuits were both resonant at the same frequency and coupled with an increased coefficient of coupling. In other words, the effect of detuning the circuits for the case of equal Q's is equivalent, as far as shape is concerned, to increasing the effective coupling. The only difference is that detuning causes the absolute magnitude of the response curve to be less than when the same shape is obtained without detuning.

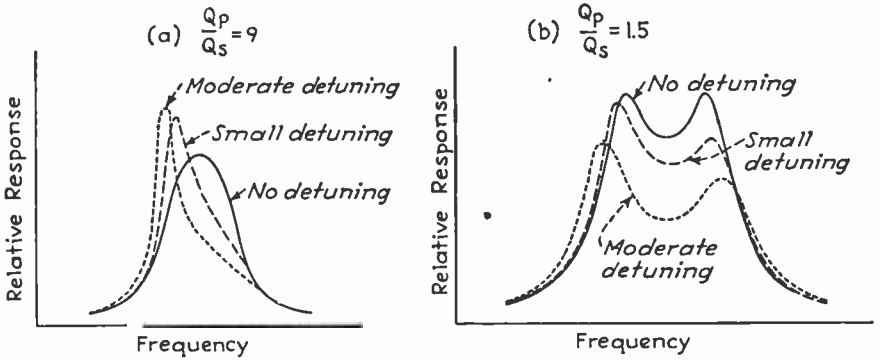


FIG. 25.—Curves illustrating the effect produced on the shape of the response curve by detuning primary and secondary circuits when the primary and secondary circuits do not have identical Q's.

Analysis shows that when $Q_p = Q_s$ the shape with detuning is as though the circuits were both tuned to the same frequency and coupled by an amount[1]

$$\left.\begin{matrix}\text{Equivalent coupling}\\ \text{corresponding to}\\ \text{detuned operation}\end{matrix}\right\} = \sqrt{k^2 + \left(\frac{\Delta}{f_0}\right)^2} \qquad (43)$$

where k = actual coefficient of coupling.

Δ = difference between resonant frequency of primary and secondary circuits.

f_0 = frequency midway between primary and secondary resonant frequencies.

In the more general case where the circuit Q's are not the same, the secondary response curve is no longer symmetrical about the mean frequency, as shown in Fig. 25. In cases where unsymmetrical peaks are obtained, the low-frequency peak will be depressed when the secondary is tuned to a higher frequency than the primary and the secondary Q is higher, or when the secondary is tuned to a lower frequency and the secondary Q is the lower. Otherwise the high frequency peak will be depressed.

Detuning produces much more effect upon the shape of the primary current curve than upon the secondary response, and if the detuning is at all large the primary

[1] See Harold A. Wheeler and J. Kelly Johnson, High Fidelity Receivers with Expanding Selectors, *Proc. I.R.E.*, Vol. 23, p. 594, June, 1935; or C. B. Aiken, *loc. cit.*

Equation (43) involves the assumption that Δ/f_0 is small compared with unity.

Ex. 1044-0171

current curve will have only a single peak, even though the secondary response shows pronounced double peaks.

7. Capacitive and Direct Inductive Coupling.—Examples of capacitive and direct inductive coupling are given in Fig. 26. The behavior of these circuits follows the same general character as that discussed for inductive coupling. Thus the secondary circuit can be considered as producing an equivalent coupled impedance in the primary circuit, and the primary circuit can be considered as inducing in the secondary a voltage that gives rise to the secondary current. The analysis for these cases can be worked out in a manner exactly analogous to the inductively coupled case. In fact, with direct inductive coupling, the same equations apply without change if the coupled inductance L_m is substituted for the mutual inductance M appearing in the inductively coupled equations.

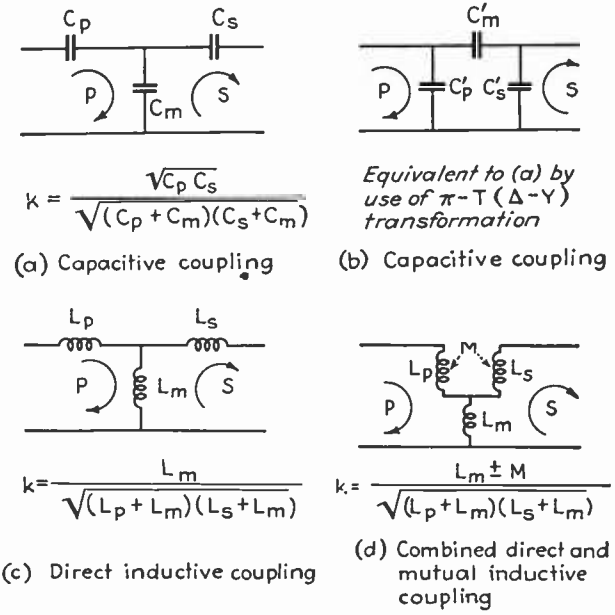


FIG. 26.—Circuits with simple capacitive and inductive coupling.

In the case of capacitive coupling, the equations already given for inductive coupling still apply, provided that one substitutes $1/j\omega C_m$ for $j\omega M$ in the equations for inductive coupling. This means that the mutual impedance varies inversely instead of directly with frequency, being now $1/\omega C_m$ instead of ωM. However, in a limited frequency range, such as that represented by the region about resonance, the difference is inconsequential. As a result, capacitively coupled circuits with a given coefficient of coupling act substantially the same as the corresponding inductively coupled circuit with the same coefficient of coupling. Hence when primary and secondary are both tuned to the same frequency, the secondary current characteristic has two humps if the coupling is large, *i.e.*, if the condenser C_m is small, whereas there is only one peak of secondary current when the coupling is small, *i.e.*, when condenser C_m is large.

8. Systems in Which Coupling Varies with Frequency.—Systems of this type are often used in arrangements involving circuits tunable over a wide-frequency range.

Ex. 1044-0172

Examples are the tuned radio-frequency circuits and antenna-coupling circuits of radio receivers, which are normally adjustable to be resonant over a three to one frequency range. In such arrangements, it is desirable to maintain substantially constant response over the entire tuning range.

Two methods are commonly used for attacking this problem. The first combines either tuning the circuits by variable condensers and using capacitive coupling or tuning by variable inductances and using inductive or direct inductive coupling. In the first of these arrangements, the coefficient of coupling is proportional to $1/f^2$; in the latter, it is proportional to f^2.

The second method is to employ combined electrostatic and magnetic coupling.[1] A simple example of such coupling is illustrated in Fig. 27. Here the coupling is capacitive at low frequencies and inductive at high frequencies, because the coupling combination of C_m in series with L_m has capacitive and inductive reactance under these respective conditions. In between, at the resonant frequency of L_m and C_m, there is zero coupling.

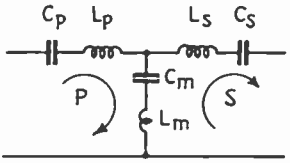


FIG. 27.—Circuit with combined capacitive and combined inductive coupling.

An almost unlimited number of arrangements for combining electrostatic and magnetic coupling are possible. A number of representative cases, and the ones most likely to be encountered in radio work, are shown in Figs. 28 to 34.[2] All these arrangements show a tuned secondary circuit LC with primary input terminals indicated by P. The part between the input terminals and the secondary can be thought of as representing the coupling system.

Analysis of Circuits with Combined Coupling.—The simplest method of analyzing these various forms of coupled circuits is to take advantage of the fact that all of them can be reduced to the simple coupled circuit of Fig. 35, provided that suitable values

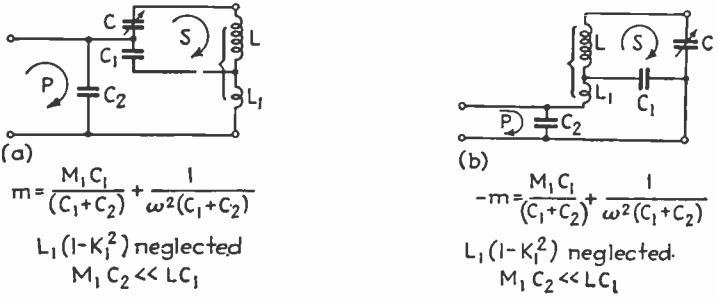


FIG. 28.—Circuits with complex coupling having coupling condensers in series with the tuning condensers.

are assigned to Z_p, Z_s, and M. The rules that determine the values of these quantities in the simple equivalent circuit are as follows:

1. The equivalent primary impedance Z_p of the equivalent circuit is the impedance that is measured across the primary terminals of the actual circuit when the secondary circuit has been opened.
2. The secondary impedance Z_s of the equivalent circuit is the impedance that is measured by opening the secondary of the actual circuit and determining the impedance between these open points when the primary is open-circuited.

[1] An exhaustive treatment of this is given by H. A. Wheeler and W. A. MacDonald. Theory and Operation of Tuned Radio-frequency Coupling Systems, *Proc. I.R.E.*, Vol. 19, p. 738, May 1931. See also Edward H Loftin and S. Young White, Combined Electromagnetic and Electrostatic Coupling and Some Uses of the Combination, *Proc. I.R.E.*, Vol. 14, p. 605, October, 1926.

[2] These are from Wheeler and MacDonald, *loc. cit.*

Ex. 1044-0173

3. The equivalent mutual inductance M is determined by assuming a current I_0 flowing into the primary circuit. The voltage that then appears across an open circuit in the secondary is equal to $-j\omega MI_0$.

In making use of the equivalent circuit of Fig. 35, it is to be remembered that the values of Z_p, Z_s, and M may all vary with frequency, so that it is generally necessary to determine a new equivalent circuit for each frequency at which calculations are to be made.

After the actual coupled circuit has been reduced by the preceding procedure to its equivalent form, shown in Fig. 35, one can then apply the formulas that have already been given for inductively coupled circuits, using the appropriate values M, Z_s, Z_p, as determined for the equivalent circuit. This procedure has the advantage of using the same fundamental formulas to handle all types of coupling, and makes it possible to carry on the analysis in the same manner for all cases.

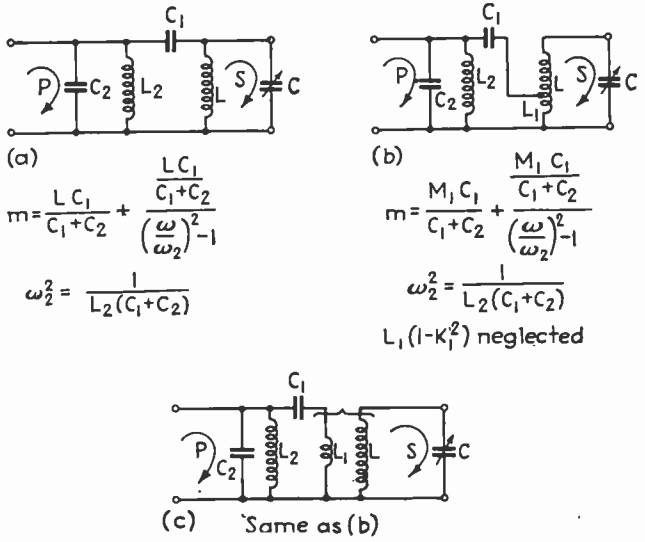


FIG. 29.—Circuits with complex coupling having high-inductance primaries not inductively coupled to the secondaries.

The quantity M that appears in the equivalent circuit represents the effective coupling that is present between the primary and secondary circuits. It is not necessarily a real mutual inductance of the inductive type, but rather a sort of mathematical fiction that gives the equivalent effect of whatever coupling is really present. If the actual coupling is capacitive, the numerical value of M will be found to be negative; if the coupling is of a complex type representing both resistive and reactive coupling, the numerical value of M will be found to have both real and imaginary parts. The proper procedure, in any case, is to take the value of M as it comes and substitute it with its appropriate sign and phase angle whenever M appears in the expressions for inductively coupled circuits.

Application of Analysis to Circuits of Figs. 28 to 34.—Equations for the equivalent mutual inductance of circuits of Figs. 28 to 34 are shown below each circuit diagram. In these equations and circuits, a uniform nomenclature has been used as follows:

L = tuned secondary coil.
C = secondary tuning condenser.
L_1 = primary coil of low inductance closely coupled to secondary coil.

Ex. 1044-0174

L_2 = primary coil of high inductance not closely coupled to secondary.
M_1 = mutual inductance between L and L_1.
M_2 = mutual inductance between L and L_2.
M_{12} = mutual inductance between L_1 and L_2.
k_1, k_2, and k_{12} = coupling coefficients corresponding to M_1, M_2, and M_{12}, respectively.
C_1 = coupling condenser.
C_2 = capacity across input terminals (such as tube capacity).
ω_2 = resonant frequency of primary circuit as viewed from input terminals P.

Relative polarity of coils is indicated by the way in which coils are drawn: thus in Fig. 30c, L and L_2 have opposite polarity from L and L_1 in Fig. 30*d*.

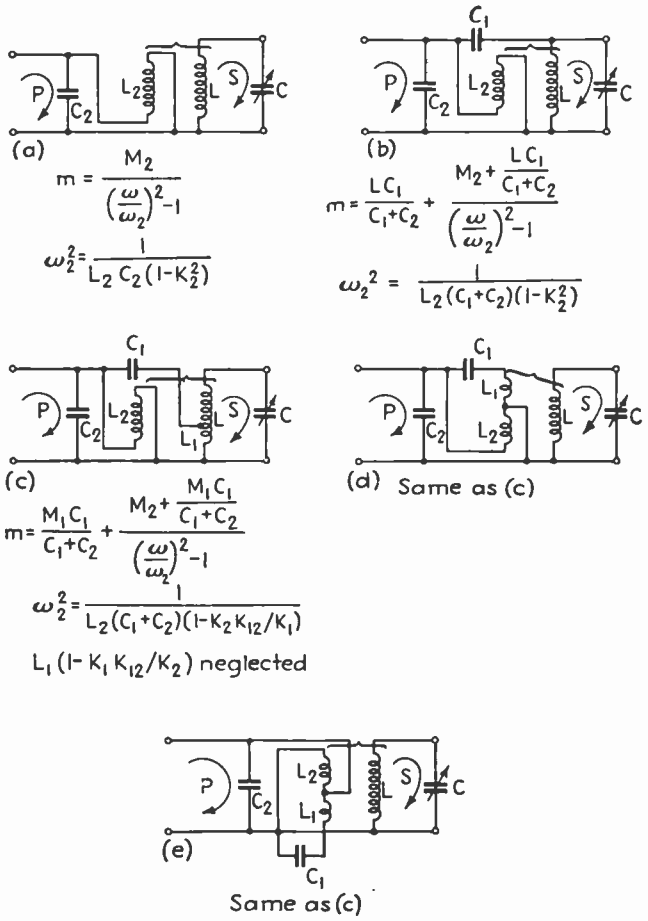
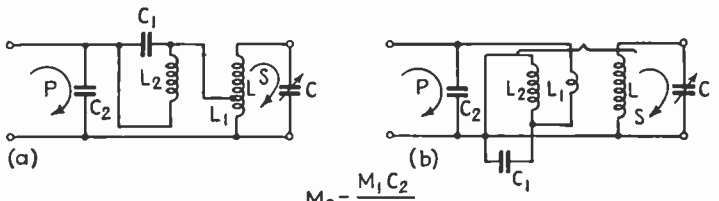


FIG. 30.—Circuits with complex coupling having high-inductance primaries inductively coupled to secondaries.

In the circuits of Figs. 28 to 34, C_1 can in some cases be supplied by the stray capacity between coils and in other cases, by the distributed capacity of the high inductance primary coil L_2. Nearly all the circuits of Figs. 28 to 33 are character-

Ex. 1044-0175

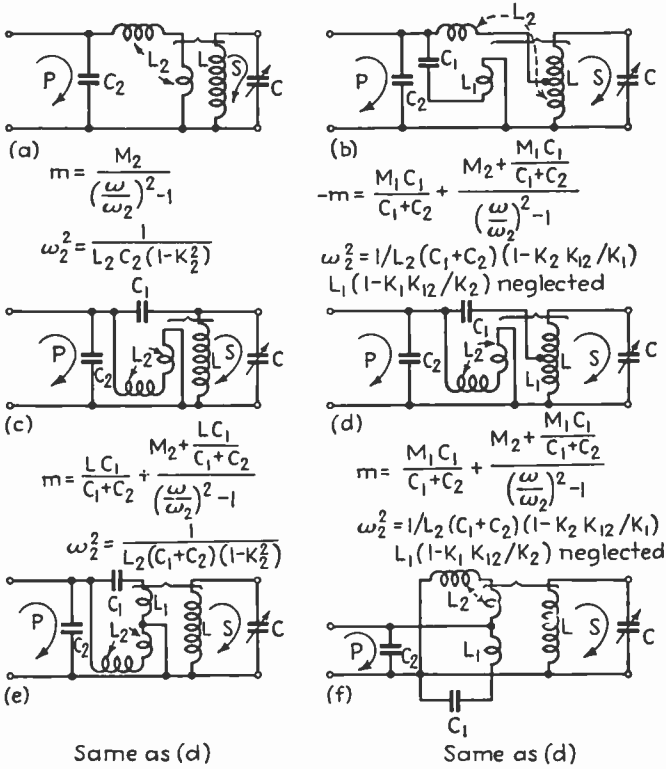


$$m = \frac{M_1 C_1}{C_1+C_2} + \frac{M_2 - \frac{M_1 C_2}{C_1+C_2}}{\left(\frac{\omega}{\omega_2}\right)^2 - 1}$$

$$\omega_2^2 = \frac{1}{L_2(C_1+C_2)(1-K_2K_{12}/K_1)}$$

$L_1(1-K_1K_{12}/K_2)$ neglected

FIG. 31.—Circuits with complex coupling having high-inductance primaries inductively coupled to secondaries.



(a) $m = \dfrac{M_2}{\left(\frac{\omega}{\omega_2}\right)^2 - 1}$

$\omega_2^2 = \dfrac{1}{L_2 C_2 (1-K_2^2)}$

(b) $-m = \dfrac{M_1 C_1}{C_1+C_2} + \dfrac{M_2 + \frac{M_1 C_1}{C_1+C_2}}{\left(\frac{\omega}{\omega_2}\right)^2 - 1}$

$\omega_2^2 = 1/L_2(C_1+C_2)(1-K_2K_{12}/K_1)$

$L_1(1-K_1K_{12}/K_2)$ neglected

(c) $m = \dfrac{LC_1}{C_1+C_2} + \dfrac{M_2 + \frac{LC_1}{C_1+C_2}}{\left(\frac{\omega}{\omega_2}\right)^2 - 1}$

$\omega_2^2 = \dfrac{1}{L_2(C_1+C_2)(1-K_2^2)}$

(d) $m = \dfrac{M_1 C_1}{C_1+C_2} + \dfrac{M_2 + \frac{M_1 C_1}{C_1+C_2}}{\left(\frac{\omega}{\omega_2}\right)^2 - 1}$

$\omega_2^2 = 1/L_2(C_1+C_2)(1-K_2K_{12}/K_1)$

$L_1(1-K_1K_{12}/K_2)$ neglected

FIG. 32.—Circuits with complex coupling having high-inductance primaries coupled to secondaries by coupling coils.

Ex. 1044-0176

ized by having an equivalent mutual inductance that is the sum of two components, one of which is independent of frequency, the other of which decreases rapidly with frequency. In all the arrangements having a high inductance primary L_2, the rate of

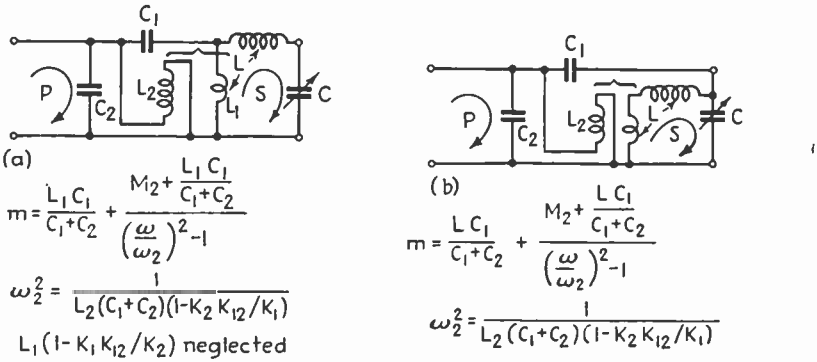


FIG. 33.—Circuits with complex coupling having high-inductance primaries coupled to coupling coils in secondary circuits.

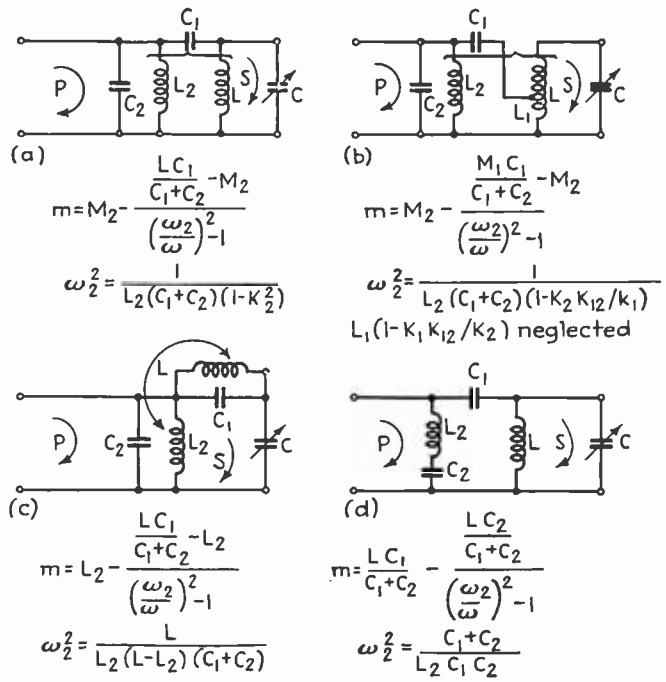


FIG. 34.—Circuits with complex coupling in which the equivalent mutual inductance varies according to a different law from that applying to Figs. 28–33.

rise of the variable component at low frequencies is determined by how close the resonant frequency $\omega_2/2\pi$ of the primary circuit is to the frequency under consideration. The circuits of Fig. 34 differ from those in Figs. 28 to 33 in that the equivalent

Ex. 1044-0177

mutual inductance equals a fixed value minus a component that increases with frequency.

Typical curves showing ways in which the equivalent mutual inductance can be made to vary with frequency with different coupling systems are shown in Fig. 36.

9. Band-pass Filters Based on Coupled Resonant Circuits.—A response curve of secondary voltage having a relatively flat top with steep sides can be obtained by properly coupling two circuits resonant at the same frequency, as is apparent by examining Fig. 22. The resulting arrangement is commonly termed a band-pass filter, and has characteristics particularly desirable for handling modulated waves, because the response can be made practically the same to side-band frequencies as to the carrier. In contrast with this, ordinary resonance curves have a rounded top and so discriminate against the higher side-band frequencies in favor of the carrier and lower side-band frequencies.

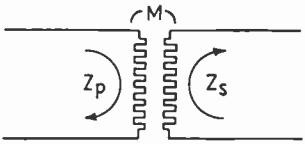


FIG. 35.—Equivalent form for any coupled circuit.

Design of Band-pass Circuits Where Primary and Secondary Are Tuned to the Same Frequency.—In order to obtain band-pass characteristics in this way, it is necessary to adjust the coefficient of coupling and the circuit Q's properly in order to obtain the desired width and flatness of the response characteristic. Whereas the width and flatness of the top of the response curve are both affected by the coefficient of coupling and the circuit Q's, the width of the top is determined primarily by the coefficient of

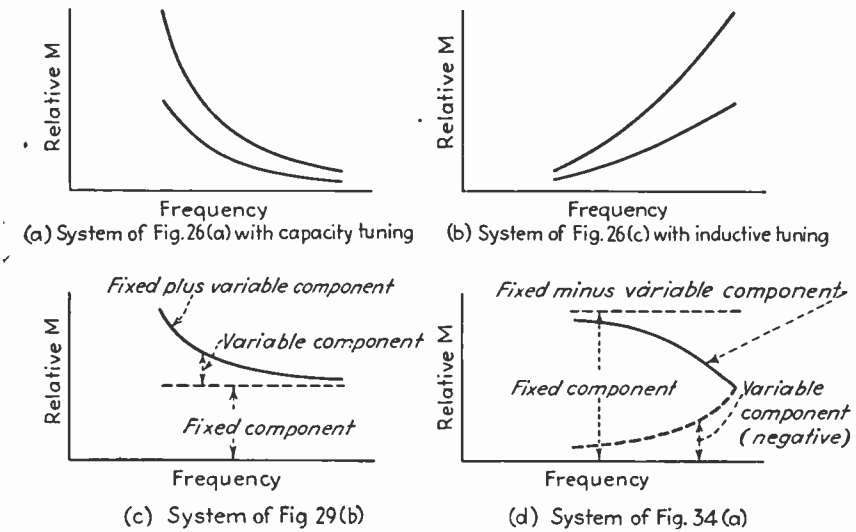


FIG. 36.—Curves showing some of the ways in which the equivalent mutual inductance can be made to vary with frequency in systems combining electrostatic and magnetic coupling.

coupling, and the flatness of the top depends mainly on the circuit Q's. Large couplings correspond to wide tops, high Q's give pronounced double peaks, and low Q's cause the top to be rounded off.

The actual design of band-pass filters to give a desired performance can be carried out by the use of Figs. 19 and 20 and Eqs. (39) to (42) The method of doing this is illustrated by the following example:

Ex. 1044-0178

Example.—It is desired to design a band-pass filter such that over a 10-kc band the output voltage is at least equal to the response at the middle of the band and does not exceed this response by a factor greater than 1.2. The pass band is to be centered on 162 kc, and the primary and secondary circuits are to have identical Q's.

The band width between peaks is $10/\sqrt{2} = 7.07$ kc, so that $\Delta f/f_0 = 7.07/162 = 0.0436$. Also, reference to Fig. 21 shows that for $Q_p = Q_s$ and a peak 1.2 times the response at resonance, k/k_c must be 1.86. If we turn to Fig. 19, we see that this corresponds to $g = 0.84$, and $k = 0.051$. Hence

$$Q = \frac{1}{k_c} = \frac{1.86}{0.051} = 36.5.$$

As a rough design rule for the case where Q_s and Q_p are approximately equal, a reasonably flat topped characteristic can be obtained over a frequency band approximately $1.2\ kf_0$, where f_0 is the frequency at the center of the band and k is the coefficient of coupling. The corresponding circuit Q's should then approximate the relation $\sqrt{Q_pQ_s} = 1.75/k$.[1]

Effect of Resonant Frequency on Band Width.—Where a band-pass filter is used for tuning purposes instead of a simple resonant circuit, the position of the pass band is ordinarily controlled by the use of identical primary and secondary variable condensers mounted on a common shaft. The width of the pass band in cycles is then approximately proportional to the resonant frequency of the circuits when inductive or direct coupling is used and approximately inversely proportional to the resonant frequency when capacitive coupling is employed. If the width of the pass band is to be constant irrespective of the frequency at which the band is located, it is necessary for the coefficient of coupling to be inversely proportional to the resonant frequency. Such a coefficient can be obtained by varying the coupling with the same control that adjusts the primary and secondary circuits, or can be approximated by the use of combined inductive and capacitive coupling.

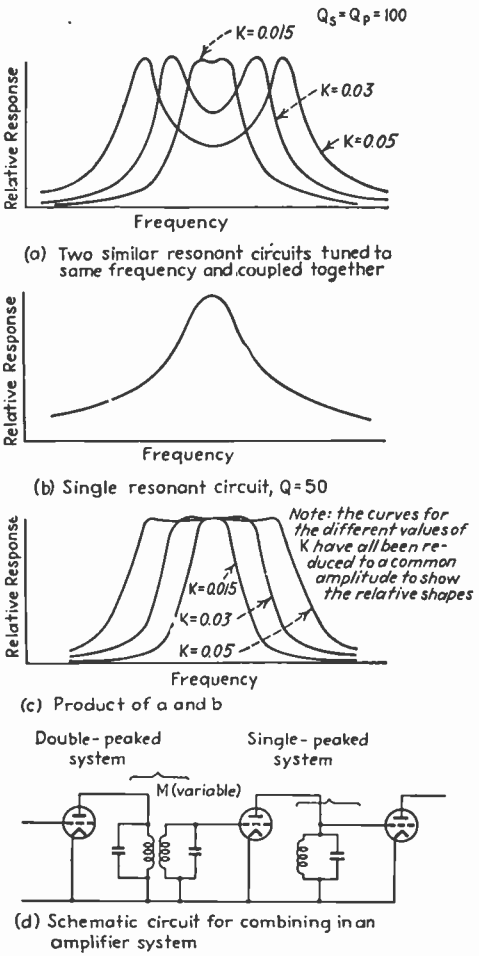


FIG. 37.—Diagram illustrating how band-pass action can be realized by combining a double-humped curve obtained from a pair of coupled circuits, with a simple resonant curve. By using proper circuit proportions, band-pass action is realized irrespective of the coefficient of coupling between the pair of coupled circuits.

Band-pass Characteristics by Detuning of Coupled Circuits.—When the primary and secondary circuits have the same Q, a slight detuning produces the same effect, as far as the shape of the secondary response curve is concerned, as increasing the coefficient

[1] Design proportions in the case where Q_s and Q_p differ appreciably are discussed in Par. 19, Sec. 5.

Ex. 1044-0179

of coupling to a value $\sqrt{k^2 + (\Delta/f_0)^2}$, where k is the actual coefficient of coupling, Δ is the difference between the resonant frequencies, and f_0 is the average of the two resonant frequencies. The only essential difference between detuning and increasing the coupling is that with detuning the magnitude of the response is less than if the same-shaped curve is obtained by increasing coupling.

When the primary and secondary circuits have appreciably different Q's, detuning causes one side of the response curve to be depressed, as illustrated in Fig. 25, with a resulting undesirable characteristic.

Detuning effects in band-pass filters are of importance primarily because such detuning may occur accidentally through improper adjustments. This method of obtaining band-pass characteristics is not generally employed because of the necessity of maintaining equal primary and secondary circuit Q's and because of the practical difficulty of adjusting the circuits to the proper resonant frequencies.

Band-pass Action Obtained by Combining a Single- and Double-peaked Resonance Curve with a Simple Resonance Curve.[1]—Band-pass action can also be obtained by combining a double-peaked curve with the resonance curve of a simple single circuit, as shown in Fig. 37. Good band-pass characteristics result when the circuits are so proportioned that the product of the double-peaked and single-peaked curve is the same at resonance and at the two peaks. Analysis shows that this condition is obtained when the circuit Q's satisfy the relation[2]

$$Q_3 = \frac{Q_1}{1 + \frac{Q_1}{Q_2}} \tag{44}$$

where Q_1 and Q_2 apply to the two circuits that are coupled to give the double-peaked curve, and Q_3 applies to the simple resonant circuit. For $Q_1 = Q_2$, Eq. (44) requires that the Q_3 of the single resonant circuit should be half that of the two resonant circuits that are coupled together.

The condition specified in Eq. (44), is independent of the coupling. Hence if the Q's of the various circuits satisfy Eq. (44), variation of the coupling between the two coupled circuits will vary the width of the pass band without destroying the band-pass action, as shown in Fig. 37.

TRANSMISSION LINES[3]

10. Basic Equations and Concepts.—The voltage and current relations that exist on a transmission line can be expressed by the equations

$$\begin{aligned} E_s &= E_r \cosh \sqrt{ZY}l + I_r Z_0 \sinh \sqrt{ZY}l \\ I_s &= I_r \cosh \sqrt{ZY}l + \frac{E_r}{Z_0} \sinh \sqrt{ZY}l \end{aligned} \tag{45}$$

where $Z_0 = \sqrt{Z/Y}$ = characteristic impedance.

$\sqrt{ZY}$ = propagation constant (or hyperbolic angle per unit length).

[1] A more detailed discussion of band-pass filters of this type is given by Harold A. Wheeler and J. Kelly Johnson, High Fidelity Receivers with Expanding Selectors, *Proc. I.R.E.*, Vol. 23, p. 594, June, 1935, and Ho-Shou Loh, On Single and Couple Tuned Circuits Having Constant Response-band Characteristics, *Proc. I.R.E.*, Vol. 26, p. 469, April, 1938.

[2] Actually, the band-pass characteristic is slightly improved if the single peak curve corresponds to a Q slightly less than the value called for by Eq. (44).

[3] More detailed information on the theory of transmission lines can be found in many books on the subject. A good treatment for the radio engineer is to be found on pp. 94–178 of W. L. Everitt, "Communication Engineering," 2d ed., McGraw-Hill, New York. Other good books are L. F. Woodruff, "Electric Power Transmission and Distribution," 2d ed., Wiley, New York; F. W. Norris and L. A. Bingham, "Electrical Characteristics of Power and Telephone Transmission Lines," International Textbook, Scranton, Pa.; A. E. Kennelly, "Hyperbolic Functions as Applied to Electrical Engineering," McGraw-Hill, New York.

Ex. 1044-0180

$Z = R + j\omega L$ = series impedance per unit length.
$Y = G + j\omega C$ = shunt admittance per unit length.
l = length from receiver in same units as Z and Y.
R, L, C, and G = resistance, inductance, capacity and leakage, respectively, of line per unit length.
E and I are voltage and current, respectively.
Subscript s denotes sending or generator-end quantity.
Subscript r denotes receiving-end or load quantity.

Characteristic Impedance.—The characteristic impedance Z_0 appearing in Eq. (45) is given by the relation

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}}\sqrt{\frac{1 + \frac{R}{j\omega L}}{1 + \frac{G}{j\omega C}}} \tag{46}$$

The characteristic impedance has the dimension of ohms, and is determined by the line proportions and the frequency.

At all radio frequencies, and also usually at all but the lowest audio frequencies, the ratios $R/\omega L$ and $G/\omega C$ are so small that the characteristic impedance can be taken as a pure resistance of $\sqrt{L/C}$ ohms. Furthermore, even when the line losses are not negligible, if $R/\omega L = G/\omega C$, the characteristic impedance is still a pure resistance equal to $\sqrt{L/C}$ ohms.

At frequencies so low that R is comparable with ωL and G comparable with ωC, the characteristic impedance varies with frequency, and approaches $\sqrt{R/G}$ as the frequency approaches zero.

Formulas and curves giving the characteristic impedance of various types of lines with air insulation are given in Figs. 38 and 39 for the case of high frequencies.[1] When dielectric insulation is employed, as with cables, the characteristic impedance is lower than the value with air insulation by the factor of $1/\sqrt{k}$, where k is the dielectric constant of the insulation.

Attenuation Constant.—The propagation constant $\sqrt{ZY}$ is commonly divided into real and imaginary parts

$$\sqrt{ZY} = \alpha + j\beta \tag{47}$$

The real part α is termed the *attenuation constant* and determines the energy dissipated in the line. The attenuation constant is determined by the resistance and leakage of the line, the frequency, and the extent to which R/L differs from G/C. At low frequencies, $\alpha = \sqrt{RG}$, which is the lowest possible value that the attenuation constant can have with a given resistance and leakage. As the frequency increases, the attenuation constant will in general become larger until at frequencies such that $\omega L >> R$ and $\omega C << G$, it reaches the limiting value

$$\left.\begin{matrix}\text{Attenuation constant} \\ \text{at high frequencies}\end{matrix}\right\} = \frac{R}{2Z_0} + \frac{GZ_0}{2} \tag{48}$$

In the special case where $R/L = G/C$, the attenuation does not increase with frequency and maintains its value $\alpha = \sqrt{RG}$ for all frequencies, even including very high frequencies, as shown by substitution in Eq. (48). A line with these particular

[1] The formulas in Fig. 38 are from J. F. Morrison, Transmission Lines, *Western Electric Pickups*, December, 1939. See also R. D. Duncan, Jr., Characteristic Impedance of Grounded and Ungrounded Open-wire Transmission Lines, *Communications*, Vol. 18, p. 10, June, 1938.

Ex. 1044-0181

LOGARITHMS TO THE BASE 10 — I_1 = GENERATOR CURRENT

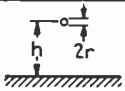
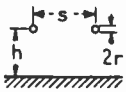
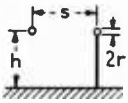
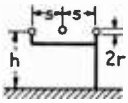
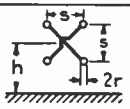
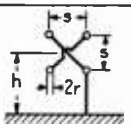
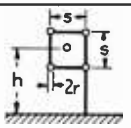
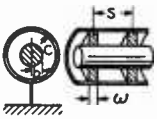
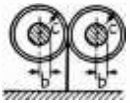
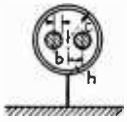
LINE CONFIGURATION		CHARACTERISTIC IMPEDANCE	NET GROUND-RETURN CURRENT
Single wire		$Z_0 = 138 \operatorname{Log} \frac{2h}{r}$	$I_{Gnd} = I_1$
2-Wire balanced		$Z_0 = 276 \operatorname{Log} \frac{s}{r}$	$I_{Gnd} = 0$
2-Wire 1 wire grounded		$Z_0 \approx 276 \frac{\operatorname{Log} \frac{s}{r} \operatorname{Log} \left[\rho^2 \frac{s}{r}\right]}{\operatorname{Log} \left[\rho^2 \left(\frac{s}{r}\right)^2\right]}$ $\rho = \frac{2h}{s}$	$I_{Gnd} \approx I_1 \frac{\operatorname{Log} \frac{s}{r}}{\operatorname{Log} \frac{2h}{r}}$
3-Wire 2 wires grounded		$Z_0 \approx 69 \left[\operatorname{Log} \frac{s^3}{2r^3} - \frac{\left(\operatorname{Log} \frac{s}{2r}\right)^2}{\operatorname{Log} \frac{2h^2}{rs}}\right]$ $\rho = \frac{2h}{s}$	$I_{Gnd} \approx I_1 \frac{\operatorname{Log} \frac{s}{2r}}{\operatorname{Log} \frac{s\rho^2}{2r}}$
4-Wire balanced		$Z_0 = 138 \left(\operatorname{Log} \frac{s}{r}\right) - 21$	$I_{Gnd} = 0$
4-Wire 2-wires grounded		$Z_0 \approx 138 \left[\frac{\operatorname{Log} \frac{s}{r\sqrt{2}} \operatorname{Log} \left[\rho^4 \frac{s}{r\sqrt{2}}\right]}{\operatorname{Log} \left[\rho^4 \left(\frac{s}{r\sqrt{2}}\right)^2\right]}\right]$ $\rho = \frac{2h}{s}$	$I_{Gnd} \approx I_1 \frac{\operatorname{Log} \frac{s}{r\sqrt{2}}}{\operatorname{Log} \frac{\rho^2 s}{r\sqrt{2}}}$
5-Wire 4 wires grounded		$Z_0 \approx 138 \left[\operatorname{Log} \frac{2h}{r} - \frac{\left[\operatorname{Log} 2\rho^2\right]^2}{\operatorname{Log} \left[\rho^3 \frac{h\sqrt{2}}{r}\right]}\right]$ $\rho \frac{2h}{s}$	$I_{Gnd} \approx I_1 \frac{\operatorname{Log} \frac{s}{r4\sqrt{2}}}{\operatorname{Log} \frac{s\rho^4}{r\sqrt{2}}}$
Concentric (coaxial)		$Z_0 = 138 \frac{\operatorname{Log} \frac{c}{b}}{\sqrt{1 + \frac{(\varepsilon - 1)\omega}{s}}}$ ε = Dielectric constant of insulating material	
Double coaxial balanced		$Z_0 = 276 \frac{\operatorname{Log} \frac{c}{b}}{\sqrt{1 + \frac{(\varepsilon - 1)\omega}{s}}}$	
Shielded pair balanced		$Z_0 = \frac{120}{\sqrt{\varepsilon}} \left[2.303 \operatorname{Log} \left(2v \frac{1-\sigma^2}{1+\sigma^2}\right) - \frac{1+4v^2}{16v^4} \left(1 - 4\sigma^2\right)\right]$ ε = Dieletric constant of medium ε = Unity for gaseous medium $v = \frac{h}{b}$; $\sigma = \frac{h}{c}$	

FIG. 38.—Characteristics of various types of transmission lines erected parallel to a perfectly conducting earth.

Ex. 1044-0182

proportions is said to be a *distortionless* line because it has the same attenuation for all frequencies.

In lines in which the insulation is primarily air, the leakage conductance G approaches zero. Also, at high radio frequencies, skin effect is so great that the

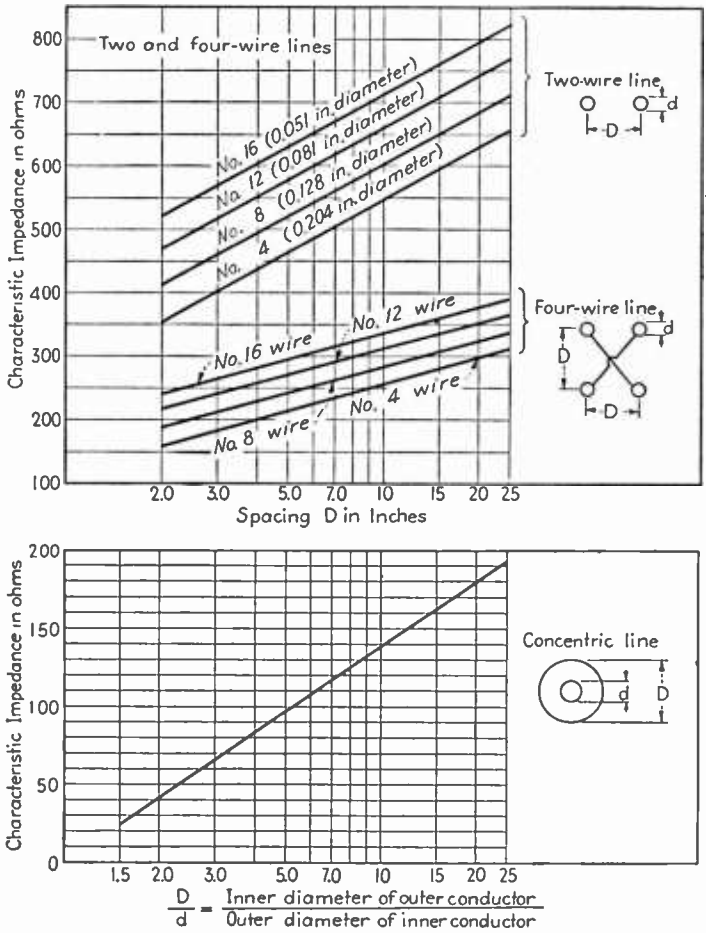


FIG. 39.—Characteristic impedance of common types of transmission lines.

resistance of the conductors is given accurately by Eq. (7), Sec. 2. Under these conditions for a copper line, one has the following:

For two-wire line:[1]

$$\alpha = 0.00362 \frac{\sqrt{f_{mc}}}{d_{in} \log_{10}\left(\frac{2D}{d}\right)} \text{ per 1,000 ft} \tag{49}$$

[1] This neglects proximity effect. For very close spacing, this value of α should be multiplied by the factor given in Fig. 6, Sec. 2.

Ex. 1044-0183

For concentric line:

$$\alpha = 0.00362 \frac{\sqrt{f_{mc}}\left(1 + \frac{D}{d}\right)}{D_{in} \log_{10}\left(\frac{D}{d}\right)} \text{ per 1,000 ft} \tag{50a}$$

where f_{mc} is the frequency in megacycles and d and D have the meanings shown in Fig. 39, and are in inches (indicated by the subscript in).

In the case of a concentric line, if D is fixed, the lowest attenuation occurs when $D/d = 3.6$, corresponding to $Z_0 = 77$ ohms and giving

$$\left.\begin{array}{l}\text{Attenuation } \alpha \text{ of optimum} \\ \text{concentric line per} \\ \text{1,000 ft}\end{array}\right\} = \frac{0.0299\sqrt{f_{mc}}}{D_{in}} \tag{50b}$$

The power that can be transmitted for a given maximum voltage gradient in a concentric line having an outer conductor diameter D is a maximum when $D/d = 1.65$, corresponding to $Z_0 = 30$ ohms.

Values of attenuation constants expressed in db per 1,000 feet are plotted in Fig. 40 for two-wire and concentric copper lines having air insulation.[1] When solid insulation is employed, the attenuations will be much greater. Thus cable with rubber insulation can be expected to have an attenuation from 10 to 50 times that of a corresponding 600-ohm air-insulated two-wire line.[2]

Phase Constant, Wave Length, and Velocity of Phase Propagation.—The imaginary part β of $\sqrt{ZY}$ is termed the *phase constant*, and determines the wave length and velocity of phase propagation.

A wave length is defined as a length of line l such that $\beta l = 2\pi$. Hence

$$\left.\begin{array}{l}\text{Length corresponding} \\ \text{to one wave length}\end{array}\right\} = \lambda = \frac{2\pi}{\beta} \tag{51}$$

In the case of lines with air insulation, the distance corresponding to a wave length will be only very slightly less than the wave length of a radio wave of the same frequency traveling in space. Accordingly

$$\left.\begin{array}{l}\text{Approximate wave length for} \\ \text{lines with air insulation}\end{array}\right\} = \frac{300{,}000{,}000}{f} \text{ meters} \tag{52}$$

where f is in cycles.

With dielectric insulation, as in the case of cables, the wave length is less than with air insulation by the factor $1/\sqrt{k}$, where k is the dielectric constant of the insulation.

The product of frequency and wave length corresponds to velocity, and is termed the *velocity of phase propagation* or simply *wave velocity*. This is not a true velocity in the sense that something tangible actually travels at a certain speed, but rather means that the relative phases at different places on the line are such as to give the appearance of velocity. With air insulation, the velocity at high frequencies is practically the same as the velocity of light; while with dielectric insulation the velocity is $1/\sqrt{k}$ as great.

Hyperbolic Functions and Complex Components.—The hyperbolic functions cosh x and sinh x appearing in Eq. (45) (where $\sqrt{ZY}l = x$) represent certain combinations of exponents as follows:

[1] The attenuation in db is 8.686 αl, and gives the energy loss in db for the line operated as a nonresonant line (see Par. 14).

[2] For further information, see C. C. Harris, Losses in Twisted Pair Transmission Lines at Radio Frequencies, *Proc. I.R.E.*, Vol. 24, p. 425, March, 1936.

$$\cosh x = \frac{\epsilon^x + \epsilon^{-x}}{2}$$
$$\sinh x = \frac{\epsilon^x - \epsilon^{-x}}{2} \tag{53}$$

The more common and useful properties of hyperbolic functions are given in Par. 15, Sec. 1. Values of the function when x is real are tabulated in Pars. 6, 7, and 8, Sec. 1.

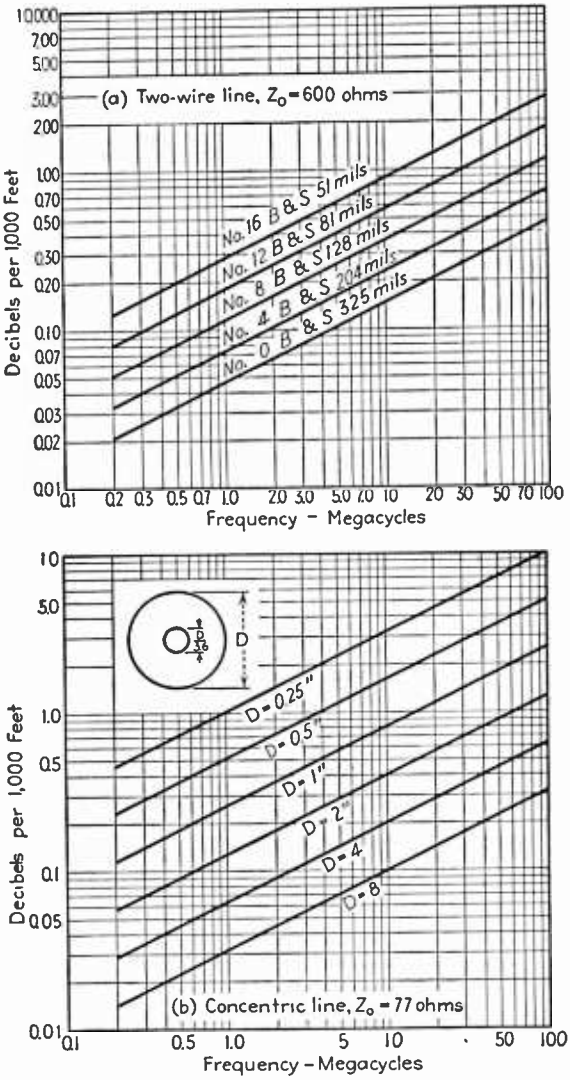


FIG. 40.—Attenuation of two-wire and concentric transmission lines.

When x is complex, as is the case in Eq. (45) where $x = \sqrt{ZY}l = \alpha l + j\beta l$, the hyperbolic function can be evaluated by any one of the following methods: (1) the

use of Kennelly's charts,[1] (2) expansion of the complex hyperbolic function into functions of real circular and real hyperbolic angles, and (3) direct evaluation of the complex exponents appearing in the definition of the function. Examples illustrating the details involved in applying the second and third methods are as follows:

Example 1.—Evaluate $\sinh \underline{2/60°}$ by the expansion method.

$$\sinh \underline{2/60°} = \sinh (1.0 + j1.732)$$

Referring to Par. 15, Sec. 1

$$\begin{aligned}\sinh (1.0 + j1.732) &= \sinh 1.0 \cos 1.732 + j \cosh 1.0 \sin 1.732 \\ &= 1.175 \times (-0.1599) + j1.547 \times 0.987 \\ &= -0.188 + j1.525 \\ &= 1.53\underline{/97°0'}\end{aligned}$$

Example 2.—Evaluate $\sinh \underline{2/60°}$ by direct evaluation of the complex exponents.

$$\sinh \underline{2/60°} = \frac{\epsilon^{\underline{2/60°}} - \epsilon^{\underline{-2/60°}}}{2}$$

But $\underline{2/60°} = 1.0 + j1.732$

$$\begin{aligned}\therefore \epsilon^{\underline{2/60°}} &= \epsilon^{(1.0+j1.732)} = \epsilon^{1.0}\,\underline{/1.732 \text{ radians}} \\ &= 2.718\underline{/99°12'} = -0.434 + j2.675\end{aligned}$$

Similarly

$$\begin{aligned}\epsilon^{\underline{-2/60°}} &= \frac{1}{\epsilon^{\underline{2/60°}}} = 0.368\underline{/-99°12'} \\ &= -0.0589 - j0.363\end{aligned}$$

Hence

$$\begin{aligned}\sinh \underline{2/60°} &= \frac{-0.434 + 0.059 + j(2.675 + 0.363)}{2} \\ &= -0.187 + j1.52 \\ &= 1.53\underline{/97°0'}\end{aligned}$$

The relationships represented by $\cosh (\alpha + j\beta)l$ and $\sinh (\alpha + j\beta)l$ are wavy curves with regular but progressively decreasing amplitude of oscillation and increasing mean amplitude as l becomes greater, as shown in Fig. 41. The difference between $\cosh x$ and $\sinh x$ is that the latter begins at zero, the former at unity. In both cases, a maximum or minimum occurs whenever βl passes through a multiple of $\pi/4$. On the other hand, the amplitude of the maxima and minima increases as αl becomes greater, whereas the range of the oscillations becomes less, as shown in Fig. 41.

Note on Handling Complex Quantities in Polar Form.—All the terms appearing in Eqs. (45) are vector quantities. In order to carry on the multiplications, divisions, and extraction of roots that are indicated, it is convenient to express Z, Y, Z_0, $\cosh \sqrt{ZY}l$, etc., in polar form. The following rules for manipulation then apply:

Products are formed by multiplying magnitudes of the vectors and adding their angles.
Quotients are formed by dividing magnitudes and subtracting angles.
Sums and differences are obtained by converting to rectangular form and adding the real and imaginary parts separately, after which conversion can be made to polar form if desired.
Square roots are obtained by taking the square root of the vector magnitude and dividing the angle by two to obtain the resulting vector.
Powers are obtained by raising the length to the appropriate power and multiplying the angle by the power.

11. Voltage and Current Distribution—Wave Trains and Their Reflection.—The expressions for the voltage and current relations of a transmission line given by Eqs. (45) can be rearranged as follows:

[1] A. E. Kennelly, Chart Atlas of Complex Hyperbolic and Circular Functions, Harvard Univ. Press, Cambridge, Mass., 1924.

Ex. 1044-0186

$$E_s = \left(\frac{E_r + I_r Z_0}{2}\right)\epsilon^{\sqrt{ZY}l} + \left(\frac{E_r - I_r Z_0}{2}\right)\epsilon^{-\sqrt{ZY}l}$$
$$I_s = \left(\frac{I_r + \frac{E_r}{Z_0}}{2}\right)\epsilon^{\sqrt{ZY}l} + \left(\frac{I_r - \frac{E_r}{Z_0}}{2}\right)\epsilon^{-\sqrt{ZY}l} \qquad (54)$$

It is to be noted that l is distance from the load. This shows that the voltages and currents existing on the line can be considered as comprising the sum of two components. The first of these consists of a voltage having a magnitude $(E_r + I_r Z_0)/2$ accompanied everywhere by a current that is equal to this voltage divided by the characteristic impedance. This first component is proportional to $\epsilon^{\sqrt{ZY}l}$, and can be termed a *wave train traveling toward the receiver.* The second component consists of a voltage $(E_r - I_r Z_0)/2$ accompanied by a current equal to the voltage divided by $-Z_0$. This component is proportional to $\epsilon^{-\sqrt{ZY}l}$, and can be termed a *wave train traveling from the receiver toward the generator.* This second wave train can be thought of as resulting from the reflection of the first wave train by the load impedance at the receiver end of the line. For this reason, the wave traveling to the receiver is often designated as the *incident wave,* while the second wave is termed the *reflected wave.*

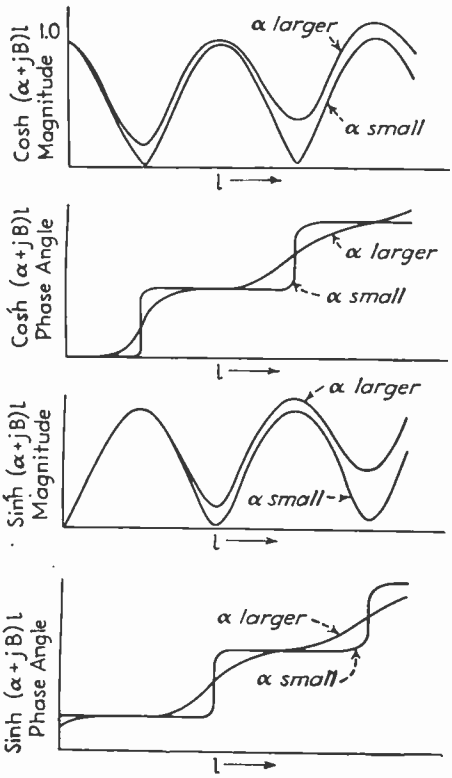


FIG. 41.—Curves showing the nature of the functions cosh $(\alpha + j\beta)$ and sinh $(\alpha + j\beta)$.

Reflection of Wave Trains.—The incident and reflected waves existing at the receiving point must be so related as to give the required receiver voltage and current, and at the same time satisfy the properties of the wave trains. The vector ratio of voltage of the reflected wave to voltage of the incident wave is termed the *coefficient of reflection,* and depends upon the vector ratio Z_L/Z_0 of load impedance to characteristic impedance according to the equation

$$\text{Coefficient of reflection} = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1} \qquad (55)$$

The vector ratio of voltage of the incident wave to the receiver voltage is

$$\frac{\text{Voltage of incident wave}}{\text{Receiver voltage}} = \frac{1 + \frac{Z_0}{Z_L}}{2} \qquad (56)$$

The magnitude and phase angle of the reflection coefficient can be conveniently obtained from Fig. 42 for a given Z_L/Z_0.[1]

Characteristics of Wave Trains.—Considering first the incident wave, as the distance from the receiver increases (*i.e.*, as $\sqrt{ZY}l$ increases), the size and phase of this wave varies by the factor $\epsilon^{\sqrt{ZY}l} = \epsilon^{(\alpha+j\beta)l} = \epsilon^{\alpha l}\angle \beta l$. The amplitude hence increases exponentially according to the factor $\epsilon^{\alpha l}$. At the same time, the phase advances by the factor βl radians, or β radians per unit length.

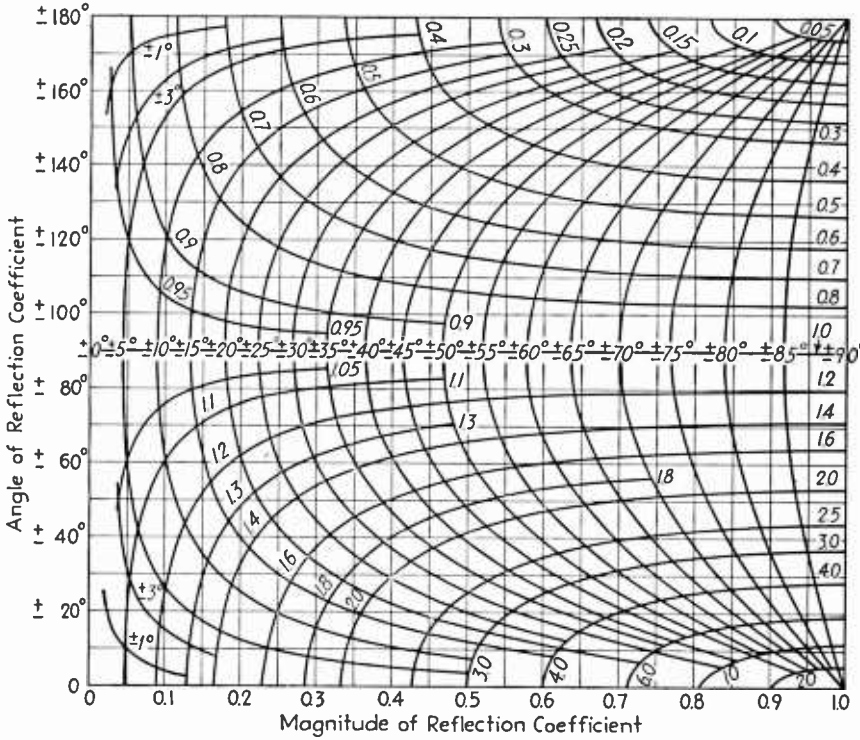


FIG. 42.—Chart giving the magnitude and phase of the reflection coefficient at the load in terms of the vector ratio Z_L/Z_0 of load to characteristic impedance. The numbers on the curvilinear system of coordinates represent the magnitude and phase of the ratio Z_L/Z_0.

If we turn next to the reflected wave, the variation of magnitude and phase with increase in l is according to the factor $\epsilon^{-\sqrt{ZY}l} = \epsilon^{-(\alpha+j\beta)l} = \epsilon^{-\alpha l}\angle -\beta l$. The amplitude now varies according to $\epsilon^{-\alpha l}$, and so decreases exponentially with increasing distance from the receiver. At the same time, the phase drops back β radians per unit length, or by a total amount βl radians.

It will be noted that the phase shift of the individual wave trains is 2π radians per wave length or 90° per quarter wave length. It is also to be noted that when αl is small there is very little change in magnitude of the individual waves.

Voltage and Current Distribution with Different Receiving-end Terminations.—The actual voltage and current existing on a transmission line is always the vector sum

[1] An equivalent chart expressed in a different form is given by P. S. Carter, Charts for Transmission-line Measurements and Computations, *R.C.A. Rev.*, Vol. 3, p. 355, January, 1939.

Ex. 1044-0188

of the two wave trains described above. The nature of the resulting distribution depends upon the relative amplitude and phase of the two wave trains at the receiving point, upon the phase shift βl, *i.e.*, upon the distance to the receiver measured in wave lengths, and upon the attenuation αl to the receiver.

With an open-circuit receiver, Eq. (56) shows that the reflection coefficient is $+1$. This means that at the reflection point, the incident and reflected waves have the

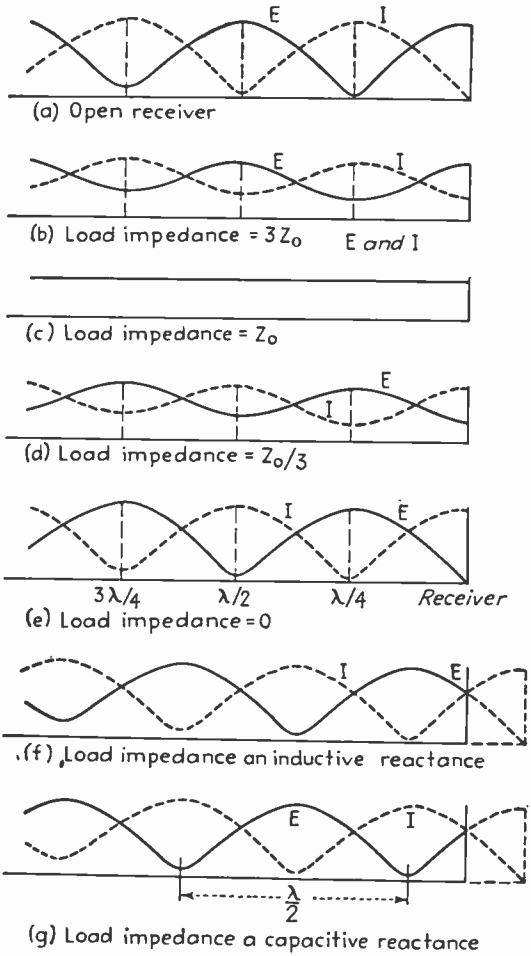
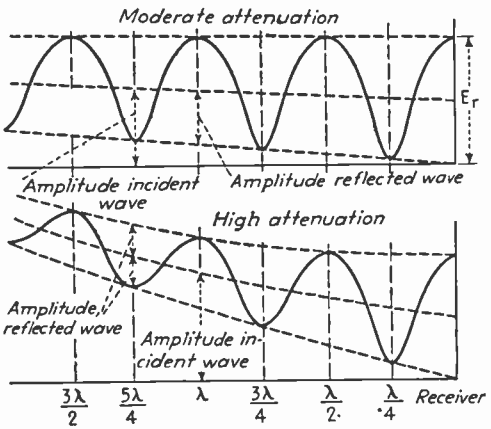


FIG. 43.—Types of voltage and current distributions produced on transmission lines by different load impedances.

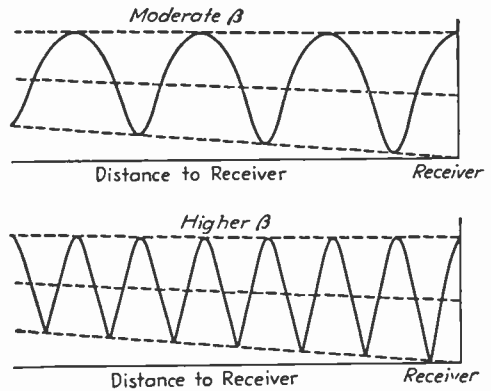
same amplitude, with both voltages in the same phase and the two currents in phase opposition. The vector sum of two such wave trains is shown in Fig. 43*a*. The voltage minima occur at distances from the receiver corresponding to an odd number of quarter wave lengths. These are points at which the phase advance βl of the incident wave and the phase retardation $-\beta l$ of the reflected wave are such as to cause the two voltages to be out of phase. The current maxima and minima coincide with voltage minima and maxima, respectively.

Ex. 1044-0189

With a short-circuited receiver, the reflection coefficient is -1. The reflection is again complete, but takes place with reversal in voltage and no change in phase of current. Accordingly, the distributions are similar to those obtained with an open-circuit receiver, except that the voltage and current curves have been interchanged, as shown in Fig. 43*e*.



(a) Effect of attenuation on voltage distribution—open circuited receiver



(b) Effect of β on voltage distribution—open circuited receiver

FIG. 44.—Curves showing the effect of the attenuation constant and phase constant on the voltage distribution in an open-circuited transmission line.

When the load impedance equals the characteristic impedance, the reflection coefficient is zero, *i.e.*, there is no reflected wave train. The current and voltage distribution then follow an exponential law, as shown in Fig. 43*c*, and no resonances are present.

With load impedances having the same phase angle as the characteristic impedance but greater magnitude, the reflection coefficient is positive but less than unity. The reflected wave then has the same phase relations as for the open-circuit receiver, but is less in magnitude. This results in a distribution of the type shown in Fig. 43*b*, which is similar to the open-circuit case, but with less pronounced resonances. The corresponding situation when the load impedance is less than the characteristic impedance resembles the short-circuit case, but again the resonances are less pronounced, as in Fig. 43*d*.

Load impedances having a phase angle differing from the characteristic impedance by 90° (corresponding to a reactive load and a resistance characteristic impedance) give a reflection coefficient that is unity, but that has a phase shift associated with it. The resonances are then just as pronounced as in the open- and short-circuit cases, but the position where the first voltage minimum occurs is displaced toward the receiver with capacitive loads and away from the receiver with inductive loads, as compared with the open-circuited line (see Figs. 43*f* and 43*g*). The amount of this displacement is $\delta/4\pi$ wave lengths, where δ is the phase angle of the reflection coefficient.

In the case where the load impedance has both a reactive and a resistive component, the reflected wave is smaller than the incident wave and also has its phase shifted. This causes the ratio of maximum to minimum amplitudes to be smaller than with complete reflection and also modifies the distance from the receiver at which the first minimum occurs.

Ex. 1044-0190

Effect of Attenuation Constant and Phase Constant (or Frequency) on Distribution Curves.—The attenuation constant determines the rate at which the amplitude of the individual wave trains varies. When the attenuation is very small, the waves change only very slightly in amplitude when traveling large distances, and the envelopes of the voltage and current distribution curves therefore change only slightly. On the other hand, when the attenuation constant is large enough so that $\epsilon^{\alpha l}$ differs appreciably from unity, the wave trains change their amplitude relatively rapidly. As the distance from the receiver is increased, the incident wave rapidly becomes larger, and the reflected wave dies out. This results in the behavior that is shown in Fig. 44*a*.

The phase constant β is almost exactly proportional to the frequency, and determines the length required to give a quarter wave length. Increasing β (as, for example, by increasing the frequency) accordingly causes the maxima and minima of the distributions to be more closely spaced, as shown in Fig. 44*b*.

Phase Shifts Produced by Lines.—In the case of a nonresonant line (load impedance = characteristic impedance), there is only a single wave train. The voltage (or current) then shifts in phase uniformly at the rate of 2π radians for each wave length, the phase advancing as the generator is approached.

When resonances occur, the phase relations are more complicated. The phase of the voltage (or current) distribution still advances 180° from one minimum in the distribution to the next minimum toward the generator, but unlike the resonant line, most of the phase shift tends to take place in the vicinity of the minima.

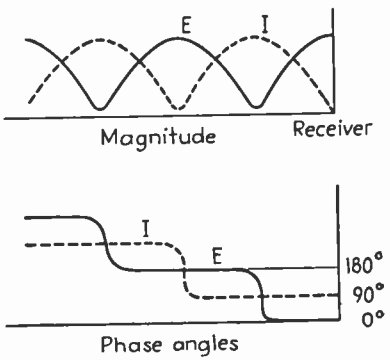


FIG. 45.—Variation of the phase of voltage and current on an open-circuited transmission line with increasing distance to the load.

Phase relations are shown in Fig. 45 for conditions corresponding to the case illustrated in Fig. 43*a*.

12. Impedance Properties of Transmission Lines.—The sending-end impedance of a transmission line is

$$\left.\begin{matrix}\text{Sending-end}\\ \text{impedance}\end{matrix}\right\} = Z_s = Z_0 \frac{\left(\frac{Z_L}{Z_0}\right)\cosh\sqrt{ZY}l + \sinh\sqrt{ZY}l}{\left(\cosh\sqrt{ZY}l + \frac{Z_L}{Z_0}\sinh\sqrt{ZY}l\right)} \tag{57}$$

In special limiting cases, this reduces to

Open-circuit receiver $\left(\frac{Z_L}{Z_0} = \infty\right)$:

$$Z_s = \frac{Z_0}{\tanh\sqrt{ZY}l} \tag{58}$$

Short-circuited receiver $\left(\frac{Z_L}{Z_0} = 0\right)$:

$$Z_s = Z_0 \tanh\sqrt{ZY}l \tag{59}$$

Nonresonant line ($Z_L = Z_0$):

$$Z_s = Z_0 = Z_L \tag{60}$$

Ex. 1044-0191

In these equations, $\tanh \sqrt{ZY}l = \sinh \sqrt{ZY}l/\cosh \sqrt{ZY}l$, and the remaining notation is the same as used above.

The variation of line impedance with frequency (or length) is given by the same factors that control the voltage and current distribution. As the length is varied, the impedance is maximum when the voltage is maximum, and is lowest when the voltage goes through a minimum.

The sending-end impedance of a line of fixed length follows a curve such as is shown in Fig. 46*a* when the frequency is varied. Here the impedance maxima are spaced at frequency intervals such that each new maximum corresponds to an extra half wave length in the line. When the receiver is open-circuited, these impedance maxima

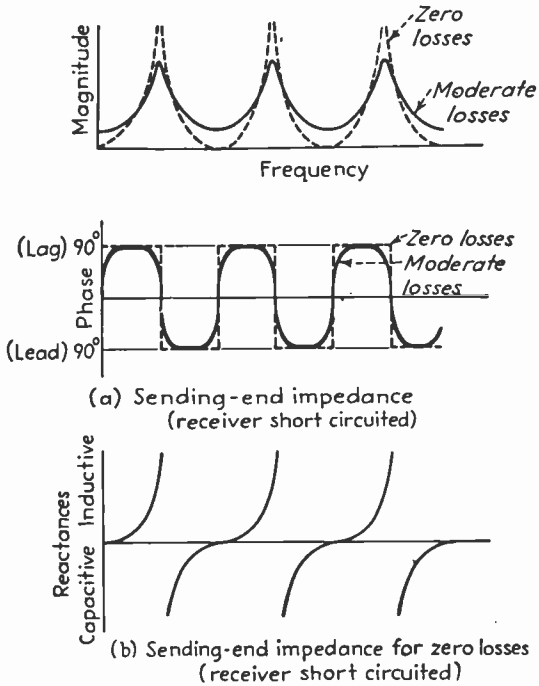


FIG. 46.—Variation of sending-end impedance of a short-circuited transmission line of fixed length as frequency is changed.

occur at frequencies such as to make the line an even number of quarter wave lengths long, while with a short-circuited receiver, the impedance maxima occur at frequencies that make the line an odd number of quarter wave lengths long.

The power factor of the line impedance is largely reactive except at the peaks and troughs, where the power factor goes through unity. The power factor alternates between leading and lagging each time the length measured in wave lengths increases a quarter of a wave length, as shown in Fig. 46.

13. Properties of Transmission Lines with Zero Losses.—In the usual line used in the transmission of radio-frequency power, the losses are so small compared with the power being transmitted that it is permissible for most purposes to assume that the line losses are zero. Also, in most radio antennas, the errors from assuming that the current distribution is as though the losses were zero introduce only a second

Ex. 1044-0192

order effect on the principal properties of the antenna, so that again the assumption of zero losses is commonly permissible.

When the resistance and leakage of a line are zero, $\alpha = 0$, $Z_0 = \sqrt{L/C}$, and $\sqrt{ZY} = j2\pi/\lambda$. Equations (45) then become

$$E_s = E_r \cos\left(2\pi \frac{l}{\lambda}\right) + jI_r \sqrt{\frac{L}{C}} \sin\left(2\pi \frac{l}{\lambda}\right)$$
$$I_s = I_r \cos\left(2\pi \frac{l}{\lambda}\right) + j \frac{E_r}{\sqrt{\frac{L}{C}}} \sin\left(2\pi \frac{l}{\lambda}\right) \tag{61}$$

where l and λ are in the same units.

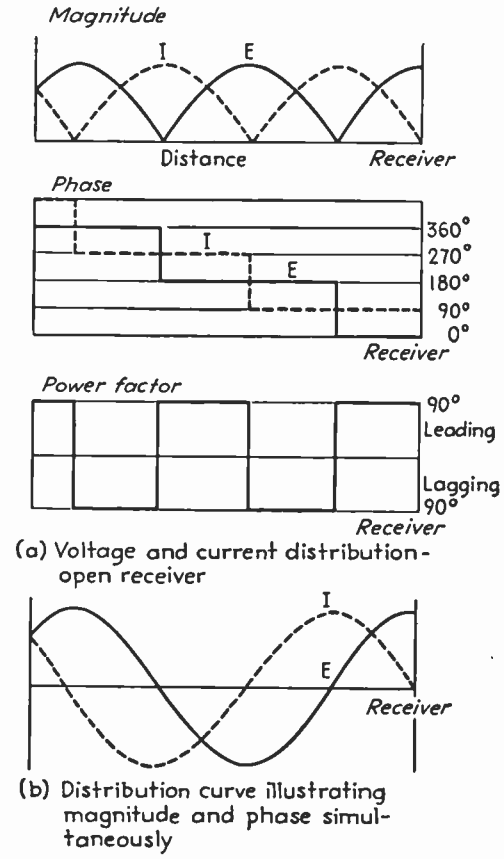


FIG. 47.—Voltage and current relations on an ideal open-circuited transmission line having zero losses.

The maxima in the distribution curves of current or voltage are similar to those of Fig. 43, except that all maxima have the same height when the attenuation is assumed to be zero, as also do the minima. The ratio of minimum to maximum is

$$\frac{(1-k)}{(1+k)},$$

Ex. 1044-0193

where k is the magnitude of the reflection coefficient. In the special case where $k = 1$, corresponding to an open- or short-circuited receiver or a reactive load, the distribution curves are sections of half sine waves, as shown in Fig. 47*a*, the phase of the voltage (or current) jumps 180° as each minimum is passed through, and the power factor jumps from 90° leading to 90° lagging (or vice versa) at each minimum, as indicated in the figure. The distribution curves for this case can then be drawn as in Fig. 47*b*, which simultaneously shows both magnitude and phase by using negative amplitudes to indicate opposite phase from positive amplitudes.

In the case of zero losses, Eq. (57), giving the impedance of the line, reduces to

$$Z_s = Z_0 \frac{\frac{Z_L}{Z_0} \cos\left(2\pi \frac{l}{\lambda}\right) + j \sin\left(2\pi \frac{l}{\lambda}\right)}{\cos\left(2\pi \frac{l}{\lambda}\right) + j \frac{Z_L}{Z_0} \sin\left(2\pi \frac{l}{\lambda}\right)} \tag{62}$$

In the special case where *the length is exactly a whole number of half wave lengths, the sending-end impedance is exactly equal to the load impedance.* Also, when the length is exactly an odd number of quarter wave lengths, then

$$Z_s = \frac{Z_0^2}{Z_L} \tag{63}$$

When the reflection is unity, corresponding to open-circuited, short-circuited, or reactive loads, the impedance goes to infinity at the frequencies that make the impedance a maximum, and the power-factor angle is everywhere 90° except at these transition points (see dotted curves, Fig. 46*a*). The curve of reactance as a function of frequency can then be drawn as in Fig. 46*b*.

14. Transmission of Radio-frequency and Audio-frequency Power by Lines.[1]— An important use of transmission lines is in carrying radio-frequency power from a transmitter to an antenna or from an antenna to a radio receiver. Lines used for this purpose are commonly said to be *resonant* or *nonresonant*, according to the method of operation.

Nonresonant Lines.—In the nonresonant lines, the terminating impedance Z_L at the end to which the line delivers the power is made equal to the characteristic impedance. There is then only the incident wave train present, and the power is transmitted at substantially unity power factor. The power loss and peak voltage to which the line is subjected are then less than with any other mode of operation.

The losses[1] in a nonresonant line depend upon the line attenuation αl, and are usually expressed in decibels, using the relation

$$\text{Decibels} = 8.686\ \alpha l \tag{64}$$

With air insulation the attenuation constant α at radio frequencies is inversely proportional to the linear dimension of the line (assuming fixed proportions) and directly to $\sqrt{f}$ (see Par. 10).

Curves giving the attenuation for two-wire copper lines spaced so that the characteristic impedance is 600 ohms, and corresponding curves for a concentric line of optimum proportions ($D/d = 3.6$) are given in Fig. 40.

Resonant Lines.—When the load impedance does not equal the characteristic impedance, a reflected wave exists, and the line is said to be resonant because of the resulting oscillatory character of the current and voltage distributions. Resonant operation has the advantage of not requiring an impedance match between the line and the load. At the same time it has the disadvantage that the average power factor

[1] An excellent discussion of the subject is given by E. J. Sterba and C. B. Feldman, Transmission Lines for Short Wave Radio Systems, *Proc. I.R.E.*, Vol. 20, p. 1163, July, 1932.

Ex. 1044-0194

along the line is lower than for nonresonant action. This results in increased power loss and greater voltage stress in proportion to transmitted power than for the corresponding nonresonant case. Curves giving the increase in loss and in voltage stress of a resonant line over a nonresonant line as a function of the reflection coefficient are given in Fig. 48. Nonresonant operation also leads in many cases to loss in power transferred, as a result of mismatching.

Transmission of Audio-frequency Power over Moderate Distances.—Transmission lines are frequently used for transmitting audio-frequency energy for short distances, such as from one part of a building to another part. Such lines differ from radio-frequency lines and ordinary telephone lines in that the length, at even the highest audio frequencies, is only a small fraction of a quarter wave length. The possibility of resonance, accordingly, does not need to be considered, and there is no advantage in making terminating impedance equal to the characteristic impedance. The impedance as viewed from the sending end is essentially equal to the terminating load

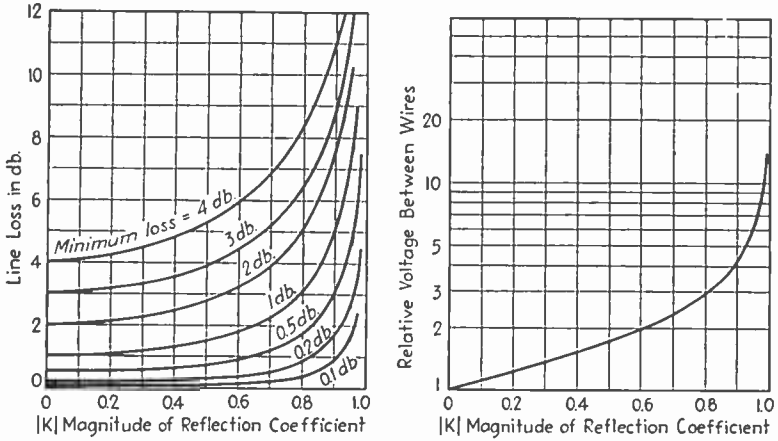


FIG. 48.—Effect of resonance on transmission line loss, and on maximum voltage between wires. The minimum loss is the loss for $k = 0$.

impedance. Such short lines are frequently referred to as 50-ohm lines, 100-ohm lines, 500-ohm lines, etc., according to the load impedance used, quite independently of the characteristic impedance.

15. Impedance Matching.—When it is desired to obtain nonresonant operation, it is necessary to make provision for coupling the load impedance to the line in such a way that the effective terminating impedance of the line equals the characteristic impedance. The methods commonly used for doing this at radio frequencies are:

Impedance Matching Network.—This arrangement makes use of a four-terminal reactive network so proportioned that the input impedance offers the desired resistance load to the line when the actual load impedance is connected to the output terminals. Impedance matching networks can also be designed to introduce any desired phase shift.

Impedance matching networks can take a variety of forms. Thus in broadcast work, unbalanced T, π, or L networks are commonly used to couple a grounded antenna to a concentric transmission line, as shown schematically in Fig. 49*a*. The design of such networks is discussed in Par. 25.

Transformers are also frequently used in impedance matching. A simple arrangement is shown in Fig. 49*b*, in which the tuning of the circuits and the mutual induct-

Ex. 1044-0195

ance are chosen to make the load impedance offered to the line a resistance of the proper magnitude. Transformers are sometimes combined with T, π, and L networks, and are always necessary, either alone or in combination with other circuit elements, when a balanced transmission line is to be coupled to a load that is unbalanced to ground.

In any impedance matching network it is to be noted that the network consumes no power, provided it is composed of reactive elements having negligible loss. Practically all the power delivered to the input terminals of such an impedance matching network finds its way to the load.

Resonant Line Coupling.—This arrangement is shown in Fig. 49c, and employs an auxiliary line as an impedance matching network. The load is connected to one end

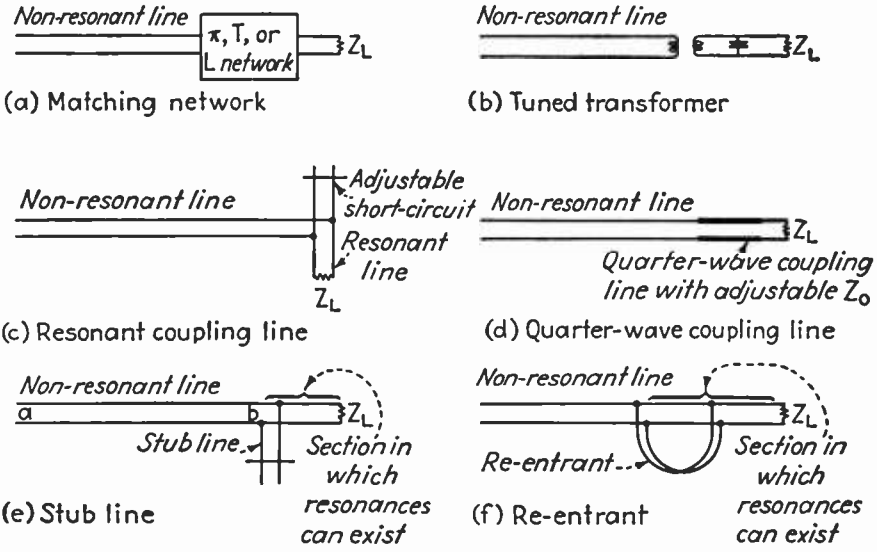


FIG. 49.—Impedance matching systems for eliminating resonances in radio-frequency transmission lines.

of this auxiliary line, and the position of an adjustable short circuit at the other end is varied until the line with the associated load is in resonance. The impedance offered to an external source of power connected between the two wires of the auxiliary line is then a resistance having a value depending upon the point at which the connection is made to the auxiliary line. The characteristic impedance of the nonresonant line can now be matched by connecting to the auxiliary line at a suitable point. The length of the auxiliary depends upon the magnitude and power factor of the actual load, and may range up to a half wave length.

Quarter-wave Coupling Line.—This arrangement, illustrated in Fig. 49d, makes use of the fact that a quarter-wave line of negligible losses will transform impedance according to Eq. (63). The reactance component of the load is first made zero by tuning the load to resonance, after which the characteristic impedance of the coupling line is varied by adjusting its spacing until the impedance at the transmission line end of the quarter wave coupling section has the required value.

Stub-line Impedance Matching System.—This arrangement is illustrated in Fig. 49e. The load is first connected directly to the transmission line, as in the case of resonant operation. Then, at a point within a wave length or less of the load, a stub

Ex. 1044-0196

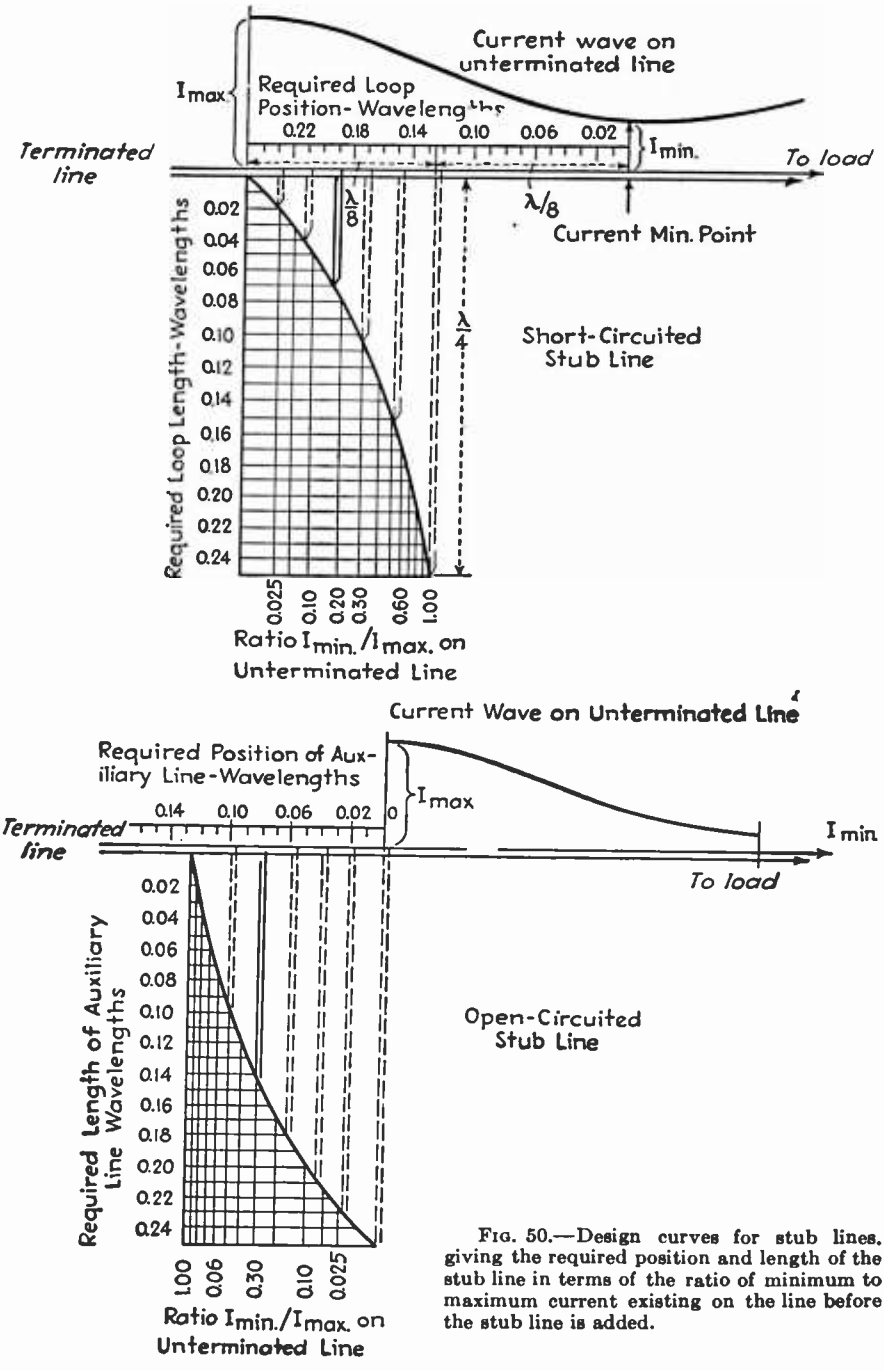


FIG. 50.—Design curves for stub lines, giving the required position and length of the stub line in terms of the ratio of minimum to maximum current existing on the line before the stub line is added.

Ex. 1044-0197

line is connected in shunt with the line. By properly proportioning the system, the impedance offered to the line *ab* at the point *b* can be made equal to the characteristic impedance of the line. The length of the stub and its proper location with respect to a current maximum are given by Fig. 50 in terms of the ratio of maximum to minimum current (or voltage) existing on the line before the stub is added.[1]

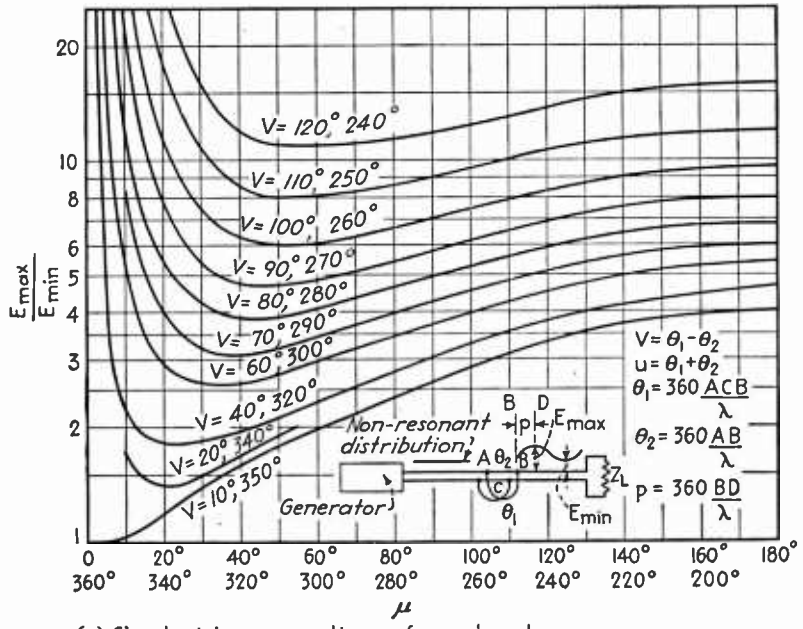


FIG. 51.—Chart for designing re-entrant sections to eliminate resonances on transmission lines. This chart gives the proportions of the re-entrant and its position with respect to a voltage maximum, in terms of the voltage maxima and minima existing on the line before the addition of the re-entrant.

Re-entrant Transmission-line Sections.[2]—This arrangement is illustrated in Fig. 49*f*, and employs an auxiliary line with both ends connected to the actual transmission line as shown. This "re-entrant" section produces an effect very similar to that of a stub line in that it can be made to introduce a compensating irregularity that causes the line on the generator side of the re-entrant to be terminated in an imped-

[1] From Sterba and Feldman, *loc. cit.*

[2] A. Alford, High-frequency Transmission Line Networks, *Elec. Comm.*, Vol. 17, p. 301, January, 1939.

Ex. 1044-0198

ance equal to the characteristic impedance of the line. The design of re-entrant sections can be carried out with the aid of Fig. 51. It will be noted that in any particular case there are an infinite number of combinations that will give a particular result.

16. Transmission Lines as Resonant Circuits.[1]—A transmission line can be used to perform the functions of a resonant circuit when properly operated. Thus a quarter wave-length line open-circuited at the receiver has a receiving-end voltage much higher than the applied voltage, and a quarter-wave line short-circuited at the receiver offers a high impedance to an applied voltage similar to the resonant impedance of a parallel circuit. Resonant lines are frequently used in place of resonant circuits at high frequencies because of their superior performance.

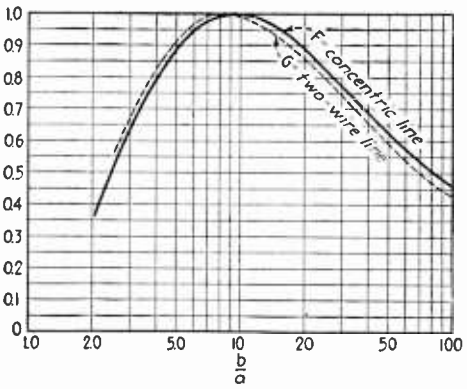


FIG. 52.—Factors F and G for use in Eqs. (66) and (67).

Lines as Parallel Resonant Circuits. A line offers a high impedance at the sending end when the receiver is short-circuited and the length is an odd multiple of a quarter wave length, or when the receiver is open-circuited and the line is an even number of quarter wave lengths long. In either case

$$Z_s = \frac{8Z_0^2 f}{Rnc} \tag{65}$$

where Z_s = sending-end impedance of line at resonance.
Z_0 = characteristic impedance of line.
f = frequency in cycles.
R = line resistance per unit length.
n = number of quarter wave lengths in line.
c = velocity of light in same units of length as R.

At high frequencies with air insulation and copper conductors, Eq. (65) becomes for the case of the concentric line

$$Z_s = \frac{11.11\sqrt{f}\,b}{n} F \tag{66}$$

where F = factor given by Fig. 52.
b = inner radius of outer conductor, cm.
a = outer radius of inner conductor, cm.

With a two-wire line (neglecting radiation resistance)

$$Z_s = \frac{23.95\sqrt{f}\,bG}{n} \tag{67}$$

where G = factor given in Fig. 52, which is a function of b/a.
b = spacing of wire centers, cm.
a = radius of wire, cm.

The curve of resonant-line impedance as a function of frequency in the region near resonance has the same shape as a resonance curve, with an equivalent Q given by the

[1] For further information see F. E. Terman, Resonant Lines in Radio Circuits, *Elec. Eng.*, Vol. 53, p. 1046, July, 1934.

Ex. 1044-0199

formula

$$Q = \frac{2\pi Z_0 f}{Rc} \tag{68}$$

For copper lines with air insulation at radio frequencies, this equation becomes

For concentric lines:

$$Q = 0.0839 \sqrt{f}\, bH \tag{69}$$

For two-wire lines (neglecting radiation resistance):

$$Q = 0.0887 \sqrt{f}\, bJ \tag{70}$$

where H and J are factors given in Fig. 53 and the remaining notation is as before. Curves giving the Q of concentric lines of optimum proportions ($b/a = 3.6$) are given in Fig. 54.

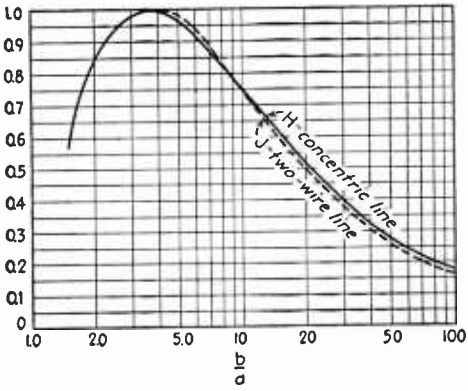


FIG. 53.—Factors H and J for use in Eqs. (69) and (70).

Resonant Lines as Series Circuits. A transmission line acts as a voltage step-up device when the receiver is open-circuited and the length is an odd number of quarter wave lengths. At frequencies near resonance, the step-up varies with frequency in exactly the same manner as a resonance curve having an equivalent Q given by Eq. (68), and the voltage step-up ratio is $Q \times 4/\pi n$ instead of Q, as in the case of the ordinary series resonant circuit.

Lines as Low-loss Reactances.—A line can serve as a low-loss inductance or capacity by employing the proper combination of length, frequency, and terminating condition. Thus a line short-circuited at the receiver will offer an inductive reactance when less than a quarter wave length long and a capacitive reactance when between a quarter and a half wave length long. With an open-circuited receiver, the conditions for inductive and capacitive reactance are interchanged. The reactances obtained are as follows:

For short-circuited receiver:

$$X_s = jZ_0 \tan\left(2\pi \frac{l}{\lambda}\right) \tag{71}$$

For open-circuited receiver:

$$X_s = -j \frac{Z_0}{\tan\left(2\pi \frac{l}{\lambda}\right)} \tag{72}$$

where l/λ is the line length measured in wave lengths and Z_0 is the characteristic impedance.

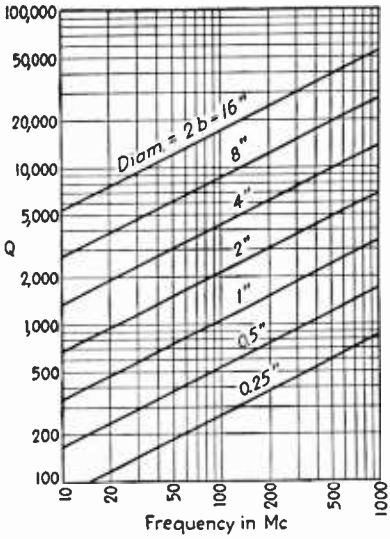


FIG. 54.—Q's of resonant concentric lines of optimum proportions ($b/a = 3.6$).

The Q of the reactance is related to the Q of the resonance curve of the corresponding quarter-wave line by the equation

Ex. 1044-0200

$$\frac{Q \text{ of reactance}}{Q \text{ of resonant quarter-wave line}} = \frac{\sin\left(4\pi\frac{l}{\lambda}\right)}{2\pi\frac{l}{\lambda}} \tag{73}$$

It will be noted that the Q of the reactance decreases with l/λ, so that for high Q the line should be less than a quarter of a wave length long.

Reactances obtained from resonant lines differ from ordinary reactances in that their impedance varies more rapidly with frequency than is the case with an inductance or a capacity.

17. Miscellaneous Characteristics. *Irregularities.*—An irregularity in a transmission line produces a reflected wave having a magnitude determined by the extent of the irregularity. Resonances are then produced on the generator side of the irregularity, even though the line is properly terminated, as shown in Fig. 55. Irregularities should generally be avoided, for they give rise to unexpected resonances, often produce unbalance in the transmission line, and in general make the behavior different from that anticipated.

Irregularities are introduced whenever the constants of the line are modified. Typical causes are leads, bends in which the two sides of the circuit are not symmetrical, poor joints, etc.

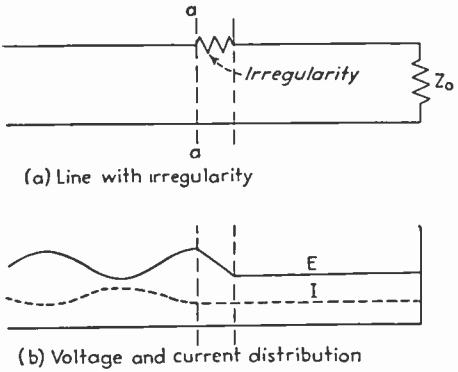


FIG. 55.—Resonances produced on a transmission line by a typical irregularity.

Balanced and Unbalanced Transmission Lines.—A line is said to be balanced when its two sides are symmetrical with respect to ground. Thus a horizontal two-wire line, and four-wire lines, are balanced structures, while a two-wire line in vertical configuration and a concentric line are unbalanced structures. Except for the concentric line, in which the outer conductor acts as a shield, unbalance causes earth currents to flow, because part of the current that should be carried by the conductor having the largest capacity to earth is diverted to the earth. This causes the current in the two sides of the transmission line to be unequal, and results in a component of current that flows out along the two transmission lines in parallel, and returns through the earth. Such unbalanced currents are to be avoided, because they give rise to extra energy loss, cause the radiation from the system to be greatly increased, and in general serve no useful purpose.

Radiation from Transmission Lines.—All transmission lines, except those of the concentric type, radiate some energy. Such radiation is often of importance, since it represents an additional energy loss, and in the case of transmission lines used in association with directional antennas, may seriously modify the directional pattern of the complete system.

The radiation from a two-wire nonresonant line is given approximately by the following formula,[1] provided that the length is at least twenty times the spacing and the spacing is not greater than one-tenth of a wave length and the line is nonresonant:

$$\frac{\text{Radiated power}}{I^2} = 160\left(\frac{\pi D}{\lambda}\right)^2 \tag{74}$$

[1] From Sterba and Feldman, *loc. cit.*

Ex. 1044-0201

where D/λ is the spacing in wave lengths, and I is the rms line current. This radiation is twice that resulting from a doublet antenna carrying the same current as the line and having a length equal to the line spacing. In addition to the radiation given by Eq. (74), the terminating connections also produce radiation, so that the total radiation from the line with its terminations will be approximately four times the power radiated from a doublet having a length equal to the line spacing, and carrying the line current.

In the case of resonant lines the amount of energy radiated in proportion to the power transmitted to the load will be somewhat greater than is the case with the nonresonant line. However, unless the reflection coefficient of the load departs greatly from unity, the increase will not be great.

Unbalanced currents are much more effective in producing radiation than the normal balanced currents because of the considerable distance between the two sides of the circuit through which the unbalanced currents flow. The amount of radiation depends both on the height of the line above ground and on the line length. The order of magnitude of the factors involved can be estimated with the aid of Fig. 56.

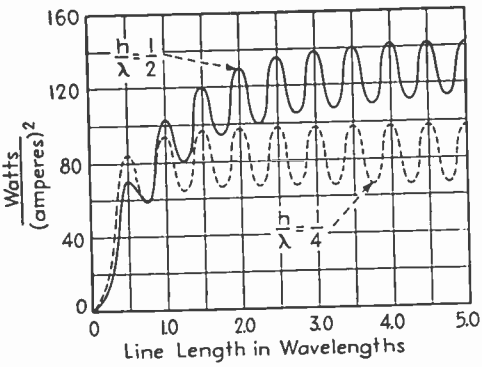


FIG. 56.—Radiation produced by unbalanced current in a transmission line.

When a two-wire line is used as a resonant circuit, radiation is quite important and may materially reduce the effective Q and parallel impedance obtained.[1]

Artificial Lines.—An artificial line is a four-terminal network composed of resistance, inductance, and capacity elements so proportioned that as far as the terminals are concerned the artificial line behaves in exactly the same way as an actual line. Artificial lines are usually in the form of symmetrical π or T networks, in which case the impedances composing the artificial line are related to the characteristic impedance Z_0 and hyperbolic angle $\sqrt{ZY}l$ of the corresponding real line, as indicated in Fig. 57*b*.[2]

Any given symmetrical T or π section is equivalent to some actual transmission line. The hyperbolic angle θ and characteristic impedance Z_0 of the real line equivalent to a given artificial line are given by the equations

For π or T line:

$$\cosh \theta = 1 + \frac{Z_1}{2Z_2} \tag{75a}$$

or

$$\sinh \frac{\theta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \tag{75b}$$

For T line:

$$Z_0 = Z_T = \sqrt{Z_1 Z_2 + \tfrac{1}{4} Z_1^2} \tag{76}$$

[1] Radiation is not present in concentric lines, provided that the outer conductor is in the form of a complete surface with closed ends. This is because the outer conductor acts as a nearly perfect shield for the fields existing within such a system.

[2] The relations given in this figure sometimes call for negative resistance elements in the artificial line, which is then physically unrealizable. This situation is likely to occur when the length of line being represented by the artificial section exceeds a quarter wave length, and can be handled by dividing the actual line into several shorter parts and representing each part by a separate artificial line.

Ex. 1044-0202

For π line.

$$Z_0 = Z_\pi = \frac{Z_1 Z_2}{\sqrt{Z_1 Z_2 + \frac{1}{4} Z_1^2}} \tag{77}$$

In these equations, Z_1 and Z_2 have the significance shown in Fig. 57*a*.

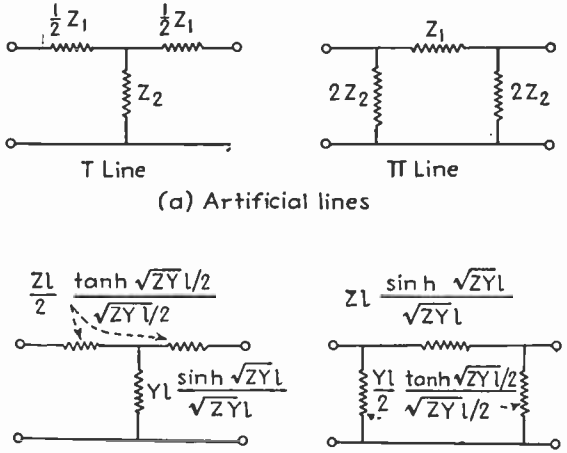


FIG. 57.—Artificial T and π lines for representing real transmission lines.

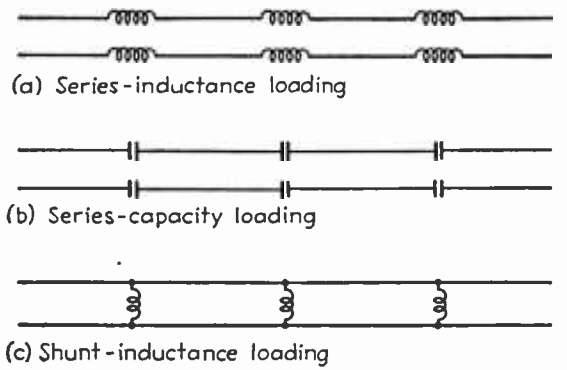


FIG. 58.—Transmission lines with various types of reactive loading.

Loaded Lines.—A loaded line is an ordinary transmission line to which lumped elements, usually capacities or inductances, are added at regular intervals. The most common use of loading is in telephone lines, particularly telephone cables, where inductance coils are commonly added at regular intervals, as in Fig. 58*a*. Such inductive loading makes the equivalent inductance per unit length of the transmission line greater than the actual inductance of the unloaded line at voice frequencies. The result is an increase in the characteristic impedance, a lowering of the phase velocity,

Ex. 1044-0203

and also, when most of the loss is in the series resistance of the line, a reduction of the attenuation constant.

Loading by the use of series capacity, or shunt inductance, as illustrated in Fig. 58, is also sometimes employed in transmission lines used in radio work. The first of these arrangements reduces the inductance per unit length of the line, and the second reduces the equivalent shunting capacity. Both arrangements reduce the imaginary part of $\sqrt{ZY}$ and so reduce β. In this way, the phase velocity can be made to exceed the velocity of light and even approach infinity.

In order that lumped loading impedances may have the same effect as though their impedance were uniformly distributed, it is necessary that the loading impedances be spaced at distances that do not appreciably exceed a quarter wave length. If the

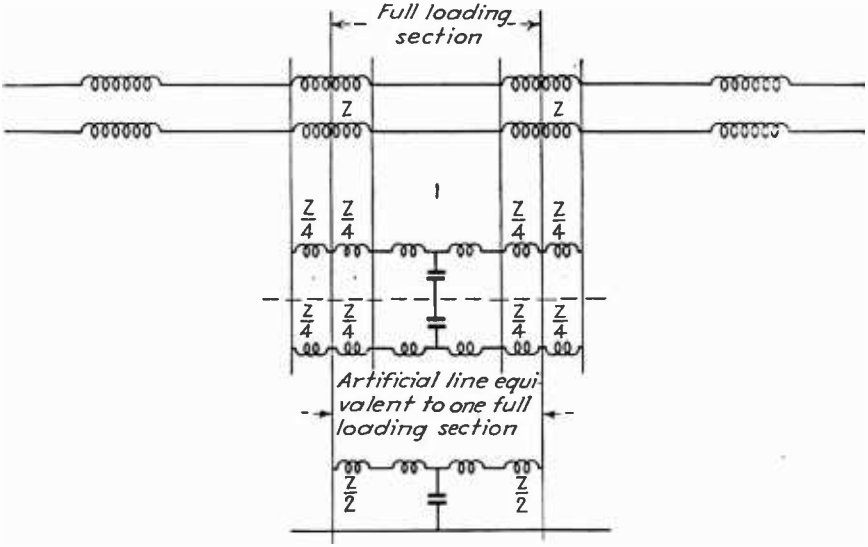


FIG. 59.—Transmission line with inductive loading, showing steps involved in determining equivalent characteristic impedance and hyperbolic angle of the loaded line.

spacing is greater, the loading impedances tend to act as irregularities or chokes and interfere with transmission.[1]

Exponentially Tapered Lines.[2]—An exponentially tapered line is one in which the spacing between conductors varies in such a way that the inductance and capacity vary exponentially with distance. Such a line acts as a combined high-pass filter and impedance-transforming device; *i.e.*, it transmits all frequencies above a certain critical or cut-off frequency with little or no attenuation (provided that the line losses are low or negligible) while attenuating lower frequencies, and at the same time has different characteristic impedances when viewed from the two ends.

[1] The effect of loading on a line can be analyzed exactly as follows: Referring to Fig. 59, the section of line between the loading points is first reduced to an equivalent artificial section. This section is then merged with the loading impedance as shown, thereby giving an artificial section that is equivalent to the actual line plus the loading corresponding to one loading section. The hyperbolic angle and characteristic impedance of this new artificial section are then the values applying to the actual loaded line per loading section.

[2] For further information see Harold A. Wheeler, Transmission Lines with Exponential Taper, *Proc. I.R.E.*, Vol. 27, p. 65, January, 1939. Charles R. Burrows, Exponential Transmission Line, *Bell System Tech. Jour.*, Vol. 17, p. 555, October, 1938. An excellent elementary summary of the latter paper is to be found in the *Bell Lab. Rec.*, Vol. 18, February, 1940.

Ex. 1044-0204

The cut-off frequency depends upon the rate of taper, and is lower the more gradual the taper, in accordance with the relation

$$\frac{L_2}{L_1} = \frac{C_1}{C_2} = \epsilon^{\left(4\pi \frac{l}{\lambda_c}\right)} \tag{78}$$

where L_1 and C_1 are the inductance and the capacity per unit length at the closely spaced end, L_2 and C_2 are the corresponding values at the wide end, l is the length of the line, and λ_c the distance corresponding to a wave length at the cut-off frequency (in the same units of length as l), calculated on the basis of the constants at the narrow end.

The ratio of impedance transformation is approximately

$$\text{Ratio of impedance transformation} = \sqrt{\frac{Z_{02}}{Z_{01}}} = \epsilon^{2\pi \frac{l}{\lambda_c}} \tag{79}$$

where Z_{02} is the characteristic impedance on the basis of the spacing at the wide end of the line, Z_{01} is the characteristic impedance based on the spacing at the other end of the line, and l and λ_c have the same meaning as above. The ratio of impedance transformation accordingly depends upon the difference in spacing at the ends of the line, *i.e.*, upon the total taper, and can be easily made as great as two to one. The impedance match obtained at the ends of the exponential line may be improved by adding terminal reactances of complexity determined by the extent the reflection coefficient at the terminals must be minimized over a frequency range.[1]

NETWORK THEORY, FILTERS, AND EQUALIZERS

18. Network Definitions.—A network is made up of resistances, inductances, capacities, and mutual inductances connected together in some manner. The resistances, inductances, mutual inductances, and capacities involved are termed *network constants* or *parameters*. When these parameters are constant, independent of the current going through them, the network is said to be *linear*.

A typical network is illustrated in Fig. 60. The junctions *a*, *b*, *c*, etc., at which the current can divide are termed *branch points*, and the sections of the network between branch points are termed *branches*. A series of branches that form a complete loop is termed a *mesh*. Examples of meshes are shown by the arrows in Fig. 60, where five separate meshes are designated.

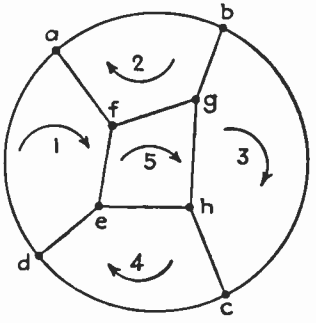


FIG. 60.—Network illustrating meshes, branches, and branch points.

A *passive network* is a network containing no source of energy, in which no energy is dissipated other than that accounted for by the resistance elements of the network. An *active network* is a network containing one or more sources of energy, or some sink of energy.

The term *two-terminal* is applied to networks operated under the conditions shown in Fig. 61*a*. Here the only applied voltage is that shown acting between terminals 1-2 obtained by opening some branch of the network. The term *four-terminal* is applied to a network operating under the conditions illustrated in Fig. 61*b*. Here a voltage E is applied in series with one branch of the network by opening up this branch to form terminals 1-2, while another branch of the network is opened up to form terminals 3-4, between which an output or load impedance is inserted. A four-terminal network is equivalent to a transmission system, and is sometimes referred to as a *transducer*.

[1] An excellent discussion of this subject is given by Wheeler, *loc. cit.*

Ex. 1044-0205

19. Network Theorems. *Superposition Theorem.—The current that flows in a linear network, or the potential difference that exists between any two points in such a network, resulting from the simultaneous application of a number of voltages distributed in any manner whatsoever throughout the network is the sum of the component currents at the first point (or the component potential differences between the two points) that would be caused by the individual voltages acting separately.*

Reciprocity Theorem.—In any network composed of linear impedances, if an electromotive force E applied between two terminals produces a current I at some branch in the network, then the same voltage E acting at the second point in the circuit, will produce the same current I at the first point.

Thévenin's Theorem.—Any linear network containing one or more sources of voltage and having two terminals behaves, insofar as a load impedance connected across the terminals is concerned, as though the network and its generators were equivalent to a simple generator having an internal impedance Z and a generated voltage E, where E is the voltage that appears across the terminals when no load impedance is connected and Z is the impedance that is measured between the terminals when all sources of voltage in the network are short-circuited.[1]

Compensation Theorem.—If an impedance ΔZ is inserted in a branch of a network, the resulting current increment produced at any point in the network is equal to the current

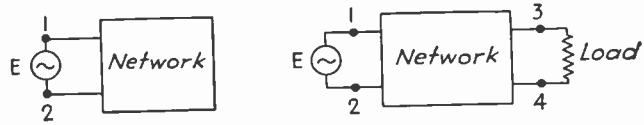


FIG. 61.—Two- and four-terminal networks.

that would be produced at that point by a compensating voltage acting in series with the modified branch, whose value is $-I\Delta Z$, where I is the original current that flowed where the impedance was inserted before the insertion was made.

20. General Mesh Equations of a Network and Their Solution.[2]—The response of a network to an applied voltage can be most readily expressed in terms of the mesh currents, *i.e.*, the currents that can be considered as circulating around the closed meshes, as indicated in Fig. 60. The branch currents can then be obtained as the vector sum of the various mesh currents that flow through the branch in question.

Mesh Equations.—The voltage and current relations in a network containing n independent meshes can be written as

$$\left.\begin{aligned} Z_{11}I_1 + Z_{12}I_2 + Z_{13}I_3 + \ldots . Z_{1n}I_n &= E_1 \\ Z_{21}I_1 + Z_{22}I_2 + Z_{23}I_3 + \ldots . Z_{2n}I_n &= E_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \\ Z_{n1}I_n + Z_{n2}I_n + Z_{n3}I_n + \ldots . Z_{nn}I_n &= E_n \end{aligned}\right\} \tag{80}$$

In these equations, I_1, I_2, etc., designate the individual mesh currents; E_1, E_2, etc., represent the vector sums of the applied voltages acting around the individual meshes numbered 1, 2, etc. Z_{11}, Z_{22}, Z_{33}, etc., represent the self-impedances of individual meshes, *i.e.*, the impedance around the mesh if all other branches of the network other than those included in the mesh in question were open-circuited. The impedance Z_{12}

[1] When the sources of energy in the network are constant-current generators, instead of constant-voltage generators, the internal impedance Z is the impedance observed between the terminals when all constant-current generators are open-circuited.

[2] An excellent discussion of the general mesh equations of a network is given in Chap. IV of E. Guillemin, "Communication Networks," Vol. I, Wiley, New York, 1931.

Ex. 1044-0206

represents the mutual impedance (coupling) between meshes 1 and 2, as a result of which, a current in mesh 2 produces a voltage drop in mesh 1, and Z_{13} represents the mutual impedance whereby current in mesh 3 produces voltage drop in mesh 1, etc. It is to be noted that reversing the order of subscripts of the Z's does not alter the value of mutual impedance. Thus $Z_{12} = Z_{21}$. Coupling may result either through mutual inductance or from impedance elements common to the two meshes.

In setting up the system of Eq. (80), care must be taken to be consistent in the matter of signs. The positive directions for the mesh currents are assigned arbitrarily. An impedance that is common to two branches is then considered to be a positive mutual impedance when the arrows representing the corresponding mesh current pass through the impedance in the same direction. If the arrows indicate that the corresponding mesh currents pass through the common impedance in opposite directions, then the mutual impedance is the negative of this common impedance. A mutual inductance is positive or negative according to whether it acts with a polarity the same as or opposite to that of a corresponding common inductance.

In setting up a system of relations such as is represented in Eq. (80), it is possible to designate the meshes in a variety of ways, subject only to the limitation that each branch of the network must be included in at least one mesh. For example, mesh 1 in Fig. 60, instead of following the configuration *afeda*, could have been defined as *abgfeda*. This would have modified the details in Eq. (80), but would have resulted in the same individual branch currents.

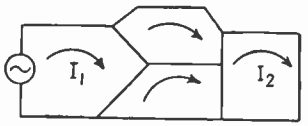


FIG. 62.—Network for illustrating meanings of driving point and transfer impedances.

In setting up mesh equations and selecting the meshes, there sometimes may be a question as to the number of independent meshes present. In such cases one can use the relation

$$\left.\begin{matrix}\text{Number of independent}\\ \text{meshes}\end{matrix}\right\} = \left.\begin{matrix}\text{number of}\\ \text{branches}\end{matrix}\right\} - \left.\begin{matrix}\text{number of}\\ \text{branch points}\end{matrix}\right\} + 1 \qquad (81)$$

A solution of the system of Eq. (80) shows that the current I_k in the kth mesh that flows as the result of the voltage E_j acting in the jth mesh is

$$I_k = E_j \frac{B_{jk}}{D} \qquad (82)$$

Here D is the determinant of the system of Eq. (80), and is given by

$$D = \begin{vmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{vmatrix} \qquad (83)$$

Methods of evaluating a determinant (as well as its more important properties) are found in mathematical texts. The quantity B_{jk} in Eq. (82) is the principal minor of D and is formed by canceling the jth row and kth column and then moving the remainder together to form a new determinant with one less row and column than D. In evaluating B_{jk}, this new determinant is prefixed with the sign $(-1)^{j+k}$.

Input and Transfer Impedance.—Consider a network having a single applied voltage, with the meshes so arranged that this voltage acts in a branch that is part only of a single mesh, as in Fig. 62. The impedance that the network offers to this applied voltage, *i.e.*, the ratio E_1/I_1, is termed the *input impedance* or *driving-point impedance*

Ex. 1044-0207

of the network. From Eq. (82) this input impedance can be expressed as

$$\text{Input impedance} = \frac{D}{B_{11}} \tag{84}$$

In an analogous manner the *transfer impedance* is defined as the ratio of the voltage E_1 applied in mesh 1 to the resulting current I_2 of mesh 2, as indicated in Fig. 62. This transfer impedance can be expressed as

$$\left.\begin{matrix}\text{Transfer impedance from} \\ \text{mesh 1 to mesh 2}\end{matrix}\right\} = \frac{D}{B_{12}} \tag{85}$$

where the meshes are so selected that I_2 is observed in a branch that is in only the second mesh of the network and the branch in which the voltage is applied is contained only in the first mesh of the system.

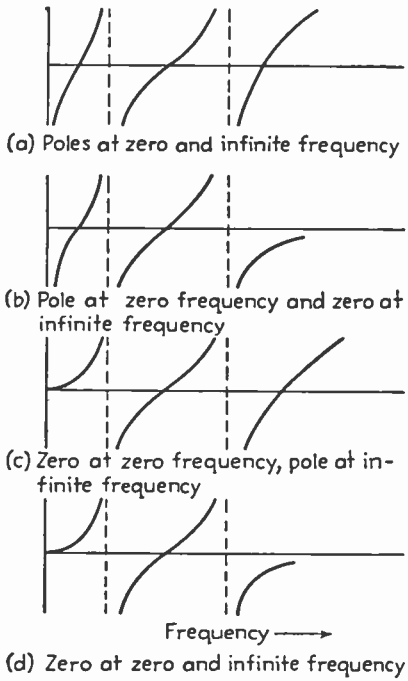


FIG. 63.—Reactance as a function of frequency for various classes of reactive networks.

21. Two-terminal Reactive Networks. The characteristics of a two-terminal network composed of ideal reactances having zero losses are important, because such networks approximate very closely the reactive arms that are used to build up filters, impedance matching networks, etc.

Foster's Reactance Theorem.[1]—The driving-point impedance of a two-terminal reactive network behaves as shown in Fig. 63. The impedance curve consists of segments going from minus infinity to plus infinity (except possibly at zero and infinite frequency where a segment may start or stop, respectively, at zero impedance). The slope of the curve is everywhere positive, and is greater than the slope of a straight line drawn to the origin. The frequencies at which the impedance is infinity are termed *poles*, and the frequencies at which the impedance is zero are termed *zeros*.

Foster has shown that the driving-point impedance of a reactive network is uniquely specified by the location of the internal zeros and poles, plus one additional piece of information.[2] Expressed analytically, the reactance function can be written as follows:

For a pole at the origin (Figs. 63*a* and 63*b*):

$$\left.\begin{matrix}\text{Driving point} \\ \text{impedance}\end{matrix}\right\} = Z = \pm j \frac{H}{\omega} \frac{(\omega^2 - \omega_1^2)(\omega^2 - \omega_3^2) \cdots (\omega^2 - \omega_p^2)}{(\omega^2 - \omega_2^2)(\omega^2 - \omega_4^2) \cdots (\omega^2 - \omega_q^2)} \tag{86a}$$

For a zero at the origin (Figs. 63*c* and 63*d*):

$$\left.\begin{matrix}\text{Driving point} \\ \text{impedance}\end{matrix}\right\} = Z = \pm j\omega H \frac{(\omega^2 - \omega_1^2)(\omega^2 - \omega_3^2) \cdots (\omega^2 - \omega_p^2)}{(\omega^2 - \omega_2^2)(\omega^2 - \omega_4^2) \cdots (\omega^2 - \omega_q^2)} \tag{86b}$$

[1] Ronald M. Foster, A Reactance Theorem, *Bell System Tech. Jour.*, Vol. 3, p. 259, April. 1924. See also Chap. IV of E. Guillemin, "Communication Networks," Vol. II, Wiley, New York, 1935.

[2] Poles or zeros at the origin, or at infinity, are referred to as external, and play no part in the specification of the reactance function.

Ex. 1044-0208

where the angular velocities $\omega_1, \omega_3, \ldots, \omega_p$ designated by odd subscripts correspond to the internal zeros of the reactance function, and the angular velocities $\omega_2, \omega_4 \ldots,$ ω_q designated by even subscripts, correspond to the internal poles of the reactance (see Fig. 63). The plus sign applies when there is a pole at infinite frequency, while the minus sign applies with a zero at infinite frequency. The sum of the number of poles and number of zeros is one less than the number of independent meshes of the network. The quantity H is a positive real constant that takes into account the fact that one additional piece of information is required to complete the specification of the reactance function.

Foster's reactance theorem shows that the impedance characteristics obtainable from a physically realizable reactance network are quite restricted. This is important, because it limits the characteristic obtainable from filters and other networks.

Synthesis of Reactive Two-terminal Networks.—Any driving-point reactance characteristic that can be obtained from any conceivable two-terminal reactive network can be realized by either one of the two networks shown in Fig. 64. The first of these

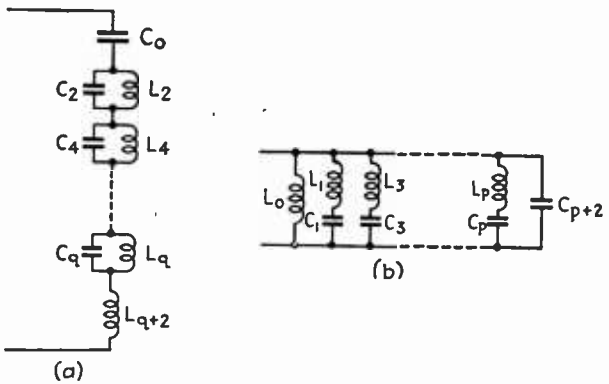


FIG. 64.—General forms of reactive networks.

consists of parallel resonant circuits connected in series, with one parallel resonant circuit corresponding to each internal pole. The series condenser C_0 is omitted in the event that the network has a zero at the origin, while the series inductance L_{q+2} is omitted if the network has a zero at infinity. The magnitudes of the circuit components required in the equivalent network of Fig. 64*a* to realize a desired impedance characteristic are given by the following relations:

$$C_k = \left|\frac{j\omega_k}{Z_k}\right| \qquad (k = 2, 4, \ldots, q) \tag{87}$$

where Z_k is the quantity obtained by omitting the term $(\omega^2 - \omega_k^2)$ from the denominator of the corresponding expression (86) for Z and evaluating the modified expression for Z with $\omega = \omega_k$. Corresponding to each C_k, one has

$$L_k = \frac{1}{\omega_k^2 C_k} \tag{88}$$

If the network has a pole at infinity, then

$$L_{q+2} = H \tag{89}$$

If the network has a pole at zero frequency

$$C_0 = \left|\frac{1}{Z_0}\right| \tag{90}$$

where Z_0 is the quantity obtained by omitting the ω in the denominator of Eq. (86*a*) under H and evaluating the modified expression for Z with $\omega = 0$.

An alternative method of synthesizing any desired impedance characteristic is to use the arrangement of Fig. 64*b*, in which the two-terminal network is built up of a number of series resonant circuits connected in parallel, with one resonant circuit for each internal zero. In the event that the poles and zeros are so arranged that the network has a pole at the origin, inductance L_0 is omitted. Similarly, if there is a pole at infinite frequency, capacity C_{p+2} is omitted. The values of the elements in the circuit of Fig. 64*b* required to give a reactance function corresponding to specified zeros, poles, and the given value of H, is obtained from the following equation which can be deduced by a partial fraction expansion of $1/Z_{11}$ as given by Eq. (86).

$$L_k = |j\omega_k Z_k'|, \qquad (k = 1, 3, \ldots, p) \tag{91}$$

where Z_k' is the quantity obtained by omitting $(\omega^2 - \omega_k^2)$ from the numerator of expression (86) for Z and evaluating the modified expression for $\omega = \omega_k$. Corresponding to each L_k, one has

$$C_k = \frac{1}{\omega^2 L_k} \tag{92}$$

If the network has a zero at infinite frequency

$$C_{p+2} = \frac{1}{H} \tag{93}$$

If the network has a zero at zero frequency

$$L_0 = Z_0' \tag{94}$$

Where Z_0' is the quantity obtained by omitting the ω that multiplies H in expression (86*b*) and then evaluating the modified expression for Z with $\omega = 0$.

The networks of Fig. 64 represent networks having the least possible number of reactive elements that can be used to realize a specified impedance characteristic. The same impedance characteristic may also be realized by many other networks other than the two shown,[1] but alternative arrangements will in most cases have additional circuit elements that are superfluous. The least number of circuit elements required is one more than the sum of the internal poles and zeros. One, or even more than one, element beyond this minimum is possible for a given number of internal zeros and poles, but such networks have exactly the same impedance characteristic as the simplest forms, such as given in Fig. 64, which are called fundamental or canonic forms because they have the least possible number of elements.

22. Inverse or Reciprocal Impedances.[2]—Two impedances Z_1 and Z_2 are said to be reciprocal with respect to an impedance Z if they are so related as to satisfy the relation

$$Z_1 Z_2 = Z^2 \tag{95}$$

Under practical conditions where reciprocal impedances are of importance, the impedance Z in Eq. (95) is always a resistance.

The process of deriving a reciprocal impedance from a given impedance is termed *reciprocation*. Reciprocation of a ladder network consisting of alternate series and shunt arms, as in Fig. 65*a*, leads to a corresponding ladder network of alternate shunt and series arms, as shown in Fig. 65*b*.

[1] Ladder networks starting with either a series impedance or a shunt impedance also can be used to develop a specified two-terminal impedance utilizing the minimum possible number of circuit elements. The circuit constants for such networks are obtained from continued fraction expansion of the reactance function. For further information, see Guillemin, *op. cit.*, pp. 198–207.

[2] See Guillemin, *op. cit.* p. 204; also, A. C. Bartlett, "Theory of Electrical Artificial Lines and Filters," pp. 53–58, Wiley, New York, 1931.

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Various special cases of reciprocation are shown in Fig. 66. These assume that reciprocation is with respect to a resistance R, *i.e.*, Z in Eq. (95) is taken as R. It will

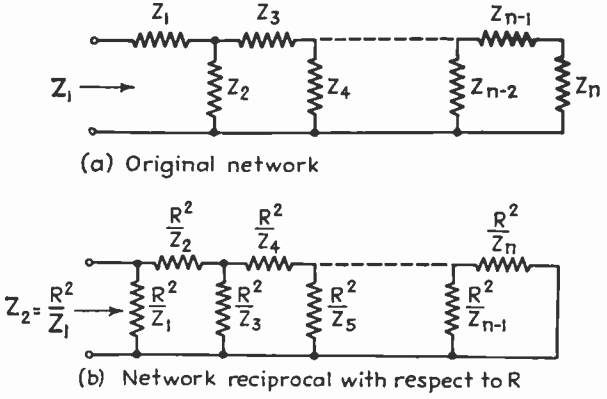


FIG. 65.—Reciprocal ladder networks.

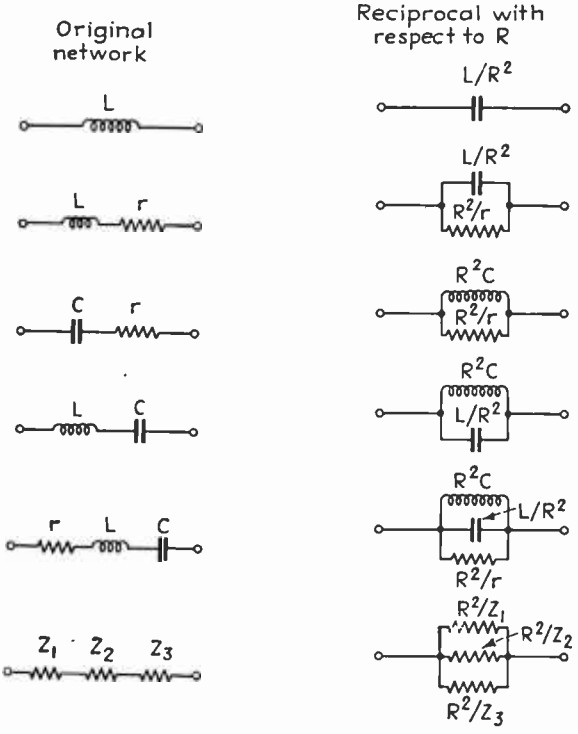


FIG. 66.—Basic examples of reciprocation.

be noted that the reciprocal of a number of impedance elements in series consists of a number of impedance elements in shunt, with each shunt element being the reciprocal of one of the series elements, and vice versa. It is also to be noted that when two

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reactive networks are reciprocal, the poles of one coincide with the zeros of the other impedance, and vice versa.

23. Fundamental Relations Existing in Four Terminal Networks.[1] *Methods of Expressing Network Characteristics.*—Insofar as the four terminals are concerned, the properties of a four-terminal network at any one frequency can be expressed in terms of any three independent properties of the network, irrespective of how complicated the network is. Although there are an unlimited number of ways in which three independent constants can be defined, the ones most commonly employed in communication networks are (1) image impedances and image transfer constant; (2) open- and short-circuit impedances; and (3) iterative impedances and iterative transfer constant.

Four-terminal Network Behavior Expressed in Terms of Image Impedances.—A network is said to be operated under image-impedance conditions when the internal impedance Z_s of the source of power acting on the input terminals of the network, and the load impedance Z_L at the output terminals, are so related to the network that the impedance looking into the network from the terminals 1-2 with the load connected

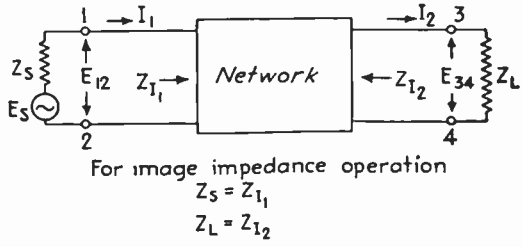


FIG. 67.—Four-terminal network operated with terminal impedances.

(see Fig. 67) is equal to the generator impedance, and, similarly, so that the impedance looking into the network at terminals 3-4 with the generator connected equals the load impedance. The generator and load impedances required to produce this condition are properties of the network, and are termed the *image impedances.* They can be designated by the symbol Z_{I_1} and Z_{I_2} for the input and output terminals, respectively (see Fig. 67).

In dealing with image impedances, the third independent property of the network required to finish the specification of the network behavior is taken as the *image transfer constant* θ, which is defined in terms of the relations

$$\frac{E_{34}}{E_{12}} = \sqrt{\frac{Z_{I_2}}{Z_{I_1}}}\epsilon^{-\theta}, \qquad \frac{I_2}{I_1} = \sqrt{\frac{Z_{I_1}}{Z_{I_2}}}\epsilon^{-\theta} \tag{96}$$

The notation is illustrated in Fig. 67. The image transfer constant θ has the same value irrespective of the direction of transmission of energy through the network.

The three network parameters Z_{I_1}, Z_{I_2}, and θ can be defined in terms of the network determinant and its minors, and also in terms of the open- and short-circuit impedance of the network, according to the equations

$$Z_{I_1} = \sqrt{\frac{DB_{22}}{B_{11}B_{1122}}} = \sqrt{Z_{oc}Z_{sc}} \tag{97}$$

$$Z_{I_2} = \sqrt{\frac{DB_{11}}{B_{22}B_{1122}}} = \sqrt{Z'_{oc}Z'_{sc}} \tag{98}$$

[1] Additional information on these subjects is given in the following books: K. S. Johnson, "Transmission Circuits for Telephonic Communication"; T. E. Shea, "Transmission Networks and Wave Filters," Van Nostrand, 1929; Guillemin, *op. cit.*

$$\tanh \theta = \sqrt{\frac{Z_{sc}}{Z_{oc}}} = \sqrt{\frac{Z'_{sc}}{Z'_{oc}}} = \sqrt{\frac{DB_{1122}}{B_{11}B_{22}}} \tag{99}$$

where Z_{oc} and Z_{sc} are the impedances at terminals 1-2 with terminals 3-4 open- and short-circuited, respectively, Z'_{oc} and Z'_{sc} are the impedances at 3-4 with 1-2 alternately open- and short-circuited, while D is the determinant for the network formed by short-circuiting terminals 1-2 and 3-4. The B's are minors of this determinant, with B_{1122} being the minor formed by striking out both first and second rows and first and second columns of the determinant.

Image-impedance operation of a network can be conveniently related to the behavior of a transmission line expressed in terms of wave trains. Although wave trains obviously cannot exist in a network having lumped constants, it is nevertheless frequently convenient to explain the behavior of a four-terminal system, insofar as the terminals are concerned, in terms of wave trains just as though these wave trains actually existed within the network instead of being hypothetical. The image impedances correspond to the characteristic impedance of the transmission line, but unless the network is symmetrical about its midpoint, there will be two image impedances because of the fact that the network can be considered as equivalent to a transmission line that is unsymmetrical and so has an impedance transforming action upon a

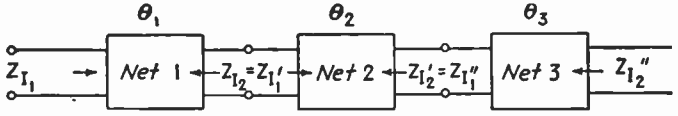


FIG. 68.—Networks connected in cascade on an image-impedance basis.

wave train. The image transfer constant θ of the network is likewise analogous to the hyperbolic angle of the transmission line. The real part of the image transfer constant is called the *attenuation constant*, and can be considered as causing the hypothetical wave train in the network to be attenuated in magnitude. The imaginary part of the image transfer constant corresponds to the phase constant of the transmission line hyperbolic angle, and causes a shift in phase of the hypothetical wave train. If the load impedance does not equal the image impedance on the output side of the network, the effect on the voltage and current relations is as though a wave train existed in the network, and was reflected by the load impedance, just as in the case of a transmission line.

When several networks are connected together in cascade on an image-impedance basis, as illustrated in Fig. 68, then the image impedances at the input and output terminals are the image impedances at the input terminals of the first network and the output terminals of the last network, respectively (Z_{I_1} and Z_{I_2}'' in Fig. 68). The image transfer constant of such a system is

$$\theta + \theta_1 + \theta_2 + \ldots + \theta_m \tag{100}$$

where θ_1, θ_2, etc., represent the image transfer constants of the first, second, etc., component networks of the system.

In a system consisting of a number of networks connected in cascade on an image-impedance basis, it is customary to refer to the image impedance existing at a particular junction point as the *impedance level* at that point.

The image-impedance method of expressing the properties of a four-terminal network is extremely important because most four-terminal networks used in communication systems are operated under conditions that approach very closely image-impedance operation. This is particularly true of filters, equalizers, and impedance matching networks.

Ex. 1044-0213

Four-terminal Networks Operated on an Iterative-impedance Basis.—A network operated on an iterative-impedance basis requires that the load impedance be such that the input impedance of the network with the load connected is equal to the load impedance. At the same time, the impedance observed by looking into the network from the output terminals toward the generator with the internal impedance of the generator connected across the input terminals of the network must equal the generator impedance. These two impedances can be designated as Z_{k_1} and Z_{k_2}, respectively, and are properties of the network. The third property necessary to specify completely the network characteristics is then taken as the iterative transfer constant P, which is defined by the equation

$$\frac{I_2}{I_1} = \epsilon^{-P} \tag{101}$$

where I_1 and I_2 are the input and output currents of the network, respectively, when operated under iterative conditions. Iterative-impedance action becomes of importance in handling problems involving L and ladder types of attenuators.

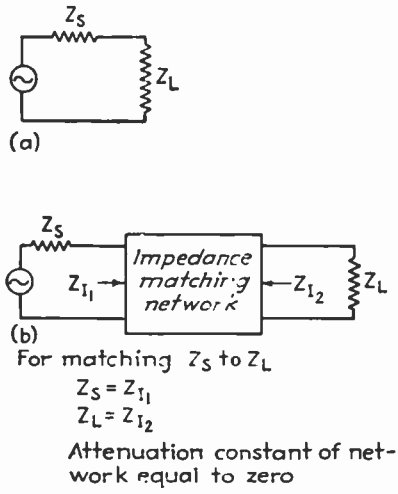


FIG. 69.—Impedance matching of generator and load with the aid of a network.

Impedance Matching.—Two impedances are said to be matched when they have the same magnitude and the same phase angle. Thus in Fig. 69*a*, the load impedance Z_L is said to be matched to the generator impedance Z_s if the load impedance equals the generator impedance. If the load and generator impedances are not matched, it is possible to obtain matching by inserting between generator and load, as shown in Fig. 69*b*, a network having $Z_{I_1} = Z_s$, $Z_{I_2} = Z_L$, and zero attenuation (*i.e.*, real part of image transfer constant θ must equal zero). The generator then sees a load impedance $Z_{I_1} = Z_s$, and the load receives its power from a source (the network output terminals) having an internal impedance $Z_{I_2} = Z_L$. Such a system has its impedances matched on an image basis at all junction points.

The ratio of load current that would be delivered by a particular generator to a particular load without matching, as in Fig. 69*a*, to the ratio of the same currents when the impedances are matched, as in Fig. 69*b*, is designated by such terms as *mismatching factor*, *reflection factor*, or *transition factor*. The absolute value of this ratio gives the loss of load current that results when no means are provided to couple the generator to the load impedance on an image-impedance basis. The value of the mismatching factor depends only upon the ratio of load to generator impedance, and is

$$\frac{\text{Load current without matching}}{\text{Load current with matching}} = k = \frac{\sqrt{\frac{4Z_s}{Z_L}}}{1 + \frac{Z_s}{Z_L}} \tag{102}$$

Expressed in decibels, the loss of load current resulting from mismatching is

$$\text{Mismatching loss in db} = 20 \log_{10} (1/k) \tag{103}$$

A chart giving loss from mismatching based on Eq. (103) is given in Fig. 70, where θ_s and θ_L are the phase angles of Z_s and Z_L, respectively.

Ex. 1044-0214

When the generator impedance Z_s is a resistance, then image-impedance matching corresponds to the condition for which the power delivered by the generator to the load is maximum, and if the load impedance fails to match the generator impedance, a loss of load power results. However, if the generator impedance has a reactive component, then failure to match the load impedance to the generator impedance can, under certain conditions, result in an increase in the load current.[1] Under these conditions, the mismatching factor will be greater than unity.

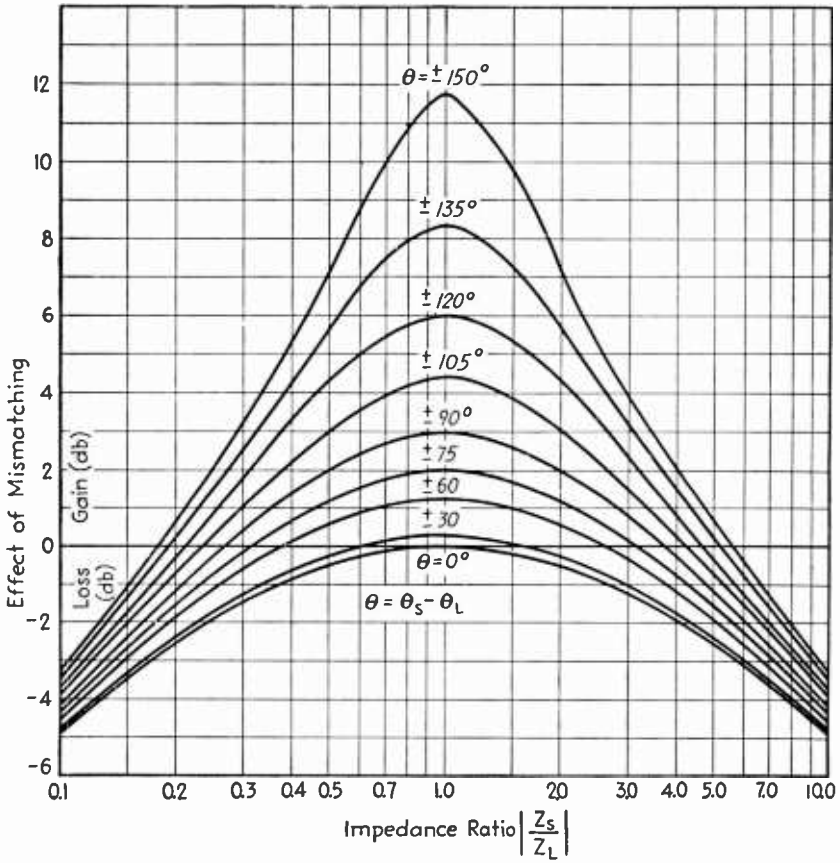


FIG. 70.—Chart giving effect of mismatching generator and load impedance.

Insertion Loss.—When a network is inserted between a sending-end impedance Z_s and a load impedance Z_L, as in Fig. 69*b*, the ratio of current in the load impedance when the network is present to the load current in the absence of the network is termed the *insertion loss*, since this is the loss in output current resulting from the insertion of the network between generator and load.

The insertion loss can be conveniently expressed by the formula

$$\text{Insertion loss} = \frac{k_1 k_2}{k} \sigma \epsilon^{-\theta} \tag{104}$$

[1] The condition for maximum possible transfer of energy to the load is realized when the resistance component of the load impedance is equal to the resistance component of the generator impedance and when at the same time the reactive component of the load impedance is equal in magnitude but opposite in sign to the reactive component of the generator impedance.

Ex. 1044-0215

where k_1 = mismatching factor of Z_s and Z_{I_1}.
k_2 = mismatching factor of Z_{I_2} and Z_L.
k = mismatching factor of Z_s and Z_L.
θ = image transfer constant.

$$\sigma = \text{interaction factor} = \frac{1}{1 - \left(\frac{Z_{I_2} - Z_L}{Z_{I_2} + Z_L}\right)\left(\frac{Z_{I_1} - Z_s}{Z_{I_1} + Z_s}\right)\epsilon^{-2\theta}}.$$

The interaction factor σ is a second-order effect representing a modification of the insertion loss that occurs when there is mismatching at *both* the input and output terminals of the network. The interaction factor takes into account the effect of a wave that is reflected from the load back to the generator through the network, and there reflected back to the load. The interaction factor becomes unity whenever at least one end of the network is matched on an image-impedance basis, or when the network attenuation is such that a wave that has traveled through the network twice will have been reduced to negligible amplitude.

Under practical conditions, the interaction factor in networks intended to be operated on an image-impedance basis is at most only a few decibels.

24. Fundamental Types of Four-terminal Networks. *π and T Networks.*—The fact that any four-terminal network can have its properties represented, insofar as the

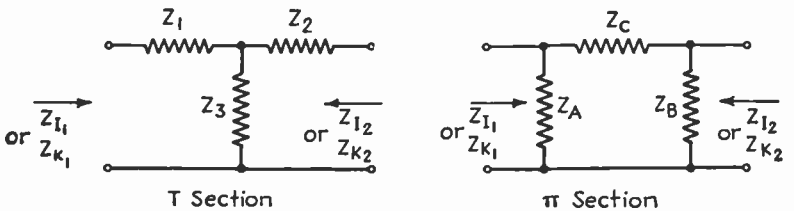


FIG. 71.—General T and π networks.

terminals are concerned, by three independent constants means that such networks can be always represented by three properly chosen independent impedances arranged in the form of a T or π, as shown in Fig. 71.

The relationship between the impedances composing such networks, and the characteristics of the system expressed on an image-impedance basis, are

For T section (see Fig. 71):

$$\left.\begin{aligned} Z_{I_1} &= \sqrt{(Z_1Z_2 + Z_1Z_3 + Z_2Z_3)\left(\frac{Z_1 + Z_3}{Z_2 + Z_3}\right)} \\ Z_{I_2} &= \sqrt{(Z_1Z_2 + Z_1Z_3 + Z_2Z_3)\left(\frac{Z_2 + Z_3}{Z_1 + Z_3}\right)} \\ \tanh\theta &= \sqrt{\frac{(Z_1Z_2 + Z_1Z_3 + Z_2Z_3)}{(Z_1 + Z_3)(Z_2 + Z_3)}} \end{aligned}\right\} \quad (105)$$

For π section (see Fig. 71):

$$\left.\begin{aligned} Z_{I_1} &= Z_A\sqrt{\frac{(Z_B + Z_C)}{(Z_A + Z_C)}\frac{Z_C}{Z_A + Z_B + Z_C}} \\ Z_{I_2} &= Z_B\sqrt{\frac{(Z_A + Z_C)}{(Z_B + Z_C)}\frac{Z_C}{Z_A + Z_B + Z_C}} \\ \tanh\theta &= \sqrt{\frac{Z_C(Z_A + Z_B + Z_C)}{(Z_A + Z_C)(Z_B + Z_C)}} \end{aligned}\right\} \quad (106)$$

Ex. 1044-0216

In the design of T and π networks, one normally knows the desired image impedances, and wishes to realize a particular transfer constant θ. The relations are then

For T section (see Fig. 71):

$$\left.\begin{aligned} Z_3 &= \sqrt{Z_{I_1} Z_{I_2} \left(\frac{1}{\tanh^2 \theta} - 1\right)} \\ Z_2 &= \frac{Z_{I_2}}{\tanh \theta} - Z_3 \\ Z_1 &= \frac{Z_{I_1}}{\tanh \theta} - Z_3 \end{aligned}\right\} \tag{107}$$

For π section (see Fig. 71):

$$\left.\begin{aligned} Z_C &= \sqrt{Z_{I_1} Z_{I_2}} \sinh \theta \\ Z_B &= \frac{1}{\frac{1}{Z_{I_2} \tanh \theta} - \frac{1}{Z_C}} \\ Z_A &= \frac{1}{\frac{1}{Z_{I_1} \tanh \theta} - \frac{1}{Z_C}} \end{aligned}\right\} \tag{108}$$

Examination of these equations shows that in the case of networks with reactive elements, the image impedances are either pure resistances or pure reactances. Furthermore, when the image impedances are resistive, the image transfer constant is a pure imaginary, while when the image impedances are reactive, the image transfer constant has a real component, and so attenuation is introduced. These relations are particularly important in the case of filters and impedance-matching networks.

L Networks.—An L network is shown in Fig. 72, and can be considered as a special case of a T or π network in which one of the impedance arms has become either zero or infinity.

The properties of an L network can be expressed in terms of image impedances and a transfer constant, as in the case of any four-terminal network. However, since there are only two impedance arms in an L network, a relationship must exist between the image impedances and image transfer constant such that if two of these are defined the third is likewise determined.

The formulas relating the image impedance and image transfer parameters of an L network with the impedance elements of the L are

$$\left.\begin{aligned} Z_1 &= \sqrt{Z_{I_1}(Z_{I_1} - Z_{I_2})} \\ Z_3 &= Z_{I_2} \sqrt{\frac{Z_{I_1}}{Z_{I_1} - Z_{I_2}}} \\ \tanh \theta &= \sqrt{\frac{Z_{I_1} - Z_{I_2}}{Z_{I_1}}} \end{aligned}\right\} \tag{109}$$

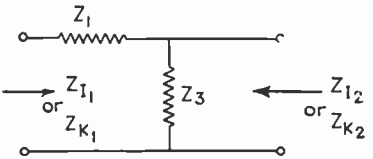


FIG. 72.—General L network.

The corresponding formulas for iterative impedance operation are

$$\left.\begin{aligned} Z_1 &= Z_{k_1} \left(\frac{\epsilon^P - 1}{\epsilon^P}\right) \\ Z_3 &= \frac{Z_{k_1}}{\epsilon^P - 1} \\ Z_{k_1} &= \frac{Z_{k_1}}{\epsilon^P} \end{aligned}\right\} \tag{110}$$

Ex. 1044-0217

where Z_{k_1} and Z_{k_2} are the iterative impedances of the two ends of the network, as shown in Fig. 72, and P is the iterative transfer constant, as defined by Eq. (101). L networks find their chief use in impedance matching systems and in attenuators.

Lattice Sections.—A lattice is a symmetrical balanced four-terminal network composed of two pairs of impedances, arranged as shown in Fig. 73. It will be noticed that the lattice is essentially a bridge in which the input is applied across one diagonal of the bridge, and the output is taken from the other diagonal.

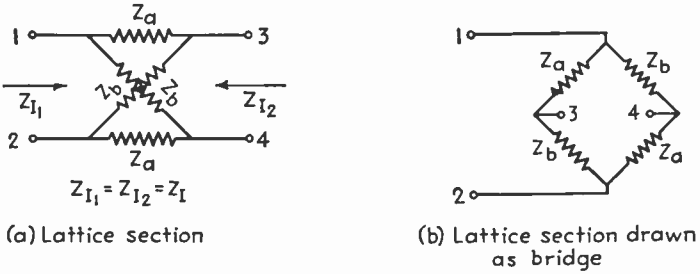


FIG. 73.—General lattice network.

The basic formulas of the lattice section in terms of image impedance and image transfer constant are

$$Z_I = Z_{I_1} = Z_{I_2} = \sqrt{Z_a Z_b}$$
$$\tanh\left(\frac{\theta}{2}\right) = \sqrt{\frac{Z_a}{Z_b}} \tag{111}$$

If one is given θ and Z_I, then

$$Z_a = Z_I \tanh\left(\frac{\theta}{2}\right)$$
$$Z_b = \frac{Z_I}{\tanh\left(\frac{\theta}{2}\right)} \tag{112}$$

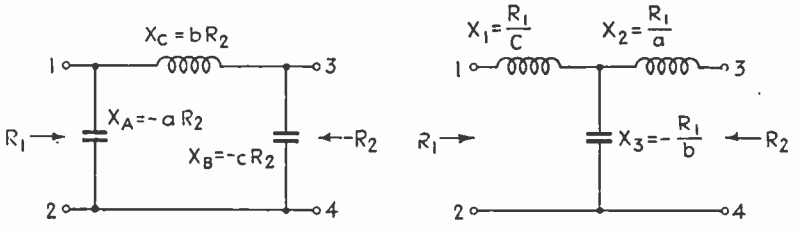
It is possible to represent any symmetrical four-terminal network by a lattice having physically realizable impedance arms. In contrast, the π or T network equivalent of a complicated four-terminal network will sometimes require negative circuit elements in some of the arms, and hence be physically unrealizable. The image impedance of a lattice depends only upon the product of the two branch impedances, whereas the image transfer constant depends only upon the ratio of these impedances. It is therefore possible in the lattice to control the transmission characteristics entirely independently of the image-impedance behavior.

Lattice networks are used in filters and equalizers.

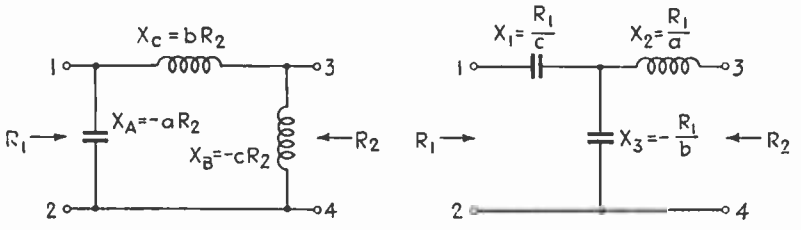
25. Reactive T, L, and π Networks for Matching Impedances.[1] *T and π Reactive Networks.*—T and π networks having impedance arms composed of reactive elements are widely used for matching an antenna to a transmission line in order to give a nonresonant characteristic impedance termination for the line. It is possible with such networks to transform any resistance load that may be offered by the antenna system to any other value of resistance that may be needed to give a characteristic impedance

[1] W. L. Everitt, Output Networks for Radio-frequency Power Amplifiers, *Proc. I.R.E.*, Vol. 19, p. 725, May, 1931; Coupling Networks, *Communications*, Vol. 18, p. 12, September, 1938; p. 12, October, 1938; Carl G. Dietsch, Terminating Concentric Lines, *Electronics*, Vol. 9, p. 16, December, 1936; Ralph P. Glover, R-f Impedance Matching Networks, *Electronics*, Vol. 9, p. 29, January, 1936.

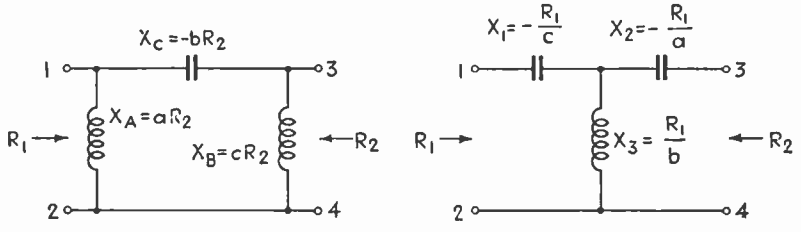
Ex. 1044-0218



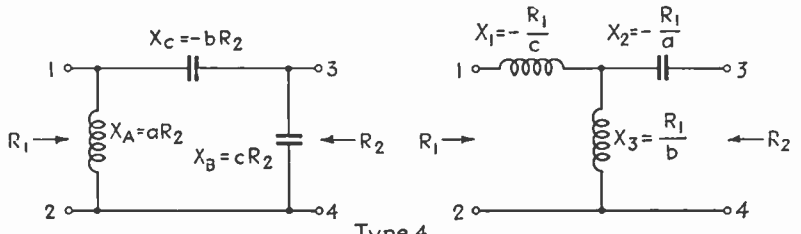
Type 1
Phase retarded by large angle (C positive)



Type 2
Phase retarded by small angle (C negative)



Type 3
Phase advanced by large angle (C positive on chart)



Type 4
Phase advanced by small angle (C negative on chart)

FIG. 74.—Three-element reactive networks which may be used for impedance matching.

load for the transmission line. At the same time, the phase shift introduced by the impedance matching network can have any desired value.

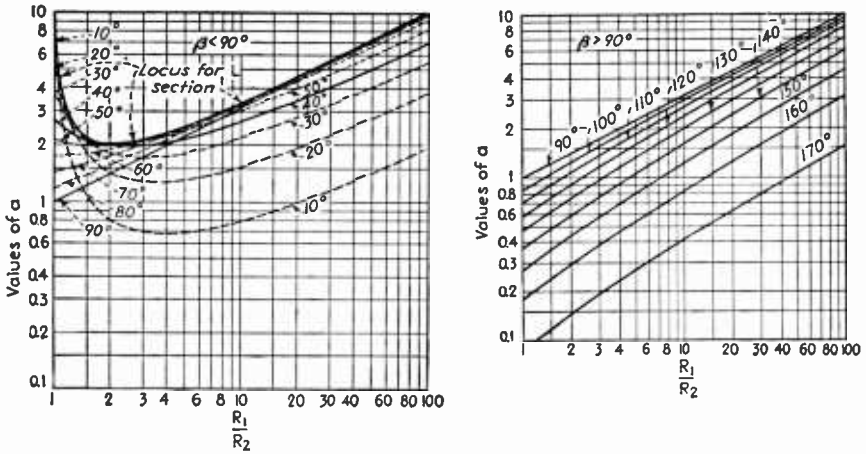


FIG. 75.—Design charts, giving values of a for different values of β for use in networks of Fig. 74.

On the assumption of an ideal network composed of reactive impedances with zero losses, the design relations represented by Eqs. (107) and (108) can be written as

For T section:

$$\left.\begin{aligned} Z_1 &= -j\,\frac{R_1 \cos\beta - \sqrt{R_1R_2}}{\sin\beta} \\ Z_2 &= -j\,\frac{R_2 \cos\beta - \sqrt{R_1R_2}}{\sin\beta} \\ Z_3 &= -j\,\frac{\sqrt{R_1R_2}}{\sin\beta} \end{aligned}\right\} \qquad (113)$$

For π section:

$$\left.\begin{aligned} Z_A &= j\,\frac{R_1R_2 \sin\beta}{R_2 \cos\beta - \sqrt{R_1R_2}} \\ Z_B &= j\,\frac{R_1R_2 \sin\beta}{R_1 \cos\beta - \sqrt{R_1R_2}} \\ Z_C &= j\sqrt{R_1R_2}\,\sin\beta \end{aligned}\right\} \qquad (114)$$

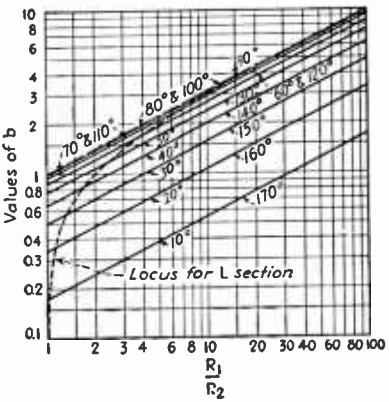


FIG. 76.—Design charts, giving values of b for different values of β for use in networks of Fig. 74.

where R_1 and R_2 are the two image impedances and β is the angle by which the phase at the output terminals lags behind the phase at the input terminals. Design charts derived from these equations are given in Figs. 75, 76, and 77, and together with Fig. 74 will give the reactances required. The charts can be used for negative as well as positive values of β. For negative values of β, the magnitudes of the constants a, b, and c, are the same as for positive, but the signs used in front of the constants are reversed, as shown in Type 3 and 4 networks of Fig. 74. In the design curves of Figs. 75 to 77, it has been assumed that R_1/R_2 is greater than unity. This is no restriction, since terminals 1-2 can be placed at either the generator or the load end of the network, according to whichever must match the higher resistance.

Ex. 1044-0220

L Reactive Networks.—An L network composed of reactive elements is able to transform a given resistance to make it look like any other resistance by making the image impedance of the network at the terminals facing the load equal the load resistance and the image impedance at the other terminal equal the desired resistance. The phase shift introduced by an L section is determined by the ratio of image impedances, and cannot be specified independently, because the L section has only two impedance arms. The L section can be considered as a special case of Fig. 74, for which constant c

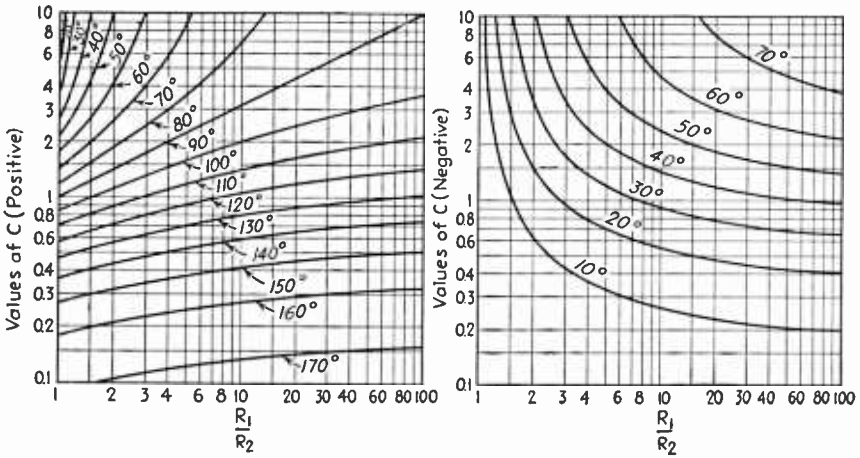


FIG. 77.—Design charts, giving values of c for different values of β for use in networks of Fig. 74.

has the value $c = \infty$, leading to sections as in Fig. 78. The loci corresponding to this condition are shown on Figs. 75 and 76. With an L section, the maximum phase shift obtainable is $\pm 90°$.

Dissipation of Power in Reactive Networks.—In practical networks, the condensers have negligible loss, but the resistance of the inductances is not entirely negligible. On the assumption that the currents in the various network branches are not appreci-

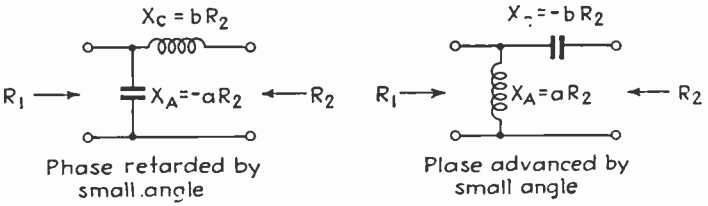


FIG. 78.—Section of L type that is obtained when c in Fig. 74 becomes infinite.

ably affected by the dissipation of the inductive elements and that the various inductive elements have a ratio of reactance to resistance (*i.e.*, Q) that is the same for all inductances, one can write[1]

$$\frac{\text{Power lost in network}}{\text{Power delivered to network}} = \frac{\delta}{Q} \tag{115}$$

where δ is a constant given by either Fig. 79 or 80, according to the type of section

[1] See Everitt, *loc. cit.*

involved. These curves apply for L as well as T and π networks, the locus for the L network being dotted on the curves.

A study of Figs. 79 and 80 shows that the efficiency of the network is implicitly determined by the impedance transformation ratio and the phase shift. There is no choice between T and π networks and between networks that advance and retard the phase, insofar as efficiency is concerned. The loss increases with increasing transformation ratio, and tends to be large when the phase shift of the network is either very

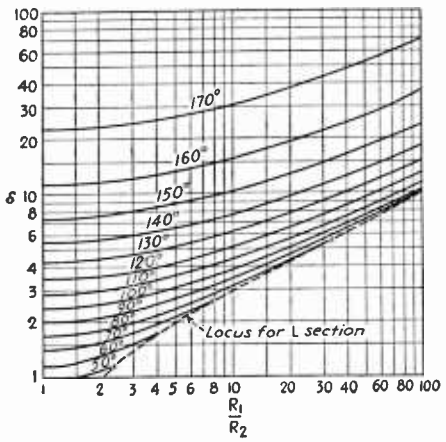


FIG. 79.—Values of δ for use in Eq. (115), applicable for Type 1 and Type 3 networks of Fig. 74. The angles are values of β.

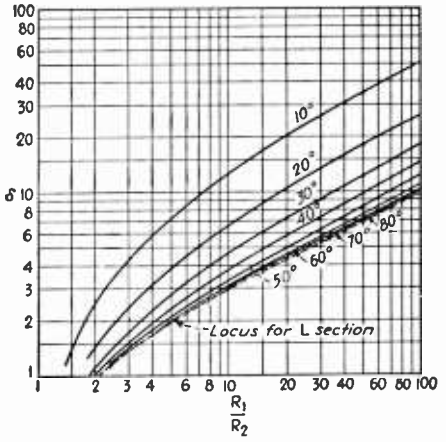


FIG. 80.—Values of δ for use in Eq. (115) applicable for Type 2 and Type 4 networks of FIG. 74. The angles are values of β.

small or very large. Finally, it will be noted that for a given transformation ratio, the L section has a lower loss than either the T or π section. In cases involving very high transformation ratios, or phase shifts that are either very small or approach 180°, an increase of efficiency can be obtained by dividing the total impedance transformation and total phase shift among two or more networks connected in tandem.

Harmonic Reduction.—Coupling networks that call for an inductance in the series arm or a capacity in a shunt arm, or both, can be readily arranged to provide dis-

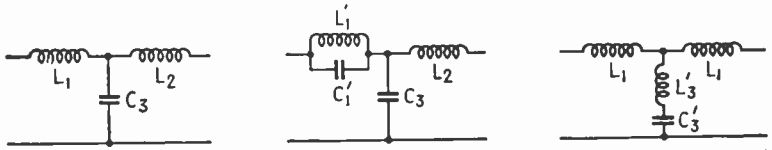


FIG. 81.—Diagram illustrating how a series inductance and a shunt capacity of an impedance matching network may be replaced by parallel and series resonant circuits respectively, to increase discrimination against an undesired harmonic.

crimination against harmonics. The discrimination against a particular harmonic can be made particularly great when the series inductive reactance is supplied by a parallel circuit, as shown in Fig. 81*b*, resonant at a frequency to be suppressed and so proportioned as to give the required inductive reactance at the frequency to be transmitted. An equivalent result is also obtainable when the shunt capacitive reactance is supplied by a suitably designed series circuit, as in Fig. 81*c*, that is resonant at the frequency to be suppressed.

Ex. 1044-0222

The design formulas are as follows:
For Fig. 81*b*:

$$L_1' = L_1(1 - \gamma^2) \tag{116a}$$

For Fig. 81*c*:

$$C_3' = C_3(1 - \gamma^2) \tag{116b}$$

where γ is the ratio of the frequency to be transmitted to the frequency to be suppressed. C_1' in Fig. 81*b* and L_3' in Fig. 81*c* are assigned values that will make the resonance occur at the frequency to be suppressed.

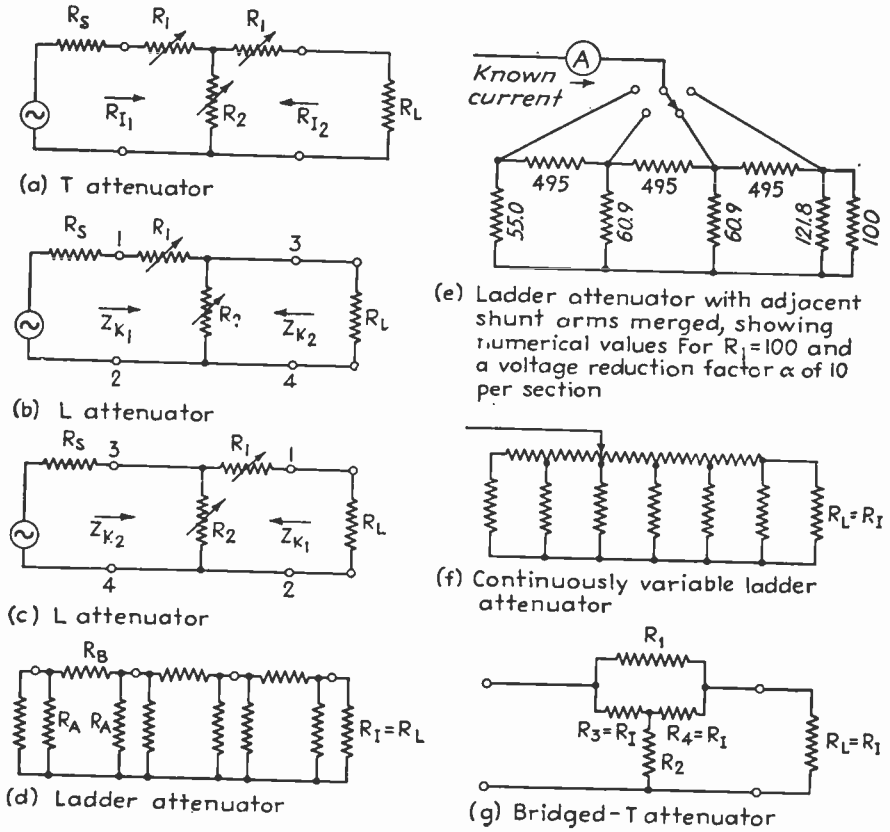


FIG. 82.—Common attenuator networks.

26. Attenuators.[1]—Attenuators are resistance networks used for the purpose of reducing voltage, current, or power in controllable and known amounts. The most commonly used types of attenuators are the *T*, *L*, ladder, and bridged-*T* types.

T Attenuators.—The *T* attenuator consists of three resistances arranged in a *T*, as shown in Fig. 82*a*. Such an attenuator is usually designed with the two series impedances of the *T* identical, thereby making the two image impedances the same.

[1] For further information on attenuators, and in particular for design information including tables giving the resistance values required in the arms for different values of attenuation, see P. K. McElroy, Designing Resistive Attenuating Networks, *Proc. I.R.E.*, Vol. 23, p. 213, March, 1935; Guy C. Omer, Jr., Lattice Attenuating Networks—Complete Design Tables, *Wireless Eng.*, Vol. 17, p. 206, May, 1940; R. E. Blakey, Network Resistances for Balanced Attenuators, *Electronics*, Vol. 8, p. 446, November, 1935.

Ex. 1044-0223

For such a symmetrical T network composed of resistance elements, the relations expressed by Eqs. (107) can be rewritten as

$$\left.\begin{aligned} R_1 &= R_I\left(\frac{\alpha - 1}{\alpha + 1}\right) \\ R_2 &= R_I\left(\frac{2\alpha}{\alpha^2 - 1}\right) \end{aligned}\right\} \tag{117}$$

where R_1 and R_2 are the attenuator resistances, as shown in Fig. 82*a*, R_I is the image impedance of the section, and α is the number whose natural logarithm is the image transfer constant (*i.e.*, $\alpha = \epsilon^{\theta}$). Thus when the generator and load impedances both equal the image impedance R_I, the insertion loss of the attenuator is α, *i.e.*, with image operation

$$\alpha = \frac{\text{load current without attenuator}}{\text{load current with attenuator}} \tag{118}$$

T attenuators are used where it is important that the presence of the attenuator in the circuit, and the amount of attenuation have no effect upon the impedance relations existing in the circuit. This is achieved by making the image impedance of the T attenuator equal either the generator or load resistance.

L Attenuators.—The L attenuator consists of two resistance arms arranged as shown in Fig. 82*b* or 82*c*. The arms are so proportioned that one of the iterative impedances of the attenuator is constant irrespective of the attenuation introduced. The corresponding design formulas are

For Z_{k_1} constant (Fig. 82*b* or *c*):

$$\left.\begin{aligned} R_1 &= Z_{k_1}\left(\frac{\alpha' - 1}{\alpha'}\right) \\ R_2 &= \frac{Z_{k_1}}{\alpha' - 1} \end{aligned}\right\} \tag{119}$$

For Z_{k_2} constant (Fig. 82*b* or *c*):

$$\left.\begin{aligned} R_1 &= Z_{k_2}(\alpha' - 1) \\ R_2 &= Z_{k_2}\left(\frac{\alpha'}{\alpha' - 1}\right) \end{aligned}\right\} \tag{120}$$

Here $\alpha' = \epsilon^{P}$, where P is the iterative transfer constant. When the impedance connected between the pair of terminals opposite from the terminals having constant iterative impedance is equal to this design value of iterative impedance, then

$$\alpha' = \frac{\text{current in load without attenuator}}{\text{current in load with attenuator}} \tag{121}$$

L attenuators are less expensive than the T type, because only two instead of three variable resistances are required to control the attenuation. At the same time the L attenuator maintains impedance independent of attenuation at only one pair of terminals as the attenuation is varied, whereas the T attenuator maintains constant impedance at both terminals. L attenuators are commonly used where a number of loads are associated with a common generator and it is necessary to control the power delivered to each load without altering the impedance offered to the source of power.

Ladder Attenuators.—Ladder attenuators consist of a series of symmetrical π sections designed so that the required ratio of voltage loss per section is obtained with image impedance operation. A typical arrangement is shown in Fig. 82*d*, where there is shown a chain of three π sections terminated at both ends with a resistance equal to the image impedance, and designed to produce the required attenuation when operated

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on an image impedance basis. The appropriate design formulas for the individual sections are

$$R_A = R_I\left(\frac{\alpha + 1}{\alpha - 1}\right)$$
$$R_B = R_I\left(\frac{\alpha^2 - 1}{2\alpha}\right) \tag{122}$$

Here R_A and R_B have the meanings shown in Fig. 82*d*, R_I is the image impedance, and α is the factor by which each section reduces the current with image-impedance conditions (*i.e.*, if $\alpha = 10$, then each section reduces the current in the load to $\frac{1}{10}$ of the load current that would be obtained with the section removed).

The impedance between any junction point in the ladder attenuator and the common side of the system is one-half the image impedance.

Ladder attenuators such as are illustrated in Fig. 82*d* are used in signal generators and in other devices requiring that voltages and currents be reduced in known ratios. A typical arrangement for producing known voltages of small magnitude is illustrated in Fig. 82*e*. Here a known current is supplied to the switch and directed so that it flows from one of the junction points to the common side of the system. This produces a known voltage across that particular branch of the shunt attenuator. The voltage appearing across the output terminals of the attenuator is then this voltage reduced in accordance with the number of attenuator sections between the input point and the load.

A continuously adjustable ladder attenuator is shown in Fig. 82*f*, and is the same as the step attenuator of Fig. 82*d*, except that the switch has been replaced by a slider that permits continuous variation of the input point. As actually constructed, the resistance shown horizontally along the top is an ordinary slide-wire potentiometer, to which suitable shunt resistances have been connected at regular intervals to form a series of π sections. Such an attenuator is inexpensive to construct, and maintains its input and output resistances constant within reasonably narrow limits for all attenuations except extreme values.

Bridged-T Attenuators.—A bridged-T attenuator consists of four resistance arms arranged as shown in Fig. 82*g*. When proportioned so that $R_3 = R_4 = R_I$, and $R_1R_2 = R_I^2$, the image impedance will be a constant value R_I, irrespective of attenuation. With image-impedance operation

$$\frac{\text{Load current with attenuator}}{\text{Load current without attenuator}} = \frac{R_I}{R_1 + R_I} \tag{123}$$

The bridged-T attenuator is equivalent to a simple T attenuator but requires only two instead of three variable resistances.

Decimal Attenuators.[1]—A decimal attenuator is a system of attenuators so arranged that a voltage or current can be reduced in decimal fractions. Such an attenuator is shown in Fig. 83, where any voltage from 0.001 to 1.0 times the input voltage can be obtained in steps of 0.001 volt. This decimal attenuator includes three similar L-type attenuators designed to operate between equal generator and load resistances and to have a constant output resistance equal to the generator resistance R_g. Each attenuator is adjustable in steps such that $1/\alpha$ can be made 0, 0.1, 0.2, 0.3, etc., up to 1.0. The second of these L attenuators feeds into a single T attenuator having an input and output resistance equal to the output resistance of the L section and having $\alpha = 10$. The third of these attenuators delivers its output to two such T sections in tandem. These three systems hence give output voltages in steps of 0.1, 0.01, and 0.001, respectively. The outputs of all three attenuating systems are connected in parallel, so that when

[1] Decimal attenuators as described here were developed by the General Radio Company.

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the output terminals are open-circuited the load resistance for any one attenuator system consists of the output resistance of the other two systems in parallel. The output voltages of the three attenuators are thus superimposed upon each other, and add up directly without being affected by the fact that the load resistance for any one attenuator is supplied by the other two attenuators, which, at the same time, produce their own output voltages. The fact that there is a mismatch of resistance at the output need not be corrected for, since this changes all output voltages by the same percentage and so does not disturb the relative outputs. The output resistance to any load connected across the output terminals is the output resistances of the three attenuator systems in parallel. The addition of such a load merely increases the mismatch on the output side of the attenuator, and changes all output voltages by the same percentage without altering the relative values at the different attenuator settings.

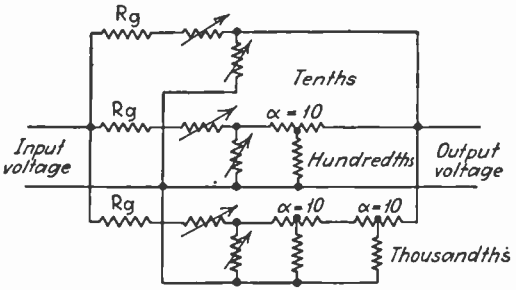


FIG. 83.—Circuit diagram of decimal attenuator.

27. Relation between Attenuation and Phase Shift in Four-terminal Networks.[1]— For any four-terminal network connected between resistive terminal impedances, there is always a minimum possible phase change that can be associated with a given transmission characteristic. In such a *minimum-phase-shift network*, the phase characteristics can be calculated if the transmission characteristic is known, and vice versa.

Any actual four-terminal network will be a minimum-phase-shift network unless (1) the network contains a transmission line or an equivalent circuit with distributed constants, or (2) the circuit includes an all-pass section, either as an individual structure or in a combination that can be replaced by an all-pass filter section plus some other physical structure. It is to be noted particularly that all ladder networks are automatically of the minimum-phase-shift type, since it is impossible to form an all-pass filter section from alternate series and shunt impedances.

Phase-area Theorem.—One of the simplest relations existing between attenuation and phase shift is

$$\int_{-\infty}^{+\infty} B d\mu = \frac{\pi}{17.37} (A_\infty - A_0) \tag{124}$$

where B = phase shift, radians.

$\mu = \log_\epsilon (f/f_0)$, where f is the actual frequency and f_0 is any convenient reference frequency.

A_∞ = attenuation, db, at infinite frequency.

A_0 = attenuation, db, at zero frequency (not at frequency f_0).

The relation described by Eq. (124), expressed in words, is to the effect that the total area under the phase characteristics, when plotted on a logarithmic frequency scale, depends only upon the difference between the transmission (or attenuation) at zero and infinite frequency, and does not depend in any way upon the way in which the

[1] H. W. Bode, Relations between Attenuation and Phase in Feedback Amplifier Design, *Bell System Tech. Jour.*, Vol. 19, p. 421, July, 1940; also, U.S. Patent No. 2,123,178; Y. W. Lee, Synthesis of Electrical Networks, *Jour. of Math. and Physics*, Vol. 11, p. 83, 1931–32.

Ex. 1044-0226

transmission varies between these limits; nor does it depend upon the physical configuration of the network, provided a minimum-phase-shift structure is employed. The equality of the phase areas under different conditions is illustrated in Fig. 84.

The relation expressed by Eq. (124) can be termed the *phase-area theorem*, and has several very important practical consequences. It means, for example, that if a network is employed to change the transmission from one fixed value to another fixed value and if, at the same time, the maximum phase shift that can be permitted at any frequency is limited to some relatively low value, then the region where the attenuation is changing must be spread out over a sufficiently wide frequency range so that the necessary area can be obtained under the phase curve without this curve having a maximum exceeding the allowable value. This fact is of considerable significance in connection with the design of feedback amplifier systems, since here the transmission around the feedback loop must have associated with it a phase shift that is always less than 180°, even when the transmission is varying.

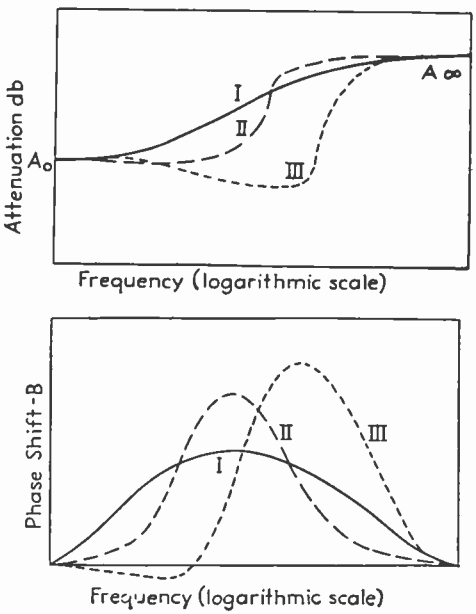


FIG. 84.—Diagram to illustrate relation between area under phase-shift curve and the change in attenuation.

Phase Shift as a Function of Attenuation Slope.—The minimum phase change associated with a given attenuation characteristic can be expressed as[1]

$$B_c = \frac{\pi}{12}\left(\frac{dA}{d\mu}\right)_c + \frac{1}{6\pi}\int_{-\infty}^{+\infty}\left[\left(\frac{dA}{d\mu}\right) - \left(\frac{dA}{d\mu}\right)_c\right]\log_e \coth\frac{|\mu|}{2}\,d\mu \qquad (125)$$

where B_c = phase shift, radians, at the frequency f_c.
$dA/d\mu$ = slope of attenuation curve, db per octave.
$\mu = \log_e (f/f_c)$, where f is frequency and f_c the frequency at which B_c is desired.
$\coth x$ = hyperbolic cotangent of $x (= 1/\tanh x)$.
subscript c denotes evaluated at $f = f_c$.

$\log \coth \left|\frac{\mu}{2}\right|$ denotes the real part of $\log \coth \frac{\mu}{2}$ (which is complex when μ is negative).

Equation (125) shows that the minimum phase change can be expressed as the sum of two components. The first of these is proportional to the slope of the attenuation characteristic when plotted on a logarithmic frequency scale, and amounts to 180° when the attenuation is varying at the rate of 12 db per octave at the reference frequency. The second term is determined by an integral, the integrand of which is proportional to the weighting function $\log_e \coth (|\mu|/2)$, which is plotted in Fig. 85, and the difference between the slope of the attenuation characteristic at the desired frequency f_c and the slope elsewhere. Because of the symmetrical character of the weighting function, this second contribution to the phase shift is determined by

[1] The π in the denominator in front of the integral represents the correction of an error in Eq. (10) of Bode's U.S. Patent No. 2,123,178, upon which the equation given here is based.

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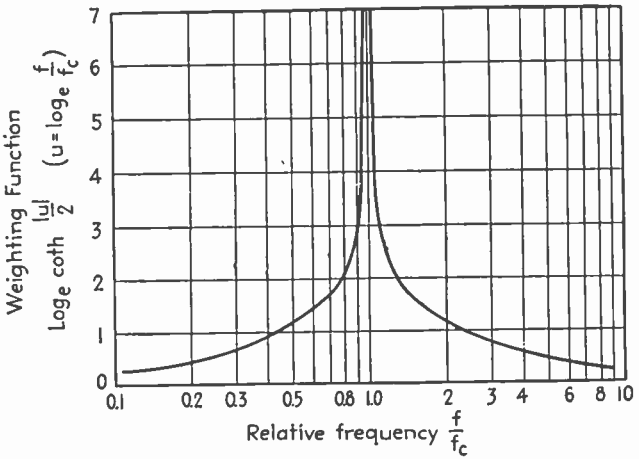


FIG. 85.—Weighting function $\log_e \coth \frac{|\mu|}{2}$ appearing in Eq. (125).

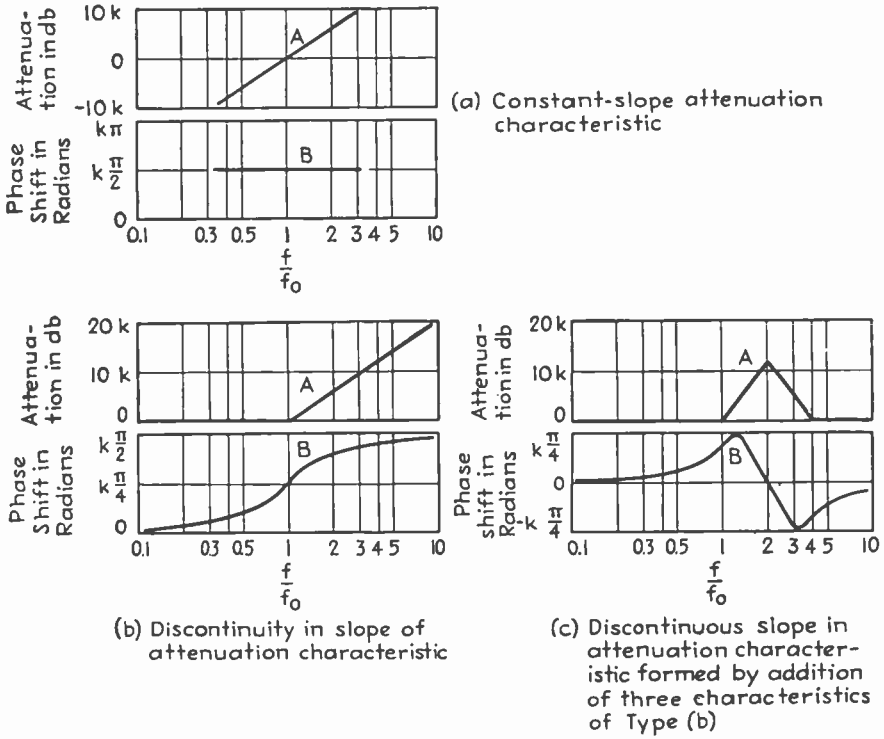


FIG. 86.—Examples of the relationship between phase and attenuation characteristic in several idealized examples.

Ex. 1044-0228

the extent that the attenuation-slope characteristic fails to have negative symmetry (*i.e.*, odd-function symmetry) about the reference frequency. Also, the shape of the weighting function is such that changes in the slope of the attenuation characteristic at frequencies close to the reference frequency have far more effect upon phase shift than do changes in the slope of the characteristic at more remote frequencies.

Examples of the relationship between attenuation characteristics and phase shift are given in Fig. 86 for several idealized cases.

Specification of Complete Phase and Attenuation Characteristics from Attenuation and Phase Fragments.—Relations have been worked out from which the complete attenuation and phase characteristics of a minimum phase-shift network can be determined by prescribing the attenuation characteristic over a portion of the frequency range and prescribing the phase-shift characteristic for the remainder of the frequency range.

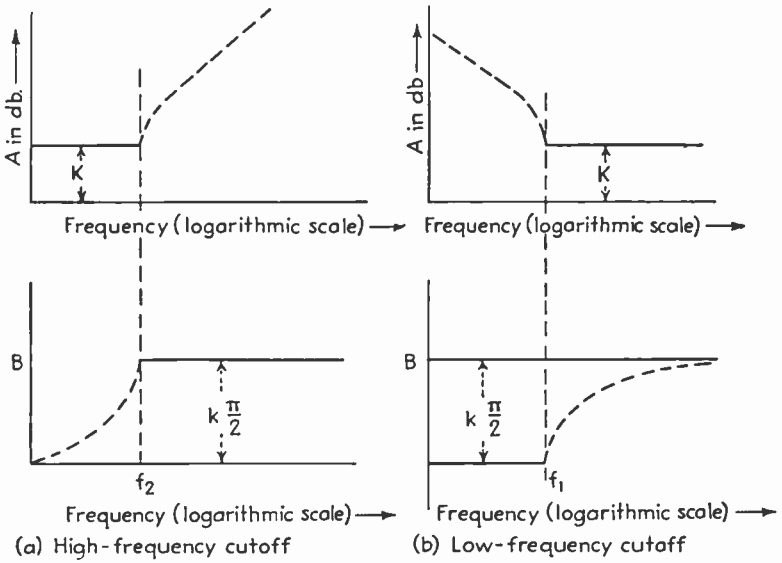


FIG. 87.—Ideal low-frequency and high-frequency cutoff characteristics.

Certain special cases of this character that are of particular importance are illustrated in Fig. 87. Here the attenuation is specified as a constant value in the useful range of the circuit, while the phase shift is specified as having a constant value (normally slightly less than 180°), outside the useful frequency range. The remainder of the characteristics are shown dotted. The essential formulas for the cases illustrated in Fig. 87 are

High-frequency cutoff (Fig. 87*a*):

$$B = k \sin^{-1} \frac{f}{f_2} \qquad f < f_2 \tag{126a}$$

$$A = K + 8.69k \log_e \left[\sqrt{\left(\frac{f}{f_2}\right)^2 - 1} + \frac{f}{f_2} \right] \qquad f > f_2 \tag{126b}$$

Low-frequency cutoff:

$$B = -k \sin^{-1} \frac{f_1}{f} \qquad f > f_1 \tag{127a}$$

$$A = K + 8.69k \log_e \left[\sqrt{\left(\frac{f_1}{f}\right)^2 - 1} + \frac{f_1}{f} \right] \qquad f < f_1 \tag{127b}$$

Ex. 1044-0229

The notation either is as illustrated in Fig. 87 or is as previously used, with the addition that B is in radians and A in db, and K is the attenuation in the region where the attenuation is constant. These expressions can be considered as representing the fundamental formulas for the high- and low-frequency cutoff characteristics of the feedback loop of the ideal feedback amplifier.

Application of Minimum Phase-shift Principles to the Design of Feedback Amplifier Circuits.—A schematic diagram of an amplifier provided with negative feedback is shown in Fig. 88. In this diagram, the portion CDE transmits a fraction of the output of the amplifier back to the input circuit, and superimposes this upon the applied signal. In the normal frequency range, the phase relations are so adjusted that this feedback action opposes the applied signal, thereby reducing the amplification and giving negative feedback. In order that such a system will not oscillate under any conditions, it is necessary that the transmission and phase characteristics of the "feedback loop" $ABCDE$ be such that the transmission around this loop is less than unity when the phase shift reaches 180°. The transmission around this loop is obtained by breaking the circuit at x, applying a signal voltage at point A, and comparing this signal voltage with the voltage that it produces at the point E.

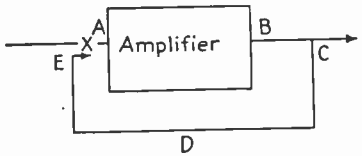


FIG. 88.—Schematic feedback amplifier diagram.

In an ideal feedback amplifier system, the transmission around the feedback loop would be constant throughout the useful band of frequencies, thereby giving constant negative feedback in the useful range. At the same time, the transmission around this loop should drop off as rapidly as possible outside the useful range of frequencies in order that the frequency range over which the transmission characteristics must be controlled accurately be no greater than absolutely necessary. Because of the phase-area theorem, this maximum rate of falling off in transmission outside the useful band will be obtained when the phase shift has a constant value that approaches as close to 180° as is practical throughout the attenuating range. The ideal characteristic for a feedback loop is accordingly that shown in Fig. 87, with k given a value that is slightly less than 2 to provide a margin of safety.

Limitations Introduced by the Asymptotic Transmission Characteristics of the Feedback Loop at Extreme Frequencies.—The ideal feedback loop characteristics cannot in many cases be realized at extremely high and extremely low frequencies, because at these extreme frequencies such factors as stray shunting capacities, grid-leak-condenser combinations, etc., take over control of the characteristics and may cause the transmission to fall off more rapidly than permissible for an ideal feedback loop. This is illustrated in Fig. 89, where ABC is the ideal characteristic, but because of the asymptotic falling off in transmission of the amplifier at high frequencies, the actual characteristic is $AB'C'$. There is then an additional phase shift at high frequencies. The excess phase shift in the region where the feedback loop transmission changes over from the ideal to the asymptotic characteristic (region C in Fig. 89), can be eliminated by the expedient shown in Fig. 90. Here a step of zero slope is introduced in the transmission characteristic just before the asymptotic characteristic is allowed to take control. The length of this step should be such that

$$\frac{f_a}{f_b} = \frac{f'_a}{f'_b} = \frac{\text{slope of ideal asymptotic characteristic in db per octave}}{\text{slope of actual asymptotic characteristic in db per octave}} \qquad (128)$$

The type of phase-shift characteristic that results is shown in Fig. 90, and the total attenuation between f'_a and f_1 (and f_2 and f_b) is very close to the largest that can possibly

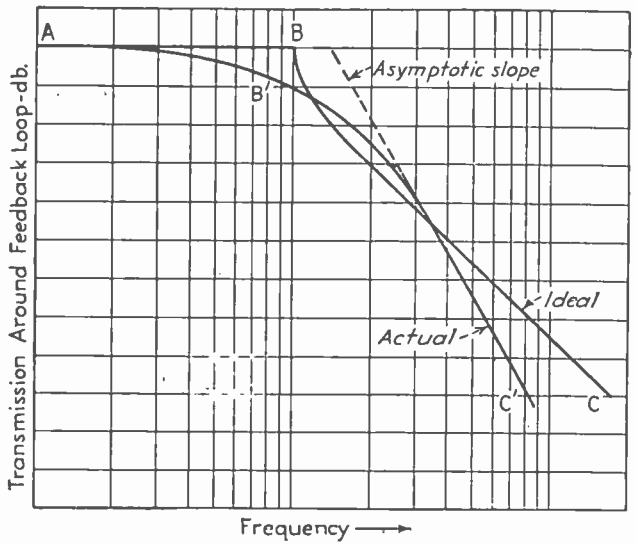


FIG. 89.—Typical amplifier cutoff characteristic, and ideal cutoff characteristic for feedback loop.

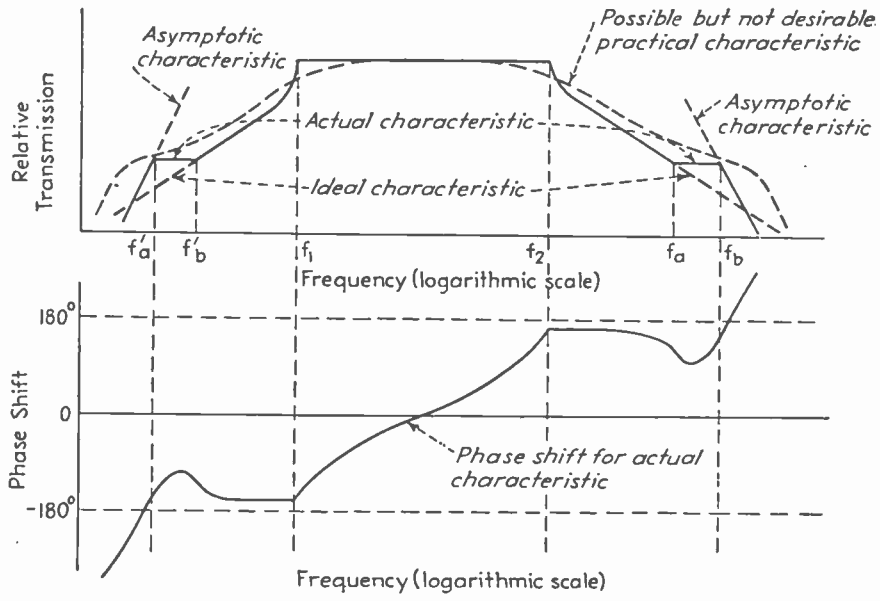


FIG. 90.—Step transition to attenuation characteristic of high slope.

Ex. 1044-0231

be obtained in this frequency interval without the maximum phase shift exceeding a fixed maximum value.

The slope of the actual asymptotic characteristic can be estimated by inspecting the circuits. Each resistance- or impedance-coupled amplifier stage will contribute 6 db per octave to the asymptotic slope at high frequencies, as will each output transformer fed from a tube and having a resistance load. An interstage coupling transformer will add 12 db per octave to the slope unless a resistance is shunted across the secondary. Similarly, at low frequencies, each grid leak-condenser combination will cause 6 db per octave slope, as will each transformer (output, input, or interstage) excited from a resistance source as a tube or line. Transformers with resonated primaries can contribute 12 db per octave at low frequencies.

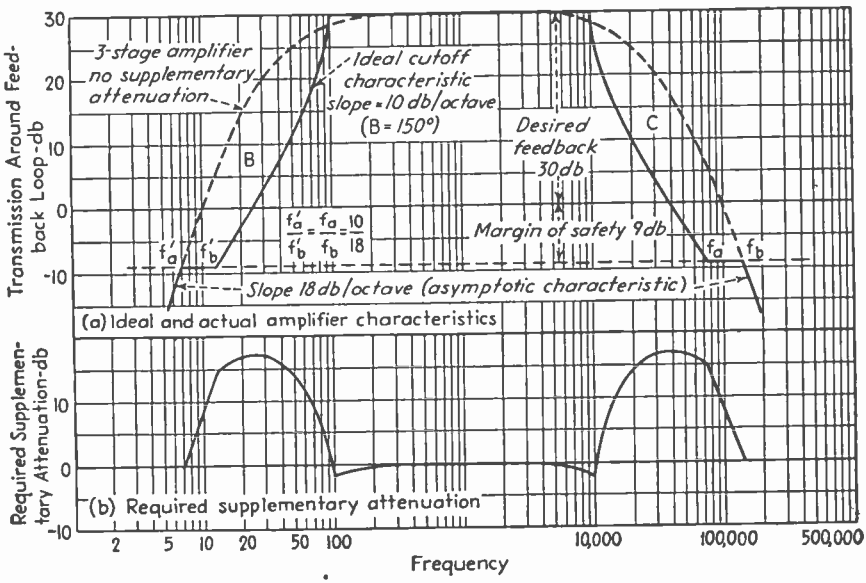


FIG. 91.—Example of requirements imposed on amplifier designed to provide 30-db feedback with 9-db margin of safety.

The asymptotic characteristic is of importance only when it has a value approaching or exceeding 12 db per octave. Thus, with many two-stage amplifiers, the asymptotic characteristic is not a design limitation, and one can theoretically hope to realize the ideal feedback loop characteristics shown in Fig. 87 instead of being forced to the step transition of Fig. 90.

Design Procedures and Considerations.—The design of a feedback amplifier under conditions where the asymptotic characteristic must be considered can be best understood by considering several specific cases.

Example 1.—Assume that a three-stage amplifier has a useful frequency range of 100 to 10,000 cycles and that 30-db feedback is desired with an amplitude margin of safety of 9 db, a phase margin of safety of 30°, and that the asymptotic slope is 18 db per octave for both high- and low-frequency cutoffs. The ideal transmission characteristic of the feedback loop can be calculated from Eqs. (126) and (127) for high and low frequencies, respectively. These characteristics are plotted in Fig. 91*a*, in which the transmission around the feedback loop in the useful frequency range is shown as being 30 db above zero level. The ideal characteristic is continued to 9 db below zero level to f_b' and f_b in order to provide the necessary amplitude margin, after which there is introduced a step having a length such that the frequencies at the two ends of the step are in the ratio 10:18 (or 18:10 as the case may be), which is the slope ratio of the ideal and asymptotic characteristics (slope of the ideal characteristic with 30° phase

Ex. 1044-0232

margin = $12 \times \frac{150}{180}$). Beyond the step, *i.e.*, below f_a' and above f_b, the asymptotic characteristic is permitted to take control, as shown. The actual amplifier would then be built so that in the absence of feedback the amplification would fall to 30 + 9 = 39 db below the amplification in the useful range at the frequencies f_a' and f_b. The frequencies f_a' and f_b for this case, when calculated as above on the basis of the ideal step characteristic, are 7 cycles and 150,000 cycles, respectively. In order to obtain the transmission actually required in order to avoid oscillation, *i.e.*, the characteristics indicated by the heavy line in Fig. 91*a*, then additional circuit elements must be added either to the amplifier or to the feedback loop in order to cause added loss in the regions marked *B* and *C* in Fig. 91. It will be noted that the amplifier must be made much broader than the actual amplification characteristic needed.

Example 2.—Consider a broadcast transmitter in which it is desired to employ 20 db feedback, with 10 db of safety margin in amplitude and a 30° phase margin of safety (k = 1.67), and is to maintain uniform response up to 10 kc. Also, assume that the transmission characteristic around the feedback loop, as observed experimentally, when no special circuit elements are introduced to control feedback characteristics, is as shown by *AHIJ* in Fig. 92*a*, with 24 db per octave asymptotic slope. The optimum transmission characteristic, as calculated by Eq. (126), is the curve *ABCDG*, but because of the asymp-

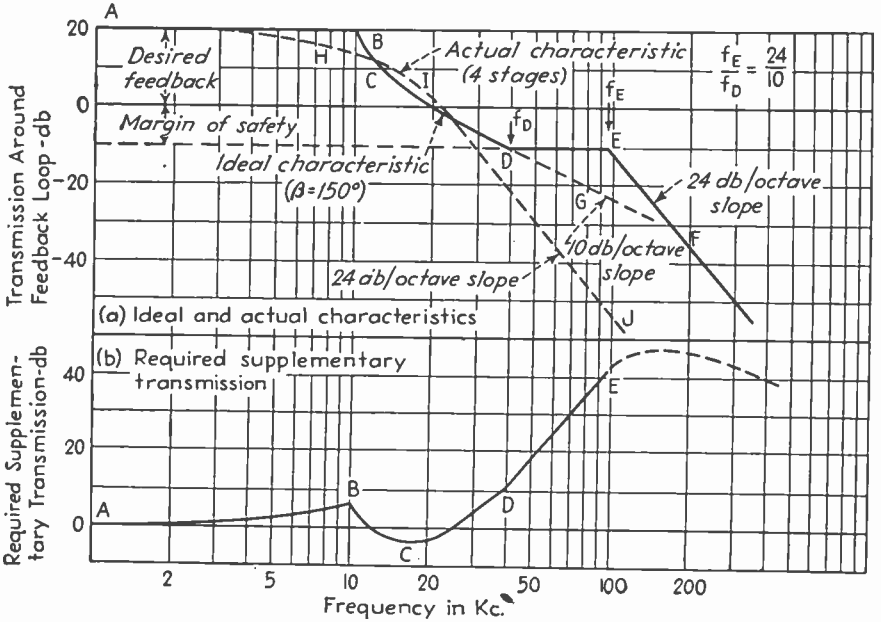


FIG. 92.—Example involving application of negative feedback to a broadcast transmitter.

totic slope near *G*, the practical curve required to give the required amplitude margin of 10 db is the step curve *ABCDEF*. When this is compared with the actual curve *AHIJ*, it is seen that in order to obtain the required characteristic, one must add circuit elements somewhere in the feedback loop that will add to the curve *AHIJ* a characteristic such as illustrated by the curve *ABCDE* of Fig. 92*b*. This can be done by suitable design of circuit elements in the feedback loop, or it may be achieved by adding further stages of amplification either in the feedback path or in the audio-frequency part of the system. These additional stages need contribute little or nothing to the gain of the system in the useful range of frequencies, but must have a rising amplification in the frequency range *DE*. Beyond the point *E*, the amplification of the extra stages can fall off in any convenient manner. If additional stages of amplification are employed to give this required supplementary rising characteristic, these additional stages must be designed so that their asymptotic characteristic does not take control until a frequency is reached that is very much higher than the frequency corresponding to *E* in Fig. 92.

Control of the way in which the transmission around the feedback loop falls off outside the useful range of frequencies can be obtained in a variety of ways. Some of the simpler and more important expedients that can be employed, together with the corresponding transmission characteristics, are illustrated in Fig. 93. The number of such networks that can be devised is of course without limit, and under some circumstances the networks involved may become very complicated.

Ex. 1044-0233

The frequency range over which the transmission characteristics of the feedback loop must be controlled is surprisingly high, and represents the price that is paid in order to obtain the benefits of negative feedback. The cost is approximately one octave for each 10 db of useful feedback plus about one or two octaves extra as a margin of safety and to take care of failure to realize exactly the optimum characteristic. Thus, if an amplifier with a useful frequency range of 60 to 15,000 cycles is to have 30 db of feedback, the characteristics of the feedback loop must be carefully controlled for at least four octaves beyond this range, or from about 4 to 240,000 cycles (compare with Fig. 91).

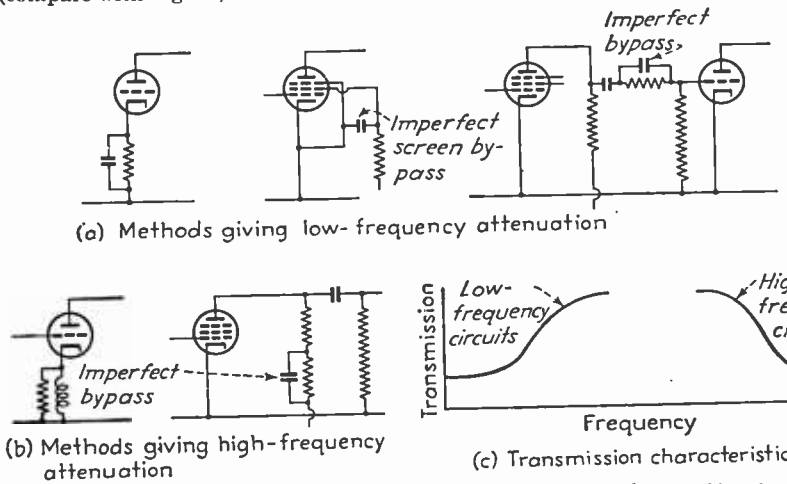


FIG. 93.—Simple methods of modifying amplifier characteristics at high and low frequencies.

28. *M*-derived (Ladder) Filters.[1]—A filter can be considered as a four-terminal network in which the image transfer constant has a value that is either small or zero in a particular range of frequencies and a value that is relatively large for other frequencies.

Fundamental Filter Equations.—Practical filters of the ladder type consist of symmetrical T or π sections composed of reactive elements. When the impedance arms are designated as in Fig. 94, then the image transfer constant θ and the image impedances Z_T and Z_π for the T and π sections are

$$\cosh \theta = 1 + \frac{Z_1}{2Z_2} \tag{129a}$$

or

$$\sinh \frac{\theta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \tag{129b}$$

$$Z_T = \sqrt{Z_1 Z_2 + \frac{1}{4} Z_1^2} \tag{130}$$

$$Z_\pi = \frac{Z_1 Z_2}{\sqrt{Z_1 Z_2 + \frac{1}{4} Z_1^2}} = \frac{Z_1 Z_2}{Z_T} \tag{131}$$

Half sections have an image transfer constant exactly half of the transfer constant of a

[1] Otto J. Zobel, Theory and Design of Uniform and Composite Electric Wave-Filters, *Bell System Tech. Jour.*, Vol. 2, p. 1, January, 1923.

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full section. The image impedances of the two sides of the half sections differ, being Z_T on one side and Z_π on the other (see Fig. 94).

An ideal filter in which the impedance arms are pure reactances with zero loss has zero attenuation (*i.e.*, the real part of θ is 0) for all frequencies that make $Z_1/4Z_2$ lie between 0 and -1. Such frequencies are termed *pass frequencies*, and the range in which they lie is termed the *pass band*. The phase constant β of the image transfer constant in the pass band is

$$\cos \beta = 1 + \frac{Z_1}{2Z_2} \tag{132}$$

The image impedance in the pass band is a pure resistance.

All frequencies other than those which make $Z_1/4Z_2$ lie between 0 and -1 suffer attenuation (*i.e.*, the real part α of θ is not zero). Such frequencies are termed *stop frequencies*, and are said to lie in the *stop band* of the filter. The phase shift in the stop

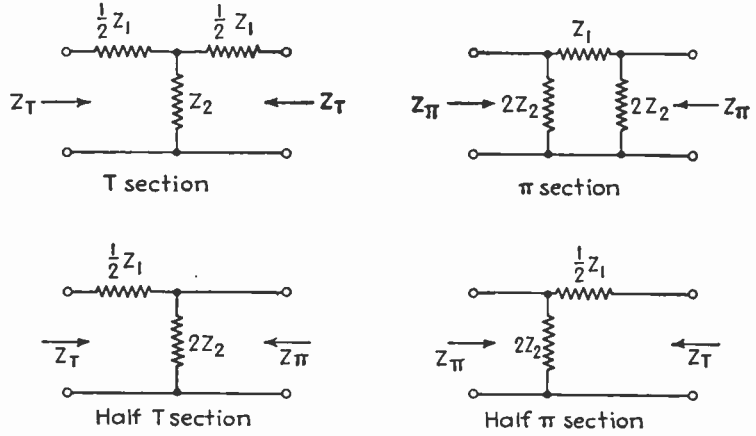


FIG. 94.—Filter sections, showing nomenclature.

bands is either zero or $\pm 180°$, and the image impedance is a pure reactance. The magnitude of the attenuation constant α for the stop frequencies is

$$\cosh \alpha = \left| 1 + \frac{Z_1}{2Z_2} \right| \tag{133}$$

Filter Design.—Practical ladder filters are built up in the manner illustrated in Fig. 95. The middle portion of the filter is composed of a series of symmetrical T or π sections connected in cascade, and at each end of the filter there is a half section as shown. All sections in the filter are matched together at each junction on an image-impedance basis.

The intermediate sections used in constructing a practical filter are of a class in which one can vary the attenuation characteristics of the section by suitably designing the series and shunt arms, without at the same time affecting in any way the image impedance. This makes it possible to arrange matters so that the frequencies for which one section gives only small attenuation are then strongly attenuated by some other section. Sections of this class, however, have an image impedance that is far from constant in the pass band of the filter. This means that a filter consisting only of such sections as might be used in the intermediate portion of a filter would not operate satisfactorily in association with a load resistance having a value inde-

Ex. 1044-0235

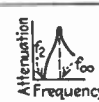
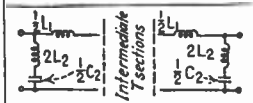
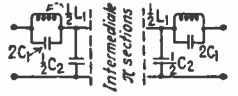
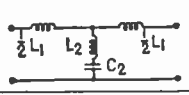
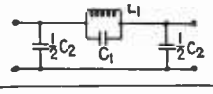
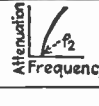
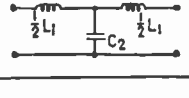
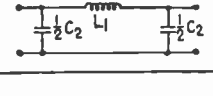
TABLE 2.—DESIGN OF LOW-PASS SECTIONS

Fundamental Relations

R = load resistance f_2 = cut-off frequency (highest frequency transmitted) f_∞ = a frequency of very high attenuation

$$L_k = \frac{R}{\pi f_2} \qquad C_k = \frac{1}{\pi f_2 R} \qquad m = \sqrt{1 - \left(\frac{f_2}{f_\infty}\right)^2}$$

Design of Sections

Type	Attenuation characteristic	A. Filters having T intermediate sections		B. Filters having π intermediate sections	
		Configuration	Formulas	Configuration	Formulas
End (m of approximately 0.6)			$L_1 = mL_k$ $L_2 = \frac{1 - m^2}{4m} L_k$ $C_2 = mC_k$		$L_1 = mL_k$ $C_1 = \frac{1 - m^2}{4m} C_k$ $C_2 = mC_k$
I			$L_1 = mL_k$ $L_2 = \frac{1 - m^2}{4m} L_k$ $C_2 = mC_k$		$L_1 = mL_k$ $C_1 = \frac{1 - m^2}{4m} C_k$ $C_2 = mC_k$
II ($f_\infty = \infty$)			$L_1 = L_k$ $C_1 = C_k$		$L_1 = L_k$ $C_2 = C_k$

Ex. 1044-0236

TABLE 3.—DESIGN OF HIGH-PASS SECTIONS

Fundamental Relations

R_1 = load resistance f_1 = cut-off frequency (lowest frequency transmitted) f_∞ = a frequency of very high attenuation

$$L_k = \frac{R}{4\pi f_1} \qquad C_k = \frac{1}{4\pi f_1 R} \qquad m = \sqrt{1 - \left(\frac{f_\infty}{f_1}\right)^2}$$

Design of Sections

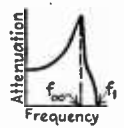
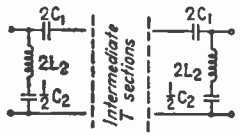
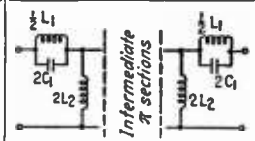
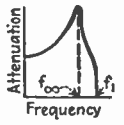
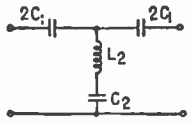
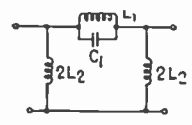
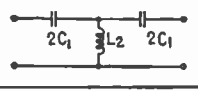
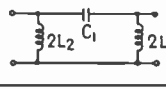
Type	Attenuation characteristic	*A.* Filters having T intermediate sections		*B.* Filters having π intermediate sections	
		Configuration	Formulas	Configuration	Formulas
End (m of approximately 0.6)			$C_1 = \frac{C_k}{m}$ $C_2 = \frac{4m}{1-m^2}C_k$ $L_2 = \frac{L_k}{m}$		$L_1 = \frac{4m}{1-m^2}L_k$ $C_1 = \frac{C_k}{m}$ $L_2 = \frac{L_k}{m}$
I			$C_1 = \frac{C_k}{m}$ $C_2 = \frac{4m}{1-m^2}C_k$ $L_2 = \frac{L_k}{m}$		$L_1 = \frac{4m}{1-m^2}L_k$ $C_1 = \frac{C_k}{m}$ $L_2 = \frac{L_k}{m}$
II $f_\infty = 0$			$C_1 = C_k$ $L_2 = L_k$		$C_1 = C_k$ $L_1 = L_k$

TABLE 4.—DESIGN OF BAND-PASS SECTIONS

Fundamental Relations

R = load resistance

f_1 = lower frequency limit of pass band

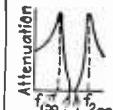
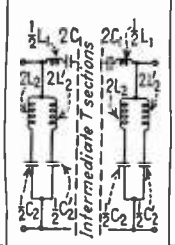
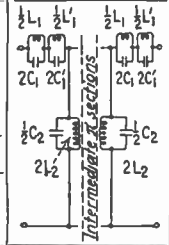
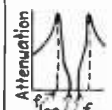
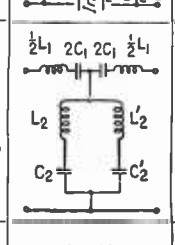
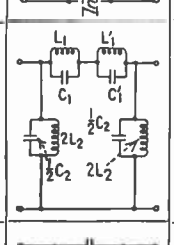
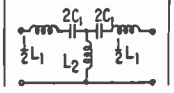
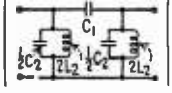
f_2 = higher frequency limit of pass band

$f_{1\infty}$ = a frequency of very high attenuation in low-frequency attenuating band

$f_{2\infty}$ = a frequency of very high attenuation in high-frequency attenuating band

$$L_{1k} = \frac{R}{\pi(f_2 - f_1)} \qquad L_{2k} = \frac{(f_2 - f_1)R}{4\pi f_1 f_2} \qquad C_{1k} = \frac{f_2 - f_1}{4\pi f_1 f_2 R} \qquad C_{2k} = \frac{1}{\pi(f_2 - f_1)R}$$

Design of Sections

Type	Attenuation characteristic	A. Filters having T intermediate sections: Configuration	A. Filters having T intermediate sections: Formulas	B. Filters having π intermediate sections: Configuration	B. Filters having π intermediate sections: Formulas	Notation for both T and π sections
End ($m_1 = m_2$ = approximately 0.6)			$L_1 = m_1 L_{1k}$ $L_2 = aL_{1k}$ $L_2' = cL_{1k}$ $C_1 = \frac{C_{1k}}{m_2}$ $C_2 = \frac{C_{1k}}{b}$ $C_2' = \frac{C_{1k}}{d}$		$L_1 = \frac{L_{2k}}{b}$ $L_1' = \frac{L_{2k}}{d}$ $L_2 = \frac{L_{2k}}{m_2}$ $C_1 = aC_{2k}$ $C_1' = cC_{2k}$ $C_2 = m_1 C_{2k}$	$g = \sqrt{\left(1 - \frac{f_1^2}{f_{1\infty}^2}\right)\left(1 - \frac{f_1^2}{f_{2\infty}^2}\right)}$ $h = \sqrt{\left(1 - \frac{f_1^2}{f_{2\infty}^2}\right)\left(1 - \frac{f_2^2}{f_{2\infty}^2}\right)}$ $m_1 = \frac{\frac{f_1 f_2}{f_{2\infty}^2} g + h}{1 - \frac{f_1^2}{f_{2\infty}^2}}$ $m_2 = \frac{g + \frac{f_1^2}{f_{1\infty}^2} h}{1 - \frac{f_1^2}{f_{2\infty}^2}}$ $a = \frac{(1-m_1^2)f_{2\infty}^2}{4gf_1f_2}\left(1 - \frac{f_{1\infty}^2}{f_{2\infty}^2}\right) = \frac{(1-m_2^2)f_1f_2}{4gf_{1\infty}^2}\left(1 - \frac{f_{1\infty}^2}{f_{2\infty}^2}\right)$ $b = \frac{(1-m_2^2)}{4g}\left(1 - \frac{f_{1\infty}^2}{f_{2\infty}^2}\right)$ $c = \frac{(1-m_1^2)}{4h}\left(1 - \frac{f_{1\infty}^2}{f_{2\infty}^2}\right)$ $d = \frac{(1-m_1^2)f_{2\infty}^2}{4hf_1f_2}\left(1 - \frac{f_{1\infty}^2}{f_{2\infty}^2}\right) = \frac{(1-m_2^2)f_1f_2}{4hf_{1\infty}^2}\left(1 - \frac{f_{1\infty}^2}{f_{2\infty}^2}\right)$ when $(m_1 = m_2)$, $g = h$, $a = d$, $b = c$, $f_{1\infty} = \frac{f_1 f_2}{f_{1\infty}}$, $m_1 = m_2 = \frac{h}{1 - \frac{f_1 f_2}{f_{2\infty}^2}}$ and $f_{2\infty}^2 = \frac{f_1^2 + f_2^2 - 2m^2 f_1 f_2}{2(1-m^2)} + \left[\left(\frac{f_1^2 + f_2^2 - 2m^2 f_1 f_2}{2(1-m^2)}\right)^2 - f_1^2 f_2^2\right]^{1/2}$ These formulas apply for both end and Type I sections
I			$L_1 = m_1 L_{1k}$ $L_2 = aL_{1k}$ $L_2' = cL_{1k}$ $C_1 = \frac{C_{1k}}{m_2}$ $C_2 = \frac{C_{1k}}{b}$ $C_2' = \frac{C_{1k}}{d}$		$L_1 = \frac{L_{2k}}{b}$ $L_1' = \frac{L_{2k}}{d}$ $L_2 = \frac{L_{2k}}{m_2}$ $C_1 = aC_{2k}$ $C_1' = cC_{2k}$ $C_2 = m_1 C_{2k}$	
II $f_{1\infty} = 0$ $f_{2\infty} = f_2$			$L_1 = \frac{f_1 R}{\pi f_2(f_2 - f_1)}$ $L_2 = \frac{(f_1 + f_2)R}{4\pi f_1 f_2}$ $C_1 = C_{1k}$		$C_1 = \frac{f_1 + f_2}{4\pi f_1 f_2 R}$ $C_2 = \frac{f_1}{\pi f_2(f_2 - f_1)R}$ $L_2 = L_{2k}$	

Ex. 1044-0238

III $f_{1\infty} = f_1$ $f_{2\infty} = \infty$	Attenuation f_1 f_2		$L_1 = L_{12}$ $\quad C_1' = \frac{\pi(f_1+f_2)R}{f_2-f_1}$ $C_1 = \frac{1}{4\pi f_1 R}$		$L_1' = \frac{R}{\pi(f_1+f_2)}$ $L_2 = \frac{(f_2-f_1)R}{4\pi f_1 f_2}$ $C_2 = C_{12}$	
IV $f_{1\infty} = 0$ $f_{2\infty} = \infty$	Attenuation $f_{1\infty}$ f_2		$L_1 = L_{12}$ $L_2 = L_{12}$ $C_1 = C_{12}$ $C_2 = C_{12}$		$L_1 = L_{12}$ $L_2 = L_{12}$ $C_1 = C_{12}$ $C_2 = C_{12}$	
V $f_{1\infty} = f_2$	Attenuation $f_{1\infty}$ f_1 f_2		$L_1 = m_1 L_{12}$ $L_2 = \frac{(1-m_1^2)L_{12}}{4m_1}$ $C_1 = \frac{C_{12}}{m_1}$ $C_2 = \frac{4m_1}{1-m_1^2}C_{12}$ See notation for m_1 and m_2		$L_1 = \frac{4m_2}{1-m_2^2}L_{12}$ $L_2 = \frac{L_{12}}{m_2}$ $C_1 = \frac{(1-m_2^2)}{4m_2}C_{12}$ $C_2 = m_2 C_{12}$ See notation for m_1 and m_2	$m_1 = \frac{f_1}{f_2} m_2 \qquad m_2 = \sqrt{\frac{1-\frac{f_1^2}{f_{2\infty}^2}}{1-\frac{f_1^2}{f_2^2}}}$
VI $f_{1\infty} = f_1$	Attenuation $f_{2\infty}$ f_1 f_2	Same circuit as above for Type V	Same formulas as above for Type V. See notation for m_1 and m_2	Same circuit as above for Type V	Same formulas as above for Type V See notation for m_1 and m_2	$m_1 = \sqrt{\frac{1-\frac{f_2^2}{f_{1\infty}^2}}{1-\frac{f_2^2}{f_1^2}}} \qquad m_2 = \frac{f_1}{f_2} m_1$
VII $f_{1\infty} = 0$	Attenuation $f_{2\infty}$ f_1 f_2 frequency		$L_1 = m_1 L_{12}$ $L_2 = a L_{12}$ $L_2' = \frac{(1-m_1^2)}{4h}L_{12}$ $C_1 = C_{12}\ C_2' = \frac{h}{a}C_{12}$		$L_1' = \frac{h}{a}L_{12}\ \ L_2 = L_{12}$ $C_1 = aC_{12}$ $C_2 = m_1 C_{12}$ $C_1' = \frac{(1-m_1^2)}{4h}C_{12}$	$h = \sqrt{\left(1-\frac{f_1^2}{f_{2\infty}^2}\right)\left(1-\frac{f_1^2}{f_{1\infty}^2}\right)} \qquad m_1 = \frac{f_1 f_2}{f_{1\infty}^2} + h$ $a = \frac{(1-m_1^2)f_{1\infty}^2}{4f_1 f_2}$
VIII $f_{2\infty} = \infty$	Attenuation $f_{2\infty}$ f_1 f_2 frequency		$L_1 = L_{12}\ \ L_2 = \frac{d}{g}L_{12}$ $C_1 = \frac{C_{12}}{m_2}$ $C_2 = \frac{4g}{1-m_2^2}C_{12}$ $C_2' = \frac{C_{12}}{d}$		$L_1 = \frac{4g}{1-m_2^2}L_{12}$ $L_2 = \frac{L_{12}}{m_2}\ \ L_1 = \frac{L_{12}}{d}$ $C_1 = \frac{d}{g}C_{12}\ \ C_2 = C_{12}$	$g = \sqrt{\left(1-\frac{f_2^2}{f_1^2}\right)\left(1-\frac{f_2^2}{f_{2\infty}^2}\right)} \qquad m_2 = g + \frac{f_{1\infty}^2}{f_1 f_2}$ $d = \frac{(1-m_2^2)f_1 f_2}{4f_{1\infty}^2}$

Ex. 1044-0239

pendent of frequency. This difficulty is overcome by employing suitably designed terminal half sections, as shown in Fig. 95, that match the intermediate portion of the filter on an image-impedance basis on one side, but offer a more desirable type of image impedance at their external terminals.

Sections suitable for use in low-pass, high-pass, and band-pass filters, together with the necessary design formulas, are given in Tables 2, 3, and 4. All the sections illustrated in these tables are suitable for use in the intermediate portion of the filter except those sections specifically designated as end sections. All intermediate sections designed from one of these tables for given cutoff frequencies, and for the same load resistance, will have identical image-impedance characteristics, and so can be connected in cascades on an image-impedance basis. The image transfer constant, *i.e.*, the attenuation and phase-shift characteristics, of the individual sections will, however, depend upon the type of section involved and upon the value assigned to the design parameter m (or m_1 and m_2).

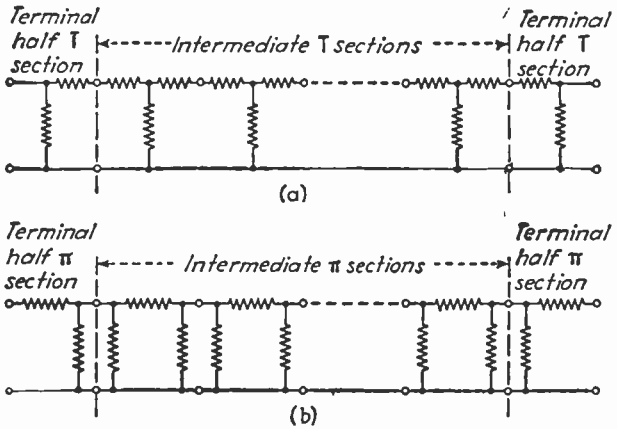


FIG. 95.—Method of building up an m-derived filter.

The terminal half sections are designed according to the formulas given in the respective tables, with the design parameter m assigned a value of approximately 0.6. When the terminal half sections are designed by the tables for the same cutoff frequencies and same load resistance as the intermediate sections, their image impedances will match the intermediate sections at one pair of terminals. Also, when the appropriate value of m is used in the design, the image impedance at the other terminals will be substantially constant over almost the entire pass band of the filter. This is illustrated in Fig. 96, where the effect of using various values of m is shown. Examples of actual filters designed according to Tables 2, 3, and 4 are given in Figs. 97, 98, and 99.

The procedure involved in the design of a filter to meet a given set of conditions falls into several well-defined steps, as follows:

1. Determine the cutoff frequencies, *i.e.*, the frequencies that mark the edge of the pass band, and the load resistance that will meet the design requirements, and decide whether T or π intermediate sections are to be used.

2. Design the terminating sections according to the proper table, using the load resistance and cutoff frequencies selected in step 1.

3. Decide on the number of intermediate sections to be used. The more sections selected the greater will be the attenuation that can be obtained in the stop band.

Ex. 1044-0240

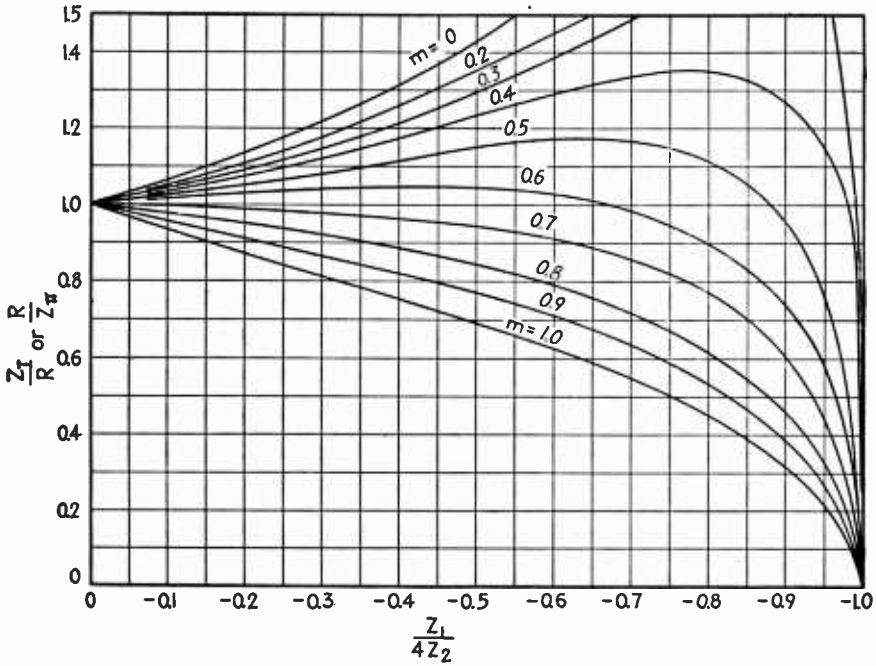
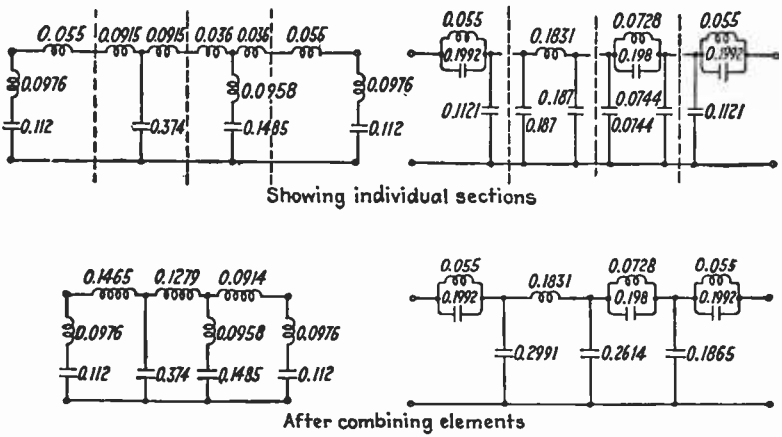


FIG. 96.—Effect of design parameter m on image-impedance characteristics in the pass band.



(a) Intermediate sections of "T" type (b) Intermediate section of "π" type

FIG. 97.—Example of a low-pass filter having a cut-off frequency of 1213 cycles, two intermediate sections, and proportioned to operate with a load resistance of 700 ohms.

Ex. 1044-0241

One intermediate section will give a moderately good filter, and will be sufficient for many requirements. More than two intermediate sections are seldom required.

4. Select the frequencies at which the different intermediate sections will have their maximum attenuation, and then design these sections for the chosen load resistance and cutoff frequencies, using formulas in the appropriate tables.

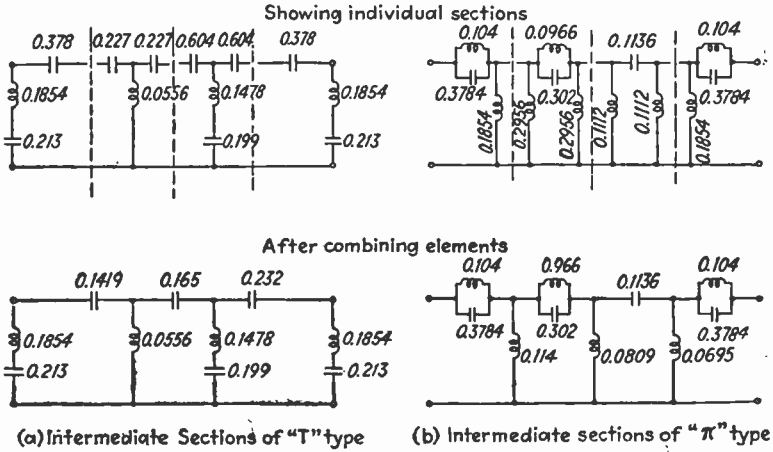


FIG. 98.—Example of a high-pass filter having a cutoff frequency of 1,000 cycles, two intermediate sections, and proportioned to operate with a load resistance of 700 ohms.

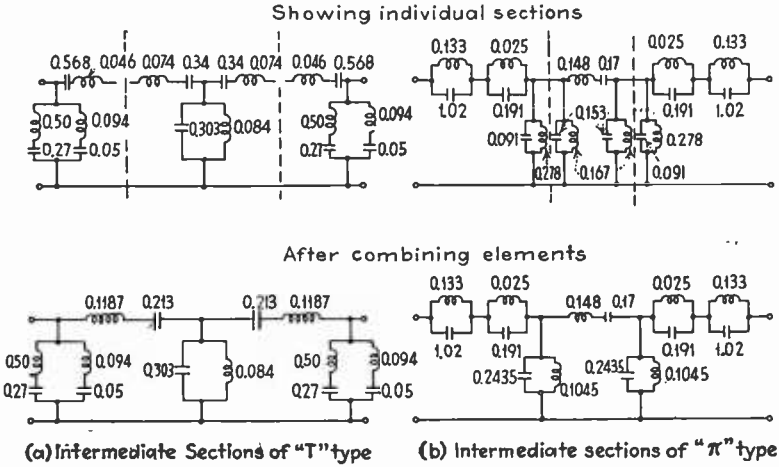


FIG. 99.—Example of a band-pass filter having cutoff frequencies of 500 and 2,000 cycles, one intermediate section, and proportioned to operate with a load resistance of 700 ohms.

The choice between T and π sections for the middle portion of the filter is primarily based upon considerations of convenience. Electrically, the performance of the two types will be identical.

The location of the frequencies for which the intermediate sections have high attenuation must be carefully selected, for upon this choice rest the attenuating properties of the filter. These frequencies of high attenuation should in general be

Ex. 1044-0242

different for the different sections, and should be so staggered throughout the attenuating bands that every frequency suffers considerable attenuation by at least one section. Inasmuch as the terminal half sections determine one frequency of high attenuation in each attenuating band, these sections should be designed before the intermediate sections in order that the attenuation of the central portion of the filter may best supplement that of the end sections.

It is sometimes desirable to obtain a very rapid rise of attenuation at the edge of the pass band of the filter. Although it is theoretically possible in the ideal case of perfect reactances to obtain a frequency of high attenuation as close to the cutoff frequency as desired, practical considerations require that such frequencies of high attenuation be not too close to the cutoff frequencies. There are two reasons for this. In the first place, as the frequency of high attenuation is moved closer and closer to the cutoff frequency, the attenuating characteristics become narrower, as shown in Fig. 100. The result is that a section designed to attenuate frequencies extremely close to cutoff will be of little use in attenuating frequencies appreciably different

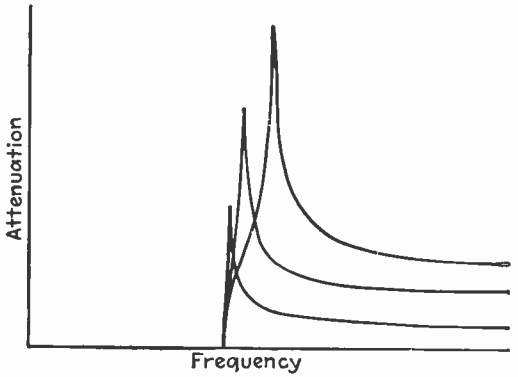


FIG. 100.—Attenuation characteristics obtained in a low-pass filter section as the frequency of high attenuation is placed closer and closer to cutoff.

from cutoff. In the second place, the reactance arms required in a filter having a frequency of high attenuation very close to cutoff assume impractical proportions, and if these impractical circuit elements are built, it will be found that their losses prevent the high attenuation that is theoretically possible from being actually realized. This is illustrated in Fig. 100. In the case of ordinary coils with reasonably high Q, the frequencies of infinite attenuation should differ from cutoff by not less than two to five per cent. Where quartz crystals are used as filter elements, the higher Q of such resonators permits this difference to be reduced greatly.

Filters with MM' and Fractional Terminations, Filters in Series and Parallel.—Filters such as are described above, employing terminating half sections designed for $m \cong 0.6$, provide an image-impedance characteristic for the external terminals of the filter that is sufficiently constant over the pass band to meet most requirements. Where a closer approximation to a constant resistance image impedance is required, more elaborate terminating sections can be employed. The next steps in complexity beyond the simple sections already discussed, are given in Fig. 101 and are termed MM' terminations.[1] These arrangements give a decided improvement in image impedance at the external terminals. Thus, in the case of a low-pass filter, the usual $m = 0.6$ termination gives an image impedance constant to within 4 per cent over

[1] Otto J. Zobel, Extensions to the Theory and Design of Electric Wave-filters, *Bell System Tech. Jour.*, Vol. 10, p. 284, April, 1931.

Ex. 1044-0243

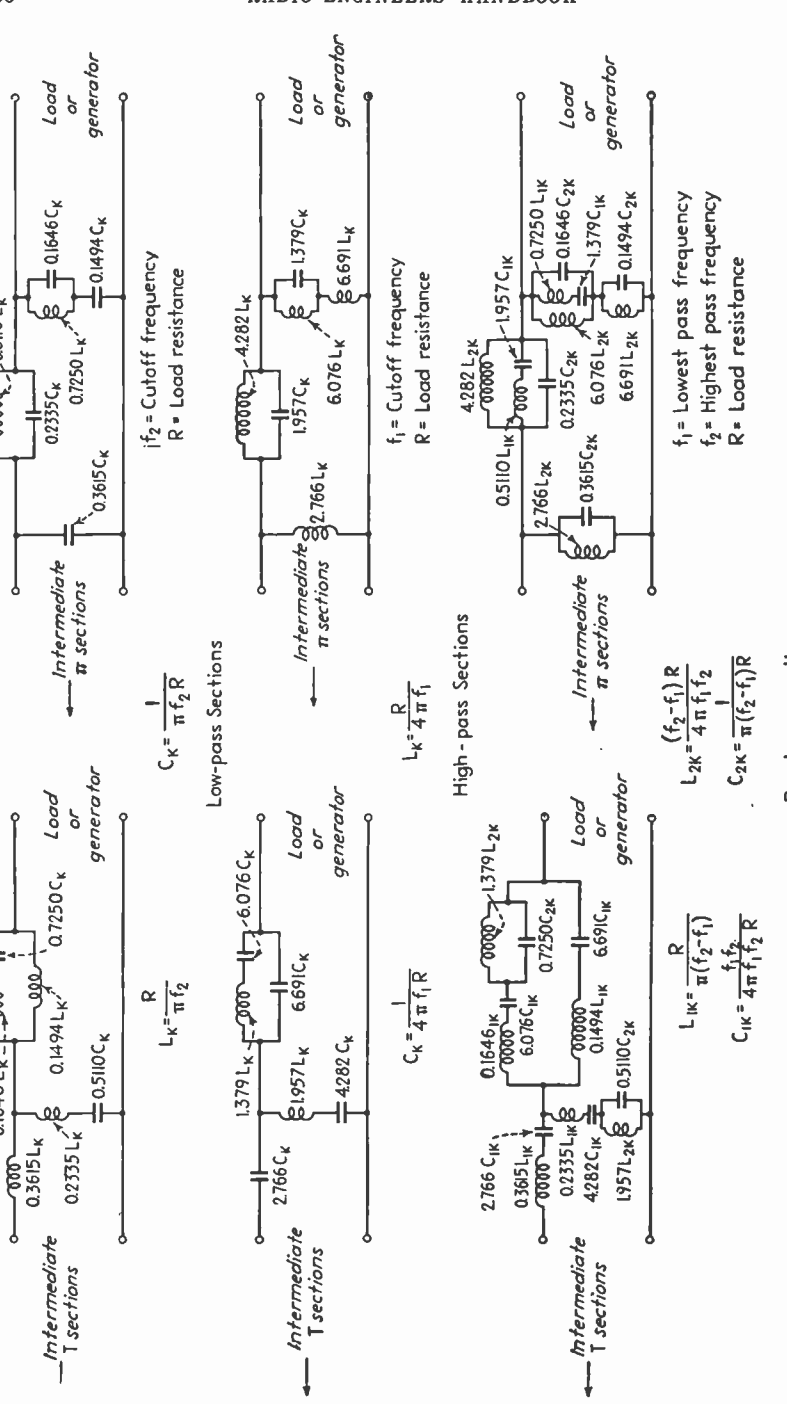


FIG. 101.—Termination sections of *M M'* type.

Ex. 1044-0244

90 per cent of the pass band, and the MM' termination gives an image impedance constant to within 2 per cent over 96 per cent of the pass band.

The MM' terminating section is characterized by having two frequencies of infinite attenuation in each stop band, as shown in Fig. 102.

It is sometimes necessary to parallel the input or the output terminals of a number of filters. In such an arrangement, each filter shunts its own image impedance across the common terminals. If one then considers the situation at a particular frequency, it is apparent that the filters that have a pass band for this frequency offer a resistance impedance, whereas the remaining filters, *i.e.*, those for which this frequency is in the stop band, provide a reactive shunting impedance that varies with frequency. This modifies the impedance presented by the combination of filters in parallel, and affects the insertion loss in an unfavorable manner.

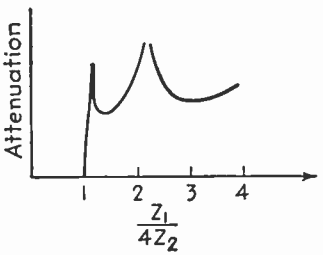


FIG. 102.—Attenuation characteristic of MM' terminating section.

This difficulty can be handled practically by employing filters in which the external terminals are shunted by a reactive arm, *i.e.*, by using a filter having T intermediate sections. The shunting reactances of all the filters in parallel then can be thought of as forming a single shunting reactance, which can then be modified as required to make the equivalent impedance seen when one is looking toward the paralleled terminals approximate a resistance in the pass band of all filters. This requires, in some instances, merely that the magnitude of the combined shunting impedance be modified in size; in other cases, an additional shunting reactive network must be placed across the common terminals.[1]

A particularly important case of filters in parallel is furnished by complementary filters, *i.e.*, where those frequencies that lie in the stop band of one filter are the pass

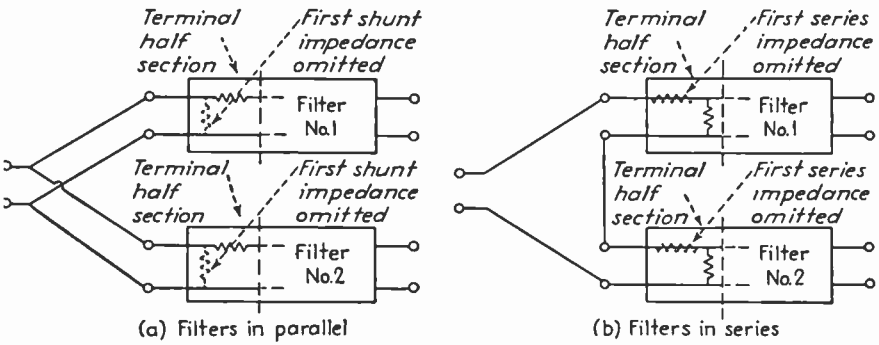


FIG. 103.—Schematic illustration of complementary filters in parallel and series (x terminations).

frequencies of the other filter. In such an arrangement, it is found that if the terminating half sections are originally both designed on the basis $m = 0.6$ and the same load resistance, then the reactive input impedance of one filter in its stop band is almost exactly equal to the reactive impedance that should be provided by the first shunting impedance of the complementary filter in its pass band, and vice versa. The result is that, if the shunting impedances at the inputs of both filters are simply

[1] Further discussion is given by Guillemin, *op. cit.*, pp. 356–366. See also O. Zobel, U.S. Patents Nos. 1,557,229 and 1,557,230.

Ex. 1044-0245

omitted, then the normal impedance relations will be maintained at all frequencies. Such an arrangement is termed a *fractional* or *x termination*, and is illustrated in Fig. 103*a*.

Filters that are connected in series at one side, such as the input, present a problem analogous to that of filters in parallel. Here each filter in its stop band places a reactance in series with the other filter or filters. The proper method of attack here consists in employing filters that have reactances in series with the external terminals *i.e.*, filters with π intermediate sections. These series reactances are then lumped together to form a single series impedance, which is then modified as required to take into account the reactances contributed by the filters in the stop bands. In the case of complementary filters in series, one designs the terminating half sections for $m = 0.6$. The series elements of the sections are then omitted, as shown in Fig. 103*b*, and the input reactance of the filter in the stop band is used to supply the series reactance required by the complementary filter in its pass band, and vice versa.

Losses.—The ideal filter with pure reactive elements discussed above can never be fully realized in practice, for although condensers can be made with negligible losses, this is never true with inductances. Experience shows, however, that when the Q of the coils is of the order of 15 or higher, the effect of the losses is only secondary and the essential conclusions and design procedures based on an ideal filter with zero losses are not seriously invalidated.

The principal effects of a moderate amount of energy loss in a filter are:

1. A small attenuation is introduced in the pass band.

2. Frequencies for which infinite attenuation would otherwise be obtained are found to have only finite attenuation.[1]

3. The abrupt transitions occurring at the cut-off frequencies in an ideal filter are rounded off by the losses.

29. Lattice Filters.[2]—*Fundamental Relations.*—A lattice will act as a filter when the impedance arms are suitably designed reactances. The lattice structure provides the most general type of symmetrical filter section that can be devised, and includes T and π sections (including m-derived types) as special cases.

The image impedance Z_I and image transfer constant θ of a lattice are

$$Z_I = \sqrt{Z_a Z_b} \tag{134}$$

$$\tanh\left(\frac{\theta}{2}\right) = \sqrt{\frac{Z_a}{Z_b}} \tag{135}$$

Here Z_a and Z_b are the lattice impedances, as shown in Fig. 73. Since the image impedance of a lattice depends upon the product of the two lattice impedances, whereas the image transfer constant depends upon the quotient of these impedances, *it is possible to specify the image impedance and the image transfer constant of a lattice filter independently of each other.*

A pass band is obtained in a lattice network whenever the two reactances Z_a and Z_b have opposite signs. Under these conditions the attenuation constant α is zero, the image impedance is a resistance, and the phase shift β is

$$\tan\frac{\beta}{2} = -j\sqrt{\frac{Z_a}{Z_b}} \tag{136}$$

[1] It is, however, possible in many cases to obtain infinite attenuation by addition of a resistance to the network in such a manner as to cancel out the residual transmission at this particular frequency. See Vernon D. Landon, M-derived Band-pass Filters with Resistance Cancellation, *R.C.A. Rev.*, Vol. 1, p. 93, October, 1936.

[2] H. W. Bode, A General Theory of Electric Wave Filters, *Jour. Math. & Phys.*, Vol. 13, p. 275, November, 1934; *Monograph* B-843 of Bell Telephone System; H. W. Bode and R. L. Dietzold, Ideal Wave Filters, *Bell System Tech. Jour.*, Vol. 14, p. 215, April, 1935.

Ex. 1044-0246

The lattice filter has a stop band whenever the two reactances Z_a and Z_b have the same sign. Under these conditions, the image impedance is a pure reactance, the phase shift is either zero or π radians, and the attenuation constant α is

For $Z_b > Z_a$ ($\beta = 0$):

$$\tanh\left(\frac{\alpha}{2}\right) = \sqrt{\frac{Z_a}{Z_b}}$$

For $Z_b < Z_a$ ($\beta = \pi$): (137)

$$\tanh\left(\frac{\alpha}{2}\right) = \sqrt{\frac{Z_b}{Z_a}}$$

The pass bands, cutoff frequencies, and stop bands of a lattice filter can be given the desired locations by properly arranging the zeros and poles of the lattice impedances Z_a and Z_b. The zeros and poles possessed by the impedance Z_a that lie within the pass band must coincide with poles and zeros, respectively, of impedance Z_b. At each cutoff frequency, one of the lattice impedances must contain either a pole or a zero that is not matched by a corresponding critical frequency in the other impedance. In the stop or attenuating band, zeros and poles of one of the lattice impedances must coincide, respectively, with the zeros and poles of the other lattice impedance. When these requirements are met, the impedances Z_a and Z_b will be of opposite sign throughout the pass band, but will have the same sign in the desired stop band. Arrangements of poles and zeros for several typical cases are shown in Fig. 104.

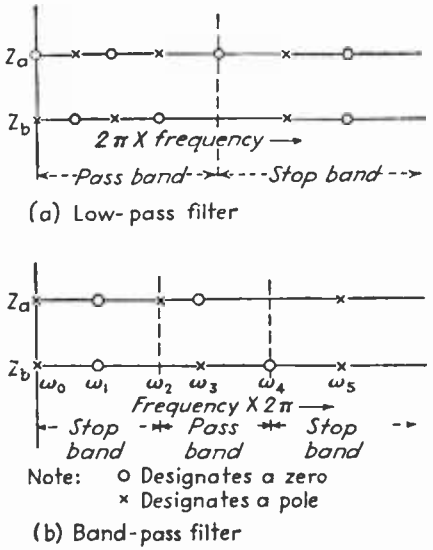


FIG. 104.—Examples of arrangements of zeros and poles in lattice filters.

It is apparent that there is a great variety of possibilities for any particular type of filter. Thus the number of critical frequencies in the pass and stop bands may be varied, as well as the position of these critical frequencies in relationship to cutoff. Likewise, the cutoff frequencies may be delineated by zeros or poles in impedance Z_a or by corresponding critical frequencies in Z_b.

Formulas for the image impedance and image transfer constant of a lattice filter can be readily derived in any particular case by substituting for the lattice impedances Z_a and Z_b that appear in Eqs. (134) and (135) the corresponding expressions for these impedances in terms of zeros and poles given by Eqs. (86*a*) and (86*b*). When such expressions are derived, it will be found that the image impedance will depend *only upon the number and location of the critical frequencies that are present in the stop band of the filter* and upon the cutoff frequencies, while the image transfer constant, *i.e.*, the phase shift in the pass band and the attenuation in the stop band, will depend *only upon the number and location of the critical frequencies that appear in the lattice impedances in the pass band* and upon the cutoff frequencies.

The procedure for setting up formulas for image transfer constant and image impedance in a particular case can be illustrated by considering Fig. 104*b*. Imped-

Ex. 1044-0247

ances of the lattice, by Eqs. (86*a*) and (86*b*), are

$$Z_a = -j\frac{H_a}{\omega}\frac{(\omega^2-\omega_1^2)(\omega^2-\omega_3^2)}{(\omega^2-\omega_2^2)(\omega^2-\omega_5^2)} \tag{138}$$

$$Z_b = -j\frac{H_b}{\omega}\frac{(\omega^2-\omega_1^2)(\omega^2-\omega_4^2)}{(\omega^2-\omega_3^2)(\omega^2-\omega_5^2)} \tag{139}$$

Substituting these values for Z_a and Z_b into Eqs. (134) and (135) gives

$$Z_I = -\sqrt{H_aH_b}\frac{(\omega^2-\omega_1^2)}{\omega(\omega^2-\omega_5^2)}\sqrt{-\frac{(\omega^2-\omega_4^2)}{(\omega^2-\omega_2^2)}} \tag{140}$$

$$\tanh\frac{\theta}{2} = \sqrt{\frac{H_a}{H_b}}(\omega^2-\omega_3^2)\frac{1}{\sqrt{(\omega^2-\omega_2^2)(\omega^2-\omega_4^2)}}$$

In these equations, H_a and H_b represent the values of the constant H in Eq. (86) for the impedances Z_a and Z_b, respectively.

If the lattice impedances Z_a and Z_b are interchanged, the only effect upon the behavior of the lattice network is to reverse the polarity of the output voltage, *i.e.*, to introduce a phase shift of π radians. If *either* one of the lattice impedances (but not both) is replaced by its reciprocal impedance with respect to a constant value R, the effect is to interchange the pass and stop bands, *i.e.*, the new filter is complementary to the original filter.

Design of Lattice Filters.—The transmission characteristics of a lattice filter, which are the phase shift in the pass band and the attenuation in the stop band, are determined by the number and location of the critical frequencies within the pass band and by the ratio H_a/H_b. By employing a sufficient number of critical frequencies and properly disposing them within the pass band, it is possible to realize almost any desired phase and attenuation characteristics. Similarly, the design factors controlling the image impedance of the filter are the critical frequencies of the lattice impedances that lie in the stop band and the value of the product $\sqrt{H_aH_b}$. As before, by employing a sufficient number of such critical frequencies and properly distributing them, it is possible to realize almost any desired image-impedance characteristic.

An ideal filter would have constant image impedance throughout the pass band, a phase shift proportional to frequency within the pass band, and a very high attenuation in the stop band. It is accordingly customary to distribute the critical frequencies within the stop band in such a manner as to maintain the image impedance as nearly constant as possible over most of the pass band. In distributing the critical frequencies within the pass band, one may either give maximum weight to linearity of the phase characteristic or place the greatest emphasis upon obtaining the highest possible attenuation within the stop band. Designs that give the most linear phase-shift characteristics with a given number of critical frequencies within the pass band do not have such desirable attenuation characteristics as can be obtained with the same number of critical frequencies and some sacrifice in linearity of the phase curve.

The attenuation of a lattice filter will be infinite whenever $\sqrt{Z_a/Z_b} = 1$, and will be smaller the farther the departure of $\sqrt{Z_a/Z_b}$ from unity. Accordingly, the attenuation characteristics for a given number of critical frequencies will approach most closely the ideal when the critical frequencies within the pass band are so distributed that the value of $\sqrt{Z_a/Z_b}$ oscillates with equal amplitudes above and below the value unity, as shown in Fig. 105*a*. A systematic procedure for selecting critical frequencies to accomplish this result has been developed by Cauer and is described in some detail by Guillemin.[1] When the design parameters are selected in the Cauer

[1] See Guillemin, *op. cit.*, pp. 394–412.

Ex. 1044-0248

manner to provide the best attenuation characteristic, the phase characteristic is far from linear.

The image impedance of a lattice filter will be constant to the extent that the quantity $\sqrt{Z_a Z_b}/R_L$ approximates unity, where R_L is the load resistance. This ideal is most closely realized for a given number of critical frequencies when $\sqrt{Z_a Z_b}/R_L$ oscillates about the value unity with equal positive and negative deviations, as shown

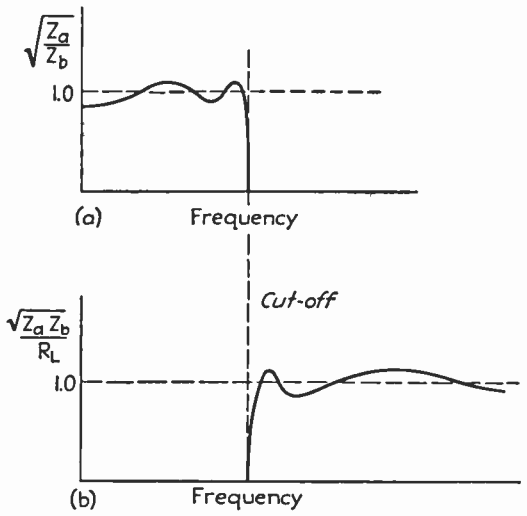


FIG. 105.—Variation of design factors $\sqrt{Z_a/Z_b}$ and $\sqrt{Z_a Z_b}/R_L$ corresponding to optimum attenuation, and optimum characteristic impedance characteristics.

in Fig. 105*b*. The procedure for locating the critical frequencies within the stop band in order to accomplish this is similar in every respect to the procedure for locating the critical frequencies in the pass band to provide an oscillatory approximation to the ideal attenuation characteristic. The detailed steps are given by Cauer and Guillemin.

A phase-shift characteristic that is substantially linear over most of the pass band, combined with an attenuation characteristic that rises rapidly beyond cutoff, can be obtained by locating the critical frequencies within the pass band according to the

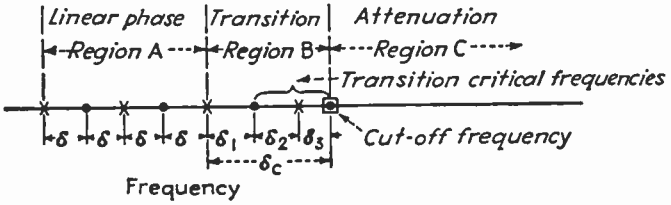


FIG. 106.—Distribution of critical frequencies in a Bode-type lattice filter.

manner devised by Bode,[1] and illustrated in Fig. 106. Here the pass band is divided into two parts, region A and region B. Region A, which comprises most of the pass band, is to have a substantially linear phase-shift characteristic; region B is a transition region in which the phase shift is not linear. In region A, critical frequencies are located at a uniform frequency interval δ as shown. Region B is provided with at

[1] Bode and Dietzold, *loc. cit.*

Ex. 1044-0249

TABLE 5.—SPACING INTERVALS*

Number of transition frequencies	δ_1/δ	δ_2/δ	δ_3/δ	δ_4/δ	δ_5/δ	δ_c/δ
1	0.500	.....	.....	.....	.....	0.500
2	0.853	0.354	.....	.....	.....	1.207
3	0.950	0.727	0.278	.....	.....	1.955
4	0.981	0.880	0.630	0.231	.....	2.723
5	0.992	0.946	0.810	0.555	0.197	3.501

* See Fig. 106.

least one critical frequency, corresponding to cutoff, and often two or more critical frequencies. In order to obtain the most favorable phase characteristic in region *A*, these critical frequencies in the transition region *B* must be located in accordance with Table 5. When this is done, the phase characteristic will be almost perfectly linear in region *A*, provided that there are not too few critical frequencies in region *A*. The associated attenuation characteristic is as shown in Fig. 107, and rises with a rapidity that increases as more transition factors are employed in region *B*. The attenuation in a Bode design never goes to infinity, although it quickly becomes quite large.

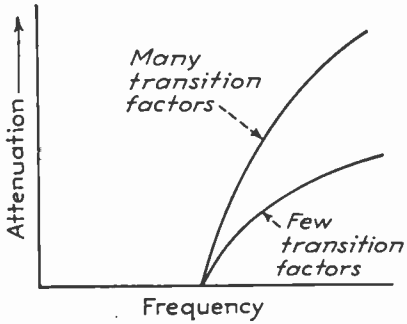
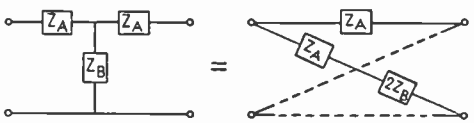


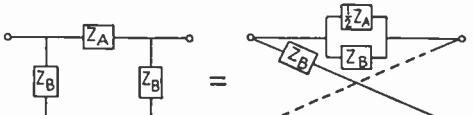
FIG. 107.—Attenuation characteristics of Bode-type lattice filter.

Relation between Ladder and Lattice Structures.—Any symmetrical T or π section can be converted to an equivalent lattice section by so selecting the lattice impedances that the open- and short-circuit impedances of the lattice are the same as for the T (or π) arrangement. The transformations necessary to accomplish this are illustrated in Fig. 108.

The reverse transformation, *i.e.*, the representation of a lattice network by a symmetrical T (or a symmetrical π) will not necessarily lead to a physically realizable T or π. Examination of Fig. 108 shows that the conversion to a symmetrical T will result in a realizable structure only when it is possible to subtract the series arm Z_a of the lattice from the lattice diagonal and have a physically realizable remainder. Similarly, the conversion from a lattice to a symmetrical π will result in a physical network only when it is physically possible to subtract the admittance $1/Z_b$ of the diagonal lattice impedance from the admittance of the series impedance arm and have a physically realizable remainder.



(a) Equivalence of lattice and T networks

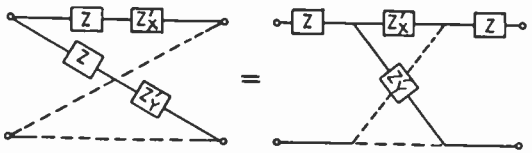


(b) Equivalence of lattice and Π networks

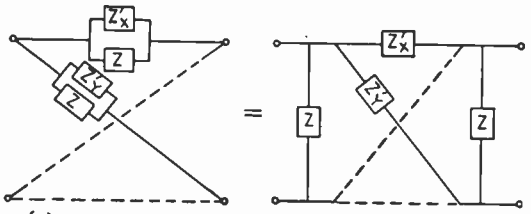
FIG. 108.—Equivalence of lattice, and T and π networks.

Ex. 1044-0250

A series impedance that is common to both arms of a lattice can be removed from the lattice and placed in series with the external terminals, as shown in Fig. 109*a*. The resulting combination of series impedance and modified lattice has exactly the same properties as the original lattice. Similarly, an impedance that is in shunt with both arms of a lattice may be removed from the lattice and placed in shunt across the external terminals, as shown in Fig. 109*b*. The validity of these transformations is



(a) Lattice structure with developed series impedances



(b) Lattice structure with developed shunt impedances

FIG. 109.—Development of lattice network into lattice plus series (or shunt) impedances.

based on the fact that the open- and short-circuit impedances possessed by the modified lattice and its external series (or shunt) impedances are exactly the same as the corresponding open- and short-circuit impedances of the original lattices.

Successive applications of these transformations can be used to transform a lattice having complicated impedance arms into a ladder network of alternate series and shunt impedances in association with a simpler lattice. In some cases, the

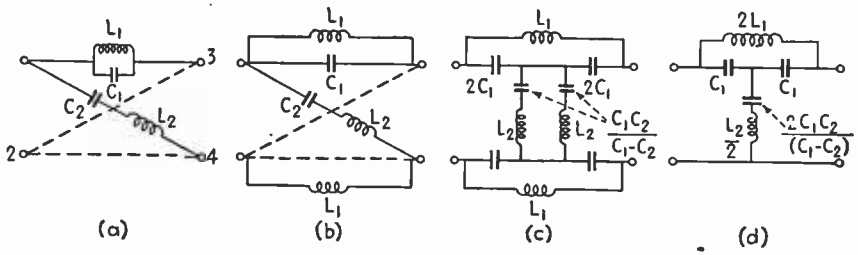


FIG. 110.—Development of a lattice which can be transformed into a bridged-T network.

residual lattice degenerates into a pair of shunt arms in parallel, or into two series arms, in which case the conversion from the original lattice to a ladder network is complete.

A lattice that cannot be converted to a ladder network can sometimes still be transformed into a bridged-*T* section. This is illustrated in Fig. 110, where by considering inductances L_1 as bridging the terminals 1-3 and 2-4, the lattice reduces to

Ex. 1044-0251

arms C_1 and C_2L_2, and when $C_1 > C_2$ can be developed into a ladder network, shown at Fig. 110*d*, that, when bridged by the inductance $2L_1$, is equivalent to the original lattice.

Comparison of Lattice and Ladder Filters.—The lattice structure provides the most general form of symmetrical section that it is possible to realize, and so provides the greatest possible choice of characteristics that can be obtained in a filter. At the same time, the lattice generally requires more coils and condensers than does the corresponding *m*-derived type of filter, and does not provide an input and output terminal at a common ground potential unless the lattice can be fully developed into a ladder or bridged-*T* network. It is found that for most communications purposes calling for filters, the *m*-derived structure is quite adequate.

30. Equalizers.—An equalizer is a network placed between the generator and load such that the current that the generator produces in the load will vary in some desired manner with frequency. An *attenuation* equalizer is an equalizer that is inserted in order to control the magnitude of the load current as a function of frequency, without any particular regard to phase relations. A *phase* equalizer is an all-pass filter designed to introduce a desired phase shift as a function of frequency in the load current.

For most purposes it is sufficient to equalize only for attenuation. If, however, circumstances require phase as well as attenuation equalization, the procedure followed is first to equalize for attenuation and then afterward to add a phase equalizer to the system to correct for any undesired features in the phase characteristic of the original system plus the attenuation equalizer.

Attenuation Equalizers.[1]—The networks commonly used as attenuation equalizers are shown in Fig. 111, which also gives the insertion loss of these networks when the load impedance R_L has the design value $R_L = R_0$.

These networks are conveniently divided into classes as indicated. The simple series and shunt equalizing networks designated as Type I find some use because of their simplicity, but have the very serious disadvantage of causing the impedance seen by the generator in looking toward the load, and also the impedance seen by the load in looking toward the generator, to depend upon the amount of attenuation introduced by the equalizer.

The *L* equalizers, designated as Type II, partially overcome this disadvantage of the Type I equalizer, since when $Z_1Z_2 = R_0^2 = R_L^2$, *i.e.*, by making Z_1 and Z_2 reciprocal networks with respect to $R_0 = R_L$, the impedance on the input side of the equalizer will be a constant resistance equal to R_L, irrespective of the amount of attenuation introduced. The resistance as seen by the load when one looks toward the generator will, however, depend upon the attenuation.

The Type III equalizer networks are the types most widely used in practical work. They are characterized by the fact that if the impedances Z_1 and Z_2 are reciprocal networks with respect to the resistance R_0, *i.e.*, if $Z_1Z_2 = R_0^2$, then the equalizer has both image impedances equal to R_0 at all frequencies. Such a network is said to be of the *constant-resistance* type. Insertion of such an equalizer in a system therefore does not disturb the impedance relations, provided that the load impedance matches the image impedance of the equalizer.

The equalizers indicated as Type IV represent more general forms of bridged-*T* networks of the constant-resistance type, and Type V represents a general lattice network of the constant-resistance type. The networks III*c* and III*d* are special cases of the more general Types IV*a* and V, respectively.

[1] Much of the material given here is based upon Chap. XVI of "Motion Picture Sound Engineering," Van Nostrand, New York, 1938.

Ex. 1044-0252

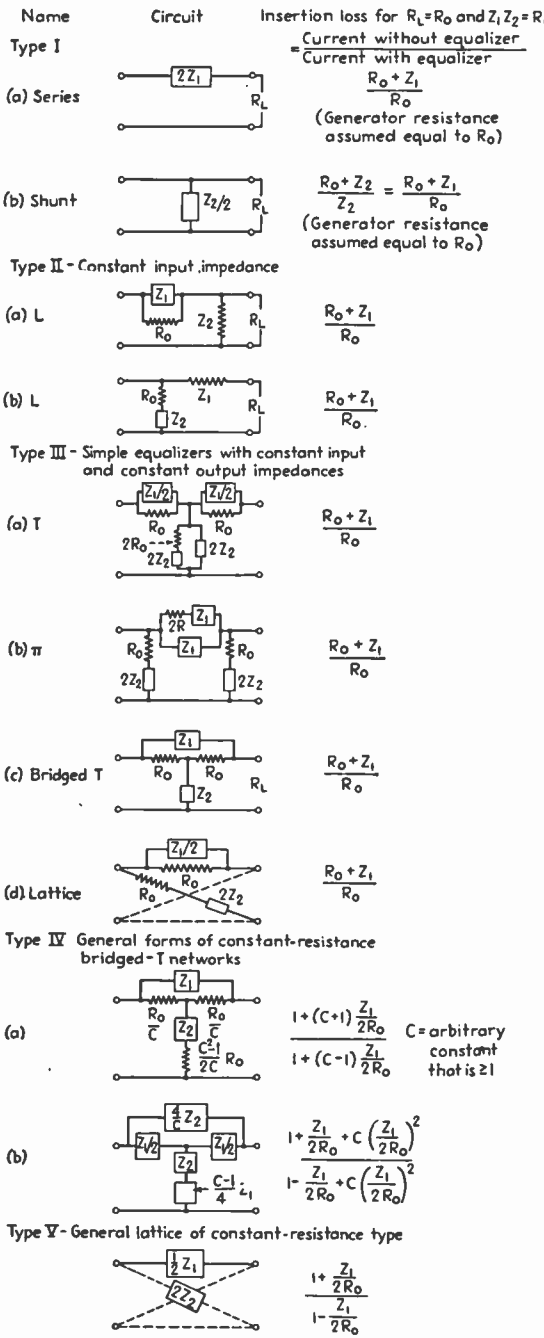


FIG. 111.—Equalizer networks and corresponding insertion loss formula.

Ex. 1044-0253

Design of Attenuation Equalizing Networks.[1]—The most important equalizers from a practical point of view are the Type III networks on Fig. 111, with the Type II sections also finding some application. The attenuation formulas are the same for

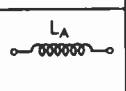
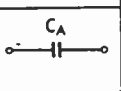
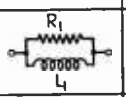
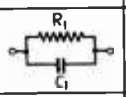
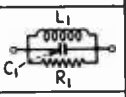
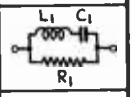
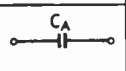
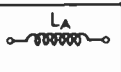
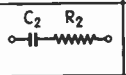
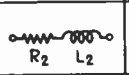
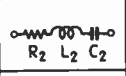
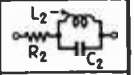
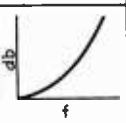
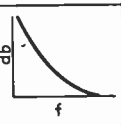
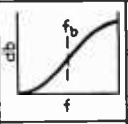
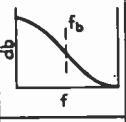
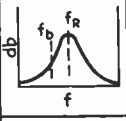
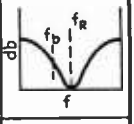
	a	b	c	d	e	f
Z_1 network						
Z_2 network						
Insertion loss characteristic in db						
Insertion loss in db	$10\log_{10}\left[1+\left(\frac{f}{f_a}\right)^2\right]$	$10\log\left[1+\left(\frac{f_a}{f}\right)^2\right]$	$10\log_{10}\left[1+\frac{K^2-1}{1+K\left(\frac{f_b}{f}\right)^2}\right]$	$10\log_{10}\left[1+\frac{K^2-1}{1+K\left(\frac{f}{f_b}\right)^2}\right]$	$10\log_{10}\left[1+\frac{K^2-1}{1+K\left(\frac{f}{f_R}\frac{f_R^2-f^2}{f^2-f_b^2}\right)^2}\right]$	$10\log_{10}\left[1+\frac{K^2-1}{1+K\left(\frac{b-\frac{1}{b}}{\frac{f}{f_R}-\frac{f_R}{f}}\right)^2}\right]$
Design formulas	$L_A=\frac{R_0}{2\pi f_a}=\frac{R_0}{\omega_a}$ $C_A=\frac{1}{2\pi f_a R_0}=\frac{1}{\omega_a R_0}$ $f_a=\frac{1}{2\pi\sqrt{L_A C_A}}$ $R_0=\sqrt{\frac{L_A}{C_A}}$		$L_1=L_B\frac{K-1}{\sqrt{K}}$ $L_2=L_B\frac{\sqrt{K}}{K-1}$ $C_1=C_B\frac{\sqrt{K}}{K-1}$ $C_2=C_B\frac{K-1}{\sqrt{K}}$		$L_1=L_B\frac{K-1}{\sqrt{K}}\frac{b^2-1}{b^2}$ $L_2=L_B\frac{\sqrt{K}}{K-1}\frac{1}{b^2-1}$ $C_1=C_B\frac{\sqrt{K}}{K-1}\frac{1}{b^2-1}$ $C_2=C_B\frac{K-1}{\sqrt{K}}\frac{b^2-1}{b^2}$	$L_1=L_B\frac{K-1}{\sqrt{K}}\frac{1}{b^2-1}$ $L_2=L_B\frac{\sqrt{K}}{K-1}\frac{b^2-1}{b^2}$ $C_1=C_B\frac{\sqrt{K}}{K-1}\frac{b^2-1}{b^2}$ $C_2=C_B\frac{K-1}{\sqrt{K}}\frac{1}{b^2-1}$
			$R_0=\sqrt{\frac{L_B}{C_B}}$ $\quad R_1=R_0(K-1)$ $\quad R_2=R_0\frac{1}{K-1}$ $L_B=\frac{R_0}{2\pi f_b}=\frac{R_0}{\omega_b}$ $\quad C_B=\frac{1}{2\pi f_b R_0}=\frac{1}{\omega_b R_0}$ $\quad f_b=\frac{1}{2\pi\sqrt{L_B C_B}}$			
Notes	f_R = Resonant frequency of Z_1 and Z_2 arms f_a = Frequency of 3db insertion loss f_b = Frequency where loss is one half maximum value (loss measured in db) f = Any frequency $b=\frac{f_R}{f_b}$ = defined as greater than unity		R_0 = Equalizer resistance Pad loss = maximum loss = $20\log_{10}K$ L = Inductance in henries C = Capacity in farads			

FIG. 112.—Design information for fundamental equalizer types.

Types I, II, and III, so that the insertion loss obtained with any one type can be duplicated in the other types by using the same R_0 and Z_1 (or Z_2). The choice between types is accordingly based on convenience in construction, cost, and the importance of maintaining the impedance relations.

[1] An excellent set of design curves is given in Chaps. XVI and XVII of "Motion Picture Sound Engineering," Van Nostrand, New York, 1938.

Ex. 1044-0254

The bridged-T structure designated as Type IIIc in Fig. 111 is the most widely used unbalanced structure; while if symmetry with respect to ground is required, Types IIIc and III*d* networks are best. The bridged T is superior to the Types III*a* and III*b* sections, since it requires fewer reactive elements. Compared with the Type II L sections, the bridged T requires only one additional fixed resistance, and, in return, gives a constant resistance irrespective of attenuation from both sides, instead of from only one side.

The types of attenuation characteristics obtainable with Type II and Type III sections when Z_1 has simple configurations are shown in Fig. 112. When Z_1 is supplied by an inductance, the attenuation rises at high frequencies, while if it is a

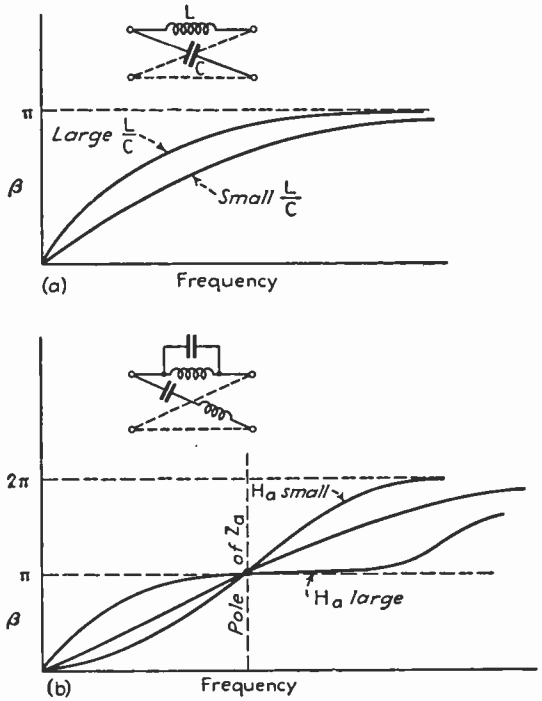


FIG. 113.—Examples of phase characteristics of simple all-pass sections (phase equalizers).

capacity, as in Fig. 112*b*, the attenuation rises at low frequencies. If the inductance is shunted with a resistance, as in Fig. 112*c*, the attenuation at high frequencies first rises and then levels off to a limiting value determined by the shunting resistance. Similarly, a resistance in shunt with a capacity as in Fig. 112*d* causes a corresponding leveling off of the rise in attenuation that would otherwise occur at low frequencies. Constructing the impedance Z_1 as a parallel resonant circuit shunted by a resistance, as in Fig. 112*e*, results in an attenuation peak at the parallel resonant frequency, with the magnitude of the peak determined by the shunting resistance. Such an arrangement is sometimes referred to as a "dip pad." When the impedance Z_1 is supplied by a series resonant circuit shunted by a resistance, as in Fig. 112*f*, the result is an attenuation at low and high frequencies determined by the shunting resistance, with negligible attenuation at the series resonant frequency.

Ex. 1044-0255

Formulas arranged in convenient form for designing equalizers having simple impedance configurations for Z_1 are given in Fig. 112 for Types I*a*, II, and III sections.

The simple Type I, II and III equalizer sections are capable of meeting most practical requirements. This is true even when relatively complicated equalization characteristics are desired, since one may break up the required attenuation characteristic into the sum of several simpler characteristics, and then obtain each simpler attenuation characteristic from a Type I, II, or III section with a relatively simple Z_1.

The general lattice network (Type V) is the most general form of symmetrical equalizer that can be devised. All other symmetrical equalizers, such as Types I*a*, III, and IV, are equivalent to special cases of the general lattice. It is accordingly possible to base the design of all equalizers upon the general lattice even though the

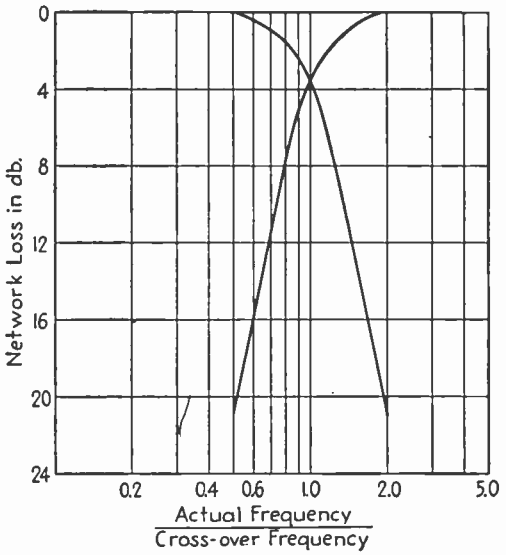


FIG. 114.—Characteristics of a typical dividing network.

network may be built in the form of a bridged T where this conversion leads to a physically possible structure.[1]

Phase Equalizers. All-pass Filters.—An all-pass filter is a filter having zero attenuation for all frequencies from zero to infinity. Such filter sections introduce phase shift without affecting attenuation, and so are employed as phase equalizers to correct phase distortion introduced by other parts of a system, and can also be used to introduce a time delay.

All-pass action can be obtained by making use of a lattice network in which the lattice impedances Z_a and Z_b are reactances that are reciprocal with respect to the desired image impedance R. This leads to an image impedance that is constant at the desired value for all frequencies.

The phase shift β introduced by an all-pass section under image-impedance operation is

[1] A discussion of this approach to equalizer design, together with design information on a considerable variety of configurations for the lattice impedances, is given by Otto J. Zobel, Distortion Correction in Electrical Circuits with Constant Resistance Recurrent Networks, *Bell System Tech. Jour.*, Vol. 7, p. 438, April, 1928. A very clear discussion of the theoretical basis of the design method developed by Zobel for the general lattice equalizer is given by Everitt, *op. cit.*, pp. 287–293.

Ex. 1044-0256

$$\tan \frac{\beta}{2} = \pm j \sqrt{\frac{Z_a^2}{R^2}} \tag{141}$$

where R is the desired image impedance.

The variation of the phase shift β with frequency can be controlled by the number and location of the internal zeros and poles in the impedance Z_a, and by the value assigned to the quantity H in the expression for Z_a given by Eq. (86). Typical phase-shift characteristics for several simple cases are illustrated in Fig. 113.

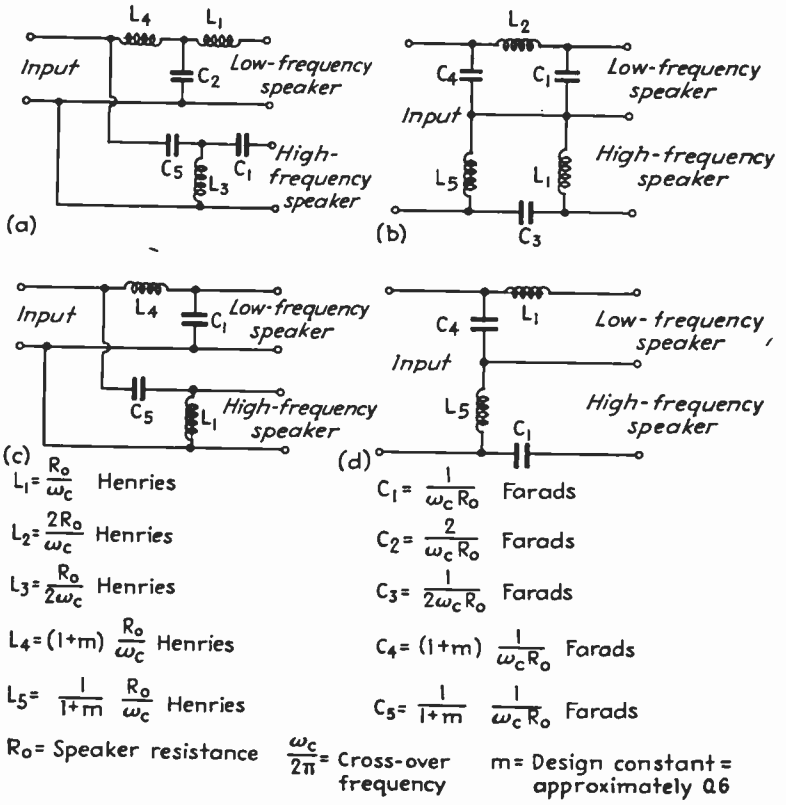


FIG. 115.—Dividing networks of the filter type.

31. Dividing Networks.[1]—The term *dividing network* is applied to a coupling system so arranged that at low frequencies power is delivered to a low-frequency loud-speaker, while at high frequencies it is delivered to a high-frequency speaker. The transmission characteristics of a typical dividing network are shown in Fig. 114. The frequency at which the power delivered to the two outputs is equal is termed the *cross-over* frequency. Experience indicates that the dividing network should provide at least twelve db attenuation one octave away from the cross-over frequency, as compared with the cross-over attenuation, whereas attenuations of more than eighteen db per octave are not necessary or desirable.

[1] See Chap. XX, "Motion Picture Sound Engineering," Van Nostrand, New York, 1938; John K. Hilliard, Loud Speaker Dividing Networks, *Electronics*, Vol. 14, p. 26, January, 1941.

Ex. 1044-0257

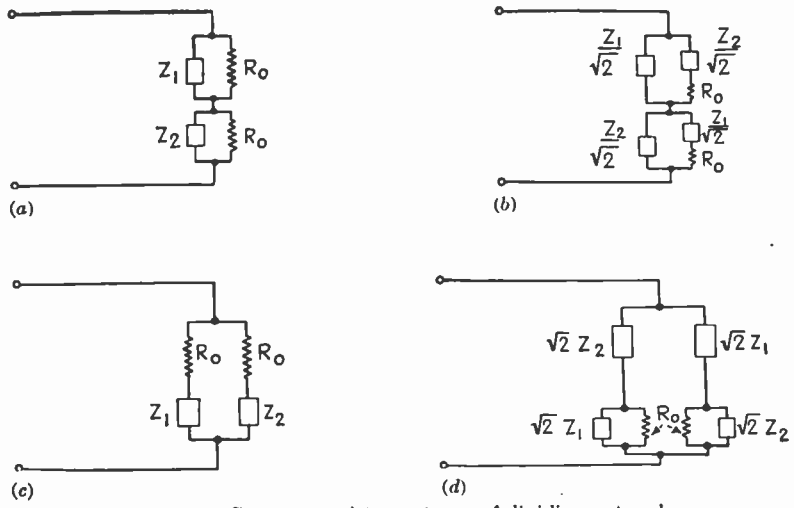


FIG. 116.—Constant-resistance types of dividing network.

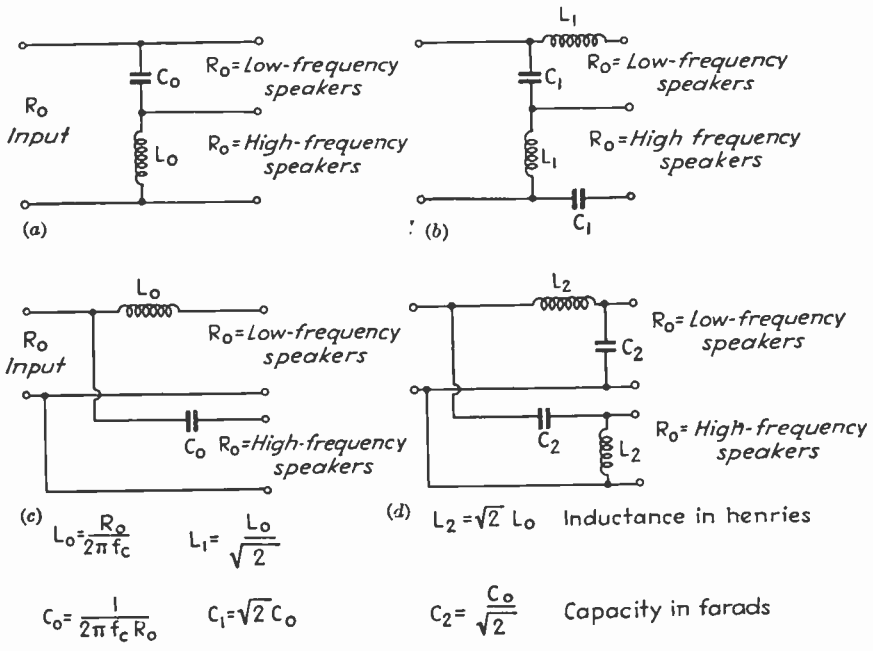


FIG. 117.—Practical constant-resistance dividing networks.

Ex. 1044-0258

There are two basic types of dividing networks. The first consists of complementary low- and high-pass filters connected with inputs either in parallel or in series. Such networks are shown in Fig. 115, which also gives the necessary design formulas. The different parts are designed as complementary low- and high-pass filters with fractional terminations. The networks shown at (*a*) and (*b*) in Fig. 115 consist of one full section of the $m = 1$ network type, with an input half section designed for $m \cong 0.6$. The input shunting impedance called for in the design of (*a*) and the series input impedance called for in the design of (*b*) are then omitted to provide the fractional termination. Such an arrangement provides an attenuation of approximately 18 db for the first octave beyond cross-over. Sections shown at Fig. 115*c* and 115*d* differ from those at (*a*) and (*b*) in that the final half section of the $m = 1$ network type is omitted. This reduces the attenuation to approximately 12 db for the first octave beyond the cross-over frequency. It would be possible to design the dividing network so that the output terminals were also provided with an $m = 0.6$ termination, but this is never done, because satisfactory performance can be obtained with the simpler arrangement.

Dividing networks of another kind, referred to as the *constant-resistance* type, are shown in Fig. 116. These are characterized by possessing an input impedance that is a constant resistance equal to R_0 when the two impedances Z_1 and Z_2 are so related that $Z_1Z_2 = R_0^2$. Practical dividing networks making use of these circuit arrangements are shown in Fig. 117, together with the necessary design formulas. The simple circuits (*a*) and (*c*) have an attenuation of approximately 6 db for the first octave beyond cutoff, and since this is smaller than desirable, such arrangements are seldom used. The two-element dividing networks of (*b*) and (*d*) are similar to those of Figs. 115*d* and 115*c*, respectively, but because of slightly different circuit proportions provide an exactly constant input resistance, whereas the circuits of Fig. 115 only approximate a constant resistance. The attenuation provided by the networks of Figs. 116*b* and *d* is about 10 db for the first octave beyond cutoff.

Dividing networks carry the full output power that the amplifier operating the loud-speaker system is capable of developing, and so must be designed to have a low transmission loss. When care is taken to employ coils of the lowest practical resistance, this loss is of the order of 0.5 db in systems providing from 12 to 18 db per octave of attenuation.

WAVES GUIDES AND RESONATORS[1,2]

32. Fundamental Concepts of Waves Guides.—The term wave guide refers to a hollow conducting tube used for the transmission of electromagnetic waves. At very high frequencies wave guides provide an alternative to the ordinary transmission line, and are comparable with or superior to a concentric line.

[1] This presentation of wave guides and cavity resonators is based on material prepared by the author's associate, Karl Spangenberg.

[2] The practical possibilities of wave guides as transmission systems for very high-frequency waves was discovered independently and almost simultaneously by W. L. Barrow and G. C. Southworth. The fundamental papers on the subject are as follows: W. L. Barrow, Transmission of Electromagnetic Waves in Hollow Tubes of Metal, *Proc. I.R.E.*, Vol. 24, p. 1298, October, 1936; G. C. Southworth, Hyper-frequency Wave Guides—General Considerations and Experimental Results, *Bell System Tech. Jour.*, Vol. 15, p. 284, April, 1936; Some Fundamental Experiments with Wave Guides, *Proc. I.R.E.*, Vol. 25, p. 807, July, 1937; John R. Carson, Sallie P. Mead, and S. A. Schelkunoff, Hyper-frequency Wave Guides—Mathematical Theory, *Bell System Tech. Jour.*, Vol. 15, p. 310, April, 1936; S. A. Schelkunoff, Transmission Theory of Plane Electromagnetic Waves, *Proc. I.R.E.*, Vol. 25, p. 1457, November, 1937; L. J. Chu and W. L. Barrow, Electromagnetic Waves of Hollow Metal Tubes of Rectangular Cross Section, *Proc. I.R.E.*, Vol. 26, p. 1520, December, 1938; L. J. Chu, Electromagnetic Waves in Elliptic Hollow Pipes of Metal, *Jour. Applied Phys.*, Vol. 9, p. 583, September, 1938.

Ex. 1044-0259

Physical Picture of Wave-guide Propagation.—A wave in order to exist in a wave guide must satisfy Maxwell's equations throughout the wave guide. It must also satisfy the boundary conditions, which are that to the extent that the walls of the wave guide are perfect conductors, there can be no tangential component of electric field at the walls of the wave guide. An exact solution for the field existing within a wave guide is therefore a relatively complicated mathematical expression.

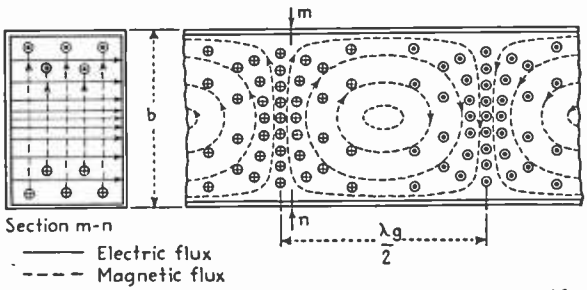


FIG. 118.—Typical field arrangement existing in a rectangular wave guide ($TE_{0,1}$ wave).

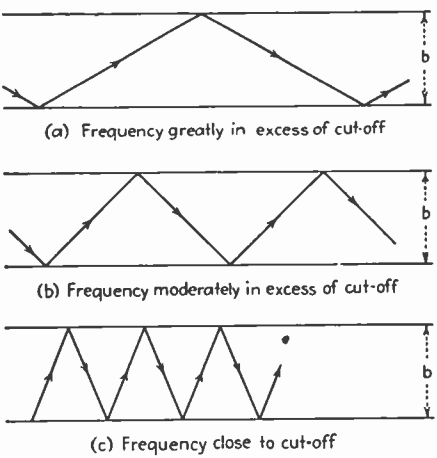


FIG. 119.—Paths followed by waves traveling back and forth between the walls of a wave guide for frequencies exceeding the cutoff frequency by varying amounts.

It is possible, however, to obtain an understanding of many of the properties of wave-guide propagation from a simple physical picture of the mechanisms involved.[1] Consider the fields shown in Fig. 118, which correspond to a typical kind of wave propagation in a rectangular guide. These can be considered as the resultant fields produced by an ordinary plane electromagnetic wave that travels back

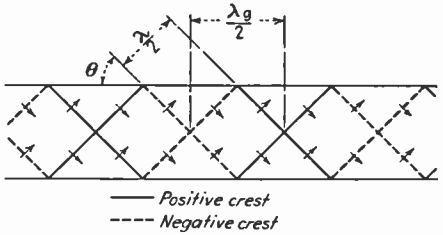
[1] This is due to L. Page and N. I. Adams, Electromagnetic Waves in Conducting Tubes, *Phys. Rev.* Vol. 52, p. 647, September, 1937; Léon Brillouin, Propagation of Electromagnetic Waves in a Tube, *Rev. gén élec.*, Vol. 40, p. 227, August, 1936. See also H. H. Skilling, "Fundamentals of Electric Waves," Wiley, 1942

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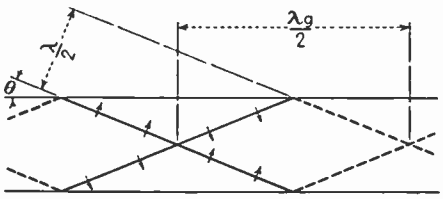
and forth between the sides of the guide, as illustrated in Fig. 119. The electric and magnetic component fields of this plane wave are in time phase, but are geometrically at right angles to each other and to the direction of propagation. Such a wave travels with the velocity of light, and upon encountering the conducting walls of the guide is reflected with a reversal of the electric field and with an angle of reflection equal to the angle of incidence.

A picture of the wave fronts involved in such propagation in a rectangular wave guide is shown in Fig. 120. When the angle θ is such that the successive positive and negative crests traveling in the same direction just fail to overlap inside the guide, as is the case in Fig. 120*a*, then it can be shown that the summation of the various waves and their reflections leads to the field distribution illustrated in Fig. 118, which travels down the wave guide and represents propagation of energy. The angle θ that the component waves must have with respect to the wave guide in order to satisfy the conditions for wave-guide propagation in a rectangular guide is given by the relation

$$\cos \theta = \frac{\lambda}{2b} \qquad (142)$$



(a) Wave fronts corresponding to Fig. 119 b



(b) Wave fronts corresponding to Fig. 119 c

FIG. 120.—Wave front corresponding to the situation illustrated in Figs. 119*b* and 119*c*.

where b is the width of the wave guide (see Fig. 118) and λ is the wave length of the wave on the basis of the velocity of light.

Because of the fact that the component waves that can be considered as building up the actual field in the wave guide all travel at an angle with respect to the axis of the guide, the rate at which energy propagates down the guide is less than the velocity of light. This velocity with which energy propagates is termed the *group velocity* and in the case of Fig. 120 is given by the relation

$$\frac{\text{Group velocity}}{\text{Velocity of light}} = \sin \theta = \sqrt{1 - \left(\frac{\lambda}{2b}\right)^2} \qquad (143)$$

At the same time, the fields of the component waves combine in such a manner that the distance λ_g, as shown in Fig. 120, is greater than the wave length λ of the waves in free space. The result is that the fields in the wave guide possess an *apparent* or *phase velocity* that is greater than the velocity of light. Thus one has

$$\frac{\text{Wave length in guide}}{\text{Wave length in free space}} = \frac{\lambda_g}{\lambda} = \frac{1}{\sin \theta} \qquad (144)$$

$$\frac{\text{Phase velocity in guide}}{\text{Velocity of light}} = \frac{\lambda_g}{\lambda} = \frac{1}{\sqrt{1 - \left(\frac{\lambda}{2b}\right)^2}} \qquad (145)$$

The notation is the same as used above. The phase velocity is of particular importance as it gives the phase shift β per unit length that a wave suffers when traveling

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down the guide. The relationship is

$$\beta = \frac{2\pi}{\lambda_g} \tag{146}$$

The phase and group velocities v_p and v_g, respectively, are related by the fact that

$$\sqrt{v_p v_g} = \text{velocity of light} \tag{147}$$

As the wave length is increased, the component waves must travel more nearly at right angles to the axis of the wave guide, as shown in Fig. 120*b*. This causes the group velocity to be lowered, and the phase velocity to be still greater than the velocity of light, until finally one has $\theta = 90°$. The component waves then bounce back and forth across the wave guide at right angles to its axis, and do not travel down the tube at all. Under these conditions the group velocity is zero, the phase velocity becomes infinite, and propagation of energy ceases. This frequency is termed the *cutoff frequency*. The wave length corresponding to cutoff is given in the case of Fig. 118 by the relation

$$\text{Wave length for cutoff} = 2b \tag{148}$$

where b is the width of the guide (see Fig. 118). It will be noted that the wave guide acts as a high-pass filter with the cutoff frequency determined by the dimensions of the guide. The fact that in order to obtain propagation, the wave guide must have a dimension comparable to a wave length limits the practical use of wave guides to extremely high frequencies.

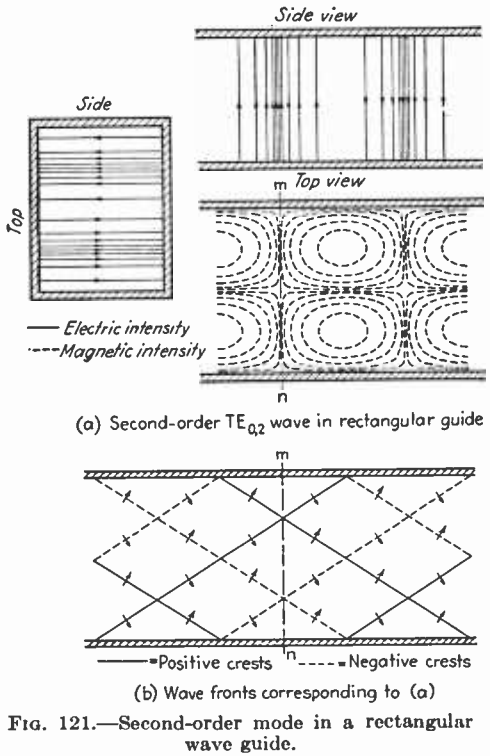


FIG. 121.—Second-order mode in a rectangular wave guide.

When the frequency is made very high, the mode of propagation shown in Fig. 118 corresponds to a component wave traveling almost parallel to the axis of the guide (see Fig. 119*a*), so that the group and phase velocities both approach the velocity of light.

At frequencies very much greater than the cutoff frequency, it is possible for higher order modes of transmission to exist in a wave guide. Thus if the frequency is high enough, propagation of energy can take place down the guide when the system of component waves that are reflected back and forth between the walls has the character shown in Fig. 121*a*. This leads to fields within the guides illustrated in Fig. 121*b*, which is a second-order mode of propagation. This second mode corresponds to a field distribution equivalent to two distributions of the fundamental mode placed side by side, but with reversed polarity. Higher order modes of propagation can exist whenever the frequency is high with respect to the cutoff frequency, or conversely, higher order modes can exist for a given frequency if the dimensions of the wave guide are unnecessarily large.

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This concept that has been presented of wave-guide propagation as involving a wave suffering successive reflections between the sides of the guide can be applied to all types of waves and to other than rectangular guides. The way in which the concept works out in these other cases is normally not as simple, however, as in the wave of Fig. 118.

Transmission Modes and Their Classification.—Waves of a variety of types may propagate down a wave guide. These include several types of fundamental modes, such as illustrated in Fig. 122 and Fig. 124, together with higher orders of each of these modes shown in Fig. 123.

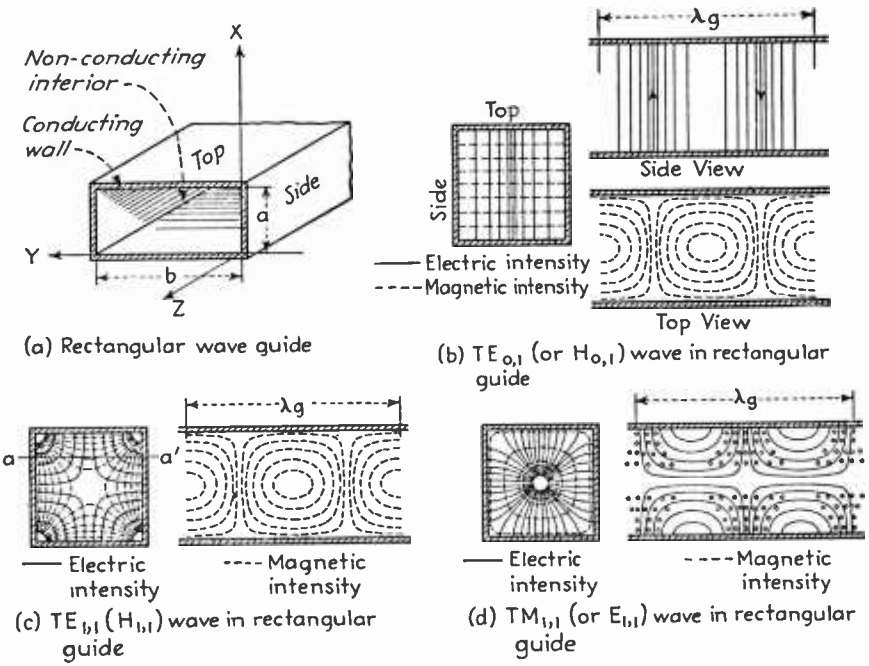


FIG. 122.—Rectangular wave guide, and field distributions inside the guide corresponding to fundamental forms of wave-guide transmission.

A study of the fields that can exist inside a closed tube shows that if the electric field has a component in the direction of propagation then the magnetic field does not. The converse of this statement is also true. Hence all modes of transmission may be put into two groups according to the axial component of field that exists. Modes of transmission having a component of electric field in the direction of propagation but no component of magnetic field in that direction are known as *E* waves or *TM* (transverse magnetic) waves. Modes of transmission having a component of magnetic field in the direction of propagation but no component of electric field in that direction are known as *H* waves or *TE* (transverse electric) waves. All possible modes that can exist in any type of hollow guide can be placed in one of these two groups. The designation in terms of the transverse field is preferred.[1]

[1] Memorandum on the Terminology of Guided Waves, Committee on Radio Wave Propagation of the Institute of Radio Engineers, Conference of Jan. 23, 1940.

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33. Properties of Guides.[1] *Rectangular Wave Guides.*[2]—The three simplest modes of wave-guide propagation that can take place in rectangular guides are shown in Fig. 122. Examples of higher order modes of these are given in Fig. 123.

A double-subscript notation is used to designate the various TE and TM waves, leading to $TE_{0,1}$, $TE_{1,1}$, etc., waves, as shown in Figs. 122 and 123. In the case of rectangular guides these subscripts denote the number of half-period variations of the fields along the x and y axes, respectively.

The properties of the three principal modes that can exist in rectangular guides are given in Table 6.[3]

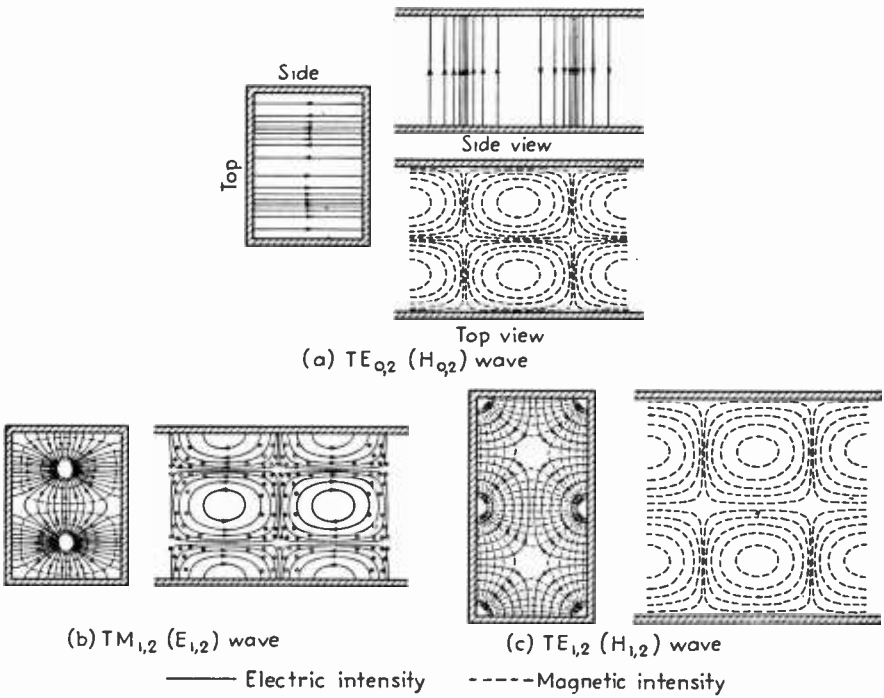


FIG. 123.—Modes of transmission in a rectangular wave guide that are of higher order than those shown in Fig. 122.

Cylindrical Wave Guides.[4]—The general relations for cylindrical guides are the same as those for the rectangular guides, though the specific form of the field equations differ. The commonest cylindrical modes are shown in Fig. 124. It will be noted that the waves of Figs. 124*a* and 124*b* are closely related to the waves of Figs. 122*b* and 124*d* of a rectangular guide.

A double-subscript notation is used to designate the various modes in a circular guide (see Fig. 124). As applied to circular guides, the first subscript denotes the number of full cycles in the radial field pattern that is traversed when the angle ϕ (see Fig. 125) passes through 360°, while the second subscript denotes the number of

[1] Information on other than rectangular and circular guides is given by Chu, *loc. cit.*, and Schelkunoff, *loc. cit.*

[2] L. J. Chu and W. L. Barrow, Electromagnetic Waves in Hollow Metal Tubes of Rectangular Cross Section, *Proc. I.R.E.*, Vol. 26, p. 1520, December, 1938.

[3] From Chu and Barrow, *loc. cit.*

[4] For further information see Southworth, Carson, *et al.*, and Chu, *loc. cit.*

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Table 6.—Properties of Rectangular Wave Guides

Notation

a = transverse dimension of guide, x dimension (see Fig. 122).
b = transverse dimension of guide, y dimension (see Fig. 122).
f = frequency, cycles per sec.
$\omega = 2\pi f$, angular velocity, radians per sec.
β = phase constant, radians per cm, based on phase velocity.
λ = wave length, cm (in free space).
c = velocity of light, 3×10^{10} cm per sec.

Operating Characteristics Common to All Rectangular Modes

Wave length in guide: $\lambda_g = 2\pi/\beta$ Wave length in free space: $\lambda = c/f$

Phase velocity: $v_p = \omega/\beta$ Group velocity: $v_g = d\omega/d\beta = c^2/v_p$

Mode	Cutoff frequency, f_0	Cutoff wave length, λ_0	Phase constant, β
$TE_{0,1}(H_{0,1})$*	$c/2b$	$2b$	$\frac{2\pi}{\lambda}\sqrt{1-\left(\frac{\lambda}{2b}\right)^2}$
$TE_{1,1}(H_{1,1})$†	$\frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}$	$\frac{2}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}}$	$\frac{2\pi}{\lambda}\sqrt{1-\left(\frac{\lambda}{2a}\right)^2-\left(\frac{\lambda}{2b}\right)^2}$
$TM_{1,1}(E_{1,1})$‡	$\frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}$	$\frac{2}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}}$	$\frac{2\pi}{\lambda}\sqrt{1-\left(\frac{\lambda}{2a}\right)^2-\left(\frac{\lambda}{2b}\right)^2}$

* See Fig. 122*b*.
† See Fig. 122*c*.
‡ See Fig. 122*d*.

Table 7.—Properties of Cylindrical Guide Modes

Notation

a = guide radius, cm.
f = frequency, cycles per sec.
$\omega = 2\pi f$, angular velocity, radians per sec.
β = phase constant, radians per cm, based on phase velocity.
λ = wave length, cm (in free space).
c = velocity of light, 3×10^{10} cm per sec.

Operating Characteristics Common to All Cylindrical Modes

Wave length in guide: $\lambda_g = 2\pi/\beta$ Wave length in free space: $\lambda = c/f$

Phase velocity: $v_p = \omega/\beta$ Group velocity: $v_g = d\omega/d\beta = c^2/v_p$

Mode	Cutoff frequency, f_0	Cutoff wave length, λ_0	Phase constant, β
$TM_{0,1}(E_0)$*	$0.383\,\frac{c}{a}$	$2.61a$	$\sqrt{\left(\frac{2\pi}{\lambda}\right)^2-\left(\frac{2.405}{a}\right)^2}$
$TE_{0,1}(H_0)$†	$0.610\,\frac{c}{a}$	$1.64a$	$\sqrt{\left(\frac{2\pi}{\lambda}\right)^2-\left(\frac{3.83}{a}\right)^2}$
$TE_{1,1}(H_1)$‡	$0.289\,\frac{c}{a}$	$3.46a$	$\sqrt{\left(\frac{2\pi}{\lambda}\right)^2-\left(\frac{1.84}{a}\right)^2}$

* See Fig. 124*b*.
† See Fig. 124*c*.
‡ See Fig. 124*a*.

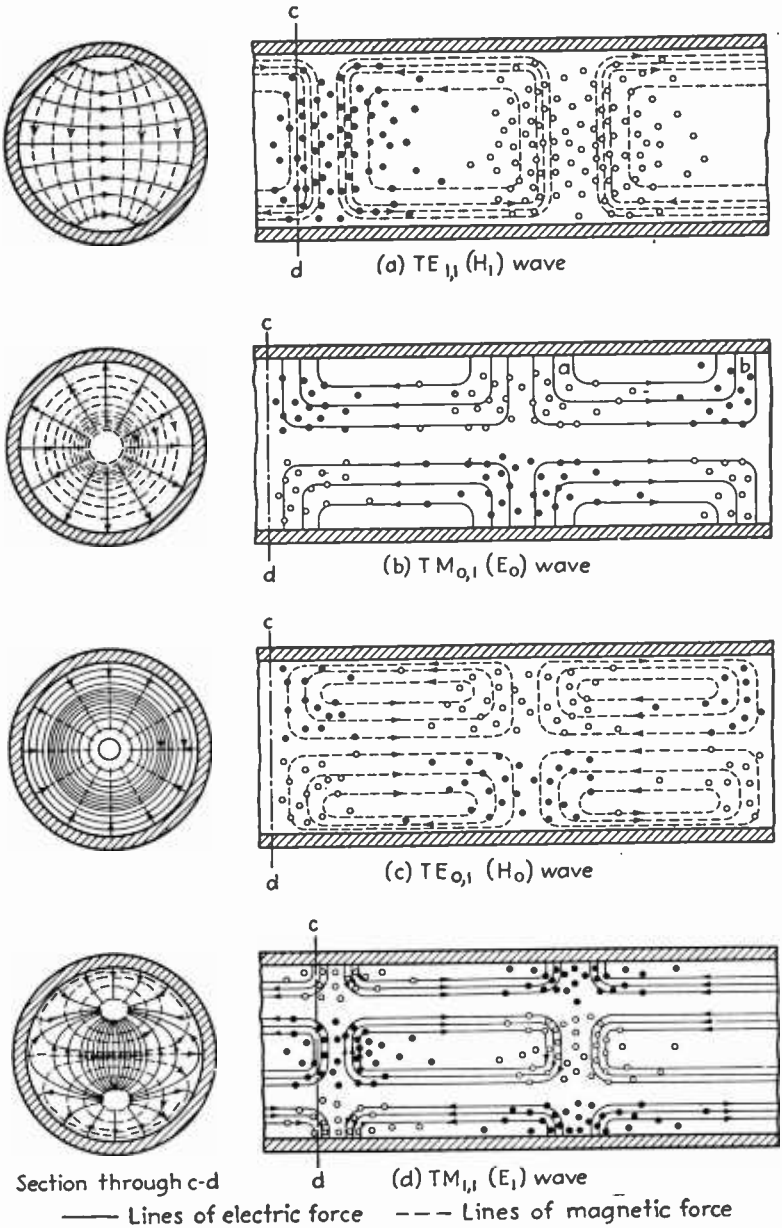


FIG. 124.—Field distribution corresponding to the fundamental modes of transmission possible in a circular wave guide.

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half cycles of field variation along a radial component between the center and the walls. Waves in circular guides are also sometimes denoted by single subscripts, as E_0, H, etc., types, as shown in Fig. 124.

The properties of the first three modes of Fig. 124 are given in Table 7.[1]

Cutoff Frequency and Properties Related Thereto.—With any mode of wave-guide propagation, as the frequency is decreased, the longitudinal loops whether magnetic or electric become longer until at the cutoff frequency they become infinite in extent, and the phase velocity is infinite. As the frequency is increased the loops become smaller and their axial length approaches a half wave length as measured in free space. The distance between corresponding points on successive waves inside the guide is always greater than the distance between corresponding points of a plane wave in free space because the phase velocity is always greater than the velocity of light.

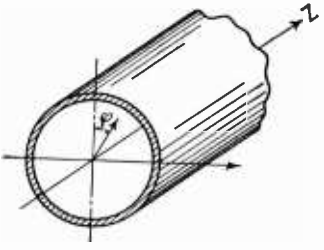


FIG. 125.—Diagram illustrating notation in circular wave guides.

The relation between the various factors, such as frequency, wave length, and velocity, is shown in Fig. 126. From this it is seen that as the frequency decreases, *i.e.*, as the wave length increases, both the phase velocity and the wave length in the guide increase to an infinite value at cutoff. This is a universal relation that holds for all modes and all shapes of guides.

All wave guides act like high-pass filters, with the cutoff frequency determined by the shape and size of the guide and by the mode of transmission. In Fig. 127 are shown the longest wave lengths passed by the simplest modes of a square and a circular wave guide. In general, the critical wave length is of the order of the cross-sectional dimension of the guide.

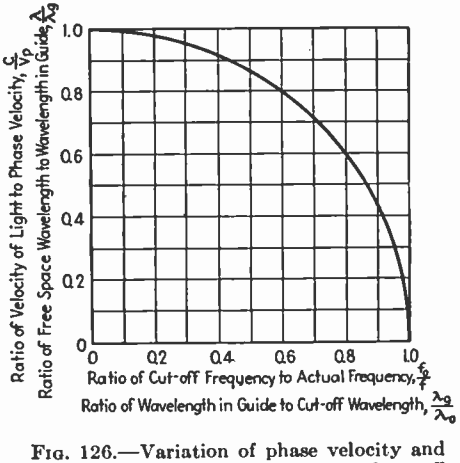


FIG. 126.—Variation of phase velocity and wave length in guide as the ratio of cutoff frequency to actual frequency is varied.

For the square guide, the lowest frequency mode has a critical wave length that is twice the width of the guide. This particular mode is unique in that its critical wave length depends only upon the width of the guide but not upon the guide height, so that a rectangular guide of a given width has the same critical wave length for any height. For a square guide, the $TE_{1,1}$ and $TM_{1,1}$ modes each have a critical wave length equal to the diagonal of the square. The critical wave length for these modes depends upon both of the dimensions of the guide if the guide shape is rectangular. It will be noted that if the dimensions of a rectangular guide are suitably chosen in relation to wave length, only the $TE_{0,1}$ mode of propagation is possible, and one is assured that the fields in the guide will correspond to those of a single pure mode.

[1] From Chu, *loc. cit.*

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Similar relations hold for the circular guide. Here the critical wave length depends upon the diameter and the mode of transmission. As frequency is increased from below the lowest cutoff frequency for a given circular guide, the guide will first pass only the cylindrical $TE_{1,1}$ mode, then also the circular magnetic mode $TM_{0,1}$, and finally the circular electric mode $TE_{0,1}$. After that, higher order modes will be passed.

Excitation of Wave-guide Modes.—Each wave-guide mode must be excited in a different way. The guide can be thought of as a region of space into which energy is radiated from the coupling or exciting circuit. Just as waves can be set up in space by straight wires and loops, so can the wave-guide modes be excited by electric probes and loops.

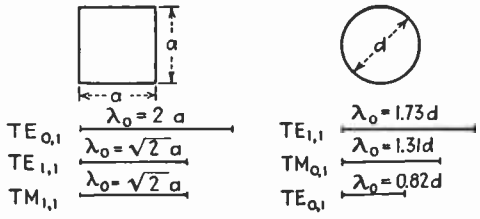


FIG. 127.—Diagram illustrating relative length of waves corresponding to cutoff frequency for the three simplest modes of propagation in square and circular wave guides.

The location of an exciting probe or loop must bear a definite relation to the fields in the guide corresponding to the desired transmission mode. For instance, if the transmission mode has an axial component of electric field and if its magnetic flux lines are transverse closed loops, then it can be excited either by an antenna probe that produces an axial component electrical field or by a loop placed so that it produces a magnetic field in the proper direction. In general, any mode can by excited either by a suitable antenna probe (or dipole) or by a suitable current loop.

Excitation of various transmission modes in a rectangular wave guide by means of antenna probes is shown in Fig. 128.[1] Corresponding arrangements for circular guides are shown in Fig. 129.[2] In all cases the probe creates an electric or magnetic field of the proper orientation.

Unless the dimensions of the guide are such that only one transmission mode can exist, any excitation device will set up several modes simultaneously, though the desired mode is usually strongest. When this is the case the wave may be purified by inserting grids or wires that will short out the electric field of the undesired mode.

The pickup device at the end of a wave guide is of the same form as the exciting device. It is rather difficult to terminate a wave guide so that there will be no reflections from the load end, unless the termination has both a reactive and a resistive adjustment.

Attenuation in Wave Guides.—Hollow-pipe guides have attenuation characteristics that are similar to those of conventional high-pass filters in which the electrical elements are lumped rather than distributed. The chief features of the attenuation characteristics of high-pass filters are that there is a high attenuation in the attenuation band (*i.e.*, for all frequencies below cutoff) and a low but finite attenuation in the pass band (*i.e.*, for frequencies above cutoff). The nature of the attenuation in the attenuation and pass bands is different. At frequencies below the cutoff frequency, power is rejected at the source because the guide dimensions and frequency do not permit the existence of the type of waves that transmit power. In this region the attenuation is so high that no effective transmission of power is possible over any appreciable distances.

[1] From Chu and Barrow, *loc. cit.*
[2] From Southworth, *loc. cit.*, and Chu, *loc. cit.*

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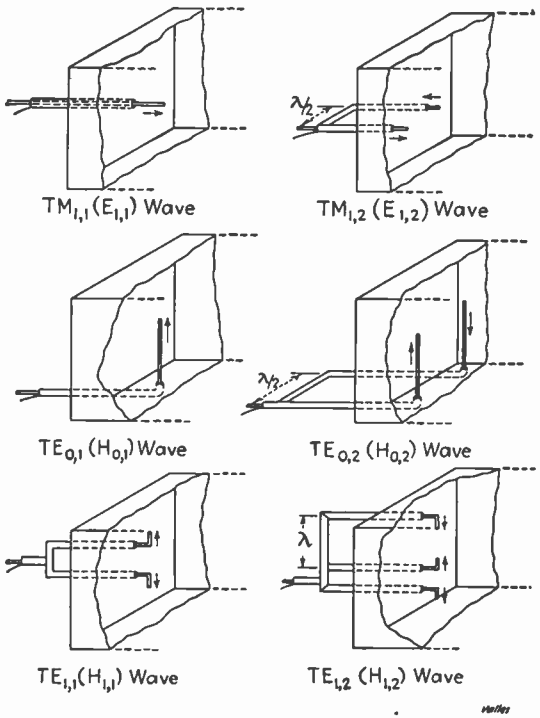


FIG. 128.—Typical probe arrangements for exciting different types of waves in rectangular guides.

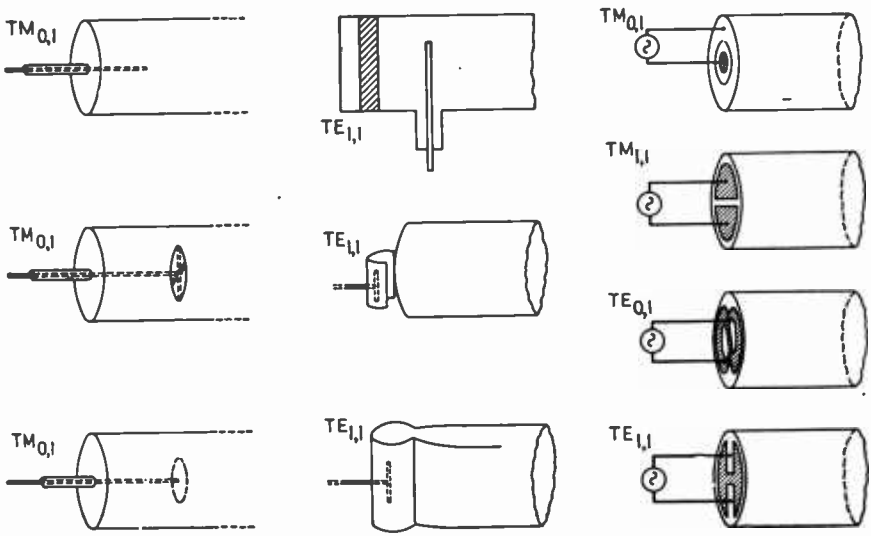


FIG. 129.—Typical methods of exciting different types of waves in circular wave guides.

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In the pass band, as frequency increases, the attenuation of a wave guide first decreases from a very large value (theoretically infinite at cutoff), then levels off at some low but finite value, and then increases as shown in Fig. 130.[1] This is the result of two opposing factors that take place as the frequency increases. First, the field configurations become more favorable for keeping the losses low, and, second, the skin effect becomes greater as the frequency is raised. This variation of attenuation with frequency is found in all shapes of guides and all modes with the single exception of the cylindrical $TE_{0,1}$ mode, for which the attenuation decreases without limit as the frequency increases.[2]

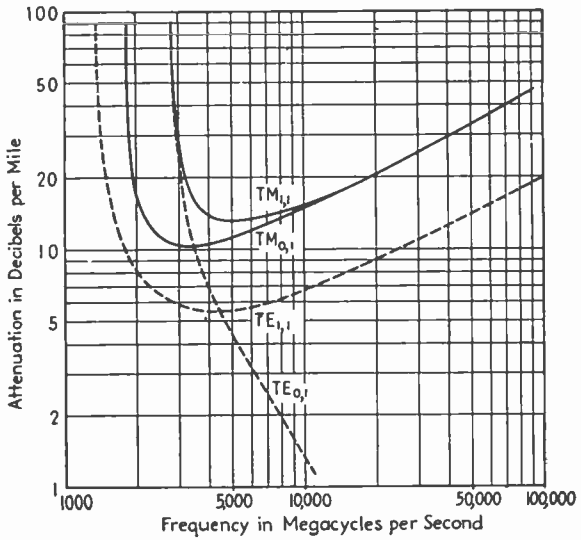


FIG. 130.—Attenuation of different types of waves in a 5-inch copper cylindrical wave guide for various frequencies.

Formulas for the attenuation of the various basic modes in cylindrical and square guides are given in Table 8.[3] Results calculated for a circular copper guide 5 inches in diameter are given in Fig. 130. Examination of Tables 8 and 9 and Fig. 130 shows that in all cases except that of the $TE_{0,1}$ mode, the attenuation decreases from a very high value at cutoff, reaches a minimum at from about 2 to 3 times the cutoff frequency, and then increases with frequency, finally increasing with the square root of frequency at very high frequencies. The minimum is quite broad, and the minimum attenuation low. Thus in Fig. 130 there is a band of frequencies several thousand megacycles wide in which the attenuation varies only 1 db, though it takes a very high frequency or a very large guide to realize this property.

[1] From Southworth, *loc. cit.*

[2] No experimental confirmation of this property, or even of the existence of the $TE_{0,2}$ mode, has been published. In fact, Chu, *loc. cit.*, and Chu and Barrow, *loc. cit.*, question the stability of this mode.

[3] These results are from Chu and Chu and Barrow, *loc. cit.*, and Carson, Mead, and Schelkunoff, *loc. cit.*

An alternative method of calculating the attenuation in a rectangular guide, based upon waves being reflected back and forth between the walls of the guide, is given by John Kemp, Electromagnetic Waves in Metal Tubes of Rectangular Cross Section, *Jour. I.E.E.*, Vol. 88, Pt. III, p. 213, September, 1941. See also a discussion of this by G. W. O. Howe, *Wireless Eng.*, Vol. 19, p. 93, March, 1942.

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TABLE 8.—WAVE-GUIDE ATTENUATION FORMULAS FOR AIR DIELECTRIC INSIDE GUIDE

Notation

r = guide radius (of circular guide), cm.
a = length of side of square guide, cm.
f = actual frequency, cycles.
f_0 = cutoff frequency, cycles.
K_1 = a constant depending on guide walls, and given at end of table.

Circular Guide

$TM_{0,1}$ (E_0) mode:

$$\alpha = \frac{0.4378K_1}{r^{3/2}} \frac{\left(\frac{f}{f_0}\right)^{3/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2 - 1}} \text{ db per mile}$$

$TM_{1,1}$ (E_1) mode:

$$\alpha = \frac{0.5520K_1}{r^{3/2}} \frac{\left(\frac{f}{f_0}\right)^{3/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2 - 1}} \text{ db per mile}$$

$TE_{0,2}$ (H_0) mode:

$$\alpha = \frac{0.5520K_1}{r^{3/2}} \frac{\left(\frac{f}{f_0}\right)^{1/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2 - 1}} \text{ db per mile}$$

$TE_{1,1}$ (H_1) mode:

$$\alpha = \frac{0.3824K_1}{r^{3/2}} \frac{\left(\frac{f}{f_0}\right)^{-1/2} + \frac{1}{2.38}\left(\frac{f}{f_0}\right)^{3/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2 - 1}} \text{ db per mile}$$

Square Guide

$TE_{0,1}$ mode:

$$\alpha = \frac{K_1}{a^{3/2}} \frac{\frac{1}{2}\left(\frac{f}{f_0}\right)^{3/2} + \left(\frac{f}{f_0}\right)^{-1/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2 - 1}} \text{ db per mile}$$

$TE_{1,1}$

$$\alpha = \frac{1.189K_1}{a^{3/2}} \frac{\left(\frac{f}{f_0}\right)^{3/2} + \left(\frac{f}{f_0}\right)^{-1/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2 - 1}} \text{ db per mile}$$

$TM_{1,1}$

$$\alpha = \frac{1.189K_1}{a^{3/2}} \frac{\left(\frac{f}{f_0}\right)^{3/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2 - 1}} \text{ db per mile}$$

Value of K_1

Copper	236.5
Aluminum	313.6
Lead	821.3
Iron ($\mu = 100$)	5,696

Ex. 1044-0271

The minimum attenuation associated with a particular mode is given by the relation

For circular guide:

$$\text{Minimum attenuation} = \frac{K_2}{r^{3\frac{1}{2}}} \text{ db per mile} \tag{149a}$$

For square guide:

$$\text{Minimum attenuation} = \frac{K_2}{a^{3\frac{1}{2}}} \text{ db per mile} \tag{149b}$$

where K_2 is a constant depending on the mode of propagation as given in Table 9 and r and a have the same meanings as in Table 8. The ratio of the frequency corresponding to minimum attenuation to the cutoff frequency is given in Table 9.

TABLE 9.—ATTENUATION PROPERTIES OF SQUARE AND CIRCULAR GUIDES

Mode	$\left(\frac{\text{Frequency of minimum attenuation}}{\text{Cutoff frequency}}\right)$	Minimum attenuation, relative value	K_2 (copper)
Circular Guide			
$TE_{1,1}$	3.150	1.00	170
$TM_{0,1}$	1.732	2.00	340
$TM_{1,1}$	1.732	2.53	430
Square Guide			
$TE_{0,1}$	2.960	1.867	265.5
$TE_{1,1}$	2.438	3.967	562.6
$TM_{1,1}$	1.732	3.20	453.3

Examination of the foregoing shows that *for a given periphery*, the $TE_{1,1}$ mode in the circular guide has the lowest possible attenuation. In all cases the attenuation for a cylindrical guide is less than that for the corresponding mode in a square guide of the same guide periphery. The lowest attenuation that can be obtained from a rectangular mode for a given guide periphery occurs for the $TE_{0,1}$ mode of Fig. 122 when the ratio of width to height is 1.18, though for this condition the attenuation is only a few per cent lower than for a guide of square cross section of the same periphery excited in the same mode.

34. Cavity Resonators.[1]—Any closed surface having conducting walls can support oscillating electromagnetic fields within it, and possesses certain resonant frequencies when excited by electrical oscillations. Resonators of this type are commonly termed *cavity resonators* or *rhumbatrons*, and find extensive use as resonant circuits at extremely high frequencies. For such use cavity resonators have the advantages of simplicity, relatively large physical size, high Q, and very high shunt impedances. At wave lengths well below one meter cavity resonators become vastly superior to the corresponding resonators with lumped circuit constants, and are fully as desirable—if not more so—as resonant concentric lines for such frequencies.

Types of Cavity Resonators and Modes of Oscillation.—Practical cavity resonators can be conveniently divided into two types. The first of these is the nonreentrant

[1] The basic paper on cavity resonators is that of W. W. Hansen, A Type of Electrical Resonator, *Jour. Applied Phys.*, Vol. 9, p. 654, October, 1938. See also E. U. Condon, Forced Oscillations in Cavity Resonators, *Jour. Applied Phys.*, Vol. 12, p. 129, February, 1940.

Ex. 1044-0272

type, which in idealized form is typified by the cylinder, prism, sphere, ellipsoid, etc., as shown in Figs. 131*a* to 131*d*. The second type is illustrated in Figs. 131*e* to 131*j* in various forms, some idealized and some practical, and is characterized by having a reentrant portion or portions.[1]

The fields corresponding to the normal or fundamental mode of operation of typical cavity resonators are shown in Fig. 131. In the case of a cylindrical resonator, the oscillations can be described crudely as involving electric charges or currents that flow up and down the sides of the cylinder and charge the electrostatic capacity

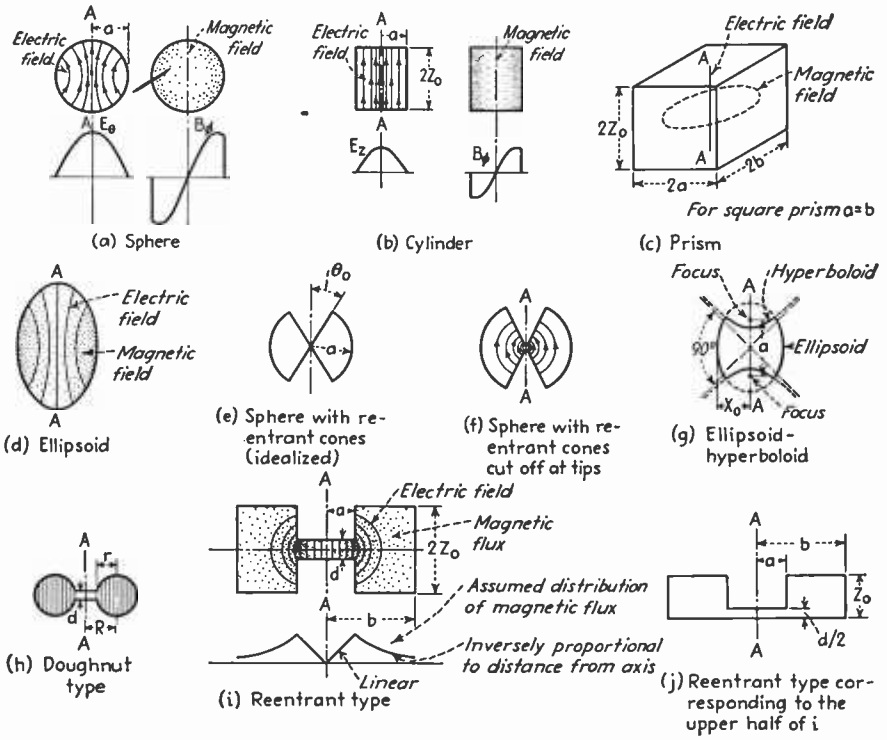


FIG. 131.—Examples of cavity resonators. All these resonators except the prism are shown as cross sections of a figure of revolution. The flux distributions shown for certain of the resonators correspond to the distributions with the normal or fundamental mode of operation.

formed by the top and bottom ends of the cylinder. The sphere is analogous. With resonators of the reentrant type operating as in Figs. 131*e* to 131*i*, the electric field is concentrated largely in the region of the reentrant portions, with most of the magnetic flux existing in the remaining regions and encircling the reentrant portions as shown. Because the electric and magnetic fields tend to concentrate in different parts of the reentrant-type cavity resonator, such a resonator is much more closely related to a circuit with lumped constants than the other forms of cavity resonators.

Higher order modes of many varieties can exist in cavity resonators. These correspond in the case of cylindrical and prism types of cavity resonators to the

[1] Characteristics of resonators of the reentrant type are discussed by W. W. Hansen and R. D. Richtmyer, On Resonators Suitable for Klystron Oscillators, *Jour. Applied Phys.*, Vol. 10, p. 189, March, 1939.

Ex. 1044-0273

higher order modes that can exist in circular and rectangular wave guides, while in the case of reentrant resonators they correspond to the higher order modes of propagation that can exist in concentric transmission lines when the conductor spacing is of the order of a half wave length or more.[1] Typical examples are shown in Fig. 132. Higher order modes correspond to a higher resonant frequency than the fundamental mode. Other properties of the resonator, such as the Q and shunt impedance, also depend upon the mode, and may be either greater or less than for the fundamental mode, according to circumstances. In particular, certain types of higher order modes have very low losses and give unusually high Q and shunt impedance. The examples of Fig. 132 are of this type. Here the fact that the magnetic flux density adjacent to the walls of the cylinder is less than for fundamental operation causes the current flowing in the walls to be small, with correspondingly lower losses and higher Q. In spite of such possible advantages obtained from higher mode operation, cavity

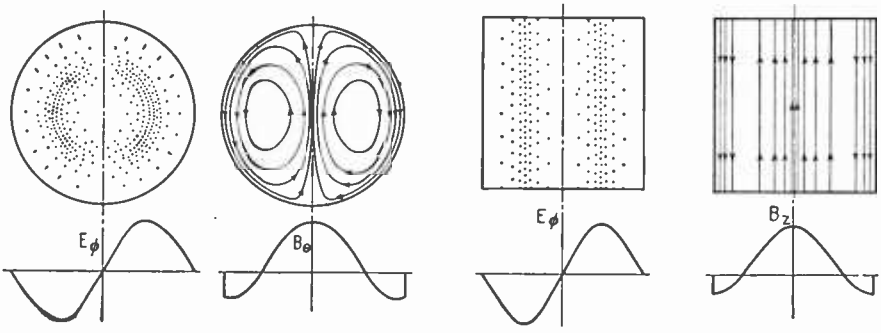


FIG. 132.—Examples of second-order mode in spherical and cylindrical cavity resonators, showing field configurations and flux distributions.

resonators are ordinarily operated in the normal or fundamental mode. This is because in the region where higher modes are possible, the frequency separation between different modes is relatively small, and there are even certain conditions under which the cavity resonator can be simultaneously resonant to more than one mode. Under these circumstances there is always the possibility that the resonator will be excited in undesired ways.

Resonant Frequency of Cavity Resonators.—The resonant frequency of a cavity resonator corresponds to a possible solution of Maxwell's equations for the electric and magnetic fields within the resonator, which will also satisfy the boundary conditions. The calculation of resonant frequency (or wave length) can be carried out exactly for geometrical shapes such as spheres, cylinders, prisms, and for certain idealized cases involving reentrant sections. Approximate solutions have been derived for certain other cases.

Formulas giving the wave length at resonance for the fundamental mode, for various simple geometrical shapes, are given in Table 10, Fig. 133, and Eqs. (150) and (151). This resonant wave length is proportional to the size of the resonator; *i.e.*, if all dimensions are doubled, the wave length corresponding to resonance will likewise be doubled. This fact simplifies the construction of resonators of shapes that cannot be calculated. To obtain a resonator operating exactly at a desired frequency, one first constructs a resonator of convenient size and of the desired

[1] See Hansen, *loc. cit.* An extensive discussion of certain types of higher modes is given by W. L. Barrow and W. W. Mieher, Natural Oscillations of Electrical Cavity Resonators, *Proc. I.R.E.*, Vol. 28, p. 184, April, 1940.

Ex. 1044-0274

TABLE 10.—PROPERTIES OF CAVITY RESONATORS

	Figure showing notation	Wave length, cm	Q	Shunt resistance, ohms	Impedance coupled into loop of area Σ sq cm, ohms
Sphere	131*a*	2.28*a*	$1.024\frac{a}{\delta}$	$81.6\frac{a}{\delta}$	$354\frac{\Sigma^2}{a^3\delta}$
Cylinder	131*b*	2.61*a*	$1.414\frac{a}{\delta}\frac{1}{1+\frac{a}{2z_0}}$	$204\frac{z_0}{\delta}\frac{1}{1+\frac{a}{2z_0}}$	$204\left(\frac{\Sigma}{az_0}\right)^2\frac{z_0}{\delta}\frac{1}{1+\frac{a}{2z_0}}$
Prism (square)	131*c*	2.83*a*	$1.41\frac{a}{\delta}\frac{1}{1+\frac{a}{2z_0}}$	$182.1\frac{z_0}{\delta}\frac{1}{1+\frac{a}{2z_0}}$	$296\left(\frac{\Sigma}{az_0}\right)^2\frac{z_0}{\delta}\frac{1}{1+\frac{a}{2z_0}}$
Spheres with reentrant cones	131*e*	4*a*	Fig. 134	Fig. 136	
Ellipsoid-hyperboloid	131*g*	See Fig. 133	Fig. 135	Fig. 137	
Toroidal type	131*h*	See Eq. (151)			
Toroidal type	131*i*	See Eq. (150)	$\frac{2.83\frac{z_0}{\lambda}\log_e\frac{b}{a}}{\frac{z_0}{a}\left(1+\frac{a}{b}\right)+\log_e\frac{b}{a}}\frac{\lambda}{\delta}$	$2{,}130\left(\frac{z_0}{\lambda}\right)^2\frac{\log_e^2\frac{b}{a}}{\frac{z_0}{a}\left(1+\frac{a}{b}\right)+\log_e\frac{b}{a}}\frac{\lambda}{\delta}$	
Toroidal type	131*j*	See Eq. (150)	Same as Fig. 131*i*	$1{,}065\left(\frac{z_0}{\lambda}\right)^2\frac{\log_e^2\frac{b}{a}}{\frac{z_0}{a}\left(1+\frac{a}{b}\right)+\log_e\frac{b}{a}}\frac{\lambda}{\alpha}$	

All dimensions in centimeters.
δ defined as in Eq. (153); for copper $\delta = 54\times10^{-8}\sqrt{\lambda_{cm}}$.
Formulas for Q and shunt resistance for type of Figs. 131*i* and 131*j* assume flux distribution of Fig. 131*i*.
Notation as in Fig. 131.

Ex. 1044-0275

proportions. The resulting resonant frequency is then measured. The ratio of this wave length to the desired resonant wave length gives a scale factor that is applied to every dimension of the final resonator to obtain the dimensions of the resonator giving the desired wave length.

With cylinders the resonant wave length is determined only by the diameter, and is not affected by the height. Similarly, the height has no effect in the case of the prism. A sphere with reentrant cones (Fig. 131*e*) has a wave length independent of the cone semiangle θ, and only moderately greater than for the same sphere without reentrant cones. These relations apply to fundamental modes.

The resonant frequency of resonators such as illustrated in Figs. 131*h* to 131*j*, which have a large lumped capacity, can be calculated crudely on the assumption of lumped constants; *i.e.*, one determines the capacity between the flat surfaces AA,

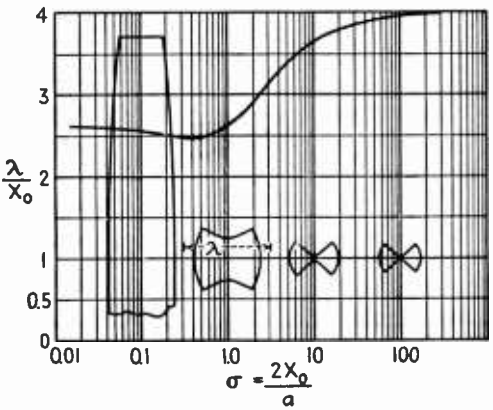


FIG. 133.—Curve giving wave length of ellipsoid-hyperboloid resonators of the type shown in Fig. 131*g* for different values of the shape factor σ_0. The little drawings show resonator cross sections corresponding to different values of σ_0, and are so scaled that all give the same wave length. Only the upper half of the drawing corresponding to $\sigma_0 = 0.1$ is shown.

and assumes that this tunes a lumped inductance corresponding to that of a one-turn toroidal coil, having a cross section such as indicated by the shading in Fig. 131*h*. For the reentrant resonator of Figs. 131*i* and 131*j* this leads to the relation[1]

$$\left.\begin{matrix}\text{Wave length in cm}\\ \text{for resonance}\end{matrix}\right\} = 2\pi\left(\frac{Z_0 a^2}{d}\log_e\frac{b}{a}\right)^{1/2} \tag{150}$$

Similarly, for Fig. 131*h*

$$\left.\begin{matrix}\text{Wave length in cm}\\ \text{for resonance}\end{matrix}\right\} = 2\pi\left\{\frac{\pi r^2 R}{d}\left(1-\sqrt{1-\left(\frac{r}{R}\right)^2}\right)\right\}^{1/2} \tag{151}$$

The notation is as shown in Fig. 131 (dimensions in cm). Results calculated by Eqs. (150) and (151) always give a wave length less than the correct value. The error depends upon the extent to which the electric field is concentrated in the narrow gap, *i.e.*, upon the smallness of the dimension d. The factor by which Eq. (150) must be multiplied to obtain the correct wave length is commonly of the order of 1.25 to 1.75.

[1] A more accurate determination can be made for this case by methods described by W. W. Hansen, On the Resonant Frequency of Closed Concentric Lines, *Jour. Applied Phys.*, Vol. 10, No. 1, January, 1939.

Ex. 1044-0276

The resonant frequency of a cavity resonator can be changed by altering the mechanical dimensions, by coupling reactance into the resonator, or by means of a copper paddle. Small changes in mechanical dimensions can be achieved by flexing walls, while large changes require some type of sliding member. The resonant frequency of the reentrant type of resonator is particularly sensitive to the gap between the reentrant portions, especially if this gap is small. Reactance can be coupled into the resonator by coupling loops or probes in the manner discussed below, and affects the resonant frequency. A copper paddle placed inside the resonator in such a manner as to intercept some of the flux lines corresponding to the normal distribution will raise the resonant frequency. Rotation of the paddle will vary this effect and so can be used to adjust the resonant frequency, exactly as a copper paddle can be used to produce small variations in the inductance of a coil.

Q of Cavity Resonators.—The Q of a cavity resonator has the same significance as for an ordinary resonant circuit, and can be defined on the basis that Q is a quantity such that when the response has dropped to 70.7 per cent of the response at resonance, the cycles off resonance is the resonant frequency divided by $2Q$. In the case of cavity resonators, it is also convenient to think of Q as expressing the ratio of the energy stored in the fields of the resonator to the energy lost per cycle, according to the relation

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy lost per cycle}} \tag{152}$$

In terms of the magnetic field H within the resonator, this is

$$Q = \frac{2\sqrt{2}}{\delta} \frac{\int H^2\, dv}{\int H^2 |da|} \tag{153}$$

where H = magnetic intensity.

$\delta = \sqrt{\rho/\pi\omega}$.

ρ = resistivity in abohms (cmu) per cm cube (for copper $\rho = 1{,}724$ abohms per cm cube).

$\omega = 2\pi$ times frequency.

The integral of the numerator is evaluated throughout the volume v of the resonator, and the integral of the denominator is taken over the area a of the inner surface of the resonator.

The Q of a resonator of any shape can be evaluated with the aid of Eq. (153) for any mode of oscillation provided that the distribution of magnetic flux is known. In simple geometrical shapes, such as spheres and cylinders, the flux distribution may be expressed analytically. The indicated integrations can then be carried out without particular difficulty, and exact formulas for Q are obtained. In other cases it is necessary to assume an approximate distribution of flux, and then to carry out the integrations either analytically or graphically as the occasion permits. The value of Q obtained in this way will not be exactly correct, but the error is small even if the assumed flux distribution differs appreciably from that actually present. This is because the flux appears both in the numerator and denominator of Eq. (153).

An important implication of Eq. (152) is that to obtain high Q, the resonator should have a large ratio of volume to surface area. The physical reason for this is apparent when it is realized that energy is stored in the volume of the resonator, whereas all the energy lost is lost at the walls. As a consequence, resonators such as spheres, cylinders, and prisms can in general be expected to have higher Q than the corresponding resonators with reentrant sections. The loss in Q resulting from reentrant sections is not excessive, however, unless the reentrant portions become very sharp spines. With resonators of the same proportions but different size, Eq. (152) shows that the Q's will be proportional to the square root of the wave length.

Ex. 1044-0277

TABLE 11.—TYPICAL CAVITY RESONATOR PROPERTIES
Copper walls — $\lambda = 40$ cm*

Type of resonator	Physical size	Q	Shunt resistance, megohms
Sphere	$a = 17.5$ cm	53,000	4.2
Cylinder (diameter = height; $z = a$)	$a = 15.3$ cm	48,800	6.1
Sphere with reentrant cones ($\theta_0 = 34°$)	$a = 10$ cm	18,200	3.5
Reentrant type (Fig. 131*i*): $\frac{b}{a} = 3.33$, $\frac{Z_0}{a} = 1.20$	$Z_0 = 2.8$ cm, $a = 2.8$ cm, $b = 3.9$ cm } approx.	10,200	0.66

* For other values of λ, the dimensions are proportional to λ, while Q and shunt resistance vary as $\lambda^{1/2}$

Values of Q for a number of types of resonators can be obtained from Table 10, Figs. 134 and 135. In the case of a sphere with reentrant cones (Fig. 131*c*) the Q passes through a broad maximum when the half angle of the cone is 34°. The Q of the ellipsoid-hyperboloid cavity resonator of Fig. 131*f* is substantially the same as that of a sphere having the diameter $2x_0$, provided that the distance a between the focuses of the ellipse is somewhat greater than the diameter. When the reverse is the case, the Q drops to about one-third that of the corresponding sphere.

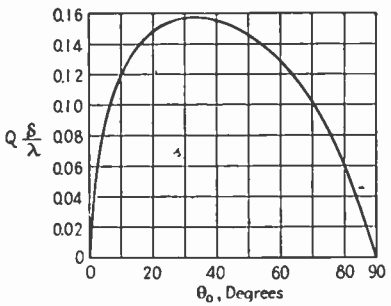


FIG. 134.—Graph showing variation in Q for the resonator of Fig. 131*e* with the angle θ_0. The notation is the same as used in Table 10 and Fig. 131*e*.

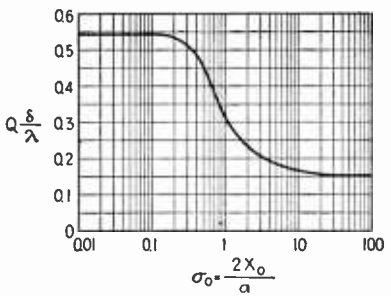


FIG. 135.—Curve giving Q of ellipsoid-hyperboloid type of cavity resonator shown in Figs. 131*g* and 133. The notation is as in Fig. 133 and Table 10.

Typical values of Q obtainable in practical cavity resonators are given in Table 11, and are extremely high compared with those encountered with ordinary resonant circuits.

Shunt Impedance of Cavity Resonators.—The shunt impedance of a cavity resonator such as those of Fig. 131 can be defined as the square of the line integral of voltage along a path such as AA, divided by the power loss in the resonator. This impedance corresponds to the parallel resonant impedance of a tuned circuit, and at resonance becomes a resistance termed the shunt resistance of the resonator.

Ex. 1044-0278

The shunt impedance of a number of types of cavity resonators can be calculated with the aid of Table 10 or Figs. 136 and 137. The values obtainable are very large, as seen from typical results given in Table 11.

The shunt resistance obtainable with cavity resonators of the reentrant type is less than without reentrants. However, this impedance is developed across a much shorter distance in the reentrant case; so the reentrant type of cavity resonator gives a much greater shunt impedance *in proportion to distance* across which the impedance

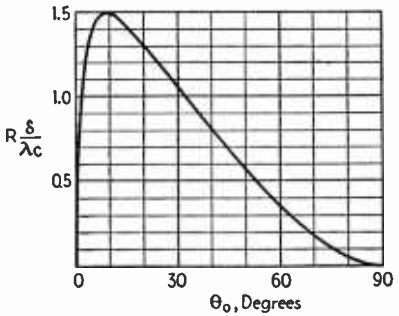


FIG. 136.—Variation of shunt impedance R of spherical resonator with reentrant cone (Fig. 131*e*) as a function of the semiangle θ of the cones (for notation see Fig. 131*e* and Table 10).

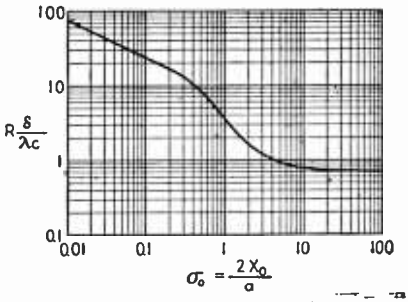


FIG. 137.—Curve giving shunt resistance for ellipsoid-hyperboloid resonators of the type shown in Fig. 131*g* for the various shapes shown in Fig. 133 (for notation see Fig. 131*g* and Table 10).

is developed. This is of importance when the resonator is associated with an electron beam, since it gives a high impedance across a distance small enough to correspond to a very short transit time for electrons.

Coupling to Cavity Resonators.—Coupling to a cavity resonator may be achieved by means of electron beams, by coupling loops, or coupling electrodes. Coupling by means of an electron beam can be accomplished by passing the beam through the resonator, as indicated in Fig. 138. In such an arrangement, it is necessary that the transit time required by the electrons to pass through the resonator be small compared with the time of a cycle. This in general requires the use of resonators of the reentrant type, in order to keep down the distance that the electrons must travel within the resonator. If the electrons in such a beam pass through the resonator in the form of bunches, one to each cycle, then the resonator will be excited by the beam, and if the resonator proportions are such that the resonant frequency approximates the frequency of the bunches, then oscillations of large amplitude will be built up in the cavity. On the other hand, if the resonator is excited by some external source of energy, and a continuous stream of electrons is passed through the resonator, then the emerging electrons will be alternately accelerated and slowed down by the oscillations within the resonator.

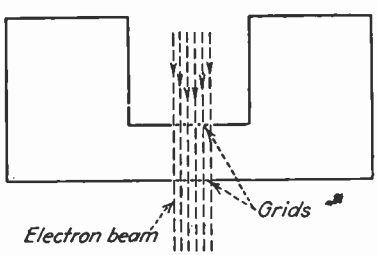


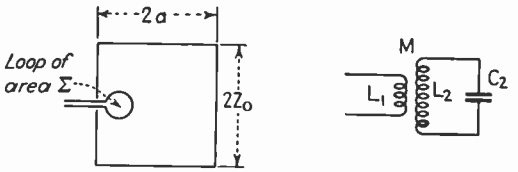
FIG. 138.—Cavity resonator of the type shown in Fig. 131*j* coupled to an electron beam that passes through the resonator.

Magnetic coupling to a cavity resonator can be obtained by means of a small coupling loop so oriented as to inclose magnetic flux lines corresponding to the desired mode of operation. A current passed through such a loop will then excite oscillations

Ex. 1044-0279

of this mode, while conversely oscillation existing in the resonator will induce a voltage in such a coupling loop.

The combination of coupling loop and cavity resonator is equivalent to the ordinary coupled circuit of Fig. 139. The impedance that the cavity resonator couples into the loop (*i.e.*, the equivalent impedance that can be considered as in



(a) Cylindrical resonator with coupling loop (b) Equivalent coupled circuit

FIG. 139.—Cylindrical resonator with coupling loop, together with equivalent coupled circuit.

series with L_1) is given in Table 10 for the case of a cylinder, square prism, and sphere, on the assumption that the coupling loop is placed close to the outer edge where the magnetic flux density is maximum. The ratio between the impedance coupled into the loop and the shunt resistance of a cavity resonator is roughly equal to the square of the ratio (loop area)/(half the cross-sectional area of the resonator). This rule may be used to estimate the size of coupling loop that will be required in resonators of types other than those given in Table 10. These relations assume that the loop is oriented so that its area is in the most favorable position for inclosing magnetic flux. The magnitude of coupling can be readily changed by simply rotating the loop, the coupling reducing to zero when the plane of the loop is parallel to the magnetic flux.

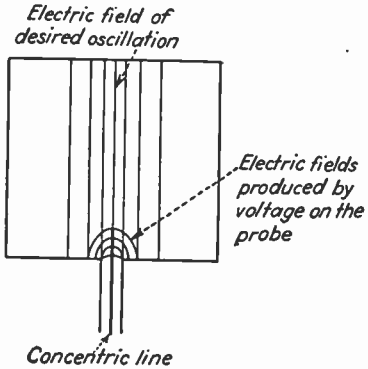


FIG. 140.—Cylindrical resonator with coupling probe, showing fields of desired oscillation and fields produced by the probe. It will be noted that the probe fields have a component in the direction of the desired field, and therefore are capable of exciting them.

Cavity resonator excitation by means of an electrode is illustrated in Fig. 140. Here there is a component of electric field produced by the presence of the electrode (or probe) that is in the same direction as the electric field of the desired oscillation in the resonator. Under these conditions a voltage applied to the electrode will excite oscillations in the resonator, and conversely oscillations in the resonator will deliver energy to the concentric line associated with the electrode.

It is also possible to couple cavity resonators to wave guides. This is accomplished by arranging so that the waves entering the cavity from the guide produce a field within the cavity that corresponds to the desired mode of cavity-resonator operation.

Miscellaneous Considerations.—A hole in a cavity resonator will cause energy to be radiated if oscillations exist inside the resonator, or, conversely, will cause oscillations to be produced within the resonator if oscillating fields external to the resonator are present. The loss of energy caused by radiation from holes is of particular importance under conditions where grids must be used in the sides of the resonator to allow for the entrance or exit of electron beams. The magnitude of energy loss through a

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hole is proportional to the cube of the hole area (sixth power of the hole radius). In the case of a square prism one has[1]

$$Q' = 2\pi \frac{\text{energy stored inside resonator}}{\text{energy lost through hole per cycle}}$$
$$= 37 \left(\frac{a^2}{\Sigma}\right)^3 \frac{z_0}{a}$$

where Σ is the area of the hole and a and z_0 are the same as in Fig. 131c and Table 10. Note that Q' is the value of Q the resonator would have if there were no losses in the resonator other than the energy leaking out of the hole. It is easy to see that the losses from a hole will be quite negligible unless this hole is of relatively large size.

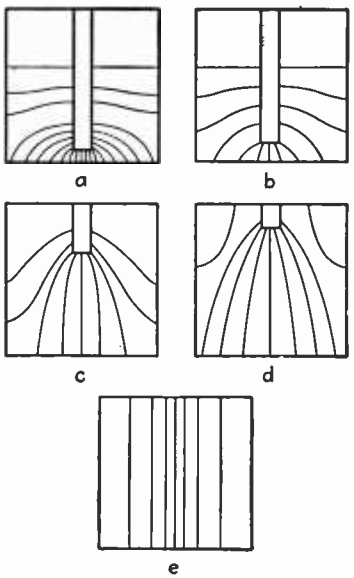


FIG. 141.—Transition of a reentrant cavity resonator of the concentric-line type to a cavity resonator that is a cylinder.

In designing a cavity resonator, it is desirable that all joints be parallel with the lines of current flow. Joints that must be traversed by currents can introduce large losses, because the currents that must be carried are extremely high and even a very low resistance in the joint will seriously reduce the Q and shunt resistance of the resonator.

The dividing line between resonant transmission lines and cavity resonators is not at all distinct. For example, one can consider a quarter-wave resonant concentric transmission line, as illustrated in Fig. 141,[2] as being a cavity resonator with a reentrant section. Furthermore, by progressively shortening this reentrant section as shown in the figure, one finally ends up with a cylindrical cavity resonator. In a like manner, it is possible to show that a cavity resonator may be also viewed as a section of wave guide in which standing waves exist.

[1] See Hansen, *loc. cit.*
[2] From Barrow and Mieher, *loc. cit.*

Ex. 1044-0281

SECTION 4

VACUUM TUBES AND ELECTRONICS[1]

LAWS OF ELECTRON BEHAVIOR

1. Physical Properties of Electrons and Ions.—For purposes of radio engineering, the most fundamental particle is the electron. The electron is the smallest particle whose mass and charge have been determined. It is the particle that constitutes most of the current flow encountered in vacuum tubes. It is the negative planetary particle of the hydrogen atom.

Another fundamental particle is the proton. The proton is the positive nuclear particle of the hydrogen atom. It has the same magnitude of charge as the electron, but its charge is positive, whereas the charge of the electron is negative. The mass of the proton is roughly 1,800 times that of the electron.

Other charged particles called ions may exist. These are usually created from a normal atom or molecule by the addition or removal of one or more electrons or protons from its structure. The mass of such ions is always much greater than that of the electron, and their charge is an integral multiple of the charge of the electron and is not restricted to the negative sign.

The principal properties of electrons and the commonest ions are given in Table 1.[2]

TABLE 1.—PHYSICAL PROPERTIES OF ELECTRONS AND IONS

Charge	
Electronic charge (e)	4.767×10^{-10} statcoulomb
	1.590×10^{-20} abcoulomb
	1.590×10^{-19} coulomb
Mass	
Mass of an electron*	9.038×10^{-28} gram
Mass of a proton	1.6608×10^{-24} gram
Mass of a hydrogen atom	1.6617×10^{-24} gram
Mass of hypothetical atom of unit atomic weight (m_0)	1.649×10^{-24} gram
Ratios	
Charge to mass of an electron*	1.769×10^{7} abcoulombs per gram
	5.303×10^{17} statcoulombs per gram
	1.769×10^{8} coulombs per gram
Mass of hydrogen atom to that of electron*	1,848.0
Atomic Weights	
Argon	39.94
Helium	4.002
Hydrogen	1.0078
Mercury	200.61
Nitrogen	14.008
Neon	20.183
Oxygen	16.000
Sodium	22.997

* Values from deflection measurements. Spectroscopic values differ by at most 0.1 per cent.

[1] This entire section was written by the author's associate, Dr. Karl Spangenberg, associate professor of electrical engineering at Stanford University.

[2] R. T. Birge, Values of the General Physical Constants, *Rev. Modern Phys.*, Vol. 1, No. 1, p. 1, July, 1929.

Ex. 1044-0282

2. Motion of Electrons. *Uniform Electrostatic Field.*—The laws of motion of an electron in a uniform electrostatic field are the same as those of a body falling freely under the influence of gravity. In the case of the electron the gravitational field is replaced by the factor eF/m, when the relations are expressed in electrostatic units, where e is the charge of the electron, F is the constant gradient of potential, and m is the mass of the electron.

The principal relations for the electron in a uniform electrostatic field are given below:

Acceleration:

$$a = \frac{eF}{m} \text{ cm per sec per sec} \qquad \text{(esu)} \tag{1}$$

Velocity (starting from rest):

$$v = \frac{eFt}{m} \text{ cm per sec} \qquad \text{(esu)} \tag{2}$$

$$v = \sqrt{\frac{2eF}{m}s} \text{ cm per sec} \qquad \text{(esu)} \tag{3}$$

Distance traveled (starting from rest):

$$s = \frac{eFt^2}{2m} \text{ cm} \qquad \text{(esu)} \tag{4}$$

where a = electron acceleration, cm per sec per sec.
v = electron velocity, cm per sec.
e = electron charge, statcoulombs.
m = electron mass, grams.
F = gradient of potential, statvolts per cm.
E = potential, statvolts.
s = distance, cm.
t = time, sec.

In practical units the velocity acquired by an electron in falling through a given potential is

$$v = 5.97 \times 10^7 \sqrt{E} \text{ cm per sec} \tag{5a}$$

where the potential E is given in volts.

Velocity is often expressed in terms of volts, the number of volts corresponding to a given velocity being given by Eq. (5*a*). Thus by a velocity of 1,000 volts is meant the velocity that an electron would acquire upon being accelerated through 1,000 volts. This is not quite correct dimensionally, since voltage to the first power is proportional to energy, but no confusion should result if the term velocity is associated with the voltage.

Other ions follow the same law as do electrons, except that the scale factor is different because of the increased mass and in some cases also because of a different charge. The relation between velocity and potential for any ion is given by

$$v = 5.97 \times 10^7 \sqrt{\frac{Enm_e}{m}} \text{ cm per sec} \tag{5b}$$

where m_e is the mass of the electron, m the mass of the ion, and n the ratio of ion charge to electron charge.

Relativity Correction for Velocity.—Equation (5) is valid until the velocity of the charged particle is no longer negligible compared with the velocity of light. Then a correction must be made for the relativistic change of mass. This change, and the resulting error in the velocity, is less than one per cent as long as the velocity of the particle is less than one-tenth of the velocity of light. The potential at which an

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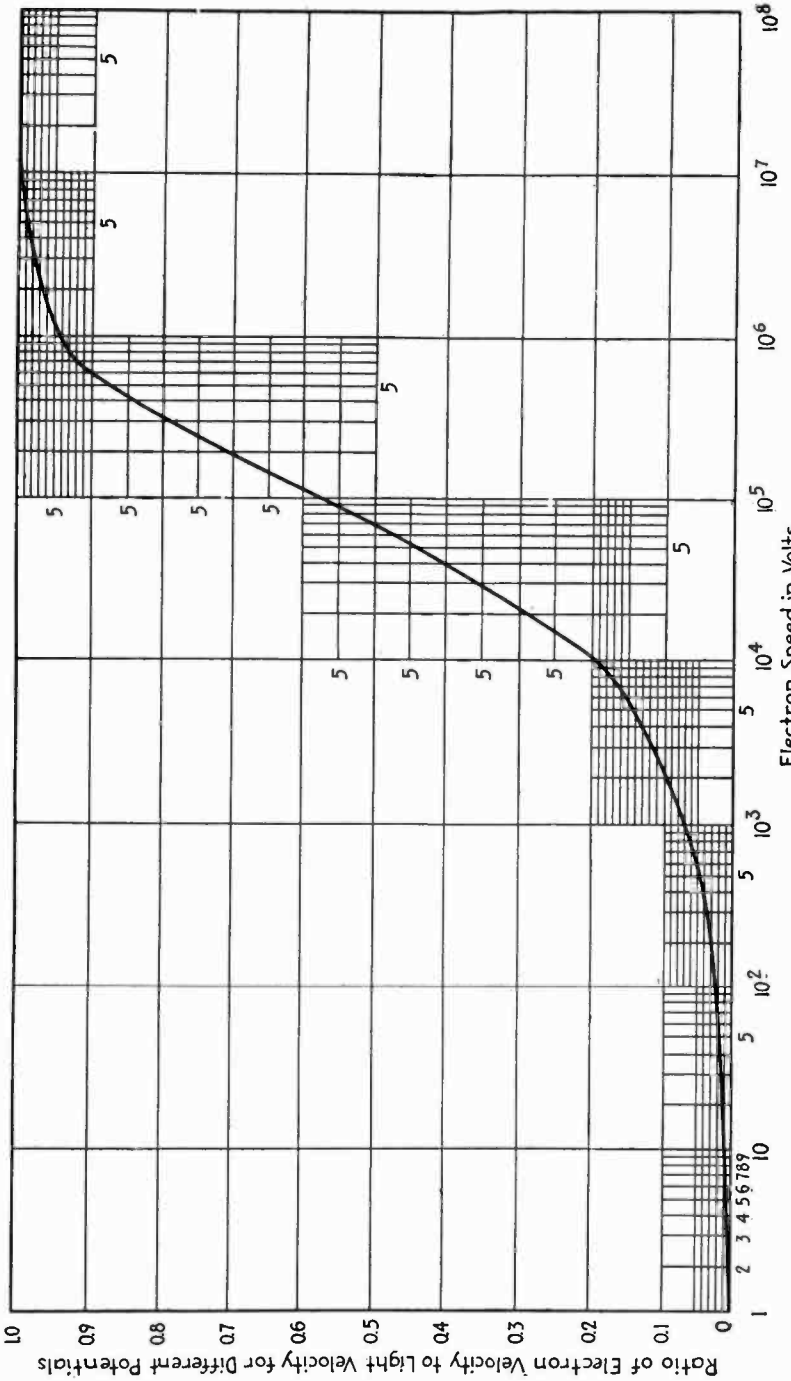


FIG. 1.—Relation between electron velocity and potential.

Ex. 1044-0284

electron reaches this velocity is about 2,500 volts. If there were no change of mass, an electron would reach the velocity of light at about 250,000 volts.

The change in mass of an electron with velocity is given by

$$m = \frac{m_e}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \tag{6}$$

which for an electron, in terms of potential is

$$m = m_e(1 + 1.94 \times 10^{-6}E) \tag{7}$$

where m is the mass in grams of the electron at velocity v or potential E, c is the velocity of light (3×10^{10} cm per sec), and E is the potential in volts

The velocity corresponding to any potential is given by

$$v = c\sqrt{1 - \frac{1}{(1 + 1.94 \times 10^{-6}E)^2}} \tag{8}$$

A plot of this equation is given in Fig. 1.

Initial Velocity at an Angle to Gradient of Uniform Field.—If an electron traveling with an initial velocity is injected into a region of uniform field, the path of the electron in the field will be parabolic and will be like that of a projectile fired from a gun in a gravitational field.

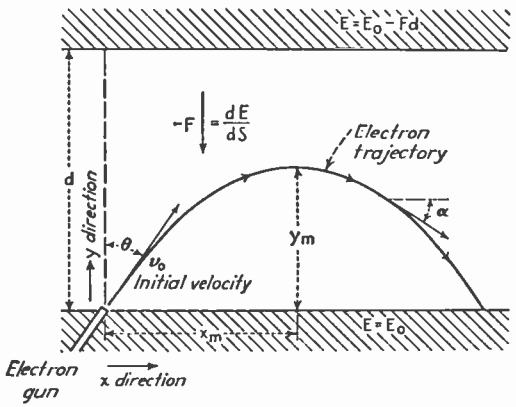


FIG. 2.—Parabolic electron trajectory in a uniform electric field.

Consider Fig. 2, which shows the path of an electron that is injected into the constant field between two electrodes. Here there is an initial component of velocity in the horizontal or x direction that is unaffected by the field, and that remains constant. The component of velocity in the vertical or y direction varies like that of a ball thrown vertically into the air.

The equation of the parabolic trajectory followed is

$$y = \frac{-Fx^2}{4E_0 \sin^2 \theta} + \frac{x}{\tan \theta} \tag{9}$$

the origin being taken at the point where the electron enters the field. The components of velocity at any point are

$$v_x = v_0 \sin \theta \text{ cm per sec} \tag{10}$$

$$v_y = \sqrt{v_0^2 \cos^2 \theta - \frac{2eFy}{m}} \text{ cm per sec} \tag{11}$$

Ex. 1044-0285

The angle that the instantaneous velocity makes with the horizontal at any point is given by

$$\tan \alpha = \frac{-xF}{2E_0 \sin^2 \theta} + \frac{1}{\tan \theta} \tag{12}$$

The maximum height to which the electron will rise, and the x component corresponding to this position, are

$$y_m = \frac{E \cos^2 \theta}{F} \tag{13}$$

$$x_m = \frac{2E \sin \theta \cos \theta}{F} \tag{14}$$

The initial components of velocity are

$$v_x = \sqrt{Ee \sin^2 \theta} \tag{15}$$

$$v_y = \sqrt{Ee \cos^2 \theta} \tag{16}$$

In the relations given above the symbols have the following significance:

x = horizontal displacement of electron, cm.
y = vertical displacement of electron, cm.
F = gradient of potential, statvolts per cm.
θ = angle that direction of original electron velocity makes with gradient (see Fig. 2).
v_x = horizontal velocity component of electron, cm per sec.
v_y = vertical velocity component of electron, cm per sec.
v_0 = original velocity of electron, cm per sec.
E_0 = potential at point where electron enters field, statvolts.
α = slope angle of parabolic path with horizontal at any point along electron trajectory.
e = electron charge, statcoulombs.

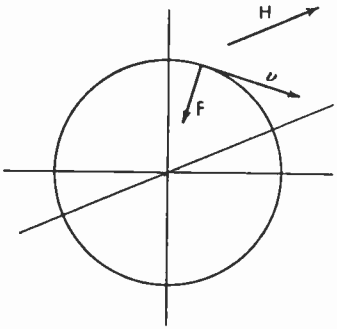


FIG. 3.—Direction of force on an electron relative to the magnetic field and the direction of velocity.

Electron in a Uniform Magnetic Field.—An electron moving at right angles to a uniform magnetic field experiences an instantaneous force whose direction is mutually perpendicular to the field and the electron velocity. This is because the electron in motion constitutes an electric current and as such is subject to the sidewise force that is experienced by a conductor in a magnetic field. The sidewise force changes the direction but not the magnitude of the velocity, with the result that the electron moves in a circular path such that the magnetic force balances the centrifugal force of the circular motion. The geometrical relation between velocity, field, and force is shown in Fig. 3. The force developed is always at right angles to both the velocity and the field.

The expression for the radius of the circular path obtained by equating the magnetic and centrifugal force is

$$r = \frac{m_e v}{eB} \tag{17}$$

Ex. 1044-0286

where r = radius of circular path, cm.
e = electron charge, abcoulombs.
m_e = electron mass, grams.
v = electron velocity, cm per sec.
B = magnetic field, lines per sq cm.

Expressing this in practical units and introducing the expression for velocity in terms of the square root of potential:

$$r = \frac{3.37\sqrt{E}}{B} \text{ cm} \tag{18}$$

where E is the electron velocity expressed in volts. The radius of the circular path is seen to be proportional to the square root of potential and inversely proportional to

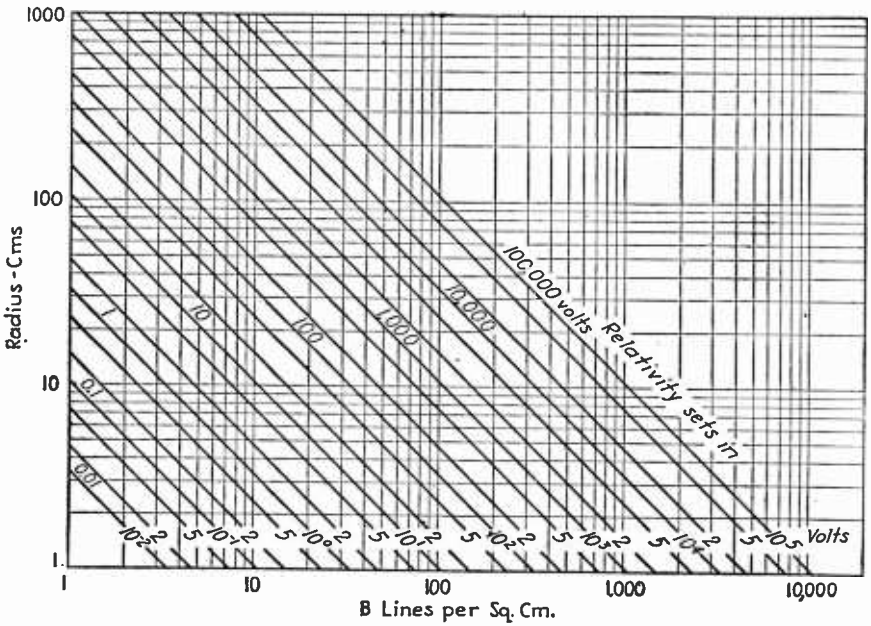


FIG. 4.—Curves of radius of circular electron path in a uniform magnetic field.

the magnetic field strength. The relation between the factors is given by the curve of Fig. 4.

If the particle is not an electron but an ion of mass m and with n times the charge of an electron, then the result of Eq. (18) must be multiplied by the factor $(m/m_e n)^{1/2}$.

The time required for the electron to make a complete circle, *i.e.*, the period, does not depend upon the voltage at which the electron is injected into the magnetic field. This is because the radius, and hence the circumference, of the circle is proportional to the velocity, thus making the period independent of either. The period T is given by

$$T = \frac{2\pi m_e}{e}\frac{1}{B} \text{ seconds} \qquad \text{(emu)} \tag{19}$$

Ex. 1044-0287

In practical units this becomes

$$T = \frac{0.355 \times 10^{-6}}{B} \text{ seconds} \tag{20}$$

The value of the period can be obtained from the curves of Fig. 4 by observing that the period in microseconds is the same as the radius in cm when the potential is 0.01122 volts.

For particles of mass m, having n times the charge of the electron, the value of Eq. (20) must be multiplied by the factor $(m/m_e n)$.

Electron at an Angle with a Uniform Magnetic Field.—If an electron is injected into a magnetic field at an angle with the field, the initial velocity can always be resolved into components parallel and perpendicular to the field. The component parallel to the magnetic field experiences no force due to the field and so is unchanged in either magnitude or direction. The component normal to the field is subject to a constant sidewise force and so undergoes the same change in direction as that described above, giving rise to a circular component of motion. The combination of the circular and linear motion gives rise to a helical path. The equations given above are directly applicable to the circular component of motion if for the velocity there be substituted the normal component of velocity.

The radius of the helix is given by

$$r = \frac{3.37 \sqrt{E} \sin \theta}{B} \text{ cm} \tag{21}$$

where θ is the angle that the initial velocity makes with the field, E is the velocity in volts, and B is the strength of the magnetic field in lines per sq cm.

The pitch p of the helix is given by

$$p = \frac{21.2 \sqrt{E} \cos \theta}{B} \text{ cm} \tag{22}$$

All the preceding remarks have assumed a uniform magnetic field. When the field is strong enough and the electron velocity low enough then the electron will move in a tightly wound helix that will follow magnetic flux lines even though these be slightly curved. Use is made of this property in the Orthicon television pickup tube.

3. Emission.[1]—The emission of electrons from metals can occur in several ways. If a metal is heated to a sufficiently high temperature emission will occur. This is known as primary or thermionic emission. Secondary emission is that which occurs when high-velocity electrons or ions strike a metal and knock out other electrons. Emission may also occur when light rays strike a material and give up energy that liberates electrons; this is known as photoemission. Thermionic and secondary emission are of most importance in ordinary vacuum tubes, and these types of emission will be the subject of this section.

Theory of Emission.—In any conductor there are a large number of free electrons moving around with various velocities. As the temperature of the conductor increases the average velocity of the free electrons increases. At the surface of the conductor there exists a restraint upon the movement of the electrons in the form of an electrostatic gravitational force. Were it not for this all the electrons in the metal would soon escape. As it is, only those having enough energy to overcome the surface forces manage to get away. The energy in electron volts that an electron must have

[1] A good introductory survey to the subject of thermionic emission is given by S. Dushman, Electron Emission, *Elec. Eng.*, Vol. 53, p. 1054, July, 1934.

Ex. 1044-0288

before it can leave a metal is known as the *work function* of the metal. It would be expected that the emission of electrons would depend upon the temperature of the metal and the work function of the particular metal involved. This is found to be the case.

A law giving the relation between emission current and temperature was first proposed by Richardson.[1] Later another emission equation was proposed by Dushman.[2] Dushman's equation is only slightly different from Richardson's, but is

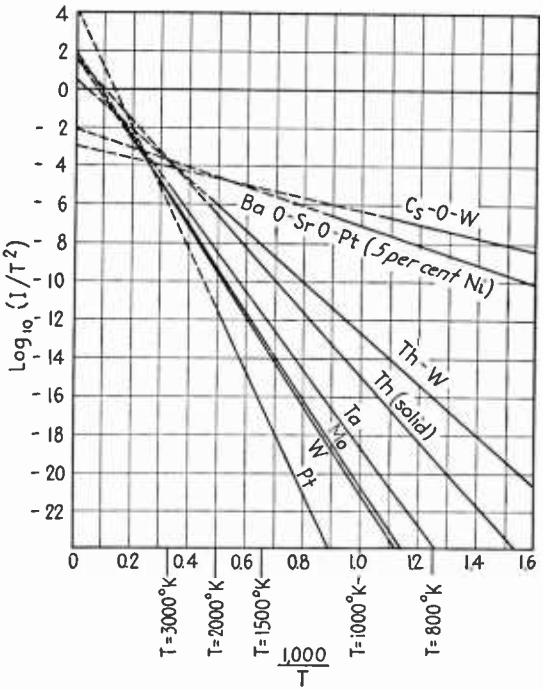


FIG. 5.—Determination of emission equation constants.

based upon better theoretical considerations and is now deemed to be the best expression available.

Dushman's equation is

$$I = AT^2\epsilon^{\frac{-b_0}{T}} \tag{23}$$

where I = current, amperes per cm².
A = 120.4 amperes per cm² deg², a universal constant.
T = absolute temperature, °K.
b_0 = temperature equivalent of the work function.
= 11,600 E_w, °K.
E_w = work function of the metal, volts.

The form of the emission equation is such that if the logarithm of I/T^2 be plotted against $1/T$, a straight line results whose slope gives the quantity b_0, and whose intercept on the I/T^2 axis gives the constant A. Such curves are shown in Fig. 5

[1] O. W. Richardson, "Emission of Electricity from Hot Bodies," Longmans, New York, 1921.
[2] S. Dushman, Thermionic Emission, *Rev. Modern Phys.*, Vol. 2, No. 4, p. 382, October, 1930. This paper gives a review of the subject up to its date, including extensive references.

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and represent the most accurate known method of determining the emission constants. Examination of the lines in Fig. 5 shows considerable variation in the intercept, with the pure metals tending to have an intercept of about 1.78, corresponding to a value of A of 60, instead of an intercept of 2.08, corresponding to a value of A of 120.4. Since the actually observed values of the constant A range from higher to lower than the theoretical value, an explanation of the effect must be found elsewhere than in incorrectness of the theory. It has been suggested that the observed value of the constant A of 60, which is about half of the theoretically predicted value of 120.4, is due to a temperature variation of the work function. A decrease in the work function of 6 parts per 100,000 per degree would effect the observed change.

Emission of Pure Metals.—Pure metals follow the form of the Dushman equation, as indicated by the straight-line nature of the curves of Fig. 5. The constants of the commonest pure metals as determined from emission measurements are given in Table 2. The next to last column of this table gives the current per square centimeter emission at the absolute temperature given in the last column.

TABLE 2.—EMISSION OF PURE METALS

Metal	A, $\frac{\text{amperes}}{\text{cm}^2\ \text{deg}^2}$	$b_0 \times 10^{-4}$, °K	E_w, volts	I_T, $\frac{\text{amperes}}{\text{cm}^2}$	T, °K
Cs	162	2.10	1.81	2.5×10^{-11}	500
Mo	60.2	5.09	4.38	2.34×10^{-2}	2,000
Pt	1.7×10^4	7.25	6.27	9.2×10^{-10}	1,600
Ta	60.2	4.72	4.07	1.38×10^{-2}	2,000
W	60.2	5.24	4.52	1.0×10^{-3}	2,000
Th	60.2	3.89	3.35	4.3×10^{-3}	1,600
Zr	330	4.79	4.13	8.5×10^{-5}	1,600

Because of their relatively low emission, pure metals are not used much, tungsten being the only pure-metal emitter of practical importance. Extensive data on the emission properties of pure tungsten have been collected, largely by Langmuir.[1] The emission constant of platinum, as reported by various investigators, shows great variation. Most recently reported values give a smaller value of A than indicated in Table 2. The variation is probably due to the difficulty of obtaining a clean surface free from occluded oxygen.

The only metal other than tungsten that has possibilities as a practical emitter is tantalum. Although tantalum cannot be made as hot as tungsten because its melting temperature is 3300°K, compared with 3655°K for tungsten, its work function is sufficiently lower, so that at any temperature less than 2500°K, its emission is at least 10 times that from tungsten. A disadvantage of tantalum is that it is easily contaminated by residual gases, which form oxides that greatly reduce the emission.

Emission of Thoriated Tungsten.—There are many metals that have a higher emission than tungsten, but it is not possible to take advantage of this by using the metals in the pure form, because most of them melt at a relatively low temperature. It is, however, possible to coat tungsten with a thin layer of other metals and thus

[1] Well summarized by E. L. Chaffee, "Theory of Thermionic Vacuum Tubes," pp. 96–102, McGraw-Hill, New York, 1933. See also I. Langmuir, The Characteristics of Tungsten Filaments as Functions of Temperature, *Phys. Rev.*, Vol. 7, p. 302; C. Davisson and L. H. Germer, The Thermionic Work Function of Tungsten, *Phys. Rev.*, Vol. 20, p. 300, April, 1922; S. Dushman, H. N. Rowe, J. W. Ewald, and C. A. Kidner, Electron Emission from Tungsten, Molybdenum, and Tantalum, *Phys. Rev.*, Vol. 25, p. 338, March, 1925; W. E. Forsythe and A. G. Worthing, The Properties of Tungsten and the Characteristics of Tungsten Lamps, *Astrophys. Jour.*, Vol. 61, p. 146, 1925.

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take advantage of the high melting temperature of tungsten and the high emission of the other metals. It has been discovered that thorium on tungsten gives the highest emission of any practical combination of this character.[1] The layers of thorium resulting from the formation procedure are molecular in thickness.

Thoriated tungsten emitters are formed by using tungsten wire that has from 1 to 2 per cent of thorium oxide mixed with it. The filament is placed in tubes in the usual way, and the tubes are degassed and baked out. It is then necessary to activate the filament to obtain the high emission. The activation procedure consists of several steps. *First*, the filament is heated to a high temperature just short of the melting temperature, such as 2800°K. This does two things: it cleans the tungsten and reduces the thorium oxide to metallic thorium. Such thorium as finds its way to the surface is immediately vaporized at this temperature. *Second*, the filament is maintained at 2100°K for a considerable time. At this temperature there is a combination action of diffusion of metallic thorium to the surface and an evaporation of some of this. The purpose of this step in the procedure is to accumulate as thick a layer of thorium as possible at the surface. The amount of thorium that gathers on the surface depends upon the relative rates of diffusion and evaporation. At 2100°K there is a pronounced maximum in the emission of thoriated filaments, indicating that at this temperature there exists the best combination of diffusion and evaporation possible for accumulating thorium on the surface. *Third*, the filament temperature is reduced to 1600°K, and some napthalene, acetylene, or other hydrocarbon gas is admitted. This causes a layer of tungsten carbide to form on the surface of the filament. Although this "carbonizing" reduces the emission somewhat, it is found that the presence of the layer reduces the subsequent evaporation of thorium by a factor of six. *Fourth*, the filament temperature is set to 1800 to 2000°K for normal operation.

Several interesting and remarkable properties of thoriated tungsten filaments should be pointed out. The thorium will adhere to the tungsten at temperatures considerably higher than that at which metallic thorium will evaporate. The work function of thoriated tungsten is lower than that of either tungsten or thorium alone. The approximate values of the work functions are

Tungsten	4.5 volts
Thorium	3.4 volts
Thoriated tungsten	2.6 volts

The reduction of the work function is apparently due to the lowering of the electrostatic surface gravitational forces by virtue of the existence of a double layer of charge caused by partially ionized electropositive metal on the surface.

Emission of Oxide-coated Cathodes.[2]—The most efficient type of emitter is the oxide-coated cathode. It consists of a metallic base of some metal or alloy with a coating of the oxides of some of the alkaline-earth metals.

The base metals, known as cores, upon which the oxide coating is formed may be of pure metal such as platinum. Better emission is, however, obtained with certain alloys such as platinum-iridium, nickel-platinum, and nickel-silicate. The best emission results from the use of an alloy of nickel, iron, cobalt, and titanium known as Konel metal.

[1] I. Langmuir, Electron Emission from Thoriated Tungsten Filaments, *Phys. Rev.*, Vol. 22, p. 357, October, 1923; S. Dushman and J. W. Ewald, Electronic Emission from Thoriated Tungsten, *Phys. Rev.*, Vol. 29, p. 857, June, 1927; W. H. Brattain and J. A. Becker, Thermionic and Absorption Characteristics of Thorium on Tungsten, *Phys. Rev.*, Vol. 43, p. 428, March, 1933.

[2] See J. A. Becker, Thermionic Electron Emission, *Bell System Tech. Jour.*, Vol. 14, p. 413, July 1935; also, *Rev. Modern Phys.*, April, 1935; J. A. Becker and R. W. Sears, Phenomena in Oxide-coated Filaments, Pt. II, Origin of Enhanced Emission, *Phys. Rev.*, Vol. 38, p. 2193, December, 1931.

Ex. 1044-0291

The best alkaline earths are barium and strontium. The coating is applied to the core in the form of barium and strontium carbonates.

It is necessary to employ an activation procedure to the carbonate coatings. This procedure consists of heating the coating to about 1500°K for a short time and applying a voltage of about 100 volts through a protective resistor. This does two things: it reduces the carbonates to oxides thermally, and bombards the oxide coating with positive ions of considerable energy. The emission will tend to rise rapidly at these temperatures, and the cathode temperature is then reduced to about 1000°K for normal operation. The exact details of this activation, or "forming" process, vary greatly in actual commercial production.

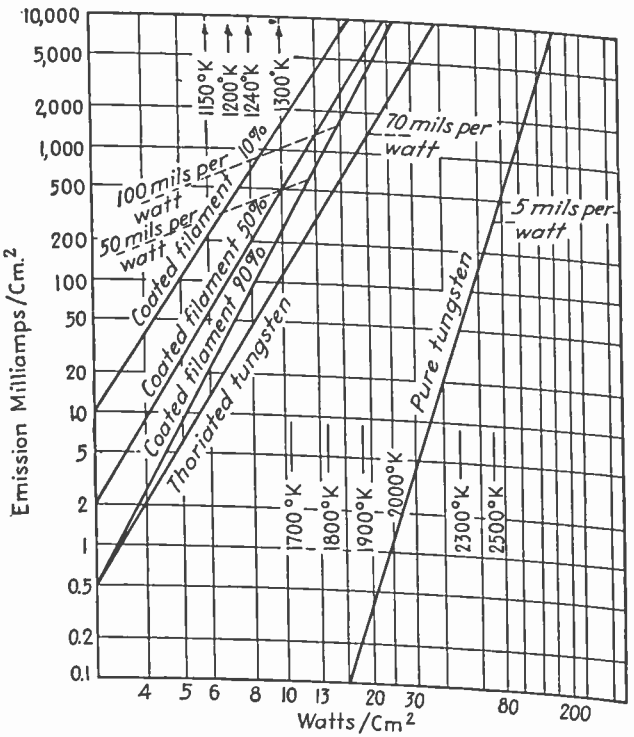


FIG. 6.—Emission efficiency of various emitters.

The mechanism of the emission of the oxide coatings is not exactly understood. Evidence exists to show that the nature of the core has a considerable influence. It is believed that there is emission from particles of the pure alkaline-earth metal that have been reduced from the oxides. The emission constants of oxide-coated cathodes are subject to some considerable variation. It is not possible to obtain the constants from curves such as those of Fig. 5 with any accuracy, because there is no sharply defined saturation current, as with tungsten and thoriated tungsten. However, the order of the work function is about 1 volt, and the emission constant A of Eq. (23) is about 0.01 ampere per cm² per degree².

Comparison of Emitters.[1]—The relative emission properties of the commonest types of emitters are shown in Fig. 6. Here are shown curves of emission against cathode heating power on a specially devised paper, known as power-emission paper, on which

[1] A good discussion is given by O. W. Pike, Cathode Design, *Communications*, Vol. 21, p. 4, October, 1941.

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these curves appear as straight lines. The curves for the oxides were obtained from tests on a large number of tubes, and the percentages indicate the relative portion of tubes having higher emission than that indicated by the particular curve. The 50 per cent curve thus corresponds to an average emission for these particular oxide-coated filaments.

The curves of Fig. 6 also indicate approximately the emission efficiency of the various types of emitters. The emission efficiency is determined by the ratio of the number of milliamperes that can be obtained per square centimeter of surface area to the heating power in watts. The emission efficiency depends upon the cathode and filament structure, since the same power into different structures will raise the emitter coating to different temperatures. Some typical emission efficiencies that can be obtained at normal operating temperature are

Pure tungsten filaments	2–10 ma per cm^2 per watt
Thoriated tungsten filaments	5–100 ma per cm^2 per watt
Oxide-coated cathodes	10–200 ma per cm^2 per watt
Oxide-coated filaments	200–1,000 ma per cm^2 per watt

Practical Emitting Structures.—The physical form that the various emitters take is influenced by practical considerations. In the case of tungsten, thoriated-tungsten

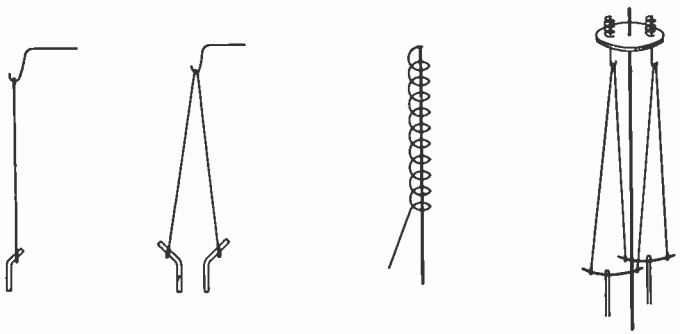


FIG. 7.—Typical filament structures.

and oxide-coated filaments, it is desired to have the emission efficiency as high as possible. This dictates a structure in which the thermal radiation is a minimum for a given wire length. It is also necessary to allow for the expansion of the wire filament. The various types of filamentary structures shown in Fig. 7 are typical. All these are arranged so that the temperature is as high as possible for a given input power. All have spring suspensions that allow for the expansion of the filament wire.

The indirectly heated cathodes consist of cylinders of thin sheet nickel on which the emitting coating is placed and in which there is contained the heater wire. The heater wire is invariably tungsten, covered with some insulating coating such as aluminum oxide. Typical arrangements are shown in Fig. 8.

Secondary Emission.—Almost all metals and some insulators will emit secondary electrons; *i.e.*, they will emit electrons when bombarded by a stream of electrons or ions.[1]

The number of secondary electrons emitted per primary electron depends upon the velocity of the primary bombarding electrons and upon the nature of the material and the condition of its surface. The number of secondaries liberated per primary electron may be greater than one. This is not a violation of the law of conservation

[1] H. E. Farnsworth, *Phys. Rev.*, Vol. 31, p. 405, 1928; Vol. 31, p. 419, 1928; Vol. 25, p. 41, 1925; Vol. 20, p. 358, 1922; L. R. G. Treolar, Measurement of Secondary Emission in Valves, *Wireless Eng.*, Vol. 15, p. 535, October, 1938; D. E. Wooldridge, Theory of Secondary Emission, *Phys. Rev.*, Ser. 2, Vol. 56, p. 562, Sept. 15, 1939.

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of energy, since the velocity of the secondary electrons is lower than that of the primary electrons. Most of the secondary electrons have velocities of about 10 volts, though a small fraction have velocities approaching that of the primary electrons. The number of secondary electrons per primary electron varies typically as shown in Fig. 9. No secondary electrons are produced for primary velocities below about

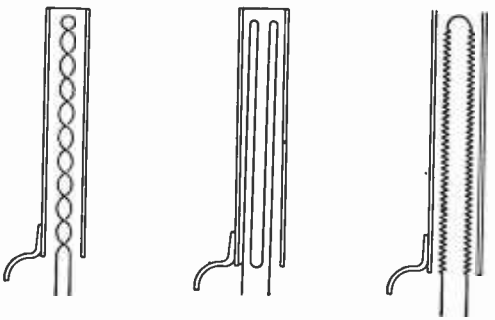


FIG. 8.—Typical heater cathode structures.

9 volts. With increasing potential of the bombarding primary electrons, the ratio of secondary to primary electrons increases, reaching a maximum in the vicinity of about 400 volts for all metals, and then decreasing. Most of the metals reach a maximum ratio of secondary to primary electrons of between one and three. Some of the complex alkali metals reach ratios of secondary to primary electrons of as high as five to eight. Maximum ratios depend greatly upon processing and surface condition.

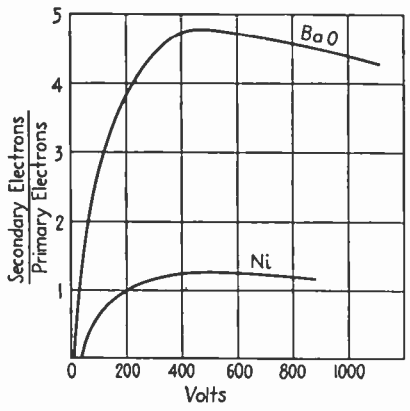


FIG. 9.—Ratio of secondary current to primary current as a function of primary potential for typical surfaces.

The number of liberated secondary electrons that succeed in escaping from a surface depends upon the potential conditions surrounding the bombarded surface. If there is an adjacent electrode at a potential higher than that of the bombarded surface, it will collect all the secondary electrons. If the adjacent electrode has a potential lower than that of the bombarded surface, the former will receive only the highest velocity secondaries, and the low-velocity electrons will be returned to the surface from which they came.

4. Space-charge Effects.—Most vacuum tubes are designed so that the emitting surface gives off a surplus of electrons, with the result that the actual current that flows is limited by the mutual repulsion between the electrons and not by the emission capabilities of the cathode. Under these conditions the current is said to be space-charge limited.

The space-charge limitation of current is brought about by the presence of electrons in the space between electrodes. These electrons introduce a negative charge that reduces the potential in the region. The reduction of potential slows down the electrons and thus increases the negative charge density, which further reduces the

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potential. The potential is finally reduced to the point at which the potential gradient at the emitting surface is zero or even slightly negative. This is a limiting equilibrium condition that sets a limit to the current that can flow for a given potential difference between electrodes.

Plane Electrode Diode.—The theory relating current density, potential, and electrode spacing for a plane electrode diode was developed by Child and Langmuir[1] and has been verified experimentally.

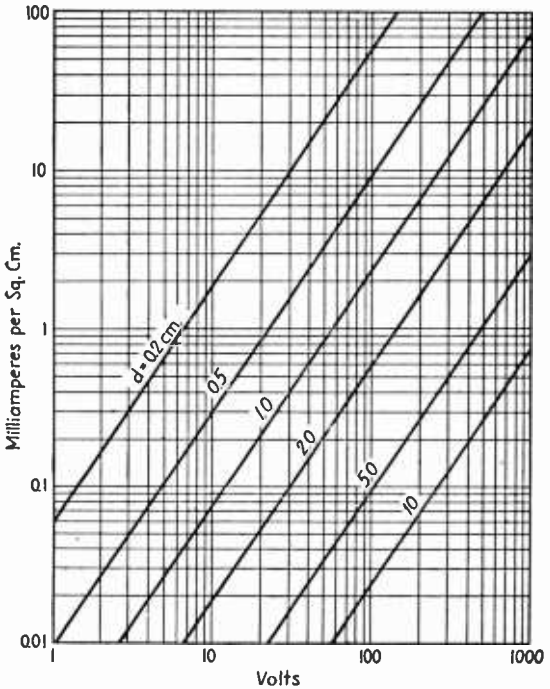


FIG. 10.—Space-charge limited current in a plane electrode diode (d = electrode spacing).

The Child-Langmuir law for parallel plane electrodes is

$$I = \frac{\sqrt{2}}{9\pi}\left(\frac{e}{m}\right)^{1/2}\frac{E^{3/2}}{d^2} \text{ statamperes per sq cm} \tag{24}$$

where e = electron charge, statcoulombs.
E = potential difference between cathode and plate, statvolts.
m = electron mass, grams.
d = cathode-plate distance, cm.

In practical units this becomes

$$I = \frac{2.331 \times 10^{-6}E^{3/2}}{d^2} \text{ amperes per sq cm} \tag{25}$$

A plot of this equation, from which numerical values are readily obtained, is given in Fig. 10.

[1] D. C. Child, Discharge from Hot CaO, *Phys. Rev.*, Vol. 32, p. 492, May, 1911; I. Langmuir, The Effect of Space Charge and Residual Gas on Thermionic Currents in High Vacuum, *Phys. Rev.*, Ser. 2, Vol. 2, p. 450, December, 1913.

Ex. 1044-0295

Equation (25) shows that the current density varies as the three-halves power of potential for a given electrode spacing. The manner in which various quantities vary in the interelectrode space may be determined by treating the current as a constant in Eq. (24). When this is done it is seen that in a given tube the potential difference between electrodes required to obtain a given current I varies as the $\frac{4}{3}$ power of the distance between electrodes. Likewise, the potential gradient in the interelectrode space varies as the one-third power of the distance between anode and cathode, and is zero at the cathode. The velocity of the electrons in the interelectrode space for a given current I varies as the two-thirds power of the distance d, assuming the velocity is zero at the cathode. The space charge density varies as the negative two-thirds power of distance from the cathode, and is theoretically infinite at the cathode because of the assumption of zero initial velocity.

Cylindrical Electrode Diode.—Relations similar to those which hold for the plane electrode diode have been worked out for the cylindrical electrode diode.[1] The current-voltage law here has the form

$$\frac{I}{l} = \frac{2}{9}\sqrt{\frac{2e}{m}}\frac{E^{3/2}}{r_p\beta^2} \qquad \text{(esu)} \tag{26}$$

where I/l is the current in statamperes per unit length of axial emitter, E is the potential difference between concentric cylindrical cathode and plate in statvolts, r_p is the plate radius, and e and m are the charge and mass of the electron. The quantity β is a dimensionless factor depending upon the ratio of the plate radius r_p to the cathode radius r_c, and is given by the series

$$\beta = u - \frac{2}{5}u^2 + \frac{11}{120}u^3 - \frac{47}{3{,}300}u^4 + \cdots \tag{27}$$

in which $u = \log_e r_p/r_c$. Values of β^2 for various values of r_p/r_c are given in Table 3.

TABLE 3

r_p/r_c	β^2	r_p/r_c	β^2	r_p/r_c	β^2
1.00	0.000	6.0	0.838	30	1.091
1.50	0.116	7.0	0.887	45	1.095
2.00	0.275	8.0	0.925	67	1.089
2.50	0.405	9.0	0.955	122	1.072
3.00	0.512	10.0	0.978	221	1.053
4.00	0.665	12.0	1.012	735	1.023
5.00	0.775	16.0	1.051	2,440	1.006
		20.0	1.072	22,026	0.999

In practical units the current-voltage relation becomes

$$\frac{I}{l} = 14.68 \times 10^{-6}\frac{E^{3/2}}{r_p\beta^2} \text{ amperes per unit length} \tag{28}$$

A plot of this equation from which numerical values are readily obtained is given in Fig. 11 for a value of $\beta^2 = 1$. If β^2 does not equal one, the value from the curves of Fig. 11 must be multiplied by the value of β^2.

[1] I. Langmuir and K. B Blodgett, Currents Limited by Space Charge between Coaxial Cylinders, *Phys. Rev.*, Ser. 2, Vol. 22, p. 347, October, 1923.

A solution taking into account the velocity of emission of the emitted electrons and giving the location and magnitude of the potential minimum near the anode is obtained by E. L. E. Wheatcroft, *Jour. I.E.E.*, Vol. 86, p. 473, 1940; also, Wireless Section, *I.E.E.*, Vol. 15, p. 94, June, 1940.

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The current-voltage law for spherical electrodes has a form similar to that for cylindrical electrodes. The current again varies as the three-halves power of potential.[1] In fact, the three-halves power relation between current and potential for a space-charge-limited condition is independent of the electrode geometry and holds for any shapes.[2]

Space Charge in the Grid-anode Region.—In screen-grid tubes it is found that under certain conditions the space charge of the electrons produces a negative gradient of

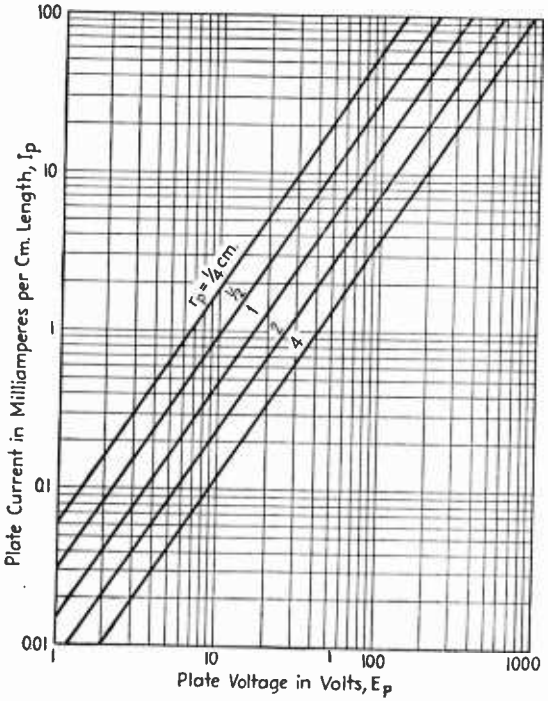


FIG. 11.—Space-charge limited current in a cylindrical electrode diode, $\beta^2 = 1$.

potential at the plate that eliminates the necessity of a suppressor grid in reducing secondary emission effects.[3] Screen-grid tubes especially designed to utilize these screen-anode space-charge effects are the *beam power tubes*. Beam tubes have aligned control and screen grids, which cause the electrons to move in sheets. They also have a screen-plate distance considerably greater than that in the conventional screen-grid tube.

A study of the effects that may occur in beam power tubes shows that the excellent plate current-voltage characteristics obtained are the result of the space-charge action

[1] I. Langmuir and K. T. Compton, Electrical Discharges in Gases, Pt. II, *Rev. Modern Phys.*, Vol. 3, p. 191, April, 1931.

[2] Langmuir and Compton, *op. cit.*

[3] J. H. O. Harries, The Anode to Accelerating Electrode Space in Thermionic Values, *Wireless Eng.*, Vol. 13, p. 190, April, 1936, "Critical Distance" Tubes, *Electronics*, Vol. 9, p. 33, May, 1936; O. H. Schade, Beam Power Tubes, *Proc. I.R.E.*, Vol. 26, p. 137, February, 1938; B. Salzberg and A. V. Haeff, Effects of Space Charge in Grid-anode Region of Vacuum Tubes, *R.C.A. Rev.*, Vol. 2, p. 336, January, 1938; C. E. Fay, A. L. Samuel, and W. Shockley, *Bell System Tech. Jour.*, Vol. 17, p. 49, January, 1938.

Ex. 1044-0297

in the screen-anode region. In this region, in addition to a possible negative gradient of potential at the anode, it is also possible to have a virtual cathode, *i.e.*, a point of zero potential and zero-potential gradient, from which electrons are partially returned to the screen grid and partially transmitted to the plate.

The different types of space-charge action that occur in the screen-anode region of a beam power tube are best classified in conjunction with the potential distributions in the interelectrode space with which they are associated. All possible potential distributions fall into one of four types. These four types are shown in Fig. 12, and have the characteristics listed below:

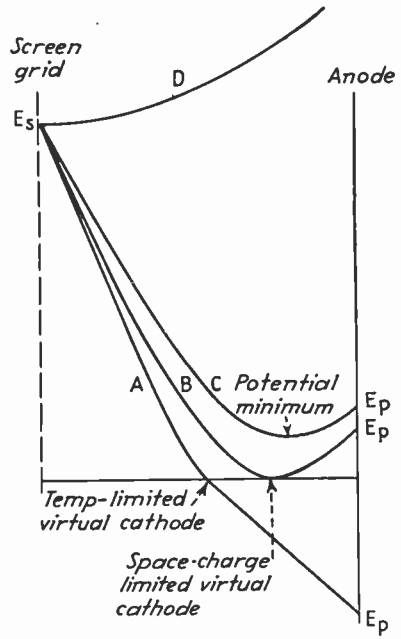


FIG. 12.—Types of potential distributions encountered in the screen-plate region of beam power tubes.

Type A.—Plate is at a negative potential. All the current injected into the region is reflected from a point at zero potential but having a negative gradient of potential. The potential distribution on the anode side of the point of zero potential is a straight line. On the screen side of the zero-potential point the potential distribution is the same as for a temperature-limited case in which the emission current is assumed to be twice the current injected into the screen-anode space through the screen.

Type B.—Both screen grid and plate are positive, with a single point at both zero potential and zero-potential gradient between them, resulting in a space-charge-limited virtual cathode. Part of the current injected through the screen is then transmitted to the plate, and the remainder is returned to the screen. The potential distribution is then proportional to the four-thirds power of distance on both sides of the virtual cathode, and the anode current is proportional to the three-halves power of anode voltage.

Type C.—Both screen grid and plate are positive, with a potential minimum at a positive potential between them. All the injected current is then transmitted to the plate.

Type D.—Both screen grid and plate are positive, with no potential minimum between them. All the injected current is transmitted to the plate.

The most important potential distribution formulas and current-voltage relations associated with the potential distributions are summarized below. The notation used is as follows:

x = distance from screen grid to a point between screen grid and plate where the potential is E.

$x_0 = 1.527 \times 10^{-3} \frac{E_s^{3/4}}{I^{1/2}}$ cm, cathode-plate spacing of an equivalent plane electrode diode, which would give a current I with a plate potential E_s.

E_s = screen-grid potential, volts.

I = current injected into the screen-grid-plate region from the screen side, amperes.

Ex. 1044-0298

b = constant appearing in the Type A distributions and related to the straight-line portion of the potential distribution curve beyond the virtual cathode by

$$\frac{dE}{dx} = \frac{-4\sqrt{2}}{3}\frac{E_0}{x_0}b^{-1/4}.$$

E = potential at point x.

E_0 = potential at point x_0.

x_1 = distance from the virtual cathode of a point at potential E in the Type B distributions, and from the potential minimum in the Type C distributions when this point lies between the screen grid and the minimum.

x_2 = distance from the virtual cathode of a point at potential E in the Type B distribution, and from the potential minimum in the Type C distribution, when this point lies between the potential minimum and the plate.

T = fraction of the current I injected through the screen grid that reaches the plate in the Type B distributions.

The potential distributions of the Type A are like those of a temperature-limited diode, and are given by

$$x = \frac{x_0}{\sqrt{2}}\left\{(1 - 2b^{1/2})(1 + b^{1/2})^{1/2} - \left[\left(\frac{E}{E_0}\right)^{1/2} - 2b^{1/2}\right]\left[\left(\frac{E}{E_0}\right)^{1/2} + b^{1/2}\right]^{1/2}\right\} \quad (29)$$

The potential distributions of the Type B are like those of a space-charge-limited diode. The potential rises from a zero value at the virtual cathode located between screen grid and plate, and follows a four-thirds power variation with distance on either side of the virtual cathode, with the scale depending upon the fraction of the current transmitted past the virtual cathode. The potential distribution between the virtual cathode and the screen grid is given by

$$x_1 = x_0\left[\frac{1}{(2 - T)^{1/2}} - \frac{\left(\frac{E}{E_0}\right)^{3/4}}{(2 - T)^{1/2}}\right] \quad (30)$$

The potential distribution between the virtual cathode and plate is given by

$$x_2 = x_0\left[\frac{1}{(2 - T)^{1/2}} + \frac{\left(\frac{E}{E_0}\right)^{3/4}}{T^{1/2}}\right] \quad (31)$$

The potential distributions of the Type C are characterized by a potential minimum between the screen grid and plate occurring at a positive potential. The distribution on the screen side of the potential minimum is given by

$$x_1 = -x_0\left\{\left[\left(\frac{E}{E_0}\right)^{1/2} + 2\left(\frac{E_{\min}}{E_0}\right)^{1/2}\right]\left[\left(\frac{E}{E_0}\right)^{1/2} - \left(\frac{E_{\min}}{E_0}\right)^{1/2}\right]^{1/2} + \left[1 + 2\left(\frac{E_{\min}}{E_0}\right)^{1/2}\right]\left[1 - \left(\frac{E_{\min}}{E_0}\right)^{1/2}\right]^{1/2}\right\} \quad (32)$$

The distribution to the right of the potential minimum, *i.e.*, between the potential minimum and the plate, is given by

$$x_2 = x_0\left\{\left[\left(\frac{E}{E_0}\right)^{1/2} + 2\left(\frac{E_{\min}}{E_0}\right)^{1/2}\right]\left[\left(\frac{E}{E_0}\right)^{1/2} - \left(\frac{E_{\min}}{E_0}\right)^{1/2}\right]^{1/2} + \left[1 + 2\left(\frac{E_{\min}}{E_0}\right)^{1/2}\right]\left[1 - \left(\frac{E_{\min}}{E_0}\right)^{1/2}\right]^{1/2}\right\} \quad (33)$$

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The potential minimum occurs at a distance

$$x_{\min} = x_0 \left[1 + 2\left(\frac{E_{\min}}{E_s}\right)^{1/2}\right]\left[1 - \left(\frac{E_{\min}}{E_s}\right)^{1/2}\right]^{1/2} \tag{34}$$

from the screen.

Potential distributions of the Type D are not of great importance, since for this distribution all the injected current is transmitted to the plate.

It is found that there is a maximum current that can be passed through the screen-grid–plate region for any combination of potentials, current, and electrode spacing. This maximum plate current $I_{p_{\max}}$ that can be passed is

$$I_{p_{\max}} = \frac{2.33 \times 10^{-6} \times (E_s^{1/2} + E_p^{1/2})^3}{a^2} \text{ amperes per cm}^2 \tag{35}$$

where a is the grid-anode distance in cm and E_p is the plate voltage.

The most important consequences of the preceding potential distributions are that tubes of the beam power type may exhibit some unusual dynamic characteristics and that the family of plate current-voltage curves has a larger region in which the curves are approximately parallel and equally spaced than is the case with the usual pentode. The plate current-voltage curves are similar to those of a pentode except that the transition between the region where the plate current increases with plate voltage and the region in which the plate current is substantially constant is smaller and occurs at a lower plate potential than for the pentode, with the result that the "shoulder" formed by this region of decreasing slope is sharper for the beam power tube than for the pentode (see Fig. 20). The reason that the plate current-voltage curves of the beam power tube are steeper at low plate voltages than are the corresponding portions of the pentode characteristics is that in the beam power tube the action of the virtual cathode between the screen grid and the plate is more uniform in the absence of the suppressor grid of the pentode, which tends to give a kind of variable-mu action.

5. Tube Noise.[1]—Vacuum tubes act as sources of noise because of the inherent electronic nature of their operation. Noise arises from the fact that the current consists of the movement of discrete particles rather than the flow of a continuous fluid, giving rise to what is known as the *shot effect*. If the emitting material is irregular in its nature, there may also be large low-frequency variations resulting from chunks of the emitting material varying in their emission, causing what is known as the *flicker effect*. Noise will also arise from variations in the secondary emission, from ionization within the tube, and from a random variation of the division of current between elements in multielectrode tubes. Of all the above-mentioned effects, the largest and most important source of noise is the shot effect.

Shot effect is noise due to the fact that electrons are discrete particles emitted from the cathode in a random way, so that any current resulting from such emission has a random or statistical variation that is termed "noise." The energy correspond-

[1] For further information on noise in tubes the reader is referred to the following series of papers published in the *R.C.A. Rev.* under the general heading Fluctuation Noise in Space-charge-limited Currents at Moderately High Frequencies: Pt. I, B. J. Thompson, General Survey, Vol. 4, p. 269, January, 1940; Pt. II, D. O. North, Diodes and Negative Grid Triodes, Vol. 4, p. 441, April, 1940; Vol. 5, p. 106, July, 1940; Pt. III, D. O. North, Multicollectors, Vol. 5, p. 244, October, 1940; Pt. IV, B. J. Thompson and D. O. North, Fluctuations Caused by Collision Ionization, Vol. 5, p. 371, January, 1941; Pt. V, W. A. Harris, Fluctuations in Vacuum Tube Amplifiers and Input Systems, Vol. 5, p. 505, April, 1941; Vol. 6, p. 114, July, 1941.

Other articles of especial importance are D. A. Bell, A Theory of Fluctuation Noise, *Jour. I.E.E.*, Vol. 82, p. 522, 1938; also, Wireless Section, *I.E.E.*, Vol. 13, p. 97, June, 1938; A. J. Rack, Effect of Space Charge and Transit Time on the Shot Noise in Diodes, *Bell System Tech. Jour.*, Vol. 17, p. 592, October, 1938.

Ex. 1044-0300

ing to these variations (*i.e.*, the energy of the resulting a-c components of the current) is found to be proportional to the magnitude of the average or d-c current. The noise energy is distributed evenly over the frequency spectrum. The shot noise in a tube, as well as depending upon the magnitude of the current, also depends upon the conditions under which this current flows. The noise for a given current is maximum when the plate is absorbing all the electrons that are liberated by the cathode, *i.e.*, when the emission is temperature limited. If the plate does not receive all the electrons emitted by the cathode, as happens when there is a very copious emission causing the current to be limited by the space-charge repulsion between electrons, then the noise is much less because of a cushioning effect upon the variations in the rate of emission produced by the great number of electrons emitted.

The rms fluctuation current from a temperature-limited diode, measured in a frequency band Δf, is given by[1]

$$i^2 = 3.18 \times 10^{-19} I \, \Delta f \tag{36}$$

where i = rms fluctuation current, amperes.
I = diode current, amperes.
Δf = frequency band in which the noise is measured in cycles per sec.

It is convenient to compare the noise from tubes with that which occurs in resistors because of thermal agitation of the electrons (see Par. 26, Sec. 5). The fluctuation current in a short-circuited resistor of resistance R at 290°K (63°F) is

$$i^2 = \frac{1.59 \times 10^{-20} \, \Delta f}{R} \tag{37}$$

where i = rms value of noise current, amperes.
Δf = width of the frequency band in which the noise is measured, cycles per sec.
R = resistance of the short-circuited resistor producing the noise, ohms.[2]

When Eq. (36) is compared with Eq. (37), it is seen that the resistance R_{eq}, which would produce the same noise as a temperature-limited diode with a current I, is given by

$$R_{eq} = \frac{0.05}{I} \text{ ohms} \tag{38}$$

In a tube in which the current to the plate is space-charge-limited, it is found that the noise is less than given by Eq. (36). Over a wide range of conditions in which only a small fraction of the emitted current reaches the plate (as is the case for all tubes operated at their rated conditions), the square of the rms fluctuation current is approximately 0.04 as great as given by Eq. (36) for the anode current in question, *i.e.*, the noise energy of a diode operating at a given current is roughly 0.04 times as great when the current flow is space-charge-limited as it is when it is temperature limited. For the diode operating under a condition of space-charge-limited current flow, the value of resistance that would produce the same noise energy as does the diode is given by[3]

$$R_{eq} = \frac{0.002}{I} \text{ ohms} \tag{39}$$

[1] W. Schottky, Spontaneous Current Fluctuations in Various Conductors, *Ann. Physik*, Vol. 57, p. 541, 1918.

[2] H. Nyquist, Thermal Agitation of Electric Charge in Conductors, *Phys. Rev.*, Vol. 32, p. 110, July, 1938.

[3] The case where part of the flow is space-charge-limited and part is temperature limited is covered by D. A. Bell, Fluctuation Noise in Partially Saturated Diodes, *Jour. I.E.E.*, Vol. 84, p. 723, 1939: also, Wireless Section, *I.E.E.*, Vol. 14, p. 177, June, 1939.

Ex. 1044-0301

where R_{eq} is an equivalent short-circuited noise-producing resistance in ohms and I is the plate current of the diode in amperes.

The equivalent noise-producing resistance of a space-charge-limited triode is[1]

$$R_{eq} = \frac{\theta}{\sigma}\frac{T}{T_0}\frac{1}{g_m} \tag{40}$$

where R_{eq} = that resistance which, if inserted in the grid circuit of the given tube, would produce as much noise energy as does the tube itself.

θ = approximately 0.667, the ratio of noise energy of tube to noise energy of a resistance equal to dynamic tube resistance and at cathode temperature.

σ = approximately 0.75, the ratio of the transconductance of the triode to the conductance of an equivalent diode (*i.e.*, the diode having a cathode-plate spacing equal to the cathode-grid spacing of the triode and having a potential of $(E_g + E_p/\mu)$ on the plate).

T = cathode temperature, °K = 273 + C°.

T_0 = room temperature, °K.

g_m = transconductance of the triode, mhos.

In a typical triode for which T = 1000°K, T_0 = 300°K, with the use of the approximate value of θ and σ indicated above, one has approximately

$$R_{eq} = \frac{3}{g_m} \text{ ohms} \tag{41}$$

where g_m is in mhos.

The noise energy in pentode, beam, and screen-grid tubes is higher than in triodes with similar characteristics, because there is an added component of noise due to the random division of current between the screen and anode. The value of the resistance that, if inserted in the grid circuit of a pentode and similar tubes, would produce as much noise energy as does the tube itself is approximately

$$R_{eq} = \frac{I_b}{I_b + I_{c2}}\left(\frac{2.5}{g_m} + \frac{20I_{c2}}{g_{m2}}\right) \text{ ohms} \tag{42}$$

where I_b = plate current, amperes.

I_{c2} = screen current, amperes.

g_m = transconductance of the pentode, mhos.

The noise energy from a pentode will be about three to five times as great as that from a triode producing an equivalent amplification.[2]

As previously indicated, there are other sources of noise in tubes, such as ionization, secondary emission, emission of positive ions, and reflection of electrons from a virtual cathode, that are not included in the foregoing discussion. These sources produce noise that is usually low compared to that from the shot effect alone. In particular, noise from ionization may generally be ignored when the grid gas current is less than a few hundredths of a microampere.[3]

VACUUM TUBES

6. Basic Tube Types. *Diodes.*—A diode is a two-element vacuum tube containing an emitter of electrons, known as the cathode, and a collector electrode, termed the anode or plate. The emitter may be either an indirectly heated cathode or a filament. The chief uses of diodes are as detectors, and as rectifiers for either control or power

[1] North, *loc. cit.*
[2] North, *loc. cit.*, and Harris, *loc. cit.*
[3] Thompson and North, *loc. cit.*

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purposes. The detector and rectifier tubes differ in size and current capacity but have the same general type of characteristics.

The plate current-voltage characteristics of a typical diode are shown in Fig. 13. For the lower portion of the curve the current is approximately proportional to the three-halves power of the anode voltage, because the current is limited by the space charge. For higher values of voltage, the current ceases to increase at the three-halves power rate and finally begins to flatten off with increasing voltage at a point determined by the emission of the cathode, which, in turn, depends primarily upon the cathode temperature. The higher the cathode temperature the higher the emission and the higher the voltage at which the current curve begins to flatten or "saturate."

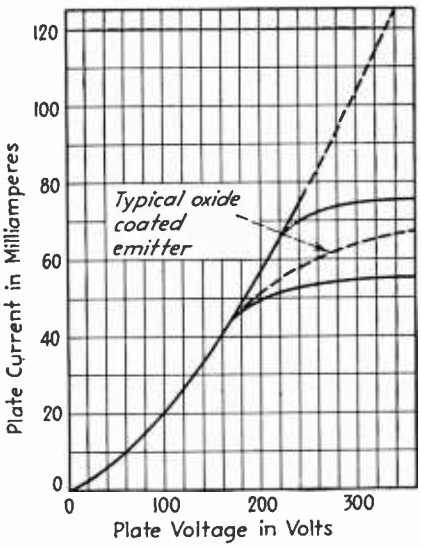


FIG. 13.—Current-voltage characteristics of a diode.

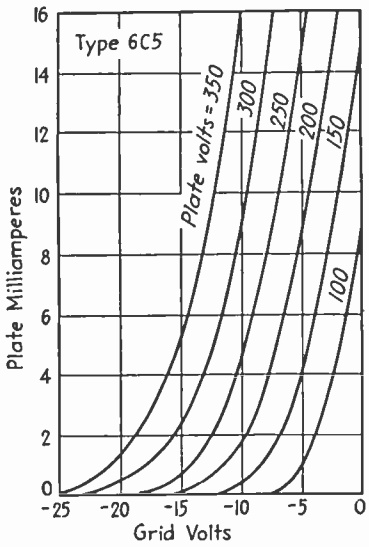


FIG. 14.—Plate-current–grid-voltage characteristics of a triode.

All current-voltage characteristics of diodes are similar at low voltages in the region in which the emission is space-charge limited. However, the portion of the plate current-voltage curves in which the current is limited by the cathode emission does not have the same shape for all types of tubes, but rather depends upon the nature of the electron-emitting surface. Thus oxide-emitting surface has less tendency to saturate sharply than a tungsten or thoriated-tungsten surface, as shown in Fig. 13.

Triodes.—A triode is a three-electrode vacuum tube containing an emitting electrode called the cathode, a control electrode called the grid, and a current-collecting electrode called the anode or plate.

The emitting electrode may be either an indirectly heated cathode, an oxide-coated filament, or a filament of tungsten or thoriated tungsten.

The control electrode, usually in the form of a grid of fine wire, surrounds the emitter and is, in turn, surrounded by the plate. The grid is usually operated at a slight negative potential and thus does not attract electrons. By virtue of its proximity to the cathode, it is able to influence the electrostatic field at the cathode to a greater extent than can the plate, and thus the grid is able to control the flow of current from the cathode without itself taking any of this current.

Ex. 1044-0303

Some typical characteristics of a triode, showing the way in which the plate current varies with grid voltage for various fixed values of plate voltage, are given in Fig. 14. The plate current is seen to increase if either the plate voltage or grid voltage is increased. The increase in plate current for a given increase in grid voltage is always much larger than the increase in plate current for the same increase in plate voltage.

The relative effectiveness of the plate and grid potentials in controlling the plate current is known as the amplification factor of the tube, symbol μ. Specifically, the *amplification factor* of a triode or any multi-electrode tube is the ratio of a small change in plate voltage to a small change in control grid voltage under the conditions that the plate current remains unchanged and that all other electrode voltages are maintained constant. The sense is usually taken as positive when the voltages are changed in opposite directions. In mathematical notation

$$\mu = -\left(\frac{dE_p}{dE_g}\right)_{I_p\text{constant}} \tag{43}$$

Observation of the curves of Fig. 14 shows that they are all similar but are only displaced from one another. This is because the plate current depends upon an equivalent voltage of value $(E_g + E_p/\mu)$ in the same way that a diode depends upon its plate voltage E_p. The current to the plate of a triode may be written as

$$I_p = k\left(E_g + \frac{E_p}{\mu}\right)^{3/2} \tag{44}$$

where I_p = plate current.
k = a constant, sometimes known as the perveance.
E_g = grid voltage.
E_p = plate voltage.
μ = amplification factor.

In work with vacuum-tube circuits it is desirable to know the resistance of the plate circuit of the tube to a small superimposed a-c voltage. This resistance is known as the *dynamic plate resistance* of the tube, and is generally designated by the symbol R_p. The mathematical definition of the dynamic plate resistance in terms of increments of plate voltage and current is

$$R_p = \left(\frac{dE_p}{dI_p}\right)_{E_g\text{constant}} \tag{45}$$

Another factor of interest in triodes and multi-electrode tubes is the relation between the a-c components of grid voltage and plate current. The ratio between these two is known as the *transconductance* or *mutual conductance* and is defined by

$$g_m = \left(\frac{dI_p}{dE_g}\right)_{E_p\text{constant}} \tag{46}$$

It will be observed from the defining relations of the tube constants mentioned above that the product of the transconductance and plate resistance is numerically equal to the amplification factor

$$g_m R_p = \mu \tag{47}$$

Other representations of the triode than the plate-current–grid-voltage characteristics of Fig. 14 are possible. In Fig. 15 is shown a typical plate-current–plate-voltage characteristic of a triode. Another possible representation of the triode

characteristics is the constant-current characteristics of Fig. 16. These curves show loci of constant current and indicate the combinations of plate and grid voltage necessary to keep the plate current constant.

For each of the three representations of the triode characteristics given, the slope of the characteristics is related to one of the tube constants. For the plate-current–

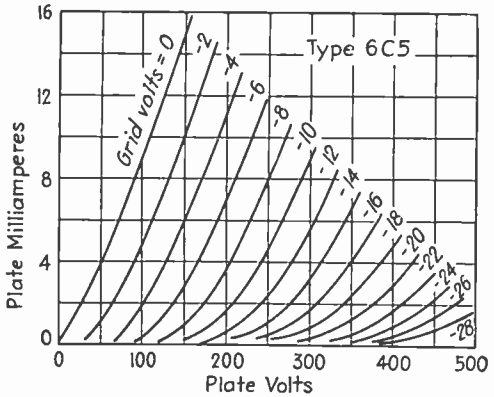


FIG. 15.—Plate-current-voltage characteristics of a triode.

grid-voltage characteristics of Fig. 14, the slope of the characteristic at any point is the transconductance of the tube at the potentials involved. In the plate-current–plate-voltage characteristics of Fig. 15, the slope of the characteristic at any point gives the reciprocal of the dynamic plate resistance. In the constant-current representation of Fig. 16 the slope of the characteristic gives the negative reciprocal of the amplification factor of the tube.

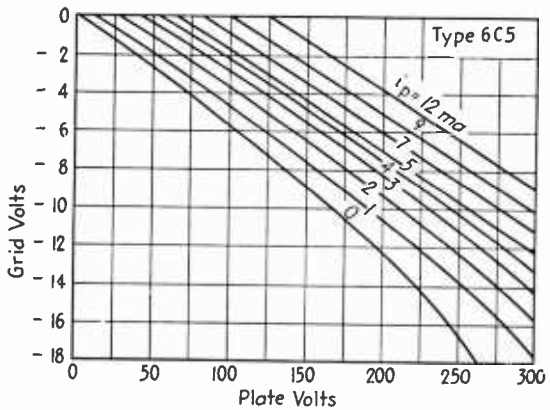


FIG. 16.—Constant-current curves of a triode.

Examination of the triode characteristics shows that the so-called tube "constants" are really not constants but vary considerably with plate current. The nature of this variation is shown in Fig. 17. It is seen that the amplification constant is most constant but tends to drop with low current. This drop occurs because of a variable-mu action that comes into play near cutoff. Some parts of the tube struc-

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ture have a lower mu than do others, and so tend to cut off at a lower voltage.[1] The transconductance varies nearly as the one-third power of plate current, and the plate resistance as the reciprocal of this.

Pentodes.—A pentode is a five-electrode high-vacuum tube. The five electrodes, in the order in which they occur in the tube, are cathode, control grid, screen grid, suppressor grid, and plate. The control grid is operated at a slight negative potential relative to the cathode. The screen grid is operated at a relatively large positive potential relative to the cathode. The suppressor grid is operated at cathode potential. The plate is operated at a large positive potential relative to cathode, and is generally but not always larger than the screen-grid potential. The control grid is usually of very fine mesh in order to obtain a high transconductance. The screen-

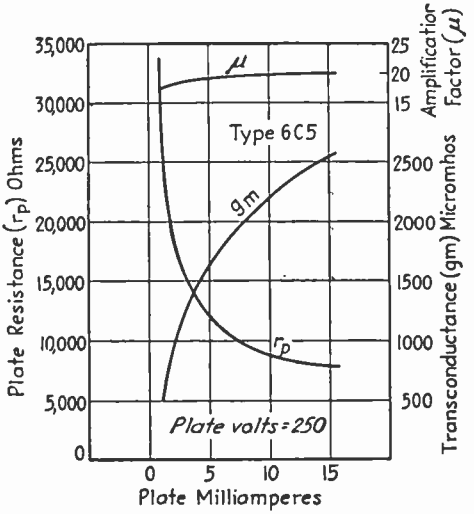


FIG. 17.—Variation of triode constants with plate current.

grid mesh is less fine than that of the control grid. The suppressor-grid mesh is very coarse so as to obstruct the flow of electrons as little as possible.

The pentode resulted from efforts to improve the characteristics of the triode and screen-grid tube. The screen grid eliminates substantially all direct capacity between control grid and plate. The suppressor grid prevents the exchange of secondary electrons between screen and plate, as occurs in the screen-grid tube. This the suppressor grid is able to do because it is operated at zero potential and hence produces a negative gradient of potential at the plate that causes secondary electrons liberated from the plate to be returned to the plate. The suppressor grid, though operated at zero potential, is able to pass electrons freely through the spaces between the wires. The suppressor grid also aids the shielding action of the screen grid by introducing some additional electrostatic shielding between the control grid and the plate.

The total current passed by the control grid to the screen grid is determined mainly by the control-grid potential, to a lesser degree by the screen-grid potential, and hardly at all by the plate potential. Thus

$$I_s = k\left(E_g + \frac{E_s}{\mu_{sg}}\right)^{3/2} \tag{48}$$

[1] See F. E. Terman and A. L. Cook, Note on the Variations in the Amplification Factor of Triodes, *Proc. I.R.E.*, Vol. 18, p. 1044, June, 1930.

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where I_s = space current, the sum of screen-grid and plate current.
k = a constant determined by the tube electrode geometry.
E_g = control-grid potential.
E_s = screen-grid potential.

$\mu_{sg} = \left(\frac{dE_s}{dE_g}\right)_{I_s \text{constant}}$, analogous to the amplification factor of a triode.

The fraction of the space current that is delivered to the plate depends upon the relative screen-grid and plate potentials. When the plate potential is larger than

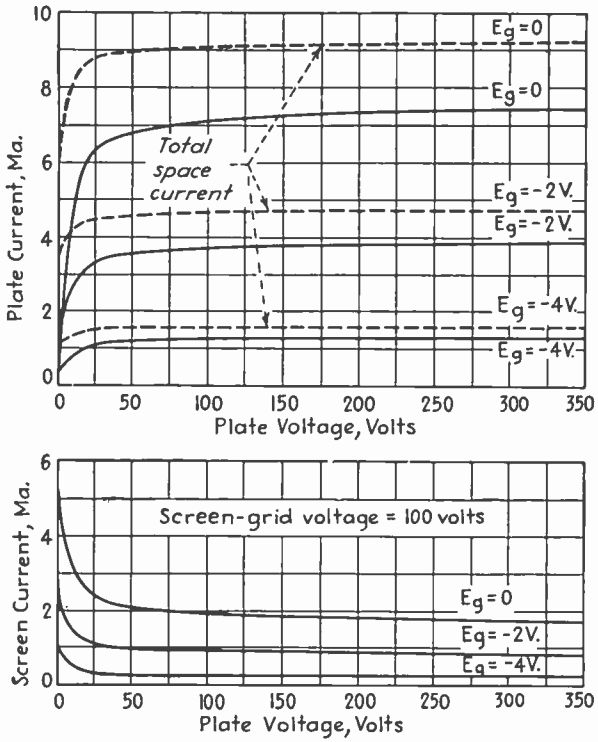


FIG. 18.—Plate-current–plate-voltage characteristics of a pentode; space-current–plate-voltage characteristics of a pentode; screen-current–plate-voltage characteristics of a pentode.

the screen potential, the screen grid intercepts a fraction of the space current that is a little larger than the ratio of projected area of the screen-grid wires to the area of the cylindrical surface in which the screen grid lies. When the plate potential is much less than the screen-grid potential, some of the electrons passed by the screen grid fail to reach the plate and are turned back to the grid, thus decreasing the fraction of the total current that is delivered to the plate.

As a result of the preceding action, both space current and plate current increase only very slightly with plate voltage when the plate voltage exceeds a certain minimum, and the plate current is then nearly a constant fraction of the space current. When the plate voltage is very low compared with the screen-grid voltage, the plate current drops as the plate voltage is decreased, and the space current drops too. The

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plate current drops because a smaller fraction of the space current is delivered to the plate, and because the space current drops somewhat. The space current drops because the current returned from the screen-grid–plate region increases the space charge around the cathode. Some typical curves showing how space current and plate current vary with plate voltage are shown in Fig. 18.

The screen-grid current at low plate voltages decreases as plate potential increases because a smaller fraction of the current originally transmitted by the screen grid is returned from the screen-grid–plate region, and because the screen intercepts a smaller fraction of the passing electrons. Current is returned from the screen-grid–plate

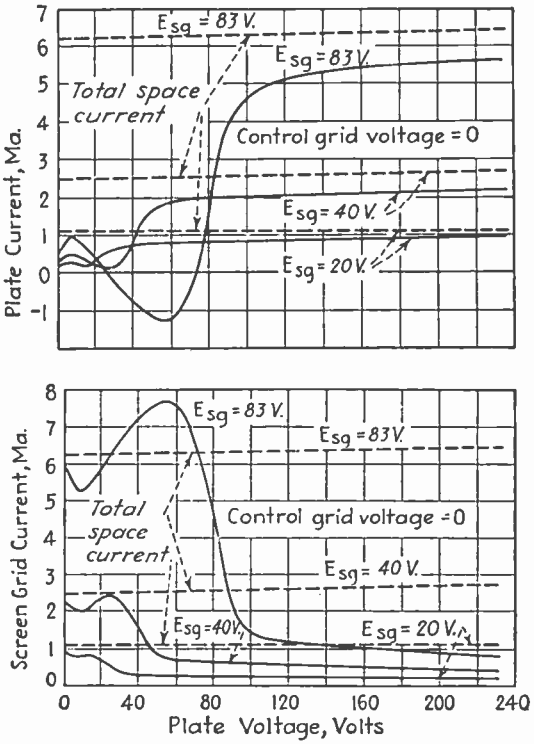


FIG. 19.—Plate and screen-grid current characteristics of a screen-grid tube.

region when the plate potential is low, because then the electrons are slowed down in the vicinity of the suppressor grid so much that some of them are actually stopped, creating a region of zero potential known as a virtual cathode. Electrons with velocity higher than average will get past this region, while those with velocity less than average will be reflected from this region back toward the screen grid. Curves of screen current as a function of plate voltage are also shown in Fig. 18.

Tetrodes.—A tetrode is a four-element vacuum tube. The four elements are cathode, control grid, screen grid, and plate. The tetrode is also known as a screen-grid tube.

The tetrode is the historical predecessor of the pentode. It was designed to have a low electrostatic capacity between control grid and plate, this low capacity being achieved by inserting an extra grid, known as the screen grid, between the control grid and plate of a triode. The tube was successful in providing the low capacity

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desired but suffered distortions in the plate-current–plate-voltage characteristics because of secondary emission effects within the tube. These detrimental effects are overcome in the pentode by the addition of the suppressor grid between screen grid and plate, and the pentode has superseded the screen-grid tube in all but a few applications.

If there were no such thing as secondary emission, the current-voltage characteristics of a screen-grid tube would look like those of a pentode. As it is, they have the form shown in Fig. 19. This is because the electrode currents are made up not only of the primary electrons, *i.e.*, the electrons emitted by the cathode, but also of secondary electrons, which are electrons knocked out of other electrodes by the impact of the primary electrons.

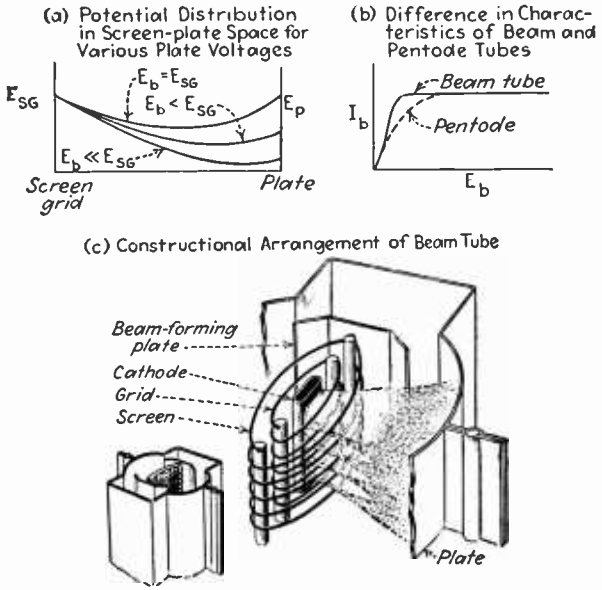


FIG. 20.—Characteristics and constructional features of the beam power tube.

Secondary electrons are always emitted when primary electrons strike an electrode with an energy of more than about 10 electron volts. The reason that the secondary-electron effects have not appeared in the triode and pentode characteristics is that the potentials within these tubes are such that under ordinary operating conditions the secondary electrons are drawn back to the electrode from which they originated.

In screen-grid tubes the secondary electrons from the screen are attracted to the plate when the plate potential is higher than the screen-grid potential, and the secondary electrons from the plate are attracted to the screen grid when the screen-grid potential is more positive than the plate potential. The latter effect is more pronounced because the plate receives more electrons and so gives rise to more secondary electrons than does the screen grid. The effect of the action indicated above results in the characteristics shown in Fig. 19. The dip in the plate-current–plate-voltage curves occurs in the region in which the plate potential is lower than the screen potential and is caused by the fact that the plate current is decreased by the amount of the secondary-electron current that flows from plate to screen grid. In this same region the screen-grid current is increased by the amount of the secondary-electron

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current that flows from plate to screen grid. The amount of the reduction of plate current in this region may be sufficient to cause the plate current to be negative.

Beam Power Tubes.—A beam power tube is a tetrode in which the effect of a suppressor grid is achieved by a special design that causes the mutual repulsion between the electrons in the screen-grid–plate region to depress the potential. This effect is achieved by using a long screen-grid–plate spacing and by causing the electrons to move in sheetlike beams, thus increasing the electron density.

The construction of a beam power tube is shown in Fig. 20. The control grid and screen grid are of the same pitch and are aligned so that the screen-grid wires lie in the shadow of the control-grid wires. End plates at cathode potential are used to keep the electrons flowing in a rectangular section of the tube. The dimensions of the grids are chosen so that the electrons are focused in the interspaces of the screen grid. This causes the screen-grid current to be much lower than the value calculated on the basis of the intercepted area.

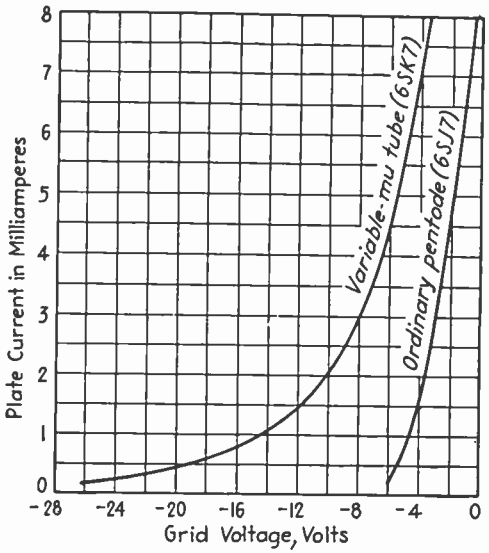


FIG. 21.—Characteristic curve of a variable-mu pentode compared with corresponding characteristic of an ordinary pentode.

The distinguishing features of the beam-power-tube characteristics are that the plate current rises very rapidly with plate voltage at low plate voltages, and then flattens off very sharply (see Fig. 20*b*). The curves show no secondary emission action except at very low currents. Here secondary-emission effects evidence themselves because the current is not high enough to depress the interelectrode potential to the point where a minimum will appear and present a negative gradient of potential to secondary electrons leaving the plate. The screen current decreases with increasing plate voltage as expected, and is relatively small in magnitude if the plate voltage is not too small.

In the screen-grid–plate region the electrons move in essentially parallel paths, and the space charge causes the potential here to be reduced. If the plate voltage is extremely low, the potential in the screen-grid–plate region is reduced to the point where a virtual cathode is formed before the plate. There is then a plane of zero potential before the plate, from which the potential rises on both sides according to

the same law as when one has the cathode of a diode whose emission is space-charge-limited (see curve *B* of Fig. 12). It is the presence of this virtual cathode that causes the plate current to rise so rapidly with plate voltage with small plate voltages. The rapid rise is due to a combination of increasing transmission of current past the virtual cathode, and movement of the virtual cathode toward the plate, causing further increase of the current. The rise in plate current with plate voltage is much more rapid in the beam power tube than in the usual pentode, and results in the characteristics shown in Fig. 20*b*.

Variable-mu Tubes.—A variable-mu tube is a tube, usually a pentode, that has a gradual cutoff as the control grid is made negative, as shown in Fig. 21. The variable-mu characteristic is obtained by employing a control-grid structure having a varying pitch between grid wires. Such a tube acts like a tube made up of a number of tubes with successively decreasing amplification factors in parallel. As the bias on such a tube is increased, the currents in the component parts are cut off one by one, until only those parts corresponding to the lowest mu portions of the grid structure are conducting. Variable-mu tubes are also known as remote cutoff tubes, and "supercontrol" tubes.

Coefficients of Screen-grid, Beam, and Pentode Tubes.—The number of possible coefficients of tubes with four or more electrodes becomes very large. These naturally divide into three types of coefficients, the dynamic resistances of the various electrodes, particularly the screen and plate, the mu-factors, and the transconductances.

The dynamic electrode resistance is the resistance that the electrode offers to a small increment in applied voltage. In the case of beam, pentode, and screen-grid tubes the resistances of the plate and screen circuits are of importance. They are defined as follows:

$$\text{Plate resistance} = R_p = \left.\frac{dE_p}{dI_p}\right|_{E_g \text{ and } E_{sg} \text{ constant}} \tag{49}$$

$$\left.\begin{matrix}\text{Screen-grid} \\ \text{resistance}\end{matrix}\right\} = R_{sg} = \left.\frac{dE_{sg}}{dI_{sg}}\right|_{E_g \text{ and } E_v \text{ constant}} \tag{50}$$

The plate resistance of these types of tubes is very high when the plate voltage is great enough to make the plate current substantially independent of plate voltage. This is particularly true with pentode tubes, and values of plate resistance of megohms are typical for small pentodes under operating conditions. On the other hand, the screen resistances are moderate, being usually about five times the plate resistance of the same tube operated as a triode by connecting the screen and plate electrodes together.

A mu factor of a tube is defined as the relative effect that some two electrodes have on some current in the tube. In the case of pentode and similar tubes one could accordingly define mu factors for the relative effectiveness of control grid and screen on the plate current, and on the total space current, for the relative effectiveness of control grid and plate on each of these currents, etc. In general, the mu factor of electrode 1 relative to electrode 2 with respect to current 3 in a tube is given by the relation

$$\text{mu factor} = \left.\frac{dE_2}{dE_1}\right|_{I_3 \text{ constant}} \tag{51}$$

With pentode, beam, and screen-grid tubes the mu factors of importance are two. *First* is the relative effectiveness of plate and control-grid voltage on the plate current, which corresponds to the amplification factor of a triode and is usually called simply the amplification factor. Under conditions where the plate resistance of the tube is

high, this amplification factor is likewise high, and for the same reason. Values of amplification factor in excess of 1,000 are typical for small pentodes. The *second* mu factor of importance in pentode, screen-grid, and beam tubes is μ_{sg} of Eq. (51), the relative effectiveness of control grid and screen on the *total* space current. This is analogous to the amplification factor of a triode, and can be called the "cutoff" amplification factor, since it determines the relative screen and control-grid potentials giving plate-current cutoff. Values for μ_{sg} of the order of 5 to 25 are typical.

The transconductances are defined in the general case as the change of current to electrode 2 as a result of a voltage increment applied to electrode 1; *i.e.*

$$\text{Transconductance} = \frac{\partial I_2}{\partial E_1} \tag{52}$$

In the case of pentodes and similar tubes one is primarily interested in the transconductance from control grid to plate, and occasionally in the transconductance from control grid voltage to screen current. The former corresponds to the transconductance of a triode and has about the same numerical value as in the corresponding triode. The transconductance to the screen current is usually only about one-fourth that to the plate, because the screen current is only about one-fourth of the plate current. The transconductance from the control-grid voltage to the plate current is the most important single coefficient of a pentode tube when operated under normal conditions.

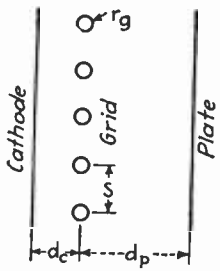


FIG. 22.—Parallel-plane triode.

7. Amplification-factor Formulas.—Formulas for the amplification factor of a triode tube may be obtained by calculating the relative effectiveness of the plate and control grid in controlling the gradient of potential at the cathode.

For a *plane electrode triode* whose electrodes comprise a plane cathode, a plane plate, and a grid of parallel, evenly spaced, equal diameter wires lying in a plane as shown in Fig. 22, the amplification factor is[1]

$$\mu = \frac{\left(\frac{2\pi d_p}{s}\right) - \log_\epsilon \cosh\left(\frac{2\pi r_g}{s}\right)}{\log_\epsilon \coth\left(\frac{2\pi r_g}{s}\right)} \tag{53}$$

where d_p = grid-plate spacing.
s = grid-wire spacing.
r_g = grid-wire radius.

For tubes in which the *screening fraction*, which is the percentage projected area that the grid occupies in its plane, is less than one-sixth, and the ratio of the grid-wire spacing to cathode-grid distance is less than one, the foregoing formula is accurate[2] to within two per cent and becomes more accurate as the two above-mentioned ratios decrease, being within one-half per cent in ordinary tubes. The amplification factor increases as the grid-plate distance is made larger, as the grid-wire radius is increased, or as the grid-wire spacing is decreased. The amplification factor is independent of the cathode-grid distance.

[1] F. B. Vodges and F. R. Elder, Formulas for the Amplification Constant for Three Element Tubes, *Phys. Rev.*, Vol. 24, p. 683, December, 1924.

[2] Formulas for cases where these limitations do not hold are given by Bernard Salzberg, Formulas for the Amplification Factor for Triodes, *Proc. I.R.E.*, Vol. 30, p. 134, March, 1942.

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For a *cylindrical triode* having a cathode and plate that are concentric cylinders, and a squirrel-cage grid of evenly spaced wires parallel to the axis and arranged in a circle as shown in Fig. 23, the amplification factor is[1]

$$\mu = \frac{N \log_{10}\left(\frac{s_p}{s_g}\right) - \log_{10} \cosh\left(\frac{N r_g}{s_g}\right)}{\log_{10} \coth\left(\frac{N r_g}{s_g}\right)} \tag{54}$$

where s_p = plate radius.
s_g = grid-wire-circle radius.
r_g = grid-wire radius.
N = number of grid wires.

The foregoing expression becomes inaccurate if the screening fraction, in this case $Nr_g/\pi s_g$, is greater than one-sixth, and if the distance between grid wires, approximately $2\pi s_g/N$, is greater than the cathode-grid distance. For the cylindrical triode, the amplification factor increases as the number of grid wires is increased, as the gridwire radius is increased, and as the radius of the grid wire circle is decreased.

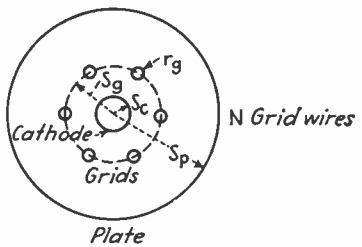


FIG. 23.—Cylindrical triode, end view.

In most tubes the idealized structures shown in Figs. 22 and 23 are only partially realized. Equation (53*a*) may, however, be generalized to include all plane electrode triodes having a grid lying in a plane, regardless of the form of the grid. This is done by recognizing that the length L_g of grid wire per unit area of grid plane is given by the reciprocal of the grid-wire spacing s

$$L_g = \frac{1}{s} \tag{55}$$

and that the screening fraction S is

$$S = \frac{2r_g}{s} \tag{56}$$

Making these substitutions in Eq. (53*a*), the *general amplification factor formula* for a *plane electrode triode* is[2]

$$\mu = \frac{(2\pi d_p L_g) - \log_e \cosh(\pi S)}{\log_e \coth(\pi S)} \tag{57}$$

where d_p = grid-plate spacing.
L_g = length of grid wire per unit area of grid plane.
S = screening fraction = ratio of projected area of grid wires to total area of grid plane.

The corresponding formula for the *amplification factor* of a *cylindrical electrode triode* is

$$\mu = \frac{L_g \log_{10}\left(\frac{s_p}{s_g}\right) - \log_{10} \cosh(\pi S)}{\log_{10} \coth(\pi S)} \tag{58}$$

[1] Vodges and Elder, *loc. cit.*
[2] Y. Kusonose, Calculation of the Characteristics and the Design of Triodes, *Proc. I.R.E.*, Vol. 17, No. 10, pp. 1706–1749, October, 1929.

where L_g = length of grid wire per unit area of grid cylinder.
S = screening fraction of grid structure.
s_p = radius of plate cylinder.
s_g = radius of grid cylinder.

The values of L_g for the most commonly encountered grid structures will now be given. In the *cylindrical triode*, if the grid is a cylindrical *screen* of *square mesh*, then

$$L_g = \frac{4\pi s_g}{d} \tag{59}$$

where s_g = radius of grid cylinder.
d = the distance between wires, as shown in Fig. 24.

In the preceding it is presumed that the wires in the grid mesh are small in diameter

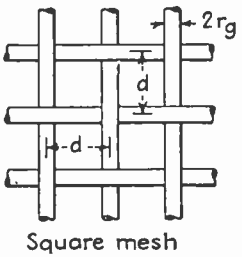


FIG. 24.—Square mesh grid.

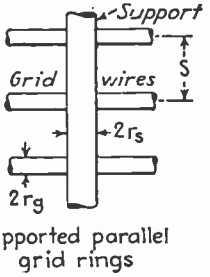


FIG. 25.—Supported parallel grid rings.

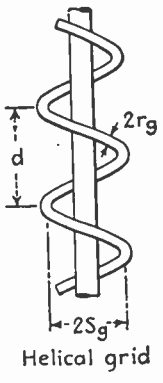


FIG. 26.—Helical grid.

compared to their spacing. If the diameter of the grid wires is not small compared to the spacing, then

$$L_g = \frac{4\pi s_g}{d}\left(1 - \frac{r_g}{d}\right) \tag{60}$$

For a cylindrical triode having a grid of *parallel rings* with supports parallel to the axis, as shown in Fig. 25,

$$L_g = \frac{2\pi s_g}{s} + n_s - \frac{2n_s r_s}{s} \tag{61}$$

where n_s = number of supports.
s = spacing of the grid rings.
r_s = radius of the support wires.
s_g = radius of the grid rings.

For a cylindrical triode with a *helical grid*, as in Fig. 26

$$L_g = \sqrt{1 + \frac{2\pi s_g^2}{d}} \tag{62}$$

where s_g = the radius of the helix.
d = distance between turns.

The *screening fractions* for the cases listed above are readily evaluated. For the cases in which the grid is a *mesh* of *wires*, a set of *parallel wires*, or a *helix*

$$S = \frac{L_g r_g}{\pi s_g} \tag{63}$$

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where r_g = radius of the grid wires.
s_g = radius of the grid wire circle.
For *parallel grid rings with supports*

$$S = \frac{2r_g(L_g - n_s) + 2n_s r_s}{2\pi s_g} \tag{64}$$

Many actual tubes have electrodes that are neither strictly plane nor cylindrical in shape. For such tubes it has been found empirically that a combination of the values of the plane and cylindrical formulas will give the correct amplification factor.[1] The combination formula that gives the actual amplification factor is

$$\mu = \mu_p - K(\mu_p - \mu_c) \tag{65}$$

where μ_p = amplification factor assuming plane electrodes, Eq. (53).
μ_c = amplification factor assuming cylindrical electrodes, Eq. (54).

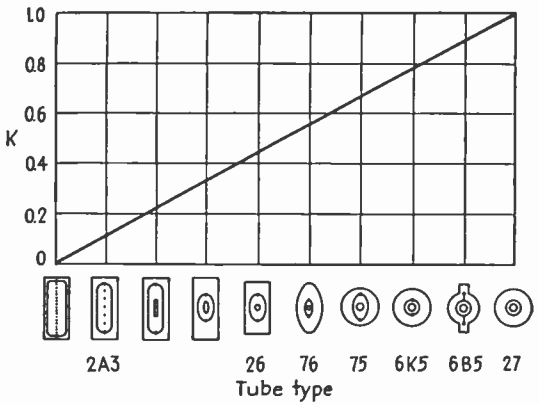


FIG. 27.—Factor K for use in Eq. (65) and associated tube types.

The factor K is a constant that depends upon the tube structure. It is an empirical constant and takes into account the effect of the grid supports. Values of K are given in Table 4 and Fig. 27.

TABLE 4.—FACTOR K FOR USE IN EQ. (65)

Tube Type	K
2A3	0.11
...	0.22
...	0.33
26	0.44
76	0.55
75	0.66
6K5	0.77
6B5	0.88
27	1.00

8. Transconductance Formulas.—The transconductance formulas of triodes and pentodes are obtained from the space-charge relations given in Par. 4 and the equivalent voltage concept discussed in Par. 6.

Triode Transconductance.—The transconductance of a triode with a plane electrode structure is

[1] E. R. Jervis, Amplification Factor Chart, *Electronics*, Vol. 12, p. 45, June, 1939.

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$$g_m = \frac{3.5\sqrt{E_g + \frac{E_p}{\mu}}}{\left[d_c^{4/3} + \frac{(d_c + d_p)^{4/3}}{\mu}\right]^{3/2}} \text{ microamperes per volt per unit area} \qquad (66)$$

where E_g = grid voltage.
E_p = plate voltage.
μ = amplification factor.
d_c = cathode-grid distance.
d_p = grid-plate distance.

and the electrode structure is that of Fig. 22. The transconductance is seen to depend upon the equivalent voltage $(E_g + E_p/\mu)$ and the electrode dimensions. Unlike the amplification factor, the transconductance does depend upon the cathode-grid distance. The transconductance of a plane electrode triode may also be expressed in terms of the plate current:

$$g_m = \frac{2.64 \times 10^{-4} i^{1/3}}{\left[d_c^{4/3} + \frac{(d_c + d_p)^{4/3}}{\mu}\right]^{3/2}} \text{ amperes per volt per unit area} \qquad (67)$$

where i is the current density in amperes per unit area and the other symbols have the same significance as in Eq. (66).[1]

The transconductance of a *cylindrical electrode triode* with the configuration of Fig. 23 is

$$g_m = \frac{2.2\left(E_g + \frac{E_p}{\mu}\right)^{1/2}}{\left[(s_g\beta_{cg}^2)^{2/3} + \frac{1}{\mu}(s_p\beta_{cp}^2)^{2/3}\right]^{3/2}} \text{ microamperes per volt per unit length of structure} \qquad (68)$$

where E_g = grid voltage.
E_p = plate voltage.
μ = amplification factor.
s_g = radius of grid-wire circle.
s_p = plate radius.
β_{cg}^2 = value of β^2 from Table 3 for argument s_g/s_c.
β_{cp}^2 = value of β^2 from Table 3 for argument s_p/s_c.
s_c = cathode radius.

9. Ultra-high-frequency Effects and Tubes.[2]—In ordinary applications of vacuum tubes the transit time of the electrons in the tube is short compared with the period of the applied voltages. As frequency is increased this becomes less and less true until finally the electron will require an appreciable fraction of a cycle to pass from one electrode to another. When this happens the behavior of the tube changes markedly.

Ultra-high-frequency Behavior of Diodes.—Even the simple diode tube experiences a difference in its behavior at ultra-high frequencies. The dynamic plate resistance of the diode drops, and at certain frequencies even becomes negative, and there are also modifications produced in the plate-cathode capacity.

When transit-time effects are of importance it is convenient for the purpose of analysis to consider that the impedance between the plate and cathode of the tube,

[1] J. H. Fremlin, Calculation of Triode Constants, *Elec. Comm.*, Vol. 18, p. 39, July, 1939.

[2] A comprehensive discussion and summary of transit-time effects is given in the book by F. B. Llewellyn, "Electron Inertia Effects," Cambridge (London), 1941. Much of this material is also given by F. B. Llewellyn, Operation of Ultra-high-frequency Tubes, *Bell System Tech. Jour.*, Vol. 15, p. 575, October, 1936.

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including the active part of the interelectrode capacity[1] as well as the resistance effects due to electron current flow, be represented by a resistance R in series with a capacitive reactance X (*i.e.*, $Z = R - jX$). In terms of this representation one has for complete space charge[2]

$$R = R_p \frac{12}{\theta^4} [2(1 - \cos\theta) - \theta \sin\theta] \tag{69}$$

$$X = -\left\{\frac{1}{\omega C} + R_p \frac{12}{\theta^4} [\theta(1 + \cos\theta) - 2\sin\theta]\right\} \tag{70}$$

where R_p = dynamic plate-cathode resistance of diode at low frequencies.
θ = transit-time angle, radians ($\theta = \omega$ times transit time, sec.).
$\omega = 2\pi$ times frequency.
C = active part of the plate-cathode capacity of the tube measured with the cathode cold.

At frequencies where the transit-time effects are small, Eqs. (69) and (70), after suitable transformations, become

$$R = R_p \left[1 - \frac{\theta^2}{15}\right] \tag{71}$$

$$X = -\frac{3\theta}{10} R_p \left[1 - \frac{\theta^2}{25.2}\right] \tag{72}$$

It will be noted that θ is the transit time in radians, on the basis that the time represented by one cycle is 2π radians. A curve showing the variation of R and X with

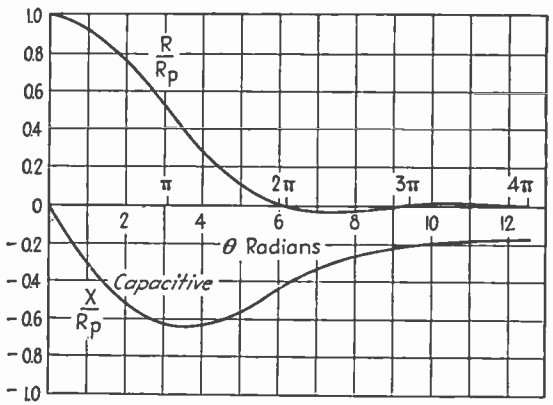


FIG. 28.—Resistance and reactance components of diode plate-cathode impedance as a function of transit angle θ (series representation).

transit angle is given in Fig. 28. It is seen that as the frequency is increased the resistance R first drops increasingly from its low-frequency value R_p until it becomes zero when the transit time is a full cycle, and with still greater transit times then

[1] The active part C of the interelectrode capacity is the capacity between parts of the tube that are effective in causing a current flow. The capacity between leads, or inactive parts of the plate or cathode structure, is not included, and represents an additional capacity that shunts the "active" impedance $R + jX$.

[2] See Llewellyn, *op. cit.*, p. 48; W. E. Benham, Electron Transit Time, *Wireless Eng.*, Vol. 16, p. 598, December, 1939. Graphical means of analyzing transit-time effects in diodes are described by R. W. Sloane and E. G. James, Transit-time Effects in Diodes in Pictorial Form, *Jour. I.E.E.*, Vol. 79, p. 291, 1936; also, Wireless Section, *I.E.E.*, Vol. 11, p. 247, September, 1936; and Rudolf Kompfner, Transit-time Phenomena in Electronic Tubes, *Wireless Eng.*, Vol. 19, p 3, January, 1942.

Ex. 1044-0317

oscillates about the zero value, being slightly negative at times. The reactive component of the impedance is always capacitive, but varies greatly. For very large transit times the tube impedance approaches that offered by the "cold" interelectrode capacity C, and the resistance component becomes very small.

In the preceding, the effect of the active part of the interelectrode capacity has been represented by a reactance in series with the equivalent plate resistance R. At frequencies where the transit-time effects are small, it is, however, more convenient to think of the tube plate-cathode impedance as representing a parallel combination of a capacity in parallel with an equivalent plate resistance. This equivalent resistance at low frequencies R is given by Eq. (69) or (71), while the equivalent low-frequency shunting capacity is $0.6C$. This low-frequency representation holds up to transit times of about $\theta = 1$. It will be noted that the presence of the electron stream in the interelectrode space causes the "hot" capacity of the tube to be only a fraction of the "cold" capacity C.

Input Resistance of Control Grids at Ultra-high Frequencies.[1]—At very high frequencies the transit time contributes a conductance component to the grid input admittance. A current will be induced in an electrode by the approach or passage of an electron even if the electron does not hit the electrode.[2] In the case of a triode or a tube with a screen grid, the currents induced in the control grid by approaching and receding electrons are different because the differences in velocities and the finite transit time impart an in-phase component to the induced current. The result of this action is that the conductance increases with frequency according to the formula[3]

$$G_g = Kg_m f^2 T^2 \tag{73}$$

where G_g = conductive component of grid input admittance.
g_m = transconductance of tube.
f = frequency.
T = transit time to a reference point near the grid plane.
K = a constant for any particular tube, a function of the cathode-grid and grid-plate transit times.

Examination of the preceding formula shows that if all the dimensions of a tube are reduced by a factor M, then the input conductance will be reduced by a factor M^2. If all the voltages are increased by a factor N, the input conductance will be reduced by a factor $N^{1/2}$.

An idea of the magnitude of the transit-time effect upon grid input conductance can be gained from the fact that the input resistance of a 57 tube is of the order of megohms at 5 megacycles, while at 30 megacycles it has dropped to 20,000 ohms and at 100 megacycles it has dropped to about 1,500 ohms. The acorn pentode 954, which makes use of a very small cathode-grid spacing, has an input resistance that is of the order of 10 times as great as that of ordinary triodes, being about 20,000 ohms at 100 megacycles. The theoretically predicted variation of conductance with the square of the frequency is borne out by experimental observations.

Ultra-high-frequency Behavior of Triodes.—The behavior of triodes at ultra-high frequencies has been worked out to include both the effects of transit time and space charge.[4] Analysis shows that it is possible to set up a circuit that is the equivalent of

[1] The discussion here assumes that the electrode on the side of the control grid away from the cathode (screen or plate, as the case may be) is at ground potential to very high frequencies.

[2] B. J. Thompson, Review of Ultra-high Frequency Vacuum Tube Problems, *R.C.A. Rev.*, Vol. 3, p. 146, October, 1938.

[3] W. R. Ferris, Input Resistance of Vacuum Tubes as Ultra-high Frequency Amplifiers, *Proc. I.R.E.*, Vol. 24, p. 82, January, 1936; D. O. North, An Analysis of the Effects of Space Charge on Grid Impedance, *Proc. I.R.E.*, Vol. 24, p. 108, January, 1936.

[4] F. B. Llewellyn, Operation of Ultra-high Frequency Tubes, *Bell System Tech. Jour.*, Vol. 14, p. 112, October, 1935.

Ex. 1044-0318

the vacuum tube and its transit-time effects, in terms of parameters that are independent of frequency and that are not complex. This is done by using a delta of impedances with the individual arms represented by the proper series or parallel combination of elements as shown in Fig. 29.[1] In this circuit the need for complex tube constants is avoided by the proper arrangements of circuit elements. The actual values applicable to the various parts of the circuit of Fig. 29 depend on the transit times, the dynamic plate resistance at low frequencies, and the capacities between the active parts of the various electrodes. The relations involved are too complicated and specialized to be given here, but are to be found in the references cited. The equivalent circuit of Fig. 29 is valid up to frequencies of 100 mc or more with ordinary tubes.

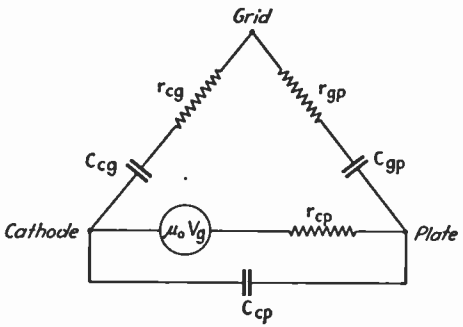


FIG. 29.—Equivalent delta of a triode at ultra-high frequencies.

The circuit of Fig. 29 is valid only for the active part of the tube, *i.e.*, the portion of the tube elements that intercept electrons. The effect of the lead impedances is not included, an important limitation to keep in mind, since the effects of the lead impedances may be as great as the effects of transit time of the electrons. Neither are the capacities between inactive parts of the electrodes and leads included. These capacities act in shunt with the equivalent circuit of Fig. 29, between the appropriate electrodes.

As far as the plate circuit of a triode is concerned, the effect of transit time can be taken into account by assuming that the ordinary equivalent plate circuit consisting of a generator voltage μe_s acting in series with the plate resistance R_p still holds (see Par. 3, Sec. 5), but that the value of amplification factor and plate resistance (and hence transconductance) is now modified in magnitude and a phase angle is associated with them. In particular, the transconductance tends to decrease in magnitude and lag by an increasingly large amount as the transit time increases,[2] while the amplification factor likewise tends to decrease and have an increasingly large phase angle.

Ultra-high-frequency Negative Grid Tubes.—Triode, beam, and other similar tubes intended for negative grid operation at very high frequencies must be especially designed for such service. The desired characteristics are those outlined in Par. 11, Sec. 6, for oscillator tubes, namely: (1) close electrode spacing to minimize transit time and give a high transconductance in proportion to electrode capacities; (2) short leads of relatively large diameter to minimize lead inductance and power losses in the leads (achieved in many cases by the use of multiple leads from the same electrode); (3) electrode and lead arrangements that facilitate operation with resonant transmission lines, the ideal being when the leads and electrodes of the tube represent extensions of the transmission line tank circuits.[3]

The tube should also be designed so that there can be no stray electrons that can shoot out of the open ends of the tube structure, and take a circuitous route in

[1] F. B. Llewellyn, Equivalent Networks of Negative-grid Vacuum Tubes at Ultra-high Frequencies, *Bell System Tech. Jour.*, Vol. 15, p. 575, October, 1936.

[2] Thus see F. B. Llewellyn, Phase Angle of Vacuum-tube Transconductance at Very High Frequencies, *Proc. I.R.E.*, Vol. 22, p. 947, August, 1934; Thompson, *loc. cit.*

[3] A tube in which this idea is carried to the limit is described by I. E. Mouromtseff and H. V. Noble, A New Type of Ultra-short-wave Oscillator, *Proc. I.R.E.*, Vol. 20, p. 1328, August, 1932.

Ex. 1044-0319

traveling from the cathode to the plate.[1] Such electrons are likely to bombard the glass envelope and thereby release secondary electrons. These indirect paths, either with or without involving secondary electrons, result in some electrons arriving at the anode with excessively long transit times, with undesirable consequences. In order to avoid difficulties of this sort, it is desirable to close the ends of the tube structure so that it will not be possible for even a few electrons to stray along indirect routes.

When high voltages are involved, as is the case with transmitter tubes, the structural arrangement must be such as to provide an adequate amount of insulation. This is particularly important as the frequency increases, because the dielectric loss in the glass is proportional to the square of the frequency, and can reach sufficiently large values at extremely large frequencies to start a cumulative heating that results in puncture of the glass envelope.

It is possible to obtain satisfactory ultra-high-frequency operation of receiving tubes up to higher frequencies than can be achieved with power tubes. This is because the close spacings that favor ultra-high-frequency operation inevitably carry with them the necessity of making the tube dimension small, and this limits the power-dissipating ability of the tube. The amount of power that can be developed in the output of an ultra-high-frequency tube is accordingly less as the frequency limit is pushed to higher values. Ingenious arrangements have been devised for increasing the ability to dissipate heat, as, for example, the use of cooling fins attached to the plate, and in some cases also to the grid, and the use of water or forced-air cooling, even in tubes of moderate power rating. Such arrangements at best, however, merely improve an otherwise unfavorable situation, but in no case do they eliminate it.

A variety of ultra-high-frequency tubes are available. These include receiving tubes of the triode, pentode, and beam types, and also triode, beam, and pentode power tubes. Most of these are air-cooled, but a few of the larger types designed for such applications as television transmitters are arranged for water or forced-air cooling. In a few instances tubes are arranged in duplex (two tubes in a single envelope) for push-pull operation.[2]

10. Special Considerations Involved in Power Tubes.[3]—Tubes intended for use as power amplifiers must be capable of dissipating plate and grid losses in proportion to the desired power. Likewise, the electron emission from the cathode and the amount of voltage that can be applied to the anode with safety must be in proportion to the power output that the tube is to develop.

[1] W. G. Wagener, The Developmental Problems and Operating Characteristics of Two New Ultra-high-frequency Triodes, *Proc. I.R.E.*, Vol. 26, p. 401, April, 1938.

[2] Tubes especially designed for ultra-high-frequency use represent the accumulation of a great deal of specialized technical experience, much of which is of interest only to the designer of tubes. Because of this, and the fact that the subject is so new that the techniques are being steadily improved and modified, detailed descriptions of the various ultra-high-frequency tubes now available on the open market are not included here. The reader wishing to obtain a background in the subject is referred to the following articles: B. J. Thompson and G. M. Rose, Jr., Vacuum Tubes of Small Dimensions for Use at Extremely High Frequencies, *Proc. I.R.E.*, Vol. 21, p. 1707, December, 1933; M. J. Kelly and A. L. Samuel, Vacuum Tubes as High Frequency Oscillators, *Elec. Eng.*, Vol. 53, p. 1504, November, 1934; A. L. Samuel and N. E. Sowers, A Power Amplifier for Ultra-high Frequencies, *Proc. I.R.E.*, Vol. 24, p. 1464, November, 1936; A. L. Samuel, A Negative Grid Triode Oscillator and Amplifier for Ultra-high Frequencies, *Proc. I.R.E.*, Vol. 25, p. 1243, October, 1937; A. K. Wing, A Push Pull Ultra-high-frequency Beam Tetrode, *R.C.A. Rev.*, Vol. 4, p. 62, July, 1939; A Transmitter for Frequency-modulated Broadcast Service Using a New Ultra-high-frequency Tetrode, *R.C.A. Rev.*, Vol. 5, p. 327, January, 1941; A. K. Wing, Jr., and J. E. Young, A New Ultra-high-frequency Tetrode and Its Use in a 1-kilowatt Television Sound Transmitter, *Proc. I.R.E.*, Vol. 29, p. 5, January, 1941; K. C. DeWalt, Three New Ultra-high-frequency Triodes, *Proc. I.R.E.*, Vol. 29, p. 475, September, 1941; Cecil B. Haller, The Design and Development of Three New Ultra-high-frequency Transmitting Tubes, *Proc. I.R.E.*, Vol. 30, p. 20, January, 1942.

[3] An outstandingly fine discussion on the construction of large tubes is given by J. Bell, J. W. Davies, and B. S. Gossling, High-power Valves: Construction, Testing, and Operation, *Jour. I.E.E.*, Vol. 83, p. 176, 1938; also, Wireless Section, *I.E.E.*, Vol. 13, p. 177, September, 1938.

Ex. 1044-0320

Small power tubes are often merely large receiving tubes constructed according to the standard receiving-tube techniques, involving oxide-coated cathodes, "getters" to obtain and maintain vacuum, etc. Such tubes are not capable of standing much abuse, however, and the maximum power that can be developed with this type of construction is limited.

Larger tubes, up to dissipations of several hundred watts, are normally still inclosed in a glass envelope but are constructed along lines entirely different from those of receiving tubes. The plates are commonly of molybdenum or tantalum, carbon sometimes being used. Grids are usually molybdenum, with tungsten and tantalum as alternates. Cathodes are sometimes oxide-coated, but more frequently are thoriated tungsten carburized for long life. The exhaust procedure is so thorough that a high degree of vacuum can be maintained without a getter. The various electrodes inside the tube are heated to high temperatures during exhaust, and the glass envelope is simultaneously baked at a temperature just below the softening point, and this process is continued until all the occluded gases have been removed. It is possible in this way to obtain a degree of vacuum such that a satisfactory life can be obtained with thoriated tungsten filaments even at high anode voltages, particularly if tantalum is used for the plate.

In tubes where the anode dissipation is of the order of kilowatts, the anodes of the tube are in the form of copper cylinders, which are part of the envelope and are cooled by means of circulating water or by being soldered to a radiating structure with many fins, which are cooled by means of an air blast. Analysis and experience indicate that water and forced-air cooling are about equally effective when used with vacuum tubes having external anodes.[1] Decision as to which type of cooling is preferable depends upon the individual factors involved in the particular case. Thus air-cooled tubes behave better in operation than water-cooled tubes with respect to "flashback." With forced-air cooling the anodes tend to run hotter, but the glass ends run cooler than with water cooling, which has some advantages and some disadvantages. With forced-air cooling the anode capacity to ground is increased by the large cooling structure, something to be avoided at very high frequencies.

The most effective way to employ water cooling is to arrange the water jacket so that the water passes over the anode in a very thin stream of high velocity. In this way any steam bubbles that may be formed on the anode tend to be scraped away by the velocity of the cooling water. The tendency for such bubbles to form limits the amount of heat that can be dissipated by a water-cooled anode, since, if the tendency of the bubble to stick to the copper is not overcome by the flow of cooling water, then that particular spot on the anode ceases to be cooled by direct contact with the water, and may readily become hot enough to cause failure of the tube.

In tubes with external anodes that are water-cooled, there is a tendency for the grid to produce a focusing effect on the electrons, which causes hot spots to be produced on the anode at the points where the electrons concentrate.[2] This is much more troublesome with water cooling than with forced-air cooling. With water cooling these hot spots tend to cause steam bubbles to form on the surface of the anode.

The cathodes of tubes employing water or forced-air cooling are always tungsten. This is because such high-power tubes are operated at very high anode voltages, and it is not practical to exhaust copper-anode tubes so thoroughly as glass-envelope tubes, where the anodes can be brought to incandescent heat during the exhaust process.

[1] Further discussion of both air and water cooling is given by I. E. Mouromtseff, Water and Forced-air Cooling of Vacuum Tubes, *Proc. I.R.E.*, Vol. 30, p. 190, April, 1942. See also E. M. Ostlund, Air Cooling Applied to External-anode Tubes, *Electronics*, Vol. 13, p. 36, June, 1940.

[2] I. E. Mouromtseff, The Influence of Grid Focusing Effect on Plate Dissipation Limit of a Vacuum Tube, *Communications*, Vol. 18, p. 9, December, 1938.

Ex. 1044-0321

Thoriated tungsten filaments accordingly do not behave satisfactorily, and pure tungsten must be employed.[1]

The thoriated-tungsten and oxide-coated cathodes should be operated at the rated voltage to obtain maximum life. With tungsten filaments, however, decreasing the filament voltage will increase the useful life at the expense of reduced electron emission (and hence lowered peak anode current). Varying the voltage applied to a tungsten filament by 5 per cent from normal will double or halve the life as the case may be, while a 10 per cent variation affects the life by a factor of 4. A study of the optimum diameter of a tungsten filament, if the costs of power and tube replacement are taken into account, indicates that the optimum diameter is one that would correspond to a life of the order of 10,000 hours or more.[2]

In water-cooled or forced-air cooled tubes operating at high anode voltages (such as 15 to 20 kv), trouble is frequently encountered from arc-backs or "flash arcs" in the tube.[3] These are ordinary high-current arcs that suddenly form between cathode and plate for no apparent reason, and short-circuit the plate supply. After such an arc has been broken by opening of the circuit breakers, and the anode voltage has been reapplied, the tube may continue to operate for many hours or days without any further trouble, while in other cases another flash arc may occur almost immediately. A tube that has a tendency to arc-back will generally improve in this respect if it is put through a conditioning process[4] that involves applying an alternating potential of about 25,000 volts between plate and filament of the tube with the filament unlighted, with suitable relays and current-limiting devices in the circuit to prevent damage from arcs. This results in the production of many flash arcs in the tube, which gradually clean up the gas that is causing the trouble, and bring the tube to a satisfactory operating condition.

Power tubes are commonly operated so that the grid goes positive during a part of the cycle. This results in grid current, and causes power dissipation at the grid of the tube, which is sometimes the limiting factor in tube operation.[5] As a consequence, the grids of power tubes often operate at relatively high temperatures, and such materials as molybdenum, tungsten, or tantalum are accordingly generally used. The fraction of the primary electrons intercepted by the grid depends upon the grid potential relative to the anode potential, and upon the grid structure. In ordinary triodes with equal grid and plate voltages the effective grid area, insofar as intercepting the flow of primary electrons is concerned, is between 120 and 180 per cent of the actual grid area.[6] The grid heating that takes place is determined by the number of primary electrons intercepted by the grid, and by the grid voltage. The actual d-c grid current as measured by a meter may differ from the number of primary electrons received by the grid as a result of secondary emission causing the grid to lose secondary electrons at the same time that it receives primary electrons. The amount of current thus lost through secondary emission will be affected by the electrode potentials, by the grid temperature, and by the character of the grid surface.

[1] It is possible by keeping a water-cooled tube continuously on a vacuum pump, to maintain a degree of vacuum about one decimal point greater than feasible when the tube is sealed off. Under these conditions thoriated-tungsten filaments are entirely practical even at very high anode voltages. Such continuously evacuated tubes have been developed by various individuals and organizations for very high-power service, but as yet have not had any extensive continuing commercial use.

[2] Thus see J. J. Vormer, Filament Design for High Power Transmitting Valves, *Proc. I.R.E.*, Vol. 26, p. 1399, November, 1938.

[3] See Bell, Davies, and Gossling, *loc. cit.*; B. S. Gossling, The Flash-arc in High-power Valves, *Jour. I.E.E.*, Vol. 71, p. 460, 1932; also, Wireless Section, *I.E.E.*, Vol. 7, p. 192, September, 1932.

[4] Further details are given in Vacuum Tube Reconditioning, *Electronics*, Vol. 14, p. 84, January, 1941.

[5] I. E. Mouromtseff and H. N. Kozanowski, Grid Temperature as a Limiting Factor in Vacuum Tube Operation, *Proc. I.R.E.*, Vol. 24, p. 447, March, 1936.

[6] Karl Spangenberg, Current Division in Plane-electrode Triodes, *Proc. I.R.E.*, Vol. 28, p. 226, May, 1940.

Ex. 1044-0322

In the case of thoriated tungsten and oxide-coated cathodes the secondary emissions may, under some conditions, become quite large as a result of cathode material that has been deposited upon the grid.

11. Receiving Tubes.—Receiving tubes can be divided into two main classes: the glass-envelope tubes, and the so-called "metal" tubes. The glass-envelope tubes, as the name implies, have a glass envelope, and the electrodes are mounted on leads running through either a stem or a glass cup, to which the bulb is sealed after the electrodes are mounted and adjusted. The metal-envelope tube employs a metal shell in place of the glass bulb, with all the leads, except perhaps a control-grid lead, brought through a glass button or glass beads in the base of the tube.

The cathodes of receiving tubes, either filaments or heaters, are practically always of the oxide-coated type. The grid and plate electrodes and side rods are usually constructed from nickel, and a "getter" is used to obtain the desired degree of vacuum. The exhaust, and also many of the other assembly operations, are performed on automatic machines.

The actual technical details of receiving tubes represent primarily a production problem, and accordingly are beyond the scope of this work. The literature available on the subject is relatively limited, and always lags behind the current practice in industry.[1]

12. "Getters."[2]—A high vacuum is obtained in receiving tubes by means of a "getter," which is volatilized inside the tube for the purpose of removing residual gas by either chemical or mechanical action. Magnesium is widely used as a getter, but other materials, such as barium, barium berylliate,[3] zirconium, phosphorus, etc., can also be employed, as well as various mixtures. A getter should be initially inert, but should be of such character that it can be made highly active by some process such as a chemical change, or vaporization of material inert at room temperature.

Some getters also act as "keepers" in that they not only remove what gas is present in the tube at the time of flashing but also combine with any gas that may subsequently be liberated within the tube.

Getters cannot be used in tubes where the anode dissipation is large. This is because the large amount of heat liberated in such tubes would vaporize the getter and destroy the vacuum.

Certain metals have the property of absorbing gases when raised to a high temperature. For example, tantalum will absorb gases when very hot, and this is one of the things that make tantalum a desirable metal to use in power tubes for plates and grids.

Zirconium is also a useful metal in this regard.[4] Zirconium at 1400° will absorb copious quantities of such gases as oxygen, nitrogen, carbon monoxide, and carbon dioxide. The optimum temperature for taking up hydrogen is, however, approximately 350°C, and at higher temperatures hydrogen begins to be given off by the metal. As a result of these properties it is possible to maintain an extremely high

[1] Useful articles on receiving tubes are M. Benjamin, C. W. Cosgrove, and G. W. Warren, Modern Receiving Valves: Design and Manufacture, *I.E.E. Wireless Proc.*, Vol. 12, p. 65, June, 1937 E. R. Wagner, Raw Materials in Vacuum Tube Manufacture, *Electronics*, Vol. 7, p. 104, April, 1934; Processes in Vacuum Tube Manufacture, *Electronics*, Vol. 7, p. 213, July, 1934; G. E. Moore, Improved Repeater Tubes, *Bell Lab. Rec.*, Vol. 18, p. 219, March, 1940; Newell R. Smith and Allen H. Schooley, Development and Production of the New Miniature Battery Tubes, *R.C.A. Rev.*, Vol. 4, p. 496, April. 1940; S. R. Mullard, The Development of the Receiving Valve, *Jour. I.E.E.*, Vol. 76, p. 10, 1935, also Wireless Section, *I.E.E.*, Vol. 10, p. 1, March, 1935.

[2] A good discussion of getters is given by E. A. Lederer and D. H. Wamsley, "Batalum," a Barium Getter for Metal Tubes, *R.C.A. Rev.*, Vol. 2, p. 117, July, 1937.

[3] See E. A. Lederer, Recent Advances in Barium Getter Technique, *R.C.A. Rev.*, Vol. 4, p. 310, January, 1940.

[4] Further information is given by J. D. Fast, Zirconium and Its Compounds with a High Melting Point, *Philips Tech. Jour.*, World's Fair Issue, 1939.

vacuum, provided that there is zirconium in the tube at these two temperatures. This can be accomplished either by employing two zirconium filaments with different filament currents or by using a single piece of zirconium in which different parts of the metal are raised to different temperatures, so that the temperature range extends from 300 to 1400°C. This expedient is frequently used in experimental tubes to clean up gases that may be evolved during use, and can also be employed in special tubes that are particularly difficult to evacuate.

13. Effect of Gas upon Tube Characteristics—Maximum Allowable Resistance in Grid Circuits.—Very small traces of gas in vacuum tubes affect the characteristics adversely in a number of ways as a result of the positive ions produced in the tube by collision between the gas molecules and the electrons flowing to the anode. The positive ions travel in the opposite direction from the electrons, and normally end their existence by falling into the cathode or the negative control grid. Electrons that bombard the cathode tend to destroy the emission of thoriated-tungsten and oxide-coated cathodes. Positive ions collected by the negative grid result in grid current, which limits the resistance that may be inserted in series with a negative grid, and which also introduces noise.

Positive-ion currents to the grid limit the d-c resistance that may safely be placed in series with the control grid and the cathode, because the voltage drop that such a grid current produces across the resistance has a polarity that makes the grid less negative than would otherwise be the case. Thus if the tube begins to liberate gas with resulting positive-ion grid current, the grid becomes less negative, thereby increasing the space current. This increases the number of positive ions produced, and will cause additional grid current, and still greater reduction in the negative grid potential. If the resistance in the grid circuit is high enough, this process can become cumulative, and in some types of tubes can easily result in the destruction of the tube as a result of excessive plate current caused from loss of grid bias. The maximum resistance that it is permissible to place in series with the grid electrode depends upon the tube characteristics and the method of obtaining bias. It is of the order of several megohms in small tubes used for voltage amplification at audio and radio frequencies. With small power tubes, such as the output tubes of radio receivers and public-address systems, the allowable grid resistance is much less, particularly if a fixed bias is employed. The use of self-bias permits an increased resistance to be employed in the grid-cathode circuit, since self-bias provides an automatic protection against excessive increase in plate current.

Positive-ion current to a control grid has a "shot" effect associated with it, since the current flow is composed of individually charged particles, and is not a uniform fluid. The noise that is produced by this current flowing through an impedance Z_g between grid and cathode can be expressed by the formula

$$R_n = 19.3\, I_g |Z_g|^2 \tag{74}$$

where $|Z_g|$ is the absolute value of the external circuit impedance between grid and cathode, I_g is the grid current in amperes, and R_n the resistance that, when connected between grid and cathode, would produce the same thermal agitation voltage as is actually produced by the grid current flowing through the impedance Z_g.

In addition to the noise effect represented by Eq. (74), the presence of gas introduces an additional noise factor through the fact that positive ions not captured by the grid travel toward the cathode and produce irregularities in the space charge around the cathode. Little is known about this effect, however, beyond the fact that it is probably small under ordinary circumstances compared with the effect represented by Eq. (74).

Ex. 1044-0324

14. Miscellaneous Types of Tubes. *Space-charge-grid Tubes.*—In the space-charge-grid tube there is an auxiliary grid, called the space-charge grid, located between the cathode and the control grid, and operated at a low positive potential. This grid increases the number of electrons drawn out of the space charge near the cathode. Although some of these electrons are immediately attracted to the space charge grid, many pass through its meshes into the region between the space-charge grid and the control grid. Here they are slowed down by the retarding field and form a space charge or virtual cathode, as shown in Fig. 30. The virtual

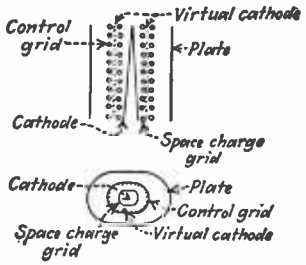


Fig. 30.—Details of a space-charge grid tube.

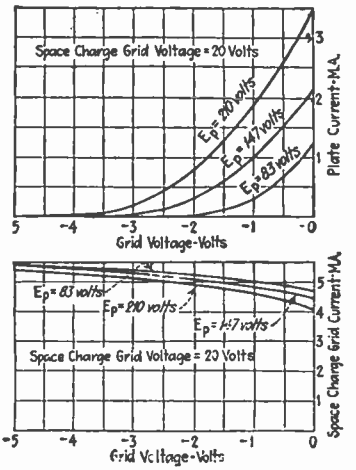


Fig. 31.—Characteristic curves of space-charge grid triode.

cathode serves as an actual cathode as far as the remainder of the tube is concerned. The characteristics that result are similar to those for conventional tubes, as is apparent from Fig. 31.

The space-charge-grid arrangement provides a virtual cathode with a large area located very close to the control grid. This gives a high transconductance in proportion to the plate potential. At the same time the characteristic curves of the tube tend to have excessive curvature when considered over an appreciable range of grid

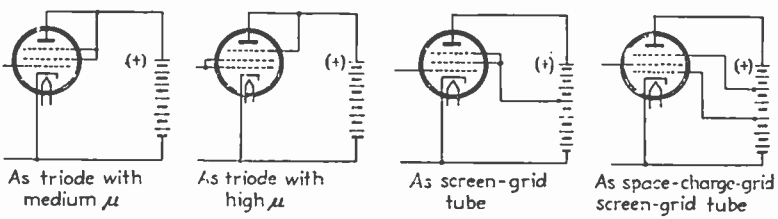


Fig. 32.—Pentode tube arranged in various ways.

voltage. Thus space-charge-grid tubes have small power-handling capacity. Furthermore, the space-charge grid draws a very heavy current, usually more than half of the total space current.

Special Connections for Conventional Tubes.—It is possible to operate conventional tubes to give special characteristics, either by employing special connections or by the proper combination of electrode voltages. Thus a pentode tube can be connected to operate as a triode with medium or high amplification factor, as a screen-grid tube, or as a space-charge-grid tube, as shown in Fig. 32.

Another rearrangement of a conventional tube is to interchange the functions of the grid and plate by making the grid the anode electrode, and by using the plate as

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the negative control electrode, as shown in Fig. 33. The operation of such an *inverted tube*[1] rests on the fundamental fact that the space current flowing to the anode, which in this case is the positive grid, depends almost solely upon the electrostatic field in the vicinity of the cathode and is substantially independent of how this field is produced. Since both plate and grid potentials affect the intensity of this electrostatic field, it is possible to use a negative plate as a control electrode to serve the same purpose as the negative grid in the usual tube. The result is an amplification factor that is quite low, the value being approximately $1/\mu$ where μ is the amplification factor of the tube operated in the normal manner. The dynamic anode resistance is also much lower than in the corresponding tube operated in the normal manner. The inverted vacuum tube is useful when it is necessary to control a large current by a very high voltage, without at the same time time consuming any energy from the high potential source.

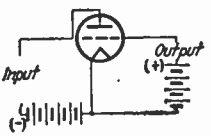


FIG. 33.—Circuit of inverted tube.

The Dynatron and Other Negative-resistance Arrangements.—In a screen-grid tube, if the screen electrode is operated at a higher voltage than the plate, and the secondary emission at the plate is sufficient, there is a range of plate voltages over which the net resultant plate current (primary electrons arriving at the plate minus secondary electrons lost to the screen) will decrease with increasing plate voltage. This effect is shown by the curve for $E_{sg} = 83$ volts in Fig. 19. The decreasing plate current with increasing plate voltage represents a negative resistance, and when a tube is used in this manner to obtain a negative resistance, it is termed a *dynatron*.[2] The negative resistance results from the fact that while the number of primary electrons that the plate receives is independent of the plate voltage, the number of secondary electrons produced at the plate increases with increasing plate voltage. With the screen more positive than the plate, all these secondary electrons flow to the screen; so if the number of secondary electrons that can be produced is considerable, the plate current not only will decrease with increasing plate voltage but will reverse in polarity at a plate potential where, on the average, each primary electron produces one secondary electron. Dynatron action is capable of giving a negative resistance as low as about 20,000 ohms, using ordinary receiving tubes in which the plates have not been specially treated to reduce secondary emission.[3] The magnitude of the negative resistance can be controlled by varying the control-grid potential. The magnitude of the negative resistance is not particularly stable with time, because secondary emission is very sensitive to the surface conditions, and these change with use.

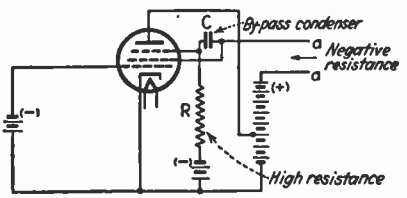


FIG. 34.—Retarding field or transitron method of producing a negative resistance.

An alternative method of obtaining negative resistance from a vacuum tube, which is generally more stable than the negative resistance obtained by dynatron action, is illustrated in Fig. 34.[4] Here a pentode tube is connected as shown and the poten-

[1] F. E. Terman, The Inverted Vacuum Tube—A Voltage Reducing Power Amplifier, *Proc. I.R.E.*, Vol. 16, p. 447, April, 1928.

[2] For further information on the dynatron, and some of its uses, see Albert W. Hull, The Dynatron—A Vacuum Tube Possessing Negative Electric Resistance, *Proc. I.R.E.*, Vol. 6, p. 5, 1918.

[3] It is generally found that some of the very early screen-grid tubes are the best for dynatron purposes. Tubes manufactured more recently commonly have the plate so treated as to reduce secondary emission. This causes the dynatron effect to be greatly reduced, or even eliminated.

[4] E. W. Herold, Negative Resistance and Devices for Obtaining It, *Proc. I.R.E.*, Vol. 23, p. 1201, October, 1935.

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tials so adjusted that there is a virtual cathode between the screen and the suppressor. Under these conditions a fraction of the space current drawn from the cathode is returned back toward the cathode, to be ultimately collected by the screen. By arranging the circuit as shown, so that the screen and suppressor are at the same alternating potential as a result of by-pass condenser C, an increase in the screen potential likewise increases the suppressor voltage. This reduces the screen current because more of the electrons entering the virtual cathode pass on to the plate, and fewer return to the screen. The result is a negative-resistance effect between the terminals *aa*, the magnitude of which can be controlled by the control grid of the tube. This method of obtaining a negative resistance is referred to as a retarding-field negative-transconductance tube, and has also been called the transitron method. As compared with the dynatron, the transitron arrangement has the advantage that the negative resistance obtained is more stable with time, and also a much lower negative resistance can be obtained.

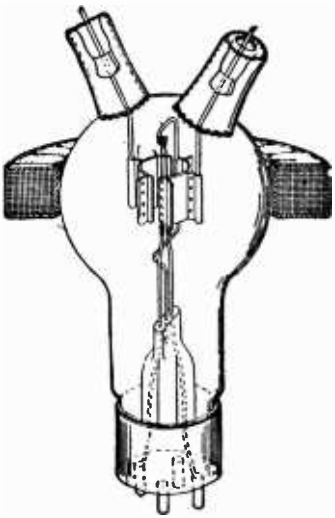


FIG. 35.—The split-anode magnetron.

It is possible to obtain negative resistance from vacuum tubes in still other ways. Some of these involve special connections of conventional vacuum tubes, such as feedback circuits and other arrangements in which the effective impedance across a pair of terminals is negative.[1] It is also possible to employ special tubes of various sorts.[2] Finally, under certain space-charge conditions in a space-charge-grid tube, it is possible for the input conductance to the control grid to be negative.[3] It is also found that associated with these negative-resistance effects obtainable through space-charge action, it is also possible to obtain a negative-capacity effect by purely electronic means involving the behavior of the virtual cathodes.

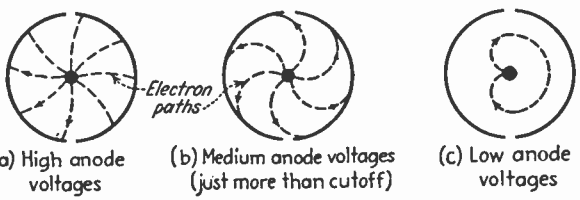


FIG. 36.—Electrode paths in split-anode magnetron under various conditions.

Magnetrons.[4]—A magnetron is a vacuum tube in which the flow of electrons from the cathode to the plate is affected by a magnetic field. The split-anode magnetron illustrated in Fig. 35 consists of a filamentary cathode and two semicylindrical plates

[1] For example, see F. E. Terman, R. R. Buss, W. R. Hewlett, and F. C. Cahill, Some Applications of Negative Feedback with Particular Reference to Laboratory Equipment, *Proc. I.R.E.*, Vol. 27, p. 647, October, 1939.

[2] Some possibilities are described by Harry C. Thompson, Electron Beams and Their Applications in Low Voltage Devices, *Proc. I.R.E.*, Vol. 24, p. 1276, October, 1936.

[3] This latter effect is occasionally encountered in multigrid tubes in which there is a virtual cathode in the tube, as, for example, in mixer tubes, and has been known to excite undesired oscillations. For further information on the impedance characteristics of space-charge-grid tubes, see Liss C. Peterson, Impedance Properties of Electron Streams, *Bell System Tech. Jour.*, Vol. 18, p. 465, July, 1939.

[4] Albert W. Hull, The Magnetron, *A.I.E.E. Jour.*, Vol. 40, p. 715, 1921; Léon Brillouin, Theory of the Magnetron, *Elec. Comm.*, Vol. 20, No. 2, p. 112, 1941.

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in an axial magnetic field, as illustrated. When the plates are positive, the electrons that are attracted to them follow curved paths, as in Fig. 36. If the plates are very positive, the curvature of these paths is small, but at lower voltages the curvature becomes greater until at a critical potential determined by the strength of the magnetic field, the electrons follow a curved path (cardioid) back to the cathode, as shown in Fig. 36c, and never get to the plate, in spite of the positive plate potential. The magnetron accordingly has a critical field strength at which plate current is cut off. This field B in lines per sq cm is

$$B = \frac{6.72}{r}\sqrt{E} \tag{75}$$

where r is the anode radius in centimeters and E is the anode voltage.

If perfect symmetry were obtained and the velocity of emission of the electrons were neglected, the cutoff would be infinitely abrupt; *i.e.*, an infinitesimal change in magnetic field would be capable of turning the entire anode current off or on. Actually, because of unavoidable imperfections, and because of the initial velocity of electrons, the cutoff is rounded off slightly.[1]

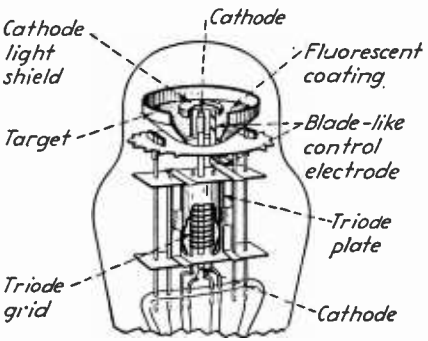


FIG. 37.—Cutaway view of visual indicator tube.

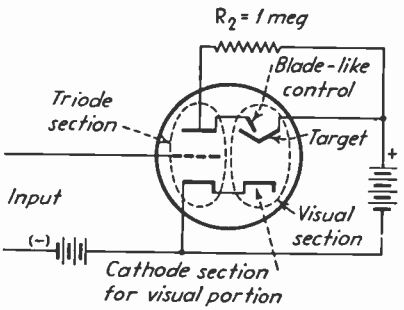


FIG. 38.—Circuit illustrating operation of visual indicator tube.

The chief value of the magnetron is in certain types of very high-frequency oscillators, such as discussed in Par. 14, Sec. 6.

Visual Indicator (Magic-eye) Tube.[2]—A typical form of visual-indicator (so-called magic-eye) tube is illustrated in Fig. 37. This consists of a high-mu triode, above which is located a cone-shaped fluorescent target connected directly to the plate-supply voltage and having its own cathode. There is also a bladelike control electrode associated with the target, which is connected directly to the triode plate. Both plate and blade are connected to the power supply through a high resistance R_2, as shown in Fig. 38. With zero potential on the grid of the triode, electrons from the cathode strike the entire cone-shaped fluorescent target except for a sector in the region of the blade electrode, which, being at a lower potential than the target, because of the voltage dropping resistance R_2, acts as a shield. However, if a negative voltage is applied to the grid of the triode the plate current becomes less. This reduces the voltage drop in resistance R_2, increasing the potential of the control electrode and reducing the sector of darkness on the fluorescent target. Thus a visual indication is obtained of the magnitude of the voltage acting on the triode tube.

[1] E. G. Linder, Effects of High Energy Electron Random Motion upon the Shape of the Magnetron Cutoff Curve, *Jour. Applied Phys.*, Vol. 9, p. 331, May, 1938.

[2] A good description of such a tube, together with some applications, is given by L. C. Waller, Applications of Visual-indicator Type Tubes, *R.C.A. Rev.*, Vol. 1, p 111, January, 1937.

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Visual-indicator tubes can be used for a wide variety of applications. Among these are tuning indicators, vacuum-tube voltmeters, overmodulation indicators, etc.

Orbital Beam Tube.[1]—An understanding of the orbital beam tube can be gained from Fig. 39. This tube is provided with a cathode of relatively flat cross section surround by a control grid G_1 and a screen grid G_2. Focusing electrodes J_1 and J_2 are employed with the outer electrode at zero potential. The inner focusing electrode J_1 is at a relatively high positive potential, so chosen that the electrons emitted from the cathode follow curved paths such as indicated by AA. The radius of these paths may be changed by varying the potential of the outer cylinder. Thus if this cylinder J_2 is made negative, the electrons will follow paths such as BB, and some of the electrons are diverted to the inner cylinder J_1. This is a feature that can be used as a method of volume control. The electrons following the path AA ultimately strike the secondary emitter electrode K_2, which is at a high positive potential, and has a surface so treated as to be a good emitter of secondary electrons. The secondary electrons

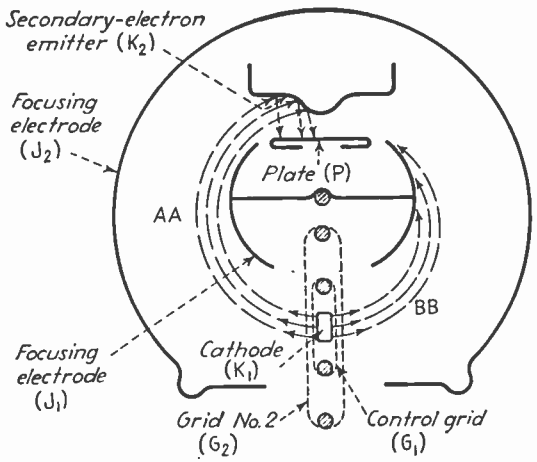


FIG. 39.—Schematic cross section of orbital-beam tube.

produced at electrode K_2 are collected by the plate P, which is at a high positive potential, and serves as the output electrode of the tube.

The orbital beam tube is able to obtain a high transconductance because of the electron multiplier action that takes place at the electrode K_2. It is possible at this electrode to obtain a number of secondary electrons for each arriving primary electron, thereby giving a large plate current in proportion to the current from the cathode. This results in a correspondingly large transconductance. Furthermore, the structural factors involved are such that this output transconductance is very large compared with the input and output capacities of the tube, so that the tube is an excellent wide-band amplifier, being superior to any of the ordinary pentode tubes in this respect.

The rather complicated arrangement of electrodes in the orbital beam tube is for the purpose of allowing the primary electrons emitted from the cathode to strike the secondary emitting surface K_2, while at the same time preventing material evaporated from the cathode from being deposited on K_2. Otherwise the secondary emitting surface would vary in its characteristics with time, and the tube behavior would be unstable.

[1] H. M. Wagner and W. R. Ferris, The Orbital-beam Secondary-electron Multiplier for Ultra-high-frequency Amplification, *Proc. I.R.E.*, Vol. 29, p. 598, November, 1941.

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ELECTRON OPTICS AND CATHODE-RAY TUBES

15. Electron Optics.—There is an analogy between the behavior of an electron in an electrostatic field and the behavior of a light ray in a medium of variable index of refraction. Electrons can be reflected, refracted, and focused just as can light rays. The term electron optics implies that the behavior of electrons can be studied in terms of the known laws of physical optics.

A study of electron and physical optics reveals that there actually is a correspondence between the quantities and laws in the two fields. The quantity in electron optics that corresponds to index of refraction is electron velocity (if it is assumed that the electron starts from rest at a point of zero potential). Corresponding to the Fermat principle of least time for physical optics, there is the Hamiltonian principle of least action that applies to electron optics.

In physical optics a ray of light is refracted upon passing a boundary between a medium of one index of refraction and a medium of another. In Fig. 40, a ray passing from a region of refractive n_1 to a region of higher index of refraction n_2 is bent toward the line normal to the surface. Similarly, in the corresponding electron optical case,

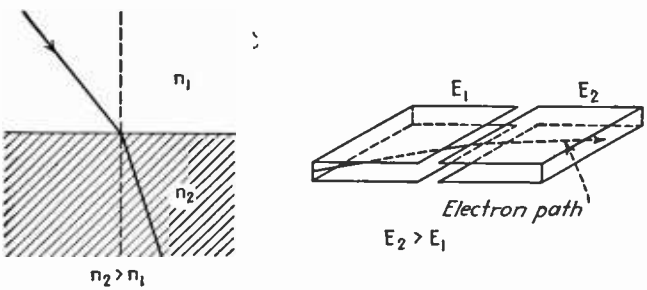


FIG. 40.—Refraction of an electron and of a light beam.

an electron passing from a region in which it has a velocity v_1, proportional to the square root of potential E_1, into a region in which it has a velocity v_2 greater than v_1 is bent toward the line normal to the boundary between the two regions. In most electron optical cases the potential changes gradually rather than abruptly, but any changing potential field is capable of causing an electron to follow a path characterized by a bend.

Any electrostatic field that has a rotational symmetry about some straight-line axis has the properties of a lens. Such a symmetrical electrostatic field is produced by any set of electrodes that themselves have rotational symmetry about the axis. The simplest type of electron lens is a circular hole in a plate, as shown in Fig. 41. If such a circular aperture is placed between planes so that there is a difference in the potential gradient on the two sides, then there is a penetration of field as shown, and electrons entering parallel to the dotted line will be uniformly deflected either toward or away from this axis. For the potentials shown in the figure the lens action is divergent. The physical lens, which is roughly the equivalent of the electron lens, is also shown in the figure.

A better electron lens is shown in Fig. 42, with its approximate physical equivalent. This lens consists of the field between two coaxial cylinders placed end to end, and at different potentials. The variation of the potential along the axis is shown. The equivalent physical lens consists of a combination of convex and concave surfaces between regions of successively higher indexes of refraction. In the equal-diameter

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cylinder lens, as this is called, the equipotentials in the first cylinder are concave in the direction of increasing potential, while those in the second cylinder are convex. As a result, an electron entering from the low potential side at a small angle with the axis first experiences a force toward the axis and then experiences a force away from the axis. Thus the first part of the lens has a convergent action, while the second part of

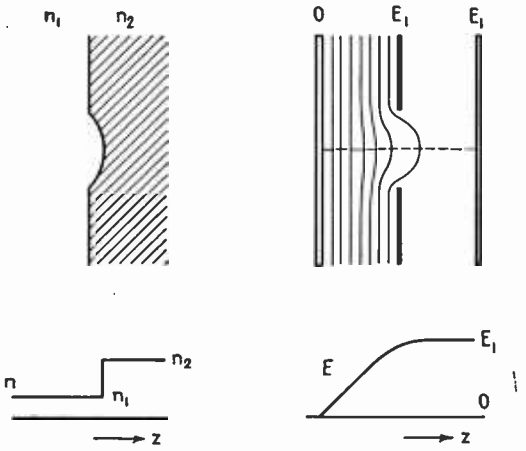


FIG. 41.—Simple electron-optical lens and physical equivalent.

the lens has a divergent action. The convergent action is stronger because the electron velocity is lower in the convergent portion of the lens and hence the electron here suffers a greater deflection for the same force.

The analogy between the electron and physical lenses indicated in Figs. 40 and 41 involves the choice of the proper segments in the physical lens. It will be observed that the front surface of the physical-lens components are curved in the same way as

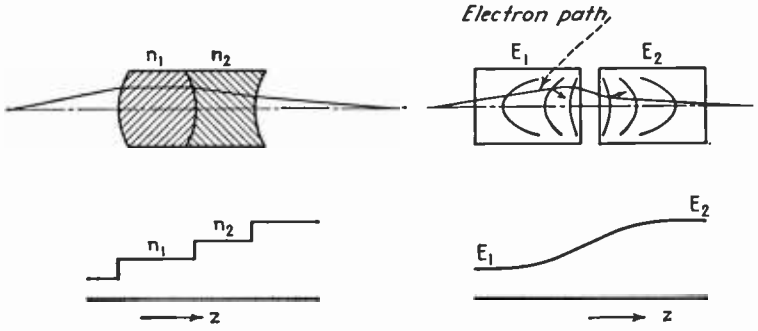


FIG. 42.—Equal-diameter cylinder lens and physical equivalent.

are the equipotentials in the electron lenses. This observation gives a means of telling by inspection how an electron lens will behave, since the property is a general one. There is also a correlation between the nature of the equipotential surfaces and the curvature of the curve of potential along the axis against distance in the axial direction. In Fig. 42 it is seen that when the curvature of the potential distribution along the axis is concave upward, then the equipotentials are concave in the direction of increasing

Ex. 1044-0331

potential and the lens has a convergent action. In this region the second derivative with distance of the axial potential is positive. In the second cylinder the second derivative of the axial potential is negative, and the lens action is divergent. This relation between the curvature of the curve of potential along the axis and the nature of the lens action always holds for lenses with rotational symmetry.

In fields with rotational symmetry the potential at any point in the field can be determined if the variation of potential with distance along the axis is known. By an application of Laplace's equation to fields of this sort, it can be shown that

$$E(r,z) = E_0(z) - \frac{r^2}{2^2}E_0''(z) + \frac{r^4}{2^2 \cdot 4^2}E_0''''(z) + \cdots + \frac{(-1)^n}{(n!)^2}\left(\frac{r}{2}\right)^{2n} E_0^{(2n)}(z) + \cdots \tag{76}$$

where $E(r,z)$ = potential at a radial distance r from the axis and at an axial distance z from some reference point on the axis.

$E_0(z)$ = potential on the axis at the point $(0,z)$.

$E_0''(z)$ = second derivative of axial potential with respect to axial distance z.

$E_0^{(2n)}$ = 2 nth derivative of axial potential with respect to axial distance z.

The axial potential can be calculated in some cases[1] but in most cases it is simpler to obtain it by a computational method or by the use of a current flow model.[2]

When the axial potential is known its derivatives are readily obtained, and thus the potential at any point in the field can be calculated from Eq. (76).

The differential equation of an electron path through a field of rotational symmetry is

$$\frac{d^2r}{dz^2} - \frac{E_0'(z)dr}{2E_0(z)dz} + E_0''\frac{r}{2} = 0 \tag{77}$$

This equation is good only for the so-called "paraxial" electrons, which are those electrons that are within a few per cent of the electrode radius from the axis, and that make an angle of a few degrees or less with the axis. The preceding equation is practically incapable of solution exactly. It is, however, possible to obtain good numerical approximations of its solution by various approximate methods.[3]

Although it is not possible to solve Eq. (77), it is still possible to make some deductions about the nature of the electron paths from it. Since the differential equation is of the second order, its complete solution may be expressed as the sum of two linearly independent solutions; *i.e.*, any electron path through the lens may be described as the sum of any two different and distinct paths. It is most convenient to use as a pair of independent paths those two which represent (1) an electron entering the lens parallel to the axis and (2) an electron leaving the lens parallel to the axis. These particular rays are known as the *principal rays* of the lens, and are indicated in Fig. 43. The ray that leaves the lens parallel to the axis is known as the *first principal ray.* It is sometimes shown as entering the lens parallel to the axis from the right as in Fig. 43, which is permissible because such a reversal of direction traces an identical path. The ray that enters the lens parallel to the axis from the

[1] P. Kirkpatrick and J. G. Beckerley, Ion Optics of Equal Coaxial Cylinders, *Rev. Sci. Instruments*, Vol. 7, p. 24, January, 1937; S. Bertram, Determination of Axial Potential Distribution in Axially Symmetric Fields, *Proc. I.R.E.*, Vol. 28, p. 418, September, 1940; S. Bertram, Calculations of Axially Symmetric Fields, *Jour. Applied Phys.*, Vol. 13, p. 496, August, 1942.

[2] I. G. Maloff and D. W. Epstein, "Electron Optics in Television," McGraw-Hill, New York, 1938; L. M. Myers, Electron Optics, Van Nostrand, New York, 1939; O. Klemperer, "Electron Optics," Cambridge (London), 1939; M. Bowman-Manifold and F. H. Nicoll, Electrolytic Field-plotting Trough for Circularly Symmetric Systems, *Nature*, Vol. 142, p. 39, July 2, 1938.

[3] Maloff and Epstein, *loc. cit.*; R. Gans, Course of the Ray in Electron Optical Systems, *Zt. Physik*, Vol. 18, p. 41, February, 1937; K. Spangenberg and L. M. Field, Some Simplified Methods of Determining the Optical Characteristics of Electron Lenses, *Proc. I.R.E.*, Vol. 30, p. 138, March, 1942.

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left is known as the *second principal ray.* The *focal points* of a lens are those points at which the principal rays cross the axis. The intersection of the initial and final straight-line portions of the principal rays when extended toward the lens between the principal planes and the focal points of the lens are known as the focal distances or *focal lengths.* These quantities are shown in Fig. 43.

A lens such as that of Fig. 43, in which the region in which the lens action takes place is comparable in length to the focal length, is known as a thick lens. The

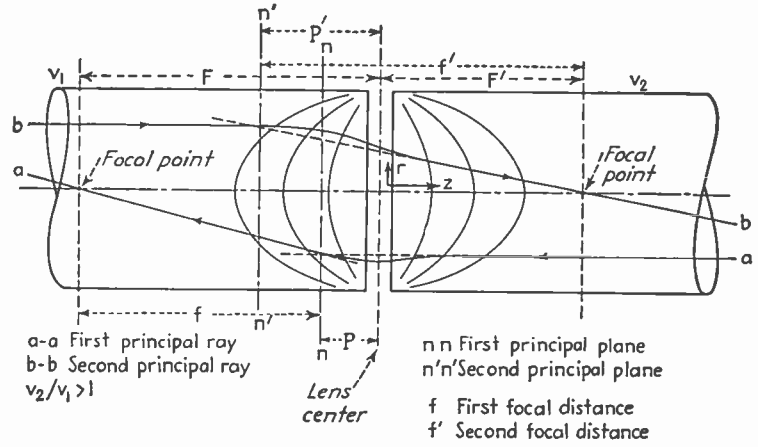


FIG. 43.—Terminology of thick electron lens.

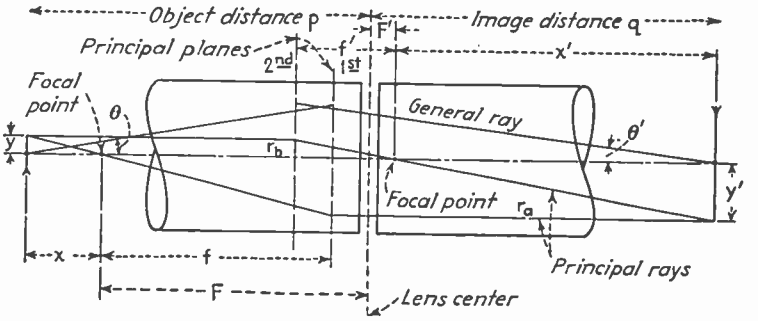


FIG. 44.—Image formation in thick electron lens.

relation between the lens characteristics and the points at which an image of a given object forms as in Fig. 44 is

$$\frac{f}{P+p}+\frac{f'}{Q+P'}=1 \tag{78}$$

where f = first focal length.
f' = second focal length.
p = distance between object and lens center.
q = distance between image and lens center.
P = distance between first principal plane and lens center.
P' = distance between second principal plane and lens center.

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This equation tells what the location of the image will be for any location of the object in terms of four fundamental quantities of the lens, f, f', P, and P'. The object and image distances as measured from their corresponding principal planes are related by Newton's formula

$$pq = ff' \tag{79}$$

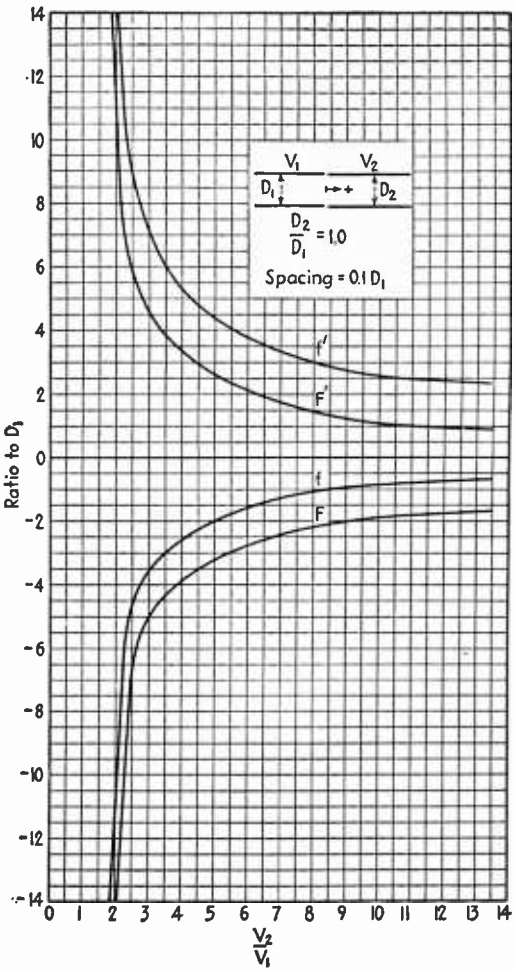


FIG. 45.—Focal characteristics of two-diameter cylinder lenses as a function of electrode voltage ratio. Figure continued on pages 327–330.

In electron lenses, the ratio of the two focal distances is related to the ratio of the electrode potentials by

$$\frac{f'}{f} = \left(\frac{V_2}{V_1}\right)^{1/2} \tag{80}$$

where V_1 = initial potential, that of first electrode.
V_2 = final potential, that of second electrode.

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There is also a relationship known as Lagrange's law between the lateral magnification, the angular magnification, and the electrode voltage ratio.

$$m_l m_a \left(\frac{E_2}{E_1}\right)^{\frac{1}{2}} = 1 \tag{81}$$

where $m_l = y'/y$ in Fig. 44, lateral magnification.
$m_a = \theta'/\theta$ in Fig. 44, angular magnification.
E_2 = second electrode potential.
E_1 = first electrode potential.

Focusing Characteristics of Lenses.—The lenses of electron optics are completely described if four quantities are known. The four quantities most convenient to use

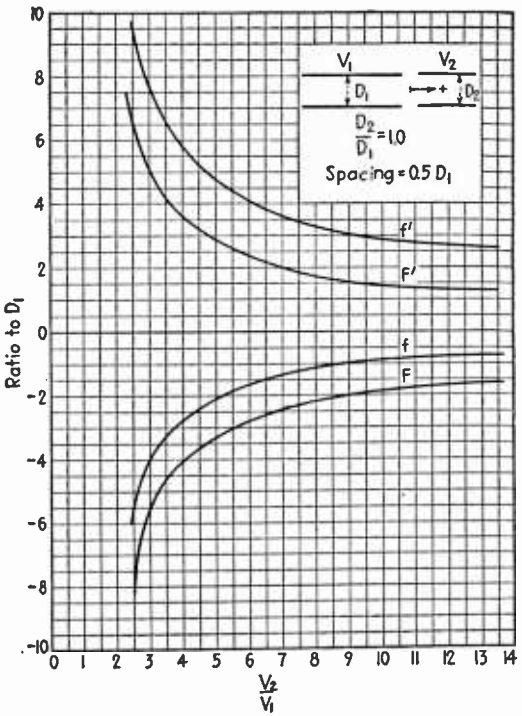


FIG. 45.—*Continued.*

are the two focal lengths and the locations of the two principal planes. Since these change with voltage ratio, it is necessary to draw curves of the four quantities listed above as a function of voltage ratio, as is done in Figs. 45 and 46.[1]

To utilize the curves of Figs. 45 and 46 use must be made of the lens relations previously given. For any voltage ratio and location of object, they enable one to calculate the location and size of the image. Since the quantities of Figs. 45 and 46 do not appear in the answers, it is desirable to present the relations between object distance, image distance, voltage ratio, and magnification in a form that does not

[1] The data presented in Figs. 45 to 49 are experimental results obtained by K. Spangenberg and L. M. Field in carrying out an investigation at Stanford University with the support of the International Standard Electric Corporation. The results are given here through the courtesy of that organization.

Ex. 1044-0335

involve the focal distances. This has been done in Figs. 47 and 48, which present what may be called "object-image-distance curves." On these curves, which are essentially a graphical presentation of all the answers to all the problems associated with one electrode configuration, there are shown the relationships between the object and image distances and the lateral magnification m as determined by the voltage ratio. Examination of these curves shows that the magnification is approximately

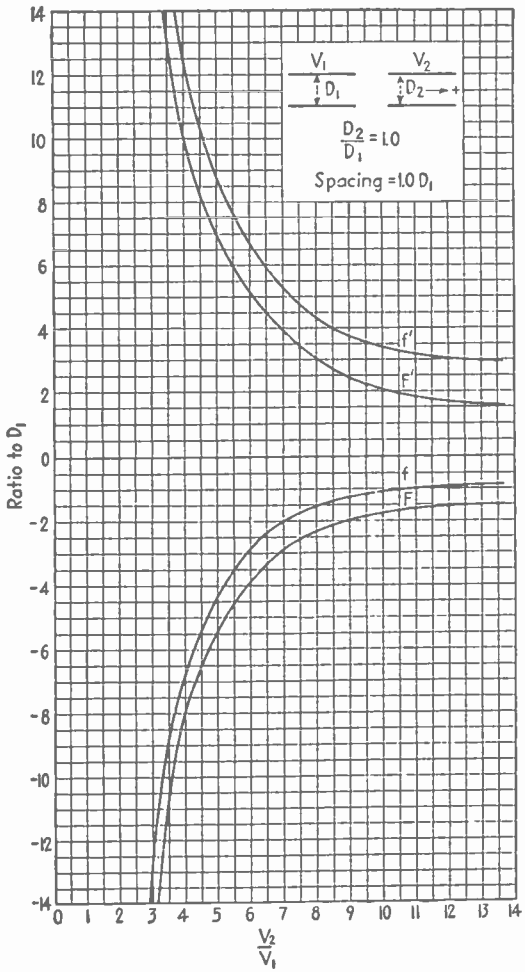


FIG. 45.—*Continued.*

0.8 of the ratio of the image to the object distance. It also shows that there is always some particular electrode configuration that makes it possible to achieve any ratio of object to image distance or magnification, provided that the proper voltage ratio is chosen.

The effect of varying the proportions of various types of lenses is illustrated in Fig. 49.

Ex. 1044-0336

A critical examination of data such as given in Figs. 45 to 49 shows that the following properties are common to all lenses:[1] (1) Focal lengths are always uniformly decreasing functions of voltage ratio. (2) Principal planes always lie on the low-voltage side of the lens center. (3) Principal planes are crossed, with exception of

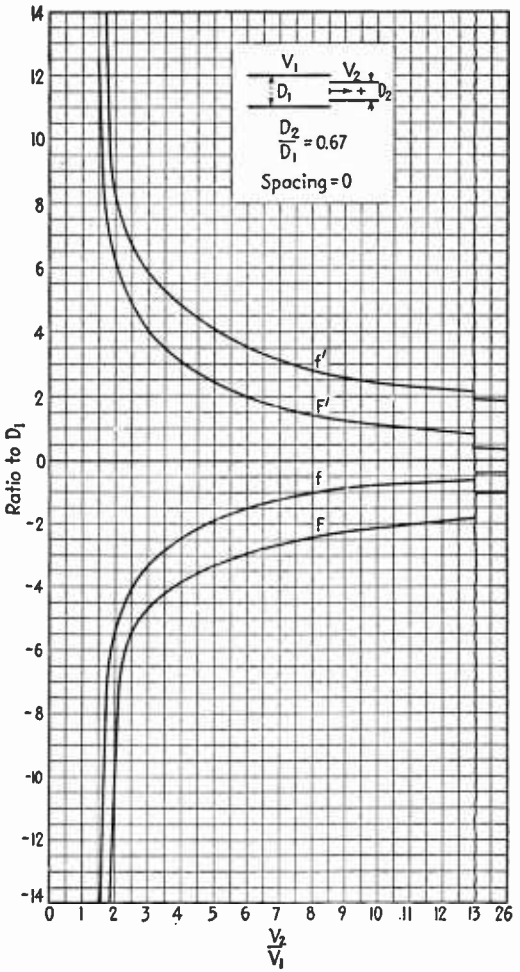


FIG. 45.—*Continued.*

large-diameter aperture lens; *i.e.*, the first principal plane lies between the second principal plane, and the lens center on the low-voltage side of the lens. (4) Focal length in the direction of increasing potential is always greater than the focal length in the other direction. (5) Position of principal planes does not change much with voltage ratio.

[1] Maloff and Epstein, *loc. cit.*; Klemperer, *loc. cit.*; Spangenberg and Field, *loc. cit.*; Frank Gray, Electrostatic Electron Optics, *Bell System Tech. Jour.*, Vol. 18, p. 1, January, 1939.

Ex. 1044-0337

The following summarizes properties of focal lengths for the particular cases indicated: (1) The focal length of two-diameter cylinder lenses increases—*i.e.*, the lens grows weaker—for all but the highest voltage ratios, as the ratio of second to first cylinder diameter increases. (2) The focal length of equal-diameter cylinder

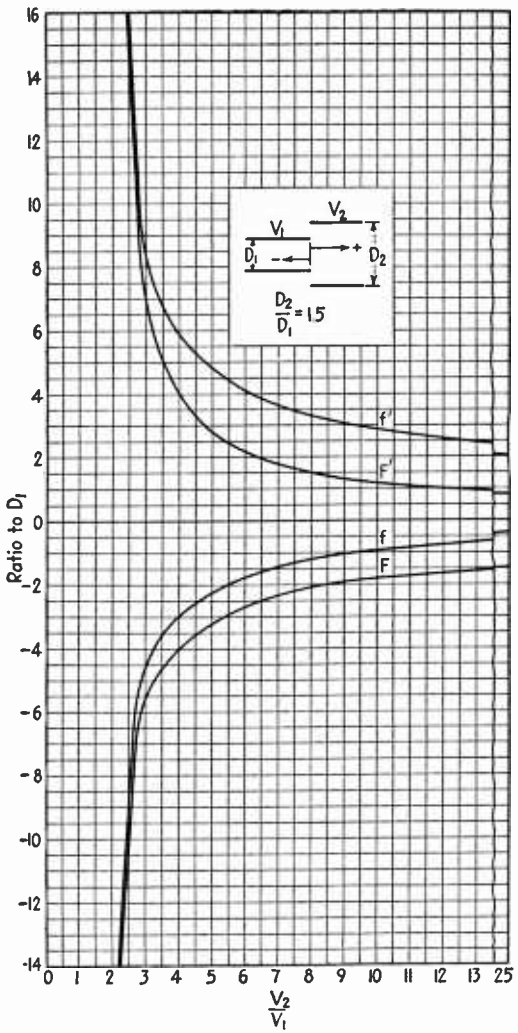


FIG. 45.—*Continued.*

lenses increases, that is, the lens grows weaker, as the axial spacing of the cylinders increases. The change is small for small spacings but increases rapidly as the spacing is increased. (3) The focal length of aperture lenses increases—*i.e.*, the lens grows weaker—as the aperture diameter increases. The change is small for small diameters but increases rapidly as the diameter increases. (4) Aperture lenses have for the most part shorter focal lengths than cylinder lenses (when the unit of length is the

Ex. 1044-0338

diameter of the first cylinder) if the aperture spacing be taken equal to first cylinder diameter. (5) Cylinder-aperture lenses have shortest focal length of all the lenses of Figs. 45 to 49. (6) Equal-diameter lenses with axial spacing of one diameter have the longest focal length of all the lenses of Figs. 45 to 49.

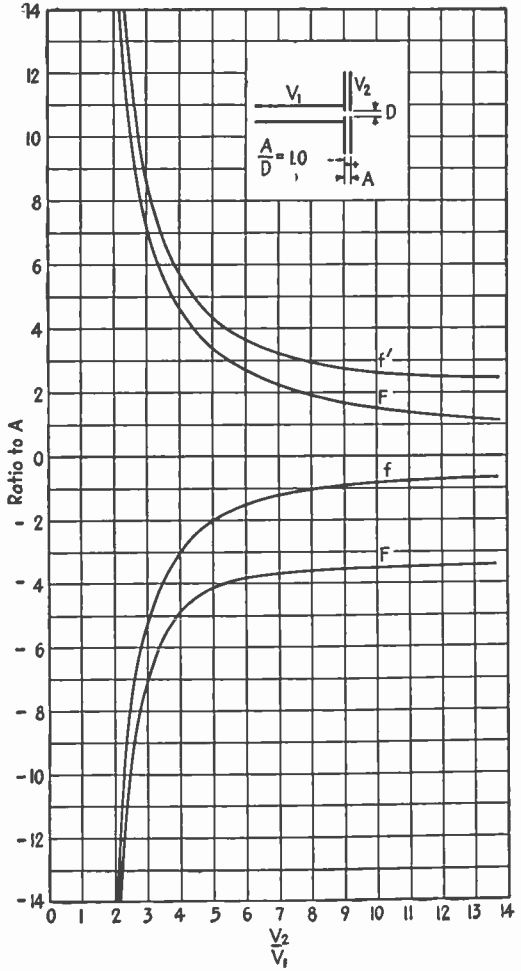


FIG. 46.—Focal characteristics of various aperture lenses as a function of electrode voltage ratio. Figure continued on pages 332–334.

The following comments apply to the magnification properties of the lenses of Figs. 45 to 49: (1) Contours of constant magnification in the object-image distance curves are approximately straight lines with a slope of one. (2) An approximate universal magnification formula that fits all lenses tested is

$$M = k\frac{q}{p} \tag{82}$$

Ex. 1044-0339

TABLE 5.—CONSTANT k IN EQ. (82)

Type of Lens	k
Cylinder lens: $D_2/D_1 = 0.667$	0.82
Cylinder lens: $D_2/D_1 = 1$, $S = 0.1$	0.78
Cylinder lens: $D_2/D_1 = 1.5$	0.76
Cylinder lens: $D_2/D_1 = 1$, $S = 0.5$	0.80
Cylinder lens: $D_2/D_1 = 1$, $S = 1$	0.60
Aperture lens: $A/D = 5$	0.95
Aperture lens: $A/D = 3$	0.80
Aperture lens: $A/D = 1$	0.78
Cylinder aperture lens	0.82

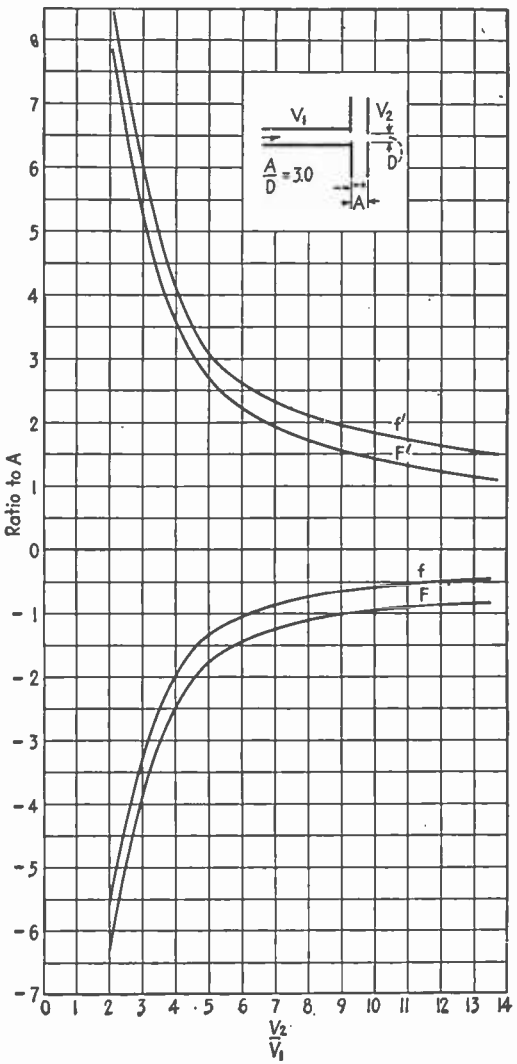


FIG. 46.—*Continued.*

where p and q are object and image distances, respectively. Values of the constant k for the lenses tested are given in Table 5, and with only two exceptions are within a few per cent of 0.8. (3) Magnification in two-diameter lenses decreases as the diameter ratio increases. The change is appreciable only for ratios less than one, however.

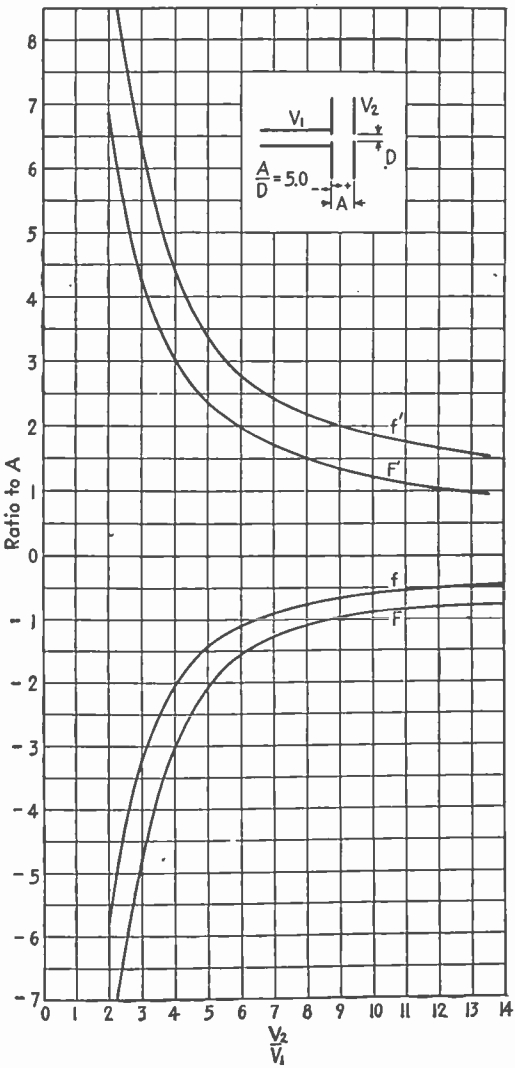


FIG. 46.—*Continued.*

(4) Magnification of equal-diameter lenses decreases as axial spacing increases. The change is inappreciable except between spacings of 0.5 and 1.0, however. (5) Magnification of aperture lenses with small apertures decreases as aperture diameter increases.

Considerations important in design that are common to all lenses of Figs. 45 to 49 are: (1) As object distance is increased at a given voltage ratio, the corresponding

Ex. 1044-0341

image distance decreases, as does also the magnification. (2) For a given object distance the image distance and magnification decrease as the voltage ratio is increased. (3) In any lens there is a minimum object distance that can be used at any given voltage ratio. This minimum object distance is determined by the vertical asymptote to the contour of constant voltage ratio.

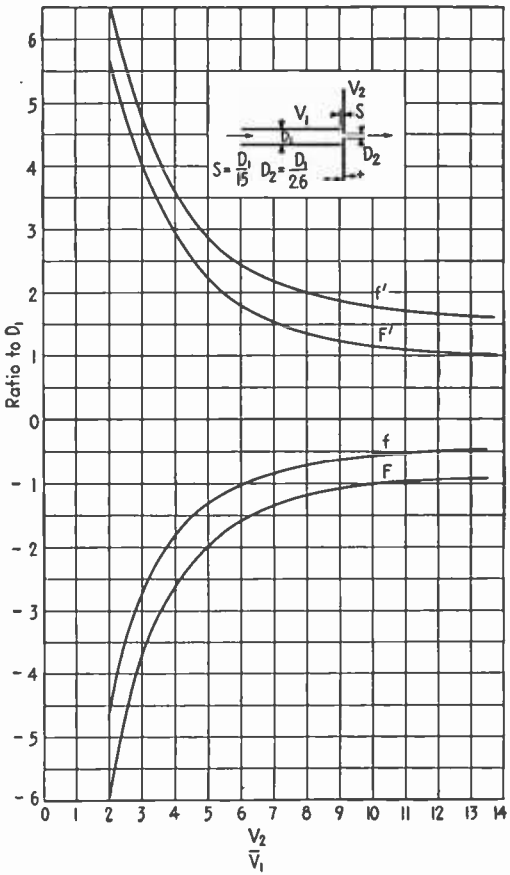


FIG. 46.—*Continued.*

The effects of object distance and voltage ratio on the relative strength of the lenses of Figs. 45 to 49 can be summarized as follows: (1) Two-diameter cylindrical lenses become stronger as diameter ratio decreases for voltage ratios between 1 and 4. This holds for all object and image distances. (2) For larger voltage ratios the lens strength is nearly independent of diameter ratio. (3) Equal-diameter lenses become stronger as the axial separation of the cylinders increases for all voltage ratios and normal values of object and image distance. (4) Aperture lenses become slightly stronger as the aperture diameter is enlarged from a very small size to a practical value, and then become greatly weaker.

Ex. 1044-0342

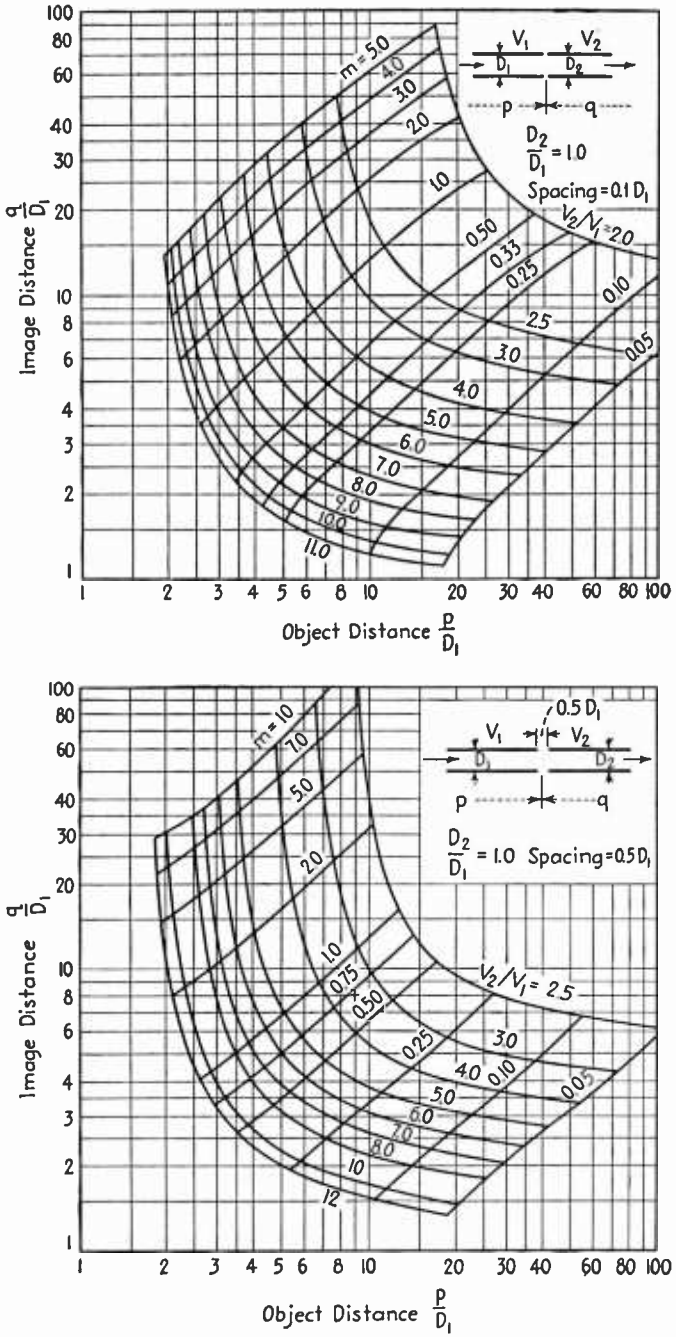
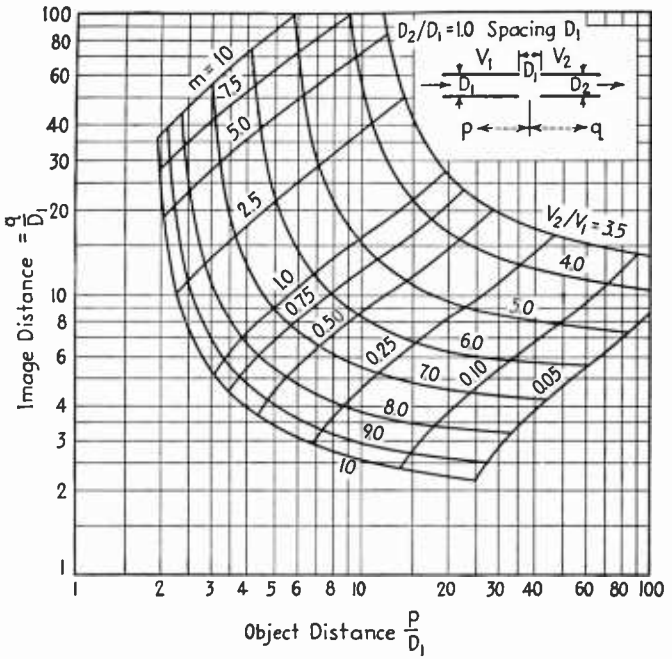


FIG. 47.—Image-object distance curves of two-diameter cylinder lenses. Figure continued on pages 336 and 337.



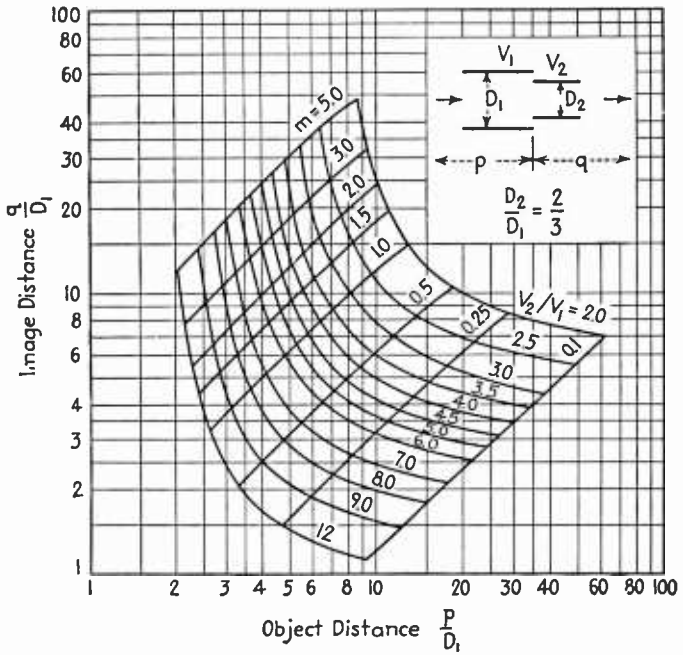


FIG. 47.—*Continued.*

Ex. 1044-0344

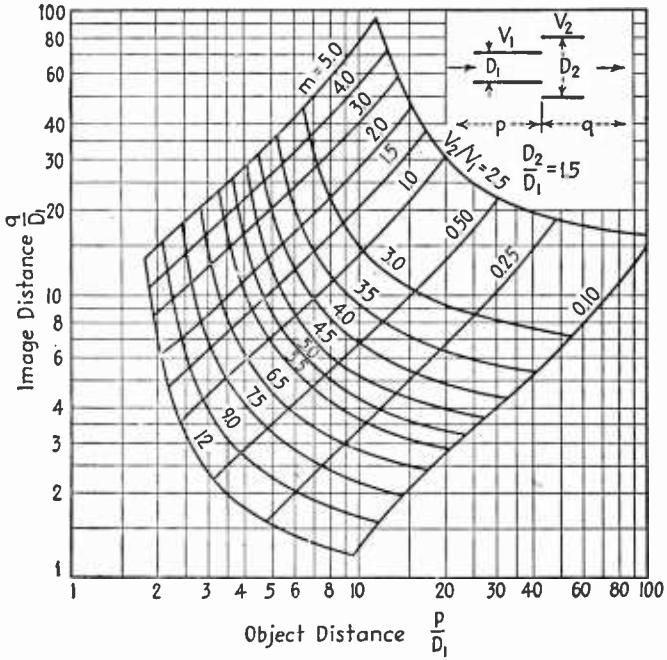


FIG. 47.—*Continued.*

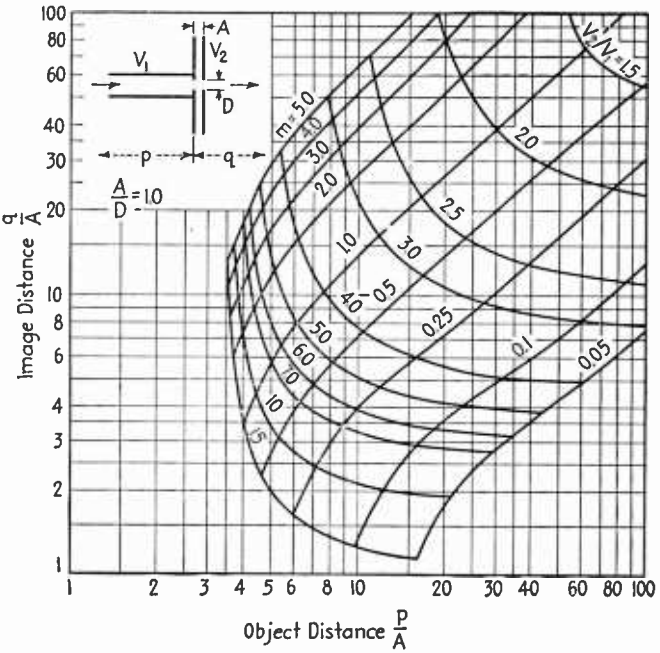
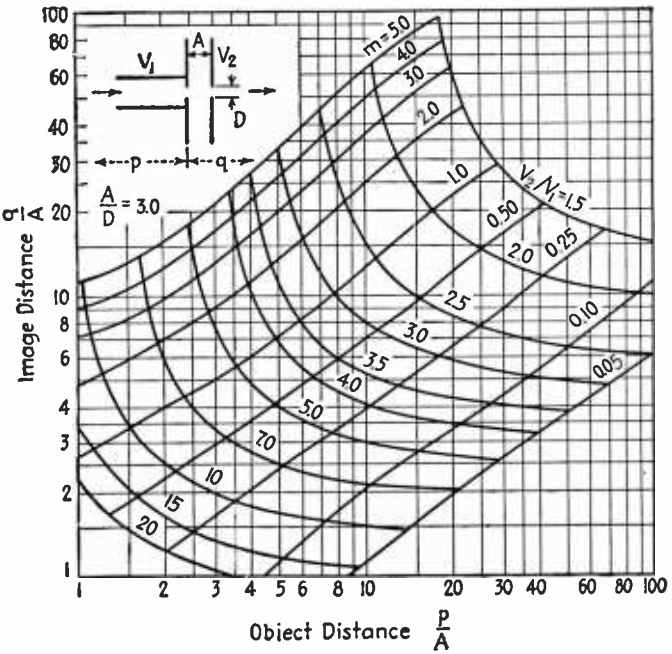


FIG. 48.—Image-object distance curves of various aperture lenses. Figure continued on pages 338 and 339.

Ex. 1044-0345



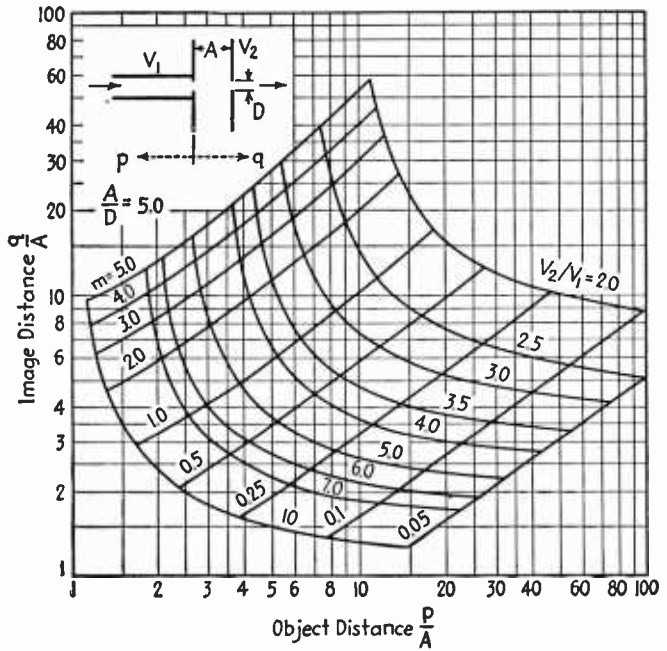


FIG. 48.—*Continued.*

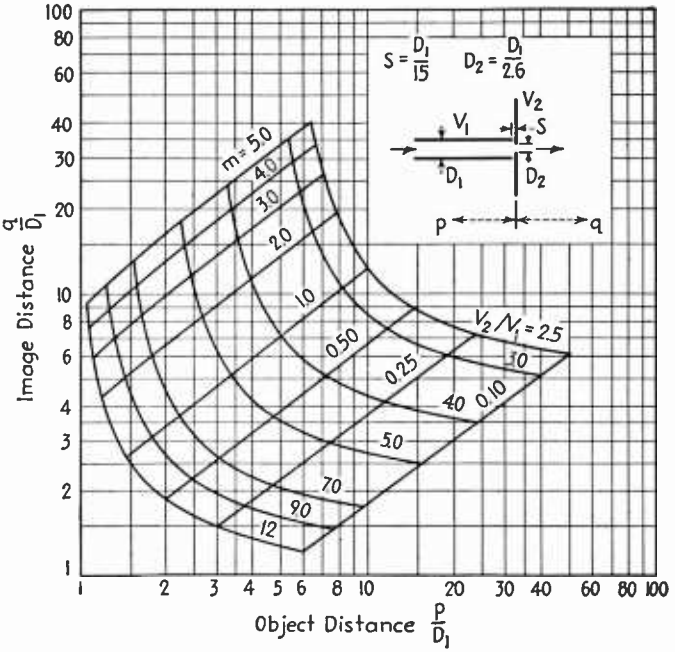
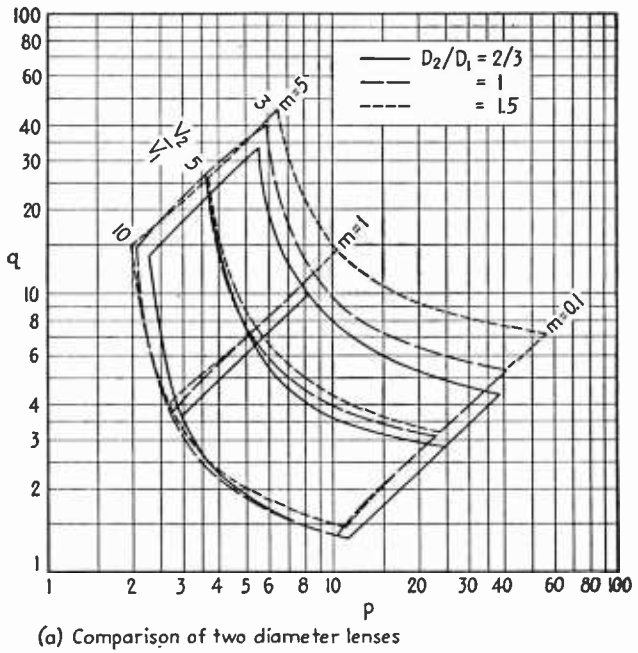


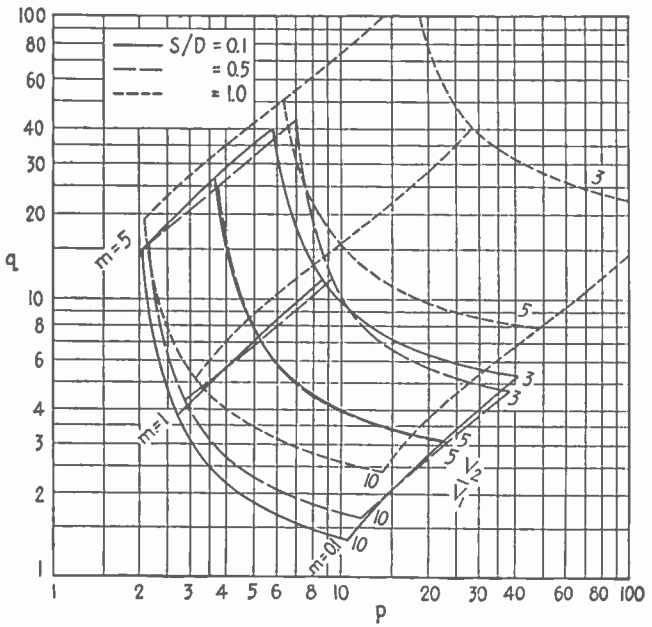
FIG. 48.—*Continued.*



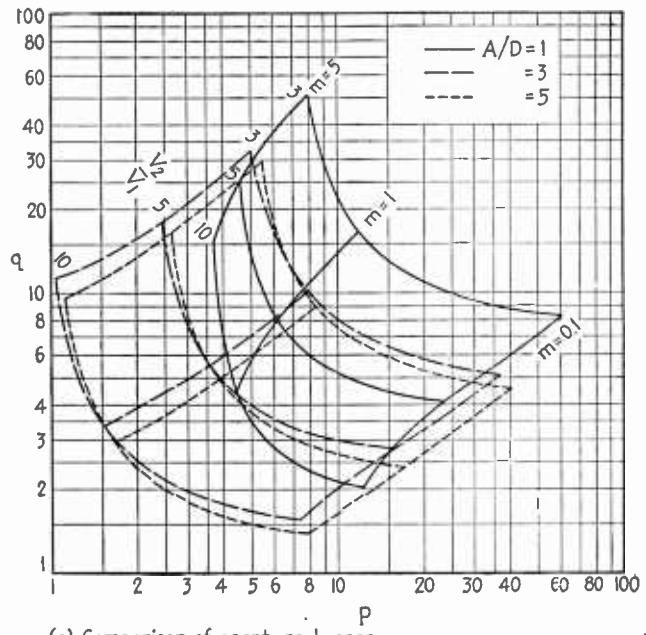
(a) Comparison of two diameter lenses

FIG. 49.—Comparison of lenses of different proportions. Figure continued on page 340.

Ex. 1044-0347



(b) Comparison of equal diameter lenses



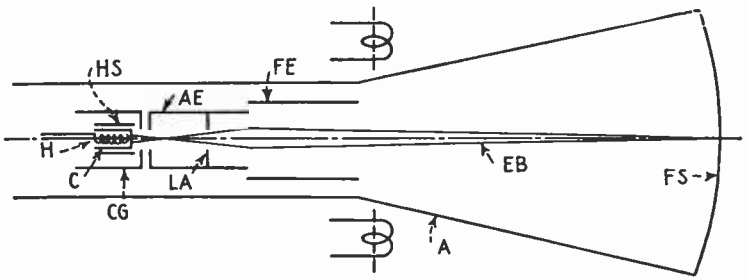
(c) Comparison of aperture lenses

FIG. 49.—*Continued.*

16. Cathode-ray Tubes.—The cathode-ray tube is an outgrowth of the modern science of electron optics. It is used extensively in engineering and scientific laboratories for such purposes as the observation of wave forms, viewing of television images, and many other applications too numerous to mention.

Every cathode-ray tube contains a source of electrons in the form of a cathode, a means for forming the emitted electrons into a pencillike beam; a control electrode for varying the beam current; a set of focusing electrodes for focusing the beam; some sort of beam-deflecting device for moving the beam about; and a fluorescent screen, which gives off light at the point where the beam strikes.

The Electron Gun.—A typical electron-gun structure is shown in Fig. 50. The cathode consists of a cylinder of sheet nickel with an oxide-coated cap on one end. The cathode is heated by a tungsten filament coated with some insulating material, such as aluminum oxide, which is inserted in the cylinder. Outside the cathode and spaced a few thousandths of an inch from it is a heat shield in the form of a nickel



H Heater
C Cathode (0.0 volts)
HS Heat shield (0.0 V)
Cg Control grid (-10 to +10 V)
AE Accelerating electrode (+200 V)
FE Focusing electrode (+800 V)
EB Electron beam
FS Fluorescent screen
LA Limiting aperture
A Aquadag coating
Potentials above are relative to cathode
Actually the focusing electrode and aquadag coating are grounded

FIG. 50.—Conventional electron gun structure with magnetic deflecting coils.

cylinder, which projects slightly beyond the end of the cathode and helps to concentrate the electrons in a beam.

The control electrode, which performs the function of a control grid in an ordinary vacuum tube, is in the form of an enveloping cylinder with aperture, properly spaced relative to the cathode.[1] The control electrode is operated at some low potential relative to the cathode, which may be either positive or negative, depending upon the electrode dimensions.

The control electrode is followed by a so-called accelerating electrode, or first anode. This usually has an aperture in the end next to the control electrode and one or two more apertures within the cylinder. The apertures within the cylinder serve to remove from the beam strongly divergent electrons that would produce a fuzzy spot.

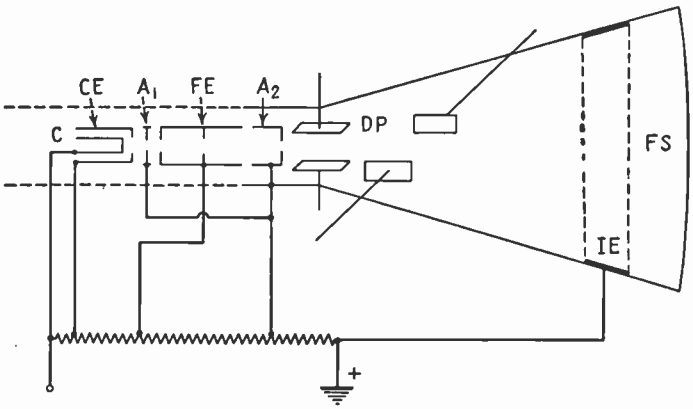
The last electrode is the second anode, or focusing electrode, which is another cylinder following the accelerating electrode. The principal focusing action occurs in the electrostatic field between the accelerating electrode and the second anode.

[1] Details of the design of the cathode and associated electrodes are given by I. G. Maloff and D. W. Epstein, "Electron Optics in Television," McGraw-Hill, New York, 1938.

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The beam is focused by changing the second anode potential, just as the beam intensity is controlled by changing the control-electrode potential. These two control actions are almost but not completely independent; *i.e.*, changing the beam intensity affects the focus somewhat and vice versa. The funnel-shaped part of the glass envelope beyond the second anode is frequently coated with aquadag, which is a conducting layer of carbon particles, and placed at the potential of this electrode. In this arrangement the deflecting devices are placed just beyond the second anode.

Another electron gun structure that can be used is shown in Fig. 51. This arrangement makes use of what is known as postdeflection acceleration.[1] Here the cathode is followed by a control electrode and then by an accelerating electrode, or anode, A_1. This accelerating electrode is the first of two at the same potential, and is followed by an electrode at a lower potential, which in this structure is called the focusing electrode.



C	Cathode	FE	Focusing electrode
CE	Control electrode	IE	Intensifier electrode
A_1	First accelerating anode	FS	Flourescent screen
A_2	Second accelerating anode	DP	Deflecting plates

FIG. 51.—Electron gun with postelectrostatic deflection acceleration.

This focusing electrode *FE* is followed by a second accelerating anode A_2 at the same potential as the first. The second accelerating electrode is followed by the deflecting devices, placed at the beginning of the funnel-shaped part of the glass envelope, that is coated with aquadag. This conducting coating extends only part way to the fluorescent screen. It is followed by a ring of conducting material just before the screen, known as the intensifier electrode, which is maintained at about twice the potential of the accelerating electrode.

The convergent focusing action on the beam occurs between the focusing electrode and the second accelerating electrode. The action between the first accelerating electrode and the focusing electrode is divergent, as is also the action between the first accelerating electrode and the control electrode.

In the postdeflection arrangement the beam is deflected at low velocity, thus giving a good deflection sensitivity, and then accelerated further to give a good photographic sensitivity. Part of the increase in deflection sensitivity gained by this arrangement is lost because the final accelerating field is somewhat convergent, thus reducing the deflection. However, a gain of 3 to 5 over the ordinary tube sensitivity is achieved.

[1] J. de Gier, A Cathode Ray Tube with Post Acceleration, *Phillips Tech. Rev.*, Vol. 5, p. 245, September, 1940; J. R. Pierce, After Acceleration and Deflection, *Proc. I.R.E.*, Vol. 29, p. 28, January, 1941.

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Deflection of Electron Beams.—The electron beam in a cathode-ray tube may be deflected either by an electrostatic or by an electromagnetic field. In the majority of applications, deflection by electrostatic means is preferred.

If a conducting plate is placed on either side of the beam at the end of the electron gun, as shown in Fig. 51, then if the mean potential of these plates is that of the last gun electrode, and yet a small potential difference is caused to appear between the plates, the electron beam will be deflected toward the more positive plate upon passing between them. This deflection occurs because there is a constant crosswise force applied to the beam as it moves through the plates with a nearly constant velocity. The result of this action is that the electrons move in a parabolic path while between the plates, and emerge at an angle with the axis that is proportional to the potential difference existing between the deflecting plates. The amount that a beam is deflected by this action is

$$d = \frac{La}{2b}\frac{E_d}{E_a} \tag{83}$$

where d = deflection at fluorescent screen.
L = length of beam from the center of deflecting plates to screen.
a = length of deflection plates.
b = spacing of deflecting plates.
E_d = deflecting-plate voltage.
E_a = beam voltage.

Ordinarily, two pairs of plates are used, one for horizontal and one for vertical deflection. To allow for the deflection of the beam, the plates are often at a small angle with the axis, so that the spacing at the end toward the fluorescent screen is greater than that at the end toward the electron gun. The second pair of plates is usually spaced farther apart than the first pair because of the deflection introduced by the first pair.

The electron beam may also be deflected by magnetic means. This is done by using the arrangement of coils shown in Fig. 50. In the region of approximately uniform magnetic field between the coils, the electrons move in portions of a circular path in a plane perpendicular to the axis of the coils. This action causes the beam to be deflected. The amount of the deflection is

$$d = \frac{La'B}{3.37\sqrt{E_a}} \tag{84}$$

where d = beam deflection at fluorescent screen, cm.
L = beam length between coil center and fluorescent screen, cm.
a' = axial length of region of uniform magnetic field, cm.
B = magnetic field strength in lines per sq cm.
E_a = beam potential, volts.

For both types of deflection, the amount of the deflection decreases as the beam potential increases. This means that if the attempt is made to increase the spot brightness, *i.e.*, the photographic sensitivity, by increasing the beam potential, then the amount of the deflection or the deflection sensitivity will be decreased. In general, any attempt to increase the photographic sensitivity decreases the deflection sensitivity, and vice versa.

The Fluorescent Screen.—Various materials will give off light upon bombardment by electrons in a vacuum. Such materials are called fluorescent in that the light they give off stops almost immediately when the excitation is removed.

Common fluorescent materials include phosphors of zinc, cadmium, and calcium. The characteristics of the principal phosphors are listed in Table 6.[1] The most com-

[1] Reprinted, by permission, from "Television," by V. K. Zworykin and G. A. Morton, John Wiley & Sons, Inc., New York, 1940.

Ex. 1044-0351

monly used material is willemite, which is zinc orthosilicate (Zn_2SiO_4). The synthetic compound used with a manganese activator gives off a bright yellow-green light having its maximum spectral energy at 5200 angstroms and having a luminous efficiency of 10 to 20 lumens per watt. This material works best when the coating contains 7 milligrams per square centimeter.

Zinc sulphide has a long-persistence characteristic that makes it useful in the study of transients. A white screen is obtained by combining a green-blue zinc sulphide with a yellow zinc-cadmium sulphide, or by a similar combination of different colors.

Coatings are made by settling out the material from a water suspension. The water must be removed very slowly after the material has settled out. This is sometimes done by means of a clock mechanism. A mild electrolyte such as ammonium carbonate is used to prevent the particles from settling nonuniformly. Screens may also be made by spraying, using a volatile organic liquid such as acetone with a small amount of binder.

TABLE 6.—PROPERTIES OF PRINCIPAL PHOSPHORS

Phosphor	Chemical composition	Color	Spectral maximum, Å	Approximate candlepower per watt*
Zinc oxide	ZnO	Violet	Ultraviolet	<0.1
Zinc sulphide	ZnS:Ag	Blue-violet	4700–4500	5T
Calcium tungstate	$CaWO_4$	Blue	4300	<1
Zinc silicate	$ZnO + SiO_2$	Blue	4200	<1
Zinc sulphide	ZnS	Light blue	4700	1–5T
Zinc aluminate	$(ZnO + Al_2O_3)$:Mn	Green-blue	5130	~1
Zinc silicate (willemite)	$(ZnO + SiO_2)$:Mn	Blue-green	5230	3T
Zinc sulphide	ZnS:Cu	Green	4700–5250	>4T
Zinc germanate	$(ZnO + GeO_2)$:Mn	Yellow-green	5370	1.5
Beta zinc silicate	$(ZnO + SiO_2)$:Mn	Green-yellow	5600–5700	3T
Zinc beryllium silicate	$(ZnO + BeO + SiO_2)$:Mn	Green to orange	5230–6500	1–2T
Zinc cadmium sulphide	(ZnS + CdS):Ag	Blue to red	4700–> 7000	5T
Calcium silicate	$(CaO + SiO_2)$:Mn	Green to orange	5500–6500	<1
Cadmium silicate	$(CdO + SiO_2)$:Mn	Orange-yellow	5850	~1
Magnesium silicate	$(MgO + SiO_2)$:Mn	Orange-red	6400–6700	<1
Zinc aluminate	$(ZnO + Al_2O_3)$:Cr	Red	> 7000	<1
Zinc beryllium zirconium silicate	$[ZnO + BeO + (Ti - Zr - Th - O_2) + SiO_2]$:Mn	White	4200 + 5500–6000	~1T
Magnesium tungstate	$MgO + WO_3$	Very light blue	4800	<1
Zinc borate	$(ZnO + B_2O_3)$:Mn	Yellow-orange	5400–6000	~1T
Cadmium borate	$(CdO + B_2O_3)$:Mn	Green-orange	5300–6300	<1
Cadmium tungstate	$CdO + WO_3$	Light blue	4900	<1T

* T = used in television.

GAS TUBES

17. Hot-cathode Gaseous Rectifier Tubes.[1]—A hot-cathode gaseous rectifier tube is an ordinary diode tube containing gas such as mercury vapor in equilibrium with liquid mercury, or argon, neon, etc. The gas pressure is ordinarily quite low, of the order of 10^{-3} mm of mercury. When a voltage is applied between emitting electrode and collector, the current-voltage relations are at first like those in a vacuum diode; *i.e.*, the current flow consists only of electrons emitted from the cathode and follows a three-halves power law. However, as the anode potential is increased, the electrons finally acquire enough energy to ionize the particles of gas by collision. The current that was formerly limited by the negative space charge of the electrons now has this space charge annulled by these positive ions, and the current rises extremely rapidly with voltage, quickly reaching the full electron emission. This current-voltage rela-

[1] For further information see A. W. Hull, Gas-filled Thermionic Tubes, *Trans. A.I.E.E.*, Vol. 47, p. 753, 1928. See also additional discussion of rectifier tubes of this type given in Par. 1, Sec. 8.

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tion is shown in Fig. 52 in comparison with the current-voltage relations of the same tube completely evacuated.

When ionization occurs in the tube there is formed a region between cathode and plate known as the "plasma," in which positive ions collect and are approximately in equilibrium with the negative electrons that are drifting through the plasma to the plate. The plasma takes up most of the room between the cathode and plate, and is at a potential slightly higher than the plate potential because of a slight surplus of positive ions in this region. The electrons liberated from the cathode strike the edge of the plasma with sufficient energy to produce ionization and then drift toward the plate. Practically all the potential drop within the tube takes place in the region immediately adjacent to the cathode. The potential distribution in the tube for a condition corresponding to the steep portion of Fig. 52 is shown in Fig. 53.

The voltage drop corresponding to the steep portion of the characteristic of Fig. 52 is approximately the ionization potential of the gas involved. In the case of mercury this is 10 to 15 volts. This voltage drop between anode and cathode is substantially constant for currents from very small values up to the full emission current of the

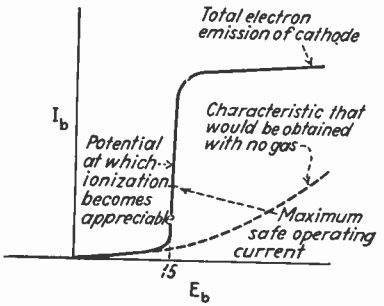


FIG. 52.—Current-voltage characteristics of a gas diode.

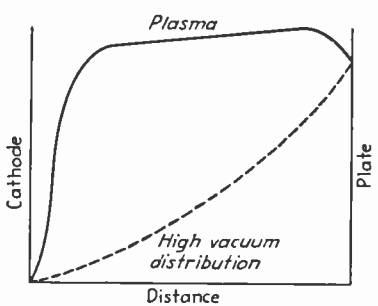


FIG. 53.—Potential distribution in a gas diode.

cathode. If there is an attempt, however, to pass more current through the tube than is represented by the cathode emission, the voltage drop will increase above the ionization potential of the gas involved. This is to be avoided because it causes cathode disintegration, and because the voltage drop in the tube, and hence the tube dissipation, is then unnecessarily large.

Study has shown that with oxide-coated and thoriated-tungsten emitters the positive ions striking the cathode in a gas-filled tube will not produce serious consequences, provided that the velocity of these positive ions does not exceed about 22 volts. However, if the velocity is greater than this, progressive disintegration of the emitting surface occurs. Damage will occur even when the voltage drop exceeds the permissible value for only a very small time. As a consequence, it is important that care be taken to arrange the circuit external to a gas tube so that the anode current in the tube is limited by the external circuit to a value safely below the cathode emission.

In the practical use of hot-cathode gas rectifier tubes, the factors that must be considered in selecting a particular tube are: (1) the allowable peak instantaneous space current; (2) the maximum average current that the tube is capable of handling; (3) the allowable peak inverse voltage. The allowable peak instantaneous current must be safely below the emitting possibilities of the cathode to ensure that at no time during life will the tube be subjected to operating conditions where the instantaneous voltage drop in the tube exceeds the ionization voltage. Every tube has a rated maximum average current that is appreciably less than the peak instantaneous current. This

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rated average current should not be exceeded, since to do so will in general cause overheating of the tube and greatly shortened life. The maximum inverse voltage applied to the anode is important, because even though when the anode is negative no space current flows, there is the possibility that an arc-back will occur through the gas if the inverse voltage is too high.

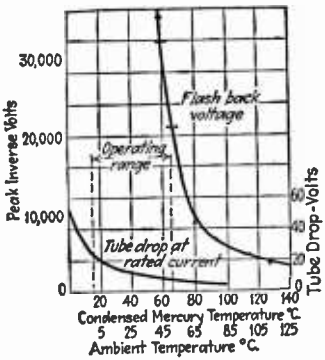


FIG. 54.—Inverse breakdown voltage and voltage-drop variation with temperature of a hot-cathode mercury-vapor diode.

An increase in the bulb temperature of a hot-cathode mercury-vapor tube causes the inverse voltage at which arc-back occurs to decrease, as indicated in Fig. 54. This same figure shows that the voltage drop in the tube increases as the temperature decreases. As a result of these characteristics, hot-cathode mercury-vapor rectifier tubes must be operated so that the temperature of the condensed mercury falls between certain limits. The highest temperature is determined by the minimum allowable inverse breakdown voltage, while the lowest allowable operating temperature is determined by the fact that the cathode disintegration potential must not be approached too closely, and the fact that the ionization in the tube is inadequate if the gas pressure (*i.e.*, temperature) is too low.

Because the current that a gas rectifier can pass is limited only by the cathode emission, and not by space charge, as in an ordinary tube, every effort is made to obtain as high an emission as possible per watt of cathode heating power. The nearly complete annulment of space charge resulting from the action of positive ions makes it possible to use cathode structures that cannot be used in vacuum tubes. Thus strongly curved surfaces, and even enclosures coated with emitting material,

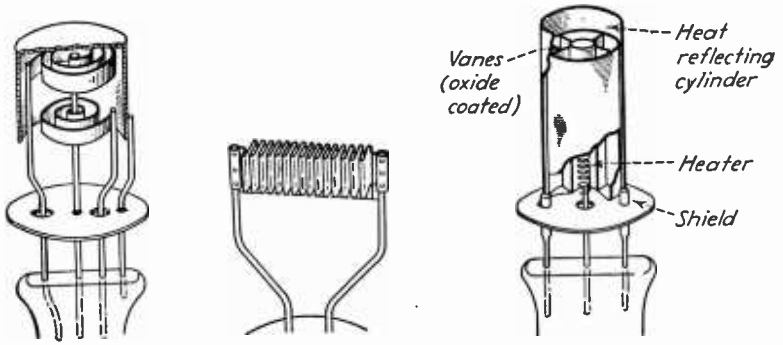


FIG. 55.—Hot-cathode emitting structures.

are permissible because the space-charge annulling positive ions are able to penetrate into these. This makes possible a higher thermal efficiency than is obtainable in vacuum tubes. Cathode structures may be either of the filamentary type or of the indirectly heated type. The filamentary type frequently makes use of a wide ribbon that is crimped and folded into a spiral to give a large emitting surface in a small volume. The indirectly heated type makes use of a heated cylinder placed within another cylinder, supported by vanes parallel to the axis. The entire interior of the

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structure is coated with emitting material, and the emitted electrons escape through the ends of the structure. All types of structures sometimes have surrounding heat shields of thin sheet metal to reduce the radiation by reflecting the heat back into the structure. Some typical emitting structures are shown in Fig. 55.

It is necessary that the peak potential difference between the ends of the filament be not only less than the ionization potential of the gas, but also be less than the difference between the ionization potential and the disintegration potential. Otherwise ionization may be caused by the filament voltage, or ions may strike part of the emitter with energy in excess of that required to cause disintegration.

A compromise must be effected between heater efficiency and heating time, since the more efficient the cathode is thermally the longer it takes to reach its operating temperature. In all cases it is necessary to bring the cathode to normal operating temperature before applying the plate potential in order to avoid damaging the emitting surface. Heating times range from 30 seconds in small tubes to many minutes for the larger structures.

The nature of the current-voltage characteristics of gas tubes makes it virtually impossible to connect such tubes directly in parallel. This is because the operating part of the characteristic is so steep that a slight difference in the characteristics of two tubes would make a very large difference in the currents taken by the tubes, and the tube with the smaller voltage drop would take practically all the current. This difficulty can be circumvented by connecting resistors in series with the tubes before connecting them in parallel.

18. Grid-controlled Gas Rectifiers (Thyratrons).[1]—Grid-controlled gas rectifier tubes, also known as thyratrons and as gas triodes, are used extensively in industrial control circuit applications. The grid-controlled gas rectifier tube can be considered as a hot-cathode gas rectifier tube in which there has been inserted a control electrode between cathode and plate. This control electrode serves a function analogous to that of the control grid of an ordinary vacuum tube, although structurally it may differ.

The grid electrode is able only to initiate the flow of current, but once current flows the magnitude of the current is not influenced by the control electrode, and neither is this electrode able to stop the flow of current. The current can be stopped only by making the plate potential zero or negative for a short period of time.

The characteristics associated with the control grid are briefly as follows. If the grid is considerably more negative than the cutoff value (*i.e.*, the grid potential at which no electrons are attracted toward the anode), and then is gradually reduced, it is found that at the point where plate current would just start to flow if the tube contained no gas, the plate current suddenly jumps from zero to a high value. This is a result of the fact that as soon as plate current begins to flow, positive ions are produced as a result of ionization by collision. Some of these are attracted toward the negative grid and surround it with a sheath of positive ions that neutralize the electrostatic effect of the grid, and so destroy its normal control action. At the same time, other positive ions are attracted toward the cathode and neutralize the space charge. Hence once ionization has started, there is no space charge to limit the current flow, and the control action of the grid has been lost. The anode current then is determined by the same factors that affect anode current in the hot-cathode gas rectifier tube.

The relations between the grid and plate potential at the moment of the initiation of the arc are similar to those which apply at cutoff in a vacuum triode. As the plate potential is made less positive, the grid potential must be made relatively more positive to start the arc. The starting voltage characteristic of a typical thyratron is

[1] A. W. Hull, Gas-filled Thermionic Tubes, *Trans. A.I.E.E.*, Vol. 47, p. 753, 1928.

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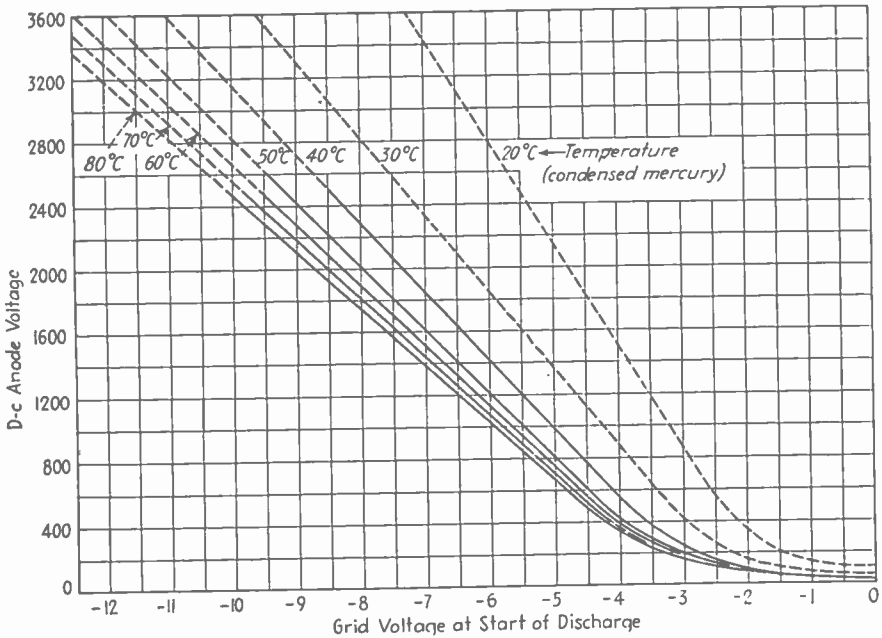


FIG. 56.—Starting characteristics of a typical thyratron using mercury.

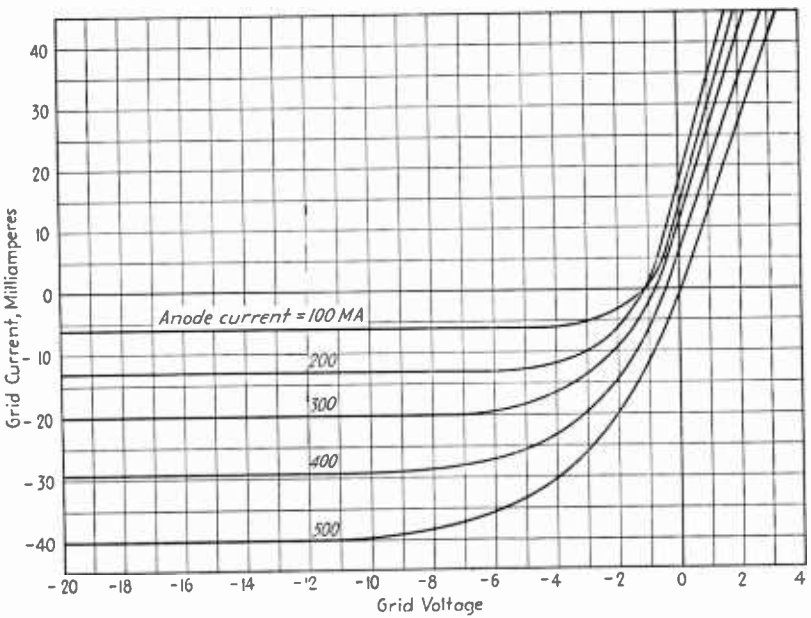


FIG. 57.—Grid-current characteristics of a typical thyratron.

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shown in Fig. 56. Over a large part of the characteristic it is seen that the ratio of positive plate potential to negative grid potential is nearly constant. This ratio is known as the "control ratio."

Although the grid loses control of the current when the arc has started, it may draw an appreciable current. This current tends to be a large negative current in the form of collected electrons when the grid is positive, the magnitude of the current rising rapidly with potential. When the grid is negative the current corresponds to a positive-ion current and tends to be small and nearly constant. Because of the possibility of large grid currents the grid must be protected by a series resistor in circuit applications. The resistor cannot, however, be too large or the voltage drop in it due to the current that does flow will interfere with the normal grid action. Some typical grid-current characteristics are shown in Fig. 57.

The arc of a thyratron tube can only be extinguished by making the plate potential zero or negative for a sufficiently long period of time. This reduction of the plate potential stops the arc because it prevents electrons from reaching the plate and causes them to be drawn back to the cathode while the positive ions are drawn back to the plate. The plate must be negative long enough for most of the positive ions to

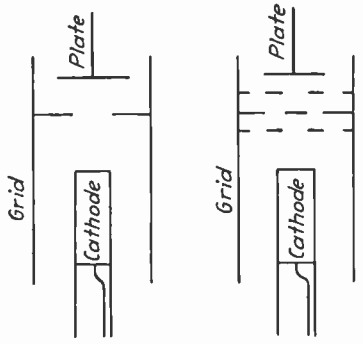


FIG. 58.—Typical thyratron electrode structures.

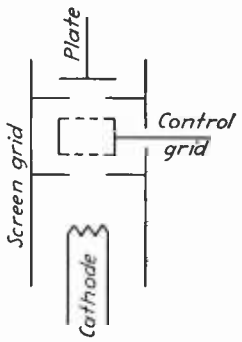


FIG. 59.—Screen-grid thyratron.

be removed from the space between electrodes, for otherwise the grid is unable to regain control of the arc. The time it takes for the grid to regain control of the arc after the plate potential has been made negative is known as the deionization time. Although the time required to initiate the arc, the ionization time, is of the order of a few microseconds, the deionization time ranges from ten to 1,000 microseconds. Hence thyratrons cannot be used in applications calling for a repetition rate of more than about fifty kilocycles.

The mode of operation of a thyratron tube makes it desirable to employ electrode structures and arrangements differing greatly from those of a vacuum triode. Cathodes are made similar to those used in gas rectifiers. The grid is frequently a nearly solid electrode surrounding the cathode, rather than a meshlike structure. Thus the usual "grid" is a cylinder having a diaphragm that separates the cathode from the plate, and that contains one or more large holes. The holes must be large enough so that they are not closed by the positive-ion sheath that forms about the grid after the initiation of the arc. The plate of the thyratron is usually just a simple disk that collects the electrons. It can be small because the low voltage drop in the tube results in a low plate heat dissipation. Some typical thyratron electrode structures are shown in Fig. 58.

Ex. 1044-0357

Although the grid current in a typical gas triode may be only 50 microamperes or less, there are many applications in which even this much current prevents the use of a high-impedance device in the grid circuit. Grid currents may be reduced one hundredfold by the use of a screen-grid structure as shown in Fig. 59. In this the screen grid is placed next to the cathode, and the control grid is placed between the screen grid and plate. In this way the control-grid current is reduced to the point where the tube may be operated directly from a photocell. The other characteristics of the screen-grid thyratron are essentially the same as those of a three-electrode thyratron.

The limitations of a thyratron are similar to those of a gas rectifier. Thus there is a certain peak current that should not be exceeded even instantaneously lest the cathode be bombarded with ions of sufficient energy to disintegrate it. Also, since the heating of the tube depends upon the average current because of the constancy of the voltage drop in the tube, there is a maximum allowable value of average current.

The allowable inverse peak voltage depends upon the temperature of the tube and upon its internal construction. Since the pressure within the tube doubles with a rise of approximately 10°C, great care must be taken to see that the tube is not overheated.

The designer must observe a number of precautions in order to keep the rated peak inverse voltage high. The electrode geometry must be such that high potential gradients within the tube are avoided. Residual gases in the tube other than the desired gas must be kept low. The electrode shapes should be such that the ionized region is confined as much as possible. This is necessary so that the positive ions removed on the first part of the negative half of the plate-voltage cycle will only have a short distance to go, for otherwise they may strike the plate with sufficient energy to produce emission, which will cause a flashback.

19. Cold-cathode Tubes. *Diodes.*—The simplest cold-cathode tube is a diode containing neon or some other inert gas at a pressure of about 0.1 mm of mercury. There is no provision for control of the arc in a diode, and since the cathode is cold, the voltage drop is high, and the current density in the discharge is low. The voltage drop is determined by the gas pressure and the cathode material. The stability of the discharge is aided by sputtering the cathode with metals of low work function, such as cerium, lathanium, and didymium, or coating with partially reduced oxides of barium and strontium. No combination of gas and cathode material that gives a stable discharge at less than 60 volts has been found.

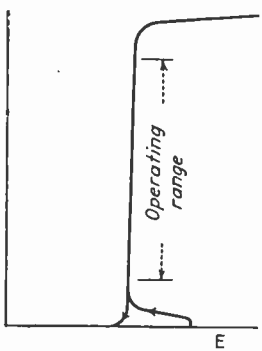


Fig. 60.—Current-voltage characteristics of a cold-cathode diode.

A small starting probe can be used as part of one electrode of the diode to give a high gradient region in which the discharge will start. As potential and current are raised the glow will spread from the starting probe until it covers the whole plate. Typical cold-cathode current-voltage characteristics are shown in Fig. 60. The striking voltage of the glow is a little higher than the normal tube drop. This is because an accumulative ionization will maintain the glow at a lower potential than that which is needed to initiate it. Cold-cathode diodes are operated in the part of the characteristic in which the voltage drop is substantially constant. If this voltage drop is exceeded the tube may be damaged because of excessive sputtering action at the cathode.

Characteristics of some typical cold-cathode diodes are given in Table 7. These tubes have all been designed to be used as voltage regulators.

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TABLE 7.—CHARACTERISTICS OF VOLTAGE-REGULATOR GAS-DISCHARGE TUBES

Tube type	Maximum current, ma	Operating voltage, volts	Igniting or striking voltage
RCA 991	2	48–87	87
RCA 874	50	90	125
VR75-30	30	75	105
VR90-30	30	90	125
VR105-30	30	105	137
VR150-30	30	150	180
WE 313C	..	60	70
¼-watt neon	5	50–60	85
3-watt neon	60	50–60	85

In the *VR* series of tubes there is a correct polarity for the applied d-c voltage. Some neon lamps have current-limiting resistors in the base that must be removed for voltage-regulator application. The WE 313C has the greatest uniformity of characteristics from tube to tube; the neon lamps and the RCA 991 have the least uniform characteristics from tube to tube.

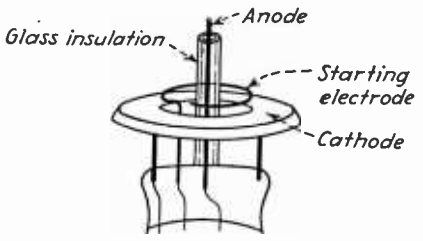


FIG. 61.—A cold-cathode discharge tube with starter electrode.

The principal uses of cold-cathode diodes are as voltage regulators, stroboscopic light sources, relaxation oscillators, polarity indicators, and surge-protection devices.

Cold-cathode Triodes.—A control type of cold-cathode tube has been developed for use as a trigger tube, especially in carrier actuated circuits.[1] This tube makes use of two fundamental principles. First, the breakdown potential between two electrodes depends upon their spacing, and second, the breakdown potential is lowered when ionization is present. Ionization is provided between cathode and a starter electrode placed close to it. The starter electrode is operated with a high resistance in series with it so that it carries little current, but the arc to it starts the main discharge to the anode, which is operated just below the starting potential corresponding to the cathode-anode spacing.

The construction of such a tube is shown in Fig. 61. The cathode is a large flat disk electrode of nickel. The control or starter electrode is a wire ring close to the cathode. The anode is a short length of wire projecting from a glass tube, which, in turn, projects through the cathode and starter electrode. An alternative construction that is also used in practice is similar, except that the cathode and starter electrodes are identical halves of a split disk. In either case, the cathode is coated with

[1] W. E. Bahls and C. H. Thomas, A New Gas-filled Triode, *Electronics*, Vol. 11, p. 14, May, 1938; S. B. Ingrahm, Cold-cathode Gas-filled Tubes as Circuit Elements, *Trans. A.I.E.E.*, Vol. 58, p. 342, July, 1939; C. N. Kimball, A New System of Remote Control, *R.C.A. Rev.*, Vol. 2, p. 303, January, 1938.

Ex. 1044-0359

oxides of barium and strontium that are partially reduced to the metals in the evacuation process.

Since a sputtering action is present while the tube is in operation, the metal is gradually used up and replaced by a further reduction of the oxide.[1] The tube accordingly has a definite life, which is determined by the rate at which the metal is used up, and so upon the time integral of the anode current. Cold-cathode triodes

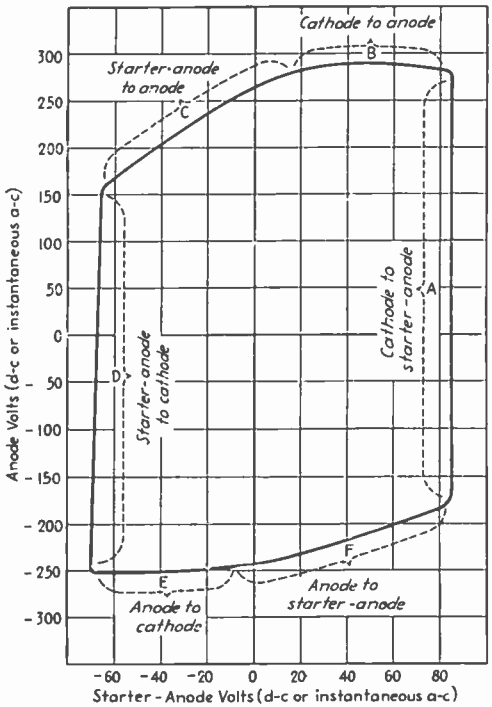


FIG. 62.—Typical breakdown characteristics of cold-cathode triode such as is shown in Fig. 61.

are designed to operate at low currents, and when possible the circuit arrangements should be such that the anode current flows as little as possible.

Since there are three electrodes in a tube such as that of Fig. 61, any one electrode may act as the control electrode to control the discharge between the other two. As a result there are six possible conditions of current flow, the breakdown characteristics of which are indicated by the closed hexagonal curve of Fig. 62. Ordinarily only the section *A* of the curve is used.

The chief uses of the cold-cathode triode are as a relay device, or trigger tube.

[1] The factors controlling life in cold-cathode tubes are well treated by G. H. Rockwood, Current Rating and Life of Cold-cathode Tubes, *Trans. A.I.E.E.*, Vol. 60, p. 901, September, 1941.

Ex. 1044-0360

SECTION 5

VACUUM-TUBE AMPLIFIERS

GENERAL CONCEPTS

1. Classification of Amplifiers.—Amplifiers are classified in a variety of ways descriptive of their character and properties. One basis is according to the frequency range covered and leads to audio-frequency, video-frequency, radio-frequency, and direct-current amplifiers. Amplifiers are also classified as voltage or power amplifiers, according to whether the primary purpose is to develop voltage or power, respectively, in the output.

Amplifiers are also often referred to as Class A, Class AB, Class B, linear, or Class C amplifiers, according to the method of adjusting the tube. A Class A amplifier is an amplifier in which the grid-bias and alternating grid voltages are such that plate current in the tube flows at all times. Again a Class AB amplifier is an amplifier in which the grid-bias and alternating grid voltages are such that plate current in the tube flows for appreciably more than half but less than the entire electrical cycle. A Class B amplifier is an amplifier in which the grid bias is approximately equal to the cutoff value so that the plate current is approximately zero when no exciting grid voltage is applied and so that plate current flows in the tube for approximately one-half of each cycle when an alternating grid voltage is applied. Class B amplifiers employing tuned load circuits are often referred to as *linear amplifiers*. A Class C amplifier is an amplifier in which the grid bias is appreciably greater than the cutoff value, so that the plate current in each tube is zero when no alternating grid voltage is applied and so that plate current in the tube flows for appreciably less than one-half of each cycle when an alternating grid voltage is applied.[1] The suffix 1 is sometimes added to indicate that grid current does not flow during any part of the cycle of input voltage, while the suffix 2 indicates that grid current flows during some part of the cycle; thus Class A_2 means a Class A amplifier with sufficient exciting voltage to drive the grid positive.

2. Distortion in Amplifiers.—The output of an amplifier may fail to reproduce the input as a result of frequency, amplitude (or nonlinear), and phase distortion. Frequency distortion occurs when different frequency components are not amplified equally. Amplitude (or nonlinear) distortion is the result of nonlinearity existing in either the grid-cathode or plate-cathode circuits of the tube, and causes frequencies to appear in the amplifier output that are not present in the voltage applied to the amplifier input.

Phase Distortion and Delay Time.—Phase distortion occurs when the relative phases of the various components being amplified are not the same in the output as in the input. Phase distortion causes the output wave shape to differ from the wave shape of the applied signal, even though both may contain exactly the same frequency components in the same relative magnitudes. The difference in phase between the output and input voltage for an individual frequency component being amplified can be expressed in terms of a time of transmission through the amplifier according to the relation

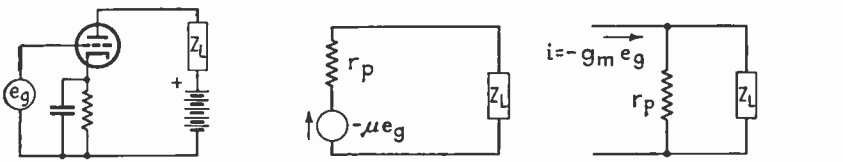
$$\text{Phase shift in radians} = \tau\omega + n\pi \tag{1}$$

[1] "Standards on Electronics," p. 7, Institute of Radio Engineers, 1938.

Ex. 1044-0361

where ω is 2π times frequency, n is an integer, and τ a quantity having the dimension of time and termed the *delay time*. In order to avoid phase distortion, τ must be the same for all frequencies; *i.e.*, the curve of phase shift as a function of frequency must be a straight line passing through an integral multiple of π at zero frequency. Any departure from such a constant delay time represents phase distortion.

3. Equivalent Amplifier Circuits.—The fundamental circuit of a vacuum-tube amplifier is shown in Fig. 1*a*. To a first-order approximation, the amplified current that flows in the plate circuit of such an amplifier tube having an amplification factor μ and transconductance g_m can be calculated by replacing the plate-cathode circuit of the tube by an equivalent generator voltage μe_g acting in series with the plate resistance of the tube. This arrangement can be referred to as the *constant-voltage generator* form by the equivalent plate circuit, and is shown in Fig. 1*b*. An alternative equivalent circuit is shown in Fig. 1*c*, in which the plate-cathode circuit of the tube is replaced by a constant-current generator supplying a current $e_g g_m$ to a circuit consist-



(a) Actual amplifier circuit (b) Equivalent amplifier circuit (constant voltage generator form (c) Equivalent amplifier circuit (constant current generator form)

FIG. 1.—Actual and equivalent plate circuits of a vacuum-tube amplifier. In the constant-voltage form the current that is produced in the plate circuit by the signal e_g acting on the grid is taken into account by postulating that the plate circuit can be replaced by an equivalent generator $-\mu e_g$ having an internal resistance equal to the plate resistance R_p. In the constant-current form the tube is considered as generating a current $-g_m e_g$ that flows through the impedance formed by the plate resistance of the tube in parallel with the load resistance. These circuits apply to all types of tubes, including pentode, beam, and screen-grid tubes as well as triodes.

ing of the plate resistance of the tube in parallel with the load resistance. This can be referred to as the *constant-current generator* form of the equivalent amplifier circuit. It expresses the same relation as the constant-voltage form, and leads to the same result as far as the load is concerned.

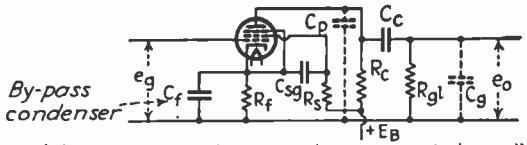
The constant-current circuit is the most convenient to use in practical calculations when plate resistance of the tube is much higher than the load resistance, as is the case with pentode and beam tubes. The constant-voltage form is most convenient when the plate resistance is of the same order of magnitude or less than the load resistance, as is commonly the case with amplifiers using triode tubes.

VOLTAGE AMPLIFIERS FOR AUDIO FREQUENCIES

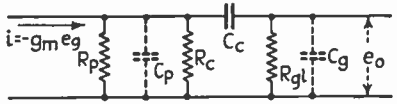
4. Resistance-coupled Audio-frequency Amplifiers.—The ordinary voltage amplifier of audio frequencies employs a pentode tube, in the plate circuit of which is a load impedance consisting of a resistance-capacity network, as shown in Fig. 2.

Amplification as a Function of Frequency When the Screen and Cathode By-pass Condensers Are Very Large.—When the impedance in the screen and cathode circuits of a resistance-coupled amplifier can be neglected the amplification varies with frequency as shown in Fig. 3. The amplification at high frequencies falls off because of the shunting capacity represented by the plate-cathode capacity of the tube plus the input capacity of the tube to which the amplified voltage is applied, plus incidental

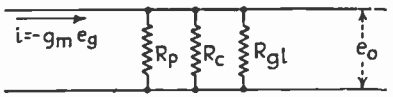
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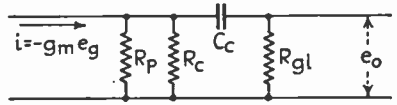
(a) Actual circuit of resistance-coupled amplifier



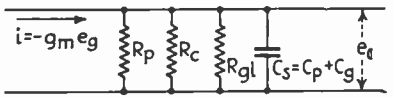
(b) Exact equivalent circuit of amplifier



(c) Simplified circuit accurate for intermediate frequencies



(d) Simplified circuit accurate for low frequencies



(e) Simplified circuit accurate for high-frequencies

e_g = signal voltage
e_o = amplified voltage
R_p = tube plate resistance
R_c = coupling resistance
R_{gl} = grid-leak resistance
R_f = bias resistance
R_s = voltage dropping resistor for screen
g_m = mutual conductance of tube
C_f = bias by-pass condenser
C_{sg} = screen by-pass condenser

C_c = coupling or blocking condenser
C_p = plate-cathode tube capacity plus stray wiring capacity to left of coupling condenser
C_g = stray wiring capacity to right of coupling condenser plus input capacity of tube or other load to which voltage is delivered
$C_s = C_p + C_g$ = total shunting capacity

FIG. 2.—Circuit of a resistance-coupled amplifier employing a pentode tube, together with equivalent plate circuits useful in making amplifier calculations, assuming large screen and bias by-pass condensers.

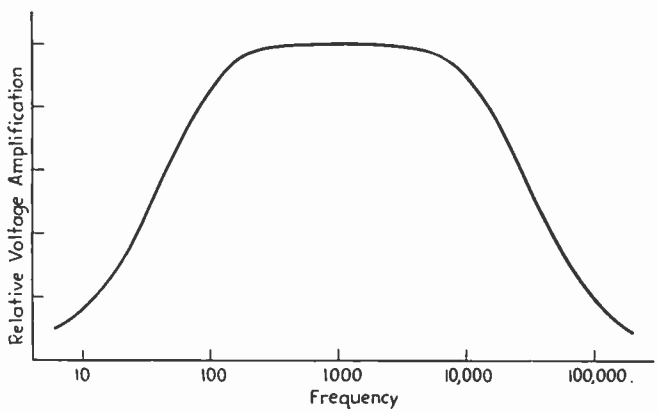


FIG. 3.—Variation of amplification with frequency in a typical resistance-coupled amplifier with large screen and bias by-pass condensers.

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capacity from the wiring. The falling off at low frequencies is caused by the loss of voltage in the coupling condenser C_c.

The amplification characteristic can be calculated with the aid of the equivalent plate circuit of Fig. 2*b*. This can be simplified in ordinary amplifiers by taking advantage of the fact that the coupling condenser C_c has negligible effect except at quite low frequencies, while the shunting capacities have negligible effect except at high frequencies.[1] This results in the equivalent circuits of Figs. 2*c* to 2*e*, each of which is applicable to a limited part of the frequency range.

In the middle range of frequencies, the amplification is maximum and depends only upon the transconductance of the tube and the equivalent resistance R_{eq} formed by plate, coupling, and grid-leak resistances, all in parallel, according to the equation

$$\text{Mid-frequency amplification} = g_m R_{eq} \tag{2}$$

where $R_{eq} = \dfrac{R_c}{1 + \dfrac{R_c}{R_{gl}} + \dfrac{R_c}{R_p}}$.

g_m = transconductance of amplifier tube, mhos.

With pentode tubes the plate resistance is very much higher than the remaining resistances involved in R_{eq}, so that the amplification is given with good accuracy by the simplified equation

$$\left.\begin{matrix}\text{Mid-frequency amplification (approximate)}\end{matrix}\right\} = g_m R_c \frac{1}{1 + \dfrac{R_c}{R_{gl}}} \tag{3}$$

In this middle-frequency range there is negligible phase shift other than a phase reversal produced as a result of the way in which the tube operates.

At low frequencies the amplification is

$$\frac{\left.\begin{matrix}\text{Amplification at low frequencies}\end{matrix}\right\}}{\left.\begin{matrix}\text{Amplification in middle range}\end{matrix}\right\}} = \frac{1}{1 - j\dfrac{f_1}{f}} \tag{4}$$

where f = actual frequency.

$$f_1 = 1/2\pi C_c R \tag{5}$$

= frequency at which the reactance of C_c equals the resistance $R\left(= R_{gl} + \dfrac{R_c}{1 + \dfrac{R_c}{R_p}}\right)$ formed by grid leak in series with the combination of plate and coupling resistances in parallel.

Under all ordinary conditions with pentodes, $R_p >> R_c$, and it is permissible to take $R = R_c + R_{gl}$. *The extent to which the amplification falls off at low frequencies is determined by the ratio of the coupling condenser reactance to the equivalent resistance R formed by combining the grid leak in series with the parallel combination of coupling resistance and plate resistance. The output falls to* 70.7 *per cent* (−3 *db*) *of the mid-frequency amplification at the frequency* f_1 *for which this reactance equals the equivalent resistance R.* The falling off in amplification and the phase shift produced at low frequencies relative to the mid-frequency conditions are given in Fig. 4.

[1] Analysis of the rather infrequently encountered case where this does not hold true is given by David C. G. Luck, A Simplified General Method for Resistance-capacity Coupled Amplifier Design, *Proc. I.R.E.*, Vol. 20, p. 1401, August, 1932; Westley F. Curtis, The Limitations of Resistance-coupled Amplification, *Proc. I.R.E.*, Vol. 24, p. 1230, September, 1936.

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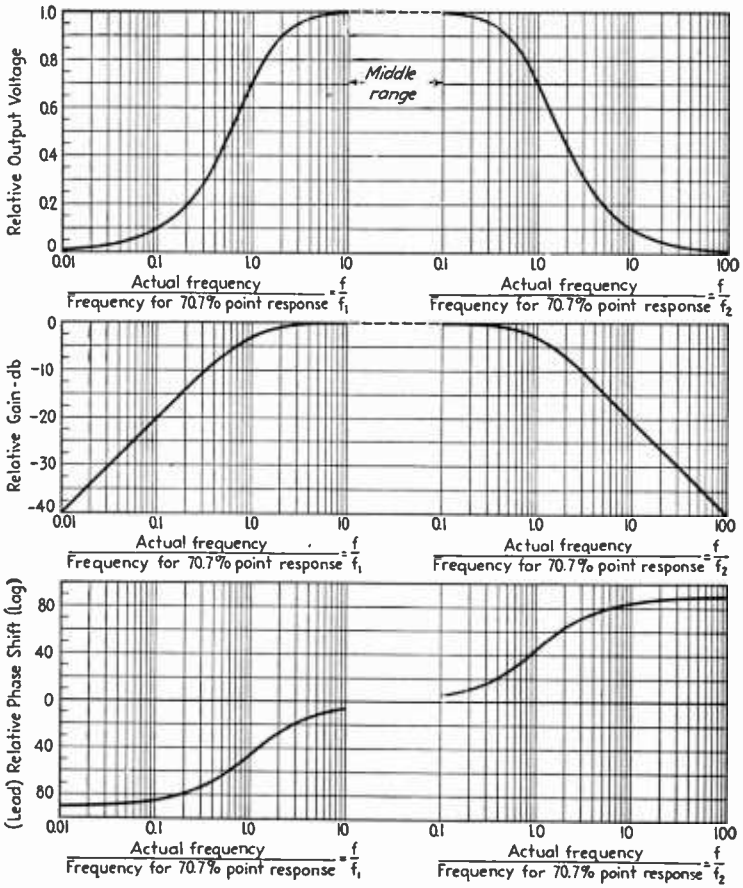


FIG 4.—Universal amplification curve showing the falling off in amplification and the phase shift that occurs in a resistance-coupled amplifier at high and low frequencies when the screen and bias by-pass condensers have negligible effect.

The relative amplification at high frequencies is

$$\frac{\left.\begin{matrix}\text{Amplification at}\\ \text{high frequencies}\end{matrix}\right\}}{\left.\begin{matrix}\text{Amplification in}\\ \text{middle range}\end{matrix}\right\}} = \frac{1}{1 + j\frac{f}{f_2}} \tag{6}$$

where f = actual frequency.

$$f_2 = 1/2\pi C_s R_{eq} \tag{7}$$

= frequency at which the reactance of the shunting condenser C_s equals the equivalent resistance R_{eq} formed by plate, grid-leak, and coupling resistances, all in parallel.

The extent to which the amplification falls off at high frequencies is determined by the ratio that the reactance of the shunting capacity C_s bears to the equivalent resistance obtained by combining coupling resistance, grid-leak resistance, and plate resistance all in parallel. The loss in amplification at high frequencies can be conveniently estimated by the fact

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that the magnitude is 70.7 per cent (−3 db) of the mid-frequency amplification at the frequency f_2 for which the reactance of the shunting capacity equals the equivalent resistance R_{eq}. The relative amplification and phase shift are given in Fig. 4.

Effect of Impedance in the Screen-grid Circuit.—Resistance-coupled amplifiers employing pentode tubes normally obtain the screen voltage from the plate-supply voltage by means of a voltage dropping resistance R_s by-passed to the cathode by condenser C_{sg}, as shown in Fig. 2*a*. Such an arrangement introduces an impedance in the screen-grid circuit that modifies the amplification, particularly at low frequencies, where the impedance is large. The effect is to reduce the output voltage, and shift its phase by the factor[1,2]

$$\left.\begin{matrix}\text{Actual output voltage}\\ \hline \text{Output voltage with zero impedance in screen circuit}\end{matrix}\right\} = \beta = \frac{\frac{K}{1+K} + j\frac{f}{f_3}}{1 + j\frac{f}{f_3}} \tag{8}$$

where $K = \dfrac{R_{sg}}{R_s} = \dfrac{\text{dynamic resistance of screen circuit of tube}}{\text{screen dropping resistance}}$.

f = actual frequency.

$$f_3 = \frac{1+K}{2\pi C_{sg} R_{sg}}$$

= frequency at which reactance of C_{sg} equals resistance formed by R_s and R_{sg} in parallel. (9)

The magnitude and phase of the factor given by Eq. (8) are plotted in Fig. 5.

Effect of Bias Impedance.—The usual voltage amplifier obtains the grid bias by means of a resistance-condenser combination between cathode and ground, as shown by $C_f R_f$ in Fig. 2*a*. This arrangement introduces an appreciable impedance in the cathode circuit at low frequencies, and since the amplified plate current must flow through this impedance, a voltage drop is produced that causes an additional voltage to be applied between grid and cathode. This reduces the amplification and also introduces an additional phase shift. The magnitude of the effect is given by[3]

$$\left.\begin{matrix}\text{Actual output voltage}\\ \hline \text{Output voltage with zero bias impedance}\end{matrix}\right\} = \gamma = \frac{1}{1 + \dfrac{g_m R_f \beta}{1 + j\dfrac{f}{f_4}}} = \frac{1}{1 + g_m R_f \eta \beta} \tag{10}$$

where β = factor given by Eq. (8) and Fig. 5.

$$\eta = \frac{1}{1 + \dfrac{f}{f_4}}$$

g_m = transconductance of the tube.

R_f = bias resistance.

f = actual frequency.

$f_4 = 1/2\pi C_f R_f$ = frequency at which reactance of C_f equals bias resistance R_f.

[1] This equation assumes that the pentode tube is operated under conditions such that the division of the total space current between the plate and screen is not affected by the plate voltage, a condition realized under ordinary conditions. For further details, see F. E. Terman, W. R. Hewlett, C. W. Palmer, W. Y. Pan, Calculation and Design of Resistance-coupled Amplifiers Using Pentode Tubes, *Trans. A.I.E.E.*, Vol. 59, p. 879, 1940.

[2] Values of R_{sg} are not yet given by the tube manufacturers, but can be approximated with sufficient accuracy for most occasions by assuming the ratio of screen resistance to plate resistance obtained when the tube is converted to a triode by connecting plate and screen together at the screen potential, is the ratio of total space current to screen current (usually about five).

[3] See Terman, Hewlett, Palmer, and Pan, *loc. cit.*

Equation (10) assumes that the a-c current flowing in the screen circuit all flows to the cathode through condenser C_{sg}, a condition closely realized in practice.

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It is to be noted that the effect of the bias impedance in reducing amplification is affected not only by the bias impedance itself but also by the impedance in the screen circuit. This is in addition to the effect of the screen circuit impedance as given by Eq. (8) and Fig. 5.

In the case where the screen circuit impedance has negligible effect (*i.e.*, when $\beta \cong 1/\underline{0}°$), the effect of the bias impedance is given by Fig. 6. Results in the more general case can be readily obtained by the use of Figs. 5, 7, and 8. The first of these figures gives the factor β, and the second gives the factor η. From these one can

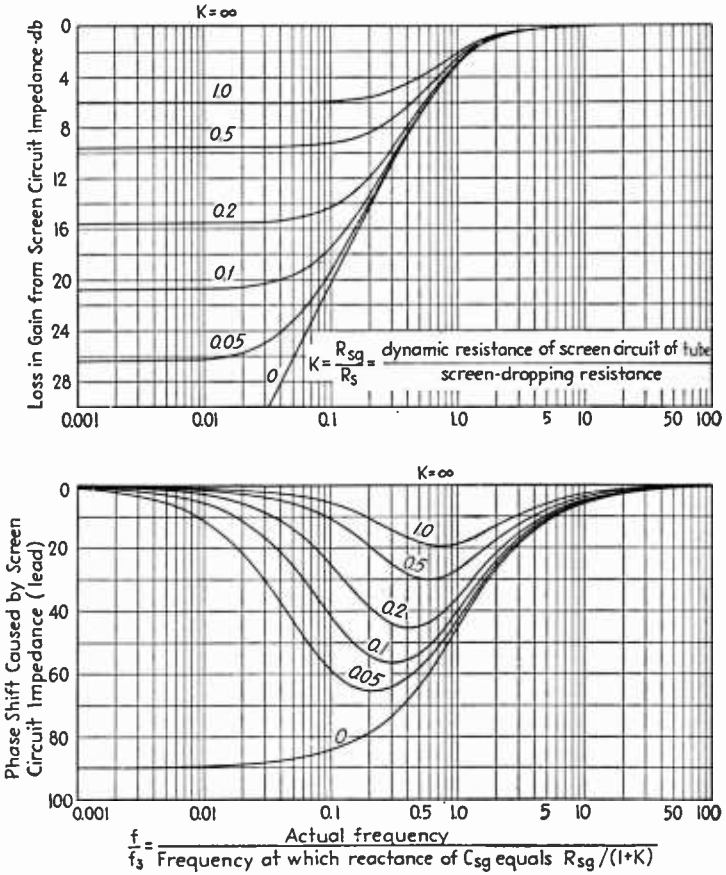


FIG. 5.—Curves giving falling off and phase shift in output voltage at low frequencies in a resistance-coupled amplifier as a result of a finite screen-cathode by-pass condenser, for various values of K.

form $R_f g_m \eta\beta$, then use Fig. 8 to give $\gamma = 1/(1 + R_f g_m \eta\beta)$. Note that the magnitudes in all these figures are given in db below unity voltage ratio, so that in forming $R_f g_m \eta\beta$, one simply adds the corresponding db values from Figs. 5 and 7 and the db value of $R_f g_m$ above unity (20 log $(R_f g_m)$ = db) to get the db magnitude $(R_f g_m)\eta\beta$, and adds angles of β and η to get the angle of $(R_f g_m)\eta\beta$.

Calculation of Actual Amplification Curve.—The actual amplification curve of an ordinary resistance-coupled amplifier such as is shown in Fig. 9 can be readily calculated by using Eq. (2) and Figs. 4, 5, 7, and 8, as illustrated by the following example:

Calculation of the Response and Phase Shift of the Amplifier Having the Constants of Fig. 9, as a Function of Frequency.—The amplification in the mid-frequency range is, according to Eq. (2),

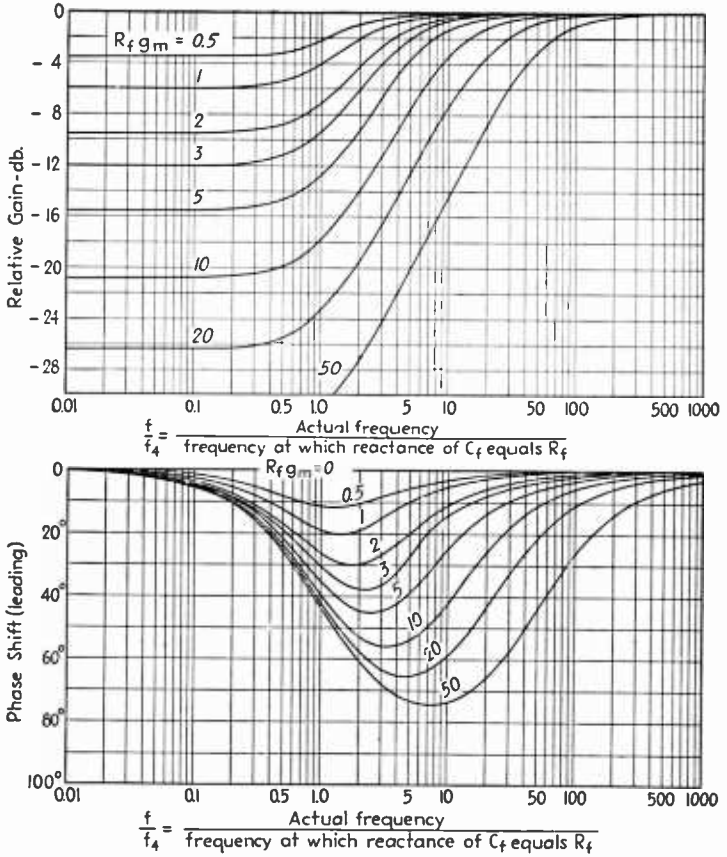


FIG. 6.—Curves giving loss in amplification and phase shift of output voltage occurring at low frequencies in a resistance-coupled amplifier as a result of a resistance-condenser bias impedance $R_f C_f$ for the case where the screen-circuit impedance has negligible effect upon the amplification, for various values of $R_f g_m$.

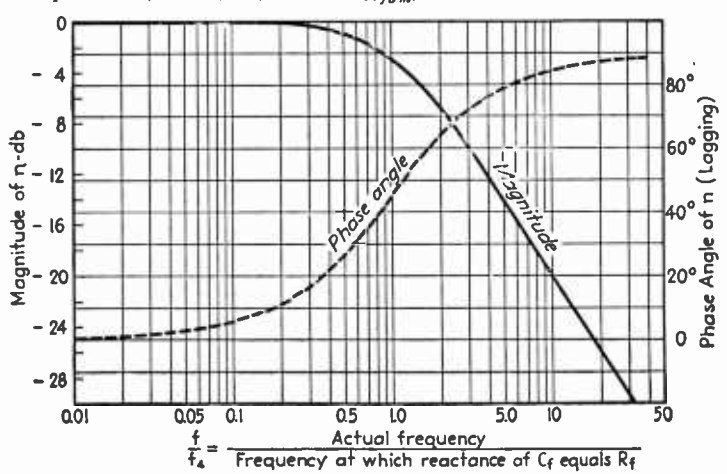


FIG. 7.—Magnitude and phase of factor η for use in Fig. 8.

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$644 \times 10^{-6} \times 250{,}000 = 161$. The -3 db point on the high-frequency response occurs at a frequency $f_2 = 1/(2\pi \times 20 \times 10^{-12} \times 250{,}000) = 32{,}000$ cycles; the corresponding point f_1 at low frequencies, assuming perfect by-passing, is $f_1 = 1/(2\pi \times 0.0025 \times 10^{-6} \times 10^{6}) = 64$ cycles. The response and phase-shift characteristic for the case of perfect by-passing is then readily obtained with the aid of Fig. 4, and is indicated by curve *a* of Fig. 9.

Considering next the modifying effect of the impedance in the screen-grid circuit, one has $K = 125{,}000/2{,}400{,}000 = 0.052$, $f_3 = 1.052/(2\pi \times 0.04 \times 10^{-6} \times 125{,}000) = 34$ cycles. The loss

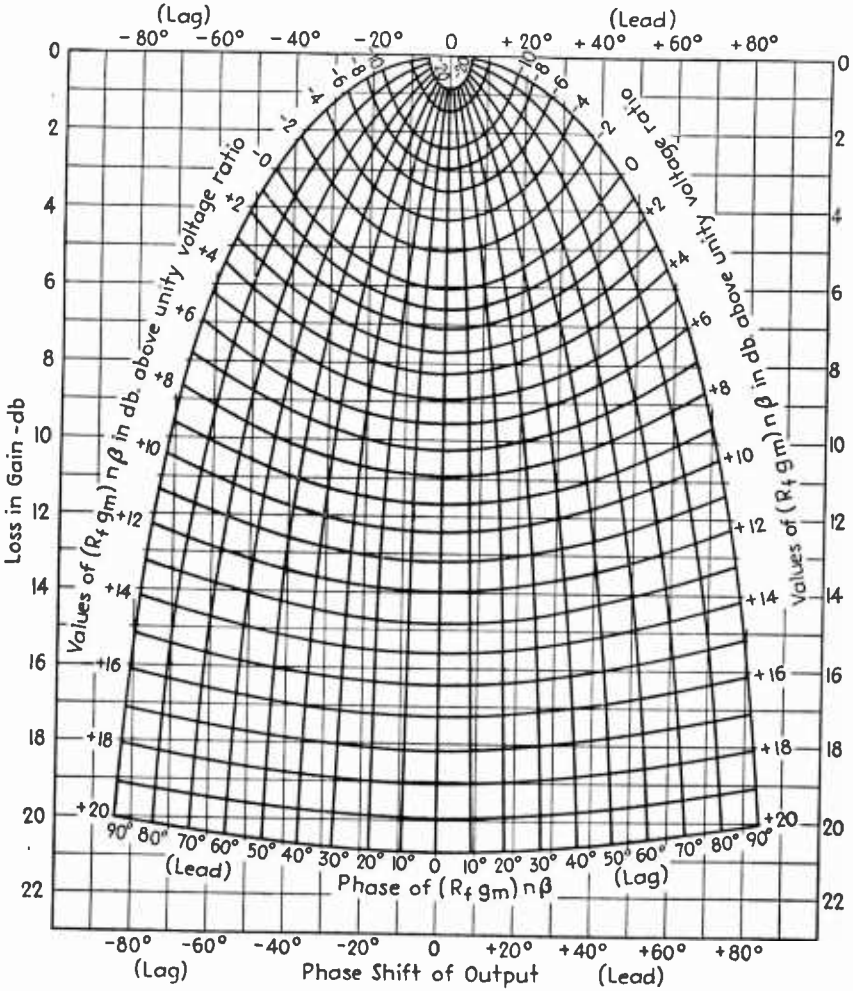


FIG. 8.—Curves from which the loss in gain and phase shift of output at low frequencies resulting from resistance-condenser bias impedance R_fC_f can be calculated in a resistance-coupled amplifier for the general case where both screen and bias impedances are of importance.

of gain and phase shift that is caused by the impedance in the screen circuit is now readily obtained from Fig. 5 and gives the results shown by curve *b* of Fig. 9. Finally considering the bias impedance, $f_4 = 1/(2\pi \times 4.2 \times 10^{-6} \times 1{,}700) = 22$ cycles, and $R_f g_m = 1{,}700 \times 644 \times 10^{-6} = 1{,}095$, or $20 \log_{10} 1.095 = 0.8$ db above unity. The corresponding effects are now obtained by using Figs. 5, 7, and 8, and are shown by curve *c* in Fig. 9.

The total falling off and phase shift in the amplification at low frequencies are then the sum *d* of curves *a*, *b*, and *c* of Fig. 9.

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Nonlinear Distortion.—The output voltage obtainable from a resistance-coupled amplifier is limited by the fact that amplitude distortion is proportionately greater as the applied signal is increased. In particular, the distortion increases very rapidly when the output exceeds a critical value determined by the conditions of operation. Avoidance of excessive amplitude distortion requires that the instantaneous plate current never reach zero during the negative half cycle of the applied voltage and that, similarly, on the positive half cycle the instantaneous potential at the plate of the

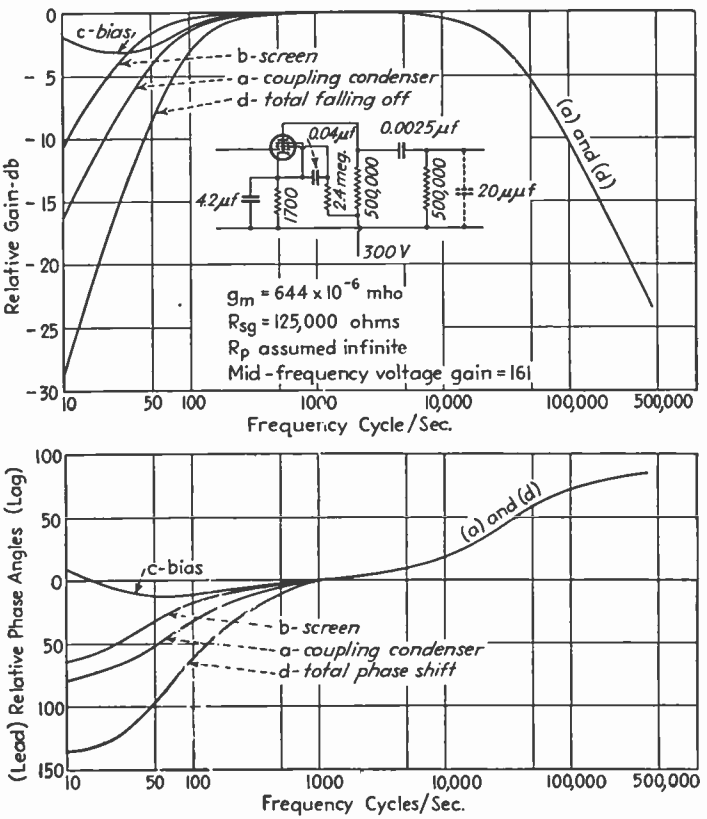


FIG. 9.—Amplification characteristics of a typical resistance-coupled amplifier, showing how the coupling condenser and the bias and screen impedances contribute to the falling off and phase shift of the output voltage at low frequencies.

tube never drops low enough to permit the formation of a virtual cathode in the vicinity of the suppressor grid. It is also necessary that the grid bias be sufficiently negative so that grid current will not be drawn at the positive peak of the applied signal. In terms of operating conditions, these requirements mean that (1) the peak alternating current I_{ac} that is available to develop voltage across the coupling resistance R_c and grid resistance R_{gl} is slightly less than the direct plate current I_{dc}; (2) the minimum potential $E_b - I_{dc}R_c - I_{ac}R_L$ reached by the plate during the cycle must exceed the plate potential at which a virtual cathode forms with a space current of $I_{dc} + I_{ac}$, where E_b is the plate-supply voltage and R_L is the load resistance formed by R_c and R_{gl} in parallel; (3) the bias voltage must exceed the peak signal amplitude by at least

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1 volt. If any of these restrictions are violated, flattening of one peak or both peaks of the output wave will occur, with consequent excessive distortion.

Methods of analyzing the distortion obtained in a particular case are given in Par. 24.

Design Procedure for Obtaining Maximum Gain in Ordinary Resistance-coupled Amplifiers.[1]—In the design of an ordinary resistance-coupled amplifier, it is necessary to compromise between the desirability, on the one hand, of high gain and, on the other hand, of being able to obtain a large output voltage without excessive distortion with a reasonably high gain. It is accordingly necessary to distinguish between designs in which the objective is maximum amplification over the required frequency range without consideration being given to the amount of undistorted output voltage available and designs in which the primary object is to obtain a large output voltage without excessive distortion.

The relations governing the amplification of a resistance-coupled amplifier using a pentode tube are incorporated in the equation

$$\text{Mid-range amplification} = nK^{\frac{1}{n}}E_d^{\frac{n-1}{n}}R_{gl}^{\frac{1}{n}}\frac{\left(\frac{R_c}{R_{gl}}\right)^{\frac{1}{n}}}{1+\frac{R_c}{R_{gl}}} \tag{12}$$

where n and K are constants in the equation $i_p = K\left(E_g + \frac{E_{sg}}{\mu_{sg}}\right)^n$, E_d is the voltage drop of d-c plate current in coupling resistance R_c, and R_{gl} and R_c are the grid-leak and coupling resistances, respectively. Equation (12) assumes that the plate resistance can be taken as infinity and that the plate voltage is great enough to prevent the formation of a virtual cathode near the suppressor. These conditions are satisfactorily realized with normal operating conditions.

From Eq. (12) it is apparent that with a given plate-supply voltage and coupling resistance, the amplification will be maximum when the largest possible fraction of the supply voltage is used up as drop in the coupling resistance. Also it can be deduced from Eq. (12) that with a given E_d and grid-leak resistance R_{gl}, the voltage gain is maximum when

$$R_p = \frac{R_{gl}}{n-1} \tag{12a}$$

where n is the exponent of the space current equation $i_p = K\left(E_g + \frac{E_{sg}}{\mu_{sg}}\right)^n$. Under practical conditions this exponent ranges from about 1.5 for operation with rated plate current to something over 2 when the plate current is quite small. The coupling resistance for maximum gain is accordingly approximately equal to the grid-leak resistance, with the gain not being very critical with respect to the ratio R_c/R_{gl}.

Design for maximum amplification in an ordinary resistance-coupled amplifier is carried out as follows: The grid-leak resistance is first chosen as the highest resistance that can be safely placed in the grid circuit of the following tube (see Sec. 4, Par. 13). The coupling resistance R_c (see Fig. 2) is then made equal to the grid-leak resistance, and the high frequency f_2 giving 70.7 per cent response in Eq. (6) calculated on the basis of an estimated shunting capacity. If this frequency is lower than desired, the coupling resistance R_c should be reduced to the value that will give the desired characteristic, and the resulting loss in gain accepted as the price required to obtain the desired high-frequency characteristic. The coupling condenser C_c is now chosen to give the desired low-frequency response [*i.e.*, a suitable value of f_1 in Eq. (4)]. The

[1] See Terman, Hewlett, Palmer, and Pan, *loc. cit.*

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low-frequency response should never be made any better than necessary, since otherwise trouble from motorboating may occur. A convenient grid bias, normally a few volts,[1] is now chosen, and a screen potential is then selected such that with the preceding coupling resistance, the potential at the plate of the tube will be approximately 20 per cent of the plate-supply voltage. These electrode voltages determine the screen and space current, which in turn fix the bias resistor R_f and screen dropping resistor R_s required. The screen by-pass condenser C_{sg} is next selected so that f_3 in Eq. (8) coincides with f_1, provided a sharp low-frequency cutoff is desired. If, on the other hand, the low-frequency cutoff should not be sharp, or if the phase shift at low frequencies is to be minimized, then C_{sg} should be large so that $f_3 << f_1$. Finally, the cathode by-pass condenser C_f is selected so that f_4 in Eq. (9) is not greater than f_3, provided sharp cutoff is desired, or so that $f_4 << f_1$ if minimum phase shift at low frequencies is important.

The foregoing procedure gives the maximum possible gain combined with the required high- and low-frequency response characteristics, either with or without sharp low-frequency cutoff as required. The output voltage is, however, limited to relatively small values because of the low d-c potential at the plate of the tube.

Design Procedure for Obtaining Maximum Output Voltage in Ordinary Resistance-coupled Amplifiers.—When it is important to obtain a large output voltage without excessive distortion, the design for maximum gain should be modified so that the coupling resistance does not exceed the grid-leak resistance, and the bias and screen potentials should be readjusted to reduce the plate current so that the direct-current voltage drop E_d in the coupling resistance R_c is approximately

$$\left.\begin{matrix}\text{Voltage drop in} \\ \text{coupling resistance}\end{matrix}\right\} = E_d = (1 - r)\left(\frac{1 + \dfrac{R_c}{R_{gl}}}{2 + \dfrac{R_c}{R_{gl}}}\right) E_B \tag{13}$$

$$\text{where } r = \frac{\left\{\begin{matrix}\text{lowest plate potential at which plate current} \\ \text{is independent of plate voltage when plate} \\ \text{current is twice the d-c value}\end{matrix}\right\}}{E_B}$$

The corresponding peak output voltage obtainable without excessive distortion is approximately

$$\left.\begin{matrix}\text{Peak a-c output} \\ \text{voltage}\end{matrix}\right\} = \frac{(1 - r)}{2 + \dfrac{R_c}{R_{gl}}} E_B \tag{14}$$

The output voltage obtainable is seen to be proportional to E_B and to decrease with R_c/R_{gl}.

The practical procedure for obtaining maximum output voltage is to start with the highest permissible value of R_{gl} and select a coupling resistance R_c that makes R_c/R_{gl} a reasonable compromise between maximum gain and maximum power output. Usual values are in the range 0.2 to 0.5, corresponding to 91 and 80 per cent, respectively, of the maximum possible output voltage. A value of r of the order of 0.1 to 0.2 is then assumed and the plate current adjusted so that the voltage drop in this coupling resistance will satisfy Eq. (13). The curves of Fig. 10 give the results of Eqs. (13) and (14) graphically. Under practical conditions it is apparent that the peak output voltage obtainable is roughly $0.35E_B$ and that the proper voltage drop in the coupling resistance will approximate $0.5E_B$.

[1] In battery-operated tubes, the tube is often operated with zero bias and a very high grid-leak resistance (5 to 10 megohms).

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A comparison of the amplification obtained when the amplifier is designed for maximum gain as contrasted with the design for large voltage output shows that in the former case the voltage amplification is about one-third greater.

Effect of Plate-supply Voltage and Grid-leak Resistance on Amplification.—Examination of Eq. (12) shows that for a fixed value of R_c/R_{gl}, the gain is proportional to $E_d^{(n-1)/n}$ (and hence to $E_B^{(n-1)/n}$), and also to $R_{gl}^{1/n}$. In practice, this means that the amplification obtainable from a resistance-coupled amplifier, irrespective of whether designed for maximum gain or large output voltage, can be expected to be

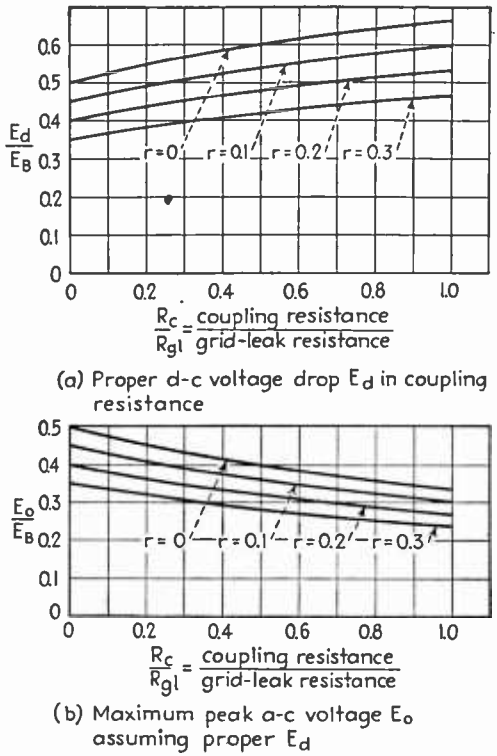


FIG. 10.—Curves giving the maximum output voltage obtainable from resistance-coupled amplifiers, and also the value of E_d required for maximum output.

proportional to something between the square root and cube root of the plate-supply voltage and between the square root and ⅔ power of the grid-leak resistance.

Miscellaneous Design Considerations.—Examples of designs recommended by tube manufacturers as suitable for audio-frequency work are given in Table 1.[1] These are proportioned with the main emphasis on large output voltage and sharp low-frequency cutoff.

The amplification and the output voltage obtainable both increase with plate-supply potential up to the point where with normal rated d-c plate current and the proper coupling resistance the d-c voltage at the plate of the tube equals the rated plate voltage.

The falling off in gain at high frequencies is determined by the equivalent resistance formed by the coupling and grid-leak resistances in parallel in relation to the shunting capacities. Since these shunting capacities are fixed, they fix the maximum

[1] These designs are from the RCA Radiotron tube manual.

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coupling resistance that can be used to give a desired frequency range. When extremely wide frequency bands are required, the coupling resistance must be lower than the optimum value for either maximum output voltage or maximum amplification. The loss resulting from somewhat less than optimum coupling resistance is not particularly great, however, since the plate current can be increased as the resistance is reduced, thereby obtaining partial compensation for the lowered impedance.

The character of the response at low frequencies can be controlled over wide limits by the amplifier circuit proportions. A very sharp cutoff at low frequencies accompanied by a large phase shift results when the critical frequencies f_1 and f_3 coincide, and $f_4 \leq f_1$ with $R_f g_m$ approximating unity. On the other hand, a gradual low-frequency cutoff with less pronounced phase shifts is obtained when f_3 and f_4 are less than f_1 (*i.e.*, when the screen and bias by-pass condensers are larger). Examples of some of the characteristics that can be obtained are shown in Fig. 11.

In ordinary amplifiers it is usually desirable to have the low-frequency response no better than absolutely necessary, with the sharpest possible cutoff. In this way motorboating and regeneration troubles are minimized. The main exception to this is in feedback amplifiers, where the sharp cutoff is to be avoided because it introduces additional phase shifts.

Amplifiers for special purposes are sometimes required to amplify down to only a few cycles. Such requirements can be most easily met by using some other biasing system than self-bias and by deriving the screen voltage from a voltage divider of low impedance. The low-frequency response is then determined by the coupling condenser C_c in accordance with Eq. (4), and is better in proportion to C_c. The largest capacity that can be used and hence the best low-frequency response obtainable is limited by the leakage resistance of the condenser. If the condenser is very large, or the dielectric poor, the leakage current will be sufficient to produce an appreciable voltage across the grid-leak resistance and bias the grid of the next stage of amplification positively. Further discussion on the amplification of very low frequencies is given below in Par. 6.

The coupling resistors used in amplifier stages operating at low signal levels must be of a type that generates relatively little "noise" voltage when carrying the d-c plate current. Otherwise the signal-to-noise ratio of the amplifier output will be poor. Wire-wound resistors and certain special composition and film types are suitable, while ordinary carbon resistors have rather poor noise characteristics (Par. 26).

The tubes used in resistance-coupled amplifiers are normally sharp cutoff general-purpose pentodes, having a rated d-c plate current of several milliamperes.

Resistance-coupled Amplifiers Using Triode Tubes.—Triode tubes are occasionally used in resistance-coupled amplifiers. Such tubes have the disadvantage of less amplification than pentodes and also offer higher input capacity to the preceding stage (see Par. 25 below). The chief advantage of using triode tubes is that the amplitude distortion is less when large voltages are being developed.

In designing triode resistance-coupled amplifiers, the coupling resistance is normally chosen so that the d-c drop in it is about one-third to one-half of the plate-supply voltage. The coupling condenser and grid-leak resistance are determined exactly as in the case of pentode tubes. A tube having a high amplification factor must be used if appreciable gain is desired, since the gain can never exceed the amplification factor.

Typical designs for resistance-coupled amplifiers employing triode tubes are shown in Table 1.

5. Transformer-coupled Audio-frequency Amplifiers.—In the transformer-coupled amplifier the load impedance in the plate circuit of the tube is supplied by a step-up transformer that delivers its secondary voltage to the grid of another amplifier tube,

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TABLE 1[1]

Pentode types: 6C6, 6J7, 57

E_b	90									180										300									E_b
R_c	0.1			0.25			0.5			0.1			0.25			0.5				0.1			0.25			0.5			R_c
R_{gl}	0.1	0.25	0.5	0.25	0.5	1	0.5	1	2	0.1	0.25	0.5	0.25	0.5	1	0.5	1	2		0.1	0.25	0.5	0.25	0.5	1	0.5	1	2	R_{gl}
R_s	0.37	0.44	0.44	1.1	1.18	1.4	2.18	2.6	2.7	0.44	0.5	0.5	1.1	1.18	1.4	2.45	2.9	2.7		0.44	0.5	0.53	1.18	1.18	1.45	2.45	2.9	2.95	R_s
R_f	1,200	1,100	1,300	2,400	2,600	3,600	4,700	5,500	5,500	1,000	750	800	1,200	1,600	2,000	2,600	3,100	3,500		500	450	600	1,100	1,200	1,300	1,700	2,200	2,300	R_f
C_{sg}	0.05	0.05	0.05	0.03	0.03	0.02	0.02	0.05	0.02	0.05	0.05	0.05	0.04	0.04	0.04	0.03	0.025	0.02		0.07	0.07	0.06	0.04	0.04	0.05	0.04	0.04	0.04	C_{sg}
C_f	5.2	5.3	4.8	3.7	3.2	2.5	2.3	2	2	6.5	6.7	6.7	5.2	4.3	3.8	3.2	2.5	2.8		8.5	8.3	8	5.5	5.4	5.8	4.2	4.1	4	C_f
C_a	0.02	0.01	0.006	0.008	0.005	0.003	0.005	0.0025	0.0015	0.02	0.01	0.006	0.008	0.005	0.0035	0.005	0.0025	0.0015		0.02	0.01	0.006	0.008	0.005	0.005	0.005	0.003	0.0025	C_a
E_o*	17	22	33	23	32	33	28	29	27	42	52	59	41	60	60	45	56	60		55	81	96	81	104	110	75	97	100	E_o*
G†	41	55	66	70	85	92	93	120	140	51	69	83	93	118	140	135	165	165		61	82	94	104	140	185	161	250	240	G†

Duplex-diode triode types: 2A6, 75

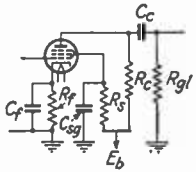
E_b	90									180									300									E_b
R_c	0.1			0.25			0.5			0.1			0.25			0.5				0.1			0.25			0.5			R_c
R_{gl}	0.1	0.25	0.5	0.25	0.5	1	0.5	1	2	0.1	0.25	0.5	0.25	0.5	1	0.5	1	2		0.1	0.25	0.5	0.25	0.5	1	0.5	1	2	R_{gl}
R_f	6,300	6,600	6,700	10,000	11,000	11,500	16,200	16,600	17,400	2,600	2,900	3,000	4,300	4,800	5,300	7,000	8,000	8,800		1,900	2,200	2,300	3,300	3,900	4,200	5,300	6,100	7,000	R_f
C_f	2.2	1.7	1.7	1.24	1.07	0.9	0.75	0.7	0.65	3.3	2.9	2.7	2.1	1.8	1.5	1.3	1.1	0.9		4	3.5	3	2.7	2	1.8	1.6	1.3	1.2	C_f
C_a	0.02	0.01	0.006	0.01	0.006	0.003	0.005	0.003	0.0015	0.025	0.015	0.007	0.015	0.007	0.004	0.007	0.004	0.002		0.03	0.015	0.007	0.015	0.007	0.004	0.007	0.004	0.002	C_c
E_o*	3	5	6	5	7	10	7	10	13	16	22	23	21	28	33	25	33	38		31	41	45	42	51	60	47	62	67	E_o*
G†	23	29	31	34	40	40	39	44	48	29	36	37	43	50	53	52	57	58		31	39	42	48	53	56	58	60	63	G†

Triode types: 56, 76

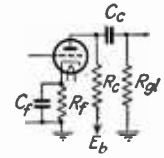
E_b	90									180									300									E_b
R_c	0.05			0.1			0.25			0.05			0.1			0.25				0.05			0.1			0.25			R_c
R_{gl}	0.05	0.1	0.25	0.1	0.25	0.5	0.25	0.5	1	0.05	0.1	0.25	0.1	0 25	0.5	0.25	0.5	1		0.05	0.1	0.25	0.1	0.25	0.5	0.25	0.5	1	R_{gl}
R_f	2,500	3,200	3,800	4,500	6,500	7,500	11,100	15,100	18,300	2,400	3,000	3,700	4,500	6,500	7,600	10,700	14,700	17,700		2,400	3,100	3,800	4,500	6,400	7,500	11,100	15,200	18,300	R_f
C_f	2	1.6	1.25	1.05	0.82	0.68	0.48	0.36	0.32	2.5	1.9	1.65	1 45	0.97	0.8	0.6	0.45	0.4		2.8	2.2	1.8	1.6	1.2	0.98	0.69	0.5	0.4	C_f
C_a	0.06	0.03	0.015	0.03	0.015	0.007	0.015	0.007	0.0035	0.06	0.035	0.015	0.035	0.015	0.008	0.015	0.007	0.0045		0.08	0.045	0.02	0.04	0.02	0.009	0.02	0.009	0.005	C_a
E_o*	16	21	23	19	23	25	21	24	28	36	48	55	45	55	57	49	59	64		65	80	95	74	95	104	82	96	108	E_o*
G†	7	7.7	8.1	8.1	8.9	9.3	9.4	9.7	9.8	7.7	8.2	9	9.3	9.5	9.8	9.7	10	10		8.3	8.9	9.4	9.5	10	10	10	10	10	G†

[1] Capacities in microfarads, resistances in megohms.
* E_o = maximum output voltage.
† G = voltage gain.

Pentode diagram



Triode diagram



as shown in Fig. 12*a*. The amplifier tube exciting the transformer is ordinarily a triode having a plate resistance of the order of 10,000 ohms and a d-c plate current of a few milliamperes.

Equivalent Circuit and Frequency Response Characteristics of Transformer-coupled Amplifier.—The exact equivalent plate circuit of a transformer-coupled amplifier is shown in Fig. 12*b* and is quite complicated. It can, however, be simplified, as shown in Fig. 12*c*, without introducing appreciable error, by neglecting primary capacity and hysteresis loss, replacing the interwinding capacity by suitable capacity across

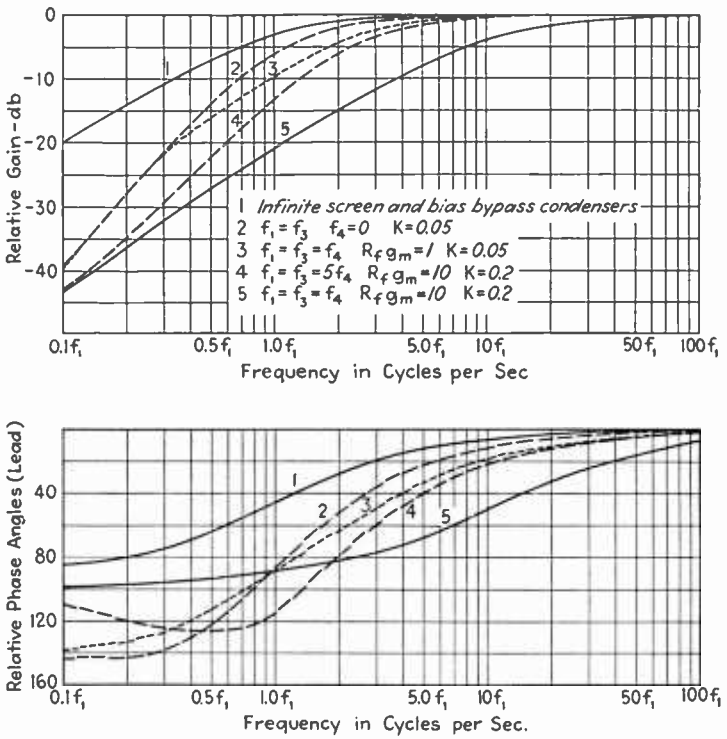


FIG. 11.—Examples of amplification characteristics that can be obtained in resistance-coupled amplifiers at low frequencies by various circuit proportions.

the secondary, lumping the primary resistance with the plate resistance of the tube, neglecting the eddy-current resistance,[1] and reducing to unity turn ratio. Still further simplifications are possible by considering only a limited range of frequencies at a time, as shown at (*d*), (*e*), and (*f*).

[1] The effect of eddy-current resistance R_e can be taken into account by using for R_p and μ values slightly lower than those actually given by the tube, according to the relations

$$\left.\begin{matrix}\text{Effective } \mu \text{ when } R_e \text{ is} \\ \text{taken into account}\end{matrix}\right\} = \frac{\text{actual } \mu \text{ of tube}}{1 + \dfrac{R_p}{R_e}} \tag{15}$$

$$\left.\begin{matrix}\text{Effective } R_p \text{ when } R_e \text{ is} \\ \text{taken into account}\end{matrix}\right\} = \frac{\text{actual value of } R_p}{1 + \dfrac{R_p}{R_e}} \tag{16}$$

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The output voltage of a transformer-coupled amplifier can be calculated by the equivalent circuits of Fig. 12, with the use of the following equations:

$$\left.\begin{matrix}\text{Amplification in middle}\\ \text{range of frequencies}\end{matrix}\right\} = \mu n \tag{17}$$

$$\frac{\text{Amplification at low frequencies}}{\text{Amplification in middle range}} = \frac{1}{1 - j\frac{f_1}{f}} \tag{18}$$

$$\frac{\text{Amplification at high frequencies}}{\text{Amplification in middle range}} = \frac{1}{(1 - \gamma^2) + j\frac{\gamma}{Q_0}} \tag{19}$$

where $f_1 = R'_p/2\pi L_p$ = frequency at which resistance of primary incremental inductance L_p equals effective plate resistance R'_p.

$$\gamma = \frac{\text{actual frequency}}{\text{series resonant frequency of } C_s \text{ and } L_s}.$$

$Q_0 = \dfrac{w_0 L}{R'_p + R_p}$ = circuit Q at frequency for which C_s and L_s are in series resonance.

The variation in relative amplification at high and low frequencies, as given by Eqs. (18) and (19), is shown graphically in Fig. 13. At low frequencies there is a falling off in amplification because of insufficient primary inductance, *with the gain being* 70.7 *per cent* (−3 *db*) *of the mid-frequency gain at the frequency* f_1, *for which the reactance of the primary incremental inductance* $2\pi f_1 L_p$ *equals the effective plate resistance* R'_p. At high frequencies the behavior is controlled by the action of the equivalent secondary capacity C_s resonating with the leakage inductance of the transformer. The frequency at which the equivalent shunt capacity (*i.e.*, actual distributed secondary capacity plus capacity of tube connected to secondary) is in series resonance with the leakage inductance of the transformer fixes the high-frequency limit of the transformer, as the gain falls well below the mid-frequency value at frequencies from 25 to 50 per cent greater than this resonant frequency. The Q of the resonant circuit formed by the shunting capacity, leakage inductance, and plate resistance (designated by Q_0 in Fig. 13) determines the character of the high-frequency response. The amplification is most nearly uniform for $Q_0 = 0.8$, with higher values resulting in a high-frequency peak, and lower values in a drooping curve.

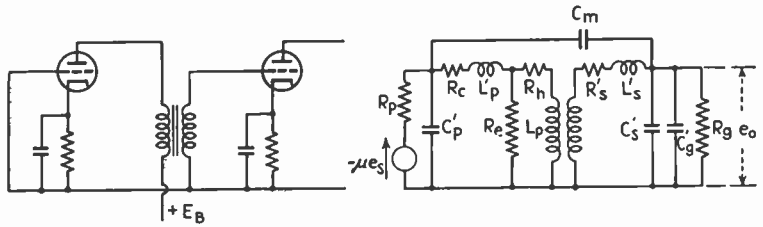
Transformer Characteristics.—The characteristics desired in an audio-frequency transformer are high primary incremental inductance to give good low-frequency response, high step-up ratio to give large amplification, and low leakage inductance and low distributed capacity to extend the response to high frequencies.

The common audio transformer employs a silicon steel core with a small air gap such as to give maximum incremental primary inductance with the normal d-c plate current of the tube passing through the primary. The primary and secondary are layer-wound, with the secondary on the outside. The turn ratio is commonly two to four, and the 70.7 per cent (−3 db) point for low frequencies is of the order of 40 to 100 cycles. The high-frequency response will depend upon whether or not the primary and secondary are interleaved and upon the way the interleaving is done. Relatively uniform response can be readily obtained up to 10,000 to 20,000 cycles. In general, the greater the high-frequency range the lower will be the step-up ratio and the less the gain.

The use of core material of high permeability, such as permalloy, increases the frequency range obtainable in a given physical size, or permits the use of a physically smaller transformer for a given range. Alloy cores are desirable where weight is a factor or where the frequency range is great.

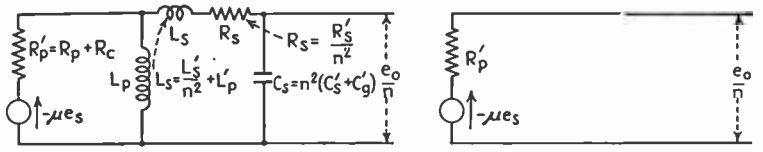
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The characteristics of audio-frequency transformers can be calculated with reasonable accuracy by the method described in Sec. 2, Par. 25. The characteristics can also be measured experimentally as described in Par. 34, Sec. 13.



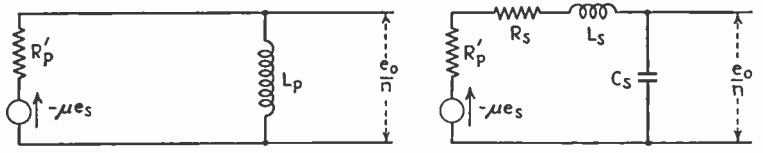
(a) Circuit of transformer-coupled amplifier

(b) Exact equivalent circuit of transformer-coupled amplifier



(c) Practical equivalent circuit of transformer-coupled amplifier reduced to unity turn ratio

(d) Simplified equivalent circuit accurate for middle range of frequencies



(e) Simplified equivalent circuit accurate at low frequencies

(f) Simplified equivalent circuit accurate at high frequencies

R_p = Plate resistance of tube
R_c = D-c resistance of primary
L'_p = Primary leakage inductance
R_e = Resistance representing eddy current loss
R_h = Resistance representing hysteresis loss
L_p = Incremental primary inductance
R'_s = Resistance of secondary winding
L'_s = Secondary leakage inductance
$L_s = L'_p + L'_s/n^2$ = total leakage inductance referred to primary side
C'_s = Secondary distributed capacity
C'_p = Capacity in shunt with primary (tube plus transformer capacity)
C_m = Capacity between primary and secondary windings of the transformer
C'_g = Input capacity of tube to which voltage e_g is delivered
R_g = Input resistance of tube to which voltage e_g is delivered
n = Ratio of secondary to primary turns
μ = Amplification factor of tube

FIG. 12.—Circuit of a transformer-coupled amplifier, together with exact and approximate equivalent plate circuits.

Miscellaneous Considerations.—For any given transformer there is a particular value of plate resistance that gives the most uniform amplification characteristic.

Ex. 1044-0378

If the plate resistance is higher, the response falls off excessively at both high and low frequencies, while if the resistance is too low, the response is excellent at low frequencies but peaked at high frequencies. The proper plate resistance can be obtained by adjusting the bias of the amplifier tube.

Pentode tubes can be used with transformer-coupled amplifiers provided a resistance is shunted across the primary of the transformer. By making this shunting resistance equal to the plate resistance of the triode tube with which the transformer was designed to operate, the frequency response characteristic is exactly the same as with the corresponding triode, and the amplifications in the two cases will be the same if the transconductance of the pentode is the same as that of the triode.

Transformer-coupled amplifiers are sometimes operated with a resistance shunted across the transformer secondary. This improves the relative response at low frequencies, reduces the relative response at high frequencies, and lowers the absolute magnitude of the amplification. The chief use of such a resistance is to eliminate the high-frequency peak that results when a transformer is associated with a plate resistance that is too low.[1]

Uses of Transformer-coupled Amplifiers.—Transformer coupling is more expensive, gives less gain, and has poorer frequency response than obtainable with resistance coupling. Its use is accordingly limited to special circumstances where resistance coupling is not entirely suitable. The most important applications of transformer coupling are in the excitation of a push-pull amplifier from a single-ended amplifier stage, and in cases where the d-c resistance in the grid circuit of the tube being excited must be low because of grid current, or because of danger of blocking (Par. 13, Sec. 4).

6. Miscellaneous Coupling Systems for Audio-frequency Voltage Amplifiers.—Although most audio-frequency voltage amplifiers use either resistance or transformer coupling, other coupling methods can be employed, and some have advantages. The most important of these miscellaneous systems are described below.

Impedance Coupling.—In this type of coupling the coupling resistance of ordinary resistance coupling is replaced by an inductance, as illustrated in Fig. 14*a*. This arrangement gives the same shape of curve of response as does resistance coupling in the usual case when the coupling condenser C_c is large enough that its reactance is not an important factor in the useful frequency range. The response at low frequencies then falls to 70.7 per cent of the mid-frequency value at the frequency for which the reactance of the coupling inductance L_c equals the resistance formed by grid leak and plate resistance in parallel. The corresponding high-frequency point is at the frequency for which the reactance of the shunting capacities equals the same equivalent resistance. The mid-frequency gain in the case of triodes approximates the amplification factor of the tube, while with pentode tubes, it is equal to the transconductance times the equivalent resistance of grid leak and plate resistance in parallel.

Transformer Coupling with Shunt (Parallel) Feed.—This arrangement, shown in Fig. 14*b*, employs an inductance-capacity combination L_cC_c that prevents direct current from flowing through the primary of the coupling transformer.[2] The response obtained from such an arrangement at high frequencies is exactly the same as with ordinary transformer coupling using the same transformer and tube. At low frequencies, however, the response may be either peaked, flat, or drooping, according to the circuit proportions, as shown in the figure. A substantially flat response can be obtained down to the frequency f_0, at which $2\pi f_0 L_p$ equals the plate resistance of the tube. This is done by selecting a value of condenser C_c that will resonate with the

[1] Methods of analyzing and designing transformer-coupled amplifiers with resistance shunted across the secondary are discussed by Paul W. Klipsch, Design of Audio Frequency Amplifier Circuits Using Transformers, *Proc. I.R.E.*, Vol. 24, p. 219, February, 1936.

[2] The inductance L_c carrying the direct current in such a combination is sometimes referred to as a *retard* coil.

Ex. 1044-0379

primary inductance L_p of the transformer at this frequency and at the same time by using a coupling inductance L_c that is at least 50 per cent greater than the primary

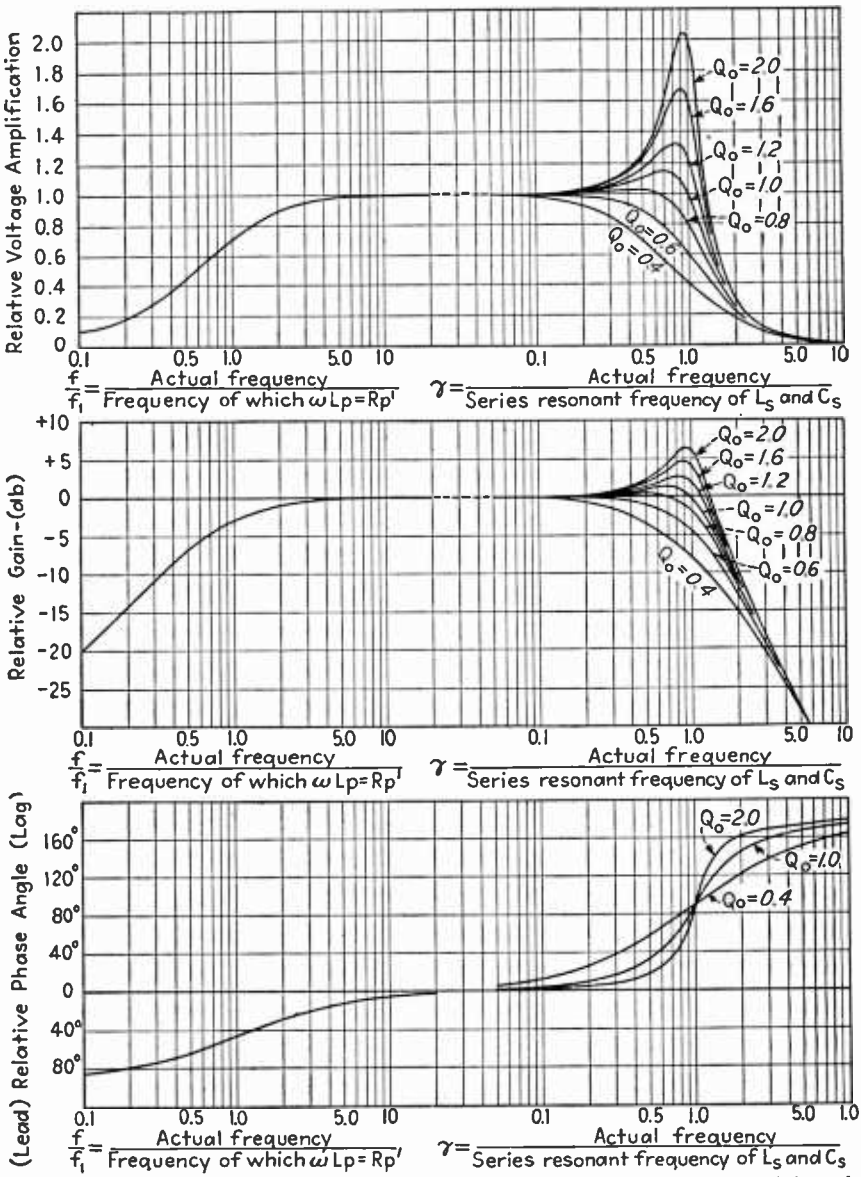


FIG. 13.—Universal amplification curve of transformer-coupled amplifier, giving the relative amplification and phase-shift characteristics.

inductance of the transformer. A smaller coupling capacity C_c gives a peaked response at low frequencies, while a very large capacity gives a falling off at low frequencies corresponding to that obtained with simple transformer coupling when the

Ex. 1044-0380

primary inductance of the transformer is the equivalent inductance formed by L_p and L_c in parallel.

Transformer coupling with shunt feed is capable of giving a better frequency response than simple transformer coupling, provided that the transformer is designed especially for this type of operation. By eliminating the direct current from the transformer primary, it is possible to assemble the transformer core with the minimum possible air gap and also, if desired, to use alloy core material having extremely high permeability. The result is sufficient increase in primary inductance to more than make up for the shunting inductance of the retard coil, with the added advantage of being able to control the character of the low-frequency response through the size of the condenser C_c. At the same time, the presence of the retard coil L_c and the increased primary inductance do not affect either the leakage reactance or the distributed capacity of the transformer. The improved low-frequency response is hence obtained without affecting the behavior at high frequencies.

Resistance Coupling with Grid Choke.—In this arrangement the grid-leak resistance is replaced by a high inductance coil illustrated in Fig. 14*c*. Such an arrangement is occasionally used where it is necessary that the direct-current resistance be low in the grid circuit of the tube to which the amplified voltage is applied. The response of such an arrangement at high frequencies is similar to that obtained with resistance coupling, with the response falling to 70.7 per cent of the mid-frequency value at the frequency for which the reactance of the shunting capacity equals the resistance formed by coupling resistance and plate resistance in parallel. The response at low frequencies depends upon the resonant frequencies of C_c and L_g and the value of $\omega L_g/R$ at this frequency, where R is the resistance formed by the plate and coupling resistances in parallel. Substantially flat response can be obtained down to a frequency f_0 by making $R = 2\pi f_0 L_g$ and selecting a capacity C_c that resonates with L_g at f_0. A smaller capacity or lower resistance gives a peak at low frequencies, while a larger capacity or higher resistance results in a drooping low-frequency response.

Input Transformers.—A transformer used to couple a transmission line, microphone, or other low impedance source of energy to the grid of a tube is termed an *input transformer*. Such an arrangement corresponds to ordinary transformer coupling, with the source impedance associated with the primary of the transformer being equivalent to the plate resistance of the tube in a transformer-coupled stage of amplification. The only difference is that with the input transformer, the source impedance is usually much lower than the plate resistance of a tube so that the primary inductance can be proportionately reduced. This permits fewer primary turns and a higher step-up ratio, which will, in general, be inversely proportional to the square root of the source impedance.

Special Coupling Systems for Modifying Low-frequency and High-frequency Response. The addition of circuit elements to the coupling network increases the variety of characteristics obtainable. A few of the many possibilities of this sort are described below.

A sharp high-frequency cutoff in resistance-coupled amplifiers, comparable with that obtainable at low frequencies, can be realized by adding inductance in series with the screen-grid and bias by-pass condensers, as shown in Fig. 14*d*. These inductances act to increase the impedance in the screen and bias circuits at high frequencies in exactly the same way as the by-pass condensers increase the impedance in these circuits at low frequencies. In fact, Eq. (8) and Fig. 5, giving the low-frequency falling off caused by the reactance of the screen condenser C_s, also give the high-frequency falling off from a screen inductance L_s if one substitutes f_{sh}/f for f/f_s, where f_{sh} is the frequency at which the reactance of the screen inductance L_s equals the resistance formed by R_s and R_{sg} in parallel. Similarly, Eq. (10) and Fig. 8, giving the

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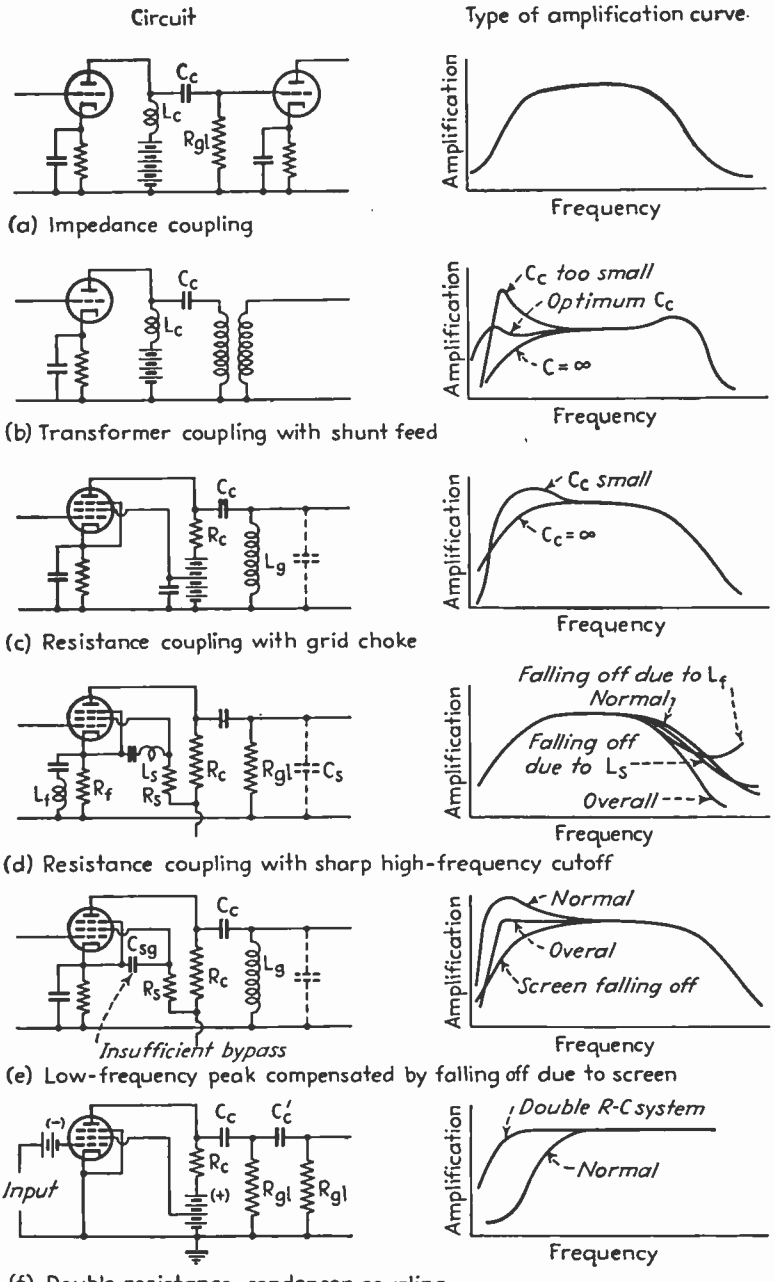


FIG. 14.—Miscellaneous coupling methods for audio-frequency amplifiers together with typical amplification curves that are obtained.

Ex. 1044-0382

loss in amplification and the phase shift resulting from the reactance of the bias by-pass condenser C_f, also gives the effect of the inductance L_f at high frequencies, provided that f_{4h}/f is substituted for f/f_4, where f_{4h} is the frequency at which the reactance of the inductance L_f equals the bias resistance R_f.

The high-frequency response can be extended to very high frequencies while substantially constant gain and negligible phase distortion are maintained by use of the coupling systems used in video amplifiers (see Par. 16).

The low-frequency response can be modified in a number of ways. Thus in coupling systems having a low-frequency peak, as, for example, with systems using a grid choke (Fig. 14*c*), it is possible to select a screen (or bias) by-pass condenser that will give a compensating falling off in gain at low frequencies. In this way a substantially uniform gain can be obtained down to a certain point, followed by a relatively sharp cutoff at lower frequencies, as illustrated in Fig. 14*e*.

The limitation to the low-frequency response in ordinary resistance-coupled systems arising from the leakage current that flows through the coupling condenser when very large coupling condensers are used can be largely eliminated by the double resistance-condenser coupling system shown in Fig. 14*f*. This arrangement makes it possible to obtain a substantially uniform response to about one-tenth of the frequency otherwise possible. Where this is not sufficient, it is possible to use direct-coupled systems such as are discussed below.

Coupling systems giving negligible phase shift and substantially constant gain down to quite low frequencies are an essential part of video amplifiers, and are discussed in detail in Par. 7.

Direct-coupled (Direct-current) Amplifiers.—A direct-coupled amplifier is one in which the grid of the tube receiving the output voltage is directly coupled to the plate of the amplifier tube. Such an arrangement is capable of amplifying direct as well as alternating voltages, and if there are no condensers in the screen circuit or in the arrangement for producing bias, the amplification at low frequencies will be uniform and without phase shift down to zero frequency. Examples of direct-coupled amplifiers are shown in Fig. 15. The arrangement at (*a*) is a simple amplifier in which the voltage developed across the coupling resistance R_c is applied directly to the grid of the output tube through a bias battery E_2, which compensates for the positive voltage from plate to ground of the amplifier tube. The circuit at (*b*) differs somewhat from that at (*a*) in that the coupling resistance R_c is the plate resistance of a pentode tube.[1] This provides an impedance that will pass the d-c plate current of the amplifier tube with only a moderate voltage drop yet that offers extremely high resistance to any change in this current. In this way a voltage amplification of several thousand is readily obtainable.

The circuits of both Figs. 15*a* and 15*b* require ungrounded batteries, which result in high shunting capacities to ground and resultant poor high-frequency response. This disadvantage can be avoided by the modification of Fig. 15(*a*) shown at (*e*) or by connecting the various stages of the amplifier across a single voltage divider,[2] as shown in Fig. 15*c*. Separate sources of filament power are required for each tube, but this is no disadvantage if heater tubes are employed. In order to prevent changes in plate current in one stage of such an arrangement from reacting unduly on the other stages, it is important that the current carried by the voltage dividing resistance be considerably greater than the plate currents of the individual tubes.

Direct-coupled amplifiers of all types tend to be troubled by slow drifts in the d-c plate current resulting from changes in battery potentials. Such variations are

[1] J. W. Horton, The Use of a Vacuum Tube as a Plate-feed Impedance, *Jour. Franklin Inst.*, Vol. 216, p. 749, December, 1933.

[2] Edward H. Loftin and S. Young White, Cascaded Direct-coupled Tube Systems Operated from Alternating Current, *Proc. I.R.E.*, Vol. 18, p. 669, April, 1930.

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amplified just as an applied signal, since the tubes have no way of distinguishing between the desired and undesired voltages. The remedy for this is to employ some form of balancing system, a simple example of which is shown in Fig. 15*d*. Here the tubes are arranged in push-pull pairs, with the signal applied in opposite phase to each set of paired tubes, while the direct-current potentials are applied in the same phase to these tubes. As a result, changes in electrode voltages balance each other out as far as the output terminals are concerned, provided that the tubes are absolutely

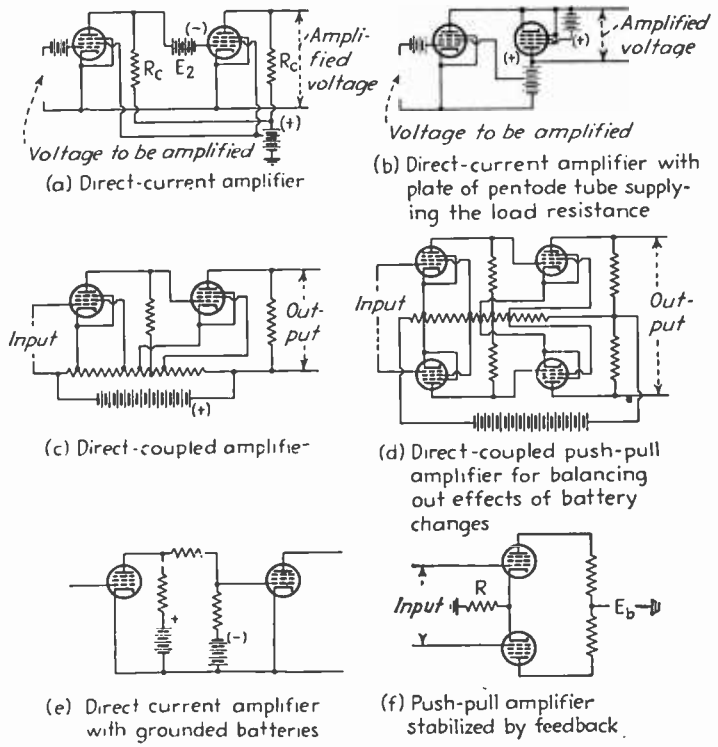


FIG. 15.—Direct-coupled amplifiers for amplifying low-frequency alternating currents without phase shift or falling off in gain, and also for amplifying direct currents.

identical. Very small d-c voltages can be successfully amplified with such balanced arrangements by employing power supplies in which the output voltage is regulated, combined with auxiliary circuits that compensate for slight differences in the characteristics of the two tubes that are paired.[1] The immunity of push-pull arrangements from drift troubles can also be greatly increased by use of negative feedback in the form of a large bias resistor R in the common cathode lead, as shown in Fig. 15*f*.[2]

[1] Discussion of a number of such balancing arrangements is given by D. B. Penick, Direct-current Amplifier Circuits for Use with the Electrometer Tube, *Rev. of Sci. Instruments*, Vol. 6, p. 115, April, 1935; G. P. Harnwell and S. N. Van Voorhis, A Balanced Electrometer Tube and Amplifying Circuit for Small Directory Currents, *Rev. Sci. Instruments*, Vol. 5, p. 244, July, 1934; J. M. Eglin, A Direct Current Amplifier for Measuring Small Currents, *Jour. O.S.A.*, Vol. 18, p. 393, May, 1929.

[2] The full design of a practical d-c amplifier giving a voltage gain of 6×10^6 and employing a push-pull system with negative feedback is given by Harold Goldberg, *Trans. A.I.E.E.*, Vol, 59, p. 60, January. 1940.

Ex. 1044-0384

AUDIO-FREQUENCY POWER AMPLIFIERS

7. Class A Power Amplifiers.—In a power amplifier, the object is to obtain as much power with only moderate distortion as is possible from a given tube, irrespective of the signal voltage required or the voltage amplification involved. In Class A power amplifiers, this output is developed under conditions such that the plate currents of the individual tubes flow continuously throughout the cycle.

Practical circuits of power amplifiers are shown in Fig. 16. In these, the load offers an impedance to the alternating components of plate current, but not to the d-c plate current.

Dynamic Characteristics and Load Lines.[1]—The proper operation of a Class A amplifier requires a careful balance between the load impedance, operating point, and

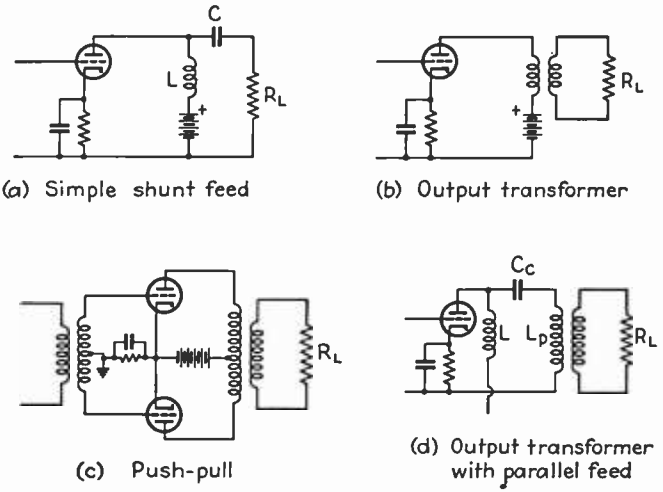


FIG. 16.—Typical Class A power-amplifier circuits.

signal voltage. The essential factors involved with resistance loads are normally determined with the aid of *dynamic characteristics* and *load lines.*

A dynamic characteristic gives the relationship between instantaneous plate current and instantaneous potential at the plate of the tube as the voltage applied to the grid of the tube is varied. A typical dynamic characteristic of a triode tube is shown in Fig. 17. Points on the dynamic characteristic can be calculated from the relation

$$I'_p = \frac{E_b - E'_b}{R_L} + I_b \tag{20}$$

where E_b = potential at plate of tube with no applied signal.
I_b = plate current with no applied signal.
E'_b = instantaneous plate potential at some point P on the dynamic characteristic.
I'_p = instantaneous current corresponding to point P.
R_L = load resistance.

[1] E. W. Kellogg, Design of Non-distorting Power Amplifiers, *Trans. A.I.E.E.*, Vol. 44, p. 302, 1925; J. C. Warner and A. V. Loughren, The Output Characteristics of Amplifier Tubes, *Proc. I.R.E.*, Vol. 14, p. 735, December, 1926.

Ex. 1044-0385

The dynamic characteristic for a given operating point and load resistance gives the relationship between the applied signal and the output voltage, as shown in Fig. 17.

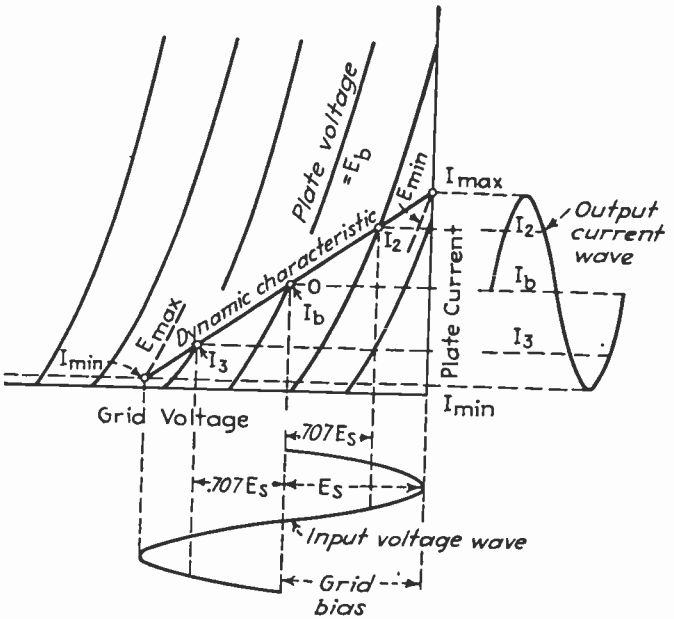


FIG. 17.—Typical dynamic characteristic for Class A power amplifier with grid driven to zero voltage, showing input and output waves and critical points.

From the dynamic characteristic one can determine the conditions for optimum operation, the wave shape of the output voltage, the output power, etc.

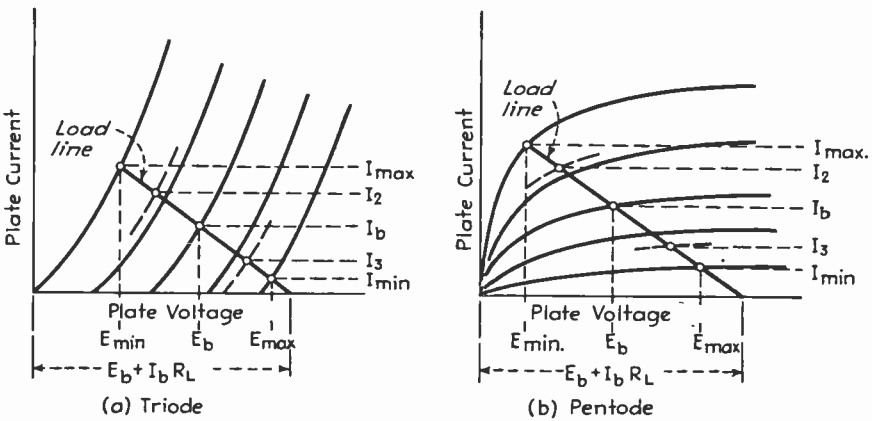


FIG. 18.—Typical load lines for triode and pentode Class A power amplifiers, showing critical points.

In the practical determination of the performance of Class A amplifiers only certain critical points on the dynamic characteristic are needed. These particular points

are most readily obtained with the aid of a *load line* drawn on the plate-voltage–plate-current characteristic curves of the tube as shown in Fig. 18. The load line is a straight line passing through the operating point (E_b, I_b) and intersecting the zero plate current axis at a plate potential of $E_b + I_bR_L$. Such a line is a dynamic characteristic plotted on the $E_b - I_b$ coordinate system.

Class A Amplifier Calculations for Resistance Loads.—The power output from a Class A amplifier with a resistance load is[1]

$$\text{Power output} = \frac{(E_{max} - E_{min})(I_{max} - I_{min})}{8} \tag{21a}$$

$$= \frac{E_bI_b}{2}\left(1 - \frac{E_{min}}{E_b}\right)\left(1 - \frac{I_{min}}{I_b}\right) \tag{21b}$$

where E_{max} and E_{min} = maximum and minimum instantaneous values of plate voltage, respectively, reached during the cycle.

I_{max} and I_{min} = maximum and minimum instantaneous values of plate current, respectively, reached during the cycle.

E_b and I_b = plate voltage and current, respectively, at the operating point.

Values of E_{max}, E_{min}, etc., are readily found from the load line for any specific operating point, load resistance, and signal voltage. In order to be able to obtain a large power output, it is obviously necessary that the d-c power input E_bI_b be large and also that the load resistance and grid bias be so selected that a signal voltage that makes I_{min}/I_b small on the negative peaks of the cycle likewise makes E_{min}/E_b small on the positive peaks.

The efficiency of a power amplifier is defined as the ratio of power output that can be developed with only moderate distortion, to the d-c power supplied to the plate. The efficiency of a Class A amplifier accordingly is

$$\text{Plate efficiency} = \frac{(E_{max} - E_{min})(I_{max} - I_{min})}{8E_bI_b} \tag{22a}$$

$$= \frac{1}{2}\left(1 - \frac{E_{min}}{E_b}\right)\left(1 - \frac{I_{min}}{I_b}\right) \tag{22b}$$

The efficiency has a theoretical maximum value of 50 per cent, and approaches this more closely as the ratios I_{min}/I_b and E_{min}/E_b are small. In practical Class A amplifiers, I_{min}/I_b is of the order of 0.1 to 0.2; while E_{min}/E_b ranges from this value for pentodes and beam tubes, and also triodes with large plate voltages or operated with the grid driven appreciably positive, to values of the order of 0.6 for triode tubes operating at low plate potentials such as 250 volts and with the grid never driven positive. The corresponding practical efficiencies are from 35 per cent down to about 20 per cent.

The load resistance R_L corresponding to specified values of E_{max}, E_{min}, I_{max}, and I_{min} is

$$R_L = \frac{(E_{max} - E_{min})}{I_{max} - I_{min}} = \frac{E_b}{I_b}\left(1 - \frac{E_{min}}{E_b}\right) \tag{23}$$

In practice, this gives load resistances of approximately E_b/I_b under conditions where the plate efficiency is reasonably high (*i.e.*, when E_{min}/E_b and I_{min}/I_b are small), and resistances about half as great for triodes operated with E_{min}/E_b approaching 0.6, as mentioned above. In triodes where the maximum instantaneous grid potential

[1] Equation (21*a*) gives the exact power output on the fundamental frequency for the case of negligible distortion, or when all the distortion is in the form of even harmonics. Equation (21*b*), however, is exact only in the absence of distortion.

Ex. 1044-0387

reached during the cycle is exactly zero, the condition for maximum power output with only moderate distortion corresponds very closely to a load resistance twice the plate resistance at the operating point.[1]

Distortion Calculation.—Nonlinear distortion is always present to some extent in a power amplifier because the dynamic characteristic is never a straight line. The character of the distortion obtained in practice depends upon the operating conditions. In triodes, when the distortion is not excessive and the grid is not driven positive, practically all of the distortion produced with a sine wave signal is second harmonic, and can be calculated from the relations[2]

$$\text{Direct-current component} = A_0 = \frac{I_{max} + I_{min} + 2I_b}{4} \tag{24a}$$

$$\text{Second harmonic} = \frac{I_{max} + I_{min} - 2I_b}{4} \tag{24b}$$

$$\frac{\text{Second harmonic}}{\text{Fundamental}} = \frac{I_{max} + I_{min} - 2I_b}{2(I_{max} - I_{min})} \tag{24c}$$

With pentodes and beam tubes, and when the distortion is large, or when the grid is driven somewhat positive, appreciable third and fourth harmonics can also be expected. In this case, the various components of the output voltage are[3]

$$\text{Direct-current component} = A_0 = \frac{\frac{1}{2}(I_{max} + I_{min}) + I_2 + I_3 + I_b}{4} \tag{25a}$$

$$\text{Fundamental} = A_1 = \frac{\sqrt{2}(I_2 - I_3) + I_{max} - I_{min}}{4} \tag{25b}$$

$$\text{Second harmonic} = A_2 = \frac{I_{max} + I_{min} - 2I_b}{4} \tag{25c}$$

$$\text{Third harmonic} = A_3 = \frac{I_{max} - I_{min} - 2A_1}{2} \tag{25d}$$

$$\text{Fourth harmonic} = A_4 = \frac{2A_0 - I_2 - I_3}{2} \tag{25e}$$

$$\text{Power output} = \frac{A_1^2 R_L}{2} \tag{25f}$$

where I_2 and I_3 are the instantaneous plate currents when the instantaneous voltage applied to the grid is the bias voltage respectively plus and minus 0.707 times the peak signal voltage acting on the grid as indicated in Figs. 17 and 18, and the remaining notation is the same as above.

[1] Maximum power sensitivity, *i.e.*, maximum output power in proportion to the applied signal is always obtained in a Class A amplifier when the load resistance equals the plate resistance. Operation under this condition does not permit as much power to be developed without excessive distortion as does operation with the load resistance called for by Eq. (23). The difference is particularly great in the case of pentodes and beam tubes.

[2] Equations (24) and (25) are approximate because they assume that any change in the average value of plate current produced as a result of distortion encounters the same load resistance as the alternating components of the plate current. With the amplifier circuits shown in Fig. 16, this is not the case, although the resulting error is small if the distortion is not excessive. The exact solution can be obtained either by modifying the load line, as described by C. E. Kilgour, Graphical Analysis of Output Tube Performance, *Proc. I.R.E.*, Vol. 19, p. 42, January, 1931, or by using the complete equivalent circuit based upon the power-series analysis of the vacuum-tube amplifier as described in Par. 24.

[3] G. S. C. Lucas, Distortion in Valve Characteristics, *Exp. Wireless and Wireless Eng.*, Vol. 8, p. 595, November, 1931.

A slightly different procedure giving the first four harmonics is described by A. Bloch, Distortion in Valves with Resistive Loads, Wireless *Engr.*, Vol. 15, p. 592, December, 1939.

An alternative method of analysis slightly more complicated and going up to the sixth harmonic is given by J. A. Hutcheson, Graphical Harmonic Analysis, *Electronics*, Vol. 9, p. 16, January, 1936.

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The "direct-current component" A_0 of Eqs. (24*a*) and (25*a*) is the d-c component of the plate current when a signal is present. When there is distortion, this will usually differ from the value I_b of d-c plate current in the absence of signal. The increase of plate current (*i.e.*, $A_0 - I_b$) caused by the application of a signal is termed the "rectified" plate current, and is usually positive. The magnitude of the rectified current is usually a fair indication of the amount of distortion present.

Equations (24) and (25) give the distortion in the output of a power amplifier when the voltage applied to the grid is sinusoidal. When the grid is driven positive, the positive peaks of the exciting waves are flattened, thereby distorting the applied voltage, as shown in Fig. 19. The amount of distortion produced in this way depends upon the internal impedance of the exciter of the power amplifier tube, the grid current, etc. The harmonics produced in the wave applied to the grid as the result of grid current are very approximately

$$\begin{aligned} \frac{\text{Second harmonic}}{\text{Fundamental}} &= \frac{Z_s I_{gm}}{4E_s} \\ \frac{\text{Third harmonic}}{\text{Fundamental}} &= \frac{Z_s i_{gm}}{4E_s} \\ \frac{\text{Fourth harmonic}}{\text{Fundamental}} &= \frac{Z_s i_{gm}}{8E_s} \end{aligned} \qquad (26)$$

where Z_s = impedance seen by grid of the tube in looking back toward source of exciting voltage.
i_{gm} = peak value of grid current.
E_s = crest value of exciting voltage in the absence of grid current.

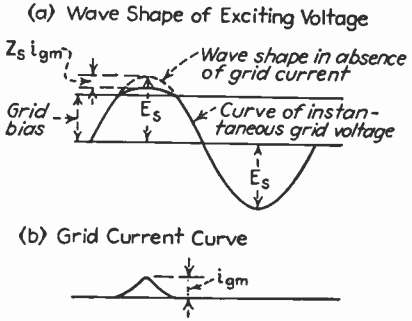


FIG. 19.—Wave shape of exciting voltage when grid is driven positive, showing flattening of the positive peaks resulting from grid current.

Selection of Operating Conditions for Class A Power Amplifiers.—Proper operating conditions for Class A operation of the commonly used power tubes are given in tube manuals, together with the amount of power that can be obtained and the accompanying distortion. It is accordingly necessary to determine operating conditions only in the case of new tubes, or tubes for which complete data are not available, or for tubes operating under special conditions.

Optimum operating conditions for triodes are selected as follows: Assume that the plate dissipation is the limiting factor, and choose a grid bias such that the d-c plate current has the maximum permissible value at the rated plate voltage. A reasonable value for I_{min} is then selected, such as one-tenth to one-fifth of the plate current at the operating point. Then to a good approximation, $I_{max} = 2I_b - I_{min}$. The minimum instantaneous plate potential E_{min} is then whatever value of plate voltage is required to give this value of I_{max} at the maximum potential to which it is desired to drive the grid. The corresponding maximum instantaneous plate potential is approximately $E_{max} = 2E_b - E_{min}$. With this information the proper load resistance and the power output can be calculated. A load line can then be drawn for this load resistance, giving the exact values of I_{min}, I_{max}, etc., and permitting calculation of the distortion. If the final results are not satisfactory, the operating conditions can then be modified as required. In particular, if the load resistance that this procedure leads to is less than twice the plate resistance of the tube, it will be found desirable to increase the negative bias on the grid until the load resistance called for is at least twice the plate resistance. This situation will sometimes be encountered with tubes operating at low plate voltages, *i.e.*, of the order of 250 volts.

Ex. 1044-0389

With pentode and beam tubes the first step in selecting proper operating conditions is to decide upon a suitable plate current at the operating plate voltage, taking into account the rated plate dissipation. Values for I_{min} and I_{max} are then determined as in triodes, and the screen potential is given a value that causes the plate current to be I_{max} at the maximum grid potential desired. This maximum grid potential is nearly always either zero or only moderately positive, since there is enough distortion in pentodes and beam tubes even without introducing further distortion from grid current. The proper grid bias at the operating point is the bias that gives the operating plate current $(I_{max} + I_{min})/2$ at this screen voltage. A load line can then be drawn by selecting a load resistance such that I_{max} corresponds to a value of E_{min} sufficient to keep just above the knee of the $E_p - I_p$ curve. The exact behavior and distortion can be checked from this load line, and small readjustments of conditions made as required.

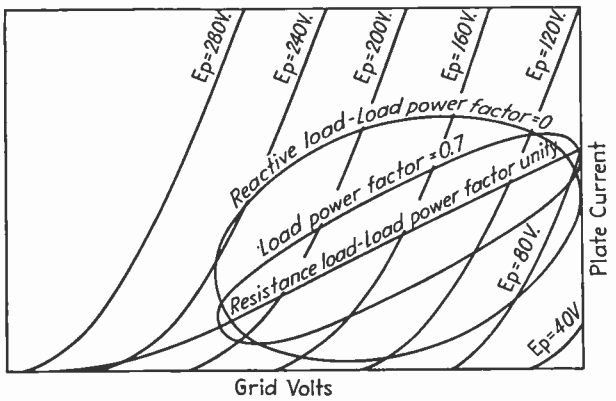


FIG. 20.—Dynamic characteristic for load impedances of varying power factors but constant magnitude, showing how reactive loads cause the dynamic characteristic to open up into an ellipse.

Reactive Loads.—When the load impedance has a reactive component, the dynamic characteristic (and also the load line) becomes a closed elliptical curve, as shown in Fig. 20.[1] Analysis is then most conveniently carried out by the power series method of Par. 24 rather than by the use of dynamic characteristics. With a reactive load, the power output obtainable is always less than with resistance load. When the load power factor is reasonably high, satisfactory results are obtained by using the same operating point as with resistance loads and making the magnitude of load impedance equal the proper resistance load. If, however, the power factor is quite low, the load impedance, bias, and applied signal may have to be modified for optimum performance.

8. Push-pull Amplifiers.—In the push-pull amplifier two tubes are arranged as shown in Fig. 16c, with the grids excited by equal voltages 180° out of phase and the outputs combined by means of a transformer having a center-tapped primary. With this arrangement even harmonics and even order combination frequencies cancel in the output, thereby permitting operation of triode and beam tubes in push-pull under conditions of high output per tube that would otherwise give excessive distortion. In addition, the push-pull arrangement avoids direct-current saturation in the core of the output transformer, because the d-c current in the two halves of the primary magnetize the core in opposite directions. There is also no a-c current of

[1] Manfred von Ardenne, On the Theory of Power Amplification, *Proc. I.R.E.*, Vol. 16, p. 193, February, 1928.

Ex. 1044-0390

fundamental frequency flowing through the source of plate potential; thus minimizing regeneration in multistage amplifiers and making it unnecessary to employ a by-pass condenser across cathode biasing resistors. Hum caused by a-c filament current or ripple in the plate-supply voltage is also balanced out by the push-pull transformer connection.

These advantages of the push-pull arrangement are such that in using triodes or beam tubes, two small tubes are commonly used in a push-pull connection in preference to a single larger tube.

Class AB Push-pull Amplifiers.—The elimination of even harmonics by the push-pull connection makes it possible in triodes and beam tubes to extend operation until the instantaneous plate current is reduced to zero for a small portion of each cycle

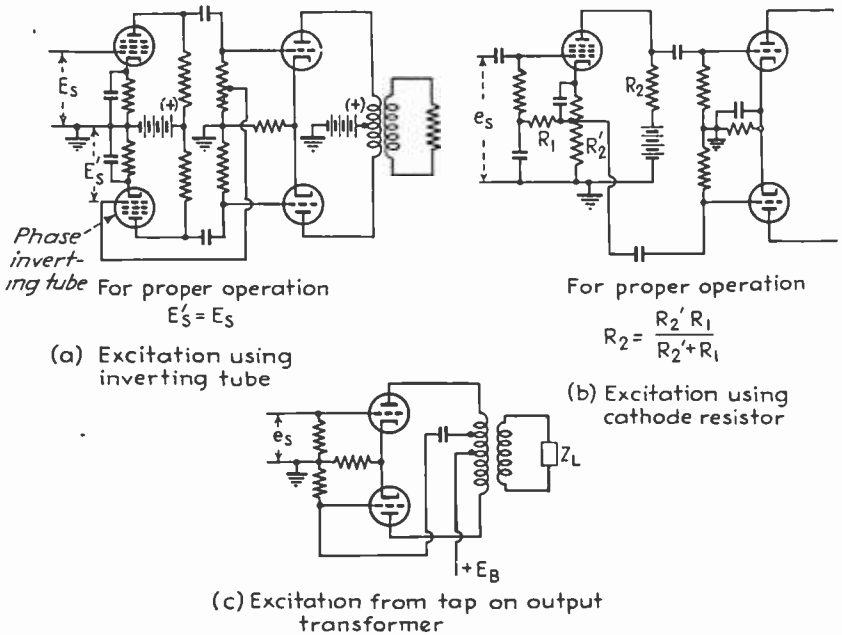


FIG. 21.—Methods of exciting push-pull amplifiers without using an input transformer.

without causing excessive distortion in the output. This is termed Class AB operation, and gives plate efficiencies of the order of 40 to 50 per cent.

Class AB operation is accompanied by a large increase in d-c plate current when the full signal voltage is applied. With fixed bias operation this is an advantage, since it permits the direct-current power input to the plate to be increased in the presence of an applied signal to a value exceeding the allowable plate dissipation. On the other hand, when self-bias is used, the full possibilities of Class AB operation cannot be realized, because if the bias keeps the plate current to an allowable value for no applied signal, the bias will be excessive when the d-c current increases in the presence of a large signal.

Excitation of Push-pull Amplifiers.—In a push-pull amplifier, the exciting voltages acting on the grids of the two tubes should be equal in magnitude and 180° out of phase. Such a voltage is usually obtained from an ordinary voltage amplifier by means of a center-tapped transformer, as shown in Fig. 16*c*, or by the use of an auxili-

Ex. 1044-0391

ary *phase inverting tube*, as shown in Fig. 21*a*. Other possible means of excitation[1] are from a cathode resistor (Fig. 21*b*), by the use of a tap on the output transformer (Fig. 21*c*), and from the a-c voltage developed across the bias resistor of the push-pull stage (see Fig. 41*c*) or across a common resistor to ground in the plate circuit (R_3 in Fig. 41*d*).

Analysis of Push-pull Amplifiers.—The behavior of a push-pull amplifier can be determined with the aid of a load line plotted on composite curves, as shown in Fig. 22.[2] To obtain these composite curves, the plate voltage-current curves of the individual tubes are placed back to back as shown, with the common operating voltage superimposed. The composite characteristic curves are then derived by averaging the plate current for grid potential curves corresponding to the same applied signal. Thus the curves for the operating grid potential $E_c = 60$ in Fig. 22

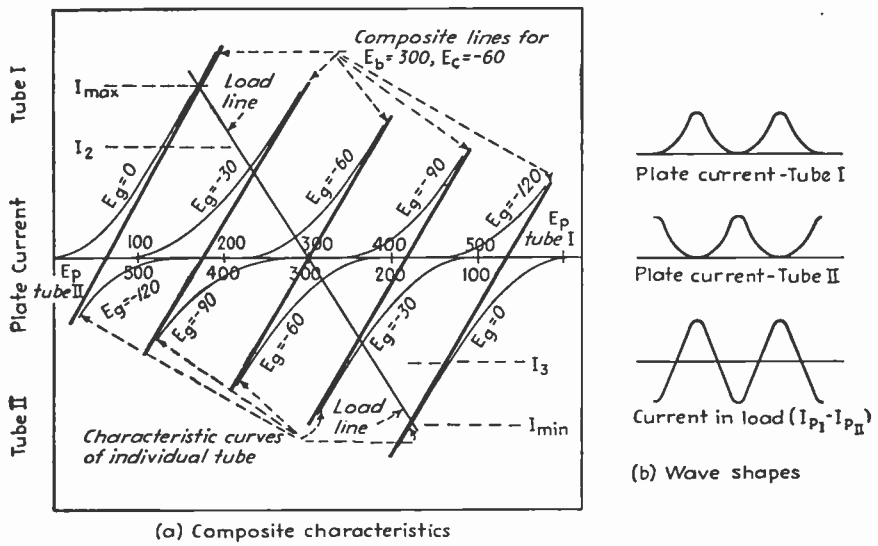


FIG. 22.—Composite characteristic curves of push-pull power amplifier, showing a load line together with the current waves for individual tubes.

are averaged because they correspond to no signal being applied, but when the signal is 30 volts on one tube it is −30 volts on the other; so one averages the grid curve for −30 volts on one tube with the curve for −90 volts on the other tube, etc. The resulting derived characteristics represent the relation between the plate-cathode voltage (which is one-half the plate-to-plate voltage) and the difference between the currents to the two anodes (which is twice the a-c current in a plate-to-plate load resistance); *i.e.*, the load line gives the voltages and currents in the output when the output transformer has a step-down ratio of 2.

The power output and distortion can be obtained in the usual way from load lines drawn on this diagram. These load lines must pass through the point on the composite characteristic corresponding to the operating plate and grid voltages, and they have a slope corresponding to one-fourth of the actual plate-to-plate load resistance. In making use of Eqs. (24) and (25) in connection with data derived from the dynamic

[1] See also Leonard Tulanskas, Bridge-type Push-pull Amplifiers, *Electronics*, Vol. 6, p. 134, May, 1933.

[2] B. J. Thompson, Graphical Determination of Performance of Push-pull Audio Amplifiers, *Proc. I.R.E.*, Vol. 21, p. 591, April, 1933.

Ex. 1044-0392

characteristic, it is to be noted that the value of I_b used in the equations has the value zero, and that since I_{min} is in the lower half of the composite diagram, it is negative when substituted into the equations. In order to calculate the plate efficiency, it is necessary to know the d-c plate current existing in the presence of the signal. This depends upon the shape of the plate-current waves of the individual tubes, which can be derived by obtaining the instantaneous plate voltage for any given instantaneous grid voltage using the load line on the composite characteristic, and then, with the aid of the characteristic curve of the tube, finding the resulting plate-current wave. Waves for a typical case of Class AB operation are shown in Fig. 22.

When the instantaneous plate current of the individual tube goes to zero, as in Class AB operation and, as is often the case, in Class A push-pull amplifiers as well, it is possible to draw an approximate load line without the necessity of first obtaining a composite characteristic.[1] This is done by noting that the load line crosses the zero plate-current axis at a potential corresponding to the operating plate voltage and also by noting that since at the peak of the exciting voltage only one tube carries current, the composite characteristic at the extreme end of the load line coincides with the actual characteristic curve of the individual tube (see Fig. 22). This determines two points on the load line, the upper half of which can then be drawn on the $E_p - I_p$ curves of the individual tube, as in Fig. 23. In this way, it is possible to determine the value of

$$I_{max},\ I_{min}\ (= -I_{max}),\ E_{min},$$

and

$$E_{max}\ (= 2E_b - E_{min})$$

corresponding to a given load resistance and instantaneous peak voltage on the grid, or one can determine the values of E_{min}, E_{max}, and load resistance corresponding to a desired I_{max} with a given instantaneous peak grid voltage.

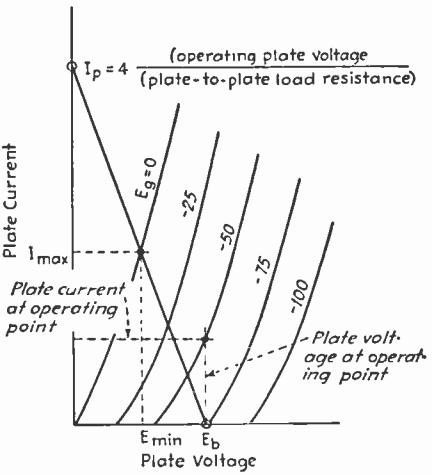


FIG. 23.—Upper half of load line for push-pull amplifier where the minimum plate current of the individual tube reaches zero. This corresponds to the part of the load line above the horizontal axis in Fig. 22.

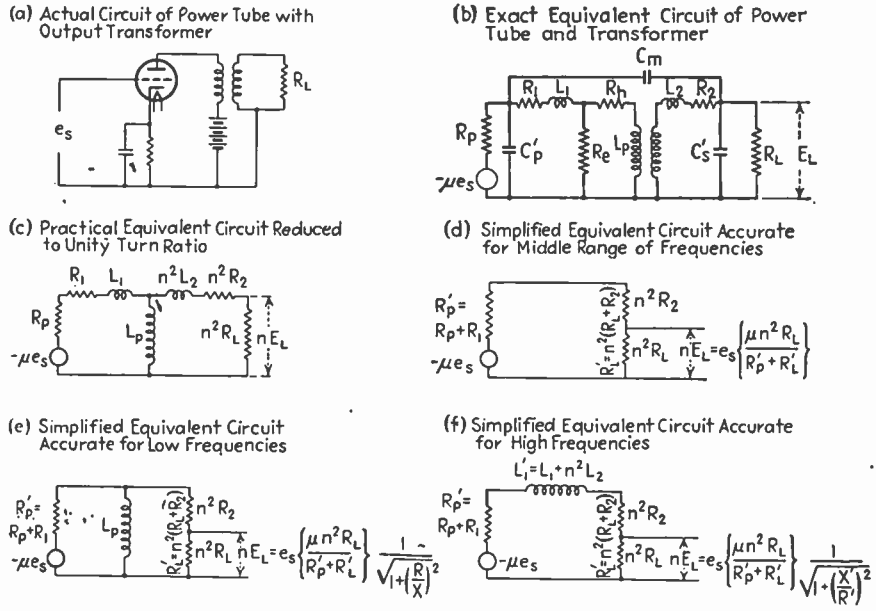
9. Output Transformers and Other Coupling Arrangements for Class A Power Amplifiers.—The load impedance of a Class A power amplifier is normally coupled to the plate circuits of the tube by means of a transformer, as shown in Figs. 16*b* and 16*c*. This makes it possible, by the use of the proper turn ratio, to make any given load offer the desired impedance to the plate circuit of the amplifier.

Frequency Response Characteristics of Power Amplifiers Using Output Transformers.[2] The variation of output voltage with frequency that results from the use of an output transformer can be analyzed with the aid of the equivalent circuits of Fig. 24. The exact equivalent circuit shown at (*b*) can for most practical purposes be simplified by neglecting the capacities associated with the transformer and also the eddy-current and hysteresis losses, giving the result at (*c*) when reduced to unity turn ratio. Further simplifications, as shown at (*d*), (*e*), and (*f*), are possible when only limited

[1] E. W. Houghton, Class AB Push-pull Calculations, *Electronics*, Vol. 10, p. 18, June, 1937.
[2] F. E. Terman and R. R. Ingebretsen, Output Transformer Response, *Electronics*, Vol. 9, p. 30, January, 1936.

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frequency ranges are considered. These equivalent circuits also apply to push-pull Class A and Class AB amplifiers, provided that the plate resistance and amplification factor used in the circuits and in the equations based on the circuits are taken as twice the values for a single tube.



μ = amplification factor of tube
R_p = plate resistance of tube (in a push-pull amplifier R_p is twice the plate resistance of one tube)
R_1 = d-c resistance of primary winding
$R_p' = R_p + R_1$ = effective plate resistance
R_2 = d-c resistance of secondary winding
R_L = load resistance
$R_L' = n^2(R_L + R_2)$ = effective load resistance reduced to unity turn ratio
$R = R_L' R_p' / (R_L' + R_p')$ = resistance formed by R_p' and R_L' in parallel
$R' = R_L' + R_p'$ = sum of effective load and effective plate resistances
n = step down voltage ratio = ratio of primary to secondary turns
L_p = primary inductance with appropriate d-c saturation
L_1 = leakage inductance of primary winding
L_2 = leakage inductance of secondary winding
$L_1' = L_1 + n^2 L_2$ = total leakage inductance reduced to unity turn ratio
$X = \omega L_p$ = reactance of transformer primary inductance
$X' = \omega L_1'$ = reactance of transformer leakage inductance
e_s = input voltage
E_L = output voltage

FIG. 24.—Equivalent circuits of power amplifier using output transformer.

In the middle range of frequency the output voltage of the transformer-coupled amplifier is

$$E_L = E_g \left(\frac{\mu n R_L}{R_p' + R_L'} \right) \tag{27}$$

The notation is shown in Fig. 24. At low frequencies the output falls off because of the shunting reactance of the primary inductance according to the relation

Ex. 1044-0394

$$\left.\begin{matrix}\text{Output voltage at}\\ \text{low frequencies}\end{matrix}\right\} = \frac{\left\{\begin{matrix}\text{Output voltage in}\\ \text{middle-frequency range}\end{matrix}\right\}}{1 - j\frac{f_1}{f}} \tag{28}$$

where f is the actual frequency, and f_1 the frequency at $R = X$. *The output voltage drops to 70.7 per cent of the mid-frequency value (output down 3 db) at the frequency* f_1

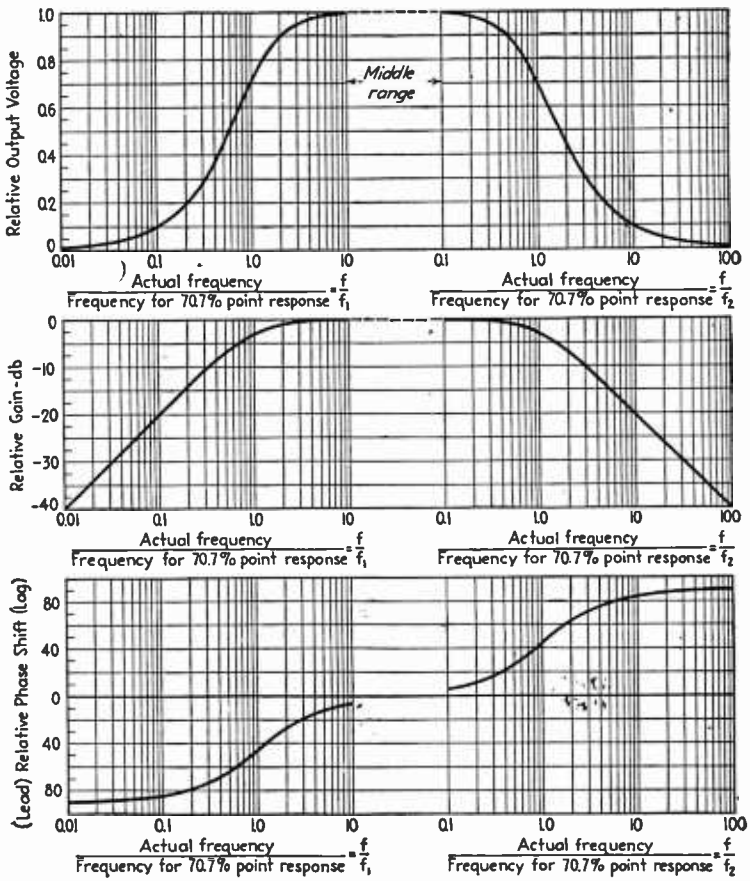


FIG. 25.—Universal amplification curve of transformer-coupled power amplifier.

for which the primary reactance X *equals the resistance* R *formed by the effective load and effective plate resistances in parallel.* At high frequencies, the output falls off because of the leakage reactance of the transformer, according to the equation

$$\left.\begin{matrix}\text{Output voltage at}\\ \text{high frequencies}\end{matrix}\right\} = \frac{\left\{\begin{matrix}\text{Output voltage in}\\ \text{middle-frequency range}\end{matrix}\right\}}{1 + j\frac{f}{f_2}} \tag{29}$$

where f_2 is the frequency at which $X' = R'$. *The 70.7 per cent response point (output*

down 3 db) occurs at a frequency f_2 such that the leakage reactance equals the sum of the effective load and effective plate resistances.

The relative magnitude and phase of the output voltage of a power amplifier using an output transformer can be readily obtained with the aid of the universal amplification curve given in Fig. 25.

In pentode and beam tubes the plate resistance is so high that the falling off in amplification calculated from Eq. (29) does not become appreciable until the frequency is very high. Under these conditions the capacities shown in the exact equivalent circuit of Fig. 24*b* may be of importance.

Transformer Characteristics.—For uniform amplification over a wide frequency range, the output transformer should have a high primary and low leakage induct-

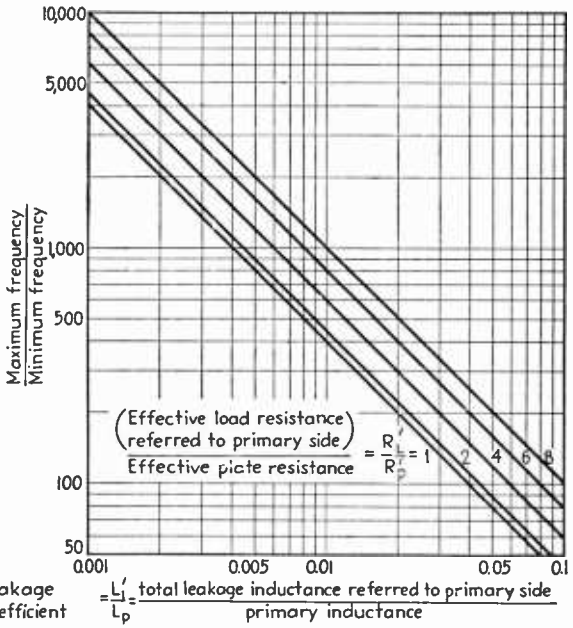


FIG. 26.—Curves giving frequency range of transformer-coupled power amplifier as a function of the leakage coefficient of the transformer.

ance. The ratio of the highest to lowest frequency over which the amplification is within 3 db of the mid-range value is given in Fig. 26 as a function of the leakage coefficient L_1'/L_p. The factors contributing to a small leakage coefficient are (1) core assembly, with a minimum possible air gap consistent with the d-c magnetization present, (2) the use of alloy cores, and (3) winding arrangements that minimize leakage inductance. In the case of transformers for push-pull operation it is also desirable that the leakage inductance between the two halves of the primary be small, as should also the leakage inductance from each half of the primary to the secondary.

The step-down ratio n for an output transformer should approximate a value such that n^2 times the load resistance equals the resistance to be presented to the tube. Actually, the turn ratio should be slightly less because of the resistances of the transformer windings. When these resistances are taken into account, the resistance offered by the transformer primary to the amplifier tube in the middle-frequency range is

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$$\left.\begin{matrix}\text{Impedance offered}\\ \text{to tube}\end{matrix}\right\} = n^2R_L + (n^2R_2 + R_1) \tag{30}$$

The notation is illustrated in Fig. 24.

An output transformer consumes a fraction of the power that the tube delivers to it as a result of the winding resistances and the core losses. Practically, the primary and secondary resistances account for virtually all the power loss except at quite low frequencies. To the extent that the core losses can be neglected, one has

$$\frac{\text{Power delivered to load}}{\text{Power delivered to transformer}} = \frac{1}{1 + \dfrac{R_1 + n^2R_2}{n^2R_L}} \tag{31}$$

The maximum current that an output transformer can carry is determined by the heating of the windings, while the maximum voltage that can be applied is limited by the permissible flux density in the core. These two factors operating together determine the power rating of the output transformer.

The relationship between the applied voltage and flux density in the core is given by the equation

$$\left.\begin{matrix}\text{Effective value of}\\ \text{applied voltage}\end{matrix}\right\} = 4.44fNBA \times 10^{-8} \tag{32}$$

where N = number of turns.
f = frequency.
B = flux density in the core in lines per unit area.
A = net area of core.

The voltage producing a given flux density is proportional to frequency, so that the voltage limit occurs at low frequencies. The allowable flux density is limited by the fact that with high densities the nonlinearity of the magnetization curve introduces distortion, and depends upon the amount of distortion that can be tolerated, the magnetic characteristics of the core material, the length of the air gap, the circuits associated with the transformer, and whether or not d-c magnetization is present. In general, the higher the flux density, the greater the d-c magnetization, and the smaller the air gap in the core, the greater will be the distortion. In ordinary output transformers operated within their power rating it is commonly found that distortion due to nonlinearity of the magnetization curve often becomes serious at frequencies for which the loss of gain is less than 3 db.

Coupling Systems Employing Parallel Feed.—The term *parallel feed* (or shunt feed) is used to denote systems in which a separate choke is provided for carrying the d-c plate current. Examples of such arrangements are shown in Figs. 16*a* and 16*d*. In such circuits the shunt-feed choke is sometimes referred to as a *retard coil*.

The direct-coupled shunt-feed circuit of Fig. 16*a* is characterized by unusually good high-frequency response, with the point at which the response drops to 70.7 per cent of the mid-range value occurring when the reactance of the shunting capacity is equal to the equivalent resistance formed by the load resistance in parallel with the plate resistance of the tube. At low frequencies the amplification characteristic depends upon the inductance L and capacity C of the coupling network, and upon the ratio of plate to load resistance. Results in critical cases are shown in Fig. 27*a*. When the coupling condenser C is extremely large, the amplification falls off at low frequencies in the same way as with simple transformer coupling, with the 70.7 per cent point occurring at the frequency for which the reactance of the inductance L equals the resistance formed by the load and plate resistances in parallel. When the ratio of plate to load resistance is somewhat less than unity, as is the case with triode

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tubes, a suitably chosen finite blocking-condenser capacity gives approximately the same falling off as does the infinite condenser. On the other hand, when the plate resistance of the tube is very high compared with the load resistance (corresponding to pentode tubes), Fig. 27*a* shows that substantially constant amplification can be

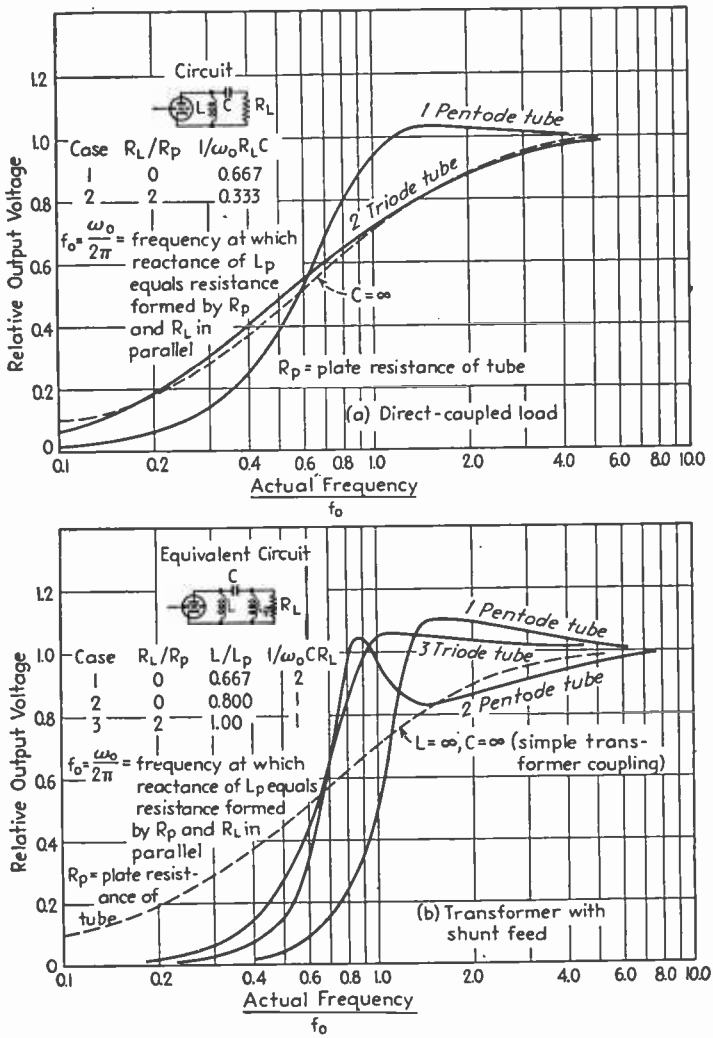
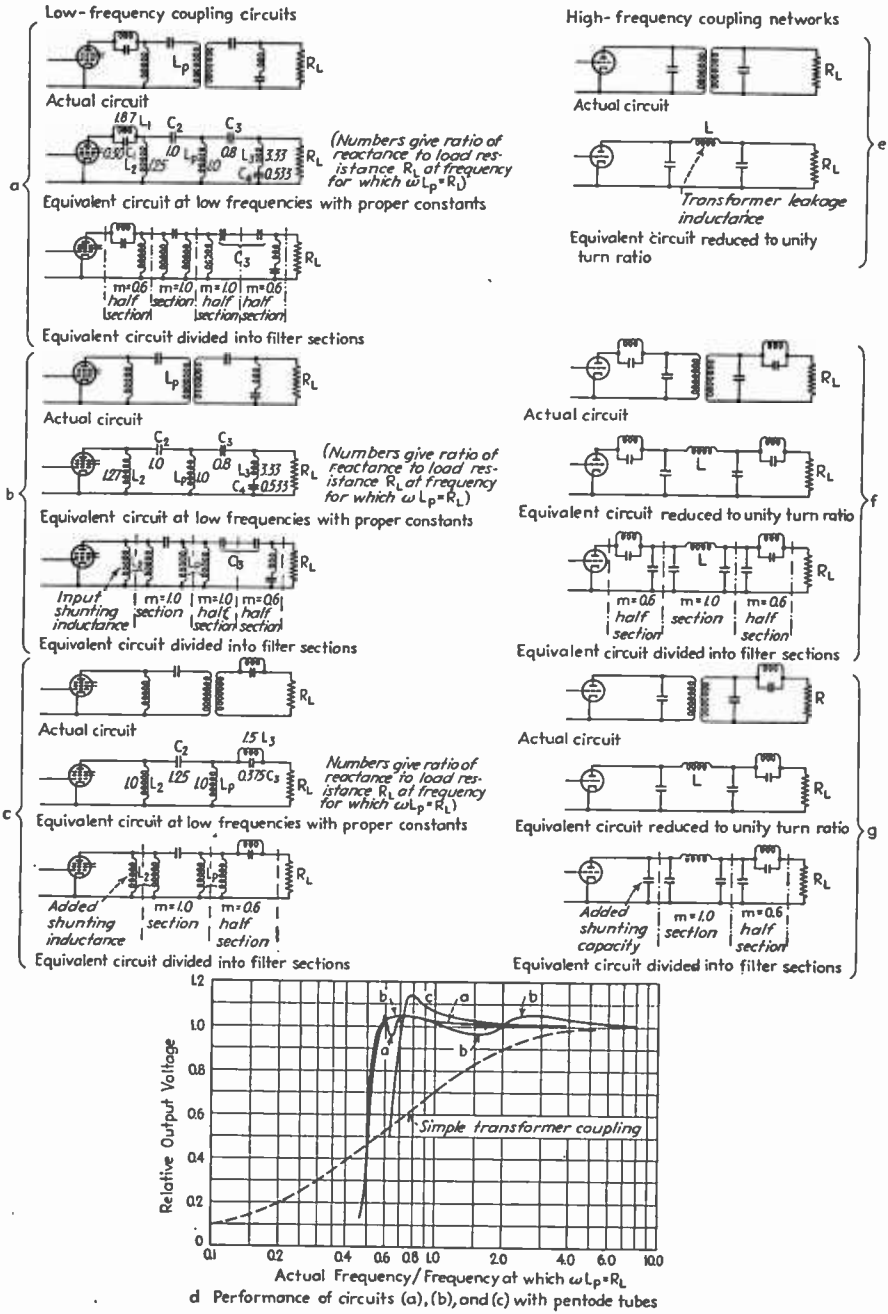


FIG. 27.—Characteristics obtainable at low frequencies by properly designed shunt-feed coupling systems.

maintained down to the frequency at which the reactance of the choke equals the load resistance, provided that the blocking capacity is such that its reactance at this same frequency is two-thirds the load resistance. The cutoff at lower frequencies is then quite sharp, however.

Transformer coupling with shunt feed gives the possibility of a better frequency response than is obtainable with a simple transformer. This is because the elimina-

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d Performance of circuits (a), (b), and (c) with pentode tubes

FIG. 28.—Filter coupling networks for improving the characteristics of power amplifiers, together with behavior of various low-frequency coupling systems.

Ex. 1044-0399

tion of the direct-current magnetization from the transformer core permits the core to be assembled with a smaller air gap, and also makes it permissible to use high permeability core material. This reduces the ratio of leakage to primary inductance of the transformer, and so increases the frequency range that can be covered. The low-frequency characteristics obtained with an output transformer having shunt feed depend upon the ratio of plate to load resistance and the inductance and capacity of the shunt-feed inductance and blocking condenser, respectively. The behavior in the critical cases is shown in Fig. 27*b*. With pentode tubes, a properly selected finite shunt-feed choke and blocking condenser will give a substantially better performance than the same transformer operated with an extremely large condenser and infinite shunt-feed choke inductance (*i.e.*, simple transformer coupling having the same transformer primary inductance L_p).

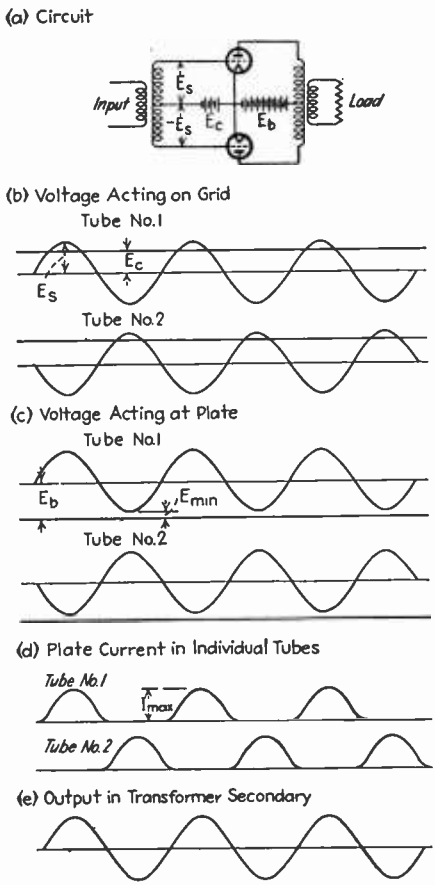


FIG. 29.—Circuit diagram of Class B audio amplifier together with oscillograms showing voltage and current relations and showing how the pulses of plate current in the individual tubes combine to produce a sinusoidal output.

Shunt-feed systems such as shown in Fig. 27 give appreciably sharper low-frequency cutoff than is obtained with simple transformer coupling.

Filter Coupling Systems.[1]—The characteristic of a coupling system involving an output transformer can be improved by incorporating the primary and leakage inductance of the transformer into a coupling network designed according to filter theory. The benefits obtained in this way are primarily an extension of the low-frequency response when pentode tubes are employed and an extension of the high-frequency response when triode tubes are used. There is also an improvement of the impedance characteristic at extreme frequencies, which increases the power-handling capacity of the amplifier under these conditions.

Filter-type coupling systems for low frequencies, together with circuit constants, are shown in Fig. 28. The circuit of Fig. 28*a* consists of ordinary filter sections arranged as shown, with the primary inductance of the transformer functioning as the full shunt inductance of an inverse-network (or constant k) section of a filter that is provided with terminating half sections. With pentode tubes, such an arrangement gives a substantially constant response down to half the frequency at which the response would be 70.7 per cent in the case of a simple transformer with the same primary inductance. With triode tubes there is little if any improvement in the

[1] Much of this material is from the thesis by C. A. Moreno, Circuit Design to Improve the Frequency Response of Output Transformers, Stanford University, 1940.

Ex. 1044-0400

low-frequency response, but a better impedance characteristic is realized at low frequencies. The network of Fig. 28*a* can be simplified as shown in Fig. 28*b*, with little loss in frequency characteristic, although at the expense of a less satisfactory impedance at low frequencies. The simplifications involved consist primarily in omitting the input terminating half section and then modifying the shunting inductance on the input side. The resulting frequency characteristic for the case of pentode tubes is shown in Fig. 28*d*. Still further simplification of the network is possible as in Fig. 28*c*, although at the expense of a low-frequency cutoff about 25 per cent higher than for the circuit of Fig. 28*b*.

Filter type of coupling networks for high frequencies are shown in Figs. 28*e* to 28*g*. These are the high-frequency analogues of the low-frequency coupling systems discussed above. With triode tubes these arrangements offer the possibility of an appreciable improvement in the high-frequency response obtainable with a given leakage inductance.

Filter coupling systems are useful under circumstances where it is otherwise difficult to obtain the required frequency response or where a very sharp cutoff is wanted or where it is important to maintain normal impedance relations at extremes in frequency. They are not otherwise justified because of the expense of the coupling network and the necessity of adjusting its constants accurately to specified values.

10. Class B Audio-frequency Power Amplifiers.[1]—The Class B audio-frequency amplifier is a push-pull amplifier in which the tubes are biased approximately to cutoff. Operated in this way, one of the tubes amplifies the positive half cycles of the signal voltage while the other amplifies the negative half cycles, with the output transformer combining these in the output, as shown in Fig. 29. Such an amplifier is characterized by high plate efficiency and relatively high output in proportion to the average dissipation in the tubes.

The output power, plate efficiency, and proper load resistance in a Class B amplifier can be determined with an accuracy sufficient for ordinary purposes by assuming that the characteristic curves of the tube are straight lines. This simplifying assumption gives

$$\left.\begin{matrix}\text{Proper load resistance}\\ \text{from plate to plate}\end{matrix}\right\} = R_L = 4\,\frac{E_b - E_{\min}}{I_{\max}} \tag{33}$$

$$\left.\begin{matrix}\text{Power output}\\ \text{from two tubes}\end{matrix}\right\} = \frac{I_{\max}(E_b - E_{\min})}{2} \tag{34}$$

$$\text{Plate efficiency} = \frac{\pi}{4}\left(1 - \frac{E_{\min}}{E_b}\right) \tag{35}$$

Here $I_{\max}$ is the peak plate current and $E_{\min}$ the minimum plate potential reached in an individual tube during the cycle, and E_b is the plate supply voltage. Exact determination of power output, efficiency, and distortion can be carried out with the aid of a load line drawn on composite characteristic curves of the push-pull combination exactly as described in connection with Fig. 22.

The maximum possible efficiency as given by Eq. (35) is 78.5 per cent. Under practical conditions, efficiencies of the order of 50 to 60 per cent can be realized.

The distortion in Class B audio amplifiers is minimized by employing a bias corresponding to the cutoff bias that would be obtained if the main part of the $I_p - E_g$ curve for the operating plate potential were projected as a straight line, as shown in Fig. 30. Such a bias is termed *projected cutoff*, and gives an output voltage that is a

[1] L. E. Barton, High Audio Output from Relatively Small Tubes, *Proc. I.R.E.*, Vol. 19, p. 1131, July, 1931; Recent Developments of the Class B Audio- and Radio-frequency Amplifiers, *Proc. I.R.E.*, Vol. 24, p. 985, July, 1936; True McLean, An Analysis of Distortion in Class B Audio Amplifiers, *Proc. I.R.E.*, Vol. 24, p. 487, March, 1936.

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substantially distortionless reproduction of the voltage acting on the grid, provided that operating conditions are such that E_{min} is not too low. Since projected cutoff is less than real cutoff, such operation causes the efficiency to be slightly lower than for true Class B operation, although much higher than for ordinary Class AB operation.

Class B audio-frequency amplifiers are ordinarily operated so that the grid is driven positive when the applied signal is sufficient to develop full output. This tends to distort the exciting voltage acting on the grids of the tubes. When operation is such that the grid is driven positive only at the peak of the cycle, the situation is the same as discussed in connection with Eq. (26). In some types of Class B amplifiers the tubes are operated with zero bias. Under these conditions the exciter must have sufficient power capacity to supply the energy dissipated at the grid of the Class B tubes, and must also have sufficiently low internal impedance so that the variations in instantaneous impedance offered by the grids during the cycle do not produce excessive distortion of the exciting voltage. In order to achieve this it is commonly necessary to couple the exciter to the grids of the push-pull Class B amplifier by means of a step-down transformer.

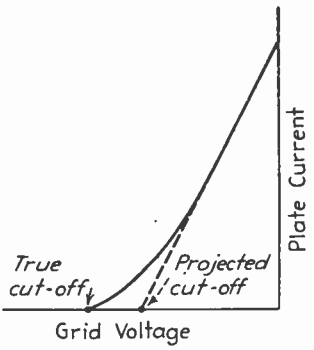


FIG. 30.—Curve illustrating meaning of projected cutoff.

The plate- and bias-supply voltages must have good regulation if best results are to be obtained in Class B operation. This is because the grid and plate currents vary with the average amplitude of the exciting voltage, and if the voltage regulation is poor the operating conditions will vary according to the amplitude of the applied signal. Regulation is much more important when the amplifier operates with a bias than when zero bias operation is used. In high-power amplifiers special compensating systems to improve the voltage regulation are often desirable.[1]

The frequency-response characteristics of Class B amplifiers depend upon the output transformer in much the same way as in Class A amplifiers. Thus the falling off in amplification at low frequencies is determined by the primary inductance of the output transformer, while the leakage inductance controls the falling off in gain at high frequencies. The exact relationships existing in Class B operation are much more complicated than in Class A amplifiers, however, because the plate currents of the individual tubes contain numerous harmonics. In particular, the intermittent character of the plate currents of the individual tubes in a Class B amplifier makes it important that the leakage inductance from one-half of the primary winding to the other half be extremely small. If this is not the case, transients will be produced in the primaries that will produce serious distortion.[2]

Class B amplifiers practically always employ triode tubes. The same tubes used in Class A and Class AB audio amplifiers and in Class C radio-frequency amplifiers are also suitable for Class B operation.[3] In addition there are special tubes designed for Class B operation at plate voltages of the order of 400, and requiring zero grid bias. These tubes have such a high amplification factor that "projected cutoff" is approximated by zero bias, and are characterized by unusually small grid current

[1] R. J. Rockwell and G. F. Platts, Automatic Compensation for Class B Bias and Plate Voltage Regulation, *Proc. I.R.E.*, Vol. 24, p. 553, April, 1936.

[2] A. Pen-Tung Sah, Quasi Transients in Class B Audio-frequency Push-pull Amplifiers, *Proc. I.R.E.*, Vol. 24, p. 1522, November, 1936.

[3] I. E. Mouromtseff and H. N. Kozanowski, Comparative Analysis of Water-cooled Tubes as Class B Audio Amplifiers, *Proc. I.R.E.*, Vol. 23, p. 1224, October, 1935.

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with positive grid voltage, and a grid-current characteristic that is reasonably linear.[1]

AUDIO-FREQUENCY AMPLIFIERS—MISCELLANEOUS

11. Feedback Amplifiers.[2]—In the feedback amplifier, a voltage derived from the amplifier output is superimposed upon the amplifier input in such a way as to oppose the applied signal in the normal frequency range (see Fig. 31). When this is properly done, the properties of the amplifier are modified in ways that are desirable for many purposes.

Effect of Feedback on Gain, Distortion, and Noise.—The amplification in the presence of feedback is given by the relation

$$\left.\begin{array}{l}\text{Voltage amplification}\\ \text{taking into account feedback}\end{array}\right\} = \frac{A}{1 - A\beta} = -\frac{1}{\beta}\frac{1}{1 - \frac{1}{A\beta}} \tag{36}$$

where A = amplification in the absence of feedback.[3]

β = fraction of the output voltage that is superimposed upon the amplifier input.

In this equation, the signs are so chosen that when the signal voltage is opposed by the voltage fed back, β is negative.

The quantity $A\beta$ can be termed the *feedback factor.* It represents the ratio of the voltage fed back and superimposed on the applied signal to the net voltage acting on the input of the amplifier (net input = applied signal plus feedback voltage). When the feedback factor $A\beta$ is large, Eq. (36) shows that the amplification becomes the reciprocal of the fraction β of the output voltage that is superimposed upon the amplifier input, *and is substantially independent of the characteristics of the amplifier itself.* When the feedback voltage is developed by a resistance network, β has zero phase angle and is independent of frequency. Under these conditions, the amplification will be substantially independent of frequency and will have negligible phase shift for all frequencies for which $A\beta$ is large. On the other hand, when the amplification is to vary with frequency in some particular way, this can be accomplished by making the feedback or β circuit have the same transmission *loss* characteristic as the desired *gain* characteristic.

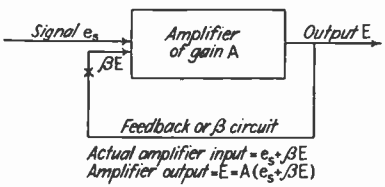


FIG. 31.—Schematic diagram of feedback amplifier.

Feedback reduces the nonlinear distortion produced in the amplifier for a given output according to the relation[4]

$$\left.\begin{array}{l}\text{Distortion}\\ \text{with feedback}\end{array}\right\} = \frac{\text{Distortion in absence of feedback}}{1 - A\beta} \tag{37}$$

[1] See L. E. Barton, Application of Class B Audio Amplifier to A-c Operated Receivers, *Proc. I.R.E.*, Vol. 20, p. 1085, July, 1931.

[2] H. S. Black, Stabilized Feed-back Amplifiers, *Elec. Eng.*, Vol. 53, p. 114, January, 1934; also, U.S. Patent 2,102,671, Dec. 21, 1937, to H. S. Black.

[3] The symbol μ is frequently used in the literature of feedback to represent amplification, but has here been replaced by A in order to avoid confusion with amplification factor.

[4] This assumes that the amplifier produces no distortion when reamplifying the distortion voltages fed back by the β circuit, and is quite accurate if the distortion in the absence of feedback is not large. For an exact analysis without this limitation, see Robert W. Sloane, Distortion in Negative Feedback Amplifiers, *Wireless Eng.*, Vol. 14, p. 259, May, 1937; also. p. 369, July, 1937; A. C. Bartlett, A Note on Negative Feedback, *Wireless Eng.*, Vol. 15, p. 90, February, 1938.

Ex. 1044-0403

If $A\beta$ is made large by employing a large amount of feedback, the result is a great percentage reduction in distortion and cross-modulation.

The signal-to-noise ratio is modified by feedback according to the relation

$$\frac{\text{Signal-to-noise ratio with feedback}}{\text{Signal-to-noise ratio without feedback}} = \frac{a_f}{a_0(1 - A\beta)} \tag{38}$$

where a_f and a_0 are the amplification between the place at which the noise is introduced and the output, with and without feedback, respectively. This equation assumes that the amount of noise introduced is the same both with and without feedback and that the output voltages are likewise the same in both cases. Feedback will greatly improve the signal-to-noise ratio when the noise is introduced in high-level parts of the amplifier, such as noise from a poorly filtered power supply in the plate circuit of the power tube.

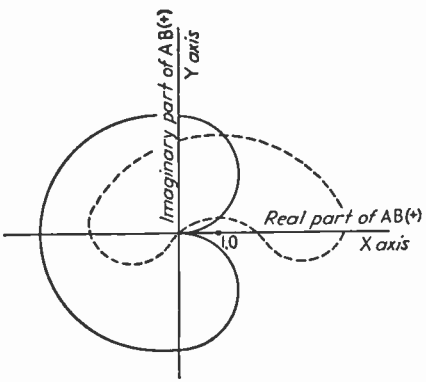


FIG. 32.—$A\beta$ curves plotted on complex plane. The two cases shown do not oscillate because the curves do not inclose the point 1, 0.

Feedback without Oscillations.[1]—The criterion for avoiding oscillations in feedback amplifiers is that when the real and imaginary parts of the feedback factor $A\beta$ and its conjugate are plotted on rectangular coordinates for frequencies from zero to infinity, with the real part along the X axis and the imaginary part along the Y axis, as shown in Fig. 32, the resulting curve must not inclose the point $X = 1$, $Y = 0$. With most amplifier circuits, this means that the feedback factor $A\beta$ should be negative in the mid-frequency range and that at high and low frequencies such that the phase of $A\beta$ is shifted $\pm 180°$ from the mid-frequency value, the magnitude of $A\beta$ must be less than unity.[2]

The fundamental factors required in the design of the feedback loop to achieve stability are discussed at length in Par. 27, Sec. 3. Briefly, the phase shift of the feedback loop (*i.e.*, phase of $A\beta$) depends upon the rate at which the magnitude of $A\beta$ varies with frequency. The more rapidly the transmission (*i.e.*, the magnitude of $A\beta$) varies with frequency, the greater tends to be the phase shift. Hence, when the phase shift is to be kept below 180°, the magnitude of $A\beta$ must not be allowed to vary too rapidly. Thus it is necessary in feedback amplifier systems to pay particular attention to the way in which the system behaves at frequencies outside the useful range, since it is here that $A\beta$ drops off in magnitude and hence it is here that large phase shifts tend to be encountered.

In the case of resistance coupling and output transformer coupling when no additional phase shift is introduced from the bias and screen impedances, the maximum phase shift that a single-stage amplifier can possibly produce will not exceed 90°.

[1] H. Nyquist, Regeneration Theory, *Bell System Tech. Jour.*, Vol. 11, p. 126, January, 1932; E. Peterson, J. G. Kreer, and L. A. Ware, Regeneration Theory and Experiment, *Proc. I.R.E.*, Vol. 22, p. 1191, October, 1934; D. G. Reid, The Necessary Conditions for Instability (or Self-oscillation) of Electrical Circuits, *Wireless Eng.*, Vol. 14, p. 588, November, 1937.

[2] The rather special case illustrated by the dotted curve in Fig. 32 violates this rule but still does not give oscillations, because the curve does not inclose the point 1, 0. Such an arrangement will, however, oscillate when the gain is reduced. Accordingly, when such an amplifier is first turned on, it will always break into oscillation as the filaments warm up and the gain is less than normal. If, however, these oscillations are momentarily stopped, as by short-circuiting one of the stages, they will not restart after the short circuit is removed.

Ex. 1044-0404

Even with additional phase shift from bias and screen impedances, or from resonances in the coupling system (as, for example, at low frequencies in the case of a resonated primary transformer or at high frequencies in the case of an interstage transformer having leakage inductance and distributed capacity), it is still rather unusual for the phase shift of a single stage amplifier to reach 180° before the magnitude of $A\beta$ has become less than unity.

With two-stage amplifiers it is still a relatively simple matter to avoid oscillation, but more care is needed in the design of the circuits. When the individual stages are such that the phase shift that each introduces does not exceed 90°, then any desired amount of feedback may be introduced by a resistance network without oscillations being produced. However, if additional phase shifts are present from the

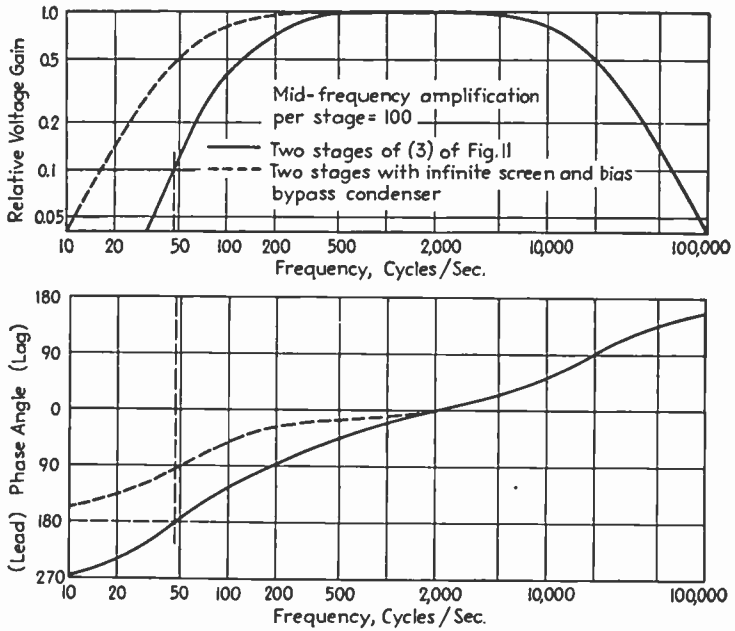


FIG. 33.—Amplifier characteristics showing different amplification and phase-shift characteristics at low frequencies.

coupling networks, from the screen and bias impedances, etc., then oscillation will occur in two-stage amplifiers when the feedback is large. This is illustrated by the solid curve in Fig. 33, where, with a two-stage amplifier of the type shown by curve 3 in Fig. 11, a 180° phase shift occurs at a frequency of 47.5 cycles, corresponding to an amplification that has dropped to 0.095 of the mid-range value. With a resistance network to develop feedback (*i.e.*, phase shift of β is zero at all frequencies), this requires that $A\beta < 10.5$ in the mid-frequency range if one is to have $A\beta < 1$ at the frequency of 47.5 cycles for which the phase shift is 180°. The corresponding value of β is 0.00105. This behavior is to be contrasted with the case of infinite bias and screen bypass condenser (shown dotted in Fig. 33) for which the phase shift does not reach 180° until the amplification has dropped to zero, so that oscillations are not possible under any conditions. If for some reason it is not possible to realize conditions corresponding to infinite bias and screen bypass condensers, and at the same time more feedback than that corresponding to $A = 10.5$ in Fig. 33 is desired, then the amplifier must be redesigned so that the amplification characteristic of one of the stages at low

frequencies is constant to a substantially lower frequency than the other stage. Doing so reduces the rate at which the amplification falls off, and so reduces the phase shift.

In amplifiers involving three or more stages the entire design revolves around controlling the amplification characteristics at high and low frequencies so that the rate at which the transmission $A\beta$ around the feedback loop falls off will not be so great as to lead to excessive phase shift and hence oscillation. When the three stages are identical, then even when the phase shift per stage has the minimum possible value

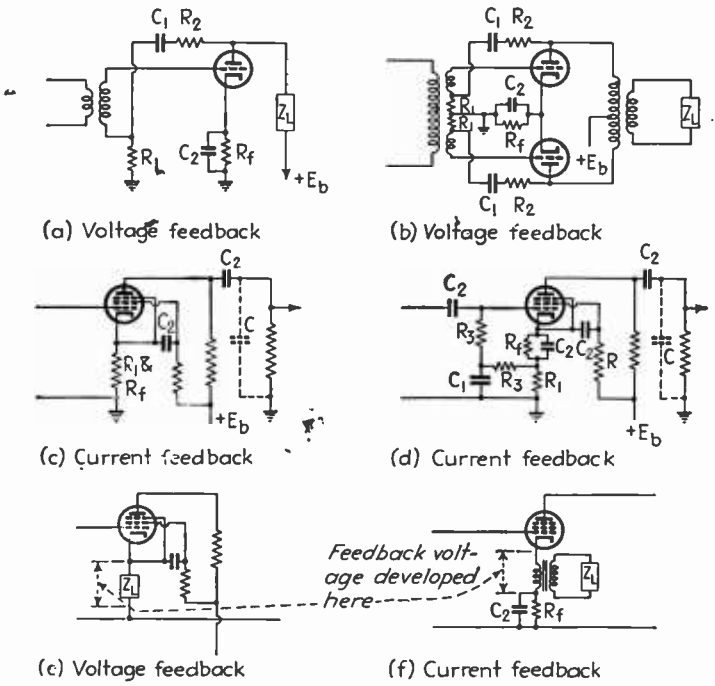


FIG. 34.—Typical feedback circuits involving a single stage of amplification.

of 90 degrees, it is necessary that $A\beta < 8$. In order to obtain a greater value of feedback it is necessary to reduce the rate at which the amplifier cuts off at high and low frequencies. A simple way of accomplishing this is to give one of the stages a flatter frequency response characteristic than the other two. Under these conditions the maximum value the feedback factor $A\beta$ can have, assuming a maximum possible phase shift of 90° per stage, is[1]

$$\left.\begin{matrix}\text{Maximum value}\\ \text{of feedback}\\ \text{factor } A\beta\end{matrix}\right\} = \frac{2(n+1)^2}{n} \tag{39}$$

$$\text{where for high frequencies } n = \frac{\left\{\begin{matrix}\text{frequency for 70.7 per cent response}\\ \text{of flat stage}\end{matrix}\right\}}{\left\{\begin{matrix}\text{frequency for 70.7 per cent response}\\ \text{of a less flat stage}\end{matrix}\right\}},$$

[1] Equation (39) was derived by Dr. R. R. Buss, former graduate student at Stanford.

Ex. 1044-0406

and for low frequencies $n = \dfrac{\left\{\begin{matrix}\text{frequency for 70.7 per cent response}\\ \text{of a less flat stage}\end{matrix}\right\}}{\left\{\begin{matrix}\text{frequency for 70.7 per cent response}\\ \text{of flat stage}\end{matrix}\right\}}$.

When n is different at low and high frequencies, the smallest value is the one to use in Eq. (39). If the phase shift per stage exceeds 90°, or if β introduces a phase shift, the allowable mid-frequency feedback factor $A\beta$ will be less than given by Eq. (39).

An alternative procedure for designing an amplifier with three stages, and the procedure that must be followed if more than three stages are involved, is described in Par. 27, Sec. 3. This consists in designing the amplifier initially to give a wider frequency response band than is actually required and then in modifying this characteristic at high and low frequencies by placing expedients such as are illustrated in Fig. 93, Sec. 3, in the amplifier network or in the feedback connection.

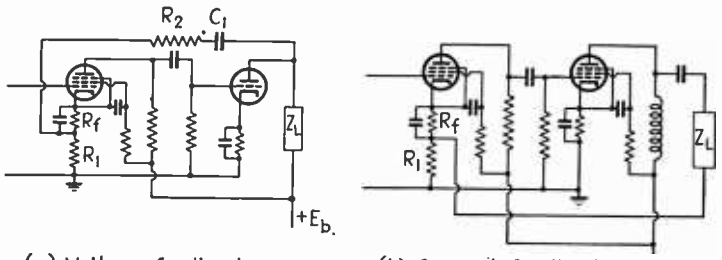


FIG. 35.—Typical feedback circuits involving two stages of amplification.

Practical Feedback Amplifiers.[1]—Typical feedback amplifier circuits are shown in Figs. 34 to 36. In these, a uniform notation is employed as follows:

R_1 = resistance across which feedback voltage is developed.
R_2 = resistance that, in conjunction with R_1, forms a voltage divider for making the feedback voltage the desired fraction of the output voltage.
R_3 = a high resistance for grid-leak or isolating purposes.
R_f = resistance across which bias voltage is developed.
C_1 = blocking condenser of such capacity as to introduce negligible reactance or phase shift at frequencies that are amplified appreciably.
C_2 = by-pass condenser.

When the feedback factor $A\beta$ is large, the feedback voltage developed across R_1 has a value almost the same in both magnitude and phase as the signal voltage applied to the amplifier input. This fact determines the way in which the feedback will affect the output voltage, since the output voltage must be whatever value is required to make the feedback voltage developed across R_1 approximate the applied signal.

The circuits of Figs. 34 to 36 can be divided into two general types, namely, those in which the feedback voltage is derived directly from the output voltage of the amplifier (*voltage feedback*), and those in which the feedback voltage is derived from the current flowing through the output of the amplifier (*current feedback*). The type of feedback involved is indicated on each of the circuits in Figs. 34 to 36. The effect of feedback upon the amplifier behavior is somewhat different with the two types of feedback. Thus voltage feedback in connection with an output transformer, as shown in Fig. 37*a*, makes the voltage across the primary reproduce accurately the signal voltage, and so improves the frequency response at low frequencies and

[1] F. E. Terman, Feedback Amplifier Design, *Electronics*, Vol. 10, p. 12, January, 1937; J. R. Day and J. B. Russell, Practical Feedback Amplifiers, *Electronics*, Vol. 10, p. 16, April, 1937.

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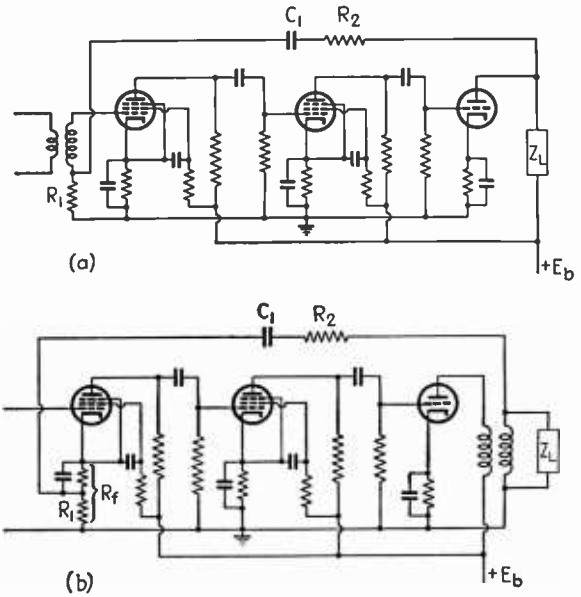


FIG. 36.—Typical feedback circuits involving three stages of amplification. In practice these circuits may be modified by expedients such as shown in Fig. 93, Sec. 3 in order to control $A\beta$ at high and low frequencies.

reduces amplitude distortion. The arrangement does not, however, improve the response at high frequencies. On the other hand, when current feedback is used with the same output transformer, as in Fig. 37*b*, the action of feedback is to tend to make the current through the transformer primary reproduce the wave shape of the applied signal and be independent of the applied frequency. The output then tends to be constant at high frequencies but falls off more at low frequencies than in the absence of feedback. Also, at low frequencies, amplitude distortion due to saturation in the core of the output transformer will be increased.

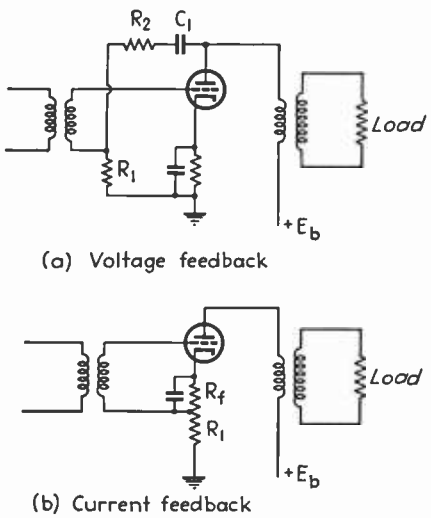


FIG. 37.—Examples of voltage and current feedback.

All the circuits shown in Figs. 34 to 36 use a resistance network to obtain feedback. It is possible to introduce reactance elements in the feedback network, and this is sometimes done, although it may lead to additional phase shifts.

Effect of Feedback on Frequency Response Characteristics.[1]—It was noted in connection with Eq. (36) that when the feedback factor $A\beta$ is large, the voltage amplifi-

[1] F. E. Terman and W. Y. Pan, Frequency Response Characteristic of Amplifiers Employing Negative Feedback, *Communications*, Vol. 19, p. 5, March, 1939; George H. Fritzinger, Frequency

Ex. 1044-0408

cation approximates $-1/\beta$. The voltage gain is hence the inverse of the loss characteristic of the β circuit. In particular, if the β network is a resistance, then substantially constant amplification with zero phase shift independent of frequency will be obtained when $A\beta$ is large. However, at high and low frequencies the amplification A tends to fall off, so that even when $A\beta$ is large in the mid-frequency range, it will become small and ordinarily have a large phase shift at extreme frequencies.

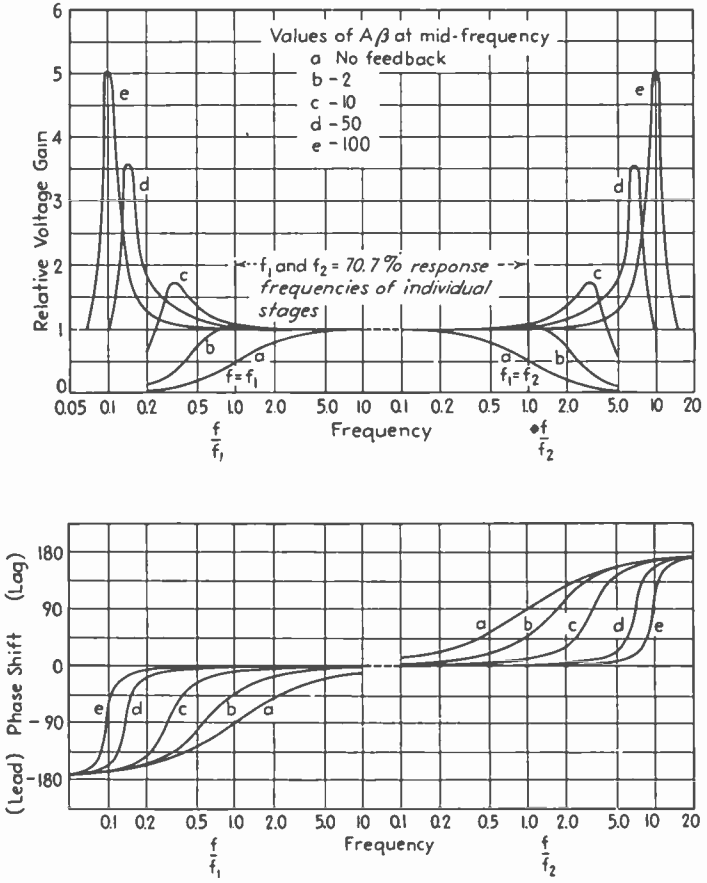


FIG. 38.—Effect of feedback factor $A\beta$ on the frequency response characteristics of a two-stage resistance-coupled amplifier having no screen grid or bias impedance.

This results in peaks of amplification at high and low frequencies if the feedback factor $A\beta$ is large in the mid-frequency range and the phase shift of $A\beta$ can approach or exceed 180° at extreme frequencies. Typical behavior of a feedback amplifier under such conditions is shown in Fig. 38. Although the use of a large amount of feedback improves the amplification and phase-shift characteristics in the middle range of frequencies, it is apparent that feedback may introduce undesirable characteristics at very low and high frequencies.

Discrimination by Inverse Feedback, *Proc. I.R.E.*, Vol. 26, p. 207, February, 1938; F. A. Everest and H. R. Johnston, The Application of Feedback to Wide-band Output Amplifiers, *Proc. I.R.E.*, Vol. 28, p. 11, February, 1940.

Ex. 1044-0409

When peaks such as shown in Fig. 38 are not too pronounced, they can be equalized at low frequencies by a resistance-condenser combination, and at high frequencies by a resistance-inductance network, as shown in Fig. 39. These circuit elements are placed outside the amplifier proper (*i.e.*, outside the feedback loop) and are so designed as to reduce the peak of amplification to the mid-frequency gain. In this way it is possible to obtain relatively flat responses up to the frequency at which the peak occurs, followed by a sharp drop in gain, as shown.

Control of Effective Internal Impedance of Amplifiers by Feedback.[1]—The internal impedance of an amplifier can be defined as the impedance that the load observes when one looks back toward the plate circuit of the amplifier tube. The effective value of this impedance can be controlled over wide limits by the use of feedback.

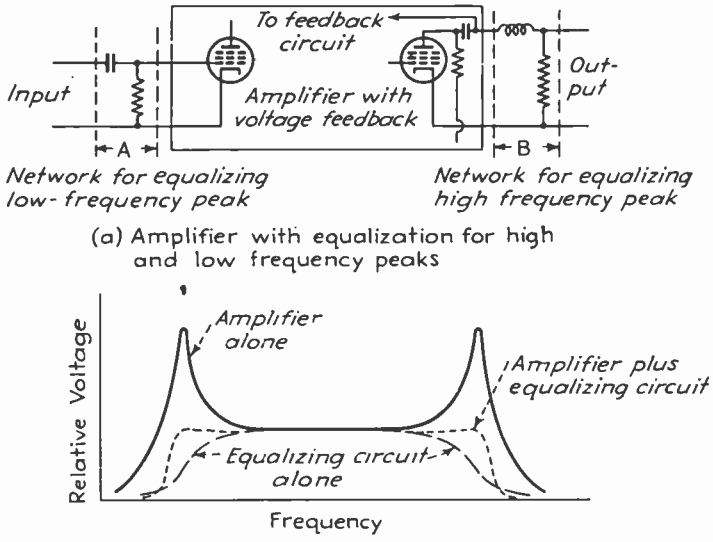


FIG. 39.—Circuit for equalizing high- and low-frequency peaks produced by feedback, together with resulting characteristics.

Thus negative voltage feedback tends to make the output voltage constant, irrespective of load impedance. This effect is equivalent to decreasing the output impedance of the amplifier. The effect of feedback on the output impedance of an amplifier is given by the equation

$$\left.\begin{matrix}\text{Effective output}\\ \text{impedance of amplifier}\end{matrix}\right\} = \frac{R_p - \mu A_1 \alpha}{1 - \mu A_1 \beta} \tag{40}$$

where R_p = plate resistance of output tube (equals output impedance in the absence of all feedback).

μ = amplification factor of output tube.

A_1 = amplification between grid of amplifier input and grid of last tube, in the absence of feedback.

$$\alpha = \text{current feedback factor} = \frac{\left\{\begin{matrix}\text{feedback voltage developed}\\ \text{by current in load}\end{matrix}\right\}}{\text{current in load}}.$$

[1] H. F. Mayer. Control of the Effective Internal Impedance of Amplifiers by Means of Feedback, *Proc. I.R.E.*, Vol. 27, p. 213, March, 1939.

Ex. 1044-0410

$$\beta = \text{voltage feedback factor} = \frac{\left\{\begin{matrix}\text{feedback voltage derived} \\ \text{from output voltage}\end{matrix}\right\}}{\text{output voltage}}.$$

The voltage and current feedback factors may be either positive or negative, according to whether the feedback aids or opposes the signal voltage, respectively.

The various effects that can be produced on the output impedance by feedback are revealed by Eq. (40). When $\alpha = \beta = 0$, there are no feedback effects, and the output impedance equals the plate resistance of the tube. When there is no current feedback ($\alpha = 0$), then the output impedance is $R_p/(1 - \mu A_1\beta)$, which will be greater or less than the plate resistance of the tube according to whether the feedback is positive or negative, respectively. On the other hand, when there is no voltage feedback ($\beta = 0$), the effective output impedance is $(R_p - \mu A_1\alpha)$. This will be less or more than the plate resistance of the tube, according to whether the feedback is positive or negative, respectively. In the special case where $\mu A_1\alpha = R_p$, it will be noted that the effective output resistance is zero, while when $\mu A_1\alpha > R_p$, the effective output impedance will be negative.

The gain of the amplifier with combined voltage and current feedback is given by the equation

$$\left\{\begin{matrix}\text{Voltage amplification} \\ \text{taking into account} \\ \text{feedback}\end{matrix}\right\} = \frac{A_0}{1 - A_0\left(\beta + \frac{\alpha}{Z_L}\right)} \tag{41}$$

where Z_L is the load impedance and A_0 the voltage amplification without feedback. This is identical with Eq. (36) except that $A\beta$ has been replaced by $A_0\left(\beta + \frac{\alpha}{Z_L}\right)$. When the two types of feedback are of opposite sign and are so related that

$$\beta = -\frac{\alpha}{Z_L} \tag{42}$$

it will be noted that the positive and negative feedbacks balance each other as far as gain is concerned, although they will still influence the output impedance.

The criterion for avoiding oscillation with combined current and voltage feedback is the same as discussed in connection with Fig. 32, except that now the effective feedback factor is $A_0\left(\beta + \frac{\alpha}{Z_L}\right)$ instead of being $A\beta$.

Modified Forms of Feedback.—A result equivalent to that given by negative feedback can be obtained with other circuit arrangements. In the arrangement shown in Fig. 40*a*, one has an ordinary negative feedback amplifier in which the first stage is designed to have as nearly ideal characteristics as possible.[1] Positive feedback is derived from the output of this first stage and is balanced against the negative feedback so that in the mid-frequency range the two feedbacks cancel. The amplification characteristic of the entire amplifier will then tend to be the same as the amplification characteristics of the carefully designed first stage, since any departure of the output from this condition produces an unbalance between the feedbacks that opposes the departure.

Another arrangement is shown schematically in Fig. 40*b*. Here A is an ordinary amplifier to which there is applied a signal voltage e that delivers an output voltage E. A fraction k of the output voltage is balanced against the signal voltage e, and a voltage proportional to the difference between e and kE is reinserted in the amplifier with such polarity that E is altered in a manner that makes $|e - kE|$ less than would

[1] E. L. Ginzton, Balanced Feedback Amplifiers, *Proc. I.R.E.*, Vol. 26, p. 1367, November, 1938.

Ex. 1044-0411

otherwise be the case. Such an amplifier acts in much the same way as one having negative feedback.[1]

Practical Use of Negative Feedback in Audio-frequency Amplifiers.[2]—Negative feedback is widely used in audio power amplifiers for the purpose of reducing nonlinear

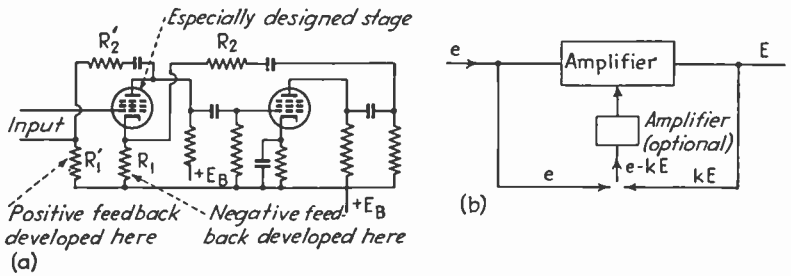
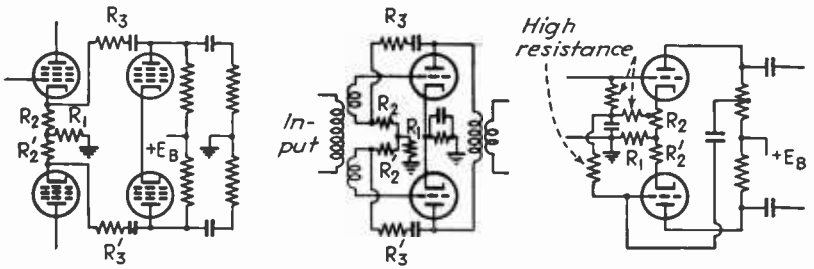


FIG. 40.—Alternative methods of obtaining behavior similar to that obtained with negative feedback.



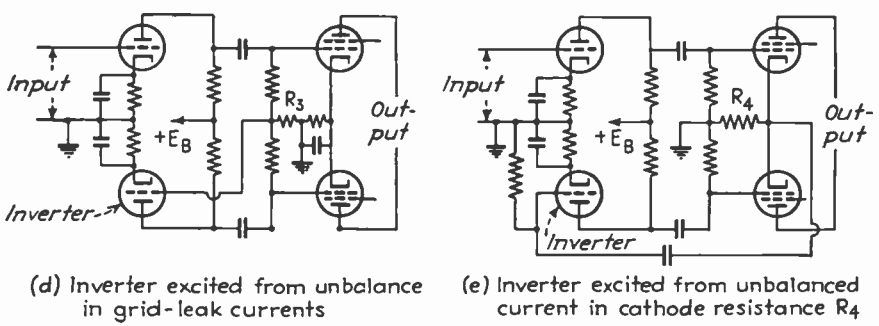


FIG. 41.—Circuits for using feedback to minimize unbalances between the two sides of a push-pull amplifier.

distortion. When pentode and beam tubes are used, negative feedback can also be used to reduce the effective output impedance of the tube to a value sufficiently low to damp out transients in loud-speakers.

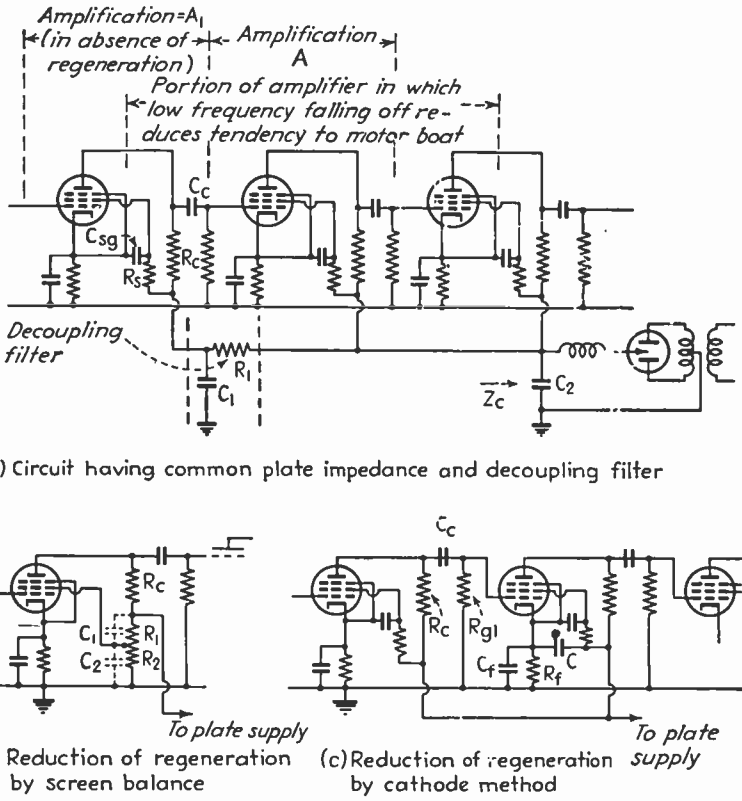
[1] Unpublished results of the author show that this circuit has all the important properties of negative feedback circuits. See also P. O. Pedersen, A Distortion-free Amplifier, *Proc. I.R.E.*, Vol. 28, p. 59, February, 1940, who gives a slightly different arrangement.

[2] The application of negative feedback to radio transmitters is given in Sec. 9, Par. 1.

Ex. 1044-0412

Negative feedback is extensively used in telephone repeaters to stabilize gain and reduce distortion and cross-modulation. Gain stability is important in long toll circuits containing many amplifiers, since the allowable variation in gain per individual amplifier with tube replacements and variations of supply voltages is very small. The reduction of cross-modulation obtainable with negative feedback is very important where a number of carrier channels are amplified simultaneously.

Feedback can be employed in push-pull amplifiers to reduce unbalance between the two sides of the push-pull system. Circuits for doing this are illustrated in Fig.



(a) Circuit having common plate impedance and decoupling filter

(b) Reduction of regeneration by screen balance

(c) Reduction of regeneration by cathode method

FIG. 42.—A three-stage amplifier with common plate supply having an internal impedance Z_c, and various means for reducing regeneration caused by Z_c.

41. In the first three of these, unbalance in the push-pull stages produces a current through the resistance R_1. This results in the development of a feedback voltage that is applied to the tubes in such a manner as to reduce the difference in the output of the two sides. In these circuits, R_2 is a resistance for developing ordinary negative feedback, and can be short-circuited or by-passed if negative feedback is not desired for other than balancing purposes. In the final two circuits, the excitation of the phase inverter tube is obtained, respectively, from a resistor R_3 used to return the two grid leaks to ground, and from the cathode resistor R_4. These resistors have a voltage developed across them corresponding to the unbalance between the two sides of the amplifier. This unbalanced voltage is then used to excite the grid of the phase-inverter tube in such a way as to make the output of the phase inverter reduce the

Ex. 1044-0413

unbalance. By employing large gain in the inverter stage, the residual unbalance between stages will normally be less than 10 per cent, even with wide variations in circuit constants.

Feedback is widely used in laboratory amplifiers and in amplifiers incorporated in measuring equipment. By making the feedback factor large and using a resistance feedback network, it is possible to obtain characteristics that approach very closely to perfection. In particular, nonlinear distortion can be made negligible, phase shift reduced virtually to zero, the amplification maintained almost absolutely constant over wide frequency bands, and the gain is substantially unaffected by tube replacements and ordinary variations in supply voltages.[1]

12. Regeneration in Multistage Amplifiers.—When several stages of amplification are connected in cascade, there is the possibility of amplified energy being transferred from stages of high power level back to stages of lower power level. Such energy transfer is termed *regeneration*, and is a form of feedback that is to be avoided, because it gives rise to undesired modifications of the frequency-response characteristic, and in some cases causes oscillations.

Regeneration Arising from a Common Plate Impedance.—Most regeneration troubles occurring in audio-frequency amplifiers arise from the internal impedance of the battery or rectifier-filter system used to supply plate voltage to several stages. The situation arising from the presence of such a common impedance is shown in Fig. 42*a*, where it is apparent that amplified current existing in the plate circuit of the final amplifier stage will develop voltage across the impedance Z_c and that this voltage will be applied to the plate circuits of the stages of lower power level and cause regeneration. This alters the amplification characteristic, and in severe cases gives rise to oscillations.

In a practical analysis of the effect of a common plate impedance, it is necessary to consider only the energy transfer from the final stage of the amplifier back to the plate circuit of the first stage. This is because energy transfer between other stages is negligible by comparison, as a result of the smaller difference in power levels involved. To the extent that this is true, the total effect of regeneration on the entire system is to alter the effective amplification between the grid of the first (input) tube and the grid of the next amplifier tube by the factor[2]

$$\frac{\left.\begin{matrix}\text{Actual amplification of}\\ \text{first stage}\end{matrix}\right\}}{\left.\begin{matrix}\text{Amplification of first}\\ \text{stage with no regeneration}\end{matrix}\right\}} = \frac{1}{1 - KA\left(\frac{A_1}{\beta_1\gamma_1}\right)\frac{Z_c}{Z_1}\frac{g_m'}{g_m}\beta'\gamma'D} \tag{43}$$

where A = amplification from grid of second tube to the grid of the last tube, as indicated in Fig. 42*a*.

A_1 = actual amplification from grid of first tube to grid of second tube in the absence of regeneration.

β_1 and γ_1 = factors taking into account loss in amplification in first stage resulting from screen and bias impedances, respectively.

$\frac{A_1}{\beta_1\gamma_1}$ = amplification of first stage when screen and bias impedances produce no falling off.

[1] F. E. Terman, R. R. Buss, W. R. Hewlett, and F. C. Cahill, Some Applications of Negative Feedback with Particular Reference to Laboratory Equipment, *Proc. I.R.E.*, Vol. 27, p. 649, October, 1939.

[2] Equation (43) assumes that the voltage developed across the common impedance produces negligible voltage upon the screen electrode of the first tube. Under practical conditions this assumption is always realized, since the impedance formed by R_1C_{sg} in the screen circuit serves as an additional decoupling filter having proportions such as effectively to isolate the screen even at very low frequencies.

Ex. 1044-0414

g_m = transconductance of first tube receiving anode voltage from the common plate supply.

g'_m = transconductance of final tube of amplifier.

K = factor such that $Kg'_m\beta'\gamma'$, multiplied by voltage on grid of last tube, gives the a-c current through the common plate impedance (see below).

Z_c = internal impedance of source of anode voltage.

D = transmission factor of decoupling filters in lead from common plate supply to the plate circuit of first tube (see below).

Z_1 = impedance between output of decoupling filter (or output of power supply when decoupling filters are not used) and the plate electrode of the first tube (in Fig. 42*a* $Z_1 = R_c$).

β' and γ' = factors taking into account loss in amplification in last stage resulting from screen and bias impedances, respectively.

The factor K in Eq. (43) is defined by the relation

$$K = \frac{\left\{\begin{matrix}\text{Admittance of circuit element} \\ \text{connecting plate supply to plate} \\ \text{electrode of final tube}\end{matrix}\right\}}{\frac{1}{Z_L} + \frac{1}{R_p}} \tag{44}$$

where Z_L is the total impedance in the plate circuit of the final tube and R_p is the plate resistance of this tube. The value of K depends upon the circuit constants, the frequency, and upon whether the tube is a triode or pentode. At low frequencies, K nearly always exceeds 0.5, and with practical amplifiers is always at least 0.2 under conditions where regeneration from the common plate impedance is important. At the same time K cannot exceed unity in practical cases.

The factor D takes into account the effect of a decoupling filter such as R_1C_1 in Fig. 42*a*. Such a filter is for the purpose of making the feedback voltage reaching the plate circuit of the first tube less than the voltage developed across the common impedance. On the assumption, usually justified, that Z_c is considerably less than R_1 and that the reactance of C_1 is considerably less than the impedance seen in looking from C_1 toward the first tube, one has for a single-stage filter

$$D = \frac{\left(\frac{1}{\omega C_1}\right)}{\sqrt{R_1^2 + \left(\frac{1}{\omega C_1}\right)^2}} \cong \frac{\left(\frac{1}{\omega C_1}\right)}{R_1} \tag{45}$$

Values of D can be obtained graphically with the aid of Fig. 43. When two filter stages are connected in tandem, the effective reduction factor is approximately the product of the reduction factors of the individual filters.

Circuit Design to Minimize Regeneration and Eliminate Motorboating.—Multistage amplifiers should be so designed that there is negligible regeneration in the useful frequency range and so that there is not sufficient regeneration at frequencies outside the normal operating range to produce oscillations.

In order for regeneration to have negligible effect, it is necessary for the factor $KA(A_1/\beta_1\gamma_1)(Z_c/Z_1)(g'_m/g_m)\beta'\gamma'D$ in Eq. (43) to be small compared with unity. To insure absence of oscillation, this quantity must have an absolute value less than unity.[1]

[1] Oscillations will not be present even when $|KA(A_1/\beta_1\gamma_1)(Z_c/Z_1)(g_m'/g_m)\beta'\gamma'D| \lesssim 1$, provided that under these conditions, the phase of the quantity is such as to give negative rather than positive feedback. For practical purposes, however, the somewhat simpler rule stated is a more convenient guide, and errs on the side of safety. The exact criterion for avoiding oscillation is the same requirement that $A\beta$ must satisfy as discussed in connection with Fig. 32.

Ex. 1044-0415

Regeneration can be easily made small or negligible for frequencies in and above the normal audio range, by shunting a capacity (C_2 in Fig. 42*a*) across the output of the common plate supply, and, where the gain is large, as is the case with more than two stages, by the use of one or two sections of decoupling filter (R_1C_1 in Fig. 42*a*). However, at low frequencies, the shunt condenser across the power supply is ineffective as a by-pass, and the isolation obtained from the decoupling filters becomes small because of the increasing reactance of condensers at low frequencies. The result in practical amplifiers is that regeneration is frequently sufficient at low frequencies to produce oscillations even when the regeneration has negligible effect upon the ampli-

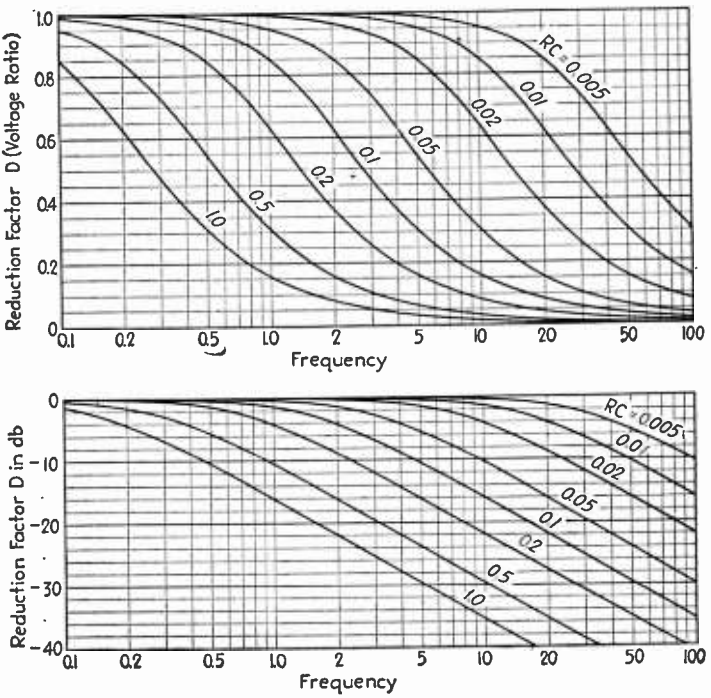


FIG. 43.—Curves giving the reduction D produced by resistance-capacity filter on the assumption that the impedance on the source side is small compared with R_1, and that the impedance on the load side is large compared with $1/\omega C_1$.

fication in the normal frequency range. Such oscillations are referred to as *motorboating*, and commonly have a frequency of only a few cycles per second.

Motorboating cannot exist if $|KA(A_1/\beta_1\gamma_1)(Z_c/Z_1)(g'_m/g_m)\beta'\gamma'D| < 1$. To accomplish this, it is necessary to rely mainly upon the low-frequency falling off in amplification produced by the tubes and circuits between the plate electrode of the first tube and the plate electrode of the final tube.[1] *Accordingly, multistage amplifiers operated from a common plate supply should never be designed to have a better low-frequency response between the plate of the first tube and the plate of the final tube than is actually required. In particular, it is very desirable that whenever possible the design of the plate*

[1] It will be noted that this includes the falling off due to screen grid and bias impedance of all tubes but the first and the effect of coupling networks in all of the plate circuits except in the plate circuit of the final tube.

Ex. 1044-0416

coupling networks and screen impedances in this section of the amplifier be so coordinated as to give maximum falling off in low-frequency amplification.

A power supply having low internal impedance at low frequencies is also helpful in eliminating motorboating. At low frequencies, the common impedance has a value approximating the equivalent internal d-c resistance corresponding to the d-c voltage regulation of the power supply; *i.e.*,

$$\text{Equivalent } Z_c \text{ at low frequencies} = \frac{\Delta E_B}{\Delta I_B} \tag{46}$$

where ΔE_B is the drop in power-supply voltage produced by increasing the output current by ΔI_B. The common impedance can be made small by using low-resistance chokes in the filter system and employing a rectifier tube with low voltage drop. The common impedance can be made negligible at low frequencies by using a power supply provided with an electronic voltage regulating system that maintains the potential across Z_c constant (see Par. 8, Sec. 8).

The tendency for regeneration and motorboating to exist can be greatly reduced by employing circuits in which the feedback from the common plate impedance to the plate circuit of the first tube is balanced by an equal and opposite feedback to some other point in the amplifier.[1] Arrangements for accomplishing this result are shown in Figs. 42*b* and 42*c*. In the first of these, the screen voltage is derived from a voltage divider R_1R_2 connected across the plate supply, so that any feedback voltage developed across this common impedance will be applied to the screen-grid electrode as well as to the plate circuit.[2] By selection of the proper ratio R_1/R_2 for the voltage divider (which can be readily done by trial), it is possible theoretically to reduce to zero all feedback (and also hum) from the common plate circuit to the amplifier stage thus balanced. This action is independent of frequency if the condenser C_2 that would normally be used to by-pass the screen to ground is balanced by a corresponding condenser C_1 so that $C_2/C_1 = R_1/R_2$. A similar result is achieved in the circuit of Fig. 42*c*, except that here the balancing voltage is applied between the cathode and ground of the following tube.[3] The required circuit proportions are

$$\begin{aligned} \frac{C_f}{C} &= \frac{R_c}{R_{gl}}\left(1 + \frac{R_{gl}}{R_p}\right) \\ \frac{C_c}{C} &= \frac{R_f}{R_{gl}}\left(1 + \frac{R_c}{R_p}\right) \end{aligned} \tag{46a}$$

The notation is illustrated in Fig. 42*c*. The circuit proportions called for by Eq. (46*a*) are of the approximate magnitude that would be used in ordinary audio-amplifier design. When carefully constructed, the balancing systems shown in Fig. 42*b* and *c* will give a reduction in the low-frequency feedback (and power-supply hum) to a particular stage of 40 db or more.

Additional means available for reducing regeneration at low frequencies include the use of push-pull amplification, particularly in the final stage, and the use of separate power supplies for different stages. With push-pull amplification in the final stage, this stage develops voltage across the source impedance only to the extent that the two sides of the amplifier are unbalanced, so that a considerable reduction in the tendency to regenerate is obtained. Where the first stage of the amplifier is also push-pull, the voltage developed across the common impedance is applied to the two

[1] Such circuit arrangements also discriminate equally against hum in the power-supply system.

[2] K. B. Gonser, A Method of Neutralizing Hum and Feedback Caused by Variations in the Plate Supply, *Proc. I.R.E.*, Vol. 26, p. 442, April, 1938.

[3] A discussion of this method of balancing regeneration, together with experimental results, is given by W. Y. Pan, Circuit for Neutralizing Low-frequency Regeneration and Power-supply Hum. *Proc. I.R.E.*, vol. 30, p. 411, September, 1942.

Ex. 1044-0417

sides of this push-pull stage in the same phase, and so cancels out to the extent that the two sides of the input stage are balanced.

In extreme cases, where the amplification involved is very great or where the amplifier must maintain its gain down to very low frequencies, it is always possible to resort to separate power supplies for the low-level and high-level stages of the amplifier, and thereby reduce the tendency toward regeneration.

Regeneration from Stray Couplings.—Although the most troublesome regeneration problems arising in audio-frequency amplifiers come from the impedance of a common power supply, it is also possible for stray electrostatic and magnetic couplings to cause regeneration. Such sources of regeneration are most effective at the higher frequencies, and can be readily taken care of by proper shielding and by proper spacing and orientation of parts.

Energy transfer between stages through electrostatic couplings can be eliminated by providing grounded shields for glass tubes or grounding the envelopes of metal tubes, by proper placing of wiring and parts that are at widely different power levels, and in severe cases by the use of supplementary electrostatic shielding.

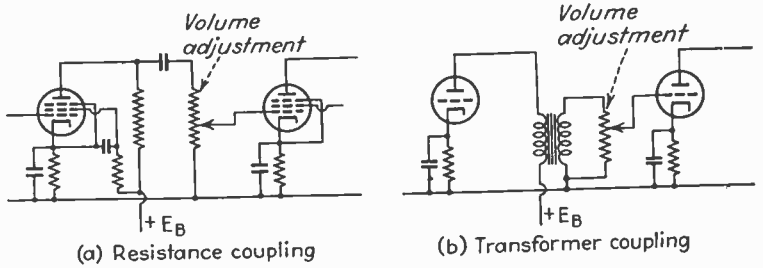


FIG. 44.—Common methods of controlling volume in resistance- and transformer-coupled amplifiers.

The chief source of magnetic coupling is the mutual inductance between transformers associated with stages operating at widely different power levels, as the input and output transformers of an amplifier. Regeneration from this source can be controlled by adequate spacing between transformers, by orientation of cores to reduce mutual inductance, and by use of transformers that are magnetically shielded or otherwise arranged to be relatively immune to external fields.

Another form of energy transfer between stages arises from the grid-plate capacity of the amplifier tube. This has an effect equivalent to modifying the equivalent input impedance of the tube, and is discussed in Par. 25.

13. Amplitude and Volume Control in Audio-frequency Amplifiers.—The volume, amplitude range, peak level, etc., in audio-frequency amplifiers can be controlled by a variety of devices,[1] the most important of which follow.

Manual Control of Volume.—Manually operated volume controls in audio-frequency amplifiers must be so arranged that the frequency response is not adversely affected by the volume setting. The control must also be located at a point of sufficiently low-power level in the amplifier that there is no possibility of stages being overloaded before the control can reduce the volume level.

The standard method of controlling volume in resistance-coupled amplifiers is shown in Fig. 44*a*, and makes use of a grid leak in the form of a high-resistance potentiometer. Such a volume control produces a slight improvement in the high-frequency response at low-volume settings.

[1] A discussion and classification of these devices is given by S. B. Wright, Amplitude Range Control, *Bell System Tech. Jour.*, Vol. 17, p. 520, October, 1938; and A. C. Norwine, Devices for Controlling Amplitude Characteristics of Telephonic Signals, *Bell System Tech. Jour.*, Vol. 17, p. 539, October, 1938.

Ex. 1044-0418

Volume control in transformer-coupled amplifiers is difficult to obtain without causing serious variations in the frequency response characteristics. When an amplifier contains both resistance- and transformer-coupled stages, it is preferable to control the volume in the resistance-coupled stages. Where this is not possible, the

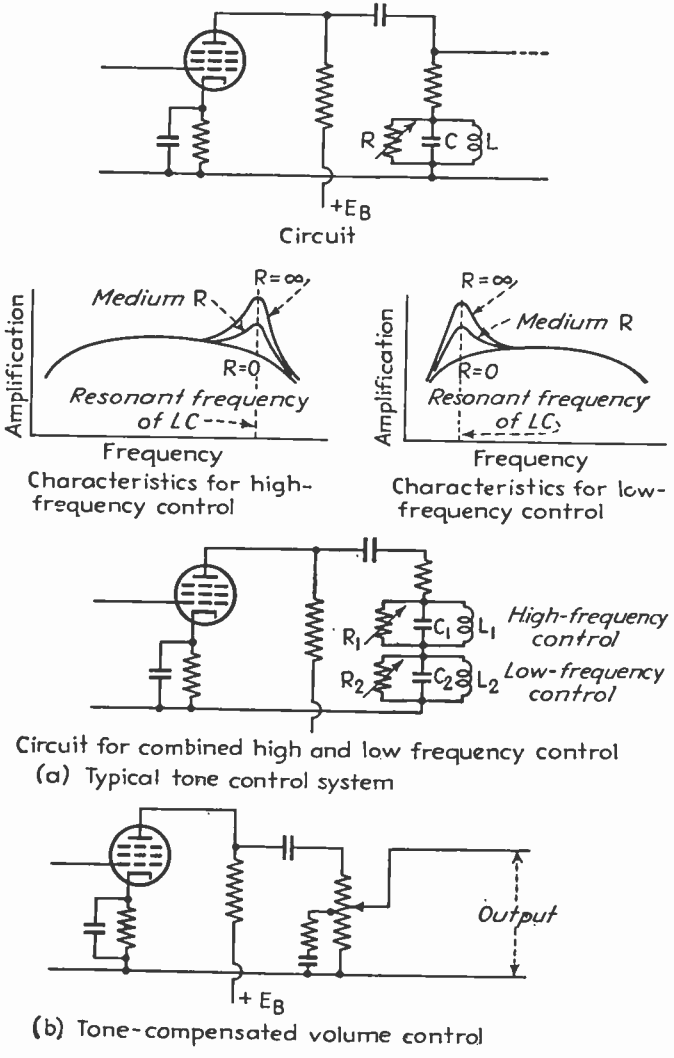


FIG. 45.—Typical tone-control circuits.

usual arrangement is to place a high-resistance potentiometer across the transformer secondary, as in Fig. 44*b*, and preferably employ a transformer designed to operate with resistance loading across its secondary.

Tone Controls and Tone-compensated Volume Controls.—Audio amplifiers are frequently provided with controls for altering the high-frequency and low-frequency characteristics. These are termed *tone controls*, and are for the purpose of modifying

Ex. 1044-0419

the character of the amplifier output to give new esthetic effects, to compensate for acoustic characteristics, reduce hum, reduce hiss or scratch, etc. Typical tone-control circuits are shown in Fig. 45*a*. Such circuits should preferably exert their control upon the high-frequency and low-frequency amplification without altering the mid-frequency gain, although in many cases this ideal is not achieved.

The characteristics of the ear are such that the apparent loudness of low-frequency tones relative to middle and high-frequency tones is less as the volume level is reduced. In order to correct for this effect, volume controls are sometimes arranged so that as the volume control reduces the gain, the reduction is less for low frequency than for middle and high-frequency components. Such an arrangement is termed a *tone-compensated volume control.* A typical example is shown in Fig. 45*b*.[1]

Volume Expanders and Compressors—Companders.—A volume compressor is a device that reduces the amplification when the signal being amplified is large and increases the amplification when the signal is small. Compressors are used to reduce the volume range in recording sound on film and disk, and are also sometimes used in radio-telephone transmission, etc.

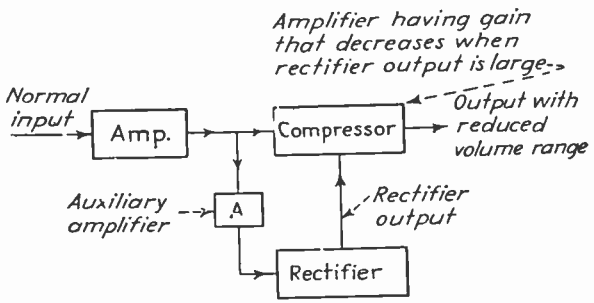


FIG. 46.—Schematic diagram of a volume compressor.

A volume expander is a device that operates in such a way as to increase the amplification when the average power level of the signal is high, and to reduce the amplification when the average power level of the signal is small, thereby causing the loud passages to be still louder and the weak passages to have their weakness accentuated. Expanders are used to counteract the compression of volume range that is necessary in sound recording, etc., and also to introduce added esthetic effects.

Volume expanders (or compressors) are essentially amplifiers in which the amplification is varied in accordance with the envelope amplitude of the signal. This control can be obtained in various ways. One method is illustrated schematically in Fig. 46. Here the signal to be compressed (or expanded) is amplified by an auxiliary amplifier *A* and then rectified in such a way that there is developed across a resistance-condenser combination a d-c voltage that tends to follow the envelope of the signal. This d-c voltage is used to vary the bias, and hence amplification, of a resistance-coupled amplifier tube using a variable-mu tube. By proper attention to circuit details, particularly the time constants, it is possible to control the character of the compression (or expansion) and also the quickness with which the amplification is increased and decreased.[2]

[1] Charts for assisting in the design of tone-compensated volume controls of this type are given by P. A. D'Orio and Rinaldo de Cola, Bass Compensation Design Chart, *Electronics*, Vol. 10, p. 38, October, 1937.

[2] Circuit diagrams (with constants) of volume expanders are given by C. M. Sinnett, Practical Volume Expansion, *Electronics*, Vol. 8, p. 14, November, 1935; Harry Paro, Public Address AVC *Electronics*, Vol. 10, p. 24, July, 1937.

Another method of obtaining volume expansion (or compression) is to introduce feedback in an

Ex. 1044-0420

A system involving a volume compressor to reduce the volume range for transmission, followed by an expander at the receiving point to restore the original range, is referred to as a compander, and is used in radio-telephone extensions of telephone toll systems to improve the signal-to-noise ratio.[1]

Automatic Volume Control—Vogads.—Automatic volume control (abbreviated A.V.C.) utilizes the rectified audio-frequency currents to control the gain of an amplifier in such a manner that the peak output is maintained at substantially constant level. A.V.C. systems are used for insuring full modulation in modulating a radio transmitter or recording sound on film, for maintaining the output of public-address systems more nearly constant when the speaker turns away from the microphone, etc.[2]

The term vogad is derived from the first letters of the phrase "voice operated gain adjusting device." A vogad is a particular form of A.V.C. system, and is used in radio-telephone extensions of wire telephone systems for the purpose of maintaining full modulation of the transmitter, irrespective of the loudness of the speaker's voice.[3]

Volume Limiter.—A volume limiter is a device that automatically reduces the amplification when the average input volume to the amplifier exceeds a predetermined value, so that the output for all inputs in excess of this value is substantially constant. Volume limiters are used in radio-telephone transmitters to prevent overmodulation.

Volume limiters ordinarily operate by rectifying a portion of the signal and using the rectified output to reduce the gain when the signal exceeds the desired level.

Peak Limiters.—A peak limiter is an amplifier so controlled by rectified voice currents that the gain is quickly reduced and then slowly restored when the instantaneous peak amplitude of the signal being amplified exceeds a predetermined value. A peak limiter differs from a volume limiter primarily in that it is controlled by the instantaneous peaks instead of by the average volume.

Peak Chopper.—A peak chopper is a device that prevents peak amplitudes from exceeding a critical value, and is ordinarily arranged to operate instantaneously.

VIDEO-FREQUENCY AMPLIFIERS

14. General Requirements of Video-frequency (Wide-band) Amplifiers.[4]—Amplifiers for the video-frequency currents of television circuits are required to handle a frequency range from about 60 cycles to about 4 megacycles with substantially uniform gain and negligible phase distortion. The maximum variation in over-all delay

ordinary amplifier by means of a resistance that is supplied by the plate resistance of a triode tube. This resistance, and hence the gain, is then controlled by a bias voltage developed by rectification of the signal. Such an arrangement has the advantage that the negative feedback reduces distortion that frequently occurs in volume expanders and compressors. See B. J. Stevens, Low Distortion Volume Expansion Using Negative Feed-back, *Wireless Eng.*, Vol. 15, p. 143, March, 1938; E. G. Cook, A Low Distortion Limiting Amplifier, *Electronics*, Vol. 12, p. 38, June, 1939; H. H. Stewart and H. S. Pollock, Compression with Feedback, Electronics, Vol. 13, p. 19, February, 1940.

[1] R. C. Mathes and S. B. Wright, The "Compandor"—An Aid Against Radio Static, *Elec. Eng.*, Vol. 53, p. 860, June, 1934; R. A. Heising, Radio Extension Links to the Telephone System, *Bell Sys. Tech. Jour.*, Vol. 19, p. 611, October, 1940.

[2] For typical A.V.C. circuits (including constants), see Harry Sohon, Supervisory and Control Equipment for Audio-frequency Amplifiers, *Proc. I.R.E.*, Vol. 21, p. 228, February, 1933; Paro, *loc. cit.*

[3] S. B. Wright, S. Doba, and A. C. Dickieson, A Vogad for Radiotelephone Circuits, *Proc. I.R.E.*, Vol. 27, p. 254, April, 1939; R. A. Heising, *loc. cit.*

[4] Following is a selected bibliography on the theory, design, and construction of video-frequency amplifiers: R. L. Freeman and J. D. Schantz, Video Amplifier Design, *Electronics*, Vol. 10, p. 22, August, 1937, and Vol. 10, p. 52, November, 1937; S. W. Seeley and C. N. Kimball, Analysis and Design of Video Amplifiers, Part II, *R.C.A. Rev.*, Vol. 3, p. 290, January, 1939; A. V. Bedford and G. L. Fredendall, Transient Response of Multistage Video-frequency Amplifiers, *Proc. I.R.E.*, Vol. 27, p. 277, April, 1939; Harold A. Wheeler, Wide-band Amplifiers for Television, *Proc. I.R.E.*, Vol. 27, p. 429, July, 1939; Albert Preisman, Some Notes on Video-amplifier Design, *R.C.A. Rev.*, Vol. 2, p. 421, April, 1938; E. W. Herold, High-frequency Correction in Resistance-coupled Amplifiers, *Communications*, Vol. 18, p. 11, August, 1938; F. A. Everest, Wide-band Television Amplifiers, *Electronics*, Vol. 11, p. 16, January, 1938, and p. 24, May, 1938; G. D. Robinson, Theoretical Notes on Certain Features of Television Receiving Circuits, *Proc. I.R.E.*, Vol. 21, p. 833, June, 1933.

Ex. 1044-0421

time tolerable in a 441-line television system is commonly of the order of 1,000 microseconds at the low-frequency end of the band and 0.1 microseconds at the high-frequency end. On the basis of 10 amplifier stages being involved, this corresponds to a phase shift per stage not exceeding 2° at 60 cycles, and linearity in the phase-shift characteristic up to the highest frequency of importance, with a deviation not exceeding 14° per stage at 4 mc. The amplification per stage should also be constant to within a few tenths of a decibel.

15. Video-frequency Voltage Amplifiers—Low-frequency Considerations.—Video-frequency voltage amplifiers employ circuit arrangements based upon the ordinary resistance-coupled circuit used for audio-frequency amplification. The severe requirements with regard to uniformity of amplification and delay time that must be met are, however, most satisfactorily realized by modifying the ordinary resistance-coupled amplifier circuit by additional circuit elements that improve the characteristics at low and high frequencies.

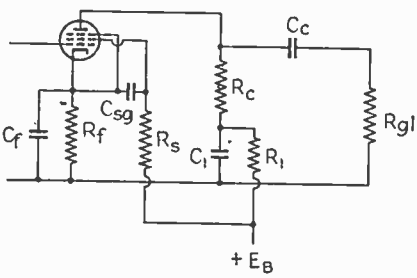


FIG. 47.—Circuit of video-frequency amplifier, showing resistance-condenser combination R_1C_1 for equalizing low-frequency response.

At low-frequencies, the circuit components causing phase shift and falling off in amplification are the coupling condenser, bias impedance, and impedance in the screen-grid circuit. Each of these must have its effect compensated for in the amplifier.

Compensation for Coupling Condenser. The coupling condenser in the plate circuit of an amplifier tube causes the output to fall off and to lead in phase at low frequencies. On the assumption that a pentode tube is used and that the grid-leak resistance is very much larger than the coupling resistance (*i.e.*, $R_{gl} >> R_c$), as is always the case in practical amplifiers, the behavior at low frequencies can be calculated from the equations

$$\text{Phase shift (lead)} = \phi = \tan^{-1}\frac{\frac{1}{\omega C_c}}{R_{gl}} \tag{47}$$

$$\text{Falling off in gain at low frequencies} = \cos\phi \tag{48}$$

where C_c is the capacity of the coupling condenser and R_{gl} the grid-leak resistance, as indicated in Fig. 47, and ω is 2π times the frequency. These equations show that the effect of the coupling condenser is more serious on phase shift than on the amplification. Thus a grid leak-condenser combination that causes the output voltage to fall only 1 per cent below the mid-frequency voltage introduces a phase shift of about 8°. Equations (47) and (48) show that if the phase shift is not to exceed 2° at 60 cycles, it is necessary that $R_{gl}C_c > 0.076$. Although this can be achieved by using a large condenser capacity and grid-leak resistance, it is usually more practicable to allow a reasonable phase shift and then provide equalization.

Compensation for the effect of the coupling condenser in the plate circuit can be obtained by the use of a resistance-capacity combination R_1C_1 in series with the plate coupling resistor, as shown in Fig. 47. In the practical case where $R_{gl} >> R_c$ the impedance Z_1 formed by the combination R_1C_1 causes the output to be increased in magnitude by the factor $(R_c + Z_1)/R_c$ and to be shifted in phase in a lagging direction by an amount corresponding to the angle by which the impedance of $Z_1 + R_c$ is leading.

Ex. 1044-0422

Perfect compensation for the effect of the coupling condenser is obtained when

$$R_c C_1 = R_{gl} C_c \tag{49}$$

and

$$R_1 = \infty \tag{50}$$

Under practical conditions an entirely satisfactory degree of compensation is obtained when $R_1 \geq 10(1/\omega C_1)$.

The effects produced by various adjustments of the compensating circuit R_1C_1 in a typical case are shown in Fig. 48. When Eq. (49) is satisfied, then the results are

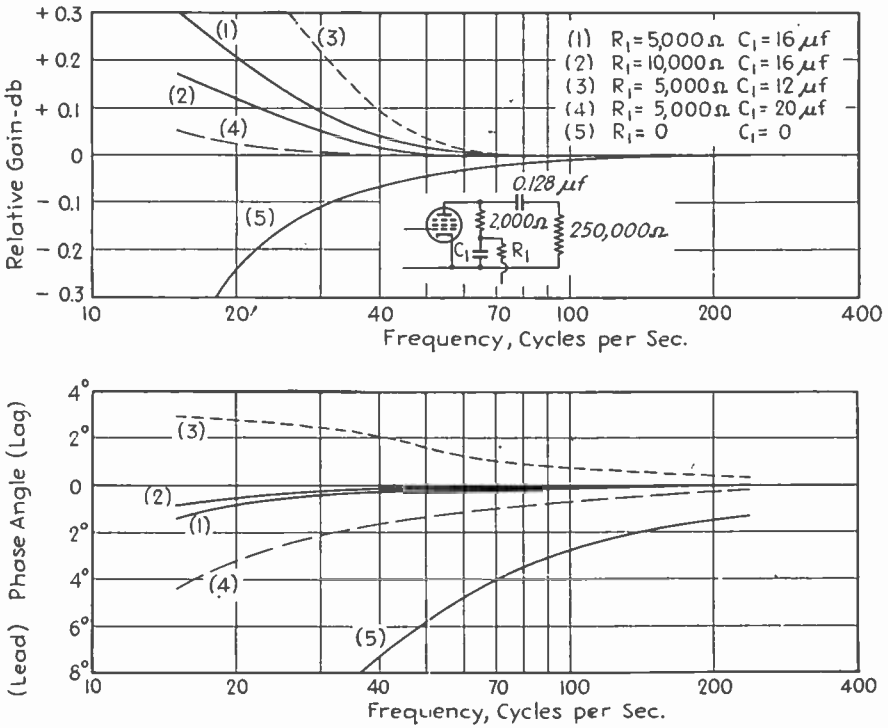


FIG. 48.—Various effects obtainable in equalizing for the effect of plate coupling condenser in video-frequency amplifiers.

more nearly perfect the higher the resistance R_1, as seen by comparison of curves (1), (2), and (5). With a fixed value for R_1, decreasing the capacity C_1 so that the time constant R_cC_1 is slightly less than $R_{gl}C_c$ will make the phase more lagging at low frequencies and tend to cause a rise in amplification, as seen by comparing curves (1) and (3) in Fig. 48. Increasing C_1 to give a larger time constant produces the opposite effect, as seen by comparing curve (4) with curves (1) and (3).

Compensation for Bias Impedance.—The bias impedance R_fC_f in Fig. 47 causes the output voltage to lead in phase and fall off in magnitude by an amount that is excessive even when the largest practicable size of by-pass condenser is used. Bias impedance can be eliminated by obtaining the bias in other ways, but this is commonly not convenient.

Ex. 1044-0423

The phase shift and change in amplification produced by the combination of the compensating network R_1C_1 and the bias network R_fC_f in Fig. 47 are given by the relation

$$\left.\begin{matrix}\text{Relative output}\\ \text{voltage at low}\\ \text{frequencies}\end{matrix}\right\} = \frac{1 + \frac{Z_1}{R_c}}{1 + g_m Z_f} \tag{51}$$

where Z_1 is the vector impedance of the combination R_1C_1, Z_f the impedance of the combination C_fR_f, R_c the coupling resistance, and g_m the transconductance. A positive phase angle in Eq. (51) represents a leading phase angle for the output voltage.

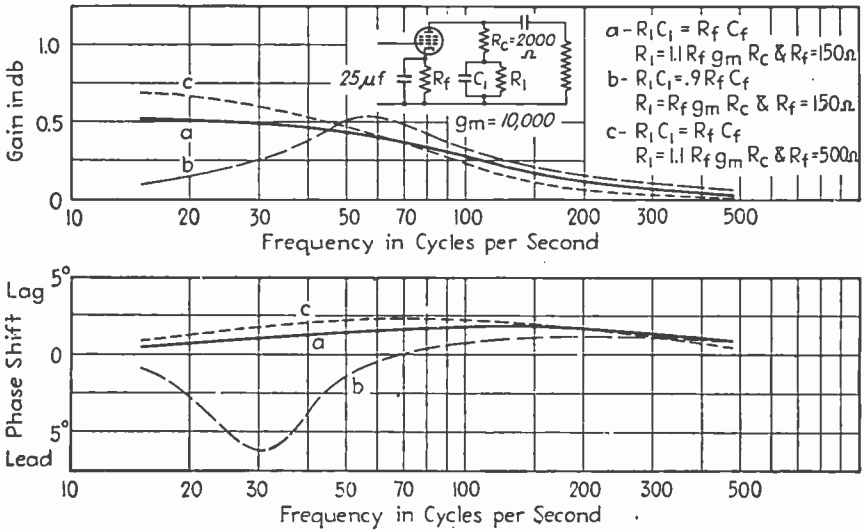


FIG. 49.—Various effects obtainable in equalizing for the cathode impedance in video-frequency amplifiers.

The right-hand side of Eq. (51) will be unity, corresponding to *perfect equalization of the bias impedance* C_fR_f, when

$$R_1 = R_f(g_m R_c) \tag{52}$$

and

$$R_1C_1 = R_fC_f \tag{53}$$

Equations (52) and (53) assume that $R_{pl} >> R_c$, a condition always satisfied in practical amplifiers. When the requirements of Eqs. (52) and (53) are not satisfied, the behavior will vary according to the character and extent of the departure from optimum proportions. The performance is shown in Fig. 49 for several typical cases.

Compensation for Impedance in Screen-grid Circuit.—The proper screen voltage is normally obtained by a screen dropping resistor R_s by-passed to the cathode by a condenser C_{sg}, as shown in Fig. 47. This arrangement places an impedance in the screen circuit that can cause the amplification at low frequencies to shift in phase and fall off in magnitude in the same manner as caused by the coupling condenser. The magnitudes of these effects can be calculated exactly by Eq. (8). Under practical conditions, they are determined mainly by the by-pass condenser C_{sg} and the dynamic screen-cathode resistance R_{sg} of the tube, and can be calculated with sufficient accu-

Ex. 1044-0424

racy by Eqs. (47) and (48) by substituting C_{sg} and R_{sg} for C_c and R_{gl}, respectively, which appear in these equations. The phase shift will not exceed 2° at 60 cycles if

$$R_{sg}C_{sg} > 0.076 \tag{54}$$

Compensation for the screen impedance under these conditions is not necessary. If Eq. (54) is not satisfied, it is necessary to introduce correction in some plate circuit, as discussed below.

Equalization for All Low-frequency Effects.—In practical amplifiers it is desirable to equalize each individual stage as completely as possible rather than attempt to equalize for the deficiencies of several stages at a single point. At the same time it is not feasible to compensate for the effects of the bias impedance and coupling condenser

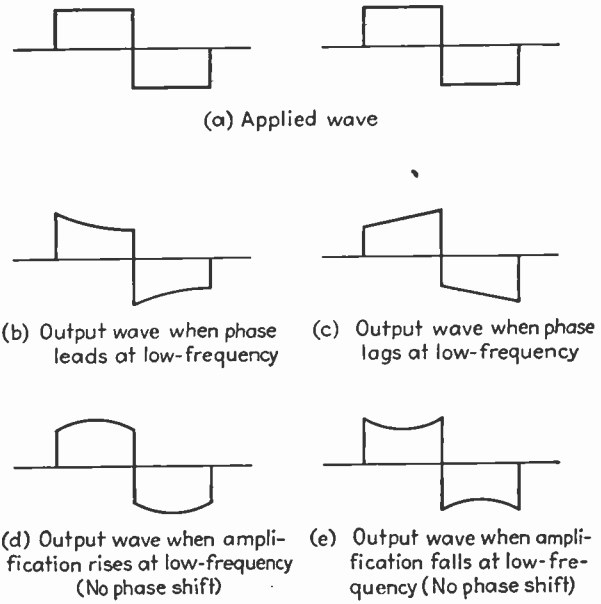


FIG. 50.—Effects produced by amplifier imperfections upon a square wave for idealized cases.

in the same stage. The practical method of meeting this situation is to make the effects from the coupling condenser (and screen impedance) small in all stages employing a bias impedance, and then for every two to five stages providing at least one stage in which there is no bias impedance. In this one stage, the resistance combination R_1C_1 in the plate circuit is then proportioned to compensate for the coupling condenser and screen impedance effects of all the stages. This arrangement operates satisfactorily as long as the total phase shift to be corrected is less than 20°.

Experimental Check of Low-frequency Compensation.—After the low-frequency compensating circuits, with proportions as calculated, have been installed in an amplifier, it is always desirable to check the over-all behavior experimentally. Adjustments can then be made to correct for residual defects in performance as necessary.

Low-frequency tests are preferably made by applying the output of a square-wave oscillator to the amplifier input (see Par. 32, Sec. 13). The shape of the resulting output wave can be interpreted in terms of amplifier characteristics at low frequencies, as indicated in Fig. 50. A tilt in the horizontal portion of the output wave is caused

Ex. 1044-0425

primarily by phase shift at the fundamental frequency; curvature of the horizontal portion is largely caused by a gain characteristic that does not remain constant down to the fundamental frequency of the square wave.

16. Video-frequency Voltage Amplifiers. *High-frequency Considerations.*—High gain over a wide frequency band with substantially constant amplification and constant delay time, as required in video-frequency voltage amplifiers, can be most readily obtained by adding further reactance elements to the ordinary resistance-coupled amplifier circuit. The resulting networks can be classified as two-terminal or four-terminal networks according to whether the plate lead of the amplifier tube and the grid lead to the succeeding tube are connected at the same or different points in the coupling network.

Two-terminal Coupling Networks—General Properties.—Types of two-terminal networks used in television amplifiers are shown in Fig. 51. These all have as their

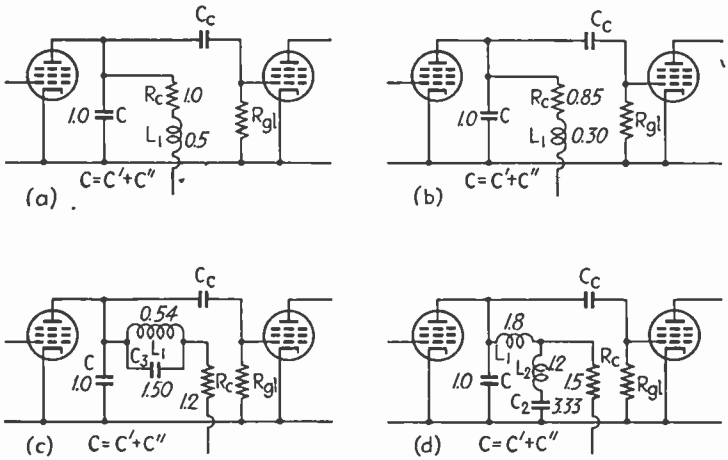


FIG. 51.—Two-terminal coupling networks used in video-amplifiers, together with recommended circuit proportions for optimum high-frequency characteristics. The numbers indicate element impedances at the top frequency f_1 that is to be amplified (reactance of C at f_1 taken as unity).

first element a shunting capacity C across the input that represents the sum of the output capacity of the amplifier tube plus the input capacity of the succeeding tube plus wiring capacity plus any padding capacity that may be added. The coupling network also includes a resistance R_c that provides the coupling impedance at the lower frequencies.

The object of the remaining reactive elements in the two-terminal network is to build up across the terminals of the condenser C the highest possible impedance that is constant in magnitude and has a phase angle proportional to frequency up to the highest frequency to be amplified. Analysis of the properties of networks shows that the highest possible constant impedance that can be built up across a capacity C is $2X_1$, where X_1 is the reactance of the capacity at the highest frequency for which the impedance is to be constant, *i.e.*, the highest frequency to be amplified.

The amplification obtained with a two-terminal coupling network having an impedance Z_c can be taken as[1]

$$\text{Voltage amplification} = g_m Z_c \tag{55}$$

[1] This equation assumes that the plate and grid-leak resistances are both large compared with Z_c, a requirement readily satisfied by all wide-band amplifiers.

Ex. 1044-0426

where g_m is the transconductance of the tube. In the region where the coupling impedance is constant, one has $Z_c = R_c$, and Eq. (55) becomes

$$\left.\begin{array}{l}\text{Amplification of flat}\\ \text{part of characteristic}\end{array}\right\} = g_m R_c \tag{56}$$

Since the maximum possible value of R_c is $2X_1$, one has

$$\left.\begin{array}{l}\text{Maximum possible voltage}\\ \text{gain with two-terminal}\\ \text{coupling network}\end{array}\right\} = 2g_m X_1 \tag{57}$$

Design of Two-terminal Coupling Networks.—The two-terminal coupling networks having greatest use in video amplifiers are shown in Fig. 51. Suitable circuit propor-

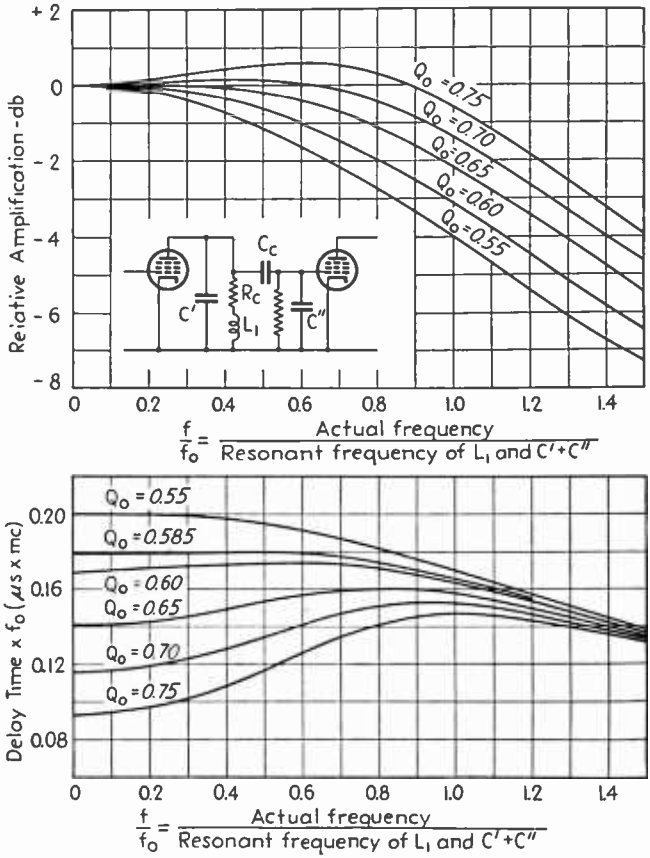


FIG. 52.—Effect of circuit proportions on the performance of the shunt-peaking circuit.

tions for good performance are indicated by the numbers associated with each circuit element. These give the impedance of the element at the top frequency f_1 to be amplified, in terms of the impedance X_1 of the shunting capacity C at this frequency. Thus for Fig. 51*b*, R_c has a resistance equal to 0.85 times the reactance of C at the frequency f_1, and the inductance L is of such size that its reactance at the frequency f_1 is $0.30X_1$.

Ex. 1044-0427

The suggested circuit proportions are a compromise between proportions giving the most uniform amplification characteristic up to the top frequency f_1 and the proportions giving the most constant delay-time characteristic. This is illustrated by Fig. 52, which shows the amplification and time-delay characteristics of the circuit of Figs. 51*a* and 51*b*. This particular arrangement, which is termed the *shunt-peaking* circuit, is a low Q resonant circuit such as is discussed in Par. 2, Sec. 3, and can have its properties expressed in terms of the resonant frequency f_0 of the inductance and capacity of the circuit, and the circuit Q at resonance (denoted by Q_0). The gain is most nearly constant up to a frequency f_1 when $f_0 = 1.41\ f_1$ (*i.e.*, $f_1/f_0 = 0.707$) and Q_0 equals 0.707, corresponding to the proportions of Fig. 51*a*. On the other hand, the

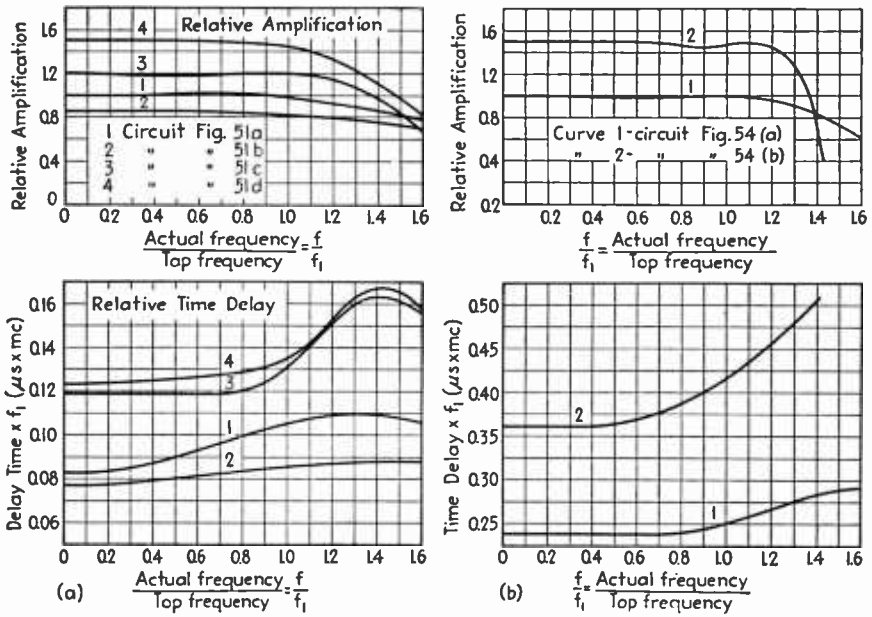


FIG. 53.—Amplification and time-delay characteristics of circuits of Figs. 51 and 54.

time-delay characteristic will be most nearly constant up to a frequency f_1 by making $f_0 = 1.67 f_1$ (*i.e.*, $f_1/f_0 = 0.6$) and $Q_0 = 0.585$. A good compromise is

$$f_0 = 1.825\ f_1 \text{ (i.e., } f_1/f_0 = 0.55)$$

and $Q_0 = 0.65$, corresponding to the proportions of Fig. 51*b*. In general, it will be found that the circuit proportions recommended by different writers will vary somewhat in accordance with the relative emphasis placed upon the importance of uniform amplification, as against uniform time delay, and may differ somewhat from those given in Fig. 51.

Amplifier performance obtained in using two-terminal coupling networks with the proportions indicated in Fig. 51 are given by curves 1 to 4 of Fig. 53, and are also summarized in Table 2. It will be noted that the gain obtainable with satisfactory constancy of amplification and delay time increases with the complexity of the coupling network, and approximates 75 per cent of the theoretical maximum possible gain in the case of the circuit of Fig. 51*d*.

Four-terminal Coupling Networks—General Properties.—Figure 54 shows types of four-terminal coupling networks used in video-frequency amplifiers to obtain desir-

Ex. 1044-0428

able performance at high frequencies. In these circuits the capacity C' associated with the plate circuit of the amplifier tube is separated from the capacity C'' associated with the grid electrode of the following tube, with the result that the gain obtainable over a given frequency band is greater than when the two capacities are paralleled, as in the case of the two-terminal coupling system.

An analysis of the properties of four-terminal coupling networks shows that the maximum possible amplification that can be obtained depends upon whichever of the capacities C' or C'' is maximum. The properties of tubes are such that normally $C'' > C'$, with the ratio being approximately 2. The effective ratio can be modified somewhat by proper disposal of circuit elements, such as blocking condensers, which have capacity to ground. Assuming that C'' is the larger capacity and denoting the reactance of this condenser as X_1'' at the highest frequency f_1 that is to be amplified, then the maximum theoretically possible amplification obtainable with the four-terminal network under practical conditions is

$$\text{Maximum possible amplification} = 2g_m X_1'' \tag{58}$$

This equation is of the same form as Eq. (57) for the two-terminal case, but since X_1'' in Eq. (58) is the reactance of C'' instead of $C' + C''$, the four-terminal network gives greater gain in accordance with the relation

$$\frac{\left.\begin{matrix}\text{Maximum practical gain}\\ \text{with four-terminal}\\ \text{coupling system}\end{matrix}\right\}}{\left.\begin{matrix}\text{Maximum possible gain}\\ \text{with two-terminal}\\ \text{coupling system}\end{matrix}\right\}} = \frac{C' + C''}{C''} \tag{59}$$

For a given total shunting capacity $C' + C''$, the most favorable division of capacities is $C' = C''$, corresponding to twice as much gain from the four-terminal system as obtainable with a two-terminal network. When $C'' = 2C'$, the four-terminal system has a possible amplification that is $\frac{3}{2}$ times the theoretical possible gain for a two-terminal network.

Design of Networks Used in Four-terminal Coupling Systems.—Four-terminal coupling networks having desirable characteristics for video-frequency voltage amplifiers are shown in Fig. 54. Network proportions that give good performance are indicated by the numbers associated with the circuit elements. These numbers give the impedances of the elements at the top frequency f_1 in terms of the impedance X_1'' of the capacity C'' at the top frequency f_1 to be amplified (assuming that $C'' > C'$). In the event that $C'' < C'$, one uses the same proportions, except that X_1'' is replaced by X_1', the reactance of C' at the frequency f_1, and the circuit is altered by exchanging the points at which the plate and grid electrodes connect to the network (as illustrated in Fig. 57). By the reciprocity theorem, the amplification and phase-shift characteristics will be unchanged by such an exchange of input and output terminals.

The circuits of Fig. 54 call for specified values of capacity ratio C''/C'. In practice, this can be obtained by padding out one or the other of the capacities by means of an adjustable trimming condenser to give the required ratio. Since the impedance level of the coupling network is determined by the largest shunting capacity, the maximum gain is obtained when the circuit is so arranged that the required capacity ratio can be achieved by padding out the smaller of the two condensers rather than by adding capacity in shunt with the larger.

The performance of the four-terminal networks of Fig. 54 with the proportions indicated is given in Fig. 53*b*. The essential properties are summarized in Table 2.

Ex. 1044-0429

Comparison of Different Coupling Systems.—Examination of Fig. 53 and Table 2 shows that the amplification obtainable with satisfactory constancy of amplification and for small delay error is greater for the four-terminal networks than the two-terminal systems. Also, the amplification obtainable with each type of coupling system

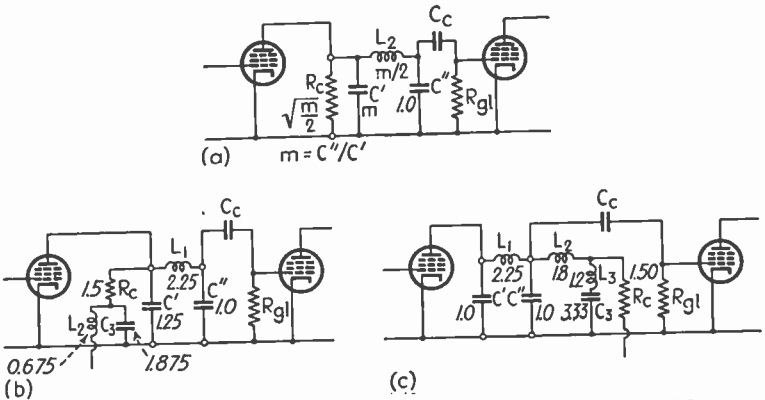


FIG. 54.—Four-terminal coupling networks used in video-frequency amplifiers, together with recommended circuit proportions giving optimum performance. The numbers shown are the element impedances at the top frequency f_1 that is to be amplified (reactance of C'' at f_1 taken as unity). The capacities C' and C''' include the tube capacities.

more closely approaches the theoretical possibilities as the number of reactive elements in the coupling network increases. Four-terminal systems give decidedly more gain.

TABLE 2.—PERFORMANCE OF SOME TYPICAL VIDEO-AMPLIFIER CIRCUITS HAVING HIGH-FREQUENCY COMPENSATION

Circuit	Relative voltage gain	Variation in amplification up to top frequency f_1	Maximum delay time error up to top frequency f_1 mc, in μs
Two-terminal cases			
Fig. 51*a*	1.0	0.02	$0.023/f_1$
Fig. 51*b*	0.85	0.035	$0.0077/f_1$
Fig. 51*c*	1.20	0.02	$0.012f_1$
Fig. 51*d*	1.50	0.035	$0.012/f_1$
Ideal	2.00	0	0
Four-terminal cases			
Fig. 54*a**	1.50	0.02	$0.012/f_1$
Fig. 54*b*†‡	2.25	0.04	$0.055/f_1$
Ideal	3.00	0	0

* For case $m = 2$.

† Figure 54*c* acts substantially the same as Fig. 54*b*, but permits the use of a larger capacity C'.

‡ Amplification and time-delay characteristics calculated on assumption of perfect filter match to load. This assumption is almost perfectly realized up to the top frequency f_1.

The four-terminal networks are, however, more complicated than the two-terminal ones, are more difficult to adjust, and require more care in construction. Also, the

Ex. 1044-0430

more complicated networks, particularly the more complicated four-terminal arrangements, have a far sharper cutoff and a much more rapid deterioration of the phase characteristics around the top frequency. This is a disadvantage in most cases, and makes it necessary in practice to use a higher top frequency than would be required with the simple shunt-peaking circuit.

Coupling Networks Considered as Filters.[1]—The two-terminal coupling networks of Fig. 51 can be considered as special cases of general types of two-terminal coupling impedances based upon the properties of low-pass filters. These general two-terminal coupling networks are shown in Fig. 55. They consist of a filter that is matched on an image-impedance basis by an m-derived half section ($m = 0.6$) to a load consisting of coupling resistance R_c. This terminating section is then built out by adding sections on an image-impedance basis so that the input of the filter is at the mid-shunt point of a Type II (see Table 2, Sec. 3) (sometimes called constant k) π section. The input is then shunted by an auxiliary condenser C_m of such size as to make the impedance across the input terminals more nearly constant with frequency than is the mid-shunt impedance of a constant-k section. If X_c is the reactance of the total capacity across the filter input (auxiliary capacity C_m plus first shunting capacity C_n of the filter) at the cutoff frequency of the filter, then, on the assumption that there is a perfect impedance match of R_c to the filter, one has

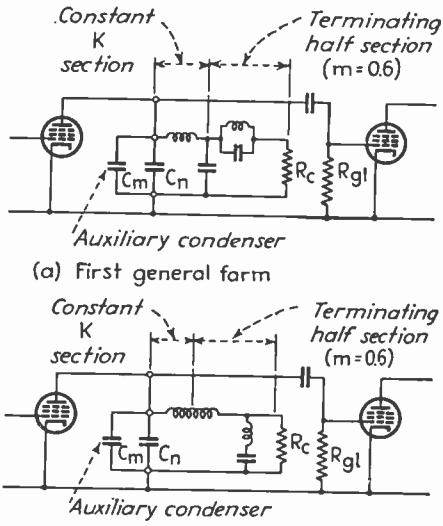


FIG. 55.—General two-terminal coupling networks based upon low-pass filters. The constant k sections are Type II sections of Table 2, Sec 3.

$$\text{Magnitude of impedance} = \frac{R_c}{\sqrt{1 - \left[1 - \left(\frac{R_c}{2X_c}\right)^2\right]\left(\frac{f}{f_c}\right)^2}} \tag{60}$$

$$\text{Phase angle of impedance} = \phi = \tan^{-1} \frac{\left(\frac{R_c}{2X_c}\right)}{\sqrt{\left(\frac{f_c}{f}\right)^2 - 1}} \tag{61}$$

where R_c = load impedance with which filter is designed to operate.

X_c = reactance of total capacity ($C_n + C_m$) across filter input terminals at the cutoff frequency f_c.

f_c = cutoff frequency of filter.

The behavior for various values of $R_c/2X_c$ is shown in Fig. 56. For $X_c = R_c/2$, the impedance that is developed (and hence the amplification obtained) *is precisely constant up to the cutoff frequency f_c of the low-pass filter, and has a value $2X_c$.*[2] *This is the highest constant impedance that can possibly be developed across a capacity C_m over this*

[1] See Wheeler, *loc. cit.*

[2] The condition $X_c = R_c/2$ is obtained when the auxiliary shunting capacity C_m equals the capacity C_n of the filter, since for a constant k section, $(1/2\pi f_c C_n) = R_c$. Hence the total shunting capacity required across the input for uniform impedance is exactly the same as the full shunt capacity of the constant k or Type II section.

Ex. 1044-0431

frequency range. The most linear phase characteristic is obtained when $X_c = 0.37R_c$, which corresponds, however, to a moderate falling off in impedance as cutoff is approached. The best compromise is when X_c is approximately $0.48R_c$.

The four-terminal networks of Fig. 54 are special cases of a general four-terminal coupling system based on the properties of wave filters, and illustrated in Fig. 57*a*. Here the impedance offered to the plate circuit of the amplifier tube is developed in the same way as in the case of the two-terminal coupling network, but now the impedance that can be developed is greater because part of the shunting capacity provided by the tubes is displaced along the filter to supply the shunting capacity

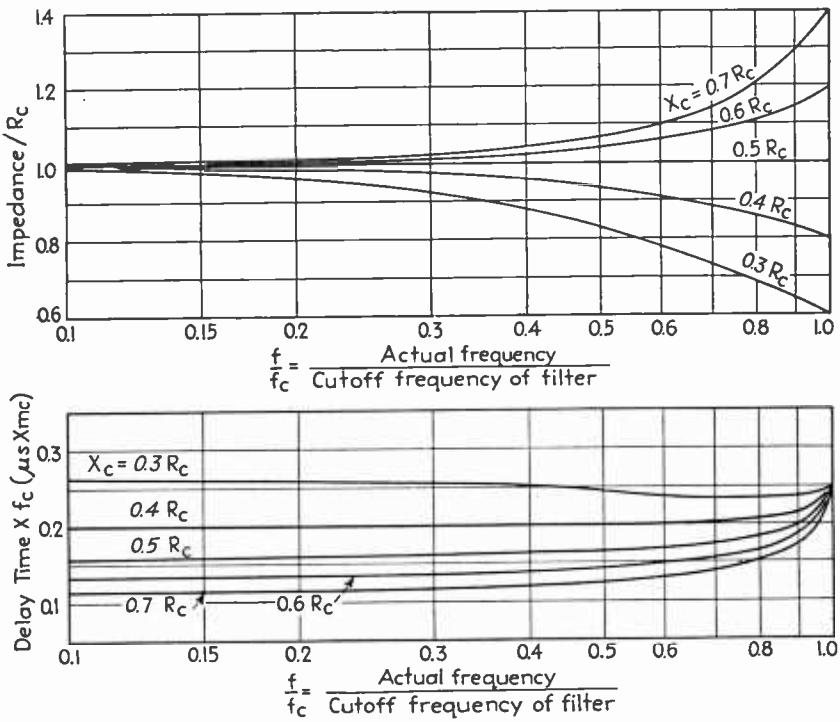


FIG. 56.—Effect of reactance X_c of the total capacity shunted across filter input at the cutoff frequency f_c upon the performance of the general coupling network of Fig. 55.

required at another shunting point. The increase in impedance level, and hence the increase in amplification, obtainable with the four-terminal arrangement depends upon the ratio of capacities C' and C'' associated with the input and output circuits of the coupling network. The impedance level is greatest when $C' = C''$, and is then exactly twice the impedance that can be developed in a two-terminal coupling system having the same total shunting capacity. The maximum possible gain obtained using a four-terminal network under such conditions is therefore exactly twice that obtainable with a two-terminal network. With other capacity ratios the relative advantage of the four-terminal arrangement is given by Eq. (59).

The separation of the grid and plate terminals existing in a four-terminal coupling system produces no effect except to introduce an extra phase shift. This is because the frequencies to be amplified are in the pass band of the filter and so are transmitted

Ex. 1044-0432

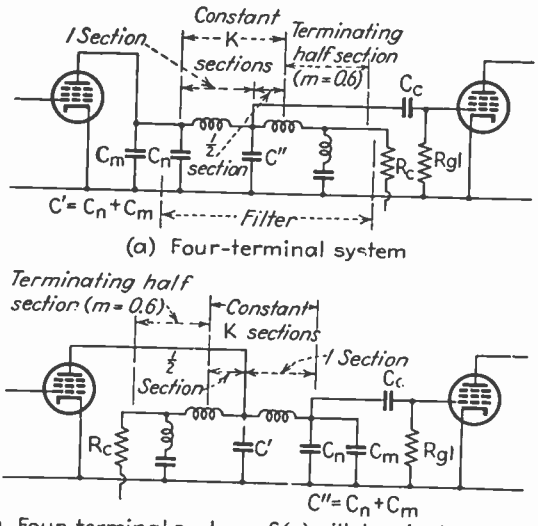


FIG. 57.—General forms of four-terminal coupling systems based upon filters. The constant k sections are Type II sections of Table 2, Sec. 3.

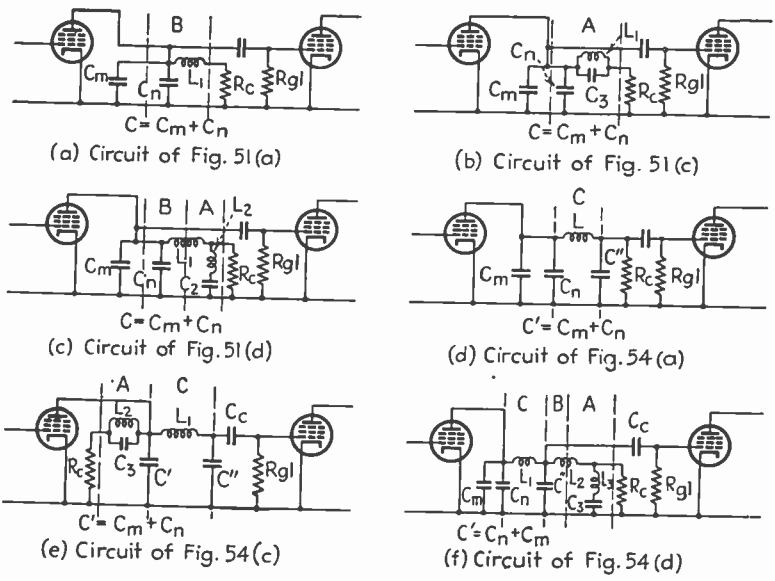


FIG. 58.—Circuits of Figs. 51 and 54 redrawn in the form of filter coupling networks. The dotted vertical lines show the division into sections (or half sections). The constant k sections are Type II sections of Table 2, Sec. 3.

Ex. 1044-0433

through the filter sections between plate and grid without attenuation, but only with phase shift.

The circuits of Figs. 51 and 54 are special cases of the general two- and four-terminal coupling systems, as is apparent when the circuits are redrawn as in Fig. 58. It will be noted in both the two- and four-terminal cases that the performance approaches more closely to the ideal as a terminating half section ($m = 0.6$) is used to match the filter to the coupling resistance, and as enough sections are added to provide full mid-shunt constant-k capacities at the points on the filter with which the input and output terminals are associated.

In four-terminal coupling systems it is theoretically possible to introduce phase-correcting sections between the input and output points, as illustrated schematically

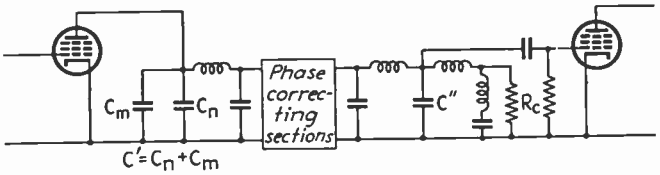


FIG. 59.—Diagram illustrating schematically the use of phase-correcting sections in a four-terminal coupling system.

in Fig. 59. In this way it is possible to obtain absolutely constant time delay, as well as absolutely constant amplification, up to the cutoff frequency of the filter. It is also possible in four-terminal systems to provide an impedance-transforming section when $C' \neq C''$ as indicated in Fig. 60. In this way full use is made of the existing shunting capacities, and maximum possible gain is obtained. Phase-correcting and impedance-transforming sections have not as yet had practical use, however.

Practical Adjustment of High-frequency Coupling Networks.—To obtain the desired high-frequency performance, it is necessary that the coupling network be adjusted in a systematic way. Trial-and-error procedure does not yield satisfactory results because of the many variables involved.

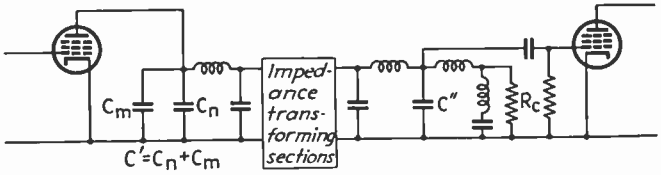


FIG. 60.—Diagram illustrating schematically the use of an impedance transforming-section in a four-terminal coupling system.

In the design of practical television amplifiers the high-frequency coupling network is calculated on the basis of shunting capacities C' and C'' slightly larger than those supplied by the tubes and wiring. Small trimming condensers are then used to pad out the capacities to the required value. In this way, it is possible to take care of variations in the capacities of individual tubes.

In four-terminal coupling systems, the network is built up by measuring the separate inductances, capacities, and resistance involved, and adjusting them separately to the calculated values, with the use of a bridge, Q meter, or other measuring device. In evaluating the capacities, the most accurate results will be obtained when the measurements are made after the coupling circuit has been mounted in final form so that the wiring capacities will be included.

Ex. 1044-0434

In the case of two-terminal wiring systems one can obtain the proper circuit proportions either by separately measuring the individual circuit elements, as in the four-terminal case, or by using an experimental method to determine when the elements have the correct value. The former method is preferable when the network is complicated; the latter, when it is simple.

The experimental procedure for the shunt-peaking circuit consists in first connecting the input and output tubes in place and biasing the output tube so that it acts as a vacuum-tube voltmeter. The coupling resistance R_c is then short-circuited, and the inductance L_1 is adjusted by trial and error until the curve of amplification versus frequency has its peak at the frequency f_0 corresponding to the desired ratio f_1/f_0. With L_1 shorted, R_c is then adjusted until the amplification at the frequency $Q_0 f_0$ is 0.707 times the amplification at a moderate frequency, where Q_0 is the desired circuit Q (see Fig. 52). The experimental adjustment of the circuit shown in Fig. 51c follows an analogous procedure. First C_3 is disconnected, after which L_1 and R_c are adjusted as in the shunt-peaking case to give $Q_0 = 0.62$, and $f_0 = 1.36 f_1$. The proper value for C_3 is then obtained by short-circuiting R_c and adjusting C_3 until the peak of amplification occurs at the frequency $0.95 f_1$.

After the proper circuit proportions have been installed according to calculations or experiment, it is desirable to check the final results by taking a curve of amplification as a function of frequency. If the resulting gain has the expected uniformity up to the top frequency f_1 to be amplified, it is then almost certain that the phase-shift characteristics will likewise be satisfactory. An alternative method of checking the high-frequency characteristics of an amplifier is to apply to the amplifier a steep wave front, such as is obtainable from a square-wave oscillator, and to observe the output wave that results.

Reproduction of High-frequency Transients by Video Amplifiers.[1]—The severe requirements that video amplifiers must meet at high frequencies arise from the need of accurately reproducing abrupt changes in amplitude of the applied voltage, such as is represented by the unit pulse in Fig. 61*a*. The analysis of amplifier behavior under the action of a unit pulse is quite involved, but the general character of the relations that exist between amplifier characteristics and the effects produced in amplifying a pulse are indicated in Fig. 61. It will be noted that the amplifier causes the pulse to rise gradually instead of abruptly, and also that there is a tendency for the amplifier output to overshoot its final value. The character of the behavior depends upon the constants of the amplifier, as indicated in Fig. 61*b*, which applies to the shunt-peaking circuit. It will be noted that the best reproduction of the pulse is when Q_0 is in the range 0.7 to 0.6, which corresponds to gain and time delay characteristics that are most nearly constant.

The rate at which the amplifier output rises as the pulse is applied is inversely proportional to the top frequency that can be amplified without serious loss in gain. This means that the greater the band width the more abruptly will the output rise, and hence the more closely will the applied pulse be reproduced.

The effect of applying a pulse to a multistage amplifier is shown in Fig. 61*c*. As the number of stagings increases, the rate of rise of output becomes more gradual, and the tendency to overshoot is more pronounced.

The result of applying a pulse to an amplifier having an ideal filter type of coupling is shown in Fig. 61*c* for the case where there are constant amplification and delay time up to the filter cutoff and no amplification at higher frequencies. For comparison, a dotted curve is shown for a shunt-peaking circuit having the top frequency equal to the cutoff frequency of the filter. The shunt-peaking circuit is superior to the filter circuit in spite of the fact that below its top frequency the shunt-peaking

[1] Bedford and Fredendall, *loc. cit.*

Ex. 1044-0435

arrangement does not have exactly constant amplification and constant time delay. This superiority is because the shunt-peaking circuit gives amplification at frequencies appreciably greater than the top frequency, whereas the ideal filter does not. This emphasizes the fact that sharp cutoff, such as is obtained with filter coupling systems, is not always desirable.

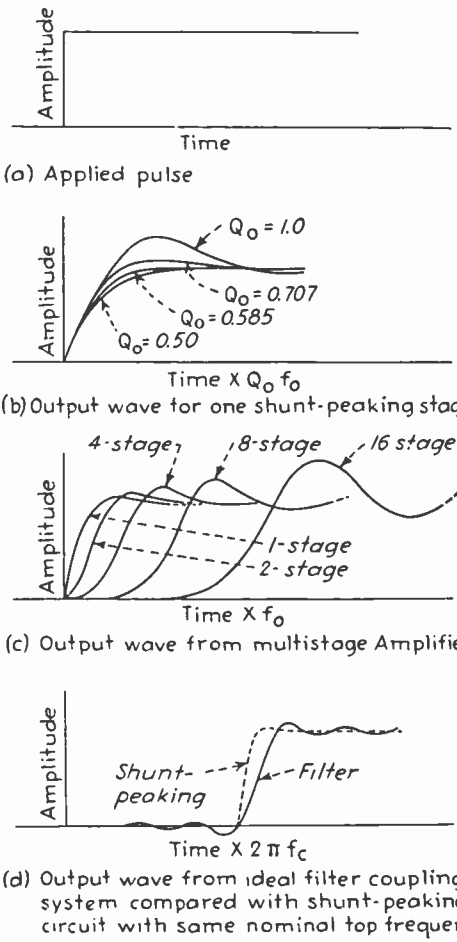


FIG. 61.—Response obtained with various amplifying systems when an impulse is applied to the amplifier input.

Combination of High-frequency Coupling Network with Low-frequency Equalizing Circuits.—In practical video amplifiers the low-frequency equalizing circuits are placed on the low potential side of the high-frequency network, as shown in Fig. 62. With this arrangement the presence of the high-frequency network has no effect upon the low-frequency equalization, because at very low frequencies the high-frequency network simply reduces to the coupling resistance R_c. At the same time, the low-frequency equalizing circuits are by-passed to ground at high frequencies by the shunting capacity that is a part of this circuit, and so become inoperative at high frequencies.

Ex. 1044-0436

Inasmuch as large condensers frequently possess a certain amount of inductance, it is desirable to shunt the equalizing condenser in the plate circuit, and also the bias and screen by-pass condensers, with small mica condensers having a capacity of the order of 0.001 to 0.01 μf, to ensure low impedance to the higher video frequencies.

17. Video-frequency Power Amplifiers.—Video-frequency power amplifiers differ from voltage amplifiers in that the objective is power developed in a load impedance rather than voltage developed across a capacity. The chief use of video-frequency power amplifiers is the production of energy to be transmitted over a cable or transmission line. Amplifiers for this purpose present special problems, since the load impedance is small, and the wide-frequency band involved makes it impossible to use transformers for impedance matching.

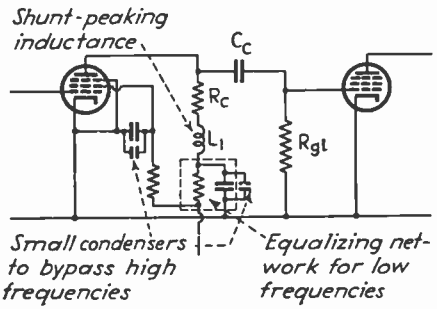


FIG. 62.—Video-amplifier circuit showing combination of high-frequency and low-frequency equalization.

Circuit arrangements for video-frequency power amplification are shown in Fig. 63. Of these, cathode coupling is generally used for reasons discussed below. The d-c plate current of the tube can, if necessary, be blocked from the load by a coupling condenser C_c and shunt-feed resistor R, as shown in Figs. 63*b* and 63*d*. This is to be avoided wherever possible, however, since the shunt-feed resistance reduces the power output obtainable, and the blocking condenser must be of tremendous size if excessive phase shift is to be avoided at low frequencies.[1]

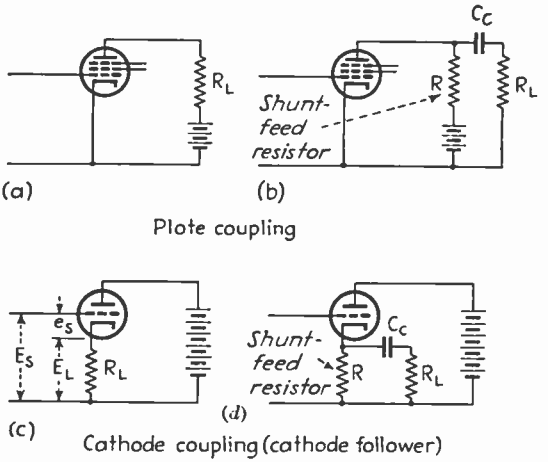


FIG. 63.—Circuits for video-frequency power amplifiers.

The maximum power that a Class A amplifier tube can develop in a load resistance R_L that is small compared with the plate resistance of the tube is

$$\left.\begin{matrix}\text{Power that can be delivered}\\ \text{to load resistance}\end{matrix}\right\} = \frac{I_p^2 R_L}{2} \qquad (62)$$

[1] Values of the order of 1,000 μf are required when the load is a cable.

Ex. 1044-0437

where I_p is the d-c plate current. Equation (62) assumes that the plate current varies from zero to twice the d-c value during operation. If amplitude distortion is to be kept small, the power actually realizable with practical tubes is of the order of one-half to three-fourths that given by Eq. (62).

Examination of Eq. (62) shows that if a large amount of energy is to be delivered to a fixed load impedance such as that represented by a cable or transmission line, it is necessary that the d-c plate current be large. When a single tube is not capable of delivering the desired power, tubes can be paralleled. The power output that is obtained will then be proportional to the *square* of the number of tubes, with the maximum number of tubes that can be employed limited by the fact that the top frequency to be amplified sets a limit to the capacity that can be allowed in shunt with the load.

Cathode-coupled Power Amplifier.[1]—The *cathode-coupled* (also called *cathode-follower*) circuit of Fig. 63*c* is generally preferred for power amplification at video frequencies. Compared with arrangements that place the load impedance in the plate circuit, the cathode-coupled amplifier has the advantages of (1) permitting one of the load terminals to be grounded; (2) better amplitude and phase characteristics at high frequencies as a result of negative feedback action; (3) higher input impedance as a result of negative feedback; (4) less amplitude distortion as a result of negative feedback; (5) a source impedance presented by the tube to the load that is small enough so that it can be readily matched to a transmission line.

The voltage developed across the load in the cathode-coupled stage of Fig. 63*c* is related to the applied voltage by the equation

$$E_L = g_m Z_{eq} E_s \tag{63}$$

where E_L = voltage developed across the load.
E_s = voltage applied to the amplifier input.
g_m = transconductance of tube.
Z_{eq} = equivalent load impedance formed by the actual load impedance in parallel with a resistance $R_p/(1 + \mu)$, where R_p is the plate resistance and μ, the amplification factor, respectively, of the tube.

Examination of Fig. 63*c* shows that the voltage E_L developed across the load opposes the applied signal E_s as far as potential between the grid and cathode is concerned, so that the circuit exhibits negative feedback characteristic. If the equivalent impedance Z_{eq} and also the transconductance g_m are not too low, the voltage E_L developed across the load will be almost but not quite equal to the applied signal voltage E_s. Under no conditions can the voltage developed across the load exceed the applied signal.

Equation (63) can also be rewritten as

$$E_L = \frac{\mu}{\mu + 1} E_s \frac{R_L}{R_L + \left(\frac{R_p}{1 + \mu}\right)} \tag{63a}$$

The notation is the same as before. Examination of Eq. (63*a*) shows that as far as the load impedance is concerned, the presence of negative feedback cause the tube to act as though it had an amplification factor $\mu/(\mu + 1)$ and a plate resistance[2] of $R_p/(1 + \mu)$.

The low equivalent tube impedance that is presented to the load in a cathode-coupled circuit is of considerable practical importance. *First*, it has the effect of improving the response characteristics at high frequencies. *Second*, it makes possible

[1] This discussion follows Albert Preisman, Some Notes on Video Amplifier Design, *R.C.A. Rev.*, Vol. 2, p. 421, April, 1938.

[2] This reduction of equivalent plate resistance by the action of negative feedback is the same here as discussed in Par. 11.

Ex. 1044-0438

an impedance match between the tube and the load presented by a cable or other transmission line. Such matching eliminates reflections at the sending end of the cable, and so reduces echoes. When a match between a tube and transmission line is required, a triode tube is selected such that with normal operation the equivalent tube impedance $R_p/(1 + \mu)$ will approximate the characteristic impedance of the transmission line. A final adjustment is then made by adding resistance in shunt with the input of the line if the tube resistance is too high or placing a resistance in series if the tube resistance is too low, as shown in Fig. 64.

The negative feedback action in the cathode-coupled stage causes the voltage e_s in Fig. 63c that exists between control grid and cathode to be considerably less than the signal E_s. As a result, any impedance Z_{gk} from cathode to control grid appears



(a) Impedance matching when equivalent tube resistance is too high

(b) Impedance matching when equivalent tube resistance is too low

FIG. 64.—Methods of obtaining exact impedance match between a transmission line and a cathode-follower stage.

to the applied voltage E_s to have a value greater than the actual impedance in accordance with the relation

$$\left.\begin{matrix}\text{Equivalent grid-cathode} \\ \text{impedance}\end{matrix}\right\} = \frac{\text{actual grid-cathode impedance}}{1 - (E_L/E_s)} \tag{64}$$

Equation (64) has three important consequences. *First*, if the grid-leak resistance of the driver stage is connected from grid to cathode, a low resistance that may be needed to carry gas currents will still appear to be a high resistance to the applied signal. *Second*, the effective capacitive loading on the driver stage is reduced according to the relation

$$\left.\begin{matrix}\text{Equivalent input} \\ \text{capacity}\end{matrix}\right\} = \left\{\begin{matrix}\text{actual input} \\ \text{capacity}\end{matrix}\right\} \times \left(1 - \frac{E_L}{E_s}\right) \tag{64a}$$

Third, placing tubes in parallel to increase the output power of a cathode-coupled stage does not make the impedance that is offered to an applied voltage E_s vary inversely as the number of tubes in parallel because with usual circuit proportions $(1 - E_L/E_s)$ is at least roughly inversely proportional to the number of tubes.[1]

High-frequency Characteristics of Video-frequency Power Amplifiers.—The amplification and phase shift of video-frequency power amplifiers at high frequencies can be calculated from the equivalent circuit of Fig. 65. Here the tube is considered as supplying a constant current $g_m E_s$ to an equivalent impedance formed by the parallel combination of the actual load impedance Z_L, the effective internal resistance of the tube, and the shunting capacity C. The internal resistance of the tube is taken as the plate resistance for all cases except cathode coupling,[2] where it is $R_p/(1 + \mu)$.

[1] Impedances between control grid and other electrodes, such as screen grid or plate, are not reduced by the negative feedback action, unless the return circuit for these electrodes is to the cathode instead of to ground.

[2] With cathode coupling, the effective transconductance is, however, taken as the actual transconductance.

Ex. 1044-0439

The shunting capacity will depend upon the method of coupling. With coupling in the plate circuit (Fig. 63*a*) it consists of the output capacity of the tube. With cathode coupling it consists of the capacity between cathode and ground, plus the cathode-heater capacity when the heater is grounded, or the capacity between the primary and secondary of the filament transformer if the heater is connected to the cathode or if a filament tube is used.

When the actual load impedance is a resistance independent of frequency, as is normally the case, the amplification and the time-delay characteristics will depend upon the ratio X/R, where X is the reactance of the shunting condenser and R is the equivalent load resistance formed by the actual load resistance in parallel with the effective internal resistance of the tube. The amplification and time-delay characteristics that result are shown in Fig. 66. For the same perfection in behavior as normally required with video-frequency voltage amplification, the top frequency should not exceed one-half of the frequency f_0 at which the reactance of the shunting condenser equals the equivalent resistance R. It is to be noted that cathode coupling employing a triode tube will give a better high-frequency characteristic than cathode coupling with a pentode tube or any form of plate coupling, because of the lower effective plate resistance in the equivalent circuit of Fig. 63*c*.

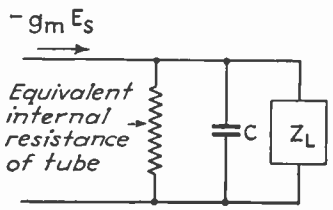


FIG. 65.—Equivalent plate circuit of a video-frequency power amplifier.

Video-frequency Amplifiers for Developing Large Voltages.[1]—Amplifiers in which the objective is to develop the maximum possible output voltage without excessive distortion, irrespective of the applied signal required, are special forms of power amplifiers of practical importance. Such amplifiers are employed to operate cathode-ray tubes in television receivers, and also for grid modulation of television transmitters.

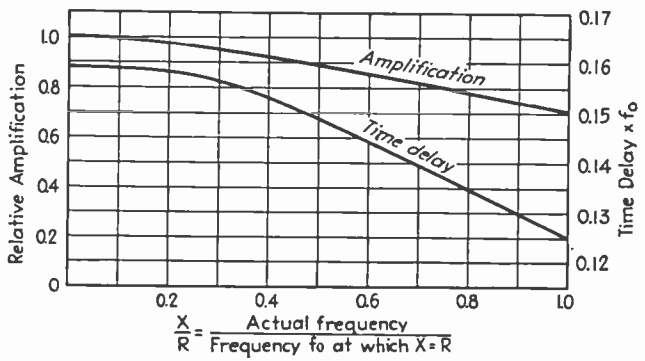


FIG. 66.—Amplitude and time-delay characteristics of a video-frequency power amplifier.

The coupling arrangements used to develop large output voltages depend upon the character of the impedance to which this voltage is applied. When this impedance is a capacity, as in the case of a cathode-ray tube, one can employ one of the usual two- or four-terminal networks, such as are used in voltage amplification, with the capacity of the load taking the place of the input capacity of the next stage in the corresponding voltage amplifier case. On the other hand, when the load across which

[1] Further discussion is given by D. E. Foster and J. A. Rankin, Video Output Systems, *R.C.A. Rev.*, Vol. 5, p. 409, April, 1941.

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the large voltage is to be developed consists of a resistance, or a resistance shunted by a capacity, then one is restricted to the circuits of Figs. 54*a* and *b*.

The maximum output voltage that can be developed by an amplifier tube is

$$\text{Peak a-c output voltage} = I_p R_L \tag{65}$$

where I_p = d-c plate current of amplifier.

R_L = load resistance used in coupling network.

When the output capacity of the tube is the biggest capacity involved in the coupling network, paralleling tubes to increase the current I_p in Eq. (65) will make it necessary to use a proportionately lower load resistance R_L, and so will not increase the output voltage obtainable. The only way in which more voltage can be realized under such restrictions is to change the type of tube to one having more favorable characteristics. On the other hand, as long as the capacity associated with the load determines the design of the coupling network, then paralleling tubes will enable increased output voltages to be obtained.

18. Tubes for Videc Amplification.—The merit of a tube as a video amplifier depends upon the particular application involved. In the case of video-frequency voltage amplifiers using two-terminal networks, the amount of amplification obtainable over a given band width is proportional to the figure of merit:

$$\left.\begin{array}{l}\text{Figure of merit for}\\ \text{voltage amplification}\\ \text{with two-terminal}\\ \text{coupling networks}\end{array}\right\} = \frac{g_m}{C_g + C_p} \tag{66a}$$

where C_g is the input capacity of the tube (sum of grid-cathode and grid-screen capacities) and C_p is the output capacity (sum of plate-screen and plate-ground capacities). With four-terminal networks, Eq. (66*a*) becomes

$$\left.\begin{array}{l}\text{Figure of merit for}\\ \text{voltage amplification}\\ \text{with four-terminal}\\ \text{coupling networks}\end{array}\right\} = \frac{g_m}{C} \tag{66b}$$

where C is either the input or the output capacity of the tube, whichever is the larger. Finally, in the ideal case of a four-terminal coupling network maintaining the same impedance level at the input and output terminals, the merit is

$$\left.\begin{array}{l}\text{Figure of merit for}\\ \text{voltage amplification}\\ \text{in ideal case}\end{array}\right\} = \frac{g_m}{\sqrt{C_g C_p}} \tag{66c}$$

Examination of these relations shows that, irrespective of the exact conditions, what is desired is a large transconductance in proportion to electrode capacities. In order to achieve this, it is necessary to make the interelectrode spacings of the tube very small.[1] It is of no avail to increase the area of the electrode structures, since doing so increases the capacities in proportion to the gain in transconductance. Commercial tubes with closely spaced electrodes (such as the 1852) are capable of giving several times as much voltage amplification as the corresponding tubes ordinarily used in resistance-coupled amplification.

The figure of merit of an amplifier tube intended to develop as large an output voltage as possible is

[1] See A. P. Kauzmann, New Television Amplifier Receiving Tubes, *R.C.A. Rev.*, Vol. 3 p. 271, January, 1939.

Ex. 1044-0441

$$\left.\begin{matrix}\text{Figure of merit}\\ \text{for large output}\\ \text{voltage}\end{matrix}\right\} = \frac{I_p}{C_p} \tag{67}$$

When tubes having the same figure of merit according to Eq. (67) differ in input capacity, then the one with the lower input capacity is to be preferred. In particular, triode tubes, because of their high effective input capacity, are always to be avoided, even though their figure of merit may be quite satisfactory. Tube characteristics favoring high merit in Eq. (67) are a large cathode, large electrode areas, and relatively large spacing. These are the characteristics of power tubes, leading to the conclusion that pentode and beam power tubes are best for developing large voltages.

Figures of merit of various representative tubes for voltage-amplifier service and for developing large output voltages are given in Table 3.

TABLE 3.—CHARACTERISTICS OF TYPICAL TELEVISION TUBES

Tube type	Rated d-c plate current, ma	Transconductance at rated plate current, μ mhos	Input capacity, $\mu\mu$f	Output capacity, $\mu\mu$f	Figures of merit for		
					Voltage gain		Large output voltage
					Two terminal	Four terminal	
Receiver pentodes:							
6D6	8.2	1,600	6	6	125	250	1.4
954	2.0	1,400	3	3	230	465	0.7
1851	10.0	9,000	15	5	450	600	2.0
1853	12.5	5,000	10	5	330	500	2.5
Transmitter pentodes:							
802	20.0	2,250	12	8.5	110	190	2.3
803	62.5	4,000	17.5	29	85	140	2.2
837	24.0	3,400	16	10	130	210	2.4
Beam power tubes:							
6L6	72.0	6,000	13	11	250	460	6.6
807	72.0	6,000	11	7	330	550	10.3

TUNED AMPLIFIERS

19. Tuned Voltage Amplifiers.—A tuned voltage amplifier is a Class A voltage amplifier in which the load impedance is supplied by a resonant circuit. Circuits of typically tuned amplifiers are shown in Fig. 67. Such amplifiers are used to amplify signal frequency voltages in radio receivers, in which case they are referred to as tuned radio-frequency amplifiers, and also to amplify intermediate-frequency voltages in superheterodyne receivers.

The important characteristics of a tuned voltage amplifier are the amplification at resonance, the variation of amplification with frequency in the immediate vicinity of resonance, and the way in which the amplification obtainable changes as the resonant frequency of the tuned circuits is varied.

Characteristics of Simple Tuned Amplifiers.—Tuned amplifiers using only a single resonant circuit to provide the load impedance, as in Figs. 67*a* and 67*b*, have a curve of amplification as a function of frequency that approximates the shape of an ordinary

Ex. 1044-0442

resonance curve having a Q somewhat lower than the actual Q of the resonant circuit (see Fig. 69). It is accordingly possible to describe the amplification characteristic of such an amplifier in terms of the amplification at resonance and the effective Q of the amplification curve. This leads to the following formulas:

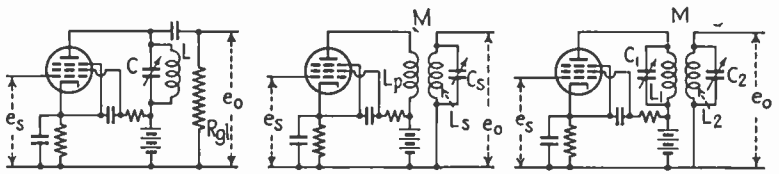


FIG. 67.—Typical tuned-amplifier circuits.

For direct-coupled circuit of Fig. 67*a*:

$$\left.\begin{matrix}\text{Amplification at}\\ \text{resonance}\end{matrix}\right\} = g_m \frac{\omega LQ}{1 + \dfrac{\omega LQ}{R_p} + \dfrac{\omega LQ}{R_{gl}}} \tag{68}$$

$$\frac{\left.\begin{matrix}\text{Effective } Q \text{ of}\\ \text{amplification curve}\end{matrix}\right\}}{\left.\begin{matrix}\text{Actual } Q \text{ of tuned}\\ \text{circuit}\end{matrix}\right\}} = \frac{1}{1 + \dfrac{\omega LQ}{R_p} + \dfrac{\omega LQ}{R_{gl}}} \tag{69}$$

For transformer-coupled circuit of Fig. 67*b*:

$$\left.\begin{matrix}\text{Amplification}\\ \text{at resonance}\end{matrix}\right\} = g_m \frac{\omega MQ}{1 + \dfrac{(\omega M)^2/R_s}{R_p}} \tag{70}$$

$$\frac{\left.\begin{matrix}\text{Effective } Q \text{ of}\\ \text{amplification curve}\end{matrix}\right\}}{\left.\begin{matrix}\text{Actual } Q \text{ of tuned}\\ \text{circuit}\end{matrix}\right\}} = \frac{1}{1 + \dfrac{(\omega M)^2/R_s}{R_p}} \tag{71}$$

In these equations, g_m is the transconductance and R_p the plate resistance of the amplifier tube; Q is the actual Q of the resonant circuit; and the remaining notation is as illustrated in the figures.

In the case of pentode tubes, the plate resistance R_p is very much greater than the parallel impedance or coupled impedance of the resonant circuit. Also, in practical amplifiers, the grid-leak resistance R_{gl} is likewise much larger than the resonant circuit impedance. Accordingly, with pentode tubes the effective Q of the amplification curve can be taken as approximating the actual Q of the resonant circuit, while the amplification at resonance is given by the following relations:

For direct-coupled circuit of Fig. 67*a*:

$$\left.\begin{matrix}\text{Approximate amplification}\\ \text{at resonance}\end{matrix}\right\} = g_m \omega LQ \tag{72}$$

For transformer-coupled circuit of Fig. 67*b*:

$$\left.\begin{matrix}\text{Approximate amplification}\\ \text{at resonance}\end{matrix}\right\} = g_m \omega MQ \tag{73}$$

Ex. 1044-0443

With triode tubes, the effective Q of the amplification curve depends greatly upon the coupling between the resonant circuit and the plate circuit of the tube. Also there is an optimum coupling that gives maximum gain. Because of this necessity of impedance matching, the transformer-coupled circuit of Fig. 67*b* is normally used with triodes. The amplification, as given by Eq. (70), is maximum when the resistance $(\omega M)^2/R_s$ coupled into the plate circuit of the tube by the tuned circuit equals the plate resistance of the tube. The effective Q under these conditions is one-half of the actual Q of the resonant circuit. With greater coupling the effective Q is reduced and the amplification becomes less, whereas with smaller coupling, the amplification drops, and the effective Q increases, as shown in Fig. 68.

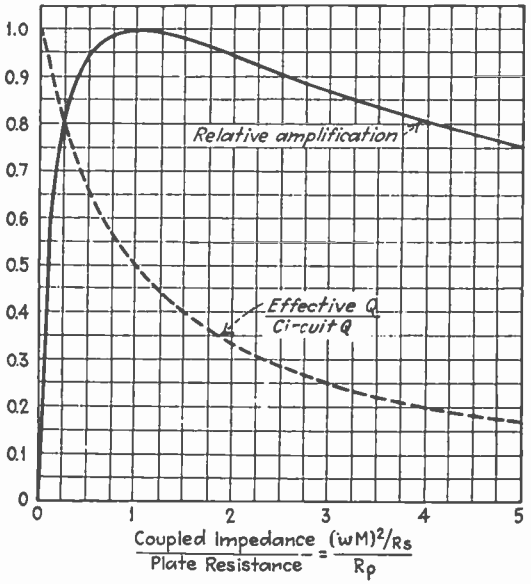


FIG. 68.—Effect of matching between load impedance and plate resistance upon the relative amplification and effective Q of a tuned amplifier.

Band-pass Amplifiers.—Band-pass coupling gives a band-pass type of amplification characteristic as shown in Fig. 69. The shape of the curve obtained, however, differs from the band-pass characteristic of the actual coupled circuits, corresponding instead to the band-pass characteristic obtained with a primary circuit having an effective Q lower than the actual Q of this circuit according to the relation[1]

$$\left.\begin{matrix}\text{Effective } Q \text{ of} \\ \text{primary circuit}\end{matrix}\right\} = \frac{\text{Actual } Q \text{ of primary}}{1 + \dfrac{(1/\omega C_1)^2}{R_1 R_p}} \tag{74}$$

where the notation is as in Fig. 67*c*, with the addition that R_1 is the series resistance of the primary tuned circuit. With pentode tubes, the effective Q closely approximates the actual Q. If Eq. (74) is taken into account, the shape of the amplification characteristic obtained with band-pass coupling can be readily determined with the aid of the principles given in Par. 9, Sec. 3. It was there shown that when the primary and secondary circuit Q's were approximately the same, then the width of

[1] For derivation, see F. E. Terman, "Radio Engineering," 2d ed., p. 216, McGraw-Hill, New York, 1937.

Ex. 1044-0444

the response band was determined by the coefficient of coupling k according to the approximate relation

$$\frac{\text{Width of response band}}{\text{Mean frequency}} = 1.2k \tag{75a}$$

Substantially constant response is then obtained over the band when the primary and secondary Q's, Q_1 and Q_2, respectively, are so related that

$$\sqrt{Q_1Q_2} = \frac{1.75}{k} \tag{75b}$$

When the primary and secondary Q's are considerably different, as, for example,

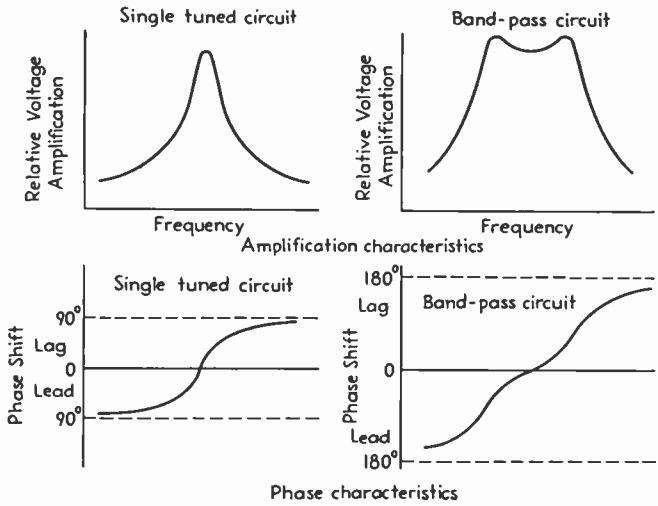


FIG. 69.—Curves showing amplification and phase-shift characteristic as a function of frequency with tuned amplifiers having a single tuned circuit and having a band-pass characteristic.

when practically all the losses of the system are in one of the resonant circuits, then the proper coefficient of coupling is given by the equation

$$\text{Desired coefficient of coupling} = k\sqrt{1 + 0.33a^2} \tag{75c}$$

where k = coefficient of coupling that would be required if the primary and secondary had equal Q's, as given by Eq. (75a).

$$a = \frac{Q_1 - Q_2}{Q_1 + Q_2}.$$

When the two circuits are resonant at the same frequency, the amplification at resonance is

$$\left.\begin{matrix}\text{Amplification at} \\ \text{resonant frequency}\end{matrix}\right\} = g_m k \frac{\omega_0\sqrt{L_1L_2}}{k^2 + \dfrac{1}{Q_1Q_2}} \tag{76}$$

where g_m = transconductance of tube.
k = coefficient of coupling between primary and secondary inductances.
ω_0 = 2π times resonant frequency.
Q_1 = effective Q of primary circuit as obtained from Eq. (74).

Ex. 1044-0445

Q_2 = Q of secondary circuit.
L_1, L_2 = primary and secondary inductances, respectively.

The amplification is maximum when the coefficient of coupling has the critical value, *i.e.*, when $k = 1/\sqrt{Q_1 Q_2}$, and for this condition becomes

$$\left.\begin{matrix}\text{Maximum possible}\\ \text{amplification}\\ \text{at resonance}\end{matrix}\right\} = g_m \frac{\omega_0 \sqrt{L_1 L_2}\sqrt{Q_1 Q_2}}{2} \tag{77}$$

Band-pass behavior can also be obtained by combining a double-humped curve of one stage having an overcoupled pair of resonant circuits, with a peaked curve of another stage having a single tuned circuit. Such systems are discussed in connection with Fig. 37, Sec. 3. Band-pass action can also be obtained by tuning primary and secondary circuits to slightly different frequencies, or by similarly detuning alternate stages of a multistage amplifier. These last arrangements are very seldom used, however, because of the difficulty of adjusting the circuits properly under field

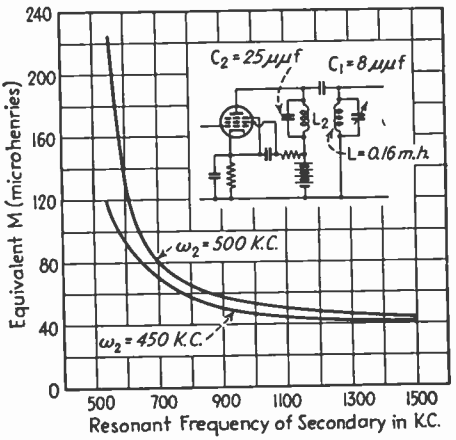


FIG. 70.—Variation of equivalent mutual inductance with frequency in a typical complex coupling system.

conditions, together with the fact that the gain with detuning is less than when the same pass band is obtained with primary and secondary resonant at the same frequency.

Variation of Amplification with Resonant Frequency—Use of Complex Coupling.—With direct and transformer coupling, the amplification tends to be proportional to resonant frequency when the circuit Q is approximately constant. In the case of tuned radio-frequency amplifiers that must be adjustable to amplify signals over a considerable frequency range, this results in greater amplification (with a consequent increased tendency toward regeneration) at the higher frequencies. This undesirable tendency can be controlled by the use of complex coupling systems such as discussed in Par. 8, Sec. 3, in which the equivalent mutual inductance varies with frequency. An example of such a circuit is shown in Fig. 70, and other possible coupling networks are given in Figs. 28 to 34 in Sec. 3. By proper proportioning of these circuits it is possible to control the variation of equivalent mutual inductance with frequency so that the amplification will be the same at two or more widely different frequencies.

Tuned amplifiers involving complex coupled systems can be reduced to an equivalent transformer coupled case by the expedient of reducing these circuits to an equiva-

Ex. 1044-0446

lent inductively coupled circuit, as discussed in connection with Fig. 35, Sec. 3, and then calculating the behavior of the resulting equivalent circuit by the formulas for transformer coupling.

Regeneration in Multistage Tuned Amplifiers.—Multistage tuned amplifiers must be carefully arranged to avoid regeneration. Coils in the tuned circuits of different stages must be either shielded, or spaced a considerable distance apart with suitable orientation. Tuning condensers must likewise be either shielded or otherwise arranged to minimize coupling between stages. With triode tubes the grid-plate capacity must be neutralized (see Par. 25). High potential leads associated with stages of widely different power levels must be carefully located, and in some cases shielded. By-pass condensers are required to prevent currents from the different stages from flowing through impedances that are common to several stages. Grounds must be arranged so that the chassis does not serve as a common return for circuits of different power levels and so become a source of coupling between stages.

The essential factors involved in locating grounds are illustrated in Fig. 71. If the coils of resonant circuits are returned to ground, as shown by b_3 and b_4, instead

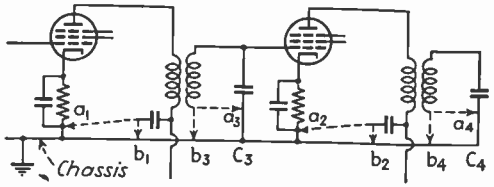


FIG. 71.—Correct and incorrect methods of locating ground connections in a multistage tuned amplifier.

of directly to the condensers, as by a_3 and a_4, then the chassis will carry tuned circuit currents from b_3 to c_3 and b_4 to c_4. The paths of these currents through the chassis will inevitably overlap to some extent, resulting in energy transfer between stages that will cause regeneration. A similar situation exists for the point at which the by-pass condenser returns to ground, the proper arrangements in Fig. 71 being a_1 and a_2 and the improper ones, b_1 and b_2.

The principal effect of regeneration in multistage tuned amplifiers is ordinarily an increase in the effective Q of one or more of the individual tuned circuits together with a slight change in the capacity required to tune these circuits to a given frequency. The increase in effective Q produces an increase in the amplification and in the selectivity.

When tuned radio-frequency amplifiers were first used, regeneration was frequently introduced intentionally in order to increase the amplification and selectivity. The development of improved amplifiers has caused this to be considered poor practice, because the regeneration varies with the resonant frequency, the volume setting, etc., is critical with respect to tube voltages, and causes oscillations when excessive.[1]

When regeneration is carried as far as possible without the production of oscillations, the amplification obtained is greater for weak than for strong signals. This is

[1] The use of negative feedback makes it possible to stabilize the regeneration, and thereby obtain selectivity corresponding to a much higher Q than that of a resonant circuit without the necessity of critical adjustments. See F. E. Terman, R. R. Buss, W. R. Hewlett, and F. C. Cahill, Some Applications of Negative Feedback with Particular Reference to Laboratory Equipment, *Proc. I.R.E.*, Vol. 27, p. 649, October, 1939; Shepard Roberts, Stabilized Regenerative Amplifier, *Rev. Sci. Instruments*, Vol. 9, p. 429, December, 1938.

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because the third-order curvature of the tube characteristic causes the effective plate resistance of the tube to be slightly greater for large signals than for small signals, thus reducing the regeneration for large signals. Analysis shows that when the regeneration is as great as possible without oscillation in the absence of a signal, the amplification is inversely proportional to the two-thirds power of the signal voltage and directly proportional to the response obtained when no regeneration is present.[1]

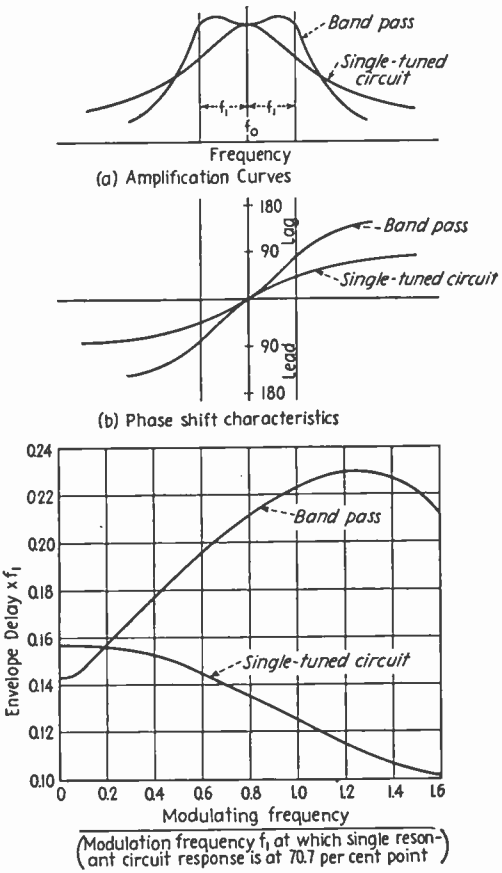


FIG. 71a.—Phase-shift and envelope-delay characteristics of typical tuned amplifiers.

Tubes.—Tuned voltage amplifiers practically always employ pentodes. Such tubes have negligible direct grid-plate capacity and so do not need be neutralized as do triodes. Also, they give more gain per stage and result in an effective Q of the amplification curve that more closely approaches the actual Q of the resonant circuit.

Volume Control.—Volume control of tuned amplifiers is normally obtained by using variable-mu tubes, and varying the bias or the screen voltage. This varies the transconductance and hence the gain, without appreciably affecting the resonant frequency of the circuits.[2] Variable-mu tubes are preferred to those having sharp

[1] Balth. van der Pol, The Effect of Regeneration on the Received Signal Strength, *Proc. I.R.E.*, Vol. 17, p. 339, February, 1929.

[2] The effect on tuning is discussed on p. 473.

Ex. 1044-0448

cutoff, because they give a less critical control, and introduce less cross-modulation with low volume settings.

Phase and Delay Characteristics of Tuned Amplifiers.—Tuned amplifiers introduce a phase shift that varies with frequency, and so give rise to time-delay effects in the same manner as audio- and video-frequency amplifiers. The characteristic of importance in the case of tuned amplifiers is the variation in phase of the envelope of the wave with modulation frequency, since this introduces a time delay in the envelope that appears as time delay in the output after demodulation. The *envelope delay* for a modulated wave is

$$\left.\begin{matrix}\text{Envelope delay in}\\ \text{seconds}\end{matrix}\right\} = \frac{\Delta\Phi}{\omega_m} \tag{78}$$

where $\Delta\Phi$ = amount side bands are shifted in phase with respect to the carrier frequency.

$\omega_m/2\pi$ = modulation frequency.

In this equation it is assumed that the upper and lower side bands suffer equal phase shifts of opposite polarity, *i.e.*, that the amplifier is adjusted so that its mid-frequency coincides with the carrier frequency.

The envelope delay will be constant only to the extent that the phase shift of the tuned amplifier is a linear function of frequency over the range of side-band frequencies involved. The phase-shift characteristics of typical tuned amplifiers are shown in Fig. 71*a*, together with the corresponding delay characteristics on the assumption of a modulated wave with the carrier centered at the middle of the pass band.

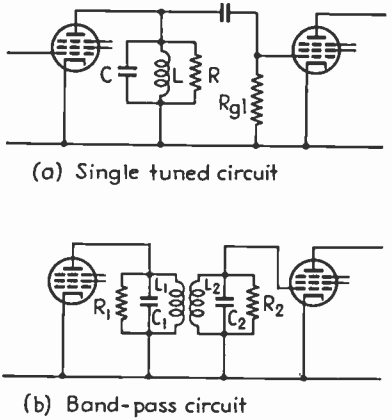


FIG. 72.—Typical circuits for tuned radio-frequency amplification of television signals.

20. Wide-band Tuned Amplifiers for Television Signals.—Tuned amplifiers for television signals differ from ordinary radio-frequency and intermediate-frequency amplifiers in that the carrier frequency is much higher and that the band width is a larger percentage of the carrier frequency. The requirements as to uniformity of amplification and linearity of phase shift within the pass band are also more severe. Intermediate-frequency amplifiers for video signals must likewise provide effective discrimination against the adjacent sound channels.

Tuned Radio-frequency Amplification of Television Signals.—The usual tuned radio-frequency wide-band amplifier consists of either a single resonant circuit or a band-pass arrangement as shown in Fig. 72, with most of the tuning capacity supplied by the tubes, and with resistance loading provided to give the wide band required. Although band-pass circuits are harder to tune to an incoming signal because two circuits must be adjusted, they are better than the single circuit from the point of view of gain, selectivity, uniformity of amplification in the pass band, phase characteristics, etc. Still more complicated coupling arrangements will give further improvement in performance, but are never used because of the added problems involved in tuning.

The behavior of tuned amplifiers employing the coupling methods shown in Fig. 72 is discussed in Par. 19. In applying these results, the effect of a resistance R shunted

Ex. 1044-0449

across a tuned circuit is to make the effective Q of the circuit have the value

$$\left.\begin{matrix}\text{Effective } Q \text{ of} \\ \text{resonant circuit}\end{matrix}\right\} = Q_1 \frac{1}{1 + \frac{Q_1}{Q_2}} \tag{79}$$

where Q_2 is the actual Q of the resonant circuit without the resistance loading and $Q_1 = R/\omega L$.

Intermediate-frequency Amplifiers.[1]—The converter of a television receiver ordinarily changes the incoming carrier to an intermediate frequency of the order of 12 mc. The band width that must be handled at this carrier frequency approximates 4.5 mc and so is an unusually large percentage of the carrier frequency. The intermediate-frequency amplifiers for television signals must also provide special means for eliminating the sound associated with the picture signal being handled and the sound of the adjacent television channel. It is also to be noted that inasmuch as ordinary television signals are of the single side-band type wherein one side band is largely suppressed, the carrier frequency is at the edge of the transmission band rather than in the middle.

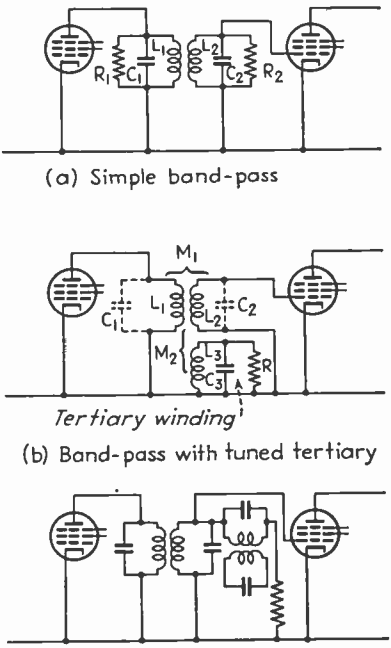


FIG. 73.—Examples of coupling networks used in intermediate-frequency amplifiers for television signals.

A great variety of circuit arrangements can be employed to give uniform amplification over the required intermediate-frequency band, and at the same time provide reasonable selectivity. Examples in practical use are shown in Fig. 73, and involve at least two resonant circuits in each of which the tuning capacity is supplied largely either by the input or output capacity of an associated tube.

The circuit of Fig. 73*a* is the same as that of Fig. 72*b* and needs no further discussion. The addition of tuned tertiary L_3C_3 containing the circuit damping and coupled to the secondary, as shown in Fig. 73*c*, gives more gain, flatter response within the pass band, and better selectivity than Fig. 73*a*. In this three-circuit system mutual inductance M_1 largely dictates the band width, and M_2 and R determine the flatness of response. The coupling system of Fig. 73*c* is a band-pass analogue of the circuits of Fig. 58, and is one of a class of band-pass circuits that can be built up on filter theory. Such band-pass filters, although having excellent characteristics, are too expensive and complicated to be generally used in television receivers.

In addition to the circuits of Fig. 73, band-pass coupling can also be obtained by suitably detuning successive stages or by combining a double-humped resonance curve of one stage with a peaked curve of the following stage, as in Fig. 37, Sec. 3.

In practical intermediate-frequency amplifiers, it is necessary to provide special trap circuits for suppressing the accompanying sound that is present at the low-

[1] Further discussion, together with designs (including all circuit constants) for intermediate-frequency amplifiers with 4 and 2.3-mc band widths, is given by Garrard Mountjoy, Simplified Television I-F Systems, *R.C.A. Rev.*, Vol. 4, p. 299, January, 1940.

Ex. 1044-0450

frequency edge of the pass band, and the sound of the adjacent television channel that is present at the other edge of the pass-band. In the case of a 12.75 mc intermediate-frequency carrier, these sound carriers have frequencies of 8.25 mc and 14.25 mc, respectively. Typical methods of providing such attenuation are illustrated in Fig. 74. At *a* suppression is accomplished with the aid of a low resistance resonant circuit *LC* tuned to the undesired frequency and coupled as shown, so that a large resistance will be introduced into the interstage network at the frequency to be suppressed. Circuit *b* employs a resonant trap in the grid lead of the amplifier tube, and circuit *c* makes use of a coupling network that is in series resonance at the undesired frequency and so gives no coupling for this frequency. The arrangement of Fig. 74*d* employs a resonant trap *LC* in the cathode circuit that reduces the amplification at the undesired frequency as a result of the large negative feedback at the resonant frequency of *LC*.

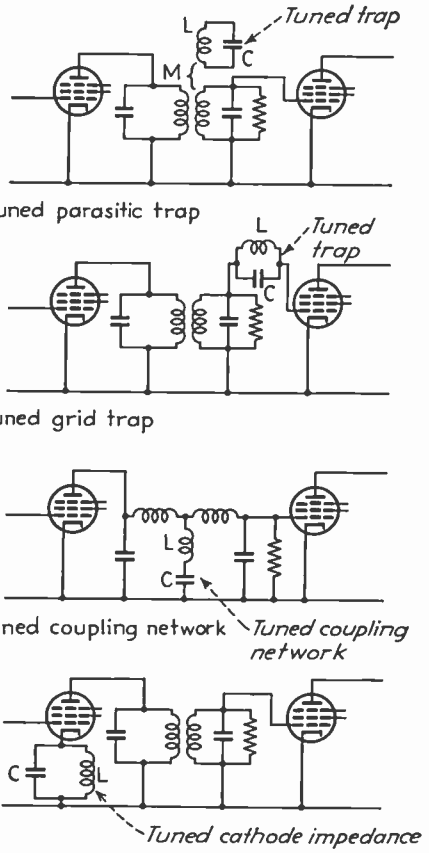


FIG. 74.—Trap circuits for discriminating against the sound signals on the edge of a television picture channel.

In practical intermediate-frequency systems, it is customary to provide about two traps for the accompanying sound, and one or two traps for the interfering sound of the adjacent channel. These can be distributed among the various intermediate-frequency coupling networks and tubes as is convenient.

Envelope Delay.—Envelope delay is of particular importance in tuned amplifiers handling television signals, because any variation in the envelope delay with modulating frequency adds to the total delay error of the system. The relation between the envelope delay and phase shift in the case of a modulated wave is given by Eq. (78). This equation also gives the envelope delay for a signal with suppressed side-band (single side-band television signal), provided that the phase shift is defined as the phase shift of the side-band frequency in question relative to the carrier.

In the case of either ordinary modulated waves or waves with one side band suppressed, the envelope-delay error produced by a single tuned circuit or a band-pass coupling network is much greater than for a stage of properly compensated video-frequency amplification, unless the band width of the tuned amplifier is appreciably greater than is necessary to accommodate the highest side-band frequency (see Fig. 71*a*). In transmitters the envelope-delay error can be kept small by using an extra

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wide pass band, although this reduces the power-handling capacity of the tuned amplifier. There is also the possibility in transmitters of compensating for errors introduced by the tuned circuits by suitably proportioning the video-frequency stages. With receivers, the problem of minimizing the delay error in the tuned circuits is more difficult, particularly in the intermediate-frequency amplifier, because of the necessity of obtaining high discrimination against the sound channels on either side of the picture band. Fortunately, only several stages of tuned amplification are required in the intermediate-frequency system of a receiver. By designing the remainder of the television system with the smallest possible delay error, a moderate phase distortion can be tolerated in the intermediate-frequency amplifier without seriously detracting from the reproduced picture.

21. Class C Amplifiers.—A Class C amplifier is an amplifier so operated that the plate current flows in the form of pulses lasting for less than half a cycle. Such

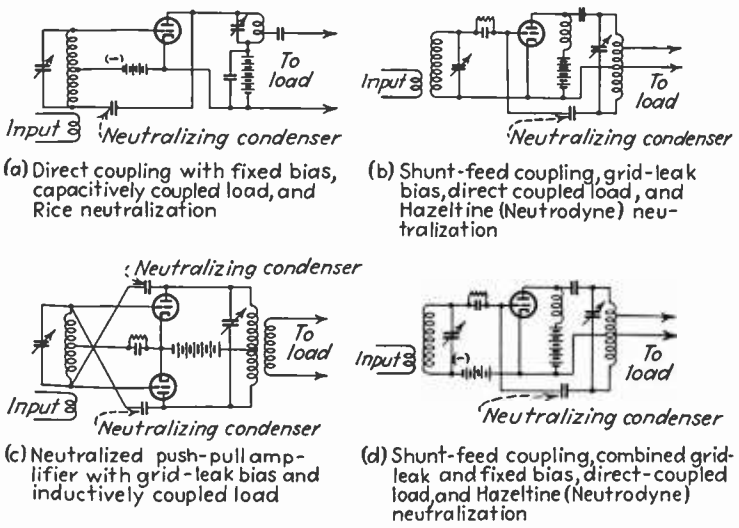


FIG. 75.—Typical circuit arrangements for Class C amplifiers.

amplifiers employ a load impedance consisting of a tuned circuit, and are characterized by relatively high plate efficiency. They are used primarily for the production of large quantities of power at a single frequency, as is necessary, for example, in radio transmitters.

Typical Class C amplifier circuits are shown in Fig. 75. These differ from each other in such respects as method of coupling the load to the tuned circuit, method of neutralization, and the way in which the grid bias is obtained.

Voltage, Current, and Power Relations in Class C Amplifiers Employing Triode Tubes.—The fundamental voltage and current relations existing in a properly adjusted Class C amplifier are shown in Fig. 76. The voltage acting on the grid electrode of the tube consists of a bias voltage E_c greater than the cutoff value, upon which is superimposed an alternating signal voltage E_s that is normally of sufficient amplitude to drive the grid positive at the peak of each cycle by an amount E_{max}, as indicated. The voltage acting on the plate electrode of the tube is the d-c plate supply voltage E_b minus the voltage drop E_L in the tuned load circuit. This drop is sinusoidal because of the resonant character of the load circuit, and with proper adjustment has a peak amplitude almost but not quite equal to the plate-supply voltage. The

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minimum instantaneous plate potential E_{min} reached during the cycle is hence small but not zero. The phase relations are such that the minimum instantaneous plate potential E_{min} occurs at the same part of the cycle as the maximum grid potential E_{max}.

The plate and grid currents that flow in the tube at any instant are the result of the combined action of the plate and grid potentials at that instant, and can be obtained from these potentials with the aid of characteristic curves of the tube. These currents are in the form of pulses having angles of flow θ_p and θ_g, respectively, that are less than 180° (Figs. 76*e* and 76*f*). These angles of current flow can be expressed as follows:

$$\cos\left(\frac{\theta_g}{2}\right) = \frac{E_s - E_{max}}{E_s} = \frac{E_c}{E_s} \qquad (80)$$

$$\cos\left(\frac{\theta_p}{2}\right) = \frac{1}{1 + \dfrac{\mu E_{max} + E_{min}}{\mu E_c - E_b}} \qquad (81a)$$

An alternative form for Eq. (81*a*) is

$$\text{Grid bias} = E_c = \frac{E_b}{\mu} + \left(E_{max} + \frac{E_{min}}{\mu}\right) \frac{\cos\left(\dfrac{\theta_p}{2}\right)}{1 - \cos\left(\dfrac{\theta_p}{2}\right)} \qquad (81b)$$

In these equations, E_{max}, E_{min}, and E_b have the definitions given above, and μ is the amplification factor of the tube. The average of the pulses of current flowing to an electrode represents the direct current drawn by that electrode.

The power input to the plate electrode of the tube at any instant is the product of plate-supply voltage and instantaneous plate current, while the corresponding power lost at the plate is the product of instantaneous plate-cathode voltage and instantaneous plate current (see Fig. 76*g*). The difference between these two quantities represents the useful output being developed at the moment. The average input, output, and loss are obtained by averaging the instantaneous powers.

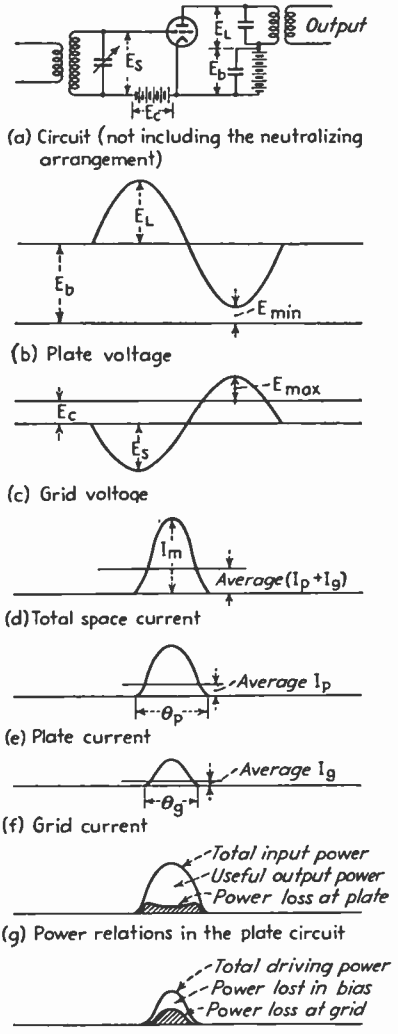


FIG. 76.—Voltage, current, and power relations in a typical Class C amplifier.

The efficiency is the ratio of average output to average input and is commonly of the order of 60 to 80 per cent. The efficiency is high in a Class C amplifier because current is permitted to flow only when most of the plate-supply voltage is used as voltage drop across the tuned load circuit, and only a small fraction is wasted as voltage drop at the plate electrode of the tube.

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The instantaneous power absorbed from the exciting voltage applied to the grid electrode of the tube is equal to the product of instantaneous exciting voltage and instantaneous grid current (Fig. 76*h*). A portion of this instantaneous power is dissipated at the grid electrode of the tube, and the remainder is dissipated in the grid-leak resistance (or is used to charge the bias battery).

Calculation of Class C Amplifier Behavior with Triode Tubes—Exact Method.—The first step in the procedure is to calculate the instantaneous plate and grid potentials at various points of the cycle according to the relations

$$\text{Instantaneous plate voltage} = e_p = E_p - (E_b - E_{\min}) \cos \beta \qquad (82a)$$

$$\text{Instantaneous grid potential} = e_g = (E_c + E_{\max}) \cos \beta - E_c \qquad (82b)$$

where β is the number of electrical degrees from the crest of the cycle at which e_p and e_g are to be evaluated and where the remaining notation is as before. For each combination of instantaneous grid and plate potentials, the corresponding instantaneous grid and plate currents can be obtained from characteristic curves of the tube. In this way the plate- and grid-current pulses can be calculated and plotted point by point. Curves of instantaneous power input to the plate circuit of the tube and of power dissipated at the plate electrode can be obtained by multiplying the instantaneous plate current by the plate-supply voltage E_b and the instantaneous plate voltage e_p, respectively. Similarly, the instantaneous power supplied by the exciting voltage E_s is equal to the instantaneous value of exciting voltage $E_s \cos \beta$ times the grid current at that instant. The part of this power that is dissipated at the grid is the product of instantaneous grid current and the grid-cathode voltage e_g at that instant.

The d-c plate and grid currents represent the average of the current pulses over the full cycle, and the mean powers are obtained in the same way by an averaging process. These averages can be obtained by planimetering the areas under the pulses, by plotting the curves on cross-section paper and counting the number of squares under the pulses, or by averaging values calculated for regular steps in β from $\beta = 0$ to $\beta = 180°$.

The exact method of calculating Class C amplifier performance results in an accuracy limited only by the extent to which the tube characteristics used actually apply to the particular tube involved under the operating conditions. The method has the disadvantage of being very laborious, even when the point-by-point calculations are carried out in a systematic manner.[1] Accordingly, the exact method of Class C amplifier analysis is used only where the approximate method given below is not satisfactory, as, for example, in some types of modulated-amplifier calculations.

Calculation of Class C Amplifier Behavior with Triode Tubes—Approximate Method.[2] An approximate calculation of Class C amplifier performance with accuracy sufficient for ordinary design purposes can be obtained without point-by-point calculations by taking advantage of the fact that the total space current $(I_p + I_g)$ can be expressed rather accurately by the relation

$$\text{Total space current} = I_p + I_g = k \left(\frac{e_p}{\mu} + e_g \right)^{\alpha} \qquad (83)$$

[1] A method of organizing the procedure so that results can be obtained with the minimum of time is given by D. C. Prince, Vacuum Tubes as Power Oscillators, *Proc. I.R.E.*, Vol. 11, p. 275, June, p. 405, August, and p. 527, October, 1923.

[2] This follows F. E. Terman and W. C. Roake, Calculation and Design of Class C Amplifiers, *Proc. I.R.E.*, Vol. 24, p. 620, April, 1936.

Other approximate methods of less accuracy but about equal complexity are given by W. G. Wagener, Simplified Methods for Computing Performance of Transmitting Tubes, *Proc. I.R.E.*, Vol. 25, p. 47, January, 1937; W. L. Everitt, Optimum Operating Conditions for Class C Amplifiers, *Proc. I.R.E.*, Vol. 22, p. 152, February, 1934.

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Here e_p and e_g are the instantaneous plate and grid potentials, respectively, μ is the amplification factor of the tube (assumed constant), k is a constant, and α is another constant normally having a value very close to $\frac{3}{2}$. To the extent that Eq. (83) holds, the pulses of plate current in a Class C amplifier have a definite form in which the direct-current and fundamental-frequency components are functions only of the angle of plate current flow θ_p, the peak value I_m of the space current, and the exponent α. This relationship has been worked out and is presented in Fig. 77 for values of α between 1 and 2.

The direct-current and fundamental frequency components of the total space currents divide between the grid and plate electrodes. The portions going to the grid electrode can be calculated from the fact that with normal amplifier adjustment, the pulses of grid current usually have a shape that approximates the square of a

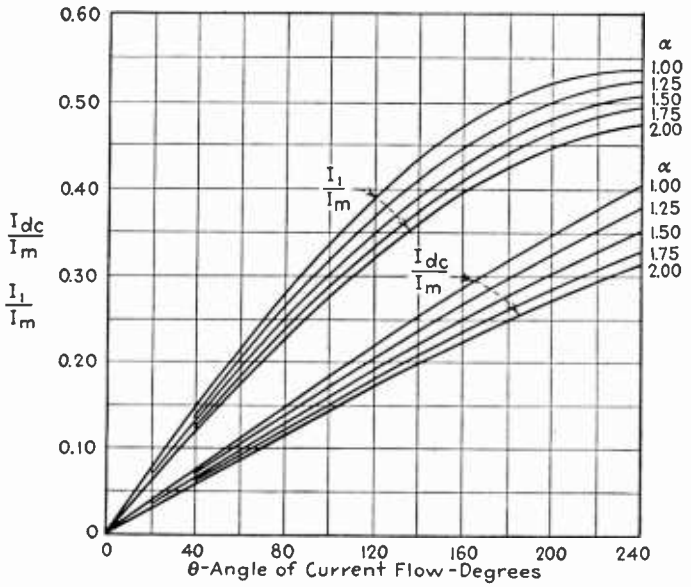


FIG. 77.—Curves giving relation of direct-current and fundamental-frequency components of the space-current pulse as a function of angle of flow θ and the peak amplitude I_m.

section of a sine wave.[1] Accordingly, by assuming that $\alpha = 2$, Fig. 77 can be used to determine the direct-current and fundamental-frequency components of the grid current when the angle of grid-current flow θ_g and the peak d-c grid current corresponding to instantaneous grid and plate electrode potentials E_{max} and E_{min} are known. The components flowing to the plate electrode are obtained by subtracting the parts going to the grid from the corresponding components of the total space current.

The power input to the Class C amplifier is now the product of battery voltage and d-c plate current, or

$$\text{Power input} = E_b I_{dc} \tag{84}$$

[1] This is the case under the usual conditions where secondary emission at the control grid is not excessive. See W. L. Everitt and Karl Spangenberg, Grid-current Flow as a Factor in the Design of Vacuum-tube Power Amplifiers, *Proc. I.R.E.*, Vol. 26, p. 612, May, 1938. With water-cooled tubes, it is sometimes found that the secondary emission may modify the shape of the grid current pulse greatly, and even make the d-c grid current slightly negative. Under these conditions, the grid current wave can be sketched and its d-c value estimated. The fundamental frequency component then has a crest value approximately twice the d-c value.

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where I_{dc} is the d-c plate current obtained with the aid of Fig. 77. Likewise, the power delivered to the load is equal to half the product of a-c plate current I_1, obtained with the aid of Fig. 77, and the alternating-current voltage developed across the load, or

$$\text{Power output} = \frac{(E_b - E_{\min})I_1}{2} \tag{85}$$

The plate dissipation is the difference between these two powers, and the efficiency is their ratio.

The required load impedance is

$$\left.\begin{matrix}\text{Load impedance between} \\ \text{plate and cathode}\end{matrix}\right\} = \frac{E_b - E_{\min}}{I_1} \tag{86}$$

The driving power is half the product of the crest values of exciting voltage E_s and the fundamental-frequency component I_{g1} of grid current, or

$$\text{Grid driving power} = \frac{E_s I_{g1}}{2} \tag{87}$$

The portion of this power delivered to the bias is equal to the product $I_g E_c$ of bias voltage and d-c grid current. The difference between the driving power and the bias power is dissipated at the grid electrode of the tube.

The only approximations involved in this analysis of Class C amplifiers is the assumption that μ and the exponent α in Eq. (83) are constant, and that the grid-current pulse follows a square law. Under ordinary conditions, the results are accurate to within about 5 per cent.

An example showing the details involved in calculating Class C amplifier performance by the approximate method is given below.

Design Considerations.—In designing a Class C amplifier, the customary objectives are high efficiency, large power output, and low exciting power. Efficiency can be made high by using a small angle of plate-current flow θ_p and a minimum instantaneous plate potential $E_{\min}$ that is small compared with the plate-supply voltage. Low exciting power is obtained by making $E_{\max}$ small and also considerably less than the minimum plate potential $E_{\min}$, and by adjusting so that the angle of plate-current flow θ_p is large. A large output power is obtained by making θ_p large, combined with $E_{\max}$ and $E_{\min}$ great enough so that the maximum space current I_m is large.

The best compromise between these various conflicting factors involved in the design is obtained when the angle of plate-current flow θ_p is of the order of 120 to 150 electrical degrees. The exciting voltage should drive the grid sufficiently positive so that the plate-current pulse will give an average plate current equal to the rated value. The load impedance should then be adjusted so that the voltage drop in the load with this plate-current pulse gives a minimum plate potential moderately greater than the maximum grid potential.

The maximum allowable peak amplitude I_m of the pulse of space current is the maximum electron emission that the filament can be depended upon to produce throughout its useful life. With tungsten filaments, it is common practice to make I_m very nearly the full emission from the filament, while in the case of thoriated-tungsten and oxide-coated filaments, the deterioration during life is such that I_m must be made much less than the emission initially obtainable from the filament. In the case of thoriated-tungsten filaments, a safety factor of 3 to 10 is typical, while with oxide-coated filaments a still larger margin of safety, such as 5 to 10, must be provided. When no information is available that gives directly the permissible peak space current, this can be taken as being four times the sum of the d-c plate and

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grid currents specified by the manufacturer as being permissible in normal Class C amplifier operation. Alternatively, one may assume that the allowable peak milliamperes per watt of heating power is 2 to 4 for tungsten (or 0.65 this value for linear and modulated amplifiers) and 10 to 40 for thoriated and oxide-coated filaments.

Design Procedure for Class C Amplifiers Employing Triode Tubes.—The design of a Class C amplifier can be systematically worked out on paper according to the following steps:

First, select the peak space current I_m on the basis of the electron emission of which the filament is capable, as discussed above.

Second, select a suitable combination of maximum grid potential E_{max} and minimum plate potential E_{min} that will draw this total space current. The minimum plate voltage must not be less than the maximum grid potential, and, if low driving power is important, the minimum plate potential should be appreciably larger than the maximum grid potential, although still relatively small compared with the plate-supply voltage.

Third, decide upon a suitable angle of plate-current flow θ_p, making a reasonable compromise between the high efficiency, small output, and large driving power obtained with small angles of flow, and the large output, small driving power, and low efficiency with large angles. Under most circumstances, the angle of flow will lie between 120 and 150°.

Fourth, calculate the grid bias by the use of Eq. (81*b*). This also determines the signal voltage required since the crest signal voltage is $E_c + E_{max}$.

Fifth, determine the d-c plate current, d-c grid current, plate dissipation, power output, efficiency, grid driving power, etc., either by the exact or by the approximate methods.

Sixth, examine the results obtained to see if they are satisfactory. If not, revise the design, and recalculate the performance.

Seventh, design the tank circuit in accordance with the principles discussed below.

The design procedure and the details involved in calculating the performance by the approximate method can be understood with the aid of the following example:

Example.—A Class C amplifier is to be designed employing a Type 800 tube operating at 1,000 volts plate potential.

The peak emission I_m will be taken as 407 ma, which is arrived at by assuming an electron emission of 100 ma per watt of filament power and a factor of safety of 6. A suitable combination of E_{max} and E_{min} for drawing this current without excessive grid current is $E_{min} = 150$, $E_{max} = 120$. The corresponding peak grid current is 85 ma. The next step is the selection of a suitable angle of flow, which will be taken as 140° as a reasonable compromise between efficiency and output. On the assumption that α equals 3½, Fig. 77 gives the direct-current and fundamental-frequency components of the total space current as 0.22 and 0.39, respectively, of the peak space current I_m. The direct-current component of the total space current is hence $407 \times 0.22 = 89.5$ ma, and the crest fundamental-frequency component of the total space current is $407 \times 0.39 = 159$ ma, crest value. It is now necessary to make allowance for the part of the total space current diverted to the grid. With the use of Eq. (80) θ_g is found to be 116°; so from Fig. 77, assuming $\alpha = 2$, the d-c and fundamental frequency components of the grid current are 0.17 and 0.31 times the peak value, or 14.4 ma and 26.3 ma, respectively. The d-c plate current is then $89.5 - 14.4 = 75.1$ ma, and the fundamental-frequency component is $159 - 26.3 = 132.7$ ma crest value. The power input to the plate is found from Eq. (84) to be

$$1{,}000 \times 0.0751 = 75.1 \text{ watts.}$$

The power delivered to the load is obtained from Eq. (85), and is

$$(1{,}000 - 150) \times 0.1327/2 = 56.4 \text{ watts.}$$

The plate loss is $75.1 - 56.4 = 18.7$ watts, and the plate efficiency $56.4 \times 100/75.1 = 75.1$ per cent. The grid bias calculated by Eq. (81*b*) is 134 volts, and can be developed by a grid-leak resistance of $134/0.0144 = 9{,}300$ ohms. The peak exciting voltage is $134 + 120 = 254$ volts, and Eq. (87) shows the exciting power to be $254 \times 0.0263/2 = 3.3$ watts, of which $134 \times 0.0144 = 1.9$ watts is dissipated in the bias, and $3.3 - 1.9 = 1.4$ watts at the grid of the tube. The load impedance is given by Eq. (86) as $(1{,}000 - 150)/0.1327 = 6{,}400$ ohms.

Tank-circuit Design.—The tuned circuit connected between the cathode and plate of the Class C amplifier, commonly called the *tank circuit*, must supply the proper

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impedance and must not consume an undue proportion of the power output of the amplifier.

The efficiency of the tank circuit is the fraction of the total power delivered to this circuit by the tube that is transferred to the load, and is

$$\left.\begin{matrix}\text{Tank-circuit} \\ \text{efficiency}\end{matrix}\right\} = \frac{Q_1 - Q_2}{Q_1} 100 \text{ per cent} \tag{88}$$

where Q_1 is the Q of the tank resonant circuit with no load coupled, and Q_2 is the effective Q of the tank circuit when the load is coupled to it. From the point of view of efficiency, it is desirable that the effective Q of the tank circuit, when the load is taken into account, be as low as possible, with the actual Q of the circuit in the absence of load as high as is practicable. A low effective Q tank circuit also reduces the reactive energy that must be stored, and so permits the use of inductances and capacities with smaller volt-ampere ratings, an important item in large amplifiers. At the same time, a low tank-circuit Q results in an appreciable impedance being offered to harmonic components of the plate-current pulse.[1]

The effective Q of the tank circuit is usually made at least 8 or 10, although in the case of some very large push-pull Class C amplifiers, values of effective Q as low as 2 or 3 are employed. With such very low values of Q it is necessary to use a tuning procedure such that the tank circuit is adjusted to offer a resistance impedance to the tube, even though the magnitude of the resulting impedance may not necessarily be maximum for this condition (see Par. 2, Sec. 3).

When the desired Q of the tank circuit has been decided upon, the inductive reactance ωL can then be determined in terms of the required load impedance. In the case of direct coupling with a tank circuit Q not too low, one has

$$\omega L = \frac{\text{required load impedance}}{\text{effective } Q} \tag{89}$$

When ωL is known, the tank-circuit inductance and capacity for a given frequency are then calculated by obvious methods.

A useful relation in the design of tank circuits is the connection that exists between the voltage across the tank circuit, the effective Q, and the power delivered to the circuit. This relation is

$$\left.\begin{matrix}\text{Effective voltage} \\ \text{across tank circuit}\end{matrix}\right\} = \sqrt{P\omega L Q_{\text{eff}}} \tag{90}$$

where P = power, watts, delivered to tank circuit.
ωL = reactance of inductive branch of tank circuit.
Q_{eff} = effective Q of tank circuit, when the effect of the coupled load resistance is taken into account.

Equation (90) applies to all types of tank circuits, including arrangements such as in Figs. 75*b*, 75*c*, and 75*d*, where the plate-cathode connection is made across only a portion of the tank circuit.

[1] The relationship between the harmonic and fundamental frequency voltages developed across the tank circuit and the effective Q for the case where the harmonic voltage is small is approximately

$$\left.\begin{matrix}\text{Per cent second} \\ \text{harmonic}\end{matrix}\right\} = \frac{67}{Q} \frac{\left(\frac{I_2}{I_m}\right)}{\frac{I_1}{I_m}} \tag{88a}$$

where I_1/I_m and I_2/I_m are coefficients obtained from Fig. 86 for the angle θ_p of plate-current flow involved. See E. H. Schulz, R-f Power Amplifier Chart, *Electronics*, Vol. 12, p. 33, December, 1939.

Ex. 1044-0458

Practical Adjustment of Class C Amplifiers.—The adjustment of Class C amplifiers to realize the design conditions is usually carried out by a cut-and-try process, using the d-c grid current, the d-c plate current, and the output power as guides.

In carrying out the adjustment, one ordinarily starts either with a complete paper design giving the conditions desired, or with data for Class C amplifier operation given by the tube manufacturer. In both cases one knows the proper grid bias, the desired d-c plate current, and the expected power output, and in most instances will also have available the necessary exciting voltage and the approximate d-c grid current. The procedure will then depend upon whether fixed or grid-leak bias is employed.

In the former case, after the bias has been given the proper value, the tank circuit is tuned to resonance with the exciting voltage. The exciting voltage and effective load impedance (*i.e.*, coupling of the load to the tank circuit) are then varied by trial until the expected d-c plate current and output power are obtained without excessive grid current. This procedure can be simplified by keeping in mind that the total space current depends primarily on the maximum grid potential E_{max} and hence upon the excitation, while with a given excitation the minimum plate potential depends upon the load impedance. The ratio of grid current to plate current is accordingly less the higher the impedance in the plate circuit of the tube. The adjustment

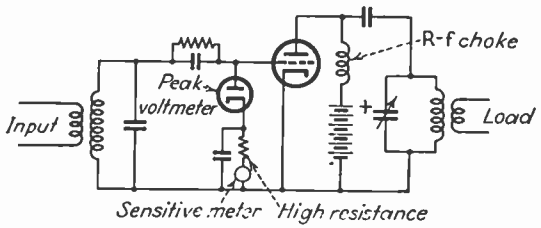


FIG. 78.—Peak voltmeter arrangement for measuring maximum positive potential reached by the grid.

procedure hence involves coupling the load to the tank circuit a reasonable amount, after which the excitation is varied until the total space current approximates the expected value. The effective load impedance is then varied by changing the coupling of the load to the tank circuit until a point is found where decreasing the coupling to the load slightly (*i.e.*, increasing the load impedance) causes the grid current to become excessive. The power output, power input, and grid current are then noted, and the entire procedure is repeated over and over again until the desired operating conditions are obtained. In carrying out the adjustments, it is necessary to pay attention to the plate dissipation (*i.e.*, difference between power input and power output), since there is always danger of damaging the tube by accidentally overheating the electrodes.

When the bias is obtained from a grid-leak resistance, the best procedure is to start with the grid-leak resistance that is required to produce the desired grid bias with the estimated grid current, and then to follow out the procedure outlined above to obtain the best operating point. The actual grid bias that results is then calculated from a knowledge of the grid-leak resistance and the grid current; if it is not the desired value, the grid-leak resistance is altered as necessary, and the process is repeated until the grid bias, space current, power output, and power input approximate the calculated values and the grid current is not excessive

Much of the cut-and-try involved in the foregoing procedure can be eliminated by the use of a peak voltmeter to measure directly the amount the grid is driven positive. Such a voltmeter is described in Par. 15, Sec. 13, and its use is illustrated in Fig. 78.

Ex. 1044-0459

In making adjustments on a Class C amplifier, it is desirable to check the tuning repeatedly, particularly after each change in the load coupling. With triodes this is done by adjusting the tuning condenser for minimum plate current. This condition normally corresponds to maximum load impedance (*i.e.*, to the lowest possible E_{min}), maximum grid current, and a unity-power-factor load. It also gives the maximum possible power output unless the effective Q of the tank circuit is so low that the points of unity power factor and maximum impedance do not coincide. In this case, the tank circuit is tuned for maximum power into the load.

Neutralization can be accomplished by turning off the plate voltage of the tube (but leaving the tube in place) and applying the exciting voltage. The neutralizing condenser is then adjusted until turning the tank circuit through resonance has no effect on the d-c plate and grid currents of the exciting stage, or until tuning the tank circuit has no reaction on the rectified grid current.

Class C Amplifiers Employing Screen-grid, Beam, and Pentode Tubes.—Screen-grid, beam, and pentode tubes can be operated as Class C amplifiers by making the grid bias greater than the cutoff value corresponding to the screen-grid potential. The performance obtained is then similar to that of triode Class C amplifiers, but with the advantage that no neutralization is required. Also, pentode and beam tubes have the added advantage of requiring smaller driving power than triode tubes. The disadvantage of pentode, beam, and screen-grid tubes is that they are, in general, more expensive than triodes, and are not available in water-cooled types.

The analysis, calculation of performance, and design are the same with screen-grid, beam, and pentode tubes as with triodes except for minor modifications introduced by the screen grid.[1] In particular, the relation between the grid bias and angle of flow is

$$\text{Grid bias} = E_c = \frac{\frac{E_{sg}}{\mu_{sg}} + E_{max} \cos\left(\frac{\theta}{2}\right)}{1 - \cos\left(\frac{\theta}{2}\right)} \tag{91}$$

where E_{sg} is the screen-grid potential and μ_{sg} is as defined by Eq. (48), Sec. 3. In selecting operating conditions, it is essential that the maximum grid potential E_{max} should not exceed the screen-grid potential.

The minimum plate potential E_{min} in the case of beam and pentode tubes must be great enough that a virtual cathode does not form in the tube when the space current has its peak value. This minimum allowable value in practical tubes is of the order of 15 to 20 per cent of the plate-supply voltage. In the case of screen-grid tubes, the minimum plate potential also must not be less than the screen-grid potential, since otherwise the plate will lose secondary electrons to the screen. Otherwise the situation is the same as with triodes, and the analysis can be carried out by either point-by-point calculations or the approximate method based upon the curves of Fig. 76.

In adjusting screen-grid and pentode Class C amplifiers, it is sometimes found that the plate potential has so little effect on the plate current that it is impossible to tune the tank circuit by adjusting for minimum plate current. Under such circumstances, the tuning adjustment can be made to give maximum current in the load.

22. Linear Amplifiers.—A linear amplifier is a tuned amplifier operated with a bias approximating the cutoff value, so that the amplified output voltage that is developed is proportional to the exciting voltage applied to the amplifier. The linear amplifier is a form of Class B amplifier, and its most important use is as a power amplifier of modulated waves.

[1] F. E. Terman and J. H. Ferns, The Calculation of Class C Amplifier and Harmonic Generator Performance of Screen-grid and Similar Tubes, *Proc. I.R.E.*, Vol. 22, p. 359, March, 1934.

Ex. 1044-0460

Distortion in Linear Amplifiers.—The most important characteristic of a linear amplifier is the relationship between output voltage and exciting voltage, since this shows the extent to which the amplifier is actually linear. A typical characteristic of this type is shown in Fig. 79 and is characterized by an approximately linear characteristic up to a critical exciting voltage, after which the output levels off or *saturates*. The saturation takes place when the alternating voltage developed in the plate circuit across the load impedance has a crest value approaching the plate-supply voltage, and results from the fact that this alternating voltage cannot be greater than the plate-supply voltage, irrespective of how large the applied signal is.

The linearity of the amplifier characteristic below saturation is greatest when the amplifier tube is biased to projected cutoff exactly, as in the case of the Class B audio amplifier (see Par. 10).

Distortion may arise in a system involving a linear amplifier as a result of the variable load that the linear amplifier presents to its exciter. The remedy for this is to employ an exciter of relatively large power capacity, or to use negative feedback or some other system of distortion correction.

The distortion in the modulation envelope that results when a modulated wave is amplified by a linear amplifier can be readily calculated when the linearity curve is known. The procedure is to determine the output voltage at certain points on the modulation cycle and then to use Eq. (24) or (25) to determine the harmonic components. These critical points are the outputs corresponding to carrier excitation, peak of the modulation cycle, trough of the modulation cycle, and when the modulation is 70 per cent of its peak and trough values, as indicated in Fig. 80.

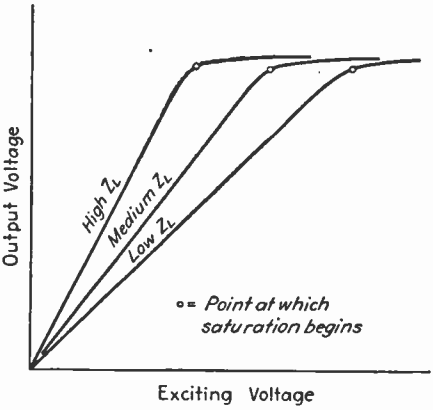


FIG. 79.—Typical linearity curves of linear amplifier with different load impedances in the plate circuit.

Design and Calculation of Linear Amplifiers.—In the design of linear amplifiers, the first step is to determine the bias voltage corresponding to projected cutoff. The bias must be constant, irrespective of the degree of modulation and of the grid current, and so must be obtained from a fixed source or by self-bias. Grid-leak arrangements are not permissible under any circumstances.

A load impedance is then selected such that saturation is just beginning to occur when there is sufficient excitation to develop full load output corresponding to the modulation peaks; *i.e.*, the load impedance is chosen so that with the rated peak power the crest voltage across the load is slightly less than the plate-supply voltage. The type of relationship existing between load impedance, exciting voltage, and output power is shown in Fig. 79. The power output corresponding to the beginning of saturation is inversely proportional to the load impedance, whereas the exciting voltage required to produce saturation is greater as the load impedance is reduced. The proper load impedance in any given case can be obtained either by experiment or by approximate calculations. In the latter case, the method of calculation is exactly that used for Class C amplifiers.

After the load impedance and exciting voltage corresponding to the peak output have been determined, the linearity is obtained by calculating the amplifier output with lesser exciting voltages. This must be done by a cut-and-try process, since the

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methods that have been developed for calculating amplifier performance, both the exact and approximate methods, give the load impedance as the answer for an

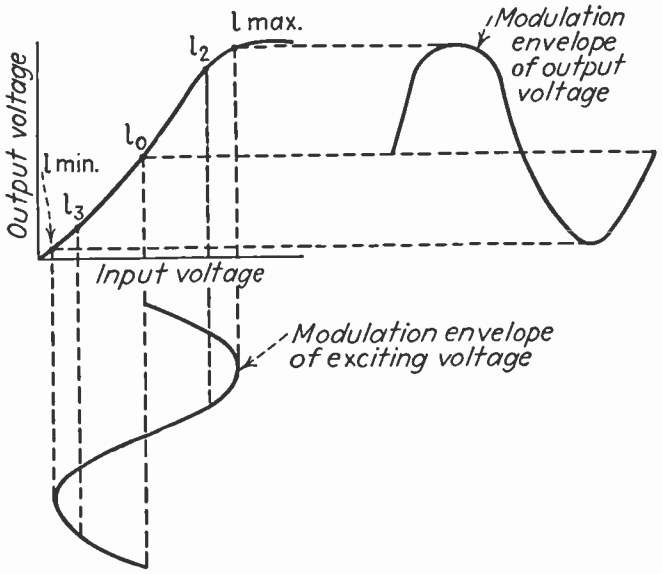


FIG. 80.—Linearity curve of linear amplifier, showing critical points on modulation cycle used in calculating distortion.

assumed value of alternating voltage between plate and cathode, but cannot be rearranged to do the converse. The procedure for calculating a linearity curve after already having determined the conditions for the peak exciting voltage is to estimate the output voltage expected for a given exciting voltage on the basis that the output voltage is approximately proportional to the exciting voltage. One then calculates the load impedance corresponding to this assumed output voltage, using either the exact or approximate methods of calculation, according to the accuracy desired. If the resulting load impedance determined in this way is greater than the actual load impedance, then one revises the estimate of output voltage to a smaller value (or, conversely, if the calculated load impedance is too small). By repeating this process two or three times, it is possible to draw a curve such as that shown in Fig. 81 and by interpolation to obtain the output voltage that would be obtained for the assumed exciting voltage with the load impedance actually present.

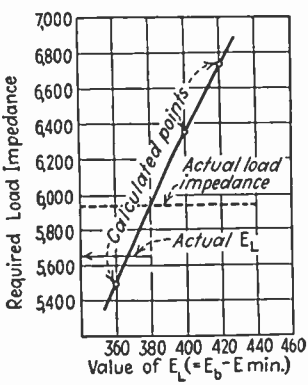


FIG. 81.—Determination of the actual output voltage for a known load impedance by plotting a curve of load impedance required for several assumed values of output voltage.

In determining linearity curves, the approximate method of calculation outlined for Class C amplifiers will give the general shape of the characteristic and detect excessive nonlinearity. However, if distortion is to be exactly evaluated, it is necessary to employ the exact method of calculation.

Ex. 1044-0462

Practical Adjustment of Linear Amplifiers.—The procedure to be followed in adjusting linear amplifiers depends upon the circumstances and the measuring equipment at hand. When ample exciting voltage is available, the combination of load impedance and exciting voltage that gives full peak output with satisfactory linearity can be readily determined experimentally, as can the bias voltage giving best linearity. When the exciting voltage available is barely sufficient to supply the desired carrier, it is then necessary to modulate this carrier, and make use of a cathode-ray oscillograph or a modulation meter to determine when satisfactory linearity is obtained.

When a linear amplifier is excited by a modulated wave, the d-c component of the plate current should not vary with the degree of modulation. Any change in the d-c current with modulation indicates either distortion or a carrier shift (*i.e.*, change in carrier amplitude), although the absence of variation does not guarantee that there is no distortion.

The fact that the d-c current of a linear amplifier should be constant means that the grid bias may be obtained by a cathode resistor.

Efficiency and Power.—The efficiency of an ideal linear amplifier having a linear characteristic and operating so that the plate current flows exactly 180° during each cycle is

$$\text{Efficiency} = \frac{\pi}{4}\left(1 - \frac{E_{\min}}{E_b}\right) 100 \text{ per cent} \tag{92}$$

where E_b is the plate-supply voltage, and $E_{\min}$ is the minimum instantaneous plate voltage during the cycle. The plate efficiency at full output is usually of the order of 50 to 65 per cent under practical conditions. With less than the full output, the efficiency is proportional to the exciting voltage.

When the signal to be amplified is a carrier wave modulated 100 per cent, the carrier amplitude is half the peak amplitude to be handled. The efficiency for the unmodulated carrier is then half of the maximum efficiency, or about 30 per cent under ordinary conditions.

The peak power that can be developed by a tube operating as a linear amplifier is approximately the same as the power that can be developed by the same tube in Class C operation. In the case of completely modulated waves, the peak power is four times the carrier power, so that the carrier power that a linear amplifier is capable of developing is approximately one-fourth of the output obtainable from the same tube operated as a Class C amplifier.

High-efficiency Linear Amplifier (Doherty Amplifier).[1]—The average plate efficiency obtained when an ordinary linear amplifier is handling a modulated wave can be increased by a system devised by Doherty and illustrated schematically in Fig. 82*a*. Here the amplifier is divided into two parts: A_1, which delivers power to the load through a quarter-wave length line, and A_2, which delivers power to the load directly. In order that the outputs of the two amplifiers will add directly at the load, a 90° phase shift, normally obtained with the aid of an artificial quarter-wave line, is introduced in the exciting voltage of one of the amplifiers to compensate for the 90° phase shift of the quarter-wave line connecting the two amplifiers.

Tube A_1 is biased to projected cutoff, as in linear amplifier operation. This tube is operated so that when the exciting voltage is the unmodulated carrier, the tube is just beginning to saturate with a load impedance that is twice the impedance that would be used to develop full output.

Tube A_2 is operated with an exciting voltage and bias adjustment such that at the peak of modulation, *i.e.*, when the exciting voltage is twice the carrier, this tube

[1] W. H. Doherty, A New High Efficiency Power Amplifier for Modulated Waves, *Proc. I.R.E.*, Vol. 24, p. 1163, September, 1936.

Ex. 1044-0463

develops an output equal to twice the carrier power when operating as an ordinary Class C amplifier, but has its output just reduced to zero when the exciting voltage is the unmodulated carrier. The tank circuit is proportioned so that its impedance is half the impedance that would be used with amplifier A_1 with this tube functioning as a conventional linear amplifier developing its full output. The characteristic impedance of the line is made twice the tank-circuit impedance.

The operation of the high-efficiency system can be described briefly as follows: When the amplitude of the exciting voltage does not exceed the carrier level, A_2 is inoperative, and A_1 functions as a conventional linear amplifier with a load impedance twice the impedance that would be used to develop full output.[1] Tube A_1 therefore gives linear amplification of the exciting voltage during the portions of the modulation cycle where the envelope of the exciting voltage does not exceed carrier level. When the exciting level is greater than carrier level, A_2 also passes current. The voltage developed across the tank circuit is then greater than previously in proportion to the

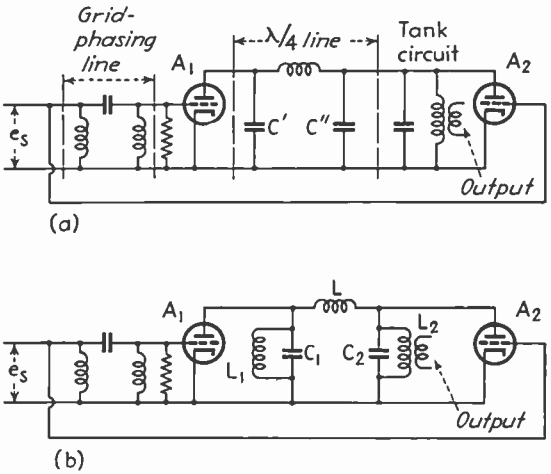


FIG. 82.—Schematic diagram of high-efficiency linear amplifier.

current delivered from A_1 over the quarter-wave line. This results in an increase in the apparent load impedance that the tank circuit offers to the transmission line. Because of the impedance inverting properties of the quarter-wave line, the impedance across the sending end of the line is reduced, thereby causing A_1 to deliver more power to the line in spite of the fact that this tube was already operating under saturation conditions. At the peak of a completely modulated wave, tube A_2 is carrying half of the power, and the apparent load impedance that the tank circuit offers to the line is increased to equality with the characteristic impedance. Tube A_1 then operates with half the impedance in its plate circuit present at carrier level, and so carries the remaining half of the peak power. Curves showing behavior as the exciting voltage varies are given in Fig. 83.

[1] The impedance offered to A_1 by the sending end of a quarter wave line is related to the load impedance by the equation

$$Z_s = \frac{Z_0^2}{Z_r} \tag{93}$$

where Z_s = sending-end impedance offered by the line.
Z_r = impedance of the load connected to the line.
Z_0 = characteristic impedance of the line.
In the case where H_2 is inactive, $Z_r = Z_0/2$, so that $Z_s = 4Z_r$.

Ex. 1044-0464

The average efficiency of the system is high for all degrees of modulation. When the exciting voltage is an unmodulated carrier, tube A_1 functions as a linear amplifier under saturation conditions corresponding to an efficiency of about 60 per cent. At the positive peaks of modulation, A_1 is still functioning as a linear amplifier under saturation conditions, and A_2 operates as a Class C amplifier with a plate efficiency of the order of 75 per cent. The result is an average efficiency for the modulation cycle that varies only slightly with the degree of modulation, and approximates the efficiency of a linear amplifier operating under peak conditions, *i.e.*, about 60 per cent.

A practical circuit for a high-efficiency amplifier is shown in simplified form in Fig. 82*b*. Here the capacity C'' of the quarter-wave line is combined with the tank circuit, and the capacity reactance of C' is obtained by resonant circuit L_1C_1 detuned a proper amount.

In setting up and tuning the circuit of Fig. 82, the first step is to determine the plate load impedance Z_L required to develop full output from A_1 with this tube

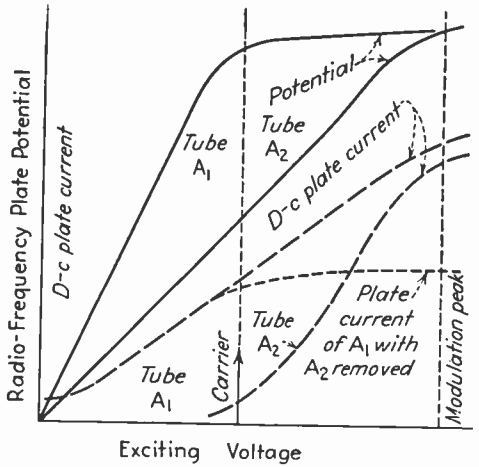


FIG. 83.—Curves showing performance of a high-efficiency linear amplifier.

functioning as a conventional linear amplifier. The characteristic impedance Z_0 of the line must then be equal to Z_L, and L should be such that $\omega L = Z_0 = Z_L$. Resonant circuit L_1C_1 is then tuned by short-circuiting L_2, applying exciting voltage to A_1, and tuning L_1C_1 for minimum plate current. Similarly, the resonant circuit L_2C_2 is tuned by short-circuiting L_1, applying voltage to A_2, and tuning L_2C_2 for minimum plate current. This ensures correct tuning and also ensures that the line is exactly one-quarter wave length long. The characteristic impedance of the line can be checked by the fact that when exciting voltage is applied to A_1 and A_2 is inoperative, the voltages across C_1 and C_2 should be in the ratio of 2:1 when the load impedance across L_2 is $Z_L/2$. An alternative way of checking the characteristic impedance of the line is to make A_2 inoperative, and observe the plate current in A_1 with the appropriate resistance $Z_L/2$ shunted across C_2. The load resistance is then transferred to shunt with C_1 and made $2Z_L$. If the characteristic impedance of the line is the proper value, the plate current of A_1 will not change.

After the circuit adjustments have been made in this way, it remains only to obtain the proper exciting and bias voltages. If these are to be obtained experimentally, a systematic procedure must be followed because of the numerous variables

Ex. 1044-0465

present. The first step is to make A_2 inoperative and determine the carrier voltage required to cause A_1 to be on the edge of saturation. The bias and excitation on A_2 are then adjusted so that when the carrier is unmodulated, A_2 just shows signs of passing plate current, while with an exciting voltage equal to twice carrier level, A_2 is operating as a normal Class C amplifier delivering half of the total power output and having a d-c plate current approximately 80 per cent of the d-c plate current of tube A_1.[1] It is to be noted that the carrier voltages applied to A_1 and A_2 will normally differ. It is also to be noted that at the peak of modulation it is not necessary for the grid of A_1 to be driven much farther positive than at carrier level. The grid of A_1 can be prevented from going excessively positive at the modulation peaks and thereby drawing an undesirably large grid current by designing the quarter-wave line in the grid circuit to have a high characteristic impedance, and also by using a grid leak with A_1 so that the bias will increase when grid current flows.

The linearity of a high-efficiency amplifier is generally not all that could be desired. Accordingly, when the distortion must be low, as is the case in broadcast transmitters, it is necessary to provide some means of distortion correction, as discussed below.

Distortion Correction by Feedback and Remodulation Systems.—Distortion produced in radio transmitters by linear amplifiers, as well as distortion in the modulator

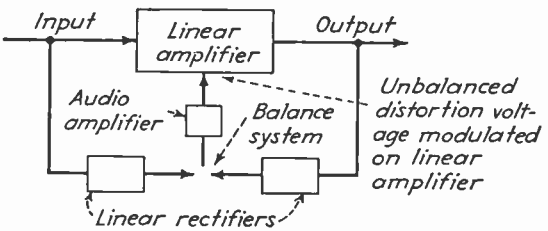


FIG. 84.—Remodulation system for reducing distortion in a linear amplifier.

and audio-frequency system, can be greatly reduced by rectifying a portion of the transmitter output, and properly feeding the resulting audio-frequency developed by the rectifier back into the audio-frequency system. This is a form of negative feedback, and is discussed in greater detail in Par. 1, Sec. 9.

The distortion produced in a linear amplifier can also be corrected by the remodulation system shown schematically in Fig. 84.[2] Here samples of the modulated waves existing in the input and output of the linear amplifier are rectified and the resulting audio-frequency voltages balanced against each other. The adjustment of the balancing device is such that when no distortion is produced in the linear amplifier the unbalanced output will be zero. Distortion, however, produces an output that is amplified and then modulated on the linear amplifier, preferably by grid modulation, in such a manner as to correct for the distortion produced in the amplifier.

Both the preceding arrangements are capable of giving substantially linear over-all performance even when there is considerable nonlinearity in the individual stages involved. These arrangements also greatly reduce hum modulation of the output wave resulting from alternating filament current and from alternating-current hum in the plate supply.

23. Harmonic Generators.—The ordinary vacuum-tube harmonic generator is essentially a Class C amplifier in which the tank circuit is tuned to an harmonic

[1] The difference in plate currents of A_1 and A_2 arises from the fact that A_1 operates as a Class B amplifier, while A_2 operates as a Class C amplifier and so has higher efficiency.

[2] This arrangement is due to F. E. Terman and R. R. Buss, Some Notes on Linear and Grid-modulated Radio-frequency Amplifiers, *Proc. I.R.E.*, Vol. 29, p. 104, March, 1941.

Ex. 1044-0466

component of the pulses of plate current instead of to the fundamental-frequency component.[1] Harmonic generators of this character are used in radio transmitters for developing appreciable amounts of power on harmonic frequencies.

Oscillograms showing voltage, current, and power relations existing in a typical harmonic generator are given in Fig. 85. These are very similar to the corresponding oscillograms for the Class C amplifiers shown in Fig. 76. The significant factors determining the performance of harmonic generators are the same as for the Class C amplifier, namely, the maximum grid potential E_{max}, the minimum plate voltage E_{min}, peak space current I_m, and the angle of plate current flow θ_p. The considerations involved in the design of harmonic generators are also the same as in the case of the Class C amplifier, except that since the harmonic content of the plate current pulse is fairly sensitive to the angle θ_p of plate-current flow, this angle must be carefully chosen. The experimental procedure followed in realizing desired operating conditions for harmonic generators is much the same as in the case of Class C amplifiers.

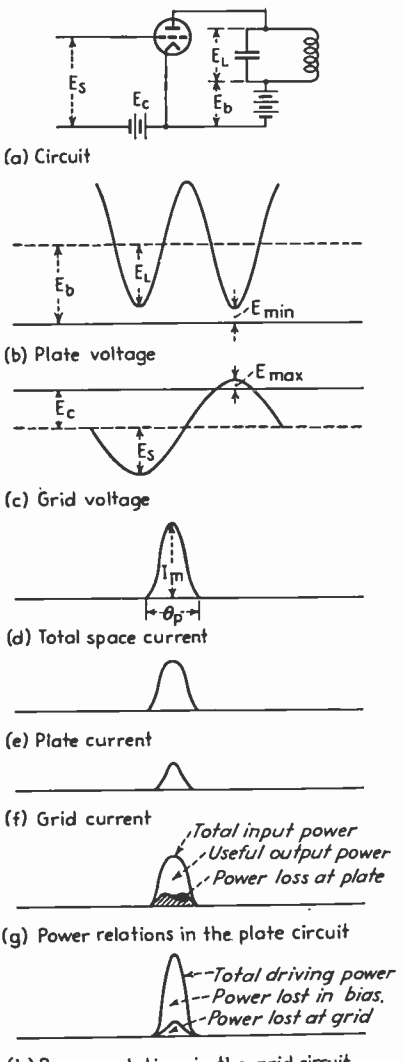


FIG. 85.—Voltage, current, and power relations in a typical Class C harmonic generator.

Factors Governing Harmonic-generator Performance.—The most desirable angle θ_p of plate-current flow is a compromise between conflicting factors. The shorter the length of current pulses in the case of a particular harmonic the higher will be the plate efficiency, but the bias, exciting voltage, and driving power are increased. Also, if the pulse is too long or too short, the output power drops off appreciably. Desirable values for θ_p under practical conditions are given in Table 4.

The harmonic power output that is obtainable decreases with the order of the harmonic. The relative harmonic output obtainable from a given tube compared with Class C output with the same peak space current is given approximately by Table 4. The load impedance is inversely proportional to the output power [Eq. (90)] and so increases with the order of the harmonic, as shown by Table 4. The exciting power required by a harmonic generator is greater than with corresponding Class C

[1] Other types of harmonic generators using vacuum tubes are described by H. J. Scott and L. J. Black, Harmonic Generation, *Proc. I.R.E.*, Vol. 26, p. 449, April, 1938; F. E. Terman, D. E. Chambers, and E. H. Fisher, Harmonic Generation by Means of Grid Circuit Distortion, *Trans. A.I.E.E.*, Vol. 50, p. 811, June, 1931.

Ex. 1044-0467

operation because of the smaller angle of current flow, and increases rapidly with the order of the harmonic.

Design and Analysis of Harmonic-generator Performance.—The maximum instantaneous grid voltage E_{max} and the peak space current I_m have the same significance and are determined by the same considerations in harmonic generators as in Class C amplifiers. The angle of current flow θ_p is selected in accordance with Table 4, and the bias required to produce a given angle of flow is

$$\text{Grid bias} = E_c = \frac{E_b\left[1 - \cos\left(\frac{n\theta}{2}\right)\right] + E_{min}\cos\left(\frac{n\theta}{2}\right)}{\mu\left[1 - \cos\left(\frac{\theta}{2}\right)\right]} + \frac{E_{max}\cos\left(\frac{\theta}{2}\right)}{1 - \cos\left(\frac{\theta}{2}\right)} \quad (94)$$

TABLE 4.—PLATE-CURRENT PULSE LENGTH AND POWER OUTPUT OF HARMONIC GENERATORS

Harmonic	Optimum length of pulse, electrical degrees at the fundamental frequency	Approximate power output, assuming that normal Class C output is 1.0	Relative load impedance, assuming that Class C case is 1.0
2	90–120	0.65	1.5
3	80–120	0.40	2.5
4	70– 90	0.30	3.3
5	60– 72	0.25	4.0

The notation is the same as in Eq. (81*b*), with the addition that n is the order of harmonic involved.

With E_{min}, E_{max}, E_b, θ_p, grid bias, and the signal voltage determined, oscillograms of plate and grid currents, instantaneous grid and plate losses, power input, etc., can be drawn from characteristic curves of the tube exactly as in the case of a Class C amplifier. Averaging these curves over a full cycle of fundamental frequency then gives power output, plate losses, d-c plate current, etc.

An approximate analysis of triode harmonic-generator behavior can be made by modifying the approximate analysis used with triode Class C amplifiers.[1] It is to be noted first that *the pulse of total space current in an harmonic generator for a given E_{max}, E_{min}, E_b, and θ_p has practically the same shape as in the case of a Class C amplifier.* To the extent that the shape is exactly the same in these two cases, the harmonic content of the space-current pulse is given by Fig. 86. The modifications in the shape of the space-current pulse resulting from the fact that the voltage drop in the plate circuit is a harmonic of the applied frequency instead of being of the same frequency can be allowed for by *subtracting* from the coefficients given by Fig. 86 a correction factor equal to k times the factor obtained from Fig. 87, where

$$k = \frac{E_b - E_{min}}{\mu E_{max} + E_{min}} \quad (95)$$

Here μ is the amplification factor of the tube. The corrected coefficients are then used to calculate the amplifier performance, exactly as the coefficients obtained from Fig. 77 were used in the Class C amplifier case. The details involved are illustrated by the following example:

[1] F. E. Terman, Analysis and Design of Harmonic Generators, *Trans. A.I.E.E.*, Vol. 57, p. 640, November, 1938.

Ex. 1044-0468

Example.—It is desired to design and calculate the performance of a third-harmonic generator employing a Type 204A tube having $\mu = 23$, a rated plate dissipation of 200 watts, and operating at a plate potential of 1,500 volts.

The first step in the design is the selection of a suitable peak space current and corresponding values of E_{min} and E_{max}. Reference to the data supplied by tube manufacturers indicates that $I_m = 1{,}260$ ma will be satisfactory, and that this could be obtained by $E_{min} = 375$, $E_{max} = 250$. With this combination, the peak grid current will be 225 ma, which is small enough to keep the driving power low, and yet at the same time E_{min} will be small compared with the plate-supply voltage.

The next step is the selection of a value of θ_p. From the discussion given above, combined with an examination of Fig. 86, it appears that $\theta_p = 100°$ would be reasonable. With this value of θ_p and assuming a $\frac{3}{2}$ power law, Fig. 86 then gives $I_0/I_m = 0.161$, and $I_n/I_m = 0.177$. The fractional corrections that Fig. 87 indicates would be applied to these for a $\frac{3}{2}$ law and for $k = 1$ are -0.33 for the d-c term and -0.060 for the third harmonic component. However, since k is

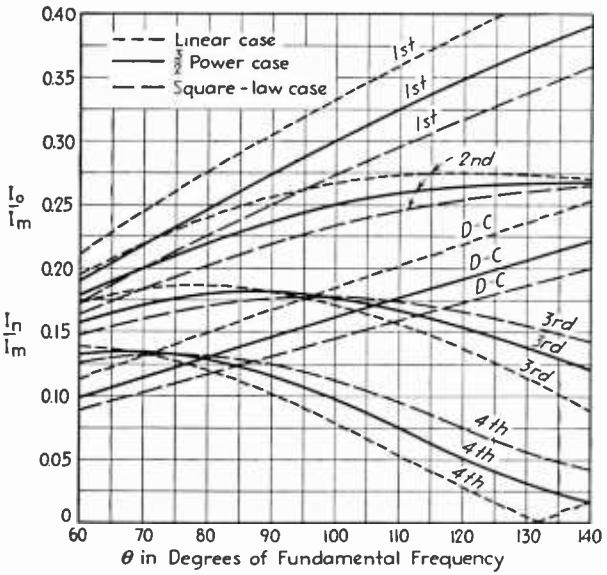


FIG. 86—Curves giving relation of direct-current and harmonic components of the space-current pulse as a function of angle of flow θ and the peak amplitude I_m.

$$\frac{[(1{,}500 - 375)]}{23 \times 250 + 375} = 0.19,$$

the actual corrections to be subtracted are -0.063 and -0.012, respectively. The d-c and third-harmonic components of the total space current are then $1{,}260 \times 0.161(1 + 0.063) = 216$ ma, and $1{,}260 \times 0.177(1 + 0.012) = 226$ ma, respectively. From Eq. (14), the grid bias is found to be 751 volts, which makes the peak signal voltage 1,001 volts. The angle θ_g of grid-current flow is then given by Eq. (80) as 82.8°. Assuming a square-law grid-current pulse, reference to Fig. 86 shows that for a peak grid current of 225 ma, the d-c, fundamental-frequency, and third-harmonic components of the grid current are 27. 52, and 39 ma, respectively. Since the peak signal voltage is 1,001 volts, the grid driving power is

$$\frac{1{,}001 \times 0.052}{2} = 26 \text{ watts.}$$

If grid-leak bias is used, the leak resistance is $751/0.027 = 28{,}000$ ohms. Next, subtracting the d-c and third-harmonic components of the grid current from the total space current gives 189 and 187 ma, respectively, as the d-c and third-harmonic components of the plate current. This calls for a tank-circuit impedance of $1{,}125/0.187 = 6{,}000$ ohms. The power input to the plate is 284 watts, while the third-harmonic power delivered to the tank circuit is $0.187 \times 1{,}125/2 = 105$ watts. The plate efficiency is hence 37 per cent and the plate dissipation is 179 watts. Inasmuch as this loss is slightly less than the rated value for normal operation, the assumed E_{min}, E_{max}, and θ_p represent a suitable operating condition.

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The approximate analysis of harmonic-generator performance can be extended to include pentode, beam, and screen-grid tubes by obvious modifications.[1]

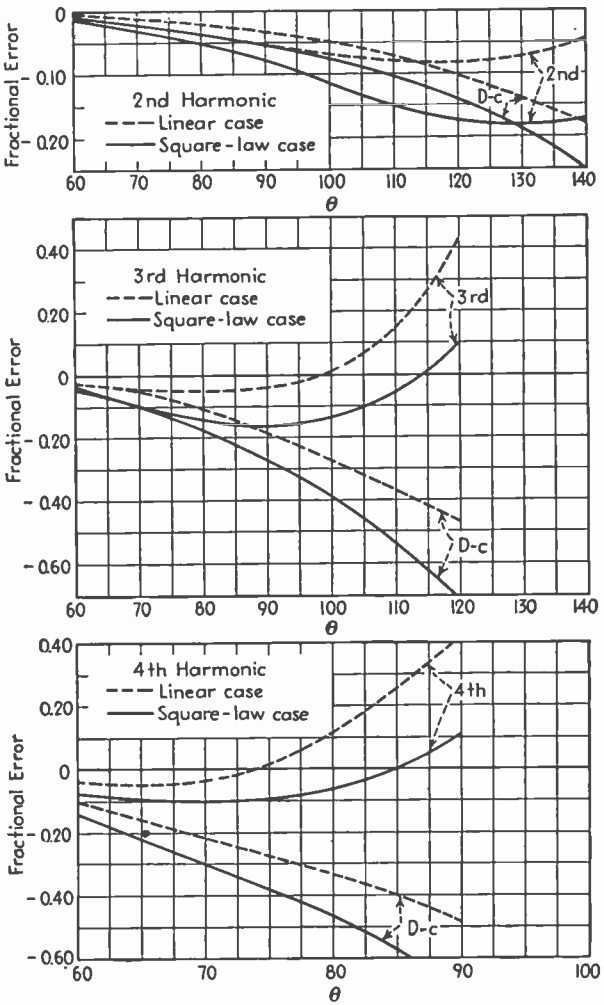


FIG. 87—Curves giving the fractional corrections that must be *subtracted* from the results obtained from Fig. 86 to obtain correct results for harmonic generator action. These curves assume $k = 1$.

MISCELLANEOUS AMPLIFIER PROPERTIES

24. Exact Equivalent Circuit of Class A Amplifier and Application to Analysis of Distortion and Cross-modulation.[2]—The exact behavior of a Class A amplifier can be determined with the aid of the equivalent circuits of Fig. 88.

[1] F. E. Terman and J. H. Ferns, Calculation of Class C Amplifier and Harmonic Generator Performance of Screen-grid and Similar Tubes, *Proc. I.R.E.*, Vol. 22, p. 359, March, 1934.

[2] For further details, see John R. Carson, A Theoretical Study of the Three-element Vacuum Tube,

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In the voltage-generator form, the current that flows in the plate circuit as a result of a signal voltage e_g applied to the control grid can be considered as being produced by a series of hypothetical voltages acting in a circuit consisting of the plate resistance of the tube in series with the load impedance, as in Fig. 88*a*. These equivalent generator

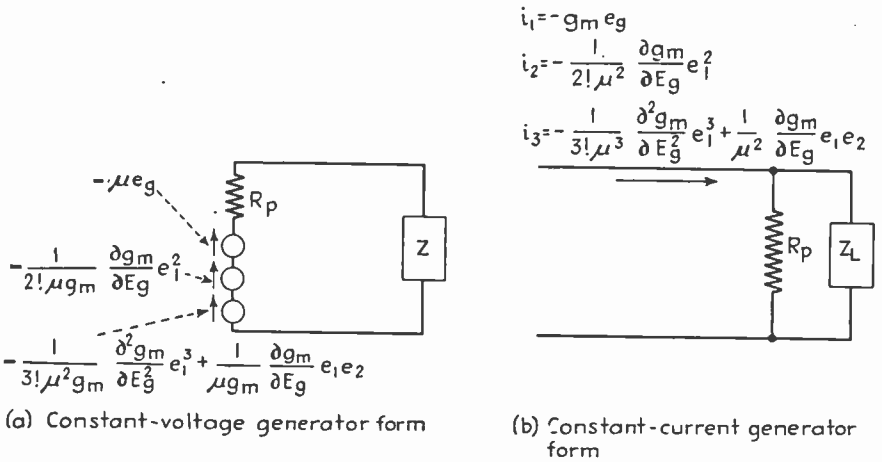


FIG. 88.—Equivalent circuits of a vacuum-tube amplifier, taking into account first-, second-, and third-order components of plate current.

voltages are proportional, respectively, to the first, second, third, etc., powers of the signal voltage and are given by the expressions

$$\left.\begin{array}{l}\text{Equivalent voltage producing}\\ \text{first-order component of}\\ \text{plate current}\end{array}\right\} = -\mu e_g \tag{96}$$

$$\left.\begin{array}{l}\text{Equivalent voltage producing}\\ \text{second-order component of}\\ \text{plate current}\end{array}\right\} = -\frac{1}{2!\mu g_m}\frac{\partial g_m}{\partial E_g}\left(\mu e_g\frac{R_p}{R_p+Z_1}\right)^2 = -\frac{1}{2!\mu g_m}\frac{\partial g_m}{\partial E_g}e_1^2 \tag{97}$$

$$\left.\begin{array}{l}\text{Equivalent voltage produc-}\\ \text{ing third-order component}\\ \text{of plate current}\end{array}\right\} = -\frac{1}{3!\mu^2 g_m}\frac{\partial^2 g_m}{\partial E_g^2}e_1^3+\frac{1}{\mu g_m}\frac{\partial g_m}{\partial E_g}e_1e_2 \tag{98}$$

where e_g = a-c voltage applied to the control grid.
μ = amplification factor of tube.
g_m = transconductance of tube.
E_g = grid bias at the operating point.
R_p = plate resistance at the operating point.

Proc. I.R.E., Vol. 7, p. 187, 1919; F. E. Terman, "Radio Engineering," 2d ed., pp. 153–157, 264–273. McGraw-Hill, New York, 1937.

The discussion in this section assumes either that the amplification factor of the tube is constant [Eqs. (96) to (101)], or that the load resistance is small compared with the plate resistance of the tube [Eqs. (100*a*) and (101*a*)]. Analyses that are more general, but correspondingly more complicated and so less useful, are given by F. B. Llewellyn, Operation of Thermionic Vacuum Tube Circuits, *Bell System Tech. Jour.*, Vol. 5, p. 433, July, 1926; E. Peterson and H. P. Evans, Modulation in Vacuum Tubes Used as Amplifiers, *Bell System Tech. Jour.*, Vol. 6, p. 442, July, 1927; J. G. Brainerd, Mathematical Theory of Four-electrode Tube, *Proc. I.R.E.*, Vol. 17, p. 1006, June, 1929.

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Z = load impedance. The subscripts 1, 2, and 3 denote the impedance offered to the first-, second-, and third-order components of the plate current. When any one order of plate current contains components of several frequencies, the appropriate value of Z must be used for each component.

$e_1 = \mu e_g \left(\frac{R_p}{R_p + Z_1}\right)$ = voltage drop produced in plate resistance by first-order component of current.

$e_2 = \left(\frac{Z_2}{R_p + Z_2}\right) \frac{1}{2!\mu g_m} \frac{\partial g_m}{\partial E_g} e_1^2$ voltage drop produced across the load impedance Z_2 by the second-order component of current.

The derivatives in these equations are all to be evaluated at the point on the tube characteristics at which the tube is operated.

In the constant-current generator form, the action taking place in the plate circuit is represented by a number of constant-current generators delivering current to an impedance consisting of the plate resistance in parallel with the load impedance, as in Fig. 88*b*. The currents produced by these constant current-generators are proportional, respectively, to the first, second, third, etc., powers of the applied signal, and are given by the equation.

$$\left.\begin{matrix}\text{Equivalent first-order} \\ \text{current}\end{matrix}\right\} = -g_m e_g \tag{99}$$

$$\left.\begin{matrix}\text{Equivalent second-} \\ \text{order current}\end{matrix}\right\} = -\frac{1}{2!\mu^2}\frac{\partial g_m}{\partial E_g}\left(\mu e_g \frac{R_p}{R_p + Z_1}\right)^2 = -\frac{1}{2!\mu^2}\frac{\partial g_m}{\partial E_g} e_1^2 \tag{100}$$

$$\left.\begin{matrix}\text{Equivalent third-} \\ \text{order current}\end{matrix}\right\} = -\frac{1}{3!\mu^3}\frac{\partial^2 g_m}{\partial E_g^2} e_1^3 + \frac{1}{\mu^2}\frac{\partial g_m}{\partial E_g} e_1 e_2 \tag{101}$$

where $R_p >> Z$, as in pentodes, Eqs. (100) and (101) can be written

$$\left.\begin{matrix}\text{Equivalent second-order} \\ \text{current in pentodes}\end{matrix}\right\} = -\frac{1}{2!}\frac{\partial g_m}{\partial E_g} e_g^2 \tag{100a}$$

$$\left.\begin{matrix}\text{Equivalent third-order} \\ \text{current in pentodes}\end{matrix}\right\} = -\frac{1}{3!}\frac{\partial^2 g_m}{\partial E_g^2} e_g^3 \tag{101a}$$

The first-order effects in the exact equivalent circuits of the Class A amplifier lead to the same results as the equivalent circuits discussed in Par. 3. The second-order effect is a correction of this simple equivalent circuit and depends upon $\partial g_m/\partial E_g$, *i.e.*, upon the curvature of the tube characteristic. The third-order effect is a second-order correction that depends upon the rate of change of curvature of the tube characteristic.

The exact equivalent circuit of the Class A amplifier is useful in calculating distortion, cross-modulation, etc. Its use is not limited to the simple case of resistance load and a sine-wave voltage, as is the method of distortion analysis involving the dynamic characteristic and load line.

Experimental Evaluation of Second- and Third-order Coefficients.—In order to make quantitative calculations based upon Eqs. (96) to (101), it is necessary to evaluate $\partial g_m/\partial E_g$ and $\partial^2 g_m/\partial E_g^2$, as well as the plate impedance, transconductance, etc., at the operating point.

These partial derivatives can be determined experimentally by applying to the grid of the tube a sine-wave voltage of known amplitude when the load impedance

Ex. 1044-0472

TABLE 5.—FIRST-, SECOND-, AND THIRD-ORDER VOLTAGES IN TWO TYPICAL CASES

	Signal $= E_a \sin \omega t$	Signal $= E_a \sin \omega_a t + E_b \sin \omega_b t$
Equivalent first-order voltage	$-\mu E_a \sin \omega t$	$-\mu(E_a \sin \omega_a t + E_b \sin \omega_b t)$
Equivalent second-order voltage	$-\frac{\mu}{2!g_m}\frac{\partial g_m}{\partial E_g}E_a^2\left(\frac{1-\cos 2\omega t}{2}\right)$	$-\frac{\mu}{2!g_m}\frac{\partial g_m}{\partial E_g}\left(\frac{E_a^2+E_b^2}{2}-\frac{E_a^2\cos 2\omega_a t+E_b^2\cos 2\omega_b t}{2}+E_aE_b\cos(\omega_a+\omega_b)t+E_aE_b\cos(\omega_a-\omega_b)t\right)$
First part of equivalent third-order voltage[1]	$-\frac{\mu}{3!g_m}\frac{\partial^2 g_m}{\partial E_g^2}E_a^3\left(\frac{3}{4}\sin\omega t-\frac{1}{4}\sin 3\omega t\right)$	$-\frac{\mu}{3!g_m}\frac{\partial^2 g_m}{\partial E_g^2}\left\{\left(\frac{3}{4}E_a^3+\frac{3E_aE_b^2}{2}\right)\sin\omega_a t+\left(\frac{3}{4}E_b^3+\frac{3E_a^2E_b}{2}\right)\sin\omega_b t-\frac{E_a^3\sin 3\omega_a t+E_b^3\sin 3\omega_b t}{4}-\frac{3}{4}E_a^2E_b[\sin(2\omega_a+\omega_b)t+\sin(-2\omega_a+\omega_b)t]-\frac{3}{4}E_aE_b^2[\sin(\omega_a+2\omega_b)t+\sin(\omega_b-2\omega_b)t]\right\}$

[1] The second part of the third-order voltage is similar in character to the first-order part, although differing in detail.

Ex. 1044-0473

in the plate circuit is negligibly small.[1] One then has

$$\frac{\partial g_m}{\partial E_g} = \frac{4\Delta I}{E^2} \tag{102}$$

$$\frac{\partial^2 g_m}{\partial E_g^2} = \frac{24 I_3}{E^3} \tag{103}$$

where ΔI is the change in d-c plate current, I_3 the crest value of the third-harmonic component of the plate current, and E is the crest value of the signal voltage applied to the grid.

The Application of Exact Equivalent Circuit to Simple Cases.—The first-, second-, and third-order components of the equivalent voltage in the exact equivalent circuit for the case of signals consisting of a sine wave and of two superimposed sine waves are summarized in Table 5. An examination of this table shows that the first-order effects account for the undistorted part of the amplified output. The second-order effects give rise to second harmonics, to sum and difference frequencies, and to a direct-current or "rectified" component, all of which are proportional to the square of the signal. The third-order effect gives rise to third harmonics and odd-order combination frequencies. There is also a third-order component having the same wave shape as the applied signal (*i.e.*, as the first-order output) but an amplitude proportional to the cube of the signal, so that when this combines with the first-order part of the output, the result is a lack of proportionality between the amplitude of the input signal and the amplitude of the output. Finally, when the applied signal consists of more than a single frequency, the third-order action causes the part of the output that is at one frequency to depend upon the amplitude of the other components present, giving rise to cross-modulation.

Fourth, sixth, and other higher even-order components give effects similar in character to those produced by the second-order action, namely, even-order harmonics, even-order combination frequencies, and a rectified d-c current. Fifth, seventh, and other higher odd-order components give effects similar to those produced by the third-order action, *i.e.*, odd harmonics, odd-order combination frequencies, lack of proportionality between input and output, and cross-modulation.

Cross-modulation (*Cross Talk*).—Cross-modulation is said to be present in a circuit when the conditions are such that when two signals of different frequencies are applied, the amplitude of the output of one frequency depends upon the amplitude of the other frequency that is present. In the case where one of the signals is a modulated wave, cross-modulation results in the amplitude of the other wave depending on the envelope amplitude of the modulated wave. This causes the modulation of the first wave to be transferred to the second wave. Cross-modulation occurs in amplifiers as a result of third-order action.

The amount of cross-modulation can be expressed in terms of a *cross-talk* or *cross-modulation* factor that is defined as the percentage modulation that a modulated wave produces on an unmodulated superimposed wave, divided by the percentage modulation of the modulated wave. In the case of pentode and other tubes where the load resistance is small compared with the plate resistance, the cross-talk factor is[2]

$$\left.\begin{matrix}\text{Cross-talk factor} \\ \text{in pentodes}\end{matrix}\right\} = \frac{E_2^2}{2g_m}\frac{\partial^2 g_m}{\partial E_g^2} \tag{104}$$

where E_2 is the crest amplitude of the carrier applied to the grid, whose modulation is

[1] An alternative procedure for measuring the third-order coefficient is given by E. W. Herold, Simple Methods for Checking R. F. Distortion or Cross-modulation of Pentode Amplifiers, *Electronics*, Vol. 13, p. 82, April, 1940.

[2] Stuart Ballantine and H. A. Snow, Reduction of Distortion and Cross Modulation in Radio Receivers by Means of Variable-mu Tetrodes, *Proc. I.R.E.*, Vol. 18, p. 2102, December, 1930.

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being transferred to the other carrier frequency. In triode tubes the cross-talk factor depends upon the load impedance to the first- and second-order currents, as well as upon the square of the carrier amplitude and the second derivative of the transconductance.

Distortion of Modulation Envelopes.—It was noted above that third-order action introduces a nonlinear relation between output and input voltage of an amplifier. When the input voltage is a modulated wave, the third-order action therefore causes the output voltage to fail to follow exactly the amplitude variations of the input voltage. This results in a distortion of the modulation envelope, and also makes the degree of modulation of the output voltage differ from that of the applied signal. When the modulation of the applied signal is sinusoidal, and the degree of modulation m is not too great, the second-harmonic distortion introduced in the modulation envelope by third-order action in the case of pentode tubes is

$$\left.\begin{array}{l}\text{Second-harmonic}\\ \text{component of modulation}\end{array}\right\} = \frac{m}{2 + \dfrac{4g_m}{E_0^2 \dfrac{\partial^2 g_m}{\partial E_g^2}}} \tag{105}$$

where E_0 is the carrier amplitude of the applied signal, and the remaining notation is as above. It will be noted that when the third-order action is large, the distortion approaches $m/2$. Practically, it is found that when modulation distortion is enough to be of significance, cross-modulation is excessive. Cross talk rather than modulation distortion accordingly limits the amount of third-order action permissible over the operating range of an amplifier under ordinary conditions.

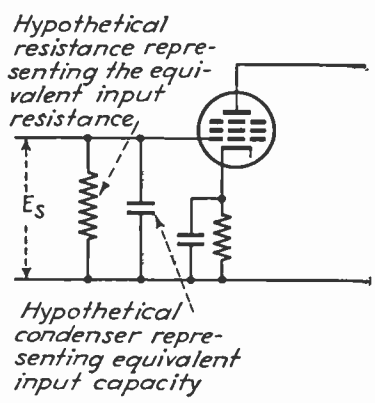


FIG. 89.—Input impedance of a vacuum-tube represented as a resistance shunted by a capacity.

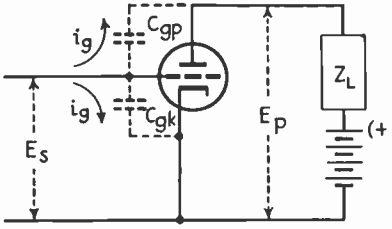


FIG. 90.—Diagrams illustrating how the interelectrode capacities permit grid current to flow when alternating voltage is applied to a negative grid.

25. Input Admittance and Output Impedance of Vacuum-tube Amplifiers. *Input Admittance.*—The input admittance of a vacuum-tube amplifier is defined as the current flowing into the control-grid electrode divided by the voltage that is applied between this electrode and the cathode. The input admittance of a vacuum-tube amplifier can be represented by a capacity shunted by a resistance, as shown in Fig. 89. The capacity is called the input capacity, and the resistance is termed either the input resistance or the input conductance.

Input Admittance of Triodes with Particular Reference to Interelectrode-capacity Effects.—The input capacity and resistance of triodes is determined primarily by the capacities existing between the electrodes of the tube. When an alternating signal voltage is applied between grid and cathode, current flows from grid to cathode and from grid to the plate as a result of the electrostatic capacity between these electrodes,

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as indicated in Fig. 90. The input admittance is equal to this total current divided by the voltage applied to the control grid. Since the part of the current that flows to the plate depends upon the difference between the signal applied to the control grid

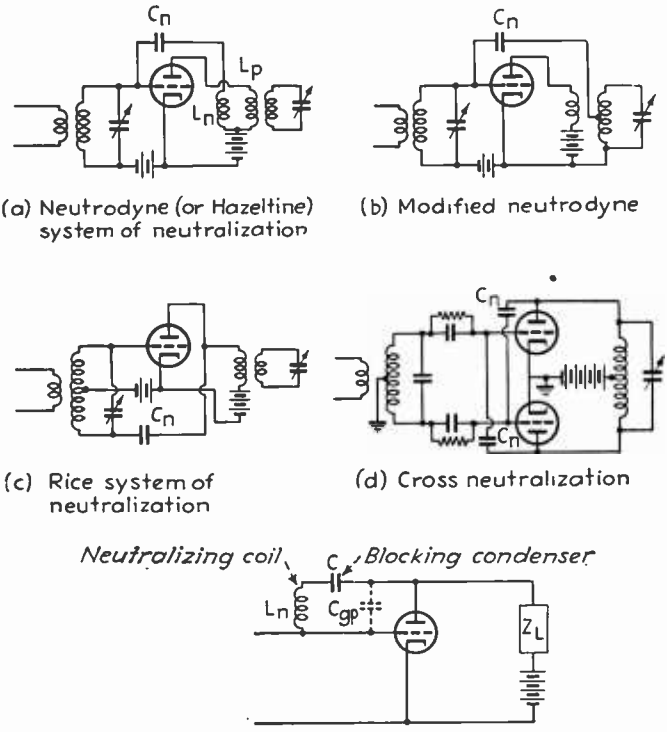


FIG. 91.—Typical neutralizing circuits.

and the amplified voltage developed in the plate circuit, the input admittance is affected by the magnitude and phase of the amplification. The exact relation is[1]

$$\text{Input capacity} = C_{gk} + C_{gp}(1 + A \cos \theta) \tag{106}$$

$$\text{Input resistance} = -\frac{\left(\frac{1}{\omega C_{gp}}\right)}{A \sin \theta} \tag{107}$$

where C_{gp} = grid-plate tube capacity.

C_{gk} = grid-cathode tube capacity.

A = voltage amplification of the tube alone, not taking into account any step-up in voltage existing in the load impedance, such as is present in the case of transformer coupling.

θ = phase angle of load impedance in the plate circuit, taken positive for inductive loads.

[1] For derivation, see F. E. Terman, "Radio Engineering," 2d ed., p. 232, McGraw-Hill, New York. See also J. M. Miller, Dependence of the Input Impedance of a Three-electrode Vacuum Tube upon the Load in the Plate Circuit, *Bur. Standards Sci. Paper* 351; M. von Ardenne and W. Stoff, On the Values and the Effects of Stray Capacities in Resistance-coupled Amplifiers, *Proc. I.R.E.*, Vol. 15, p. 895, November, 1927; E. L. Chaffee, Equivalent Circuits of an Electron Tube and the Equivalent Input and Output Admittances, *Proc. I.R.E.*, Vol. 17, p. 1633, September, 1929.

Ex. 1044-0476

The input admittance of a triode amplifier depends largely upon the magnitude A and phase θ of the amplification, because with practical tubes the grid-plate capacity has approximately the same magnitude as the grid-filament capacity, and the amplification is normally considerably greater than unity. Under these conditions the input capacity is approximately proportional to the amplification, and is much greater than the interelectrode capacities themselves.

The input resistance takes into account the energy transferred between the plate and grid electrodes through the grid-plate capacity. With capacitive load impedances in the plate circuit (θ negative), energy is transferred directly from the grid to the plate circuit, with the result that the input resistance is positive. On the other hand, with inductive load impedances in the plate circuit (θ negative), amplified energy is transferred from the plate circuit of the tube back to the grid circuit, with the result that the equivalent input resistance is negative. A resistance load impedance in the plate circuit ($\theta = 0$) gives rise to infinite input resistance, corresponding to zero energy transfer between input and output electrodes. The input resistance for a given phase shift θ is inversely proportional to the product of frequency and amplification.

Neutralization of Grid-plate Capacity in Triode Tubes.—Most of the input capacity and practically all the input resistance of triode tubes results from the current that flows through the grid-plate capacity of the tube. The effect of this capacity can be eliminated or neutralized by the arrangements illustrated in Fig. 91. In the neutrodyne circuit the coil L_n is closely coupled to the primary, and so polarized that it applies a voltage to the neutralizing condenser C_n that is of opposite phase from the a-c voltage between plate and cathode. The current through the neutralizing condenser is hence of opposite phase from the current through the grid-plate capacity C_{gp} of the tube, and by proper adjustment of C_n the energies represented by the two currents can be given the same magnitude. The input capacity of such an amplifier with perfect neutralization is $C_{gk} + C_{gp} + C_n$, and the input resistance is infinite. If the neutralizing coil L_n is closely coupled to its primary and the leads have negligible inductive reactance, the neutralization is substantially independent of frequency over a wide frequency band.[1]

An alternative neutralizing arrangement is the Rice circuit (Fig. 91*c*). Here the neutralizing capacity C_n is adjusted so that the current through it neutralizes the effect of the current through the grid-plate capacity as far as the tuned circuit associated with the grid of the tube is concerned. This form of neutralization is theoretically independent of frequency, just as is the neutrodyne type, but has the disadvantage that only half of the signal voltage developed across the input circuit is applied to the grid of the tube, and that neither side of the tuning condenser in the input circuit can be grounded. Also, if several stages are in cascade, Rice neutralization is more likely to cause trouble from parasitic oscillations.

Cross neutralization is used with push-pull amplifiers and requires the addition of no special circuits other than the neutralizing condensers. It can be thought of as a form of neutrodyne that takes advantage of the fact that the voltages on the two sides of a push-pull amplifier are of opposite polarity, thus giving the phase relations required for neutralizing.

In coil neutralization, the neutralizing inductance L_n is resonated with the grid-plate capacity C_{gp} at the frequency for which neutralization is to be effective. In this way the current flowing from control grid to plate is reduced practically to zero, thus eliminating the grid-plate capacity as far as input admittance effects are concerned. In order that the neutralizing coil will not short-circuit the plate-supply

[1] At very high frequencies the variation in the reactance of the leads cannot be neglected. If neutralization is to be maintained over a wide frequency band under these conditions, it is necessary to use modified circuits (see Fig. 20, Sec. 6).

Ex. 1044-0477

battery, it is necessary to place the blocking condenser C in series with the inductance. The circuit proportions are then arranged so that the combination L_nC offers the inductive reactance required to resonate with C_{gp} at the frequency of neutralization. Coil neutralization is simpler than the other types, but is effective for only one frequency. It is used extensively with transmitters, such as broadcast transmitters, that operate only at a single frequency.

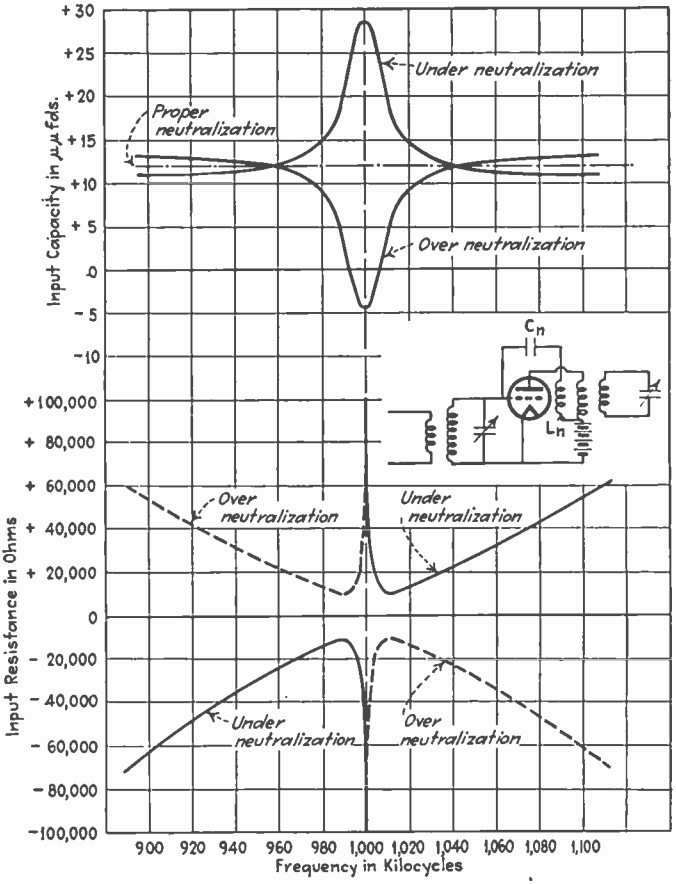


FIG. 92.—Curves showing the effect of an improper neutralizing capacity C_n on input resistance and capacity of a tuned radio-frequency amplifier.

In all neutralizing systems it is necessary to adjust the circuit proportions so that neutralization is exactly correct. Overneutralization produces results similar in character to and just as bad as underneutralization, as illustrated in Fig. 92.

Grounded-grid Self-neutralized Circuit.—The necessity of neutralization can be avoided by means of the grounded-grid circuit of Fig. 93. Here the grid is grounded, the input voltage is applied between cathode and ground, and the output impedance is placed between plate and ground. With this arrangement, capacity currents flowing between plate and grid as a result of the output voltage E_0 developed in the plate circuit do not flow through the input circuit. There is hence no interaction with the source of exciting voltage E_s as a result of grid-plate tube capacity.

Ex. 1044-0478

Analysis of the voltage and current relations in the grounded-grid circuit shows that since the driving voltage E_s is in series with the external circuit connecting plate and cathode, the tube acts, as far as the output E_0 is concerned, as though it were excited in the usual manner and had an amplification factor $(\mu + 1)$ instead of μ. Also, the exciting voltage delivers some energy directly to the plate circuit, because the amplified plate current flows through E_s in the same polarity as E_s. This places a load on the exciting voltage E_s. This load reduces the gain obtained as compared with a neutrodyne circuit, for example, although appreciable amplification is still possible.

The grounded-grid circuit finds its chief use under conditions where triode tubes must be used, and either (1) it is impractical to obtain the degree of capacity neutralization required for satisfactory operation or (2) the tube or circuit design is such that the natural method of operation is with the grid at ground potential. These conditions are most likely to exist where the tube is to be operated over an enormous frequency range or at a very high frequency, or with special tubes designed for operation at extremely high frequencies.[1]

Input Conductance of Pentodes.—The input conductance of a pentode tube, if negative grid operation is assumed, arises primarily from the transit time of the

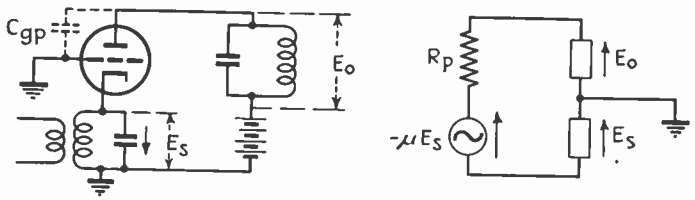


FIG. 93.—Grounded-grid self-neutralized circuit.

electrons and from the fact that the lead from the cathode electrode to the cathode pin on the base of the tube is common to the grid and plate circuits of the tube. Dielectric losses in the grid-cathode and grid-screen capacities are occasionally of importance.[2] The grid-plate capacity contributes very little to the input conductance of a pentode tube, because although this capacity acts the same in pentodes as in triodes, its magnitude is so small (0.005 $\mu\mu$f for a typical voltage amplifier pentode) that the energy transfer taking place between plate and grid is too small to be important.

The input resistance arising from the fact that the electrons require a finite time to travel from cathode to plate is given by[3]

$$\left.\begin{matrix}\text{Input resistance resulting}\\ \text{from finite transit time}\end{matrix}\right\} = \frac{1}{K g_m f^2 \tau^2} \qquad (108)$$

where g_m = transconductance of the tube.

f = frequency.

τ = time required for the electron to travel from the cathode to the grid plane.

The constant K is determined by the grid and plate voltages and by the ratio of transit times from cathode to grid plane and grid plane to anode. The component of input resistance arising from transit time is directly proportional to the square root of the electrode voltages and inversely proportional to the square of the linear dimension of the tube.

[1] Discussion of the grounded grid circuits, with particular reference to high-frequency high-power amplifiers, is given by C. E. Strong, The Inverted Amplifier, *Electronics*, Vol. 13, p. 14, July, 1940.

[2] A discussion of dielectric losses is given by C. J. Franks, Measured Input Losses of Vacuum Tubes, *Electronics*, Vol. 8, p. 222, July, 1935.

[3] W. R. Ferris, Input Resistance of Vacuum Tubes as Ultra-high-frequency Amplifiers, *Proc. I.R.E.*, Vol. 24, p. 82, January, 1936. See Par. 9, Sec. 4 for further discussion of transit time effects.

Ex. 1044-0479

The inductance of the cathode lead causes the voltage existing across the grid-cathode capacity to differ from the signal voltage applied to the tube by an amount equal to the voltage developed across the lead inductance by the amplified plate current (see Fig. 94). This fact causes the current flowing through the grid-cathode capacity to have a component in phase with the applied voltage.[1] The corresponding input resistance, on the assumption that the reactance of the lead inductance L_K is small compared with the grid-cathode capacity C_{gk}, is

$$\left.\begin{array}{l}\text{Input resistance resulting}\\ \text{from inductance of cathode}\\ \text{lead}\end{array}\right\} = \frac{1}{\omega^2 g_m L_K C_{gk}} \tag{109}$$

It will be noted that the input resistance caused by the inductance of the cathode lead varies with frequency and transconductance in exactly the same manner as does the input resistance resulting from transit time action. In ordinary pentode tubes, these two effects are of the same order of magnitude. The combined action of cathode lead inductance and transit time is of great importance at the higher radio frequencies. This is evident from Table 6, which gives results of measurements on input resistance of a pentode tube.

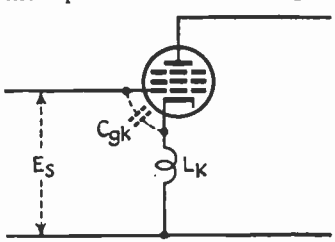


FIG. 94.—The equivalent input circuit of a tube taking into account the inductance of the cathode lead.

The input conductance resulting from the combined effects of cathode-lead inductance and transit time can be neutralized by the arrangement in Fig. 95*a*.[2] Here a small inductance L_n is placed between the cathode terminal on the tube socket and the ground. The grid-return lead is brought directly to the tube socket, and a capacity C_{gg} is connected between grid and ground. Neutralization is obtained when

$$\frac{L_n}{L_k} = \frac{C_{gk}}{C_{gg}} \tag{110}$$

With these proportions, the in-phase component of the current through C_{gg} is equal in magnitude but opposite in phase to the in-phase component of current through C_{gk}, resulting in complete neutralization of the input resistance independently of frequency, provided that $\omega L_k << 1/\omega C_{gk}$. In practical amplifiers, the neutralizing inductance L_n can be a short length of wire of the order of one inch long connecting the cathode terminal of the tube socket to the ground lug, as shown in Fig. 95*b*.

An alternative method of neutralizing the input conductance is to place a small inductance (about 0.1μh) in the screen lead. This method is especially adaptable to converter tubes.

The effect of the cathode lead inductance can be eliminated by using especially constructed tubes having two cathode leads, as shown in Fig. 95*c*.[3] In this way the voltage developed in the cathode lead by the plate current does not appear in series with the voltage applied to the control grid.

Input Capacity of Pentodes, with Particular Reference to the Effect of Control-grid Bias.—The input capacity of a pentode is given by the equation

$$\text{Input capacity} = C_{gk} + C_{ge} + C_{gp}(1 + A\cos\theta) \tag{111}$$

[1] M. J. O. Strutt and A. van der Ziel, The Causes for the Increase of the Admittances of Modern High-frequency Amplifier Tubes on Short Waves, *Proc. I.R.E.*, Vol. 26, p. 1011, August, 1938.

[2] R. L. Freeman, Input Conductance Neutralization, *Electronics*, Vol. 12, p. 22, October, 1939.

[3] Such tubes are discussed by F. Preisach and I. Zakarias, Input Conductance, *Wireless Eng.*, Vol. 17, p. 147, April, 1940.

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where C_{gk} = grid-cathode capacity.
C_{gs} = grid-screen capacity.
C_{gp} = grid-plate capacity.
A = voltage amplification (magnitude) of tube alone (not including any voltage step-up in plate coupling network).
θ = phase angle of amplification.

The component $C_{gp}(1 + A \cos \theta)$ is the smallest of the three because of the small grid-plate capacity in pentode tubes, but may be of the order of $1 \mu\mu f$ in radio-frequency

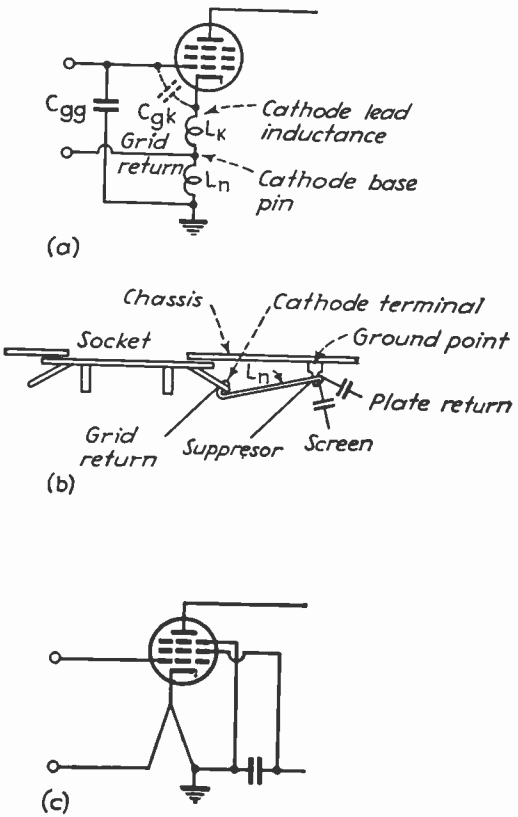


FIG. 95.—Methods of neutralizing the effects of cathode lead inductance and transit time on input resistance.

pentodes when the amplification is large, and somewhat more in audio-frequency pentodes.

The input capacity of a pentode varies somewhat with the grid bias.[1] This is in part due to a change in the component $C_{gp}(1 + A \cos \theta)$ as the grid bias varies the amplification, and in part due to the fact that as the bias varies the plate current the effective position of the space charge surrounding the cathode changes. The total variations produced by varying the grid bias are of the order of 1 to 3 $\mu\mu f$ in typical

[1] R. L. Freeman, Use of Feedback to Compensate for Vacuum-tube Input-capacitance Variations with Grid Bias, *Proc. I.R.E.*, Vol. 26, p. 1360, November, 1938; T. Iowerth Jones, The Dependence of the Inter-electrode Capacitances of Valves upon the Operating Conditions, Wireless Section, *I.E.E.*, Vol. 13, p. 11, March, 1938; also, *Jour. I.E.E.*, Vol. 81, p. 658, 1937.

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radio-frequency pentodes. This will detune an ordinary 450-kc intermediate-frequency coupling circuit by about 5 kc.

The change in input capacity is approximately proportional to the change in the transconductance of the tube, and so can be neutralized by the arrangements shown in Fig. 96. Here a resistance R_n is placed between cathode and ground and is so related to the capacity C_{gk} between grid and cathode as to satisfy the relation

$$R_n = \frac{\Delta C}{C_{gk} g_m} \tag{112}$$

where g_m is the transconductance for some particular operating point and ΔC is the increase in input capacity of the tube when the transconductance is g_m over the input capacity when the tube biased to cutoff. When the secondary trimmer condenser C_i is returned to ground, as in Fig. 96*b*, the value of resistance R_n required approximates that normally used for self-bias. On the other hand, if C_{gk} is made large by returning the trimmer condenser of the input circuit to the cathode, instead of to ground, as in Fig. 96*a*, the neutralizing resistance R_n is then of the order of 20 ohms with ordinary r-f pentodes used as intermediate-frequency amplifiers.[1]

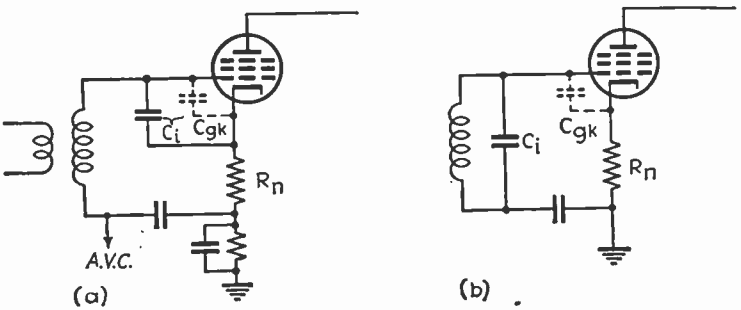


FIG. 96.—Circuits for neutralizing the variation of input capacity of a Class A pentode amplifier with variation in grid bias.

Input Admittance of Amplifier Tube Subjected to a Negative Feedback Voltage.—When a negative feedback voltage is superimposed upon the signal applied to an amplifier, the equivalent input admittance offered by the system to the applied signal voltage is less than in the absence of negative feedback according to the factor

$$\frac{\left\{\begin{matrix}\text{Input admittance with} \\ \text{negative feedback}\end{matrix}\right\}}{\left\{\begin{matrix}\text{Input admittance without} \\ \text{negative feedback}\end{matrix}\right\}} = \frac{1}{1 - A\beta} \tag{113}$$

where $A\beta$ is the feedback factor as defined in connection with Eq. (36). The reduction in input admittance is considerable when $A\beta$ is large, and arises from the fact that most of the signal voltage is used to overcome the negative feedback voltage and only a small fraction is applied across the grid-cathode terminals of the tube.

Practical Importance of Input Admittance in Audio-frequency and Radio-frequency Amplifiers.—In audio-frequency amplifiers, the input capacity is one of the largest shunting capacities in the coupling network in the plate circuit of the preceding ampli-

[1] Another means of compensating for variations in input capacity caused by variation in the control-grid bias by the A.V.C. system is to apply A.V.C. voltage in appropriate amount to the No. 3 grid. See J. F. Farrington, Compensation of Vacuum Tube Input Capacitance Variation by Bias Potential Control, *R.M.A. Eng.*, Vol. 4, p. 13, November, 1937.

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fier stage. A pentode is accordingly preferable to a triode tube from the point of view of response obtainable at high frequencies unless the triode is neutralized. In the case of push-pull triodes, neutralization can be easily applied and will reduce the input capacity to the same order of magnitude as that obtained with pentodes, resulting in considerable improvement in high-frequency response.[1] The resistance component of the input admittance is of negligible importance at audio frequencies, since the input resistance is so large as to produce only slight modification of the amplification characteristic, even at the higher audio frequencies with unneutralized triodes.

In tuned radio-frequency amplifiers, the input capacity of the tube is an important factor in fixing the minimum circuit capacity. With unneutralized or partially neutralized triodes, the input capacity varies with frequency, as illustrated in Fig. 92. This dependence of the capacity on frequency can distort the resonance curve of the tuned circuit developing the signal voltage applied to the grid of the tube.

The input resistance at broadcast and lower frequencies is normally caused by the action of the grid-plate tube capacity. With a load impedance supplied by a resonant circuit, the input resistance depends primarily upon frequency, being infinite at resonance and minimum at the 70.7 per cent points of the amplification curve. This minimum value for the case of a simple tuned load circuit is

$$\text{Minimum input resistance} = \frac{(1/\omega C_{gp})}{(A_0/2)} \tag{114}$$

where $1/\omega C_{gp}$ is a reactance of the grid-plate capacity C_{gp} and A_0 is the amplification from grid to plate electrodes at resonance.[2] The input resistance is negative at frequencies below resonance and positive for frequencies greater than resonance, and so tends to distort the resonance curve of the input tuned circuit. If the input resistance is low, oscillations will be produced when the input tuned circuit is resonant at a slightly lower frequency than the output circuit. The effects of the input resistance are so great in tuned amplifiers using triode tubes that such amplifiers must always be neutralized. With pentode tubes, however, the minimum input resistance at broadcast frequencies is so high because of the small grid-plate capacity that it can ordinarily be ignored unless it becomes cumulative through a series of stages.

In the high-frequency and ultra-high-frequency bands, the input resistance is determined primarily by the transit time and cathode lead effects, since these produce effects proportional to the square of frequency, whereas the action of the grid-plate capacity is proportional to frequency. The input resistance at very high frequencies is low enough to produce serious loading, with consequent loss of gain and selectivity.

Output Impedance of Vacuum Tubes.—The output impedance of a vacuum tube is defined as the impedance that the plate circuit of the tube offers to an external voltage applied between the plate and cathode electrodes of the tube. This impedance can be represented by a resistance shunted by a capacity and is the impedance seen by a load looking toward the tube.

In triode tubes the resistance component of the output impedance approximates the plate resistance of the tube up to very high radio frequencies. The output capacity also approximates the sum of the grid-plate and plate-cathode interelectrode capacities. These approximations neglect the effect of dielectric losses, inductance in leads, and energy transfer between the input and output circuits of the tube through the grid-plate capacity, but give satisfactory results for the usual triode operating conditions. In particular, the energy transfer between input and output circuits

[1] P. W. Klipsch, Applying Neutralization to A. F Amplifiers, *Electronics*, Vol. 7, p. 252, August, 1934.

[2] When the tuned load circuit is coupled to the plate circuit of the tube, as with transformer coupling, the value of A_0 is less than the actual voltage amplification of the stage by the factor M/L_s, where M is the mutual inductance between primary and secondary and L_s the inductance of the secondary.

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has a relatively small effect upon the output circuit as compared with the effect on the input circuit because of the difference in power levels.

In pentode and beam tubes the output capacity is equal to the sum of the plate-cathode capacity and the plate-screen capacity. The output resistance, however, depends very greatly upon frequency, being equal to the plate resistance of the tube only at audio and the lower radio frequencies. As the frequency is increased, the output resistance becomes less, first because of dielectric losses associated with the interelectrode capacities of the tube, and at still higher frequencies as a result of the inductance of the lead wires from the electrodes to the base pins of the tube and capacities between electrodes. The equivalent shunting resistance as measured for a particular pentode tube is shown in Table 6,[1] and it is seen that the output resistance at the higher radio frequencies is far less than the plate resistance of the tube.

26. Tube and Circuit Noise in Amplifiers.—The term *noise* as applied to vacuum-tube amplifiers is commonly used to designate spurious voltages of a random character that represent energy more or less uniformly distributed over an appreciable frequency band. In the case of audio-frequency amplifiers, such noise produces a characteristic hiss.

Thermal Agitation Noise.[2]—The random motion of free electrons in a conductor causes small potential differences to be developed across the terminals of the conductor.

TABLE 6.—INPUT AND OUTPUT RESISTANCE OF A PENTODE AMPLIFIER*

Wave length, meters	Input resistance		Wave length, meters	Output resistance	
	Tube cold, megohms	Normal, megohms		Tube cold, megohms	Normal, megohms
230.0	3.9	3.3	62.5	0.75	0.43
39.5	1.6	0.38	20.4	0.35	0.19
21.2	0.74	0.11	5.05	0.045	0.022
12.4	0.40	0.036			
5.6	0.19	0.0086			

* Data from M. J. O. Strutt and A. Van der Ziel, The Causes for the Increase of the Admittances of Modern High-frequency Amplifier Tubes on Short Waves, *Proc. I.R.E.*, Vol. 26, p. 1011, August, 1938.

This action is termed *thermal agitation*, and the resulting voltage is

$$\left.\begin{array}{l}\text{Square of effective value of}\\ \text{voltage components lying be-}\\ \text{tween frequencies } f_1 \text{ and } f_2\end{array}\right\} = E^2 = 4kT\int_{f_1}^{f_2} R\,df \qquad (115)$$

where k = Boltzmann's constant = 1.374×10^{-23} joule per °K.
T = absolute temperature, °K.
R = resistance component of impedance across which the thermal agitation is developed (a function of frequency).
f = frequency.

[1] From Strutt and van der Ziel, *loc. cit.*

[2] J. B. Johnson, Thermal Agitation of Electricity in Conductors, *Phys. Rev.*, Vol. 32, p. 97, July, 1928; H. Nyquist, Thermal Agitation of Electronic Change in Conductors, *Phys. Rev.*, Vol. 32, p. 110, July, 1928; J. B. Johnson and F. B. Llewellyn, Limits to Amplification, *Elec. Eng.*, Vol. 53, p. 1449, November, 1934; F. C. Williams, Thermal Fluctuations in Complex Networks, Wireless Section, *I.E.E.*, Vol. 13, p. 53, March, 1938, *Jour. I.E.E.*, Vol. 81, p. 751, 1937; F. C. Williams, Coexistent Thermal and Thermionic Fluctuations in Complex Networks, Wireless Section, *I.E.E.*, Vol. 13, p. 327, September, 1938; *Jour. I.E.E.*, Vol. 83, p. 76, 1938.

Ex. 1044-0484

In the special case where the resistance component of the impedance is constant over the range of frequencies from f_1 to f_2, Eq. (114) reduces to the much simpler form

$$E^2 = 4kTR(f_2 - f_1) \tag{116}$$

In a network involving two or more conductors at different temperatures, the combined thermal agitation effect can be obtained by considering that each impedance involved acts as a generator having the mean square voltage specified by Eq. (115) in series with the impedance.[1] The mean square thermal agitation voltage is by Eq. (116) proportional to the *resistance component* of the impedance across which the voltage is developed, and to be proportional to the band width. The peak amplitude is of the order of 3 to 4 times the equivalent rms value.[2] The energy represented by the thermal agitation voltages developed across a given resistance is uniformly distributed over the entire frequency spectrum from zero frequency to frequencies well above the highest used in communication.

The voltages developed by thermal agitation set a limit to the smallest voltage that can be amplified without being lost in a background of noise. This limit is determined by the band width being amplified and not by the position in the frequency spectrum at which the band is located. The magnitude of the effect can be estimated from the fact that the rms thermal agitation voltage developed across a ½-megohm resistance at 300°K is 6.4 μv for a frequency band of 5,000 cycles.

Noise from Granular Resistances.[3]—Resistances composed of carbon granules (such as an ordinary carbon resistor) generate noise far in excess of the thermal agitation noise when a direct current is passed through the resistance. This high noise arises from fluctuations in the contact resistance between adjacent granules. The noise voltage that results is proportional to the current, and also tends to increase somewhat faster than the resistance.

This effect makes the ordinary carbon resistor unsatisfactory as a plate-coupling resistance in low-level stages of a resistance-coupled amplifier. Carbon resistors are, however, always suitable for bias and screen-grid voltage dropping resistors where by-pass condensers are used.

Tube Noise.—Noise voltages are generated within tubes as a result of a number of actions, the most important of which are (1) random variations in electron emission from the cathode; (2) random variations in the current division between the plate and other positive electrodes, such as the screen grid (this effect is absent in triodes operated with the grid negative); (3) variations in the grid current resulting from positive-ion current. These effects are discussed in Pars. 5 and 13, Sec. 4, where formulas for calculating their magnitudes are also given. Under practical conditions the smallest voltage that can be amplified is sometimes limited by tube noise rather than thermal agitation.

27. Hum.—The term hum is applied to alternating currents appearing in the output of an amplifier as a result of the effect of power-frequency voltages, currents, and fields.

Hum in Audio-frequency Amplifiers.—In audio-frequency amplifiers, hum results from the introduction into the amplifier circuits of currents of the power frequency and its harmonics that are amplified directly by the amplifier. Hum is particularly

[1] F. C. Williams, The Representation and Computation of Fluctuation Voltages, Wireless Section, *I.E.E.*, Vol. 14, p. 325, September, 1939, *Jour. I.E.E.*, Vol. 85, p. 280, 1939.

[2] V. D. Landon, A Study of the Characteristic of Noise, *Proc. I.R.E.*, Vol. 24, p. 1514, November, 1936; The Distribution of Amplitude with Time in Fluctuation Noise, *Proc. I.R.E.*, Vol. 29, p. 50, February, 1941.

[3] C. J. Christensen and G. L. Pearson, Spontaneous Fluctuations in Carbon Microphones and Other Granular Resistances, *Bell System Tech. Jour.*, Vol. 15, p. 181, April, 1936.

Ex. 1044-0485

troublesome in high-gain audio-frequency amplifiers, since any effects introduced in low-level stages are subjected to large amounts of amplification.

The possible causes of hum in audio-frequency amplifiers are stray electrostatic and magnetic fields, alternating current in the filaments or heaters of the tubes, and poorly filtered power-supply systems. These last two sources of hum are considered in Pars. 9 and 5, Sec. 8. The types of hum that are usually most troublesome in audio-frequency amplifiers are those arising from the presence of electrostatic and magnetic fields.

The chief sources of magnetic fields in audio-frequency amplifiers are the filament leads and the power transformer. These fields induce voltages in coupling transformers, particularly transformers in low-level parts of the amplifier, may induce voltages in closed loops in improperly arranged wiring, and in severe cases may even affect the flow of electrons within the tubes. The most effective way to minimize hum from magnetic fields is to arrange the filament leads in the form of a twisted pair, employ adequate spacing between the low-level parts of the amplifier and the power transformer, and avoid as far as possible the use of coupling transformers in low-level stages. Where coupling transformers cannot be avoided, induced voltages from magnetic fields can be minimized by use of adequate spacing between coupling and power transformers, orientation of the coupling transformers to give minimum mutual inductance, and the use of shielded coupling transformers. It is also helpful to employ a nonmagnetic chassis, or at least to isolate the coupling and power transformers from a sheet-iron chassis by mounting them upon a nonmagnetic base that is then fastened to the chassis.

Electrostatic fields may arise in an amplifier from unshielded wires carrying alternating currents of power frequency, but are more commonly the result of power wires external to the apparatus itself. Hum arising from electrostatic fields can be eliminated by electrostatically shielding all leads that have a high impedance to ground, such as grid leads, by shielding the tubes and the filament wires of low-level stages, and by grounding the chassis to a metal stake driven in wet earth or a water pipe. The use of electrostatic shielding between primaries and secondaries of all transformers (power, input, filament, output, etc.) on the amplifier chassis is also helpful.

Hum in Radio-frequency Amplifiers.—Hum is less troublesome in radio-frequency amplifiers than in audio-frequency amplifiers, since the hum voltages are not amplified directly. However, the same effects that produce hum in audio-frequency amplifiers also will cause any radio-frequency voltage that is being amplified to be modulated at least slightly by the hum.

28. Microphonic Effects.[1]—The term *microphonic*, as applied to vacuum-tube amplifiers, refers to effects produced by mechanical vibration, and may arise from the vibration either of tubes or of the associated circuits.

Microphonic Action in Tubes.—The most important source of microphonic action in amplifiers is the amplifier tube. Vibrations of the tube elements produce variations in plate current that are amplified directly in audio- and video-frequency amplifiers, and that modulate any signal being amplified by radio-frequency amplifiers.

Microphonic effects in tubes can be minimized by preventing vibrations from reaching the tube, by selection of individual tubes for low microphonic action, and by the use of special tubes of unusually rigid construction. The chief way in which vibrations reach a tube is through the base, and this can be greatly reduced by supporting the tube on a shockproof mounting. Individual tubes of the same type may vary as much

[1] D. B. Penick. Measurement and Reduction of Microphonic Noise in Vacuum Tubes, *Bell System Tech. Jour.*, Vol. 13, p. 614, October, 1934; Alan C. Rockwood and Warren R. Ferris, Microphonic Improvements in Vacuum Tubes, *Proc. I.R.E.*, Vol. 17, p. 1621, September, 1929.

Ex. 1044-0486

as 70 db in their tendency to microphonic action, so that there is considerable gain in selecting the least noisy tubes from a large number. A very low susceptibility to vibrations can be obtained by tube constructions that provide unusual rigidity and bracing of the tube electrodes, particularly the filaments and heaters. Improvements of the order of 20 db over standard tubes are obtainable with such special low microphonic tubes.

Microphonic action in tubes sometimes produces a harsh crackling or sputtering type of noise. This is caused by poor contacts between conducting members, or by intermittent electrical leaks across insulation. Such sputter noise contains frequency components in the radio-frequency band, whereas ordinary microphonic noises do not.

Microphonic Action in Circuits.—Mechanical vibration of circuits associated with an amplifier tube will modulate the voltages being amplified. This effect is particularly important in radio-frequency amplifiers and oscillators. The circuit elements that are the most frequent offenders are the coils and variable condensers of resonant circuits. Microphonic effects of this sort can be controlled by rigid construction of the affected circuit elements, by mounting the entire unit on rubber, and by protection against the direct action of sound waves

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SECTION 6

OSCILLATORS

POWER OSCILLATORS

1. Circuits for Power Oscillators.[1]—A vacuum-tube oscillator is essentially a vacuum-tube amplifier arranged so that an exciting voltage of the proper magnitude and phase to produce the amplified output is obtained from this output. The tube in such an arrangement acts as an inverter that changes d-c plate power into a-c energy. In power oscillators, the tube is usually operated under conditions of bias, exciting

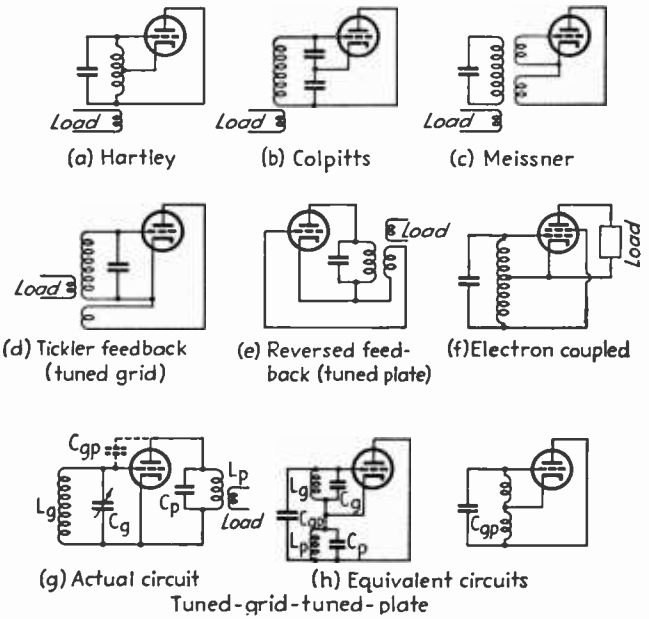


FIG. 1.—Schematic circuits of common types of power oscillators.

voltage, etc., corresponding to Class C amplifier conditions in order to give a large output with good efficiency.

Circuits.—Typical oscillator circuits are shown schematically in Fig. 1. Of these, the ones most frequently used in power oscillators are the Hartley, Colpitts, and the tuned-grid tuned-plate arrangements. In the Hartley and Colpitts circuits an exciting voltage of the appropriate phase is obtained by connecting the grid and plate electrodes to opposite ends of the tank circuit with respect to the cathode connection, with the ratio of exciting voltage to alternating plate-cathode voltage determined by the relative reactances on the two sides of the cathode connection.

[1] For a general discussion of power oscillators, see R. A. Heising, The Audion Oscillator, *A.I.E.E. Jour.*, Vol. 39, p. 365, April; p. 471, May, 1920; D. C. Prince, Vacuum Tubes as Power Oscillators, *Proc. I.R.E.*, Vol. 11, p. 275, June, p. 405, August, and p. 527, October, 1923.

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In the tuned-grid tuned-plate circuit, the grid tuned circuit L_gC_g and the plate tuned circuit L_pC_p are both adjusted to offer an inductive reactance at the frequency to be generated. The circuit can accordingly be redrawn as shown in Fig. 1*h*, and is seen to be the equivalent of a Hartley circuit in which the ratio of exciting voltage to alternating plate-cathode voltage is determined by the relative amounts of detuning of the plate and grid circuits. The capacity C_{gp} is usually supplied by the interelectrode capacity of the tube, although at lower frequencies added shunting capacity is helpful.

In the electron-coupled circuit, the cathode, grid, and screen grid of a pentode or screen-grid tube are operated as a triode oscillator, with the screen serving as the anode.[1] Only a small fraction of the space current is intercepted by the screen, but the oscillator circuits are so designed that this will maintain the oscillations properly. The remaining electrons, which represent most of the space current, go on to the plate and produce power output by flowing through the load impedance that is connected in series with the plate electrode. This arrangement has the advantage of making the oscillations independent of the load, and is used where frequency stability is important (see Par. 3).

The various oscillator circuits shown in Fig. 1 are all capable of giving the same power output, efficiency, etc., when used with a particular tube. The choice between them is accordingly one of convenience, when such factors as circuit details, ease of adjustment, tendency for parasitic oscillations to be produced, etc., are taken into account.

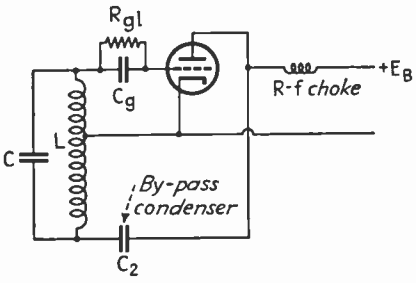


FIG. 2.—Hartley oscillator circuit with shunt feed.

The d-c plate voltage can be introduced in the oscillator either by shunt feed or series feed exactly as in the case of the Class C amplifier. Shunt feed is required in the Colpitts circuit, and is usually preferable in the Hartley circuit.

Bias in power oscillators is practically always obtained by means of a grid-leak grid-condenser arrangement, as shown in Fig. 2, since this makes the oscillator self-starting, and is more likely to give a stable operating point under conditions corresponding to high efficiency operation. The use of a grid leak also tends to make the oscillators self-adjusting at conditions corresponding to good efficiency, and improves the frequency stability.

The grid condenser should have a low enough reactance at the frequency of oscillation so that the alternating voltage drop is small. The condenser should not be too large, however, since then intermittent oscillators may be produced (see below). Also, some capacitive reactance between the grid electrode and the tank circuit is useful in improving the frequency stability (see Par. 3).

Oscillators developing considerable power often have resistances or chokes or both placed in series with leads going to the grid and plate electrodes of the tube in order to eliminate parasitic oscillations (see Par. 5).

Design Considerations.—The voltage and current relations existing in the tube of a power oscillator are exactly the same as in the case of the corresponding Class C amplifier. The performance of the tube can accordingly be determined by calculation as outlined in Par. 21, Sec. 5, except that since the driving power is now obtained from the amplified output, the net output of the oscillator is less than that of the corresponding Class C amplifier by the amount of the driving power.

[1] J. B. Dow, A Recent Development in Vacuum Tube Oscillator Circuits, *Proc. I.R.E.*, Vol. 19, p. 2095, December, 1931.

Ex. 1044-0489

The circuits of a power oscillator are designed in the same manner as for the Class C amplifier, with the addition that provision must be made so that the grid of the tube obtains from the amplified output an exciting voltage of the proper magnitude and phase. The procedure for laying out the circuits of a power oscillator is illustrated by the following example:

Example.—An oscillator is to be designed to operate under the same conditions as the Class C amplifier in the example of Par. 21, Sec. 5, using a Hartley circuit having an effective Q of 50. Assuming the grid and plate taps are to be at the ends of the tank-circuit inductance, the total alternating voltage across the tank circuit will be the sum of the exciting and alternating plate voltages, or

$$254 + 850 = 1{,}104 \text{ volts.}$$

The filament tap is located so that the ratio between grid and plate voltages is $^{254}/_{850}$. The tank-circuit inductive reactance ωL required is found by Eq. (90), Sec. 5, to be

$$(1{,}104/\sqrt{})^2/(56 \times 50) = 218 \text{ ohms}$$

for a tank-circuit power of 56 watts. From this the required tank-circuit inductance and capacity for any frequency can be calculated. The tank-circuit circulating current is $1{,}104/218 = 5.06$ amp crest. Since the grid bias required is 134 volts, with a grid current of 14 ma, the grid-leak resistance can be estimated as $134/0.014 = 9{,}600$ ohms. The grid-condenser capacity should be larger than the grid-cathode tube capacity and also large enough to have a low reactance at the operating frequency, but not so large as to cause intermittent oscillations (see below).

If shunt feed is used, as in Fig. 2, the tank-circuit inductance L required to obtain 218 ohms of reactance will be influenced by the fact that the reactance of the radio-frequency choke is in parallel with a portion of the tank circuit. The blocking condenser C_2 for the shunt-feed arrangement must be capable of withstanding the d-c plate voltage E_B and should, at the same time, be large enough to have relatively low reactance compared with the impedance seen by the plate of the tube in looking toward the tank circuit.

If the grid and plate connections were not made to the ends of the coil, the voltage across the tank circuit would have to be increased accordingly, and a higher tank-circuit inductance would be required to obtain the required conditions with the same effective circuit Q. If other oscillator circuits than the Hartley had been employed, minor modifications of this general procedure would be necessary.

Practical Adjustment of Oscillator Circuits.—The procedure to be followed in placing an oscillator in operation depends upon the completeness with which the preliminary design has been worked out. When every detail has been predetermined, including even such things as the proper points on the tank circuit to connect the plate, grid, and cathode leads and the appropriate coupling to the load, it is necessary merely to apply voltage to the oscillator, check the resulting grid bias, power output, frequency, etc., and then make minor readjustments as required to realize exactly the desired behavior.

In the more usual case, the paper design has been carried only to the point where the tank-circuit inductance and capacity, the expected power output, operating grid bias, d-c plate current, etc., are known, while such things as the coupling to the load, the grid-leak resistance, the cathode connection to the tank circuit in Hartley circuits, and the exact tuning in a tuned-grid tuned-plate circuit, are to be determined by trial and error. Under such conditions the adjustment process should start with relatively loose coupling between the load and tank circuit and with the other circuit adjustments at estimated values. After oscillations are obtained, the coupling to the load is increased until the output approximates the rated value and the input power, efficiency, grid bias, etc., noted. Readjustments are then made of grid-leak resistance, exciting voltage, load coupling, etc., as required.

The trial-and-error adjustment of oscillators can be systematized by keeping in mind the fact that oscillators adjusted for reasonable efficiency have the following characteristics: (1) The bias is determined primarily by the exciting voltage. Increasing the excitation increases the bias and reduces the angle of flow while at the same time increasing the maximum grid potential somewhat. (2) The maximum positive grid potential E_{max} is determined both by the exciting voltage and by the grid-leak resistance, being greater as the grid-leak resistance is reduced and as the excitation is increased. (3) The tank-circuit current is determined primarily by the coupling

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between the tank circuit and plate circuit of the tube, since the tank current is normally such that the crest alternating voltage developed between plate and cathode is just less than the plate-supply voltage. Hence more coupling between the plate and tank circuits reduces the tank-circuit current. (4) The minimum plate potential E_{min} is determined largely by the coupling between the load and the tank circuit, as is also the d-c plate current. Close coupling increases the dissipation in the load for a given tank-circuit current. This tends to reduce the amplitude of oscillations, thereby increasing the minimum plate potential and increasing the d-c plate current that the plate draws. (5) The grid current is determined by the exciting voltage, grid-leak resistance, and load coupling, and for practical purposes is fixed by the maximum grid voltage E_{max} and the minimum plate potential E_{min}.

2. Miscellaneous Considerations in Power Oscillators. *Starting of Oscillations.*—Oscillators employing grid-leak bias are always self-starting. This is because when the plate voltage is first applied, the grid is at zero potential, and the resulting transconductance is so large that if oscillations will exist at all they will build up, with thermal agitation voltages in the circuit acting as the initiating impulse.[1]

Oscillators employing a fixed bias that places the initial operating point close to or beyond cutoff may not be self-starting, because the plate current, and hence the transconductance, will either be low or zero when the plate voltage is first applied. In such cases oscillations may persist if once started by a momentary reduction in the bias or by a sufficiently large transient voltage induced in the circuits.

Amplitude of Oscillations.—After being started, the oscillations increase in amplitude until the power generated by the tube is equal to the energy dissipated in the load and tank circuits. In practical power oscillators employing grid-leak bias and operating under normal full-load conditions, this equilibrium occurs when the peak alternating voltage developed by the tank circuit between plate and cathode is just slightly less than the plate-supply voltage, corresponding to a small minimum plate potential E_{min}. As the load on the tank circuit is reduced, the output voltage will increase, but even with zero load will never exceed the d-c plate voltage, since this would reduce the minimum plate potential to zero and give zero plate current. Varying the load on an oscillator within reasonable limits therefore has only a small effect upon the amplitude of oscillations. The grid and plate currents are, however, quite sensitive to load, the former increasing and the latter decreasing as the load becomes less.

Intermittent Operation.—When a grid leak is used to develop the bias in an oscillator, it is sometimes found that the oscillations obtained are periodically interrupted at an audio- or radio-frequency rate. Such behavior results when the time constant $R_{gl}C_g$ of the grid leak-condenser combination is too large to permit the bias to readjust itself with the rapidity necessary to follow changes in amplitude produced by random effects. When interrupted oscillations are produced, the rate of interruption is of the order of $1/R_{gl}C_g$ times per second, where R_{gl} and C_g are in ohms and farads, respectively.

Interrupted oscillations can be eliminated by reducing the time constant $R_{gl}C_g$. In practice this means reducing the grid-condenser capacity, since the grid-leak resistance is fixed by the bias requirements.

The tendency for interrupted oscillations to be produced is greatest when the exciting voltage and grid bias are both large, corresponding to conditions with a small angle of plate-current flow and high efficiency.

Blocking.—The phenomenon of blocking appears as a sudden stoppage of oscillations, accompanied by a reversal of grid current, and an increase of plate current to a

[1] Actually, the tendency for oscillations to start will be increased slightly if a few volts of negative bias is applied to the control grid, since in this way the grid current will be initially zero and less energy will be required to sustain a small amplitude of oscillations than when the bias is zero.

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value much higher than can be obtained with the full direct-current supply voltage and zero grid potential. A high-power tube is usually destroyed by blocking, since the energy dissipated at the plate is enormous. Blocking is caused by operating conditions that permit secondary electron emission to take place at the grid to such an extent that the grid loses more electrons by secondary emission than it gains from the cathode by direct flow. This causes a reversal of the grid current, resulting in the development of a positive grid-bias voltage by the grid leak, and consequently an excessive plate current flow. In order that blocking may exist, it is necessary that the minimum instantaneous plate voltage and the maximum instantaneous grid potential obtained during the cycle both be high and that the grid leak have a high resistance.

Blocking, when it occurs, is the result of attempting to force the output of the oscillator by increasing the load resistance coupled into the plate circuit. This results in a reduction in the amplitude of oscillations, which increases the minimum plate voltage. If the grid excitation is then increased, the secondary electron emission at the grid will be increased because of the increased positive potential reached by the grid. Under unfavorable conditions this will cause the net grid current to become less, which reduces the grid bias and makes the maximum positive grid potential still greater, causing a further reduction in the grid current, etc.

In order for blocking to occur, the grid must emit secondary electrons. In modern air-cooled tubes the grid structures have usually been so treated that the grid current will never reverse under conditions encountered in ordinary operation, so that blocking cannot occur in these tubes. However, with water-cooled tubes, or where very high electrode voltages are employed, it is not always possible to construct the tube in such a way that negative grid current can be avoided. Such tubes are therefore susceptible to blocking, and either must be operated with considerable care or must be provided with a grid rectifier similar to that shown in Fig. 21, which has the resistance R adjusted so that the rectified current always exceeds the most negative grid current obtainable. In this way the current through the grid leak can never reverse, and all possibility of blocking is avoided.

Wave Shape.—The output of a power oscillator contains harmonics as a result of the fact that the plate and grid currents of the tube flow in the form of pulses. The factors controlling the amplitude of the harmonic voltages developed in the load are the same as in the Class C amplifier. In general it may be said that the harmonics will be reduced if the angle of plate current flow is large (corresponding to low efficiency) and if the effective Q of the tank circuit is large, corresponding to low L/C.

Tubes.—The same tubes are used for power oscillators as for Class C amplifiers.

FREQUENCY STABILITY OF OSCILLATORS—CRYSTAL OSCILLATORS

3. Frequency and Frequency Stability of Ordinary Oscillators.—The frequency generated by a vacuum-tube oscillator adjusts itself to a value such that the voltage that the oscillations apply to the grid of the tube is of exactly the proper phase to produce the oscillations that supply this grid-exciting voltage. This frequency approximates the resonant frequency of the tank circuit,[1] but is also influenced by such factors as the voltages acting on the tube, the effective Q of the tank circuit, the harmonics generated, the resistance and reactance coupled into the tank circuit by the load, etc. All these modifying factors tend to produce small phase shifts between the exciting voltage and the output voltage of the tube, and in order to compensate for these, the oscillator must operate slightly off the resonant frequency of the tank circuit in order to introduce a compensating phase shift.

[1] When there is more than one resonant circuit, as in the tuned-grid tuned-plate arrangement, the frequency tends to be controlled by the circuit in which the circulating reactive volt-amperes are greatest.

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Tank-circuit Design for Stable Frequency Operation—Resonant Line Oscillators.—The resonant frequency and the effective Q of the tank circuit are the two most important factors affecting the frequency generated by an oscillator. The generated frequency will vary with the resonant frequency of the tank circuit, while the effect of the tube voltages, load impedance, etc., on the frequency are inversely proportional to the effective Q of the tank circuit.

The resonant frequency of the tank circuit will vary as a result of the temperature coefficient of the inductance and capacity. Included in the temperature effect is the capacity of the tubes that are associated with the resonant circuit and so affect the frequency. In considering the effect of temperature, one must include not only changes in ambient temperature but also the heating of the inductance, capacity, and tubes that occurs during normal operation.

The effect on frequency of unavoidable variations in tube capacity can be minimized by arranging the oscillator circuits so that the electrostatic energy stored in the tube capacities is as small as possible compared with the total electrostatic energy stored in the resonant circuit.[1] This means a high circuit Q, the highest possible ratio of transconductance to changes in tube capacities, and the smallest possible coupling from the tank circuit to the grid and plate electrodes of the tube that will permit oscillations to be maintained. In practical oscillators the ratio of electrostatic energy in the resonant circuit to the electrostatic energy stored in the tube capacities can be made greater the lower the resonant frequency. As a result, an oscillator directly generating the desired frequency can be expected to have poorer frequency stability with respect to temperature changes in the tube, and to tube replacements, than a combination consisting of a harmonic generator and an oscillator operating at a fraction of the desired frequency.

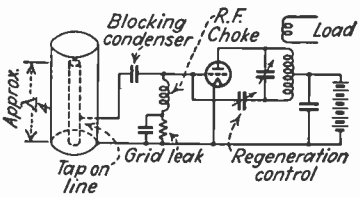


FIG. 3.—Tuned-grid tuned-plate oscillator using resonant line for the tuned-grid circuit.

The effective Q of the tank circuit is important because the percentage change in frequency required to produce a given compensating phase shift between input and output voltages of the tube is inversely proportional to the effective Q. A high effective Q corresponds to a load not too closely coupled, and either to a low L/C ratio in the tank circuit, or an arrangement in which the plate is coupled to only a small part of the tank circuit so that the voltage across the circuit is much greater than the plate-cathode voltage.[2]

High tank-circuit efficiency with high effective Q requires a low loss (high Q) resonant circuit. At frequencies greater than 15 mc to 20 mc, this can be most conveniently realized by the use of resonant lines for the tank circuit.[3] Such lines have high Q (see Par. 16, Sec. 3), are simple to construct, and are mechanically stable. They are also able to dissipate their losses with only a small temperature rise, and where desired can be compensated for temperature changes. An example of a resonant-line oscillator is shown in Fig. 3. This is a tuned-grid tuned-plate arrangement with the line supplying the grid tuned circuit. High frequency stability is achieved by designing the line to have high Q and then connecting the grid so that the section of the line

[1] Walter Van B. Roberts, The Limits of Inherent Frequency Stability, *R.C.A. Rev.*, Vol. 4, p. 478, April, 1940.

[2] G. F. Lampkin, An Improvement in Constant Frequency Oscillators, *Proc. I.R.E.*, Vol. 27, p. 199, March, 1939.

[3] J. W. Conklin, J. L. Finch, and C. W. Hansell, New Methods of Frequency Control Employing Long Lines, *Proc. I.R.E.*, Vol. 19, p. 1918, November, 1931; C. W. Hansell, Resonant Lines for Frequency Control, *Elec. Eng.*, Vol. 54, p. 852, August, 1935; Clarence W. Hansell and Philip S. Carter, Frequency Control by Low Power Factor Line Circuits, *Proc. I.R.E.*, Vol. 24, p. 597, April, 1936.

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between grid and cathode is only a small part of the total, so that the reactive energy stored in the line will be large.

Effect of Load on Frequency—Master-oscillator and Electron-coupled Arrangements.—The load affects the frequency of an oscillator as the result of the resistance and reactance coupled into the tank circuit. For maximum stability the load should be a resistance, and the coupling to the tank circuit small, so that the oscillator operates only lightly loaded.

The influence of the load on frequency can be minimized by using the oscillator to excite a power amplifier that develops the power required by the load. Such an arrangement is termed a *master-oscillator power-amplifier* system (abbreviated MOPA) and is most effective when the power amplifier consists of several stages employing pentodes, screen-grid tubes, or neutralized triodes. The stage following the master oscillator is then operated as a "buffer amplifier," *i.e.*, with relatively low grid current, so that the loading on the master-oscillator stage is a minimum. Changes in the grid current of the buffer amplifier such as might be produced by variations in the plate-circuit conditions in the amplifier then have minimum effect on the frequency.

The electron-coupled oscillator is equivalent to an oscillator and power amplifier combined in one tube, and can be made to give a frequency practically independent of

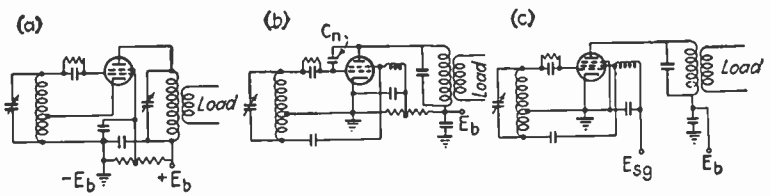


FIG. 4.—Electron-coupled oscillator circuits.

the load impedance that receives the output. To achieve this result, the minimum instantaneous plate potential must be great enough so that there is no tendency for a virtual cathode to form in the tube and return electrons to the screen and cathode, or for secondary electrons to be lost by the plate. This means that the load impedance must not be too great. Further, in the case of screen-grid tubes, the screen grid must be operated at ground potential as at Fig. 4*a*, or the plate-screen capacity must be neutralized as in Fig. 4*b*, in order to prevent capacitive coupling from the plate or output portion to the oscillator section of the tube.

Electron-coupled oscillators using screen-grid tubes generate a frequency practically independent of plate-supply voltage, provided that the ratio of screen to plate voltage is maintained at a suitable value. This desirable result can be obtained by deriving the screen voltage from a voltage divider connected across the plate voltage as in Fig. 4 and adjusting the screen tap by trial until the frequency does not change with a small variation in supply voltage.

Effect of Tube Voltages on Frequency.—The plate and filament voltages acting on the tube affect the frequency by influencing the alternating currents that flow between the tank circuit and the grid and plate electrodes of the tube. This can be minimized by using tank circuits having a high effective Q (low L/C), by the use of grid-leak bias, particularly a high-resistance grid leak, and by employing the closest possible coupling between the different parts of tapped coils, such as are employed in the Hartley circuit, or between the separate coils used in arrangements such as the reversed feedback.

The frequency can be made substantially independent of plate and filament voltages by inserting suitable reactances in series with the leads going to the plate and grid

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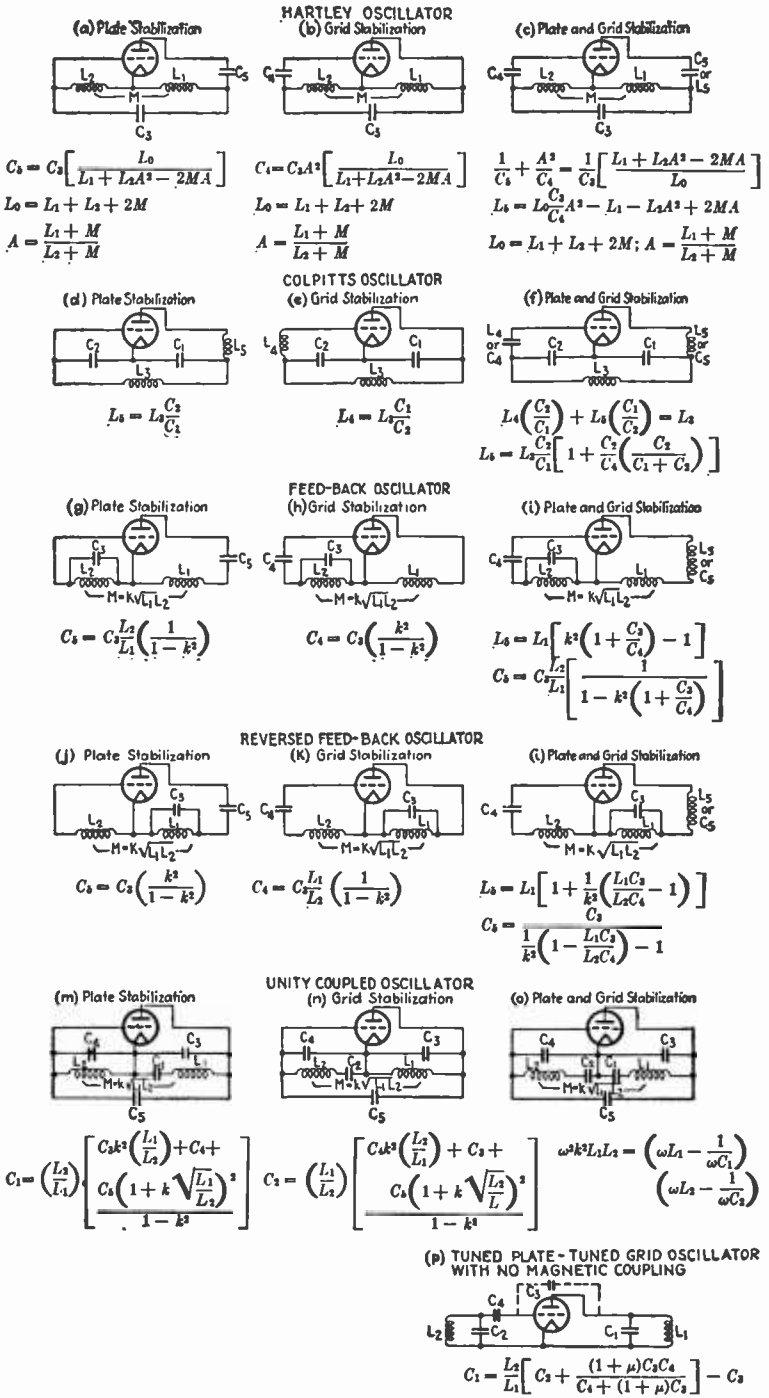


FIG. 5.—Circuits and circuit proportions that will make the frequency independent of tube constants, provided that the Q of the resonant circuit is very high (theoretically infinite).

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electrodes.[1] This is shown in Fig. 5, which also gives the formulas for calculating the required reactance in the ideal case of infinite effective Q. Under actual conditions these idealized formulas give only approximate results and the required reactance must be determined by trial. Examination of the circuits of Fig. 5 shows that the grid condenser associated with the grid-leak resistance, or the plate blocking condenser used in shunt-feed arrangements, can be used to supply the required reactance. Thus the frequency of any ordinary oscillating circuit can be made substantially independent of plate and filament voltage for one frequency by a proper choice of these condensers.

When circuits are compensated in the manner illustrated in Fig. 5, the frequency will also be independent of the load, provided that the load is a resistance and is connected either from grid to cathode or from plate to cathode.

The frequency can also be made independent of electrode voltages by means shown in Fig. 6.[2] Here the tap a on the inductance L divides the tank-circuit voltage in the same ratio as do the two condensers C_1 and C_2. A resistance (or coil) is connected between the junction of the condensers and the point a as shown, and the cathode return brought to a point on this resistance that trial shows gives the best frequency stability.

Effect of Harmonics on Frequency Stability.—Harmonic voltages in the circuits of an oscillator adversely affect the frequency stability. This arises from the fact that the harmonics cross-modulate with each other and with the fundamental to produce fundamental frequency currents that are not necessarily in phase with the fundamental frequency currents arising from the normal mode of operation. The phase of the resultant fundamental frequency current affects the frequency of operation, which accordingly becomes sensitive to the amount and distribution of the harmonics in the circuits. The effect of harmonic voltages on frequency stability can be minimized by employing a tank circuit having high effective Q, so that the impedance to the harmonics will be a minimum, and by selecting operating conditions that keep the amplitude of the harmonic voltages and currents at a minimum.

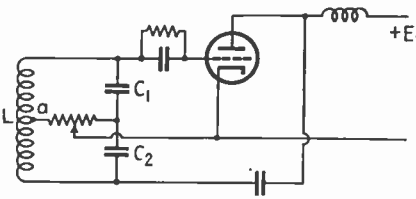


FIG. 6.—Method of improving frequency stability of an oscillator.

4. Crystal Oscillators.[3]—The frequency stability of an oscillator can be made very high by replacing the usual resonant circuit that controls the frequency by a mechanically vibrating piezoelectric quartz crystal and utilizing the piezoelectric effect to give energy transfer between the electric circuits and the mechanical vibrations.

Properties of Piezoelectric Quartz Crystals.—A piezoelectric quartz crystal in its natural state has a hexagonal cross section and pointed ends, as illustrated in Fig. 7.

[1] F. B. Llewellyn, Constant Frequency Oscillators, *Proc. I.R.E.*, Vol. 19, p. 2063, December, 1931; G. H. Stevenson, Stabilized Feedback Oscillators, *Bell System Tech. Jour.*, Vol. 17, p. 458, July, 1938.

[2] Y. Kusunose and S. Ishikawa, Frequency Stabilization of Radio Transmitters, *Proc. I.R.E.*, Vol. 20, p. 310, February, 1932.

[3] Following is a small bibliography on crystal oscillators selected from the very extensive literature on the subject: W. G. Cady, The Piezo-electric Resonator, *Proc. I.R.E.*, Vol. 10, p. 83, April, 1922; G. W. Pierce, Piezo-electric Crystal Resonators and Crystal Oscillators Applied to the Precision Calibration of Wavemeters, *Proc. Am. Acad.*, Vol. 59, p. 81, 1923; W. G. Cady, Bibliography on Piezoelectricity, *Proc. I.R.E.*, Vol. 16, p. 521, April, 1928, F. R. Lack, Observations on Modes of Vibration and Temperature Coefficient of Quartz Crystal Plates, *Proc. I.R.E.*, Vol. 17, p. 1123, July, 1929, F. R. Lack, G. W. Willard, and I. E. Fair, Some Improvements in Quartz Crystals Circuit Elements, *Bell System Tech. Jour.*, Vol. 13, p. 453, July, 1934; W. P. Mason, Electrical Wave Filters Employing Quartz Crystals as Elements, *Bell System Tech. Jour.*, Vol. 13, p. 405, July, 1934; W. P. Mason, Low Temperature Coefficient Quartz Crystals, *Bell System Tech. Jour.*, Vol. 19, p. 74, January, 1940; Isaac Koga, Notes on Piezoelectric Quartz Crystals, *Proc. I.R.E.*, Vol. 24, p. 510, March, 1936; C. F. Booth, The Application and Use of Quartz Crystals in Telecommunications, *Jour. I.E.E.*, Vol. 88, Part III, p. 97, June, 1941.

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The properties of such a crystal can be conveniently referred to three sets of axes as shown in the figure. The first of these, designated as the optical or Z axis, passes through the apexes of the crystal. The second set, termed the electrical or X axes, consists of the three axes that lie in a plane perpendicular to the optical axis and pass through the corners of the crystal. The third set, termed the mechanical or Y axes, consists of three axes that lie in the same plane as the X axes, but are perpendicular to the faces of the crystal. The exact location of the axes can be determined even when the crystal is imperfectly formed, or when a complete crystal is not available, by using a polariscope or an X-ray spectrometer.

When an electric stress is applied to a quartz crystal in the direction of the X axis, a mechanical stress is produced in the direction of the Y axis at right angles. Conversely, a mechanical stress along the Y axis will cause electrical charges to appear on the faces of the crystal perpendicular to the X axis that goes at right angles to the Y axis involved. The polarity of the electric stress and the direction of the corresponding mechanical force are directly related, a reversal in one causing a reversal in the other. This relationship between electrical stress and mechanical force is termed the *piezoelectric effect*, and provides a means of relating mechanical vibrations to electrical circuits. The piezoelectric effect is present to some extent with electrical stress applied in any direction, since there will nearly always be a component of the stress parallel to an X axis. Similarly, any mechanical force will usually produce some mechanical stress in the direction of a Y axis.

An alternating voltage applied across a quartz crystal will cause the crystal to vibrate, and if the frequency of the applied alternating voltage approximates a frequency at which mechanical resonance can exist in the crystal, the amplitude of vibrations will be very large. Any crystal has a number of such resonant frequencies that depend upon the crystal dimensions, the type of oscillation involved, and the orientation of the plate cut from the natural crystal. Various modes of oscillation are possible in crystal resonators. Those most commonly used are simple longitudinal and shear oscillations, but it is also possible to generate flexural and torsional vibrations and various harmonic modes.[1]

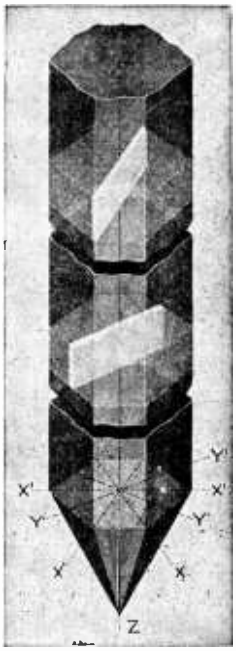


FIG. 7.—Illustration showing the natural quartz crystal and the relation of the electric or X, the mechanical or Y, and the optical or Z axes to the crystal structure. The upper section shows a Y (or 30°) cut plate, while the plate in the center section is X- (or Curie-) cut. The third Y axis, $Y''Y''$, is not shown because the perspective of the drawing makes it coincide with the ZZ axis.

The desirable properties for a piezoelectric resonator to possess are a low temperature coefficient of resonant frequency, a high piezoelectric activity, and a frequency spectrum containing only one resonant frequency in the vicinity of the desired oscilla-

[1] Some examples of crystal oscillators employing such special modes of vibration are given by J. R. Harrison, Piezo-electric Resonance and Oscillatory Phenomena with Flexural Vibrations in Quartz Plates, *Proc. I.R.E.*, Vol. 15, p. 1040, December, 1927; August Hund and R. B. Wright, New Piezo Oscillations with Quartz Cylinders Cut Along the Optical Axis, *Jour. Research*, Vol. 4, p. 383, March, 1930; *Proc. I.R.E.*, Vol. 18, p. 741, May, 1930; N. H. Williams, Modes of Vibration of Piezo-electric Crystals, *Proc. I.R.E.*, Vol. 21, p. 990, July, 1933; Harold Osterberg and John W. Cookson, Piezoelectric Stabilization of High Frequencies, *Rev. Sci. Instruments*, Vol. 5, p. 281, August, 1934; Tsi-Ze Ny, Long-Chao Tsien, and Sun-Hung Fang, Oscillations of Hollow Quartz Cylinders Cut Along the Optic Axis. *Proc. I.R.E.*, Vol. 24, p. 1484, November, 1936.

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tion. Temperature can alter the frequency of mechanical resonance through its effects on the density, the linear dimensions, and the moduli of elasticity of the crystal. Inasmuch as some of the elastic constants of a crystal are positive, while others are negative, the temperature coefficient of frequency may be either positive or negative or zero, according to the mode of oscillation, the orientation of the crystal plate, and the shape of the plate.

The term activity applied to a quartz crystal indicates the ease with which the desired oscillations can be excited, and is affected by the orientation of the crystal plate with respect to the axes. The activity will be high when the orientation is such that an electric stress applied across the faces of the crystal will produce a large mechanical stress of the type required to maintain the oscillations.

The resonance properties of a vibrating crystal are determined by the crystal dimensions, orientation, and six elastic constants, and can be calculated for those shapes that are sufficiently simple to permit of mathematical solution. The temperature coefficient of frequency can also be calculated from the crystal orientation and the temperature coefficient of the elastic constants, and conditions giving low or zero temperature coefficients thereby predicted.

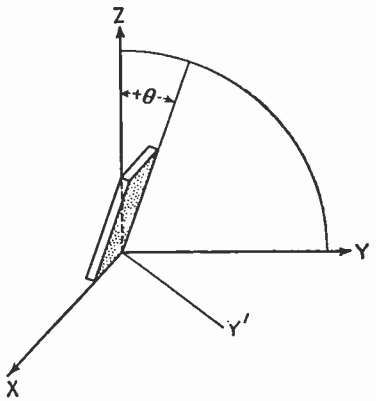


FIG. 8.—Diagram showing how crystal plane is rotated about the X axis to obtain the AT cut.

Common Crystal Cuts.—An infinite variety of shapes and orientations of crystals can be used in crystal oscillators. The crystals used in practice are, however, practically always in the form of plates that are either circular or approximately square if the frequency is high, or sometimes in the form of bars if the frequency is low.

The commonly used orientations of plates with respect to the crystal axes are designated as AT, CT, GT, V, X, Y, etc., cuts.

The AT-cut crystal[1] is extensively used for frequencies ranging from about 500 kc upward to about 10 mc. This type of plate is cut from a plane that is rotated about an X axis so that the angle θ made with the Z axis, as indicated in Fig. 8, is approximately 35.5°. When voltage is applied across the flat sides of such a plate, shear vibrations are set up, causing displacements indicated by Fig. 9*a*. The resonant frequency is related to the thickness of the crystal by the equation

$$\text{Frequency, kc, for } AT \text{ cut} = \frac{1{,}675}{\text{thickness, mm}} \tag{1}$$

The AT-cut crystal is characterized by substantially zero temperature coefficient at a temperature determined by the exact value of θ, and has a high piezoelectric activity. Furthermore, the elastic coupling between the desired mode of oscillation and harmonics of the width vibration is almost zero, so that the frequency spectrum is very simple. This is in contrast with the complicated frequency spectrum of Y-cut plates.

The CT-cut plate is suitable for operation in the frequency range of 50 to 100 kc.[2] It is essentially an AT-cut plate operating in a mode such that vibrations cause displacements as shown in Fig. 9*b*. The frequency of vibration is accordingly determined

[1] See Lack, Willard, and Fair, *loc. cit.*

[2] S. C. Hight and G. W. Willard, A Simplified Circuit for Frequency Substandards Employing a New Type of Low-frequency Zero-temperature-coefficient Quartz Crystal, *Proc. I.R.E.*, Vol. 25, p. 549, May, 1937.

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by the large dimension of the plate, and, in the case of plates, x mm square is given by the equation

$$\text{Frequency, kc, for } CT\text{-cut plates} = \frac{3{,}070}{x} \tag{2}$$

When the plates are square and are cut so that the angle θ in Fig. 8 is approximately 37.5°, the temperature coefficient of vibration will be zero at a temperature approximating room temperature. The CT plate has the very desirable feature that its

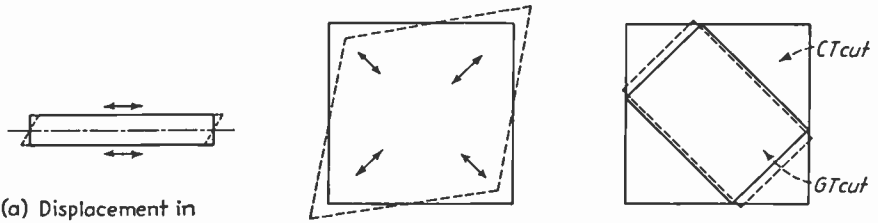


FIG. 9.—Displacements produced by the vibrations in various crystal cuts.

frequency can be decreased slightly by grinding in the center of the plate and increased slightly by grinding near the edges, as indicated by D and I, respectively, in Fig. 10*a*. Also, the temperature coefficient can be adjusted either upward or downward by grinding on edges P or N, respectively, as indicated in Fig. 10*b*.

The AT- and CT-cut plates are particular cases of a general class of shear vibrations having zero temperature coefficient, and obtained by rotating the plane of the plate appropriately about two axes.[1] Most of these zero temperature cuts are not particularly useful, however, either because of low activity or because of a complicated

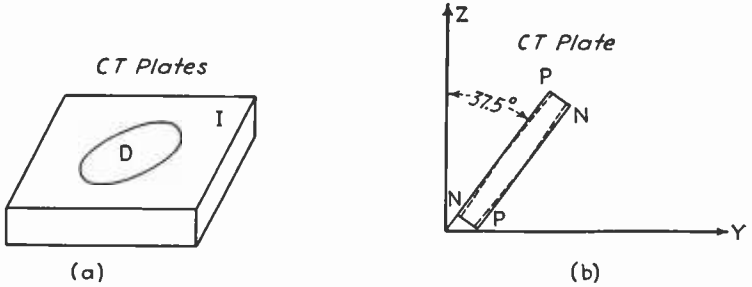


FIG. 10.—Diagram illustrating how the temperature coefficient and resonant frequency of CT plates may be controlled by edge grinding.

frequency spectrum. An exception to this is the V-cut crystal, which has high activity combined with low temperature coefficient.

The frequency of most low-temperature-coefficient cuts depends upon the temperature in the manner illustrated by the AT and CT plates in Fig. 11. The temperature coefficient of frequency is zero only in the vicinity of a particular temperature that is determined by the exact orientation of the plate. An exception

[1] See W. P. Mason, Low Temperature Coefficient Quartz Crystals, *Bell System Tech. Jour.*, Vol. 19, p. 74, January, 1940.

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is the *GT* cut, for which the temperature coefficient of frequency is zero over a wide temperature range, as illustrated in Fig. 11.

The *GT* plate[1] is a modified *CT* type in which the plane of the crystal in Fig. 9 corresponds to $\theta = 51.5°$ instead of 37.5°. The crystal is rectangular instead of square with a ratio of width to length of 0.855. The length is in the direction of the diagonal of the *CT* plate, as indicated in Fig. 9*c*. Such a crystal has two principal modes of vibration, both of which are longitudinal, as indicated in Fig. 9*c*, and correspond to resonant frequencies determined by the length and width, respectively. The *GT* crystal uses the width vibration and has a temperature coefficient that depends upon the angle θ of the crystal plane and the ratio of width to length. For each value of θ over a wide range, there is a particular ratio of width to length at which the temperature coefficient at a given temperature is zero. For the particular value of θ corresponding to the *GT* cut, this coefficient is substantially zero over a wide temperature range, as indicated in Fig. 11. The frequency of a *GT* plate is determined primarily

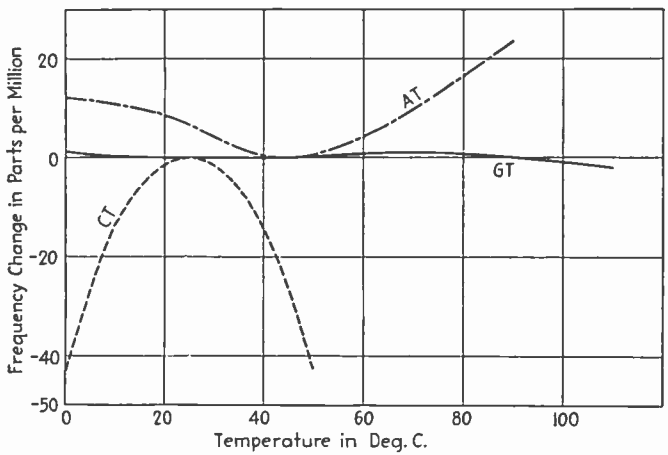


FIG. 11.—Variation of resonant frequency with temperature with various types of crystals.

by the width, while the temperature coefficient is controlled by the ratio of width to length. The frequency and temperature coefficient can hence be adjusted independently.

Y-cut crystal plates[2] were once widely used, but have now largely been displaced by other cuts having superior characteristics. The orientation of a *Y*-cut plate is shown in Fig. 7. Such a plate has one resonant frequency determined by the thickness, and involves the same shear mode of vibration used in *AT*-cut plates. This resonant frequency is given approximately by the equation

$$\left.\begin{matrix}\text{Thickness frequency, kc} \\ \text{of } Y\text{-cut crystal}\end{matrix}\right\} = \frac{1{,}960}{\text{thickness, mm}} \tag{3}$$

Y-cut plates also possess a low-frequency resonance corresponding to a single longitudinal (or width) vibration, having the frequency

[1] W. P. Mason, A New Quartz Crystal Plate, Designated the GT, Which Produces a Very Constant Frequency over a Wide Temperature Range, *Proc. I.R.E.*, Vol. 28, p. 220, May, 1940.

[2] A good discussion of *X*- and *Y*-cut plates is given by Lack, *loc. cit.* Also see Geoffrey Builder and J. E. Benson, Contour Mode Vibrations in *Y*-cut Quartz-crystal Plates, *Proc. I.R.E.*, Vol. 29, p. 182, April, 1941.

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$$\left.\begin{matrix}\text{Width frequency,}\\ \text{kc, of } Y\text{-cut crystal}\end{matrix}\right\} = \frac{2{,}860}{\text{width, mm, along electric axis}} \qquad (4)$$

Y-cut plates operating at the thickness frequency possess a very complicated frequency spectrum as a result of elastic couplings to harmonics of the width vibration. In a limited frequency range, there are commonly a series of resonant frequencies, as illustrated in Fig. 12. These are affected by the width of the crystal, and although they can be changed by edge grinding to change the width, such grinding merely causes a new series of resonant frequencies to move in to replace those thereby moved out. Thickness vibrations of Y-cut crystals are characterized by a discontinuous temperature-frequency characteristic, as illustrated in Fig. 13. This is a result of elastic couplings to harmonics of the width vibration, and arises from the fact that the width vibration has a negative temperature coefficient of approximately -20 parts in a million per°C, whereas a pure thickness vibration has a positive coefficient. The resultant temperature coefficient of the thickness vibration may vary between -20 and $+100$ parts in a million per°C, with the exact value depending upon the temperature and the ratio of width to thickness. It will be noted from Fig. 13 that in limited temperature ranges the temperature coefficient may be zero. This was the earliest method discovered for obtaining a temperature coefficient of zero.[1]

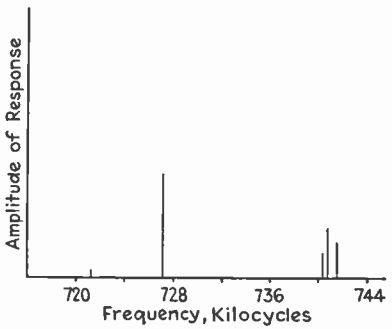


FIG. 12.—Typical frequency spectrum of thin quartz plate showing how several closely spaced resonant frequencies sometimes occur. The length of each vertical line indicates in a rough way the relative tendency to oscillate at that frequency.

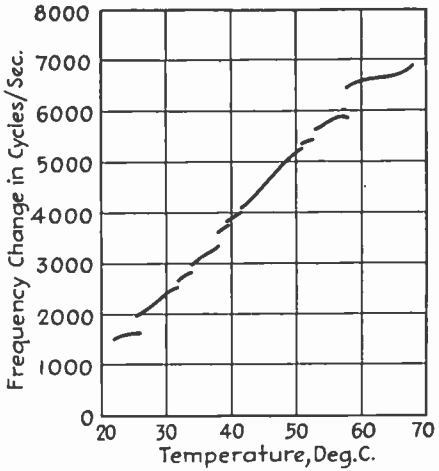


FIG. 13.—Temperature-frequency characteristic of a Y plate showing the discontinuities in frequency and the dependence of temperature coefficient upon the exact temperature.

The X-cut crystal plate is oriented in relation to the crystal axis as shown in Fig. 7. When voltage is applied across the two faces of the crystal, it vibrates with simple longitudinal vibrations, with two possible resonant frequencies corresponding to the thickness of the crystal, or the width in the direction of the Y axis, according to the relation

$$\left.\begin{matrix}\text{Resonant frequency,}\\ \text{kc, of } X\text{-cut plate}\end{matrix}\right\} = \frac{2{,}860}{\text{thickness or width, mm}} \qquad (5)$$

The X-cut crystal tends to have a complicated frequency spectrum similar to that obtained with Y cut, and is not quite so active as the Y or AT cuts. The temperature coefficient under ordinary conditions is approximately -20 parts in a million per°C. However, if the width vibration is used, and the length in the direction of the Z axis is very small compared

[1] W. A. Marrison, A High Precision Standard of Frequency, *Proc. I.R.E.*, Vol. 17, p. 1103, July, 1929.

Ex. 1044-0501

with the width, then the temperature coefficient of frequency is small, and will be zero when the bar is square and has sides that are 0.272 times the width along the mechanical axis.

Equivalent Electric Circuit of Crystal Oscillator.[1]—As far as an electrical circuit associated with a vibrating crystal is concerned, the crystal can be replaced by the electrical network of Fig. 14*b*, in which C_1 represents the electrostatic capacity between the crystal electrodes when the crystal is in place but not vibrating, and the series combination L, C, and R represents the equivalent mass, compliance, and frictional loss of the vibrating crystal, respectively. The impedance offered by the crystal to the electrical circuits accordingly is of the character shown in Fig. 14*c*, being large at the resonant frequency of L and $C + C_1$ and low at a near-by frequency for which L and C are in series resonance. The behavior of electrical circuits containing a crystal can be analyzed by replacing the crystal by its equivalent electrical network and then determining the behavior of the resulting system.

The magnitudes of L, C, and C_1, used to represent the crystal in Fig. 14, depend upon the type of vibration involved, the dimensions of the crystal, and the orientation

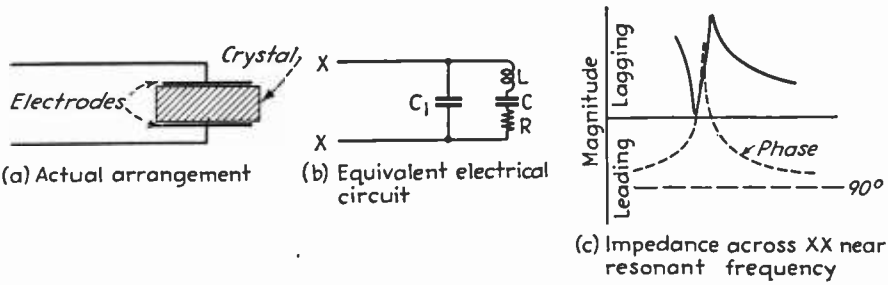


FIG. 14.—Circuit of crystal oscillator together with equivalent circuit in which the crystal is represented by an electric network.

of the crystal plate with respect to the axes. Numerical values for an X-cut plate can be calculated from the equations

$$C = 0.0029 \frac{lw}{t} \mu\mu f \tag{6a}$$

$$C_1 = 0.40 \frac{lw}{t} \mu\mu f \tag{6b}$$

$$L = 118 \frac{t^3}{lw} \text{ henrys (thickness vibration)} \tag{6c}$$

$$= 118 \frac{wl}{l} \text{ henrys (width vibration)} \tag{6d}$$

where w, l, and t are the dimensions in centimeters measured in the direction of the Y axis parallel to the surface of the crystal, in the direction of the X axis perpendicular to this Y axis, and in the direction of the Z axis, respectively. Typical numerical results are shown in Table 1. It will be noted that since the ratio C/C_1 is very much less than unity, the coupling between the crystal and an external electrical circuit associated with the crystal is quite small. The equivalent Q of a crystal resonator is affected by the method of mounting, etc., and is generally very high. Thus etched

[1] K. S. Van Dyke, The Piezo-electric Resonator and Its Equivalent Network, *Proc. I.R.E.*, Vol. 16, p. 742, June, 1928; D. W. Dye, Piezo Electric Quartz Resonator and Its Equivalent Electrical Circuit, *Proc. Phys. Soc. (London)*, Vol. 38, p. 399, 1926; P. Vigoreaux, *Phil. Mag.*, p. 1140, December, 1928; W. P. Mason, An Electromechanical Representation of a Piezo-electric Crystal Used as a Transducer, *Proc. I.R.E.*, Vol. 23, p. 1252, October, 1935.

Ex. 1044-0502

crystals with plated electrodes, mounted in a vacuum, have been found to give Q's of the order of 500,000.[1]

Crystal Oscillator Circuits.—A quartz crystal can be used to control the frequency of an oscillator by utilizing the crystal to provide the resonant circuit that determines the oscillator frequency. The commonest such circuit arrangement employed with crystals is that shown in Fig. 15*a*. This is equivalent to a tuned-grid tuned-plate circuit, with the crystal supplying the grid tuned circuit. To obtain oscillations, the resonant circuit in the plate is tuned to a frequency higher than that of the crystal, so that the reactance in the plate circuit is inductive at the crystal resonance. The amplitude of oscillation is determined by the amount of inductive reactance in the

TABLE 1.—CHARACTERISTICS OF TYPICAL QUARTZ CRYSTAL

Dimensions:	
Thickness	0.636 cm
Width	3.33 cm
Length	2.75 cm
Resonant frequency (thickness vibration)	430 kc
Equivalent electrical characteristics:	
L = 3.3 henrys	
C = 0.042$\mu\mu$f	
C_1 = 5.8$\mu\mu$f	
R = 4,500 ohms {very approx.}	
Q = 2,300 {very approx.}	

plate circuit, and by the grid-plate tube capacity. In some instances, such as with pentodes, or with crystals operating at low frequencies, it is desirable to supplement the grid-plate tube capacity by an auxiliary condenser C_2, as in Fig. 15*b*. The frequency of oscillation in Fig. 15*b* can be made insensitive to circuit adjustments and to the electrode voltages of the tube by proper choice of the capacities C_2 and C_p. The optimum conditions can be readily realized by first adjusting C_2 so that the generated frequency is maximum and then adjusting C_p for maximum generated frequency, after which the process is repeated in the same order to take care of slight interactions that may be present.[2]

In the circuits of Fig. 15*c* and 15*d*, the crystal is placed in the network that couples the output of the vacuum-tube amplifier to the input, and thereby controls the frequency through its effect on the magnitude and phase of the feedback.[3] In these arrangements, the resonant frequency approximates very closely the series resonant frequency of LC in the equivalent crystal circuit, and so is lower than in the arrangements of Figs. 15*a* and 15*b*.

The circuit of Fig. 15*e* utilizes the crystal as one arm of a bridge, in which the other arms are the resistances R_1, R_2, and R_3.[4] These resistances are arranged so that, together with the equivalent series resistance R of the crystal, the bridge would be in balance except for R_1. This last resistance is provided by a lamp in which the resistance of the filament is sensitive to temperature and hence to the current passing through the bridge. For proper operation, the value of R_1 with the lamp cold is lower than required to balance the bridge, while with the lamp at full brilliancy R_1 is higher than needed for balance. Output from the amplifier tube is applied across

[1] Karl S. Van Dyke, A Determination of Some of the Properties of the Piezo-electric Quartz Resonator, *Proc. I.R.E.*, Vol. 23, p. 386, April, 1935.

[2] See Hight and Willard, *loc. cit.*

[3] An analysis of the circuit of Fig. 15*d* is given by M. Boella, Performance of Piezo Oscillators and the Influence of Decrement of Quartz on the Frequency of Oscillation, *Proc. I.R.E.*, Vol. 19, p. 1252, July, 1931.

[4] See L. A. Meachem, The Bridge-Stabilized Oscillator, *Proc. I.R.E.*, Vol. 26, p. 1278, October, 1938.

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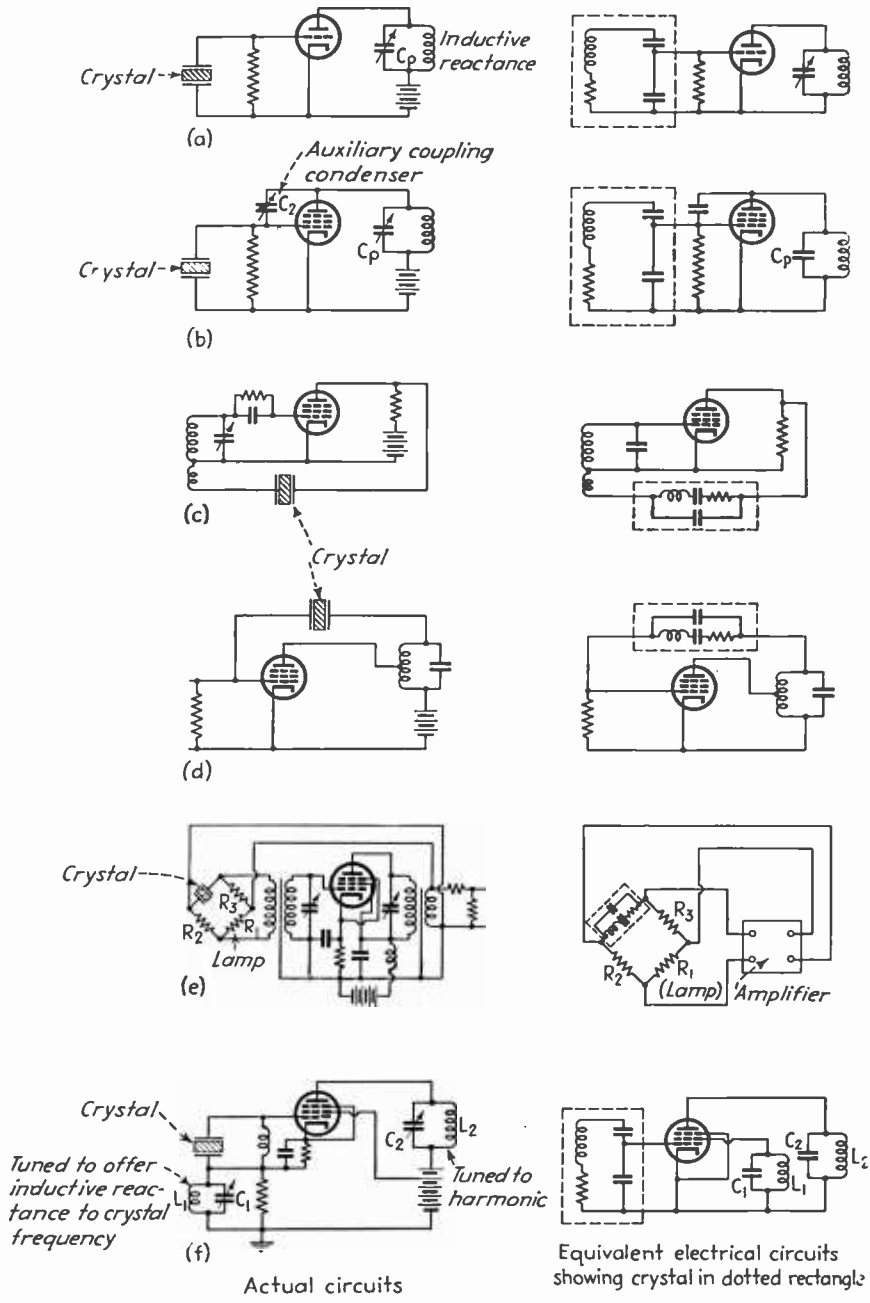


FIG. 15.—Common crystal circuits, together with the equivalent electrical circuits.

Ex. 1044-0504

one diagonal of the bridge, and input to the same amplifier is derived from the other diagonal. When plate voltage is first applied, the bridge is out of balance because R_1 is too low. Energy is then transferred from the plate of the tube, through the bridge, and back to the grid, thereby starting oscillations at the series resonant frequency of the crystal. As these oscillations build up in amplitude, the resulting current through R_1 increases its resistance, bringing the bridge more nearly in balance and reducing the tendency to oscillate until finally an equilibrium amplitude is reached with a current that causes the bridge to be almost but not quite balanced. If the amplification of the tubes is large, the generated frequency will be substantially independent of the tube constants, the voltages applied to the tube, or the electrical circuits outside the bridge.

One of the factors contributing to the very high frequency stability obtained with the circuit of Fig. 15*e* is that the equilibrium amplitude can be limited so that the tubes are able to function as Class A amplifiers. The frequency stability of any of the other oscillator circuits of Fig. 15 can likewise be increased by using some form of automatic amplitude control that limits the operation to small amplitudes, corresponding to Class A conditions. The disadvantage of doing this, however, is that the power output obtained from the crystal is then quite small.

A circuit combining a crystal oscillator and harmonic generator is shown in Fig. 15*f*. This is commonly used by amateurs, and is known as the tritet circuit. Here the control grid, screen grid, and cathode of a pentode or screen-grid tube function as a triode crystal oscillator with the screen serving as the plate, while the plate tank circuit is tuned to a harmonic. This arrangement is convenient where it is not necessary to attain the utmost in frequency stability.

The frequency of a crystal oscillator can be controlled to the extent of a few parts in a million without regrinding the crystal by adding inductance or capacity in series with the crystal in circuits such as Fig. 15*e*, where the crystal operates at its series resonant frequency, or in shunt in cases where the crystal operates at its parallel resonant frequency as in Fig. 15*a*. The extent of the control obtained in this way over frequency can be analyzed by replacing the crystal by its equivalent resonant circuit and then determining the resonant frequency of the combination of crystal and associated reactances.

Crystal Mountings.[1]—The crystals used in piezo electric oscillators are normally mounted between two electrodes to which the voltage developed by the vacuum tube is applied. In some instances these electrodes are deposited directly on the crystal faces by sputtering or by chemical means. More commonly, however, the electrodes are metal plates. The upper plate in some cases will be in contact with the crystal, either resting lightly or under spring pressure, while in other cases a clearance of a few thousandths of an inch is provided between the upper plate and the crystal. When an air gap is used, it is necessary either that it be small in order to avoid standing supersonic waves or that the entire mounting be evacuated.[2]

The method of holding the crystal in place depends upon the type of crystal, mode of vibration, and the kind of service. For ordinary commercial applications where the frequency stability must be good but ruggedness is also important, it is desirable to have a rigid mounting. This can be obtained with Y- and AT-cut crystals vibrating in their thickness mode by arranging the upper plate so that it touches the crystal only at a few points around the periphery, and then is held firmly against the

[1] O. M. Hovgaard, Application of Quartz Plates to Radio Transmitters, *Proc. I.R.E.*, Vol. 20, p. 767, May, 1932; Vincent E. Heaton and E. G. Lapham, Quartz Plate Mountings and Temperature Control for Piezo Oscillators, *Proc. I.R.E.*, Vol. 20, p. 261, February, 1932; R. C. Hitchcock, Mounting Quartz Oscillator Crystals, *Proc. I.R.E.*, Vol. 15, p. 902, November, 1927.

[2] August Hund, Notes on Quartz Plates, Air Gap Effect, and Audio-frequency Generation, *Proc. I.R.E.*, Vol. 16, p. 1072, August, 1928.

Ex. 1044-0505

crystal by a spring. Crystals vibrating at low frequencies, such as the *CT* and *GT* cuts, and *X*- and *Y*-cut plates using longitudinal (width) vibrations, possess nodal points at which rigid clamping is possible without introducing appreciable damping.

In primary and secondary standards it is usually preferable to arrange the mounting so that the crystal rests on the lower plate and does not touch the upper plate. This reduces the damping and avoids the possibility of irregularities. The air gap with such mountings is preferably controlled by the use of pyrex or other spacers having the same temperature coefficient of expansion as quartz. When a spaced upper plate is used, it is necessary to employ a retainer to hold the crystal in place, for otherwise it will creep. In some cases the crystal holder is sealed and evacuated.

When the utmost in frequency stability is required, it is customary to use a mounting provided with a heater and thermostat that maintain the crystal at approximately constant temperature. This is desirable even with crystals such as the *AT*- and *AC* cuts having a low temperature coefficient, even though under these conditions the temperature need not be regulated very closely. If high frequency stability is to be obtained with *X*- and *Y*-cut crystals, it is necessary that the design of the thermostat and heater be such as to maintain crystal temperature to 0.01°C or less.[1]

Performance of Crystal Oscillators.—Crystal oscillators are capable of operating in the frequency range 50 to 15,000 kc. The low-frequency limit arises from the difficulty of obtaining large quartz plates, while the high-frequency limit comes from the fact that if the plate is extremely thin it becomes too fragile to be practicable.

The power that can be controlled by a crystal oscillator is limited at low frequencies by the tendency of the crystal to crack from mechanical stress due to large amplitudes of vibration, and at high frequencies by the tendency of the crystal to overheat as a result of dissipation of radio-frequency energy in the crystal. In high-frequency work, the radio-frequency current passing through the crystal is commonly taken as a measure of the load on the crystal, with 100 ma generally considered as a reasonable upper limit. Practical crystal oscillators operating in the range 1.5 to 10 mc commonly develop 5 to 15 watts output. Although larger powers can be obtained, particularly if pentode tubes are used, this causes the frequency stability to be lowered and introduces danger of damaging the crystal. When frequency stability is very important, operation at still smaller outputs is desirable.

The frequency of a crystal oscillator such as is used in commercial broadcast stations will normally remain constant to at least 10 parts in a million over long periods of time, and not infrequently the variations will not exceed one part in a million. When the utmost care is taken to minimize every possible cause of frequency variation, stabilities of the order of one part in 10^7 to one part in 10^8 can be maintained over long time intervals.

MISCELLANEOUS TYPES OF OSCILLATORS AND OSCILLATIONS

5. Parasitic Oscillations.—The term *parasitic* is applied to undesired oscillations occurring in ordinary power amplifiers, oscillators, etc. Parasitic oscillations are very likely to occur when large tubes are employed because of the long leads, large interelectrode capacities, and relatively high values of transconductance involved. Parasitics cause reduction in the power at the desired mode of operation, introduce spurious frequencies, give rise to distortion in linear amplifiers and modulators, may produce spurious side bands, can cause flashovers, etc.

[1] The design of thermostats and heaters is discussed by James K. Clapp, Temperature Control for Frequency Standards, *Proc. I.R.E.*, Vol. 18, p. 2003, December, 1930; W. A. Marrison, Thermostat Design fdr Frequency Standard, *Proc. I.R.E.*, Vol. 16, p. 976, July, 1928; W. F. Diehl, A New Piezoelectric Quartz Crystal Holder with Thermal Compensation, *R.C.A. Rev.*, Vol. 1, p. 86, October, 1936.

Ex. 1044-0506

Examples of Parasitic Oscillations.[1]—Parasitic oscillations can be classed as high-frequency, *i.e.*, higher in frequency than the desired mode of operation, low-frequency, and dynatron types.

In the high-frequency parasitic oscillations the leads between the tube and tank circuit serve as the inductance for the parasitic circuits, and the interelectrode capacities of the tube, sometimes augmented by the neutralizing condensers, act as the

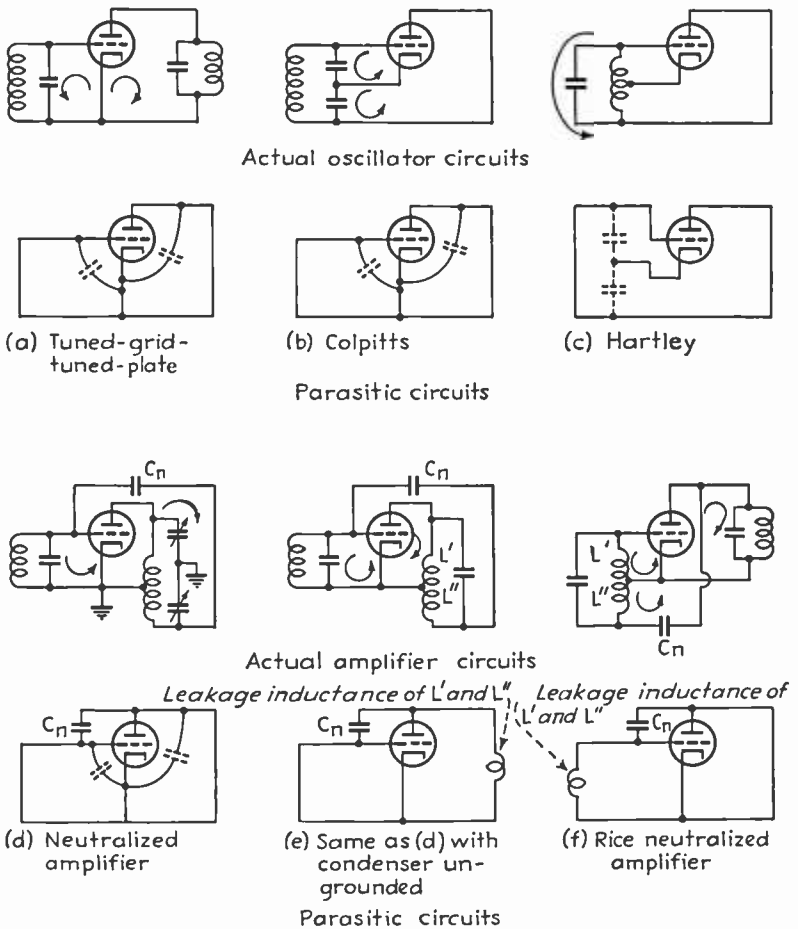


Fig. 16.—Actual oscillator and amplifier circuits, together with high-frequency parasitic circuits that can exist.

capacities. At these high frequencies the normal tank-circuit tuning capacity can be considered as a short circuit, while the tank-circuit inductance acts approximately as an open circuit.

Typical examples of such high-frequency parasitic oscillations are given in Fig. 16. Thus the tuned-grid tuned-plate oscillator circuit (which may be a crystal oscillator circuit) can have a high-frequency parasitic oscillation as shown, in which the induct-

[1] Additional examples of parasitics are given by G. W. Fyler, Parasites and Instability in Radio Transmitters, *Proc. I.R.E.*, Vol. 23, p. 985, September, 1935.

Ex. 1044-0507

ances of the tuned circuits are supplied by the one-turn loops from the tube electrodes through the tuning capacities back to the cathode. The Colpitts circuit can likewise produce a high-frequency parasitic of the tuned-grid tuned-plate variety, as shown, while the Hartley oscillator circuit can give parasitic operation by acting as a Colpitts circuit with the inductance supplied by the one turn lead from grid through the tuning condenser and back to the plate electrode.

An ordinary Class C amplifier will act as a tuned-grid tuned-plate oscillator circuit, as shown at Fig. 16*d*. It will be noted that the neutralizing condenser not only fails

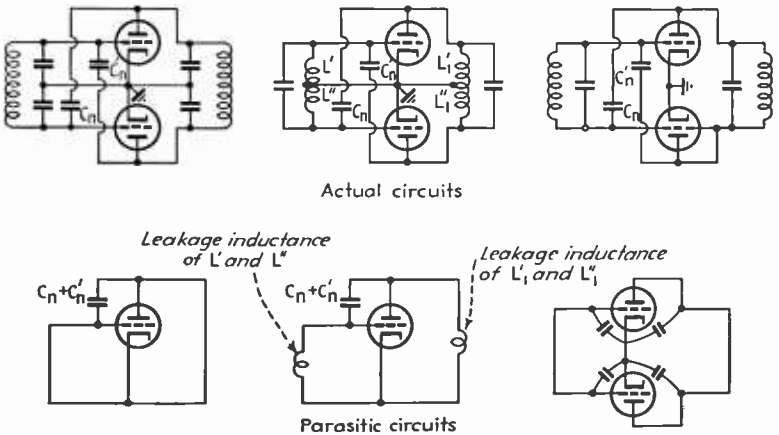


FIG. 17.—Push-pull amplifier circuits and high-frequency parasitic circuits that can exist.

to reduce the energy transfer through the grid-plate capacity of the tube but, as far as the parasitic oscillation is concerned, acts to increase the effective capacity between these electrodes. Even when a different system of neutralization is used, as, for example, in Fig. 16*f*, the circuit can still give a high-frequency parasitic by acting as a tuned-grid tuned-plate oscillator. A push-pull amplifier employing cross-neutralization, with either the coil or condenser center-grounded, has a tendency to develop oscillations of the tuned-grid tuned-plate type, as shown in Fig. 17, with the two tubes acting in parallel. Under these conditions, the neutralizing condensers do not prevent energy transfer from grid to plate, but actually act to increase this transfer. When neither coil nor condenser center is grounded, a high-frequency tuned-grid tuned-plate parasitic is possible unless the neutralization is maintained at the parasitic frequency. Similarly, a neutralized power amplifier employing tubes connected in parallel, as shown in Fig. 18, has a tendency to produce parasitic oscillations of the tuned-grid tuned-plate type, with the two tubes acting in push-pull.

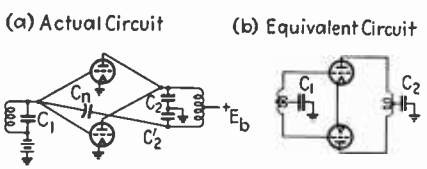


FIG. 18.—Amplifier with parallel tubes showing how the tubes may act in push-pull to produce a high-frequency parasitic.

Parasitic oscillations of relatively low frequency are caused by radio-frequency chokes. Examples are shown in Fig. 19. At *a*, it is seen that a push-pull power amplifier with grid and plate chokes can produce a low-frequency parasitic in which the grid and plate tuning inductances are the corresponding radio-frequency chokes and in which the tubes act in parallel in a tuned-grid tuned-plate circuit. It will be

Ex. 1044-0508

noted that the neutralization is not effective in reducing this type of parasitic. It is shown at *b* how a Colpitts oscillator with grid and plate chokes can have a low-frequency parasitic.

During the portion of the cycle when the grid of a power oscillator or amplifier is positive, the grid-cathode circuit of some tubes will display negative resistance characteristics as a result of secondary electron emission at the grid. During this portion of the cycle, the negative grid-cathode resistance shunts any parasitic resonant circuit existing between grid and ground, and may introduce high-frequency oscillations of the dynatron type.

Elimination of Parasitic Oscillations.—Parasitic oscillations can be expected as a matter of course in any new design of power amplifier or power oscillator involving large tubes. In the case of Class C amplifiers, the simplest method of investigating the presence of parasitic oscillations of all types except those produced by dynatron action is to remove the exciting voltage, make the grid bias small or even zero, and

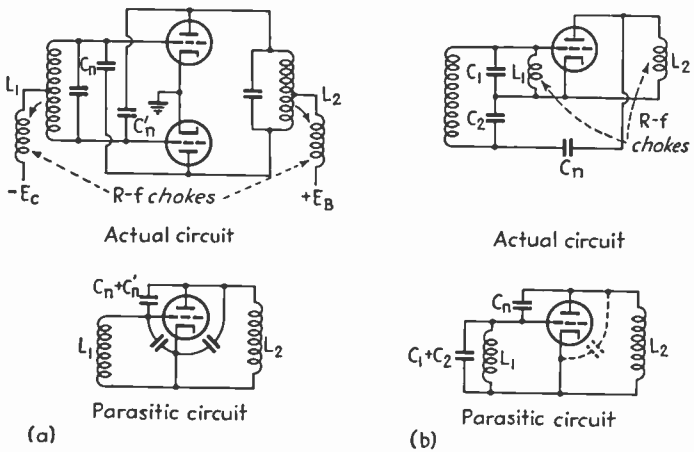


FIG. 19.—Amplifier circuits together with low-frequency parasitic circuits that can exist.

operate the tube at a lowered plate voltage so that the rated plate dissipation will not be exceeded. This ensures a high transconductance and so gives conditions favorable for the excitation of the parasitics, which can then be searched for by means of a neon tube on a stick or simply by drawing a spark to a wire on the end of a stick. If oscillations are present, their character is found by determining the points on the circuit that are at highest voltage and measuring the frequency. The equivalent circuit can then be deduced and remedial means devised. It is frequently found that upon elimination of one parasitic oscillation, another oscillation of a different type will appear, etc.

Various expedients are available for eliminating parasitics. Where the parasitics are of the tuned-grid tuned-plate type, the use of short grid leads and long plate leads, or the insertion of a small choke in the plate lead next to the tube, will cure the trouble. In this way the plate circuit is tuned to a lower resonant frequency than is the grid tank circuit, and oscillations cannot occur. The use of resistors connected in the grid and plate leads next to the tube is also effective, since these resistors are in series with the tank circuits of the parasitics. At the same time they are not in the tank circuit for normal operation, and so have little effect on the desired oscillations.

The use of neutralizing arrangements that maintain neutralization over wide frequency bands will eliminate certain types of parasitics. A schematic circuit of

Ex. 1044-0509

an ordinary neutralizing system showing how the inductances of the leads will prevent neutralization from being obtained except at a single frequency is given in Fig. 20, together with two symmetrical arrangements in which the lead inductances are so arranged that the neutralization is effective over a wide frequency band.

Low-frequency parasitics normally arise from radio-frequency chokes, and can be eliminated by circuit arrangements that do not require chokes. Where it is not possible to eliminate all chokes, it is desirable to avoid the use of chokes in *both* the

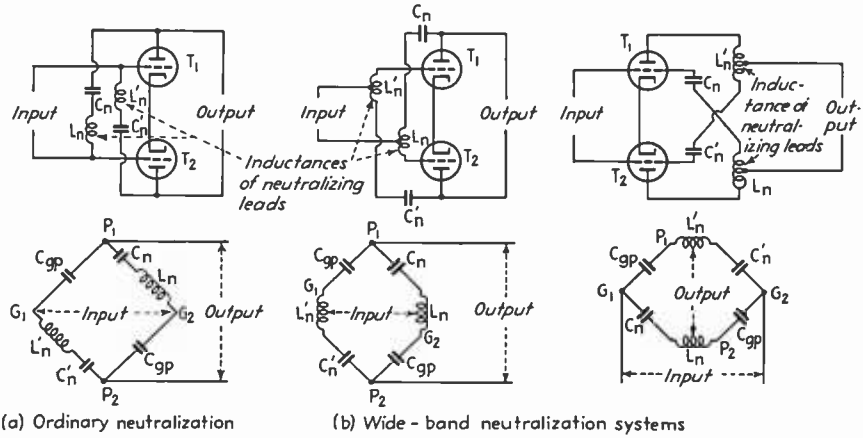


FIG. 20.—Neutralizing systems with push-pull tubes showing how bringing the input and output leads to the proper points will make the neutralization effective over very wide frequency bands.

grid and plate sides of the same tube. Where two chokes must be employed, low-frequency parasitics of the tuned-grid tuned-plate type can be avoided by arranging matters so that the resonant frequency produced in the plate circuit by the plate choke is less than the corresponding resonant frequency resulting from the grid choke.

Parasitics of the dynatron type can be eliminated by connecting a rectifier between grid and cathode, as illustrated in Fig. 21, and adjusting the resistance R in this rectifier so that the equivalent resistance formed by the parallel combination of rectifier and the negative tube resistance will always be positive.

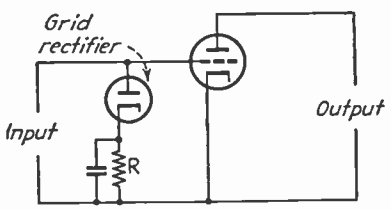


FIG. 21.—Grid rectifier for neutralizing dynatron effect at grid of tube.

Screen-grid and pentode power-amplifier tubes are practically immune from most types of parasitic oscillations, provided that the screen grid is really at ground potential, since then no energy transfer through the tube is possible. However, the circuit from the screen-grid electrode through the by-pass condenser to the cathode has inductance, and it is possible for parasitic oscillations of the tuned-grid tuned-plate type to exist in such tubes, with the screen grid serving the same function as the plate in the ordinary triode circuit.

Parasitic Oscillations in Audio Amplifiers.—Audio-frequency amplifiers, particularly those using large tubes, are frequently troubled with parasitics. Such amplifiers commonly employ triode tubes and are seldom neutralized. This, together with the fact that the input and output transformers are complicated networks involving

Ex. 1044-0510

inductance and distributed capacities, introduces the possibility of a variety of parasitic oscillations.

6. Laboratory Oscillators. *Special Considerations in Laboratory Oscillators.*—In oscillators used in testing audio amplifiers, etc., the most important considerations are: (1) frequency stability with tube voltage changes, tube replacements and changes in load impedance, (2) good wave shape, and (3) constant output over a wide frequency range. The efficiency is only of secondary importance, since the power required ordinarily does not exceed a few watts, and is usually obtained from a power amplifier that isolates the load from the oscillator proper.

Ordinary oscillator circuits such as the Hartley, electron-coupled, etc., are used extensively in radio-frequency laboratory oscillators. However, many laboratory oscillators, particularly those used in testing audio and video amplifiers, are special types not used for other purposes. The more important of these are described below.

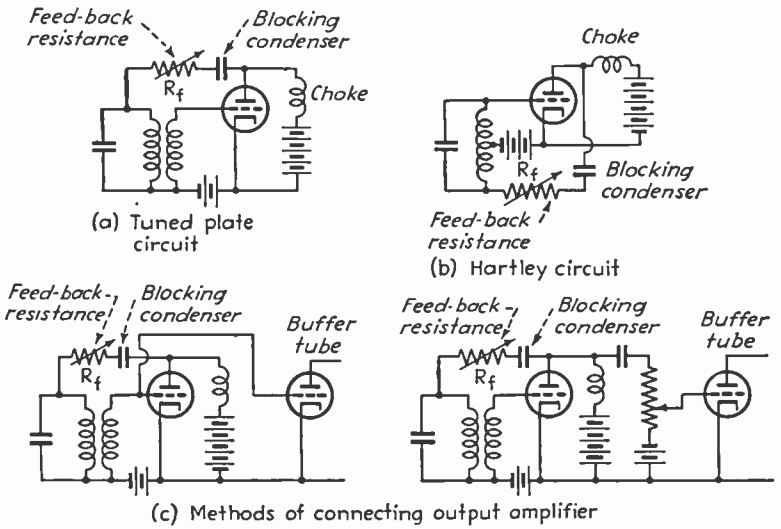


FIG. 22.—Typical laboratory oscillators with resistance stabilization.

Resistance-stabilized Oscillator.[1]—The resistance-stabilized oscillator is a conventional oscillator, usually of the tuned-plate or Hartley types, with the addition of a resistance between the plate and the tuned circuit as shown by R_f in Fig. 22. Such oscillators are widely used in laboratory work at audio and the lower radio frequencies. When properly designed, they are characterized by excellent wave shape, a frequency that is substantially independent of tube voltages and tube replacements, and an output that is substantially constant over wide frequency ranges.

The tube in a resistance-stabilized oscillator is adjusted to operate as a Class A amplifier with fixed bias (usually self-bias) and with the "feedback" resistance R_f in Fig. 22 so high that oscillations are just barely able to start. Upon the application of plate voltage, oscillations then build up until the grid goes positive at the peak of the cycle, when the added losses resulting from the grid current thus drawn prevent further increase in amplitude. For good wave shape, the tube, when considered as an amplifier, must be so adjusted that it will amplify without distortion an alternating voltage

[1] See F. E. Terman, Resistance Stabilized Oscillators, *Electronics*, Vol. 6, p. 190, July, 1933; J. W. Horton, Vacuum Tube Oscillators—A Graphical Method of Analysis, *Bell System Tech. Jour.*, Vol. 3, p. 508, July, 1924.

Ex. 1044-0511

on the grid having a crest value several volts greater than the grid bias. This corresponds to a bias slightly less than would be used at the same plate voltage for normal Class A operation without grid current. The frequency generated will be substantially independent of the voltages applied to the tube and will not change with tube replacements, provided that the feedback resistance R_f is considerably greater than the plate resistance of the tube.

The design of a resistance-stabilized oscillator is built around the fact that when the impedance of the plate choke is much greater than the plate resistance, oscillations are just barely able to start when the feedback resistance has the value

$$\text{Starting feedback resistance} = R_L(\mu n - 1) - R_p \qquad (7)$$

where R_L = parallel resonant impedance of tuned circuit between feedback resistance and cathode.
μ = amplification factor of tube.
n = ratio of voltage transformation from plate to grid of coil of tuned circuit.
R_p = plate resistance of tube.

In order to obtain certainty of starting, it is desirable to make R_f 5 to 15 per cent less than given by Eq. (7). For good frequency stability the feedback resistance should

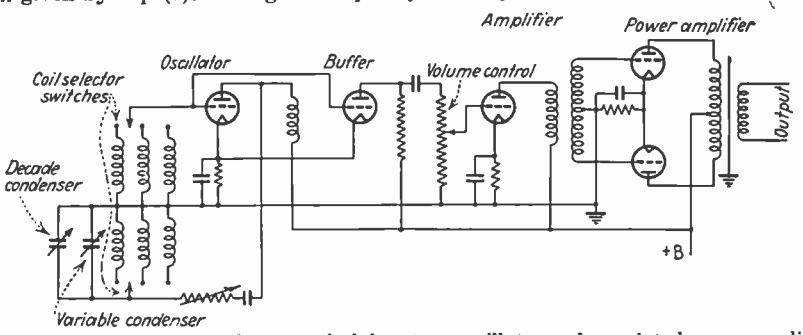


FIG. 23.—Schematic circuit diagram of a laboratory oscillator and associated power amplifier for covering the entire audio-frequency range.

also be at least twice and preferably not less than five times the plate resistance of the tube, and at the same time is preferably not greater than 500,000 ohms. Most satisfactory designs are obtained with triode tubes having an amplification factor of the order of 5 to 8 and coil ratios n of unity. The grid bias should be at least 3 to 5 volts, and the plate potential should be so selected that with a signal having a crest value several volts greater than the bias, the tube will not approach too closely to cutoff on the negative peaks.

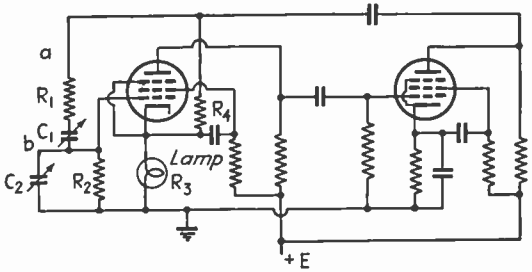
The circuit diagram of a complete audio-frequency oscillator of the resistance-stabilized type is shown in Fig. 23. Continuous variation of frequency is obtained by using a variable tuning condenser supplemented by a decade condenser, together with provision for switching coils in and out of the circuit. The feedback resistance is adjustable for each particular frequency to a value slightly less than that for which oscillations start. The proper setting of this resistance will depend upon the frequency and can be given on the frequency calibration chart, or the resistance can be adjusted to a value such that a microammeter in the grid circuit of the oscillator will show a small predetermined grid current.

Oscillators with Resistance-capacity Tuning.[1]—In this type of oscillator, the frequency is determined by a resistance-capacity network that provides regenerative

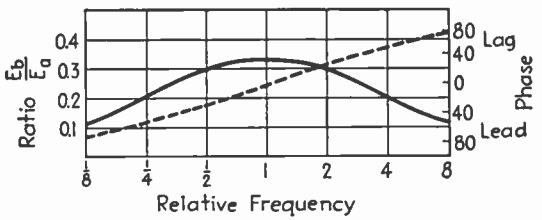
[1] F. E. Terman, R. R. Buss, W. R. Hewlett, and F. C. Cahill, Some Applications of Negative Feedback with Particular Reference to Laboratory Equipment, *Proc. I.R.E.*, Vol. 10, p. 649, October, 1939;

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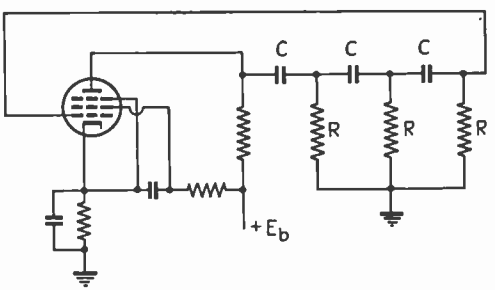
coupling between the output and input of a feedback amplifier, as shown in Fig. 24*a*. Here the network consists of $R_1C_1R_2C_2$, and by proportioning so that $R_1C_1 = R_2C_2$ under all conditions, the ratio of the voltage developed across the input (point *b*) of the amplifier to the voltage existing across the output (point *a*) varies with frequency as shown in Fig. 24*b*. Oscillations tend to take place at the maximum of this curve, which is at a frequency of $1/(2\pi\sqrt{R_1R_2C_1C_2})$ cycles per second.



(a) Circuit of resistance-capacity oscillator



(b) Amplitude and phase characteristics



(c) Phase-shift oscillator

FIG. 24.—Circuit of resistance-capacity tuned oscillator and phase-shift oscillator.

For proper results the amplifier associated with the resistance-capacity network must have negligible phase shift, and provision must be made for controlling the amplitude of the oscillations. The phase shift can be made negligible down to very low frequencies by using as few by-pass condensers as possible, making these of very large capacity, and then employing a large amount of negative feedback. Phase shift at high frequencies can be minimized by designing the amplifier for wide-band response.

H. H. Scott, A New Type of Selective Circuit and Some Applications, *Proc. I.R.E.*, Vol. 26, p. 226, February, 1938.

The amplitude can be most satisfactorily controlled by using a lamp to provide the feedback resistance R_3. When the amplitude of oscillations is small, the alternating current through R_3 is likewise small, making the resistance and hence the amount of feedback low. This increases the amplification and allows oscillations to build up. However, increased output increases the current through R_3, which increases the resistance and hence the amount of negative feedback, and reduces the tendency to oscillate. The result is an amplitude that can be made to stabilize at a value substantially constant under widely varying conditions and at a value that does not overload the tubes and distort the wave shape.

Resistance-capacity tuned oscillators have many advantages over beat frequency, resistance-stabilized, and other audio- and video-frequency oscillators used in general laboratory work. The frequency generated is inversely proportional to tuning capacity instead of inversely proportional to the square root of the tuning capacity, as in ordinary oscillators. A 10:1 frequency range can therefore be conveniently obtained from an ordinary variable condenser, with decimal multiplying factors available by changing the resistances R_1 and R_2 in decimal values. The wave shape is extremely good, frequency stability is high, and the output virtually constant over enormous frequency ranges. The oscillations will also readily synchronize in harmonic relation with voltages injected almost anywhere in the circuit, a useful property for certain applications.

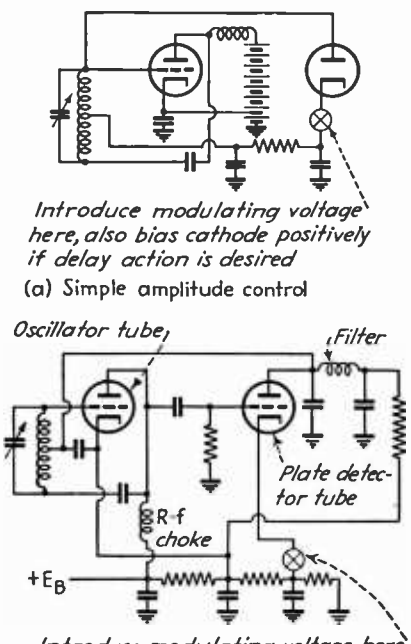


FIG. 25.—Conventional oscillator with amplitude control.

Phase-shift Oscillators.[1]—The phase-shift oscillator is a special type of resistance-capacity tuned oscillator that operates with a single tube. The basic circuit arrangement is illustrated in Fig. 24*c*. Here a three-mesh resistance-capacity phase-shifting network RC is connected between the output and input of an amplifier tube, with the circuit so proportioned that the total phase shift between plate and grid terminals is 180° at the frequency of oscillation desired. When the gain of such an amplifier is adjusted either manually or by an automatic-volume-control circuit barely to maintain oscillation, almost pure sine-wave output is obtained. The frequency stability of the circuit with respect to voltage changes is then also very good.

Phase-shift oscillators are particularly suitable for operation at fixed frequencies. and probably represent the simplest satisfactory audio oscillator for this purpose, Variable frequency operation can be obtained by simultaneously adjusting all condensers, or all resistances, from a common control, but the complications thus introduced are such that the arrangement of Fig. 24*a* is usually preferable for variable-frequency operation.

[1] Further information on phase-shift oscillators, including a discussion of modified circuit arrangements, design information, and a sample design, is given by E. L. Ginzton and L. M. Hollingsworth, Phase-shift Oscillators, *Proc. I.R.E.*, Vol. 29, p. 43, February, 1941.

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