



Foundations of Analog and Digital Electronic Circuits

ANANT AGARWAL AND JEFFREY H. LANG



Publisher: Denise E. M. Penrose
Publishing Services Manager: Simon Crump
Editorial Assistant: Valerie Witte
Cover Design: Frances Baca
Composition: Cepha Imaging Pvt. Ltd., India
Technical Illustration: Dartmouth Publishing, Inc.
Copyeditor: Eileen Kramer
Proofreader: Katherine Hasal
Indexer: Kevin Broccoli
Interior printer: King Printing Co., Inc.
Cover printer: King Printing Co., Inc.

Morgan Kaufmann Publishers is an imprint of Elsevier.
500 Sansome Street, Suite 400, San Francisco, CA 94111

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Library of Congress Cataloging-in-Publication Data
ISBN: 1-55860-735-8

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Printed in the United States of America
Last digit is the print number: 20 19 18 17 16 15 14

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illustrative circuit. We will then apply the same systematic method to solve more complicated examples, including the one shown in Figure 2.1.

2.1 TERMINOLOGY

Lumped circuit elements are the fundamental building blocks of electronic circuits. Virtually all of our analyses will be conducted on circuits containing two-terminal elements; multi-terminal elements will be modeled using combinations of two-terminal elements. We have already seen several two-terminal elements such as resistors, voltage sources, and current sources. Electronic access to an element is made through its *terminals*.

An electronic *circuit* is constructed by connecting together a collection of separate elements at their terminals, as shown in Figure 2.2. The junction points at which the terminals of two or more elements are connected are referred to as the *nodes* of a circuit. Similarly, the connections between the nodes are referred to as the edges or *branches* of a circuit. Note that each element in Figure 2.2 forms a single branch. Thus an element and a branch are the same for circuits comprising only two-terminal elements. Finally, circuit *loops* are defined to be closed paths through a circuit along its branches.

Several nodes, branches, and loops are identified in Figure 2.2. In the circuit in Figure 2.2, there are 10 branches (and thus, 10 elements) and 6 nodes.

As another example, *a* is a node in the circuit depicted in Figure 2.1 at which three branches meet. Similarly, *b* is a node at which two branches meet. *ab* and *bc* are examples of branches in the circuit. The circuit has five branches and four nodes.

Since we assume that the interconnections between the elements in a circuit are perfect (i.e., the wires are ideal), then it is not necessary for a set of elements to be joined together at a single point in space for their interconnection to be considered a single node. An example of this is shown in Figure 2.3. While the four elements in the figure are connected together, their connection does not occur at a single point in space. Rather, it is a distributed connection.

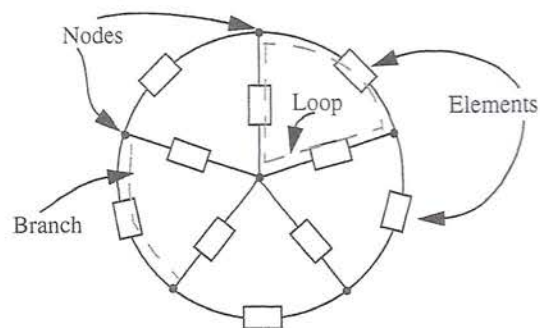


FIGURE 2.2 An arbitrary circuit.

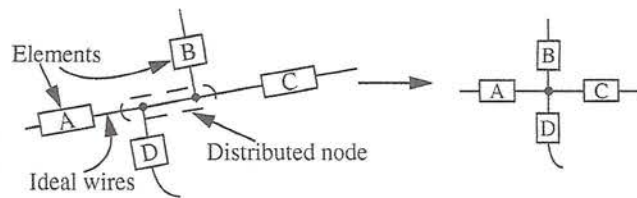


FIGURE 2.3 Distributed interconnections of four circuit elements that nonetheless occur at a single node.

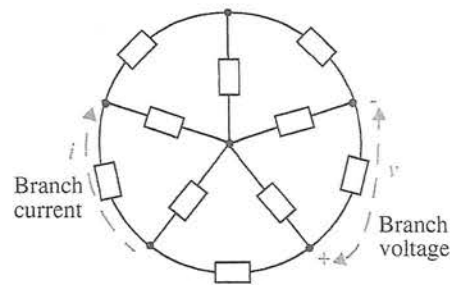


FIGURE 2.4 Voltage and current definitions illustrated on a branch in a circuit.

Nonetheless, because the interconnections are perfect, the connection can be considered to be a single node, as indicated in the figure.

The primary signals within a circuit are its currents and voltages, which we denote by the symbols i and v , respectively. We define a *branch current* as the current along a branch of the circuit (see Figure 2.4), and a *branch voltage* as the potential difference measured across a branch. Since elements and branches are the same for circuits formed of two-terminal elements, the branch voltages and currents are the same as the corresponding terminal variables for the elements forming the branches. Recall, as defined in Chapter 1, the terminal variables for an element are the voltage across and the current through the element.

As an example, i_4 is a branch current that flows through branch bc in the circuit in Figure 2.1. Similarly, v_4 is the branch voltage for the branch bc .

2.2 KIRCHHOFF'S LAWS

Kirchhoff's current law and Kirchhoff's voltage law describe how lumped-parameter circuit elements couple at their terminals when they are assembled into a circuit. KCL and KVL are themselves lumped-parameter simplifications of Maxwell's Equations. This section defines KCL and KVL and justifies that they are reasonable using intuitive arguments.¹ These laws are employed in circuit analysis throughout this book.

¹ The interested reader can refer to Section A.2 in Appendix A for a derivation of Kirchhoff's laws from Maxwell's Equations under the lumped matter discipline.

2.2.1 KCL

Let us start with Kirchhoff's current law (KCL).

KCL The current flowing out of any node in a circuit must equal the current flowing in. That is, the algebraic sum of all branch currents flowing into any node must be zero.

Put another way, KCL states that the net current that flows into a node through some of its branches must flow out from that node through its remaining branches.

Referring to Figure 2.5, if the currents through the three branches *into* node *a* are i_a , i_b , and i_c , then KCL states that

$$i_a + i_b + i_c = 0.$$

Similarly, the currents into node *b* must sum to zero. Accordingly, we must have

$$-i_b - i_4 = 0.$$

KCL has a simple intuitive justification. Referring to the closed box-like surface depicted in Figure 2.5, it is easy to see that the currents i_a , i_b , and i_c must sum to zero, for otherwise, there would be a continuous charge buildup at node *a*. Thus, KCL is simply a statement of the conservation of charge.

Let us now illustrate the different interpretations of KCL with the help of Figure 2.6. Which interpretation you use depends upon convenience and the specific circuit you are trying to analyze. Figure 2.6 shows a node joining *N* branches. Each of the branches contains some two-terminal element, the specifics of which are not relevant to our discussion. Note that all branch currents are defined to be positive into the node. Since KCL states that no net

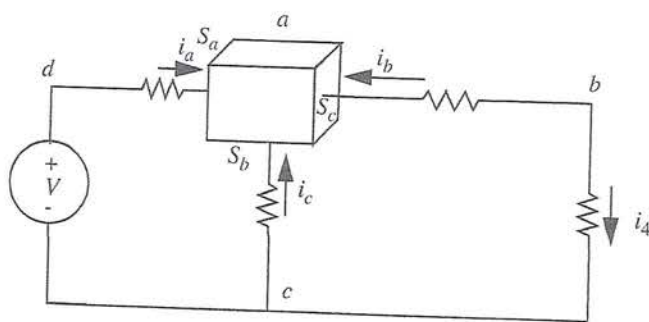


FIGURE 2.5 Currents into a node in the network.

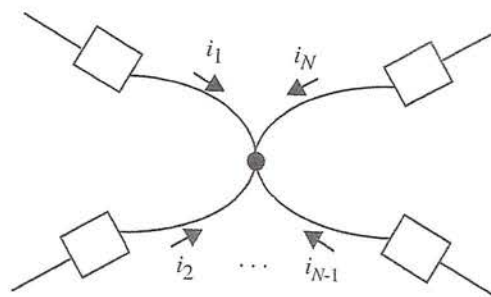


FIGURE 2.6 A node at which *N* branches join.

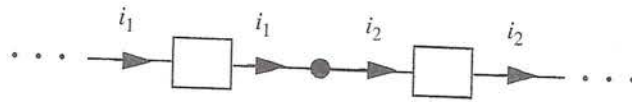


FIGURE 2.7 Two series-connected circuit elements.

current can flow into a node, it follows for the node in Figure 2.6 that

$$\sum_{n=1}^N i_n = 0. \quad (2.1)$$

Next, by negating Equation 2.1, KCL becomes

$$\sum_{n=1}^N (-i_n) = 0. \quad (2.2)$$

Since $-i_n$ is a current defined to be positive out from the node in Figure 2.6, this second form of KCL states that no net current can flow out from a node. Finally, Equation 2.1 can be rearranged to take the form

$$\sum_{n=1}^M i_n = \sum_{n=M+1}^N (-i_n), \quad (2.3)$$

which demonstrates that the current flowing into a node through one set of branches must flow out from the node through the remaining branches.

An important simplification of KCL focuses on the two series-connected circuit elements shown in Figure 2.7. Taking KCL to state that no net current can flow into a node, the application of KCL at the node between the two elements yields

$$i_1 - i_2 = 0 \Rightarrow i_1 = i_2. \quad (2.4)$$

This result is important because it shows that the branch currents passing through two series-connected elements must be the same. That is, there is nowhere for the current i_1 to go as it enters the node connecting the two elements except to exit that node as i_2 . In fact, with multiple applications of KCL, this observation is extendible to a longer string of series-connected elements. Such an extension would show that a common branch current passes through a string of series-connected elements.

EXAMPLE 2.1 A MORE GENERAL USE OF KCL To illustrate the more general use of KCL consider the circuit in Figure 2.8, which has six branches

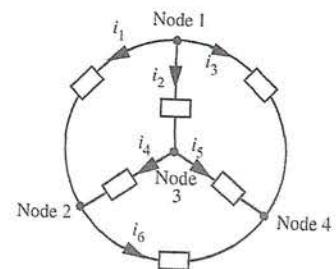


FIGURE 2.8 A circuit illustrating a more general use of KCL.