

ANTENNA THEORY AND DESIGN

SECOND EDITION

**WARREN L. STUTZMAN
GARY A. THIELE**

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Chapter 1

Antenna Fundamentals and Definitions

1.1 INTRODUCTION

Communication between humans was first by sound through voice. With the desire for long distance communication came devices such as drums. Then, visual methods such as signal flags and smoke signals were used. These optical communication devices, of course, utilized the light portion of the electromagnetic spectrum. It has been only very recent in human history that the electromagnetic spectrum, outside the visible region, has been employed for communication, through the use of radio.

The radio antenna is an essential component in any radio system. An antenna is a device that provides a means for radiating or receiving radio waves. In other words, it provides a transition from a guided wave on a transmission line to a “free-space” wave (and vice versa in the receiving case). Thus, information can be transferred between different locations without any intervening structures. The possible frequencies of the electromagnetic waves carrying this information form the electromagnetic spectrum (the radio frequency bands are given in Appendix A). One of humankind’s greatest natural resources is the electromagnetic spectrum and the antenna has been instrumental in harnessing this resource. A brief history of antenna technology [1–4] and a discussion of the uses of antennas follow.

Perhaps the first radiation experiment was performed in 1842 by Joseph Henry of Princeton University, the inventor of wire telegraphy. He “threw a spark” in a circuit in an upper room and observed that needles were magnetized by the current in a receiving circuit located in the cellar. This experiment was extended to a distance of over a kilometer. Henry also detected lightning flashes with a vertical wire on the roof of his house. These experiments marked the beginning of wire antennas.

Based on his observations in 1875 that telegraph key closures radiate, Thomas Edison patented a communication system in 1885 that employed top-loaded, vertical antennas.

The theoretical foundations for antennas rest on Maxwell’s equations, which James Clerk Maxwell (1831–1879) presented before the Royal Society in 1864, that unify electric and magnetic forces into a single theory of electromagnetism. Maxwell also predicted that light is explained by electromagnetics and that light and electromagnetic disturbances both travel at the same speed.

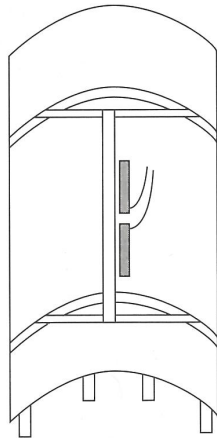
In 1887 the German physicist Heinrich Hertz (1857–1894) was able to verify

experimentally the claim of Maxwell that electromagnetic actions propagate through air. Hertz discovered that electrical disturbances could be detected with a single loop of the proper dimensions for resonance that contains an air gap for sparks to occur. The primary source of electrical disturbances studied by Hertz consisted of two metal plates in the same plane, each with a wire connected to an induction coil; this early antenna is similar to the capacitor-plate dipole antenna described in Section 2.1 and was called a “Hertzian dipole.” Hertz also constructed loop antennas. Motivated by the need for more directive radiation, he also invented reflector antennas. In 1888 he constructed a parabolic cylinder reflector antenna from a sheet of zinc; see Fig. 1-1*a*. It was fed with a dipole along the focal line and operated at 455 MHz.

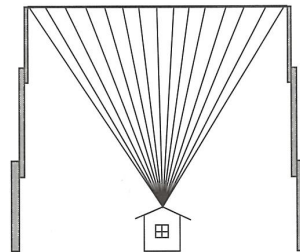
Guglielmo Marconi (1874–1937), an Italian inventor, also built a microwave parabolic cylinder reflector in 1895 for his original code transmission at 1.2 GHz. But his subsequent work was at lower frequencies for improved communication range. The transmitting antenna for the first transatlantic radio communication in 1901 consisted of a 70-kHz spark transmitter connected between the ground and a system of 50 wires, forming a 48-m tall fan monopole; see Fig. 1-1*b*. The antenna resembles a variation of the discane antenna described in Sec. 6.3. The receiving antenna was supported by kites.

Although Marconi is credited as the pioneer of radio, Mahlon Loomis (1826–1886), a dentist and inventor in Washington, DC, received a U.S. patent in 1872 for an “Improvement in Telegraphying” in which he described the use of an “aerial” to radiate and receive “pulsations.” In October 1866, Loomis demonstrated his wireless signaling system to U.S. senators in the Blue Ridge Mountains of Virginia using wire supported by kites at both the transmitting and receiving antennas about twenty miles apart.

The Russian physicist Alexander Popov (1859–1905) also recognized the importance of Hertz’s discovery of radio waves and began working on ways of receiving



(a) The 455-MHz cylinder reflector antenna invented by Hertz in 1888



(b) The monopole transmitting antenna used by Marconi at 70 kHz for the first transatlantic radio communication

Figure 1-1 Examples of early antennas.

them a year before Marconi. He is sometimes credited with using the first antenna in the first radio system by sending a signal over a 3-mile ship-to-shore path in 1897. However, it was Marconi who developed radio commercially and also pioneered transoceanic radio communication. Marconi may be considered to be the father of what was then called wireless. Since then the term "radio" has been used, but "wireless" has also returned to popular use.

In 1912 the Institute of Radio Engineers was formed by the merger of the Wireless Institute and the Society of Radio Engineers. The importance of antennas is punctuated by the fact that the first article of the first issue of *Proceedings of the I.R.E.* was on antennas [5].

Antenna developments in the early years were limited by the availability of signal generators. Resonant length antennas (e.g., a half-wavelength dipole) of manageable physical size were possible about 1920 after the De Forest triode tube was used to produce continuous wave signals up to 1 MHz. Just before World War II, microwave (about 1 GHz) klystron and magnetron signal generators were developed along with hollow pipe waveguides. These led to the development of horn antennas, although Chunder Bose (1858–1937) in India produced the first electromagnetic horn antenna many years earlier. The first commercial microwave radiotelephone system in 1934 was operated between England and France at 1.8 GHz. The need for radar during the war spawned many "modern" antennas, such as large reflectors, lenses, and waveguide slot arrays [6].

Let us now direct our attention to the uses of antennas. Electromagnetic energy can be transported using a transmission line. Alternatively, no guiding structure is needed if antennas are used. For a transmitter-receiver spacing of R , the power loss of a transmission line is proportional to $(e^{-\alpha R})^2$, where α is the attenuation constant of the transmission line. If the antennas are used in a line of sight configuration, the power loss is proportional to $1/R^2$. Many factors enter into the decision of whether to use transmission lines or antennas. Generally speaking, at low frequencies and short distances transmission lines are practical. But high frequencies are attractive because of the available bandwidth. As distances become large and frequency increases, the signal losses and the costs of using transmission lines become large, and thus the use of antennas is favored. A notable exception to this is the fiber optic transmission line, which has very low loss. Transmission lines offer the advantages of not being subject to interference that is often encountered in radio systems and added bandwidth is achieved by laying new cable. However, there are significant costs and construction delays associated with cable.

In several applications, antennas *must* be used. For example, mobile communications involving aircraft, spacecraft, ships, or land vehicles require antennas. Antennas are also popular in broadcast situations where one transmit terminal can serve an unlimited number of receivers, which can be mobile (e.g., car radio). Non-broadcast radio applications such as municipal radio (police, fire, rescue) and amateur radio also require antennas. Also, personal communication devices such as pagers and cellular telephones are commonplace.

There are also many noncommunication applications for antennas. These include remote sensing and industrial applications. Remote sensing systems are either active (e.g., radar) or passive (e.g., radiometry) and receive scattered energy or inherent emissions from objects, respectively. The received signals are processed to infer information about the objects or scenes. Industrial applications include cooking and drying with microwaves.

Other factors that influence the choice of the type of transmission system include

historical reasons, security, and reliability. Telephone companies began interconnecting multiple transmit-receive terminals with transmission lines before radio technology was available. Currently, domestic telephone companies employ microwave radio and fiber optic transmission lines for long distance telephone calls. Satellite radio links are used heavily for international telephone calls. In addition, satellite-based communication systems are the primary means of distributing television program material to affiliate stations. Television programs distributed directly to consumers by satellite is increasing worldwide. Also, very small aperture terminals (VSAT) for satellite radio systems are widely used in private data networks to interconnect, for example, chain retail stores. Transmission lines inherently offer more security than radio. However, radio links using digital communications can be secured with coding techniques. Also, additional security is unnecessary for most communication systems. Another factor to be considered is reliability. For example, radio signals are affected by environmental conditions such as structures along the signal path, the ionosphere, and weather. Furthermore, interference is always a threat to radio systems. On the other hand, cables are vulnerable to being damaged by earthquakes or being accidentally dug up. All these factors must be examined together with the costs associated with using transmission lines and antennas. Cable systems often require expensive land purchase or lease. Every year radio equipment decreases in cost and increases in reliability. This tends to tip the scale in favor of radio systems. Cable and radio communication systems will continue to be used in the future with the choice depending on the specific application. For high reliability, both cable and radio are employed to provide diversity.

As we shall see, antennas cannot be miniaturized and replaced by a chip as often happens in electronics. Although long-used types of antennas will remain in use far into the future, new applications will require innovative antenna systems. For example, the demand for more communications is leading to personal communications systems (PCS) where each person will be freed of wire connections by carrying a small radio that can be used anywhere on the globe. The first of the seven IEEE New Technology Directions Committee grand challenges for electrotechnology is, "To make any person anywhere in the world reachable at his or her discretion, at any time by communication methods independent of connecting wires and cables." A "wireless" society is possible only through the use of antennas. Indeed, the future of antennas is very bright.

The next two sections of this first chapter provide a basic understanding of how antennas operate and an overview of the types of antennas encountered in practice. The remainder of Chap. 1 is devoted to developing the fundamental principles and terminology used throughout the book.

References to specific literature works are found at the end of the chapter where the citation occurs. In addition, a complete bibliography is found in Appendix H. About 150 books are listed by their topical coverage to aid the student in locating further details. The IEEE definitions of antenna terms [Ref. 1 in App. H] are followed very closely in this book.

1.2 HOW ANTENNAS RADIATE

Before we proceed with a mathematical development of antennas that is necessary for engineering design, it is instructive to explain the basic principles of radiation. Radiation is a disturbance in the electromagnetic fields that propagates away from the source of the disturbance so that the total power associated with the wave in a

lossless medium is constant with radial distance. This disturbance is created by a time-varying current source that has an accelerated charge distribution associated with it. We, therefore, begin our discussion of radiation with a single accelerated charge.

Consider a single electric charge moving with constant velocity in the z -direction of Fig. 1-2. Prior to arrival at point A , the static electric field lines extend radially away from the charge to infinity and move with the charge. At point A , the charge begins to be accelerated (i.e., velocity is increased) until reaching point B , where it continues on at the acquired velocity. The static electric field (often called the Coulomb field) originates at the charge and is directed radially away from the charge. The radial field lines outside the circle of radius r_A in Fig. 1-2 originated when the charge was at point A . The circle of radius r_B is centered on point B , which is the charge position at the end of the acceleration period Δt . Interior to r_B , the electric field lines extend radially away from point B . The distance between the circles is that distance light would travel in time Δt , or $\Delta r = r_B - r_A = \Delta t c$. Since the charge moves slowly compared to the speed of light, $\Delta z \ll \Delta r$ and the circles are nearly concentric; the distance Δz in Fig. 1-2 is shown large relative to Δr for clarity. The electric field lines in the Δr region are joined together because of the required continuity of electric field lines in the absence of charges. This region is obviously one of disturbed field structure. This disturbance was caused by acceleration of the charge, which ended a time r_B/c earlier than the instant represented in Fig. 1-2. This

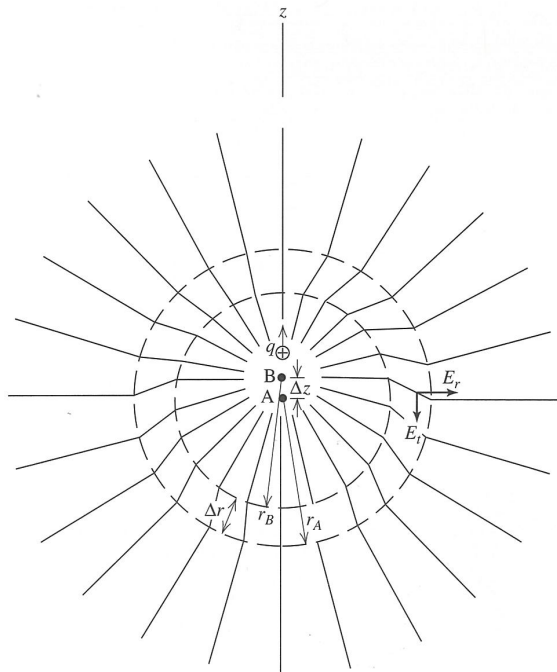


Figure 1-2 Illustration of how an accelerated charged particle radiates. Charge q moves with constant velocity in the $+z$ -direction until it reaches point A (time $t = 0$), after which it accelerates to point B (time $t = \Delta t$) and then maintains its velocity. The electric field lines shown here are for a time r_B/c after the charge passed point B .

disturbance expands outward and has a transverse component E_r , which is the radiated field that persists as the disturbance propagates to infinity.

This example illustrates that radiation is a disturbance. It is directly analogous to a transient wave created by a stone dropped into a calm lake, where the disturbance of the lake surface continues to propagate radially away from the impact point long after the stone has disappeared. If charges are accelerated back and forth (i.e., oscillate), a regular disturbance is created and radiation is continuous. Antennas are designed to support charge oscillations.

The directional properties of radiation are evident in the accelerated charge example. The disturbance in Fig. 1-2 is maximum in a direction perpendicular to the charge acceleration direction, and we shall see in this chapter that maximum radiation occurs perpendicular to a straight wire antenna.

We can now explain how an actual antenna operates. To do this, we begin with the open-circuited transmission line of Fig. 1-3, which has a standing wave pattern with a zero current magnitude at the wire end and nulls every half wavelength from the end. The currents are in opposite directions on the wires, as indicated by arrows in Fig. 1-3a. In transmission lines, the conductors guide the waves and the power resides in the region surrounding the conductors as manifested by the electric and magnetic fields. The fields for the open-ended transmission line are shown in Fig. 1-3a. The electric fields originate from or terminate on charges on the wires and are

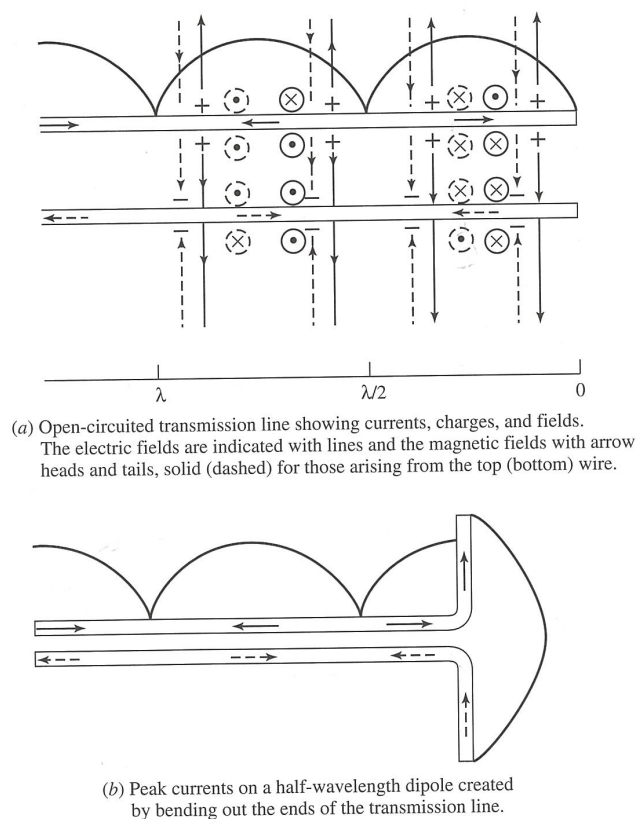


Figure 1-3 Evolution of a dipole antenna from an open-circuited transmission line.

perpendicular to the wires. The magnetic fields encircle the wires. Note that all fields reinforce between the wires and cancel elsewhere. This is true for a wire spacing that is much smaller than a wavelength, as is usually the case. If the ends of the wires are bent outward as shown in Fig. 1-3b, the reinforced fields between the wires are exposed to space. Note that the currents on the vertical wire halves, which are each a quarter wavelength in this case, are no longer opposed as with the transmission line, but are both upwardly directed. In reality, the currents on the dipole are approximately sinusoidal as shown, but the transmission line currents are not pure standing waves due to the improved impedance match presented by the antenna compared to the open circuit. The situation of Fig. 1-3b is for a peak current condition. As time proceeds and current oscillations occur, disturbances are created that propagate away from the wire, much as the accelerated single charge.

The time dynamics of the fields associated with an oscillating dipole charge distribution are shown in Fig. 1-4 [7]. This is similar to the electrostatic dipole with equal, but opposite signed, separated charges. In this case, the charge distributions oscillate at frequency f . As the charge distributions at the ends oscillate, a current flows between them that is uniform with distance. This is the ideal dipole of Sec. 1.6. In Fig. 1-4, an oscillating current of frequency f (and period $T = 1/f$) was turned on a quarter period before $t = 0$. The upward-flowing current creates an excess of charges on the upper half of the dipole and a deficit of charges on the lower half. Peak charge buildup occurs at $t = 0$ as shown in Fig. 1-4a and produces a voltage between the dipole halves. The positive charges on the top are attracted to the negative charges on the bottom half of the dipole, creating a current. The current

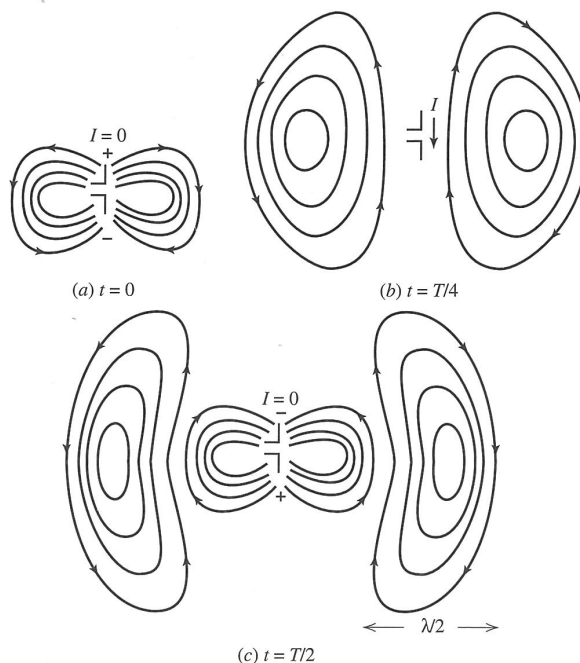


Figure 1-4 Electric fields of an oscillating dipole for various instants of time. The oscillations are of frequency f with a period of $T = 1/f$.

is maximum at $t = T/4$ as shown in Fig. 1-4b, at which time the charges have been neutralized and there are no longer charges for the termination of electric field lines, which form closed loops near the dipole. During the next quarter cycle, negative charges accumulate at the top end of the dipole as shown in Fig. 1-4c. Near the dipole, the fields are most intense normal to the oscillating charges on the dipole, just as we found with the single accelerated charge. As time progresses, the electric field lines detach from the dipole, forming closed loops in space. Viewed in terms of current, the conduction current on the antenna converts to a displacement current in space, consisting of the longitudinal fields near the antenna and solenoidal (loops) fields away from the antenna. Thus, current continuity is satisfied. This process continues, producing radiation via electric field components that are transverse to the radial direction and propagate to large distances from the antenna. We shall see in Sec. 1.6 that the mathematical solution of the (oscillating) dipole produces the property required for successful radiation: fields decrease with distance as $1/r$. In contrast, the electric field of an electrostatic dipole decreases as $1/r^3$. The time-space behavior of fields from an antenna is revisited in Sec. 11.8 and 11.9.

1.3 OVERVIEW OF ANTENNAS

An antenna acts to convert guided waves on a transmission structure into free space waves. Figure 1-3 illustrates a parallel wire transmission line feeding a half-wave dipole antenna. The official IEEE definition of an antenna follows this concept: "That part of a transmitting or receiving system that is designed to radiate or receive electromagnetic waves." Most antennas are reciprocal devices and behave the same on transmit as on receive. Antennas are treated as transmitting or receiving as appropriate for the particular situation. In the receiving mode, antennas act to collect incoming waves and direct them to a common feed point where a transmission line is attached. In some cases, antennas focus radio waves just as lenses focus optical waves. In all cases, antennas have directional characteristics; that is, electromagnetic power density is radiated from a transmitting antenna with intensity that varies with angle around the antenna.

In this section, we introduce the parameters used to evaluate antennas and then discuss the four types of antennas. The parameters are defined and developed in more detail in the remainder of this chapter after the brief overview given here.

Antenna performance parameters are listed in Table 1-1. The *radiation pattern* (or simply, *pattern*) gives the angular variation of radiation at a fixed distance from an antenna when the antenna is transmitting. Radiation is quantified by noting the value of power density S at a fixed distance r from the antenna. When receiving, the antenna responds to an incoming wave from a given direction according to the pattern value in that direction. The typical pattern in Fig. 1-5 shows the pattern main beam and side lobes. This directive antenna, with a single narrow main beam, is used in point-to-point communications. In some applications, the shape of the main beam is important. On the other hand, an omnidirectional antenna with constant radiation in one plane is used in broadcast situations.

An antenna is essentially a spatial amplifier and *directivity* expresses how much greater the peak radiated power density is for an antenna than it would be if all the radiated power were distributed uniformly around the antenna. Fig. 1-5 shows the radiation pattern of a real antenna compared to an *isotropic* spatial distribution; also see Fig. 1-20. The spatial enhancement that can be achieved by an antenna is evident. *Gain* G is directivity reduced by the losses on the antenna.

Table 1-1 Antenna Performance Parameters

- **Radiation Pattern $F(\theta, \phi)$:** Angular variation of radiation around the antenna, including:
 - Directive, single or multiple narrow beams
 - Omnidirectional (uniform radiation in one plane)
 - Shaped main beam
- **Directivity D :** Ratio of power density in the direction of the pattern maximum to the average power density at the same distance from the antenna.
- **Gain G :** Directivity reduced by the losses on the antenna.
- **Polarization:** The figure traced out with time by the instantaneous electric field vector associated with the radiation from an antenna when transmitting. Antenna polarizations: Linear, Circular, Elliptical
- **Impedance Z_A :** Input impedance at the antenna terminals.
- **Bandwidth:** Range of frequencies over which important performance parameters are acceptable.
- **Scanning:** Movement of the radiation pattern in space. Scanning is accomplished by mechanical movement or by electronic means such as adjustment of antenna current phase.
- **System Considerations:** Size, weight, power handling, radar cross section, environmental operating conditions, etc.

The third parameter, *polarization*, describes the vector nature of electric fields radiated by an antenna and is discussed in detail in Sec. 1.10. The figure traced out with time by the tip of the instantaneous electric field vector determines the polarization of the wave. A straight wire antenna radiates a wave with linear polarization parallel to the wire. Another popular polarization is circular. In general, polarization is elliptical. *Dual polarized* antennas enable the doubling of communication capacity by carrying separate information on orthogonal polarizations over the same physical link at the same frequency.

The *input impedance* of an antenna is the ratio of the voltage to current at the antenna terminals. The usual goal is to match antenna input impedance to the characteristic impedance of the connecting transmission line. *Bandwidth* is the range of frequencies with acceptable antenna performance as measured by one or more of the performance parameters; see Chap. 6 for commonly used definitions of bandwidth. Finally, it is often desired to scan the main beam of an antenna over a region of space. This can be accomplished by moving an antenna, by electronic scanning or by a combination of mechanical and electronic scanning.

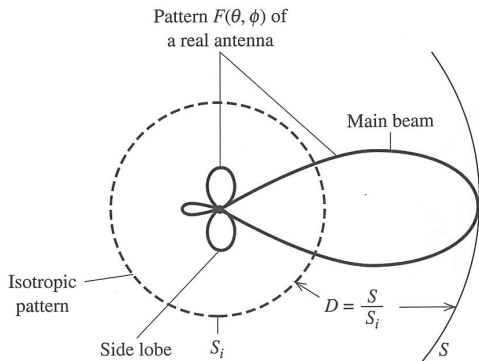


Figure 1-5 Illustration of radiation pattern $F(\theta, \phi)$ and directivity D . The power densities at the same distance are S and S_i for the real and isotropic antennas, respectively.

10 Chapter 1 Antenna Fundamentals and Definitions

- **Electrically small antennas:** The extent of the antenna structure is much less than a wavelength λ .

Properties:

- Very low directivity
- Low input resistance
- High input reactance
- Low radiation efficiency

Examples:



Short dipole



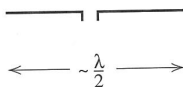
Small loop

- **Resonant antennas:** The antenna operates well at a single or selected narrow frequency bands.

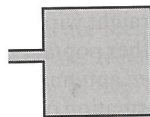
Properties:

- Low to moderate gain
- Real input impedance
- Narrow bandwidth

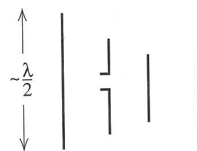
Examples:



Half-wave dipole



Microstrip patch



Yagi

- **Broadband antennas:** The pattern, gain, and impedance remain acceptable and are nearly constant over a wide frequency range, and are characterized by an active region with a circumference of one wavelength or an extent of a half-wavelength, which relocates on the antenna as frequency changes.

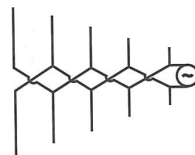
Properties:

- Low to moderate gain
- Constant gain
- Real input impedance
- Wide bandwidth

Examples:



Spiral



Log periodic dipole array

Figure 1-6 Types of antennas.

- **Aperture antennas:** Has a physical aperture (opening) through which waves flow.

Properties:

High gain

Gain increases with frequency

Moderate bandwidth

Examples:

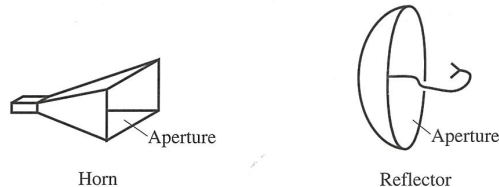


Figure 1-6 (continued).

There are trade-offs between parameter values. Usually, performance cannot be improved significantly for one parameter without sacrificing one or more of the other parameter levels. This is the antenna design challenge.

Antennas can be divided into four basic types by their performance as a function of frequency. These are introduced so that the common features can be grasped early in the study of antennas. When you encounter a new antenna, try to determine which type it is. The antenna types in Fig. 1-6 are listed in the order that they are commonly used across the radio spectrum; see Appendix A for lists of frequency bands. This discussion serves as an overview and should be referred to from time to time as your knowledge of antennas builds. Electrically small antennas are used at VHF frequencies and below. Resonant antennas are mainly used from HF to low GHz frequencies. Broadband antennas are mainly used from VHF to middle GHz frequencies. Aperture antennas are mainly used at UHF and above.

Electrically small (or simply, *small*) *antennas* are much less than a wavelength in extent. Recall from electromagnetics that wavelength λ is related to frequency f through the speed of light c as $\lambda = c/f$. Electrically small antennas are simple in structure and their properties are not sensitive to construction details. The vertical monopole used for AM reception on cars is a good example. It is about 0.003λ long and has a pattern that is nearly omnidirectional in the horizontal plane. This is often a desirable property, but its low input resistance and high input reactance are serious disadvantages. Also, small antennas are inefficient because of ohmic losses on the structure.

Resonant antennas are popular when a simple structure with good input impedance over a narrow band of frequencies is needed. It has a broad main beam and low or moderate (a few dB) gain. The half-wavelength long dipole is a prominent example.

Many applications require an antenna that operates over a wide frequency range. A *broadband antenna* has acceptable performance as measured with one or more parameters (pattern, gain, and/or impedance) over a 2:1 bandwidth ratio of upper to lower operating frequency. A broadband antenna is characterized by an active region. Propagating (or, traveling) waves originate at the feed point and travel without radiation to the active region where most of the power is radiated. A broadband antenna with circular geometry has an active region where the circumference is one wavelength and produces circular polarization. An example is the spiral antenna

illustrated in Fig. 1-6 that can have a 20:1 bandwidth. A broadband antenna made up of linear elements or straight edges has an active region where these elements are about a half-wavelength in extent and produces linearly polarized radiation parallel to the linear elements. Since only a portion of a broadband antenna is responsible for radiation at a given frequency, the gain is low. But it may be an advantage to have gain that is nearly constant with frequency, although low. Also, the traveling wave nature of a broadband antenna means that it has a real-valued input impedance.

Aperture antennas have an opening through which propagating electromagnetic waves flow. A horn antenna is a good example; it acts as a "funnel," directing the waves into the connecting waveguide. The aperture is usually several wavelengths long in one or more dimensions. The pattern usually has a narrow main beam, leading to high gain. The pattern main beam narrows with increasing frequency for a fixed physical aperture size. Bandwidth is moderate (as much as 2:1).

1.4 ELECTROMAGNETIC FUNDAMENTALS

This and the next section present a brief review of electromagnetic field principles and the solution of Maxwell's equations for radiation problems. Any basic electromagnetic fields textbook can be consulted for the details on these topics.

The fundamental electromagnetic equations are¹

$$\nabla \times \mathcal{E} = -\frac{\partial \mathcal{B}}{\partial t} \quad (1-1)$$

$$\nabla \times \mathcal{H} = \frac{\partial}{\partial t} \mathcal{D} + \mathcal{J}_T \quad (1-2)$$

$$\nabla \cdot \mathcal{D} = \rho_T(t) \quad (1-3)$$

$$\nabla \cdot \mathcal{B} = 0 \quad (1-4)$$

$$\nabla \cdot \mathcal{J}_T = -\frac{\partial}{\partial t} \rho_T(t) \quad (1-5)$$

The first four of these differential equations are frequently referred to as Maxwell's equations and the last as the continuity equation. The curl equations together with the continuity equation are equivalent to the curl and divergence equations. In time-varying field problems, the curl equations with the continuity equation is the most convenient formulation. Each of these differential equations has an integral counterpart.

If the sources $\rho_T(t)$ and $\mathcal{J}_T(t)$ vary sinusoidally with time at radian frequency ω , the fields will also vary sinusoidally and are frequently called time-harmonic fields. The fundamental electromagnetic equations and their solutions are considerably simplified if phasor fields are introduced as follows:²

$$\mathcal{E} = \text{Re}(\mathbf{E}e^{j\omega t}), \quad \mathcal{H} = \text{Re}(\mathbf{H}e^{j\omega t}), \quad \text{etc.} \quad (1-6)$$

where phasor quantities $\mathbf{E}$, $\mathbf{H}$, $\mathbf{D}$, $\mathbf{B}$, ρ_T , and $\mathbf{J}_T$ are complex-valued functions of spatial coordinates only (i.e., time dependence is not shown). Using the phasor

¹Time-varying quantities will be denoted with script quantities, for example, $\mathcal{E} = \mathcal{E}(x, y, z, t)$.

²The student is cautioned that some authors use $e^{-j\omega t}$, which leads to sign differences in subsequent developments.

definitions of the electromagnetic quantities from (1-6) in (1-1) to (1-5) and eliminating the $e^{j\omega t}$ factors that appear on both sides of the equations yields

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B} \quad (1-7)$$

$$\nabla \times \mathbf{H} = j\omega \mathbf{D} + \mathbf{J}_T \quad (1-8)$$

$$\nabla \cdot \mathbf{D} = \rho_T \quad (1-9)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (1-10)$$

$$\nabla \cdot \mathbf{J}_T = -j\omega \rho_T \quad (1-11)$$

The time derivatives in (1-1) to (1-5) have been replaced by a $j\omega$ factor in (1-7) to (1-11) and time-varying electromagnetic quantities have been replaced by their phasor counterpart. This process is similar to the solution of network equations where the time-dependent differential equations are Laplace-transformed and the time derivatives are thus replaced by $j\omega$ (or s). Equations (1-7) to (1-10) are often referred to as the time-harmonic form of Maxwell's equations because they apply to sinusoidally varying (i.e., time-harmonic) fields.

If more than one frequency is present, the time-varying forms of the electromagnetic quantities can be found by inverse transforms after (1-7) to (1-11) have been solved for the phasor quantities as a function of radian frequency ω . This is again analogous to the procedure used to solve network problems. Fortunately, this is not usually necessary in antenna problems since the bandwidth of the signals is usually very small. In the typical case, a carrier frequency is accompanied by some form of modulation giving a spread of frequencies around the carrier. For analysis purposes, we use a single frequency equal to the carrier frequency. Thus, all subsequent material in this book (except in Chap. 11) will assume time-harmonic fields.

The total current density $\mathbf{J}_T$ is composed of an impressed, or source, current $\mathbf{J}$ and a conduction current density term $\sigma \mathbf{E}$, which occurs in response to the impressed current:

$$\mathbf{J}_T = \sigma \mathbf{E} + \mathbf{J} \quad (1-12)$$

The role played by the impressed current density is that of a known quantity. It is quite frequently an assumed current density on an antenna, but as far as the field equations are concerned, it is a known function. The current density $\sigma \mathbf{E}$ is a current density flowing on a nearby conductor due to the fields created by source $\mathbf{J}$ and can be computed after the field equations are solved for $\mathbf{E}$. In addition to conductivity σ , a material is further characterized by permittivity ϵ and permeability μ , where³

$$\mathbf{D} = \epsilon \mathbf{E} \quad (1-13)$$

and

$$\mathbf{B} = \mu \mathbf{H} \quad (1-14)$$

We now rewrite the field equations in preparation for their solution. Substituting (1-12) and (1-13) into (1-8) gives

$$\nabla \times \mathbf{H} = j\omega \left(\epsilon + \frac{\sigma}{j\omega} \right) \mathbf{E} + \mathbf{J} = j\omega \epsilon' \mathbf{E} + \mathbf{J} \quad (1-15)$$

³In general, ϵ and μ can be complex, but in most antenna problems they can be approximated as real constants.

where we have defined $\varepsilon' = \varepsilon - j(\sigma/\omega)$. For antenna problems, we are usually solving for the fields in air surrounding an antenna where $\sigma = 0$ and $\varepsilon' = \varepsilon$. We therefore use ε instead of ε' in subsequent developments. However, if the conductivity is nonzero, ε can be replaced by $\varepsilon' = \varepsilon - j(\sigma/\omega)$. Note also that $\mathbf{E}$ and $\mathbf{H}$ are the fields of primary interest in antennas. They are properly referred to as electric and magnetic field intensities and have units of V/m and A/m, respectively. For conciseness, it is common to refer to them simply as electric and magnetic fields.

Let ρ be the source charge corresponding to the source current density $\mathbf{J}$. Then using (1-12) to (1-14) in (1-7) and (1-9) to (1-11), and repeating (1-15), we have [see Prob. 1.4-2 (for 1-18)]

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \quad (1-16)$$

$$\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} + \mathbf{J} \quad (1-17)$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon} \quad (1-18)$$

$$\nabla \cdot \mathbf{H} = 0 \quad (1-19)$$

$$\nabla \cdot \mathbf{J} = -j\omega\rho \quad (1-20)$$

These are the time-harmonic electromagnetic field equations with source current density $\mathbf{J}$ and source charge density ρ shown explicitly. Sometimes, it is convenient to introduce a fictitious magnetic current density $\mathbf{M}$. Then (1-16) becomes

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} - \mathbf{M} \quad (1-21)$$

Magnetic currents are useful as equivalent sources that replace complicated electric fields.

The solution of the fundamental electromagnetic equations is not complete until the boundary conditions are satisfied. A sufficient set of boundary conditions in the time-harmonic form is

$$\hat{\mathbf{n}} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{J}_s \quad (1-22)$$

$$(\mathbf{E}_2 - \mathbf{E}_1) \times \hat{\mathbf{n}} = \mathbf{M}_s \quad (1-23)$$

where the electric and magnetic surface currents $\mathbf{J}_s$ and $\mathbf{M}_s$ flow on the boundary between two homogeneous media with constitutive parameters ε_1 , μ_1 , σ_1 , and ε_2 , μ_2 , σ_2 . $\mathbf{M}_s$ is zero unless an equivalent magnetic current sheet is used. The unit normal to the boundary surface $\hat{\mathbf{n}}$ is directed from medium 1 into medium 2. The cross products with the unit normal form the tangential components to the boundary, and these equations can be written as

$$H_{\tan 2} = H_{\tan 1} + J_s \quad (1-24)$$

$$E_{\tan 2} = E_{\tan 1} + M_s \quad (1-25)$$

These boundary conditions are derived from the integral form of (1-17) and (1-21). If one side is a perfect electrical conductor, the boundary conditions become

$$H_{\tan} = J_s \quad (1-26)$$

$$E_{\tan} = 0 \quad (1-27)$$

The tangential boundary conditions on the magnetic field intensity are illustrated in Fig. 1-7 for the general case and for the case where medium 1 is a perfect con-

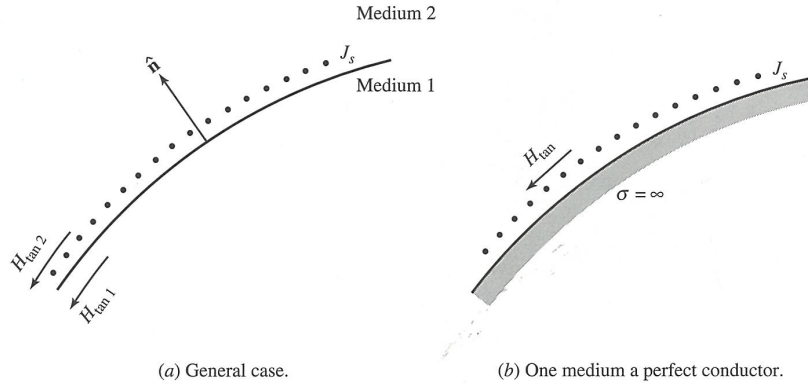


Figure 1-7 Magnetic field intensity boundary condition.

ductor. It is important to note that all field quantities in the boundary condition equations are evaluated at the boundary and the equations apply to each point along the boundary.

Also derivable from Maxwell's curl equations is a conservation of power equation, or Poynting's theorem. Consider a volume v bounded by a closed surface s . The complex power P_s delivered by the sources in v equals the sum of the power P_f flowing out of s , the time-average power $P_{d_{av}}$ dissipated in v , plus the time-average stored power in v :

$$P_s = P_f + P_{d_{av}} + j2\omega(W_{m_{av}} - W_{e_{av}}) \quad (1-28)$$

The complex power flowing out through closed surface s is found from

$$P_f = \frac{1}{2} \oint_s \mathbf{E} \times \mathbf{H}^* \cdot d\mathbf{s} \quad (1-29)$$

where $d\mathbf{s} = ds\hat{\mathbf{n}}$ and $\hat{\mathbf{n}}$ is the unit normal to the surface directed *out* from the surface. Note that $\mathbf{E}$ and $\mathbf{H}$ are peak phasors, not rms, leading to 1/2 in power expressions. The integrand inside this integral is defined as the *Poynting vector*:

$$\mathbf{S} = \frac{1}{2} \mathbf{E} \times \mathbf{H}^* \quad (1-30)$$

which is a power density with units of W/m². The time-average dissipated power in volume v bounded by closed surface s is

$$P_{d_{av}} = \frac{1}{2} \iiint_v \sigma |E|^2 dv \quad (1-31)$$

The time-average stored magnetic energy is

$$W_{m_{av}} = \frac{1}{2} \iiint_v \frac{1}{2} \mu |H|^2 dv \quad (1-32)$$

The time-average stored electric energy is

$$W_{e_{av}} = \frac{1}{2} \iiint_v \frac{1}{2} \epsilon |E|^2 dv \quad (1-33)$$

If the source power is not known explicitly, it can be calculated from the volume current density as follows:

$$P_s = -\frac{1}{2} \iiint_v \mathbf{E} \cdot \mathbf{J}^* dv \quad (1-34)$$

If magnetic current density is used, the term $\mathbf{H}^* \cdot \mathbf{M}$ is added to the integrand in the preceding equation.

From (1-29), we see that the integral of the complex Poynting vector $\frac{1}{2}\mathbf{E} \times \mathbf{H}^*$ over a closed surface s gives the total complex power flowing out through the surface s . It is assumed that the complex Poynting vector represents the complex power density in watts per square meter at a point. Then the complex power through any surface s (not necessarily closed) can be found by integrating the complex Poynting vector over that surface. We are particularly interested in real power (the real component of the complex power that represents the electric and magnetic field intensities being in-phase). The real power flowing through surface s is

$$P = \operatorname{Re} \left(\iint_s \mathbf{S} \cdot d\mathbf{s} \right) = \frac{1}{2} \operatorname{Re} \left(\iint_s \mathbf{E} \times \mathbf{H}^* \cdot d\mathbf{s} \right) \quad (1-35)$$

The reference direction for this average power flow is that of the specified unit normal $\hat{\mathbf{n}}$ contained in $d\mathbf{s} = ds\hat{\mathbf{n}}$.

1.5 SOLUTION OF MAXWELL'S EQUATIONS FOR RADIATION PROBLEMS

This section develops procedures for finding fields radiated by an antenna based on Maxwell's equations. Subsequent antenna analysis in this book begins with these basic relations and it is not necessary to return to Maxwell's equations.

The antenna problem consists of solving for the fields that are created by an impressed current distribution $\mathbf{J}$. In the simplest approach, this current distribution is obtained during the solution process. How to obtain the current distribution will be discussed at various points in the book, but for the moment suppose we have the current distribution and wish to determine the fields $\mathbf{E}$ and $\mathbf{H}$. As mentioned in the previous section, we need only work with the two curl equations of Maxwell's equations as given by (1-16) and (1-17). These are two coupled, linear, first-order differential equations. They are coupled because the unknown functions $\mathbf{E}$ and $\mathbf{H}$ appear in both equations. Thus, these equations must be solved simultaneously. In order to simplify the solution for $\mathbf{E}$ and $\mathbf{H}$ with a given $\mathbf{J}$, we introduce the scalar and vector potential functions Φ and $\mathbf{A}$.

The vector potential is introduced by noting from (1-19) that the divergence of $\mathbf{H}$ is zero:

$$\nabla \cdot \mathbf{H} = 0 \quad (1-36)$$

Therefore, the vector field $\mathbf{H}$ has only circulation; for this reason, it is often called a solenoidal field. Because it possesses only a circulation, it can be represented by the curl of some other vector function as follows:

$$\mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A} \quad (1-37)$$

where $\mathbf{A}$ is the (*magnetic*) *vector potential*. To be more precise, (1-37) is possible because it satisfies (1-36) identically; that is, from (C-9) $\nabla \cdot \nabla \times \mathbf{A} \equiv 0$ for any $\mathbf{A}$. The curl of $\mathbf{A}$ is defined by (1-37), but its divergence is yet to be specified for a complete definition of $\mathbf{A}$.

The scalar potential is introduced by substituting (1-37) into (1-16), which gives

$$\nabla \times (\mathbf{E} + j\omega\mathbf{A}) = 0 \quad (1-38)$$

The expression in parentheses is an electric field, and since its curl is zero, it is a conservative field and behaves as a static electric field. The (*electric*) *scalar potential* Φ is defined from

$$\mathbf{E} + j\omega\mathbf{A} = -\nabla\Phi \quad (1-39)$$

because this definition satisfies (1-38) identically, that is, from (C-10) $\nabla \times \nabla\Phi \equiv 0$ for any Φ . Solving (1-39) for the total electric field gives

$$\mathbf{E} = -j\omega\mathbf{A} - \nabla\Phi \quad (1-40)$$

which may be a familiar result.

The fields $\mathbf{E}$ and $\mathbf{H}$ are now expressed in terms of potential functions by (1-37) and (1-40). If we knew the potential functions, then the fields could be obtained. We now discuss the solution for the potential functions. Substituting (1-37) into (1-17) gives

$$\nabla \times \mathbf{H} = \frac{1}{\mu} \nabla \times \nabla \times \mathbf{A} = j\omega\epsilon\mathbf{E} + \mathbf{J} \quad (1-41)$$

Using the following vector identity, from (C-17),

$$\nabla \times \nabla \times \mathbf{A} \equiv \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A} \quad (1-42)$$

and (1-40) in (1-41) yields

$$\nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A} = j\omega\mu\epsilon(-j\omega\mathbf{A} - \nabla\Phi) + \mu\mathbf{J} \quad (1-43)$$

or

$$\nabla^2\mathbf{A} + \omega^2\mu\epsilon\mathbf{A} - \nabla(j\omega\mu\epsilon\Phi + \nabla \cdot \mathbf{A}) = -\mu\mathbf{J} \quad (1-44)$$

As we mentioned previously, the divergence of $\mathbf{A}$ is yet to be specified. A convenient choice would be one that eliminates the third term of (1-44). It is the Lorentz condition (perhaps more properly attributed to L. Lorenz rather than H. Lorentz [8]):

$$\nabla \cdot \mathbf{A} = -j\omega\mu\epsilon\Phi. \quad (1-45)$$

Then (1-44) reduces to

$$\boxed{\nabla^2\mathbf{A} + \omega^2\mu\epsilon\mathbf{A} = -\mu\mathbf{J}} \quad (1-46)$$

The choice of (1-45) leads to a decoupling of variables: that is, (1-46) involves $\mathbf{A}$ and not Φ . This is the *vector wave equation*. It is a differential equation that can be solved for $\mathbf{A}$ after the impressed current $\mathbf{J}$ is specified. The fields are easily found then from (1-37) and

$$\boxed{\mathbf{E} = -j\omega\mathbf{A} - j \frac{\nabla(\nabla \cdot \mathbf{A})}{\omega\mu\epsilon}} \quad (1-47)$$

where this equation was obtained from (1-40) and (1-45). Notice that only a knowledge of $\mathbf{A}$ is required. A more cumbersome approach would be to solve the scalar wave equation

$$\nabla^2 \Phi + \omega^2 \mu \epsilon \Phi = -\frac{\rho}{\epsilon} \quad (1-48)$$

in addition to the vector wave equation. It is left as a problem to derive (1-48). If this approach is used, $\mathbf{E}$ is found from (1-40). Note that ρ in (1-48) is related to $\mathbf{J}$ in (1-46) by the continuity equation of (1-20).

The vector wave equation (1-46) is solved by forming three scalar equations. This begins by decomposing $\mathbf{A}$ into rectangular components using (C-18):

$$\nabla^2 \mathbf{A} = \hat{x} \nabla^2 A_x + \hat{y} \nabla^2 A_y + \hat{z} \nabla^2 A_z \quad (1-49)$$

Rectangular components are used because the unit vectors in rectangular components can be factored out of the Laplacian since they are not themselves functions of coordinates. This feature is unique to the rectangular coordinate system. Although $\mathbf{A}$ is always decomposed into rectangular components, the Laplacian of each component of $\mathbf{A}$ is expressed in a coordinate system appropriate to the geometry of the problem. The solution proceeds by substituting (1-49) into (1-46) and equating rectangular components:

$$\begin{aligned} \nabla^2 A_x + \beta^2 A_x &= -\mu J_x \\ \nabla^2 A_y + \beta^2 A_y &= -\mu J_y \\ \nabla^2 A_z + \beta^2 A_z &= -\mu J_z \end{aligned} \quad (1-50)$$

where $\beta^2 = \omega^2 \mu \epsilon$. The real-valued constant

$$\beta = \omega \sqrt{\mu \epsilon} \quad (1-51)$$

is recognized as the phase constant for a plane wave.

The three equations in (1-50) are identical in form. After solving one of these equations, the other two are easily solved. We first find the solution for a point source. This unit impulse response solution can then be used to form a general solution by viewing an arbitrary source as a collection of point sources. The differential equation for a point source is

$$\nabla^2 \psi + \beta^2 \psi = -\delta(x) \delta(y) \delta(z) \quad (1-52)$$

where ψ is the response to a point source at the origin, and $\delta(\)$ is the unit impulse function, or Dirac delta function (see Appendix F.1). In spite of the fact that the point source is of infinitesimal extent, its associated current has a direction. This is because in solving practical problems, the point source represents a small subdivision of current that does have a direction. If the point source current is taken as z -directed, then

$$\psi = A_z \quad (1-53)$$

Since the point source is zero everywhere except at the origin, (1-52) becomes

$$\nabla^2 \psi + \beta^2 \psi = 0 \quad (1-54)$$

away from the origin.

This is the complex *scalar wave equation* or *Helmholtz equation*. Because of spherical symmetry, the Laplacian is written in spherical coordinates and ψ has only radial dependence. The two solutions to (1-54) are $e^{-j\beta r}/r$ and $e^{+j\beta r}/r$. These correspond to waves propagating radially outward and inward, respectively. The physically meaningful solution is the one for waves traveling away from the point source. Evaluating the constant of proportionality (see Prob. 1.5-2), we have for the point source solution:

$$\psi = \frac{e^{-j\beta r}}{4\pi r} \quad (1-55)$$

This is the solution to (1-52) and is the magnitude and phase variation with distance r away from a point source located at the origin. If the source were positioned at an arbitrary location, we must compute the distance R between the source location and observation point P (see Fig. 1-8). Then

$$\psi = \frac{e^{-j\beta R}}{4\pi R} \quad (1-56)$$

The point source serves as a starting point for the ideal dipole antenna solution, which is discussed in the next section.

For an arbitrary z -directed current density, the vector potential is also z -directed. If we consider the source to be a collection of point sources weighted by the distribution J_z , the response A_z is a sum of the point source responses of (1-56). This is expressed by the integral over the source volume v' :

$$A_z = \iiint_{v'} \mu J_z \frac{e^{-j\beta R}}{4\pi R} dv' \quad (1-57)$$

Similar equations hold for the x - and y -components. The total solution is then the sum of all components, which is

$$\mathbf{A} = \iiint_{v'} \mu \mathbf{J} \frac{e^{-j\beta R}}{4\pi R} dv' \quad (1-58)$$

This is the solution to the vector wave equation (1-46). The geometry is shown in Fig. 1-8. The coordinate system shown is used to describe both the source point and field point. $\mathbf{r}'$ is the vector from the coordinate origin to the source point, and $\mathbf{r}_p$ is the vector from the coordinate origin to the field point P . The vector $\mathbf{R}$ is the vector

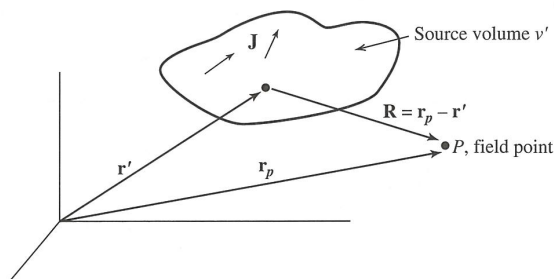


Figure 1-8 Vectors used to solve radiation problems.

from the source point to the field point and is given by $\mathbf{r}_p - \mathbf{r}'$. This geometry is standard and will be used here.

We can summarize rather simply the procedure for finding the fields generated by a current distribution $\mathbf{J}$. First, $\mathbf{A}$ is found from (1-58). The $\mathbf{H}$ field is found from (1-37). The $\mathbf{E}$ field can be found from (1-47), but frequently it is simpler to find $\mathbf{E}$ from (1-17) as

$$\mathbf{E} = \frac{1}{j\omega\epsilon} (\nabla \times \mathbf{H} - \mathbf{J}) \quad (1-59)$$

in the source region, or from

$$\mathbf{E} = \frac{1}{j\omega\epsilon} \nabla \times \mathbf{H} \quad (1-60)$$

if the field point is removed in distance from the source; that is, if $\mathbf{J} = 0$ at point P .

1.6 THE IDEAL DIPOLE

The principles presented in the previous section are used in this section to find the fields of an infinitesimal element of current. We shall use the term **ideal dipole** for a uniform amplitude current that is electrically small with $\Delta z \ll \lambda$. It is ideal in the sense that the current is uniform in both magnitude and phase over the radiating element extent. Such a discontinuous current is difficult to realize in practice; practical realizations that approximate the ideal dipole are presented in Sec. 2.1. The term *current element* is often used for the ideal dipole to describe its application as a section of a larger current associated with an actual antenna. Thus, any practical antenna can first be decomposed into filaments of continuous current that are then subdivided into ideal dipoles. The fields from the antenna are then found by summing contributions from the ideal dipoles. Other terms used for the ideal dipole are *Hertzian electric dipole*, *electric dipole*, *infinitesimal dipole*, and *doublet*. An electrically small, center-fed wire antenna has a current distribution that tapers to zero from the center to the ends of the wire. This *short dipole* antenna has the same pattern as an ideal dipole and is discussed further in Secs. 1.9 and 2.1.

Consider an element of current of length Δz along the z -axis centered on the coordinate origin. It is of constant amplitude I . In this case, the volume integral of (1-58) for vector potential reduces to the one-dimensional integral⁴

$$\mathbf{A} = \hat{\mathbf{z}}\mu I \int_{-\Delta z/2}^{\Delta z/2} \frac{e^{-j\beta R}}{4\pi R} dz' \quad (1-61)$$

The length Δz is very small compared to a wavelength and to the distance R ; see Fig. 1-9. Since Δz is very small, the distance R from points on the current element

⁴The result in (1-61) could also be obtained by representing the current density on the dipole as

$$\mathbf{J} = I \delta(x') \delta(y') \hat{\mathbf{z}} \quad \text{for} \quad -\frac{\Delta z}{2} < z' < \frac{\Delta z}{2}$$

Substituting this into (1-58) yields

$$\mathbf{A} = \hat{\mathbf{z}}\mu I \int_{-\infty}^{\infty} \delta(x') dx' \int_{-\infty}^{\infty} \delta(y') dy' \int_{-\Delta z/2}^{\Delta z/2} \frac{e^{-j\beta R}}{4\pi R} dz'$$

from which (1-61) follows.

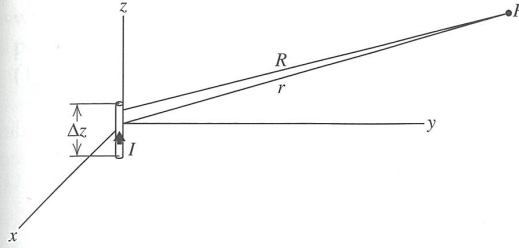


Figure 1-9 The ideal dipole. The current I is uniform, $\Delta z \ll \lambda$, and $R \approx r$.

to the field point approximately equals the distance r from the origin to the field point. Substituting r for R in (1-61) and integrating gives

$$\mathbf{A} = \frac{\mu I e^{-j\beta r}}{4\pi r} \Delta z \hat{\mathbf{z}} \quad (1-62)$$

This is exactly true for a point current element and is approximately true for a small ($\Delta z \ll \lambda$ and $\Delta z \ll R$) but finite uniform current element. The vector potential A_z for a point source was also derived in the previous section; see (1-55) in which $I \Delta z = 1$. For many current sources, we can readily make the substitution of r for R in the denominator of the integrand in (1-61), but usually cannot make the same substitution in the exponent. However, in the case of a very small source, we can use r for R in both the denominator and exponent.

We are now ready to calculate the electromagnetic fields created by the ideal dipole. The magnetic field is found from (1-37) as

$$\mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A} = \frac{1}{\mu} \nabla \times (A_z \hat{\mathbf{z}}) \quad (1-63)$$

If we apply the vector identity (C-16), the preceding equation becomes

$$\mathbf{H} = \frac{1}{\mu} (\nabla A_z) \times \hat{\mathbf{z}} + \frac{1}{\mu} A_z (\nabla \times \hat{\mathbf{z}}) = \frac{1}{\mu} (\nabla A_z) \times \hat{\mathbf{z}} \quad (1-64)$$

The second term is zero because the curl of a constant vector is zero. Substituting (1-62) into (1-64), we have

$$\mathbf{H} = \nabla \left(\frac{I \Delta z e^{-j\beta r}}{4\pi r} \right) \times \hat{\mathbf{z}} \quad (1-65)$$

Applying the gradient in spherical coordinates from (C-33) gives

$$\begin{aligned} \mathbf{H} &= \frac{I \Delta z}{4\pi} \frac{\partial}{\partial r} \left(\frac{e^{-j\beta r}}{r} \right) \hat{\mathbf{r}} \times \hat{\mathbf{z}} \\ &= \frac{I \Delta z}{4\pi} \left[\frac{-j\beta e^{-j\beta r}}{r} - \frac{e^{-j\beta r}}{r^2} \right] \hat{\mathbf{r}} \times \hat{\mathbf{z}} \end{aligned} \quad (1-66)$$

From (C-3), we have

$$\hat{\mathbf{r}} \times \hat{\mathbf{z}} = \hat{\mathbf{r}} \times (\hat{\mathbf{r}} \cos \theta - \hat{\boldsymbol{\theta}} \sin \theta) = -\hat{\boldsymbol{\phi}} \sin \theta \quad (1-67)$$

Substituting (1-67) into (1-66) gives

$$\mathbf{H} = \frac{I \Delta z}{4\pi} \left[\frac{j\beta}{r} + \frac{1}{r^2} \right] e^{-j\beta r} \sin \theta \hat{\boldsymbol{\phi}} \quad (1-68)$$