The electric field can be obtained from (1-60) as

$$\mathbf{E} = \frac{I\,\Delta z}{4\pi}\left[\frac{j\omega\mu}{r} + \sqrt{\frac{\mu}{\varepsilon}}\frac{1}{r^2} + \frac{1}{j\omega\varepsilon r^3}\right]e^{-j\beta r}\sin\theta\,\hat{\boldsymbol{\theta}} + \frac{I\,\Delta z}{2\pi}\left[\sqrt{\frac{\mu}{\varepsilon}}\frac{1}{r^2} + \frac{1}{j\omega\varepsilon}\frac{1}{r^3}\right]e^{-j\beta r}\cos\theta\,\hat{\mathbf{r}} \tag{1-69}$$

Here, β is given by (1-51) and is related to wavelength as

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda} \tag{1-70}$$

Note that if the medium surrounding the dipole is air or free space, $\beta = \omega\sqrt{\mu_o\varepsilon_o}$, where μ_o and ε_o are the permeability and permittivity of free space.

Eqs. (1-68) and (1-69) can be written as

$$\mathbf{H} = \frac{I\,\Delta z}{4\pi}\,j\beta\left(1 + \frac{1}{j\beta r}\right)\frac{e^{-j\beta r}}{r}\sin\theta\,\hat{\boldsymbol{\phi}} \tag{1-71a}$$

$$\mathbf{E} = \frac{I\,\Delta z}{4\pi}\,j\omega\mu\left[1 + \frac{1}{j\beta r} - \frac{1}{(\beta r)^2}\right]\frac{e^{-j\beta r}}{r}\sin\theta\,\hat{\boldsymbol{\theta}} + \frac{I\,\Delta z}{2\pi}\,\eta\left[\frac{1}{r} - j\,\frac{1}{\beta r^2}\right]\frac{e^{-j\beta r}}{r}\cos\theta\,\hat{\mathbf{r}} \tag{1-71b}$$

If βr is large (i.e., $\beta r \gg 1$, or $r \gg \lambda$ since $\beta = 2\pi/\lambda$), then all terms having inverse powers of $j\beta r$ are small compared to unity, and (1-71) reduces to

$$\mathbf{E} = \frac{I\,\Delta z}{4\pi}\,j\omega\mu\,\frac{e^{-j\beta r}}{r}\sin\theta\,\hat{\boldsymbol{\theta}} \tag{1-72a}$$

$$\mathbf{H} = \frac{I\,\Delta z}{4\pi}\,j\beta\,\frac{e^{-j\beta r}}{r}\sin\theta\,\hat{\boldsymbol{\phi}} \tag{1-72b}$$

These are the fields of an ideal dipole at large distances from the dipole. The ratio of these electric and magnetic field components is

$$\frac{E_\theta}{H_\phi} = \frac{\omega\mu}{\beta} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \eta \tag{1-73}$$

where $\eta = \sqrt{\mu/\varepsilon}$ is the intrinsic impedance of the medium (for free space $\eta_o = 376.7\ \Omega \approx 120\pi\ \Omega$). This is a property of plane waves. Also, as we shall see, at large distances from any antenna the fields are related in this manner.

Using the fields of (1-72) in (1-30) gives an expression for the complex power flowing density out of a sphere of radius r surrounding the ideal dipole:

$$\begin{aligned}\mathbf{S} &= \frac{1}{2}\,\mathbf{E}\times\mathbf{H}^* \\ &= \frac{1}{2}\left(\frac{I\,\Delta z}{4\pi}\right)^2 j\omega\mu\,\frac{e^{-j\beta r}}{r}\sin\theta\,\hat{\boldsymbol{\theta}}\times(-j\beta)\,\frac{e^{+j\beta r}}{r}\sin\theta\,\hat{\boldsymbol{\phi}} \\ &= \frac{1}{2}\left(\frac{I\,\Delta z}{4\pi}\right)^2 \omega\mu\beta\,\frac{\sin^2\theta}{r^2}\,\hat{\mathbf{r}}\end{aligned} \tag{1-74}$$

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which is real-valued and radially directed, both characteristics of radiation. The total power flowing out through a sphere of radius r surrounding the ideal dipole using (1-29) is

$$P_f = \iint \mathbf{S} \cdot d\mathbf{s} = \frac{1}{2}\left(\frac{I\,\Delta z}{4\pi}\right)^2 \omega\mu\beta \int_0^{2\pi} d\phi \int_0^{\pi} \sin^3\theta\, d\theta$$
$$= \frac{1}{2}\left(\frac{I\,\Delta z}{4\pi}\right)^2 \omega\mu\beta 2\pi \frac{4}{3}$$
$$= \frac{\omega\mu\beta}{12\pi}(I\,\Delta z)^2 \tag{1-75}$$

This is a real quantity, and real power indicates dissipated power. It is dissipated in the sense that it travels away from the source. In fact, the average power going out through a sphere of radius r can be found as indicated in (1-35) by taking the real part of (1-75), which leaves it unchanged. This power expression is independent of r, and thus if we integrate over a sphere of larger radius, we still have the same total power streaming through it. We refer to this type of power as *radiated power*. The fields in (1-72) are called *radiation fields*.

The general field expressions of (1-71) are valid at any distance from an ideal dipole and are important in some applications and for understanding the input impedance of a dipole. For distances so close to the dipole that $\beta r \ll 1$, or $r \ll \lambda$, only the dominant terms with the largest inverse powers of r need be retained in each component of (1-71):

$$\mathbf{H^{nf}} = \frac{I\,\Delta z e^{-j\beta r}}{4\pi r^2}\sin\theta\,\hat{\boldsymbol{\phi}} \tag{1-76a}$$

$$\mathbf{E^{nf}} = -j\eta\frac{I\,\Delta z}{4\pi\beta}\frac{e^{-j\beta r}}{r^3}\sin\theta\,\hat{\boldsymbol{\theta}} - j\eta\frac{I\,\Delta z}{2\pi\beta}\frac{e^{-j\beta r}}{r^3}\cos\theta\,\hat{\mathbf{r}} \tag{1-76b}$$

These are referred to as the *near fields* of the antenna. Actually, the magnetic field of (1-76a) which varies as $1/r^2$ is that of a short, steady or slowly oscillating current, that is, an induction field. The electric fields of (1-76b) vary as $1/r^3$ and are those of an electrostatic or quasi-static dipole with charges of $+q$ and $-q$ spaced Δz apart. Note that the electric field components E_θ^{nf} and E_r^{nf} are in-phase, but are in phase-quadrature with the magnetic field H_ϕ^{nf}, indicating reactive power. This can be shown directly using these near fields in the complex Poynting vector expression of (1-30):

$$\mathbf{S^{nf}} = \frac{1}{2}[E_\theta^{nf}H_\phi^{nf*}\hat{\mathbf{r}} - E_r^{nf}H_\phi^{nf*}\hat{\boldsymbol{\theta}}]$$
$$= -\frac{j\eta}{2\beta}\left(\frac{I\,\Delta z}{4\pi}\right)^2\frac{1}{r^5}(\sin^2\theta\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}) \tag{1-77}$$

Note that this power density vector is imaginary and therefore has no time-average radial power flow. The radiation fields, in contrast, are in-phase giving a real-valued Poynting vector that is radially directed; see (1-72) and (1-74). The imaginary power density corresponds to standing waves, rather than traveling waves associated with radiation, and indicate stored energy as in any reactive device. The quadrature phase relationship between the electric and magnetic field components of (1-76)

indicates that energy is interchanged between these fields with time. That is, at one instant of time electric fields near the dipole are strong close to maximum charge regions and a quarter-period later energy is stored in the magnetic field, primarily close to the center of the dipole where the current is maximum.

The imaginary power density in the near field is manifested by a reactive component of the antenna input impedance. The real part of the input impedance represents radiation if ohmic losses on the antenna structure can be neglected. Antenna impedance will be discussed further in Sec. 1.9. The power density associated with radiation exists everywhere and passes through the near field. The radiated power density from (1-74) and the near-field power density from (1-77) are both maximum for $\theta = 90°$. The distance for which the maximum radiated and reactive powers are equal for the ideal dipole is $r = \lambda/2\pi$. That is, interior to this radius reactive power dominates. This region is sometimes referred to as the *radiansphere*.

At large distances from an antenna, called the *far-field region*, all power is radiated power. The far field is further characterized by the fact that the angular distribution around the antenna (i.e., the radiation pattern) is independent of distance from the antenna. Field regions and the distance away from an antenna where the far field begins are discussed further in Sec. 1.7.3.

1.7 RADIATION PATTERNS

We briefly introduce the radiation pattern in Sec. 1.3 as a description of the angular variation of radiation level around an antenna. This is perhaps the most important characteristic of an antenna. In this section, we present several definitions associated with patterns and develop the general procedures for calculating radiation patterns.

1.7.1 Radiation Pattern Basics

A *radiation pattern* (*antenna pattern*) is a graphical representation of the radiation (far-field) properties of an antenna. We have seen that the radiation fields from a transmitting antenna vary inversely with distance, e.g., $1/r$. The variation with observation angles (θ, ϕ), however, depends on the antenna.

Radiation patterns can be understood by examining the ideal dipole. The fields radiated from an ideal dipole are shown in Fig. 1-10*a* over the surface of a sphere of radius r that is in the far field. The length and orientation of the field vectors follow from (1-72); they are shown for an instant of time for which the fields are peak. The angular variation of E_θ and H_ϕ over the sphere is sin θ. An electric field probe antenna moved over the sphere surface and oriented parallel to E_θ will have an output proportional to sin θ; see Fig. 9-7. Any plane containing the z-axis has the same radiation pattern since there is no ϕ variation in the fields. A pattern taken in one of these planes is called an *E-plane pattern* because it contains the electric vector. A pattern taken in a plane perpendicular to an E-plane and cutting through the test antenna (the xy-plane in this case) is called an *H-plane pattern* because it contains the magnetic field H_ϕ. The E- and H-plane patterns, in general, are referred to as *principal plane patterns*. The E- and H-plane patterns for the ideal dipole are shown in Figs. 1-10*b* and 1-10*c*. These are polar plots in which the distance from the origin to the curve is proportional to the field intensity; they are often called polar patterns or polar diagrams.

The complete pattern for the ideal dipole is shown in isometric view with a slice removed in Fig. 1-10*d*. This solid polar radiation pattern resembles a "doughnut"

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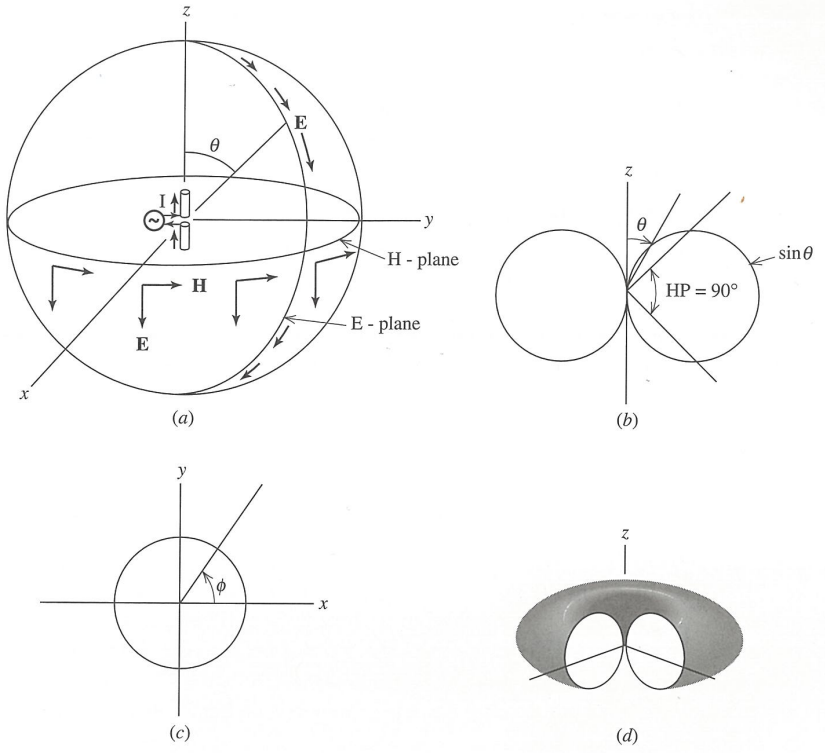


Figure 1-10 Radiation from an ideal dipole. (*a*) Field components. (*b*) *E*-plane radiation pattern polar plot of $|E_\theta|$ or $|H_\phi|$. (*c*) *H*-plane radiation pattern polar plot of $|E_\theta|$ or $|H_\phi|$. (*d*) Three-dimensional plot of radiation pattern.

with no hole. It is referred to as an *omnidirectional pattern* since it is uniform in the *xy*-plane. Omnidirectional antennas are very popular in ground-based applications with the omnidirectional plane horizontal. When encountering new antennas, the reader should attempt to visualize the complete pattern in three dimensions.

1.7.2 Radiation from Line Currents

Radiation patterns in general can be calculated in a manner similar to that used for the ideal dipole if the current distribution on the antenna is known. This is done by first finding the vector potential given in (1-58). As a simple example, consider a filament of current along the z-axis and located near the origin. Many antennas can be modeled by this **line source**; straight wire antennas are good examples. In this case, the vector potential has only a z-component and the vector potential integral is one-dimensional[5]:

$$A_z = \mu \int I(z') \frac{e^{-j\beta R}}{4\pi R} dz' \tag{1-78}$$

[5]This result could also be obtained by using $J_z(r') = I(z')\,\delta(x')\,\delta(y')$ in (1-57), where $dv' = dx'\,dy'\,dz'$.

Due to the symmetry of the source, we expect that the radiation fields will not vary with ϕ. This is because as the observer moves around the source such that r and z are constant, the appearance of the source remains the same; thus, its radiation fields are also unchanged. Therefore, for simplicity we will confine the observation point to a fixed ϕ in the yz-plane ($\phi = 90°$) as shown in Fig. 1-11. Then from Fig. 1-11, we see that

$$r^2 = y^2 + z^2 \tag{1-79}$$

$$z = r\cos\theta \tag{1-80}$$

$$y = r\sin\theta \tag{1-81}$$

If we apply the general geometry of Fig. 1-8 to this case, $\mathbf{r}_p = \mathbf{r} = y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$ and $\mathbf{r}' = z'\hat{\mathbf{z}}$ lead to $\mathbf{R} = \mathbf{r}_p - \mathbf{r}' = y\hat{\mathbf{y}} + (z - z')\hat{\mathbf{z}}$, and then

$$R = \sqrt{y^2 + (z - z')^2} = \sqrt{y^2 + z^2 - 2zz' + (z')^2} \tag{1-82}$$

Substituting (1-79) and (1-80) into (1-81), to put all field point coordinates into the spherical coordinate system, gives

$$R = \{r^2 + [-2r\cos\theta\, z' + (z')^2]\}^{1/2} \tag{1-83}$$

In order to develop approximate expressions for R, we expand (1-83) using the binomial theorem (F-4):

$$\begin{aligned} R &= (r^2)^{1/2} + \frac{1}{2}(r^2)^{-1/2}[-2r\cos\theta\, z' + (z')^2] + \frac{\frac{1}{2}(-\frac{1}{2})}{2}(r^2)^{-3/2} \\ &\quad \cdot [-2r\cos\theta\, z' + (z')^2]^2 + \cdots \\ &= r - z'\cos\theta + \frac{(z')^2\sin^2\theta}{2r} + \frac{(z')^3\sin^2\theta\cos\theta}{2r^2} + \cdots \end{aligned} \tag{1-84}$$

The terms in this series decrease as the power of z' increases if z' is small compared to r. This expression for R is used in the radiation integral (1-78) to different degrees of approximation. In the denominator (which affects only the amplitude), we let

$$R \approx r \tag{1-85}$$

We can do this because in the far field r is very large compared to the antenna size, so $r \gg z' \geq z'\cos\theta$. In the phase term $-\beta R$, we must be more accurate when computing the distance from points along the line source to the observation point.

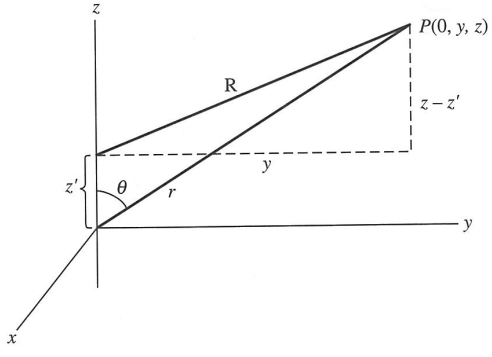


Figure 1-11 Geometry used for field calculations of a line source along the z-axis.

The integral (1-78) sums the contributions from all the points along the line source. Although the amplitude of waves due to each source point is essentially the same, the phase can be different if the path length differences are a sizable fraction of a wavelength. We, therefore, include the first two terms of the series in (1-84) for the R in the numerator of (1-78), giving

$$R \approx r - z' \cos\theta \tag{1-86}$$

Using the far-field approximations (1-85) and (1-86) in (1-78) yields

$$A_z = \mu \int I(z') \frac{e^{-j\beta(r - z'\cos\theta)}}{4\pi r}\, dz' = \mu \frac{e^{-j\beta r}}{4\pi r} \int I(z') e^{j\beta z'\cos\theta}\, dz' \tag{1-87}$$

where the integral is over the extent of the line source. This may be recognized as a Fourier-transform-type integral; see Sec. 4.3. Next, the magnetic field is found using (1-37):

$$\begin{aligned} \mathbf{H} &= \frac{1}{\mu} \nabla \times \mathbf{A} = \frac{1}{\mu} \nabla \times (A_z \hat{\mathbf{z}}) \\ &= \frac{1}{\mu} \nabla \times (-A_z \sin\theta \hat{\boldsymbol{\theta}} + A_z \cos\theta \hat{\mathbf{r}}) \end{aligned} \tag{1-88}$$

where (C-3) was used. Since A_z is a function of r and θ, the curl in spherical coordinates, as given by (C-35), leads to

$$\mathbf{H} = \hat{\boldsymbol{\phi}} \frac{1}{\mu} \frac{1}{r} \left[\frac{\partial}{\partial r} (-rA_z \sin\theta) - \frac{\partial}{\partial\theta} (A_z \cos\theta) \right] \tag{1-89}$$

Substitution of (1-87) into the above gives

$$\begin{aligned} \mathbf{H} = \hat{\boldsymbol{\phi}} \frac{1}{\mu} \Bigg\{ & \mu \frac{-\sin\theta}{4\pi r} \int I(z') e^{j\beta z'\cos\theta}\, dz' \frac{\partial}{\partial r} e^{-j\beta r} - \frac{e^{-j\beta r}}{4\pi r^2} \frac{\partial}{\partial\theta} \\ & \cdot \left[\mu \cos\theta \int I(z') e^{j\beta z'\cos\theta}\, dz' \right] \Bigg\} \\ = \hat{\boldsymbol{\phi}} \frac{1}{\mu} \frac{e^{-j\beta r}}{4\pi r} \Bigg\{ & j\beta\, \mu \sin\theta \int I(z') e^{j\beta z'\cos\theta}\, dz' \\ & - \frac{1}{r} \frac{\partial}{\partial\theta} \left[\mu \cos\theta \int I(z') e^{j\beta z'\cos\theta}\, dz' \right] \Bigg\} \end{aligned} \tag{1-90}$$

The ratio of the first term to the second term above is of the order βr. If $\beta r \gg 1$, the second term is small compared to the first and can be neglected, as we did for the far-field approximation of the ideal dipole in Sec. 1.6. Thus, (1-90) becomes

$$\mathbf{H} = \hat{\boldsymbol{\phi}} \frac{j\beta}{\mu} \sin\theta\, \mu \frac{e^{-j\beta r}}{4\pi r} \int I(z') e^{j\beta z'\cos\theta}\, dz' = \frac{j\beta}{\mu} \sin\theta\, A_z \hat{\boldsymbol{\phi}} \tag{1-91}$$

The electric field is found from (1-47), which is

$$\mathbf{E} = -j\omega\mathbf{A} - j \frac{\nabla(\nabla \cdot \mathbf{A})}{\omega\mu\epsilon} \tag{1-92}$$

Using (1-78) in (1-79) and retaining only the r^{-1} term (and assuming $\beta r \gg 1$) lead to the far-field approximation

$$\mathbf{E} = -j\omega A_\theta \hat{\boldsymbol{\theta}} = j\omega \sin\theta\, A_z \hat{\boldsymbol{\theta}} \tag{1-93}$$

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Note that this result is the portion of the first term of (1-92) that is transverse to $\hat{\mathbf{r}}$ because $-j\omega\mathbf{A} = -j\omega(-A_z \sin\theta\hat{\boldsymbol{\theta}} + A_z \cos\theta\hat{\mathbf{r}})$. This is an important general result for z-directed sources that is not restricted to line sources.

The radiation fields from a z-directed line source (any z-directed current source in general) are H_ϕ and E_θ, and are found from (1-91) and (1-93). The only remaining problem is to calculate A_z, which is given by (1-57) in general and by (1-87) for z-directed line sources. The calculation of A_z is the focal point of antenna analysis. We will return to this topic after pausing to further examine the characteristics of the far-field region.

The ratio of the radiation field components as given by (1-91) and (1-93) yields

$$E_\theta = \frac{\omega\mu}{\beta} H_\phi = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} H_\phi = \eta H_\phi \tag{1-94}$$

where $\eta = \sqrt{\mu/\varepsilon}$ is the intrinsic impedance of the medium. Thus, the radiation fields are perpendicular to each other and to the direction of propagation $\hat{\mathbf{r}}$ and their magnitudes are related by (1-94). These are the familiar properties of a plane wave. They also hold for the general form of a "transverse electromagnetic (TEM) wave" that has both the electric and magnetic fields transverse to the direction of propagation. In general, radiation from a finite antenna is a special case of a TEM wave, called a "spherical wave," that propagates radially outward from the antenna and the radiation fields have no radial components. Spherical wave behavior is also characterized by the $e^{-j\beta r}/4\pi r$ factor in the field expressions; see (1-91). The $e^{-j\beta r}$ phase factor indicates a traveling wave propagating radially outward from the origin and the $1/r$ magnitude dependence leads to constant power flow just as with the infinitesimal dipole. In fact, the radiation fields of all antennas of finite extent display this dependence with distance from the antenna.

Another way to view radiation field behavior is to note that spherical waves appear to an observer in the far field to be a plane wave. This "local plane wave behavior" occurs because the radius of curvature of the spherical wave is so large that the phase front is nearly planar over a local region.

1.7.3 Far-Field Conditions and Field Regions

The results for the line current from the previous section are easily generalized to an arbitrary, but finite-sized, antenna. In the far field of an antenna, the fields exhibit local plane wave behavior and have $1/r$ magnitude dependence. In this section, we develop the conditions for determining the minimum distance from an antenna for far-field behavior. This begins with a geometric interpretation for far-field approximations.

If parallel lines (or rays) are drawn from each point on a line current as shown in Fig. 1-12, the distance R to the far field is geometrically related to r by (1-86), which was derived by neglecting high-order terms in the expression for R in (1-84). The parallel ray assumption is exact only when the observation point is at infinity, but it is a good approximation in the far field. Radiation calculations often start by assuming parallel rays and then determining R for the phase by geometrical techniques. From the general source shown in Fig. 1-13, we see that

$$R = r - r' \cos\alpha \tag{1-95}$$

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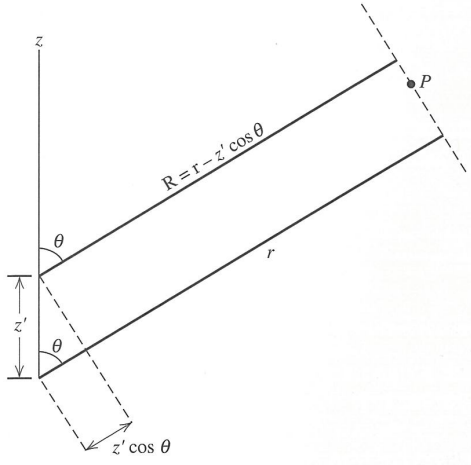


Figure 1-12 Parallel ray approximation for far-field calculations of a line source.

Using the definition of dot product, we have

$$R = r - r' \frac{\mathbf{r} \cdot \mathbf{r}'}{rr'}$$

or

$$\boxed{R = r - \hat{\mathbf{r}} \cdot \mathbf{r}'} \tag{1-96}$$

This is a general approximation to R for the phase factor in the radiation integral. Notice that if $\mathbf{r}' = z'\hat{\mathbf{z}}$, as for line sources along the z-axis, (1-96) reduces to (1-86).

The definition of the distance from the source where the far field begins is where errors due to the parallel ray approximation become insignificant. The distance

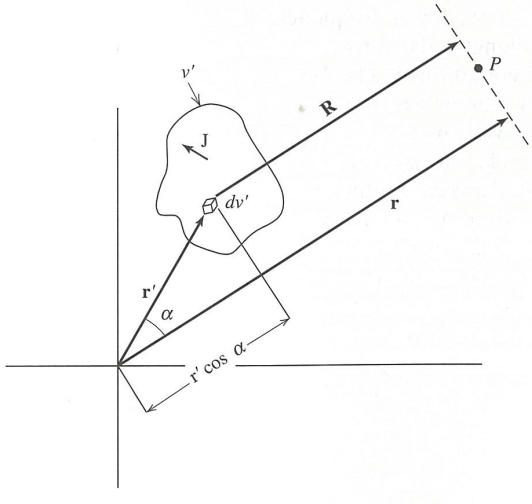


Figure 1-13 Parallel ray approximation for far-field calculations of a general source.

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where the far field begins, r_{ff}, is taken to be that value of r for which the path length deviation due to neglecting the third term of (1-84) is a sixteenth of a wavelength. This corresponds to a phase error (by neglecting the third term) of $2\pi/\lambda \cdot \lambda/16 = \pi/8$ rad $= 22.5°$.

If D is the length of the line source, r_{ff} is found by equating the maximum value of the third term of (1-84), which occurs for $z' = D/2$ and $\theta = 90°$, to a sixteenth of a wavelength:

$$\frac{(D/2)^2}{2r_{ff}} = \frac{\lambda}{16} \tag{1-97}$$

Solving for r_{ff} gives

$$\boxed{r_{ff} = \frac{2D^2}{\lambda}} \tag{1-98}$$

The far-field region is $r \geq r_{ff}$ and r_{ff} is called the *far-field distance*, or Rayleigh distance.

The far-field conditions are summarized as follows:

$$\boxed{\begin{aligned} r &> \frac{2D^2}{\lambda} && \text{(1-99a)} \\ r &\gg D && \text{(1-99b)} \\ r &\gg \lambda && \text{(1-99c)} \end{aligned}} \quad \textit{far-field conditions}$$

The condition $r \gg D$ was mentioned in association with the approximation $R \approx r$ of (1-85) for use in the magnitude dependence. The condition $r \gg \lambda$ follows from $\beta r = (2\pi r/\lambda) \gg 1$ that was used to reduce (1-90) to (1-91). Usually, the far field is taken to begin at a distance given by (1-98), where D is the maximum dimension of the antenna. This is usually a sufficient condition for antennas operating in the UHF region and above. At lower frequencies, where the antenna can be small compared to the wavelength, the far-field distance may have to be greater than $2D^2/\lambda$ in order that all conditions in (1-99) are satisfied. See Prob. 1.7-4.

The concept of field regions was introduced in Sec. 1.6 and illustrated with the fields of an ideal dipole. We can now generalize that discussion to any finite antenna of maximum extent D. The distance to the far field is $2D^2/\lambda$. This zone was historically called the *Fraunhofer region* if the antenna is focused at infinity; that is, if the rays at large distances from the antenna when transmitting are parallel. In the far-field region, the radiation pattern is independent of distance. For example, the $\sin\theta$ pattern of an ideal dipole is valid anywhere in its far field. The zone interior to r_{ff}, called the near field, is divided into two subregions. The *reactive near-field region* is closest to the antenna and is that region for which the reactive field dominates over the radiative fields. This region extends to a distance $0.62\sqrt{D^3/\lambda}$ from the antenna (see Prob. 1.7-7), as long as $D \gg \lambda$. We mentioned in Sec. 1.6 that for an ideal dipole, for which $D = \Delta z \ll \lambda$, this distance is $\lambda/2\pi$. Between the reactive near-field and far-field regions is the *radiating near-field region* in which the radiation fields dominate and where the angular field distribution depends on distance

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from the antenna. For an antenna focused at infinity, the region is sometimes referred to as the *Fresnel region*. We can summarize the field region distances for cases where $D \gg \lambda$ as follows:

Region	Distance from antenna (r)	
Reactive near field	0 to $0.62\sqrt{D^3/\lambda}$	(1-100a)
Radiating near field	$0.62\sqrt{D^3/\lambda}$ to $2D^2/\lambda$	(1-100b)
Far field	$2D^2/\lambda$ to ∞	(1-100c)

1.7.4 Steps in the Evaluation of Radiation Fields

The derivation for the fields radiated by a line source in Sec. 1.7.2 can be generalized for application to any antenna. Fortunately, the derivation itself need not be repeated each time an antenna is analyzed. That is, it is not necessary to return to Maxwell's equation with each new antenna system. Instead, we work from the results of the line source and its generalizations, which can be reduced to the three-step procedure detailed below:

1. **Find A.** Select a coordinate system most compatible with the geometry of the antenna, using the notation of Fig. 1-8. In general, use (1-58) with $R \approx r$ in the magnitude factor and the parallel ray approximation of (1-96) for determining phase differences over the antenna. These yield

$$\mathbf{A} = \mu \frac{e^{-j\beta r}}{4\pi r} \iiint_{v'} \mathbf{J} e^{j\beta \hat{\mathbf{r}}\cdot\mathbf{r}'} \, dv' \tag{1-101}$$

For z-directed sources,

$$\mathbf{A} = \hat{\mathbf{z}}\mu \frac{e^{-j\beta r}}{4\pi r} \iiint_{v'} J_z e^{j\beta \hat{\mathbf{r}}\cdot\mathbf{r}'} \, dv' \tag{1-102}$$

For z-directed line sources on the z-axis,

$$\mathbf{A} = \hat{\mathbf{z}}\mu \frac{e^{-j\beta r}}{4\pi r} \int I(z') e^{j\beta z' \cos\theta} \, dz' \tag{1-103}$$

which is (1-87).

2. **Find E.** In general, use the component of

$$\mathbf{E} = -j\omega\mathbf{A} \tag{1-104}$$

which is transverse to the direction of propagation $\hat{\mathbf{r}}$. This is expressed formally as

$$\mathbf{E} = -j\omega\mathbf{A} - (-j\omega\mathbf{A} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} = -j\omega(A_\theta\hat{\boldsymbol{\theta}} + A_\phi\hat{\boldsymbol{\phi}}) \tag{1-105}$$

which arises from the component of $\mathbf{A}$ tangent to the far-field sphere. For z-directed sources,

$$\mathbf{E} = j\omega \sin\theta \, A_z\hat{\boldsymbol{\theta}} \tag{1-106}$$

which is (1-93)

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3. Find H. In general, use the plane wave relation.

$$\mathbf{H} = \frac{1}{\eta}\hat{\mathbf{r}} \times \mathbf{E} \tag{1-107}$$

This equation expresses the fact that in the far field, the directions of **E** and **H** are perpendicular to each other and to the direction of propagation, and also that their magnitudes are related by η. For z-directed sources,

$$H_\phi = \frac{E_\theta}{\eta} \tag{1-108}$$

which is (1-73).

The most difficult step is the first, calculating the radiation integral. This topic will be discussed many times throughout the book, but to immediately develop an appreciation for the process, we will present an example. This uniform line source example also provides a specific setting for introducing general radiation pattern concepts and definitions.

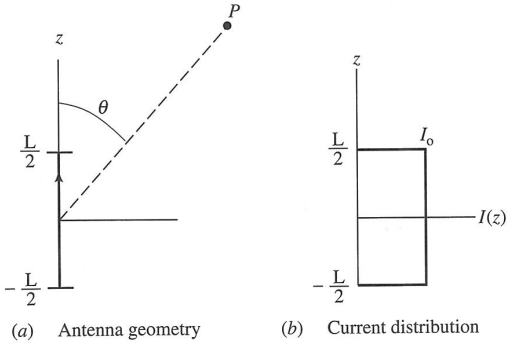
EXAMPLE 1-1 *The Uniform Line Source*

The **uniform line source** is a line source for which the current is constant along its extent. If we use a z-directed uniform line source centered on the origin and along the z-axis, the current is

$$I(z') = \begin{cases} I_o & x' = 0, \quad y' = 0, \quad |z'| \le \dfrac{L}{2} \\ 0 & \textit{elsewhere} \end{cases} \tag{1-109}$$

where L is the length of the line source; see Fig. 1-14. We first find A_z from (1-103) as follows:

$$\begin{aligned} A_z &= \mu \frac{e^{-j\beta r}}{4\pi r} \int_{-L/2}^{L/2} I_o e^{j\beta z' \cos\theta}\, dz' \\ &= \mu \frac{e^{-j\beta r}}{4\pi r} I_o \left[\frac{e^{j\beta(L/2)\cos\theta} - e^{-j\beta(L/2)\cos\theta}}{j\beta \cos\theta} \right] \\ &= \mu \frac{I_o L e^{-j\beta r}}{4\pi r} \frac{\sin[(\beta L/2)\cos\theta]}{(\beta L/2)\cos\theta}. \end{aligned} \tag{1-110}$$



(*a*) Antenna geometry (*b*) Current distribution

Figure 1-14 The uniform line source (Example 1-1).

The electric field from (1-106) is then

$$\mathbf{E} = j\omega \sin\theta\, A_z\hat{\boldsymbol{\theta}} = \frac{j\omega\mu I_o L e^{-j\beta r}}{4\pi r} \sin\theta \frac{\sin[(\beta L/2)\cos\theta]}{(\beta L/2)\cos\theta}\hat{\boldsymbol{\theta}} \tag{1-111}$$

The magnetic field is simply found from this using $H_\phi = E_\theta/\eta$.

1.7.5 Radiation Pattern Definitions

Since the radiation pattern is the variation over a sphere centered on the antenna, r is constant and we have only θ and ϕ variation of the field. It is convenient to normalize the field expression such that its maximum value is unity. This is accomplished as follows for a z-directed source that has only a θ-component of $\mathbf{E}$:

$$F(\theta, \phi) = \frac{E_\theta}{E_\theta(\max)} \tag{1-112}$$

where $F(\theta, \phi)$ is the *normalized field pattern* and $E_\theta(\max)$ is the maximum value of the magnitude of E_θ over a sphere of radius r.

In general, E_θ can be complex-valued and, therefore, so can $F(\theta, \phi)$. In this case, the phase is usually set to zero at the same point the magnitude is normalized to unity. This is appropriate since we are only interested in relative phase behavior. The pattern magnitude is obtained by taking the absolute value of (1-112).

An element of current on the z-axis has a normalized field pattern from (1-72a) of

$$F(\theta) = \frac{(I\,\Delta z/4\pi)j\omega\mu(e^{-j\beta r}/r)\sin\theta}{(I\,\Delta z/4\pi)j\omega\mu(e^{-j\beta r}/r)} = \sin\theta \tag{1-113}$$

and there is no ϕ variation. The normalized field pattern for the uniform line source is from (1-111) in (1-112)

$$F(\theta) = \sin\theta \frac{\sin[(\beta L/2)\cos\theta]}{(\beta L/2)\cos\theta} \tag{1-114}$$

and again there is no ϕ variation. The second factor of this expression is the function $\sin(u)/u$ and we will encounter it frequently. It has a maximum value of unity at $u = 0$; this corresponds to $\theta = 90°$, where $u = (\beta L/2)\cos\theta$. Substituting $\theta = 90°$ in (1-114) gives unity and we see that $F(\theta)$ is properly normalized.

In general, a normalized field pattern can be written as the product

$$\boxed{F(\theta, \phi) = g(\theta, \phi)f(\theta, \phi)} \tag{1-115}$$

where $g(\theta, \phi)$ is the *element factor* and $f(\theta, \phi)$ the *pattern factor*. The pattern factor comes from the integral over the current and is strictly due to the distribution of current in space. The element factor is the pattern of an infinitesimal current element in the current distribution. For example, we found for a z-directed current element that $F(\theta) = \sin\theta$. This is, obviously, also the element factor, so

$$g(\theta) = \sin\theta \tag{1-116}$$

for a z-directed current element. Actually, this factor originates from (1-93) and can be interpreted as the projection of the current element in the θ-direction. In other words, at $\theta = 90°$ we see the maximum length of the current, whereas at $\theta = 0°$ or

180° we see the endview of an infinitesimal current that yields no radiation. The sin θ factor expresses the fraction of the size of the current as seen from the observation angle θ. On the other hand, the pattern factor $f(\theta, \phi)$ represents the integrated effect of radiation contributions from the current distribution, which can be treated as being made up of many current elements. The pattern value in a specific direction is then found by summing the parallel rays from each current element to the far field with the magnitude and phase of each included. The radiation integral of (1-101) sums the far-field contributions from the current elements and when normalized yields the pattern factor. Antenna analysis is usually easier to understand by considering the antenna to be transmitting as we have here. However, most antennas are reciprocal and thus their radiation properties are identical when used for reception; see Sec. 9.4.

A typical power pattern is shown in Fig. 1-15 as a polar plot. The rays from various parts of an antenna arrive in the far field with different magnitude and phase due to variations in the current where the ray originated on the antenna and due to phase changes arising from path length differences to the far field. These rays interfere, as computed through the radiation integral, and produce a "lobing" effect.

The radiation lobe containing the direction of maximum radiation is the *major lobe*, *main lobe*, or *main beam*. It is the most intense portion of the radiation pattern and is caused by the fact that the rays from various parts of the antenna arrive in the far field more nearly in-phase than they do for other directions. For a source with constant phase, all rays arrive in-phase in the direction normal to the antenna and the pattern is maximum there. For the ideal dipole, we have said that the source is so small that there are essentially no phase differences for rays along the source and thus the pattern factor is unity.

For the z-directed uniform line source pattern of (1-114), we can identify the factors as

$$g(\theta) = \sin \theta \tag{1-117}$$

and

$$f(\theta) = \frac{\sin[(\beta L/2) \cos \theta]}{(\beta L/2) \cos \theta} \tag{1-118}$$

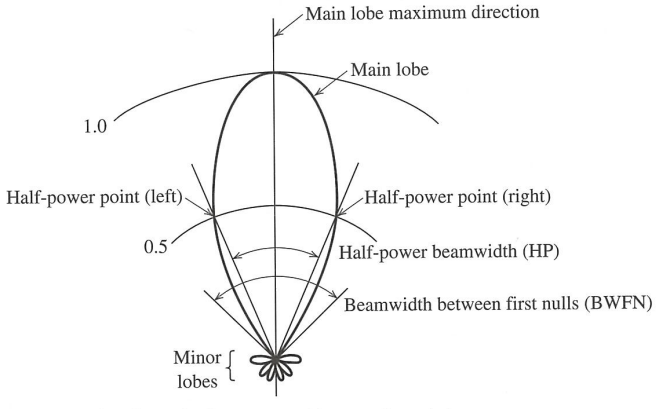


Figure 1-15 A typical power pattern polar plot.

For long line sources ($L \gg \lambda$), the pattern factor of (1-118) is much sharper than the element factor sin θ, and the total pattern is approximately that of (1-118); that is, $F(\theta) \approx f(\theta)$. Hence, in many cases we need only work with $f(\theta)$, which is obtained from (1-103). If we allow the beam to be scanned (this will be discussed in Sec. 1.7.6), the element factor becomes important as the pattern maximum approaches the z-axis.

Frequently, the directional properties of the radiation from an antenna are described by another form of radiation pattern, the power pattern. The power pattern gives angular dependence of the power density and is found from the θ, ϕ variation of the r-component of the Poynting vector. For z-directed sources, $H_\phi = E_\theta/\eta$ so the r-component of the Poynting vector is $(1/2)E_\theta H_\phi^* = |E_\theta|^2/2\eta$ and the normalized power pattern is simply the square of its field pattern magnitude $P(\theta) = |F(\theta)|^2$. The general normalized *power pattern* is

$$\boxed{P(\theta, \phi) = |F(\theta, \phi)|^2} \tag{1-119}$$

The normalized power pattern for a z-directed current element is

$$P(\theta) = \sin^2 \theta \tag{1-120}$$

and for a z-directed uniform line source is

$$P(\theta) = \left\{ \sin \theta \frac{\sin[(\beta L/2) \cos \theta]}{(\beta L/2) \cos \theta} \right\}^2 \tag{1-121}$$

Frequently, patterns are plotted in decibels. *It is important to recognize that the field (magnitude) pattern and power pattern are the same in decibels.* This follows directly from the definitions. For field intensity in decibels,

$$|F(\theta, \phi)|_{\text{dB}} = 20 \log|F(\theta, \phi)| \tag{1-122}$$

and for power in decibels,

$$P(\theta, \phi)_{\text{dB}} = 10 \log P(\theta, \phi) = 10 \log|F(\theta, \phi)|^2 = 20 \log|F(\theta, \phi)| \tag{1-123}$$

and we see that

$$\boxed{P(\theta, \phi)_{\text{dB}} = |F(\theta, \phi)|_{\text{dB}}} \tag{1-124}$$

1.7.6 Radiation Pattern Parameters

A typical antenna power pattern is shown in Fig. 1-15 as a polar plot in linear units (rather than decibels). It consists of several *lobes*. The *main lobe* (or main beam or major lobe) is the lobe containing the direction of maximum radiation. There is also usually a series of lobes smaller than the main lobe. Any lobe other than the main lobe is called a *minor lobe*. Minor lobes are composed of *side lobes* and *back lobes*. Back lobes are directly opposite the main lobe, or sometimes they are taken to be the lobes in the half-space opposite the main lobe. The term side lobe is sometimes reserved for those minor lobes near the main lobe, but is most often taken to be synonymous with minor lobe; we will use the latter convention.

The radiation from an antenna is represented mathematically through the radiation pattern function, $F(\theta, \phi)$ for field and $P(\theta, \phi)$ for power. This angular distribution of radiation is visualized through various graphical representations of the

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pattern, which we discuss in this section. Graphical representations also are used to introduce definitions of pattern parameters that are commonly used to quantify radiation pattern characteristics.

A three-dimensional plot as in Fig. 1-10*d* gives a good overall impression of the entire radiation pattern, but cannot convey accurate quantitative information. Cuts through this pattern in various planes are the most popular pattern plots. They usually include the *E*- and *H*-plane patterns; see Figs. 1-10*b* and 1-10*c*. Pattern cuts are often given various fixed ϕ values, leaving the pattern a function of θ alone; we will assume that is the case here. Typically, the side lobes are alternately positive- and negative-valued; see Fig. 4-1*a*. In fact, a pattern in its most general form can be complex-valued. Then we use the magnitude of the field pattern $|F(\theta)|$ or the power pattern $P(\theta)$.

A measure of how well the power is concentrated into the main lobe is the (relative) *side lobe level*, which is the ratio of the pattern value of a side lobe peak to the pattern value of the main lobe. The largest side lobe level for the whole pattern is the *maximum (relative) side lobe level*, frequently abbreviated as SLL. In decibels, it is given by

$$\text{SLL}_{\text{dB}} = 20 \log \frac{|F(\text{SLL})|}{|F(\text{max})|} \tag{1-125}$$

where $|F(\text{max})|$ is the maximum value of the pattern magnitude and $|F(\text{SLL})|$ is the pattern value of the maximum of the highest side lobe magnitude. For a normalized pattern, $F(\text{max}) = 1$.

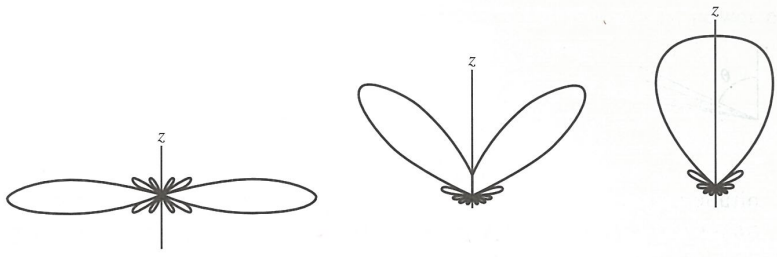
The width of the main beam is quantified through *half-power beamwidth* HP, which is the angular separation of the points where the main beam of the power pattern equals one-half the maximum value:

$$\text{HP} = |\theta_{\text{HP left}} - \theta_{\text{HP right}}| \tag{1-126}$$

where $\theta_{\text{HP left}}$ and $\theta_{\text{HP right}}$ are points to the "left" and "right" of the main beam maximum for which the normalized power pattern has a value of one-half (see Fig. 1-15). On the field pattern $|F(\theta)|$, these points correspond to the value $1/\sqrt{2}$. For example, the sin θ pattern of an ideal dipole has a value of $1/\sqrt{2}$ for θ values of $\theta_{\text{HP left}} = 135°$ and $\theta_{\text{HP right}} = 45°$. Then $\text{HP} = |135° - 45°| = 90°$. This is shown in Fig. 1-10*b*. Note that the definition of HP is the magnitude of the difference of the half-power points and the assignment of left and right can be interchanged without changing HP. In three dimensions, the radiation pattern major lobe becomes a solid object and the half-power contour is a continuous curve. If this curve is essentially elliptical, the pattern cuts that contain the major and minor axes of the ellipse determine what the IEEE defines as the *principal half-power beamwidths*.

Antennas are often referred to by the type of pattern they produce. An *isotropic antenna*, which is hypothetical, radiates equally in all directions giving a constant radiation pattern. An *omnidirectional antenna* produces a pattern that is constant in one plane; the ideal dipole of Fig. 1-10 is an example. The pattern shape resembles a "doughnut." We often refer to antennas as being broadside or endfire. A *broadside antenna* is one for which the main beam maximum is in a direction normal to the plane containing the antenna. An *endfire antenna* is one for which the main beam is in the plane containing the antenna. For a linear current on the z-axis, the broadside direction is $\theta = 90°$ and the endfire directions are 0° and 180°. For example, an ideal dipole is a broadside antenna. For z-directed line sources, several patterns are possible. Figure 1-16 illustrates a few $|f(\theta)|$ patterns. The entire pattern

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(a) Broadside (b) Intermediate (c) Endfire

Figure 1-16 Polar plots of uniform line source patterns $|f(\theta)|$.

in three dimensions is imagined by rotating the pattern about the z-axis. The full pattern can then be generated from the E-plane patterns shown. The broadside pattern of Fig. 1-16*a* is called a *fan beam*. The full three-dimensional endfire pattern for Fig. 1-16*b* has a single lobe in the endfire direction. This single lobe is referred to as a *pencil beam*. Note that the sin θ element factor, which must multiply these patterns to obtain the total pattern, will have a significant effect on the endfire pattern.

1.8 DIRECTIVITY AND GAIN

One very important description of an antenna is how much it concentrates energy in one direction in preference to radiation in other directions. This characteristic of an antenna is called its directivity and is equal to its power gain if the antenna is 100% efficient. Usually, power gain is expressed relative to a reference such as an isotropic radiator or half-wavelength dipole.

Toward the definition of directivity, let us begin by recalling that the power radiated by an antenna from (1-29) is

$$P = \iint \mathbf{S} \cdot d\mathbf{s} = \frac{1}{2} \operatorname{Re} \iint (\mathbf{E} \times \mathbf{H}^*) \cdot d\mathbf{s} \tag{1-127}$$

$$= \frac{1}{2} \operatorname{Re} \int_0^{2\pi} \int_0^{\pi} (E_\theta H_\phi^* - E_\phi H_\theta^*) r^2 \sin\theta \, d\theta \, d\phi \tag{1-128}$$

In general, there will be both θ- and ϕ-components of the radiation fields. From (1-107), we find that

$$H_\phi = \frac{E_\theta}{\eta} \quad \text{and} \quad H_\theta = -\frac{E_\phi}{\eta} \tag{1-129}$$

Using these in (1-128) gives

$$P = \frac{1}{2\eta} \iint (|E_\theta|^2 + |E_\phi|^2) r^2 \, d\Omega \tag{1-130}$$

where $d\Omega$ = element of solid angle = $\sin\theta \, d\theta \, d\phi$, which is shown in Fig. 1-17. The integral can be evaluated over any surface enclosing the antenna; however, for simplicity a spherical surface centered on the origin is usually used. Since the mag-

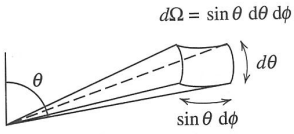


Figure 1-17 Element of solid angle $d\Omega$.

nitude variations of the radiation fields are $1/r$, we find it convenient to introduce *radiation intensity*, which is defined from

$$U(\theta, \phi) = \tfrac{1}{2}\,\mathrm{Re}(\mathbf{E} \times \mathbf{H}^*) \cdot r^2\hat{\mathbf{r}} = \mathrm{S}(\theta, \phi)r^2 \tag{1-131}$$

Radiation intensity is the power radiated in a given direction per unit solid angle and has units of watts per square radian (or steradian, sr). The advantage to using radiation intensity is that it is independent of distance r. Radiation intensity can be expressed as

$$U(\theta, \phi) = U_m|F(\theta, \phi)|^2 \tag{1-132}$$

where U_m is the maximum radiation intensity, and $|F(\theta, \phi)|^2$ is the power pattern normalized to a maximum value of unity in the direction $(\theta_{\max}, \phi_{\max})$, and

$$U_m = U(\theta_{\max}, \phi_{\max}) \tag{1-133}$$

The total power radiated is obtained by integrating the radiation intensity over all angles around the antenna:

$$P = \iint U(\theta, \phi)\, d\Omega = U_m \iint |F(\theta, \phi)|^2\, d\Omega \tag{1-134}$$

An isotropic source with uniform radiation in all directions is only hypothetical but is sometimes a useful concept. The radiation intensity of an isotropic source is constant over all space, at a value of U_{ave}. Then $P = \iint U_{\text{ave}}\, d\Omega = U_{\text{ave}} \iint d\Omega = 4\pi U_{\text{ave}}$ since there are 4π sr in all space (see Prob. 1.8-1). For nonisotropic sources, the radiation intensity is not constant throughout space, but an average power per steradian can be defined as

$$U_{\text{ave}} = \frac{1}{4\pi} \iint U(\theta, \phi)\, d\Omega = \frac{P}{4\pi} \tag{1-135}$$

The average radiation intensity U_{ave} equals the radiation intensity $U(\theta, \phi)$ that an isotropic source with the same input power P would radiate.

As an example, consider the ideal dipole again; we find from (1-72) and (1-131) that

$$U(\theta, \phi) = \frac{1}{2}\left(\frac{I\,\Delta z}{4\pi}\right)^2 \beta\omega\mu \sin^2\theta \tag{1-136}$$

so

$$U_m = \frac{1}{2}\left(\frac{I\,\Delta z}{4\pi}\right)^2 \beta\omega\mu \tag{1-137}$$

and

$$F(\theta, \phi) = \sin\theta \tag{1-138}$$

The average radiation intensity follows from the total radiated power expression (1-75) for an ideal dipole as

$$U_{\text{ave}} = \frac{P}{4\pi} = \frac{(\beta\omega\mu/12\pi)(I\,\Delta z)^2}{4\pi} = \frac{1}{3}\left(\frac{I\,\Delta z}{4\pi}\right)^2 \beta\omega\mu$$

$$= \frac{2}{3}\,U_m \qquad \textit{ideal dipole} \tag{1-139}$$

Thus, $U_m = 1.5U_{\text{ave}}$ for the ideal dipole, which means that in the direction of maximum radiation, the radiation intensity is 50% more than that which would occur from an isotropic source radiating the same total power.

Directivity. *Directivity* is defined as the ratio of the radiation intensity in a certain direction to the average radiation intensity, or

$$D(\theta, \phi) = \frac{U(\theta, \phi)}{U_{\text{ave}}} \tag{1-140}$$

If we divide the numerator and denominator by r^2, then we have power densities. So, directivity is also the ratio of the power density in a certain direction at a given range r to the average power density at that range, or

$$D(\theta, \phi) = \frac{U(\theta, \phi)/r^2}{U_{\text{ave}}/r^2} = \frac{\frac{1}{2}\,\text{Re}(\mathbf{E} \times \mathbf{H}^*) \cdot \hat{\mathbf{r}}}{P/4\pi r^2} \tag{1-141}$$

Substitution of (1-135) for U_{ave} in (1-140) yields

$$D(\theta, \phi) = \frac{U(\theta, \phi)}{\dfrac{1}{4\pi}\displaystyle\iint U(\theta, \phi)\, d\Omega} = \frac{|F(\theta, \phi)|^2}{\dfrac{1}{4\pi}\displaystyle\iint |F(\theta, \phi)|^2\, d\Omega}$$

$$= \frac{4\pi}{\Omega_A}\,|F(\theta, \phi)|^2 \tag{1-142}$$

where Ω_A is the *beam solid angle* defined by

$$\boxed{\Omega_A = \iint |F(\theta, \phi)|^2\, d\Omega} \tag{1-143}$$

This result shows that directivity is entirely determined by pattern shape. Beam solid angle is the solid angle through which all the power would be radiated if the power per unit solid angle (radiation intensity) equaled the maximum value over the beam area. This is illustrated in Fig. 1-18. From (1-134) and (1-143), we see that

$$P = U_m \Omega_A \tag{1-144}$$

This can also be inferred from Fig. 1-18*b*.

When directivity is quoted as a single number without reference to a direction, maximum (peak) directivity is usually intended. Maximum directivity follows from (1-140) as

$$D = \frac{U_m}{U_{\text{ave}}} \tag{1-145}$$

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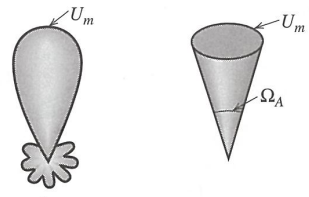


Figure 1-18 Antenna beam solid angle Ω_A. (*a*) Plot of radiation intensity $U(\theta, \phi)$ from an actual antenna. (*b*) Plot of radiation intensity with all radiation from the actual antenna concentrated into a cone of solid angle Ω_A with constant radiation intensity equal to the maximum of the actual pattern.

Using (1-135) and (1-144) in (1-145) gives

$$D = \frac{U_m}{P/4\pi} = \frac{4\pi U_m}{P} = \frac{4\pi U_m}{U_m \Omega_A} \tag{1-146a}$$

or

$$\boxed{D = \frac{4\pi}{\Omega_A}} \tag{1-146b}$$

Also from (1-132) in (1-140) we see that

$$D(\theta, \phi) = \frac{U_m |F(\theta, \phi)^2|}{U_{\text{ave}}} = D|F(\theta, \phi)|^2 \tag{1-147}$$

and since $|F(\theta, \phi)|^2$ has a maximum value of unity, the maximum value of directivity is D.

The concept of directivity is illustrated in Fig. 1-19. If the radiated power were distributed isotropically over all of space, the radiation intensity would have a maximum value equal to its average value as shown in Fig. 1-19*a*; that is, $U_m = U_{\text{ave}}$ or $\Omega_A = 4\pi$. Thus, the directivity of this isotropic pattern is unity. The distribution of radiation intensity $U(\theta, \phi)$ for an actual antenna is shown in Fig. 1-19b. It has a maximum radiation intensity in the direction $(\theta_{\max}, \phi_{\max})$ of $U_m = DU_{\text{ave}}$ and an average radiation intensity of $U_{\text{ave}} = P/4\pi$. By directing the radiated power P in a preferred direction, we can increase the radiation intensity in that direction by a factor of D over what it would be if the same radiated power had been isotropically radiated.

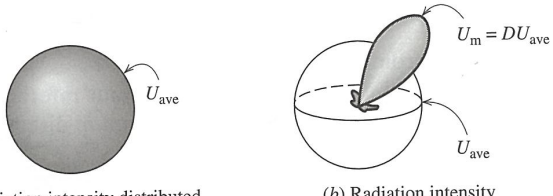


Figure 1-19 Illustration of directivity.

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EXAMPLE 1-2 ***Directivity of an Ideal Dipole***

The directivity of an ideal dipole can now be easily calculated using (1-139) in (1-145):

$$D = \frac{U_m}{U_{\text{ave}}} = \frac{U_m}{\frac{2}{3}U_m} = \frac{3}{2} \qquad \textit{ideal dipole} \tag{1-148}$$

Usually directivity is calculated directly from (1-146b), and the directivity calculation reduces to one of finding the beam solid angle. To illustrate, we use the ideal dipole. Substituting (1-138) in (1-143) leads to

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} |\sin\theta|^2 \sin\theta \, d\theta \, d\phi = 2\pi\frac{4}{3} = \frac{8\pi}{3} \tag{1-149}$$

and we obtain the same value of directivity from

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{8\pi/3} = \frac{3}{2} \tag{1-150}$$

Thus, the directivity of an ideal dipole is 50% greater than that of an isotropic source, which has a directivity of 1.

EXAMPLE 1-3 ***Directivity of a Sector Omnidirectional Pattern***

An ideal omnidirectional antenna would have constant radiation in the horizontal plane ($\theta = 90°$) and would fall rapidly to zero outside that plane. Suppose that the pattern in the vertical plane is constant out to $\pm\frac{\pi}{6}$ ($\pm 30°$) from horizontal. The pattern expression is then written as

$$F(\theta) = \begin{cases} 1 & \frac{1}{3}\pi < \theta < \frac{2}{3}\pi \\ 0 & \text{elsewhere} \end{cases} \tag{1-151}$$

The solid angle of the pattern from (1-143) is

$$\begin{aligned} \Omega_A &= \iint |F(\theta, \phi)|^2 \, d\Omega = \int_0^{2\pi}\int_{\pi/3}^{2\pi/3} 1^2 \sin\theta \, d\theta \, d\phi \\ &= (2\pi)[-\cos\theta]_{\pi/3}^{2\pi/3} = (2\pi)(0.5 + 0.5) = 2\pi \end{aligned} \tag{1-152}$$

The directivity from (1-146b) is

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{2\pi} = 2 \tag{1-153}$$

Gain. As noted above, directivity is solely determined by the radiation pattern of an antenna. When an antenna is used in a system (say, as a transmitting antenna), we are actually interested in how efficiently the antenna transforms available power at its input terminals to radiated power, together with its directive properties. *Power gain* (or simply *gain*) is used to quantify this and is defined as 4π times the ratio of the radiation intensity in a given direction to the net power accepted by the antenna from the connected transmitter, or

$$G(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{P_{\text{in}}} \tag{1-154}$$

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where $G(\theta, \phi)$ is the gain and $U(\theta, \phi)$ is the radiation intensity of the antenna in the direction (θ, ϕ) including the effect of any losses on the antenna, and P_{in} is the input power accepted by the antenna. This definition does not include losses due to mismatches of impedance or polarization, which are discussed in Sec. 9.1. The maximum value of gain is the maximum of (1-154), so

$$G = \frac{4\pi U_m}{P_{\text{in}}} \tag{1-155}$$

Thus, gain can be expressed as a function of θ and ϕ and can also be given as a value in a specific direction. If no direction is specified and the gain value is not given as a function of θ and ϕ, it is assumed to be the maximum gain.

Directivity can be written from (1-146a) as $D = 4\pi U_m/P$. Comparing this with (1-155), we see that the only difference between maximum gain and directivity is the power value used. Directivity can be viewed as the gain an antenna would have if all input power appeared as radiated power; that is, $P_{\text{in}} = P$. Gain reflects the fact that real antennas do not behave in this fashion and some of the input power is lost on the antenna. The portion of input power P_{in} that does not appear as radiated power is absorbed on the antenna and nearby structures. This prompts us to define *radiation efficiency* e_r as

$$e_r = \frac{P}{P_{\text{in}}} \tag{1-156}$$

Note that

$$0 \le e_r \le 1 \tag{1-157}$$

Using (1-156) in (1-54) gives

$$G(\theta, \phi) = e_r \frac{4\pi U(\theta, \phi)}{P} = e_r \frac{U(\theta, \phi)}{U_{\text{ave}}} = e_r D(\theta, \phi) \tag{1-158}$$

Similarly, for maximum gain

$$\boxed{G = e_r D} \tag{1-159}$$

Thus, the maximum gain of an antenna is equal to its purely directional characteristic of maximum directivity multiplied by radiation efficiency.

The terminology found in the literature is inconsistent and often incorrect on the topics of directivity and gain. Directivity and gain can be functions of angle or be maximum values, that is, $D(\theta, \phi)$ or D, and $G(\theta, \phi)$ or G. Formerly, the term "directive gain" was used for directivity as a function of angle, but its use is no longer recommended by the IEEE. If no other information is given during a discussion of directivity or gain, it can safely be assumed that the maximum value is intended.

Units for Directivity and Gain. Since gain is a power ratio it can be calculated in decibels as follows:

$$G_{\text{dB}} = 10 \log G \tag{1-160}$$

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Similarly for directivity,

$$D_{\text{dB}} = 10 \log D \tag{1-161}$$

For example, the directivity in decibels of an ideal dipole is

$$D_{\text{dB}} = 10 \log 1.5 = 1.76 \text{ dB} \qquad \textit{ideal dipole} \tag{1-162}$$

Frequently, gain is used to describe the performance of the antenna relative to some standard reference antenna. This *relative gain* is defined as the ratio of the maximum radiation intensity from the antenna U_m to the maximum radiation intensity from a reference antenna $U_{m,\text{ref}}$ with the same input power, or

$$G_{\text{ref}} = \frac{U_m}{U_{m,\text{ref}}} \tag{1-163}$$

This is a convenient definition from a measurement standpoint. The formal definition of gain employs a hypothetical lossless isotropic antenna as a reference antenna. This can be shown by noting that the lossless isotropic reference antenna has a maximum radiation intensity of $P_{\text{in}}/4\pi$ since all its input power is radiated, and substituting this into (1-163) for $U_{m,\text{ref}}$ leads to (1-155).

It is common at frequencies below 1 GHz to quote gain values relative to that of a half-wave dipole. The directivity of a half-wave dipole is 1.64, or 2.15 dB; see Sec. 5.1. Gain relative to a half-wave dipole carries the units of dBd. The unit dBi is often used instead of dB to emphasize that an isotropic antenna is the reference. In addition, the term *absolute gain*, which is synonymous with gain, is sometimes used. As a numerical example, consider an antenna with a gain of 6.1 dB; its gain can be written in the following ways:

$$G = 6.1 \text{ dB} = 6.1 \text{ dBi} = 3.95 \text{ dBd} \tag{1-164}$$

1.9 ANTENNA IMPEDANCE, RADIATION EFFICIENCY, AND THE SHORT DIPOLE

The input impedance of an antenna is the impedance presented by the antenna at its terminals. Thus, suitable terminals must be defined for an antenna. The input impedance will be affected by other antennas or objects that are nearby, but for this discussion we assume that the antenna is isolated. Input impedance is composed of real and imaginary parts:

$$Z_A = R_A + jX_A \tag{1-165}$$

The input resistance R_A represents dissipation, which occurs in two ways. Power that leaves the antenna and never returns (i.e., radiation) is a form of dissipation. There are also ohmic losses associated with heating on the antenna structure, but on many antennas ohmic losses are small compared to radiation losses. However, ohmic losses are usually significant on electrically small antennas, which have dimensions much less than a wavelength. The input reactance X_A represents power stored in the near field of the antenna. As a consequence of reciprocity, the impedance of an antenna is identical during reception and transmission.

First, we discuss the input resistance. The average power dissipated in an antenna is

$$P_{\text{in}} = \frac{1}{2} R_A |I_A|^2 \tag{1-166}$$

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where I_A is the current at the input terminals. Note that a factor of $\frac{1}{2}$ is present because current I_A is the peak value in the time waveform. Separating the dissipated power into radiative and ohmic losses gives

$$\begin{aligned} P_{\text{in}} &= P + P_{\text{ohmic}} \\ \frac{1}{2} R_A |I_A|^2 &= \frac{1}{2} R_r |I_A|^2 + \frac{1}{2} R_{\text{ohmic}} |I_A|^2 \end{aligned} \tag{1-167}$$

where we define the radiation resistance of an antenna referred to the input terminals as

$$R_r = \frac{2P}{|I_A|^2} \tag{1-168}$$

It follows from (1-167) that

$$R_A = R_r + R_{\text{ohmic}} \tag{1-169}$$

where R_{ohmic} is resistance associated with ohmic losses that include the directly driven part of the antenna as well as losses in other portions of the antenna structure such as a ground plane. Ohmic resistance of an antenna is defined as

$$R_{\text{ohmic}} = \frac{2P_{\text{ohmic}}}{|I_A|^2} = \frac{2(P_{\text{in}} - P)}{|I_A|^2} \tag{1-170}$$

The radiated power is found using (1-35):

$$P = \frac{1}{2} \iint_{s_{\text{ff}}} (\mathbf{E} \times \mathbf{H}^*) \cdot d\mathbf{s} \tag{1-171}$$

where s_{ff} is a surface in the far field, usually spherical. P is real-valued because the power density $\mathbf{S} = \frac{1}{2}\mathbf{E} \times \mathbf{H}^*$ is real-valued in the far field.

Radiation resistance can be defined relative to the current at any point on the antenna, but we reserve R_r for radiation resistance referred to the input terminals. Radiation resistance relative to the maximum current I_m that occurs on the antenna R_{rm} is obtained by using I_m in place of I_A in (1-168). In this section, we discuss center-fed electrically short antennas, which always have a current maximum at the input, so $R_r = R_{rm}$. We discuss this topic again in Sec. 5.1.

The power radiated from an ideal dipole of length $\Delta z \ll \lambda$ and input current $I_A = I$ is given by (1-75), which together with (1-168) gives the radiation resistance:

$$\begin{aligned} R_r &= \frac{2P}{|I_A|^2} = \frac{2}{I^2} \frac{\omega\mu\beta}{12\pi} (I\,\Delta z)^2 = \frac{\sqrt{\mu\omega}\sqrt{\mu}\sqrt{\varepsilon}}{\sqrt{\varepsilon}\,6\pi} \beta(\Delta z)^2 \\ &= \eta \frac{\beta^2}{6\pi} (\Delta z)^2 = \eta \frac{2}{3} \pi \left(\frac{\Delta z}{\lambda}\right)^2 \\ R_r &= 80\pi^2 \left(\frac{\Delta z}{\lambda}\right)^2 \ \Omega \qquad \textit{ideal dipole} \end{aligned} \tag{1-172}$$

For ideal dipoles, R_r is very small since $\Delta z \ll \lambda$.

The relative amounts of input power dissipated by radiation and ohmic losses determine the efficiency of the antenna. This is expressed by the radiation efficiency

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e_r that was introduced in Sec. 1.8 and defined in (1-156) as the ratio of total radiated power to the net power accepted by the antenna, so

$$e_r = \frac{P}{P_{\text{in}}} = \frac{P}{P + P_{\text{ohmic}}} \tag{1-173}$$

Substituting (1-167) and (1-168) into (1-173) yields

$$e_r = \frac{\frac{1}{2} R_r |I_A|^2}{\frac{1}{2} R_r |I_A|^2 + \frac{1}{2} R_{\text{ohmic}} |I_A|^2} = \frac{R_r}{R_r + R_{\text{ohmic}}} = \frac{R_r}{R_A} \tag{1-174}$$

where (1-169) was used. Except for low frequencies, the skin depth $\delta = \sqrt{2/\omega\mu\sigma}$ is much smaller than the conductor radius and then the ohmic resistance for an antenna of length L that carries an axially uniform current is

$$R_{\text{ohmic}} \approx \frac{L}{2\pi a} R_s \tag{1-175}$$

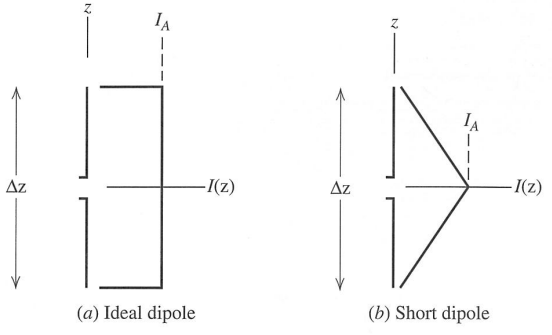
where L is the length of the wire, a the wire radius, and R_s the surface resistance:

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}} \tag{1-176}$$

For many antennas, radiation efficiency is nearly 100%. For electrically small antennas, however, the radiation efficiency can be low. We, therefore, take a closer look at them.

The ideal dipole has a uniform current as shown in Fig. 1-20*a*. In reality, the current on a straight wire antenna must smoothly go to zero at the wire ends. The current distribution on a center-fed wire dipole of length $\Delta z \ll \lambda$, called a **short dipole**, is approximately triangular in shape as illustrated in Fig. 1-20*b*. If end loading such as with metal plates (see Fig. 2-3) is added to the short dipole, the radial current reduces to zero at the edge of the plates, giving a nearly uniform current on the vertical portion of the dipole, which permits use of the ideal dipole model. More will be said about short dipoles in Sec. 2.1.

Pattern calculations for the ideal dipole were performed in Sec. 1.6 assuming that the magnitude and phase differences of rays coming from points on the wire due to different path lengths were negligible. Since the short dipole also satisfies $\Delta z \ll \lambda$, the pattern will also be the same sin θ radiation pattern as the ideal dipole. In



(*a*) Ideal dipole (*b*) Short dipole

Figure 1-20 The ideal dipole and short dipole with current distributions; $\Delta z \ll \lambda$. I_A is the value of the input current at the terminals in the center of each antenna. The short dipole of (*b*) is that which is encountered in practice.

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addition, the ideal dipole and short dipole will have the same directivity value of 1.5 because pattern shape completely determines directivity.

The triangular current distribution of the short dipole leads to an equivalent length that is one-half that of its physical length. This is because the equivalent length is proportional to the area under the current versus distance curves shown in Fig. 1-20, which follows from the radiation integral of (1-103) with $\exp(j\beta z' \cos\theta) \approx 1$ for short dipoles. The radiated fields are, in turn, proportional to this equivalent length. Since the radiation resistance is proportional to the integral of the far-zone electric field squared and the patterns of the ideal and short dipoles are the same, the radiation resistances are proportional to the equivalent lengths squared. The area of the triangle shape current on the short dipole is one-half that of the uniform current shape, so the radiation resistance is one fourth that of the ideal dipole. Dividing (1-171) by 4 gives

$$R_r = 20\pi^2\left(\frac{\Delta z}{\lambda}\right)^2 \ \Omega \qquad \textit{short dipole} \tag{1-177}$$

The ohmic resistance for the short dipole is also reduced from that of the ideal dipole. The ohmic resistance of a short dipole is found by first determining the power dissipation from ohmic losses, which at any point along the antenna is proportional to the current squared. In fact, in general the total power dissipated is evaluated by integrating the current squared over the wire antenna, which together with (1-170) yields

$$R_{\text{ohmic}} = \frac{2P_{\text{ohmic}}}{|I_A|^2} = \frac{1}{|I_A|^2}\frac{R_s}{2\pi a}\int_{-L/2}^{L/2} |I(z)|^2\, dz \tag{1-178}$$

It is easy to show that this reduces to (1-175) for a uniform current of length $L = \Delta z$. The short dipole triangular current of Fig. 1-20*b* can be written as a function of position along the wire as

$$I(z) = I_A\left(1 - \frac{2|z|}{\Delta z}\right), \qquad |z| \le \frac{\Delta z}{2} \tag{1-179}$$

Using this in (1-178) yields

$$R_{\text{ohmic}} = \frac{\Delta z}{2\pi a}\frac{R_s}{3} \qquad \textit{short dipole} \tag{1-180}$$

Notice that this is one-third that for an ideal dipole of the same length Δz. Since the radiation resistance for the short dipole is one-fourth that of an ideal dipole, the radiation resistance is decreased more relative to the ohmic resistance, and thus the efficiency is lower for the short dipole than it is for an ideal dipole of the same length.

EXAMPLE 1-4 *Radiation Efficiency of an AM Car Radio Antenna*

Most fender-mount car radio antennas are 31 in. long. We assume that the fender images the monopole antenna, forming a dipole 62 in. long (L = 1.575 m) and 1/8 in. in diameter

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(a = 0.159 cm). Monopoles are treated in Sec. 2.3, but here we use a dipole model. For an operating frequency of 1 MHz (λ = 300 m), the electrical length is 0.00525λ, indicating an electrically small antenna. If a short dipole model is assumed, the radiation resistance follows from (1-177) as

$$R_r = 20\pi^2\left(\frac{1.575}{300}\right)^2 = 0.00545\ \Omega \tag{1-181}$$

Using the conductivity of steel (see App. B.1) in (1-176) gives

$$R_s = \sqrt{\frac{4\pi \times 10^{-7} \cdot 2\pi \times 10^6}{2 \cdot 2 \times 10^6}} = 1.40 \times 10^{-3}\ \Omega \tag{1-182}$$

The ohmic resistance from (1-180) is

$$R_{\text{ohmic}} = \frac{L}{2\pi a}\frac{R_s}{3} = \frac{1.575}{2\pi \cdot 1.59 \times 10^{-3}}\frac{1.40 \times 10^{-3}}{3} = 0.0736\ \Omega \tag{1-183}$$

The radiation efficiency from (1-174), (1-181), and (1-183) is

$$e_r = \frac{R_r}{R_r + R_{\text{ohmic}}} = \frac{0.00545}{0.00545 + 0.0736} = 6.7\% \tag{1-184}$$

The 6.7% radiation efficiency of the above example is low. Low efficiency in receiving antennas for broadcast applications is overcome by using high-power transmitters operating into tall antennas that are efficient. Thus, cost and complexity are concentrated into a few transmitting stations, allowing inexpensive and simple receiving antennas.

In addition to loss of efficiency, ohmic losses on antennas have another undesirable effect. As with any resistive element in an electrical system, ohmic losses on antennas are noise sources. This can be a problem for receiving applications when the signal is low. For frequencies around 1 MHz and below, external noise, mainly due to lightning, is significant and always present. The external noise picked up by the antenna is proportional to the antenna radiation resistance and is usually larger than the noise arising from internal ohmic resistance. Antenna noise is discussed further in Sec. 9.2.

We now return to the reactive part of the input impedance. In contrast to radiated power that contributes to the real part of the input impedance, the reactive part of the input impedance represents power stored in the near field. This behavior is very similar to a complex load impedance in circuit theory. Antennas that are electrically small (i.e., much smaller than a wavelength) have a large input reactance, in addition to a small radiation resistance. For example, the short dipole has a capacitive reactance, whereas an electrically small loop antenna has an inductive reactance. This is an expected result from low-frequency circuit theory. The reactance of a short dipole is approximated by [9]:

$$X_A = -\frac{120}{\pi\dfrac{\Delta z}{\lambda}}\left[\ln\left(\frac{\Delta z}{2a}\right) - 1\right]\Omega \qquad \textit{short dipole} \tag{1-185}$$

This gives a large capacitive reactance for very short dipoles. The total input impedance of the short dipole is $R_r + R_{\text{ohmic}} + jX_A$, where X_A is given above, R_r is given by (1-177), and R_{ohmic} is given by (1-180).

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EXAMPLE 1-5 ***Input Reactance of an AM Car Radio Antenna***

We now return to Example 1-4 and calculate the reactive portion of the input impedance. Using $\Delta z/\lambda = 0.00525$ and $\Delta z/2a = 157.5/2 \cdot 0.159 = 495.3$ in (1-185) gives $X_A = -37{,}870\ \Omega$. This is a very large capacitive reactance, leading to a severe impedance mismatch. Also see Fig. 5-6.

Antenna impedance is important to the transfer of power from a transmitter to an antenna or from an antenna to a receiver. For example, to maximize the power transferred from a receiving antenna, the receiver impedance should be a conjugate match to the antenna impedance (equal resistances, equal magnitude and opposite sign reactances). Receivers have real-valued impedance, typically 50 Ω, so it is necessary to "tune out" the antenna reactance with a matching network. There are two disadvantages to using matching networks: Ohmic losses in the network components such as tuning coils reduce efficiency, and second, a matching network provides a match only over a narrow band of frequencies, which reduces the operational bandwidth. Impedance-matching techniques are discussed in Section 5.3.

1.10 ANTENNA POLARIZATION

A monochromatic electromagnetic wave, which varies sinusoidally with time, is characterized at an observation point by its frequency, magnitude, phase, and polarization. The first three of these are familiar parameters, but polarization is often not well understood by students and practicing engineers. The polarization of an antenna is the polarization of the wave radiated in a given direction by the antenna when transmitting. In this section, we first discuss the possible polarizations of an electromagnetic wave, and then antenna polarization will follow directly from wave polarization. A complete discussion of wave and antenna polarization is found in [10].

The phase front (surface of constant phase) of a wave radiated by a finite-sized radiator becomes nearly planar over small observation regions. This wave is referred to as a *plane wave* and its electric and magnetic fields lie in a plane. The *polarization* of a plane wave is the figure the instantaneous electric field traces out with time at a fixed observation point. An example is the vertical, linearly polarized wave in Fig. 1-21, which shows the spatial variation of the electric field at a fixed instant of time.

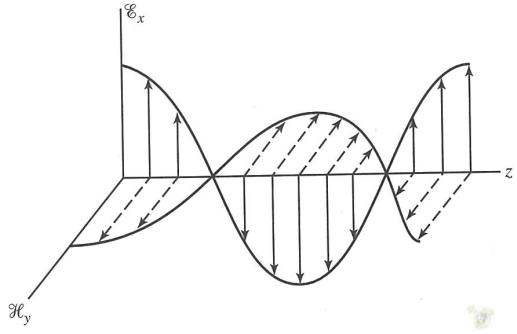


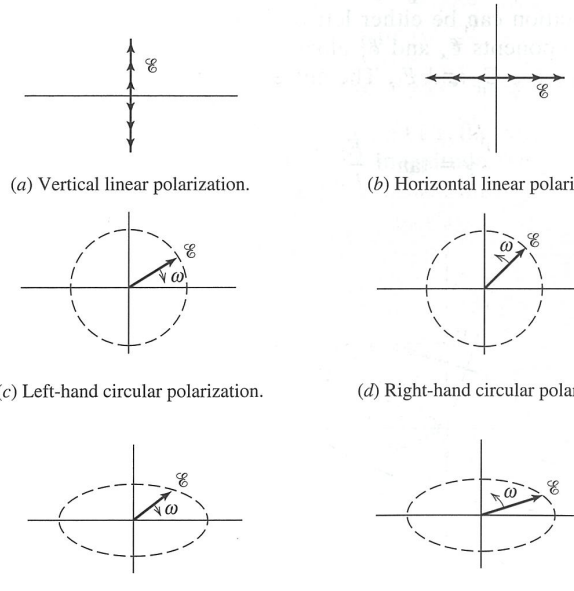
Figure 1-21 The spatial behavior of the electric (solid) and magnetic (dashed) fields of a linearly (vertical) polarized wave for a fixed instant of time. (From [10]. Reprinted by permission of Artech House, Inc., Boston, MA.)

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As time progresses, the electric field ($\mathscr{E}_x$) at a fixed point oscillates back and forth along a vertical line. For a completely polarized wave, the figure traced out is, in general, an ellipse. As indicated in Fig. 1-21, the temporal and spatial variations of the magnetic field are similar to those for the electric field, except that the magnetic field is perpendicular to the electric field. Waves can have a nonperiodic behavior, but we will not consider such randomly polarized wave components because antennas cannot generate them.

There are some important special cases of the polarization ellipse. If the electric field vector moves back and forth along a line, it is said to be *linearly polarized*; see Figs. 1-22*a* and 1-22*b*. An example is the electric field from an ideal dipole or any linear current. If the electric field vector remains constant in length but rotates around in a circular path, it is *circularly polarized*. Rotation at radian frequency ω is in one of two directions, referred to as the sense of rotation. If the wave is traveling toward the observer and the vector rotates clockwise, it is *left-hand polarized*. The left-hand rule applies here: With the thumb of the left hand in the direction of propagation, the fingers will curl in the direction of rotation of the instantaneous electric field $\mathscr{E}$. If it rotates counterclockwise, it is *right-hand polarized*. Left- and right-hand circularly polarized waves are shown in Figs. 1-22*c* and 1-22*d*. A helix antenna produces circularly polarized waves and the sense of rotation of the wave is the same as that of the helix windings; for example, a right-hand wound helix produces a right-hand circularly polarized wave; see Sec. 6.2. Finally, Figs. 1-22*e* and 1-22*f* show the most general cases of left-hand and right-hand sensed *elliptical polarizations*.

The time-space behavior of the important special case of circular polarization is



(*a*) Vertical linear polarization. (*b*) Horizontal linear polarization.

(*c*) Left-hand circular polarization. (*d*) Right-hand circular polarization.

(*e*) Left-hand elliptical polarization. (*f*) Right-hand elliptical polarization.

Figure 1-22 Some wave polarization states. The wave is approaching.

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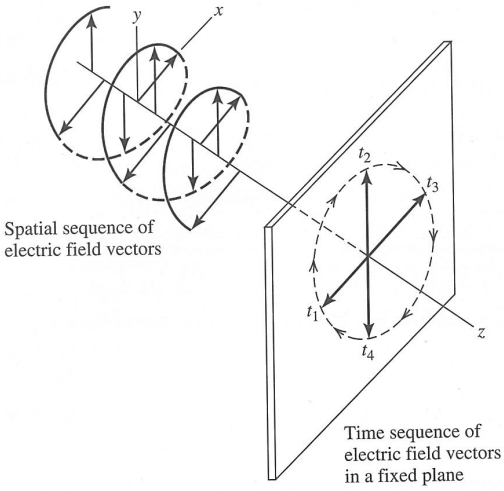


Figure 1-23 Perspective view of a left-hand circularly polarized wave shown at a fixed instant of time and the time sequence of electric field vectors as the wave passes through a fixed plane in the $+z$-direction. (From [10]. Reprinted by permission of Artech House, Inc., Boston, MA.)

difficult to visualize. Figure 1-23 provides a space perspective view of a left-hand circularly polarized wave. As the vector pattern translates along the $+z$-axis, the electric field at a fixed point appears to rotate clockwise in the xy-plane (yielding a left-hand circularly polarized wave). This is illustrated with the time sequence of vectors in the fixed plane shown in Fig. 1-23.

A general polarization ellipse is shown in Fig. 1-24 with a reference axis system. The wave associated with this polarization ellipse is traveling in the $+z$-direction. The sense of rotation can be either left or right. The instantaneous electric field vector $\mathscr{E}$ has components $\mathscr{E}_x$ and $\mathscr{E}_y$ along the x- and y-axes. The peak values of these components are E_1 and E_2. The angle γ describes the relative values of E_1 and E_2 from

$$\gamma = \tan^{-1}\frac{E_2}{E_1}, \qquad 0° \le \gamma \le 90° \tag{1-186}$$

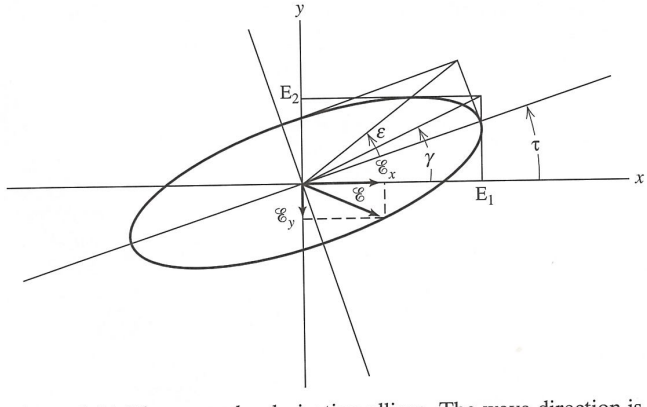


Figure 1-24 The general polarization ellipse. The wave direction is out of the page in the $+z$-direction. The tip of the instantaneous electric field vector $\mathscr{E}$ traces out the ellipse.

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The tilt angle of the ellipse τ is the angle between the x-axis (horizontal) and the major axis of the ellipse. The angle ε is seen from Fig. 1-24 to be

$$\varepsilon = \cot^{-1}(-AR), \qquad 1 \le |AR| \le \infty, \qquad -45° \le \varepsilon \le 45° \tag{1-187}$$

where the axial ratio of the ellipse $|AR|$ is the ratio of the major axis electric field component to that along the minor axis. The sign of AR is positive for right-hand sense and negative for left-hand sense. Axial ratio is often expressed in dB as 20 log $|AR|$.

The instantaneous electric field for the wave of Fig. 1-24 can be written as (with $z = 0$ for simplicity)

$$\mathscr{E} = \mathscr{E}_x\hat{\mathbf{x}} + \mathscr{E}_y\hat{\mathbf{y}} = E_1 \cos \omega t\,\hat{\mathbf{x}} + E_2 \cos(\omega t + \delta)\hat{\mathbf{y}} \tag{1-188}$$

where δ is the phase by which the y-component leads the x-component. This representation describes the ellipse shape as time t progresses. If the components are in-phase ($\delta = 0$), the net vector is linearly polarized. The orientation of the linear polarization depends on the relative values of E_1 and E_2. For example, if $E_1 = 0$, vertical linear polarization results; if $E_2 = 0$, horizontal linear results; if $E_1 = E_2$, the polarization is linear at 45° with respect to the axes. Linear polarization is a collapsed ellipse with infinite axial ratio. If δ is nonzero, the axial ratio is finite. When $\delta > 0$, $\mathscr{E}_y$ leads $\mathscr{E}_x$ in-phase and the sense of rotation is left-hand. For $\delta < 0$, the sense is right-hand. If $E_1 = E_2$ (thus, $\gamma = 45°$) and $\delta = \pm 90°$, the polarization is circular (+90° is left-hand and −90° is right-hand). The axial ratio magnitude of a circularly polarized wave is unity.

The phasor form of (1-188) is

$$\mathbf{E} = E_1\hat{\mathbf{x}} + E_2 e^{j\delta}\hat{\mathbf{y}} \tag{1-189}$$

which can be written as (see Prob. 1.10-3)

$$\mathbf{E} = \sqrt{E_1^2 + E_2^2}(\cos \gamma\,\hat{\mathbf{x}} + \sin \gamma\, e^{j\delta}\hat{\mathbf{y}}) = |E|\hat{\mathbf{e}} \tag{1-190}$$

The factor $|E|$ is the field magnitude and $\hat{\mathbf{e}}$ is the complex vector representation for the field and is normalized to unity magnitude. Thus, γ and δ completely specify the polarization state of the wave. In fact, either pair of angles (ε, τ) or (γ, δ) uniquely define the polarization state of a wave. The transformations between these angles are

$$\gamma = \frac{1}{2}\cos^{-1}(\cos 2\varepsilon \cos 2\tau) \tag{1-191}$$

$$\delta = \tan^{-1}\left(\frac{\tan 2\varepsilon}{\sin 2\tau}\right) \tag{1-192}$$

The *polarization of an antenna* is the polarization of the wave radiated by the antenna when transmitting. Therefore, all the discussions on wave polarization apply directly to antenna polarization. The polarization of waves radiated by an antenna will vary with direction. Usually, the polarization characteristics of an antenna remain relatively constant over its main beam and the polarization on the main beam peak is used to describe the antenna polarization. However, the radiation from side lobes can differ greatly in polarization from that of the main beam. When measuring the radiation from an antenna, both E_θ and E_ϕ should be measured to be complete. The principal plane patterns of a perfectly linearly polarized antenna,

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such as a line source on the z-axis, are completely specified when a linearly polarized probe antenna is oriented to respond to E_θ.

Reciprocal antennas have identical radiation patterns on transmit and receive. This extends to the vector nature of the radiation that includes polarization. A transmit antenna is polarization matched to a receive antenna if its polarization ellipse axial ratio, sense, and major axis orientation are the same as those of the receive antenna (in the direction of the transmit antenna). For example, a right-hand circularly polarized receiving antenna is polarized matched to a right circularly polarized wave. As a mechanical analogy, consider a right-hand threaded rod corresponding to a right-hand circularly polarized (RHCP) wave and a right-hand tapped hole representing a RHCP antenna. The rod and hole are matched when screwed in or out, corresponding to reception or transmission.

It is interesting to examine the polarizations used in the U.S. broadcast FM radio and TV industry. Historically, TV and FM broadcast transmitting antennas were horizontally polarized. In recent years, CP has been used for TV and FM since the FCC now allows transmitters to have the maximum EIRP in both horizontal polarization and vertical polarization. This was permitted because nearly all vehicle receiving antennas are VP.

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PROBLEMS

1.3-1 *What is this antenna?* (a) Locate an antenna that you would like to know more about. It can be one you see in your community or one you find in a catalog or magazine. Investigate and learn more about the antenna as you move through this book. For now make a sketch of the antenna and describe its location and surroundings. (b) After covering a substantial part of this book, provide an explanation of the antenna in (a) including the type of antenna it is, its operating frequency, how it is being used, and performance parameter values that you can estimate such as beamwidth and gain.

1.4-1 Use (1-6) in (1-1) to derive (1-7).

1.4-2 Use (1-13) in (1-19) together with (1-11), (1-12), and (1-20) to derive (1-18).

1.4-3 Assuming ε and μ are real and $\mathbf{M} = 0$, derive (1-28) through (1-34) using the identity (C-19).

1.4-4 Write the complex power equation for a series RLC network driven by a voltage generator in a form analogous to the Poynting theorem.

1.5-1 Derive (1-48) starting with (1-18).

1.5-2 a. Show that $\psi = Ce^{-j\beta r}/r$ satisfies (1-54) at all points except the origin.

b. By integrating (1-52) over a small volume containing the origin, substituting $\psi = Ce^{-j\beta r}/r$, and letting r approach zero, show that $C = (4\pi)^{-1}$, thus proving (1-55).

1.6-1 Show that (1-71b) follows from (1-69).

1.6-2 The expression for the electric field intensity of an ideal dipole can be derived in two ways.

a. Derive (1-71b) using the magnetic field intensity expression (1-71a) in (1-60).

b. Derive (1-71b) using the vector potential expression (1-62) in (1-45).

1.6-3 For a z-directed current element $I\,\Delta z$ in free space and located at the origin of a spherical coordinate system:

a. Calculate the complex Poynting vector in the general case, where r can be in the near-field region. Use the fields of (1-71).

b. Then find the expression for the time-average power flowing out through a sphere of radius r enclosing the current element. Your answer will be that of (1-75). Why?

1.6-4 Show that the electric field for the ideal dipole in (1-71b) satisfies Maxwell's equation $\nabla \cdot \mathbf{E} = 0$.

1.7-1 Prove (1-93) by using (1-87) in (1-92) and retaining only $1/r$ terms; that is, using $\beta r \gg 1$.

1.7-2 *Uniform line source.*

a. Find the half-power beamwidth of the uniform line source pattern factor $|f(\theta)|$ of (1-118). Your answer should be in the form

$$\text{HP} = K\lambda/L \quad \text{for} \quad L \gg \lambda.$$

Determine the constant K. *Hint:* First find the values u_{HP} of $u = (\beta L/2)\cos\theta$ for which $|f(u_{\text{HP}})| = 1/\sqrt{2}$. Then use the approximation $\cos^{-1}(\pm x) \approx \pi/2 \mp x$ for x small.

b. Calculate the maximum side lobe level for the pattern in decibels relative to the main beam maximum. The side lobe maximum can be located by differentiating $f(u)$ with respect to u, setting equal to zero, and solving for u.

c. Suppose now that the current has a linear phase taper across it so that

$$I(z') = I_o e^{j\beta_o z'}$$

What is $f(\theta)$ now? If we let $\beta_o = -\beta\cos\theta_o$ where is the pattern maximum (main beam pointing direction)? This is how the scanned beams of Fig. 1-16 are generated.

1.7-3 Equation (1-96) can be derived without initially assuming that the rays are parallel. Derive (1-96) by writing $R = [(\mathbf{r} - \mathbf{r}') \cdot (\mathbf{r} - \mathbf{r}')]^{1/2}$, expanding, factoring out an r, neglecting the smallest term, and using the first two terms of the binomial expansion.

1.7-4 Using the inner boundary of the far-field to be $r_{\text{ff}} = 2L^2/\lambda$ for a linear antenna of length L, find r_{ff} for the following three antennas: $L = 5\lambda$, a half-wave dipole ($L = \lambda/2$), and a short dipole ($L = 0.01\lambda$). Is the far-field boundary you have computed valid for each of these; if not, why not?

1.7-5 It can be shown that criteria for the far-field distance corresponding to (1-99b) and (1-99c) are more accurately given by $r > 5D$ and $r > 1.6\lambda$. Using these together with (1-99a), plot a single graph of r/λ (vertical axis) versus D/λ for the far-field boundary. Indicate which region of the graph corresponds to the far field.

1.7-6 A car radio antenna is 1 m long and operates at a frequency of 1 MHz. Use the graph of Prob. 1.7-5 to find the far-field distance.

1.7-7 Derive the near-field region boundary expression in (1-100) of $0.62\sqrt{D^3/\lambda}$. Do this by finding the maximum error of the fourth term in (1-84) with $z' = D/2$ and equating to $\pi/8$.

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1.8-1 Show that there is $4\pi\ sr$ in all space by integrating $d\Omega$ over a sphere.
1.8-2 A *power* pattern is given by $|\cos^n \theta|$ for $0 \le \theta \le \pi/2$ and is zero for $\pi/2 \le \theta \le \pi$. (a) Calculate the directivity for $n = 1$, 2, and 3, (b) Find the HP values in degrees for each n, (c) Sketch the patterns for the n values on one polar plot and comment on them, (d) Explain the directivity value for the case of $n = 0$.
1.8-3 An antenna has a far-field pattern which is independent of ϕ but which varies with θ as follows:

$$\begin{aligned} F &= 1 \quad \text{for} \quad 0° \le \theta \le 30°. \\ F &= 0.5 \quad \text{for} \quad 60° \le \theta \le 120°. \\ F &= 0.707 \quad \text{for} \quad 150° \le \theta \le 180°. \\ F &= 0 \quad \text{for} \quad 30° < \theta < 60° \quad \text{and} \quad 120° < \theta < 150°. \end{aligned}$$

Find the directivity. Also find the directivity in the direction $\theta = 90°$.
1.8-4 For a single-lobed pattern the beam solid angle is approximately given by

$$\Omega_A \approx \text{HP}_E \text{HP}_H$$

where HP_E and HP_H are the half-power beamwidths in radians of the main beam in the E and H planes. Show that

$$D \approx \frac{41{,}253}{\text{HP}_{E°}\text{HP}_{H°}}$$

where $\text{HP}_{E°}$ and $\text{HP}_{H°}$ are the E and H plane half-power beamwidths in degrees.
1.8-5 A horn antenna with low side lobes has half-power beamwidths of 29° in both principal planes. Use the approximate expression in Prob. 1.8-4 to compute the directivity of the horn in decibels.
1.8-6 A sector pattern has uniform radiation intensity over a specified angular region and is zero elsewhere. An example is

$$F(\theta) = \begin{cases} 1 & \dfrac{\pi}{2} - \alpha < \theta < \dfrac{\pi}{2} + \alpha \\ 0 & \text{elsewhere} \end{cases}$$

Derive an expression for the directivity corresponding to this pattern.
1.8-7 An airplane is flying parallel to the ground (in the z-direction). For a surface search radar an antenna is required that uniformly illuminates the ground over some region. The so-called cosecant pattern will do this. From the figure we see that $h = r \cos\left(\dfrac{\pi}{2} - \theta\right)$ or

$$r = \frac{h}{\sin \theta} = h \csc \theta.$$

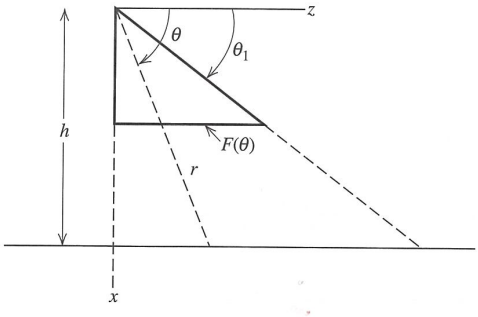
This expresses how much farther the radiation must travel to reach the ground as θ is decreased. The radiation pattern

$$F(\theta) = \csc \theta,$$

will just compensate for the $1/r$ field variation with distance. If, in addition, the ϕ variation is a sector pattern of small angular extent ϕ_0, then

$$F(\theta, \phi) = \begin{cases} \csc \theta & \theta_1 < \theta < \dfrac{\pi}{2},\ 0 < \phi < \phi_0 \\ 0 & \text{elsewhere.} \end{cases}$$

Derive an expression for the directivity.



1.8-8 *Gaussian Pattern.* A circularly symmetric, narrow beam antenna pattern is frequently modeled by a Gaussian shape given by

$$F(\theta) = e^{-4 \ln \sqrt{2} \, (\theta/HP)^2}$$

Derive expressions for the directivity associated with this pattern in terms of the half-power beamwidth HP in radians and in degrees. Do this by approximating sin θ by θ in the integration for Ω_A and extending the integration limits to infinity.

1.8-9 An antenna has a directivity of 20 and a radiation efficiency of 90%. Compute the gain in dB.

1.8-10 Compute the gain of an antenna which has a radiation efficiency of 95% and the following radiation pattern

$$F(\theta) = \begin{cases} 1 & 0 \le \theta < 20^\circ \\ 0.707 & 20^\circ \le \theta < 120^\circ \\ 0 & 120^\circ \le \theta < 180^\circ \end{cases}$$

1.9-1 A 2-m-long dipole made of 6.35-mm (0.25-in.) diameter aluminum is operated at 500 kHz. Compute its radiation efficiency, assuming

a. The current is uniform
b. The current is triangular.

1.9-2 A citizen's band radio at 27 MHz uses a half-wavelength long antenna that has a radiation resistance of 70 Ω. Compute the radiation efficiency if the antenna is made with 6.35-mm-diameter aluminum. As a rough approximation assume that the current is triangular.

1.9-3 Use the ohmic resistance formula of (1-178) to verify the expression for R_{ohmic} for

a. A uniform current given by (1-175)
b. A triangular current given by (1-180).

1.9-4 A cordless telephone operating at 50 MHz has a 38-cm long monopole antenna made of 4-mm diameter aluminum tubing. Compute the radiation efficiency.

1.10-1 The instantaneous electric field components of an elliptically polarized wave are $\mathscr{E}_x = E_1 \cos(\omega t - \beta z)$ and $\mathscr{E}_y = E_2 \cos(\omega t - \beta z + \delta)$. Specify E_1, E_2, and δ for the following polarizations:

a. Linear with $E_1 \neq 0$ and $E_2 \neq 0$.
b. Right circular.
c. Left circular.
d. Elliptical with $E_1 = E_2$.
e. Elliptical with $\delta = 90^\circ$.

1.10-2 Write the frequency domain form of the total vector electric fields given in Prob. 1.10-1.

1.10-3 Start with (1-189) and prove (1-190). Use the fact that the magnitude of $\mathbf{E}$ follows from $|\mathbf{E}|^2 = \mathbf{E} \cdot \mathbf{E}^*$. Also note that γ in Fig. 1-24 is in a triangle with sides E_1 and E_2 and hypotenuse $|\mathbf{E}|$.

1.10-4 Prove that a RHCP wave normally incident on a plane perfect conductor changes to LHCP upon reflection.

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