

Signal Processor Implementation of Variable Digital Filters

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Abstract—The implementation of two recently introduced variable digital filter schemes using a TMS320 series digital signal processor is presented. One is a method for updating the coefficients of an FIR filter in a simple manner such that the cutoff frequency can be controlled through a single parameter. The other is a method for tuning the cutoff frequency of an IIR filter with one parameter using a series expansion of the low-pass-low-pass frequency transformation. The measured frequency responses compare well with the theory.

I. INTRODUCTION

VARIABLE cutoff frequency filters are often used as laboratory instruments, or as a part of a laboratory instrument. In this paper, we review variable digital filter schemes, both recursive and nonrecursive, which can be used in such applications. We also present examples of the implementation of these filters using a TMS320 series digital signal processor.

The theory of variable digital filters has received considerable attention over the last decade. Oppenheim *et al.* [1] proposed a transformation method for linear phase finite impulse response (FIR) filters. The transformation varies the filter cutoff frequency by distorting the frequency axis, while preserving the frequency characteristics. This approach has been further developed [2], [3] and extended to bandpass filters [4], [5]. For infinite impulse response (IIR) filters, Constantinides [6] provides the theoretical background for varying the filter cutoff frequency. This method utilizes stable all-pass transformation functions which transform the low-pass prototype characteristics to the desired low-pass, high-pass, bandpass, or bandstop characteristics. The use of frequency transformations for the design of tunable filters has been studied by several authors [7]–[11]. Recently, Mitra *et al.* [12] presented an efficient method for real time frequency transformation of IIR digital filters. In this method, the low-pass-low-pass transformation is expressed as a Taylor series expansion. By truncating the series expansion after the linear term, an extremely simple transformation is obtained, and the filter coefficients can be calculated as a linear function of the tuning parameter.

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A straightforward way for tuning a filter is to vary all the filter coefficients in such a way that the filter characteristics vary in the desired manner. Mutluay and Fahmy [13] proposed a method for IIR filters where the filter coefficients are presented as simple analytic functions of the tuning parameters by fitting the functions to a set of predesigned filter coefficients. For FIR filters, an efficient method for controlling the filter coefficients was presented by Jarske *et al.* [14]. Their method is based on the basic properties of ideal band selective filters. The filter coefficients are calculated from an analytic function of the tuning parameters which is of the same form as for the ideal infinite length filter.

In the following discussion, we review the methods of Mitra *et al.* [12] and Jarske *et al.* [14] more closely and consider the signal processor implementation of variable digital filters using these methods. Examples of both methods have been implemented with the TMS32010 digital signal processor, and the results from frequency response measurements are presented.

II. THE TUNING METHODS

In this section, the variable digital filter schemes introduced in [12] and [14] are reviewed.

A. Recursive Variable Digital Filters

The basic idea of transformation methods in digital filter design is to generate the desired transfer function from a prototype transfer function by means of the substitution

$$z^{-1} = T(z). \quad (1)$$

For low-pass-low-pass transformation of IIR filters, $T(z)$ is a first-order all-pass function given by [6]

$$T(z) = \frac{z^{-1} - \beta}{1 - \beta z^{-1}} \quad (2)$$

where

$$\beta = \frac{\sin\left(\frac{\omega_{co} - \omega_c}{2}\right)}{\sin\left(\frac{\omega_{co} + \omega_c}{2}\right)}. \quad (3)$$

Here, ω_{co} is the cutoff frequency of the prototype filter and ω_c is the desired cutoff frequency. Similarly, the low-pass-bandpass transformation can be done with a second-order all-pass transformation function.

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A wide variety of band selective IIR transfer functions can be realized as a parallel connection of two all-pass sections, which in turn can be implemented as a cascade of second- and first-order all-pass blocks. This filter structure has also been utilized in [12]. First- and second-order all-pass blocks can be realized in a number of computationally efficient ways [15].

When the low-pass-low-pass transformation of (2) is applied to a prototype filter, the coefficients of the transformed filter can be expressed as functions of the parameter β and the prototype filter coefficients. For a first-order all-pass transfer function

$$A_1(z) = \frac{z^{-1} + a_0}{1 + a_0 z^{-1}} \quad (4)$$

the coefficient of the transformed function becomes [12]

$$a_0 = \frac{a_{00} + \beta}{1 + a_{00}\beta} \quad (5)$$

where a_{00} is the prototype coefficient. For a second-order all-pass section

$$A_2(z) = \frac{z^{-2} + a_1 z^{-1} + a_0}{1 + a_1 z^{-1} + a_0 z^{-2}} \quad (6)$$

the coefficients can be calculated from

$$a_0 = \frac{\beta^2 + a_{10}\beta + a_{00}}{1 + a_{10}\beta + a_{00}\beta^2} \quad (7)$$

$$a_1 = \frac{2\beta + a_{10} + a_{10}\beta^2}{1 + a_{10}\beta + a_{00}\beta^2} \quad (8)$$

where a_{10} and a_{00} are the prototype coefficients. In the approach of [12], (5), (7), and (8) are expressed as Taylor series expansions with respect to the parameter β , and the series expansions are truncated after the linear term. The coefficients can then be approximated with

$$a_i \approx a_{i0} + \beta c_i \quad (9)$$

where

$$c_0 = 1 - a_{00}^2 \quad (10)$$

for the first-order all-pass section, and

$$c_0 = a_{10}(1 - a_{00}) \quad (11)$$

$$c_1 = 2 + 2a_{00} - a_{10}^2 \quad (12)$$

for the second-order all-pass section. In the implementation of the variable filter, we can either calculate the coefficients using (9) or replace each filter coefficient with a tuning branch in parallel with the prototype coefficient, as shown in Fig. 1. In a signal processor implementation, both approaches are almost equal from a computational point of view.

B. Nonrecursive Variable Digital Filters

The method of variable FIR filters [14] is based on the basic properties of low-pass digital filters. The impulse

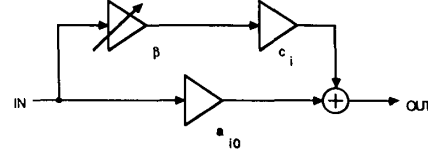


Fig. 1. A tunable coefficient according to (9).

response of an ideal low-pass digital filter is given by

$$h_{ID}(n) = \frac{\omega_c}{\pi} \text{sinc}\left(\frac{\omega_c n}{\pi}\right) \quad (13)$$

where ω_c is the cutoff frequency, $\text{sinc}(x) = \sin \pi x / \pi x$ and $\text{sinc}(0)$ is defined to be equal to one. This expression is a relatively simple function of the filter cutoff frequency, and it can be used for tuning the filter with a single parameter. The length of the ideal impulse response is, naturally, infinite. However, if a finite length filter is designed by truncating the ideal unit sample response with any symmetric window, the simple form of (13) is not changed.

It is also shown in [14] that optimal filters with equal ripples in the passband and stopband can be approximated successfully with a function of the form

$$h_0(n) = \begin{cases} c(n)\omega_c + d, & \text{for } n = 0 \\ c(n) \sin(\omega_c n), & \text{for } 1 \leq |n| \leq N \end{cases} \quad (14)$$

where ω_c is the variable 6 dB cutoff frequency, $c(-n) = -c(n)$ for $|n| \geq 1$, and d is zero for ideal low-pass filters but can be adjusted for better performance or for obtaining high-pass or notch responses. This allows more flexible design than the traditional window methods. It is also possible to design variable FIR filters with unequal ripples in the passband and stopband by using a slightly more complicated expression for the filter impulse response. Variable high-pass, bandpass, and notch filters can be obtained from a low-pass prototype with simple methods [14].

The main problem in the implementation of variable filters using this approach is the sine function. Standard methods for implementing circular functions are table look-up and series expansion. The values of a sinusoid can also be calculated recursively using a digital sine-wave generator [16].

III. THE HARDWARE

Several typical variable digital filters, designed using the above schemes, were implemented using the Texas Instruments TMS32010 assembler. The processor has 16-bit fixed point 2's complement arithmetic, a 32-bit accumulator, 4K program memory address space and 144 words of internal data memory. The TMS32010 digital signal processor evaluation module (EVM) provides the ability to develop, debug, and test programs [17]. When used in conjunction with the TMS32010 analog interface board (AIB) [18], digital filter algorithms can be tested in

real time. The programs can be developed also in a host computer, and downloaded to the EVM program memory.

The AIB has a parallel 16-bit expansion port which is useful in testing variable filters. The expansion port is seen in port address PA3 by the TMS32010 processor on the EVM board. In the examples, we wanted to input a parameter (β , for instance) to the processor without interrupting the actual filtering. For this purpose, 16 DIP switches were connected to the expansion port such that a 16-bit word can be set to the port and read by the processor. The EOC2 signal (see [18, appendix A]) is required to latch the data to the expansion port. In our tests, the EOC2-pin was connected to the end of conversion (EOC)-pin of the A/D converter on the AIB. This way, every time a new sample is available at the A/D converter, the parameter in the expansion port is also available.

The example filters were analyzed with a Hewlett Packard HP 3580A spectrum analyzer. The analyzer provides a frequency sweep signal which can be input to the filter, and the frequency response of the filter can be measured. The spectrum analyzer also has x- and y-outputs for plotting the responses. A block diagram of the hardware configuration used in the measurements is shown in Fig. 2.

IV. EXAMPLES

A. IIR Example

A fifth-order variable cutoff low-pass IIR filter was implemented using the fifth-order elliptic prototype filter given in [12]. The filter was realized as a cascade of a first- and second-order all-pass block connected in parallel with a second-order all-pass block (Fig. 3). The all-pass networks of Fig. 4 were used. Because of the selected structure (Fig. 3), the complementary high-pass output is also available without any additional computational effort. The filter routine proceeds as follows.

- 1) Initialize AIB and read prototype coefficients.
- 2) Wait for A/D conversion.
- 3) Read signal sample and parameter β .
- 4) Update coefficients using (9).
- 5) Execute the filter algorithm (Figs. 3 and 4).
- 6) Write output to D/A converter, and go to 2).¹

The filter algorithm was realized according to Fig. 3. The structure of Fig. 4(a) was used for A_{11} and the structure of Fig. 4(b) for A_{22} and A_{23} . The frequency response of this filter was measured with $\beta = -0.125, 0.0, 0.125, \dots, 0.75$, and the plotted results are presented in Fig. 5(b). For comparison, the responses calculated using (4), (6), and (9)–(12) are shown in Fig. 5(a). Because of the truncated series expansion approach used in this method, the range of cutoff frequencies is somewhat limited which can be seen from the theoretical responses, as well as the measured responses. The stopband error increases as the

¹Program codes implementing the examples presented in this paper are available from the authors.

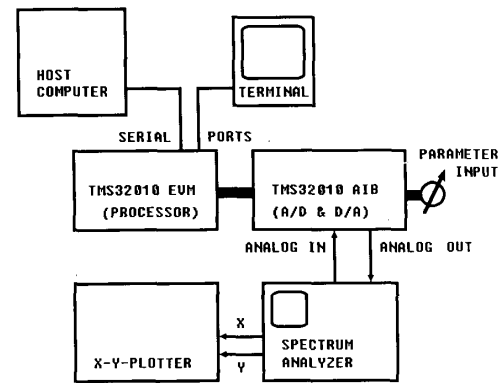


Fig. 2. The hardware configuration used in the measurements.

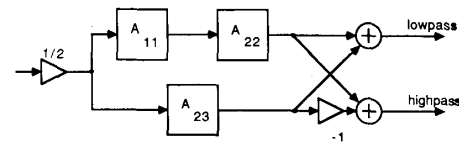


Fig. 3. A fifth-order IIR filter implemented using first- and second-order all-pass blocks.

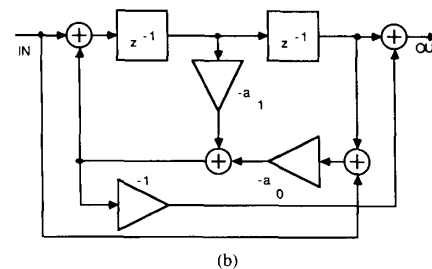
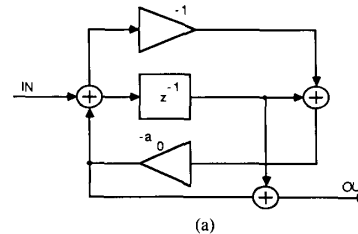


Fig. 4. A first-order (a) and a second-order (b) all-pass network.

value of β diverges from zero. The filter of this example works with sampling rates up to 50 kHz provided that the A/D converter is fast enough. Higher order filters require correspondingly more execution time. In the measurements, a 10-kHz sampling rate was used because the analog filters on the AIB have a fixed cutoff frequency 4.7 kHz. The input signal was swept from zero to 5 kHz at the speed 0.5 kHz/s.

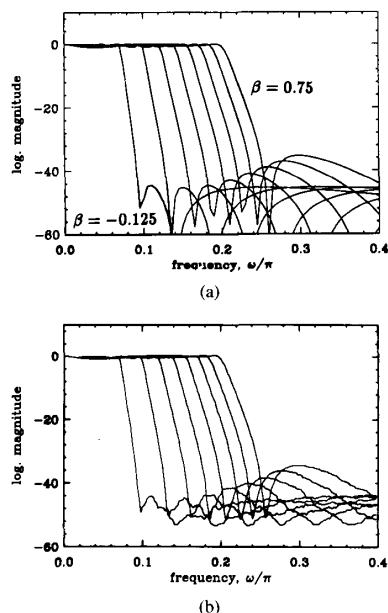


Fig. 5. (a) Calculated frequency responses of the fifth-order variable IIR filter. (b) Measured frequency responses of the same fifth-order variable IIR filter.

B. FIR Example

A variable linear phase low-pass FIR filter of length 59 was implemented using (14). The filter has a transition bandwidth of 0.1π and a stopband attenuation of over 45 dB. The sine function required in this method was realized as a 1024-word lookup table in the program memory. It contains one whole cycle of the sine function. The filter structure used in this example is a simple direct form FIR filter. The filter routine proceeds as follows.

- 1) Initialize AIB.
- 2) Read cutoff frequency.
- 3) Wait for A/D conversion.
- 4) Read signal sample and execute filter routine using old coefficients. Write output to D/A converter.
- 5) Calculate one new coefficient using (14).
- 6) Repeat 3)–5) until the whole set of new coefficients is calculated. Store them as old coefficients.
- 7) Go to 2).

The frequency response of this filter was measured with several different cutoff frequencies in the way described in the IIR example, and the plotted results are presented in Fig. 6(b). For comparison purpose, the frequency responses, calculated using the coefficients of (14) and floating point arithmetic, are shown in Fig. 6(a). One can see from the measured responses, as well as the theoretical responses, that the tuning range covers practically the whole frequency range from zero to half the sampling frequency.

The filter of this example works with sampling rates up to 30 kHz. The highest sampling rate is inversely propor-

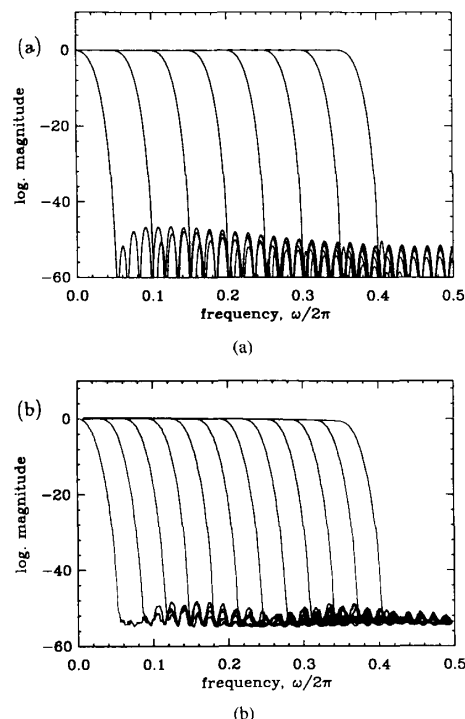


Fig. 6. (a) Calculated frequency responses of the 58th-order variable FIR filter. (b) Measured frequency responses of the same filter.

tional to the filter length. The filter length is limited to 59 in this example by the on-chip data memory of the signal processor. The 144-word data memory of the TMS32010 can store 59 signal samples plus two sets of coefficients (30 coefficients each). Longer filters can be realized with processors having more internal data memory, such as TMS320C25. Naturally, it is also possible to use external data memory at the expense of slower operation and lower sampling frequency.

The program of this FIR example can be easily converted into a variable center frequency bandpass filter by replacing the prototype coefficients with the coefficients of a narrow-band low-pass filter, and the sine function in the look-up table with the cosine function. Also, the center coefficient of the bandpass filter is constant (not a function of the center frequency as in the low-pass case). An example of the measured frequency responses of a variable center bandpass filter of length 59 is presented in Fig. 7.

V. CONCLUDING REMARKS

The TMS32010 digital signal processor implementation of two recently introduced efficient variable digital filter methods have been outlined. The tuning range of the reviewed variable filters is relatively wide, especially in the FIR filter case, which makes them useful for many applications. The measured variable filter frequency responses compare well with the theoretical responses calculated with "infinite" precision.

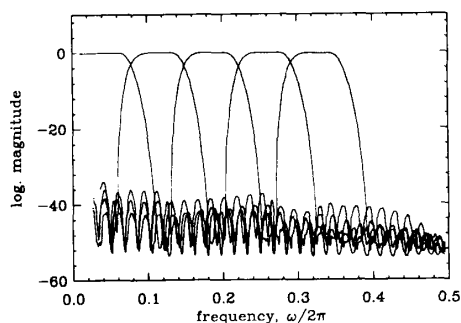


Fig. 7. Measurement frequency responses of a 58th-order variable center frequency FIR bandpass filter.

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