

MICROMERITICS

THE TECHNOLOGY of FINE PARTICLES

by

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SECOND EDITION



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CHAPTER 5

THEORY OF SIEVING AND GRADING OF MATERIALS

THE most rapid and convenient method for size separation of particles is by sieves. Sieves are constructed of wire cloth with definitely spaced openings so that material may be graded uniformly. Any size of opening may be obtained by proper choice of wire size and appropriate spacing, although it is customary to use openings and wire sizes which, on the basis of experience, have been proved to give satisfactory size separation. In this chapter we are concerned chiefly with the theory of sieving. Sieve-sizes, where stated, refer to standard series. The results apply generally to test sieves, but it is obvious that they have wide application to industrial conditions.

All crushing and pulverizing equipment has a point of maximum efficiency which is a function of particle-size. If we exceed this size or fall below it, we fail to obtain the highest capacities of the equipment. Moreover, it is desirable so to utilize such equipment that the maximum amount of material produced is of the size desired. This is but one example; other examples are the necessary fineness of cement and sizing of aggregates to produce concrete mixtures for maximum strength, the production of uniform-grade ceramic ware, and the manufacture of abrasives with definite cutting qualities.

It is customary to use square openings in the various types of sieves for industrial and test purposes. However, round and rectangular openings are frequently used, the former in cylindrical-type screens. As will be shown later, neither the square opening nor the round opening is capable of sharply differentiating sizes. Much depends upon the shape of the particles and how they pass through the openings. The average size of particles passing one screen and retained upon another (next in size) can be determined only approximately from the size of the openings. As already explained, this is partly due to the shape of the particles; however, to some degree it is also a function of the shape of the opening itself. Thus, given a square opening of a side equal to l , any needle-shaped particle can be made to pass diagonally if it has dimensions less than $l\sqrt{2}$. Sieves and screens have contributed much to industrial progress even though particle-size differences do occur in carefully graded

TABLE 11—THE TYLER STANDARD SCREEN SCALE SIEVES

This screen scale has as its base an opening of 0.0029 in. which is the opening in 200-mesh 0.0021-in. wire, the standard sieve, as adopted by the Bureau of Standards of the United States Government, the openings increasing in the ratio of the square root of 2 or 1.414.

Where a closer sizing is required Column 5 shows the Tyler Standard Screen Scale with intermediate sieves. In this series the sieve openings increase in the ratio of the fourth root of 2 or 1.189.

1	2	3	4	5	6	7	8	9
Tyler Standard Screen Scale $\sqrt{2}$ or 1.414 openings (in.)	Every other sieve from 0.0029 in. to 0.742 in. Ratio of 2 to 1	Every other sieve from 0.0041 in. to 1.050 in. Ratio of 2 to 1	Every 4th sieve from 0.0029 in. to 0.742 in. Ratio of 4 to 1	For closer sizing sieves from 0.0029 in. to 1.050 in. Ratio of $\sqrt[4]{2}$ or 1.189	Open- ings (mm)	Open- ing in frac- tions of inch (ap- prox.)	Mesh	Diam- eter of wire
1.050		1.050		1.050	26.67	1	...	0.148
....				0.883	22.43	$7/8$	...	0.135
0.742	0.742		0.742	0.742	18.85	$3/4$	...	0.135
....				0.624	15.85	$5/8$	...	0.120
0.525		0.525		0.525	13.33	$1/2$	...	0.105
....				0.441	11.20	$7/16$	...	0.105
0.371	0.371	...		0.371	9.423	$3/8$	...	0.092
....				0.312	7.925	$5/16$	$2^{1/2}$	0.088
0.263		0.263		0.263	6.680	$1/4$	3	0.070
....				0.221	5.613	$7/32$	$3^{1/2}$	0.065
0.185	0.185		0.185	0.185	4.699	$3/16$	4	0.065
....				0.156	3.962	$5/32$	5	0.044
0.131		0.131		0.131	3.327	$1/8$	6	0.036
....				0.110	2.794	$7/64$	7	0.0328
0.093	0.093			0.093	2.362	$3/32$	8	0.032
....				0.078	1.981	$5/64$	9	0.033
0.065		0.065		0.065	1.651	$1/16$	10	0.035
....				0.055	1.397	..	12	0.028
0.046	0.046		0.046	0.046	1.168	$3/64$	14	0.025
....				0.0390	0.991	..	16	0.0235
0.0328		0.0328		0.0328	0.833	$1/32$	20	0.0172
....				0.0276	0.701	..	24	0.0141
0.0232	0.0232			0.0232	0.589	..	28	0.0125
....				0.0195	0.495	..	32	0.0118
0.0164		0.0164		0.0164	0.417	$1/64$	35	0.0122
....				0.0138	0.351	..	42	0.0100
0.0116	0.0116		0.0116	0.0116	0.295	..	48	0.0092
....				0.0097	0.246	..	60	0.0070
0.0082		0.0082		0.0082	0.208	..	65	0.0072
....				0.0069	0.175	..	80	0.0056
0.0058	0.0058			0.0058	0.147	..	100	0.0042
....				0.0049	0.124	..	115	0.0038
0.0041		0.0041		0.0041	0.104	..	150	0.0026
....				0.0035	0.088	..	170	0.0024
0.0029	0.0029		0.0029	0.0029	0.074	..	200	0.0021

material. For large particles and aggregates there is no simpler method of grading material than sieving. Even the production of fine materials, of the order of $70\ \mu$ and less, depends on the use of wire-cloth screens. In fact no industry, whether it be road building, ceramics, or chemical engineering, can succeed without the necessary sieve equipment to size materials. In spite of great reliance on sieves for grading of materials, little is known about the rate of sieving for a mass of material of different sizes through a given screen.

TYPES OF SIEVES

Tyler Sieve Series—Because of the economic importance of sieving operations, it was necessary to standardize sizes of sieve openings. In this country recognition is given two types of wire-cloth screen: The so-called “Tyler” sieves and the U. S. Sieve Series. British practice calls for use of the British Engineering Standards Association (B.S.A.) Series. The Tyler Standard Screen Scale consists of openings whose sizes are in the fixed ratio of $\sqrt{2}$. These screens were brought out by the W. S. Tyler Company in 1910, and for many years were the only standardized sieves used in the United States. Various bases have been proposed as the starting point of screen scales; some advocated the use of 1 in., and others of 1-mm openings. The Tyler series is based on the size of the opening in 200-mesh wire cloth. The standard Tyler 200-mesh sieve is made from 0.001-in. wire and has an opening of 0.0029 in. Table 11 gives the Tyler Standard Screen Scale. It will be noted that the openings vary from 1.050 to 0.0029 in. Screens larger and smaller than those given in the table are available, but for sizing of materials, laboratory practice, and industrial grading according to a specified scale, the sizes shown in Table 11 are most practicable.

U. S. Sieve Series—A series of sieves, differing but slightly from the Tyler Standard Screen Scale, was proposed in 1919 by the National Bureau of Standards. This has been called the U. S. Sieve Series and in general the sizes advance in the same ratio as the Tyler Screen Scale, but a 1-mm opening is used as a base rather than the opening for the 200-mesh cloth in the Tyler series. There appears to be no reason for preferring one type of series over the other. Table 12 shows the various characteristics of the U. S. Sieve Series adopted by the A.S.T.M. which may be contrasted with those of the Tyler Screen Series given in Table 11.

The I.M.M. Series and British Engineering Standards Association Series—The I.M.M. Series was developed in 1907 by the Institute of Mining and Metallurgy of England. This series is mainly for laboratory use, and was developed on the theory that the diameter of the wire

should be equal to the size of the screen opening, keeping as nearly as possible to 25 percent opening and 75 percent dead space. The object

TABLE 12—NOMINAL DIMENSIONS, PERMISSIBLE VARIATIONS, AND LIMITS FOR WOVEN WIRE CLOTH OF STANDARD SIEVES, U. S. SERIES. A.S.T.M. STANDARD^a

Size or sieve designation		Sieve opening		Per- missible varia- tions in av. opening (per- cent $\pm$)	Permissible variations in max opening (percent $\pm$)	Wire diameter	
Micron	No	Mm	Inch (approx. equiv- alents)			Mm	Inch (approx. equivalents)
5660	3 $\frac{1}{2}$	5.66	0.233	3	10	1.28 -1.90	0.050 -0.075
4760	4	4.76	0.187	3	10	1.14 -1.68	0.045 -0.066
4000	5	4.00	0.157	3	10	1.00 -1.47	0.039 -0.058
8360	6	3.36	0.132	3	10	0.87 -1.32	0.034 -0.052
2330	7	2.83	0.111	3	10	0.80 -1.20	0.031 -0.047
2380	8	2.38	0.0937	3	10	0.74 -1.10	0.0291-0.0433
2000	10	2.00	0.0787	3	10	0.68 -1.00	0.0268-0.0394
1680	12	1.68	0.0661	3	10	0.62 -0.90	0.0244-0.0354
1410	14	1.41	0.0555	3	10	0.56 -0.80	0.0220-0.0315
1190	16	1.19	0.0469	3	10	0.50 -0.70	0.0197-0.0276
1000	18	1.00	0.0394	5	15	0.43 -0.62	0.0169-0.0244
840	20	0.84	0.0331	5	15	0.38 -0.55	0.0150-0.0217
710	25	0.71	0.0280	5	15	0.33 -0.48	0.0130-0.0189
590	30	0.59	0.0232	5	15	0.29 -0.42	0.0114-0.0165
500	35	0.50	0.0197	5	15	0.26 -0.37	0.0102-0.0146
420	40	0.42	0.0165	5	25	0.23 -0.33	0.0091-0.0130
350	45	0.35	0.0138	5	25	0.20 -0.29	0.0079-0.0114
297	50	0.297	0.0117	5	25	0.170-0.253	0.0067-0.0100
250	60	0.250	0.0098	5	25	0.149-0.220	0.0059-0.0087
210	70	0.210	0.0083	5	25	0.130-0.187	0.0051-0.0074
177	80	0.177	0.0070	6	40	0.114-0.154	0.0045-0.0061
149	100	0.149	0.0059	6	40	0.096-0.125	0.0038-0.0049
125	120	0.125	0.0049	6	40	0.079-0.103	0.0031-0.0041
105	140	0.105	0.0041	6	40	0.063-0.087	0.0025-0.0034
88	170	0.088	0.0035	6	40	0.054-0.073	0.0021-0.0029
74	200	0.074	0.0029	7	60	0.045-0.061	0.0018-0.0024
62	230	0.062	0.0024	7	90	0.039-0.052	0.0015-0.0020
53	270	0.053	0.0021	7	90	0.035-0.046	0.0014-0.0018
44	325	0.044	0.0017	7	90	0.031-0.040	0.0012-0.0016
37	400	0.037	0.0015	7	90	0.023-0.035	0.0009-0.0014

^a For sieves from the 1000-micron (No. 18) to the 37-micron (No. 400) size, inclusive, not more than 5 percent of the openings shall exceed the nominal opening by more than one-half of the permissible variation in the maximum opening.

of maintaining this relation between wire diameter and sieve opening seems to have been that it would prevent the wires from shifting. In

modern sieving practice there is no need for concern about shifting or slipping of wires since crimping forces them to stay in place. Due to selection of openings in terms of wire diameters, the I.M.M. series is made of heavy wire and is not always suitable for test sieves. Some question also arises on the desirability of limiting the open area of the sieve to a ratio of 1:3. In spite of these objections, however, the screen has found wide use in England for more than 30 years. The chief characteristics of this type of series are shown in Table 13.

TABLE 13—THE BRITISH ENGINEERING STANDARDS ASSOCIATION AND I.M.M. SIEVE SERIES

B. S. A. sieve	Nominal size of aperture	Nominal diameter of wire	Equivalent Tyler mesh	Equivalent I.M.M. sieve
5	0.1320	0.068	6	...
6	0.1107	0.056	7	...
7	0.0949	0.048	8	...
8	0.0810	0.044	9	...
10	0.0660	0.034	10	...
12	0.0553	0.028	12	...
14	0.0474	0.024	14	...
16	0.0395	0.023	16	...
18	0.0336	0.022	20	...
22	0.0275	0.018	24	...
25	0.0236	0.0164	28	...
30	0.0197	0.0136	32	...
36	0.0166	0.0112	35	30
44	0.0139	0.0088	42	...
52	0.0116	0.0076	48	...
60	0.0099	0.0068	60	50
72	0.0083	0.0056	65	60
85	0.007	0.0048	80	70
100	0.006	0.004	100	80
120	0.0049	0.0034	115	100
150	0.0041	0.0026	150	120
170	0.0035	0.0024	170	...
200	0.003	0.002	200	150
240	0.0026	0.0016	250	200

The I.M.M. Series has been superseded by the British Engineering Standards Association Series (1932). This sieve series is also shown in Table 13. It will be noted that the openings follow closely those of the Tyler series, the differences being due to the desire of specifying British Standard Wire diameters.

German and French Series—Because foreign literature often lists experiments in which powdered material is sized by sieves peculiar to a

given country, for completeness we also list in Table 14 the screens used in France and Germany. Those for France are designated AFNOR (*Association française de normalisation*, 1938). The German series, DIN, refers to the *Deutsche Industrie-Normen*, 1171, 1934. It is very much similar to the French series in many respects.

TABLE 14—FRENCH AND GERMAN STANDARD SIEVE OPENINGS (MM)

French Series AFNOR (1938)	German series DIN (1934)	Equivalent Tyler mesh number	Equivalent U. S. standard mesh number
...	...	...	...
5.00	5.00	...	...
4.00	4.00	5	5
3.15	3.00	...	...
2.50	2.50	...	...
2.00	2.00	9	10
1.60	1.50 (4)	...	...
1.25	...	...	...
...	1.20 (5)	14	16
1.00	1.00 (6)	16	18
0.80	...	...	...
...	0.75 (8)	...	...
0.63	...	...	...
...	0.60 (10)	28	30
0.50	0.50 (12)	32	35
0.40	0.43 (14)	35	40
...	0.40 (16)	...	...
0.315	...	...	...
...	0.30 (20)	48	50
0.25	0.25 (24)	60	60
0.16	...	...	...
...	0.15 (40)	100	100
0.125	0.12 (50)	115	120
0.10	0.10 (60)	150	140
...	0.09 (70)	170	170
0.08	...	...	...
...	0.075 (80)	200	200
0.063	0.060 (100)	250	230
0.05	...	270	270

Large-Opening Screens—For industrial sizing, screens may be more than 4 in. in size and may have either square openings, or round openings which are approximately equivalent to the stated sizes of square openings in testing sieves. The relation between square and round open-

ings is shown in Table 15. The numbered sieves are those of the U. S. Standard Sieve Series. Typical uses for the sizes listed are given in Table 16. The size numbers and nominal size square openings are those developed by the U. S. Bureau of Standards.

Size Ratio of Openings—In this country the ratio between the different sieve openings has been taken as $\sqrt{2}$, a practice recommended by Rittinger (1867). This, of course, is only approximately true in the I.M.M. Series, as indicated. In this latter series, the ratio is determined entirely by the proper selection of wire diameters for the size of opening to be produced. In the Tyler series, developed from a 200-mesh sieve as the base (0.0029-in. opening), the width of opening in each successive screen is $\sqrt{2}$ times the opening in the preceding sieve. The choice of this ratio makes the area of each successive opening double that of the

TABLE 15—APPROXIMATELY EQUIVALENT ROUND- AND SQUARE-OPENING TESTING SCREENS

(This table shows the sizes of round openings which are approximately equivalent to the stated sizes of square openings in testing sieves. Numbered sieves are those of the United States Standard Sieve Series.)

Square openings	Round openings (in.)	Square openings (in.)	Round openings (in.)
No. 8	$1/8$	$1 1/4$	$1 1/2$
No. 4	$1/4$	$1 1/2$	$1 3/4$
$3/8$ in.	$1/2$	$1 3/4$	2
$1/2$ in.	$5/8$	2	$2 3/8$
$3/8$ in.	$3/4$	$2 1/4$	$2 3/4$
$5/4$ in.	$7/8$	$2 1/2$	3
$7/8$ in.	1	3	$3 1/2$
1 in.	$1 1/4$	$3 1/2$	$4 1/4$
$1 1/8$ in.	$1 3/8$	4	$4 3/8$

next finer, or one-half that of the next coarser sieve. Another advantage of the Tyler series, due to selection of the $\sqrt{2}$ ratio, is that by omitting every other screen a ratio width of 2 to 1 is obtained. Skipping two sizes the ratio is approximately 3 to 1, and by skipping three sizes the ratio is 4 to 1, etc. Thus, for many important industrial operations it is possible to obtain $\sqrt{2}$, 2, 3, or 4 to 1 ratios of openings from the Tyler series.

The U. S. Sieve Series does not progress by the same ratio as the Tyler series in every instance. This may be checked by reference to Tables 11 and 12. The U. S. Sieve Series has been widely used in this country, and accepted as the standard by the American Society for Testing Materials.

TABLE 16—TYPICAL APPLICATIONS OF SIEVES FOR SIZING COARSE AGGREGATES

Use	Size number and nominal size ^a																				
	Size number	1	2	24	3	357	4	467	5	57	6	67	68	7	79	8	9	10	G1	G2	G3
	Nominal size	3 1/2 to 1 1/2	2 1/2 to 1 1/2	2 1/2 to 3/4	2 to 1	2 to No. 4	1 1/2 to 3/4	1 1/2 to No. 4	1 to 3/8	1 to No. 4	3/4 to 3/8	3/4 to No. 4	3/4 to No. 8	1/2 to No. 4	1/2 to No. 8	3/8 to No. 8	No. 4 to No. 16	No. 4 to 0	1 1/2 to No. 50	1 1/2 to No. 8	1 1/2 to No. 4
Water-bound macadam:																					
Coarse aggregate.....		×	×		×																
Filler.....																		×			
Bituminous macadam, penetra- tion method:																					
Coarse aggregate.....		×	×		×																
Choke.....									×		×	×			×						
Seal.....																×	×				
Bituminous plant mixes.....																					
Base or surface courses:																					
Base, open mix.....					×		×														
Base, closed mix.....						×		×		×											
Binder course.....										×		×									
Surface course, coarse grad- ing.....									×	×		×									
Surface course, fine grading.....																×	×				
Seal.....																×	×				
Bituminous road mix:																					
Mixing course.....									×			×									
Choke.....																×	×				
Seal.....																×	×				
Drag leveling course:																					
Leveling course.....												×	×		×						
Seal.....																×	×				
Bituminous surface treatment.....											×		×		×	×	×				
Seal for airport construction.....																	×				
Portland cement concrete.....		×	×		×	×	×	×		×		×		×							
Railroad ballast:																					
Stone or slag.....				×	×		×		×												
Gravel.....																			×	×	×
Roofing.....																×					

^a In inches, except where otherwise indicated; numbered sieves are those of the United States Standard Sieve Series.

For closer sizing it is possible to provide a ratio between sieve openings of $\sqrt[3]{2}$. The ratio between sizes of the openings is then $\sqrt[3]{2}$ or 1.189, and the difference in areas between two successive openings is about $1\frac{1}{2}$ times instead of 2, as with the Tyler and U. S. series. Reference to Table 11 shows the refinements possible with the use of the $\sqrt[3]{2}$ series.

In addition to the $\sqrt{2}$ and $\sqrt[3]{2}$ series, it is also possible to utilize a series whose successive differences are in the ratio of $\sqrt[3]{3}$. This series includes both the English inch and the metric millimeter, and should have had a certain amount of international acceptance. Unfortunately, it has never been practical to work out a series of such meshes and sizes of wire that could be woven satisfactorily. In addition to this objection, the cube root of the series does not include any of the sieves now in general use.

SIEVING EFFICIENCY

Factors Affecting Efficiency—To obtain a gradation of material passing from one sieve to another or to a container, some motion must be given to the particles contained on the screens so they will pass through the openings. This motion may be imparted by hand or by some apparatus which either vibrates or shakes the material. The method chosen should be one in which a maximum amount of material will pass a given screen in the shortest period of time. There are several mechanical shaker devices available such as the Tyler Ro-Tap, vibrators of the Hummer electric type, or the Leahy No-Blind screen, which depend on a high-frequency vibration imparted to the material being screened. Fahrenwald and Stockdale (1929) made a careful study of the effect of sieve motion on sieving efficiency. These investigators point out that 100 percent screening efficiency is unattainable inasmuch as the element of time is an important factor. Thus, a single difficult grain placed on a vibrating sieve may pass the sieve almost at once, but on a second trial may remain for a considerable time. The mechanics of sieving were aptly described by Stephenson (1926-1927), who compared sieving to floating timber logs down a river. The logs pile up against boulders and jam, then break loose and travel on for some time before jamming again. He indicated this periodicity in the case of sieving quartz. The quartz was originally passed through a 183-mesh silk screen and then made to pass a 200-mesh copper screen. The data obtained in a test are shown in Figure 21.

Effect of Screen Motion—It will be seen that from the first to the fourth minute the amount passed ranged from 2.272 to 2.314 g per min, but after the seventeenth minute there was considerable variation in the

percentages which passed. The nature of the jamming is clearly indicated by the inconsistent character of the amounts passing from minute to minute. Stephenson did not describe the method by which the sieving was done, but in later paragraphs this will be shown to be extremely important. However, assuming that whatever the method used it was constant throughout the experiment, the results indicate clearly the importance of the time element in sieving. The experiments of Fahrenwald and Stockdale were more extensive than those described by Stephenson. These investigators utilized two types of materials: Pure

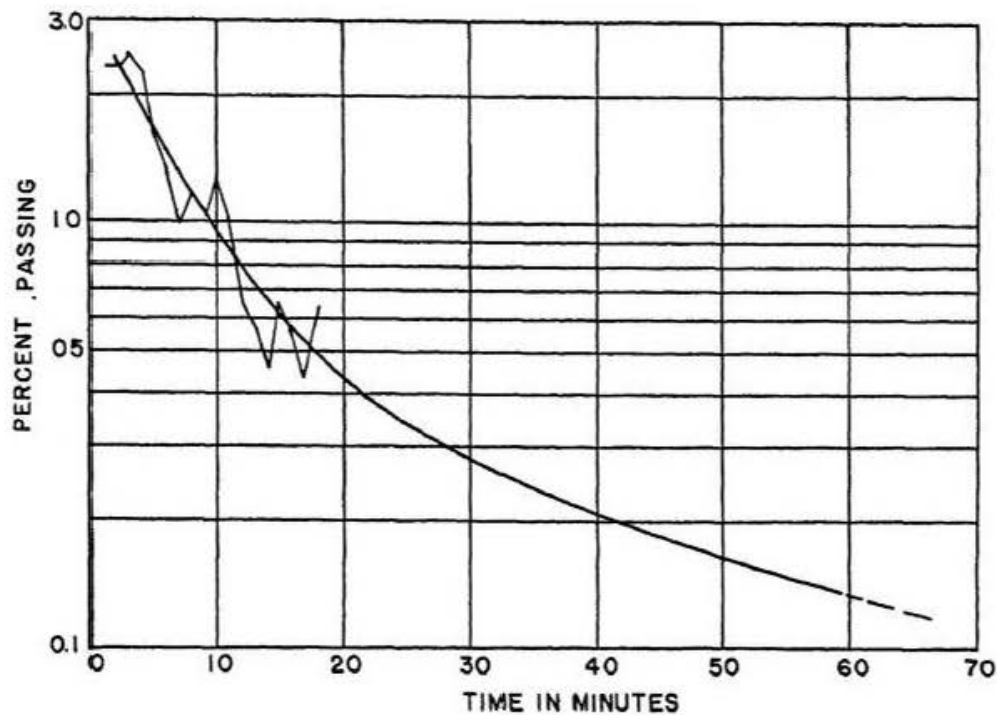


FIGURE 21. RATE-OF-SIEVING CURVE.

quartz (specific gravity 2.65) crushed to pass a 10-mesh sieve, and galena (specific gravity 7.5) prepared in a like manner. The quartz and galena were screened into six sizes ranging from 14/20-mesh to 65/100-mesh Tyler sieve. The sieving was done with the Tyler Ro-Tap sieve shaker, and shaken by hand until practically no more material passed the limiting sieve.

The following sieve motions were employed:

1. Horizontal rotary motion produced by a Ro-Tap sieve shaker, without the aid of the hammer or bumper.
2. Horizontal rotary motion combined with a vertical bumping or jarring motion such as produced by the Ro-Tap shaker with the aid of the hammer, which struck the screen frame about 155 times per min.

3. Vertical jarring motion produced by striking the bottom of a screen, held in one hand, with the palm of the other hand, using sufficient force to make the particles jump about.

4. Horizontal jarring or shaking motion produced by tapping with the palm of one hand, while holding a screen in the other—these taps were imparted to the screen at the rate of about 240 per min.

5. High-frequency vibrating motion similar to that produced by the Hummer electric, or the Leahy No-Blind screen.

Efficiency tests with sand and galena were made in the intermediate-size sieves of the Tyler $\sqrt{2}$ series. In this way the average of the limiting sizes (Tyler $\sqrt{2}$ series) was almost equal to the size of the opening of the testing sieve, and the ratio of undersize to oversize was about one to one. Thus the condition of the feed in each test was made as difficult as possible.

In their experiments with high-frequency vibrating sieves, Fahrenwald and Stockdale constructed a special apparatus with which the frequency and amplitude of vibration could be adjusted at will. The frequencies used in the experiments ranged from 500 to 2500 vibrations per min, while the amplitude of vibrations extended from 0.1 mm to approximately 0.7 mm.

Tests were also made with thin and thick beds of material, with the motions previously described. In the thin-bed tests conducted sufficient material was weighed out to cover the screen surface 1 particle in depth. For quartz this was approximately 25 g and for galena approximately 75 g. The sample was placed in the testing sieve and the sieve was shaken until all the material was scattered evenly over the screen surface. The material that passed through the screen in this operation was returned to the sieve surface. The sieve was then subjected to the motions previously described. When tests on deep beds of material were made, 250-g samples were used, giving a depth of approximately $\frac{1}{4}$ in. The material was subjected to the same motions as for the thin bed. Typical results obtained from various experiments are shown in Figure 22. It will be noted that the maximum amount of material sieved through each series of screens during a given interval of time does not follow any definite law. The figures also indicate that in so far as the relative efficiencies of the various methods are concerned, vibratory motion is first (provided the best combination of amplitude and frequency is used), followed successively by side-tap (hand) motion, bottom-tap (hand) motion, Ro-Tap (with hammer) motion, and Ro-Tap (without hammer) motion.

The curves of Figure 22 obtained with thick beds of material are not consistent with those obtained for thin beds of material. No data are

available on the effect of varying the load on the screens. Blinding is excessive in each motion used, except with the vibrator, in which case it is at a minimum. The data show further that with large loads the

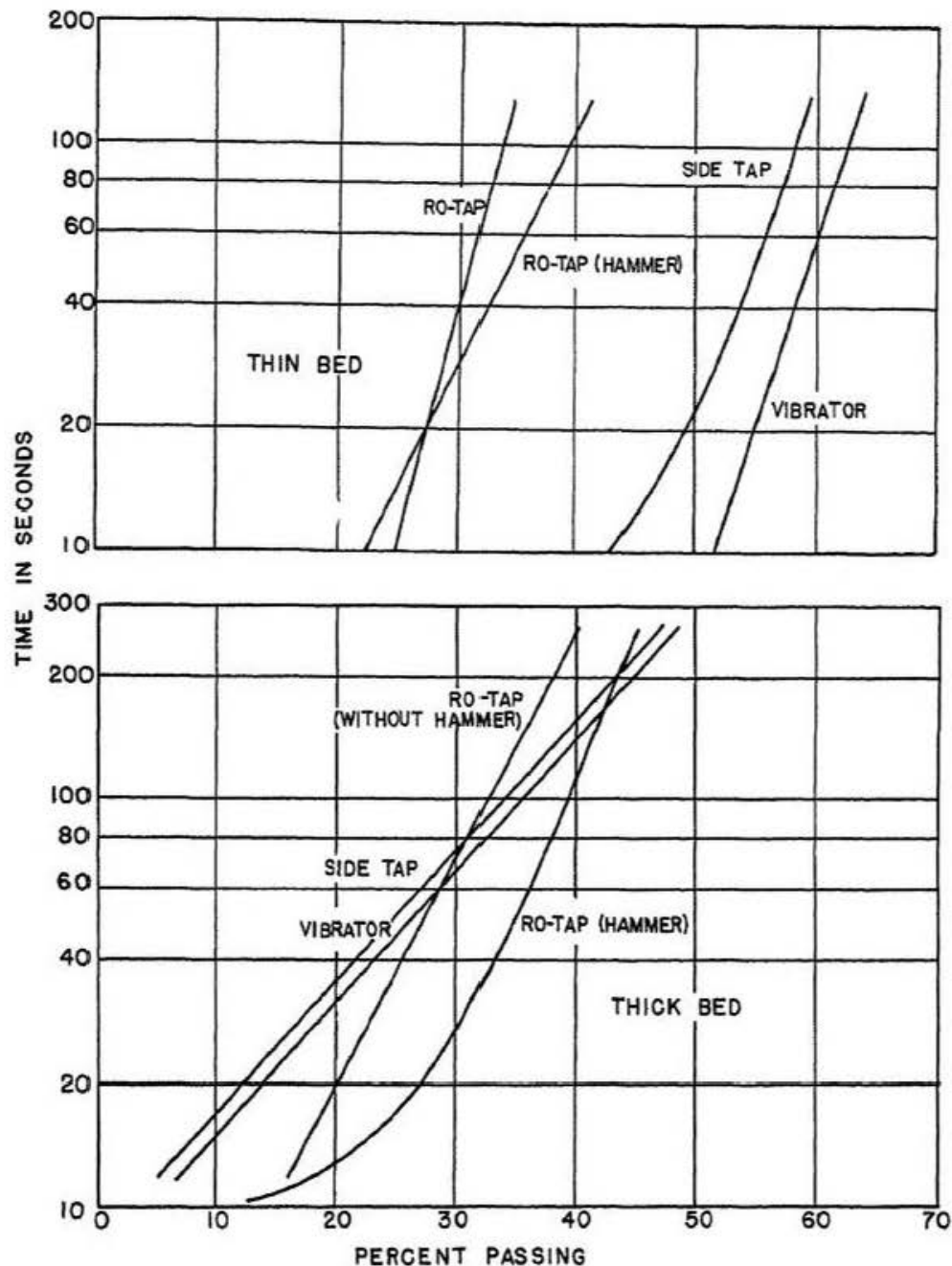


FIGURE 22. EFFECT OF VARIOUS MOTIONS ON RATE OF SIEVING.

amount of material passing a given series of sieves is approximately 25 percent less than with smaller amounts. Even after sieving for a relatively long period of time the amount of material passing for a given screen motion is never equal to that for the smaller bed loads. In general

the data suggest that if a set of screens can be standardized for a given load, the time required to screen additional material is almost proportional to the amount of material placed upon the screen. In other words if 2 min are required to screen 10 g of a given material, 100 g will require approximately 20 min. This, of course, assumes that the materials concerned are similar and that the sieve motions are the same in each case.

The results of studies made by varying the frequency and amplitude of the vibration are shown in Figure 23. The curves of the figure show that amplitude of vibration has a greater effect on the rate of screening than frequency. A change in amplitude of approximately 0.20 mm

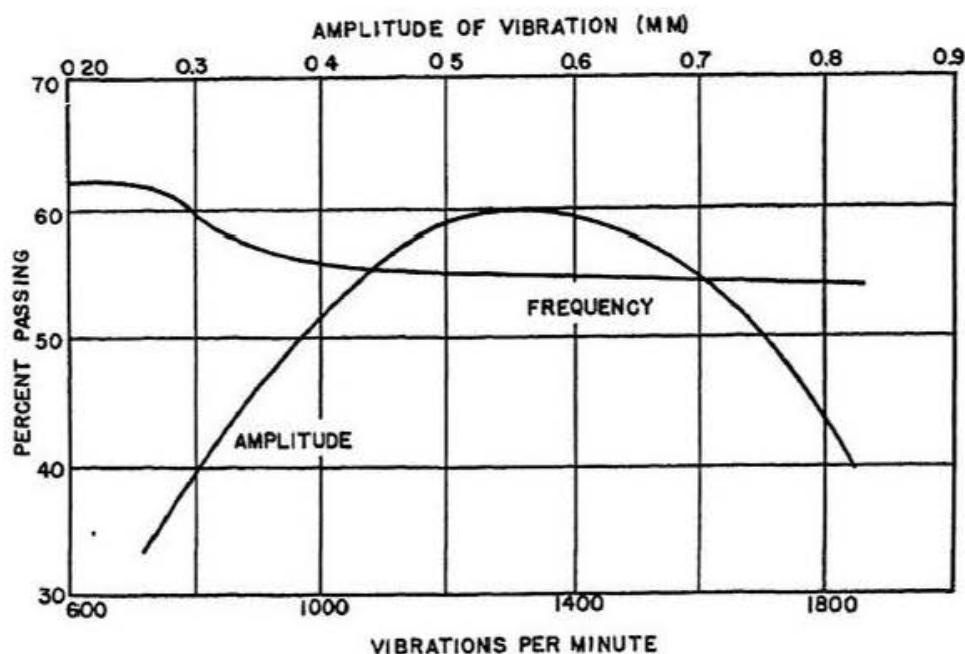


FIGURE 23. RELATION OF FREQUENCY AND AMPLITUDE OF VIBRATION TO EFFECTIVENESS OF SIEVING.

produces a greater change in the screening rate than a change in frequency of 300 or 400 vibrations per min. The condensed data showing amplitudes of vibration and frequencies, taken from the peaks of the amplitude and frequency curves for different sizes of quartz particles, are shown in Table 17.

The "efficiency" of the various sieve motions may be expressed by the formula

$$\text{efficiency} = \frac{\text{percentage material actually passing}}{\text{total percentage capable of passing}}$$

Fahrenwald and Stockdale considered the total material capable of passing a given sieve as that amount of material passing a sieve in 120

TABLE 17—COMBINATION OF AMPLITUDES AND FREQUENCIES OF VIBRATION FOUND MOST EFFICIENT FOR QUARTZ OF VARIOUS SIZES

Tyler mesh	Amplitude of vibration (mm)	Frequency of vibration (vpm)
14/20	0.60	580
20/28	0.40	1100
28/35	0.35	1350
35/48	0.30	1350
48/65	0.30	1500
65/100	0.20	1700

sec when subjected to vibratory motion. In comparing the effectiveness of various sieve motions, the total amount passing in this time was used as a criterion. Data using this criterion are presented in Table 18.

TABLE 18—EFFECTIVENESS OF VARIOUS SIEVE MOTIONS

Material used	Size mesh	Ro-Tap (without hammer) motion (percentage through)	Ro-Tap (with hammer) motion (percentage through)	Bottom-Tap (hand) motion (percentage through)	Side-Tap (hand) motion (percentage through)	Vibratory motion (percentage through)	Maximum difference (percentage through)
Quartz	14/20	30.2	36.7	55.9	56.2	54.8	26.0
Quartz	20/28	46.8	45.4	76.3	73.4	76.0	29.5
Quartz	28/35	43.8	37.0	60.2	62.7	64.0	27.0
Quartz	35/48	41.3	45.9	56.3	67.8	67.1	26.5
Quartz	48/65	32.6	35.3	32.5	40.9	43.0	10.5
Quartz	65/100	43.8	54.8	55.6	56.8	57.4	13.6
Galena	14/20	44.2	50.5	..	60.0	63.2	19.0
Galena	20/28	32.3	36.0	..	53.6	54.7	22.4
Galena	28/35	25.1	25.6	..	39.6	43.1	18.0
Galena	35/48	34.3	37.7	..	52.3	53.1	18.8
Galena	48/65	27.1	35.3	..	42.3	43.3	16.2
Galena	65/100	33.4	38.1	..	42.6	44.7	11.3

Lister (1924) found that the rapidity of screening through round-opening screens varied approximately in proportion to the square root of the diameters of the holes. Expressed on a weight basis Lister gives the following formula

$$w = k \sqrt{D} \quad \text{Eq (5-1)}$$

where w is the weight of material passing in a unit-time, D the diameter of the screen opening, and k a constant to be determined by experiment. Lister gave no reasoning for the derivation of this formula.

Fagerholt's Analysis—Fagerholt (1945) gives the following with regard to the laws governing sieving action. If we assume that the weight of particles of a given size passing a sieve in a unit time is proportional to the weight of the particles of the same size remaining on the sieve, we have

$$\frac{dw(d,t)}{dt} = -w'(d)w(d,t)$$

where $w(d,t)$ denotes the weight of particles which are retained on the sieve at the time t , and $w'(d)$ is a constant greater than zero. Integrating from time $t = t_0$ to $t = t$

$$w(d,t) = w(d,t_0) \exp [-w'(d)|t]$$

If d_{max} denotes the diameter of the largest particle capable of passing the sieve, then

$$\begin{aligned} w'(d) &\rightarrow 0 \text{ when } d \rightarrow d_{max} \\ w'(d) &= 0 \text{ when } d \geq d_{max} \end{aligned}$$

Thus, $w'(d)$ is a measure of the probability of passage.

The probability p of a particle passing a sieve, assuming the particle is thrown at right angles to the opening and that it does not touch the wire, has been determined by Mogensen (cited by Fagerholt) as

$$p = \frac{(l - d)^2}{(l + \delta)^2}$$

where l is the width of the mesh, d the diameter of the particle, and δ the thickness of the mesh. For a given sieve we may write

$$p = \alpha(l - d)^2 = \beta(d_{max} - d)^2$$

or as indicated above

$$w'(d) = \gamma(d_{max} - d)^2$$

where α , β and γ are suitable constants. Thus, for values of $d \leq d_{max}$

$$w(d,t) = w(d,t_0) \exp [-\gamma t(d_{max} - d)^2]$$

For values of $d \geq d_{max}$

$$w(d,t) = w(d,t_0)$$

If $C(d_{max})$ denotes the weight of particles capable of passing a given sieve and $R(t)$ denotes the residue remaining at the time t , then

$$C(d_{max}) = 1 - R(t)$$

If $F(d)$ denotes the weight-frequency distribution of the particles being sieved, then the residue $R(t)$ will be expressed by

$$\begin{aligned} R(t) &= \int_0^{\infty} F(d) \frac{w(d,t)}{w(d,t_0)} \cdot d(d) \\ &= \int_0^{d_{max}} F(d) \cdot \exp[-Ct(d_{max} - d)^2] dt + \int_{d_{max}}^{\infty} F(d) \cdot d(d) \\ &= \int_0^{d_{max}} F(d) \cdot \exp[-Ct(d_{max} - d)^2] dt + 1 - C(d_{max}) \end{aligned}$$

Since for practical purposes we may say that between two sieve sizes, the particle-weight distribution is linear:

$$C(d) = a + bd = a + F(d) \cdot d$$

and that particles smaller than d_0 have disappeared from the residue in time t . Thus

$$\begin{aligned} &b \int_0^{d_{max}} \exp[-\gamma t(d - d_{max})^2] \cdot d(d) = \\ &b \int_0^{d_0} \exp[-\gamma t(d - d_{max})^2] d(d) + b \int_{d_0}^{d_{max}} \exp[\gamma t(d - d_{max})^2] \cdot d(d) \end{aligned}$$

and following the assumption just made the first integral on the right is approximately zero. Hence we have

$$R(t) = 1 - C(d_{max}) + b \int_{d_0}^{d_{max}} \exp[-\gamma t(d_{max} - d)^2] \cdot d(d)$$

and writing $\Phi(d)$ for the probability integral

$$R(t) = 1 - C(d_{max}) + \frac{b\sqrt{\pi}}{\sqrt{\gamma t}} \{ \Phi(0) - \Phi[(d_0 - d_{max}) \sqrt{2\gamma t}] \}$$

If $(d_{max} - d_0)\sqrt{2\gamma t}$ is very large the corresponding Φ -function can be discarded, so that

$$R(t) = 1 - C(d_{max}) + \frac{b}{2} \sqrt{\frac{\pi}{\gamma t}} \quad (\text{approximately})$$

Replacing $1 - C(d_{max})$ by $R(\infty)$ which denotes the residue corresponding to complete separation

$$R(t) = R(\infty) + \frac{b}{2} \sqrt{\frac{\pi}{\gamma t}}$$

The weight-distribution being linear between two sieve sizes, we may write

$$b[d_{max} - dR(t)] = \frac{b}{2} \sqrt{\frac{\pi}{\gamma t}}$$

$$dR(t) = d_{max} - \frac{1}{2} \sqrt{\frac{\pi}{\gamma t}}$$

where $dR(t)$ denotes a particle size corresponding to the residue obtained by sieving for a time t and is dependent on particle shape and conditions of sieving. The value of d_{max} cannot be ascertained directly, though reasonable approximations can be made as to its value.

Fagerholt has shown that the average weight diameter d_w of particles may be determined from the weights of residues after two sieving periods t_1 and t_2 , ($t_2 > t_1$), and from corresponding expressions of the number of particles passing. Thus, he obtains

$$d_w = d_{max} - \frac{1}{\sqrt{\pi\gamma}} \left(\frac{1}{\sqrt{t_1}} + \frac{1}{\sqrt{t_2}} \right)$$

$R(t_1) - R(t_2)$ = weight of particles passing. For determining $dR(t)$ above, it is convenient to have the sieving time t_2 adjusted so that

$$d_{R(t_1)} = d_w$$

so that

$$\frac{1}{2} \sqrt{\frac{\pi}{\gamma t_1}} = \frac{1}{\sqrt{\pi\gamma}} \left(\frac{1}{\sqrt{t_1}} + \frac{1}{\sqrt{t_2}} \right)$$

$$\sqrt{\frac{t_1}{t_2}} = \frac{\pi}{2} - 1$$

or

$$t_2 = 3t_1 \quad (\text{approximately})$$

Thus, the particle size $dR(t)$ corresponding to the sieving residue $R(t)$ obtained by sieving in the time t is determined as the average particle

size d_w of the fraction passing the sieve by continued sieving under the same conditions up to the time $3t$.

PARTICLE-SIZE DETERMINATION

Average Size Based on Openings—The use of screens for particle-size determination was mentioned briefly in Chapter 4. When material is passed through one sieve and retained on the next smaller, the size assigned to the particles retained is a matter of choice. We may assign a diameter to the particles which is equal to the size of the opening in the larger or smaller sieve, the arithmetic or geometric mean of the openings of the two sieves, or a statistical average diameter determined by microscopic or other methods of size-measurement for particles retained on the smaller sieve. Averages obtained by the last-mentioned

TABLE 19—SIZE-WEIGHT DISTRIBUTION OF 435.2 G SAMPLE OF CRUSHED QUARTZ (TYLER SIEVE SERIES)

Sieve numbers (passing, retained) (1)	Arithmetic- mean size of openings (mm) (2)	Weight of quartz retained on smaller sieve (g) (3)	Percent re- tained on smaller sieve (4)	Weight- size (2) × (4) (5)
4/6	4.013	8.7	2.0	8.0
6/8	2.845	36.6	8.4	23.9
8/10	2.014	65.3	15.0	30.5
10/14	1.422	103.1	23.7	33.7
14/20	1.000	91.0	20.9	20.9
20/28	0.711	77.5	17.8	12.7
28/35	0.503	33.4	7.7	3.9
35/48	0.356	19.6	4.5	1.6
		435.2	100.0	135.2
Arithmetic mean = $135.2/100 = 1.35$ mm.				

method are dependent upon the character of the material sieved as well as the size of the sieve openings. In practice, it is customary to express the size of sieved material in terms of sieve-sizes used. Particular attention should always be given to stating which method is used in deriving the average size of sieved material. If, for example, a given material passes a 14-mesh and is retained on 20-mesh Tyler sieves (expressed symbolically 14/20), it should be stated clearly whether the average diameter of the particles is based on (a) the opening size of the 14-mesh sieve, (b) arithmetic average of 14- and 20-mesh sieve openings, or (c)

geometric average of these openings. The opening of a 14-mesh sieve is 1.17 mm, that of a 20-mesh 0.83 mm, and the average of the two sieves $(1.17 + 0.83)/2 = 1.00$ mm. The geometric mean is $\sqrt{1.17 \times 0.83} = 0.97$ mm. The differences are significant, thus emphasizing the need for stating clearly how particle-size averages of screened materials are derived.

Weight-Size Distribution—When a given material is passed through a series of sieves much as the Tyler or U. S. series, and the amount of material retained on each sieve determined, a weight-size distribution of the material is obtained. An example of such a size-analysis is shown in Columns 1, 2, and 3 of Table 19. The table also illustrates the method of computing the average diameter of the material. The geometric mean or the median may be obtained by methods outlined in Chapter 3, when weight of material passing or retained is substituted for number of particles.

TABLE 20—MOMENT METHOD FOR COMPUTING AVERAGE MESH SIZE OF CRUSHED QUARTZ. (TYLER SIEVE SERIES)

Sieve numbers (passing, retained) (1)	Percent quartz passing (g) (2)	Weight assigned to sieve (3)	Moment (2) × (3)
4/6	2.0	1	2.0
6/8	8.4	2	16.8
8/10	15.0	3	45.0
10/14	23.7	4	94.8
			4.4 ^a
14/20	20.9	5	104.5
20/28	17.8	6	106.8
28/35	7.7	7	53.9
35/48	4.5	8	20.0
	100.0		443.8
Mesh size = sieve corresponding to weight 4.4			
= mesh between numbers 14 and 20 sieves.			

^a Position of mean mesh.

A more simple method of sizing material, based on determining average "mesh-size," was developed by Coghill (1928). Since, as already indicated, standard-sieve series progress in size according to a fixed ratio, each sieve starting with the largest or smallest may be assigned a weight 1, the next in size 2, the next 3, and so on. The method of application is shown in Table 20 for data given in the preceding table. The weight assigned to each sieve is multiplied by the corresponding weight percent of the material. These are simply the moments of the weights.

The sum of the moments divided by the total weight (100 in the present case) is the mean "mesh-size."

RELATION BETWEEN COUNT AND WEIGHT DISTRIBUTIONS

T Hatch (1933) extended the basic size-distribution equation (Eq 3-17) to apply to weight-distribution curves in sieve-analysis. Thus

$$\rho \alpha_v n d^3 = \frac{\Sigma \rho \alpha_v n d^3}{\log \sigma_g' \sqrt{2\pi}} \cdot \exp [-(\log d - \log d_g')^2 / 2 \log^2 \sigma_g'] \quad \text{Eq (5-2)}$$

where ρ , d , α_v , and n are the density, diameter, volume-shape factor, and number of particles retained on each sieve, respectively, and d_g' and σ_g' are the geometric mean diameter and geometric standard deviation of the curve, by weight. The total weight of the sample is of course the term $\Sigma \rho \alpha_v n d^3$ on the right of the equation. Hence $\rho \alpha_v n d^3$ is the weight retained on each sieve.

The arithmetic mean diameter is the sum of the weights retained on each sieve multiplied by the corresponding diameter and divided by the total weight, thus

$$d_w = \Sigma (\rho \alpha_v n d^3) d / \Sigma \rho \alpha_v n d^3 = \frac{\Sigma n d^4}{\Sigma n d^3} \quad \text{Eq (5-3)}$$

This may be developed by methods outlined in Chapter 3, so that

$$\ln d_w = \ln \frac{\Sigma n d^4}{\Sigma n d^3} = d_g' + 0.5 \ln^2 \sigma_g' \quad \text{Eq (5-4)}$$

But by Eq (3-24)

$$\ln d_w^3 = \ln \left(\frac{\Sigma n d^4}{\Sigma n} \right) = 3 \ln d_g + 4.5 \ln^2 \sigma_g \quad \text{Eq (5-5)}$$

where d_g and σ_g are the statistical parameters of the size-frequency curve, by count. Adding Eqs (5-4) and (5-5)

$$\ln \left(\frac{\Sigma n d^4}{\Sigma n} \right) = \ln d_g' + 0.5 \ln^2 \sigma_g' + 3 \ln d_g + 4.5 \ln^2 \sigma_g \quad \text{Eq (5-6)}$$

Again using the development given in Chapter 3, Eq (5-6) may be expressed by the equation

$$\ln \left(\frac{\Sigma n d^4}{\Sigma n} \right) = 4 \ln d_g + 8 \ln^2 \sigma_g \quad \text{Eq (5-7)}$$

Substituting in Eq (5-6)

$$\ln d_g' = \ln d_g + 3.5 \ln^2 \sigma_g - 0.5 \ln^2 \sigma_g' \quad \text{Eq (5-8)}$$

The values of σ_g and σ_g' are the same, that is, the geometric standard deviations by count and weight are equal. This is easily proved by substitution of Eq (5-8) and the value of n given by the size-frequency distribution by count in Eq (5-2). Hence,

$$\ln d_g = \ln d_g' - 3 \ln^2 \sigma_g$$

or using logs to the base 10

$$\log d_g = \log d_g' - 6.908 \log^2 \sigma_g' \quad \text{Eq (5-9)}$$

This relation permits us to transfer size-distributions from a weight to a size basis.

Corresponding formulas for other statistical diameters discussed in Chapter 3 are as follows:

$$\log d_{av} = \log d_g' - 5.7565 \log^2 \sigma_g \quad \text{Eq (5-10)}$$

$$\log d_h = \log d_g' - 8.0591 \log^2 \sigma_g \quad \text{Eq (5-11)}$$

$$\log d_s = \log d_g' - 4.6052 \log^2 \sigma_g \quad \text{Eq (5-12)}$$

$$\log d_v = \log d_g' - 3.4539 \log^2 \sigma_g \quad \text{Eq (5-13)}$$

$$\log d_{rs} = \log d_g' - 1.1513 \log^2 \sigma_g \quad \text{Eq (5-14)}$$

The above equations apply only when the weight-in-size distribution is known to follow the log-probability distribution. Their application to a complete sieve-analysis also depends upon "calibration" of each sieve of the series used as explained in subsequent paragraphs.

CALIBRATION OF SIEVES

Hatch Method of Calibration—For the production of materials with uniform size, it is essential that sieve openings be calibrated. Specifications for many industrial products require that sizes be within specified limits. Unless screen openings are of uniform dimensions, specified limits may be exceeded. When fine material is made to pass through two sieves, the size of the material retained on the sieve with smaller openings is usually taken as the arithmetic or the geometric mean of the openings of the two screens. Actually the material is widely distributed as to size, as may be seen by reference to Figure 24, which shows material passing a 65-mesh Tyler sieve and retained on a 100-mesh. The arith-

metic or geometric mean of the openings does not indicate the true mean size of the material. Figure 24 illustrates the fact that particles of a given diameter may be found on several sieves. Clean-cut and definite size separation, therefore, is not possible by use of sieves. It will be seen in Figure 24 that the median size (count basis = 290μ) is closer to the opening size of a 48-mesh sieve than the 65-mesh Tyler sieve through which the material passed. Material obtained from any single source should always give the same distribution, but other materials may give much different distributions.

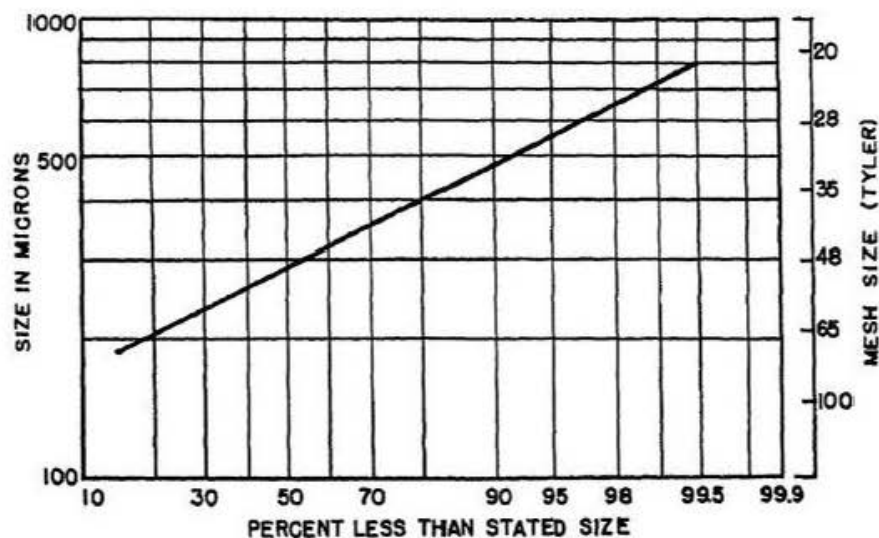


FIGURE 24. SUMMATION CURVE OF MATERIAL PASSING 65- AND RETAINED ON 100-MESH TYLER SIEVES.

T. Hatch (1933) applied this type of calibration to samples of limestone. The particles were measured microscopically as outlined in Chapter 4, and the median diameter *by weight* determined from Eq (5-9). The calibrated size of a sieve was taken as the median or geometric average size *by weight* of the material retained on the sieve. Thus, applying Eq (5-9) to the data plotted in Figure 24, we obtain the calibrated size of the sieve as follows:

$$\begin{aligned}\log d_g' &= \log d_g + 6.098 \log^2 \sigma_g \\ &= 2.462 + 6.098 \times 0.162 \times 0.162 \\ &= 2.478\end{aligned}$$

so that

$$d_g' = 302 \mu$$

Calibration sizes of four sieves, determined by Hatch for crushed lime-

stone, are given in Table 21. The data presented in the table show that the calibrated sizes are larger than the sieve openings.

TABLE 21—HATCH'S (1933) CALIBRATION OF FOUR SIEVES (TYLER SERIES) USING CRUSHED LIMESTONE

Material retained on (mesh)	Size of sieve opening (μ)	Calibration size (μ)
100	147	282
150	104	199
200	74	136
325	37	100

Application of the Hatch equations to a screen analysis is as follows: A summation curve is plotted on log-probability grid. The "percentage less than *calibrated* sieve-size" is plotted instead of the usual "percentage less than stated sieve-size." If the summation curve is a straight line, Eq (5-9) applies. The method is useful where precise information is desired on particle-size of a screened product.

Hatch extended his method of analysis to size-distribution curves ranging from coarse-screen analysis through fine particles measured microscopically. While excellent results were obtained by using this technique on laboratory samples, the method cannot be generalized to cover all types of distributions encountered in practice. As already explained in Chapter 3, size-frequency distributions may assume a variety of shapes. The Hatch development applies only to distributions which follow the normal or log-probability law. When size-distributions are hyperbolic in the lower extremes and follow normal log-probability laws in the upper extremes, the Hatch analysis must necessarily fail. Nevertheless, the relationships developed by Hatch have a far-reaching practical importance.

Weber and Moran Method of Calibration—Another method of calibrating sieves consists in measuring random openings in the sieve and obtaining size-frequency distributions as outlined in Chapter 3. The openings are measured by filar micrometer or by projection methods. Two screens are then identical if the mean size of the openings and the standard deviations are the same. Weber and Moran (1938) use the coefficient of variation

$$v = 100\sigma/l_{av} \quad \text{Eq (5-15)}$$

as a measure of sieve equivalence, where σ is the standard deviation and l_{av} the arithmetic average of the sieve openings. Their data indicate that

the difference between the average opening of a sieve with a high coefficient of variation and that of a sieve having the same retention but a smaller coefficient increases as the standard of the former increases above 6 percent. Sieves with high coefficients of variation behave as if their average opening were larger than that calculated in terms of the average of sieve openings. This difference is due to the relatively larger portion of oversized openings in screens possessing high coefficients of variation. However, since shaking time is also an important variable, it too must be considered. With longer shaking times a relatively small number of oversize openings would tend to increase the average size of material passed. Weber and Moran have proposed the following empirical formula for determining the effective opening of a sieve in terms of statistical constants and shaking time

$$l_e = l_{av} \left[1 + 0.002t \left(\frac{\nu - 6}{0.06} \right)^{1/2} \right] \quad \text{Eq (5-16)}$$

where l_e is the effective opening for a shaking time of t min and ν is the coefficient of variation as defined by Eq (5-15).

FINENESS MODULUS

The fineness modulus was developed in an effort to correlate strength of concrete in terms of its component sizes. The fineness modulus is simply the sum of the percentages in the sieve-analysis divided by 100. In concrete work, fineness modulus is applied to the aggregate and the following Tyler Standard Scale sieves are used: 100, 48, 28, 14, 8, 4, and $3/8$ -in., $3/4$ -in., and $1\frac{1}{2}$ -in. mesh. By using this series of sieves each sieve has a clear width opening double that of the next finest sieve, or one-half that of the next coarsest sieve.

Problems

1. The following is a sieve-analysis of a certain sand:

Sieve-size (Tyler)	Weight (percent)
6	0.5
8	3.5
10	12.6
14	25.3
20	38.4
28	48.9
35	65.3

Obtain the geometric mean size by plotting on a log-probability grid, and by means of Eq (5-10) determine the average size on a count basis.

2. Calculate the calibration size (weight basis) for a sand ($\rho = 2.64$) passing a 200-mesh and retained on a 325-mesh Tyler sieve, given the following size-frequency distribution:

Average group size (μ)	Percent less than stated size
60	0.7
70	5.0
80	30.0
90	50.0
100	82.0
110	90.3
120	96.1
130	99.1
140	99.5
150	99.8

3. Given a screen which is vibrated and assume that N of the particles contained on it are capable of passing with equal facility. If Φ denotes the probability of a single particle passing in a given interval of time, and assuming that Φ remains unchanged during the screening process, show that after ν time intervals the number of particles which has passed the screen is given by the expression

$$N\Phi[1 + (1 - \Phi)(1 - \Phi)^2 + \dots (1 - \Phi)^{\nu-1}]$$

or, what is equivalent, that the amount passing during the ν th interval is

$$N\Phi(1 - \Phi)^{\nu-1}$$

Hence also prove that the number of particles of the same kind left on the screen after the ν th interval of time is

$$N(1 - \Phi)^{\nu}$$

4. In a study of screening capacity, Herried (1935) obtained the following relation expressing certain screening variables when separating dry coke fines in $1/4$ -in. square opening screen

$$[(L/0.038) - 0.88](\omega - 868) = 1020$$

where L is the length of the screen and ω the shaft speed in rpm. The formula gives the maximum capacity of the screen. Plot the curve and discuss possible limitations of the formula. Note: The value of L should range from $1/32$ in. to $21/32$ in.

5. Given a screen of n -squares, each of area a . If the whole area of the screen is A , determine the probability of a particle whose largest dimension is less than $\sqrt{a}$ falling through the screen.

Note: The chance of a particle falling through a given opening a_1 is determined

as follows: Since there are n equally probable openings through which the particle may fall, the probability p_1 for this opening is

$$p_1 = a_1/A,$$

and since the whole probability is $p_1 + p_2 + \dots + p_n$,

$$p = \frac{1}{n} \cdot \frac{a_1}{A}$$

For squares of equal size $a = a_1 = a_2 = \dots$ etc.

$$p = a/A = 1/n$$

6. Given a screen with openings $2a$ by $2b$ on which is thrown at random a rodlike particle of length $2c$ less than either side of the opening, determine the probability that the particle will pass without touching the sides of the opening.

Note: The solution given by LaPlace for the probability q of crossing one side of the rectangular opening is

$$q = (c/\pi ab)(2a + 2b - c)$$

Hence the probability of not crossing either side is

$$p = 1 - (c/\pi ab)(2a + 2b - c)$$

If one of the sides, say b , is infinite, then the solution reduces to Buffon's problem of parallel lines on which a rod of length c less than the distance between the lines is thrown. In particular, when the openings of a screen are square

$$p = 1 - 4c/\pi a - c^2/\pi a^2$$

Thus, p is a minimum when $c = a$, or $p = 0.045$. (Compute p for various values of c/a .) For a wire screen, the wire thickness being δ , Fagerholt (1940) cites Andreasen as having determined the probability of a particle of diameter d passing through the opening as $[(a - d)/(a + \delta)]^2$.

7. Particles are screened on two sieves the size of whose openings are D_1 and D_2 . Show that the average weight diameter of the particles passing the larger and retained on the smaller is

$$d_w = \alpha \sqrt{\frac{2D_1^3 D_2^3}{D_1 + D_2}}$$

Note: Write the average particle passing D_1 and D_2 as $d_1 = \alpha D_1$ and $d_2 = \alpha D_2$, where the α 's are shape factors. If the $C(d)$ denotes the weight of particles less than d_w and if in the interval d_1 and d_2 the size distribution is linear

$$C(d) = a + bd$$

The constant b is therefore the weight of the particles. If there are $N(d)$ particles of size d_w

$$dN(d) = bd^{-3}d(d)/\rho$$

$$N(d_1) - N(d_2) = b(d_1^{-2} - d_2^{-2})/2\rho$$

Since the weight distribution of these particles may be denoted by $C(d)$

$$C(d_1) - C(d_2) = b(d_2 - d_1)$$

$$d_w = \sqrt{\frac{2d_1^2 d_2^2}{d_1^3 + d_2^3}} = \alpha \sqrt{\frac{2D_1^2 D_2^2}{D_1^3 + D_2^3}}$$