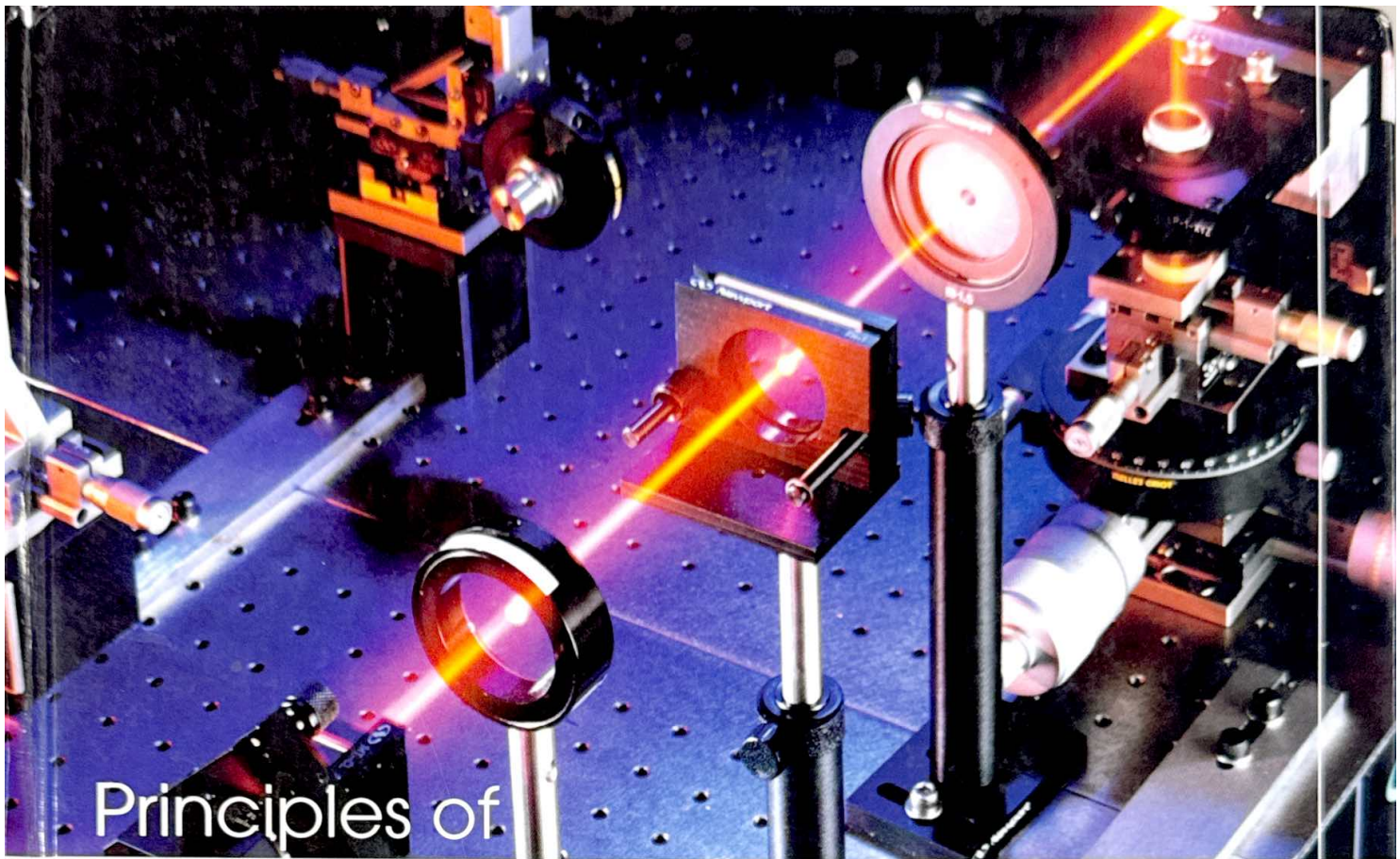




Principles of

PHYSICAL OPTICS



C.A. Bennett

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PRINCIPLES OF PHYSICAL OPTICS

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CHAPTER 3

REFLECTION AND REFRACTION

And perhaps, posterity will thank me for having shown it that the ancients did not know everything.
—Fermat

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3.1 INTRODUCTION

This chapter investigates the propagation of electromagnetic waves at interfaces between different optical media. The direction of refracted beams was first described by Snell in the early 1600s; reflection was described much earlier by the Greeks. Both phenomena were explained first variationally in 1657 by Fermat, then later by Maxwell with his field

equations of electromagnetism. Fresnel was first to describe the *quantity* of light reflected and refracted at a boundary with the equations that bear his name. His accomplishment was all the more remarkable in that he derived these results well before the availability of Maxwell's equations. The chapter concludes with an introduction to *dispersion* in optical materials that provides a conceptual foundation for refraction and absorption.

3.2 OVERVIEW OF REFLECTION AND REFRACTION

Reflection and *refraction* occur whenever light enters into a new medium where its speed changes value. From the previous chapter, we found that the speed of light within a material is given by

$$v = \frac{1}{\sqrt{\epsilon\mu_0}} = \frac{1}{\sqrt{(K_E\epsilon_0)\mu_0}} = \frac{1}{\sqrt{K_E}} \frac{1}{\sqrt{\epsilon_0\mu_0}} = \frac{c}{n} \quad (3.1)$$

where c is the speed of light in vacuum, K_E is the dielectric constant, and n is the index of refraction

$$n = \sqrt{K_E} \quad (3.2)$$

Thus, reflection and refraction occur whenever light enters into a new medium where the value of n changes value. Table 3.1 gives the indices of refraction for some common optical materials.

The *law of reflection* has been known experimentally since antiquity: *the angle of incidence is equal to the angle of reflection*. In order to define these angles, we make use of the concept of a *light ray*. For a plane wave with plane-parallel wavefronts, the associated ray is a directed line segment pointing perpendicular to the wavefronts in the direction of propagation of the wave.

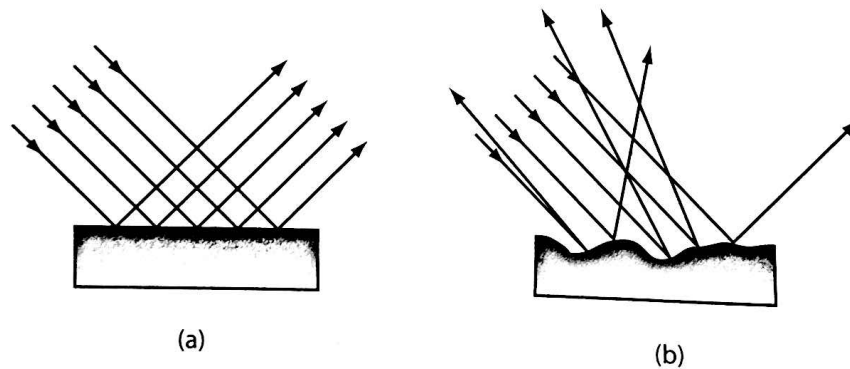


Figure 3.1. (a) Specular reflection. (b) Diffuse reflection.

An entire beam will reflect according to the law of reflection only if the surface of the interface between the two media is smooth. Reflection from a smooth surface is called *specular reflection*. Reflection from a rough surface is called *diffuse reflection*. Specular and diffuse reflection are illustrated in Figure 3.1.

Figure 3.2 shows a beam of light reflected from an interface separating two media. The incident beam is indicated by the incident ray, and the reflected beam is indicated by the reflected ray. The angle of incidence and angle of reflection are measured from the *normal to the interface*. Using these angles, the law of reflection is stated as follows:

$$\theta_i = \theta_r \quad (3.3)$$

Table 3.1. Range of transparency and indices of refraction for selected optical materials.

Material	Nominal Range of Transparency (μm)	n_r ^a
Vacuum	—	1
Air	0.2 – 5.0, 9.0 – 14.0	1.00029
Water	—	1.33
Ice	—	1.31
BK-7 Glass	0.35 – 2.0	1.50
Optical Crown Glass	0.35 – 2.5	1.52
Fused Silica (SiO_2) UV Grade	0.175 – 2.2, 2.9 – 3.6	1.46
Fused Silica (SiO_2) IR Grade	0.2 – 3.9	1.46
Quartz Crystal (SiO_2)	0.2 – 2.4	1.46
Zirconium Dioxide (ZrO_2)	0.36 – 7.0	2.40
Sapphire (Al_2O_3)	0.15 – 5.0	1.76
Diamond (C)	0.22 – 4.3, 5.4 – FIR^b	2.42
Calcium Fluoride (CaF_2)	0.18 – 8.0	1.40
Magnesium Oxide (MgO)	0.4 – 8.0	1.72
Barium Fluoride (BaF_2)	0.2 – 11	1.47
Strontium Fluoride (SrF_2)	0.15 – 11	1.44
Strontium Titanate (SrTiO_3)	0.4 – 6	2.31
Sodium Chloride (NaCl)	0.35 – 15	1.53
Potassium Bromide (KBr)	0.3 – 25	1.54
Potassium Chloride (KCl)	0.3 – 20	1.48
Cesium Iodide (CsI)	0.5 – 70	1.76
Potassium Iodide (KI)	0.4 – 40	1.64
Zinc Selenide (ZnSe)	0.5 – 20	2.49
Zinc Sulfide (ZnS)	0.45 – 14	2.27
Silicon (Si)	1.0 – 100	3.42
Germanium (Ge)	2 – 17	4.02
Gallium Arsenide (GaAs)	1.0 – 17	3.14
Cadmium Telluride (CdTe)	1.0 – 28	2.56
Polyethylene, High Density	16 – FIR	1.55

^aNominal, over the range of transparency. See Section 3.7.

^bFar-infrared.

The *law of refraction*, also called *Snell's law*, was found experimentally by Snell early in the seventeenth century. In Figure 3.2, the light ray is incident within the *incident medium* and the light beam transmits into the *transmitted medium*. The index of refraction of the incident medium is n_i and the index of refraction of the transmitted medium is n_t . According to Snell's law

$$n_i \sin \theta_i = n_t \sin \theta_t \quad (3.4)$$

Finally, it is implicit in Figure 3.2 that the incident, reflected, and refracted rays all lie within a single plane. This plane is called the *plane of incidence*.

We will refer to the case where $n_i < n_t$ as *external incidence*. An example of external incidence occurs at an air-glass interface. *Internal incidence* occurs when $n_i > n_t$; for example, when a light beam passes from glass to air.

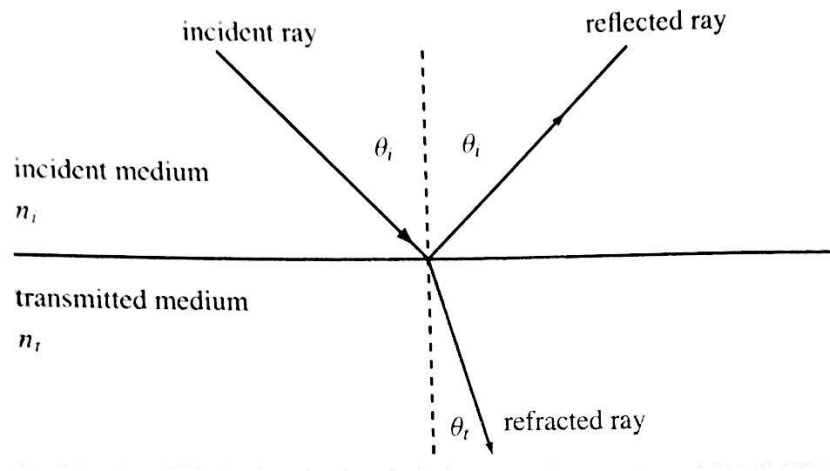


Figure 3.2. An interface between two media, showing the incident, reflected, and refracted rays.

■ EXAMPLE 3.1

Show that a light beam passing through a plane slab emerges parallel to its incident direction displaced laterally by a distance d given by

$$d = \frac{t}{\cos \theta_2} \sin (\theta_1 - \theta_2) \quad (3.5)$$

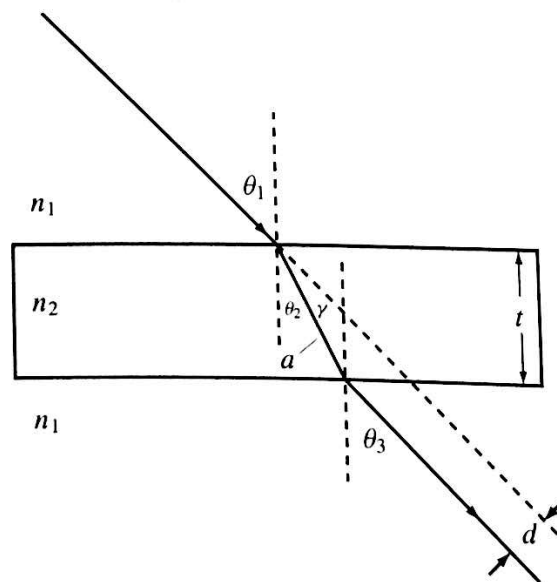


Figure 3.3. Light rays refracting through a slab of material with parallel sides.

Solution

Let the slab have index of refraction n_2 and the incident and transmitted medium

have index of refraction n_1 as shown in Figure 3.3. To show that the incident and transmitted rays are parallel, begin with Snell's law at the first interface:

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1$$

The beam transmitted by the first interface is incident internally on the second interface. Since the two interfaces are parallel, the angle of incidence for the second interface is θ_2 :

$$\sin \theta_3 = \frac{n_2}{n_1} \sin \theta_2$$

Combining these results gives

$$\sin \theta_3 = \frac{n_2}{n_1} \left(\frac{n_1}{n_2} \sin \theta_1 \right) = \sin \theta_1$$

Since θ_1 and θ_3 are both positive and less than 90° , $\theta_1 = \theta_3$, and thus the incident and transmitted beams are parallel.

To find the offset distance d , refer to Figure 3.3, and note that

$$d = a \sin \gamma$$

where

$$a = \frac{t}{\cos \theta_2}$$

and $\gamma = \theta_1 - \theta_2$. Combining these results gives Equation 3.5:

$$d = \frac{t}{\cos \theta_2} \sin (\theta_1 - \theta_2)$$

The amount of offset is a linear function of the slab thickness.

The laws of reflection and refraction may be derived from Maxwell's equations, as we show in Section 3.3.2. Before doing so, it will be instructive to derive these laws using a variational approach first described by Fermat.

3.2.1 Fermat's Principle of Least Time

Hero of Alexandria discovered that a light ray traveling from an object to a mirror and then to an observer's eye takes a path that is *shorter* than any other available path. In 1657, the French mathematician Fermat deduced the law of refraction from a similar principle: that light travels between two points along the path that requires the smallest *transit time*¹.

Consider first the case of reflection as illustrated in Figure 3.4, where light from a source S travels to an observer at D . The total distance from S to D is

$$L = \sqrt{s^2 + x^2} + \sqrt{d^2 + (\rho - x)^2}$$

¹For a modern derivation of this principle from a completely different perspective, see Section 6.5. See also: *QED: The Strange Theory of Light and Matter*, by R. P. Feynman [5]