

A Hybrid Analytical-Simulation Procedure for Performance Evaluation in M-QAM-OFDM Schemes in Presence of Nonlinear Distortions

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Abstract—This paper presents a hybrid analytical-simulation procedure for performance evaluation in M-ary quadrature amplitude modulation (M-QAM) orthogonal frequency-division multiplexing (OFDM) digital radio systems in the presence of nonlinear distortions caused by high-power amplifiers (HPA's). The present analysis is carried out considering an additive white Gaussian noise (AWGN) transmission channel. It is shown that, in the case of an OFDM system with a large number of subcarriers, the distortion on the received symbol caused by the amplifier can be modeled, with good approximation, as a "Gaussian nonlinear noise" added to the received symbol. This important result allows a hybrid analytical-simulation approach to solve the problem of performance evaluation. In practice, the simulation aspect is only used to estimate means and variances of the "nonlinear noise." Such estimated parameters are subsequently used to evaluate analytically the system bit-error rate (BER) using an expression, which takes into account both AWGN and "nonlinear noise" effects. The advantage of the proposed method lies in the strongly reduced computational time. In fact, an accurate estimate of the "nonlinear noise" parameters requires only few iterations when compared with a classical semianalytical approach. This is especially true when low BER values ($<10^{-4}$) have to be estimated. The proposed procedure will be applied to evaluate M-QAM-OFDM performance in the presence of nonlinear distortions caused by traveling-wave tube (TWT) and solid-state-power (SSP) amplifiers.

Index Terms—Amplifier nonlinearities, M-QAM modulation, OFDM.

I. INTRODUCTION

DUE TO ITS robustness in environments affected by high interference and multipath and to its spectral efficiency, orthogonal frequency-division multiplexing (OFDM) is considered as an effective modulation technique for high-speed digital transmission of many services such as messages, facsimile, cordless telephone, and image to mobile receivers. In addition, OFDM has been standardized for the European digital terrestrial television broadcasting system [1] to both fixed and portable receivers. With properly chosen parameters, this system could be appropriate for digital satellite broadcasting as well. OFDM is a multicarrier transmission scheme, which uses frequency-division multiplexing [efficiently implemented through inverse fast Fourier transform (IFFT)] to transport

an input data stream on N orthogonal subcarriers within the usable frequency band of the channel. This technique is effective in a multipath environment since each subchannel carries a low bit rate, thus, resulting in a symbol period much longer than typical echo delays. In addition, owing to the use of a guard interval (inserted before each modulation period), echo equalization simply reduces to a scalar multiplication of the detected decision variables by a corrective coefficient [2]–[6].

The major drawback of the OFDM technique is its large signal dynamic caused by the high (several hundreds) number of subcarriers with random phase and amplitude, which are summed in the modulator.¹ To keep acceptable performance in presence of nonlinear amplifiers, high-output backoff (OBO) are required, thus, resulting in low-power efficiency. This fact is especially important in the case of a satellite system or portable transmitters, where power is a costly resource. For this reason, a preliminary study of the system is required to find a good tradeoff between transmitted power and degradation caused by nonlinear distortions. Some results about OFDM performance in presence of nonlinearity are given in [7] and [9].

Generally, it is very difficult to mathematically analyze a M-QAM radio system with a nonlinear transmit amplifier and error-correction decoding. Thus, the application of numerical simulations is practically inevitable. In particular, in the case of OFDM systems, where a large number N of subcarriers (several hundreds) are considered, the use of a Monte Carlo procedure leads to long computation times.

A "classical" semianalytical approach allows a considerable reduction in simulation times. It consists in generating the random data symbols at the input of the system, treating only the nonlinear effects by simulation and considering the effect of the Gaussian channel noise analytically. However, for a multicarrier system, even a semianalytical approach becomes time expensive when low bit-error rate (BER) have to be simulated and a lot of system parameters have to be optimized.

In this paper, a procedure where the simulation is only used to estimate some parameters that describe the channel nonlinearity is suggested. The estimated parameters are subsequently used to analytically evaluate the BER taking into account both AWGN and "nonlinear noise" effects. In the following this approach will be referred to as hybrid analytical-simulation approach. A similar procedure was introduced in [10] and [11]

¹ Comparable dynamics are experienced by the frequency-division multiplex (FDM) amplifiers in the telephony analog multiplexing.

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for single-carrier systems. The advantage of this method is that, to obtain an accurate estimate of nonlinear parameters only few iterations are required compared with a classical semianalytical approach. As an example, this procedure is applied in this paper to evaluate the OBO value, which minimizes the “total system degradation” for 4-PSK-, 16-QAM-, and 64-QAM-OFDM systems.

The paper is organized as follows. In Section II, the principles of OFDM are reviewed. The models of traveling-wave tube amplifier (TWTA) and a solid-state-power amplifier (SSPA) are described in Section III and are used to simulate the effects of nonlinearities on the OFDM signals. The system analysis is outlined in Section IV, where the proposed procedure for performance evaluation is derived. This analysis is carried out considering a nonfading AWGN channel and no channel coding. Finally, M-QAM-OFDM performance results in presence of a high-power amplifier (HPA), obtained by applying the proposed method, are reported in Section V.

II. THE OFDM MODULATION

The complex envelope of the OFDM signal can be written as [2]–[6]

$$x(t) = \sum_{n=-N/2}^{N/2-1} \sum_{i=-\infty}^{+\infty} a_n(i) s_n(t - iT') \quad (1)$$

where $T' = T + T_g$ is the OFDM symbol period, N is the number of subcarriers (N is assumed as an even integer power of two)

$$s_n(t) = \begin{cases} \sqrt{2\varepsilon/T'} e^{j2\pi f_n t}, & \text{for } -T_g \leq t < T \\ 0, & \text{elsewhere} \end{cases} \quad (2)$$

where ε is the transmitted pulse energy, T_g is the guard interval, and $a_n(i) = a_{In}(i) + ja_{Qn}(i)$ is the emitted symbol in the i th time slot on the n th subchannel with subcarrier frequency $f_n = n\Delta f$ ($\Delta f = 1/T$ is the subcarrier distance). For sake of clarity, the temporal index i in the emitted symbols will not be indicated in the following. In the case of a M-QAM constellation

$$a_{In} \in A, a_{Qn} \in A, \\ A = \{2u - 1 - \sqrt{M}, u = 1, 2, \dots, \sqrt{M}\}. \quad (3)$$

The introduction of the guard interval leads to a reduction of the bandwidth efficiency by the factor $r = 1/\eta$, where $\eta = T/T'$ and the signal-to-noise ratio is degraded by $D = 10 \log(\eta)$. It is well known [2], [3], [6] that the OFDM signal can be digitally obtained through IFFT. A baseband scheme of the FFT-based OFDM system is reported in Fig. 1. In this figure, the samples of the OFDM signal are generated (at rate N/T) at the output of the parallel to serial (P/S) converter at the emission-end. These samples are then processed by a low-pass filter $H_e(f)$ to generate the analog complex signal $x(t)$. The signal is then amplified and corrupted by additive white Gaussian noise $z(t)$ with double-sided spectral power density $N_0/2$. In the following analysis, a nonfrequency selective channel is assumed since attention is focused on the effects

of nonlinearity. The filter $H_r(f)$ at the receiver end is used to reduce interchannel interference. The demodulation scheme is symmetrical to the transmission scheme apart from the presence of a complex automatic gain control (CAGC) applied to all decision variables. It should be mentioned that in the following analysis the subsystems for carrier recovery and symbol synchronization are considered ideal and are not shown in Fig. 1.

III. HPA MODELS

In this section, the memoryless model for the nonlinear HPA is briefly recalled. The complex envelope of the input signal to the HPA is

$$x(t) = \rho(t) \cdot e^{j\varphi(t)} \quad (4)$$

and the complex envelope of the output signal can be expressed by

$$\tilde{x}(t) = A[\rho(t)] \cdot e^{j\{\varphi(t) + \Phi[\rho(t)]\}} \quad (5)$$

where $A(\rho)$ and $\Phi(\rho)$ represent, respectively, the AM/AM and AM/PM conversion characteristics of the nonlinear amplifier. Two nonlinear HPA models have been adopted:

- 1) a travelling-wave tube with strong AM/PM conversion;
- 2) a solid-state amplifier without AM/PM conversion.

In addition, the AM/AM conversion characteristics are normalized so that the small-signal amplification is unitary.

A. TWTA

According to the memoryless Saleh TWT model [12], the AM/AM and AM/PM conversion characteristics can be expressed as

$$A[\rho(t)] = A_{\text{sat}}^2 \frac{\rho(t)}{\rho^2(t) + A_{\text{sat}}^2} \\ \Phi[\rho(t)] = \frac{\pi}{3} \frac{\rho^2(t)}{\rho^2(t) + A_{\text{sat}}^2} \text{ rad.} \quad (6)$$

In these expressions, A_{sat} represents the amplifier input saturation voltage.

B. SSPA

The AM/AM and AM/PM conversion characteristics for SSPA amplifier [7], [8] can be approximated by

$$A[\rho(t)] = \frac{\rho}{\left[1 + \left(\frac{\rho}{A_0}\right)^{2p}\right]^{1/2p}} \\ \Phi[\rho(t)] = 0 \quad (7)$$

where A_0 is the maximum output amplitude and the parameter p controls the smoothness of the transition from the linear region to the limiting region.

C. Effects of Nonlinearities on OFDM Signal

When a multicarrier signal is amplified, the effects of distortions are:

- 1) interference between the in-phase and quadrature (I/Q) components due to AM/PM conversion (if present);
- 2) intermodulation effects, which occur between the OFDM subchannels causing spectral spreading of the transmitted signal and adjacent subchannel interference.

If more than one OFDM signal is transmitted, intermodulation will also cause adjacent-channel interference (ACI).

In general, even if the source of nonlinearity is memoryless, the presence of filtering before it makes the overall transmission chain acting as a nonlinear system with memory. However, in this case, no intersymbol interference (ISI) between OFDM symbols is present due to the rectangular shape of the transmitted pulses.

The operating point of the amplifier is usually identified by the "backoff." In the remainder of the paper, the following definitions for the input backoff (IBO) and the output backoff (OBO) are adopted:

$$\begin{aligned} \text{IBO} &= 10 \log_{10} \frac{P_{0,IN}}{P_{IN}} \quad \text{in decibels} \\ \text{OBO} &= 10 \log_{10} \frac{P_{0,OUT}}{P_{OUT}} \quad \text{in decibels} \end{aligned} \quad (8)$$

where P_{IN} is the mean power of the signal at the input of the HPA, P_{OUT} is the mean power of the transmitted signal, $P_{0,OUT}$ is the maximum output power (saturation power), and $P_{0,IN}$ is the input power corresponding to the maximum output power. The effects of the nonlinearities can be reduced by working with high backoff, which corresponds to moving the operating point of the amplifier to the linear region. Unfortunately, this leads to a loss in power efficiency of the HPA.

In this context, a useful performance measure is the total degradation (TD) as a function of the HPA output backoff. This parameter is defined as follows:

$$\text{TD}_{\text{dB}} = \text{SNR}_{\text{dB}} - \text{SNR}'_{\text{dB}} + \text{OBO} \quad (9)$$

where SNR_{dB} is the required signal-to-noise ratio in decibels at the input of the threshold detector to obtain a fixed BER (say, e.g., 10^{-4}) for a given value of the OBO and SNR'_{dB} is the required SNR to obtain the same BER in the absence of nonlinearity. The optimum operating point (i.e., the point which provides a good tradeoff between output power and degradation due to nonlinear distortions) corresponds to the minimum of function (9).

Generally, it is very difficult to mathematically analyze an M-QAM radio system with a nonlinear transmit amplifier and error correction decoding. Thus, the application of numerical simulations is practically inevitable. A simulation program of an OFDM transmission scheme has been developed to produce the symbol samples at the detection input (both in the I and Q

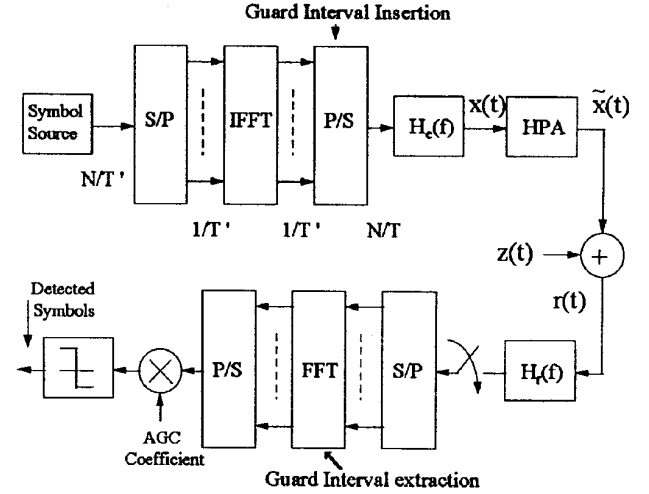


Fig. 1. Baseband representation of the OFDM transmission scheme with FFT modem implementation (all variables are complex valued).

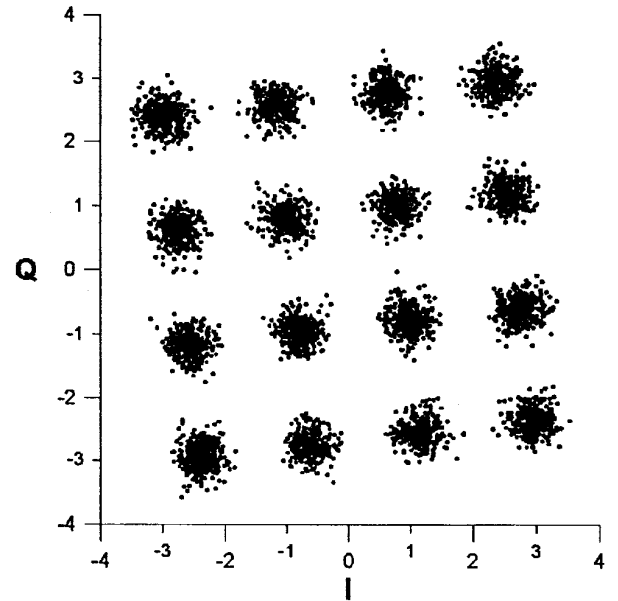


Fig. 2. Constellation at the input of the CAGC for 16-QAM OFDM, TWTA, and OBO = 7 dB.

channels) in the presence of the nonlinearity according to the equivalent baseband scheme shown in Fig. 1. Since it is only desired to analyze nonlinearity effects, no Gaussian noise has been added to the received signal.

In the following, the transmission of a single OFDM channel is assumed, however, the present analysis can be extended to include the ACI contribution due to other transmitted OFDM signals.

The overall digital system was simulated by using the oversampling and interpolation filtering method. An oversampling factor (E) equal to eight was chosen. Considering that the minimum required sampling frequency is $f_c = N/T$ (when N is large the OFDM signal may be assumed band limited), then the number of samples per block (i.e., in a symbol period)

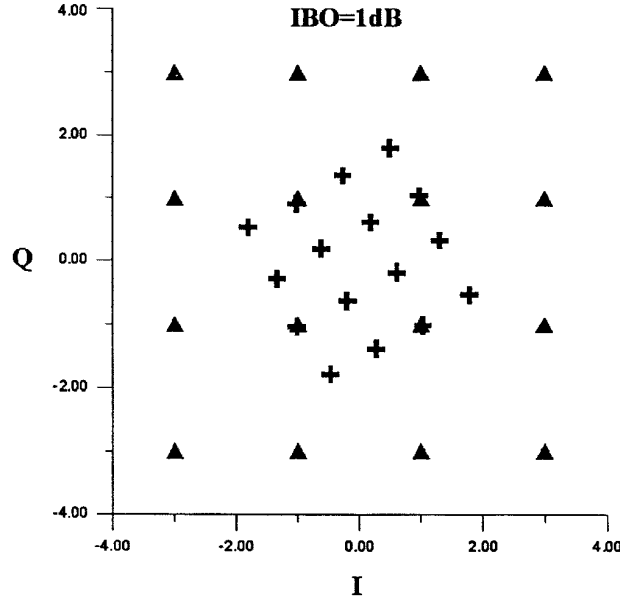


Fig. 3. Comparison between transmitted (triangles) and received (crosses) averaged constellation at the CAGC input with IBO = 1 dB and TWTA.

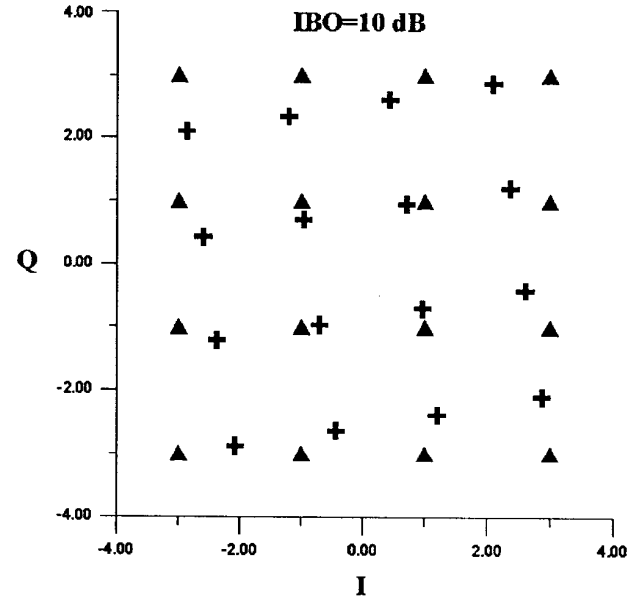


Fig. 4. Comparison between transmitted (triangles) and received (crosses) averaged constellation at the CAGC input with IBO = 10 dB and TWTA.

used in the simulation is Tf_cE (e.g., if $T_g = 0$, $N = 128$, and $E = 8$, it follows that 1024 samples/block are required). As an example, Fig. 2 shows the received constellation (output of the receiver S/P converter) before the CAGC for a 16-QAM-OFDM scheme using TWTA with OBO = 7 dB and $N = 128$ subcarriers.

From the analysis of many of such examples, it was deduced that the HPA determines the following effects on the received constellations at the input of CAGC: cloud-like shape, rotation, attenuation, and, for low IBO, warping. No relevant translation was observed. These effects are pointed out for the case of a TWTA in the two examples reported in Figs. 3 and 4, where the clouds have been wiped out by averaging the received symbols belonging to the same decision region over a large number of samples (in Figs. 3 and 4, we shall refer this to as “averaged constellation”).

It can be observed that a slight distortion is evident only in the case of IBO = 1 dB, whereas the main effect on the received constellation consists in a rotation and attenuation.

Before going further in the analysis, it is helpful to quantize the aforementioned effects on the received constellation. For the reason that will be clear in the following, it is helpful to subdivide the received decision variables at the output of k th subchannel into M subgroups, each one corresponding to a constellation point of the signal set. The constellation points have been ordered with an index i ($i = 1, \dots, M$) according to the following correspondence between order number i and constellation symbol: $i = (u-1)\sqrt{M} + \nu$, where u, ν ($u, \nu = 1, 2, \dots, \sqrt{M}$) address the symbol as indicated in (3). The averaged i th decision variable (i.e., after cloud removal) at the k th FFT output before CAGC correction can be modeled as

$$y_k^i = c_k^i \cdot a_k^i \quad \text{with} \quad i = 1 \dots M \quad (10)$$

where c_k^i is a complex coefficient, which takes into account the rotation and attenuation, caused by the HPA, of the i th constellation point a_k^i , while any translation turns out to be negligible. The constellation is warped only if c_k^i is dependent on the emitted constellation point. In order to assess this aspect, the averaged received constellation for a fixed IBO was simulated. All the simulated results reported in the following will refer to a TWTA, although the qualitative considerations that follow can be extended to different HPA's. To estimate the complex coefficient c_k^i the clouds were eliminated by averaging the received symbols belonging to the same decision region over a large number of samples. Figs. 5 and 6 point out, respectively, the variation of the normalized absolute value and phase of c_k^i versus i for different values of the IBO in the case of a 16-QAM constellation, $k = N/2$ and $N = 128$. To outline the dependence of this distortion effect on the IBO, a normalization was made with respect to the absolute value and phase of the following coefficient

$$\bar{c}_k = \frac{1}{M} \sum_{i=1}^M c_k^i. \quad (11)$$

Figs. 5 and 6 show a slight dependence of c_k^i on the constellation point i . This dependence increases when the IBO is reduced. In order to obtain a quantitative evaluation of c_k^i variation with the index i , the following moments were introduced:

$$\sigma_{\angle c, k}^2 = \frac{1}{M} \sum_{i=1}^M [\arg(c_k^i) - \arg(\bar{c}_k)]^2 \quad (12)$$

$$\sigma_{c, k}^2 = \frac{1}{M} \sum_{i=1}^M |c_k^i - \bar{c}_k|^2. \quad (13)$$

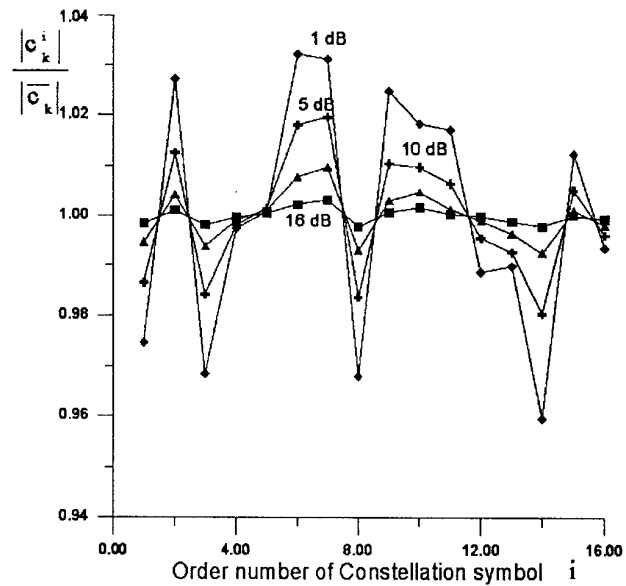


Fig. 5. Variation of the amplitude of coefficient c_k^i as a function of the constellation point index for different IBO values, $k = N/2$, $N = 128$, 16 QAM, and TWTA.

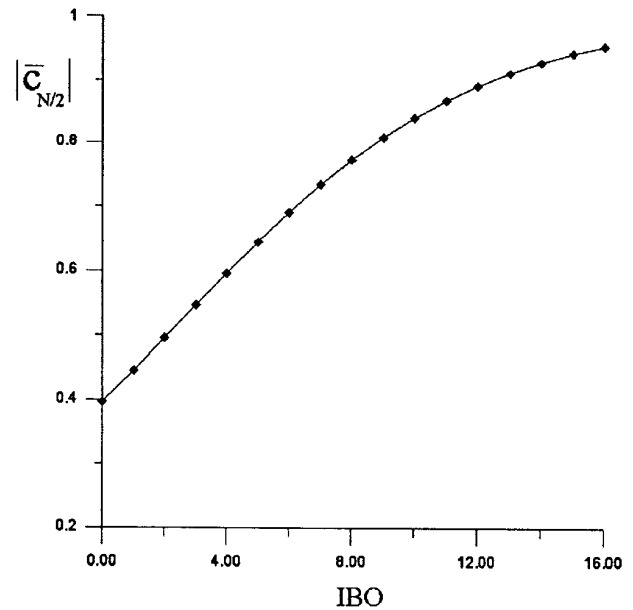


Fig. 7. Absolute value of (11) versus IBO for the central subchannel ($k = N/2$), $N = 128$, 16 QAM, and TWTA.

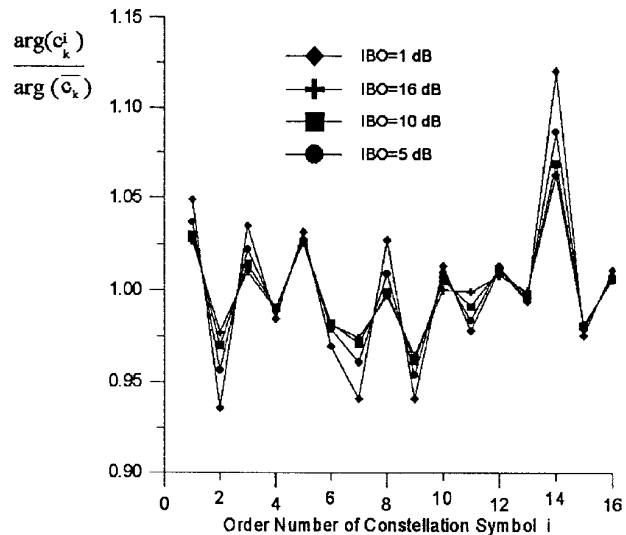


Fig. 6. Variation of the normalized phase of coefficient c_k^i as a function of the constellation point index for different IBO values, $k = N/2$, $N = 128$, 16 QAM, and TWTA.

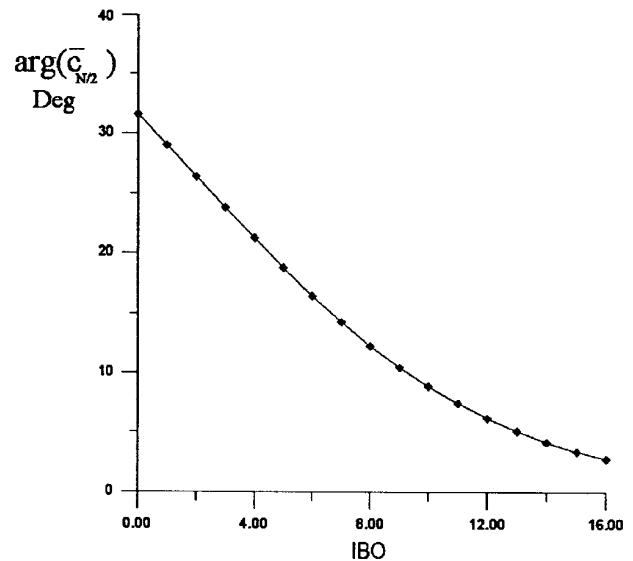


Fig. 8. Phase (degree) of (11) versus IBO for the central subchannel ($k = N/2$), $N = 128$, 16 QAM, and TWTA.

Notice that magnitude and phase of (11) are, respectively, related to the average attenuation and rotation of the received constellation caused by the amplifier nonlinearity. In Figs. 7 and 8, absolute value and phase of (11) versus IBO for the subchannel $k = N/2$ ($N = 128$) are reported. Fig. 8 gives a relationship between the average rotation of the constellation and the amount of AM/PM conversion as a function of the IBO adopted.

Equations (12) and (13) give a measure of the constellation warping. Some simulation results of (12) and (13) are reported in Figs. 9 and 10 as a function of IBO for $k = N/2$, 16 QAM, and TWTA.

From the results of Figs. 7–10, it can be concluded that, for all practical values of the IBO, the constellation is mostly rotated and attenuated while warping can be assumed as a secondary effect.

It must be remarked that parameters (11)–(13) have been defined for a fixed subchannel. In order to assess whether these parameters are a function of the subchannel considered, the received constellation has been simulated for different values of the IBO and amplitude and phase of (11) have been evaluated as a function of subchannel k . Some results are reported in Figs. 11 and 12.

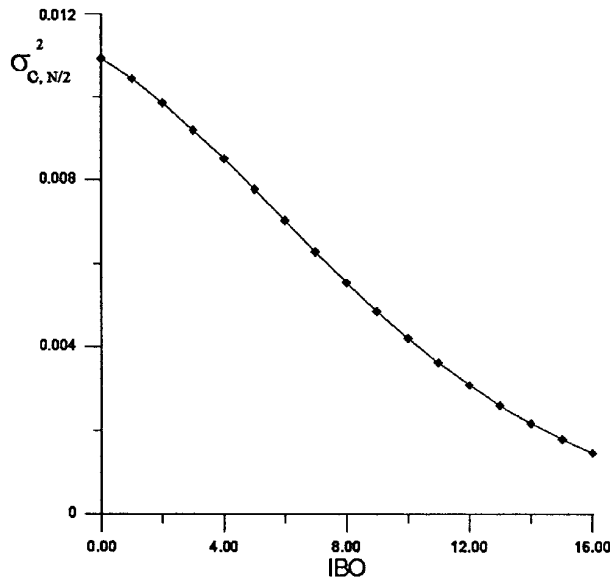


Fig. 9. Variance (13) versus IBO for $k = N/2$, $N = 128$, 16 QAM, and TWTA.

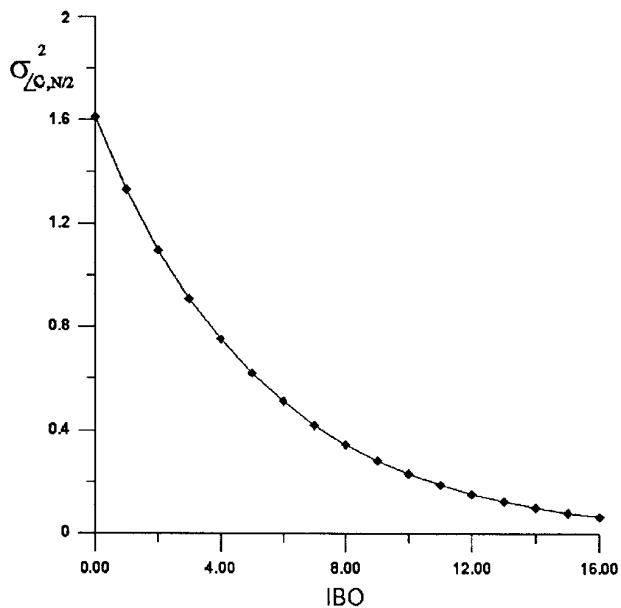


Fig. 10. Variance (12) versus IBO for $k = N/2$, $N = 128$, 16 QAM, and TWTA.

In Figs. 11 and 12, it can be observed that, apart from irrelevant fluctuations (whose amplitude depends on the IBO), the effects on the received constellation (rotation, etc.) are substantially independent of the considered subchannel.

IV. ANALYSIS

In the previous section, a qualitative and quantitative description of nonlinearity effects on the OFDM signal has been carried out. Problems arise when evaluation of BER and TD is dealt with. High simulation time has been found to be

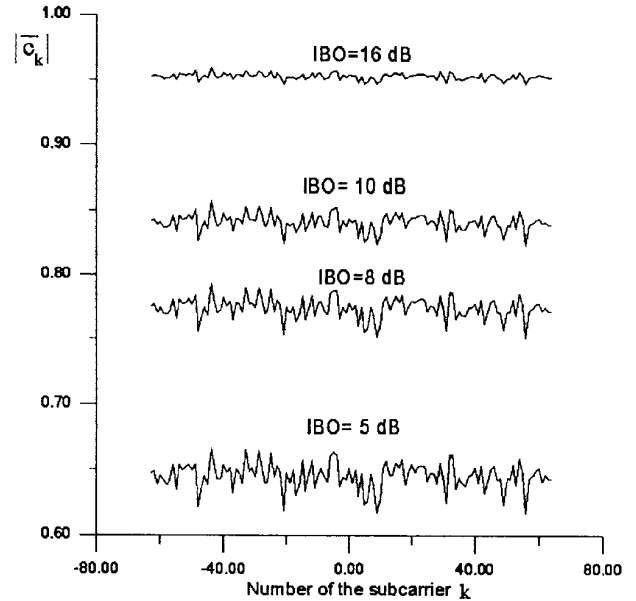


Fig. 11. Absolute value of (11) as a function of subcarrier number (k) for different IBO values, 16 QAM, and TWTA.

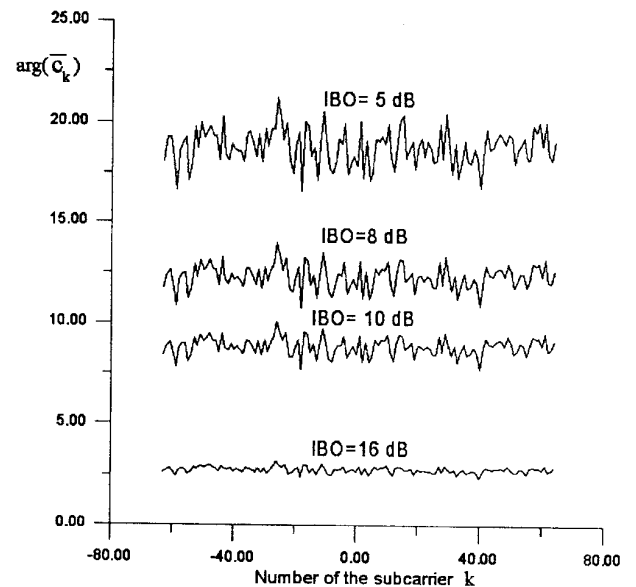


Fig. 12. Phase (degree) of (11) as a function of subcarrier number (k) for different IBO values, 16 QAM, and TWTA.

necessary in order to obtain reliable results when low BER have to be estimated. In addition, the required simulation time increases with the number of subchannels.

To reduce computational time, in this section a hybrid analytical-simulation procedure is suggested, where the simulation approach is only used to estimate some parameters that statistically describe the nonlinear channel effects on the received signal. Such estimated parameters are subsequently used to evaluate BER analytically. The advantage of the proposed method lies in the fact that an accurate estimate of these parameters requires only few iterations compared with a classical semianalytical approach.

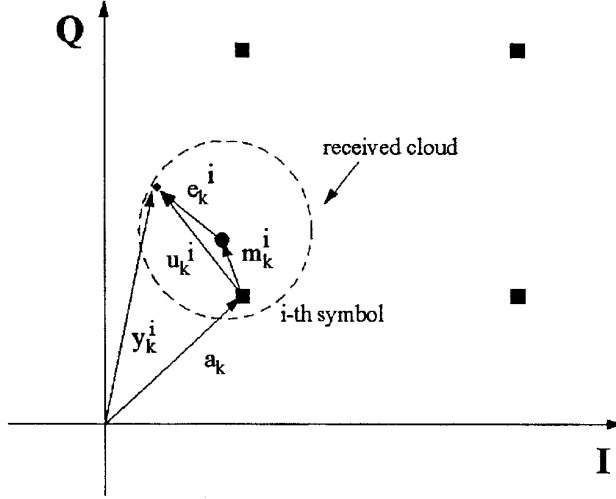


Fig. 13. Signal-space representation of the received symbol on the k th subchannel (the receiver Gaussian noise is neglected).

In order to explain the principles of this procedure, the received decision variables at the output of k th subchannel are subdivided into M subgroups, each one corresponding to a symbol (i.e., a constellation point) of the signal set. Thus, the decision variable at the k th FFT output, after CAGC correction, can be written as

$$y_k^i = a_k^i + u_k^i + \nu_k^i \quad \text{where} \quad \nu_k^i = \frac{\nu_k}{c}, \quad i = 1 \dots M \quad (14)$$

where c is the complex CAGC coefficient, integer i indexes the constellation point (as indicated in Section III-C), the term u_k^i caused by the nonlinear HPA takes into account the effect of I/Q crosstalk and adjacent subchannel interference on the received symbol, and ν_k is a zero-mean Gaussian variable, which denotes the receiver additive noise at the k th filter bank output [additive Gaussian noise (AGN)]. The decision variables for the I-Q branches can be written as

$$\begin{aligned} y_{k,I}^i &= a_{k,I}^i + u_{k,I}^i + \nu_{k,I}^i \\ y_{k,Q}^i &= a_{k,Q}^i + u_{k,Q}^i + \nu_{k,Q}^i, \quad i = 1, \dots, M. \end{aligned} \quad (15)$$

It should be noted that (differently from what happens in single-carrier systems) all the received clouds have a circular shape with a high density of points just in the cloud center. This fact suggests modeling the interference term u_k^i as (see Fig. 13)

$$u_k^i = m_k^i + e_k^i \quad (16)$$

where m_k^i is a constant displacement (due to nonlinear distortion) with respect to the ideal i th constellation point and e_k^i is approximated by a complex zero-mean Gaussian variable with uncorrelated components. In the following, we shall refer this variable to as “nonlinear noise.”

In addition, simulation results showed that the variance $\sigma_{e_k^i}^2$ of the variable e_k^i is practically independent of the subchannel (k) and of the constellation point (i) considered. Thus, as a variance of the nonlinear noise, it is possible to adopt, with negligible error, a single value (σ_e^2) that we chose as the

average cloud variance over all the subchannels and symbols, that is

$$\begin{aligned} \sigma_e^2 &= \frac{1}{N \cdot M} \sum_{i,k} \sigma_{e_k^i}^2 = \frac{1}{2} (\sigma_I^2 + \sigma_Q^2) \cong \sigma_I^2 \cong \sigma_Q^2 \\ \sigma_k^2 &= \frac{1}{2} E[c_k^i c_k^{i*}] = \frac{1}{2} (\sigma_{k,I}^2 + \sigma_{k,Q}^2) \cong \sigma_{k,I}^2 \cong \sigma_{k,Q}^2. \end{aligned} \quad (17)$$

Since, as shown in the previous section, also the mean constellation rotation resulted to be practically independent of k , the average of (m_k^i) over all the subchannels can be assumed as mean cloud displacement of i th point, that is

$$m^i = \frac{1}{N} \sum_k m_k^i. \quad (18)$$

Finally, considering that the “nonlinear noise” is independent of the AWGN noise, the overall noise variance of each quadrature decision variable can be expressed by

$$\begin{aligned} \sigma_T^2 &= \frac{\sigma_\nu^2}{|c|^2} + \sigma_e^2 \\ \sigma_\nu^2 &= \frac{1}{2} E[\nu_k \nu_k^*] = \frac{M-1}{3 \log_2 M (E_b/N_0) \eta} \end{aligned} \quad (19)$$

where E_b is the average bit energy per subchannel and σ_ν^2 is the AGN power. The normalization used in the expression of σ_ν^2 assumes a signal constellation with distance two between adjacent symbols.

The mean symbol rotations (m^i) and the nonlinear noise variance (σ_e^2) are estimated through (17) and (18) by simulating a distorted signal constellation at the input of the detector with no AWGN added. Once the “nonlinear noise parameters” are estimated, the symbol error probability conditioned to the i th symbol and to the IBO is given, for each signal component, by

$$P_I(i) = \begin{cases} \frac{1}{2} \operatorname{erfc}\left(\frac{1+m_I^i}{\sqrt{2}\sigma_T}\right); & i = M - \sqrt{M} + 1, M - \sqrt{M} + 2, \dots, M \\ \frac{1}{2} \operatorname{erfc}\left(\frac{1-m_I^i}{\sqrt{2}\sigma_T}\right); & i = 1, 2, \dots, \sqrt{M} \\ \frac{1}{2} \operatorname{erfc}\left(\frac{1+m_I^i}{\sqrt{2}\sigma_T}\right) + \frac{1}{2} \operatorname{erfc}\left(\frac{1-m_I^i}{\sqrt{2}\sigma_T}\right); & \text{otherwise} \end{cases} \quad (20a)$$

$$P_Q(i) = \begin{cases} \frac{1}{2} \operatorname{erfc}\left(\frac{1+m_Q^i}{\sqrt{2}\sigma_T}\right); & i = \sqrt{M}, 2\sqrt{M}, \dots, \sqrt{M}\sqrt{M} \\ \frac{1}{2} \operatorname{erfc}\left(\frac{1-m_Q^i}{\sqrt{2}\sigma_T}\right); & i = 1, 1+\sqrt{M}, 1+2\sqrt{M}, \dots, 1+(\sqrt{M}-1)\sqrt{M} \\ \frac{1}{2} \operatorname{erfc}\left(\frac{1+m_Q^i}{\sqrt{2}\sigma_T}\right) + \frac{1}{2} \operatorname{erfc}\left(\frac{1-m_Q^i}{\sqrt{2}\sigma_T}\right); & \text{otherwise} \end{cases} \quad (20b)$$

where $m_I^i = \operatorname{Re}\{m^i\}$ and $m_Q^i = \operatorname{Im}\{m^i\}$.

The total symbol error probability P_s can be calculated by averaging the conditional probabilities in (20a) and (20b) over

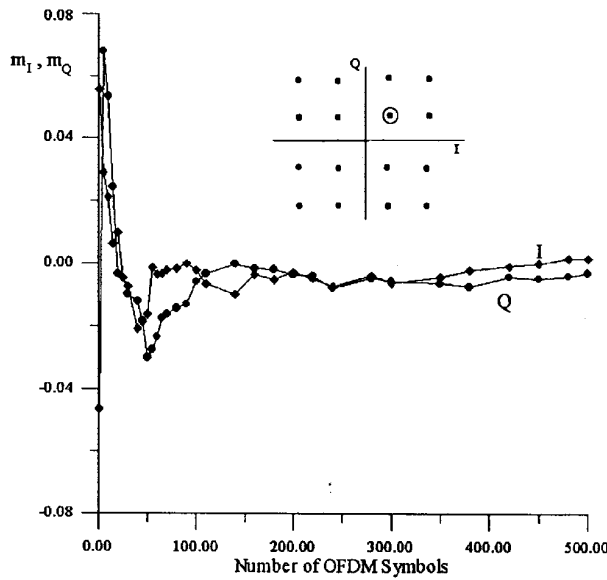


Fig. 14. I/Q components of (18) versus number of OFDM symbols for constellation symbol ($i = 11$) and OBO = 5.4 dB (TWTA and 16 QAM).

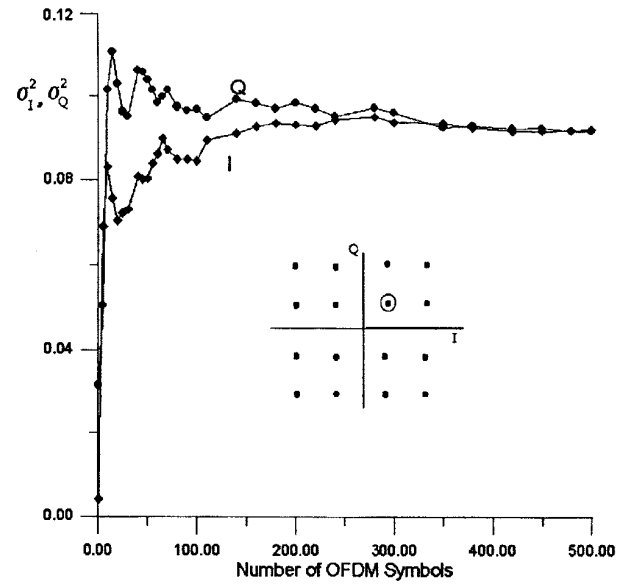


Fig. 15. I/Q mean square values (17) versus number of OFDM symbols for constellation symbol ($i = 11$) and OBO = 5.4 dB (TWTA and 16 QAM).

the input alphabet set of symbols. If the transmitted symbols are statistically independent and equiprobably distributed, then

$$P_s = \frac{1}{M} \sum_{i=1}^M [P_I(i) + P_Q(i) - P_I(i) \cdot P_Q(i)]. \quad (21)$$

If the nonlinear distortion process has appropriate symmetries, the averaging over all data symbols in (21) can be reduced to an average over just one quadrant. Finally, the bit-error probability, assuming a Gray-coded signal set and neglecting the double and multiple errors, is given by

$$P_b = P_s / \log_2 M. \quad (22)$$

It should be mentioned that, in general, the symbol error probability for a multicarrier system is defined as the average of symbol error probabilities over all subchannels. However, since in this analysis the channel is assumed nonfrequency selective and the overall nonlinear noise contribution has been considered practically independent of the subchannel, the OFDM symbol error probability coincides with (21).

Notice that the noise variance (σ_e^2) and the mean symbol rotations (m^i) depend on the working point of the amplifier and are decreasing functions of the absolute system IBO.

In the following, some data about the computational effort required to obtain stable values for the statistical parameters (17) and (18) are given. As an example, a 16-QAM constellation and TWTA are considered. In Figs. 14–16, convergence curves, from a blind start, of (17) and (18) are reported. Such curves have been evaluated after CAGC for the constellation point corresponding to $i = 11$ and IBO = 10 dB (OBO = 5.4 dB).

From analogous simulations for $i = 12, 15$, and 16 (points representative of the first quadrant), similar results were obtained.

In the above reported examples, it can be observed that the estimate of the nonlinearity parameters becomes stable after

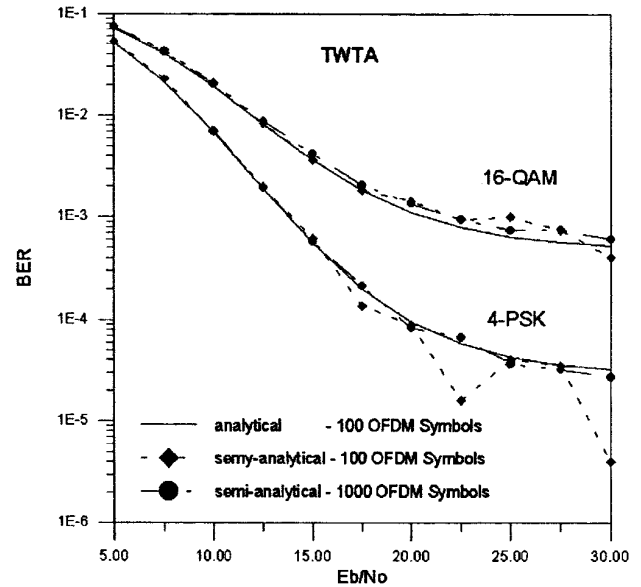


Fig. 16. Comparison between hybrid simulation-analytical and classical semianalytical BER evaluation for 4-PSK and 16-QAM-OFDM schemes (4 PSK: OBO = 3 dB; 16 QAM: OBO = 5.4 dB; CAGC included; no guard interval).

about 100 OFDM symbols. The computational effort required by the suggested method will be compared to the classical semianalytic approach in the next section.

The simulation results also confirmed that the variances σ_I^2 and σ_Q^2 are independent of the considered constellation point (i) and that the cloud is symmetrical ($\sigma_I^2 = \sigma_Q^2$). Notice that the effect of CAGC is that of compensating for average constellation rotation and attenuation so that m_I and m_Q are minimized (see Fig. 14).

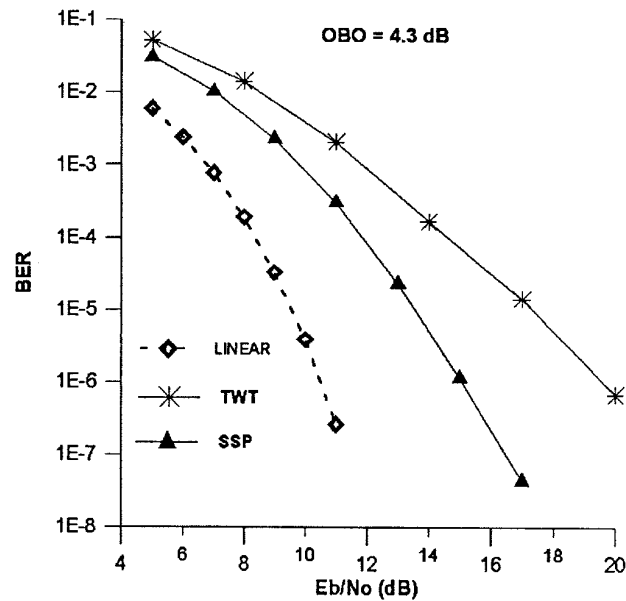


Fig. 17. The BER versus signal-to-noise ratio for TWTA and SSPA (4-PSK OFDM, $N = 512$, and CAGC included).

V. PERFORMANCE RESULTS

In this section, some performance results obtained by applying the proposed hybrid analytical-simulation procedure are reported. In Fig. 16, the BER obtained with a classical semianalytical procedure and with the proposed method are compared in the case of a 4-PSK and 16-QAM-OFDM systems. The semianalytical BER was evaluated by generating 100 and 1000 OFDM symbols while only 100 were used to estimate the nonlinear noise parameters (17), (18). It is evident that, with the semianalytical method, at least 1000 symbols are required to obtain comparable accuracy when BER is in the order of 10^{-4} . In addition, the lower the BER to be estimated the higher is the improvement obtainable in term of computational time since the effort required to estimate the parameters (17) and (18) is independent of the BER to be evaluated. Moreover, in the suggested approach a computational effort is only required to estimate the nonlinear parameters; subsequently, if a fixed IBO is considered, any BER value is analytically obtained without any additional simulation time required.

The good agreement between the results obtained with the classical semianalytical approach and the proposed hybrid analytical-simulation in Fig. 16 confirms that the Gaussian assumption is a valid model to describe the statistics of the "nonlinear noise."

In Fig. 17, the BER versus E_b/N_0 , evaluated with the proposed method (20a)–(22), for TWTA and SSPA and for a fixed output backoff is plotted. The curves are bounded by the system operating without nonlinearity. The results refer to a 4-PSK-OFDM system with 512 subcarriers, guard interval equal to 1/4 of the symbol period, and use of CAGC system. In Figs. 18 and 19, the TD_{dB} versus the OBO is reported for 4-, 16-, and 64-QAM OFDM systems adopting TWT or SSP amplifiers. The TD values were obtained applying the hybrid analytical-simulation approach (20a)–(22). The results

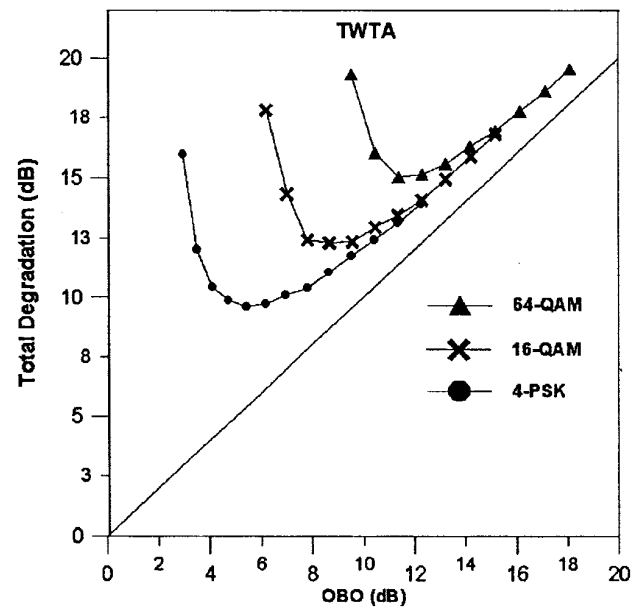


Fig. 18. Total degradation versus output backoff at $BER = 10^{-4}$ for OFDM system using TWT amplifier (CAGC included).

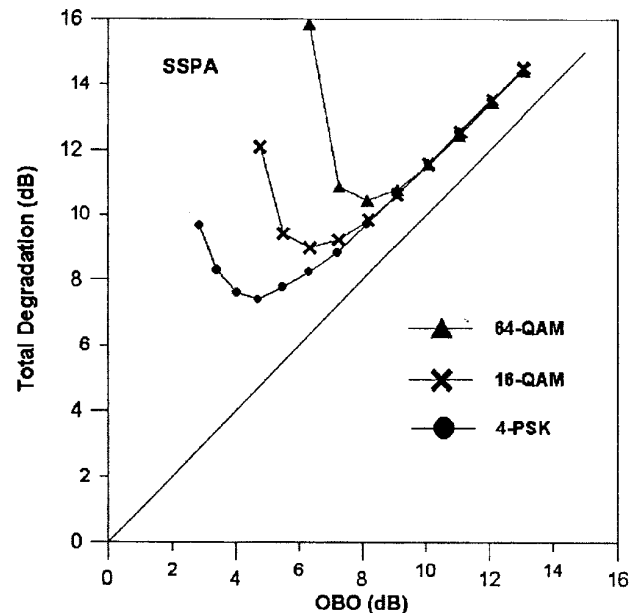


Fig. 19. Total degradation versus output backoff at $BER = 10^{-4}$ for OFDM system using SSP amplifier (CAGC included).

show, as expected, less severe degradation caused by the SSPA owing to the lack of phase distortion and that about 1 dB of degradation is due to the adoption of a guard interval with $\eta = 0.75$. From these figures, it is possible to infer the optimal OBO values for each system configuration considered.

In Fig. 16, it can be observed that BER for the 16-QAM case ($OBO = 5.4$ dB) never reaches 10^{-4} since irreducible BER occurs. This result is confirmed by Fig. 18, which shows that for the 16-QAM case, the TD asymptotically tends to infinity when OBO approaches 6 dB.

VI. CONCLUSIONS

In this paper, the effects of nonlinearities on the OFDM signal were evaluated. It was shown that the main effect of HPA on the received constellation is the generation of clouds, an uniform attenuation, rotation, and a slight warping. Moreover, all these factors are quite independent of the considered subchannel.

In addition, a model for evaluation of error probability of an OFDM system in presence of high-power nonlinear amplifier was proposed. It was shown that, in the case of a large number of subcarriers, the effect of nonlinear distortions can be taken into account just introducing an equivalent Gaussian "nonlinear noise" added to the received symbol. The means and variances of this noise are evaluated by simulation and used in an analytical BER expression. It was found that few iterations (some tens) are required to obtain a good estimate of the nonlinear process statistics against the several hundred iterations that are required to simulate low BER with a classical semianalytical approach. Thus, the proposed method greatly reduces the computational time.

In addition, it was found that the Gaussian approximation, differently from what happens in the case of single-carrier systems, where it is limited to weak nonlinearities [10], [11], holds for OFDM also with low OBO values.

Finally, the proposed hybrid analytical-simulation procedure was applied to an M-QAM-OFDM system ($N = 512$ and 1024 and $M = 4, 16$, and 64) to determine the system TD as a function of the OBO.

REFERENCES

- [1] ETS 300 744, "Digital broadcasting systems for television, sound and data services; framing structure, channel coding and modulation for digital terrestrial television," 1996.
- [2] S. B. Weinstein and P. M. Ebert, "Data transmission by frequency-division multiplexing using the discrete Fourier transform," *IEEE Trans. Commun.*, vol. COM-19, pp. 628–634, Oct. 1971.
- [3] L. J. Cimini, "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. Commun.*, vol. COM-33, pp. 665–675, July 1985.
- [4] J. A. C. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Commun. Mag.*, vol. 28, no. 5, pp. 5–14, May 1990.
- [5] N. J. Fliege, "Orthogonal multiple carrier data transmission," *Europ. Trans. Telecommun.*, vol. 3, pp. 35–44, May/June 1992.
- [6] R. Lassalle and M. Alard, "Principles of modulation and channel coding for digital broadcasting for mobile receivers," *EBU Rev.—Tech.*, no. 224, Aug. 1987.
- [7] C. Rapp, "Effects of HPA-nonlinearity on 4-DPSK-OFDM-signal for a digital sound broadcasting system," in *Proc. 2nd European Conf. Satellite Communications*, Liege, Belgium, vol. ESA-SP-332, Oct. 22–24, 1991, pp. 179–184.
- [8] E. Bogenfeld, R. Valentin, K. Metzger, and W. Sauer-Greff, "Influence on nonlinear HPA on trellis-coded OFDM for terrestrial broadcasting of digital HDTV," in *ICC'93*, pp. 1433–1438.
- [9] G. Santella and F. Mazzenga, "A model for performance evaluation in M-QAM-OFDM schemes in presence of nonlinear distortions," in *45th IEEE Vehicular Technology Conf., VTC'95*, Chicago, IL, vol. 2, July 1995, pp. 830–834.
- [10] K. J. Friederichs and J. Kovats, "Performance of underlying hamming codes in M-QAM digital radio systems with nonlinear distortions," *ETT*, vol. 3, no. 1, pp. 31–38, Jan./Feb. 1992.
- [11] S. Pupolin and L. J. Greenstein, "Performance analysis of digital radio links with nonlinear transmit amplifiers," in *IEEE Select. Areas Commun.*, vol. SAC-5, pp. 534–546, Apr. 1987.
- [12] A. A. M. Saleh, "Frequency-independent and frequency-dependent nonlinear models of TWT amplifiers," *IEEE Trans. Commun.*, vol. COM-29, no. 11, pp. 1715–1720, 1981.
- [13] J. G. Proakis, *Digital Communications*, 2nd ed. New York: McGraw-Hill, 1989.



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