

SYNCHRONIZATION FOR SINGLE CARRIER MODULATION WITH FREQUENCY DOMAIN EQUALIZATION

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Abstract — This paper deals with the synchronization of single carrier transmission with frequency domain equalization (FDE) for broadband mobile radio applications. For such applications carrier frequency and temporal position of the FFT (fast Fourier transform) window have to be synchronized. Furthermore, an estimation of the channel transfer function is necessary.

I. INTRODUCTION

In this paper the transmission of digital signals via wideband frequency-selective radio channels is investigated. Mobile radio channels in a microcellular environment can exhibit a large time dispersion up to about 5 μ s. In case of high symbol rates, intersymbol interference (ISI) smears the individual symbols over a long period of time. For example, a time dispersion of 5 μ s at a symbol rate of 10 MHz causes ISI over 50 symbols. Today, adaptive time domain equalizers which can cope with such a strong ISI cannot be realized for high data rates. Only transmission systems with frequency domain signal processing exhibit a practicable complexity.

The main advantage of single carrier modulation compared with multicarrier modulation is the fact that the energy of individual symbols is distributed over the whole available frequency spectrum. Therefore, narrowband notches in the transfer function have only a small impact on error probability [1]. Furthermore, the output signal of a single carrier transmitter shows a small crest factor (depending on the modulation scheme) whereas a multicarrier signal exhibits approximately a Gaussian distribution.

The aim of this paper is to propose synchronization methods for single carrier modulation schemes with FDE. The paper is organized as follows: In section II single carrier transmission with FDE is reviewed and the parameters which have to be synchronized are identified. Carrier frequency synchronization as well as coarse timing synchronization are considered in sections III and IV. Next, in section V channel estimation is investigated. Finally, a conclusion completes the paper.

II. SINGLE CARRIER TRANSMISSION AND SYNCHRONIZATION REQUIREMENTS

The block diagram of the considered single carrier transmission system is sketched in Fig. 1. The FFT (fast Fourier transform) and its inverse are used at the input and output of the frequency domain equalizer in the receiver. The figure shows very clearly that the basic concept of single carrier

modulation with FDE is very similar to OFDM (orthogonal frequency division multiplexing) transmission. The same blocks are needed, the main difference is that the block “inverse FFT” is moved from the transmitter to the receiver [2]. Therefore, single carrier modulation with FDE and OFDM exhibit approximately the same complexity.

Since the FFT algorithm is used, a blockwise signal transmission has to be carried out. Each block consists of $N_{\text{FFT}} = 2^m$ complex data symbols and will be called FFT symbol. This definition will be used throughout this paper. Like in an OFDM system, a periodic extension (guard interval) is added in order to mitigate interblock interference (extended FFT symbol).

There is however, a basic difference between the single and multicarrier modulation schemes: In case of the single carrier system, the decision is carried out in time domain, whereas in case of the multicarrier system the decision is carried out in frequency domain. In the single carrier system, an inverse FFT operation is located between equalization and decision. This inverse FFT operation spreads the noise contributions of all the individual subcarriers on all the samples in time domain. Since the noise contributions of highly attenuated subcarriers can be rather large, a zero-forcing equalizer shows a poor noise performance. Because of this reason, a minimum mean square error (MMSE) equalizer is used for the single carrier system.

The transfer function of the equalizer $H_{\text{eq}}(\omega, t)$ depends on the frequency-dependent signal-to-noise ratio (SNR) $S/N(\omega, t)$ at the input of the receiver:

$$H_{\text{eq}}(\omega, t) = \frac{1}{H(\omega, t)} \cdot \frac{S/N(\omega, t)}{S/N(\omega, t) + 1}, \quad (1)$$

where $H(\omega, t)$ denotes the time-variant transfer function of the radio channel. For large SNRs the MMSE equalizer turns into the zero-forcing equalizer which multiplies with the inverse transfer function.

Next, synchronization aspects are considered. In general, for a digital transmission system the following parameters have to be synchronized: *carrier frequency*, *carrier phase*, *clock frequency*, *clock phase*, as well as the *temporal position of the transmission frame*. Additionally, for OFDM and related modulation schemes which use the FFT algorithm, a synchronization of the *temporal position of the FFT window* is required. “FFT window” denotes the interval of samples

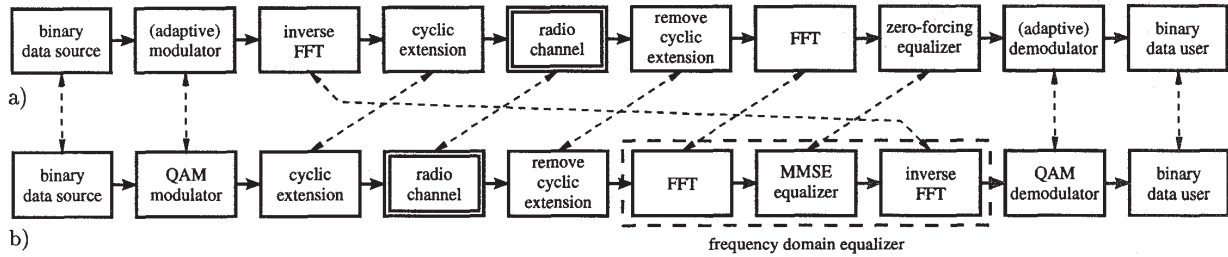


Figure 1: Block diagram of a) an OFDM and b) a single carrier transmission system with frequency domain equalization.

which are processed within one FFT operation. Furthermore, for equalization an estimate of the channel transfer function is required in many cases.

Next, the parameters which have to be synchronized for single carrier transmission with FDE via broadband mobile radio channels are identified. Although single carrier transmission is less sensitive to carrier phase noise than OFDM [3], a powerful synchronization of carrier frequency is required. The synchronization of carrier phase is necessary for coherent detection and is carried out by the channel estimation and equalization.

In many cases, the precision and stability of clock oscillators is sufficient so that no clock frequency synchronization is needed. Furthermore, clock and carrier frequency are often coupled so that by synchronization of carrier frequency also the clock frequency is synchronized. The clock phase need not to be synchronized – its effect on system performance is compensated by channel estimation and equalization. Only a coarse timing synchronization is needed which optimizes the position of the FFT window. The synchronization of the FFT window and the frame synchronization can be combined. For the equalization in frequency domain as well as the synchronization of carrier and clock phase an estimation of the channel transfer function is necessary.

From the above considerations it is concluded that only the following parameters have to be synchronized:

- carrier frequency,
- coarse timing of FFT symbols and transmission frame.
- Furthermore, for equalization an estimation of the channel transfer function is required.

Because of the similarity of single carrier and OFDM transmission, similar requirements have to be met for the synchronization.

Only pilot-aided synchronization methods are considered in this paper since a robust synchronization performance shall be obtained. Unlike in OFDM systems, for single carrier systems it is not possible to distribute pilot symbols individually in time and frequency domain since single carrier signals require the whole available frequency spectrum. Therefore, it is

only possible to multiplex pilot symbols into the data stream in time domain. Multiplexing of single pilot symbols within an FFT symbol does not make sense because the ISI of the transmission channel spreads the pilot symbols over a large number of symbols. Because of this reason, only complete FFT symbols are used as pilot symbols for channel estimation and frequency synchronization.

In this paper a transmission frame with two pilot FFT symbols is used (see Fig. 2). It is similar to the frame structure in [4]. The frame consists of extended FFT symbols: a synchronization symbol, a reference symbol for channel estimation, and FFT symbols for data transmission. The synchronization symbol is used for synchronization of carrier frequency, FFT window, and transmission frame.

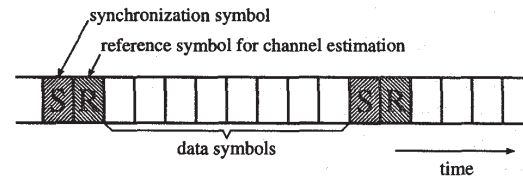


Figure 2: Frame structure of the transmitted signal – each block corresponds to an extended FFT symbol.

In the following, a single carrier system with robust open loop synchronization is analyzed. The block diagram of the considered system is shown in Fig. 3.

III. CARRIER FREQUENCY SYNCHRONIZATION

For carrier frequency synchronization it is assumed that the stability of the radio frequency oscillators is sufficiently high so that only a fine frequency offset estimation has to be carried out. This means that the frequency offset δf is smaller than half of the spacing between the samples in frequency domain. For an OFDM system this is less than half of the subcarrier spacing $\Delta f = 1/(N_{\text{FFT}}\Delta T)$, where N_{FFT} denotes the length of the FFT and ΔT the duration of a data symbol. If the frequency synchronization is carried out only once within a transmission frame, the phase error at the ends of the frame due to a carrier frequency offset must be much smaller than the minimum phase shift caused by the modulation with data symbols.

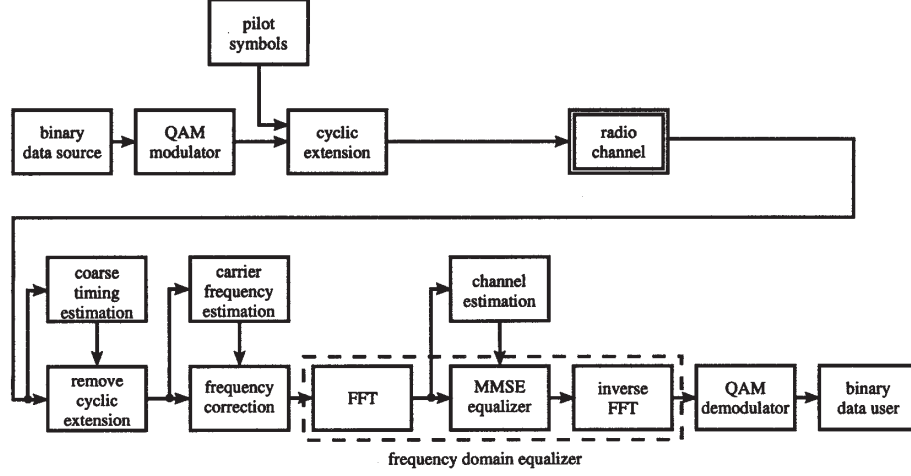


Figure 3: Single carrier transmission system with pilot-aided open loop synchronization.

Most known methods for fine frequency estimation are based on the observation of the phase shift in a periodical signal interval. Denoting the period with N_{sync} and the complex envelope samples of the received signal with r_i , the following correspondence holds during the periodic signal interval (noise is neglected):

$$r_{i+N_{\text{sync}}} = r_i. \quad (2)$$

If a frequency offset δf is present, we have:

$$r_{i+N_{\text{sync}}} = r_i \cdot e^{j 2 \pi \delta f N_{\text{sync}} \Delta T}. \quad (3)$$

A maximum likelihood estimator for the frequency offset has been proposed [5, 6, 7, 4], which calculates the following autocorrelation:

$$\hat{z} = \sum_{i=1}^{N_{\Sigma}} r_{i+k+N_{\text{sync}}} \cdot r_i^* \quad (4)$$

where N_{Σ} denotes the number of terms in the sum and k the time shift from coarse timing synchronization. The estimated frequency offset is obtained from:

$$\delta \hat{f} = \frac{1}{2 \pi N_{\text{sync}} \Delta T} \cdot \varphi(\hat{z}) \quad (5)$$

where $\varphi(\hat{z})$ is the phase of $\hat{z}$. Due to the periodicity of the phase φ , the maximum detectable frequency deviation equals $\pm 1/(2 N_{\text{sync}} \Delta T)$.

For high SNR the variance of the estimation error in an additive Gaussian noise channel equals:

$$\langle (\delta \hat{f} - \delta f)^2 \rangle = \sigma_{\delta f}^2 = \frac{1}{(2 \pi N_{\text{sync}} \Delta T)^2} \cdot \frac{1}{N_{\Sigma}} \cdot \frac{\sigma_N^2}{\sigma_S^2}, \quad (6)$$

where σ_S^2 denotes the average signal power and σ_N^2 the average noise power. The variance of the estimation error does not depend on the frequency offset if it is sufficiently smaller than

the maximum detectable frequency deviation. For a mobile radio channel this variance is increased by residual interblock interference and the temporal variation of the channel.

The performance of the carrier frequency offset estimator can be improved by low-pass filtering (averaging) of the estimated frequency deviation and in case of a fast varying carrier frequency by interpolation between the estimates.

The periodical signal which is required for frequency offset estimation, can be obtained with the following methods:

1. The inherent periodicity of the signal caused by the cyclic extension during the guard interval can be utilized [8, 9, 10]. But it has to be taken care that in any case the guard interval is longer than the duration of the impulse response of the radio channel. The received signal exhibits only in that case a (generally short) periodicity. The drawback of this method is that the estimation error depends on the temporal dispersion of the radio channel.
2. Synchronization symbols can be transmitted which contain two (or more) periods of a test signal. Such a signal can be generated easily in frequency domain from an FFT symbol with vanishing odd frequency samples [4].
3. A further possibility is to transmit two identical FFT symbols subsequently.

For better robustness of the synchronization in a frequency-selective and time-variant radio channel, a pilot-aided synchronization is considered in the following. A system is investigated which uses a synchronization symbol (see Fig. 2) with two periods of a constant envelope signal. The constant envelope signal is generated from DFT (discrete Fourier transform) of a CAZAC (constant amplitude zero autocorrelation) sequence. Also in time domain these sequences exhibit a constant amplitude, which is desirable for single carrier modu-

lation. For a synchronization symbol with two periods, we have $N_{\text{sync}} = N_{\Sigma} = N_{\text{FFT}}/2$. The estimated frequency offset is obtained from

$$\delta \hat{f} = \Delta f \cdot \frac{\varphi(\hat{z})}{\pi}. \quad (7)$$

The frequency estimator is unambiguous for $-\Delta f < \delta f < \Delta f$.

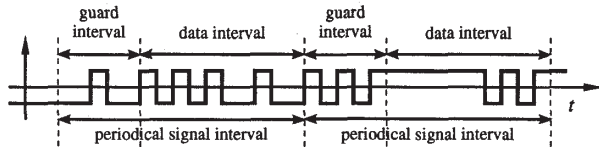
In addition to the calculation in time domain, it is also possible to carry out carrier frequency estimation in frequency domain. Equation (3) shows that the frequency offset causes a constant phase difference between both considered periods. If both periods are transformed separately into frequency domain, the same phase difference results also in frequency domain for all samples, since the same constant phase term is contained in all samples of the second period in time domain.

IV. COARSE TIMING SYNCHRONIZATION

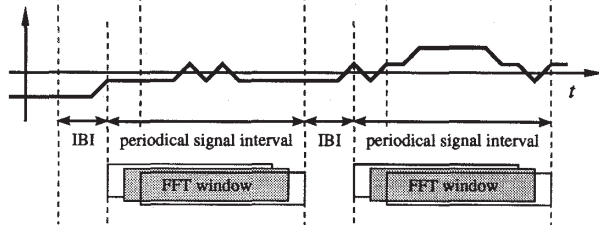
The first step in synchronization is a timing synchronization which finds and optimizes the position of the FFT window. Like in an OFDM system, only a coarse synchronization of the FFT symbols is necessary. For the FFT operation in the receiver, the FFT window has to be positioned to that time interval where the transients have died out and where a periodical signal is found.

The problem of timing synchronization is illustrated in Fig. 4 for a simplified example. In the upper part the transmitted signal is shown. The band limitation is neglected. It is assumed that the signal is transmitted via a channel with the displayed rectangular impulse response. The time intervals in the received signal with interblock interference (IBI) and the periodical time intervals are indicated. The task of the coarse timing synchronization is to position the FFT window any-

transmitted signal:



received signal (without noise):



chanal impulse response:



Figure 4: Illustration of the task of the coarse timing synchronization: optimization of the temporal position of the FFT window.

where into the periodical time interval. The exact position of the FFT window does not make any difference.

A transmission frame may contain a null (FFT) symbol where the transmitted amplitude equals zero. In [11, 12] it has been proposed to use a null symbol for the coarse timing synchronisation. Since besides the null symbol also other zero amplitude intervals can exist, the null symbol has to be confirmed e.g. by specific properties of adjacent FFT symbols.

For detection of the null symbol, the average power of the received signal during a given gliding time interval is calculated. The duration of the interval can be the FFT length. The position of the null symbol can be found by comparing the average power with one or two thresholds.

A second approach for coarse timing synchronization is to detect a periodicity within a synchronization symbol. It is most efficient to use the above described synchronization symbol not only for carrier frequency synchronization but also for coarse timing synchronization [4]. A periodical signal interval can be searched by minimizing the periodicity metric [13, 14, 15] which calculates the squared average distance between the two considered signal intervals:

$$M_p = \frac{1}{N_{\Sigma}} \sum_{i=1}^{N_{\Sigma}} |r_{i+k+N_{\text{sync}}} - r_{i+k} \cdot e^{j\vartheta}|^2. \quad (8)$$

The phase term $e^{j\vartheta}$ has the task to compensate the effect of the carrier frequency offset. The optimum choice of ϑ is $\vartheta = -\varphi(\hat{z})$ from Eq. (4). Inserting the optimum phase into (8) yields [13]:

$$M_p = \frac{1}{N_{\Sigma}} \cdot \hat{E} - \frac{2}{N_{\Sigma}} \cdot |\hat{z}|, \quad (9)$$

where $\hat{E}$ denotes the signal energy of both periods:

$$\hat{E} = \sum_{i=1}^{N_{\Sigma}} (|r_{i+k+N_{\text{sync}}}|^2 + |r_{i+k}|^2). \quad (10)$$

At time intervals where no signal is present, a false lock-in can be avoided using the normalized periodicity metric:

$$M_{\text{pn}} = \frac{\hat{E} - 2 \cdot |\hat{z}|}{\hat{E}}. \quad (11)$$

The timing jitter can be reduced by lowpass filtering of the periodicity metric with respect to the position k of the FFT window. Additionally, the timing jitter is further reduced by averaging the position of the FFT window over several frames.

V. CHANNEL ESTIMATION

Blind channel estimators are inappropriate for mobile radio channels because of the time-variant behaviour of such channels. Therefore, a pilot-aided channel estimation should be used. The required frequency of reference symbols depends on the velocity of temporal variation of the channel. A coarse

estimate of the required frequency of reference symbols can be obtained from the condition that the tolerable phase error (depends on modulation scheme) equals the phase rotation due to the maximum Doppler shift during the period between two reference symbols.

Different types of reference symbols can be used. In order to get a frequency-independent estimation error, the amplitude spectrum of the reference symbols should not depend on frequency. Therefore, the reference symbol is generated in frequency domain with a CAZAC sequence:

$$X_R(\nu \Delta\omega) = e^{j\varphi_\nu} \quad \text{for } \nu = 1, \dots, N \quad (12)$$

with $\Delta\omega = 2\pi \Delta f$. At the channel estimator, the received reference symbols y_R are transformed into frequency domain and divided by the spectrum of the original reference symbols:

$$H_{\text{est}}(\nu \Delta\omega) = \frac{Y_R(\nu \Delta\omega)}{X_R(\nu \Delta\omega)} = Y_R(\nu \Delta\omega) \cdot X_R^*(\nu \Delta\omega). \quad (13)$$

In a first step the estimated channel transfer function can be assumed to remain constant during a complete frame. Because of this assumption, estimation errors are caused by the additive receiver noise and the temporal variation of the channel transfer function. Different methods for reduction of the estimation error can be applied:

- Multiple transmission of reference symbols per frame allows to average the expression (13) over some reference symbols [16].
- A frame structure where the reference symbol is located at the center rather than at the beginning of the frame, exhibits smaller estimation errors at the edges of the frame.
- In case of fast fading channels an interpolation between estimated channel transfer functions increases the accuracy of channel estimation at time instants between reference symbols.
- The effect of additive noise can be reduced by low-pass filtering (with respect to observation time) of the channel transfer function. Wiener filtering minimizes the estimation error [17].

VI. Conclusion

Open loop synchronization methods have been discussed for a single carrier system with FDE. Similar synchronization methods can be used for single carrier systems with FDE as for OFDM systems. The main difference is that pilot symbols cannot be distributed individually in time and frequency domain as in OFDM systems. A small transmission overhead is obtained if the synchronization symbol is used for both carrier frequency and timing synchronization.

The transmission overhead could be further reduced by using the synchronization symbol also as reference symbol for

channel estimation (so that no separate reference symbol is necessary). In that case, the estimate for the channel transfer function has to be interpolated at odd frequencies what implies a higher computational effort.

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