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FOURIER TRANSFORM COMMUNICATION SYSTEM

by

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ABSTRACT

The development of rapid algorithms for computation of the discrete Fourier transform has encouraged the use of this transform in the design of communication systems. Here we describe and analyze a data transmission system in which the transmitted signal is the Fourier transform of the original data sequence and the demodulator is a discrete Fourier transformer. This system is a realization of the frequency division multiplexing strategy known as "parallel data transmission", and it is constructed in this manner so that the data demodulator, after analog to digital conversion, may be a computer program employing one of the fast Fourier transform algorithms. The system appears attractive in that it may be entirely implemented by digital circuitry. We study the performance of this system in the presence of typical linear channel characteristics. It is shown, via computer simulation and computation of the variances of errors, how the system corrects linear channel distortion.

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INTRODUCTION

A digital data communication system can be conceived in which the demodulator is a discrete Fourier transformer (1). The desired data is retrieved from computation of the discrete Fourier transform of received signal samples. This method of demodulation is applicable whenever the data can be easily extracted from the coefficients in a Fourier series representation of the transmitted signal. A natural application is in parallel data transmission where the data is sent in frequency-multiplexed parallel channels.

Parallel systems have been studied and comparisons with serial transmission made (2),(3). The principal objections to conventional parallel systems are a) equipment complexity, and b) severe mutual interference among channels when the transmission channel contains nonlinearities. The present system reduces equipment complexity greatly by using all digital processing of the received data. It is also possible to use a general purpose computer programmed with the fast Fourier transform (4),(5) as the heart of the receiver. Unfortunately, crosstalk, if it is a problem, cannot be eliminated by this demodulation method.

This paper is intended as an introduction to the concept of Fourier transform and more general linear transform receivers and will not argue the merits of one particular system. We will, for purposes of illustration, explain the operation of

a particular system and develop mathematical models from which performance can be deduced.

In the next section we give a physical explanation of the operation of the Fourier transform communication system. Section II presents a general mathematical model and indicates the effects of various channel impairments. The last section provides preliminary analytical results for a typical system.

I. SYSTEM OPERATION

Basically, the Fourier transform data communication system transmits the Fourier transform of the original data signal, and the receiver computes the inverse transform to retrieve the data. Consider a binary pulse sequence

$$x(t) = \sum_k d_k \delta(t - k\Delta t),$$

where Δt is the interval between pulses, for example, the sequence pictured in Figure 1. The Fourier transform of N successive pulses is

$$X(f) = 2 \sum_{n=0}^{N-1} d_n e^{-j2\pi n f \Delta t}, \quad (1)$$

where the factor of 2 is inserted for later convenience. If this Fourier transform is transmitted as a complex time signal, with the frequency variable replaced by $f = t/N(\Delta t)^2$ (arbitrarily selected for later convenience), we have

$$\begin{aligned}
 X_1(t) &= X\left(\frac{t}{N(\Delta t)^2}\right) = 2 \sum_{n=0}^{N-1} d_n e^{-\frac{j2\pi nt}{N\Delta t}} \\
 &= 2 \sum_{n=0}^{N-1} d_n [\cos 2\pi f_n t - j \sin 2\pi f_n t], \\
 0 &\leq t \leq (N-1)\Delta t, \quad (2)
 \end{aligned}$$

where $f_n = n/N\Delta t$. Clearly, $X_1(t)$ is a sum of time-limited sinusoids, each of which is multiplied by one of the data symbols d_k . The data which was originally in serial (one after the other) form is now in parallel (simultaneous transmission) form. The total time required for transmission of the N data is $N\Delta t$ in both cases.

In practice we are constrained to transmit the real part of $X_1(t)$:

$$\begin{aligned}
 y(t) = \text{Re } X_1(t) &= 2 \sum_{n=0}^{N-1} [a_n \cos 2\pi f_n t + b_n \sin 2\pi f_n t], \\
 0 &\leq t \leq (N-1)\Delta t, \quad (3)
 \end{aligned}$$

where $d_n = a_n + jb_n$. The existence of a quadrature carrier signal is seen to permit transmitting $2N$ real data instead of N in the time interval $N\Delta t$ making the most efficient use of the frequency space.

If the channel through which $y(t)$ is transmitted is distortionless as illustrated in Figure 2, then $y(t)$ is received and the receiver must retrieve the data $\{a_n\}$ and $\{b_n\}$. This can be done by use of a sampler followed by a discrete Fourier transformer. Because of the presence of two sets of N data on orthogonal carriers, it is necessary to sample twice as often as expected,[†] i.e., at intervals $\Delta t/2$ instead of Δt , and perform a discrete Fourier transform on the $2N$ samples so obtained. The samples are

$$\begin{aligned} y_k = y\left(k \frac{\Delta t}{2}\right) &= 2 \sum_{n=0}^{N-1} \left[a_n \cos 2\pi f_n \frac{k\Delta t}{2} + b_n \sin 2\pi f_n \frac{k\Delta t}{2} \right] \\ &= 2 \sum_{n=0}^{N-1} \left[a_n \cos \frac{2\pi nk}{2N} + b_n \sin \frac{2\pi nk}{2N} \right], \end{aligned}$$

where

$$f_n k\Delta t = \frac{n}{N\Delta t} \cdot k\Delta t = \frac{nk}{N}. \quad (4)$$

These $2N$ samples are then used as input to a discrete Fourier transformer, yielding

[†] When sampling is done at Δt intervals and the discrete Fourier transform is taken of the N samples, the resulting complex numbers are functions of a_k and b_k from which a_k and b_k cannot be separately obtained.

$$\begin{aligned}
 z_\ell &= \frac{1}{2N} \sum_{k=0}^{2N-1} y_k e^{j \frac{2\pi \ell k}{2N}} \\
 &= \frac{2}{2N} \sum_{n=0}^{N-1} \left[a_n \sum_{k=0}^{2N-1} e^{j \frac{2\pi \ell k}{2N}} \cos \frac{2\pi n k}{2N} + b_n \sum_{k=0}^{2N-1} e^{j \frac{2\pi \ell k}{2N}} \sin \frac{2\pi n k}{2N} \right] \\
 &= \begin{cases} 2a_0, & \ell = 0 \\ a_\ell - j b_\ell, & \ell = 1, 2, \dots, N-1, \\ \text{irrelevant}, & \ell > N-1 \end{cases} \quad (5)
 \end{aligned}$$

where we have used the fact that

$$\frac{1}{2N} \sum_{k=0}^{2N-1} e^{j \frac{2\pi m k}{2N}} = \delta(m-k) \bmod 2N$$

in producing the second equality. The discrete Fourier transformer receiver does indeed retrieve the original data exactly when there is no channel impairment. As the introduction pointed out, the instrumentation of the receiver can be entirely digital or, equivalently, the discrete Fourier transform can be accomplished by a programmed digital computer using the fast Fourier transform.

When there are channel impairments, this receiver may, with small modification, still perform very well. In particular linear channel distortion can be largely corrected by multiplication of the complex output numbers $\{z_\ell\}$ by appropriate compensation

factors. Figure 3 illustrates the system. We now describe the Fourier transform receiver in the presence of linear channel distortion.

The transmitted waveform is again $y(t)$, given by equation 3. Assuming a sequence long enough so that the difference $\Delta f = 1/N\Delta t$ in adjacent subcarrier frequencies is small, the variation of the channel transfer function over a subcarrier channel[†] is negligible, as indicated in Figure 4. Thus a good approximation to the signal after passing through the channel filter is

$$r(t) = 2 \sum_{n=0}^{N-1} H_n [a_n \cos(2\pi n \Delta f t + \theta_n) + b_n \sin(2\pi n \Delta f t + \theta_n)], \quad (6)$$

where H_n and θ_n are the magnitude and phase respectively of the channel filter $H(f)$ at frequency $f_n = n/N\Delta t$. Sampling, as before, at intervals $\Delta t/2$, we have

$$\begin{aligned} r_k &= r\left(k \frac{\Delta t}{2}\right) \\ &= 2 \sum_{n=0}^{N-1} \left[a_n H_n \cos\left(\frac{2\pi n k}{2N} + \theta_n\right) + b_n H_n \sin\left(\frac{2\pi n k}{2N} + \theta_n\right) \right], \\ k &= 0, 1, \dots, 2N-1 \quad (7) \end{aligned}$$

[†] The transmitted energy does not exist only at the subcarrier frequencies $\{f_n\}$ but is spread out because the subcarriers are on for a time $N\Delta t$, not infinity.

The discrete Fourier transform yields

$$z_\ell = \frac{1}{2N} \sum_{k=0}^{2N-1} r_k e^{-\frac{j2\pi\ell k}{2N}}$$

$$= \begin{cases} 2a_0 H_0 \cos \theta_0 + 2b_0 H_0 \sin \theta_0, & \ell = 0 \\ H_\ell e^{j\theta_\ell} (a_\ell - jb_\ell), & \ell = 1, 2, \dots, N-1 \\ \text{irrelevant,} & \ell > N-1 \end{cases} \quad (8)$$

Since z_0 is all real, a_0 and b_0 cannot be separated. For ℓ greater than 0 define

$$x_{1\ell} = \frac{1}{H_\ell} [\operatorname{Re}(z_\ell) \cos \theta_\ell + \operatorname{Im}(z_\ell) \sin \theta_\ell]$$

$$x_{2\ell} = \frac{1}{H_\ell} [\operatorname{Re}(z_\ell) \sin \theta_\ell - \operatorname{Im}(z_\ell) \cos \theta_\ell] \quad (9)$$

Then by simple trigonometry,

$$\begin{aligned} x_{1\ell} &= a_\ell \\ &, \quad \ell = 1, 2, \dots, N-1. \\ x_{2\ell} &= b_\ell \end{aligned} \quad (10)$$

Recall that the data are so neatly retrieved only if the use of values of $H(f)$ at N discrete frequencies is a valid approximation.

Mutual Interference of Successive Blocks

An important degradation which, in the interest of simplicity, has not yet been introduced is that resulting from interference among blocks of data. This interference occurs when the phase $\theta(f)$ of the channel filter is not a constant, so that different frequency components are delayed by different amounts. This causes some frequency components of one data block to spill over onto the demodulation interval of later data blocks, and if the n^{th} digit of the $m-1^{\text{th}}$ block is of opposite polarity from the n^{th} digit of the m^{th} block, the demodulation of the m^{th} block will introduce some error. The variance of the magnitude of this error has been computed for several channel filters and is reported in Section III. Mutual interference of blocks is not expected to be a serious practical problem.

II. SYSTEM MODELING

In the previous section we demonstrated heuristically how linear channel characteristics enter to modify the received signal and its subsequent processing in the Fourier transform receiver. The assumption was made that the finite block length $T' = N\Delta t$ had a minor effect on the received signal, permitting use of a finite number of points on the channel characteristic

for correction purposes. Of course, in real-life situations some intersymbol interference will result from this time truncation and from channel bandlimiting. The ideal linear receiver is then not the Fourier transform receiver but a more general linear transform receiver. In this section we will write the mathematical formulation for this more general receiver.

We begin by arranging the data symbols to be transmitted during the m^{th} block and the carrier vector signal in column matrices as follows:

$$\vec{A}_m = \begin{bmatrix} a_1^{(m)} \\ a_2^{(m)} \\ \vdots \\ a_N^{(m)} \\ b_1^{(m)} \\ \vdots \\ b_N^{(m)} \end{bmatrix} \quad \begin{array}{c} \uparrow \\ 2N \text{ entries} \\ \downarrow \end{array} \quad \vec{S}(t) = \begin{bmatrix} \cos 2\pi\Delta ft \\ \cos 2\pi 2\Delta ft \\ \vdots \\ \cos 2\pi N\Delta ft \\ \sin 2\pi\Delta ft \\ \sin 2\pi 2\Delta ft \\ \vdots \\ \sin 2\pi N\Delta ft \end{bmatrix} \quad \begin{array}{c} \uparrow \\ 2N \text{ signals} \\ \downarrow \end{array} \quad (11)$$

Here Δf is the frequency spacing between subcarriers such that $\Delta f \cdot N = F$ the channel bandwidth. In terms of the above two vectors, the data signal admitted to the channel may be represented as

$$D(t) = \vec{S}^+(t) \sum_m \vec{A}_m g(t-mT') \quad (12)$$

where $g(t)$ is a scalar rectangular pulse of T' seconds duration and unit height and the symbol $(\cdot)^+$ indicates the transpose. The representation (12) is possible since parameters in the carrier vector are so chosen as to make $\vec{S}(t)$ periodic in T' . To accomplish this periodicity we must have T' (the duration of a block) = $N\Delta t$ where $\Delta t/2$ is the receiver sampling period and $\Delta t \cdot F = 1$. Now consider the data wave at the output of the linear medium possessing impulse response $h(t)$. Because of linearity it is sufficient to know the medium response to $g(t)\vec{S}(t)$. Denote this vector response by

$$\vec{W}(t) = \int_{-\infty}^{\infty} \vec{S}(\tau) g(\tau) h(t-\tau) d\tau. \quad (13)$$

Since $\vec{S}(t)$ is periodic in T' , $\vec{W}(t-mT')$ is the response to $\vec{S}(t)g(t-mT')$ and therefore we can represent the output data wave by

$$D_o(t) = \sum_m \vec{W}^+(t-mT') \vec{A}_m \quad (14)$$

To derive the data vectors, the receiver operates on $D_o(t)$. The first operation consists of periodically sampling the wave. Then by operating on successive blocks of $2N$ samples estimates of the respective data vectors are made. Thus consider arranging the $2N$ samples at the k^{th} block in a column vector

$$\begin{aligned} \vec{Q}(kT') &= \begin{bmatrix} D_o(kT' + t_1) \\ \vdots \\ D_o(kT' + t_N) \\ \vdots \\ D_o(kT' + t_{2N}) \end{bmatrix} = \begin{bmatrix} \sum_m \vec{W}^+(kT' - mT' + t_1) \vec{A}_m \\ \vdots \\ \sum_m \vec{W}^+(kT' - mT' + t_2) \vec{A}_m \\ \vdots \\ \sum_m \vec{W}^+(kT' - mT' + t_{2N}) \vec{A}_m \end{bmatrix} \quad (15) \\ &= \sum_m \tilde{R}(kT' - mT') \vec{A}_m \end{aligned}$$

where $t_\ell = \frac{\Delta t \ell}{2}$ $\ell = 1, 2, \dots, 2N$ and the $2N \times 2N$ matrix $\tilde{R}(t)$ is

$$\tilde{R}(t) = \begin{bmatrix} \vec{W}^+(t + t_1) \\ \vec{W}^+(t + t_2) \\ \vdots \\ \vec{W}^+(t + t_{2N}) \end{bmatrix} \quad (16)$$

A very useful analogy with baseband serial data transmission can now be made. In a conventional baseband serial system the data symbols sequentially modulate a pulse $r(t)$ giving rise to a received wave

$$q(t) = \sum_m a_m r(t-nT') \quad (17)$$

In the system considered here, the symbols become vectors, the pulse is a matrix time function and the received vector data wave is

$$\vec{Q}(t) = \sum_m \tilde{R}(t-nT') \vec{A}_m. \quad (18)$$

Of course in our application the time parameter in (18) has physical meaning only at discrete values $t = kT'$. Thus for analytical purposes the mathematical model of the system is depicted in Figure (5). Based on this model, we are in a position to formulate certain optimization problems.

The receiver detects the respective data vectors by manipulating the received signal vector $\vec{Q}(t)$. Let us suppose that only linear operations on $\vec{Q}(t)$ are allowed. This immediately restricts the operators on $\vec{Q}$ to be matrices. As the first optimization problem, let us attempt to detect, say, the $\vec{A}_0$ data vector in the presence of an additive zero mean Gaussian noise vector $\vec{N}(t)$ with covariance matrix

$$\tilde{\rho}(\tau) = E\{\vec{N}^+(t)\vec{N}(t+\tau)\} \quad (19)$$

where $E\{\cdot\}$ denotes the expectation operator. One approach might be to determine a transformation matrix $\tilde{M}(2N \times 2N)$ such that the mean-squared detection error is a minimum. To detect $\vec{A}_0$ we observe $\vec{Q}$ plus $\vec{N}$ at time $t = 0$ (this is not restrictive since we assume the data to be stationary). Thus

$$\begin{aligned} \vec{V}(0) &= \vec{Q}(0) + \vec{N}(0) \\ &= \tilde{R}(0)\vec{A}_0 + \sum'_m \tilde{R}(-mT')\vec{A}_m + \vec{N}(0) \end{aligned} \quad (20)$$

where $\sum'_m$ denotes deletion of the zeroth term in the sum. To form the error vector we first operate on $\vec{V}$ by $\tilde{M}$ and then subtract the true value $\vec{A}_0$ to obtain

$$\vec{\epsilon} = \tilde{M}\vec{V}(0) - \vec{A}_0 \quad (21)$$

We next proceed to calculate $\tilde{M}_0$ such that

$$\begin{aligned} f[\tilde{M}_0] &= \min_{\tilde{M}} E\{\vec{\epsilon}^+ \vec{\epsilon}\} \\ &= \min_{\tilde{M}} E\{\text{Tr} \vec{\epsilon} \vec{\epsilon}^+\} \end{aligned} \quad (22)$$

where Tr denotes the trace of the matrix.

This is a straightforward mathematical problem and the solution for $\tilde{M}_0$ which minimizes (22) can be shown to be

$$\tilde{M}_0 = \tilde{R}_0^+ \left[\sum_m \tilde{R}_m \tilde{R}_m^+ + \tilde{\rho}(0) \right]^{-1} \quad (23)$$

where $\tilde{R}_m = \tilde{R}(-mT')$ and $[\cdot]^{-1}$ stands for the inverse. Clearly in the absence of noise and intersymbol interference, $\tilde{M}_0 = \tilde{R}_0^{-1}$. In order to evaluate the resulting mean-squared error we must know the matrix $\tilde{R}(t)$ which is a function of channel characteristics.

The optimization just carried out is clearly restrictive. The intersymbol interference was given the same treatment as the additive noise. Since intersymbol interference may be viewed as a kind of deterministic interference we might do better to use equalization (6) just as is done in serial systems. In this vector model, the equalizer tap weights must be matrices. Let us therefore define a tapped delay-line network as indicated in Figure (6). The tap gains are the matrices $\tilde{H}_n$, $n = 1, \dots, P$ and the respective delays are of the same duration as a data block. The vector impulse response of this network is

$$\vec{h}(t) = \sum_{n=1}^P \tilde{H}_n \vec{\delta}(t-nT') \quad (24)$$

When $\vec{Q}(t)$ is the input to this network the output is

$$\begin{aligned}\vec{Q}(t) &= \sum_{n=1}^P \tilde{H}_n \vec{Q}(t-nT') \\ &= \sum_{n=1}^P \tilde{H}_n \sum_m \tilde{R}(t-nT'-mT') \vec{A}_m\end{aligned}\quad (25)$$

Thus at $t = 0$ we obtain

$$\vec{Q}(0) = \sum_{n=1}^P \tilde{H}_n \tilde{R}(-mT') \vec{A}_0 + \sum_n \tilde{H}_n \left\{ \sum'_m \tilde{R}(-nT'-mT') \vec{A}_m \right\} \quad (26)$$

Let us for the moment neglect the added noise and focus attention only on the intersymbol interference. The question we might pose now is the following: What is an optimum way of choosing the set of matrices $\{\tilde{H}_n\}$ such that the mean-squared error is a minimum? We proceed as before and form the error vector

$$\vec{\epsilon} = \vec{Q}(0) - \vec{A}_0 \quad (27)$$

and determine a set of matrices $\{\tilde{H}_n\}$ which minimize $\text{tr } E\{\vec{\epsilon}\vec{\epsilon}^+\}$. The calculus here is considerably more complicated than in the previous situation. However it can be shown that the optimum set of matrices $\{\tilde{H}_n\}$ satisfies the linear set of matrix equations

$$\tilde{R}_{\lambda}^{+} = \sum_{n=1}^P \tilde{H}_n \tilde{W}_{n\lambda} \quad (28)$$

$$\lambda = 1, \dots, P$$

where

$$\tilde{W}_{n\lambda} = \sum_m \tilde{R}_{n+m} \tilde{R}_{\lambda+m}^{+}.$$

In the event that there is no intersymbol interference this result reduces to the previous result.

We must bear in mind that the discrete Fourier transformer is just a special case of the matrix $\tilde{M}_0$ calculated before. In fact, when the channel is ideal and there is no added noise, $\tilde{M}_0$ is precisely the Fourier transform matrix discussed in the previous section. In practice it is hoped that no equalization will be required. Otherwise the system becomes hopelessly complicated. In the next section we shall calculate the mean-squared error for a typical channel when the matrix $\tilde{M}_0$ is the discrete Fourier transformer. Even though the use of the discrete Fourier transformer is not optimum, in certain applications it may be adequate resulting in a mean-squared error sufficiently small.

III. NUMERICAL RESULTS

The Fourier transform receiver has been evaluated for some representative linear channels. Under the assumption that the data $\{a_n\}$ and $\{b_n\}$ are all mutually independent ± 1 random variables with equally probable levels, it is possible to derive an expression for the variance of the error of each output in the set $\{x_{1\ell}, x_{2\ell}, \ell = 1, 2, \dots, N-1\}$ from the desired ideal value $x_{1\ell} = a_\ell$ or $x_{2\ell} = b_\ell$. Essentially we compute the diagonal elements of the mean-squared error matrix $\vec{\epsilon}$ (equation 21) for the special Fourier transform version of the matrix M .

If the error variances are much less than 1, then the receiver is operating properly. For a nondispersive channel there should be no error at all, and as the phase distortion increases the error variances should also increase. As a model for the transmission system we examine a Fourier transform receiver operating on a 32-channel parallel data signal, described by equations 8-10 of Section I. The subcarrier frequencies are at multiples of 100 Hz, so that the highest subcarrier frequency is 3100 Hz. Because of the finite duration of each data block, there is a band of occupied spectrum around each subcarrier frequency, as already noted in Section I.

Figure 7 presents the error variances for a distortionless channel. The numbers are believed to be largely computer error and not indicative of degradation in the transmission system.

Figures 8-10 show the error variances for three highly dispersive channels, identical except for the cutoff points. As expected, the outermost subcarrier channel suffers severely when the cutoff is at the 3100 Hz center frequency of this channel. The difference in delay between 0 and 1550 Hz components is 0.01 second or a whole block length. By beginning to sample at a properly delayed time, which is equivalent to beginning at the old undelayed time with a channel characteristic including negative delays, as pictured in Figures 8-10, the effect of the delay distortion is minimized. Note that even with this severe distortion the error variance is tolerable and the Fourier transform receiver can operate.

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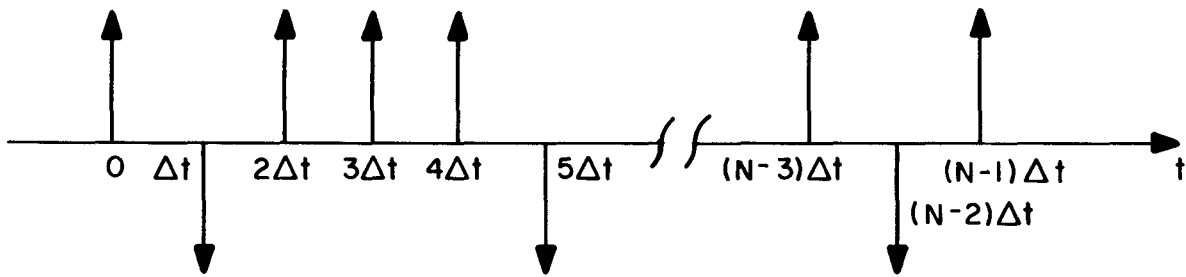


FIG. 1 A TYPICAL SEQUENCE OF DATA PULSES

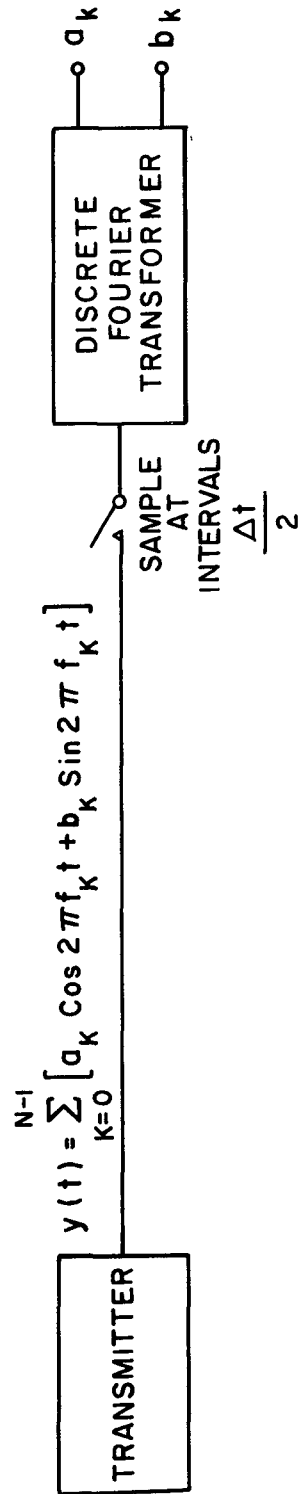


FIG. 2 THE FOURIER TRANSFORM SYSTEM WITH A DISTORTIONLESS CHANNEL.

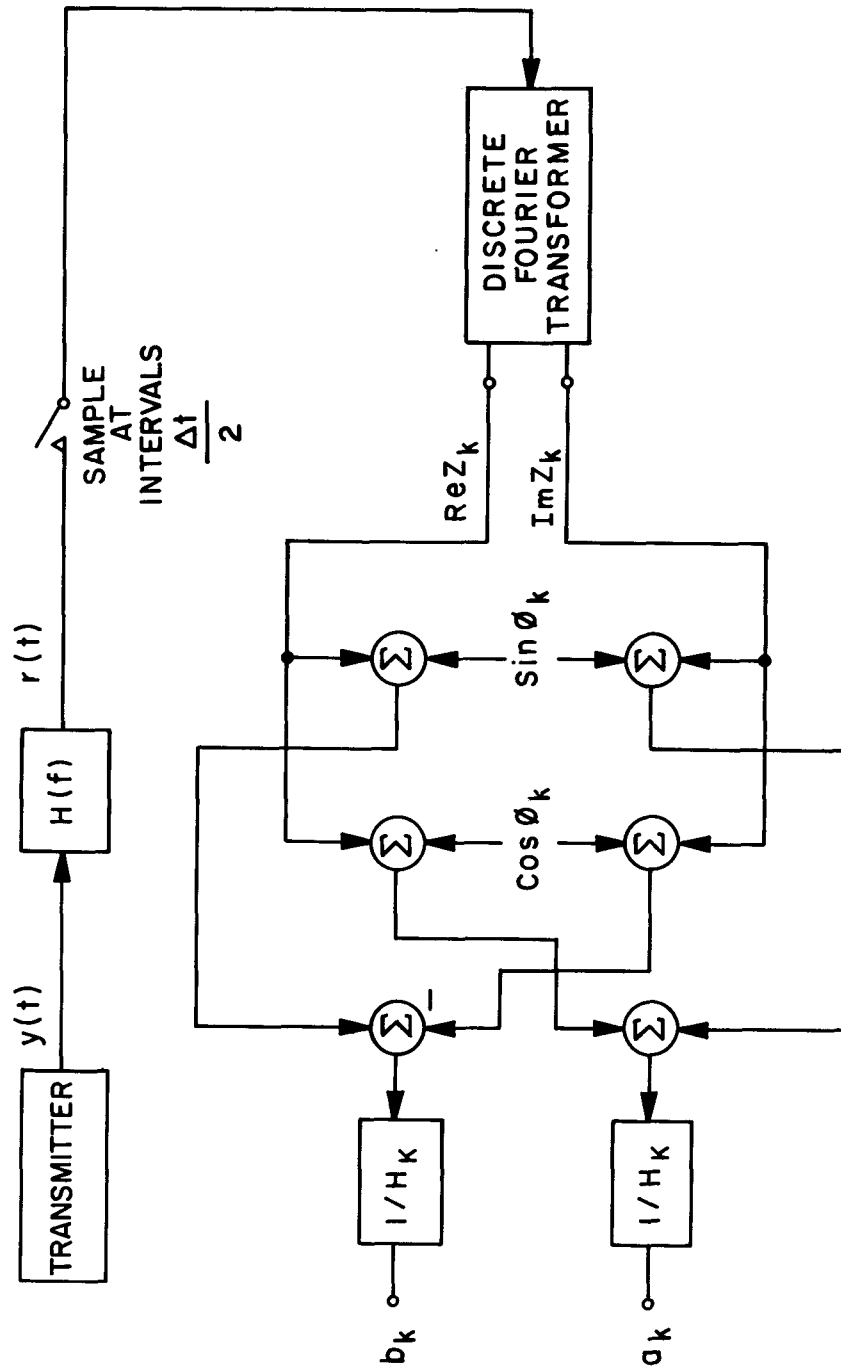


FIG. 3 FOURIER TRANSFORM COMMUNICATION SYSTEM WITH CORRECTION FOR LINEAR DISTORTION.

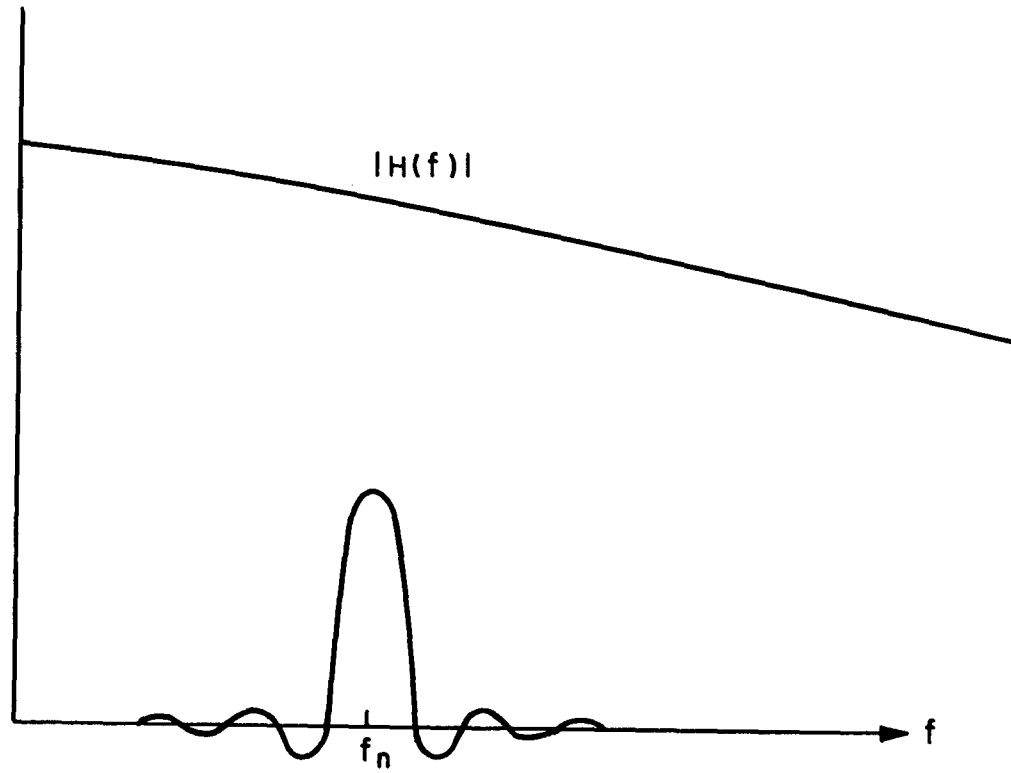


FIG. 4 MAGNITUDE $|H(f)|$ OF TYPICAL CHANNEL TRANSFER FUNCTION COMPARED WITH FOURIER TRANSFORM OF SIGNAL IN n^{th} SUBCARRIER CHANNEL.

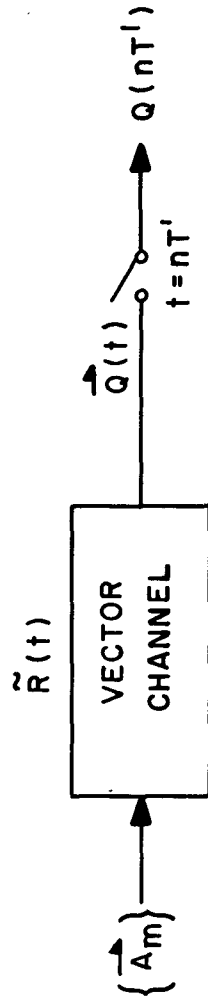


FIG.5 THE VECTOR MODEL

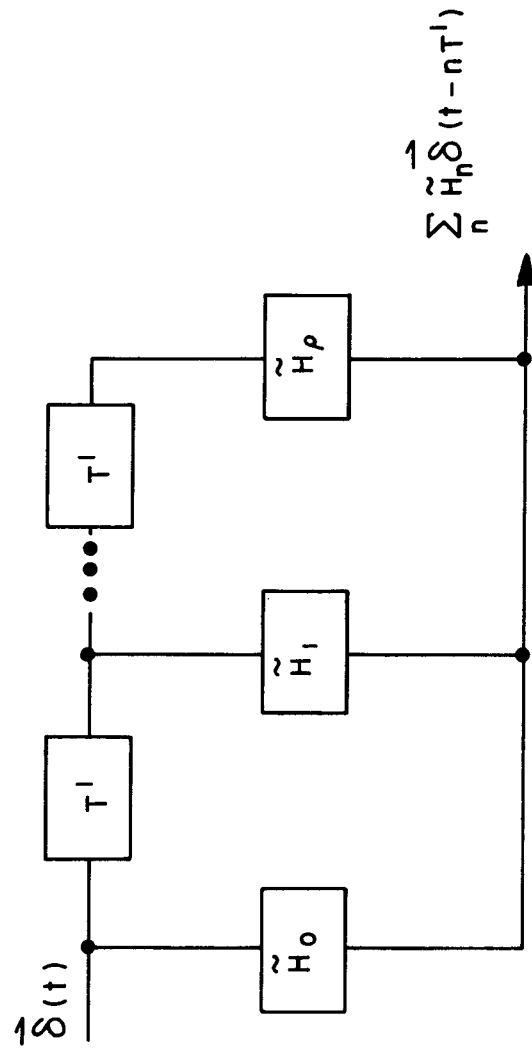


FIG. 6 MATRIX TAP - DELAY LINE EQUALIZER

CHANNEL CHARACTERISTIC

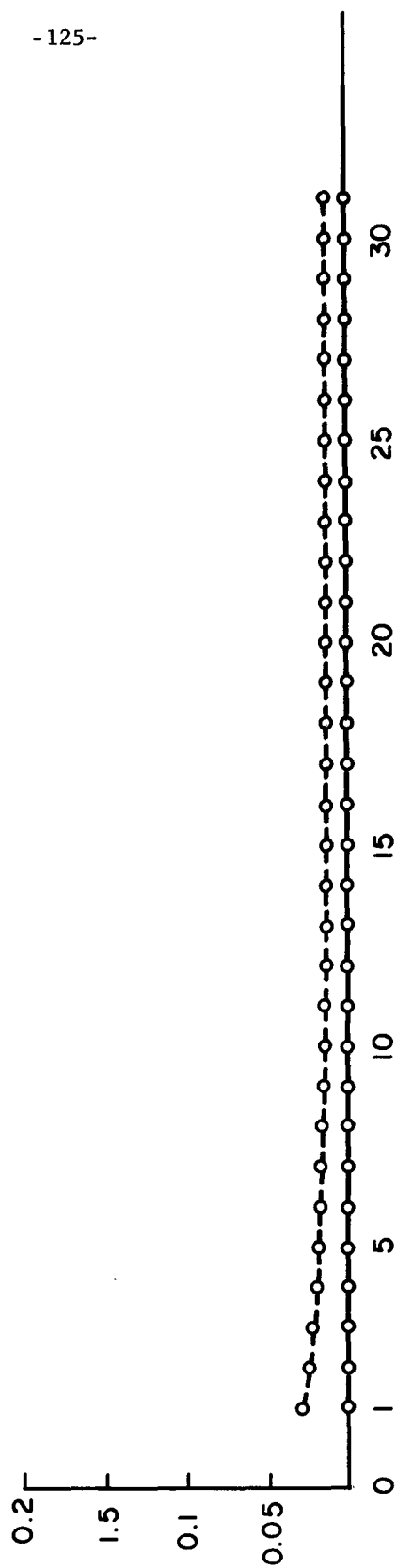
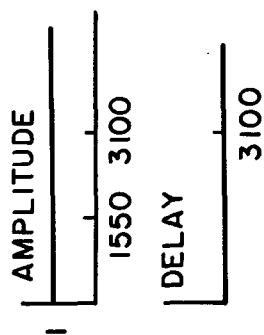


FIG. 7 REFERENCE LEVELS FOR ERROR VARIANCES OF OUTPUTS OF 32- FREQUENCY
FOURIER TRANSFORM DATA TRANSMISSION SYSTEM WITH MULTIPLICATIVE
EQUALIZATION WHEN CHANNEL IS DISTORTIONLESS.

CHANNEL CHARACTERISTIC

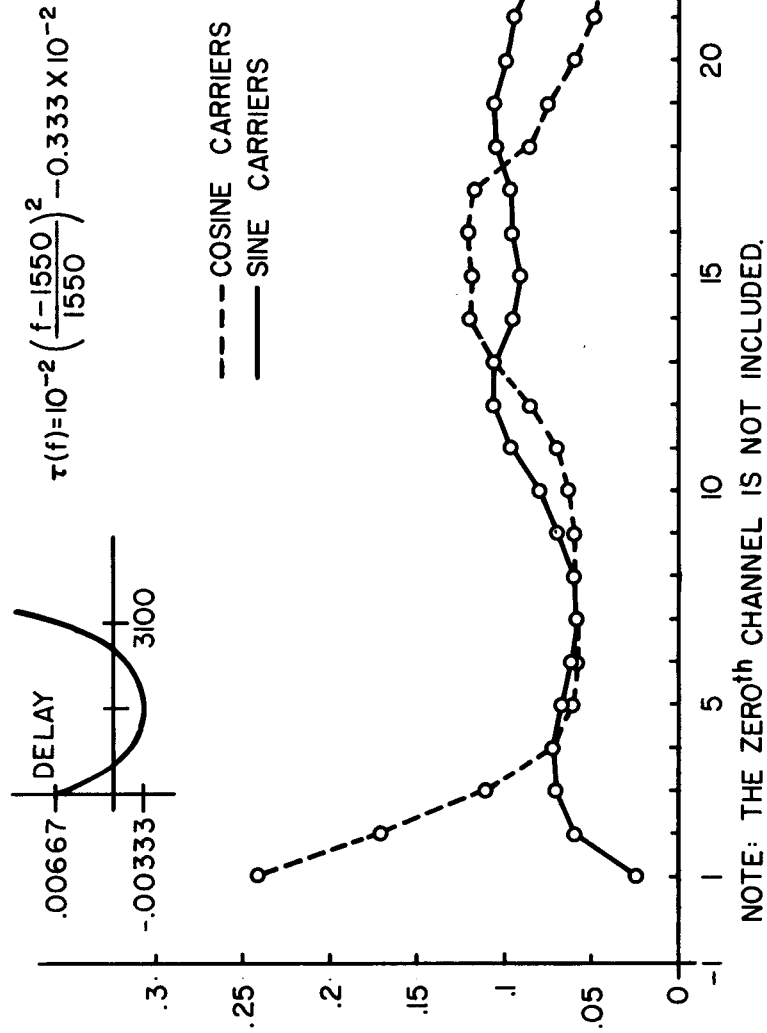
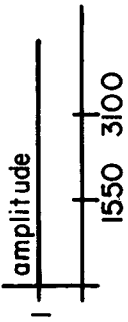


FIG. 8 ERROR VARIANCES OF 32-FREQUENCY FOURIER TRANSFORM DATA
TRANSMISSION SYSTEM WITH MULTIPLICATIVE EQUALIZATION; TYPICAL CHANNEL
WITH INFINITE CUTOFF.

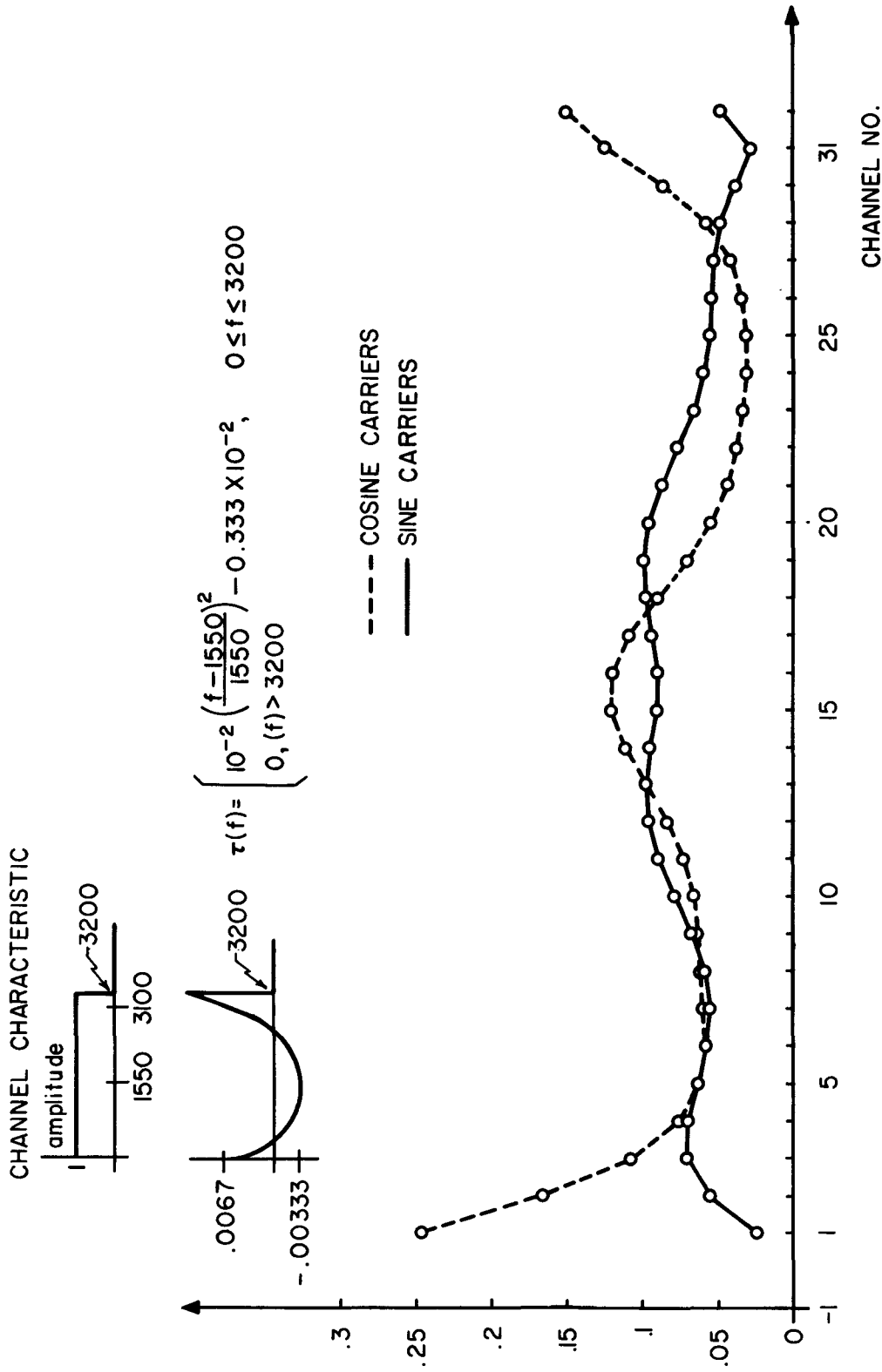


FIG. 9. ERROR VARIANCES OF 32-FREQUENCY FOURIER TRANSFORM DATA TRANSMISSION SYSTEM WITH MULTIPLICATIVE EQUALIZATION; TYPICAL CHANNEL WITH 3200 HZ CUTOFF.

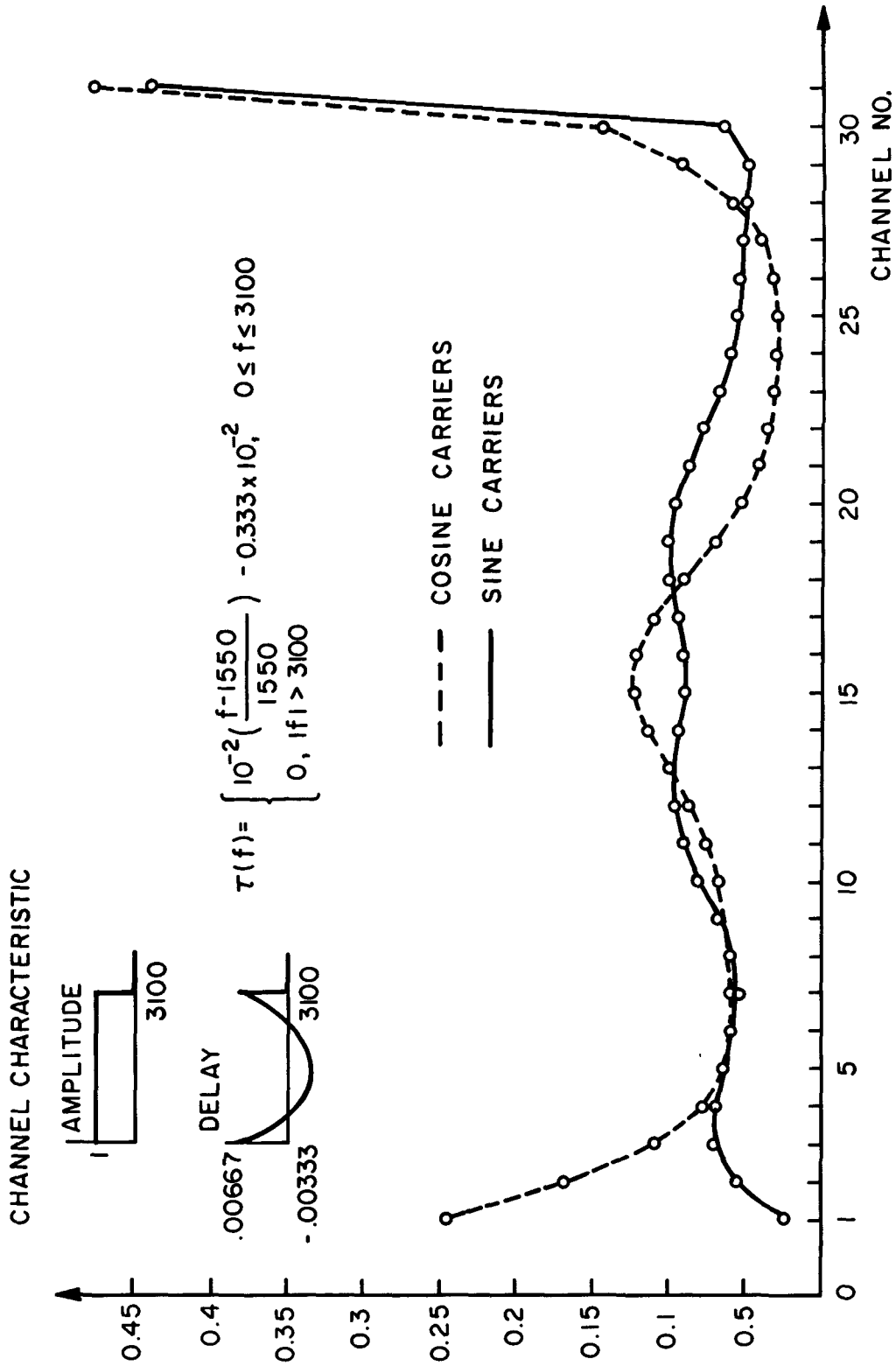


FIG. 10 ERROR VARIANCES OF 32-FREQUENCY FOURIER TRANSFORM DATA TRANSMISSION SYSTEM WITH MULTIPLICATIVE EQUALIZATION; TYPICAL CHANNEL WITH 3100HZ CUTOFF.