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1. The sinusoidal carrier wave $\cos(2\pi f_c t)$ is delayed by τ_p seconds; hence τ_p represents the *phase delay*. Sometimes, τ_p is also referred to as the *carrier delay*.
2. The envelope $m(t)$ is delayed by τ_g seconds; hence, τ_g represents the *envelope* or *group delay*.

Note that τ_g is related to the slope of the phase $\beta(f)$, measured at $f = f_c$, as in Eq. (2.171). Note also that when the phase response $\beta(f)$ varies linearly with frequency and $\beta(f_c)$ is zero, the phase delay and group delay assume a common value.

2.15 NUMERICAL COMPUTATION OF THE FOURIER TRANSFORM

The material presented in this chapter clearly testifies to the importance of the Fourier transform as a theoretical tool for the representation of deterministic signals and linear time-invariant systems. The importance of the Fourier transform is further enhanced by the fact that there exists a class of algorithms called fast Fourier transform algorithms¹² for the numerical computation of the Fourier transform in a highly efficient manner.

The fast Fourier transform algorithm is derived from the discrete Fourier transform in which, as the name implies, both time and frequency are represented in discrete form. The discrete Fourier transform provides an *approximation* to the Fourier transform. In order to properly represent the information content of the original signal, we have to take special care in performing the sampling operations involved in defining the discrete Fourier transform. A detailed treatment of the sampling process is presented in Chapter 6. For the present, it suffices to say that given a band-limited signal, the sampling rate should be greater than twice the highest frequency component of the input signal. Moreover, if the samples are uniformly spaced by T_s seconds, the spectrum of the signal becomes periodic, repeating every $f_s = (1/T_s)$ Hz. Let N denote the number of frequency samples contained in an interval f_s . Hence, the *frequency resolution* involved in the numerical computation of the Fourier transform is defined by

$$\Delta f = \frac{f_s}{N} = \frac{1}{NT_s} = \frac{1}{T} \quad (2.174)$$

where T is the total duration of the signal.

Consider then a *finite data sequence* $\{g_0, g_1, \dots, g_{N-1}\}$. For brevity, we refer to this sequence as g_n , in which the subscript is the *time index* $n = 0, 1, \dots, N-1$. Such a sequence may represent the result of sampling an *analog signal* $g(t)$ at times $t = 0, T_s, \dots, (N-1)T_s$, where T_s is the sampling interval. The ordering of the data sequence defines the sample time in that $g_0, g_1, \dots, g_{N-1}$ denote samples of $g(t)$ taken at times $0, T_s, \dots, (N-1)T_s$, respectively. Thus we have

$$g_n = g(nT_s) \quad (2.175)$$

We formally define the *discrete Fourier transform* (DFT) of g_n as

$$G_k = \sum_{n=0}^{N-1} g_n \exp\left(-\frac{j2\pi}{N} kn\right) \quad k = 0, 1, \dots, N-1 \quad (2.176)$$

The sequence $\{G_0, G_1, \dots, G_{N-1}\}$ is called the *transform sequence*. For brevity, we refer to this sequence as G_k , in which the subscript is the *frequency index* $k = 0, 1, \dots, N - 1$. Correspondingly, we define the *inverse discrete Fourier transform* (IDFT) of G_k as

$$g_n = \frac{1}{N} \sum_{k=0}^{N-1} G_k \exp\left(\frac{j2\pi}{N} kn\right) \quad n = 0, 1, \dots, N - 1 \quad (2.177)$$

The DFT and the IDFT form a transform pair. Specifically, given a data sequence g_n , we may use the DFT to compute the transform sequence G_k ; and given the transform sequence G_k , we may use the IDFT to recover the original data sequence g_n .

A distinctive feature of the DFT is that for the finite summations defined in Eqs. (2.176) and (2.177), there is no question of convergence.

When discussing the DFT (and algorithms for its computation), the words “sample” and “point” are used interchangeably to refer to a sequence value. Also, it is common practice to refer to a sequence of length N as an *N-point sequence*, and refer to the DFT of a data sequence of length N as an *N-point DFT*.

Interpretation of the DFT and the IDFT

We may visualize the DFT process, described in Eq. (2.176), as a collection of N *complex heterodyning* and *averaging* operations, as shown in Fig. 2.37a. We say that the heterodyning is complex in that samples of the data sequence are multiplied by *complex exponential sequences*. There are a total of N complex exponential sequences to be considered, corresponding to the frequency index $k = 0, 1, \dots, N - 1$. Their periods have been selected in such a way that each complex exponential sequence has precisely an integer number of cycles in the total interval 0 to $N - 1$. The zero-frequency response, corresponding to $k = 0$, is the only exception.

For the interpretation of the IDFT process, described in Eq. (2.177), we may use the scheme shown in Fig. 2.37b. Here we have a collection of N *complex signal generators*, each of which produces a complex exponential sequence:

$$\begin{aligned} \exp\left(\frac{j2\pi}{N} kn\right) &= \cos\left(\frac{2\pi}{N} kn\right) + j \sin\left(\frac{2\pi}{N} kn\right) \\ &= \left\{ \cos\left(\frac{2\pi}{N} kn\right), \sin\left(\frac{2\pi}{N} kn\right) \right\} \end{aligned} \quad (2.178)$$

where $k = 0, 1, \dots, N - 1$.

Thus, each complex signal generator, in reality, consists of a pair of generators that outputs a cosinusoidal and a sinusoidal sequence of k cycles per observation interval. The output of each complex signal generator is weighted by the complex Fourier coefficient G_k . At each time index n , an output is formed by summing the weighted complex generator outputs.

It is noteworthy that although the DFT and the IDFT are similar in their mathematical formulations, as described in Eqs. (2.176) and (2.177), their interpretations, as depicted in Figs. 2.37a and 2.37b, are so completely different.

Also, the addition of harmonically related periodic signals, as in Figs. 2.37a and 2.37b, suggests that their outputs G_k and g_n must be both periodic. Moreover,

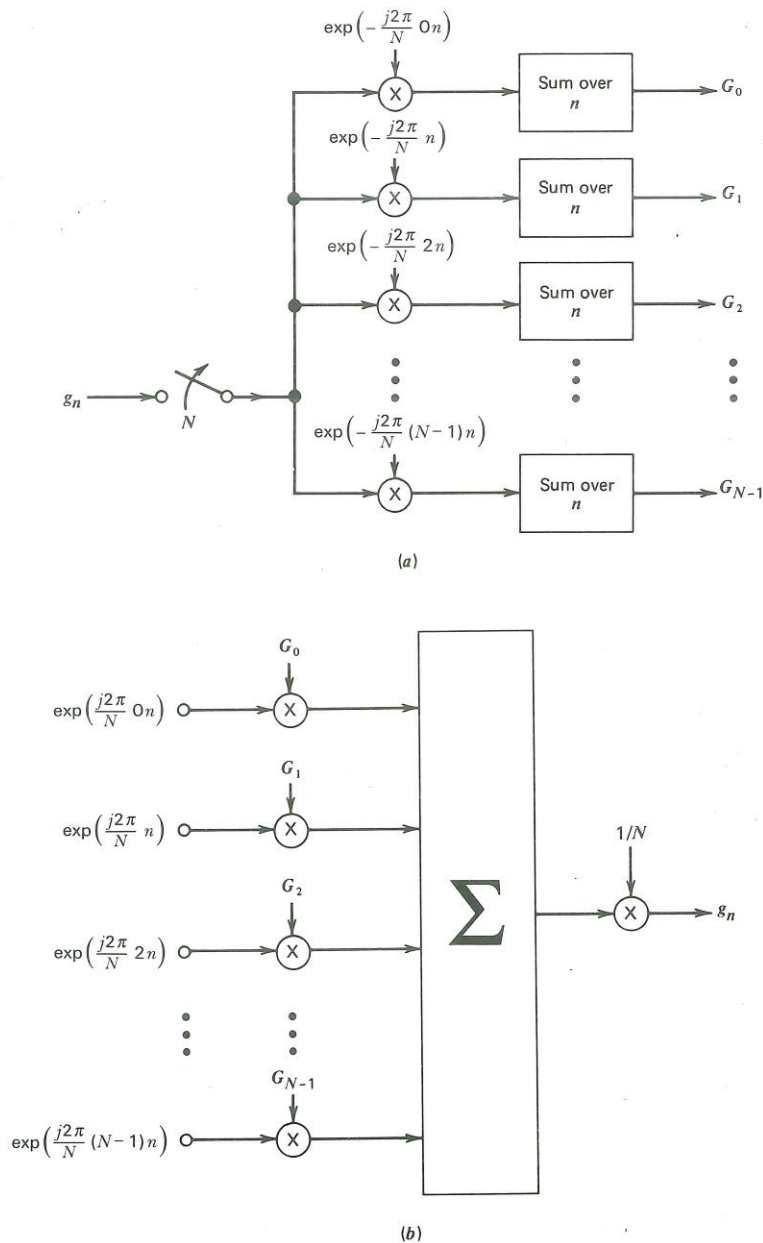


Figure 2.37 Interpretation of (a) the DFT and (b) the IDFT.

the processors shown in Figs. 2.37a and 2.37b must be linear, suggesting that the DFT and IDFT are both linear operations. This important property is also obvious from the defining equations (2.176) and (2.177).

Fast Fourier Transform Algorithms

In the discrete Fourier transform (DFT) both the input and the output consist of sequences of numbers defined at uniformly spaced points in time and fre-

quency, respectively. This feature makes the DFT ideally suited for direct numerical evaluation on a digital computer. Moreover, the computation can be implemented most efficiently using a class of algorithms, called *fast Fourier transform (FFT) algorithms*. An algorithm refers to a "recipe" that can be written in the form of a computer program.

FFT algorithms are efficient because they use a greatly reduced number of arithmetic operations as compared to the brute force computation of the DFT. Basically, an FFT algorithm attains its computational efficiency by following a "divide and conquer" strategy, whereby the original DFT computation is decomposed successively into smaller DFT computations. In this section, we describe one version of a popular FFT algorithm, the development of which is based on such a strategy.

To proceed with the development, we first rewrite Eq. (2.176), defining the DFT of g_n in the convenient form

$$G_k = \sum_{n=0}^{N-1} g_n W^{nk} \quad k = 0, 1, \dots, N-1 \quad (2.179)$$

where

$$W = \exp\left(-\frac{j2\pi}{N}\right) \quad (2.180)$$

We readily see that

$$\begin{aligned} W^N &= 1 \\ W^{N/2} &= -1 \\ W^{(k+lN)(n+mN)} &= W^{kn} \quad m, l = 0, \pm 1, \pm 2, \dots \end{aligned}$$

That is, W^{kn} is periodic with period N . The periodicity of W^{kn} is a key feature in the development of FFT algorithms.

Let N , the number of points in the data sequence, be an integer power of two, as shown by

$$N = 2^L$$

where L is an integer. Since N is an even integer, $N/2$ is an integer, and so we may divide the data sequence into the first half and the last half of the points. Thus, we may rewrite Eq. (2.179) as

$$\begin{aligned} G_k &= \sum_{n=0}^{(N/2)-1} g_n W^{kn} + \sum_{n=N/2}^{N-1} g_n W^{kn} \\ &= \sum_{n=0}^{(N/2)-1} g_n W^{kn} + \sum_{n=0}^{(N/2)-1} g_{n+N/2} W^{k(n+N/2)} \\ &= \sum_{n=0}^{(N/2)-1} (g_n + g_{n+N/2} W^{kN/2}) W^{kn} \quad k = 0, 1, \dots, N-1 \end{aligned} \quad (2.181)$$

Since $W^{N/2} = -1$, we have

$$W^{kN/2} = (-1)^k$$

Accordingly, the factor $W^{kN/2}$ in Eq. (2.181) takes on only one of two possible values, $+1$ or -1 , depending on whether the frequency index k is even or odd, respectively. These two cases are considered in what follows.

First, let k be *even*, so that $W^{kN/2} = 1$. Also let

$$k = 2l, \quad l = 0, 1, \dots, \frac{N}{2} - 1$$

and let

$$x_n = g_n + g_{n+N/2} \quad (2.182)$$

Then, we may put Eq. (2.181) into the form

$$\begin{aligned} G_{2l} &= \sum_{n=0}^{(N/2)-1} x_n W^{2ln} \\ &= \sum_{n=0}^{(N/2)-1} x_n (W^2)^{ln} \quad l = 0, 1, \dots, \frac{N}{2} - 1 \end{aligned} \quad (2.183)$$

From the definition of W given in Eq. (2.180), we readily see that

$$\begin{aligned} W^2 &= \exp\left(-\frac{j4\pi}{N}\right) \\ &= \exp\left(-\frac{j2\pi}{N/2}\right) \end{aligned}$$

Hence, we recognize the sum on the right-hand side of Eq. (2.183) as the $(N/2)$ -point DFT of the sequence x_n .

Next, let k be *odd*, so that $W^{kN/2} = -1$. Also, let

$$k = 2l + 1, \quad l = 0, 1, \dots, \frac{N}{2} - 1$$

and let

$$y_n = g_n - g_{n+N/2} \quad (2.184)$$

Then, we may put Eq. (2.181) into the corresponding form

$$\begin{aligned} G_{2l+1} &= \sum_{n=0}^{(N/2)-1} y_n W^{(2l+1)n} \\ &= \sum_{n=0}^{(N/2)-1} [y_n W^n] (W^2)^{ln} \quad l = 0, 1, \dots, \frac{N}{2} - 1 \end{aligned} \quad (2.185)$$

We recognize the sum on the right-hand side of Eq. (2.185) as the $(N/2)$ -point DFT of the sequence $y_n W^n$. The parameter W^n associated with y_n is called a *twiddle factor*.

Equations (2.183) and (2.185) show that the even- and odd-valued samples of the transform sequence G_k can be obtained from the $(N/2)$ -point DFTs of the sequences x_n and $y_n W^n$, respectively. The sequences x_n and y_n are themselves related to the original data sequence g_n by Eqs. (2.182) and (2.184), respectively. Thus, the problem of computing an N -point DFT is reduced to that of computing two $(N/2)$ -point DFTs. The procedure is repeated a second time, whereby an $(N/2)$ -point is decomposed into two $(N/4)$ -point DFTs. The decomposition procedure is continued in this fashion until (after $L = \log_2 N$ stages) we reach the trivial case of N single-point DFTs.

Figure 2.38 illustrates the computations involved in applying the formulas of Eqs. (2.183) and (2.185) to an 8-point data sequence; that is, $N = 8$. In constructing the left-hand portions of the figure, we have used signal-flow graph notation. A *signal-flow graph* consists of an interconnection of *nodes* and *branches*. The *direction* of signal transmission along a branch is indicated by an arrow. A branch multiplies the variable at a node (to which it is connected) by the branch *transmittance*. A node sums the outputs of all incoming branches. The convention used for branch transmittances in Fig. 2.38 is as follows. When no coefficient is indicated on a branch, the transmittance of that branch is assumed to be unity. For other branches, the transmittance of a branch is indicated by -1 or an integer power of W , placed alongside the arrow on the branch.

Thus, in Fig. 2.38a the computation of an 8-point DFT is reduced to that of two 4-point DFTs. The procedure for the 8-point DFT may be mimicked to simplify the computation of the 4-point DFT. This is illustrated in Fig. 2.38b, where the computation of a 4-point DFT is reduced to that of two 2-point DFTs. Finally, the computation of a 2-point DFT is shown in Fig. 2.38c.

Combining the ideas described in Fig. 2.38, we obtain the complete signal-flow graph of Fig. 2.39 for the computation of the 8-point DFT. A repetitive structure, called a *butterfly*, can be discerned in the FFT algorithm of Fig. 2.39; a butterfly has two inputs and two outputs. Examples of butterflies (for the three stages of the algorithm) are shown by the bold-faced lines in Fig. 2.39.

For the general case of $N = 2^L$, the algorithm requires $L = \log_2 N$ stages of computation. Each stage requires $(N/2)$ butterflies. Each butterfly involves one complex multiplication and two complex additions (to be precise, one addition and one subtraction). Accordingly, the FFT structure described here requires $(N/2)\log_2 N$ complex multiplications and $N\log_2 N$ complex additions. (Actually, the number of multiplications quoted is pessimistic, because we may omit all twiddle factors $W^0 = 1$ and $W^{N/2} = -1$, $W^{N/4} = j$, $W^{3N/4} = -j$.) This computational complexity is significantly smaller than that of the N^2 complex multiplications and $N(N-1)$ complex additions required for the *direct* computation of the DFT. The computational savings made possible by the FFT algorithm become more substantial as we increase the data length N .

We may establish two other important features of the FFT algorithm by carefully examining the signal-flow graph shown in Fig. 2.39:

1. At each stage of the computation, the new set of N complex numbers resulting from the computation can be stored in the same memory locations used to store the previous set. This kind of computation is referred to as *in-place computation*.

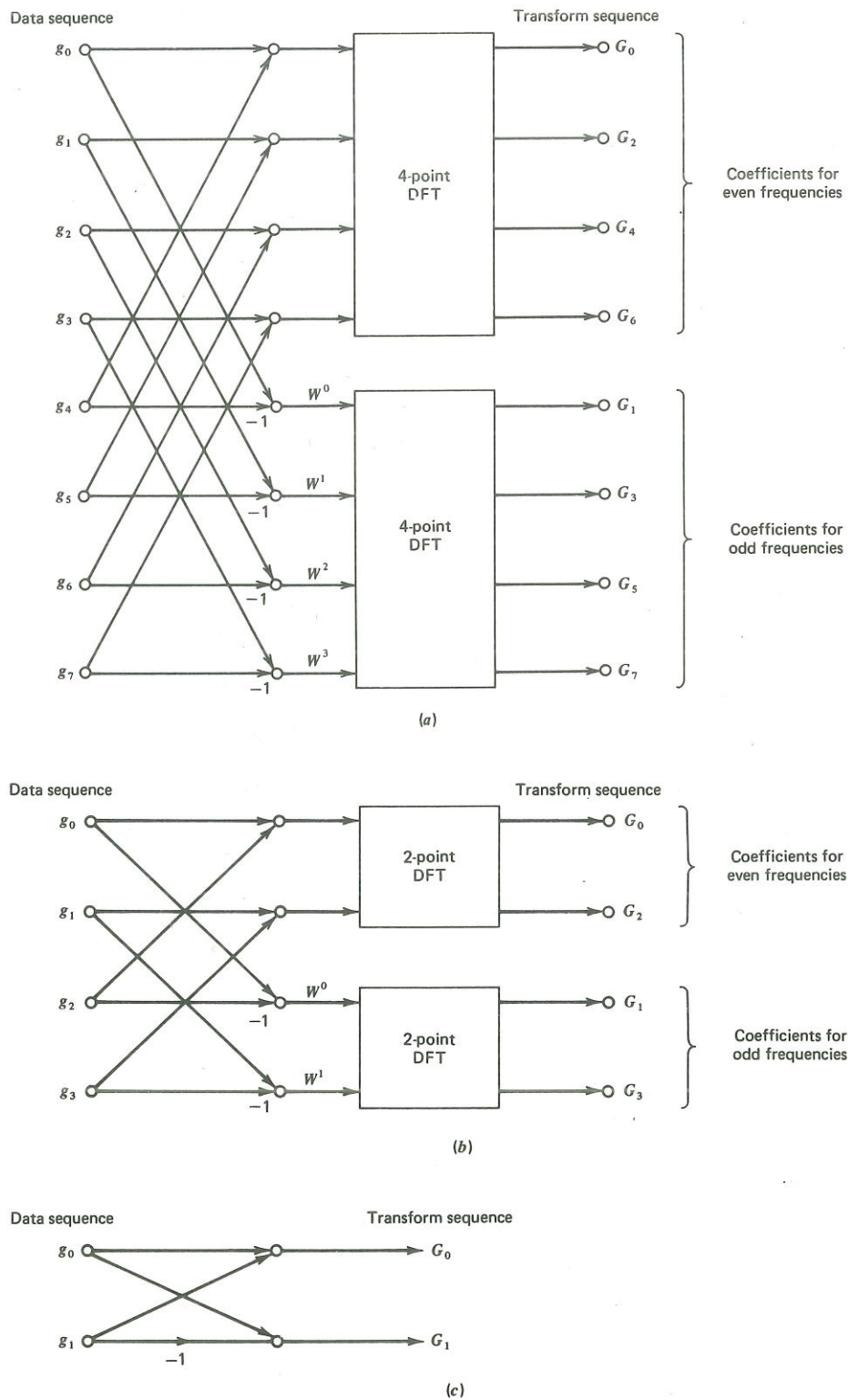


Figure 2.38 (a) Reduction of 8-point DFT into two 4-point DFTs. (b) Reduction of 4-point DFT into two 2-point DFTs. (c) Trivial case of 2-point DFT.

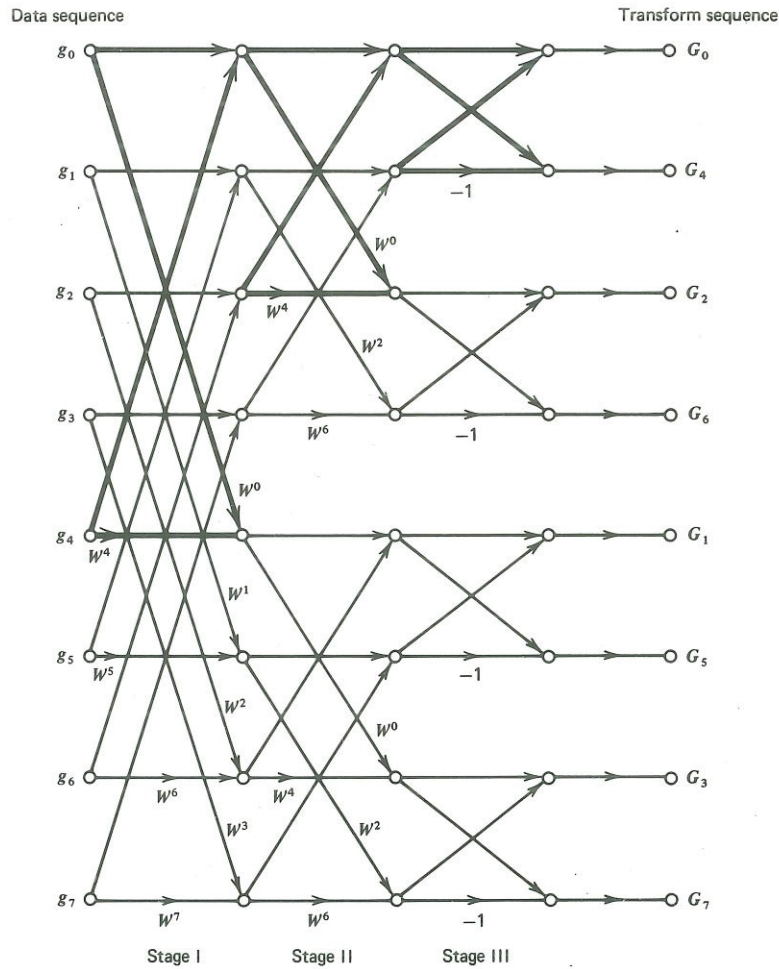


Figure 2.39 Decimation-in-frequency FFT algorithm.

2. The samples of the transform sequence G_k are stored in a *bit-reversed order*. To illustrate the meaning of this terminology, consider Table 2.2 constructed for the case of $N = 8$. At the left of the table, we show the eight possible values of the frequency index k (in their natural order) and their 3-bit binary representations. At the right of the table, we show the corresponding bit-reversed binary representations and indices. We observe that the bit-reversed indices in the right-most column of Table 2.2 appear in the same order as the indices at the output of the FFT algorithm in Fig. 2.39.

The FFT algorithm depicted in Fig. 2.39 is referred to as a *decimation-in-frequency algorithm*, because the transform (frequency) sequence G_k is divided successively into smaller subsequences. In another popular FFT algorithm, called a *decimation-in-time algorithm*, the data (time) sequence g_n is divided successively into smaller subsequences. Both algorithms have the same computational complexity. They differ from each other in two respects. First, for decimation-in-frequency, the input is in natural order, whereas the output is in bit-reversed

Table 2.2 Illustrating Bit Reversal

Frequency Index, k	Binary Representation	Bit-Reversed Binary Representation	Bit-Reversed Index
0	000	000	0
1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7

order. The reverse is true for decimation-in-time. Second, the butterfly for decimation-in-time is slightly different from that for decimation-in-frequency. The reader is invited to derive the details of the decimation-in-time algorithm using the divide-and-conquer strategy that led to the development of the algorithm described in Fig. 2.39.

Computation of the IDFT

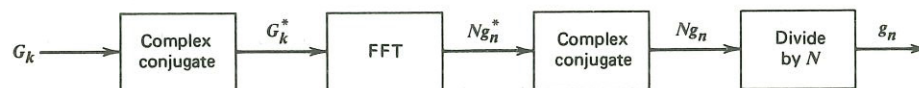
The IDFT of the transform G_k is defined by Eq. (2.177). We may rewrite this equation in terms of the complex parameter W as

$$g_n = \frac{1}{N} \sum_{k=0}^{N-1} G_k W^{-kn} \quad n = 0, 1, \dots, N-1 \quad (2.186)$$

Taking the complex conjugate of Eq. (2.186) and multiplying by N , we get

$$Ng_n^* = \sum_{k=0}^{N-1} G_k^* W^{kn} \quad n = 0, 1, \dots, N-1 \quad (2.187)$$

The right-hand side of Eq. (2.187) is recognized as the N -point DFT of the complex-conjugated sequence G_k^* . Accordingly, Eq. (2.187) suggests that we may compute the desired sequence g_n using the scheme shown in Fig. 2.40, based on an N -point FFT algorithm. Thus, the same FFT algorithm can be used to handle the computation of both the IDFT and the DFT.

**Figure 2.40** Use of the FFT algorithm for computing the IDFT.

2.16 SUMMARY

In this chapter we have described the Fourier transform as a fundamental tool for relating the time-domain and frequency-domain descriptions of a deterministic signal. The signal of interest may be an energy signal or a power signal. The Fourier transform includes the exponential Fourier series as a special case, provided that we permit the use of the Dirac delta function.

An inverse relationship exists between the time-domain and frequency-domain descriptions of a signal. Whenever an operation is performed on the waveform of a signal in the time domain, a corresponding modification is applied to the spectrum of the signal in the frequency domain. An important consequence of this inverse relationship is the fact that the time-bandwidth product of an energy signal is a constant; the definitions of signal duration and bandwidth merely affect the value of the constant.

An important signal processing operation frequently encountered in communication systems is that of linear filtering. This operation involves the convolution of the input signal with the impulse response of the filter or, equivalently, the multiplication of the Fourier transform of the input signal by the transfer function (i.e., Fourier transform of the impulse response) of the filter. Low-pass and band-pass filters represent two commonly used types of filters. Band-pass filtering is usually more complicated than low-pass filtering. However, through the combined use of a complex envelope for the representation of an input band-pass signal and the complex impulse response for the representation of a band-pass filter, we may formulate a low-pass equivalent for the band-pass filtering problem and thereby replace a difficult problem with a much simpler one. It is also important to note that there is no loss of information in establishing this equivalence. The concepts of complex envelope and complex impulse response are rooted in Hilbert transformation.

The final part of the chapter was concerned with the discrete Fourier transform and its numerical computation. Basically, the discrete Fourier transform is obtained from the standard Fourier transform by uniformly sampling both the input and the output. The fast Fourier transform algorithm provides a practical means for the efficient implementation of the discrete Fourier transform on a digital computer. This makes the fast Fourier transform algorithm a powerful computational tool for spectral analysis and linear filtering.

NOTES AND REFERENCES

1. The origin of the theory of Fourier series and Fourier transform is to be found in J. B. J. Fourier, *The Analytical Theory of Heat*, translated from French by A. Freeman (Cambridge University Press, 1878). The books by Bracewell (1986) and Champeney (1973) provide detailed treatments of the Fourier transform with emphasis on the physical aspects of the subject.
2. For an extensive list of Fourier-transform pairs, see the book by Campbell and Foster (1948).
3. If a time function $g(t)$ is such that the value of the energy $\int_{-\infty}^{\infty} |g(t)|^2 dt$ is defined and finite, then the Fourier transform $G(f)$ of the function $g(t)$ exists and

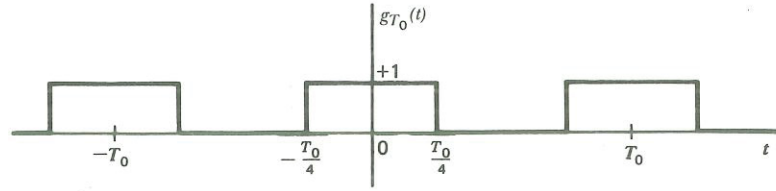


Figure 3.6 Periodic pulse train.

The unwanted terms are removed from the load voltage $v_2(t)$ by means of a band-pass filter with mid-band frequency f_c and bandwidth $2W$, provided that $f_c > 2W$. This latter condition ensures that the frequency separations between the desired AM wave and the unwanted components are large enough for the band-pass filter to suppress the unwanted components.

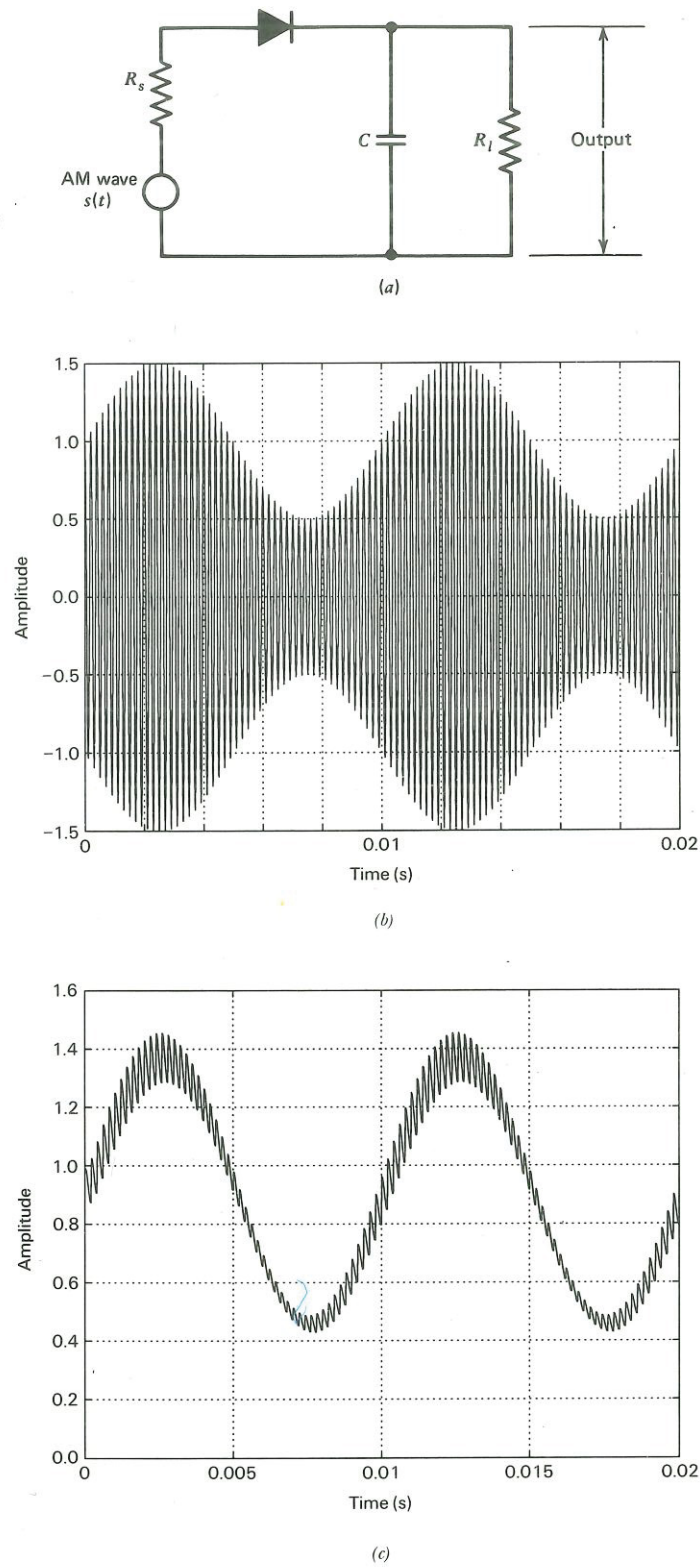
Envelope Detector

As mentioned in the introductory section, the process of *demodulation* is used to recover the original modulating wave from the incoming modulated wave; in effect, demodulation is the reverse of the modulation process. As with modulation, the demodulation of an AM wave can be accomplished using various devices; here, we describe a simple and yet highly effective device known as the *envelope detector*. Some version of this demodulator is used in almost all commercial AM radio receivers. For it to function properly, however, the AM wave has to be narrow-band, which requires that the carrier frequency be large compared to the message bandwidth. Moreover, the percentage modulation must be less than 100 percent.

An envelope detector of the series type is shown in Fig. 3.7a, which consists of a diode and a resistor–capacitor (RC) filter. The operation of this envelope detector is as follows. On a positive half-cycle of the input signal, the diode is forward-biased and the capacitor C charges up rapidly to the peak value of the input signal. When the input signal falls below this value, the diode becomes reverse-biased and the capacitor C discharges slowly through the load resistor R_L . The discharging process continues until the next positive half-cycle. When the input signal becomes greater than the voltage across the capacitor, the diode conducts again and the process is repeated. We assume that the diode is ideal, presenting resistance r_f to current flow in the forward-biased region and infinite resistance in the reverse-biased region. We further assume that the AM wave applied to the envelope detector is supplied by a voltage source of internal impedance R_s . The charging time constant $(r_f + R_s)C$ must be short compared with the carrier period $1/f_c$, that is,

$$(r_f + R_s)C \ll \frac{1}{f_c} \quad (3.12)$$

so that the capacitor C charges rapidly and thereby follows the applied voltage up to the positive peak when the diode is conducting. On the other hand, the discharging time constant $R_L C$ must be long enough to ensure that the capacitor discharges slowly through the load resistor R_L between positive peaks of the car-



rier wave, but not so long that the capacitor voltage will not discharge at the maximum rate of change of the modulating wave, that is

$$\frac{1}{f_c} \ll R_t C \ll \frac{1}{W} \quad (3.13)$$

where W is the message bandwidth. The result is that the capacitor voltage or detector output is nearly the same as the envelope of the AM wave, as demonstrated next.

COMPUTER EXPERIMENT I Sinusoidal Amplitude Modulation

Consider the sinusoidal AM wave shown in Fig. 3.7*b*, assuming 50 percent modulation. The envelope detector output is shown in Fig. 3.7*c*. This latter waveform is computed assuming that the diode is ideal, having a constant resistance r_f when forward-biased and infinite resistance when reverse-biased. The numerical values used in the computation of Fig. 3.7*c* are as follows:

Source resistance	$R_s = 75 \Omega$
Forward resistance	$r_f = 25 \Omega$
Load resistance	$R_t = 10 \text{ k}\Omega$
Capacitance	$C = 0.01 \mu\text{F}$
Message bandwidth	$W = 1 \text{ kHz}$
Carrier frequency	$f_c = 20 \text{ kHz}$

Figure 3.7*c* shows that the envelope detector output contains a small ripple at the carrier frequency; this ripple is easily removed by low-pass filtering.

3.3 VIRTUES, LIMITATIONS, AND MODIFICATIONS OF AMPLITUDE MODULATION

Amplitude modulation is the oldest method of performing modulation. Its biggest virtue is the ease with which it is generated and reversed. Modulation is accomplished rather simply in the transmitter using a switching modulator (described earlier) or a square-law modulator (described in Problem 3.4). Demodulation is accomplished just as easily in the receiver using an envelope detector (described earlier) or a square-law detector (described in Problem 3.6). The net result is that an amplitude modulating system is relatively cheap to build, which is the reason that AM radio broadcasting has been popular for so long and is quite likely to remain so well into the future.

However, from Chapter 1 we recall that transmitted power and channel bandwidth are our two primary communication resources and they should be used efficiently. In this context, we find that the standard form of amplitude modulation defined in Eq. (3.2) suffers from two major limitations:

1. *Amplitude modulation is wasteful of power.* The carrier wave $c(t)$ is completely independent of the information-bearing signal or baseband signal $m(t)$. The transmission of the carrier wave therefore represents a waste of power, which means that in amplitude modulation only a fraction of the total transmitted power is actually affected by $m(t)$.

formance. This is followed by three sections on noise in AM receivers, namely, double sideband-suppressed carrier, single sideband, and standard amplitude modulation receivers. Next, we discuss noise in FM receivers, the analysis of which is a more difficult task. The chapter concludes with a comparative evaluation of the noise performance of AM and FM systems.

5.2 RECEIVER MODEL

The idea of *modeling* is fundamental to the study of all physical systems, including communication systems. Through modeling, we improve our understanding of the capabilities and limitations of a system. In formulating a receiver model for the study of noise in CW modulation systems, we need to keep the following points in mind:

- The model provides an adequate description of the form of receiver noise that is of common concern.
- The model accounts for the inherent filtering and modulation characteristics of the system.
- The model is simple enough for a statistical analysis of the system to be possible.

For the situation at hand, we propose to use the *receiver model* of Fig. 5.1, shown in its most basic form. In this figure, $s(t)$ denotes the incoming *modulated signal* and $w(t)$ denotes *front-end receiver noise*. The *received signal* is therefore made up of the sum of $s(t)$ and $w(t)$; this is the signal that the receiver has to work on. The *band-pass filter* in the model of Fig. 5.1 represents the combined filtering action of the tuned amplifiers used in the actual receiver for the purpose of signal amplification prior to demodulation. The bandwidth of this band-pass filter is just wide enough to pass the modulated signal $s(t)$ without distortion. As for the *demodulator* in the model of Fig. 5.1, its details naturally depend on the type of modulation used.

In performing the noise analysis of a communication system, the customary practice is to assume that the noise $w(t)$ is *additive*, *white*, and *Gaussian*. We thus let the power spectral density of the noise $w(t)$ be denoted by $N_0/2$, defined for both positive and negative frequencies; that is, N_0 is the *average noise power per unit bandwidth measured at the front end of the receiver*. We also assume that the band-pass filter in the receiver model of Fig. 5.1 is ideal, having a bandwidth equal to the transmission bandwidth B_T of the modulated signal $s(t)$ and a mid-band frequency equal to the carrier frequency f_c . The latter assumption is justified for double sideband-suppressed carrier (DSB-SC) modulation, full amplitude modulation (AM), and frequency modulation (FM); the cases of single sideband

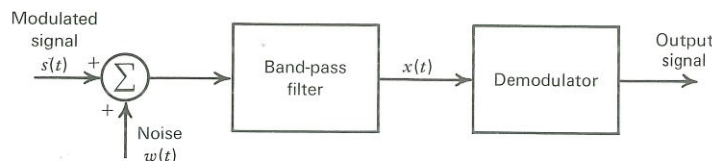


Figure 5.1 Noisy receiver model.

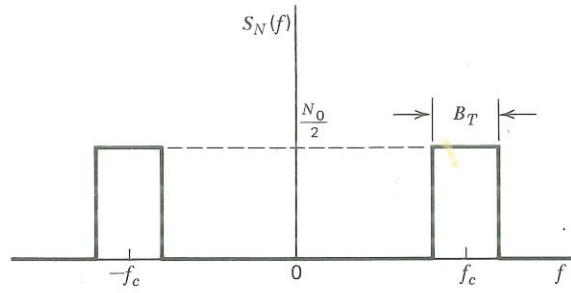


Figure 5.2 Idealized characteristic of band-pass filtered noise.

(SSB) modulation and vestigial sideband (VSB) modulation require special considerations. Taking the mid-band frequency of the band-pass filter to be the same as the carrier frequency f_c , we may model the power spectral density $S_N(f)$ of the noise $n(t)$, resulting from the passage of the white noise $w(t)$ through the filter, as shown in Fig. 5.2. Typically, the carrier frequency f_c is large compared to the transmission bandwidth B_T . We may therefore treat the *filtered noise* $n(t)$ as a narrow-band noise represented in the canonical form

$$n(t) = n_I(t) \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t) \quad (5.1)$$

where $n_I(t)$ is the *in-phase noise component* and $n_Q(t)$ is the *quadrature noise component*, both measured with respect to the carrier wave $A_c \cos(2\pi f_c t)$. The filtered signal $x(t)$ available for demodulation is defined by

$$x(t) = s(t) + n(t) \quad (5.2)$$

The details of $s(t)$ depend on the type of modulation used. In any event, the average noise power at the demodulator input is equal to the total area under the curve of the power spectral density $S_N(f)$. From Fig. 5.2 we readily see that this average noise power is equal to $N_0 B_T$. Given the format of $s(t)$, we may also determine the average signal power at the demodulator input. With the modulated signal $s(t)$ and the filtered noise $n(t)$ appearing additively at the demodulator input in accordance with Eq. (5.2), we may go on to define an *input signal-to-noise ratio*, $(\text{SNR})_I$, as the ratio of the average power of the modulated signal $s(t)$ to the average power of the filtered noise $n(t)$.

A more useful measure of noise performance, however, is the *output signal-to-noise ratio*, $(\text{SNR})_O$, defined as the ratio of the average power of the demodulated message signal to the average power of the noise, both measured at the receiver output. The output signal-to-noise ratio provides an intuitive measure for describing the fidelity with which the demodulation process in the receiver recovers the message signal from the modulated signal in the presence of additive noise. For such a criterion to be well defined, the recovered message signal and the corruptive noise component must appear *additively* at the demodulator output. This condition is perfectly valid in the case of a receiver using coherent detection. On the other hand, when the receiver uses envelope detection as in full AM or frequency discrimination as in FM, we have to assume that the average power of the filtered noise

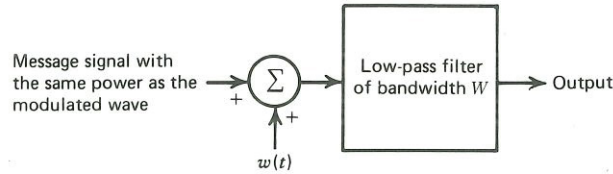


Figure 5.3 The baseband transmission model, assuming a message signal of bandwidth W , used for calculating channel signal-to-noise ratio.

$n(t)$ is relatively low to justify the use of output signal-to-noise ratio as a measure of receiver performance.

The output signal-to-noise ratio depends, among other factors, on the type of modulation used in the transmitter and the type of demodulation used in the receiver. Thus it is informative to compare the output signal-to-noise ratios for different modulation–demodulation systems. However, for this comparison to be of meaningful value, it must be made on an equal basis as described here:

- The modulated signal $s(t)$ transmitted by each system has the same average power.
- The front-end receiver noise $w(t)$ has the same average power measured in the message bandwidth W .

Accordingly, as a frame of reference we define the *channel signal-to-noise ratio*, $(\text{SNR})_C$, as the ratio of the average power of the modulated signal to the average power of noise in the message bandwidth, both measured at the receiver input. This ratio may be viewed as the signal-to-noise ratio that results from *baseband (direct) transmission* of the message signal $m(t)$ without modulation, as modeled in Fig. 5.3. Here it is assumed that (1) the message power at the low-pass filter input is adjusted to be the same as the average power of the modulated signal, and (2) the low-pass filter passes the message signal and rejects out-of-band noise.

For the purpose of comparing different continuous-wave (CW) modulation systems, we *normalize* the receiver performance by dividing the output signal-to-noise ratio by the channel signal-to-noise ratio. We thus define a *figure of merit* for the receiver as follows:

$$\text{Figure of merit} = \frac{(\text{SNR})_O}{(\text{SNR})_C} \quad (5.3)$$

Clearly, the higher the value of the figure of merit, the better will the noise performance of the receiver be. The figure of merit may equal one, be less than one, or be greater than one, depending on the type of modulation used.

In the next four sections, we use the ideas described herein to perform a noise analysis of (1) DSB-SC receivers using coherent detection, (2) SSB receivers using coherent detection, (3) AM receivers using envelope detection, and (4) FM receivers using frequency discrimination; we also consider related issues that arise under high noise levels. These receivers pertain to typical examples of CW modulation systems, which exhibit different noise behavior.

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