
Roark's Formulas for Stress and Strain

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Seventh Edition

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Loadings: Structural Cross-Sections

The shape of a structural cross-section has a huge effect on its strength and stiffness. As shown, the formulas can be quite complex, but they are easily encoded in a spreadsheet to enable a design engineer to play “what-if” scenarios. This represents a basic starting set used by most design engineers. As a design engineer’s focus and experience grows, other structural sections unique to their application can be added.¹

The spreadsheets *Section_Round_Rectangle.xls*, *Section_angle.xls*, *Section_channel.xls*, and *Section_I.xls* are provided for the reader to experiment with different cross-sections. In general, if torsional stiffness or strength is desired, then a closed section, preferably circular, should be used. If maximum bending strength is to be obtained, then an I-beam or rectangular tube is best. It is a simple exercise for the reader to create spreadsheets for the generic open and closed sections shown. Furthermore, these spreadsheets present just the starting point, and the reader can customize them to include calculations for different bending or torsional boundary conditions.

In creating the spreadsheets for various sections, the formula assume that the inscribed circle diameter D is known, but this is not typically given by the manufacturer of structural shapes. A good geometry refresher is to calculate the diameter of the inscribed circle in the corners of angles, channels, and I-beams. As with any problem, start with a sketch, add known dimensions and boundary conditions, and label the unknowns. As shown in the small figure, this quickly leads to an equation based on the Pythagorean theorem: The diameter is then found using the quadratic formula:

$$\left(d + r - \frac{D}{2}\right)^2 + \left(b + r - \frac{D}{2}\right)^2 = \left(r + \frac{D}{2}\right)^2 \quad \text{let } R = \frac{D}{2}$$

$$R^2 + R(-6r - 2d - 2b) + (b^2 + d^2 + r^2 + 2r(b + d)) = 0$$

1. The formula for the effective torsional moment of inertia were developed from equations given in R. Roark and W. Young *Formulas for Stress and Strain*. Fifth Edition, McGraw-Hill Book Company, NY, pp 290-296. Additional formulas for various bending and torsional beam end constraints can also be found. A slightly more advanced topic, not covered here, but easy to investigate, is the *shear center* of a non-symmetric section. For example, if a channel section is loaded with the force normal to its flange, the strong direction, it will also twist unless the load is applied through the shear center. See page 194 of Roark. No design engineer should be without a Roark!

Finite element analysis is a great tool, but until the structure is designed, you need analytical and practical intuition of how shapes’ geometry affects strength and stiffness. For example, use the spreadsheet *round_rectangle_strength.xls* to conduct a design study: what to use, a round or square tube for a waterjet cutter’s main structural axes?² If we pick a constant size, and vary the wall thickness to maintain constant area (weight):

Tube	Size (mm)	Wall thickness	Area	I _{bending}	I _{torsion}
Round	100	10	2827	2898119	5796238
Square	100	7.65	2826	4044369	6025209
ratio	1.00	1.31	1.00	0.72	0.96

Whereas if we are constrained by size, such as by a maximum bellows size, what is the biggest section that will fit inside a given circular bellows diameter:

Tube	Size (mm)	Wall thickness	Area	I _{bending}	I _{torsion}
Round	100	10	2827	2898119	5796238
Square	70.7	12.05	2827	1689839	2432178
ratio	1.41	0.83	1.00	1.72	2.38

On the other hand, if the wall thickness is made the same and the size of the steel tube selected to once again obtain the same cross sectional area:

Tube	Size (mm)	Wall thickness	Area	I _{bending}	I _{torsion}
Round	100	10	2827	2898119	5796238
Square	80.7	10	2827	2400573	3530185
ratio	1.24	1.00	1.00	1.21	1.64

Is it surprising that in many cases, a round beam is actually better than a square beam? What if an I-beam were used? It all really depends on the functional requirements of the system. Many a design opportunity was missed because of reliance on old or inappropriate generalizations!

2. This question confronted the author when he created the design concept for the OMAX 2652 Abrasive Waterjet Machining Center. See www.omax.com

TABLE A.1 Properties of sections (Continued)

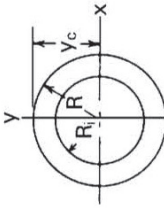
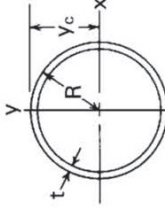
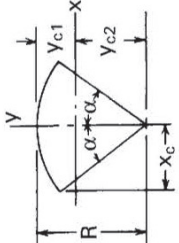
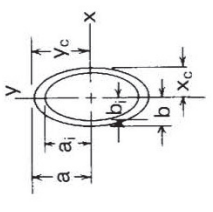
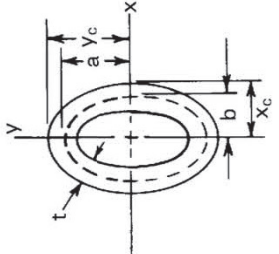
16. Hollow circle		$A = \pi(R^2 - R_i^2)$ $y_c = R$	$I_x = I_y = \frac{\pi}{4}(R^4 - R_i^4)$ $r_x = r_y = \frac{1}{2}\sqrt{R^2 + R_i^2}$	$Z_x = Z_y = 1.333(R^3 - R_i^3)$ $SF_x = 1.698 \frac{R^4 - R_i^4}{R^4 - R_i^2}$
17. Very thin annulus		$A = 2\pi Rt$ $y_c = R$	$I_x = I_y = \pi R^3 t$ $r_x = r_y = 0.707R$	$Z_x = Z_y = 4R^2 t$ $SF_x = SF_y = \frac{4}{\pi}$
18. Sector of solid circle		$A = \alpha R^2$ $y_{c1} = R \left(1 - \frac{2 \sin \alpha}{3\alpha} \right)$ $y_{c2} = \frac{2R \sin \alpha}{3\alpha}$ $x_c = R \sin \alpha$	$I_x = \frac{R^4}{4} \left(\alpha + \sin \alpha \cos \alpha - \frac{16 \sin^2 \alpha}{9\alpha} \right)$ $I_y = \frac{R^4}{4} (\alpha - \sin \alpha \cos \alpha)$ (Note: If α is small, $\alpha - \sin \alpha \cos \alpha = \frac{2}{3}\alpha^3 - \frac{2}{15}\alpha^5$) $r_x = \frac{R}{2} \sqrt{1 + \frac{\sin \alpha \cos \alpha}{\alpha} - \frac{16 \sin^2 \alpha}{9\alpha^2}}$ $r_y = \frac{R}{2} \sqrt{1 - \frac{\sin \alpha \cos \alpha}{\alpha}}$	If $\alpha \leq 54.3^\circ$, then $Z_x = 0.6667R^3 \left[\sin \alpha - \left(\frac{\alpha^3}{2 \tan \alpha} \right)^{1/2} \right]$ Neutral axis x is located a distance $R(0.5\alpha/\tan \alpha)^{1/2}$ from the vertex. If $\alpha \geq 54.3^\circ$, then $Z_x = 0.6667R^3(2 \sin^3 \alpha_1 - \sin \alpha)$ where the expression $2\alpha_1 - \sin 2\alpha_1 = \alpha$ is solved for the value of α_1. Neutral axis x is located a distance $R \cos \alpha_1$ from the vertex. If $\alpha \leq 73.09^\circ$, then $SF_x = \frac{Z_x y_{c2}}{I_x}$ If $73.09^\circ \leq \alpha \leq 90^\circ$, then $SF_x = \frac{Z_x y_{c1}}{I_x}$ $Z_y = 0.6667R^3(1 - \cos \alpha)$ If $\alpha \leq 90^\circ$, then $SF_y = 2.6667 \sin \alpha \frac{1 - \cos \alpha}{\alpha - \sin \alpha \cos \alpha}$ If $\alpha \geq 90^\circ$, then $SF_y = 2.6667 \frac{1 - \cos \alpha}{\alpha - \sin \alpha \cos \alpha}$

TABLE A.1 Properties of sections (Continued)

Form of section	Area and distances from centroid to extremities	Moments and products of inertia and radii of gyration about central axes	Plastic section moduli, shape factors, and locations of plastic neutral axes
25. Hollow ellipse 	$A = \pi(ab - a_i b_i)$ $y_c = a$ $x_c = b$	$I_x = \frac{\pi}{4}(ba^3 - b_i a_i^3)$ $I_y = \frac{\pi}{4}(ab^3 - a_i b_i^3)$ $I_x = \frac{1}{2} \sqrt{\frac{ba^3 - b_i a_i^3}{ab - a_i b_i}}$ $I_y = \frac{1}{2} \sqrt{\frac{ab^3 - a_i b_i^3}{ab - a_i b_i}}$	$Z_x = 1.333(a^2 b - a_i^2 b_i)$ $Z_y = 1.333(b^2 a - b_i^2 a_i)$ $SF_x = 1.698 \frac{a^3 b - a_i^3 b_i}{a^3 b - a_i^3 b_i}$ $SF_y = 1.698 \frac{b^3 a - b_i^3 a_i}{b^3 a - b_i^3 a_i}$
26. Hollow ellipse with constant wall thickness t . The midthickness perimeter is an ellipse (shown dashed). $0.2 < a/b < 5$ 	$A = \pi t \left[1 + K_1 \left(\frac{a-b}{a+b} \right)^2 \right]$ where $K_1 = 0.2464 + 0.002222 \left(\frac{a}{b} + \frac{b}{a} \right)$ $y_c = a + \frac{t}{2}$ $x_c = b + \frac{t}{2}$	$I_x = \frac{\pi}{4} t a^2 (a + 3b) \left[1 + K_2 \left(\frac{a-b}{a+b} \right)^2 \right]$ $I_y = \frac{\pi}{16} t^3 (3a + b) \left[1 + K_3 \left(\frac{a-b}{a+b} \right)^2 \right]$ where $K_2 = 0.1349 + 0.1279 \frac{a}{b} - 0.01284 \left(\frac{a}{b} \right)^2$ $K_3 = 0.1349 + 0.1279 \frac{b}{a} - 0.01284 \left(\frac{b}{a} \right)^2$ For I_y interchange a and b in the expressions for I_x , K_2 , and K_3	$Z_x = 1.333 t a (a + 2b) \left[1 + K_4 \left(\frac{a-b}{a+b} \right)^2 \right] + \frac{t^3}{3}$ where $K_4 = 0.1835 + 0.895 \frac{a}{b} - 0.00978 \left(\frac{a}{b} \right)^2$ For Z_y interchange a and b in the expression for Z_x and K_4 .

Note: For this case the inner and outer perimeters are both ellipses and the wall thickness is not constant. For a cross section with a constant wall thickness see case 26.

See the note on maximum wall thickness in case 27.