

Energy Management in Hybrid Electric Vehicles

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Abstract

A hybrid electric vehicle is propelled with stored energy from a battery or flywheel, plus energy produced by burning fuel in an engine. The cost of the energy consumed, as well as the quantity of air pollutants released, can be reduced by optimizing (1) the ratings of the battery and engine, and (2) the power output that will be delivered by each source under the expected driving conditions. Life-cycle cost can be minimized by running the engine at a constant speed and power, and by avoiding deep discharges of the battery.

An on-board "energy manager," which contains an embedded computer, can track the energy content of the battery and optimize the load division between the battery and engine when given a travel time by the driver over a specified route. It can command the engine to deliver more power whenever the alternative is a life-shortening deep discharge of the battery. The vehicle designer needs to perform a system-engineering analysis to optimize the ratings of the engine and battery. For this he needs to understand the power required to move the vehicle at desired speeds over hills on the expected routes, and through headwinds that vary from day to day.

In this supporting analysis we modeled the hybrid vehicle by using Lagrange's calculus. We applied the model to a battery-powered electric bicycle which travelled a 200-mile route over a variety of hills. An unexpected result from this model is the energy consumed while travelling over a hilly route. With a specified travel time,

no energy could be saved by climbing hills slowly and going down them rapidly. We did not consider the propulsion efficiency in this calculation. The next step will be to incorporate into the model variations in efficiency of the engine and the propulsion motor as speed is varied.

Introduction

In a hybrid electric car the fuel-burning engine can run at a speed and output power which gives the highest efficiency. Transient power for start-ups and hill-climbing can be supplied by a battery. Prototype hybrid vehicles have achieved fuel economies of over 100 miles per gallon.

Modeling is the key route to high efficiency in a hybrid electric vehicle. In aerospace engineering we have developed computational fluid dynamic models of airplanes. A scaled version of the modeled airplane is built and tested in the wind tunnel. The mathematical model is adjusted to make it reproduce the wind-tunnel results. The designer can then input small changes in the engine performance, aerodynamic form, and weight of his airplane. The model will predict the airplane's new performance in terms of fuel consumption and flight time for pertinent flight routes. The old DC-10 airplane delivered 30 passenger miles per gallon of jet fuel. The new 777 airplane, which was configured with computational fluid dynamics, delivers up to 90 passenger miles per gallon.

The hybrid-vehicle variables that can be optimized by modeling are many. Components that can be optimized include

engine rating, motor rating, and battery energy capacity. Variables to be considered in minimizing life-cycle cost include the size and type of battery. Deeply discharging most batteries reduces their lifetime as measured in charge/discharge cycles. Adopting a bigger battery reduces the depth of discharge and lengthens battery life, but at the cost of fuel for hauling the heavier battery over hills.

On-board energy management, which is now possible with small high-capacity embedded computers, is a new tool that can make hybrid electric vehicles even better. The energy manager can perform simple functions, such as informing the driver when the battery's energy content is falling below a designated minimum. By using digital data that is now available the energy manager can perform even more sophisticated tasks. We have digital data bases that contain the elevations of highways at 0.1-mile intervals. After the driver designates the route and destination of a trip and the desired travel time, the energy manager could calculate the energy required in each segment of the travel. For example, there might be a mountain pass on the route. Upon arrival at the beginning of the climb to the pass the energy manager would (1) calculate the required energy, (2) determine the energy that will be generated by the constant-power engine, (3) get the battery's energy from its charge/discharge history, and (4) advise the driver on the maximum speed with which he can drive up to the pass without excessively discharging his battery.

We used a method based on Lagrange's calculusⁱ to show that it is possible to determine, using an on-board computer, the minimum energy required to travel between designated points. The

method solves, by using the calculated travel energy and specified trip time the individual speeds one should travel, with available power, over each road grade. This method determines the minimum energy by setting to zero the derivative of a created function, using the operator's desired travel time.

Formulation of the Minimization Technique

We determine the minimum energy use by first writing the equations of the energy used for propelling a ground vehicle. The four components of energy use are acceleration, friction, air drag, and hill climbing. Energy recoverable from deceleration and hill descent in this study is ignored. The equations for consumed energy (**E**) for air drag, climbing hills and friction areⁱⁱ, respectively:

For air drag

$$E_{\text{drag}} = (C_d DA/2)(V_g + V_a)^2 s_i \quad \text{Eq.1a}$$

for hills

$$E_{\text{hill}} = 9.81 W G s_i \quad \text{Eq.1b}$$

for friction

$$E_{\text{fric}} = 9.81 W C_r s_i \quad \text{Eq.1c}$$

The terms in these equations are: C_d is coefficient of aerodynamic drag, D is air density (gm/m^3), A is frontal area (m^2), V_g is ground speed (m/s), V_a is component of wind speed head-on to vehicle velocity (m/s), s_i is the distance traveled (m), W is the vehicle weight including riders (kg), G is the road surface grade (a number less than 1) C_r is the coefficient of rolling resistance (a number less than 1). These equations give the energy in joules (watt seconds) For

ⁱ Kaplan, W, Advanced Calculus, Addison Wesley, 1952.

ⁱⁱ Morchin, W.C., "Battery-Powered Electric Bicycles", IEEE Technical Applications Conference, NORTHCON 94, Seattle, WA, October 12-14, 1994, Conference Record pp 269-274,

purposes of brevity we let

$$C_1 = (C_d DA/2), C_2 = 9.81WG,$$

$$\text{And } C_3 = 9.81WC_r$$

To illustrate the problem, we consider the minimization of $E = E(t,s)$ where $g(t,s)=0$ and $h(t,s)=0$ are given and where t represents time and s distance. A subscript refers to a particular segment of time and distance.

$$g(t,s) = t_1 + t_2 + t_3 + \dots + t_i - T = 0 \quad \text{Eq.2}$$

$$h(t,s) = s_1 + s_2 + s_3 + \dots + s_i - S = 0 \quad \text{Eq.3}$$

The equations $g=0$ and $h=0$ describe two "surfaces" in space and the problem is thus one of minimizing $E(t,s)$ as (t,s) varies on the curve of intersection of these "surfaces". At a minimum point, the derivative of E along the curve, i.e., the directional derivative along the tangent to the curve, must be 0. This directional derivative is the component of the vector σE along the tangent. It follows that σE must lie in a plane normal to the curve at the point. This plane also contains the vectors σg and σh . Hence (Kaplan section 1-5) there must exist scalars λ_1 and λ_2 such that

$$\sigma E - \lambda_1 \sigma g - \lambda_2 \sigma h = 0 \quad \text{Eq.4}$$

at the critical point. This is equivalent to the two scalar equations:

$$\delta_t E - \lambda_1 \delta_t g - \lambda_2 \delta_t h = 0 \quad \text{Eq.5}$$

$$\delta_s E - \lambda_1 \delta_s g - \lambda_2 \delta_s h = 0 \quad \text{Eq.6}$$

where δ_t and δ_s are the partial derivatives with respect to t and s . These two equations, together with the equations $g(t,s) = 0$ and $h(t,s) = 0$ serve as four equations in the four unknowns: t, s, λ_1 , and λ_2 .

Using the previously defined terms for the energy of traveling up a grade, overcoming air drag and friction we get:

$$E_i = C_{1i}s_i + C_{2i}(s_i/t_i + V_{ai})^2 s_i + C_{3i}s_i \quad \text{Eq.7}$$

where the subscript i is used to denote a portion, s_i , of the total traveled distance S that has unique physical characteristics from other portions of the traveled distance.

Working with each i^{th} portion of the traveled distance in equation 5 and performing the derivatives of equation 7 we obtain:

$$-3C_{2i}S_i^3/t_i^3 - 4C_{2i}V_{ai}S_i^2/t_i^2 - \lambda_1 = 0 \quad \text{Eq.8}$$

and repeating the process with equations 6 and 7 we get:

$$C_{1i} + 3C_{2i}S_i^2/t_i^2 + 4C_{2i}V_{ai}S_i/t_i + C_{2i}V_{ai}^2 + C_{3i} - \lambda_2 = 0 \quad \text{Eq.9}$$

To proceed with the solution we will simplify our work by assuming the wind speed denoted by V_{ai} is zero.

Equations (5), (6), (8), and (9) are the basis for determining the unknowns noted above. We note that the terms s_i are predetermined when one has knowledge of the route. However we will proceed for the purpose of arriving at a solution for the other unknowns. Starting with equations (5) and (8), we will solve for $t_2, t_3, \dots, t_i$ in terms of t_1 :

$$-3C_{21}S_1^3/t_1^3 = \lambda_1 = -3C_{22}S_2^3/t_2^3 \quad \text{Eq.10}$$

Taking note of the term C_{2i} , which equals $0.5C_d DA$, we will make a reasonable assumption that the term is constant for all the portions of the traveled distance. The assumption may not hold up well if there is a significant elevation change, such as from 0 to 10,000 feet, throughout the distance

traveled. With this assumption we find that $t_2 = t_1 s_2 / s_1$ and similarly $t_3 = t_1 s_3 / s_1, \dots, t_i = t_1 s_i / s_1$. Substituting these terms into **Equation (2)** we obtain:

$$t_1 + t_1 s_2 / s_1 + t_1 s_3 / s_1 + \dots + t_1 s_i / s_1 = T \quad \text{Eq.11}$$

Simplifying we obtain: $t_1 [s_1 / s_1 + (S - s_1) / s_1] = T$

so that then

$$t_1 = Ts_1 / S, \quad t_2 = Ts_2 / S, \quad t_3 = Ts_3 / S, \quad t_i = Ts_i / S \quad \text{Eq.12}$$

This is the solution we seek. We find that solving **equations (3)** and **(9)** results in the same solution for the t_i terms.

We have applied these results to a limited trip scenario to illustrate the important results of this technique. To construct the scenario we used the distribution of grades determined for each 0.1 mile interval along the Seattle-to-Portland Bicycle Classic 200 mile routeⁱⁱⁱ. The distribution is shown in **Figure 1**. We simplify this to a table of values:

Table 1. Simplified Distribution of Segments of Distance Traveled

Grade, %	Portion of distance, %
0	86
2	7
3	4
4	3

We analytically determined the energy use, assuming 100 percent

ⁱⁱⁱ Morchin, W.C. "Trip Modeling for Electric-Powered Bicycles", IEEE Technical Applications Conference, Northcon95, Seattle, WA, November 4-6, 1996, Conference Record pp 373-377

efficiency, of an electric-powered bicycle for a certain distance that was proportioned into segments according to Table 1. The analysis used a specified travel time to complete the specified route. The travel time was allocated to the travel segments according to the time algorithm of **Equation (12)**. The outcome of the analysis showed that minimum energy is used for a user specified total travel time if a constant speed is applied no matter if going up various road grades or not. We illustrate this with **Figure 2**. The figure shows a certain percent change in energy use for preset changes in travel speed over the various road grades. We see that for the combination of speeds there was always more energy used than if a constant speed was used. We attempted to achieve less energy use by going slower for increased road grades. It was not possible to find other combinations of speed distributions that could better the constant speed use. We interpret this as meaning one uses more energy to overcome aerodynamic drag on the flat terrain (going faster to make up for lost time for going slower on the higher road grades) than overcoming gravity on the hills. This seems counter-intuitive. However, recall that this is for a 100% efficient system.

When we consider motor losses and presumably other mechanical losses we find that the above energy minimization algorithm of **Equation (12)** does not provide for minimum energy use. It is very difficult to analytically model motor efficiency in a general sense and therefore hard to incorporate into the mathematical models. Motor efficiency depends upon the materials used, motor configurations, and the motor controllers. In addition the efficiency as a function of motor power is highly nonlinear. **Figure 3** shows an example of one such motor. It is the motor that the Zap Bicycle Company uses. We have incorporated this

motor efficiency curve into our computer model for determining minimum energy use.

Figure 4 shows the results of using the motor efficiency characteristics. We found that about 1 ½ percent energy could be saved compared to the previously determined constant speed operation. We found that for this particular situation that reducing ones speed on the 4% grades by about 25%, on the 3% grades by about 11%, and on the 2% grades by about 7% along with increasing ones speed on flat terrain by about 2% lead to the best results.

Conclusion

We believe the results described here can be applied to any type of motor or engine propelled ground vehicle. However, our studies are limited to electric-powered bicycles. Much more energy can be saved for larger vehicles than bicycles. The present study should be extended to include a wide variety of terrain and the effects of system losses. The algorithms need to be developed to include these system losses. The mathematical difficulties introduced by the effects of head winds should be solved. And finally a hardware demonstration program is needed to validate and demonstrate the energy savings that can be made possible.

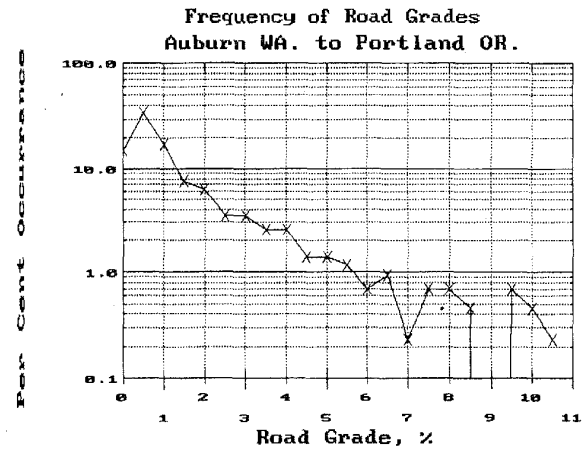
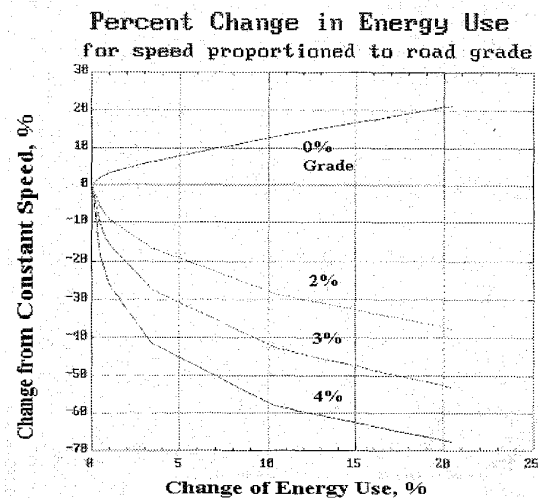


Figure 1. Distribution of frequency of occurrence for road grades on Seattle-to-Portland Bicycle Classic route. 180 mile



distance from Auburn WA to Portland Or.

Figure 2. Illustration of percent change in energy use over hypothetical route with road grades of 0%, 2%, 3%, and 4%. Example conditions are for an electric-powered bicycle without pedal assistance and assuming 100% system efficiency. Weight of bicycle and rider is 142 kg, coefficient of aerodynamic drag is 1, frontal area is 0.42 m², air density is 1.209 kg/m³, and coefficient of rolling resistance is 0.005.

Data shown are normalized to a bicycle ground speed of 10.4 m/s (23.3 mph).

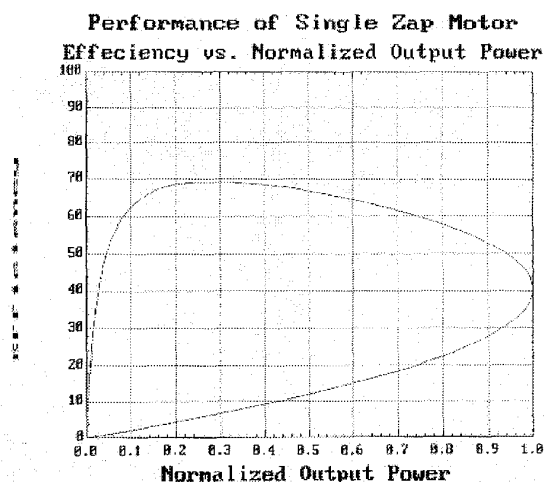


Figure 3. Example of efficiency variation of a permanent magnet field brushed motor with output motor power. Motor power shown is normalized to maximum motor output power. Example motor is that of ZAP Bicycle Co. on an electric-powered bicycle.

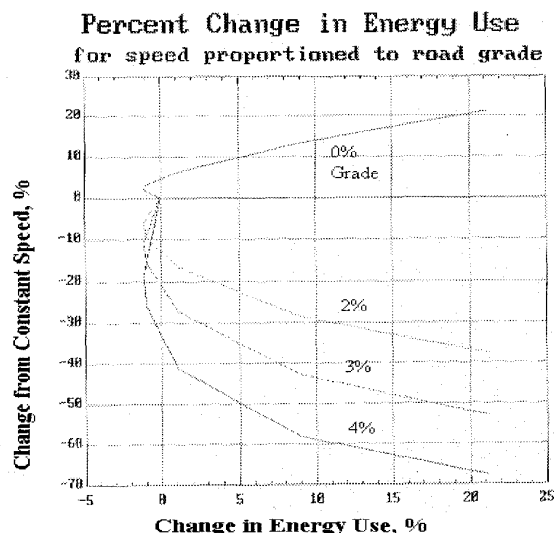


Figure 4. Illustration of energy usage when motor efficiency is used. Curves are calculated for the same conditions as for Figure 2 except motor efficiency is 67.1% for 0% grades, 67.5% for 2% grades, 64% for 3% grades, and 59% for 4% grades. We found that about 1 ½ percent energy could be saved, compared to that shown in Figure 1. We found that for this particular situation that reducing ones speed on the 4% grades by 25%, on the 3% grades by 11%, and on the 2% grades by about 7% along with increasing speed on flat terrain by about 2% lead to the best results.