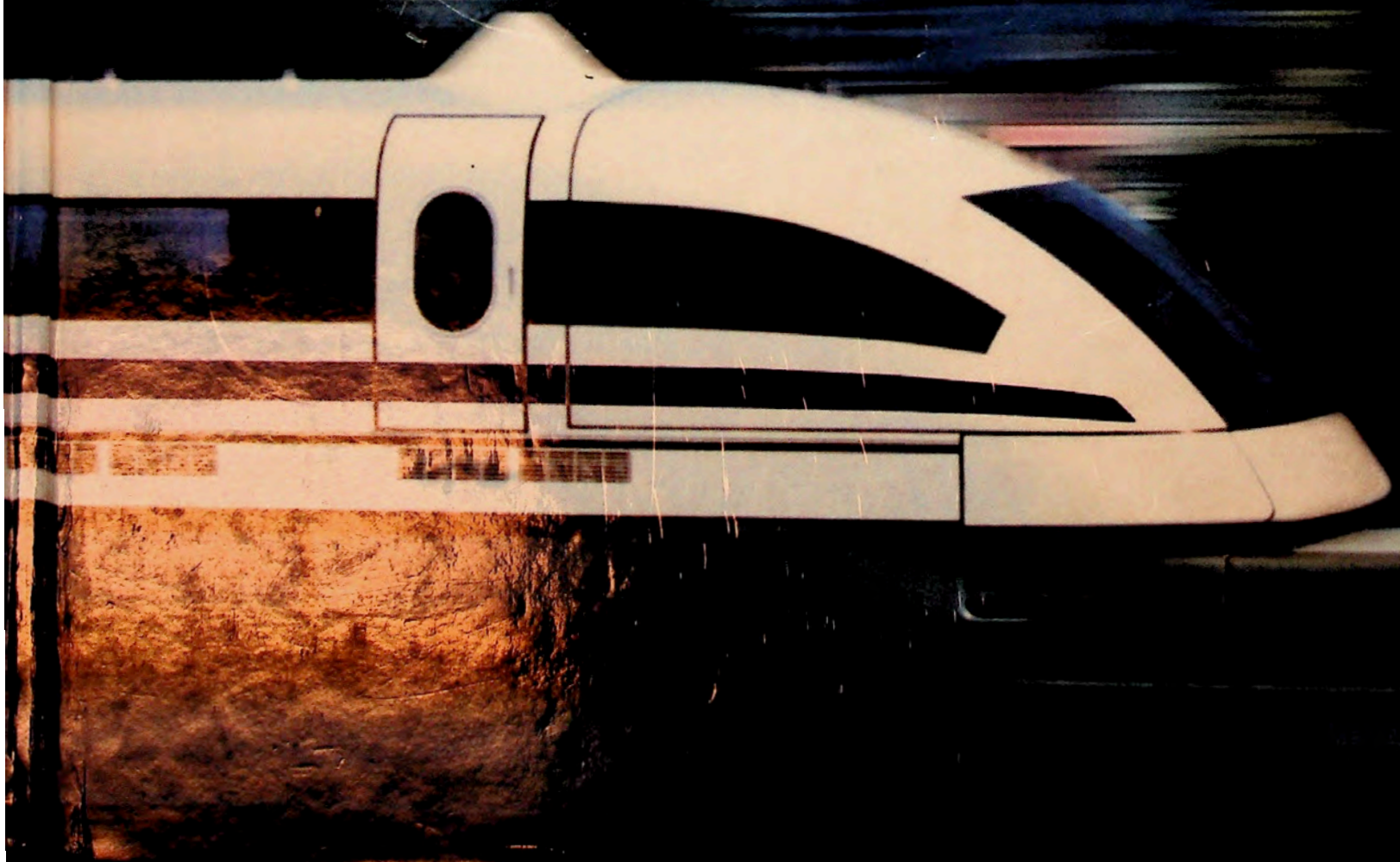


INSTRUCTOR'S EDITION

# PRINCIPLES OF PHYSICS

A CALCULUS-BASED TEXT FOURTH EDITION

SERWAY & JEWETT



# PRINCIPLES OF PHYSICS

A CALCULUS-BASED TEXT **FOURTH EDITION**

**Raymond A. Serway**

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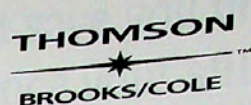
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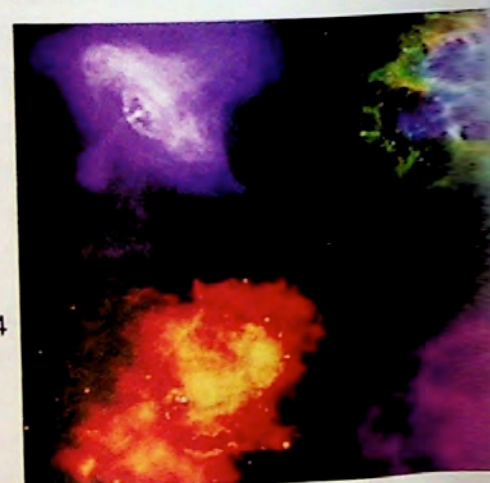
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Mass should not be confused with weight. **Mass and weight are two different quantities.** As we shall see later in this chapter, the weight of an object is equal to the magnitude of the gravitational force exerted on the object and varies with location. For example, a person who weighs 180 lb on the Earth weighs only about 30 lb on the Moon. On the other hand, the mass of an object is the same everywhere. An object having a mass of 2 kg on Earth also has a mass of 2 kg on the Moon.

■ Mass and weight are different quantities

## 4.4 NEWTON'S SECOND LAW—THE PARTICLE UNDER A NET FORCE

Newton's first law explains what happens to an object when no force acts on it: It either remains at rest or moves in a straight line with constant speed. This law allows us to define an inertial frame of reference. It also allows us to identify force as that which changes motion. Newton's second law answers the question of what happens to an object that has a nonzero net force acting on it, based on our discussion of mass in the preceding section.

Imagine you are pushing a block of ice across a frictionless horizontal surface. When you exert some horizontal force  $\vec{F}$ , the block moves with some acceleration  $\vec{a}$ . Experiments show that if you apply a force twice as large to the same object, the acceleration doubles. If you increase the applied force to  $3\vec{F}$ , the original acceleration is tripled, and so on. From such observations, we conclude that **the acceleration of an object is directly proportional to the net force acting on it.** We alluded to this proportionality in our discussion of acceleration in Chapter 2. We are now ready to extend that discussion.

These observations and those in Section 4.3 relating mass and acceleration are summarized in **Newton's second law**:

The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

### PITFALL PREVENTION 4.2

**FORCE IS THE CAUSE OF CHANGES IN MOTION** Be sure that you are clear on the role of force. Many times, students make the mistake of thinking that force is the cause of motion. We can, though, have motion in the absence of forces, as described in Newton's first law. Be sure to understand that force is the cause of *changes* in motion.

■ Newton's second law

### PITFALL PREVENTION 4.3

**$m\vec{a}$  IS NOT A FORCE** Equation 4.2 does *not* say that the product  $m\vec{a}$  is a force. All forces on an object are added vectorially to generate the net force on the left side of the equation. This net force is then equated to the product of the mass of the object and the acceleration that results from the net force. Do *not* include an " $m\vec{a}$  force" in your analysis.

We write this law as

$$\vec{a} \propto \frac{\sum \vec{F}}{m}$$

where  $\sum \vec{F}$  is the **net force**, which is the vector sum of *all* forces acting on the object of mass  $m$ . If the object consists of a system of individual elements, the net force is the vector sum of all forces *external* to the system. Any *internal* forces—that is, forces between elements of the system—are not included because they do not affect the motion of the entire system. The net force is sometimes called the *resultant force*, the *sum of the forces*, the *total force*, or the *unbalanced force*.

Newton's second law in mathematical form is a statement of this relationship that makes the preceding proportionality an equality:<sup>1</sup>

$$\sum \vec{F} = m\vec{a} \quad [4.2]$$

Note that Equation 4.2 is a *vector* expression and hence is equivalent to the following three component equations:

$$\sum F_x = ma_x \quad \sum F_y = ma_y \quad \sum F_z = ma_z \quad [4.3]$$

Newton's second law introduces us to a new analysis model, the particle under a net force. If a particle, or an object that can be modeled as a particle, is under the

■ Mathematical representation of Newton's second law

■ Newton's second law in component form

<sup>1</sup>Equation 4.2 is valid only when the speed of the object is much less than the speed of light. We will treat the relativistic situation in Chapter 9.

influence of a net force, Equation 4.2, the mathematical statement of Newton's second law, can be used to describe its motion. The acceleration is constant if the net force is constant. Therefore, the particle under a constant net force will have its motion described as a particle under constant acceleration. Of course, not all forces are constant, and when they are not, the particle cannot be modeled as one under constant acceleration. We shall investigate situations in this chapter and the next involving both constant and varying forces.

**QUICK QUIZ 4.2** An object experiences no acceleration. Which of the following *cannot* be true for the object? (a) A single force acts on the object. (b) No forces act on the object. (c) Forces act on the object, but the forces cancel.

**QUICK QUIZ 4.3** You push an object, initially at rest, across a frictionless floor with a constant force for a time interval  $\Delta t$ , resulting in a final speed of  $v$  for the object. You repeat the experiment, but with a force that is twice as large. What time interval is now required to reach the same final speed  $v$ ? (a)  $4 \Delta t$  (b)  $2 \Delta t$  (c)  $\Delta t$  (d)  $\Delta t/2$  (e)  $\Delta t/4$

## Unit of Force

The SI unit of force is the **newton**, which is defined as the force that, when acting on a 1-kg mass, produces an acceleration of  $1 \text{ m/s}^2$ .

From this definition and Newton's second law, we see that the newton can be expressed in terms of the fundamental units of mass, length, and time:

■ Definition of the newton

$$1 \text{ N} \equiv 1 \text{ kg} \cdot \text{m/s}^2 \quad [4.4]$$

The units of mass, acceleration, and force are summarized in Table 4.1. Most of the calculations we shall make in our study of mechanics will be in SI units. Equalities between units in the SI and U.S. customary systems are given in Appendix A.

## Thinking Physics 4.1

In a train, the cars are connected by *couplers*. The couplers between the cars exert forces on the cars as the train is pulled by the locomotive in the front. Imagine that the train is speeding up in the forward direction. As you imagine moving from the locomotive to the last car, does the force exerted by the couplers *increase*, *decrease*, or *stay the same*? What if the engineer applies the brakes? How does the force vary from locomotive to last car in this case? (Assume that the only brakes applied are those on the engine.)

**Reasoning** The force *decreases* from the front of the train to the back. The coupler between the locomotive and the first car must apply enough force to accelerate all the remaining cars. As we move back along the train, each coupler is accelerating less mass behind it. The last coupler only has to accelerate the last car, so it exerts the smallest force. If the brakes are applied, the force decreases from front to back of the train also. The first coupler, at the back of the locomotive, must apply a large force to slow down all the remaining cars. The final coupler must only apply a force large enough to slow down the mass of the last car. ■

**TABLE 4.1** Units of Mass, Acceleration, and Force

| System of Units | Mass (M) | Acceleration ( $\text{L/T}^2$ ) | Force ( $\text{ML/T}^2$ )                     |
|-----------------|----------|---------------------------------|-----------------------------------------------|
| SI              | kg       | $\text{m/s}^2$                  | $\text{N} = \text{kg} \cdot \text{m/s}^2$     |
| U.S. customary  | slug     | $\text{ft/s}^2$                 | $\text{lb} = \text{slug} \cdot \text{ft/s}^2$ |

**EXAMPLE 6.2** The Scalar Product

The vectors  $\vec{A}$  and  $\vec{B}$  are given by  $\vec{A} = 2\hat{i} + 3\hat{j}$  and  $\vec{B} = -\hat{i} + 2\hat{j}$ .

**A** Determine the scalar product  $\vec{A} \cdot \vec{B}$ .

**Solution** We can evaluate the scalar product directly using the unit vector notation:

$$\begin{aligned}\vec{A} \cdot \vec{B} &= (2\hat{i} + 3\hat{j}) \cdot (-\hat{i} + 2\hat{j}) \\ &= -2\hat{i} \cdot \hat{i} + 2\hat{i} \cdot 2\hat{j} - 3\hat{j} \cdot \hat{i} + 3\hat{j} \cdot 2\hat{j} \\ &= -2 + 6 = 4\end{aligned}$$

where we have used that  $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = 1$  and  $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{i} = 0$ . The same result is obtained using Equation 6.9 directly, where  $A_x = 2$ ,  $A_y = 3$ ,  $B_x = -1$ ,

and  $B_y = 2$ . Note that the result has no units because no units were specified on the original vectors  $\vec{A}$  and  $\vec{B}$ .

**B** Find the angle  $\theta$  between  $\vec{A}$  and  $\vec{B}$ .

**Solution** The magnitudes of  $\vec{A}$  and  $\vec{B}$  are given by

$$\begin{aligned}A &= \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13} \\ B &= \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}\end{aligned}$$

Using Equation 6.3 and the result from part A gives

$$\begin{aligned}\cos \theta &= \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}} = 0.496 \\ \theta &= \cos^{-1}(0.496) = 60.3^\circ\end{aligned}$$

**6.4 WORK DONE BY A VARYING FORCE**

Consider a particle being displaced along the  $x$  axis under the action of a force with an  $x$  component  $F_x$  that varies with position, as in the graphical representation in Figure 6.7. The particle is displaced in the direction of increasing  $x$  from  $x = x_i$  to  $x = x_f$ . In such a situation, we cannot use Equation 6.1 to calculate the work done by the force because this relationship applies only when  $\vec{F}$  is constant in magnitude and direction. As seen in Figure 6.7, we do not have a *single* value of the force to substitute into Equation 6.1. If, however, we imagine that the point of application of the force undergoes a *small* displacement in the  $x$  direction so that  $\Delta r = \Delta x$ , as shown in Figure 6.7a, the  $x$  component  $F_x$  of the force is approximately constant over this interval. We can then approximate the work done by the force on the particle for this small displacement as

$$W_1 \approx F_x \Delta x \quad [6.10]$$

This quantity is just the area of the shaded geometric model rectangle in Figure 6.7a. If we imagine that the curve described by  $F_x$  versus  $x$  is divided into a large number of such intervals, the total work done for the displacement from  $x_i$  to  $x_f$  is approximately equal to the sum of a large number of such terms:

$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$

If the displacements  $\Delta x$  are allowed to approach zero, the number of terms in the sum increases without limit, but the value of the sum approaches a definite value equal to the area under the curve bounded by  $F_x$  and the  $x$  axis in Figure 6.7b. As you probably have learned in calculus, this limit of the sum is called an *integral* and is represented by

$$\lim_{\Delta x \rightarrow 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$

The limits on the integral  $x = x_i$  to  $x = x_f$  define what is called a **definite integral**. (An *indefinite integral* is the limit of a sum over an unspecified interval. Appendix B.7 gives a brief description of integration.) This definite integral is numerically

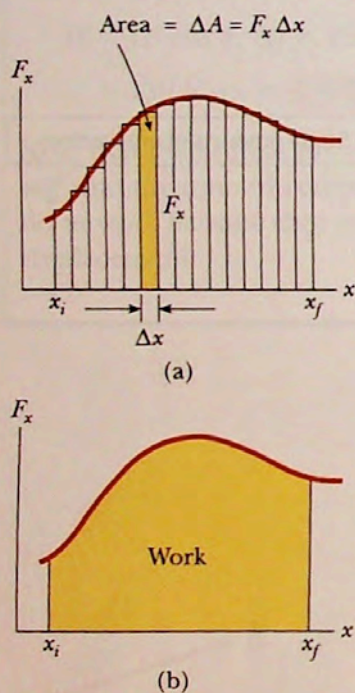


**FIGURE 6.7** (a) The work done

by a force of magnitude  $F_x$  for the small displacement  $\Delta x$  is  $F_x \Delta x$ , which equals the area of the shaded rectangle. The total work done for the displacement from  $x_i$  to  $x_f$  is approximately equal to the sum of the areas of all the rectangles. (b) The work done by the variable force  $F_x$  as the particle moves from  $x_i$  to  $x_f$  is exactly equal to the area under this curve.

equal to the area under the curve of  $F_x$  versus  $x$  between  $x_i$  and  $x_f$ . Therefore, we can express the work done by  $F_x$  for the displacement from  $x_i$  to  $x_f$  as

$$W = \int_{x_i}^{x_f} F_x dx \quad [6.11]$$

This equation reduces to Equation 6.1 when  $F_x = F \cos \theta$  is constant and  $x_f - x_i = \Delta x$ .

If more than one force acts on a system *and the system can be modeled as a particle*, the total work done on the system is just the work done by the net force. If we express the  $x$  component of the net force as  $\Sigma F_x$ , the total work, or *net work*, done on the particle as it moves from  $x_i$  to  $x_f$  is

$$\Sigma W = W_{\text{net}} = \int_{x_i}^{x_f} (\Sigma F_x) dx$$

For the general case of a particle moving along an arbitrary path while acted on by a net force  $\Sigma \vec{F}$ , we use the scalar product:

$$\Sigma W = W_{\text{net}} = \int (\Sigma \vec{F}) \cdot d\vec{r} \quad [6.12]$$

■ Work done by a variable net force

where the integral is calculated over the path that the particle takes through space.

If the system cannot be modeled as a particle (for example, if the system consists of multiple particles that can move with respect to each other), we cannot use Equation 6.12 because different forces on the system may move through different displacements. In that case, we must evaluate the work done by each force separately and then add the works algebraically.

## Work Done by a Spring

A common physical system for which the force varies with position is shown in Active Figure 6.8. A block on a horizontal, frictionless surface is connected to a spring. If the block is located at a position  $x$  relative to its equilibrium position  $x = 0$ , the stretched or compressed spring exerts a force on the block given by

$$F_s = -kx \quad [6.13]$$

■ Hooke's law

where  $k$  is a positive constant called the *force constant* (or *spring constant* or *stiffness constant*) of the spring. This force law for springs is known as **Hooke's law**. For many springs, Hooke's law can describe the behavior very accurately provided that the displacement from equilibrium is not too large. The value of  $k$  is a measure of the stiffness of the spring. Stiff springs have larger  $k$  values, and weak springs have smaller  $k$  values. We shall employ a simplification model in which all springs obey Hooke's law unless specified otherwise.

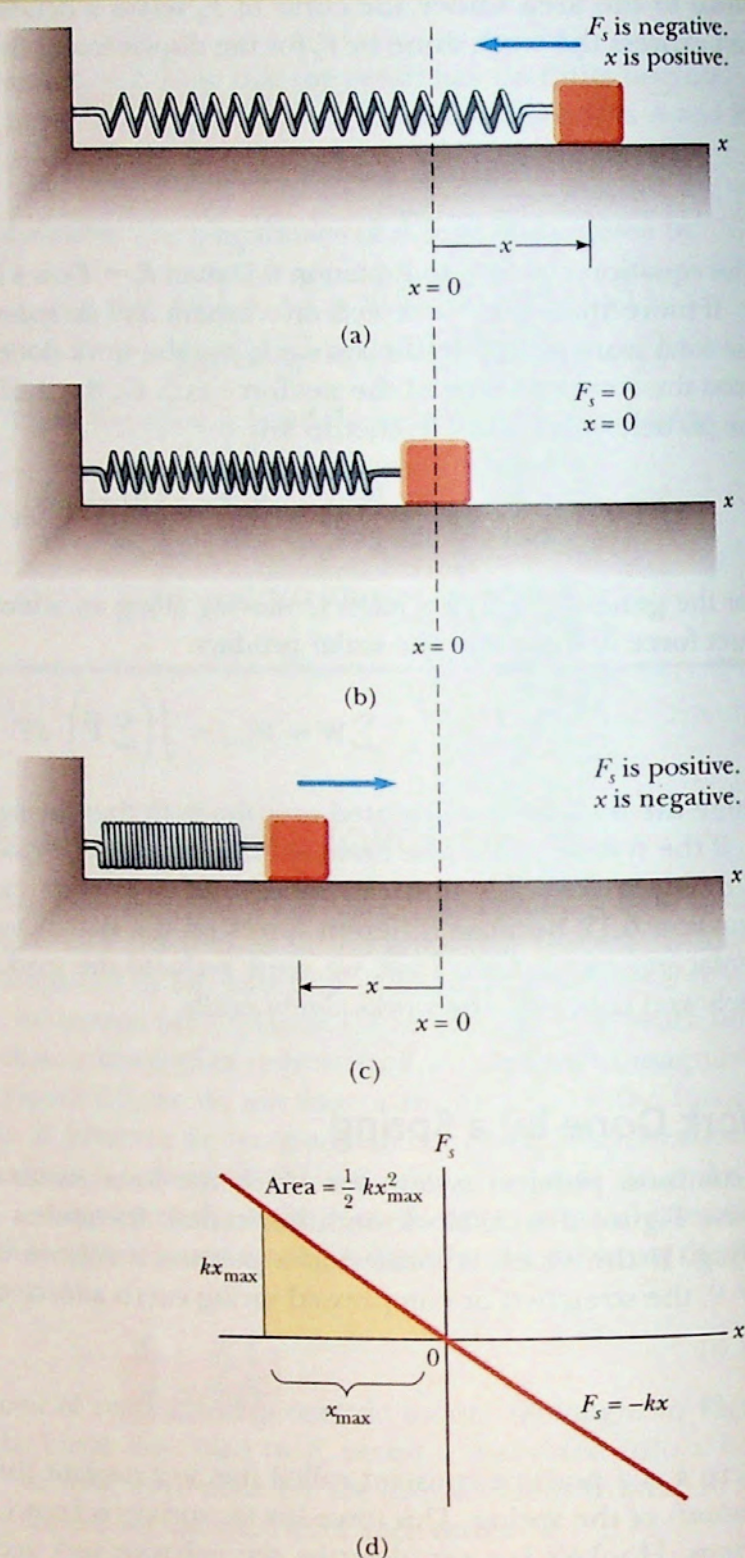
The negative sign in Equation 6.13 signifies that the force exerted by the spring on the block is always directed *opposite* the displacement from the equilibrium position  $x = 0$ . For example, when  $x > 0$ , such that the block is pulled to the right and the spring is stretched as in Active Figure 6.8a, the spring force is to the left, or negative. When  $x < 0$ , and the spring is compressed as in Active Figure 6.8c, the spring force is to the right, or positive. Of course, when  $x = 0$ , as in Active Figure 6.8b, the spring is unstretched and  $F_s = 0$ . Because the spring force always acts toward the equilibrium position, it is sometimes called a *restoring force*.

If the block is displaced to a position  $-x_{\text{max}}$  and then released, it moves from  $-x_{\text{max}}$  through zero to  $+x_{\text{max}}$  (assuming a frictionless surface) and then turns around and returns to  $-x_{\text{max}}$ . The details of this oscillating motion will be

**ACTIVE FIGURE 6.8**

The force exerted by a spring on a block varies with the block's displacement from the equilibrium position  $x = 0$ . (a) When  $x$  is positive (stretched spring), the spring force is to the left. (b) When  $x$  is zero (natural length of the spring), the spring force is zero. (c) When  $x$  is negative (compressed spring), the spring force is to the right. (d) Graph of  $F_s$  versus  $x$  for the block-spring system. The work done by the spring force as the block moves from  $-x_{\max}$  to 0 is the area of the shaded triangle,  $\frac{1}{2}kx_{\max}^2$ .

**PhysicsNow™** Observe the block's motion for various maximum displacements and spring constants by logging into PhysicsNow at [www.pop4e.com](http://www.pop4e.com) and going to Active Figure 6.8.



discussed in Chapter 12. For our purposes here, let us calculate the work done by the spring force on the block as the block moves from  $x_i = -x_{\max}$  to  $x_f = 0$ . Applying the particle model to the block and using Equation 6.11, we have

$$W_s = \int_{x_i}^{x_f} F_s dx = \int_{-x_{\max}}^0 (-kx) dx = \frac{1}{2}kx_{\max}^2 \quad [6.14]$$

The work done by the spring force on the block is positive because the spring force is in the same direction as the displacement (both are to the right).

If we consider the work done by the spring force on the block as the block continues to move from  $x_i = 0$  to  $x_f = x_{\max}$ , we find that  $W_s = -\frac{1}{2}kx_{\max}^2$ . This work is negative because for this part of the motion the displacement is to the right and the spring force is to the left. Therefore, the *net* work done by the spring force on the block as it moves from  $x_i = -x_{\max}$  to  $x_f = x_{\max}$  is zero.

If we plot  $F_s$  versus  $x$ , as in Active Figure 6.8d, we arrive at the same results. The work calculated in Equation 6.14 is equal to the area of the shaded triangle in Active Figure 6.8d, with base  $x_{\max}$  and height  $kx_{\max}$ . This area is  $\frac{1}{2}kx_{\max}^2$ .

If the block undergoes an *arbitrary* displacement from  $x = x_i$  to  $x = x_f$ , the work done by the spring force is

$$W_s = \int_{x_i}^{x_f} (-kx) dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 \quad [6.15]$$

From this equation we see that the work done by the spring force on the block is zero for any motion that ends where it began ( $x_i = x_f$ ). We shall make use of this important result in Chapter 7, where we describe the motion of this system in more detail. Equation 6.15 also shows that the work done by the spring force is zero when the block moves between any two symmetric locations,  $x_i = -x_f$ . Consider the curve representing the spring force in Active Figure 6.8d; if the block moves from  $x = -x_{\max}$  to  $x = +x_{\max}$ , the total work is zero because we are adding a positive area (for  $-x_{\max} < x < 0$ ) to a negative area (for  $0 < x < +x_{\max}$ ) of equal magnitude.

Equations 6.14 and 6.15 describe the work done by the spring force on the block. Now consider the work done by an *external agent* on the block as the agent applies a force to the spring and stretches it *very slowly* from  $x_i = -x_{\max}$  to  $x_f = 0$  as in Figure 6.9. This work can be easily calculated by noting that the applied force  $\vec{F}_{\text{app}}$  is of equal magnitude and opposite direction to the spring force  $\vec{F}_s$  at any value of the position (because the block is not accelerating), so that  $F_{\text{app}} = -(-kx) = +kx$ . The work done by this applied force (the external agent) on the block is therefore

$$W_{F_{\text{app}}} = \int_{-x_{\max}}^0 F_{\text{app}} dx = \int_{-x_{\max}}^0 kx dx = -\frac{1}{2}kx_{\max}^2$$

Note that this work is equal to the negative of the work done by the spring force on the block for this displacement (Eq. 6.14). The work is negative because the external agent must push to the left on the spring in Figure 6.9 to prevent it from expanding, and this direction is opposite the direction of the displacement as the block moves from  $-x_{\max}$  to 0.

#### Work done by a spring

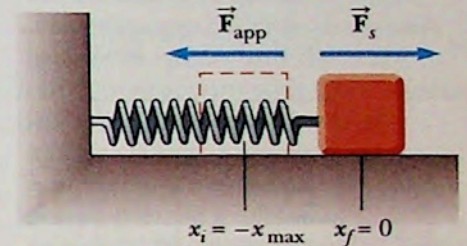


**FIGURE 6.9** A block moves from  $x_i = -x_{\max}$  to  $x_f = 0$  on a frictionless surface as a force  $\vec{F}_{\text{app}}$  is applied to the block. If the process is carried out very slowly, the applied force is equal in magnitude and opposite in direction to the spring force at all times.

**QUICK QUIZ 6.3** A dart is loaded into a spring-loaded toy dart gun by pushing the spring in by a distance  $d$ . For the next loading, the spring is compressed a distance  $2d$ . How much work is required to load the second dart compared to that required to load the first? (a) four times as much (b) two times as much (c) the same (d) half as much (e) one-fourth as much

#### EXAMPLE 6.3 Work Required to Stretch a Spring

One end of a horizontal spring ( $k = 80 \text{ N/m}$ ) is held fixed while an external force is applied to the free end, stretching it slowly from  $x_A = 0$  to  $x_B = 4.0 \text{ cm}$ .

**A** Find the work done by the external force on the spring.

**Solution** Because we have not been told otherwise, we assume that the spring obeys Hooke's law. We place the zero reference of the coordinate axis at the free end of the unstretched spring. The applied force is  $F_{\text{app}} = kx = (80 \text{ N/m})(x)$ . The work done by  $F_{\text{app}}$  is the area of the triangle from 0 to 4.0 cm in Figure 6.10:

$$W = \frac{1}{2}kx_B^2 = \frac{1}{2}(80 \text{ N/m})(0.040 \text{ m})^2 = 0.064 \text{ J}$$

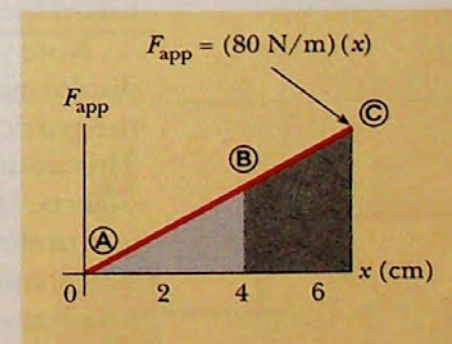


**FIGURE 6.10** (Example 6.3) A graph of the applied force required to stretch a spring that obeys Hooke's law versus the elongation of the spring.

## 10.5 TORQUE AND THE VECTOR PRODUCT

Recall our stationary exercise bicycle from the preceding section. We caused the rotational motion of the wheel by applying forces to the pedals. When a net force is exerted on a rigid object pivoted about some axis and the line of action<sup>2</sup> of the force does not pass through the pivot, the object tends to rotate about that axis. For example, when you push on a door, the door rotates about an axis through the hinges. The tendency of a force to rotate an object about some axis is measured by a vector quantity called **torque**. Torque is the cause of changes in rotational motion and is analogous to force, which causes changes in translational motion. Consider the wrench pivoted about the axis through  $O$  in Figure 10.10. The applied force  $\vec{F}$  generally can act at an angle  $\phi$  with respect to the position vector  $\vec{r}$  locating the point of application of the force. We define the torque  $\tau$  resulting from the force  $\vec{F}$  with the expression<sup>3</sup>

$$\tau \equiv rF \sin \phi \quad [10.18]$$

It is very important to recognize that **torque is defined only when a reference axis is specified**, from which the distance  $r$  is determined. We can interpret Equation 10.18 in two different ways. Looking at the force components in Figure 10.10, we see that the component  $F \cos \phi$  parallel to  $\vec{r}$  will not cause a rotation of the wrench around the pivot point because its line of action passes right through the pivot point. Similarly, you cannot open a door by pushing on the hinges! Therefore, only the perpendicular component  $F \sin \phi$  causes a rotation of the wrench about the pivot. In this case, we can write Equation 10.18 as

$$\tau = r(F \sin \phi)$$

so that the torque is the product of the distance to the point of application of the force and the perpendicular component of the force. In some problems, this method is the easiest way to interpret the calculation of the torque.

The second way to interpret Equation 10.18 is to associate the sine function with the distance  $r$  so that we can write

$$\tau = F(r \sin \phi) = Fd$$

The quantity  $d = r \sin \phi$ , called the **moment arm** (or *lever arm*) of the force  $\vec{F}$ , represents the perpendicular distance from the rotation axis to the line of action of  $\vec{F}$ . In some problems, this approach to the calculation of the torque is easier than that of resolving the force into components.

If two or more forces are acting on a rigid object, as in Active Figure 10.11, each has a tendency to produce a rotation about the pivot at  $O$ . For example, if the object is initially at rest,  $\vec{F}_2$  tends to rotate the object clockwise and  $\vec{F}_1$  tends to rotate the object counterclockwise. We shall use the convention that the sign of the torque resulting from a force is positive if its turning tendency is counterclockwise around the rotation axis and negative if its turning tendency is clockwise. For example, in Active Figure 10.11, the torque resulting from  $\vec{F}_1$ , which has a moment arm of  $d_1$ , is *positive* and equal to  $+F_1 d_1$ ; the torque from  $\vec{F}_2$  is *negative* and equal to  $-F_2 d_2$ . Hence, the *net* torque acting on the rigid object about an axis through  $O$  is

$$\tau_{\text{net}} = \tau_1 + \tau_2 = F_1 d_1 - F_2 d_2$$

<sup>2</sup>The line of action of a force is an imaginary line colinear with the force vector and extending to infinity in both directions.

<sup>3</sup>In general, torque is a vector. For rotation about a fixed axis, however, we will use italic, nonbold notation and specify the direction with a positive or a negative sign as we did for angular speed and acceleration in Section 10.1. We will treat the vector nature of torque briefly in a short while.

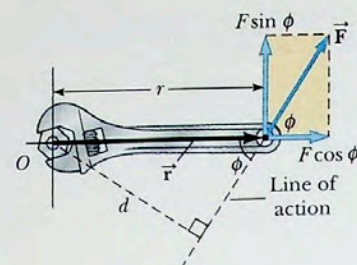
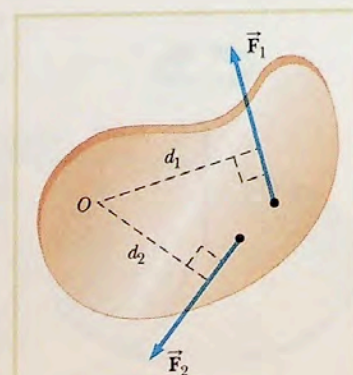


**FIGURE 10.10** A force  $\vec{F}$  is applied to a wrench in an effort to loosen a bolt. The force has a greater rotating tendency about  $O$  as  $F$  increases and as the moment arm  $d$  increases. The component  $F \sin \phi$  tends to rotate the system about  $O$ .

### PITFALL PREVENTION 10.5

**TORQUE DEPENDS ON YOUR CHOICE** Like moment of inertia, torque has no unique value. Its value depends on your choice of rotation axis.



### ACTIVE FIGURE 10.11

The force  $\vec{F}_1$  tends to rotate the object counterclockwise about an axis through  $O$ , and  $\vec{F}_2$  tends to rotate the object clockwise.

**PhysicsNow™** Change the magnitudes, directions, and points of application of forces  $\vec{F}_1$  and  $\vec{F}_2$  to see how the object accelerates under the action of the two forces by logging into PhysicsNow at [www.pop4e.com](http://www.pop4e.com) and going to Active Figure 10.11.

From the definition of torque, we see that the rotating tendency increases as  $F$  increases and as  $d$  increases. For example, we cause more rotation of a door if (a) we push harder or (b) we push at the doorknob rather than at a point close to the hinges. **Torque should not be confused with force.** Torque *depends* on force, but it also depends on *where the force is applied*. Torque has units of force times length, or newton · meters ( $\text{N} \cdot \text{m}$ ) in SI units.<sup>4</sup>

So far, we have not discussed the vector nature of torque aside from assigning a positive or negative value to  $\tau$ . Consider a force  $\vec{F}$  acting on a particle of a rigid object located at the vector position  $\vec{r}$  (Active Fig. 10.12). The *magnitude* of the torque due to this force relative to an axis through the origin is  $|rF \sin \phi|$ , where  $\phi$  is the angle between  $\vec{r}$  and  $\vec{F}$ . The axis about which  $\vec{F}$  would tend to produce rotation of the object is perpendicular to the plane formed by  $\vec{r}$  and  $\vec{F}$ . If the force lies in the  $xy$  plane, as in Active Figure 10.12, the torque is represented by a vector parallel to the  $z$  axis. The force in Active Figure 10.12 creates a torque that tends to rotate the object counterclockwise when we are looking down the  $z$  axis. We define the direction of torque such that the vector  $\vec{\tau}$  is in the positive  $z$  direction (i.e., coming toward your eyes). If we reverse the direction of  $\vec{F}$  in Active Figure 10.12,  $\vec{\tau}$  is in the negative  $z$  direction. With this choice, the torque vector can be defined to be equal to the **vector product**, or **cross product**, of  $\vec{r}$  and  $\vec{F}$ :

■ Definition of torque using the cross product

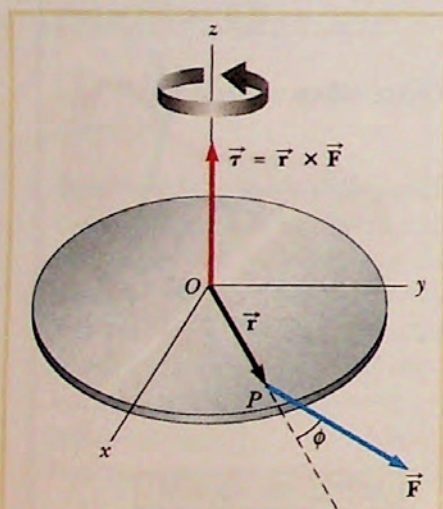
$$\vec{\tau} \equiv \vec{r} \times \vec{F} \quad [10.19]$$

We now give a formal definition of the vector product, first introduced in Section 1.8. Given any two vectors  $\vec{A}$  and  $\vec{B}$ , the vector product  $\vec{A} \times \vec{B}$  is defined as a third vector  $\vec{C}$ , the *magnitude* of which is  $AB \sin \theta$ , where  $\theta$  is the angle between  $\vec{A}$  and  $\vec{B}$ :

$$\vec{C} = \vec{A} \times \vec{B} \quad [10.20]$$

$$C = |\vec{C}| \equiv AB \sin \theta \quad [10.21]$$

Note that the quantity  $AB \sin \theta$  is equal to the area of the parallelogram formed by  $\vec{A}$  and  $\vec{B}$ , as shown in Figure 10.13. The *direction* of  $\vec{A} \times \vec{B}$  is perpendicular to the plane formed by  $\vec{A}$  and  $\vec{B}$  and is determined by the right-hand rule illustrated in Figure 10.13. The four fingers of the right hand are pointed along  $\vec{A}$  and then “wrapped” into  $\vec{B}$  through the angle  $\theta$ . The direction of the upright thumb is the direction of  $\vec{A} \times \vec{B}$ . Because of the notation,  $\vec{A} \times \vec{B}$  is often read “ $\vec{A}$  cross  $\vec{B}$ ,” hence the term *cross product*.

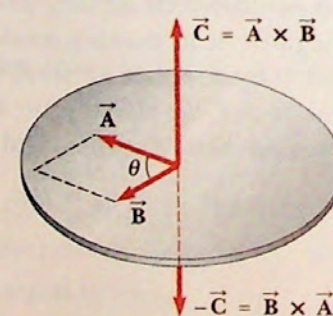


ACTIVE FIGURE 10.12

The torque vector  $\vec{\tau}$  lies in a direction perpendicular to the plane formed by the position vector  $\vec{r}$  and the applied force vector  $\vec{F}$ .

**PhysicsNow™** By logging into PhysicsNow at [www.pop4e.com](http://www.pop4e.com) and going to Active Figure 10.12 you can move point  $P$  and change the force vector  $\vec{F}$  to see the effect on the torque vector.

**FIGURE 10.13** The vector product  $\vec{A} \times \vec{B}$  is a third vector  $\vec{C}$  having a magnitude  $AB \sin \theta$  equal to the area of the parallelogram shown. The vector  $\vec{C}$  is perpendicular to the plane formed by  $\vec{A}$  and  $\vec{B}$ , and its direction is determined by the right-hand rule.



Right-hand rule



<sup>4</sup>In Chapter 6, we saw the product of newtons and meters when we defined work and called this product a *joule*. We do not use this term here because the joule is only to be used when discussing energy. For torque, the unit is simply the newton · meter, or  $\text{N} \cdot \text{m}$ .

Some properties of the vector product follow from its definition:

- Unlike the case of the scalar product, the vector product is not commutative; in fact,

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \quad [10.22]$$

Therefore, if you change the order of the vector product, you must change the sign. One can easily verify this relation with the right-hand rule (see Fig. 10.13).

- If  $\vec{A}$  is parallel to  $\vec{B}$  ( $\theta = 0^\circ$  or  $180^\circ$ ), then  $\vec{A} \times \vec{B} = 0$ ; therefore, it follows that  $\vec{A} \times \vec{A} = 0$ .
- If  $\vec{A}$  is perpendicular to  $\vec{B}$ , then  $|\vec{A} \times \vec{B}| = AB$ . It is left to Problem 10.25 to show, from Equations 10.20 and 10.21 and the definition of unit vectors, that the vector products of the unit vectors  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$  obey the following expressions:

$$\begin{aligned} \hat{i} \times \hat{i} &= \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0 \\ \hat{i} \times \hat{j} &= -\hat{j} \times \hat{i} = \hat{k} \\ \hat{j} \times \hat{k} &= -\hat{k} \times \hat{j} = \hat{i} \\ \hat{k} \times \hat{i} &= -\hat{i} \times \hat{k} = \hat{j} \end{aligned} \quad [10.23]$$

Signs are interchangeable. For example,  $\hat{i} \times (-\hat{j}) = -\hat{i} \times \hat{j} = -\hat{k}$ .

**QUICK QUIZ 10.5** If you are trying to loosen a stubborn screw from a piece of wood with a screwdriver and fail, should you find a screwdriver for which the handle is (a) longer or (b) fatter? If you are trying to loosen a stubborn bolt from a piece of metal with a wrench and fail, should you find a wrench for which the handle is (c) longer or (d) fatter?

### EXAMPLE 10.6 The Net Torque on a Cylinder

A one-piece cylinder is shaped as in Figure 10.14, with a core section protruding from the larger drum. The cylinder is free to rotate around the central axis shown in the drawing. A rope wrapped around the drum, of radius  $R_1$ , exerts a force  $\vec{T}_1$  to the right on the cylinder. A rope wrapped around the core, of radius  $R_2$ , exerts a force  $\vec{T}_2$  downward on the cylinder.

**A** What is the net torque acting on the cylinder about the rotation axis (which is the  $z$  axis in Fig. 10.14)?

**Solution** The torque due to  $\vec{T}_1$  is  $-R_1 T_1$ . It is negative because it tends to produce a clockwise rotation from the point of view in Figure 10.14. The torque due to  $\vec{T}_2$  is  $+R_2 T_2$  and is positive because it tends to produce a counterclockwise rotation. Therefore, the net torque about the rotation axis is

$$\tau_{\text{net}} = \tau_1 + \tau_2 = R_2 T_2 - R_1 T_1$$

**B** Suppose  $T_1 = 5.0$  N,  $R_1 = 1.0$  m,  $T_2 = 6.0$  N, and  $R_2 = 0.50$  m. What is the net torque about the rotation axis and which way does the cylinder rotate if it starts from rest?

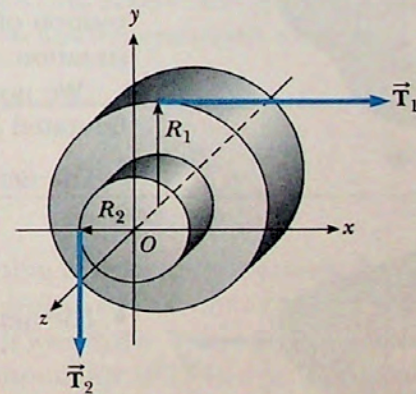


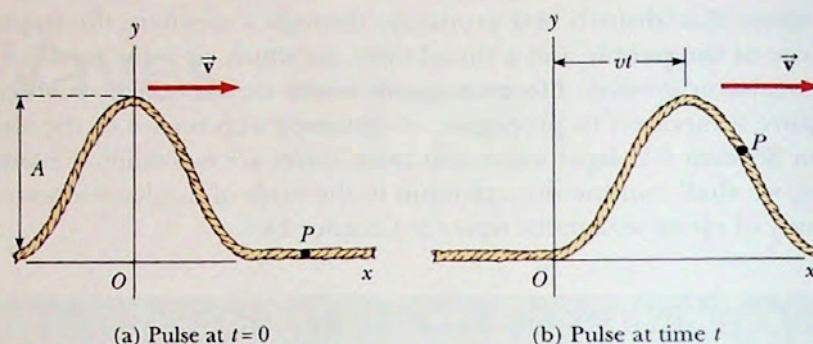
**FIGURE 10.14** (Example 10.6) A solid cylinder pivoted about the  $z$  axis through  $O$ . The moment arm of  $\vec{T}_1$  is  $R_1$ , and the moment arm of  $\vec{T}_2$  is  $R_2$ .

**Solution** We substitute numerical values in the result from part A:

$$\tau_{\text{net}} = (6.0 \text{ N})(0.50 \text{ m}) - (5.0 \text{ N})(1.0 \text{ m}) = -2.0 \text{ N} \cdot \text{m}$$

Because the net torque is negative, the cylinder rotates **clockwise** from rest.

**FIGURE 13.4** A one-dimensional pulse traveling to the right with a speed  $v$ . (a) At  $t = 0$ , the shape of the pulse is given by  $y = f(x)$ . (b) At some later time  $t$ , the shape remains unchanged and the vertical position of any element of the medium is given by  $y = f(x - vt)$ .



develop a mathematical representation for the propagation of this pulse. Consider a pulse traveling to the right with constant speed  $v$  on a long, stretched string as in Figure 13.4. The pulse moves along the  $x$  axis (the axis of the string), and the transverse (up-and-down) displacement of the elements of the string is described by means of the position  $y$ .

Figure 13.4a represents the shape and position of the pulse at time  $t = 0$ . At this time, the shape of the pulse, whatever it may be, can be represented by some mathematical function that we will write as  $y(x, 0) = f(x)$ . This function describes the vertical position  $y$  of the element of the string located at each value of  $x$  at time  $t = 0$ . Because the speed of the pulse is  $v$ , the pulse has traveled to the right a distance  $vt$  at time  $t$  (Fig. 13.4b). We adopt a simplification model in which the shape of the pulse does not change with time.<sup>1</sup> Therefore, at time  $t$ , the shape of the pulse is the same as it was at time  $t = 0$ , as in Figure 13.4a. Consequently, an element of the string at  $x$  at this time has the same  $y$  position as an element located at  $x - vt$  had at time  $t = 0$ :

$$y(x, t) = y(x - vt, 0)$$

In general, then, we can represent the position  $y$  for all values of  $x$  and  $t$ , measured in a stationary frame with the origin at  $O$ , as

$$y(x, t) = f(x - vt) \quad [13.1a]$$

If the pulse travels to the left, the position of an element of the string is described by

$$y(x, t) = f(x + vt) \quad [13.1b]$$

The function  $y$ , sometimes called the **wave function**, depends on the two variables  $x$  and  $t$ . For this reason, it is often written  $y(x, t)$ , which is read “ $y$  as a function of  $x$  and  $t$ .”

It is important to understand the meaning of  $y$ . Consider a point  $P$  on the string, identified by a particular value of its  $x$  coordinate as in Figure 13.4. As the pulse passes through  $P$ , the  $y$  coordinate of this point increases, reaches a maximum, and then decreases to zero. **The wave function  $y(x, t)$  represents the  $y$  position of any element of string located at position  $x$  at any time  $t$ .** Furthermore, if  $t$  is fixed (e.g., in the case of taking a snapshot of the pulse), the wave function  $y$  as a function of  $x$ , sometimes called the **waveform**, defines a curve representing the actual geometric shape of the pulse at that time.

#### QUICK QUIZ 13.1

In a long line of people waiting to buy tickets, the first person leaves and a pulse of motion occurs as people step forward to fill the gap. As each person steps forward, the gap moves through the line. Is the propagation of this gap (a) transverse or (b) longitudinal? Consider “the wave” at a baseball game when people stand up and shout as the wave arrives at their location and the resultant pulse moves around the stadium. Is this wave (c) transverse or (d) longitudinal?

<sup>1</sup>In reality, the pulse changes its shape and gradually spreads out during the motion. This effect, called *dispersion*, is common to many mechanical waves, but we adopt a simplification model that ignores this effect.

**EXAMPLE 13.1** A Pulse Moving to the Right

A pulse moving to the right along the  $x$  axis is represented by the wave function

$$y(x, t) = \frac{2.0}{(x - 3.0t)^2 + 1}$$

where  $x$  and  $y$  are measured in centimeters and  $t$  is in seconds. Let us plot the waveform at  $t = 0$ ,  $t = 1.0$  s, and  $t = 2.0$  s.

**Solution** First, note that this function is of the form  $y = f(x - vt)$ . By inspection, we see that the speed of the wave is  $v = 3.0$  cm/s. The location of the peak of the pulse occurs at the value of  $x$  for which the denominator is a minimum, that is, where  $(x - 3.0t) = 0$ . Therefore, the peaks occur at  $x = 0.0$  cm at  $t = 0$ , at  $x = 3.0$  cm at  $t = 1.0$  s, and  $x = 6.0$  cm at  $t = 2.0$  s.

At times  $t = 0$ ,  $t = 1.0$  s, and  $t = 2.0$  s, the wave function expressions are

$$y(x, 0) = \frac{2.0}{x^2 + 1} \quad \text{at } t = 0$$

$$y(x, 1.0) = \frac{2.0}{(x - 3.0)^2 + 1} \quad \text{at } t = 1.0 \text{ s}$$

$$y(x, 2.0) = \frac{2.0}{(x - 6.0)^2 + 1} \quad \text{at } t = 2.0 \text{ s}$$

We can now use these expressions to plot the wave function versus  $x$  at these times. For example, let us evaluate  $y(x, 0)$  at  $x = 0.50$  cm:

$$y(0.50, 0) = \frac{2.0}{(0.50)^2 + 1} = 1.6 \text{ cm}$$

Likewise,  $y(1.0, 0) = 1.0$  cm,  $y(2.0, 0) = 0.40$  cm, and so on. A continuation of this procedure for other values of  $x$  yields the waveform shown in Figure 13.5a. In a similar manner, one obtains the graphs of  $y(x, 1.0)$  and  $y(x, 2.0)$ , shown in Figures 13.5b and 13.5c, respectively. These snapshots show that the pulse moves to the right

without changing its shape and has a constant speed of 3.0 cm/s.

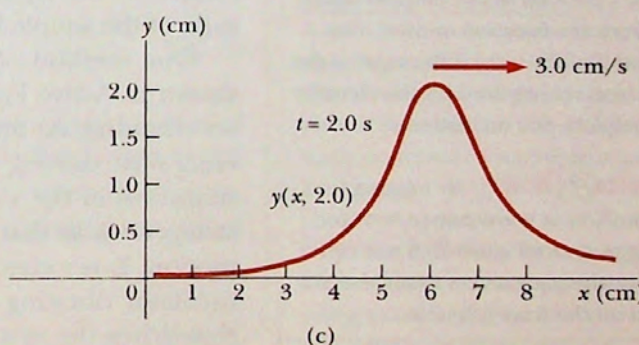
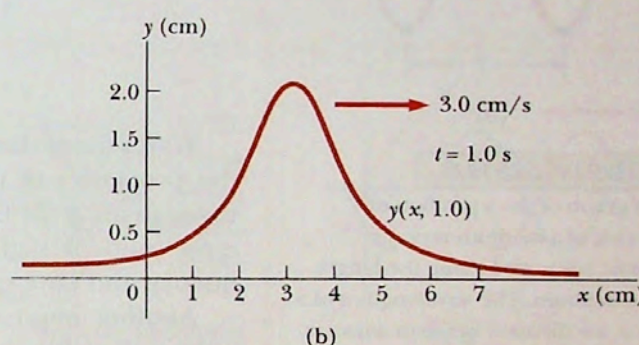
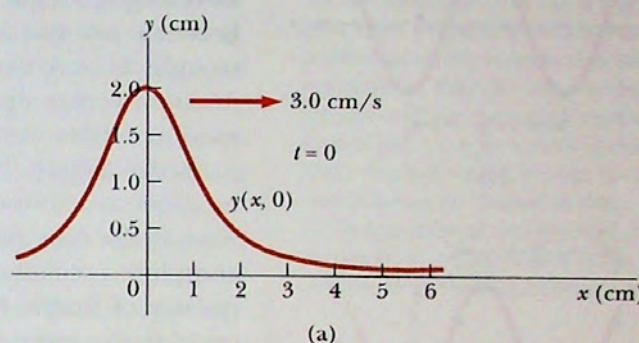
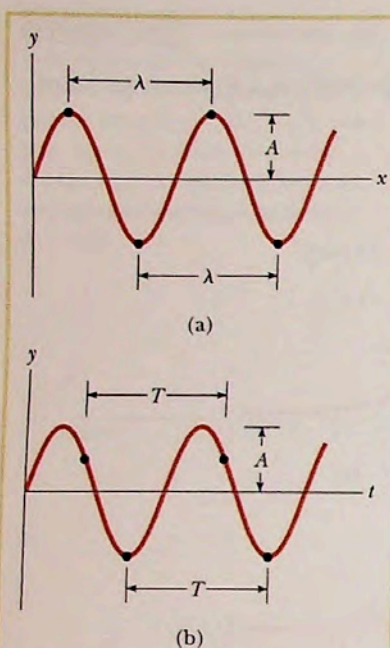


**FIGURE 13.5** (Example 13.1) Graphs of the function  $y(x, t) = 2.0/[(x - 3.0t)^2 + 1]$  at (a)  $t = 0$ , (b)  $t = 1.0$  s, and (c)  $t = 2.0$  s.

**13.2 THE WAVE MODEL**

We have discussed creating a disturbance moving through a medium such as a stretched string by a simple up-and-down displacement of the end of the string. This action results in a pulse moving along the medium. A continuous wave is created by shaking the end of the string in simple harmonic motion, which we studied in Chapter 12. If we do that, the string will take on the shape shown by the curve in the graph in Active Figure 13.6a, with this shape remaining the same but moving toward the right. This shape is what we call a **sinusoidal** wave because the waveform in Active Figure 13.6a is that of a sine wave. The point with the largest positive displacement of the string is called the **crest** of the wave. The lowest point is called the **trough**. The crest and trough move along with the wave, and a particular point on

**ACTIVE FIGURE 13.6**

(a) A graph of the  $y$  position of elements of a medium versus  $x$  position, measured along the length of the medium. The wavelength  $\lambda$  of a wave is the distance between adjacent crests or adjacent troughs. (b) A graph of the  $y$  position of one element of the medium as a function of time. The period  $T$  of the wave is the same as the time interval required for the element to complete one oscillation.

**PhysicsNow™** By logging into PhysicsNow at [www.pop4e.com](http://www.pop4e.com) and going to Active Figure 13.6 you can change the parameters to see the effect on the wave function.

the string alternates between locations on a crest and a trough. In idealized wave motion in an idealized medium, each element of the medium undergoes simple harmonic motion around its equilibrium position.

Three physical characteristics are important in describing a sinusoidal wave: **wavelength**, **frequency**, and **wave speed**. **One wavelength is the minimum distance between any two identical points on a wave** such as adjacent crests or adjacent troughs as in Active Figure 13.6a, which is a graph of  $y$  position of elements of the medium versus  $x$  position for a sinusoidal wave at a specific time. The symbol  $\lambda$  is used to denote wavelength.

Active Figure 13.6b shows  $y$  position versus time for a single element of the medium as a sinusoidal wave is passing through its position  $x$ . The **period**  $T$  of the wave is the time interval required for an element of the medium to undergo one complete oscillation. The **frequency**  $f$  of sinusoidal waves is the same as the frequency of simple harmonic motion of an element of the medium. The period is equal to the inverse of the frequency:

$$T = \frac{1}{f} \quad [13.2]$$

Waves travel through the medium with a specific **wave speed**, which depends on the properties of the medium being disturbed. For instance, sound waves travel through air at 20°C with a speed of about 343 m/s, whereas the speed of sound in most solids is higher than 343 m/s. We will learn more about wavelength, frequency, and wave speed in the next section.

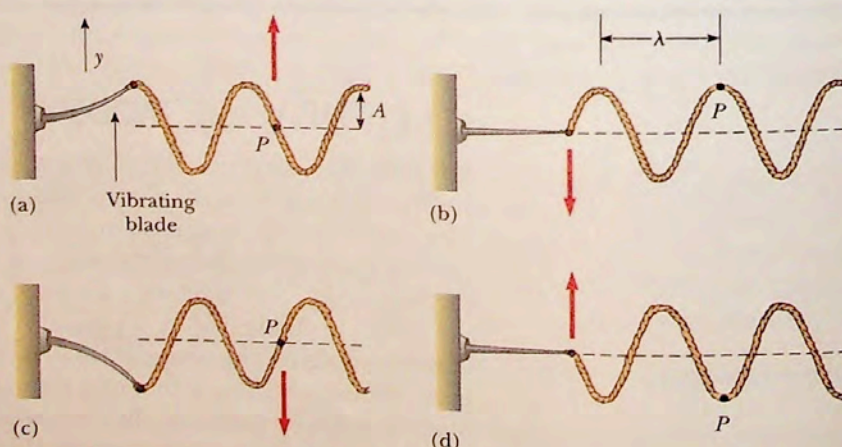
Another important parameter for the wave in Active Figure 13.6 is the **amplitude** of the wave. Amplitude is the maximum position of an element of the medium relative to the equilibrium position. It is denoted by  $A$  and is the same as the amplitude of the simple harmonic motion of the elements of the medium.

One method of producing a traveling sinusoidal wave on a very long string is shown in Active Figure 13.7. One end of the string is connected to a blade that is set vibrating. As the blade oscillates vertically with simple harmonic motion, a traveling wave moving to the right is set up on the string. Active Figure 13.7 represents snapshots of the wave at intervals of one quarter of a period. **Each element of the string, such as that at  $P$ , oscillates vertically in the  $y$  direction with simple harmonic motion.** Every element of the string can therefore be treated as a simple harmonic oscillator vibrating with a frequency equal to the frequency of vibration of the blade that drives the string. Although each element oscillates in the  $y$  direction, the wave (or disturbance) travels in the  $x$  direction with a speed  $v$ . Of course, this situation is the definition of a transverse wave. In this case, the energy carried by the traveling wave is supplied by the vibrating blade.

**ACTIVE FIGURE 13.7**

One method for producing a sinusoidal wave on a continuous string. The left end of the string is connected to a blade that is set into vibration. Every element of the string, such as the one at point  $P$ , oscillates with simple harmonic motion in the vertical direction.

**PhysicsNow™** Log into PhysicsNow at [www.pop4e.com](http://www.pop4e.com) and go to Active Figure 13.7 to adjust the frequency of the blade.



In the early chapters of this book, we developed several analysis models based on the particle model. With our introduction to waves, we can develop a new simplification model, the **wave model**, which will allow us to explore more analysis models for solving problems. An ideal particle has zero size. We can build physical objects with nonzero size as combinations of particles. Thus, the particle can be considered a basic building block. An ideal wave has a single frequency and is infinitely long; that is, the wave exists throughout the Universe. (It is beyond the mathematical scope of this text at this point to prove this fact, but a wave of finite length must necessarily have a mixture of frequencies.) We will find that we can combine ideal waves, just as we combined particles, and thus the ideal wave can be considered as a basic building block. We will explore this concept in Section 14.7.

The wave model starts with an ideal wave having a single frequency, wavelength, wave speed, and amplitude. From this beginning, we describe waves in a variety of situations that serve as analysis models to help us solve problems. In the next section, we further develop our mathematical representation for the wave model.

### 13.3 THE TRAVELING WAVE

Let us investigate further the mathematics of a sinusoidal wave (Active Fig. 13.8). The brown curve represents a snapshot of a sinusoidal wave at  $t = 0$ , and the blue curve represents a snapshot of the wave at some later time  $t$ . In what follows, we will develop the principal features and mathematical representations of the model of a **traveling wave**. This analysis model is used in situations in which a wave moves through space without interacting with any other waves or particles.

At  $t = 0$ , the brown curve in Active Figure 13.8 can be described mathematically as

$$y = A \sin \left( \frac{2\pi}{\lambda} x \right) \quad [13.3]$$

where the amplitude  $A$ , as usual, represents the maximum value of the position of an element relative to the equilibrium position and  $\lambda$  is the wavelength as defined in Active Figure 13.6a. Therefore, we see that the value of  $y$  is the same when  $x$  is increased by an integral multiple of  $\lambda$ . If the wave moves to the right with a speed of  $v$ , the wave function at some later time  $t$  is

$$y = A \sin \left[ \frac{2\pi}{\lambda} (x - vt) \right] \quad [13.4]$$

That is, the sinusoidal wave has moved to the right a distance of  $vt$  at time  $t$  as in Active Figure 13.8. Note that the wave function has the form  $f(x - vt)$  and represents a wave traveling to the right. If the wave were traveling to the left, the quantity  $x - vt$  would be replaced by  $x + vt$ , just as in the case of the traveling pulse described by Equations 13.1a and 13.1b.

Because the period  $T$  is the time interval required for the wave to travel a distance of one wavelength, the speed, wavelength, and period are related by

$$v = \frac{\lambda}{T} \quad [13.5]$$

Substituting Equation 13.5 into Equation 13.4, we find that

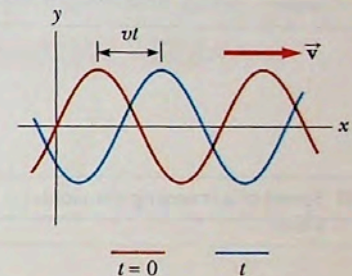
$$y = A \sin \left[ 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right) \right] \quad [13.6]$$

This form of the wave function shows the periodic nature of  $y$  in both space and time. That is, at any given time  $t$  (a snapshot of the wave),  $y$  has the same value at the positions  $x$ ,  $x + \lambda$ ,  $x + 2\lambda$ , and so on. Furthermore, at any given position  $x$  (at

#### PITFALL PREVENTION 13.1

##### WHAT'S THE DIFFERENCE BETWEEN ACTIVE FIGURES 13.6a AND 13.6b?

The curves in both parts of the figure are the same, but (a) is a graph of vertical position versus horizontal position and (b) is vertical position versus time. Part (a) can also be interpreted as a pictorial representation of the wave for a series of elements of the medium, which is what you would see at an instant of time. Part (b) is a graphical representation of the position of one element of the medium as a function of time.



ACTIVE FIGURE 13.8

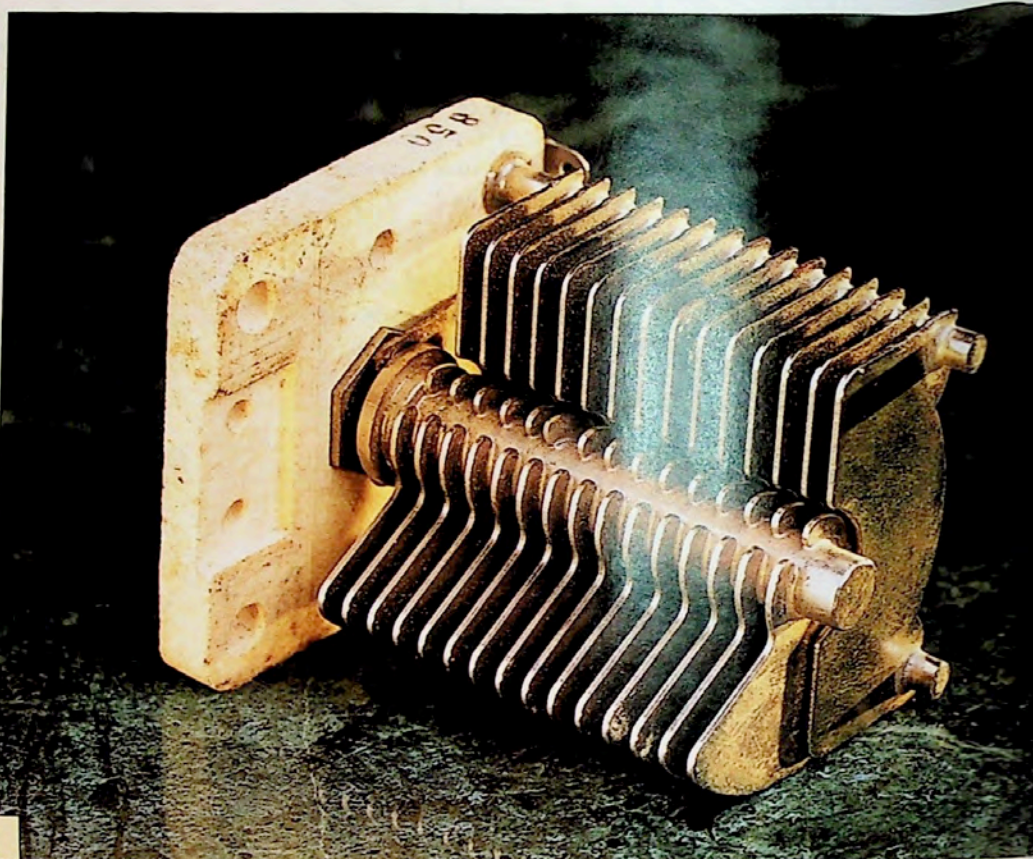
A one-dimensional sinusoidal wave traveling to the right with a speed  $v$ . The brown curve represents a snapshot of the wave at  $t = 0$ , and the blue curve represents a snapshot at some later time  $t$ .

**PhysicsNow™** By logging into PhysicsNow at [www.pop4e.com](http://www.pop4e.com) and going to Active Figure 13.8 you can watch the wave move and take snapshots of it at various times.

# Electric Potential and Capacitance

This device is a *variable capacitor*, used to tune radios to a selected station. When one set of metal plates is rotated so as to lie between a fixed set of plates, the *capacitance* of the device changes. Capacitance is a parameter that depends on *electric potential*, the primary topic of this chapter.

(George Sempie)



## CHAPTER OUTLINE

- 20.1 Potential Difference and Electric Potential
- 20.2 Potential Differences in a Uniform Electric Field
- 20.3 Electric Potential and Electric Potential Energy Due to Point Charges
- 20.4 Obtaining Electric Field from Electric Potential
- 20.5 Electric Potential Due to Continuous Charge Distributions
- 20.6 Electric Potential of a Charged Conductor
- 20.7 Capacitance
- 20.8 Combinations of Capacitors
- 20.9 Energy Stored in a Charged Capacitor
- 20.10 Capacitors with Dielectrics
- 20.11 Context Connection—The Atmosphere as a Capacitor

## SUMMARY

The concept of potential energy was introduced in Chapter 7 in connection with such conservative forces as gravity and the elastic force of a spring. By using the principle of conservation of mechanical energy in an isolated system, we are often able to avoid working directly with forces when solving mechanical problems. In this chapter, we shall use the energy concept in our study of electricity. Because the electrostatic force (given by Coulomb's law) is conservative, electrostatic phenomena can conveniently be described in terms of an *electric potential* energy function. This concept enables us to define a quantity called *electric potential*, which is a scalar quantity and which therefore leads to a simpler means of describing some electrostatic phenomena than the electric field method. As we shall see in subsequent chapters, the concept of electric potential is of great practical value in many applications.

This chapter also addresses the properties of capacitors, devices that store charge. The ability of a capacitor to store charge is measured by its *capacitance*. Capacitors are used in common

applications such as frequency tuners in radio receivers, filters in power supplies, dampers to eliminate unwanted sparking in automobile ignition systems, and energy-storing devices in electronic flash units.

## 20.1 POTENTIAL DIFFERENCE AND ELECTRIC POTENTIAL

When a point charge  $q_0$  is placed in an electric field  $\vec{E}$ , the electric force on the particle is  $q_0\vec{E}$  (Eq. 19.4). This force is the vector sum of the individual forces exerted on  $q_0$  by the various source charges producing the field  $\vec{E}$ . It follows that the force  $q_0\vec{E}$  is conservative because the individual forces governed by Coulomb's law are conservative. (See Section 7.3 for a review of conservative forces.) Let us consider a system consisting of the point charge and all the source charges creating the electric field. Because the field represents the effect of the source charges, we can also consider the system to be the electric field and the charge  $q_0$  that we place in the field, without referring specifically to the source charges. When the point charge moves in response to the electric force within the electric field, work is done on the particle by the field. For an infinitesimal displacement  $d\vec{s}$  of a point charge  $q_0$ , the work done by the electric field on the charge is  $\vec{F}_e \cdot d\vec{s} = q_0\vec{E} \cdot d\vec{s}$ .

The work done by the field on a point charge is similar to the work done by a gravitational field on a falling object. We found in Chapter 7 that the gravitational potential energy of an isolated object–field system changes by an amount equal to the negative of the work done within the system by the field on the object (Eq. 7.3). Similarly, the work done within the system by the electric field on a charged particle changes the potential energy of the isolated charge–field system by an amount  $dU = -dW = -q_0\vec{E} \cdot d\vec{s}$ . For a finite displacement of a test particle of charge  $q_0$  between points  $A$  and  $B$ , the **change in potential energy** of the charge–field system is

$$\Delta U = U_B - U_A = -q_0 \int_A^B \vec{E} \cdot d\vec{s} \quad [20.1]$$

The integral in Equation 20.1 is performed over the path along which the particle moves from  $A$  to  $B$ . It is called either a **path integral** or a **line integral**. Because the force  $q_0\vec{E}$  is conservative, **this integral does not depend on the path taken between  $A$  and  $B$ .**

In Chapter 19, we recognized that the force between a test charge and a distribution of source charges depends on all the charges, whereas the electric field is defined as a quantity established only by the source charges. We do something similar in this discussion. The potential energy of the system of a test charge  $q_0$  in an electric field  $\vec{E}$  depends on the test charge and all the source charges establishing the electric field. Let us remove the effect of the test charge by dividing the potential energy of the system by the test charge. The potential energy  $U$  of the system per unit charge  $q_0$  is independent of the value of  $q_0$  and has a unique value at every point in an electric field. The quantity  $U/q_0$  is called the **electric potential**  $V$  (or simply the **potential**):

$$V \equiv \frac{U}{q_0} \quad [20.2]$$

Because potential energy is a scalar, electric potential is also a scalar quantity. Note that potential is not a property of the charge–field system because we have divided the potential energy of the system by the charge. It is a property only of the field. Therefore, in the physical situation, we can imagine removing the test charge from the field. The potential still exists at the point the test charge occupied and is due to the source charges that establish the electric field.

The **potential difference**  $\Delta V = V_B - V_A$  between the points  $A$  and  $B$  is defined as the change in potential energy of the charge–field system when the test particle is

■ **Change in potential energy for a charge–field system**

### PITFALL PREVENTION 20.1

**POTENTIAL AND POTENTIAL ENERGY**  
The potential is characteristic of the field only, independent of a charged test particle that may be placed in the field. Potential energy is characteristic of the charge–field system due to an interaction between the field and a charged particle placed in the field.

■ **Definition of electric potential**

■ Potential difference between two points in an electric field

moved between the points divided by the charge  $q_0$  on the test particle:

$$\Delta V = \frac{\Delta U}{q_0} = - \int_A^B \vec{E} \cdot d\vec{s} \quad [20.3]$$

**Potential difference should not be confused with potential energy difference.** The potential difference between two points in an electric field is *proportional* to the potential energy difference of the charge-field system when the charge is at the two points, and we see from Equation 20.3 that the two are related by  $\Delta U = q_0 \Delta V$ .

Equation 20.3 defines potential difference only. The potential is often taken to be zero at some convenient point, sometimes called a *ground*. We usually set the potential due to one or more source charges at zero for a point at infinity (i.e., a point infinitely remote from the source charges producing the electric field). With this choice, we can say that **the electric potential at an arbitrary point due to source charges equals the work required by an external agent to bring a test particle from infinity to that point divided by the charge on the test particle**. Therefore, if we take  $V_A = 0$  at infinity in Equation 20.3, the potential at any point  $P$  is

$$V_P = - \int_{\infty}^P \vec{E} \cdot d\vec{s} \quad [20.4]$$

where  $\vec{E}$  is the electric field established by the source charges. In reality,  $V_P$  represents the potential difference between the point  $P$  and a point at infinity. (Note that Eq. 20.4 is a special case of Eq. 20.3.) When discussing potentials in an electric circuit, we shall set  $V = 0$  at some selected point in the circuit.

Because potential is a measure of energy per unit charge, the SI units of potential are joules per coulomb, called the **volt (V)**:

$$1 \text{ V} \equiv 1 \text{ J/C}$$

That is, if we release a particle with a charge of 1 C in an electric field and it moves from a point of high potential to a point of low potential through a potential difference of  $-1 \text{ V}$ , it will have 1 J of work done on it by the field and therefore will attain a kinetic energy of 1 J. (From the continuity equation for energy, Eq. 6.20, for the system of the particle,  $W = \Delta K$ .) Alternatively, 1 J of work must be done by an external agent to take a particle with a charge of 1 C through a potential difference of  $+1 \text{ V}$  at constant velocity. (From the continuity equation for energy, for the particle-field system,  $W = \Delta U$ .) Equation 20.3 shows that the potential difference also has the same units as the product of electric field and displacement. It therefore follows that the SI units of electric field, newtons per coulomb, can be expressed as volts per meter:

$$1 \text{ N/C} = 1 \text{ V/m}$$

which suggests that the electric field can be interpreted as the rate of change in space of the electric potential. A strong electric field corresponds to a potential that changes rapidly in space, whereas a weak field represents a slowly changing potential.

As we learned in Section 9.7, a unit of energy commonly used in physics is the **electron volt (eV)**:

$$1 \text{ eV} = (1e)(1 \text{ V}) = (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) = 1.60 \times 10^{-19} \text{ J} \quad [20.5]$$

One eV is the kinetic energy gained by a particle with charge  $e$  being accelerated by an electric field through a potential difference of magnitude 1 V. Equation 20.5 can be used to convert any energy in joules to electron volts. For instance, an electron in the beam of a typical TV picture tube may have a speed of  $3.0 \times 10^7 \text{ m/s}$ . This speed corresponds to a kinetic energy of  $4.1 \times 10^{-16} \text{ J}$ , which is equivalent to  $2.6 \times 10^3 \text{ eV}$ . Such an electron has to be accelerated from rest through a potential difference of 2.6 kV to reach this speed.

### PITFALL PREVENTION 20.2

**VOLTAGE** In practice, a variety of phrases are used to describe the potential difference between two points, the most common being **voltage**, arising from the unit for potential. A voltage *applied* to a device, such as a television, or *across* a device has the same meaning as the potential difference across the device. For example, if we say that the voltage applied to a lightbulb is 120 V, we mean that the potential difference between the two electrical contacts on the lightbulb is 120 V.

■ The electron volt

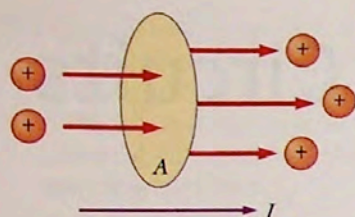


**FIGURE 21.1** Charges in motion through an area  $A$ . The time rate at which charge flows through the area is defined as the current  $I$ . The direction of the current is the direction in which positive charges flow when free to do so.

#### ■ Electric current

### ❏ PITFALL PREVENTION 21.1

**CURRENT FLOW IS REDUNDANT** The phrase *current flow* is commonly used, although it is strictly incorrect, because current is a flow (of charge). This terminology is similar to the phrase *heat transfer*, which is also redundant because heat is a transfer (of energy). We will avoid the phrase *current flow* and speak of *charge flow* or *flow of charge*.

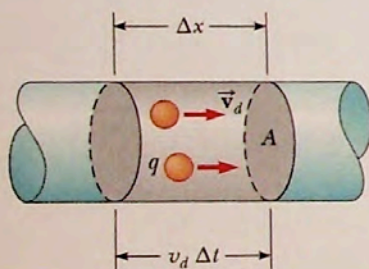


**FIGURE 21.2** A section of a uniform conductor of cross-sectional area  $A$ . The mobile charge carriers move with an average speed  $v_d$  along the wire, and the displacement they experience in this direction in a time interval  $\Delta t$  is  $\Delta x = v_d \Delta t$ . If we choose  $\Delta t$  to be the time interval during which the charges are displaced, on the average, by the length of the cylinder, the number of carriers in the section of length  $\Delta x$  is  $nAv_d \Delta t$ , where  $n$  is the number of carriers per unit volume.

## 21.1 ELECTRIC CURRENT

Whenever charge is flowing, an **electric current** is said to exist. To define current mathematically, suppose charged particles are moving perpendicular to a surface of area  $A$  as in Figure 21.1. (This area could be the cross-sectional area of a wire, for example.) The current is defined as **the rate at which electric charge flows through this surface**. If  $\Delta Q$  is the amount of charge that passes through this area in a time interval  $\Delta t$ , the average current  $I_{\text{avg}}$  over the time interval is the ratio of the charge to the time interval:

$$I_{\text{avg}} = \frac{\Delta Q}{\Delta t} \quad [21.1]$$

It is possible for the rate at which charge flows to vary in time. We define the **instantaneous current**  $I$  as the limit of the preceding expression as  $\Delta t$  goes to zero:

$$I \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \quad [21.2]$$

The SI unit of current is the **ampere** (A):

$$1 \text{ A} = 1 \text{ C/s} \quad [21.3]$$

That is, 1 A of current is equivalent to 1 C of charge passing through a surface in 1 s.

The particles flowing through a surface as in Figure 21.1 can be charged positively or negatively, or we can have two or more types of particles moving, with charges of both signs in the flow. **Conventionally, we define the direction of the current as the direction of flow of positive charge**, regardless of the sign of the actual charged particles in motion.<sup>1</sup> In a common conductor such as copper, the current is physically due to the motion of the negatively charged electrons. Therefore, when we speak of current in such a conductor, **the direction of the current is opposite the direction of flow of electrons**. On the other hand, if one considers a beam of positively charged protons in a particle accelerator, the current is in the direction of motion of the protons. In some cases—gases and electrolytes, for example—the current is the result of the flow of both positive and negative charged particles. It is common to refer to a moving charged particle (whether it is positive or negative) as a mobile **charge carrier**. For example, the charge carriers in a metal are electrons.

We now build a structural model that will allow us to relate the macroscopic current to the motion of the charged particles. Consider identical charged particles moving in a conductor of cross-sectional area  $A$  (Fig. 21.2). The volume of a section of the conductor of length  $\Delta x$  (the gray region shown in Fig. 21.2) is  $A \Delta x$ . If  $n$  represents the number of mobile charge carriers per unit volume (in other words, the charge carrier density), the number of carriers in the gray section is  $nA \Delta x$ . Therefore, the total charge  $\Delta Q$  in this section is

$$\Delta Q = \text{number of carriers in section} \times \text{charge per carrier} = (nA \Delta x)q$$

where  $q$  is the charge on each carrier. If the carriers move with an average velocity component  $v_d$  in the  $x$  direction (along the wire), the displacement they experience in this direction in a time interval  $\Delta t$  is  $\Delta x = v_d \Delta t$ . The speed  $v_d$  of the charge carrier along the wire is an average speed called the **drift speed**. Let us choose  $\Delta t$  to be the time interval required for the charges in the cylinder to move through a displacement whose magnitude is equal to the length of the cylinder. This time

<sup>1</sup>Even though we discuss a direction for current, current is not a vector. As we shall see later in the chapter, currents add algebraically and not vectorially.

interval is also that required for all the charges in the cylinder to pass through the circular area at one end. With this choice, we can write  $\Delta Q$  in the form

$$\Delta Q = (nAv_d \Delta t)q$$

If we divide both sides of this equation by  $\Delta t$ , we see that the average current in the conductor is

$$I_{\text{avg}} = \frac{\Delta Q}{\Delta t} = nqv_d A \quad [21.4]$$

Equation 21.4 relates a macroscopically measured average current to the microscopic origin of the current: the density of charge carriers  $n$ , the charge per carrier  $q$ , and the drift speed  $v_d$ .

**QUICK QUIZ 21.1** Consider positive and negative charges moving horizontally through the four regions shown in Figure 21.3. Rank the currents in these four regions, from lowest to highest.

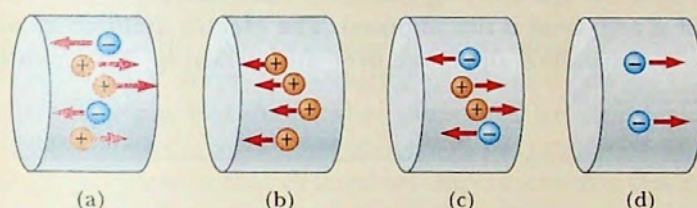


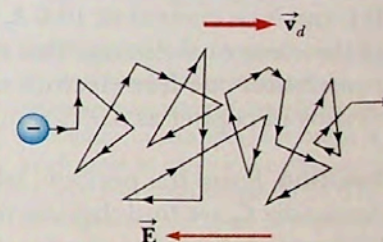
**FIGURE 21.3** (Quick Quiz 21.1) Four groups of charges move through a region.

Let us investigate further the notion of drift speed. We have identified drift speed as an average speed along the wire, but the charge carriers are by no means moving in a straight line with speed  $v_d$ . Consider a conductor in which the charge carriers are free electrons. In the absence of a potential difference across the conductor, these electrons undergo random motion similar to that of gas molecules in the structural model of kinetic theory that we studied in Chapter 16. This random motion is related to the temperature of the conductor. The electrons undergo repeated collisions with the metal atoms, and the result is a complicated zigzag motion (Active Fig. 21.4). When a potential difference is applied across the conductor, an electric field is established in the conductor. The electric field exerts an electric force on the electrons (Eq. 19.4). This force accelerates the electrons and hence produces a current. The motion of the electrons due to the electric force is superimposed on their random motion to provide an average velocity whose magnitude is the drift speed.

When electrons make collisions with metal atoms during their motion, they transfer energy to the atoms. This energy transfer causes an increase in the vibrational energy of the atoms and a corresponding increase in the temperature of the conductor.<sup>2</sup> This process involves all three types of energy storage in the continuity equation for energy, Equation 6.20. If we consider the system to be the electrons, the metal atoms, and the electric field (which is established by an external source such as a battery), the energy at the instant when the potential difference is applied across the conductor is electric potential energy associated with the field and the electrons. This energy is transformed by work done by the field on the electrons to kinetic energy of electrons. When the electrons strike the metal atoms, some of the kinetic energy is transferred to the atoms, which adds to the internal energy of the system.

<sup>2</sup>This increase in temperature is sometimes called *joule heating*, but that term is a misnomer because there is no heat involved. We will not use this wording.

■ Current in terms of microscopic parameters



**ACTIVE FIGURE 21.4**

A schematic representation of the zigzag motion of a charge carrier in a conductor. The changes in direction are due to collisions with atoms in the conductor. Note that the net motion of electrons is opposite the direction of the electric field. Because of the acceleration of the charge carriers due to the electric force, the paths are actually parabolic. The drift speed, however, is much smaller than the average speed, so the parabolic shape is not visible on this scale.

**PhysicsNow**™ Log into PhysicsNow at [www.pop4e.com](http://www.pop4e.com) and go to Active Figure 21.4 to adjust the electric field to see the resulting effect on the motion of an electron.

The **current density**  $J$  in the conductor is defined as the current per unit area. From Equation 21.4, the current density is

$$J \equiv \frac{I}{A} = nqv_d$$

where  $J$  has the SI units amperes per square meter.

### Thinking Physics 21.1

In Chapter 19, we claimed that the electric field inside a conductor is zero. In the preceding discussion, however, we have used the notion of an electric field in a conducting wire that exerts electric forces on electrons, causing them to move with drift velocity. Is this notion inconsistent with Chapter 19?

**Reasoning** The electric field is zero only in a conductor in *electrostatic equilibrium*, that is, a conductor in which the charges are at rest after having moved to equilibrium positions. In a current-carrying conductor, the charges are not at rest, so the requirement for a zero field is not imposed. The electric field in a conductor in a circuit is due to a distribution of charge over the surface of the conductor that can be quite complicated.<sup>3</sup> ■

#### EXAMPLE 21.1 Drift Speed in a Copper Wire

The 12-gauge copper wire in a typical residential building has a cross-sectional area of  $3.31 \times 10^{-6} \text{ m}^2$ . If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. Take the density of copper as  $8.95 \text{ g/cm}^3$ .

**Solution** From the periodic table of the elements in Appendix C, we find that the molar mass of copper is 63.5 g/mol. Knowing the density of copper enables us to calculate the volume occupied by 1 mol of copper:

$$V = \frac{M}{\rho} = \frac{63.5 \text{ g/mol}}{8.95 \text{ g/cm}^3} = 7.09 \text{ cm}^3/\text{mol}$$

Recall that one mole of any substance contains Avogadro's number of atoms,  $6.02 \times 10^{23}$  atoms. Because

each copper atom contributes one free electron to the current, the density of charge carriers is

$$\begin{aligned} n &= \frac{6.02 \times 10^{23} \text{ electrons}}{7.09 \text{ cm}^3} \left( \frac{1.00 \times 10^6 \text{ cm}^3}{1 \text{ m}^3} \right) \\ &= 8.49 \times 10^{28} \text{ electrons/m}^3 \end{aligned}$$

From Equation 21.4, we find that the drift speed is

$$\begin{aligned} v_d &= \frac{I}{nqA} \\ &= \frac{10.0 \text{ C/s}}{(8.49 \times 10^{28} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})(3.31 \times 10^{-6} \text{ m}^2)} \\ &= 2.22 \times 10^{-4} \text{ m/s} \end{aligned}$$

#### PITFALL PREVENTION 21.2

**ELECTRONS ARE AVAILABLE EVERYWHERE** Let us emphasize the point being made here: *Electrons do not have to travel from the light switch to the light for the light to operate.* Electrons already in the filament of the lightbulb move in response to the electric field set up by the battery. Note also that the role of a battery is not to provide electrons to the circuit. It establishes the electric field that exerts a force on electrons already in the wires and elements of the circuit.

Example 21.1 shows that typical drift speeds in conductors are very small. In fact, the drift speed is much smaller than the average speed between collisions. For instance, electrons traveling with the drift speed calculated in Example 21.1 would take about 75 min to travel 1 m! In view of this low speed, you might wonder why a light turns on almost instantaneously when a switch is thrown. In a conductor, the electric field that drives the free electrons is established in the conductor almost instantaneously. Therefore, when you flip a light switch, the electric force that causes the electrons to start moving in the wire with a drift speed begins immediately. Electrons already in the filament of the lightbulb begin to move in response to this force, and the lightbulb begins to emit light.

<sup>3</sup>See Chapter 6 in R. Chabay and B. Sherwood, *Electric and Magnetic Interactions*, (New York: Wiley, 1996) for details on this charge distribution.

## 22.1 HISTORICAL OVERVIEW

Many historians of science believe that the compass, which uses a magnetic needle, was used in China as early as the 13th century B.C., its invention being of Arab or Indian origin. The phenomenon of magnetism was known to the Greeks as early as about 800 B.C. They discovered that certain stones, made of a material now called *magnetite* ( $\text{Fe}_3\text{O}_4$ ), attracted pieces of iron.

In 1269, Pierre de Maricourt (c. 1220–?) mapped out the directions taken by a magnetized needle when it was placed at various points on the surface of a spherical natural magnet. He found that the directions formed lines that encircled the sphere and passed through two points diametrically opposite each other, which he called the **poles** of the magnet. Subsequent experiments have shown that every magnet, regardless of its shape, has two poles, called **north** (N) and **south** (S), that exhibit forces on each other in a manner analogous to electric charges. That is, similar poles (N–N or S–S) repel each other and dissimilar poles (N–S) attract each other. The poles received their names because of the behavior of a magnet in the presence of the Earth's magnetic field. If a bar magnet is suspended from its midpoint by a piece of string so that it can swing freely in a horizontal plane, it rotates until its “north” pole points to the north geographic pole of the Earth (which is a south magnetic pole) and its “south” pole points to the Earth's south geographic pole. (The same idea is used to construct a simple compass.)

In 1600, William Gilbert (1544–1603) extended these experiments to a variety of materials. Using the fact that a compass needle orients in preferred directions, Gilbert suggested that magnets are attracted to land masses. In 1750, John Michell (1724–1793) used a torsion balance to show that magnetic poles exert attractive or repulsive forces on each other and that these forces vary as the inverse square of their separation. Although the force between two magnetic poles is similar to the force between two electric charges, an important difference exists. Electric charges can be isolated (witness the electron and proton), whereas magnetic poles cannot be isolated. That is, **magnetic poles are always found in pairs**. No matter how many times a permanent magnet is cut, each piece always has a north pole and a south pole. (Some theories speculate that magnetic monopoles—isolated north or south poles—may exist in nature, and attempts to detect them currently make up an active experimental field of investigation. None of these attempts has yet proven successful, however.)

The relationship between magnetism and electricity was discovered in 1819 when, while preparing for a lecture demonstration, Danish scientist Hans Christian Oersted found that an electric current in a wire deflected a nearby compass needle. Shortly thereafter, André-Marie Ampère (1775–1836) deduced quantitative laws of magnetic force between current-carrying conductors. He also suggested that electric current loops of molecular size are responsible for *all* magnetic phenomena.

In the 1820s, Faraday and, independently, Joseph Henry (1797–1878) identified further connections between electricity and magnetism. They showed that an electric current could be produced in a circuit either by moving a magnet near the circuit or by changing the current in a nearby circuit. Their observations demonstrated that a changing magnetic field produces an electric field. Years later, theoretical work by James Clerk Maxwell showed that the reverse is also true: a changing electric field gives rise to a magnetic field.

In this chapter, we shall investigate the effects of constant magnetic fields on charges and currents, and study the sources of magnetic fields. In the next chapter, we shall explore the effects of magnetic fields that vary in time.



**HANS CHRISTIAN OERSTED**  
(1777–1851)

Oersted, a Danish physicist, is best known for observing that a compass needle deflects when placed near a wire carrying a current. This important discovery was the first evidence of the connection between electric and magnetic phenomena. Oersted was also the first to prepare pure aluminum.

North Wind Picture Archives

## 22.2 THE MAGNETIC FIELD

In earlier chapters, we described the interaction between charged objects in terms of electric fields. Recall that an electric field surrounds any stationary electric charge. The region of space surrounding a *moving* charge includes a **magnetic field**

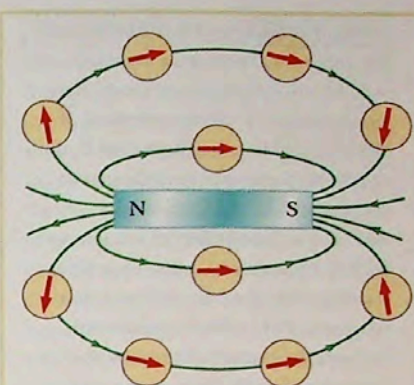
in addition to the electric field. A magnetic field also surrounds any material with permanent magnetism. We find that the magnetic field is a vector field, as is the electric field.

To describe any type of vector field, we must define its magnitude and its direction. The direction of the magnetic field vector  $\vec{B}$  at any location is the direction in which the north pole of a compass needle points at that location. Active Figure 22.1 shows how the magnetic field of a bar magnet can be traced with the aid of a compass, defining a **magnetic field line**, similar in many ways to the electric field lines we studied in Chapter 19. Several magnetic field lines of a bar magnet traced out in this manner are shown in the two-dimensional pictorial representation in Active Figure 22.1. Magnetic field patterns can be displayed by small iron filings placed in the vicinity of a magnet, as in Figure 22.2.

We can quantify the magnetic field  $\vec{B}$  by using our model of a particle in a field. The existence of a magnetic field at some point in space can be determined by measuring the **magnetic force**  $\vec{F}_B$  exerted on an appropriate test particle placed at that point. This process is the same one we followed in defining the electric field in Chapter 19. Our test particle will be an electrically charged particle such as a proton. If we perform such an experiment, we find the following results:

- The magnetic force  $\vec{F}_B$  is proportional to the charge  $q$  of the particle as well as to the speed  $v$  of the particle.
- When a charged particle moves parallel to the magnetic field vector, the magnetic force  $\vec{F}_B$  on the charge is zero.
- When the velocity vector makes an angle  $\theta$  with the magnetic field, the magnetic force acts in a direction perpendicular to both  $\vec{v}$  and  $\vec{B}$ ; that is, the magnetic force is perpendicular to the plane formed by  $\vec{v}$  and  $\vec{B}$  (Fig. 22.3a).
- The magnetic force on a negative charge is directed opposite to the force on a positive charge moving in the same direction (Fig. 22.3b).
- If the velocity vector makes an angle  $\theta$  with the magnetic field, the magnitude of the magnetic force is proportional to  $\sin \theta$ .

These results show that the magnetic force on a particle is more complicated than the electric force. The magnetic force is distinctive because it depends on the



**ACTIVE FIGURE 22.1**

A small compass can be used to trace the magnetic field lines of a bar magnet.

**PhysicsNow™** Log into PhysicsNow at [www.pop4e.com](http://www.pop4e.com) and go to Active Figure 22.1 to move the compass around and trace the field lines for yourself.

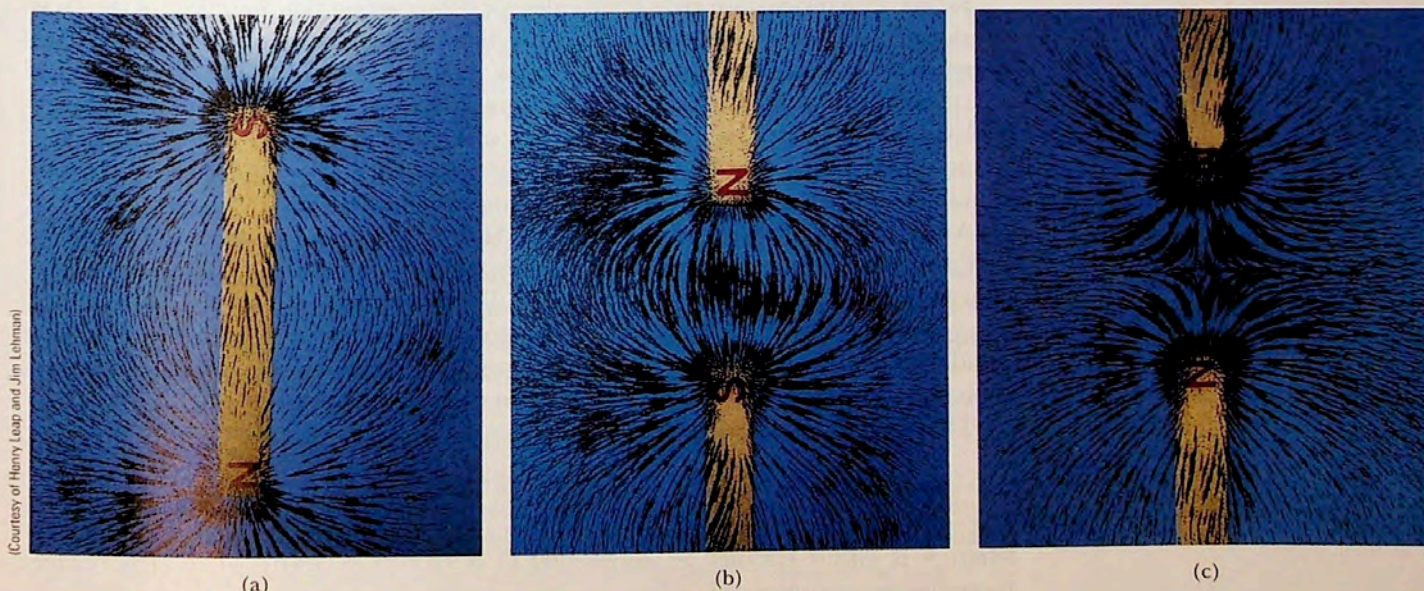
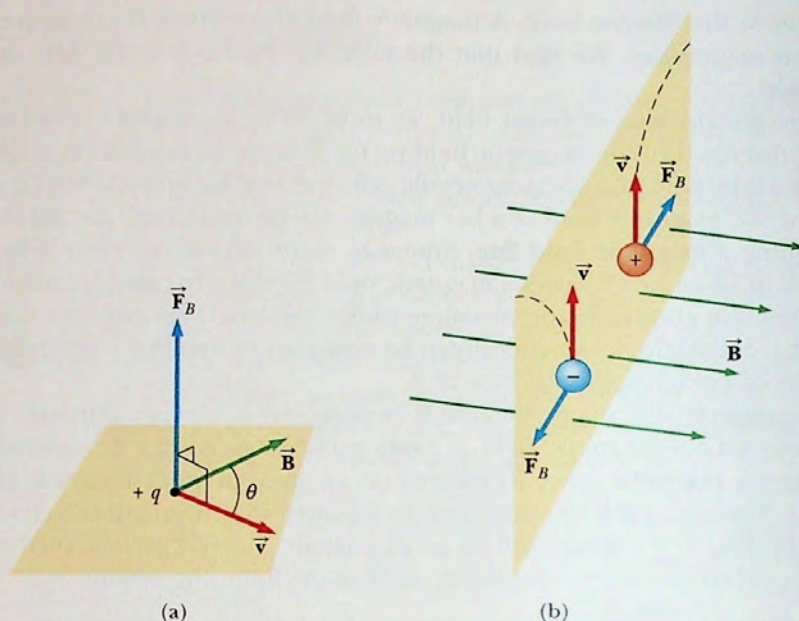


**FIGURE 22.2**

- (a) Magnetic field patterns surrounding a bar magnet as displayed with iron filings. (b) Magnetic field patterns between *dissimilar* poles of two bar magnets. (c) Magnetic field pattern between *similar* poles of two bar magnets.

**FIGURE 22.3** The direction of the magnetic force on a charged particle moving with a velocity  $\vec{v}$  in the presence of a magnetic field  $\vec{B}$ . (a) When  $\vec{v}$  is at an angle  $\theta$  to  $\vec{B}$ , the magnetic force is perpendicular to both  $\vec{v}$  and  $\vec{B}$ . (b) Oppositely directed magnetic forces are exerted on two oppositely charged particles moving with the same velocity in a magnetic field. The broken lines suggest the paths followed by the particles after the instant shown in the figure.



velocity of the particle and because its direction is perpendicular to both  $\vec{v}$  and  $\vec{B}$ . Despite this complicated behavior, these observations can be summarized in a compact way by writing the magnetic force in the form

■ Magnetic force on a charged particle moving in a magnetic field

$$\vec{F}_B = q\vec{v} \times \vec{B} \quad [22.1]$$

where the direction of the magnetic force is that of  $\vec{v} \times \vec{B}$ , which, by definition of the cross product, is perpendicular to both  $\vec{v}$  and  $\vec{B}$ . Equation 22.1 is analogous to Equation 19.4,  $\vec{F}_e = q\vec{E}$ , but is clearly more complicated. We can regard Equation 22.1 as an operational definition of the magnetic field at a point in space. The SI unit of magnetic field is the **tesla** (T), where

$$1 \text{ T} = 1 \text{ N} \cdot \text{s} / \text{C} \cdot \text{m}$$

Figure 22.4 reviews two right-hand rules for determining the direction of the cross product  $\vec{v} \times \vec{B}$  and determining the direction of  $\vec{F}_B$ . The rule in Figure 22.4a depends on our right-hand rule for the cross product in Figure 10.13. You point the four fingers of your right hand along the direction of  $\vec{v}$  with the palm facing  $\vec{B}$  and curl them toward  $\vec{B}$ . The extended thumb, which is at a right angle to the fingers, points in the direction of  $\vec{v} \times \vec{B}$ . Because  $\vec{F}_B = q\vec{v} \times \vec{B}$ ,  $\vec{F}_B$  is in the direction of your thumb if  $q$  is positive and opposite the direction of your thumb if  $q$  is negative.

A second rule is shown in Figure 22.4b. Here the thumb points in the direction of  $\vec{v}$  and the extended fingers in the direction of  $\vec{B}$ . Now, the force  $\vec{F}_B$  on a positive charge extends outward from your palm. The advantage of this rule is that the force on the charge is in the direction that you would push on something with your hand, outward from your palm. The force on a negative charge is in the opposite direction. Feel free to use either of these two right-hand rules.

The magnitude of the magnetic force is

$$F_B = |q|vB \sin \theta \quad [22.2]$$

where  $\theta$  is the angle between  $\vec{v}$  and  $\vec{B}$ . From this expression, we see that  $F_B$  is zero when  $\vec{v}$  is either parallel or antiparallel to  $\vec{B}$  ( $\theta = 0$  or  $180^\circ$ ). Furthermore, the force has its maximum value  $F_B = |q|vB$  when  $\vec{v}$  is perpendicular to  $\vec{B}$  ( $\theta = 90^\circ$ ).

There are important differences between electric and magnetic forces on charged particles:

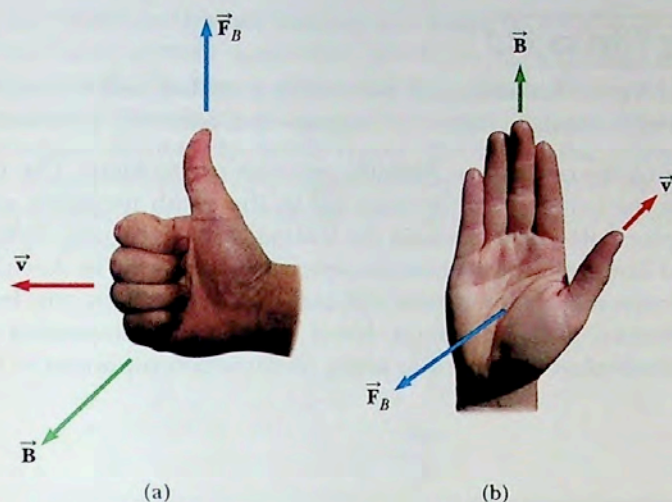


**FIGURE 22.4** Two right-hand rules for determining the direction of the magnetic force  $\vec{F}_B = q\vec{v} \times \vec{B}$  acting on a particle with charge  $q$  moving with a velocity  $\vec{v}$  in a magnetic field  $\vec{B}$ . (a) In this rule, the fingers point in the direction of  $\vec{v}$ , with  $\vec{B}$  coming out of your palm, so that you can curl your fingers in the direction of  $\vec{B}$ . The direction of  $\vec{v} \times \vec{B}$ , and the force on a positive charge, is the direction in which the thumb points. (b) In this rule, the vector  $\vec{v}$  is in the direction of your thumb and  $\vec{B}$  is in the direction of your fingers. The force  $\vec{F}_B$  on a positive charge is in the direction of your palm, as if you are pushing the particle with your hand.

- The electric force is always parallel or antiparallel to the direction of the electric field, whereas the magnetic force is perpendicular to the magnetic field.
- The electric force acts on a charged particle independent of the particle's velocity, whereas the magnetic force acts on a charged particle only when the particle is in motion and the force is proportional to the velocity.
- The electric force does work in displacing a charged particle, whereas the magnetic force associated with a constant magnetic field does *no* work when a charged particle is displaced.

This last statement is true because when a charge moves in a constant magnetic field, the magnetic force is always *perpendicular* to the displacement. That is, for a small displacement  $d\vec{s}$  of a particle, the work done by the magnetic force on the particle is  $dW = \vec{F}_B \cdot d\vec{s} = (\vec{F}_B \cdot \vec{v})dt = 0$  because the magnetic force is a vector perpendicular to  $\vec{v}$ . From this property and the work-kinetic energy theorem, we conclude that the kinetic energy of a charged particle *cannot* be altered by a constant magnetic field alone. In other words, when a charge moves with a velocity of  $\vec{v}$ , an applied magnetic field can alter the direction of the velocity vector, but it cannot change the speed of the particle.

In Figures 22.3 and 22.4, we used green arrows to represent magnetic field vectors, which will be the convention in this book. In Active Figure 22.1, we represented the magnetic field of a bar magnet with green field lines. Studying magnetic fields presents a complication that we avoided in electric fields. In our study of electric fields, we drew all electric field vectors in the plane of the page or used perspective to represent them directed at an angle to the page. The cross product in Equation 22.1 requires us to think in three dimensions for problems in magnetism. Thus, in addition to drawing vectors pointing left or right and up or down, we will need a method of drawing vectors into or out of the page. These methods of representing the vectors are illustrated in Figure 22.5. A vector coming out of the page is represented by a dot, which we can think of as the tip of the arrowhead representing the vector coming through the paper toward us (Fig. 22.5a). A vector going into the page is represented by a cross, which we can think of as the tail feathers of an arrow going into the page (Fig. 22.5b). This depiction can be used for any type of vector we will encounter: magnetic field, velocity, force, and so on.

**QUICK QUIZ 22.1** An electron moves in the plane of this paper toward the top of the page. A magnetic field is also in the plane of the page and directed toward the right. What is the direction of the magnetic force on the electron? (a) toward the top of the page (b) toward the bottom of the page (c) toward the left edge of the page (d) toward the right edge of the page (e) upward out of the page (f) downward into the page

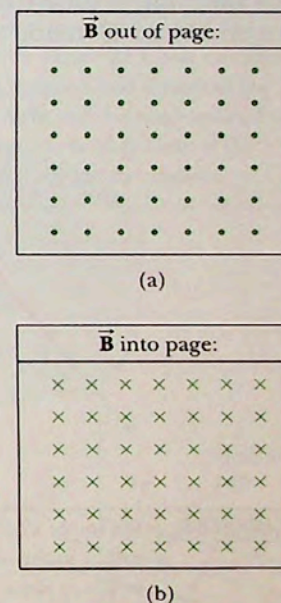


**FIGURE 22.5** (a) Magnetic field lines coming out of the paper are indicated by dots, representing the tips of arrows coming outward. (b) Magnetic field lines going into the paper are indicated by crosses, representing the feathers of arrows going inward.

### Thinking Physics 22.1

On a business trip to Australia, you take along your U.S.-made compass that you used in your Boy Scout days. Does this compass work correctly in Australia?

**Reasoning** Using the compass in Australia presents no problem. The north pole of the magnet in the compass will be attracted to the south magnetic pole near the north geographic pole, just as it was in the United States. The only difference in the magnetic field lines is that they have an upward component in Australia, where they have a downward component in the United States. When you hold the compass in a horizontal plane, it cannot detect the vertical component of the field; however, it only displays the direction of the horizontal component of the magnetic field. ■

### EXAMPLE 22.1 An Electron Moving in a Magnetic Field

An electron in a television picture tube moves toward the front of the tube with a speed of  $8.0 \times 10^6$  m/s along the  $x$  axis (Fig. 22.6). The neck of the tube is surrounded by a coil of wire that creates a magnetic field of magnitude 0.025 T, directed at an angle of  $60^\circ$  to the  $x$  axis and lying in the  $xy$  plane. Calculate the magnetic force on and acceleration of the electron.

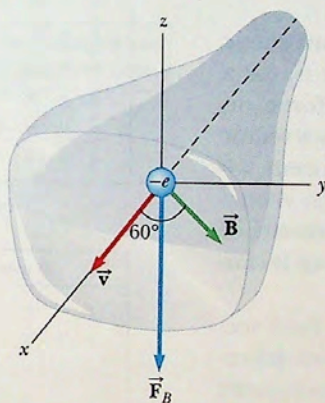


**FIGURE 22.6** (Example 22.1) The magnetic force  $\vec{F}_B$  on the electron is in the negative  $z$  direction when  $\vec{v}$  and  $\vec{B}$  lie in the  $xy$  plane.

**Solution** Using Equation 22.2, we find the magnitude of the magnetic force:

$$\begin{aligned} F_B &= |q|vB \sin \theta \\ &= (1.60 \times 10^{-19} \text{ C})(8.0 \times 10^6 \text{ m/s})(0.025 \text{ T})(\sin 60^\circ) \\ &= 2.8 \times 10^{-14} \text{ N} \end{aligned}$$

Because  $\vec{v} \times \vec{B}$  is in the positive  $z$  direction (from the right-hand rule) and the charge is negative,  $\vec{F}_B$  is in the negative  $z$  direction.

Once we have determined the magnetic force, we have a Chapter 4 problem because the electron is a particle under a net force and the acceleration is determined from Newton's second law. The mass of the electron is  $m_e = 9.1 \times 10^{-31}$  kg, and so its acceleration is

$$a = \frac{F_B}{m_e} = \frac{2.8 \times 10^{-14} \text{ N}}{9.1 \times 10^{-31} \text{ kg}} = 3.1 \times 10^{16} \text{ m/s}^2$$

in the negative  $z$  direction.

## 22.3 MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

In Section 22.2, we found that the magnetic force acting on a charged particle moving in a magnetic field is perpendicular to the velocity of the particle and that, consequently, the work done on the particle by the magnetic force is zero. Consider now the special case of a positively charged particle moving in a uniform magnetic field when the initial velocity vector of the particle is perpendicular to the field. Let us assume that the direction of the magnetic field is into the page. Active Figure 22.7 shows that the particle moves in a circular path whose plane is perpendicular to the magnetic field.

**Solution** To calculate the field inside the toroid, we evaluate the line integral of  $\vec{B} \cdot d\vec{s}$  over the circular amperian loop of radius  $r$  in the plane of Figure 22.33. By symmetry, we see that conditions 1 and 2 apply: the magnetic field is constant in magnitude on this circle and tangent to it, so  $\vec{B} \cdot d\vec{s} = B ds$ .

Furthermore, note that the closed path surrounds a circular area through which  $N$  loops of wire pass, each of which carries a current  $I$ . The right side of Equation 22.29 is therefore  $\mu_0 NI$  in this case. Ampère's law

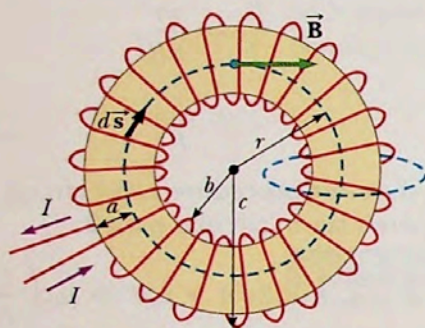


**FIGURE 22.33** (Example 22.8) A toroid consisting of many turns of wire wrapped around a doughnut-shaped structure (called a torus). If the coils are closely spaced, the field in the interior of the toroid is tangent to the dashed circle and varies as  $1/r$ . The dimension  $a$  is the cross-sectional radius of the torus. The field outside the toroid is very small and can be described by using the amperian loop at the right side, perpendicular to the page.

applied to the circular path gives

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r} \quad [22.31]$$

This result shows that  $B$  varies as  $1/r$  and hence is nonuniform within the coil. If  $r$  is large compared with the cross-sectional radius  $a$  of the torus, however, the field is approximately uniform inside the coil.

For an ideal toroid in which the turns are closely spaced, the external magnetic field is close to zero. It is not exactly zero, however. In Figure 22.33, imagine the radius  $r$  of the amperian loop to be either smaller than  $b$  or larger than  $c$ . In either case, the loop encloses zero net current, so  $\oint \vec{B} \cdot d\vec{s} = 0$ . We might be tempted to claim that this expression proves that  $\vec{B} = 0$ , but it does not. Consider the amperian loop on the right side of the toroid in Figure 22.33. The plane of this loop is perpendicular to the page and the toroid passes through the loop. As charges enter the toroid as indicated by the current directions in Figure 22.33, they work their way counterclockwise around the toroid. Thus, a current passes through the perpendicular amperian loop! This current is small, but it is not zero. As a result, the toroid acts as a current loop and produces a weak external field of the form shown in Figure 22.26a. The reason that  $\oint \vec{B} \cdot d\vec{s} = 0$  for the amperian loops of radius  $r < b$  and  $r > c$  in the plane of the page is that the field lines are perpendicular to  $d\vec{s}$  (condition 3), not because  $\vec{B} = 0$  (condition 4).

## 22.10 THE MAGNETIC FIELD OF A SOLENOID

A solenoid is a long wire wound in the form of a helix. If the turns are closely spaced, this configuration can produce a reasonably uniform magnetic field throughout the volume enclosed by the solenoid, except close to its ends. Each of the turns can be modeled as a circular loop, and the net magnetic field is the vector sum of the fields due to all the turns.

If the turns are closely spaced and the solenoid is of finite length, the field lines are as shown in Figure 22.34a. In this case, the field lines diverge from one end and converge at the opposite end. An inspection of this field distribution exterior to the solenoid shows a similarity to the field of a bar magnet (Fig. 22.34b). Hence, one end of the solenoid behaves like the north pole of a magnet and the opposite end behaves like the south pole. As the length of the solenoid increases, the field within it becomes more and more uniform. When the solenoid's turns are closely spaced and its length is large compared with its radius, it approaches the case of an *ideal solenoid*. For an ideal solenoid, the field outside the solenoid is negligible and the field inside is uniform. We will use the ideal solenoid as a simplification model for a real solenoid.

If we consider the amperian loop perpendicular to the page in Figure 22.35, surrounding the ideal solenoid, we see that it encloses a small current as the charges in the wire move coil by coil along the length of the solenoid. Thus, there is a

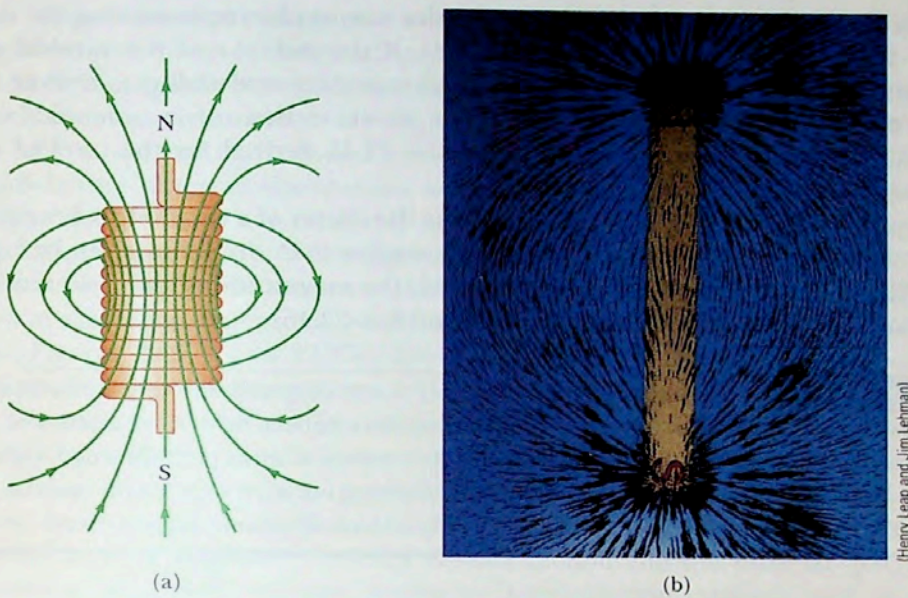


**FIGURE 22.34** (a) Magnetic field lines for a tightly wound solenoid of finite length carrying a steady current. The field in the space enclosed by the solenoid is nearly uniform and strong. Note that the field lines resemble those of a bar magnet and that the solenoid effectively has north and south poles. (b) The magnetic field pattern of a bar magnet, displayed with iron filings.

nonzero magnetic field outside the solenoid. It is a weak field, with circular field lines, like those due to a line of current as in Figure 22.23. For an ideal solenoid, it is the only field external to the solenoid. We can eliminate this field in Figure 22.35 by adding a second layer of turns of wire outside the first layer. If the first layer of turns is wrapped so that the turns progress from the bottom of Figure 22.35 to the top and the second layer has turns progressing from the top to the bottom, the net current along the axis is zero.

We can use Ampère's law to obtain an expression for the magnetic field inside an ideal solenoid. A longitudinal cross-section of part of our ideal solenoid (Fig. 22.35) carries current  $I$ . Here,  $\vec{B}$  inside the ideal solenoid is uniform and parallel to the axis. Consider a rectangular path of length  $\ell$  and width  $w$  as shown in Figure 22.35. We can apply Ampère's law to this path by evaluating the integral of  $\vec{B} \cdot d\vec{s}$  over each of the four sides of the rectangle. The contribution along side 3 is zero because the magnetic field lines are perpendicular to the path in this region, which matches condition 3 in Section 22.9. The contributions from sides 2 and 4 are both zero because  $\vec{B}$  is perpendicular to  $d\vec{s}$  along these paths, both inside and outside the solenoid. Side 1, whose length is  $\ell$ , gives a contribution to the integral because  $\vec{B}$  along this portion of the path is constant in magnitude and parallel to  $d\vec{s}$ , which matches conditions 1 and 2. The integral over the closed rectangular path therefore has the value

$$\oint \vec{B} \cdot d\vec{s} = \int_{\text{side 1}} \vec{B} \cdot d\vec{s} = B \int_{\text{side 1}} ds = B\ell$$

The right side of Ampère's law involves the *total* current that passes through the surface bounded by the path of integration. In our case, the total current through the rectangular path equals the current through each turn of the solenoid multiplied by the number of turns enclosed by the path of integration. If  $N$  is the number of turns in the length  $\ell$ , the total current through the rectangle equals  $NI$ . Ampère's law applied to this path therefore gives

$$\begin{aligned} \oint \vec{B} \cdot d\vec{s} &= B\ell = \mu_0 NI \\ B &= \mu_0 \frac{N}{\ell} I = \mu_0 nI \end{aligned} \quad [22.32]$$

where  $n = N/\ell$  is the number of turns *per unit length* (not to be confused with  $N$ , the number of turns).

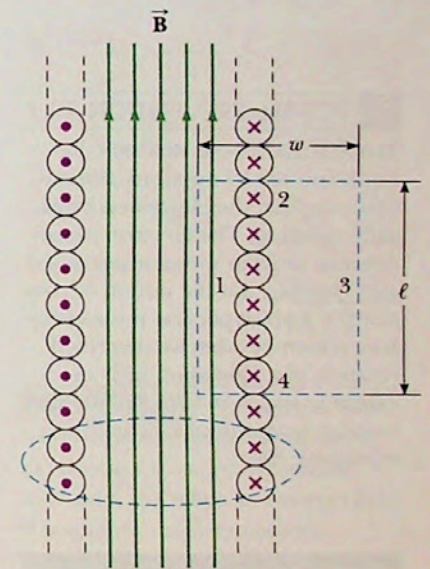


**FIGURE 22.35** Cross-sectional view of an ideal solenoid, where the interior magnetic field is uniform and the exterior field is close to zero. Ampère's law applied to the circular path near the bottom whose plane is perpendicular to the page can be used to show that there is a weak field outside the solenoid. Ampère's law applied to the rectangular dashed path in the plane of the page can be used to calculate the magnitude of the interior field.

■ Magnetic field inside a long solenoid

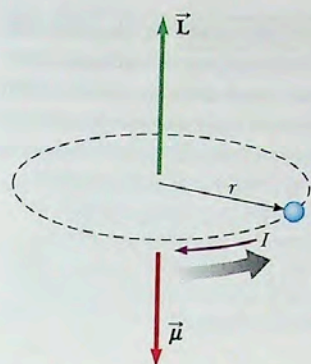


**FIGURE 22.36** An electron moving in a circular orbit of radius  $r$  has an angular momentum  $\vec{L}$  in one direction and a magnetic moment  $\vec{\mu}$  in the opposite direction. The motion of the electron in the direction of the gray arrow results in a current in the direction shown.

### PITFALL PREVENTION 22.3

**THE ELECTRON DOES NOT SPIN** Do not be misled by the word *spin* into believing that the electron is physically spinning. The electron has an intrinsic angular momentum *as if it were spinning*, but the notion of rotation for a point particle is meaningless; remember that we described rotation of a *rigid object*, with an extent in space, in Chapter 10. Spin angular momentum is actually a relativistic effect.

**TABLE 22.1**

**Magnetic Moments of Some Atoms and Ions**

| Atom or Ion      | Magnet Moment per Atom or Ion ( $10^{-24}$ J/T) |
|------------------|-------------------------------------------------|
| H                | 9.27                                            |
| He               | 0                                               |
| Ne               | 0                                               |
| Fe               | 2.06                                            |
| Co               | 16.0                                            |
| Ni               | 5.62                                            |
| Gd               | 65.8                                            |
| Dy               | 92.7                                            |
| Co <sup>2+</sup> | 44.5                                            |
| Ni <sup>2+</sup> | 29.7                                            |
| Fe <sup>2+</sup> | 50.1                                            |
| Ce <sup>3+</sup> | 19.8                                            |
| Yb <sup>3+</sup> | 37.1                                            |

We also could obtain this result in a simpler manner by reconsidering the magnetic field of a toroidal coil (Example 22.8). If the radius  $r$  of the toroid containing  $N$  turns is large compared with its cross-sectional radius  $a$ , a short section of the toroidal coil approximates a short section of a solenoid with  $n = N/2\pi r$ . In this limit, we see that Equation 22.31 derived for the toroid agrees with Equation 22.32.

Equation 22.32 is valid only for points near the center of a very long solenoid. As you might expect, the field near each end is smaller than the value given by Equation 22.32. At the very end of a long solenoid, the magnitude of the field is one-half that of the field at the center (see Problem 22.46).

**QUICK QUIZ 22.7** Consider a solenoid that is very long compared with the radius. Of the following choices, the most effective way to increase the magnetic field in the interior of the solenoid is to (a) double its length, keeping the number of turns per unit length constant, (b) reduce its radius by half, keeping the number of turns per unit length constant, or (c) overwrap the entire solenoid with an additional layer of current-carrying wire.

## 22.11 MAGNETISM IN MATTER

The magnetic field produced by a current in a coil of wire gives a hint about why some materials are magnetic. To understand why some materials are magnetic, it is instructive to begin this discussion with the Bohr model of the atom, in which electrons are assumed to move in circular orbits about the much more massive nucleus. Figure 22.36 shows the angular momentum associated with the electron. In the Bohr model, each electron, with a charge of magnitude  $1.6 \times 10^{-19}$  C, circles the atom once in about  $10^{-16}$  s. If we divide the electronic charge by this time interval, we find that the orbiting electron is equivalent to a current of  $1.6 \times 10^{-3}$  A. Each orbiting electron is therefore viewed as a tiny current loop with a corresponding magnetic moment. Because the charge of the electron is negative, the magnetic moment is directed opposite to the angular momentum as shown in Figure 22.36.

In most substances, the magnetic moment of one electron in an atom is canceled by that of another electron in the atom, orbiting in the opposite direction. The net result is that **the magnetic effect produced by the orbital motion of electrons is either zero or very small for most materials.**

In addition to its orbital angular momentum, an electron has an **intrinsic angular momentum**, called **spin**, which also contributes to its magnetic moment. The spin of an electron is an angular momentum separate from its orbital angular momentum, just as the spin of the Earth is separate from its orbital motion about the Sun. Even if the electron is at rest, it still has an angular momentum associated with spin. We shall investigate spin more deeply in Chapter 29.

In atoms or ions containing multiple electrons, many electrons are paired with their spins in opposite directions, an arrangement that results in a cancellation of the spin magnetic moments. An atom with an odd number of electrons, however, must have at least one “unpaired” electron and a corresponding spin magnetic moment. The net magnetic moment of the atom leads to various types of magnetic behavior. The magnetic moments of several atoms and ions are listed in Table 22.1.

### Ferromagnetic Materials

Iron, cobalt, nickel, gadolinium, and dysprosium are strongly magnetic materials and are said to be **ferromagnetic**. Ferromagnetic substances, used to fabricate permanent magnets, contain atoms with spin magnetic moments that tend to align parallel to each other even in a weak external magnetic field. Once the moments are aligned, the substance remains magnetized after the external field is removed.

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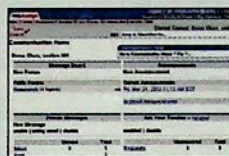
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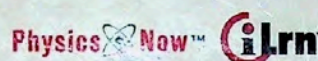


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