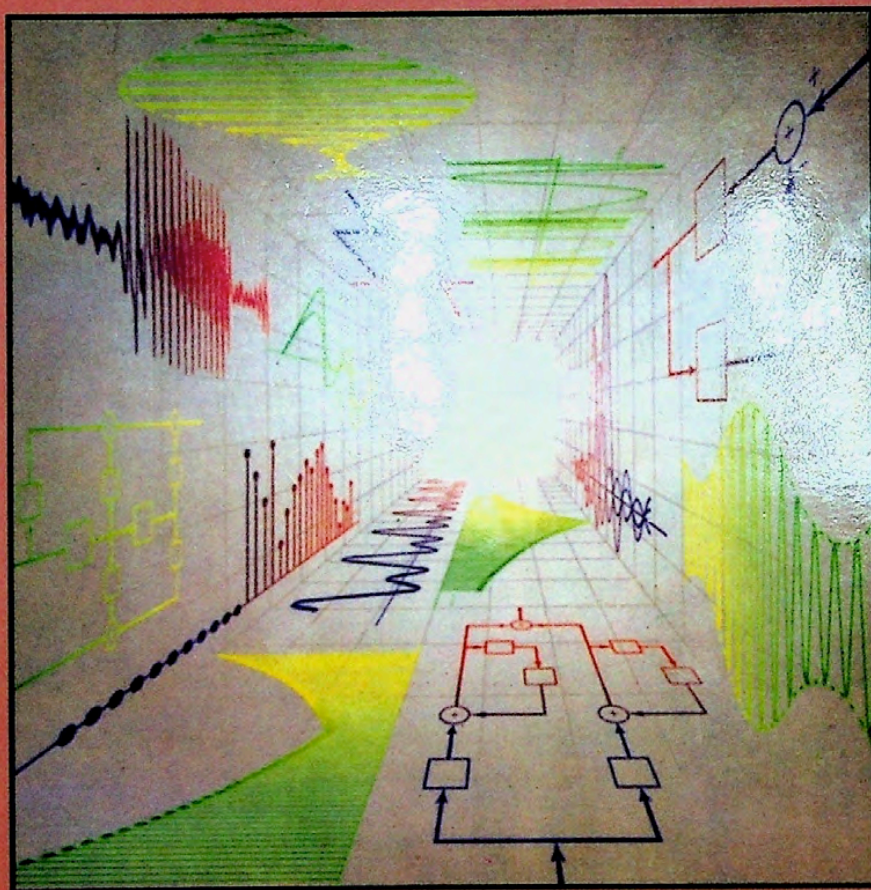


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Second Edition

SIGNALS & SYSTEMS



**ALAN V. OPPENHEIM
ALAN S. WILLSKY**

WITH S. HAMID NAWAB



SECOND EDITION

SIGNALS & SYSTEMS

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by Alan V. Oppenheim and Alan S. Willsky with S. Hamid Nawab

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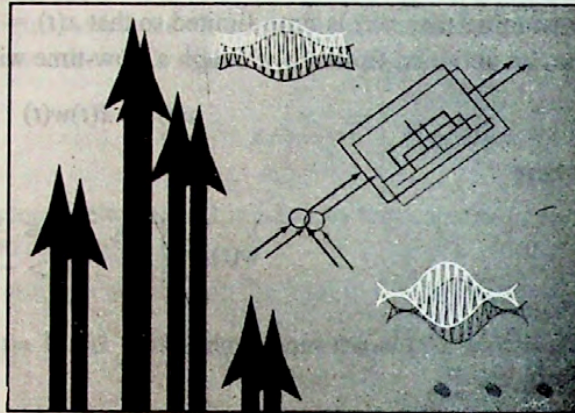
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8

COMMUNICATION SYSTEMS



8.0 INTRODUCTION

Communication systems play a key role in our modern world in transmitting information between people, systems, and computers. In general terms, in all communication systems the information at the source is first processed by a transmitter or modulator to change it into a form suitable for transmission over the communication channel. At the receiver, the signal is then recovered through appropriate processing. This processing is required for a variety of reasons. In particular, quite typically, any specific communication channel has associated with it a frequency range over which it is best suited for transmitting a signal and outside of which communication is severely degraded or impossible. For example, the atmosphere will rapidly attenuate signals in the audible frequency range (10 Hz to 20 kHz), whereas it will propagate signals at a higher frequency range over longer distances. Thus, in transmitting audio signals such as speech or music over a communication channel that relies on propagation through the atmosphere, the transmitter first embeds the signal through an appropriate process into another, higher frequency signal.

Many of the concepts and techniques we have developed in the earlier chapters of this text play a central role in the analysis and design of communication systems. As with any concept that is closely tied to a wide variety of important applications, there are a large number of detailed issues to be considered, and, as indicated in the bibliography, there are many excellent texts on the subject. While a full and detailed analysis of communication systems is well beyond the scope of our discussions here, with the background of the previous chapters we are now in a position to introduce some of the basic principles and issues encountered in the design and analysis of these systems.

The general process of embedding an information-bearing signal into a second signal is typically referred to as *modulation*. Extracting the information-bearing signal

is known as *demodulation*. As we will see, modulation techniques not only allow us to embed information into signals that can be transmitted effectively, but also make possible the simultaneous transmission of more than one signal with overlapping spectra over the same channel, through a concept referred to as *multiplexing*.

There are a wide variety of modulation methods used in practice, and in this chapter we examine several of the most important of these. One large class of modulation methods relies on the concept of *amplitude modulation* or AM in which the signal we wish to transmit is used to modulate the amplitude of another signal. A very common form of amplitude modulation is *sinusoidal amplitude modulation*, which we explore in some detail in Sections 8.1–8.4 together with the related concepts of frequency-division multiplexing. Another important class of AM systems involves the modulation of the amplitude of a pulsed signal, and in Sections 8.5 and 8.6 we examine this form of modulation as well as the concept of time-division multiplexing. In Section 8.7 we then examine a different form of modulation, namely *sinusoidal frequency modulation* in which the information-bearing signal is used to vary the frequency of a sinusoidal signal.

All of the discussion up through Section 8.7 focuses attention on continuous-time signals, since most transmission media, such as the atmosphere, are best thought of as continuous-time phenomena. Nevertheless, not only is it possible to develop analogous techniques for discrete-time signals, but it is of considerable practical importance to consider modulation concepts involving such signals, and in Section 8.8 we examine some of the basic ideas behind the communication of discrete-time signals.

8.1 COMPLEX EXPONENTIAL AND SINUSOIDAL AMPLITUDE MODULATION

Many communication systems rely on the concept of sinusoidal amplitude modulation, in which a complex exponential or sinusoidal signal $c(t)$ has its amplitude multiplied (modulated) by the information-bearing signal $x(t)$. The signal $x(t)$ is typically referred to as the *modulating signal* and the signal $c(t)$ as the *carrier signal*. The modulated signal $y(t)$ is then the product of these two signals:

$$y(t) = x(t)c(t)$$

As we discussed in Section 8.0, an important objective in modulation is to produce a signal whose frequency range is suitable for transmission over the communication channel to be used. In telephone transmission systems, for example, long-distance transmission is often accomplished over microwave or satellite links. The individual voice signals are in the frequency range 200 Hz to 4 kHz, whereas a microwave link requires signals in the range 300 megahertz (MHz) to 300 gigahertz (GHz), and communication satellite links operate in the frequency range from a few hundred MHz to over 40 GHz. Thus, for transmission over these channels, the information in a voice signal must be shifted into these higher ranges of frequency. As we will see in this section, sinusoidal amplitude modulation achieves such a shift in frequency in a very simple manner.

8.1.1 Amplitude Modulation with a Complex Exponential Carrier

There are two common forms of sinusoidal amplitude modulation, one in which the carrier signal is a complex exponential of the form

$$c(t) = e^{j(\omega_c t + \theta_c)} \quad (8.1)$$

and the second in which the carrier signal is sinusoidal and of the form

$$c(t) = \cos(\omega_c t + \theta_c). \quad (8.2)$$

In both cases, the frequency ω_c is referred to as the *carrier frequency*. Let us consider first the case of a complex exponential carrier, and for convenience, let us choose $\theta_c = 0$, so that the modulated signal is

$$y(t) = x(t)e^{j\omega_c t}. \quad (8.3)$$

From the multiplication property (Section 4.5), and with $X(j\omega)$, $Y(j\omega)$, and $C(j\omega)$ denoting the Fourier transforms of $x(t)$, $y(t)$, and $c(t)$, respectively,

$$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * C(j\omega). \quad (8.4)$$

For $c(t)$ a complex exponential as given in eq. (8.1),

$$C(j\omega) = 2\pi\delta(\omega - \omega_c), \quad (8.5)$$

and hence,

$$Y(j\omega) = X(j\omega - j\omega_c). \quad (8.6)$$

Thus, the spectrum of the modulated output $y(t)$ is simply that of the input, shifted in frequency by an amount equal to the carrier frequency ω_c . For example, with $X(j\omega)$ band limited with highest frequency ω_M (and bandwidth $2\omega_M$), as depicted in Figure 8.1(a), the output spectrum $Y(j\omega)$ is that shown in Figure 8.1(c).

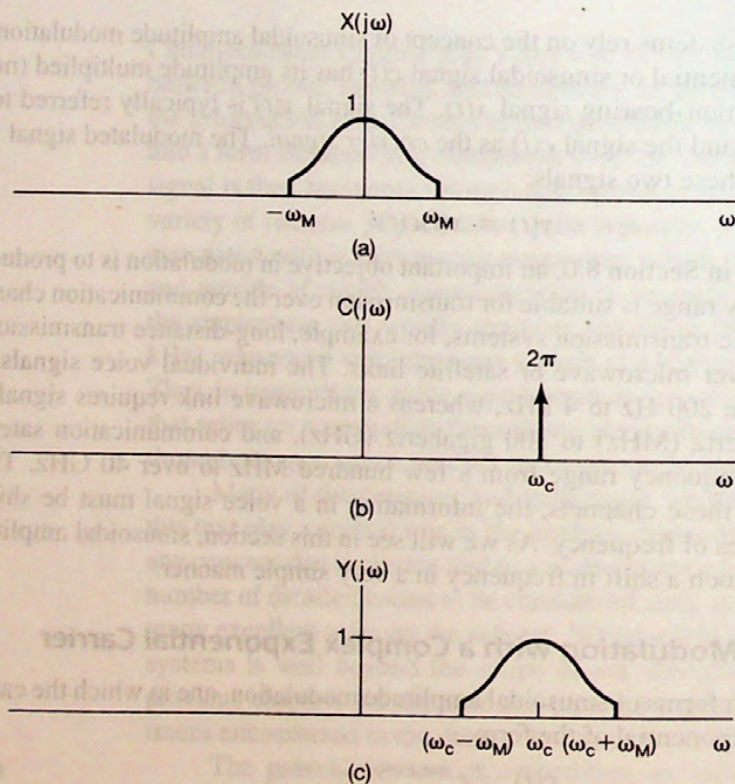


Figure 8.1 Effect in the frequency domain of amplitude modulation with a complex exponential carrier: (a) spectrum of modulating signal $x(t)$; (b) spectrum of carrier $c(t) = e^{j\omega_c t}$; (c) spectrum of amplitude-modulated signal $y(t) = x(t)e^{j\omega_c t}$.

From eq. (8.3), it is clear that $x(t)$ can be recovered from the modulated signal $y(t)$ by multiplying by the complex exponential $e^{-j\omega_c t}$; that is,

$$x(t) = y(t)e^{-j\omega_c t}. \quad (8.7)$$

In the frequency domain, this has the effect of shifting the spectrum of the modulated signal back to its original position on the frequency axis. The process of recovering the original signal from the modulated signal is referred to as *demodulation*, a topic we discuss at more length in Section 8.2.

Since $e^{j\omega_c t}$ is a complex signal, eq. (8.3) can be rewritten as

$$y(t) = x(t) \cos \omega_c t + jx(t) \sin \omega_c t. \quad (8.8)$$

Implementation of eq. (8.7) or (8.8) with $x(t)$ real utilizes two separate multipliers and two sinusoidal carrier signals that have a phase difference of $\pi/2$, as depicted in Figure 8.2 for $c(t)$ given by eq. (8.1). In Section 8.4 we give an example of one of the applications in which there are particular advantages to using a system, such as in Figure 8.2, employing two sinusoidal carriers with a phase difference of $\pi/2$.

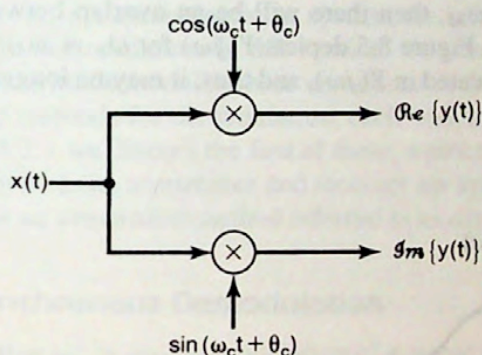


Figure 8.2 Implementation of amplitude modulation with a complex exponential carrier $c(t) = e^{j(\omega_c t + \theta_c)}$.

8.1.2 Amplitude Modulation with a Sinusoidal Carrier

In many situations, using a sinusoidal carrier of the form of eq. (8.2) is often simpler than and equally as effective as using a complex exponential carrier. In effect, using a sinusoidal carrier corresponds to retaining only the real or imaginary part of the output of Figure 8.2. A system that uses a sinusoidal carrier is depicted in Figure 8.3.

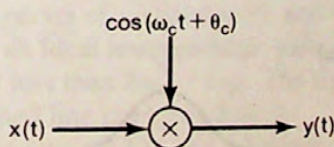


Figure 8.3 Amplitude modulation with a sinusoidal carrier.

The effect of amplitude modulation with a sinusoidal carrier in the form of eq. (8.2) can be analyzed in a manner identical to that in the preceding subsection. Again, for

convenience we choose $\theta_c = 0$. In this case, the spectrum of the carrier signal is

$$C(j\omega) = \pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)], \quad (8.9)$$

and thus, from eq. (8.4),

$$Y(j\omega) = \frac{1}{2}[X(j\omega - j\omega_c) + X(j\omega + j\omega_c)]. \quad (8.10)$$

With $X(j\omega)$ as depicted in Figure 8.4(a), the spectrum of $y(t)$ is that shown in Figure 8.4(c). Note that there is now a replication of the spectrum of the original signal, centered around both $+\omega_c$ and $-\omega_c$. As a consequence, $x(t)$ is recoverable from $y(t)$ only if $\omega_M \leq \omega_c$, since otherwise the two replications will overlap in frequency. This is in contrast to the case of a complex exponential carrier, for which a replication of the spectrum of the original signal is centered only around ω_c . Specifically, as we saw in Section 8.1.1, in the case of amplitude modulation with a complex exponential carrier, $x(t)$ can always be recovered from $y(t)$ for any choice of ω_c by shifting the spectrum back to its original location by multiplying by $e^{-j\omega_c t}$, as in eq. (8.7). With a sinusoidal carrier, on the other hand, as we see from Figure 8.4, if $\omega_c < \omega_M$, then there will be an overlap between the two replications of $X(j\omega)$. For example, Figure 8.5 depicts $Y(j\omega)$ for $\omega_c = \omega_M/2$. Clearly, the spectrum of $x(t)$ is no longer replicated in $Y(j\omega)$, and thus, it may no longer be possible to recover $x(t)$ from $y(t)$.

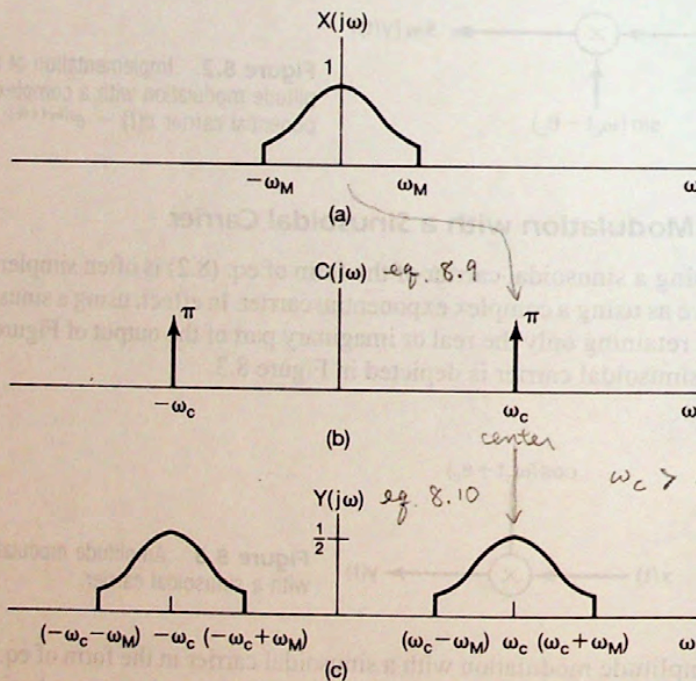


Figure 8.4 Effect in the frequency domain of amplitude modulation with a sinusoidal carrier: (a) spectrum of modulating signal $x(t)$; (b) spectrum of carrier $c(t) = \cos \omega_c t$; (c) spectrum of amplitude-modulated signal.

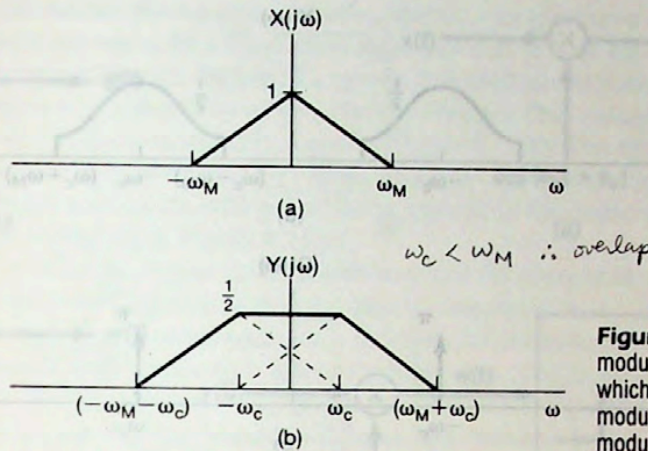


Figure 8.5 Sinusoidal amplitude modulation with carrier $\cos \omega_c t$ for which $\omega_c = \omega_M/2$: (a) spectrum of modulating signal; (b) spectrum of modulated signal.

8.2 DEMODULATION FOR SINUSOIDAL AM

At the receiver in a communication system, the information-bearing signal $x(t)$ is recovered through demodulation. In this section, we examine the process of demodulation for sinusoidal amplitude modulation, as introduced in the previous section. There are two commonly used methods for demodulation, each with its own advantages and disadvantages. In Section 8.2.1 we discuss the first of these, a process referred to as *synchronous demodulation*, in which the transmitter and receiver are synchronized in phase. In Section 8.2.2, we describe an alternative method referred to as *asynchronous demodulation*.

8.2.1 Synchronous Demodulation

Assuming that $\omega_c > \omega_M$, demodulation of a signal that was modulated with a sinusoidal carrier is relatively straightforward. Specifically, consider the signal

$$y(t) = x(t) \cos \omega_c t. \quad (8.11)$$

As was suggested in Example 4.21, the original signal can be recovered by modulating $y(t)$ with the same sinusoidal carrier and applying a lowpass filter to the result. To see this, consider

$$w(t) = y(t) \cos \omega_c t. \quad (8.12)$$

Figure 8.6 shows the spectra of $y(t)$ and $w(t)$, and we observe that $x(t)$ can be recovered from $w(t)$ by applying an ideal lowpass filter with a gain of 2 and a cutoff frequency that is greater than ω_M and less than $2\omega_c - \omega_M$. The frequency response of the lowpass filter is indicated by the dashed line in Figure 8.6(c).

The basis for using eq. (8.12) and a lowpass filter to demodulate $y(t)$ can also be seen algebraically. From eqs. (8.11) and (8.12), it follows that

$$w(t) = x(t) \cos^2 \omega_c t,$$

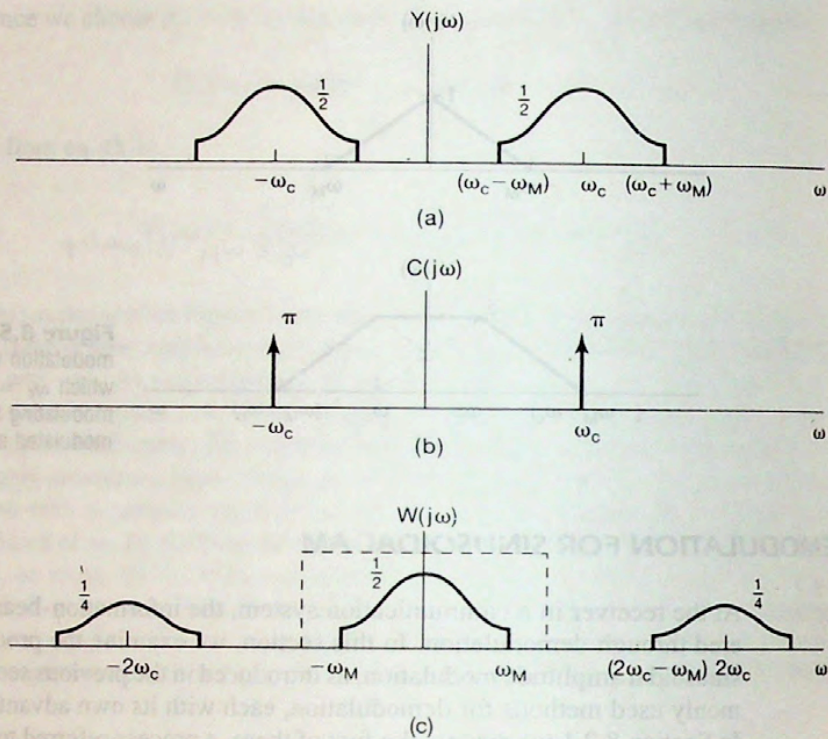


Figure 8.6 Demodulation of an amplitude-modulated signal with a sinusoidal carrier: (a) spectrum of modulated signal; (b) spectrum of carrier signal; (c) spectrum of modulated signal multiplied by the carrier. The dashed line indicates the frequency response of a lowpass filter used to extract the demodulated signal.

or, using the trigonometric identity

$$\cos^2 \omega_c t = \frac{1}{2} + \frac{1}{2} \cos 2\omega_c t,$$

we can rewrite $w(t)$ as

$$w(t) = \frac{1}{2} x(t) + \frac{1}{2} x(t) \cos 2\omega_c t. \quad (8.13)$$

Thus, $w(t)$ consists of the sum of two terms, namely one-half the original signal and one-half the original signal modulated with a sinusoidal carrier at twice the original carrier frequency ω . Both of these terms are apparent in the spectrum shown in Figure 8.6(c). Applying the lowpass filter to $w(t)$ corresponds to retaining the first term on the right-hand side of eq. (8.13) and eliminating the second term.

The overall system for amplitude modulation and demodulation using a complex exponential carrier is depicted in Figure 8.7, and the overall system for modulation and demodulation using a sinusoidal carrier is depicted in Figure 8.8. In these figures, we have indicated the more general case in which, for both the complex exponential and the sinusoidal carrier, a carrier phase θ_c is included. The modification of the preceding analysis so as to include θ_c is straightforward and is considered in Problem 8.21.

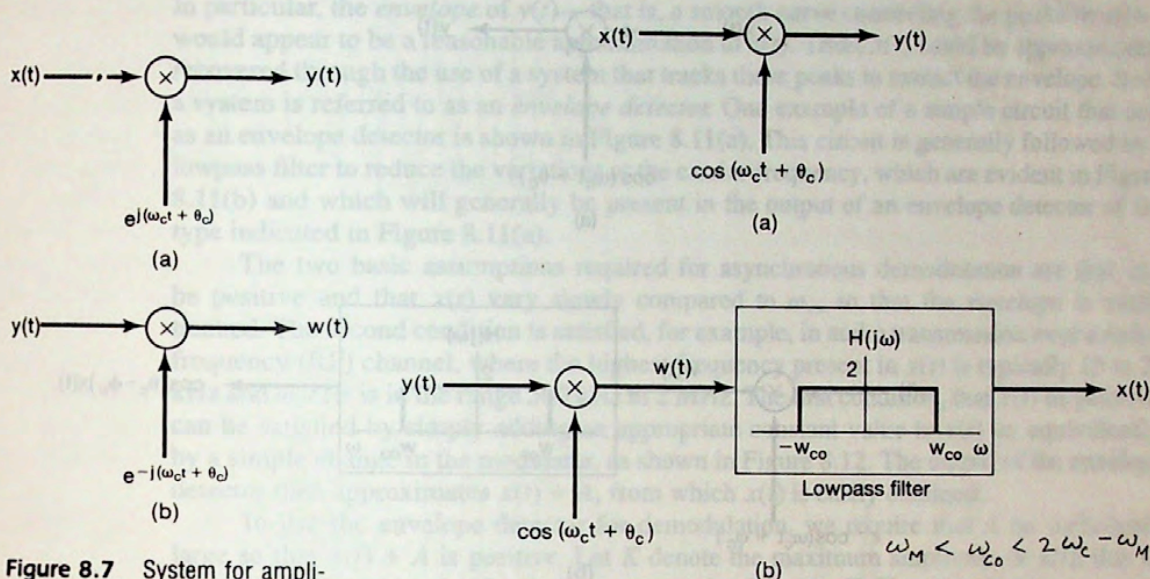


Figure 8.7 System for amplitude modulation and demodulation using a complex exponential carrier: (a) modulation; (b) demodulation.

Figure 8.8 Amplitude modulation and demodulation with a sinusoidal carrier: (a) modulation system; (b) demodulation system. The lowpass filter cut-off frequency ω_{co} is greater than ω_M and less than $2\omega_c - \omega_M$.

In the systems of Figures 8.7 and 8.8, the demodulating signal is assumed to be synchronized in phase with the modulating signal, and consequently the process is referred to as *synchronous demodulation*. Suppose, however, that the modulator and demodulator are not synchronized in phase. For the case of the complex exponential carrier, with θ_c denoting the phase of the modulating carrier and ϕ_c the phase of the demodulating carrier,

$$y(t) = e^{j(\omega_c t + \theta_c)} x(t), \quad (8.14)$$

$$w(t) = e^{-j(\omega_c t + \phi_c)} y(t), \quad (8.15)$$

and consequently,

$$w(t) = e^{j(\theta_c - \phi_c)} x(t). \quad (8.16)$$

Thus, if $\theta_c \neq \phi_c$, $w(t)$ will have a complex amplitude factor. For the particular case in which $x(t)$ is positive, $x(t) = |w(t)|$, and thus $x(t)$ can be recovered by taking the magnitude of the demodulated signal.

For the sinusoidal carrier, again let θ_c and ϕ_c denote the phases of the modulating and demodulating carriers, respectively, as indicated in Figure 8.9. The input to the lowpass filter is now

$$w(t) = x(t) \cos(\omega_c t + \theta_c) \cos(\omega_c t + \phi_c), \quad (8.17)$$

or, using the trigonometric identity

$$\cos(\omega_c t + \theta_c) \cos(\omega_c t + \phi_c) = \frac{1}{2} \cos(\theta_c - \phi_c) + \frac{1}{2} \cos(2\omega_c t + \theta_c + \phi_c), \quad (8.18)$$

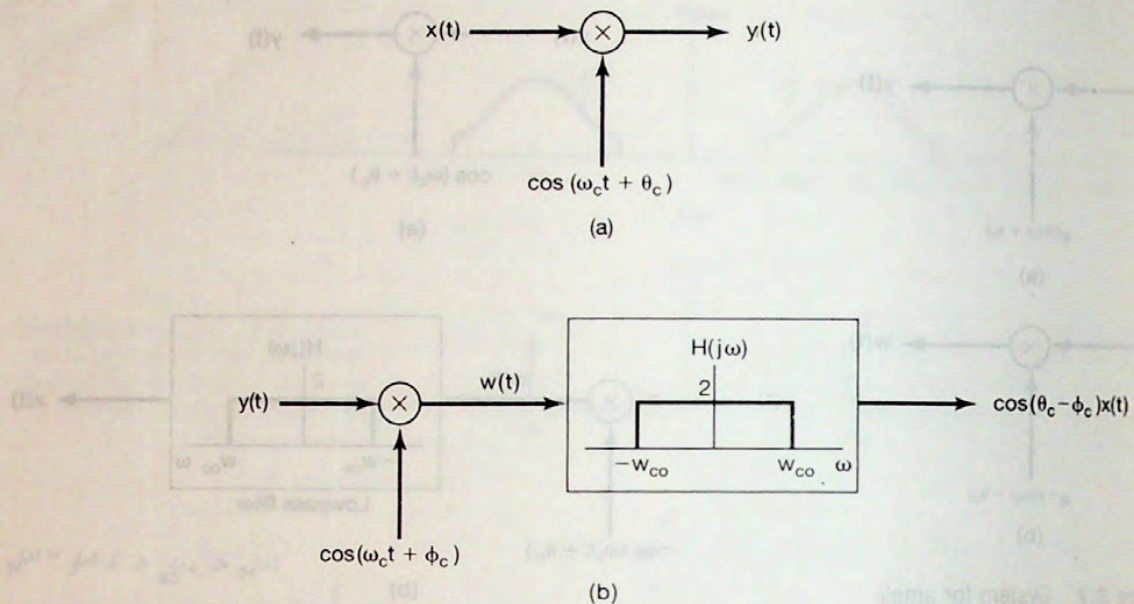


Figure 8.9 Sinusoidal amplitude modulation and demodulation system for which the carrier signals and the modulator and demodulator are not synchronized: (a) modulator; (b) demodulator.

we have

$$w(t) = \frac{1}{2} \cos(\theta_c - \phi_c) x(t) + \frac{1}{2} x(t) \cos(2\omega_c t + \theta_c + \phi_c), \quad (8.19)$$

and the output of the lowpass filter is then $x(t)$ multiplied by the amplitude factor $\cos(\theta_c - \phi_c)$. If the oscillators in the modulator and demodulator are in phase, $\theta_c = \phi_c$, and the output of the lowpass filter is $x(t)$. On the other hand, if these oscillators have a phase difference of $\pi/2$, the output will be zero. In general, for a maximum output signal, the oscillators should be in phase. Of even more importance, the phase relation between the two oscillators must be maintained over time, so that the amplitude factor $\cos(\theta_c - \phi_c)$ does not vary. This requires careful synchronization between the modulator and the demodulator, which is often difficult, particularly when they are geographically separated, as is typical in a communication system. The corresponding effects of, and the need for, synchronization not only between the phase of the modulator and demodulator, but between the frequencies of the carrier signals used in both, are explored in detail in Problem 8.23.

8.2.2 Asynchronous Demodulation

In many systems that employ sinusoidal amplitude modulation, an alternative demodulation procedure referred to as *asynchronous demodulation* is commonly used. Asynchronous demodulation avoids the need for synchronization between the modulator and demodulator. In particular, suppose that $x(t)$ is always positive and that the carrier frequency ω_c is much higher than ω_M , the highest frequency in the modulating signal. The modulated signal $y(t)$ will then have the general form illustrated in Figure 8.10.

In particular, the *envelope* of $y(t)$ —that is, a smooth curve connecting the peaks in $y(t)$ —would appear to be a reasonable approximation to $x(t)$. Thus, $x(t)$ could be approximately recovered through the use of a system that tracks these peaks to extract the envelope. Such a system is referred to as an *envelope detector*. One example of a simple circuit that acts as an envelope detector is shown in Figure 8.11(a). This circuit is generally followed by a lowpass filter to reduce the variations at the carrier frequency, which are evident in Figure 8.11(b) and which will generally be present in the output of an envelope detector of the type indicated in Figure 8.11(a).

The two basic assumptions required for asynchronous demodulation are that $x(t)$ be positive and that $x(t)$ vary slowly compared to ω_c , so that the envelope is easily tracked. The second condition is satisfied, for example, in audio transmission over a radio-frequency (RF) channel, where the highest frequency present in $x(t)$ is typically 15 to 20 kHz and $\omega_c/2\pi$ is in the range 500 kHz to 2 MHz. The first condition, that $x(t)$ be positive, can be satisfied by simply adding an appropriate constant value to $x(t)$ or, equivalently, by a simple change in the modulator, as shown in Figure 8.12. The output of the envelope detector then approximates $x(t) + A$, from which $x(t)$ is easily obtained.

To use the envelope detector for demodulation, we require that A be sufficiently large so that $x(t) + A$ is positive. Let K denote the maximum amplitude of $x(t)$; that is, $|x(t)| \leq K$. For $x(t) + A$ to be positive, we require that $A > K$. The ratio K/A is commonly referred to as the *modulation index* m . Expressed in percent, it is referred to as the *percent modulation*. An illustration of the output of the modulator of Figure 8.12 for $x(t)$ sinusoidal and for $m = 0.5$ (50% modulation) and $m = 1.0$ (100% modulation), is shown in Figure 8.13.

In Figure 8.14, we show a comparison of the spectra associated with the modulated signal when synchronous demodulation and when asynchronous demodulation are used. We note in particular that the output of the modulator for the asynchronous system in Figure 8.12 has an additional component $A \cos \omega_c t$ that is neither present nor necessary in the synchronous system. This is represented in the spectrum of Figure 8.14(c) by the presence of impulses at $+\omega_c$ and $-\omega_c$. For a fixed maximum amplitude K of the modulating signal, as A is decreased the relative amount of carrier present in the modulated output decreases. Since the carrier component in the output contains no information, its presence

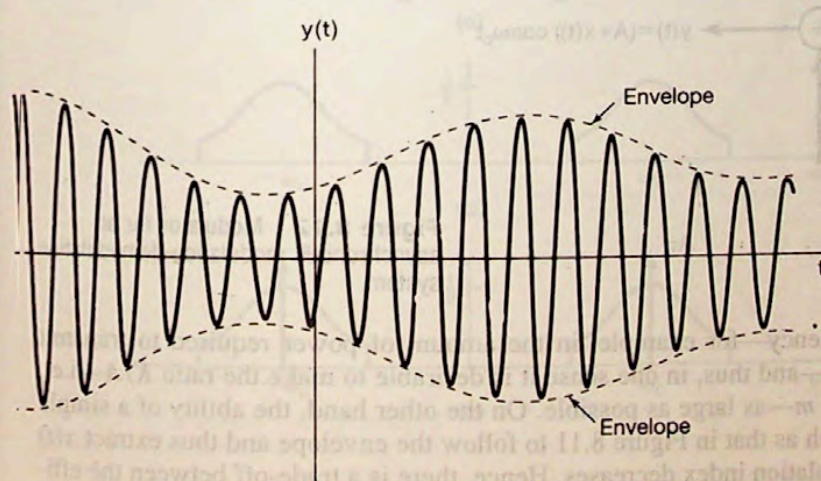


Figure 8.10 Amplitude-modulated signal for which the modulating signal is positive. The dashed curve represents the envelope of the modulated signal.

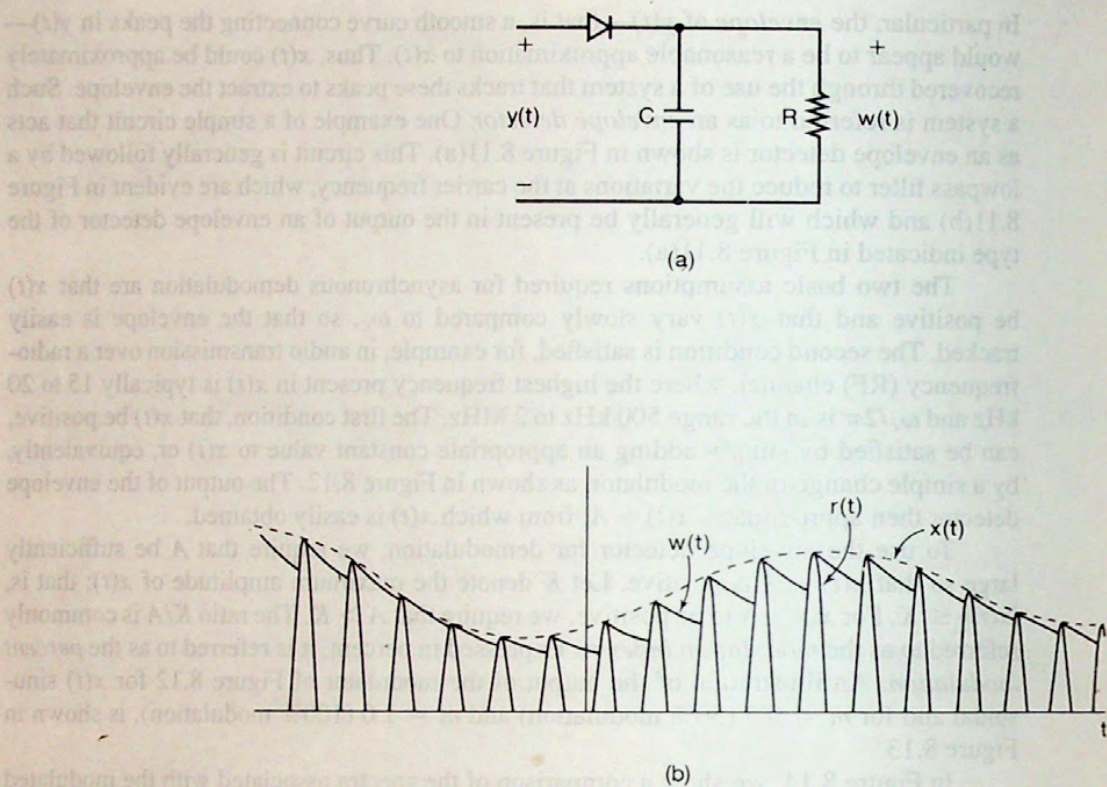


Figure 8.11 Demodulation by envelope detection: (a) circuit for envelope detection using half-wave rectification; (b) waveforms associated with the envelope detector in (a): $r(t)$ is the half-wave rectified signal, $x(t)$ is the true envelope, and $w(t)$ is the envelope obtained from the circuit in (a). The relationship between $x(t)$ and $w(t)$ has been exaggerated in (b) for purposes of illustration. In a practical asynchronous demodulation system, $w(t)$ would typically be a much closer approximation to $x(t)$ than depicted here.

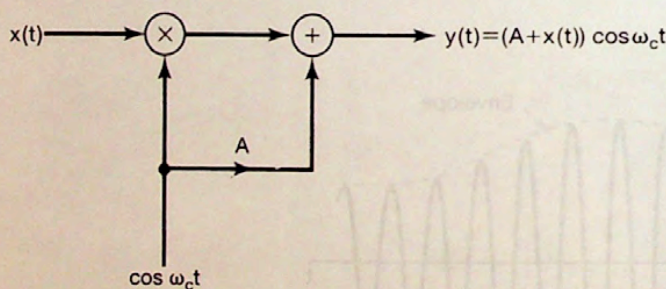


Figure 8.12 Modulator for an asynchronous modulation-demodulation system.

represents an inefficiency—for example, in the amount of power required to transmit the modulated signal—and thus, in one sense it is desirable to make the ratio K/A —i.e., the modulation index m —as large as possible. On the other hand, the ability of a simple envelope detector such as that in Figure 8.11 to follow the envelope and thus extract $x(t)$ improves as the modulation index decreases. Hence, there is a trade-off between the effi-

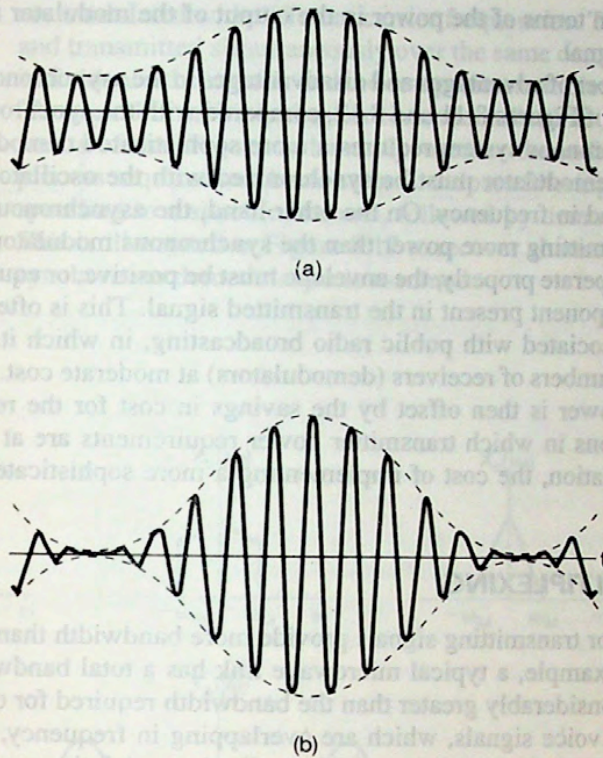


Figure 8.13 Output of the amplitude modulation system of Figure 8.12: (a) modulation index $m = 0.5$; (b) modulation index $m = 1.0$.

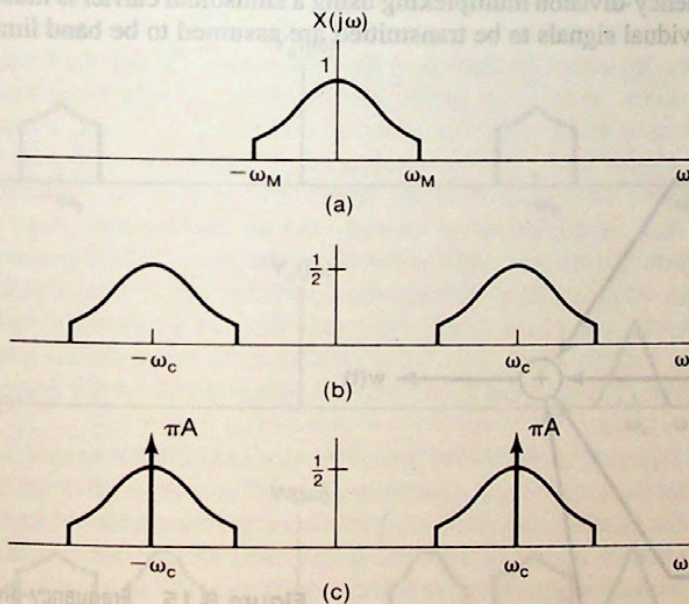


Figure 8.14 Comparison of spectra for synchronous and asynchronous sinusoidal amplitude modulation systems: (a) spectrum of modulating signal; (b) spectrum of $x(t) \cos \omega_c t$ representing modulated signal in a synchronous system; (c) spectrum of $[x(t) + A] \cos \omega_c t$ representing modulated signal in an asynchronous system.

ciency of the system in terms of the power in the output of the modulator and the gain of the demodulated signal.

There are a number of advantages and disadvantages to the asynchronous modulation demodulation system of Figures 8.11 and 8.12, compared with the synchronous system of Figure 8.8. The synchronous system requires a more sophisticated demodulator because the oscillator in the demodulator must be synchronized with the oscillator in the modulator, both in phase and in frequency. On the other hand, the asynchronous modulator in general requires transmitting more power than the synchronous modulator, since, for the envelope detector to operate properly, the envelope must be positive, or equivalently, there must be a carrier component present in the transmitted signal. This is often preferable in cases such as that associated with public radio broadcasting, in which it is desirable to mass-produce large numbers of receivers (demodulators) at moderate cost. The additional cost in transmitted power is then offset by the savings in cost for the receiver. On the other hand, in situations in which transmitter power requirements are at a premium, as in satellite communication, the cost of implementing a more sophisticated synchronous receiver is warranted.

8.3 FREQUENCY-DIVISION MULTIPLEXING

Many systems used for transmitting signals provide more bandwidth than is required by any one signal. For example, a typical microwave link has a total bandwidth of several gigahertz, which is considerably greater than the bandwidth required for one voice channel. If the individual voice signals, which are overlapping in frequency, have their frequency content shifted by means of sinusoidal amplitude modulation so that the spectra of the modulated signals no longer overlap, they can be transmitted *simultaneously* over a single wideband channel. The resulting concept is referred to as *frequency-division multiplexing* (FDM). Frequency-division multiplexing using a sinusoidal carrier is illustrated in Figure 8.15. The individual signals to be transmitted are assumed to be band limited as

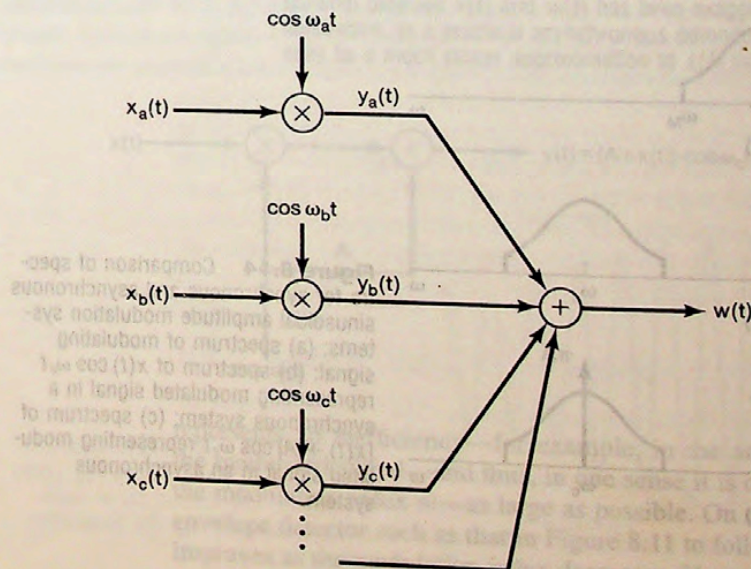
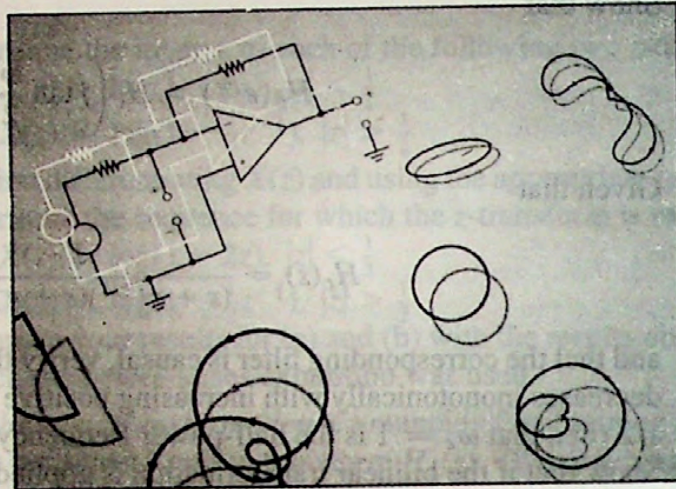


Figure 8.15 Frequency-division multiplexing using sinusoidal amplitude modulation.

11

LINEAR FEEDBACK SYSTEMS



11.0 INTRODUCTION

It has long been recognized that in many situations there are particular advantages to be gained by using feedback—that is, by using the output of a system to control or modify the input. For example, it is common in electromechanical systems, such as a motor whose shaft position is to be maintained at a constant angle, to measure the error between the desired and the true position and to use this error in the form of a signal to turn the shaft in the appropriate direction. This is illustrated in Figure 11.1, where we have depicted the use of a dc motor for the accurate pointing of a telescope. In Figure 11.1(a) we have indicated pictorially what such a system would look like, where $v(t)$ is the input voltage to the motor and $\theta(t)$ is the angular position of the telescope platform. The block diagram for the motor-driven pointing system is shown in Figure 11.1(b). A feedback system for controlling the position of the telescope is illustrated in Figure 11.1(c), and a block diagram equivalent to this system is shown in Figure 11.1(d). The external, or *reference*, input to this feedback system is the desired shaft angle θ_D . A potentiometer is used to convert the angle into a voltage $K_1\theta_D$ proportional to θ_D . Similarly, a second potentiometer produces a voltage $K_1\theta(t)$ proportional to the actual platform angle. These two voltages are compared, producing an error voltage $K_1(\theta_D - \theta(t))$, which is amplified and then used to drive the electric motor.

Figure 11.1 suggests two different methods for pointing the telescope. One of these is the feedback system of Figures 11.1(c) and (d). Here, the input that we must provide is the desired reference angle θ_D . Alternatively, if the initial angle, the desired angle, and the detailed electrical and mechanical characteristics of the motor-shaft assembly were known exactly, we could specify the precise history of the input voltage $v(t)$ that would first accelerate and then decelerate the shaft, bringing the platform to a stop at the desired

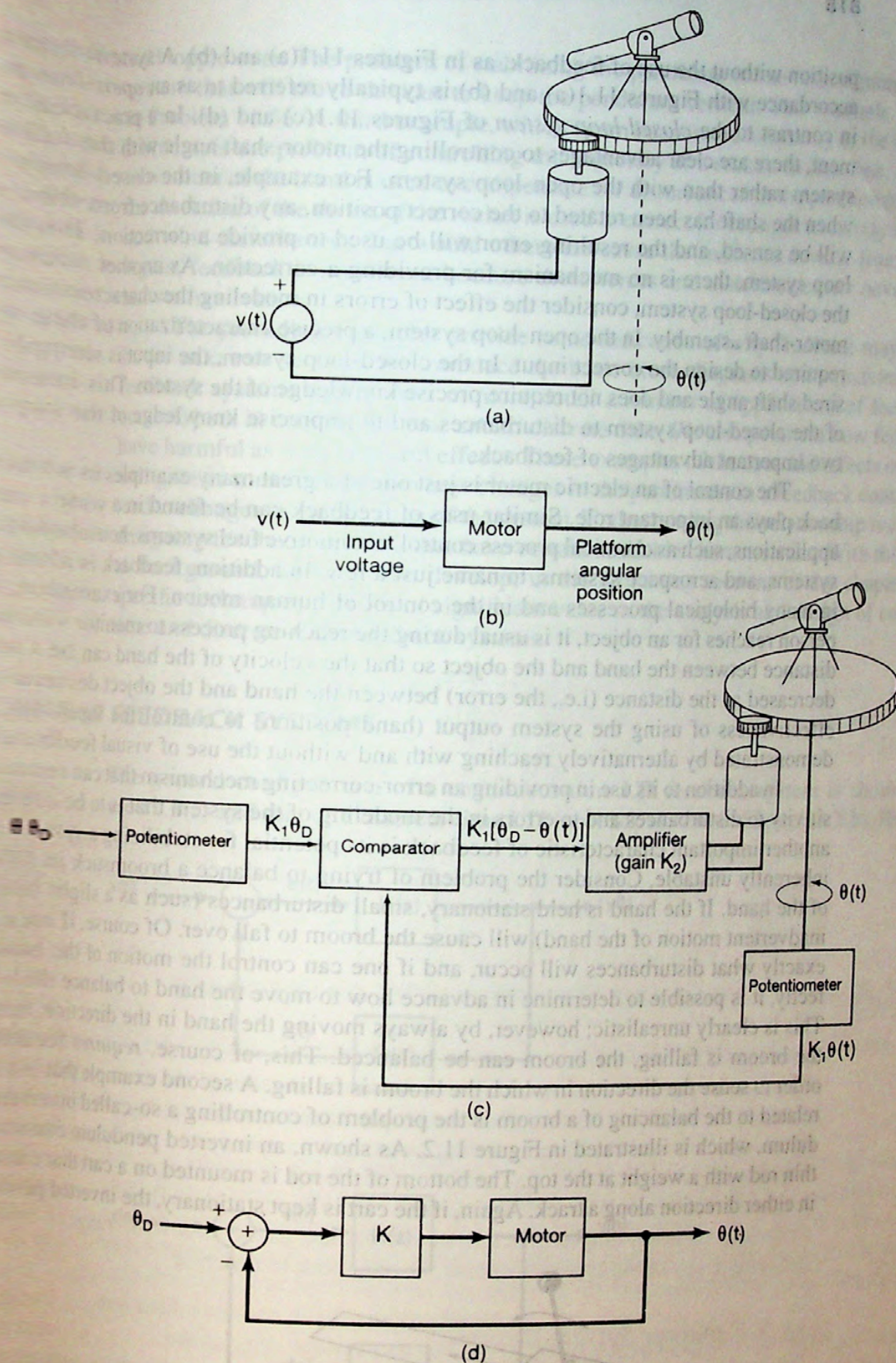


Figure 11.1 Use of feedback to control the angular position of a telescope: (a) dc motor-driven telescope platform; (b) block diagram of the system in (a); (c) feedback system for pointing the telescope; (d) block diagram of the system in (c) (here, $K = K_1K_2$).

position without the use of feedback, as in Figures 11.1(a) and (b). A system operating in accordance with Figures 11.1(a) and (b) is typically referred to as an *open-loop system*, in contrast to the *closed-loop system* of Figures 11.1(c) and (d). In a practical environment, there are clear advantages to controlling the motor-shaft angle with the closed-loop system rather than with the open-loop system. For example, in the closed-loop system, when the shaft has been rotated to the correct position, any disturbance from this position will be sensed, and the resulting error will be used to provide a correction. In the open-loop system, there is no mechanism for providing a correction. As another advantage of the closed-loop system, consider the effect of errors in modeling the characteristics of the motor-shaft assembly. In the open-loop system, a precise characterization of the system is required to design the correct input. In the closed-loop system, the input is simply the desired shaft angle and does not require precise knowledge of the system. This insensitivity of the closed-loop system to disturbances and to imprecise knowledge of the system are two important advantages of feedback.

The control of an electric motor is just one of a great many examples in which feedback plays an important role. Similar uses of feedback can be found in a wide variety of applications, such as chemical process control, automotive fuel systems, household heating systems, and aerospace systems, to name just a few. In addition, feedback is also present in many biological processes and in the control of human motion. For example, when a person reaches for an object, it is usual during the reaching process to monitor visually the distance between the hand and the object so that the velocity of the hand can be smoothly decreased as the distance (i.e., the error) between the hand and the object decreases. The effectiveness of using the system output (hand position) to control the input is clearly demonstrated by alternatively reaching with and without the use of visual feedback.

In addition to its use in providing an error-correcting mechanism that can reduce sensitivity to disturbances and to errors in the modeling of the system that is to be controlled, another important characteristic of feedback is its potential for stabilizing a system that is inherently unstable. Consider the problem of trying to balance a broomstick in the palm of the hand. If the hand is held stationary, small disturbances (such as a slight breeze or inadvertent motion of the hand) will cause the broom to fall over. Of course, if one knows exactly what disturbances will occur, and if one can control the motion of the hand perfectly, it is possible to determine in advance how to move the hand to balance the broom. This is clearly unrealistic; however, by always moving the hand in the direction in which the broom is falling, the broom can be balanced. This, of course, *requires* feedback in order to sense the direction in which the broom is falling. A second example that is closely related to the balancing of a broom is the problem of controlling a so-called inverted pendulum, which is illustrated in Figure 11.2. As shown, an inverted pendulum consists of a thin rod with a weight at the top. The bottom of the rod is mounted on a cart that can move in either direction along a track. Again, if the cart is kept stationary, the inverted pendulum

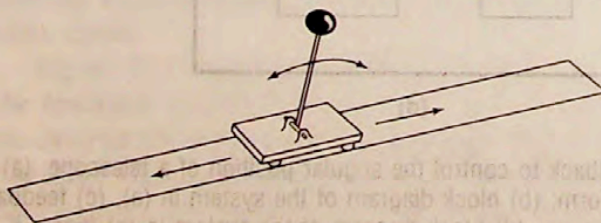


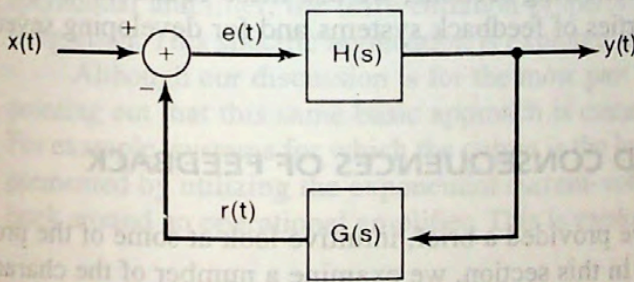
Figure 11.2 An inverted pendulum.

will topple over. The problem of stabilizing the pendulum is one of designing a feedback system that will move the cart to keep the pendulum vertical. This example is examined in Problem 11.56. A third example, which again bears some similarity to the balancing of a broom, is the problem of controlling the trajectory of a rocket. In this case, much as the movement of the hand is used to compensate for disturbances in the position of the broom, the direction of the thrust of the rocket is used to correct for changes in aerodynamic forces and wind disturbances that would otherwise cause the rocket to deviate from its course. Again, feedback is important, because these forces and disturbances are never precisely known in advance.

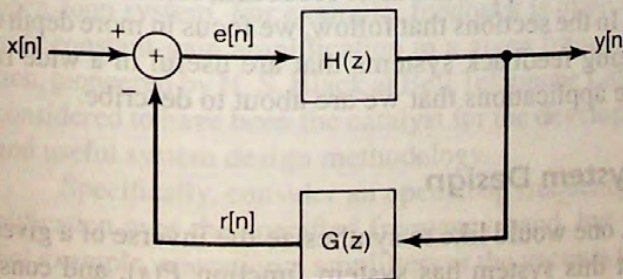
The preceding examples provide some indication of why feedback may be useful. In the next two sections we introduce the basic block diagrams and equations for linear feedback systems and discuss in more detail a number of applications of feedback and control, both in continuous time and in discrete time. We also point out how feedback can have harmful as well as useful effects. These examples of the uses and effects of feedback will give us some insight into how changes in the parameters in a feedback control system lead to changes in the behavior of the system. Understanding this relationship is essential in designing feedback systems that have certain desirable characteristics. With this material as background, we will then develop, in the remaining sections of the chapter, several specific techniques that are of significant value in the analysis and design of continuous-time and discrete-time feedback systems.

LINEAR FEEDBACK SYSTEMS

The general configuration of a continuous-time LTI feedback system is shown in Figure 11.3(a) and that of a discrete-time LTI feedback system in Figure 11.3(b). Because of



(a)



(b)

Figure 11.3 Basic feedback system configurations in (a) continuous time and (b) discrete time.

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