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Micromachined Dual Input Axis Rate Gyroscope

by

Thor Nelson Juneau

**B.S. (Carnegie Mellon University) 1991
M.S. (University of California, Berkeley) 1993**

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in

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of the

UNIVERSITY of CALIFORNIA, BERKELEY

Committee in charge:

**Professor Albert P. Pisano, Chair
Professor Roger T. Howe
Professor R. Horowitz**

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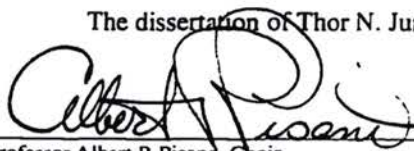
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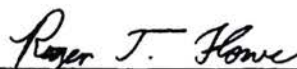
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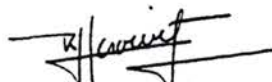
The dissertation of Thor N. Juneau is approved:


Professor Albert P. Pisano, Chair

10 DEC 97
Date


Professor Roger T. Howe

16 December 1997
Date


Professor Roberto Horowitz

12 December 1997
Date

University of California, Berkeley

1997

To my Mother, Sister, and Father
who are both family and dear friends

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1. INTRODUCTION

1.1 PRELUDE

Modern aviation, space travel, and navigation would be impossible without gyroscopes. Gyroscopes are used to measure the rotation angles or rotation rates between a body-fixed axis and an inertially-fixed axis. This allows estimation of pointing direction or vehicle heading. In this respect a gyroscope is akin to a magnetic compass except that a gyroscope can reference to any direction (not only North) and is not dependent on magnetic field lines. Whereas a compass may be inaccurate in the face of magnetic anomalies or helpless in space, a shielded gyroscope can provide measurement independent of outside magnetic environment. Until now, gyroscopes have been bulky, expensive devices. However, micromachining has raised the possibility of revolutionizing the field by providing inexpensive, miniature gyroscopes with good performance.

Conventional macro-technology gyroscopes are large, heavy, and costly. Adding all support circuitry for operation, filtering, trimming, compensation, and analog-to-digital conversion multiplies the volume, the power consumption, and the cost many fold. In fact, typical miniature mechanical and quartz gyroscopes [1], [2] when all mounting brackets and support circuitry are included cost thousands of dollars and require a dozen cubic inches of volume. More accurate Ring Laser Gyroscopes and Interferometric Fiber Optical Gyroscopes are even larger and more expensive. Principally, size and cost drawbacks have precluded gyroscopes from competing in virtually all high volume military and consumer product fields. Further disadvantages including poor shock survival, low bandwidths, and excessive power consumption have further limited gyroscope usage. The bottom line is that conventional gyroscopes cannot meet the demands of many applications.

Recognizing this unfulfilled need, microelectromechanical system (MEMS) researchers have pursued the goal of devising a micromachined gyroscope. The inherent size, weight, and power advantages of MEMS should allow micromachined rate gyroscopes to fill the void left by conventional technology. Batch fabrication utilizing

standard VLSI compatible micromachining techniques should yield at least an order of magnitude in price reduction. The solid state sensor reliability and robustness both to shock and to vibration make micromachining even more attractive.

The majority of published micro-gyroscope designs are single input axis sensors based on either translational vibration [3], [4] or structural mode vibration [5]. Few of these designs have been demonstrated in a micromachining process integrated with circuitry. In contrast, this research focused on fabricating a dual-axis rate gyroscope using the fully integrated Analog Devices Inc. (ADI) [6] and Sandia National Labs (SNL) surface micromachining processes [7]. An advantage of dual-axis operation is that the two sense axes are pre-aligned to each other by lithography. Placing interface and signal processing circuits on chip allows improved noise performance, extreme miniaturization, and inexpensive manufacture. In addition, integrated surface micromachining enables multiple sensors on the same substrate. Combining this dual-axis rate gyroscope with a z-axis gyroscope [8] yields angular rate measurement for all orthogonal axes without the need to align individual sensors. By also adding a triad of micro-accelerometers [9], [10] a thumb-nail sized Inertial Measurement Unit (IMU) with accompanying signal processing has been realized. The micrograph in Fig. 1-1 shows the final dual-axis rate gyroscope version as part of an IMU designed by Berkeley Sensor and Actuator Center (BSAC) personnel and fabricated by Sandia National Laboratories.

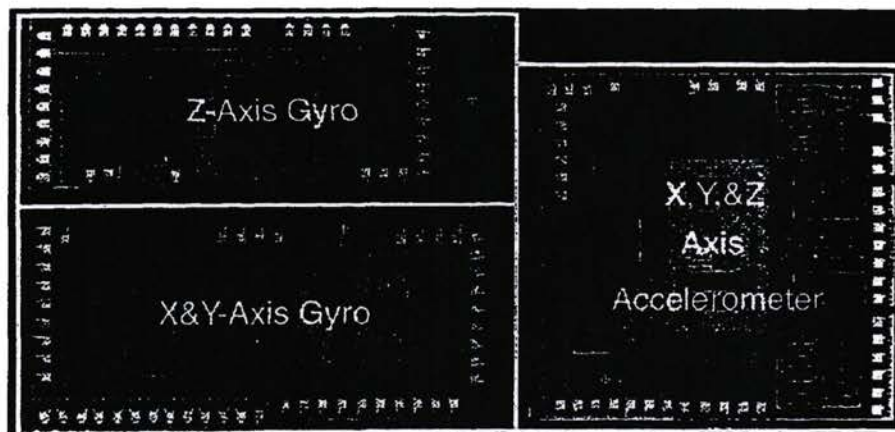


Figure 1-1. Micrograph of the μ IMU sensor array designed by BSAC and fabricated by Sandia National Labs.

1.2 WHAT IS A MICROMACHINED DUAL-AXIS RATE GYROSCOPE?

As discussed previously, the principle use of gyroscopes is to measure orientation, heading, or pointing direction. Gyroscopes can be divided into two broad categories. One category measures orientation angles directly, while the other category actually measures rotation *rate* and for this second type true orientation angles must be estimated indirectly by integration. Free gyroscopes or spinning wheel gyroscopes mounted on gimbals are included in the first direct measurement category. These were the mainstay of aviation and navigation for decades [11]. However, the advent of inexpensive, miniature computers allowed the rise of strap-down navigation. The term strap-down navigation refers to inertial measurement units which have no gimbal mounted gyroscopes, but rather use non-gimbal rate gyroscopes. This computationally intensive method uses rate gyroscopes of the second broad category. In essence, the angular rate measurements from rate gyroscopes are integrated to estimate orientation angles. Although this approach requires more computation, advantages in gyroscope cost and reliability have allowed strap-down navigation to become dominant in many applications. Some applications,

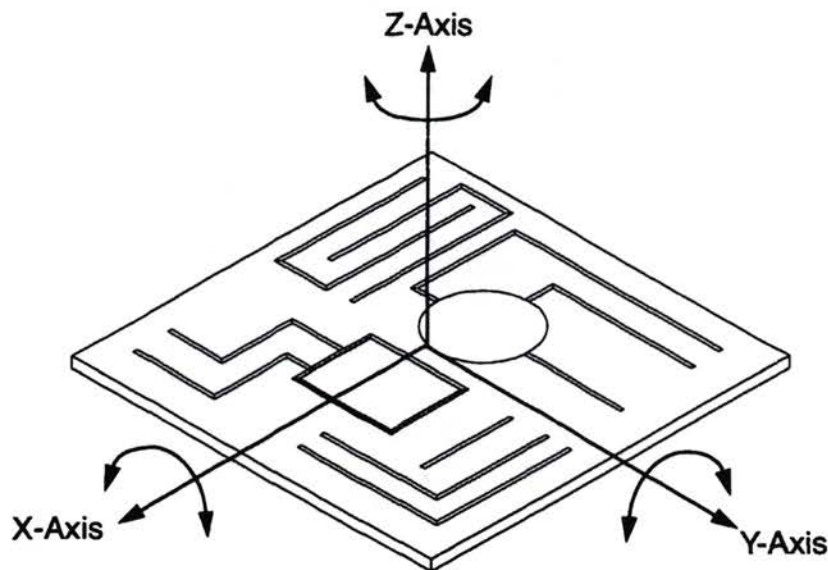


Figure 1-2. *Perspective view of the micromachined dual-axis rate gyroscope fabricated on a computer chip.*

such as platform stabilization and missile guidance, require angular rate in addition to true orientation angle rendering rate gyroscopes the natural choice.

Micromachine technology lends itself well to the fabrication of strap-down rate gyroscopes. Fabrication of rotary micromachines with long lifetime, low-friction bearings is exceedingly difficult, so a free gyroscope or spinning wheel gyroscope which measures orientation directly was not attempted. Instead a spring mounted, vibratory gyroscope requiring no bearing was developed. This resulted in a rate gyroscope which measured angular rate in degrees/second [12], [13]. To achieve dual-axis operation, the circular mechanical sensor was designed to have two identical axes. This allowed simultaneous measurement of rotation rate about the two axes parallel to the substrate. The final result of this work was a fully functional micromachined dual-axis rate gyroscope with accompanying support circuitry fabricated on a single silicon substrate as shown in Fig. 1-2.

1.3 SURVEY OF CONVENTIONAL GYROSCOPE TECHNOLOGY

Conventional gyroscope technology ranges from high-end inertial navigation instruments to low-end consumer products sensors [14]. This diversity in performance and operational requirements has spawned a vast diversity in gyroscope technology. Spinning wheel, vibrating tuning fork, solid state laser, and magnetohydrodynamic gyroscopes are but a few examples of present technology. Unfortunately, what almost all these technologies have in common is high cost, large size, and appreciable power consumption.

The first technology developed was the purely mechanical spinning wheel gyroscope. Mechanical gyroscope designs span the entire performance range from nuclear submarine navigation to simple consumer products [11]. Typical tactical grade spinning wheel gyroscope made by Humphrey [15] and Allied Signal [16] have performance in the range of degrees per hour bias stability. As the name suggests, spinning wheel gyroscopes have a gimbal-mounted inertial wheel which spins at a high rate often above 10,000 RPM. Due to conservation of angular momentum, the spinning wheel will attempt to remain pointed

in a constant direction despite the arbitrary motion of the gyroscope housing. Measuring the angle between the spinning wheel and the outside housing which is mounted to the vehicle gives the vehicle heading or attitude. Many variations on this theme exist, including force balanced rate gyroscopes and dynamically tuned gyroscopes. The highest performance gyroscopes are still mechanical in the form of electrostatically levitated solid metal spheres used in nuclear submarine navigation. The keys to mechanical gyroscope manufacture are precision machining, low friction bearings, extremely fast spin speeds, highly stable force transducers, and electrical compensation. All these requirements increase size, cost, and power consumption.

Most military and commercial inertial navigation systems rely on either ring laser or interferometric fiber-optic gyroscopes instead of mechanical designs. Both approaches are based on clockwise and counterclockwise laser light experiencing rotation, but the underlying physics is different for each type [11]. High-end applications generally incorporate ring laser gyroscopes in which counter-rotating laser beams have different laser cavity lengths due to rotation of the entire assembly and hence different light frequencies. On the other hand, navigation in the 1 nautical mile per hour range often incorporates interferometric fiber-optic gyroscopes for this type is beginning to challenge tradition ring laser gyroscopes in terms of accuracy [17]. These instruments operate via the Sagnac effect where identical, counter-rotating laser beams have different travel lengths and are therefore out of phase when recombined. Hence the name interferometric because the phase difference between the two laser beams creates an interference pattern which can be measured. Both laser gyroscope types are well established, but are far from inexpensive and miniaturized.

Perhaps the most unconventional gyroscope is the magnetohydrodynamic rate sensor sold by Applied Technology Associates [18]. An annulus (doughnut shape) of highly conductive fluid is placed in a magnetic field. During rotation, the relative motion between the conducting fluid and the magnetic field creates a voltage measured by two electrodes which form the walls of the annulus. The sensor has good wide band noise performance; however, interactions between the conductive fluid and back electromotive

force roll off sensitivity below 1 Hertz confining usage to dynamic and short duration applications.

Recently, there have been great strides towards developing inexpensive, miniaturized gyroscopes. Systron Donner manufactures a novel vibrating tuning fork gyroscope fashioned from quartz [1]. This relatively small tactical gyroscope is very robust and incorporated in many military applications. One of the first gyroscopes developed for the consumer market is the Murata piezoelectric vibrating gyroscope [19]. This gyroscope provides performance of 3 °/second at constant temperature. Another company named Gyration has developed a spinning wheel gyroscope costing less than \$20 in quantity for use in 3-D computer mice and presentation pointers [20]. This has not been an exhaustive list of technology, but rather a summary of the most common approaches. A comparison of performance versus cost and size is summarized by Fig. 1-3. As always, the price of higher performance is higher cost and larger size for conventional macro technology.

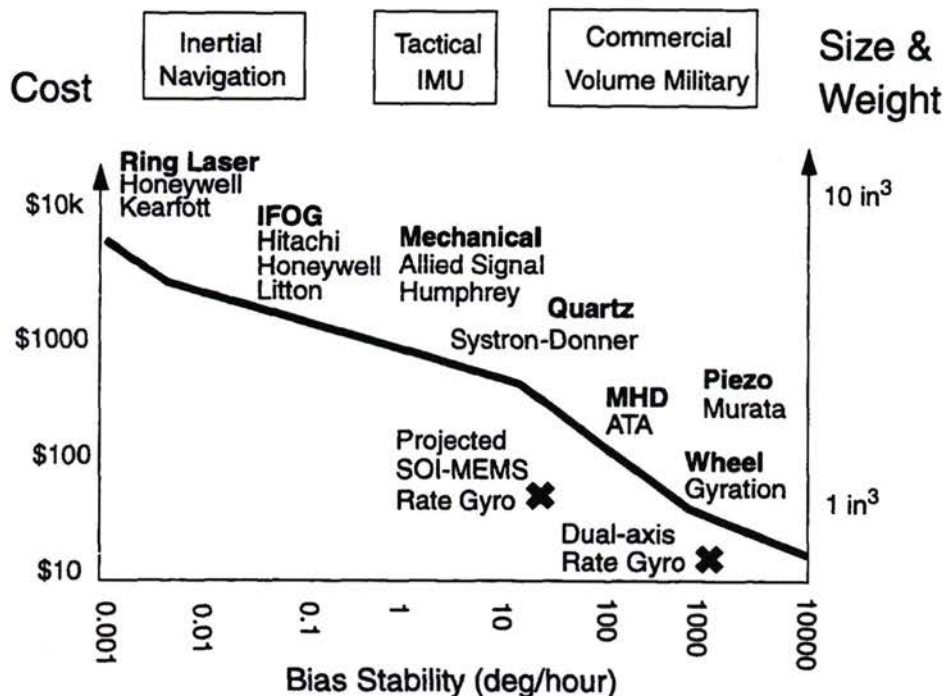


Figure 1-3. Graph of price versus performance trade-offs with broad application areas. Size includes support circuitry.

All of the above technology is wanting in terms of size, cost, power consumption, and performance. The advent of silicon micromachining allows the possibility of developing gyroscopes which are miniature, inexpensive, and low power yet exhibit adequate performance. Currently, no silicon micromachined gyroscopes are commercially available. However, firms around the globe including Delco [22], Samsung [23], Murata [24], and Bosch [25] to name a few have published research results involving micromachined gyroscope development. As Fig. 1-3 depicts, the micromachined dual-axis rate gyroscope described in this thesis is in the commercial and high volume military application range of $1^\circ/\text{second}$. The next generation BSAC developed sensors using a new SOI-MEMS [29] technology are projected to reduce bias noise to near the tactical inertial sensor level of $10^\circ/\text{hour}$. A Bay Area based company Integrated Micro Instruments intends to commercialize performance gyroscopes and accelerometers using this type of technology. But the question of whether there is a market for such technology must be answered. As the next section describes, there appears to be a gigantic future market for micromachined gyroscopes.

1.4 MICROMACHINED GYROSCOPE APPLICATIONS

The serious drawbacks of conventional technology invite micromachine technology to supplant conventional gyroscopes in many present application and to provide enabling technology for new applications beyond the capabilities of present designs. Micromachine sales are experiencing explosive growth. Market surveys by Systems Planning Corporation [31] and Semiconductor Equipment and Materials International [32] forecast annual growth of 50%-60% a year compounding to a \$10 billion market by the year 2000. Both market surveys expect the inertial micromachine segment to encompass roughly 20% of this huge, fast growth market. The market is very broad, ranging from automotive to aerospace. Market niches exist for single sensors, multi-sensor modules, and full micro-inertial measurement units (μIMUs). Markets conducive to micromachined gyros demand low cost and small size while requiring only low to medium performance. A recent Samsung paper [26] projected that micromachined accelerometer

and gyroscope sales will grow to \$700 million for the automobile market, \$500 million for the consumer electronic market, and \$300 million for the industrial market by the year 2002.

The strong Department of Defense support for MEMS research stems from the many vital applications spanning all branches of the military [33]. Micromachined gyroscopes could dramatically reduce the purchase prices and launch weight of short flight tactical weapons such as smart bombs and tactical missiles. One of the most promising applications is augmented Global Positioning System (GPS) navigation. Coupled with GPS, inertial sensors will provide backup in the face of enemy jamming and satellite blockage by buildings or terrain [34], [35]. While GPS is fully functional, offsets and scale factors of the inertial sensors can be calibrated. Many applications such as platform stabilization, antenna pointing, and auto-pilots require less stringent drift performance commensurate with micromachined gyroscopes. Finally, the robustness and low cost of solid-state sensors can facilitate munitions safing, arming, and fusing [33].

With 470 million cars on the road and 50 million cars annually sold world-wide, the automobile market promises to be the largest inertial MEMS market segment. A growing number of automobile manufacturers are switching to silicon micromachined accelerometers such as the Analog Devices ADXL-50 [36], [37]. Micromachined gyroscopes will enable safety advances such as affordable active suspension, adaptive anti-skid, and anti-roll over options [38]. Perhaps more importantly, computer map matching and navigation systems using gyroscopes for dead reckoning are arriving on the market. In fact, more than million automobile navigation systems have been sold in Japan.

Another potentially high volume market is consumer electronics [39]. For years virtual reality has been heralded as the next great entertainment leap. Yet without miniature, inexpensive sensors to measure head orientation, the market has not been ignited. Micromachined gyroscopes with magnetic update to cancel drift could change the industry. Some conventional gyroscope products are already on the market. For example, the firm Gyration has a line of 3-D computer mice and presentation pointers using an inexpensive gyroscope which could be replaced by higher performance micromachined gyroscopes [20].

Space exploration and commercial satellite fields are also ripe for MEMS gyroscope usage [41]. Inertial sensors are crucial for spacecraft missions. Attitude correction, maneuver control, tumble recovery, health monitoring, and large structure stabilization can all be implemented using accurate inertial measurements from micro-gyroscopes [40]. Inertial sensors allow stabilization and pointing of instruments such as cameras, antennas, solar panels, and detectors. Future high performance inertial sensors could provide backup and "fill in the gaps" during Global Position System (GPS) outages due to physical or electrical interference, sunsensor loss due to solar eclipses, and star tracker down time between star acquisitions. As with all space technology, inertial sensor size, weight, and power consumption must be minimized. Every added watt of power requires added solar panel area which in turn adds more weight and cost. The need for miniature sensors is accentuated by the goal of designing microspacecraft which are several orders of magnitude smaller and cheaper than present generation spacecraft exemplified by Galileo. Many of these applications do not require inertial navigation grade sensors but rather require tactical grade sensors. Gyroscopes with $1\text{-}10^\circ/\text{hour}$ bias stability and accelerometers with $10\text{-}100\text{ }\mu\text{g}$ stability are adequate [40]. In summary, there is a clear and present need for MEMS gyroscopes. This need has sparked prolific research as the next section reveals.

1.5 SURVEY OF MICROMACHINED GYROSCOPE DESIGNS

There are a number of research centers investigating silicon micromachined gyroscopes. Each group has different design philosophies and fabrication techniques. Most work has focused on variations of surface micromachining with structural films between 2 and $20\text{ }\mu\text{m}$ thick. The basic operating principle of all vibrating gyroscopes is based on the generation and detection of a Coriolis acceleration. For a Coriolis acceleration to be generated, a proof mass must be put in motion. Thus far, micromachined wheel bearings have displayed inadequate lifetime, so most designs utilize spring mounted vibratory motion. Some of the typical proof masses for micromechanical

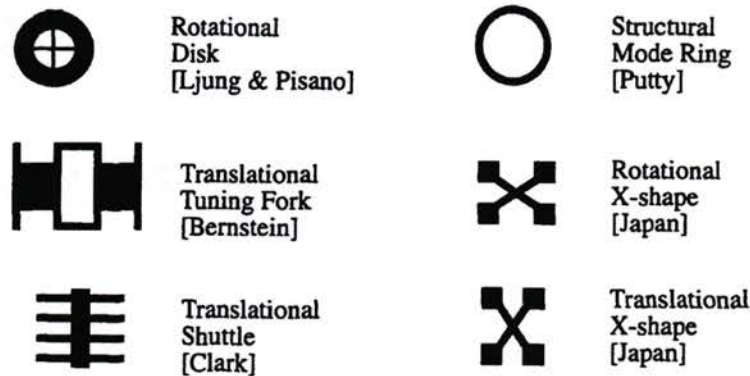


Figure 1-4. Silhouettes of common micromachined gyroscope designs.

gyroscopes are pictured in Fig. 1-4. These broad types include translational tuning fork, rotary disk, translational shuttle, structural mode ring, rotational X-shape, and translational X-shape. Because the automobile market offers massive sales volume and requires only low performance, most research has focused on micromachining automotive grade gyroscopes.

Draper Labs was the first organization to develop a micromachined gyroscope [44]. More recent Draper Labs designs include a tuning fork gyroscope [4] developed for both the automotive and aerospace market several years ago, but the silicon on glass fabrication process used is not conducive to circuit integration and commercialization has been slow. Delco automotive has invested in a vibrating ring derived from the "wine glass" gyro for the automotive market [22], [5], [21]. This design has good attenuation of undesirable translation and rotational disturbances. However, this excellent robustness also results in relatively low sensitivity making improvement to inertial grade performance difficult. Several Japanese manufacturers including Murata [24] and Matsushita [45] have published gyroscope announcements involving both translational shuttle and translational X-shape designs. Again these are aimed directly at the low accuracy, high volume automobile market. The Jet Propulsion Lab has produced a novel rotational X-shape gyroscope with a brass pillar for added inertia [46]. This gyroscope has exhibited fine drift performance, but the gyroscope is not designed for mass production and the lack of circuit integration

results in automotive grade noise performance. William Clark at the Berkeley Sensor & Actuator Center has also developed a fully integrated, translational shuttle-type microgyroscope which measures rotation rates perpendicular to the substrate [3]. This design ports naturally into high aspect ratio processes and is the basis for commercial gyroscope development by Integrated Micro Instruments. A similar shuttle design was reported by Samsung [23]. Although this is clearly not an exhaustive list of researchers, the most recent and most common types of micromachined gyroscopes have been presented.

The micromachined dual-axis rate gyroscope presented in this thesis utilizes a rotational disk proof mass [47]. This design is first mentioned in a 1992 internal BSAC document [48] written by Per Ljung and Albert Pisano. The circular design is the key to dual-axis operation. Due to quad symmetry, the two orthogonal axes parallel with the substrate are identical. This allows simultaneous measurement of rate inputs about these two identical axes. As discussed in later chapters, this rotational disk design has many advantages stemming from the circular proof mass.

1.6 THESIS OUTLINE

This thesis is divided into seven chapters. The first chapter is of course the introduction which motivates the research and provides an outline for the rest of thesis. The second chapter describes the general theory of operation and overall system architecture. The following three chapters detail aspects of the gyroscope design. Chapter Three discusses the mechanical structure design. The electrical interface and signal processing are explained in Chapter Four. The key mechanical and electrical design issues are brought together in Chapter Five which explores system wide design trade-offs and optimization. Next, Chapter Six discusses the fabrication plus the experimental methods and results. As always, there are some performance issues stemming from the open loop operation of the original prototypes. The final chapter projects solutions to these imperfections and provides conclusions to bring the thesis to a close.

2. THEORY OF OPERATION

The classical mechanical gyroscope relies on a spinning wheel or rotor exhibiting gyroscopic motion in the form of Coriolis accelerations [11]. Unfortunately, fabricating microscopic, low friction bearings needed for this classical approach is challenging. Instead, this thesis presents a rotor mounted on springs rather than bearings. Hence, the rotor motion is a counterclockwise and then clockwise angular oscillation rather than a constant spinning motion. This oscillatory motion still exhibits gyroscopic dynamics which can be used to measure angular rate.

2.1 MECHANICAL STRUCTURE

The vast majority of micromachined gyroscopes use *translational* displacement motion. In contrast, the dual-axis rate gyroscope discussed herein is driven into *rotational* oscillatory motion. The proof mass is actually an inertial rotor comprised of a $2\text{ }\mu\text{m}$ to $2.3\text{ }\mu\text{m}$ thick polysilicon disk. As depicted in Fig. 2-1, this inertial rotor is suspended $1.6\text{ }\mu\text{m}$ to $2\text{ }\mu\text{m}$ above the substrate by four symmetrically placed beams anchored to the

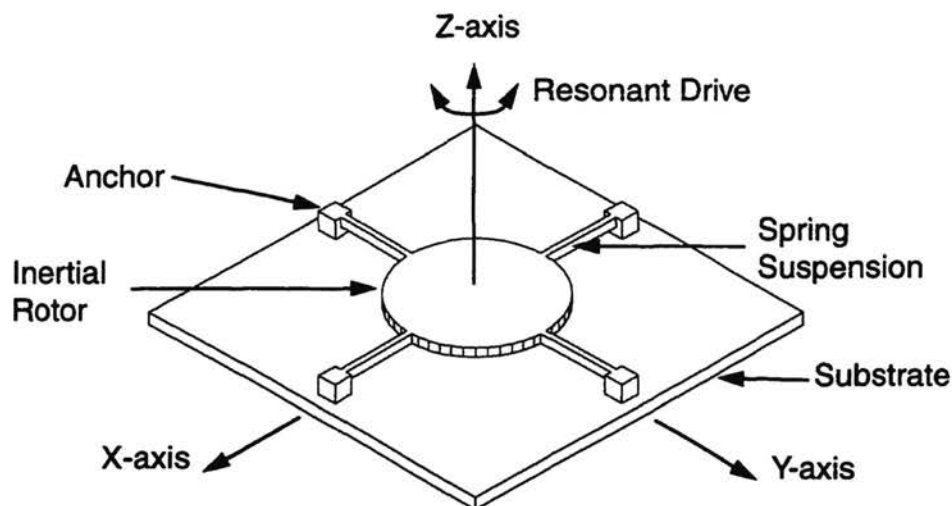


Figure 2-1. Conceptual illustration of the mechanical sensor element for the dual-axis rate gyroscope.

substrate. The beams provide a torsional suspension allowing rotational compliance about all three axes. The polysilicon thickness and inertial rotor height above the substrate varied depending on the surface micromachining process used.

The basic operating principle of all vibratory gyroscopes relies on the generation and detection of a Coriolis acceleration. In gyroscopic dynamics, there is a distinct motion about all three orthogonal axes. First, the proof mass is put into oscillatory motion about the z-axis perpendicular to the substrate. Once in motion, the proof mass is sensitive to angular rates induced by the substrate being rotated by outside forces. This input rotation rate is on a second axis (say the x-axis) perpendicular to the first drive axis. The input rate induces a Coriolis acceleration about the third axis (y-axis) which is perpendicular to both the z-axis drive and the rate input x-axis. This Coriolis acceleration induces a Coriolis motion with an amplitude proportional to the angular rate of the substrate. Detecting the Coriolis motion induced by the Coriolis acceleration allows the original rotation of the gyroscope to be inferred.

The key to dual-axis operation is the circular symmetry of the device. The mechanical sensor is identical about the x- and y-axis in the plane of the substrate. When the inertial rotor is resonating, any rotation rate of the substrate about the x-axis will

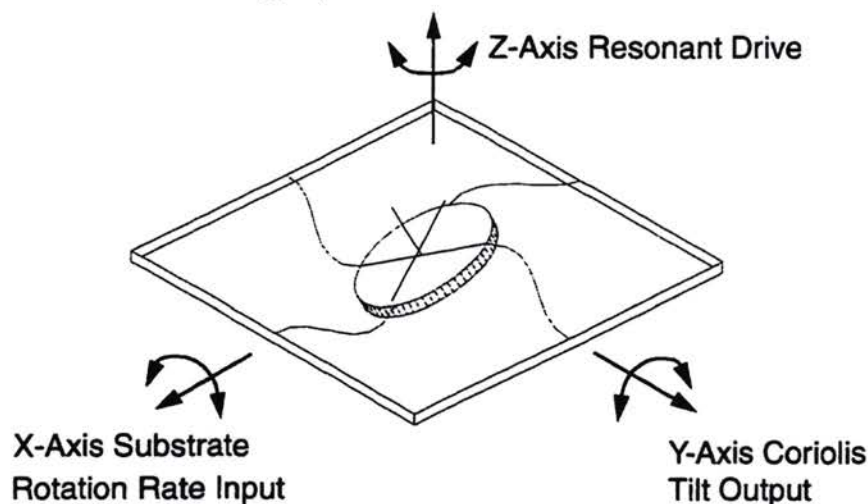


Figure 2-2. Conceptual view of the Coriolis tilting motion invoked by rotation of the substrate.

induce a Coriolis angular acceleration about the y-axis which in turn induces a tilting oscillation of the rotor about the y-axis. Because the mechanical gyroscope is symmetric in two orthogonal axes, any rotation rate about the y-axis likewise invokes a tilting oscillation about the x-axis, thereby allowing dual-axis rotation rate measurement. Hence the rotation rate about both these axes can be measured simultaneously resulting in a dual-axis rate gyroscope. The axes are orthogonal, so the small tilting deflections associated with Coriolis motion will not mix or combine. However as the Chapter Six regarding experimental results reports, the designer must be vigilant to ensure low cross-axis sensitivity. The conceptual illustration in Fig. 2-2 shows the out of plane tilting motion of the inertial rotor in response to a rotation rate input.

2.2 SIMPLIFIED GYROSCOPE DYNAMICS

These intuitive dynamics are reflected in the simplified dynamics of gyroscopic motion. The linearized, first-order dynamics are described in Eq. 2-1 and Eq. 2-2 where α_x and α_y are Coriolis accelerations for each axis, Ω_x and Ω_y are the input rotation rates to be measured, and $\dot{\theta}_z$ is the resonant drive angular rate.

$$\text{X-AXIS: } \alpha_x = 2 \cdot \Omega_y \cdot \dot{\theta}_z \quad (2-1)$$

$$\text{Y-AXIS: } \alpha_y = -2 \cdot \Omega_x \cdot \dot{\theta}_z \quad (2-2)$$

The Coriolis acceleration is proportional to the product of the drive motion and the input rotation rate to be measured. As discussed earlier, the drive motion is an oscillation at the natural frequency ω_z of the drive axis. Therefore the tilting oscillations about the x- and y-axis are at the same frequency as the resonant drive frequency ω_z . These tilting oscillations are amplitude modulated signals with amplitude proportional to the respective rotation rate inputs. Therefore, the rotation rate can be inferred by measuring the rotor tilt oscillation and demodulating this measurement at the resonant drive frequency.

This is very much like amplitude modulated (AM) radio. The dual-axis rate gyroscope oscillation drive frequency can be thought of as the radio station frequency. At the broadcast station, the sound is modulated with the station frequency and transmitted.

Loud sounds have large oscillation amplitude, while soft sounds have small amplitude. In the gyroscope, this is analogous to large rotation rate inputs generating large amplitude inertial rotor tilt, and small rate inputs generating small tilt amplitude. A listener uses a radio receiver to pick up and demodulate the station signal, leaving the original sound or music. For the dual-axis rate gyroscope, sense circuitry measures the rotor Coriolis tilt motion and demodulates, leaving a voltage signal proportional to the original angular rate input.

For an AM radio receiver to operate, the radio must be tuned to the proper frequency. A similar analogy exists for vibratory gyroscopes. Performance can be much improved by matching drive oscillation frequency with the natural frequencies of both sense axes. This is because the gyroscope sense axes have large resonant peaks, so several orders of magnitude improvement in sensitivity is gained through matching modes. This is analogous to tuning a radio receiver until the best reception for a particular station is found. In a gyroscope, tuning using electrostatic force is used to "down-tune" (i.e., reduce the natural frequency) both sense axis natural frequencies until they nearly match the drive frequency. The principle drawback of electrostatic tuning is that mechanical shock survivability is reduced due to the electrostatic attraction from tuning voltages. With the basic dynamics illuminated, the problem of actually setting the gyroscope in motion and sensing Coriolis tilting can be tackled.

2.3 RESONANT DRIVE

There can be no rotation rate sensing unless the inertial rotor is driven into rotational resonance. This task is accomplished using a highly linear electrostatic comb drive [49]. Pairs of alternating differential combs, half of which are drive and half of which are sense, can be seen surrounding the rotor in Fig. 2-3. A voltage difference between the inertial rotor and the stationary drive combs around the rotor circumference induces attractive electrostatic forces on the rotor. By varying this voltage difference, alternating clockwise and counter clockwise moments can be applied to the rotor thereby exciting oscillation. The key to exciting oscillation is to use this drive moment to cancel out the natural air

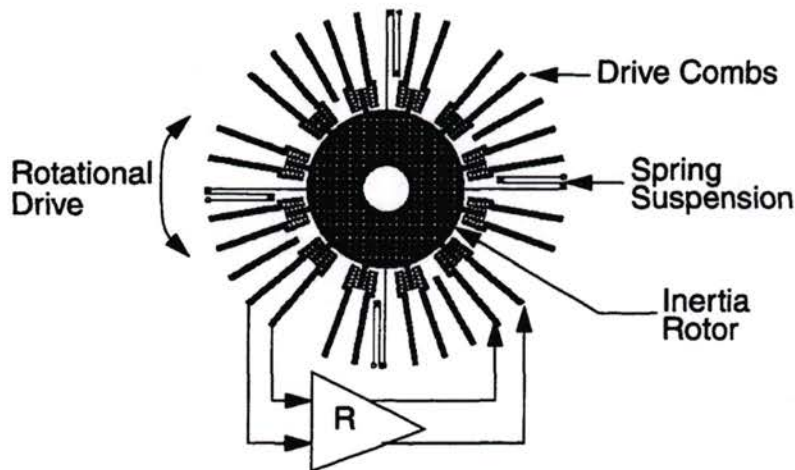


Figure 2-3. Diagram of the rotational resonance drive scheme. Sense and drive electrostatic combs are used in conjunction with a trans-resistance amplifier to induce self oscillation.

damping of the rotor. With no damping, the system becomes unstable and the inertial rotor self oscillates at its natural frequency. A differential trans-resistance amplifier (i.e., current-to-voltage converter) measures rotor rotation rate using the sense combs and provides positive feedback to the drive combs to effectively cancel viscous damping [50]. Since the amplitude of resonance directly determines the scale factor, an automatic gain control loop is used to ensure constant oscillation amplitude [51]. The details of resonant drive and amplitude control are saved for Section 4.1.

2.4 CORIOLIS MOTION DETECTION

With the rotor in oscillatory motion, the gyroscope is sensitive to substrate rotation rates. As previously noted, a rotation rate input induces a Coriolis acceleration, which in turn induces a tilting oscillation. Using a differential capacitance measurement scheme to detect the tilt oscillation amplitude, the original rotation rate input can be inferred. In Fig. 2-4 is shown the rotor with an underlying pair of quadrant shaped electrodes. These electrodes form a capacitive divider with the inertial rotor. If the rotor tilts about the axis perpendicular to the page, then the capacitance of one sense capacitor increases while the

capacitance of the other decreases [47]. This differential change in capacitance is detected via an integrator electrically connected to the inertial rotor in conjunction with a modulated sense voltage applied between the pair of quadrant shaped electrodes. If the sense capacitors are equal, then the modulation voltage causes charge to flow between the sense capacitors with no charge escaping through the integrator. If the sense capacitors are unbalanced due to inertial rotor tilt, then some charge flows through the integrator producing an measurement voltage proportional to tilt displacement and modulated at the sense voltage frequency. Hence, the Coriolis motion is measured and the desired input rotation rate measurement can be derived through signal processing.

Dual-axis operation is achieved by placing four quadrant-shaped electrodes beneath the rotor. This allows Coriolis tilt detection about both orthogonal axes. A different voltage modulation frequency is used for each pair of sense electrodes thereby allowing the signal processing circuitry to discriminate between the two axes. Separate demodulation circuits for each sense axis provide two voltage outputs proportional to the

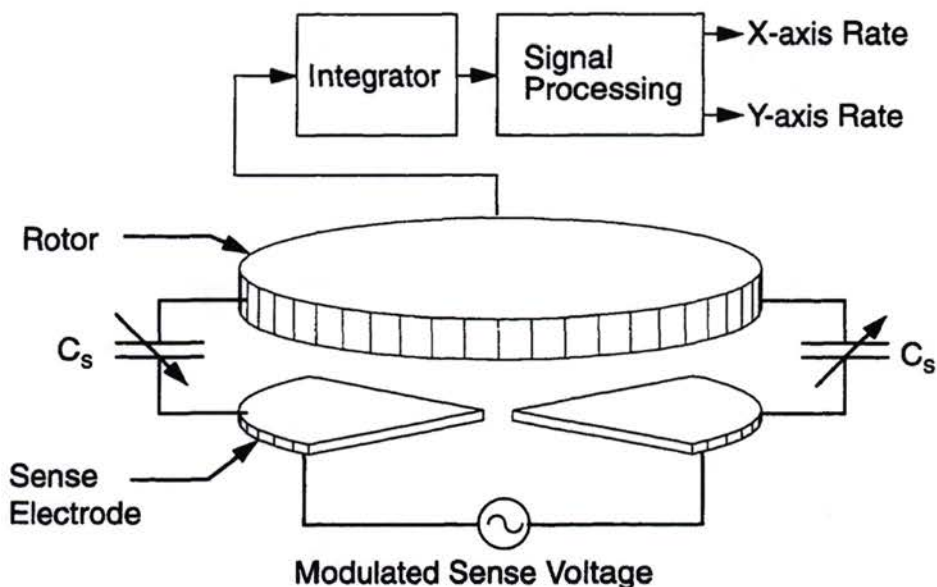


Figure 2-4. System level schematic of the Coriolis tilt motion sense electronics for a single axis.

two angular rate inputs. The output voltages must be demodulated twice: first a demodulation removes the sense modulation frequency, and second a demodulation removes the drive resonant frequency leaving the base band rate input measurements.

2.5 FABRICATED PROTOTYPES

In order to prove design concepts, prototypes of the dual-axis rate gyroscope were fabricated in two different integrated micromachining processes. One set of dual-axis rate gyroscopes was fabricated using the ADI BiMEMS process [6], while a second set was fabricated using the Sandia National Labs (SNL) experimental process [7]. The micrograph in Fig 2-5 shows the first working dual-axis rate gyroscope which was fabricated by ADI. The ADI design included minimal interface circuitry and no on-chip signal processing.

After the completion of the ADI design, an entirely new design was completed using SNL as a foundry. The SNL process provided advantages such as improved structural polysilicon, higher density interconnect, and smaller minimum circuit sizes. This allowed better mechanical structure optimization and added electrical functionality in terms of on-

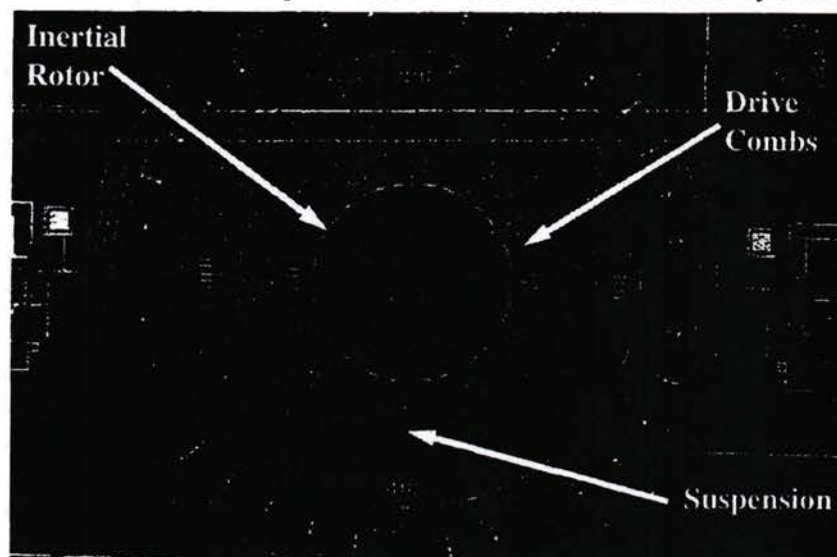


Figure 2-5. Micrograph of the first generation dual-axis gyroscope fabricated by Analog Devices.

chip signal processing and closed-loop feedback electrodes. The obvious difference between the designs in each technology is that the ADI versions had the drive combs and beam suspension on the outside of the inertial rotor, where as the SNL version had the drive combs and beam suspension on the inside of the inertial rotor as shown in Fig. 2-6. The reasoning behind these design choices is detailed in Section 3.4 regarding inertial rotor and suspension design.

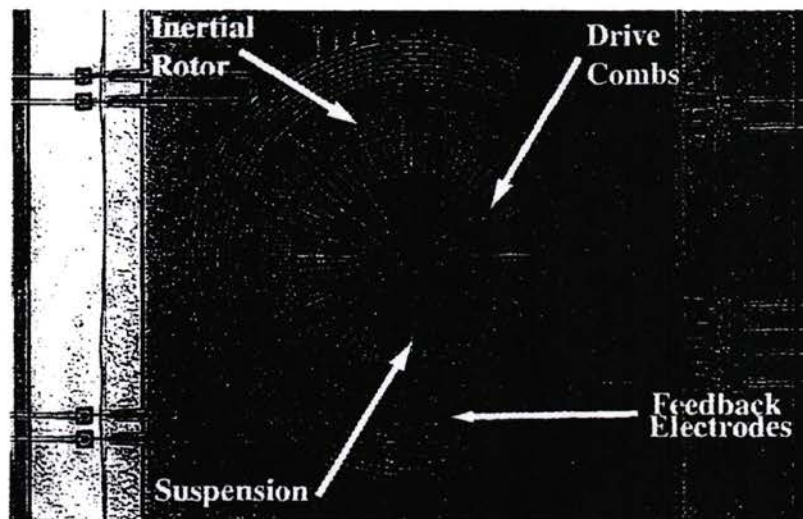


Figure 2-6. Micrograph showing the Sandia National Labs version of the dual-axis rate gyroscope.

2.6 SUMMARY

This completes a brief overview of the gyroscope system architecture. The following two chapters give details regarding the mechanical structure and the electronics. Design optimization depends on both mechanical and electrical aspects. Therefore, only after these aspects are described can Chapter Five discuss design optimization. After the dual-axis rate gyroscope is described in detail, the experimental results of several fabricated devices are discussed. In particular, the matching of natural frequencies via suspension design and electrostatic frequency tuning is explained. Matching modes provides

enhanced noise performance, but also decreased scale factor stability and dynamical performance. Solutions to these difficulties are explored in Chapter Seven regarding closed loop control and future directions.

3. MECHANICAL STRUCTURE

This chapter describes the mechanical structure in detail. As the previous chapter mentioned, the mechanical structure is comprised of an inertial rotor secured to the substrate by four symmetrically placed beams. The derivation of system kinematics and in turn the system dynamics are summarized. The underlying dynamics provide the foundation upon which the entire sensor design is based. In particular, the dynamical equations reveal how Coriolis accelerations can be created and then detected. The mechanical sensitivity to angular rate inputs is derived and the method of frequency matching to greatly enhance sensitivity is highlighted. With the governing equations formulated, attention is focused on the physical design of the inertial rotor and attached four beam suspension. The dynamical parameters such as moment of inertial, damping, and spring constants must be related to actual structure geometry and material characteristics. The electrostatic fine tuning of sense axis frequencies vital for enhanced mode matching is explained with focus placed on deriving maximum tuning range. Finally, the absolute minimum detectable signal limit due to Brownian noise is explored. Optimization of the mechanical structure is closely coupled with the electrical design. For this reason, the actual choice of all mechanical dimensions is left until Chapter Five: Design Trade-offs and Optimization which follows Chapter Four: Electrical Interface and Signal Processing.

3.1 DYNAMICAL EQUATIONS OF MOTION

The literature devoted to gyroscope dynamics is truly vast. Rather than repeat previous work, this derivation will summarize the key steps that lead to the ideal, linearized equations of motion. Theory and experimentation have shown that deflections are rather small and within the range of linear approximation. This level of approximation is imperative for design as the full nonlinear equations of motion become unwieldy. A more detailed description of the dual-axis rate gyroscope dynamics can be found in the doctoral thesis completed by P. B. Ljung [52].

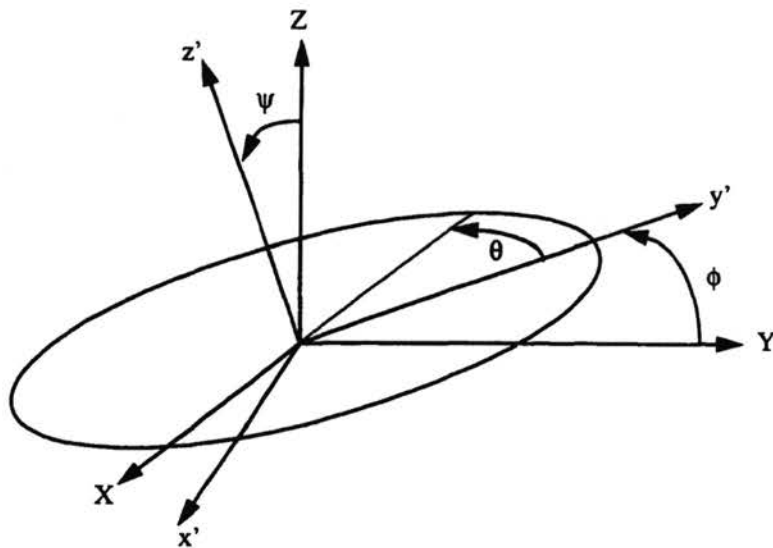


Figure 3-1. Euler coordinates used to define inertial rotor position with respect to a fixed inertial reference axes.

3.1.1 Kinematic Formulation

The first step in developing the dynamical equations of motion for any system is to define coordinates which fully characterize the kinematics of the system. These coordinates are not unique, but every effort should be made to choose a set of coordinates which simplifies the problem. As discussed in the introduction, the micromachined dual-axis gyroscope is comprised of an inertial disk mounted upon a four beam suspension which allows rotation about all three axes. This suspension is designed to attenuate translational accelerations by placing translation natural frequencies far from rotation natural frequencies. Hence, the system can be satisfactorily modeled using only angular rotation coordinates with the assumption that the inertial rotor stays centered. Three rotation coordinates are needed to measure the inertial rotor position with respect to the substrate. The conventional Euler angle coordinates were used [53]. As is shown in Fig. 3-1, the tilted plane on which the inertial rotor spins can be defined by the first two standard Euler coordinates ϕ and ψ , while the rotation angle of the spinning rotor about the z -axis can be defined by an angular coordinate θ . The first angular coordinate ϕ measures

the rotation angle about the x-axis. Next the coordinates ψ measures the rotation angle about the pivoted y' axis. These two coordinates define a plane in which the rotor spins, and the last coordinate θ measures the rotor spin angle about the z' axis.

Dynamical equation formulation requires the time derivative of angular coordinates, so the angular rates must be tackled next. The total system kinetic energy is a function of total body angular rate with respect to inertial space. Although rotations are not vector quantities, angular rates are vector quantities and so superposition holds. Therefore, all sources of angular rotation rate can be transformed to one set of coordinate axes and then summed to produce total angular rotation rate. This summation includes the angular rate of the rotor relative to the substrate plus the angular rate of the substrate relative to fixed inertial space. Normally, all rates are referenced to the body center coordinates. The body center coordinates are fixed in the body such that moments of inertia do not change with respect to the three axes. This is vital as variable moments of inertia are to be avoided. The inherent symmetry of the circular rotor allows some simplification. The motion of inertial rotor is constant at every angle about the z' -axis, so pure body centered axes are not required. Instead, a set of constant moment of inertia axes denoted x', y', z' are used.

The first step in developing total angular rate is to find the angular rate of the rotor relative to the substrate in terms of the three angular coordinates. The angular rate vector due *only* to coordinate changes, denoted by ω , is given in Eq. 3-1. These are derived by converting rotation rate vectors $\dot{\phi}, \dot{\psi}, \dot{\theta}$ into the x', y', z' coordinates [53].

$$\omega = \begin{bmatrix} \omega_{x'} \\ \omega_{y'} \\ \omega_{z'} \end{bmatrix} = \begin{bmatrix} \dot{\phi} \cos \psi \\ \dot{\psi} \\ \dot{\phi} \sin \psi + \dot{\theta} \end{bmatrix} \quad (3-1)$$

Next, the angular rate of the substrate relative to inertial space must be derived and converted to constant moment of inertia coordinates. In general, the rotation rates the dual-axis gyroscope is to measure are referenced to the substrate. These angular rates are relative to inertial space but are referenced to substrate coordinates X, Y, Z . Hence a rotation matrix must be derived to convert all vectors in substrate fixed coordinates to the

constant moment of inertia coordinates $x'y'z'$. The matrix is given in Eq. 3-2 and can be derived using simple trigonometry.

$$\Theta = \begin{bmatrix} \cos\psi & -\sin\phi\sin\psi & -\cos\phi\sin\psi \\ 0 & \cos\phi & \sin\phi \\ \sin\psi & -\sin\phi\cos\psi & \cos\phi\cos\psi \end{bmatrix} \quad (3-2)$$

Deriving the total angular rate vector for the inertial rotor requires summing the angular rates relative to the substrates $\bar{\omega}$ with substrate angular rates relative to inertial space $\Omega_x, \Omega_y, \Omega_z$. The substrate rates are converted to constant moment of inertia coordinates via the rotation matrix Θ . This conversion and summation are embodied in Eq. 3-3 where Ω is the vector describing substrate rotation rates $\Omega_x, \Omega_y, \Omega_z$ about respective substrate axes.

$$\omega = \bar{\omega} + \Theta\Omega \quad (3-3)$$

Hitherto, only the kinematics of the rotor have been discussed. Only geometry and calculus has been used to derive vector quantities need for dynamical modeling. Now at last, dynamical modeling can begin.

3.1.2 Lagrangian Dynamics Formulation

Once a suitable set of coordinates has been settled upon, the equations governing dynamics can be derived. The Lagrangian method is a system energy based technique which provides the full equations of motion. This is a very systematic procedure which requires only the kinetic energy, potential energy, and energy loss scalar functions as a starting point. These quantities are derived beginning with the kinetic energy T . The short hand matrix form of the kinetic energy is written in terms of the ideal diagonal inertial matrix I and the total angular rotation rate vector of the rotor. The elements of I represent the angular moment of inertia about the respective axis subscript [54].

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \quad (3-4)$$

$$T = \frac{1}{2}(\omega^T \cdot I \cdot \omega) \quad (3-5)$$

Next, an approximate potential energy function must be settled upon. Since the angular deflections are expected to be quite small (less than one degree), the assumption of linear suspension spring coefficients is justified. For simplicity, the spring coefficients are assumed to be uncoupled, but coupling will be added later in Chapter Six. This results in the approximate potential energy function V given in Eq. 3-6 with K_{ii} representing the spring rate coefficients of each respective axis.

$$V = \frac{1}{2}(K_{xx}\phi^2 + K_{yy}\psi^2 + K_{zz}\theta^2) \quad (3-6)$$

Now the energy loss due to viscous friction must be accounted for. Typically, an approximate Rayleigh dissipation function must be used. Again, for simplicity a linear viscous damping term is assumed with the resulting Rayleigh dissipation function R given in Eq. 3-7. The damping force is assumed to be proportional to rotor angular rate relative to the substrate $\dot{\omega}$ and a viscous friction coefficients C_{ii} for each axis [53].

$$R = \frac{1}{2}(C_{xx}\dot{\phi}^2 + C_{yy}\dot{\psi}^2 + C_{zz}\dot{\theta}^2) \quad (3-7)$$

With all necessary scalar energy functions derived, the mechanics of Lagrangian dynamics can be used to reveal the equations of motion. First, the Lagrangian L is formed from the kinetic and potential energy functions.

$$L = T - V \quad (3-8)$$

Second, Lagrange's method coupled with the Rayleigh dissipation function R is used to produce the equations of motion. This produces the three nonlinear, coupled equations of motion. The left hand side of each equation models the gyroscope dynamics, while the right hand side represents the external torques. The right hand side is zero for the x- and

y-axis because there are no external torques of interest. In contrast, the z-axis has rotor drive torque M_z which induces rotor oscillation [53].

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \frac{\partial L}{\partial \phi} + \frac{\partial R}{\partial \dot{\phi}} = 0 \quad (3-9)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\psi}}\right) - \frac{\partial L}{\partial \psi} + \frac{\partial R}{\partial \dot{\psi}} = 0 \quad (3-10)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} + \frac{\partial R}{\partial \dot{\theta}} = M_z \quad (3-11)$$

3.1.3 Ideal Linearized Equations of Motion

Unfortunately, the dynamical equations produced are nonlinear, coupled, and lengthy. As such they are wholly unsuitable for design synthesis. However, the assumed small motions allow linearization which results in the much simplified and decoupled equations of motion [54]. After linearization, most terms are negligible and may be removed. Many terms are removed by assuming that the z-axis drive motions are far larger than the Coriolis induced motions in the x- and y-axes. Although the dual-axis rate gyroscope is sensitive to angular accelerations about all axes, these terms are small and typically far below the Coriolis motion bandwidth of interest. Therefore, no angular acceleration terms are included. The key to dual-axis operation is revealed in Eq. 3-12 and 3-13 for the angular motion about the x- and y-axes. These two equations are nearly identical except that the subscripts x and y are exactly reversed and the Coriolis acceleration changes sign. The z-axis equation (Eq. 3-14) models the resonant drive dynamics which are central to setting the inertial rotor in motion.

$$I_{xx}\ddot{\phi} + C_{xx}\dot{\phi} + K_{xx}\phi = I_{zz}\Omega_y\dot{\theta} \quad (3-12)$$

$$I_{yy}\ddot{\psi} + C_{yy}\dot{\psi} + K_{yy}\psi = -I_{zz}\Omega_x\dot{\theta} \quad (3-13)$$

$$I_{zz}\ddot{\theta} + C_{zz}\dot{\theta} + K_{zz}\theta = M_z \quad (3-14)$$

3.2 CORIOLIS MOTION INDUCED BY RATE INPUT

In order to generate a Coriolis acceleration, the inertial rotor is driven into angular resonance about the z-axis (perpendicular to the substrate). This results in both z-axis rotation θ and angular rate $\dot{\theta}$ being sinusoidal time varying signals with a frequency of precisely the z-axis natural frequency ω_z .

$$\theta = \theta_0 \sin(\omega_z t) \quad (3-15)$$

$$\dot{\theta} = \theta_0 \omega_z \cos(\omega_z t) \quad (3-16)$$

The gyroscopic dynamics are embodied in the x- and y-axis equations of motion Eq. 3-17 and 3-18. These equations were derived from Eq. 3-12 and 3-13 by dividing both sides by the respective moment of inertia I_{xx} or I_{yy} and converting the equations to the standard frequency format. The axis mode frequencies are ω_x and ω_y with respective quality factors Q_x and Q_y for each axis. Note the gyroscope duality is visible in the mirror image symmetry of the two equations of motion.

$$\ddot{\phi} + \frac{\omega_x}{Q} \dot{\phi} + \omega_x^2 \phi = 2\Omega_y \theta_0 \omega_z \cos(\omega_z t) \quad (3-17)$$

$$\ddot{\psi} + \frac{\omega_y}{Q} \dot{\psi} + \omega_y^2 \psi = -2\Omega_x \theta_0 \omega_z \cos(\omega_z t) \quad (3-18)$$

When the inertial rotor is resonating, any rotation rate Ω_x of the substrate about the x-axis will induce a Coriolis angular acceleration about the y-axis which in turn induces a tilting oscillation of the rotor about the y-axis. Because the mechanical gyroscope is symmetric in two orthogonal axes, any rotation rate Ω_y about the y-axis likewise invokes a tilting oscillation about the x-axis, thereby allowing dual-axis rotation rate measurement.

The tilting oscillations about the x- and y-axis are at the same frequency as the resonant drive ω_z . These tilting oscillations are amplitude modulated signals with amplitude proportional to the respective rotation rate inputs. This is represented in Fig. 3-2 where underlying high frequency drive is modulated by a low frequency sinusoidal rate input Ω_x to be measured. The amplitude or envelope of the signal is proportional to the

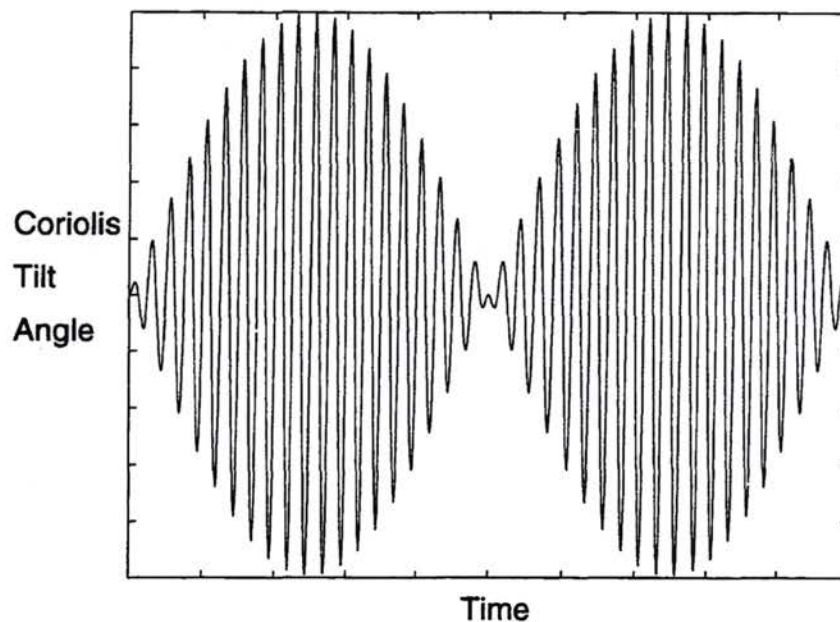


Figure 3-2. Graph representation of the amplitude modulated motion induced by Coriolis acceleration.

rate input which is to be measured. Therefore, the rotation rate can be inferred by capacitively detecting the rotor tilt oscillation and demodulating with the resonant drive signal.

3.3 MECHANICAL SENSITIVITY AND MODE MATCHING

3.3.1 Angular Deflection Output to Rate Input

Mechanical sensitivity is of paramount importance because it directly determines the minimum detectable signal in the face of interface electronics noise. Electronic noise will limit both the smallest possible rotor tilt deflection which can be detected and the measurement resolution. Hence high sensitivity is desirable for low minimum detectable signal. The mechanical sensitivity relating Coriolis rotor tilt amplitude to substrate angular rate input for each sense axis can be derived from Eq. 3-17 with the final form

shown below for the x-axis. Assuming the x- and y-axis are nearly identical, then each axis will have equivalent magnitude sensitivity as approximated in Eq. 3-19.

$$\frac{\|\theta_x\|}{\|\Omega_y\|} \approx \left\| \frac{2\theta_0\omega_z}{\omega_x^2 + \frac{j\omega_x\omega_z}{Q_x} - \omega_z^2} \right\| \quad (3-19)$$

This is the standard second order frequency response with θ_0 the resonant drive amplitude, ω_z the resonant drive frequency, ω_x the sense axis frequency, and Q_x the quality or inverse damping coefficient. The sensitivity equations can be used to improve sensitivity and hence noise performance. Clearly increasing drive amplitude is advantageous. Also, because the sensitivity is a function of drive and sense natural frequencies, an optimal set of frequencies can be used as discussed in the next subsection.

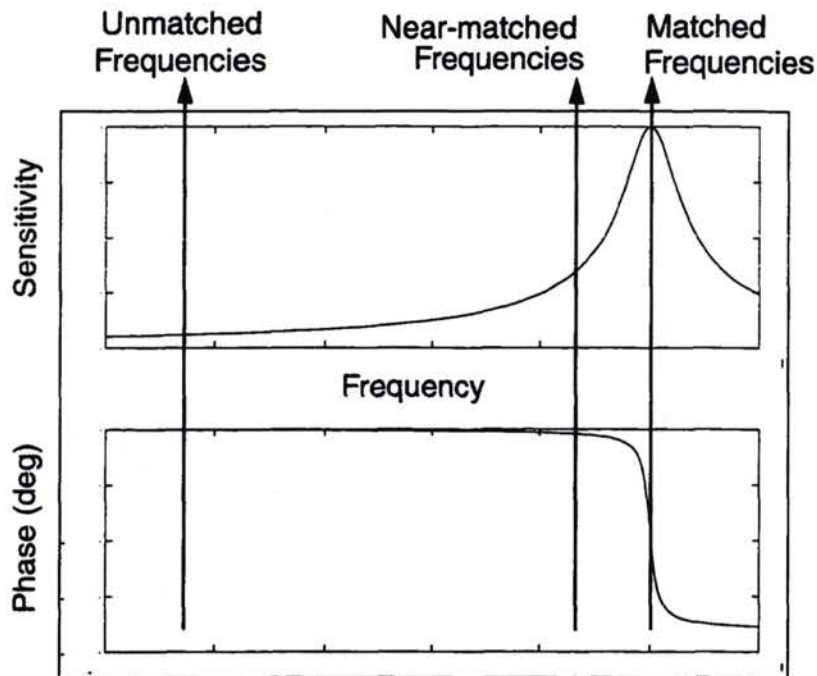


Figure 3-3. Bode plot showing simulated sensitivity and output phase versus sense axis natural frequency.

3.3.2 Improving Sensitivity Via Mode Matching

Since the rate sensor is operated in partial vacuum, the quality factors Q_x and Q_y are generally above 900 with exact values given in Chapter Six regarding experimental results. Thus, a large resonant peak exists which provides the possibility of excellent gain when drive and sense frequencies match [55]. Sensitivity is plotted against sense axis natural frequency in Fig. 3-3. Unmatched natural frequencies give poor signal gain, whereas exactly matched natural frequencies give maximum gain. To improve sensitivity, the four beam suspension is designed to match sense and drive modes closely. However, process variation and manufacturing tolerances make extremely close matching impossible in practice. A later section will discuss using suspension design for coarse frequency tuning while using electrostatic frequency tuning for fine matching after fabrication. But first the method of sensing the Coriolis rotor tilt must be addressed.

3.4 PARAMETRIC MODELING OF ROTOR AND SUSPENSION

The equations of motion use lumped parameters for moment of inertial, suspension stiffness, and damping. These parameters must be derived from mechanical sense element dimensions and material properties. The parametric derivations were intentionally kept simple for two principal reasons. First, a complex derivation including all possible effects becomes intractable and useless as a design tool. Second, parameter errors due to manufacturing process variation often outweigh second order theoretical effects. Hence, a balance must be struck between accuracy and usability.

3.4.1 Rotor Moment of Inertia

The total system moment of inertia is the sum of the rotor moment of inertia and the moment of inertia of the electrostatic combs. Although the notion is counter intuitive, the gyroscope has better performance if the center is hollowed out. Leaving the center hollow does in fact reduce performance enhancing parameters such as the moment of inertia and sensitivity, but only slightly. That is because moment of inertia is proportional to the fourth power of radius and sensitivity is proportional to radius cubed. On the other hand, increased rotor radius increases surface area. Larger surface area implies larger parasitic

capacitance which increases the input referred electrical noise and larger electrostatic forces impinging on the rotor which may pull the rotor toward the substrate. Rotor mass also increases with larger radius. This lowers translational natural frequencies which in turn diminishes both shock and vibration survival. Hence, an optimum exists balancing performance improving inertia and sensitivity with performance degrading parasitic capacitance and mass. This optimization encompasses both mechanical and electrical system aspects of the design, and so will be left until Chapter Five.

The rotor body can be modeled using the outer radius R_o and inner radius R_i . The exact integral for moment of inertia is complicated by etch holes and gaps between resonant drive combs. Instead, estimates based on effective polysilicon density were used. As Eq. 3-20 suggests, the total z-axis moment of inertia is the sum of rotor inertia I_{zr} and drive comb inertia I_{zc} . Using these radii in conjunction with structural polysilicon thickness h , and rotor polysilicon effective density ρ_e ; the rotor moment of inertia about the drive axis I_{zr} can be derived [55]. Likewise, the electrostatic comb moment of inertia I_{zc} can be estimated with the inner comb radius R_{ci} , and outer comb radius R_{co} , and the effective comb density ρ_{ce} .

$$I_{zz} = I_{zr} + I_{zc} \quad (3-20)$$

$$I_{zz} = \frac{\pi}{2} \rho_e h (R_o^4 - R_i^4) + \frac{\pi}{2} \rho_{ec} h (R_{oc}^4 - R_{ic}^4) \quad (3-21)$$

Likewise, the moment of inertia about the sense axis can be calculated. As Eq. 3-22 shows, the sense axis moment of inertia is exactly half the drive axis moment of inertia.

$$I_{xx} = I_{yy} = \frac{1}{2} I_{zz} \quad (3-22)$$

The effective density correctly discounts the rotor polysilicon density for release etch holes and the comb density for comb air gaps. The effective density is the true bulk material density multiplied by the actual polysilicon area and divided by the sum of polysilicon area and area missing due to etch holes or comb gaps. For example, in the Sandia National Labs (SNL) process rotor density ρ_e was discounted to 92% and comb density ρ_{ce} was discounted to 25%. Typical moment of inertia estimates for ADI designs were 5×10^{-18} kg/m and for SNL designs 2.5×10^{-17} kg/m.

3.4.2 Quality Factor due to Rotor Air Damping

The system damping is composed of both structural damping intrinsic to the mechanical element and air damping provided by the surroundings. The structural damping is extremely small with a quality factor Q of perhaps more than 75,000. Although the dual-axis rate gyroscope is operated in moderate vacuum, the air damping is still dominant. For small motions, the air damping torque is approximately linear with respect to rotor rotation rate, so linear damping coefficients are used for dynamical modeling. The damping may be representing in non-dimensional form by using quality factor as shown earlier in Eq. 3-17 and 3-18. The quality factor Q_z for the drive mode is much higher than the sense mode because the drive mode is almost purely incompressible viscous damping, while the sense mode is compressible squeeze film damping.

The drive axis quality factor Q_z can be approximated analytically by using viscous flow equations with an effective viscosity. First the damping torque is calculated, and then this torque is converted into a quality factor Q_z . The air damping on the top of the rotor is small compared to the small gap damping beneath the rotor and therefore ignored for simplicity. Although in vacuum the viscous continuum assumption breaks down, an effective viscosity coefficient μ_{eff} can be used to approximate a linear correspondence between fluid shear stress and fluid velocity gradient [58]. Hence the differential force dF on a differential area element da can be approximated by Eq. 3-23 with μ_{eff} as the *effective* viscosity coefficient, g as the gap between rotor and substrate, and v as the rotor velocity at that point [56].

$$dF = \frac{\mu_{eff}}{g} v \cdot da \quad (3-23)$$

Of course the gyroscope system is rotary, so this differential force must be converted to a differential moment about the rotor center by replacing velocity with $R\dot{\theta}$ where R is radius from the rotor center and $\dot{\theta}$ is rotor angular rate at that point. The total viscous damping torque T can be found by integrating over the entire area A of the rotor as in Eq. 3-24. This is an approximation because the damping at comb tine tips and on the rotor top is ignored. Furthermore, the effective viscosity approximation of the transition from fluid continuum to rare gas particles is imperfect. The factor β is used to account for comb

tine sidewall area which increases damping. In the ADI design with combs on the outside perimeter, comb drag dominates and β was estimated to be 1.8. This estimate considers comb tine overlap, the smaller 1 μm gap between combs, and that each comb tine has two sidewalls.

$$T = \beta \int_A \left(\frac{\mu_{eff}}{g} R^2 \dot{\theta} \cdot da \right) = \frac{\mu_{eff} \beta}{g} \left(\int_A (R^2 \cdot da) \right) \dot{\theta} \quad (3-24)$$

The damping torque is now converted into an equivalent quality factor Q . The quality factor Q is a function of the damping torque T , the rotor angular rate $\dot{\theta}$, the rotor moment of inertia about the drive axis I_{zz} , and the drive mode natural frequency ω_z .

$$Q = \frac{I_{zz} \omega_z \dot{\theta}}{T} \quad (3-25)$$

By substituting the analytical function for drive axis moment of inertia I_{zz} , a simple approximation for Q can be derived. Invoking the definition of moment of inertia about a given axis, the term I_{zz} can be rewritten in terms of radius R , total rotor area A , the mass density of polysilicon ρ , and the polysilicon thickness or height h as in Eq. 3-26.

$$I_{zz} = \rho h \int_A (R^2 \cdot da) \quad (3-26)$$

Now this exact representation of moment of inertia is substituted into Eq. 3-25 and the integral terms cancel revealing a much simplified Eq. 3-28.

$$Q = \frac{\rho \left(\int_A (R^2 \cdot da) \right) \omega_z \dot{\theta}}{\frac{\mu_{eff} \beta}{g} \left(\int_A (R^2 \cdot da) \right) \dot{\theta}} \quad (3-27)$$

$$Q = \frac{\rho h \omega_z g}{\beta \mu_{eff}} \quad (3-28)$$

At first glance, the quality factor Q appears independent of rotor radius or the beam suspension; however, the drive axis natural frequency ω_z is dependent upon both rotor radius and suspension design. A quality factor Q of 71,000 is calculated using typical values 2300 Kg/m^3 for mass density, 28 kHz for drive frequency, 2 μm for polysilicon film

thickness, $1.6\ \mu\text{m}$ for the air gap, and $1\text{e}10^{-8}\ \text{Kg/m/s}$ effective viscosity at 60 mTorr pressure. This method overestimates the Q as verified by the actual Q value of $>30,000$ measured experimentally.

Unfortunately, the quality factor of the sense mode is not simple to calculate. No analytical solution was found for compressible squeeze film damping of a circular rotor riddled with etch holes. Some recent researchers have investigated squeeze film damping in a rectangular plate [59-60], but all results are arrived at through numerical computation of specific cases which cannot be generalized to the current gyroscope problem. However, experimentation revealed an approximate empirical relationship of sense axis Q being an order of magnitude lower than drive axis Q in the pressure range of interest.

3.4.3 Suspension Spring Constants

Attention is now focused on the suspension design and the resulting spring constants. There are several key design constraints which the suspension design must satisfy. First, any mechanical structure deformation due to fabrication induced residual stress gradients must be accommodated by the suspension. This was particularly true for gyroscopes fabricated by Analog Devices Inc. (ADI) because relatively large stress gradients in the structural polysilicon were present. Second, all natural frequencies, both rotational and translational, must be above a given threshold for shock and vibration robustness. This work attempted to keep all frequencies above high audio frequencies of 10-20 kHz. Preferably, the rotational frequencies will be lower than the translational frequencies. Finally, the suspension should allow close but not exact matching of the rotational drive mode and both the rotational sense modes to enhance sensitivity. Ideally, the sense modes will be slightly higher than the drive mode so electrostatic down-tuning can equalize all rotational natural frequencies. The fact that the sense axis moment of inertia is half the drive axis moment of inertia implies that the sense axis spring constant must be approximately half the drive axis spring constant for effective mode matching. The following subsections layout the fundamental considerations and governing equations constraining beam suspension design. The actual design optimization and final dimensions are left until Chapter Five.

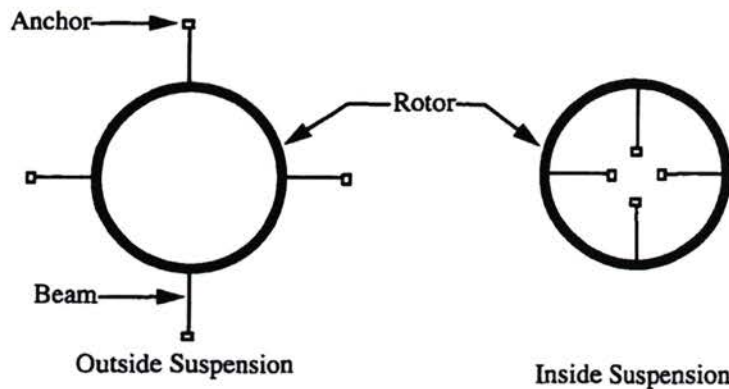


Figure 3-4. Conceptual illustration of both inside and outside suspension designs. Inside suspension provides more compact designs, but excessive residual stress gradient can force the use of outside suspension.

3.4.3.1 Inside Versus Outside Suspension

The overriding design consideration which must be tackled is whether the suspension should be placed inside the rotor or outside the rotor as depicted in Fig. 3-4. Placing the suspension inside the rotor can reduce sensor area and force all translation modes to be higher relative to the rotational modes. Unfortunately, the inside suspension can be disastrous if the residual stress gradient causes the polysilicon structure to warp into a convex shape with the center at the high point. This allows the outer edge of the gyroscope to drag on the substrate and halts all motion. For this reason, the outside suspension had to be used for the ADI fabricated gyroscopes, despite area and mode frequency drawbacks.

3.4.3.2 Rotational Spring Constant Derivation

The linear spring constants for both inside and outside suspension designs were derived using simple Euler beam bending theory. A spring constant represents the linear relationship between angular rotor displacement versus suspension torque and is accurate for the case of small deflections. All spring constants were assumed uncoupled for this simplified analysis. The spring constants can be found by calculating the torque required to deflect the rotor a given angular displacement. Each beam can be thought of as a

cantilever with a force and an angular moment applied at the end attached to the rotor. For a given rotor angle, the beam must satisfy both a deflection and a slope constraint at the end attached to the rotor. Hence, there are two linear equations (one for end deflection and one for end slope) with two unknowns (force and moments applied by the rotor). Converting the force and moment applied upon the beam end to a total moment about the rotor center provides the spring constant for a single beam. The resulting moment about the rotor center for a single beam M_R is the sum of the single beam moment M and the single beam force F multiplied by the radius from beam end to rotor center R .

$$M_R = M + F \times R \quad (3-29)$$

Simply multiplying the spring constant calculated for a single beam by four gives the total angular spring constant for all four suspension beams in the drive mode. This is mathematically represented by Eq. 3-30 where K_{zz} total drive axis spring constant and K_{bz} is the spring constant for a single beam.

$$K_{zz} = 4K_{bz} \quad (3-30)$$

The sense mode spring constants K_{xx} and K_{yy} can be approximated by twice the single beams spring constant K_{bx} as in Eq. 3-31. This is because only two beams exhibit bending, while the other two beams exhibit weak torsion which produces negligible torque.

$$K_{xx} = K_{yy} = 2K_{bx} \quad (3-31)$$

Remembering that in order to match mode frequencies the sense axis spring constant must be half the drive axis spring constant. This requirement can be proven by assuming the natural frequencies ω_x , ω_y , and ω_z are equal and then substituting expressions based on total spring constants and moments of inertia.

$$\omega_x = \omega_y = \omega_z \quad (3-32)$$

$$\sqrt{\frac{K_{xx}}{I_{xx}}} = \sqrt{\frac{K_{yy}}{I_{yy}}} \approx \sqrt{\frac{K_{zz}}{I_{zz}}} \quad (3-33)$$

Now the total spring constants can be replaced by single beams expressions while z-axis moment of inertia can be replaced by twice the x-axis moment of inertia and both the x- and y-axis moments of inertia are assumed equal.

$$\sqrt{\frac{2K_{bx}}{I_{xx}}} = \sqrt{\frac{2K_{by}}{I_{yy}}} \approx \sqrt{\frac{4K_{bz}}{2I_{xx}}} \quad (3-34)$$

$$K_{bx} = K_{by} = K_{bz} \quad (3-35)$$

Therefore, all frequencies are ideally matched if all single beam spring constants are identical. In practice process variation forces electrostatic tuning to be used, so in fact the sense x- and y-axis spring rates will be designed slightly larger as Subsection 3.4.3.3 regarding frequency ratios reveals. However, setting all single beam spring constants *approximately* equal is a good rule of thumb.

Thus the height and width of the suspension beams should be approximately equal. This results in each *individual* beam having spring constants approximately equal in both the drive and sense bending directions. The sense axis spring constant (with only two beams in bending) is then naturally one half the drive axis spring constant (with all four beams in bending).

Serpentine or folded suspensions may be used to conserve die area and provide some stress relief. This can be mathematically accommodated easily by simplifying the problem into a single beam bending problem with modified deflection and slope constraints. In order to facilitate drive and sense mode matching, the end connections

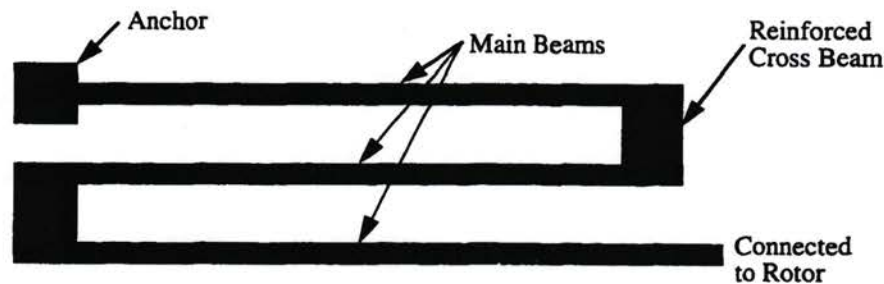


Figure 3-5. Typical serpentine or folded suspension with reinforced cross beams at connected ends for better mode matching.

between parallel beams are designed to minimize flexure. As Fig. 3-5 shows, this is accomplished by designing wide, reinforced cross beams connecting folded beams at the extreme ends. Therefore, the spring constants in both sense and drive direction depend only on the main beams and not the connecting ends. This allows more precise mode matching because spring constants depend mostly on beam aspect ratio and length, not complex torsion and deflections at the connection ends.

The problem is further simplified by making all serpentine beams approximately the same length. Because each beam is approximately identical in both dimension and end conditions, each beam must exhibit identical deflection and slope change. Hence, a two fold suspension can be simplified into a single beam problem with the beam undergoing half the total deflection and half the slope change of the total suspension. Generalizing this concept to multiple folds simply requires a single beam to exhibit deflection and slope divided by the number of folds. Each serpentine suspension was oriented such that the gyroscope was symmetrical about the 45 degree diagonal. This symmetry ensured that no initial rotor displacement occurred due to fabrication induced residual stress gradient in the beams.

The previous assertions and the following calculations rest on several key assumptions for an idealized gyroscope [62]. First, all deflections are assumed small enough so as to remain in the linear range with beam elongation negligible so Euler beam theory is applicable. Second, all beams are identical in dimension and material properties. Third, the rotor can be considered absolutely rigid compared to the suspension beams, and hence rotor bending is negligible. This assumption is reasonable because the rotor is almost two orders of magnitude wider than the individual beams. Finally, although large residual stress are present after fabrication, the serpentine suspensions provide some stress relief. The stress relief was intentionally implemented to lower natural frequencies and allow all critical natural frequencies to be dependent on beam dimensions.

Calculation of the spring constant for outside beams begins with the formulation of the deflection in Eq. 3-36 and deflection slope in Eq. 3-37 at the rotor attached end of each beam [61]. If the rotor rotates through an angle θ , then the beam end attached to the rotor

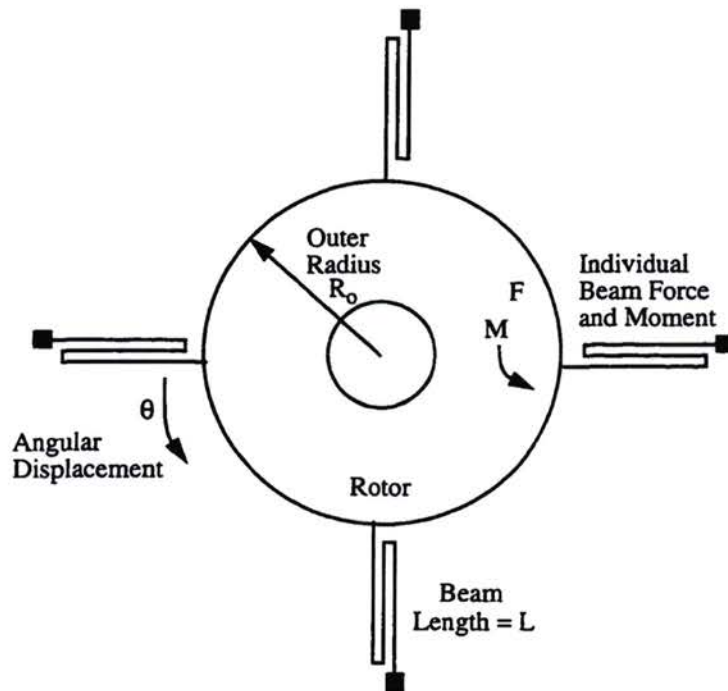


Figure 3-6. Top view of polysilicon rotor showing dimensions and variable definitions for an outside suspension design.

must have slope θ and deflect a distance $R_o\theta$. Both these equations depend on the unknown force F and moment M applied by the rotor. Since Euler beam theory is purely linear, the deflection and slope induced by both applied force and moment can be combined linearly.

$$\frac{FL^3}{3EI} + \frac{ML^2}{2EI} = \frac{R_o\theta}{N} \quad (3-36)$$

$$\frac{FL^2}{2EI} + \frac{FL}{EI} = -\frac{\theta}{N} \quad (3-37)$$

Beam length L , outer rotor radius R_o , beam cross sectional moment of inertia I , number of serpentine folds N , and modulus of elasticity E all play a role. Although the thin film polysilicon can exhibit large residual stresses, these are not included because all suspension designs implemented stress relief. Incomplete stress relief would result in higher spring constants than calculated.

The drive and sense axis may have different cross sectional moments of inertia to intentionally mismatch natural frequencies. This is accomplished by changing the aspect ratio which is defined as the ratio between beam height h and beam width b . The corresponding beam moments of inertia can be calculated as in Eq. 3-38 and 3-39 [61].

$$I_{bz} = \frac{hb^3}{12} \quad (3-38)$$

$$I_{bx} = \frac{bh^3}{12} \quad (3-39)$$

Solving these two simultaneously results in the following equation for drive and sense mode spring constants in terms of material and geometric parameters. Interestingly, increasing the radius with respect to suspension beam length results in dramatically higher rotational stiffness.

$$\text{DRIVE: } K_{zz} = 16 \frac{EI_{bz}}{LN} \left\{ 3 \left\langle \frac{R_o}{L} \right\rangle^2 + 3 \left\langle \frac{R_o}{L} \right\rangle + 1 \right\} \quad (3-40)$$

$$\text{SENSE: } K_{xx} = K_{yy} = 8 \frac{EI_{bx}}{LN} \left\{ 3 \left\langle \frac{R_o}{L} \right\rangle^2 + 3 \left\langle \frac{R_o}{L} \right\rangle + 1 \right\} \quad (3-41)$$

Calculation of the inside suspension linear spring constant proceeds the same way, except that the original deflection and slope equations are altered. The constraint for beam deflection, Eq. 3-42, changes polarity. In this case, the beam deflection and slope have the same sign unlike the case of outside suspension in which the deflection and beam slope have opposite signs. Also, it is the inner radius R_i of the rotor which affects stiffness [61].

$$\frac{FL^3}{3EI} + \frac{ML^2}{2EI} = -\frac{R_i\theta}{N} \quad (3-42)$$

$$\frac{FL^2}{2EI} + \frac{FL}{EI} = -\frac{\theta}{N} \quad (3-43)$$

Again these equations are simultaneously solved to reveal the final drive and sense spring constants. Note the equations for the inside and outside spring constants are nearly identical save for a sign change in the center term.

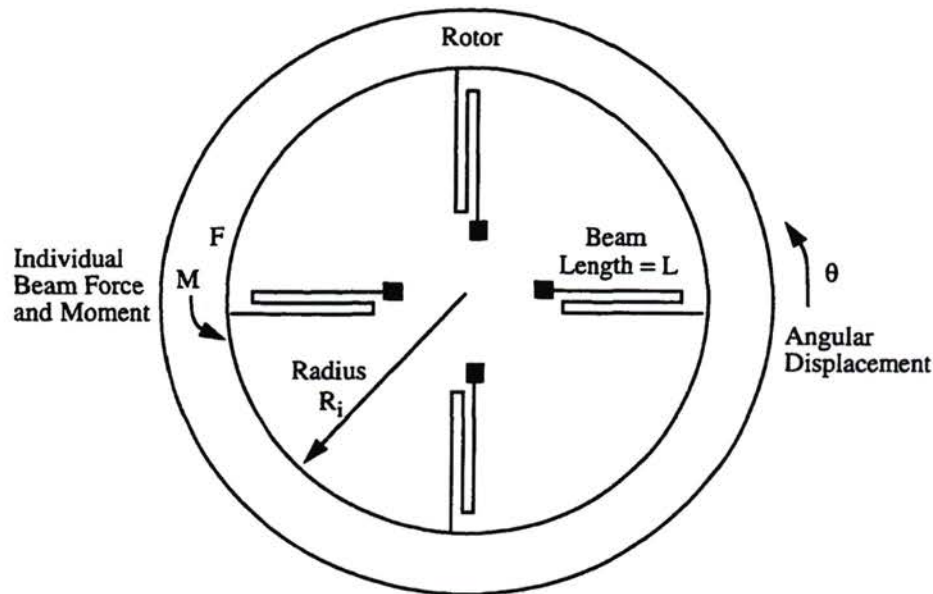


Figure 3-7. Top view of polysilicon rotor showing dimensions and variable definitions for an inside suspension design.

$$\text{DRIVE: } K_{zz} = 16 \frac{EI_{bz}}{LN} \left\{ 3 \left\langle \frac{R_i}{L} \right\rangle^2 - 3 \left\langle \frac{R_i}{L} \right\rangle + 1 \right\} \quad (3-44)$$

$$\text{SENSE: } K_{xx} = K_{yy} = 8 \frac{EI_{bx}}{LN} \left\{ 3 \left\langle \frac{R_i}{L} \right\rangle^2 - 3 \left\langle \frac{R_i}{L} \right\rangle + 1 \right\} \quad (3-45)$$

This negative term allows inside suspension designs to produce lower rotational modes frequencies than outside suspension designs given equivalent radius and beam lengths. This is very important as subsection 3.4.3.4 will demonstrate that the translational modes for both inside and outside suspensions are equal.

3.4.3.3 Ratio Between Sense and Drive Mode Frequencies

As stated previously, the ratio of sense spring constant to drive spring constant should be half for successful mode matching. Unfortunately, in practice the modes are never truly equal due to manufacturing tolerances. The fact that polysilicon deposition sets beam height h while lithography sets beam width b makes manufacturing well matched modes extremely difficult. To overcome this difficulty, electrostatic tuning can be used to down

tune sense modes. Sense and drive modes are course tuned using beam geometry and then fine tuned using electrostatics. The only rotation mode frequency tuning which can occur is down tuning of the sense axis rotation mode. For this reason, it is advantageous to purposely mismatch modes with the sense mode 5-10% higher than the drive mode. This allows electrostatic down tuning to equalize modes in the face of small fabrication deviations. The frequency ratio can be shown to depend only on beam cross-section moment of inertia which in turn is dependent only on the aspect ratio of the beam.

$$\frac{\omega_{Drive}}{\omega_{Sense}} = \frac{\sqrt{\frac{K_{Drive}}{I_{zz}}}}{\sqrt{\frac{K_{Sense}}{I_{xx}}}} = \frac{\sqrt{I_{bz}}}{\sqrt{I_{bx}}} = \frac{b}{h} \quad (3-46)$$

The ideal target mode matching ratio between sense axis and drive axis can be selected using the aspect ratio b/h . For example, the ADI BiMEMS process had a fixed polysilicon thickness or height of $2 \mu\text{m}$, so the beam width was made $1.8 \mu\text{m}$ to attempt a sense mode mismatch of plus 10%. The SNL process had a structural polysilicon thickness of $2.3 \mu\text{m}$ with a width of $2.1 \mu\text{m}$ to produce a mode mismatch of 9%. More details regarding choosing a frequency ratio are discussed in Chapter Five.

3.4.3.4 Translational Spring Constant Derivation

With all rigid rotational mode frequencies investigated, attention is now focused on the translational modes. Ideally, the translational mode frequencies will be much higher than the rotational mode frequencies. Higher frequencies attenuate rectilinear accelerations such as shocks and vibrations which will alter scale factor or interrupt operation. In addition, the structure must have strength enough to withstand electrostatic forces due to sensing voltages. As discussed in the next sections, it is the z-axis translation mode frequency which dictates maximum modulation voltage on the Coriolis sense electrodes and maximum electrostatic frequency tuning range.

The z-axis perpendicular to the substrate is the most critical for surface micromachines. One of the serious problems plaguing surface micromachining is stiction between the substrate and the polysilicon structure. Unfortunately, the thin film polysilicon used in a typical surface micromachining process provides little stiffness in the z-axis direction to prevent the mechanical structure from contacting the substrate. Once in contact, the structures often adhere to the substrate, rendering the device inoperable. Chapter Four points out that external z-axis accelerations should not produce false Coriolis outputs due to the differential sensing design. However, changing rotor height caused by accelerations will change scale factor. Many methods including the addition of small bumps to reduce contact area and anti-stiction monolayers have been utilized, but a stiffer z-axis mode increases the shock input needed to instigate contact between structure and substrate.

The x-axis and y-axis translation modes parallel to the substrate are naturally stiff because these involve beam compression and not beam bending. However, the z-axis mode is essentially a pure beaming mode, and therefore must be calculated. The same assumptions regarding linearity, rotor rigidity, identical beams, and negligible residual stress effects used to calculate rotational spring constants are also assumed for z-axis spring constant calculation. The identical beam assumption is important because this implies symmetry which requires that the rotor remain parallel to the substrate.

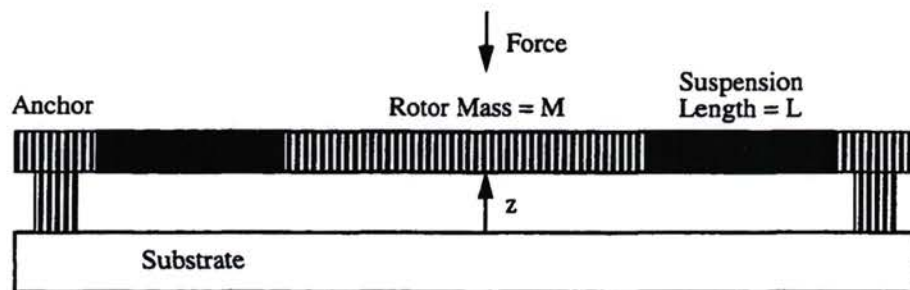


Figure 3-8. Cross sectional side view of the rotor with suspension.

The exact same approach will be used for translational spring constant calculation as was used for rotational spring constant calculation. The beams are modeled as cantilever beams rigidly mounted to the substrate. All four suspension beams are identical, so the

total z-axis spring constant is simply four times the spring constant of each individual beam. For a given rotor z-axis deflection, the deflection and slope constraints of the individual beam ends attached to the rotor are derived. Clearly, the beam end displacement is equal to the rotor axial displacement z . The slope is constrained to zero because the rotor must be parallel to the substrate due to symmetry and the rotor cannot deflect appreciably due to relative rigidity. Again, serpentine or folded suspension is accommodated by noting that each beam is constrained to deflect a fraction of the total z-axis rotor motion as given by $1/N$ where N is the number of folds. Writing the beam end motion constraints in terms of applied moment M and force F results in Eq. 3-47 and 3-48 with E representing modulus of elasticity and I_{bx} representing the beam cross section moment of inertia.

$$\frac{FL^3}{3EI_{bx}} + \frac{ML^2}{2EI_{bx}} = \frac{z}{N} \quad (3-47)$$

$$\frac{FL^2}{2EI_{bx}} + \frac{FL}{EI_{bx}} = 0 \quad (3-48)$$

These equations can be solved simultaneously to find the moment M and force F . The moments of diametrically opposed beams cancel out leaving only the force component. Multiplying this by four, since there are four beams, yields the total z-axis spring constant K_{Tz} . Typical calculated values were 1 N/m for the ADI devices and 6 N/m for SNL devices.

$$K_{Tz} = \frac{48EI_{bx}}{NL^3} \quad (3-49)$$

The actual z-axis translation mode frequency can be calculated as follows with M representing the total rotor mass.

$$\omega_{Tz} = \sqrt{\frac{K_{Tz}}{M}} \quad (3-50)$$

Note that the translational spring constant does not depend on rotor radius. Hence, both inside and outside suspensions have the same z-axis mode frequency if suspension designs and rotor mass are equivalent. Typical calculated values were 12 kHz for ADI devices

and 22 kHz for SNL fabricated devices. This implies that the ADI devices should withstand a static 9000 g acceleration before touching the substrate, while the SNL design should withstand a 38000 g acceleration. In general, the outside suspension will have higher rotational frequencies than equivalent inside suspension. This implies that inside suspensions are advantageous because the z-axis translation mode frequency will be higher for a given sense axis rotation frequency. This is important as a higher z-axis frequency provides better acceleration attenuation as well as increased sense and tuning voltages as revealed in the next sections. On the other hand, care must be taken since inside suspensions prove less attenuation of angular accelerations.

3.5 IMPROVING SENSITIVITY VIA ELECTROSTATIC FREQUENCY TUNING

3.5.1 Electrostatic Down Tuning of Sense Mode Frequencies

As stated previously in section 3.3.2, sensitivity can be significantly improved by matching sense mode frequencies with drive mode frequency. Unfortunately, process variations during manufacture make precise mode matching via mechanical structure design impossible in practice. Under etching, over etching, and polysilicon thickness all change suspension beam aspect ratio and thereby change frequency matching. To solve this difficulty, electrostatic tuning can be used to compensate for random process variations. The main drawback of this electrostatic tuning approach is a significant decrease in maximum mechanical shock survival if significant frequency tuning is needed.

The rotor is essentially a parallel plate suspended by the beam springs above the substrate. Any voltage applied between the rotor and the four quarter-pie shaped electrodes beneath the rotor will result in an electrostatic force. Because electrostatic force is always attractive, the electrical force behavior is opposite that of the beam suspension. As the rotor moves closer to the substrate, the electrostatic force becomes stronger. Likewise, as the rotor moves away from the substrate the electrostatic force becomes weaker. Tilting motion also exhibits the same behavior as the edge tilted closer to the substrate is pulled down, while the edge tilted away from the substrate has less force acting upon it. Hence, the electrostatic force acts like an unstable or 'negative' spring

working to force the rotor away from steady state equilibrium. For small motions, the forces may be linearized about the rotor equilibrium position. The total x-axis torsional spring constant $\hat{K}_{xx}$ is the sum of the positive mechanical constant K_{xx} and the negative electrostatic spring constant K_{ex} as shown in Eq. 3-51. The second equation shows that the y-axis torsional spring constant $\hat{K}_{yy}$ complies with the same equation formulation due to symmetry.

$$\hat{K}_{xx} = K_{xx} - K_{ex} \quad (3-51)$$

$$\hat{K}_{yy} = K_{yy} - K_{ey} \quad (3-52)$$

This shows that the sense springs and therefore the sense natural frequencies can be down tuned. Since the effective electrostatic spring can never be positive, the sense frequencies must be designed via beam aspect ratio to be higher than the drive frequency. After fabrication, the voltage applied between the sense electrodes and the rotor can be adjusted to down tune the sense frequencies until they nearly match the drive frequency.

The linearized spring constant is defined as the derivative of the electrical torque M_x with respect to rotor tilt angle ϕ . This can be rewritten as Eq. 3-53 in terms of the physical parameters applied voltage V and the second derivative of each quadrant sense electrode capacitance C_s . The y-axis follows suit in Eq. 3-54. Calculation of the sense capacitance partial derivatives will be explored in Chapter Four. The calculation is strongly dependent on the air gap between rotor and substrate. This gap decreases with applied voltage, so the second derivative of sense capacitance is also a function of applied voltage.

$$K_{ex} = \frac{\partial M_x}{\partial \phi} = \frac{\partial^2 C_s}{\partial \phi^2} V^2 \quad (3-53)$$

$$K_{ey} = \frac{\partial M_y}{\partial \psi} = \frac{\partial^2 C_s}{\partial \psi^2} V^2 \quad (3-54)$$

3.5.2 Tuning Range Set by Pull-down Voltage

From the previous equations, it appears that the total spring constant can be down tuned to any value simply by increasing the applied voltage. However, this is not the case

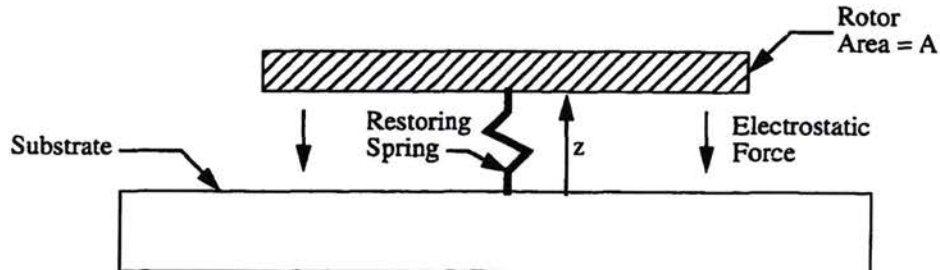


Figure 3-9. Conceptual side view of the rotor acted upon by both the suspension spring restoring force and the electrostatic attractive force.

because there is a limit to how high the voltage can be set. As the voltage is increased, the rotor will also deflect down towards the substrate in the z -axis translation direction until the suspension force exactly balances the electrostatic force. However, there exists a voltage, termed the pull-down voltage, beyond which the suspension z -axis restoring force is too weak to balance the electrostatic force. At the pull-down voltage the electrostatic force overpowers the z -axis restoring force of the springs, and the rotor snaps down to the substrate [63].

As illustrated in Fig. 3-9, there are two opposing forces acting on the gyroscope rotor. One is the suspension spring restoring force F_{Spring} which works to keep the rotor at equilibrium, and the second is the electrostatic force F_{Electro} which works to pull the rotor to the substrate. The strength of these forces can be approximated using the rotor distance from the substrate z , the voltage applied between rotor and sense electrodes V_s , and physical characteristics of the rotor. Eq. 3-55 shows the suspension spring force with the original gap g and the z -axis translation spring constant K_{Tz} .

$$F_{\text{Spring}} = K_{Tz}(g - z) \quad (3-55)$$

Eq. 3-56 shows the electrostatic force impinging on the rotor in terms of rotor area A and the dielectric coefficient of vacuum ϵ . This relation assumes that the inertial rotor is ideally flat and that no outside accelerations are altering rotor position.

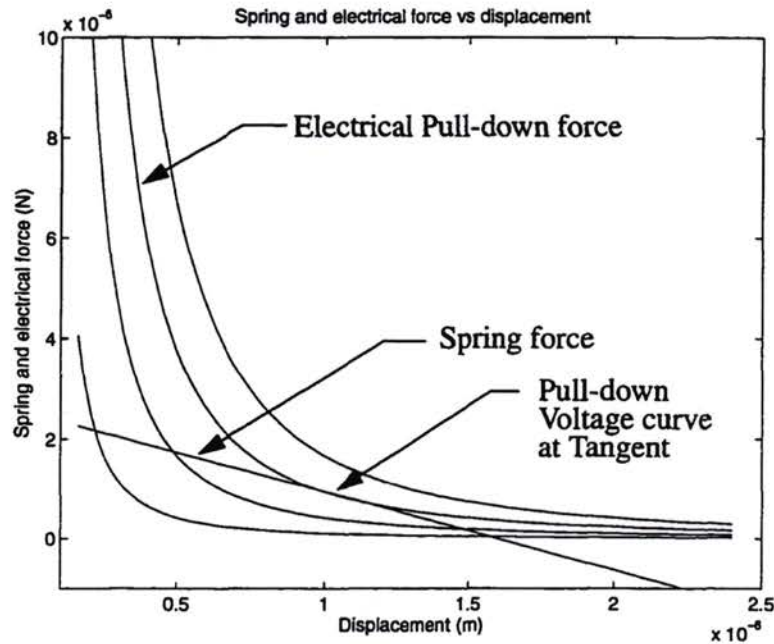


Figure 3-10. Suspension spring and electrostatic force curves plotted with respect to z -axis displacement.

$$F_{Electro} = -\frac{1}{2} \frac{A\epsilon V^2}{z^2} \quad (3-56)$$

These two opposing forces can be plotted against rotor distance from the substrate z as in Fig. 3-10. Changing the voltage applied between the rotor and the sense electrodes shifts the electrostatic force upward for higher voltages. For low voltages, the force curves intersect at two points where suspension force and electrostatic force are equal. One equilibrium point is stable and the rotor stays at this point unless perturbed. The other equilibrium point is unstable. If the rotor is moved to this unstable point close to the substrate, the electrostatic force dominates and pulls the rotor down. Above this critical distance, the rotor will snap back to the stable equilibrium point. At very high voltages, there is no force curve intersection, and the electrostatic force always dominates pulling the rotor to the substrate. In between these two extremes lies a critical pull-down voltage where the suspension can just balance the spring. At the pull-down voltage, only one

equilibrium point exists and the two force curves are tangent. This results in a metastable equilibrium point with zero spring constant and a zero eigenvalue.

This critical pull-down voltage is the absolute maximum voltage DC voltage which can be applied between the rotor and the sense electrodes. Hence, the pull-down voltage limits the rotational x- and y-axis frequency tuning range as well as the maximum sense modulation voltage. This critical pull-down voltage can be calculated by solving two equations simultaneously for two independent variables. The first equation is the force balance Eq. 3-57 which enforces equal and opposite forces at equilibrium and can be rewritten in terms of key variables as in Eq. 3-58. This formulation assumes that the mechanical suspension is linear over the range of motion and that the rotor acts as an ideal parallel capacitor ignoring fringing fields.

$$F_{Spring} + F_{Electro} = 0 \quad (3-57)$$

$$K_{Tz}(g - z) = \frac{1}{2} \frac{A\epsilon V^2}{z^2} \quad (3-58)$$

The second equation embodies the important observation that the two force curves are tangent at the pull-down voltage. Therefore, the slopes of the two curves are equal as enforced by in Eq. 3-59 with substitutions to yield Eq. 3-60.

$$\frac{\partial F_{Spring}}{\partial z} = \frac{\partial F_{Electro}}{\partial z} \quad (3-59)$$

$$K_{Tz} = \frac{A\epsilon V^2}{z^3} \quad (3-60)$$

All of the variables in the above equations are known except for the pull-down voltage and the rotor distance from the substrate when pull-down occurs. Solving the equations using substitution recovers the rotor distance above the substrate z in terms of the original as fabricated gap g . Increasing voltage pulls the rotor down until the gap is $2/3$ the original zero voltage gap. At this point, pull down occurs. This critical distance $(2/3)g$ can be substituted back into the force balance equation and the pull-down voltage can be derived as in Eq. 3-62.

$$z = \frac{2}{3}g \quad (3-61)$$

$$V_{pull} = \sqrt{\frac{8g^3 K_{Tz}}{27A\epsilon}} \quad (3-62)$$

This pull-down voltage sets the absolute upper limit on voltage which can be applied for frequency tuning. In practice, a lower voltage must be used since at the critical pull-down voltage, the total spring constant is zero and any perturbation will result in the rotor snapping down. Therefore it is advantageous to choose a desired restoring total spring constant K_d which provides good stability robustness and good survival to outside accelerations. Then the voltage which results in this desired spring constant must be calculated. This can be accomplished by solving two equations simultaneously for rotor distance z and desired voltage V_d . The first equation is Eq. 3-63 which simply states the desired spring constant is the original z -axis translation spring constant minus the effective electrostatic spring constant. The effective electrostatic spring constant is simply the derivative of electrostatic force with respect to rotor height z .

$$K_d = K_{Tz} - \frac{A\epsilon V^2}{z^3} \quad (3-63)$$

The second equation is the familiar force balancing equation ensuring forces are equal at the equilibrium point.

$$K_{Tz}(g - z) = \frac{A\epsilon V^2}{z^2} \quad (3-64)$$

Solving through substitution results in an expression for the new gap between rotor and substrate z in terms of the original manufactured gap g . This can be substituted to finally reveal the desired voltage required to provide the desired spring constant in Eq. 3-66.

$$\frac{z}{g} = \frac{2}{3 - \frac{K_d}{K_{Tz}}} \quad (3-65)$$

$$V^2 = \frac{8g^3 K_{Tz}^2}{\epsilon A (3K_{Tz} - K_d)} \left[1 - \frac{2K_{Tz}}{(3K_{Tz} - K_d)} \right] \quad (3-66)$$

Utilizing these equations, the ratio of new versus old or original spring constant can be plotted as in Fig. 3-11. The first ADI dual-axis gyroscope design, which will be explained in detail in a later chapter, was modeled. The horizontal axis shows the voltage applied between rotor and sense electrodes. The top graph shows the z-axis translation spring constant drop with increased voltage until it reaches zero. At this zero point, pull-down occurs with applied voltage equaling approximately 2.4 volts for the ADI designs and 4 volts for the SNL designs. Experimental data showed that the actual pull-down voltage was in fact 3.4 volts for ADI designs. The SNL design were much lower than expected at 2.5 volts partially due to over-etching of the suspension beams. Hence, this calculation is an approximate estimate with spring hardening and fringe capacitances ignored. The lower graph shows the ratio between new gap and the old or original manufactured gap

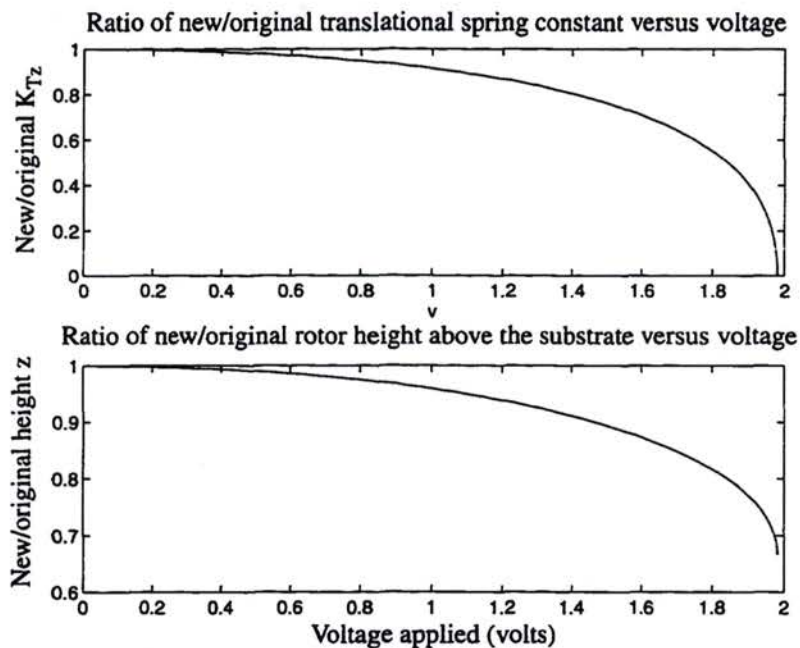


Figure 3-11. The top graph shows the ratio between new and original translational spring constants versus applied voltage. The bottom graph shows new versus original rotor height versus applied voltage.

between rotor and substrate. As expected, the gap decreases with increased voltage as the spring deflects to balance the electrostatic force. Eventually, electrostatic force dominates and pull-down occurs when the new gap measures 2/3 the original gap.

Using the desired voltage required for the desired spring constant, the maximum tuning range can be calculated. This actually overstates the tuning range as the sense voltage required for electrical interface circuitry must be added. However, this approximates the maximum possible frequency tuning range of the rotational spring constant as given in the following equations.

$$\frac{\hat{\omega}_x}{\omega_x} = \frac{\hat{K}_x}{K_x} = \sqrt{1 - \frac{K_{ex}}{K_x}} \quad (3-67)$$

$$K_{ex} = \frac{\partial^2 C_s}{\partial \phi^2} V_d^2 \quad (3-68)$$

3.6 MINIMUM DETECTABLE SIGNAL DUE TO BROWNIAN NOISE

Maximizing sensitivity by frequency tuning is important for performance in the face of electrical interface noise. However even if the circuits were absolutely noiseless, the ultimate minimum detectable signal is limited by mechanical damping or Brownian noise [64]. Just as electrical resistors have thermal or Johnson noise, the mechanical 'resistors' or dampers have Brownian noise. In this medium vacuum case, viscous air damping is the white noise source as air molecules randomly impinge on the mechanical structure. The ultimate limit in hard vacuum where air damping becomes negligible is set by the structural damping of polysilicon. In either case, the resulting angular torque or moment root power density M_n can be derived in terms of the damping coefficient C_{xx} , temperature T , and Boltzmann constant k_B as revealed in Eq. 3-69.

$$M_n = \sqrt{4k_B T C_{xx}} \quad (3-69)$$

This torque noise appears as a false Coriolis acceleration noise which sets the absolute minimum detectable rate signal for the gyroscope. Converting the Brownian noise torque

into an equivalent false rate input is accomplished in Eq. 3-70 by setting the Brownian torque noise equal to the Coriolis acceleration term from previous Eq. 3-12.

$$I_{zz}\Omega_n\dot{\theta} = M_n \quad (3-70)$$

The expression for drive oscillation velocity $\dot{\theta}$ from equation 3-16 can be substituted. The sinusoid term spreads the input rate signal over twice the original bandwidth while reducing magnitude by a factor of 2. This allows calculation of the equivalent false rate input caused by Brownian noise [65] in Eq. 3-72.

$$I_{zz}\Omega_n\theta_0\omega_z\cos(\omega_z t) = M_n \quad (3-71)$$

$$\Omega_n = 4 \frac{\sqrt{k_B T C_{xx}}}{I_{zz}\theta_0\omega_z} \quad (3-72)$$

This equation can be converted into a more useful form involving quality factor Q as shown below.

$$\Omega_n = \frac{4}{\theta_0} \sqrt{\frac{k_B T}{I_{zz}\omega_z Q_x} \left(\frac{\omega_x}{\omega_z} \right)} \quad (3-73)$$

Despite operation in a moderate vacuum (50-100 mTorr), the main source of damping is still due to air viscosity and not structural damping. At lower pressures the damping would be reduced, and therefore Brownian noise would be reduced. However, at very low pressures the structural damping of the polysilicon becomes dominant and no more improvement can be achieved without structural or material alteration. As quality factor Q versus pressure data contained in Chapter Six shows, the pressure level at which structural damping becomes dominant was not reached using available test apparatus. Actual gyroscope testing occurred at approximately room temperature with an operational pressure of 60 mTorr resulting in a Q of approximately 1000. This resulted in an estimated Brownian noise level of approximately $36^\circ/\text{hour}/\sqrt{\text{Hz}}$ for the first generation dual-axis gyroscope fabricated by ADI and $22^\circ/\text{hour}/\sqrt{\text{Hz}}$ for the larger dual-axis gyroscope fabricated by SNL. Unfortunately, the Brownian noise performance level was never reached because electrical noise from the interface circuitry was the dominant noise source. However, future design generations should approach the Brownian noise level.

3.7 CONCLUSIONS

This chapter has described design of the mechanical structure in detail. The chapter began with formulation of the decoupled, linearized equations of motion. These equation revealed that sensitivity could be significantly enhanced by matching drive and sense mode frequencies. The narrative proceeded to then find analytical expressions for the parameters contained in the equations of motion. This related lumped dynamical parameters and mode matching criteria to actual rotor and beam geometries and material properties. No manufacturing process is perfect, so electrostatic tuning is required to truly match modes. However, the design optimization and final mechanical structure dimensions were not presented. This is because interface electronics are an integral part of the mechanical design, and optimization is impossible without a detailed understanding of the electronics. The electronics required to set the mechanical structure in motion and detect the ensuing Coriolis tilt motion are discussed in the next chapter. Then and only then can design optimization be explored in Chapter Five.

4. ELECTRICAL INTERFACE AND SIGNAL PROCESSING

The dual-axis gyroscope requires both interface and signal processing circuitry. The overall system level schematic is pictured in Fig. 4-1. First and foremost, the gyroscope must be driven into rotational oscillation with a constant amplitude. This is the duty of the trans-resistance amplifier in combination with Automatic Gain Control (AGC). Next, a capacitive measurement scheme must detect Coriolis motion in both orthogonal input axes and then produce useful electrical signals. High frequency digital sense voltages applied to the quadrant sense electrodes in conjunction with an integrator attached to the rotor provide Coriolis sensing. These electrical sense signals must undergo signal processing to produce the final rotation rate output signals. Finally, all of the signal processing, modulated voltages, and AGC must be coordinated by a master clock. Therefore, a phase-locked-loop is used to slave a voltage controlled oscillator to the gyroscope drive frequency. Using basic digital circuit building blocks, all of the required digital signal processing and modulation signals are created. While the ADI fabricated versions had much of this circuitry off-chip, a later section discusses the Sandia National Labs (SNL) fabricated versions which integrated nearly everything on-chip. In fact, only passive

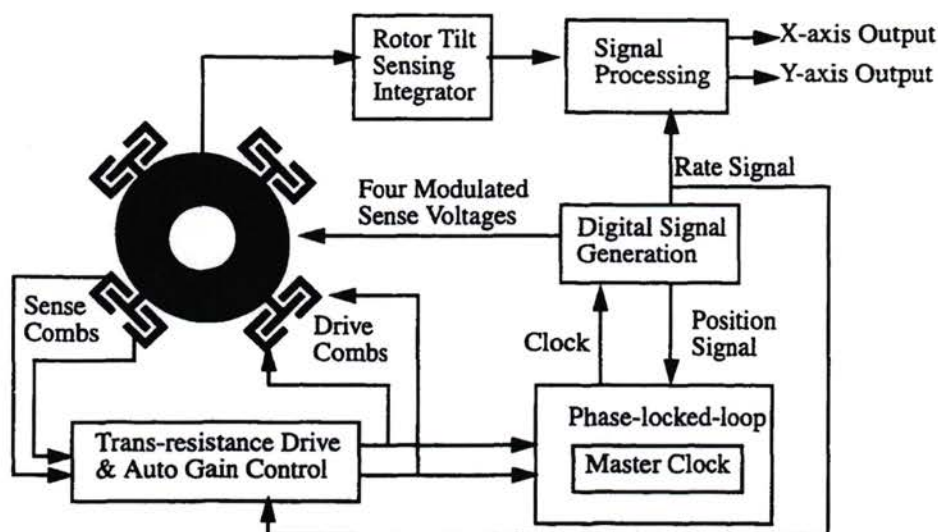


Figure 4-1. System level schematic showing entire dual-axis gyroscope electronics.

components and sources are needed for operation of the dual-axis rate gyroscopes fabricated by SNL. Important effects such as quadrature error, drive feedthrough, motion current, and distortion which impact performance are also discussed.

4.1 RESONANT DRIVE

There can be no rate sensing unless the inertia rotor is in motion. To this end, the inertial rotor is driven into rotational resonance about the z-axis perpendicular to the substrate. An electrostatic comb drive was used as it has the distinct advantage of remaining linear despite large displacements. As discussed by Clark Nguyen [50], a trans-resistance amplifier provides positive feedback which essentially cancels viscous damping and thereby induces self resonance. A phase-locked-loop locks on to the electrical drive voltage to provide a clean signal for amplitude control and signal processing. The resonant amplitude is carefully controlled using automatic gain control (AGC) because scale factor is directly proportional to resonant amplitude. The diagram in Fig. 4-2 depicts the entire drive system. Each of these important subsystems will now be discussed in turn.

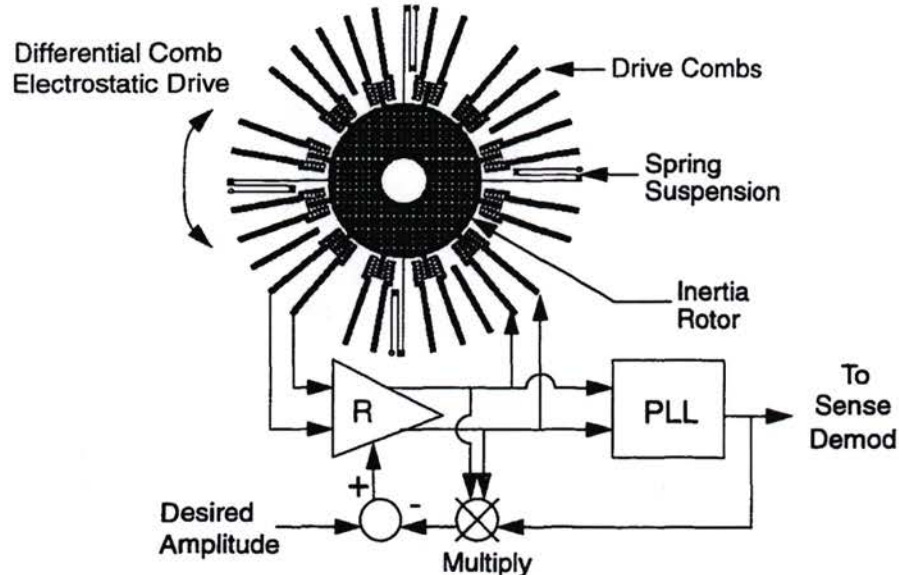


Figure 4-2. System level schematic of the trans-resistance resonant drive with automatic gain control plus phase-locked-loop for clean signal generation.

4.1.1 Electrostatic Comb Drive

Comb electrodes are used to both impart a torque to the rotor and to sense the rotation rate of the rotor. One set of combs is dedicated to providing driving torque, while a second set of combs is dedicated to providing rotation rate measurement. Electrostatic comb drive was chosen over other electrode configurations such as parallel plate because comb electrode sensing and forcing remain linear despite a degree or more of angular displacement [49]. This is vital for gyroscope actuation since the target angular displacement of one degree results in several micrometers of rotor motion.

The drive subsystem was fully differential. In other words, half the drive combs pointed clockwise, while the other half pointed counterclockwise. Likewise, half the sense combs pointed clockwise, while the other half pointed counterclockwise. This provided a differential measurement of rotation rate and a differential torque applied to the rotor. Differential comb drive circuitry was crucial for functionality and performance. The usual arguments involving less susceptibility to outside interference and power supply variations are important but are not the primary reasons for differential design.

The most important differential argument stems from the fact that the rotor is used as the sense node for Coriolis motion detection as detailed in Section 4.2. Therefore, the sensitive measurement node is affected by the comb drive voltages via the air gap capacitance between drive comb fingers and the rotor. This results in drive voltage feedthrough and extraneous motion current in the critical Coriolis sensing circuitry. If these parasitic signals become large, then the Coriolis sensing circuitry can be corrupted due to distortion and amplifier saturation. Differential drive reduces the size of these parasitic signals (although they cannot be totally eliminated as discussed in Subsection 4.1.7).

To ensure that the rotor will indeed oscillate, two key comb electrode parameters must be calculated. These two parameters are the torque generated by a given drive voltage and the current output generated by a given rotation rate. The generated torque can be calculated by considering the potential energy E of the rotor and drive combs with a voltage potential V_c between them. This can be written as Eq. 4-1 in terms of the applied

voltage and the air gap capacitance between the rotor and the clockwise C_{cw} plus counterclockwise C_{ccw} drive combs.

$$E = \frac{1}{2}(C_{cw}V_c^2 + C_{ccw}V_c^2) \quad (4-1)$$

Invoking the definition of potential energy, the resulting torque T can be calculated by differentiating the potential energy with respect to rotation position as in Eq. 4-2. The dependence upon the square of voltage can be linearized by assuming a large constant DC voltage V_{DC} and a smaller, variable AC drive voltages v_{cw} and v_{ccw} .

$$T = \frac{dE}{d\theta} = \frac{1}{2} \left[\frac{\partial C_{cw}}{\partial \theta} V_{DC} v_{cw} + \frac{\partial C_{ccw}}{\partial \theta} V_{DC} v_{ccw} \right] \quad (4-2)$$

Noting that the counter clockwise and clockwise combs are identical except for physical direction, the torque T in Eq. 4-2 can be simplified. As the rotor rotates θ , the clockwise combs increase in capacitance while the counter clockwise combs decrease in capacitance. Hence, the derivative of air gap capacitance with respect to θ is equal and opposite for the two combs sets. Further simplification results from using a differential drive signal such that $v_{cw} = -v_{ccw}$. The negative voltage and negative partial differential cancel and torque simplifies to Eq. 4-3.

$$T = \left[\frac{\partial C_{cw}}{\partial \theta} V_{DC} \right] v_{cw} \quad (4-3)$$

The air gap capacitance can be estimated using the parallel plate approximation. The total plate area is determined by rotor and comb finger overlap, while the capacitor gap is simply the distance between comb fingers and the rotor. This ignores fringe capacitance and therefore results in a conservative estimate of torque. The air gap capacitance is estimated in Eq. 4-4 where N is the number of air gaps on clockwise combs, ϵ is the permittivity of vacuum, h is the polysilicon height, d is the comb air gap between rotor and stationary combs, R_i is the radius from gyroscope center to the inner most air gap, R_o is the radius from gyroscope center to the outer most air gap, and θ_0 is the nominal angle of overlap.

$$C_{cw} = \frac{N\epsilon h}{d} \left(\frac{R_o + R_i}{2} \right) (\theta_0 + \theta) \quad (4-4)$$

Taking the partial derivative with respect to rotation angle (Eq. 4-5) and substituting back into the torque equation (Eq. 4-3) finally reveals the torque generated by a given drive voltage in Eq. 4-6. The key to comb drive linearity is that the partial derivative of overlap capacitance with respect to rotation angle is constant. Assuming adequate time overlap and travel, drive torque has no dependence upon rotor angular position.

$$\frac{\partial C_{cw}}{\partial \theta} = \frac{N\epsilon h}{d} \left(\frac{R_o + R_i}{2} \right) \quad (4-5)$$

$$T = \left[\frac{N\epsilon h}{d} \left(\frac{R_o + R_i}{2} \right) V_{DC} \right] v_{cw} \quad (4-6)$$

Next the current output generated by rotor rotation rate is found from basic principles. The starting relationship is the simple linear capacitor Eq. 4-7 modeling the total stored charge Q given the capacitor size and applied voltage. In this case, the capacitance is the overlap capacitance between the rotor and stationary sense combs and the voltage is the DC sense voltage. The voltages and comb dimensions are assumed to be identical for both electrostatic sense and drive combs.

$$Q = (C_{cw} + C_{ccw}) V_{DC} \quad (4-7)$$

Since current is the time derivative of charge, the output current can be determined by taking the total time derivative of the charge stored due to the voltage applied between the rotor and stationary sense combs as in Eq. 4-8.

$$i = \frac{dQ}{dt} = 2 \left[\frac{\partial C_{cw}}{\partial \theta} V_{DC} \right] \dot{\theta} \quad (4-8)$$

As shown previously, the partial derivative of overlap capacitance with respect to rotor rotation angle is constant. By fixing the sense voltage constant in time, the output current is directly proportional to rotor rotation rate $\dot{\theta}$. With the basic comb electrode sense and drive characteristics illuminated, attention is now focused on using the combs to induce rotor oscillation.

4.1.2 Positive Feedback for Resonance

Positive feedback is used to induce rotor self oscillation at the drive mode natural frequency. This technique is used in many oscillators including quartz crystals and micromachined tuning forks [51]. In short, the positive feedback cancels the inherent viscous damping of the rotor. This leaves the system unstable and the rotor begins to self oscillate. In this case, the feedback path starts with the sense combs generating a current proportional to rotation rate. This current is converted to a voltage by a trans-resistance amplifier. The trans-resistance approach was chosen over lower noise topologies such as the Pierce oscillator scheme because the trans-resistance method was previously proven operational by Nguyen [50] and allowed variable gain. Finally, the voltage is applied to the drive combs which induce rotor rotation.

The key to successful operation is ensuring that the combs and trans-resistance amplifier are designed with enough loop gain to overcome viscous damping. The calculation begins with Eq. 4-9 which represents the obvious requirement that the viscous damping in the drive axis must be cancelled by the resonant drive torque.

$$T = C_{zz}\dot{\theta} = \frac{I_{zz}Q_{zz}\dot{\theta}}{\omega_z} \quad (4-9)$$

Now the resonant drive torque given a rotation rate must be calculated. As stated previously, the rotation rate generates a sense comb output current which is converted to a voltage by the trans-resistance amplifier then fed back to the drive combs. Assuming the rotor resonant frequency is well below the first pole of the amplifier and that feedthrough is minimal, the transfer functions are all frequency independent algebraic expressions. Substituting the comb transduction expression and adding trans-resistance ohmic gain R_{Ω} the torque is shown to be as in Eq. 4-10.

$$T = \left[\frac{\partial C_{cw}}{\partial \theta} V_{DC} \right] R_{\Omega} i = 2 \left[\frac{\partial C_{cw}}{\partial \theta} V_{DC} \right]^2 R_{\Omega} \dot{\theta} \quad (4-10)$$

Replacing resonant torque in Eq. 4-9 with the above expression gives the combined design equation for comb structures and trans-resistance amplifier in Eq. 4-11 [50]. This

can be converted into Eq. 4-12 involving only geometry, comb finger numbers, physical constants, and trans-resistance for inspection of design trade-offs.

$$R_{\Omega} = \frac{C_{zz}}{2 \left[\frac{\partial C_{cw}}{\partial \theta} V_{DC} \right]^2} \quad (4-11)$$

$$R_{\Omega} = \frac{C_{zz}}{2 \left[\frac{N\epsilon h}{d} \left(\frac{R_o - R_i}{2} \right) V_{DC} \right]^2} \quad (4-12)$$

4.1.3 Trans-resistance Amplifier Design

The trans-resistance amplifier must satisfy several design criteria for successful operation and improved performance. As afore mentioned, the trans-resistance amplifier must have adequate gain at the rotor drive natural frequency to induce self oscillation. This implies that a high DC gain with a first electrical pole well above the drive mode natural frequency is required. It is also important to have minimal phase lag at the drive frequency. Since the trans-resistance output is used to synchronize the master PLL clock, any phase lag will reduce the performance of the final signal processing. As discussed in a later in Subsection 4.1.6, this will exacerbate offset drift due to rotor wobble or quadrature error. Typically, higher gain results in lower bandwidth and greater phase lag, so there is a maximum practical trans-resistance gain for any given technology. The gain may also be limited due to destabilizing capacitive feedthrough from drive to sense combs.

An important feature of the trans-resistance amplifier is that a variable gain must be provided. Once the rotor is self oscillating, oscillation amplitude will grow unchecked until a nonlinearity fixes amplitude. This results in a very large, distorted, and uncontrolled amplitude. Because the gyroscope mechanical scale factor is directly proportional to oscillation amplitude, it is of paramount importance to accurately fix oscillation amplitude constant. Hence, an automatic gain controller is needed to measure amplitude and adjust the trans-resistance amplifier gain.

The first stage of the trans-resistance amplifier utilizes a high gain op-amp with a variable resistor in feedback. This approach had the advantage that is was very simple and

was previously proven operational by Clark Nguyen [50]. The first drive circuitry for the ADI dual-axis rate gyroscope was designed by Bill Clark [3] and used the scheme depicted in Fig. 4-3. The variable feedback resistor was accomplished by using a triode PMOS [66] having a high length to width ratio. This technique produced a very simple, area efficient design. Unfortunately, the trans-resistance is not stable with temperature which complicates resonant amplitude measurement. An off-chip voltage on the gate provided the means to vary the feedback resistive gain R_v . A second stage provided added gain A in the form of a voltage amplifier.

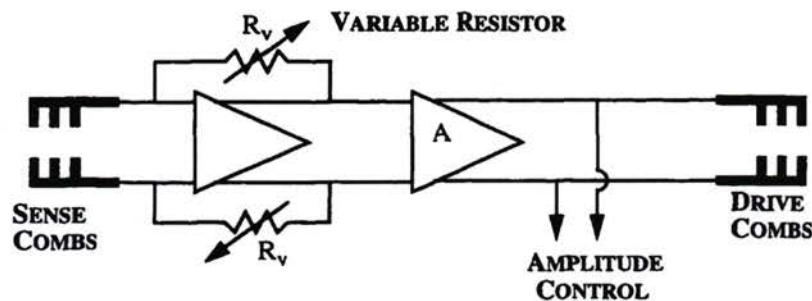


Figure 4-3. Conceptual resonant drive design used on early gyroscope fabricated by ADI.

The total differential trans-resistance R_Ω can be calculated by multiplying the first stage equivalent resistance with the second stage gain as in Eq. 4-13.

$$R_\Omega = R_v \times A \quad (4-13)$$

The trans-resistance of the first stage can be calculated using the formula for a PMOS operating in the triode region with negligible voltage difference between source and drain. In Eq. 4-14, μ is hole mobility, C_{ox} is the gate capacitance, W is transistor width, L is transistor length, V_{GS} is the voltage from source to gate, and V_t is the transistor threshold voltage. By varying V_{GS} using an off-chip voltage source, the trans-resistance could be adjusted. The approximate maximum trans-resistance attained by this particular amplifier was 20 M Ω .

$$R_v = \frac{1}{\mu C_{ox}(V_{GS} - V_t)} \quad (4-14)$$

The main drawback of this very simple approach is that the variable gain trans-resistance amplifier output is used for oscillation amplitude control. As the variable gain changes, so does the scale factor between physical oscillation amplitude and trans-resistance voltage output. It is very difficult to guarantee a constant oscillation amplitude if the scale factor is changing. A solution to this drawback is detailed later in Section 4.3 explaining the fully integrated design fabricated by SNL.

4.1.4 Phase-locked-loop Lock-in to Resonance

Both Coriolis signal processing and automatic gain control require demodulation with the resonant drive signal. Analog multiplication of critical signals with the raw resonant drive signal is not preferable due to small signal size and the large noise inherent to trans-resistance amplifiers. Phase-locked-loops or PLLs are often used to surmount these difficulties and provide a clean digital signal. An example of this technique is shown in the experimental data in Fig. 4-4 where the PLL produces a clean square wave exactly 90 degrees out of phase with an irregular, noisy sinusoid emanating from the trans-resistance amplifier.

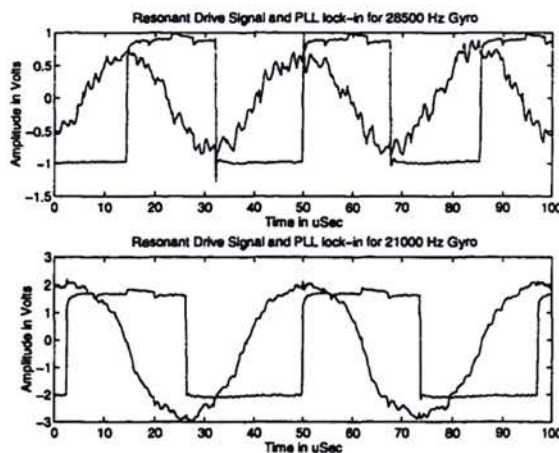


Figure 4-4. Oscilloscope camera pictures of sinusoidal drive signals and locked-in PLL digital square wave signals.

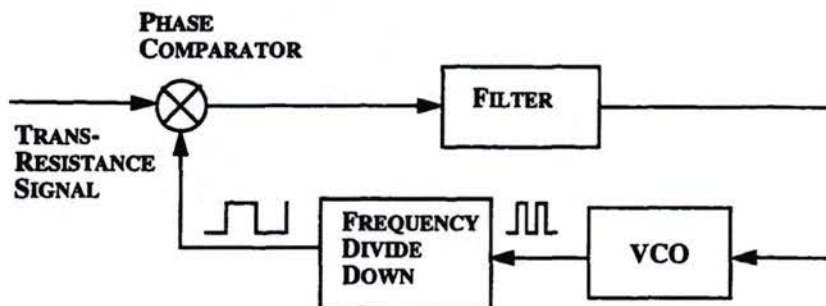


Figure 4-5. System level diagram of the phase-locked-loop (PLL).

The heart of the PLL is the voltage control oscillator or VCO which produces the clean digital output squarewave at the same frequency as the incoming trans-resistance signal [67]. A feedback loop compares the VCO signal to the incoming trans-resistance signal and feeds back a correction voltage to the VCO. Assuming the drive mode natural frequency is within the frequency lock-in range of the PLL system, the feedback loop reaches a steady state with the VCO producing a digital output at the exact frequency of the trans-resistance drive signal but 90 degrees out of phase. The PLL system is schematically shown in Fig. 4-5. The incoming trans-resistance signal is multiplied or chopped with the VCO digital signal. The resulting signal is then filtered and fed back as the voltage control for the VCO. In the actual system, the VCO runs at a frequency much higher than drive resonance and is divided down to the resonance frequency. This allows generation of several digital waveforms for signal processing and amplitude control. One waveform is called the rate signal because it is in phase with the rotor rotation rate. Hence, this signal has the same frequency as the resonant frequency and is in phase with the trans-resistance drive signal. Since the Coriolis tilt oscillation is also in phase with rotation rate, the rate signal is used for demodulating to recover the Coriolis signal amplitude. The rate signal is also used for oscillation amplitude control since it is in phase with the trans-resistance amplifier. A second waveform is called the position signal because it is in phase with rotor position. The position signal is the PLL comparison signal which is at the drive natural frequency, but is 90 degrees out of phase with the trans-resistance drive. Finally, two high frequency signals used for Coriolis motion sensing are also generated as described in Section 4.2.

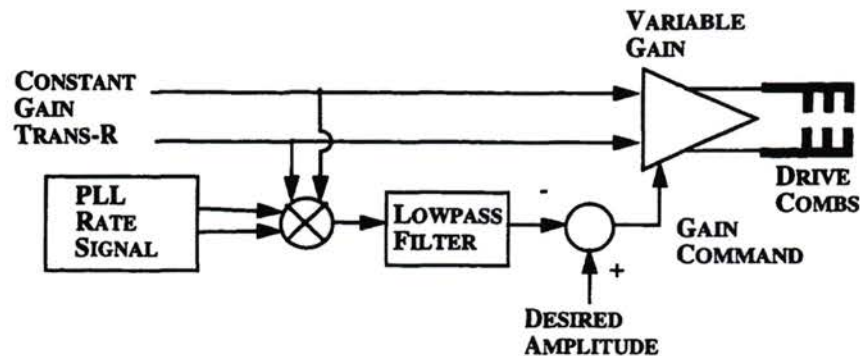


Figure 4-6. Overview diagram of the automatic gain control (AGC) circuitry used to fix rotor resonant amplitude constant.

4.1.5 Resonant Amplitude Control Via AGC

As discussed earlier, the gyroscope mechanical sensitivity and hence scale factor is directly proportional to rotor oscillation amplitude. To ensure constant scale factor, the amplitude of oscillation must be held constant via automatic gain control (AGC). The underlying principle of AGC is that if oscillation amplitude is above the desired level, then trans-resistance gain is decreased. Likewise, if the oscillation amplitude is below the desired level, then trans-resistance gain is increased. The feedback system which accomplishes this task is the subject of Fig. 4-6.

The trans-resistance amplifier produces a sinewave with an amplitude proportional to physical rotor oscillation amplitude. This sinusoid is converted into a usable DC voltage proportional to oscillation amplitude by chopping with the PLL generated rate signal and lowpass filtering. The resulting DC voltage is proportional to oscillation amplitude and is compared with the desired amplitude voltage. The error signal from this comparison is fed back into the variable gain trans-resistance amplifier to correct the oscillation amplitude. Typically, the drive voltage amplitude as measured at the drive combs was set at 50 mV for ADI designs and 120 mV for SNL designs.

4.1.6 Quadrature Error

As with all rotating bodies, the dual-axis gyroscope rotor will exhibit some wobble due to rotor moment of inertia imbalance as well as spring asymmetry and imperfections. This wobble is in phase with rotor position, and so out of phase with the critical Coriolis motion. Therefore, theoretically the wobble can be removed using demodulation with the orthogonal PLL generated rate signal. However, in practice the rate signal will exhibit some phase lag. This allows a component of the wobble motion to masquerade as a false Coriolis motion. This false signal is termed quadrature error. Typically there might be only 1-5 degrees of phase lag, but because the wobble is a gigantic motion compared to minute Coriolis tilt, the quadrature error can be significant. Rough experimental measurement showed the wobble to be approximately 100-500 ppm (parts per million) of the drive motion which translates into a quadrature error of 2-10 °/sec if electronic phase lag is merely 1°. The quadrature error is a DC offset which theoretically could be calibrated out. Yet, the quadrature error magnitude depends on second order manufacturing imperfections and as such may tend to drift. The designs tested hence far have attempted to overcome quadrature error through careful trans-resistance amplifier design, but future designs may rely on directly cancelling wobble as discussed in Section 7.4.

4.1.7 Feedthrough and Residual Motion Current

Attaching the sense integrator to the rotor has some side effects in terms of residual resonant drive signals appearing on the integrator output. The differential topology of the trans-resistance drive eliminates these residual signals to a large extent, but some remnants remain. The main two signals are feedthrough and motion current. Both have their origin in the interaction between drive combs and drive voltage.

In general, the overlap capacitance between the rotor fingers and the drive combs will have two components: a constant capacitance and a sinusoidally varying overlap capacitance due to the rotor oscillator motion. Clockwise and counterclockwise capacitance change in opposite directions as represented in Eq. 4-15 and Eq. 4-16.

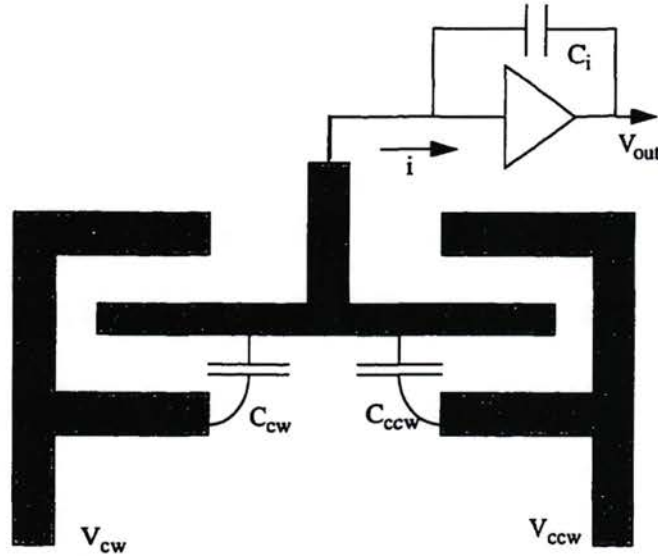


Figure 4-7. Simplified view of air gap capacitance between stationary drive combs and the inertial rotor.

$$C_{cw} = C_{cw0} + C \sin(\omega t) \quad (4-15)$$

$$C_{ccw} = C_{ccw0} - C \sin(\omega t) \quad (4-16)$$

Likewise, the drive voltages can be represented by a constant voltage term and a varying sinusoidal voltage as in Eq. 4-17 and 4-18.

$$V_{cw} = V_{cw0} + V \cos(\omega t) \quad (4-17)$$

$$V_{ccw} = V_{ccw0} - V \cos(\omega t) \quad (4-18)$$

The current passing into the integrator can be calculated by taking the total time derivative of Eq. 4-19 which represents the charge stored across the rotor to comb drive overlap capacitance. Rearranging yields Eq. 4-20 which predicts both the feedthrough and motion current.

$$Q = V_{cw} C_{cw} + V_{ccw} C_{ccw} \quad (4-19)$$

$$i = \frac{dQ}{dt} = [C_{cw0} - C_{ccw0}] V \cos(\omega t) + CV \omega \sin(2\omega t) + \Delta_{dc} \quad (4-20)$$

Direct signal feedthrough stems from the constant capacitance component in the first term on the right-hand side of Eq. 4-20. The differential drive voltage should theoretically cancel out entirely since one drive comb voltage rises as the other comb voltage falls. But mismatch between the clockwise and counterclockwise drive capacitance can result in slight feedthrough causing a sense integrator signal 10-20 mV amplitude. This is especially true for large voltage drive or when a large number of combs are used. The sinusoidally varying overlap capacitance is different because even with perfect matching, a residual motion current is produced. This can be seen in the second term on the right-hand side of Eq. 4-20. This term can become quite large and care must be taken to ensure the sense integrator does not saturate or severely distort. The ADI design has a residual motion current which produced a 100-200 mV output from the integrator. Theoretically the residual motion current could be used for PLL lock-in and resonant amplitude control. However, in practice difficulties with oscillation start-up and precise amplitude measurement make this less attractive.

4.2 CORIOLIS MOTION SENSING

With the resonant drive system fully explained, attention is turned toward the actual sensing of Coriolis tilt motion. This requires a capacitive measurement scheme to transduce mechanical motion into a capacitance change, sense interface electronic to detect the change in capacitance, and signal processing to create the final output rate measurement voltage.

4.2.1 Capacitive Detection of Coriolis Motion

It has been shown previously that a rotation rate input induces a Coriolis acceleration, which in turn induces a tilting oscillation. Using a differential capacitance measurement scheme to detect the tilt oscillation amplitude, the original rotation rate input can be inferred. To this end, four quadrant electrodes are patterned underneath the rotor. One pair of diametrically-opposed electrodes provides x-axis tilt sensing, while the second pair provides y-axis tilt sensing. In Fig. 4-8 is shown the rotor with only one pair of quadrant

electrodes for simplicity. These electrodes form a capacitive divider with the inertial rotor. If the rotor tilts about the axis perpendicular to the page, then the capacitance of one sense capacitor increases while the capacitance of the other decreases. This differential change in capacitance is detected via an integrator electrically connected to the inertial rotor in conjunction with a modulated sense voltage applied between the pair of quadrant electrodes.

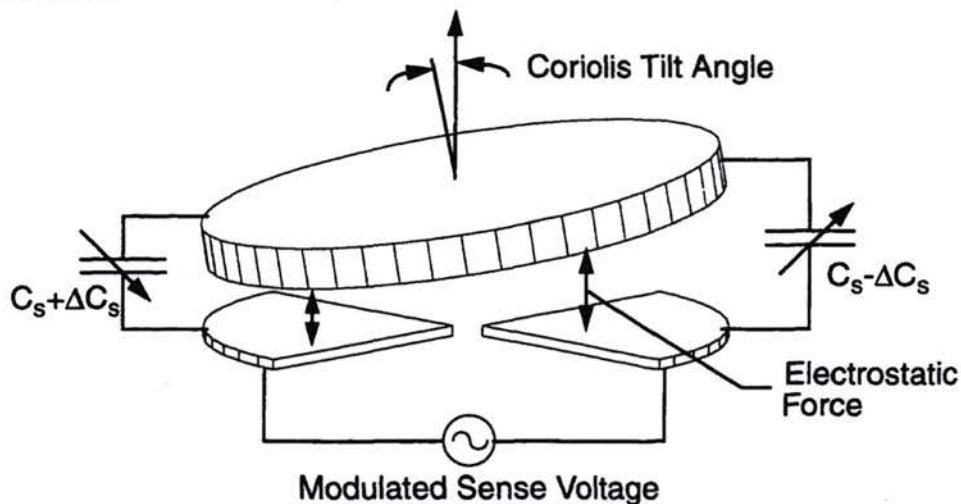


Figure 4-8. Side view showing rotor with underlying diffusion sense electrodes for one sense axis.

The dual-axis rate gyroscope is equally sensitive to rotation rates about both the x- and y-axes. Differentiating between x- and y-axis input rotation rates is accomplished by differentiating between the orthogonal x- and y-axis inertial rotor tilt oscillations due to Coriolis acceleration. Two pairs of separate quadrant electrodes are patterned beneath the inertial rotor. As illustrated in Fig. 4-9, there is a pair of diametrically-opposed quadrant electrodes for each axis. One pair measures x-axis tilt oscillation while the other pair measures y-axis tilt oscillation.

Because all tilt oscillation detection is completed using a single integrator that is electrically connected to the structure, electrical differentiation between x- and y-axis tilt oscillation is accomplished by using a different sense modulation frequency for each axis. Separate demodulation circuits for each axis provide two separate output signals proportional to the two orthogonal rotation rate inputs.

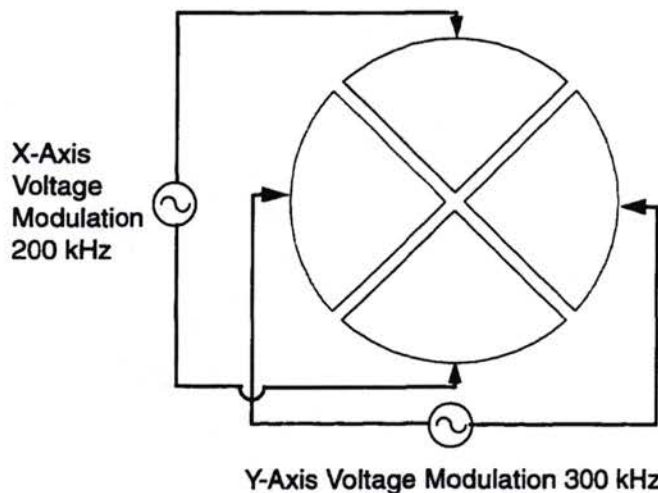


Figure 4-9. View of the quadrant sense electrodes beneath the inertial rotor. Note the different modulation frequencies for each axes.

As the multiple signal frequency bands shown in Fig. 4-10 suggest, choosing sense modulation frequencies demands great care. Clearly, the sense modulation voltages should have a far higher frequency than the inertial rotor resonance to avoid mixing with drive feed-through, double frequency motion current, and any drive signal distortion. Since the integrator output signals for each axis are actually double modulated, the detection signals for each axis appear as double side bands spaced equally distant about each respective modulation voltage frequency. The frequency difference between each side band and the original sense modulation voltage is equal to the drive frequency (28 kHz in this case) because the tilt oscillation induced by Coriolis acceleration is at the drive frequency. These side bands should never mix, so the sense voltage modulation frequencies have a minimum separation of greater than twice the drive resonant frequency. In addition, higher harmonics resulting from distortion can interfere with signal purity, so frequencies must be chosen such that even multiples of the important signal frequencies do not coincide. The sense modulation frequencies 200 kHz and 300 kHz were chosen because they satisfied all these requires and were within the electrical circuitry bandwidth.

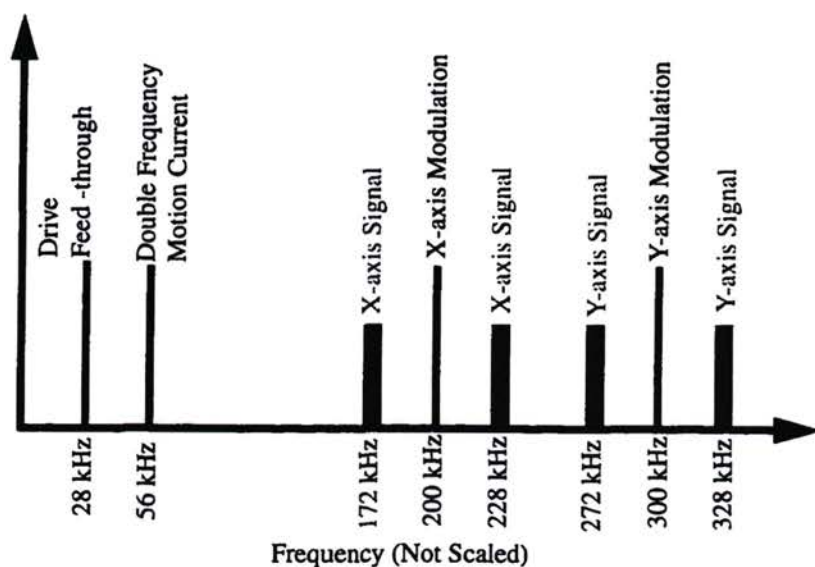


Figure 4-10. Conceptual power spectral density of all important dual-axis rate gyroscope signals. Note signal frequencies are chosen to avoid overlapping.

Both of these sense modulation voltages are created from the high frequency VCO signal generated in the phase-locked-loop. The base VCO signal is approximately 2.4 MHz during normal operation. This high frequency signal is divided down to produce the sense modulation signals. For example, dividing the VCO signal down by 12 yields a 200 kHz signal, while dividing down by 8 yields the 300 kHz signal.

4.2.2 Interface Circuitry and Signal Processing

As previously noted, the key to dual-axis gyroscope operation is sensing a differential capacitance change between substrate mounted electrodes and the inertial rotor. Two diametrically opposed electrodes form a capacitive divider with the inertial rotor. If the rotor tilts about the axis perpendicular to the page, then the capacitance of one sense capacitor increases while the capacitance of the other decreases. An integrator holds the rotor at a constant voltage rendering the rotor a virtual ground. If the sense capacitors are equal, then the modulation voltage causes charge to flow between the sense capacitors

with no charge escaping through the integrator. If the sense capacitors are unbalanced due to inertial rotor tilt, then some charge flows through the integrator producing a measurement voltage proportional to tilt displacement and modulated at the sense voltage frequency. The integrator is realized by placing a small integrating capacitor of approximately 50 fF in feedback around a high-gain operational amplifier.

Capacitive sensing using an integrator demands an operational amplifier with very high input impedance and no significant input bias current. Hence, an amplifier with MOSFET input transistors was used because MOSFET input impedance is extremely high ($>10^8 \Omega$). Unfortunately, this poses a problem for DC biasing the inertial rotor at a set voltage since there is no DC current path to ground. This problem is further exacerbated by any minute leakage currents which may cause the rotor voltage to drift. To solve this problem, a resistive path was created by placing a subthreshold MOSFET in parallel with the integrating capacitor. This provided a high enough input impedance for effective capacitive measurement, while providing a low enough resistive path to ground for setting of the rotor bias voltage.

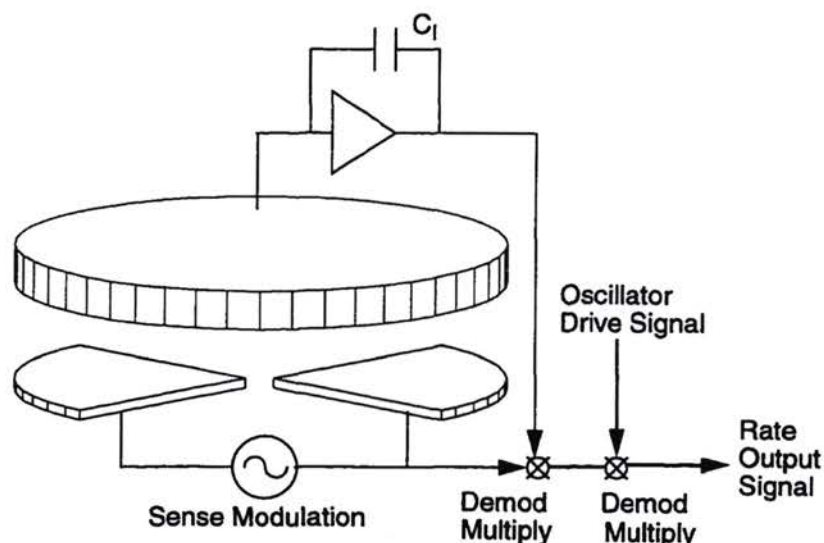


Figure 4-11. Side view and schematic showing the sense integrator and the signal processing required to produce final rotation rate measurement for one axis.

As shown in Fig 4-11, the voltage output from the integrator must be demodulated twice to recover the desired voltage output signal. The first demodulation removes the sense voltage modulation frequency leaving a voltage proportional to inertial rotor tilt position. The second demodulation removes the inertial rotor drive frequency leaving a base band voltage signal proportional to rotation rate input. The second demodulation uses the PLL generated rate signal which is in phase with both the trans-resistance drive signal and the desired Coriolis signal. In practice, this double demodulation was accomplished using a single chopping of the integrator signal and a lowpass filter. This could be done because the demodulation multiplication is associative, so the sense modulation signal and the PLL generated rate signal could be pre-multiplied. This requires a digital multiply which is completed using a simple XOR circuit rather than an analog multiplier [68]. The resulting digital signal was then used to chop the integrator output thereby realizing both demodulations with only a single analog chopper circuit.

Because all tilt oscillation detection is completed using a single integrator electrically connected to the structure, electrical differentiation between x- and y-axis tilt oscillation is accomplished by using a different sense modulation frequency for each axis. Two nearly exact copies of the demodulation signal processing circuitry are integrated on-chip. The only difference between the circuit copies is that one demodulates with the x-axis sense modulation frequency, while the other demodulates with the y-axis frequency. Demodulation circuits for each axis provide two output voltage signals proportional to the two orthogonal rotation rate inputs.

4.2.3 Electrical Sensitivity to Rotation Rate Inputs

The electrical sensitivity to rotation rate input relates the voltage output to a rotation rate input. The calculation of this quantity can be separated into two distinct sensitivities. Mechanical sensitivity and electrical sensitivity to rotor displacement combine to give the electrical sensitivity to rotation rate inputs. It is important to find the combined sensitivity because mechanical sensitivity or electrical sensitivity to Coriolis displacement alone can be misleading.

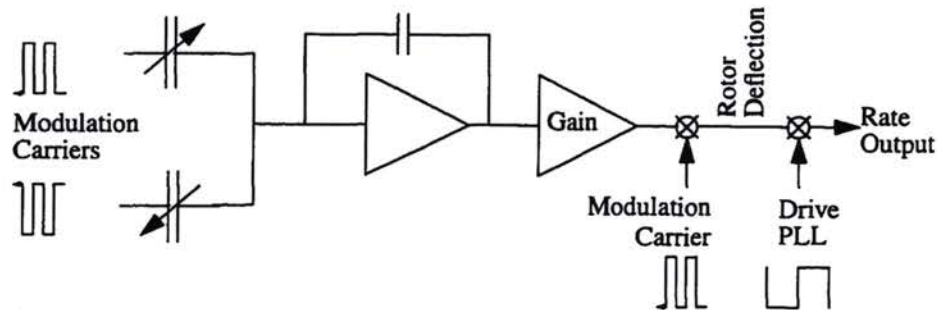


Figure 4-12. Schematic showing circuit equivalent of the entire interface circuitry and signal processing combination.

The mechanical sensitivity was calculated in Chapter Three, so only the electrical sensitivity to rotor displacement need be addressed here. The electrical sensitivity to rotor displacement relates voltage output to rotor angular deflection. The x-and y-axis are identical and ideally decoupled, so only the x-axis electrical sensitivity will be explored. The calculation begins with the change in sense capacitance given an angular displacement of the rotor. Assuming a small angular displacement ϕ , the electrode sense capacitance can be approximated by a constant value C_{s0} and the linear term of the Taylor series. Any positive angular deflection of the rotor causes one capacitor C_{sp} to increase in capacitance, while the other capacitor C_{sn} decreases in capacitance. Thus the two separate sense capacitances can be written as Eq. 4-21 and 4-22.

$$C_{sp} = C_{s0} + \frac{\partial C_s}{\partial \phi} \phi \quad (4-21)$$

$$C_{sn} = C_{s0} - \frac{\partial C_s}{\partial \phi} \phi \quad (4-22)$$

Because the integrator attempts to hold the rotor voltage constant, any imbalance in the sense capacitors due to rotor displacement will force current onto the integrator. The foundation of determining this sense current is the relationship between charge on a capacitor given capacitance and voltage. The charge in question is the sense charge q on the rotor.

In this case, the capacitance is the capacitance between the two electrodes and the rotor (C_{sp} and C_{sn}) while the voltage is the modulated sense voltage V_s assuming the rotor is held at ground. The two other sense electrodes which measure rotation about the orthogonal y-axis direction can be ignored because they exhibit no net change in capacitance due to rotation about the x-axis. The sense voltage is differential, so the sense voltage on each sense capacitor is equal and opposite as depicted by Eq. 4-23.

$$q = C_{sp}V_s - C_{sn}V_s \quad (4-23)$$

Substituting the Taylor series approximation of the sense capacitance (Eq. 4-21 and 4-22) into Eq. 4-23 gives the much simplified total charge on the rotor due to the x-axis sense voltages in Eq. 4-24. Note the constant sense capacitance terms are cancelled.

$$q = 2\left(\frac{\partial C_s}{\partial \phi}\right)\phi V_s \quad (4-24)$$

The definition of current is change in charge over time. Therefore, the current flowing through the integrator is simply the time derivative of the charge on the rotor as defined in Eq. 4-25.

$$i = \frac{dq}{dt} = \frac{d}{dt}\left[2\left(\frac{\partial C_s}{\partial \phi}\right)\phi V_s\right] \quad (4-25)$$

The integrator of course integrates this current on to a capacitor C_I which thereby produces a voltage. Therefore the voltage out of the integrator V_{int} can be written as shown in Eq. 4-26 and finally Eq. 4-27. This assumes the frequencies of the modulation sense voltage are high enough such that the interface circuitry behaves as an idea integrator.

$$V_{int} = \frac{1}{C_I} \int i \quad (4-26)$$

$$V_{int} = \frac{2\left(\frac{\partial C_s}{\partial \phi}\right)\phi V_s}{C_I} \quad (4-27)$$

The voltage output of the integrator undergoes several transformations including voltage gain, two demodulations, and final filtering. Both the demodulators and the filter have individual gains. To simplify representation without losing any important nuances, the effects of these final stages can be captured in an effective gain A_e . The final form of the electrical sensitivity to rotor displacement is given in Eq. 4-28.

$$\frac{V_{out}}{\phi} = \left[2 \left(\frac{\partial C_s}{\partial \phi} \right) \frac{V_s}{C_I} \right] A_e \quad (4-28)$$

Now electrical sensitivity to rotor deflection can be combined with mechanical sensitivity to reveal overall electrical sensitivity. The mechanical sensitivity is reduced by 1/2 because the multiplicative Coriolis motion reduces the amplitude by 1/2 while doubling the bandwidth.

The combined mechanical system, sense electrodes, and integrator can be summarized in Eq. 4-29 with sense voltage V_s , integrating capacitor C_I , and electrode capacitive sensitivity $(\partial C_s)/(\partial \phi)$.

$$\left| \frac{V_{out}}{\Omega_y} \right| = \left| \frac{\phi}{\Omega_y} \right| \left| \frac{V_{out}}{\phi} \right| = \left| \frac{\theta_0 \omega_z}{\omega_x^2 + \frac{j\omega_z \omega_x}{Q_x} - \omega_z^2} \right| \left[2 \left(\frac{\partial C_s}{\partial \phi} \right) \frac{V_s}{C_I} \right] A_e \quad (4-29)$$

It is clear that increasing the sense voltage differential V_s between the rotor and the quarter circle sense electrodes will increase overall sensitivity. Unfortunately, this sense voltage also results in an electrostatic force which tends to pull the inertial rotor down to the substrate. The critical pull down voltage limits the maximum applicable sense voltage and is proportional to the natural frequency for z-axis translation perpendicular to the substrate. Conventional wisdom would suggest one should maximize mechanical sensitivity by lower the rotational natural frequencies. However, lower rotational natural frequencies result in lower z-axis translational frequency also. For this reason, the sense voltage for lower frequency suspensions must be reduced, nearly eliminating all mechanical sensitivity gains. In general, the integrator capacitor is set by minimum process capability or minimum size to avoid amplifier saturation due to feedthrough and

motion currents. That leaves the derivative of sense capacitance with respect to tilt angle as the an important factor for improving sensitivity, and it is discussed next.

Hitherto, the electrical sensitivity has been documented in general terms. In order to find the numerical electrical sensitivity of the dual-axis rate gyroscope, the change in capacitance for a given change in rotor tilt angle $(\partial C_s)/(\partial \phi)$ must be derived. To estimate this quantity, first the actual capacitance between to rotor and a quadrant electrode is calculated ignoring fringing fields. Since small tilt angles are assumed, a parallel plate capacitance estimate is found for a differential element on the sense electrode. Integrating over the entire electrode area yields the total sense capacitance C_s for a single electrode. The discrete differential capacitance dC_s can be written in terms of ϵ the dielectric constant for air, dA the differential area element, and $g(\phi)$ the angle dependent capacitor gap as in Eq. 4-30.

$$dC_s = \frac{\epsilon dA}{g(\phi)} \quad (4-30)$$

The total capacitance is the integral sum over the entire quadrant electrode area. This integral is best tackled by dividing the integral into two separate integrals. One integrates the area of a triangle and the other integrates the area of a circular portion as exhibited in Fig. 4-13.

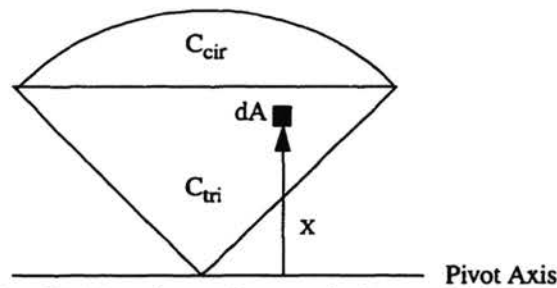


Figure 4-13. Division of capacitance calculation into two separate integrals.

Because integration is a linear operation, the two separate integrals can be added together to form the total capacitance as Eq. 4-31 shows. The differential area dA and the gap are written in terms of tilt angle ϕ , distance from the pivot axis x , electrode radius r , and original gap g_0 . This yields the integral in Eq. 4-32.

$$C_s = C_{tri} + C_{cir} \quad (4-31)$$

$$C_s = 2\epsilon \int_0^{\frac{1}{\sqrt{2}}} \frac{xdx}{g_0 + x \tan(\phi)} + 2\epsilon \int_{\frac{1}{\sqrt{2}}}^r \frac{\sqrt{r^2 - x^2} dx}{g_0 + x \tan(\phi)} \quad (4-32)$$

Although this integral can be found exactly, the answer becomes very cumbersome. However, the assumption of small tilt angles justifies linear approximation using a Taylor series expansion. Finally, the capacitance sensitivity to tilt angle can be approximated as shown in Eq. 4-33. The sensitivity is additive, so calculating the differential for a solid rotor of outside radius R_o and then subtracting a smaller inner radius R_i hollow center will give the capacitance sensitivity for a hollow rotor design. The key results to note are the cubic dependence on radius and the square dependence on the height of the rotor above the sense electrodes z . Thus capacitance sensitivity can be greatly enhanced by increasing rotor radius or by decreasing gap size.

$$\frac{\partial C_s}{\partial \phi} \equiv \left(-\frac{0.471\epsilon}{z^2} \right) (R_o^3 - R_i^3) \quad (4-33)$$

The calculated change in sense capacitance given a change in rotor angle for the ADI fabricated gyroscope was 5.3 pF/rad, but for the larger SNL fabricated gyroscope it was 8 pF/rad.

As noted earlier in Section 3.5, both the sense frequency tuning and maximum tuning range depend on the second derivative of sense capacitance. This can be calculated using the same approach with the final results given in Eq. 4-34.

$$\frac{\partial^2 C_s}{\partial \phi^2} \equiv \frac{0.264\epsilon}{z^3} (R_o^4 - R_i^4) \quad (4-34)$$

4.2.4 Electrical Noise Resolution Limit

Although the absolute minimum detectable signal is set by Brownian motion of the rotor, the actual minimum resolution of the dual-axis gyroscope is limited by electrical

noise in the interface circuitry. Specifically, the input voltage noise at the front end of the sense integrator was calculated to be the dominant noise source. In this case, the dominant noise originates from the thermal noise of the polysilicon interconnect rather than the inherent noise voltage of the input MOSFET pair. There are many other noise sources including trans-resistance drive noise capacitively coupling onto the structure and PLL phase noise allowing corruption of the Coriolis signal by quadrature error to name a few. However, these sources were calculated to be below the dominant resistive interconnect source. Later electrical stages in the Coriolis signal path add negligible noise as the signal magnitude is greatly increased by the initial integrator gain.

The minimum resolution can be found by equating the voltage output due to electrical noise with the voltage output of a hypothetical white spectrum angular rate input Ω_n . This hypothetical rate input noise power integrated over bandwidth will give gyroscope minimum detectable signal. Converting a voltage output to a hypothetical rate input is completed easily [65] by dividing the noise density of the output voltage by the electrical sensitivity as in Eq. 4-35.

$$\Omega_n = \frac{v_n}{\left| \frac{V_{out}}{\Omega_y} \right|} \quad (4-35)$$

With electrical sensitivity already calculated in the previous subsection, the voltage noise output must be derived. Mark Lemkin has shown [69] that for his accelerometer designs the dominant noise source in both ADI and SNL technology is the thermal noise generated by the resistance of the interconnect between mechanical structure and interface circuitry. The same was calculated for the dual-axis rate gyroscope. This resistance includes both the polysilicon suspension beams which the signals must flow through and the signal runners from suspension anchor to the integrator input transistors. For reasonable transistor sizes and bias currents, the noise contribution of the interface amplifier is relatively small compared to the interconnect resistance based noise. Although ADI employed n+ diffusion connections and SNL employed polysilicon connections, both resulted in about the same order of magnitude resistance between 8 k Ω

and 12 k Ω . The schematic in Fig. 4-14 shows the sense circuitry with the resistance and equivalent noise source v_r in front of the interconnect resistance R . The parasitic capacitance C_{para} between the rotor and the substrate and sense electrodes also has a direct effect on noise performance.

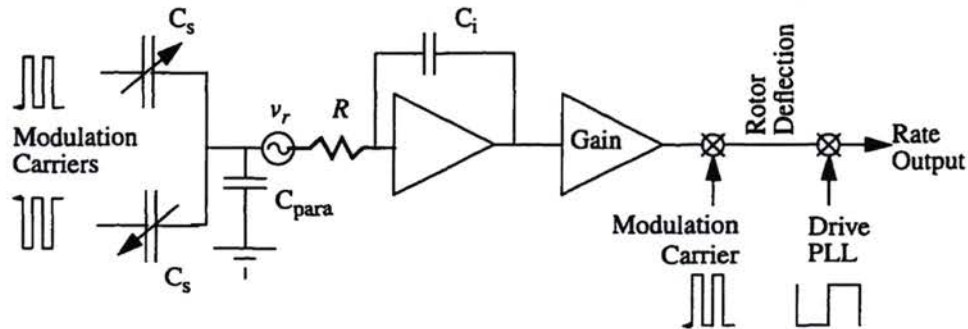


Figure 4-14. Schematic diagram showing the dominant electrical noise source with all interface and signal processing circuitry degraded by this noise.

Mark Lemkin [69] showed that the sense circuit schematic can be rearranged into the simplified equivalent circuit in Fig. 4-15. This allows direct calculation of the output voltage v_n by considering the v_r induced noise current which is integrated on capacitor C_i as shown in Eq. 4-36.

$$v_n = \frac{(2C_s + C_{para})A_e v_r}{C_i} \quad (4-36)$$

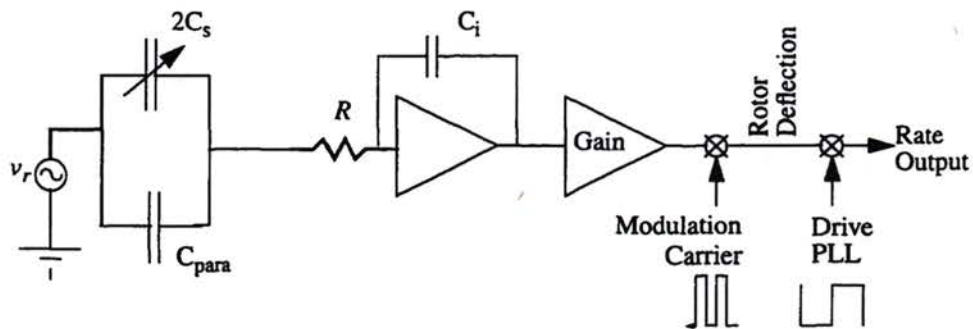


Figure 4-15. Schematic diagram showing equivalent electrical noise source after rearrangement of the circuit to simplify analysis.

The thermal noise induced by any resistive load can be calculated [67] as in Eq. 4-37 where R is the interconnect resistance, T is temperature, and k_B is the Boltzmann constant. The typical noise calculated for ADI versions was approximately $11 \text{ nV}/\sqrt{\text{Hz}}$.

$$v_r = \sqrt{4k_B TR} \quad (4-37)$$

The equivalent angular rate noise power density can be calculated by substituting Eq. 4-36 and 4-37 into Eq. 4-35 with the results in Eq. 4-38.

$$\Omega_n = \frac{(2C_s + C_{para})}{\left(\frac{\partial C_s}{\partial \phi}\right) V_s} \left| \frac{\omega_x^2 + \frac{j\omega_z \omega_x}{Q_x} - \omega_z^2}{2\theta_0 \omega_z} \right| \sqrt{4k_B TR} \quad (4-38)$$

Eq. 4-38 implies that equivalent angular rate noise caused by electrical interconnect resistance can be reduced by decreasing connection resistance R , decreasing all capacitances, increasing capacitive sensitivity, and increasing sense modulation voltage V_s . The middle term in the absolute value brackets models the mechanical dynamics. This is the inverse of the second order dynamical model of gyroscopic motion. This term is minimized by matching sense and drive mode frequencies and increasing resonant drive amplitude θ_0 . Assuming poor frequency matching, the ADI version rate noise power density was calculated to be $0.09^\circ/\text{sec}/\sqrt{\text{Hz}}$. Improving the matching between drive and sense axis frequencies gives a reduced noise density. Actual experiments yielded noise levels several times higher than calculated. Lower drive oscillation amplitude and inexact capacitive sensitivity estimation are among the most likely contributors to error. However, wide band noise is but one performance measurement which must be addressed. Offset drift must also be considered, as in the next subsection.

4.2.5 Effects of Distortion

Quadrature error (first discussed in Subsection 4.1.6) originates from rotor wobble caused by imbalance and suspension spring imperfections. The only difference between the desired Coriolis motion and the corrupting wobble motion is that they are 90 degrees out of phase. As stated previously, phase lag in the electronics can increase quadrature

error offset. Distortion can also alter phase and induce quadrature error offsets. In a perfectly linear system, the 90° phase difference is retained and demodulation can remove the wobble and thereby eliminate quadrature error. Unfortunately, the electronics are not perfectly linear; the gain stage and the demodulation circuits following the integrator may exhibit distortion. This distortion may eliminate the phase difference between Coriolis and wobble signals and thus corrupt the output signal.

To see how wobble can corrupt Coriolis output, the effects of distortion must be modeled. The starting point is the rotor motion itself. The simplified rotor motion consists of a combination of Coriolis motion and rotor wobble as Eq. 4-39 denotes. These two components are 90° out of phase, and so can be symbolically represented by a sine and a cosine with Coriolis amplitude C and wobble amplitude W as in Eq. 4-40.

$$\text{Motion} = \text{Coriolis} + \text{Wobble} \quad (4-39)$$

$$\phi = C \sin(\omega t) + W \cos(\omega t) \quad (4-40)$$

The sense integrator output is the motion modulated at the sense modulation voltage frequency. This integrator output signal V_{int} then passes through the distorting gain stage. Low order effects of distortion can be modeled using the Taylor series in Eq. 4-41.

$$V_{gain} = A_1 V_{int} + A_2 V_{int}^2 + A_3 V_{int}^3 \quad (4-41)$$

After the double demodulation, the final voltage output V_{out} can be derived. As expected, the output voltage is proportional to the Coriolis motion as the first term on the right-hand side of Eq. 4-42 shows.

$$V_{out} \propto C + A_3 W^2 K \quad (4-42)$$

However, the output voltage is also proportional to the square of the wobble with a proportionality constant K . The odd order harmonic term in the gain stage altered the wobble phase such that the Coriolis signal was corrupted. The wobble motion is massive compared to the Coriolis motion, so odd order distortion should be avoided at all costs. The use of square law MOS and feedback helped alleviate odd harmonic distortion in each design.

4.3 FULLY INTEGRATED SNL FABRICATED DESIGN

Two different surface micromachining processes were used to fabricate different versions of the dual-axis rate gyroscope. The ADI fabricated versions had only circuitry requiring low noise and low phase loss on chip. All other circuitry was built on a PC board. Clearly, a higher level of integration provides improved miniaturization and reduces part count. The more compact circuitry made possible by smaller minimum dimensions in the SNL process enabled a higher level of integration. For the most part, integration was simply a matter of moving comparable circuit blocks from off-chip to on-chip and will not be detailed. However, the trans-resistance amplifier will be discussed as important changes improving scale factor stability were added.

4.3.1 Trans-resistance Amplifier and AGC

All versions used a trans-resistance amplifier to generate rotor drive oscillation. However, there are important differences between the ADI and SNL versions. The main drawback of the simple approach used by ADI fabricated versions is that the variable gain trans-resistance amplifier output is used for oscillation amplitude control. As the variable gain changes, so does the scale factor between physical oscillation amplitude and trans-resistance voltage output. It is very difficult to guarantee a constant oscillation amplitude if the scale factor is changing. A better approach would be to use a constant gain trans-resistance amplifier to produce the signals for amplitude control. Then use a separate variable gain amplifier to adjust the signal driving the structure. Such a topology was used for the SNL dual-axis gyroscope design as shown in Fig. 4-16.

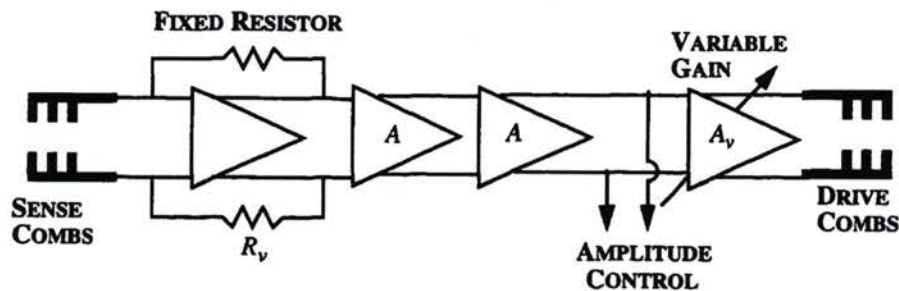


Figure 4-16. Conceptual resonant drive design used for gyroscope fabricated by Sandia National Lab.

Due to large structure size, a very high trans-resistance gain was required. Therefore, several stages were used to complete the design. The first stage was the traditional trans-resistance amplifier with an NMOS as a variable feedback resistor R_v giving an approximate maximum resistance of 100 k Ω . This resistor was set to a fixed value during operation to ensure constant gain. Next two fast amplifiers having medium gain A equal to approximately 22 were cascaded to provide adequate gain. These three sub-circuits comprised the constant gain trans-resistance amplifier. The output drive signal amplitude is proportional to oscillation amplitude and is used by the AGC to fix oscillation amplitude constant. A final variable gain amplifier (gain A_v) then acts upon the constant gain signal to provide the actual drive signals applied to the drive combs. This amplifier had a variable voltage gain from 4 through zero down to -4. The total maximum trans-resistance gain R_Ω was therefore positive and negative 200 M Ω as calculated using Eq. 4-43.

$$R_\Omega = R_v \times A^2 \times A_v \quad (4-43)$$

As with all electronics, the electrical system must remain stable and not ring. The trans-resistance amplifier is designed to self oscillate the structure, not self oscillate itself without the structure moving. Theoretically, the total trans-resistance amplifier has no electrical feedback path, so it is nominally stable by definition. However, in practice there will always be some capacitive feedthrough from the drive combs to the sense combs. The size the phase of the feedthrough signal is determined by wire routing and mechanical structure layout. For exceedingly large trans-resistance values, positive capacitive feedback of less than 1 fF can induce instability. Carefully planned wire routing and structure layout can alleviate much of the feedthrough capacitance.

For added protection, compensation capacitors were added to the SNL designs. The standard single dominant pole compensation method was implemented. This method relies on a very low frequency dominant pole to reduce loop gain below one before secondary poles reduce phase to -180 degrees. All the trans-resistance circuit stages have higher poles above 50 MHz, except the first stage which had a single lower pole provided by a compensation capacitor in parallel with the variable resistance NMOS. The first stage was chosen as the location of the compensation capacitor for two distinct reasons.

Firstly, the first stage has an NMOS in feedback which provides a possible destabilizing feedback path and compensation can add to local stability. Secondly, the stabilizing dominant pole frequency decreases as the equivalent resistance of the feedback NMOS increases. Hence for larger trans-resistance values which might be susceptible to feedthrough instability, the stabilizing dominant pole is automatically lower in frequency thereby improving stabilization.

The wideband amplifier used in the SNL design was designed to have an approximate gain of 22 while providing a bandwidth of 50 MHz. In addition, the wideband amplifier was designed to operate despite large variations in backgate effect and input common mode voltage. A variation of the folded cascode was used with resistor loads for improved linearity. The wideband amplifier schematic is shown in Fig. 4-17. Differential input signals enter at nodes *inp* and *inn* with the gained differential output exiting at nodes *outp* and *outn*.

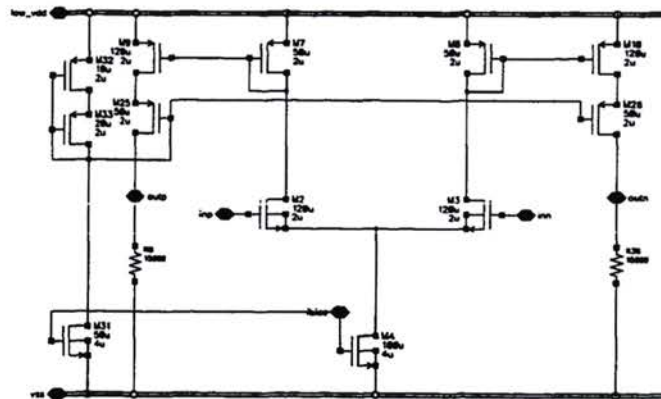


Figure 4-17. Cadence schematic of one of the wideband amplifier used to increase trans-resistance gain. Each amplifier increases trans-resistance by a factor of 22.

The final stage of the trans-resistance amplifier is the variable gain amplifier shown in Fig. 4-18. The automatic gain controller adjusts the gain of this stage to control rotor resonant amplitude. The variable gain amplifier is loosely based on the Gilbert multiplier [67]. In short, the constant trans-resistance signal entering at nodes *inn* and *inp* are

multiplied by a variable voltage entering at node agc thereby setting the gain between 4 and -4. Resistive loads placed the common mode output within less than one volt of the bottom rail for maximum drive voltage difference between stationary drive combs and the inertial rotor.

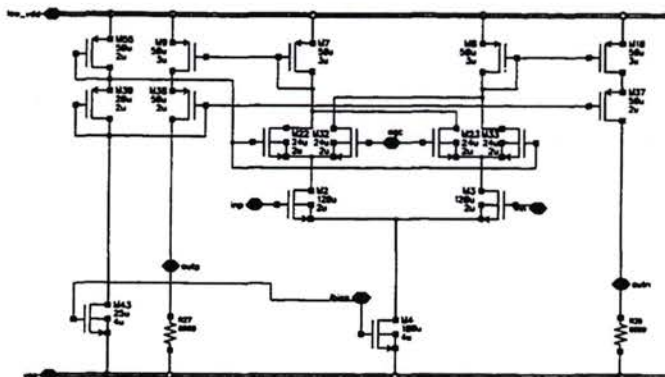


Figure 4-18. Cadence schematic of the variable gain amplifier used to vary drive trans-resistance from 200 M Ω to -200 M Ω

4.4 ELECTRICAL SYSTEM SUMMARY

Thus far, this thesis has delved into all the dual-axis gyroscope subsystems. All these subsystems are interrelated, and these connections have been touched upon throughout the last four chapters. The next chapter pulls all these subsystems together into a full system. Then system wide design trade-offs and optimization are explored. A full system approach is imperative as optimizing certain subsystems may degrade overall system performance. Many features, both electrical and mechanical, must be considered together to develop a truly optimized design.

5. DESIGN TRADE-OFFS AND OPTIMIZATION

The above chapters describe the important aspects of the mechanical and electrical design. Now the task remains to combine all these aspects into an optimized system which will perform to specifications. In general the fabrication process, for example Analog Devices Inc. (ADI) or Sandia National Laboratory (SNL), will be given. Hence there are a number of process constraints dictated by process capability. These include structural material properties, minimum mask resolution, and electrical capacitance to the substrate. Along with these process constraints are performance constraints which represent robustness criteria, performance specifications, and interface circuitry limitations. A simplified optimal design problem can be formulated where a set of design parameters are optimally chosen to satisfy performance constraints while remaining within process constraints and minimizing rate measurement noise [72]. In most industrial situations, it would be the noise which is a given constraint and the die area would be minimized. However, for this work the rotor outside radius was a constrained due to residual stress or die usage limitations, so rate measurement noise was minimized instead.

The circuitry can be optimized separately from the mechanical structure. Only the circuitry which directly interfaces to the structure has serious impact on performance specifications, so the majority of the signal processing circuitry can be decoupled from the optimal design problem. Furthermore, key interface electronics can be optimized purely in electrical terms or optimized based on the mechanical structure. For example, the trans-resistance amplifier can be designed to produce the maximum resistance with acceptable phase lag for a given technology. On the other hand, the sense integrator which is attached to the structure can be optimized for minimum noise given structure parasitic capacitance and interconnect resistivity. The bottom line is that the structure can and will be treated as a stand alone design problem.

5.1 FABRICATION PROCESS LIMITATIONS

First, the limitations inherent to the process must be represented mathematically using constrain equations and inequalities. These include material properties, film thicknesses, parasitic capacitance, minimum dimensions, and maximum dimensions. Table 1 below shows the approximate constraints for both the ADI and the SNL surface micromachine processes.

Table 1: Process Constraints

Variable	Symbol	ADI	SNL
Structural Polysilicon Thickness	h	$2\ \mu\text{m}$	$2.3\ \mu\text{m}$
Nominal gap between poly and substrate	g	$1.6\ \mu\text{m}$	$2\ \mu\text{m}$
Minimum comb finger gap	d	$1\ \mu\text{m}$	$1\ \mu\text{m}$
Polysilicon modulus of elasticity	E	$150\ \text{GPa}$	$150\ \text{GPa}$
Polysilicon mass density	ρ	$2300\ \text{kg/m}^3$	$2300\ \text{kg/m}^3$
Air viscosity at operation vacuum pressure	μ	$2 \times 10^{-8}\ \text{Kg/m/s}$	$2 \times 10^{-8}\ \text{Kg/m/s}$
Residual stress gradient	$d\sigma/dz$	$4.4\ \text{MPa}/\mu\text{m}$	0
Maximum allowable voltage	V_{max}	15 V	6 V
Parasitic capacitance of interconnect	C_{para}	2 pF	1 pF
Maximum size of structure	radius	500	1000

5.2 PERFORMANCE CONSTRAINTS

Next the design criteria and performance specifications must be written as design constraints which must be satisfied. One obvious constraint stems from the need to put the inertial rotor in motion. Therefore, the gyroscope must be designed such that the trans-resistance amplifier with maximum resistance R_{Ω} can induce drive oscillation. The maximum trans-resistance value depends on transistor speed, transistor gain, and feedthrough as described earlier. This suggests that the amplifier maximum gain is dependent on circuit technology and optimization can be completed separately from the

mechanical design. The gyroscope clearly cannot operate if the rotor is pulled to the substrate by electrostatic voltages. To ensure survival during start-up transients and possible electrostatic discharges (ESD), a minimum pull-down voltage V_{pull} should be established. Survival in the face of mechanical shock can be enhanced by determining a minimum natural frequency for all modes. This also aids in operational robustness to outside accelerations and vibrations. The minimum frequencies are either the drive frequency ω_z or the x-axis translation frequency ω_{Tz} .

Other design constraints follow directly from performance specifications. The most obvious is minimum detectable signal Ω_n , but there are several other interrelated factors. For example, the electrostatic tuning range should be chosen in an effort to provide frequency matching despite process variations. Ideally, the tuning range will be broad enough to allow mode matching despite sense axis mode frequency tolerance $\pm\Delta\omega_x$ and drive frequency tolerance $\pm\Delta\omega_z$. Closely connected to the tuning range is the ratio between sense and drive mode frequencies. Mode separation should be wide enough to ensure the sense mode is above the drive mode frequency regardless of process variation. Yet the mode separation should not be so wide that frequency tuning cannot match modes. The tuning range and mode separation assuming exact matching is desired are illustrated in Fig. 5-1.

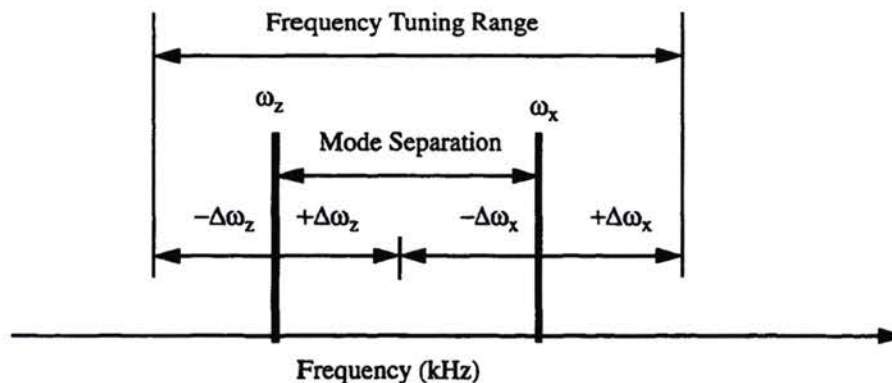


Figure 5-1. The minimum mode separation and frequency tuning range for optimal design.

Sense mode tunability can be defined as the percentage change in sense mode frequency derived from electrostatic tuning. Assuming the mode frequency tolerances are not large, then the Taylor series approximation can be invoked and tunability can be expressed as a relationship between mechanical spring constant K_{xx} and electrostatic spring constant K_{ex} as in Eq. 5-1. The original sense mode frequency is ω_x and the frequency after maximum tuning is $\tilde{\omega}_x$. The bottom line is that the rotational spring tuning range can be defined as the ratio between the maximum electrostatic spring constant K_{ex} and the original mechanical spring constant K_x .

$$\frac{K_{ex}}{K_{xx}} \equiv 2 \frac{\omega_x - \tilde{\omega}_x}{\omega_x} \quad (5-1)$$

As Fig. 5-1 suggests, the tuning range should span the full tolerance range of both sense and drive mode frequencies. Again assuming tolerances are small and therefore sense and drive frequencies should be nearly equal, the tunability constraint can be expressed in terms of percentage frequency variation of the sense and drive mode in Eq. 5-2.

$$\frac{K_{ex}}{K_{xx}} \equiv 4 \left(\frac{\Delta\omega_x}{\omega_x} + \frac{\Delta\omega_z}{\omega_z} \right) \quad (5-2)$$

Likewise, the nominal mode frequency separation between sense and drive axis can be derived from Fig. 5-1. The mode separation should be the sum of sense mode tolerance and drive mode tolerance. With the same small tolerance assumptions, the ratio constraint place upon drive axis to sense axis mode frequencies can be expressed in Eq. 5-3.

$$\frac{\omega_z}{\omega_x} \equiv 1 - \frac{\Delta\omega_x}{\omega_x} - \frac{\Delta\omega_z}{\omega_z} \quad (5-3)$$

Finally, all of the above constraints should be satisfied efficiently through the use of minimum die area or a pre-determined die area. The structural die area is dependent on rotor radius R , so in fact radius becomes the key design constraint. In this case, maximum rotor radius was used as a constraint. All of these design constraints are enumerated in the Table 2 below for the ADI fabricated version.

Table 2: Performance Derived Design Constraints for ADI Version

Parameter	Limit
Trans-impedance	$R_{\Omega} < 20M\Omega$
Pull-down Voltage	$V_{pull} > 2V$
Tunability	$\frac{K_{ex}}{K_x} > 4\left(\frac{\Delta\omega_x}{\omega_x} + \frac{\Delta\omega_z}{\omega_z}\right)$
Minimum frequency	$\min(\omega_z, \omega_{Tz}) > 12kHz$
Detectable Signal	$\Omega_n < 1(^{\circ}/s)$ or $\text{Min}(\Omega_n)$
Ratio between Drive/Sense Natural Frequency	$\frac{\omega_{drive}}{\omega_{sense}} = 1 - \frac{\Delta\omega_x}{\omega_x} - \frac{\Delta\omega_z}{\omega_z}$
Die area	$\text{Min}(R)$ or $R < R_{\max}$

5.3 INDEPENDENT DESIGN PARAMETERS AND DESIGN TRADE-OFFS

A set of independent design parameters must be chosen to fully characterize critical aspects of the mechanical structure. The inertial rotor, beam suspension, and comb drive must all be represented. The rotor itself can be compactly described by the inside radius R_i and the outside radius R . The suspension can efficiently modeled using the average beam length, polysilicon film thickness or height h , and the beam width b . The general type of suspension, such as single beam or serpentine multiple fold, must be determined before hand. Large residual stress, low desired frequencies, or limited radial space often dictate the need for serpentine suspension. Full characterization of the drive combs requires inner most comb radius, outer most comb radius, and number of comb gaps; but only number of comb gaps N will be considered for simplicity. The definitions of these design parameters for both inside suspension designs and outside suspension designs are showing in Fig. 5-2 and 5-3 respectively.

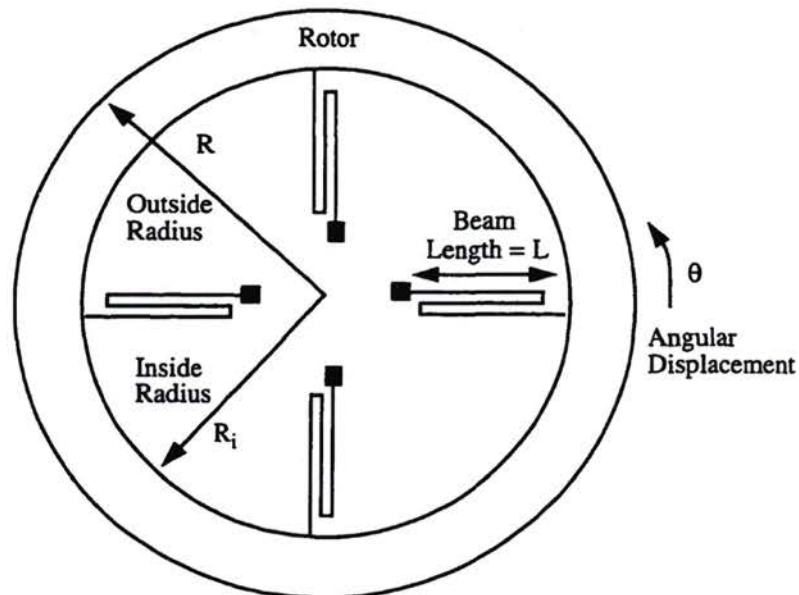


Figure 5-2. Top view of polysilicon rotor showing dimensions and design variable definitions for inside suspension dual-axis gyroscope.

The design variables can be further simplified by choosing a single dimensional parameter and converting all other design parameters to dimensionless parameters. The outside rotor radius R was chosen as the dimension parameter because it has the greatest effect on performance and best embodies overall mechanical structure size. The ratio of inside rotor radius to outside rotor radius characterizes the inertial rotor. The suspension can be characterized by the ratio between beam length L and inertial rotor outside radius R in conjunction with the beam cross-section aspect ratio b/h . Including the already dimensionless comb gap number N , the design parameters are as follows.

$$R, \frac{R_i}{R}, \frac{L}{R}, \frac{b}{h}, N$$

The most important design trade-offs involve R , R_i/R , L/R , and b/h . A sufficient number of comb teeth gaps is vital for achieving resonance, but the only real trade-off is increasing the number of teeth consumes more die area. Increasing or decreasing each of the design parameters provides interrelated advantages and disadvantages. This yields a

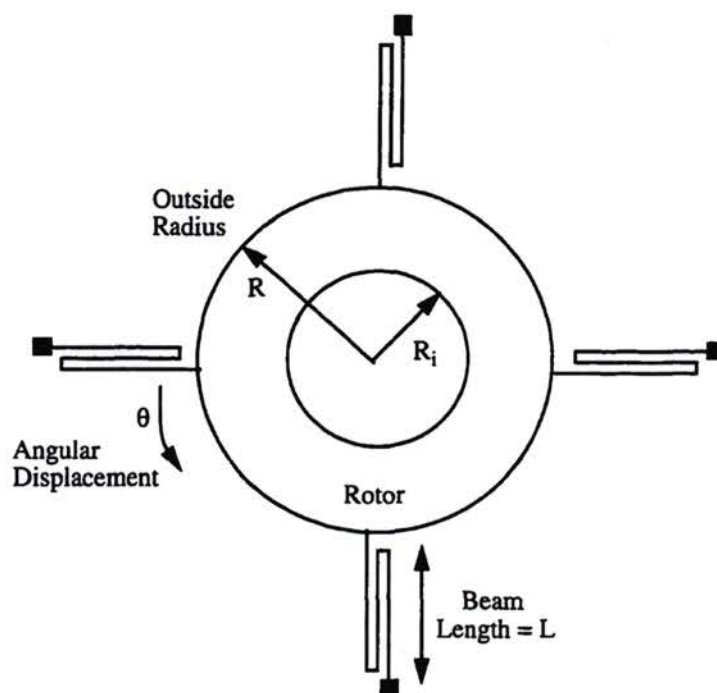


Figure 5-3. Top view of polysilicon rotor showing dimensions and design variable definitions for outside suspension dual-axis gyroscope.

complex, coupled optimization problem. The following paragraphs illuminate the important trade-offs which must be considered during the design process.

The outer rotor radius has paramount effect upon gyroscope performance and operation. A large radius obviously provides greater capacitive sensitivity (see Eq. 4-33) thereby improving noise performance. A larger radius improves sense axis frequency tuning range (Eq. 4-34 provides the dominant trend) which also implies better noise performance. Another benefit might be increased moment of inertia as this tends to lower the natural frequencies and increase sensitivity. On the other hand, there are many drawbacks to increased rotor radius. Aside from increased die area, the larger rotor area caused by a larger rotor radius has several detrimental side effects. A larger area exposed to the substrate and sense electrodes beneath the rotor produces higher parasitic capacitance on the rotor which in turn increases noise level. More area lowers pull-down

voltage the requires lower sense modulation voltages thus reducing sensitivity. A larger rotor naturally implies a large mass which reduces translational natural frequencies. The large rotor clearly requires a more powerful drive to overcome viscous friction. Finally, in technologies with residual stress gradients, a larger rotor leads to increased warpage. In general, the requirement for higher noise performance must be balanced with the drawbacks of utilizing a larger radius rotor.

Many of the drawbacks plaguing a larger rotor radius can be partially surmounted by hollowing out the rotor center or in other words increasing the ratio R_i/R . It is true that hollowing out the rotor center will reduce capacitive sensitivity. However, the vast majority of sensitivity is derived from the outer rotor edge where the sense electrode area is largest and the motion during tilting is greatest. Thus, hollowing out the center reduces sensitivity only slightly while imparting many crucial advantages. This is represented mathematically by the fact that sensitivity is proportional to R^3 (see Eq. 4-33) while rotor area and the related drawbacks are proportional to R^2 . With lower rotor area, the rotor parasitic capacitance is lower thus reducing noise. Reduced area increases pull-down voltage which in turn increases both allowable sense modulation voltage and tuning range. The hollow center also reduces mass which increases the natural frequencies of translational modes. There is of course a limit to how large a hole can be etched in the rotor center, for at some point the mechanical rigidity of the rotor will be compromised.

The ratio L/R embodies the complex relationship between the rotor and the beam suspension. Shorter beams and thus lower L/R represents a stiffening of the mechanical structure. Clearly both rotational and translational frequencies will be increased with lower L/R . Interestingly, translational frequencies increase more relative to sense natural frequencies (compare Eq. 3-41 to 3-49) in the design region of interest. Hence, for a given sense mode frequency, the translational modes are higher. This provides better mechanical shock protection, higher pull-down voltages, and larger frequencies tuning range. Disadvantages include the fact that shorter beams will exhibit nonlinearity for smaller oscillation amplitudes. In addition, short beams have less capacity for stress relief.

The final design parameter is the beam cross-section aspect ratio b/h . Since the polysilicon thickness or height h is given for a given process, only beam width b is alterable. Increasing b increases the ratio of drive to sense frequency. If the sense and drive frequency are too far apart, then the electrostatic frequency tuning range may not be sufficient to match modes and maximize sensitivity. On the other hand, very closely matched mode designs may leave the drive mode below the sense modes due to process variation. This would render electrostatic tuning useless as the sense modes can only be down tuned. The end result would be attenuated sensitivity and a reversal of polarity because Coriolis tilt motion would be 180° out of phase.

All of these trade-offs are tabulated in the Table 3. For those advantages and disadvantages where a clean relationship to a design variable exists, the order of proportionality is given.

Table 3: Mechanical Design Trade-offs

Variable	Advantage	Disadvantage
Larger R	More capacitive sensitivity $\propto R^3$ More sense frequency tuning range $\propto R^4$ More inertia $\propto R^4$	More parasitic capacitance $\propto R^2$ More drive trans-R $\propto R^4$ More die area $\propto R^2$ More warping $\propto R$ Lower pull-down voltage $\propto R^2$ Lower allowable sense voltage $\propto R$ More mass yielding lower translational natural frequency $\propto R^2$

Table 3: Mechanical Design Trade-offs

Variable	Advantage	Disadvantage
Smaller L/R	Higher translational natural frequencies Higher pulldown voltage Higher sense voltage without pull-down or excessive translational frequency down-tuning More sense frequency tuning range	Higher rotational natural frequencies Lower resonant amplitude due to onset of nonlinearity Less strain relief
Larger Ri/R	Reduced parasitic capacitance Less exposed rotor area yielding: Increased pull-down voltage Increased allowable sense voltage Decreased mass yielding higher translational frequencies More sense frequency tuning range Allows inside combs	Reduces capacitive sensitivity Reduces inertial rotor rigidity
Decrease b/h	Larger immunity to process variations causing the drive mode to be above the sense mode.	Requires increased electrostatic tuning range

5.4 DESIGN OPTIMIZATION

With the design problem formally outlined and the trade-offs considered, some form of optimization scheme must be utilized to produce an operative design. The first generation ADI design was semi-optimized by hand through a design parameter space search. Due to residual stress induced warpage, the rotor was limited to 150 μm . The SNL version used computer optimization through flooding the design space [72] with parameter combinations and searching for global minimum noise. This performance was constrained by a maximum outer radius of 250 μm for die area utilization reasons.

Unfortunately, these radius sizes and the given technology forced abridged electrostatic tuning ranges because the pull-down voltages were low. However, all other relevant constraints were met. This resulted in dual-axis gyroscope designs tabulated in Table 4.

Table 4: Final Design Parameters

Design Parameters	ADI High Frequency	ADI Low Frequency	SNL
R	150 μm	150 μm	250 μm
R _i /R	0.33	0.33	0.82
L/R	1.2 Outside	0.73 Outside	0.48 Inside
b/h	0.90	0.90	0.91
N	384	384	512

The micrographs in Fig. 5-4 and 5-5 show these dimensions for the high frequency ADI version and the SNL version respectively. These dimensions satisfied all design constraints except full tuning range. The next chapter gives more detail regarding fabrication and highlights the performance attained using these designs.

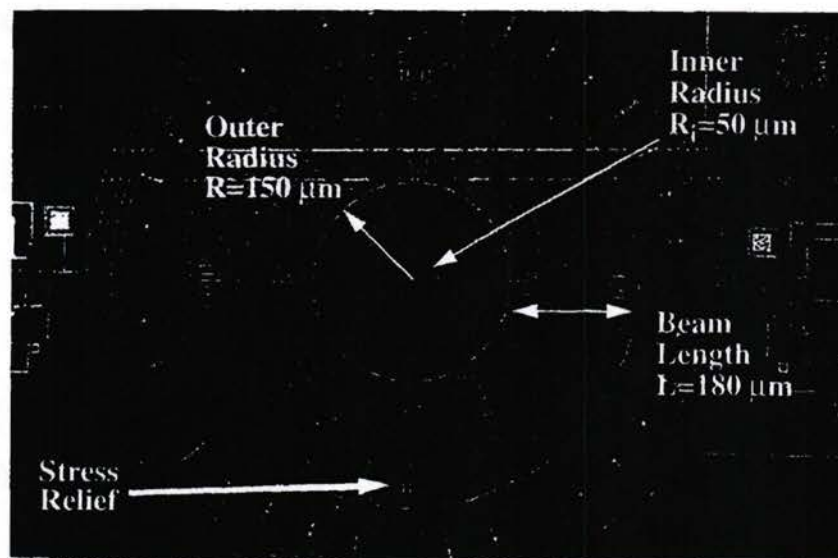


Figure 5-4. Micrograph of the first generation dual-axis gyroscope fabricated by Analog Devices showing key design dimensions.

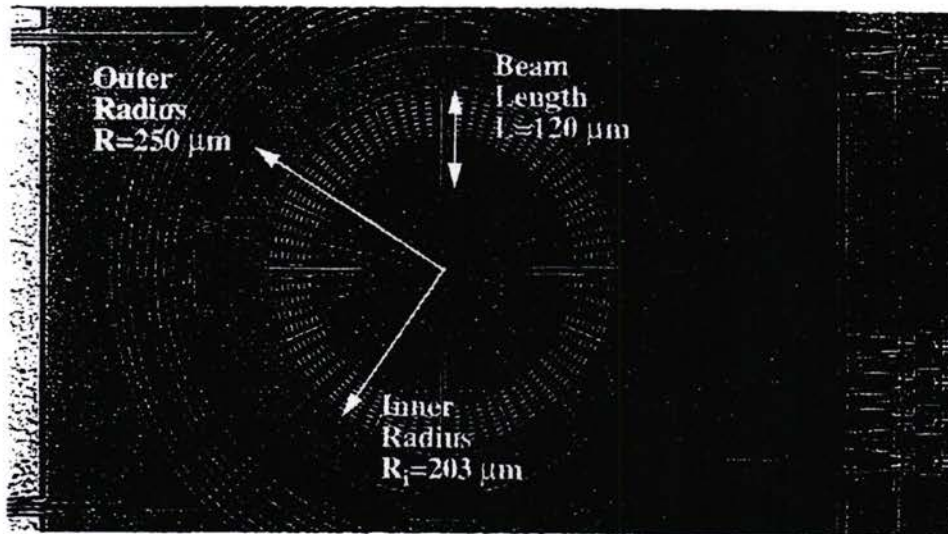


Figure 5-5. Micrograph of the dual-axis rate gyroscope fabricated by Sandia National Labs showing critical design dimensions.

6. FABRICATION AND EXPERIMENTAL RESULTS

6.1 DESCRIPTION OF FABRICATED DEVICES

All of the dual-axis gyroscope designs were fabricated using surface micromachine technology. However, two different surface micromachining processes, one developed by Analog Devices Inc. (ADI) and the other developed by Sandia National Labs (SNL), were used. The key difference between the fabrication methods is that SNL fabricates the micromechanical structure first and then forms the circuitry, whereas ADI interleaves the structure fabrication and the circuitry formation. These fabrication methods were inspired by the fact that the deposition and annealing of the structural polysilicon requires temperatures above the aluminum interconnect melting temperature. Hence, the aluminum interconnect must be deposited and patterned after the structural polysilicon is deposited.

6.1.1 Fabrication by Analog Devices Inc.

The ADI BiMEMS surface micromachining process [6] used for the dual-axis gyroscope is an industrial production process used to produce the ADXL-50 [36] and ADXL-05 micromachined accelerometers. The process begins with the fabrication of all electrical circuit layers except the interconnect aluminum. As stated previously, the polysilicon which embodies the micromechanical structure must be deposited before the aluminum due to the high deposition temperatures.

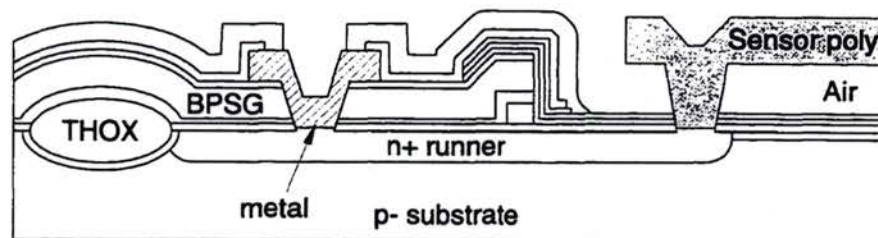


Figure 6-1. Cross-section of the ADI surface micromachining process. Figure courtesy of ADI.

Once all the BiCMOS circuitry layers except the aluminum are fabricated, the micromachines are fashioned in a moat region located at the die center. ADI implements the classic sacrificial oxide surface micromachine process reported by Howe and Muller [73]. First a sacrificial oxide is deposited. Second, holes are etched through the sacrificial oxide to anchor the mechanical structures to the substrate and allow electrical connection via n+ diffusions embedded in the substrate. Next, the structural polysilicon is deposited and annealed. With all high temperature operations complete, the aluminum circuit interconnect can be sputtered and patterned. Finally, the mechanical structures are released by etching the sacrificial oxide from beneath the polysilicon layer. This results in free standing structures secured to the substrate at the anchor points as shown in Fig. 6-1.

The micrograph in Fig. 6-2 shows one of the two operational designs fabricated by ADI. The circular disk at the center is the inertial rotor with the drive combs projecting radially from the circumference. The four beam suspension also extends radially from the inertial rotor. Unfortunately, the ADI process results in a high residual stress gradient which tends to cause the mechanical structure to warp concave downward with the center at the highest point. For this reason, an outside suspension with stress relief serpentine

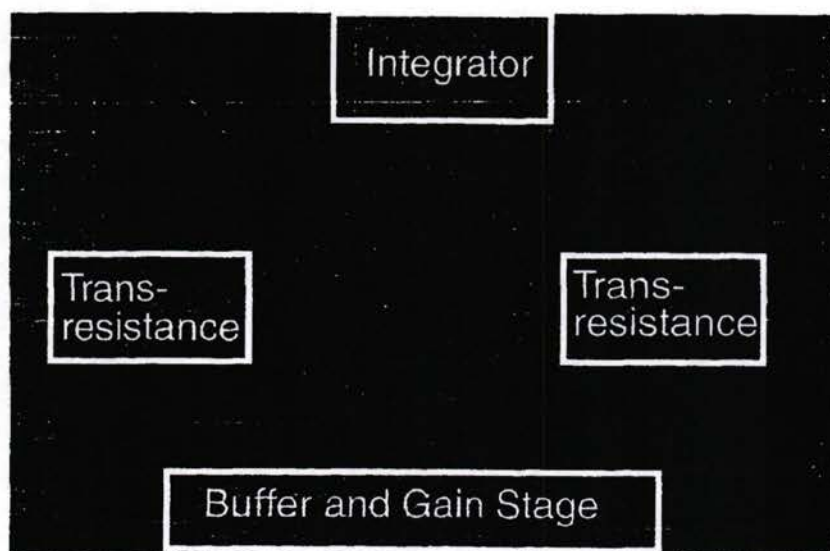


Figure 6-2. Micrograph of the first generation dual-axis gyroscope mechanical structure and interface electronics.

springs was used. As can be seen in Fig. 6-2, only vital interface circuitry was placed on chip to simplify design and allow maximum signal processing flexibility. Only the sense integrator and trans-resistance drive amplifiers were integrated.

6.1.2 Fabricated by Sandia National Labs

The overriding disadvantage of the ADI process is that the circuit and mechanical fabrication is interleaved. Therefore, standard circuit fabrication cannot be used. This limits the ability to contract outside foundries to fabricate the circuitry and hampers the ability to adopt successive generations of circuitry as technology evolves. To surmount these drawbacks, SNL has developed an embedded process which effectively divides and isolates the mechanical structure fabrication from the electrical circuit formation [7]. The term 'embedded process' is used because the mechanical structures are fabricated in a trench and then embedded in oxide to allow standard circuitry fabrication. This theoretically allows many surface micromachine processes to be married to any standard circuit process.

The cross-section of Fig. 6-3 reveals the how the embedded process is accomplished. First, a trench is etched into the silicon wafer to form the mechanical structure area. Second, the mechanical structure are fashioned using essentially the same sacrificial surface micromachined process described earlier. The key difference is that ADI

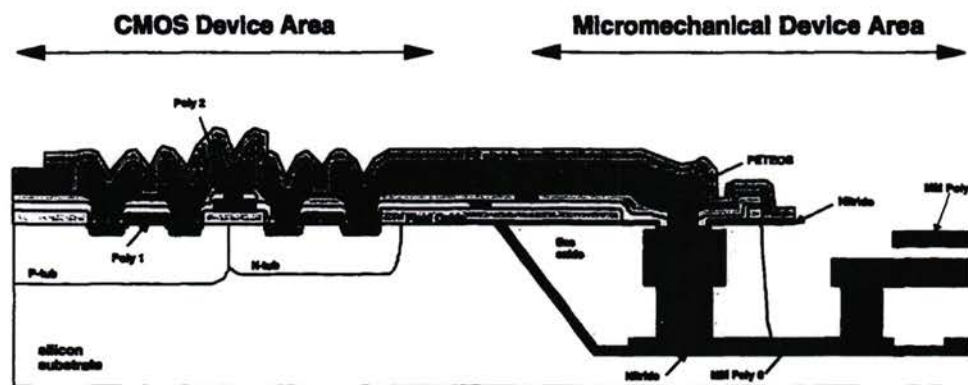


Figure 6-3. Cross-section of Sandia National Labs fully integrated process with embedded micromachines. Figure courtesy of Sandia National Labs.

interconnect was a diffusion path, while SNL actually utilizes an extra signal polysilicon layer atop the trench floor. Next the entire trench is filled with oxide to embed the mechanical structure and protect it from later circuit fabrication. The entire wafer is then chemical mechanical polished (CMP) to expose the silicon substrate and remove all topology associated with mechanical structure fabrication. This flat wafer can now enter a standard circuitry process with the electronics placed around the outside of the trench. A 2 μm CMOS technology was used for the SNL dual-axis gyroscope designs. Electrical connection to the mechanical structure is realized using a bridge connected to a pedestal which carries signals down to the polysilicon interconnect at the bottom of the trench. Finally, the embedding oxide and sacrificial oxide are etched from the trench releasing the final mechanical structure.

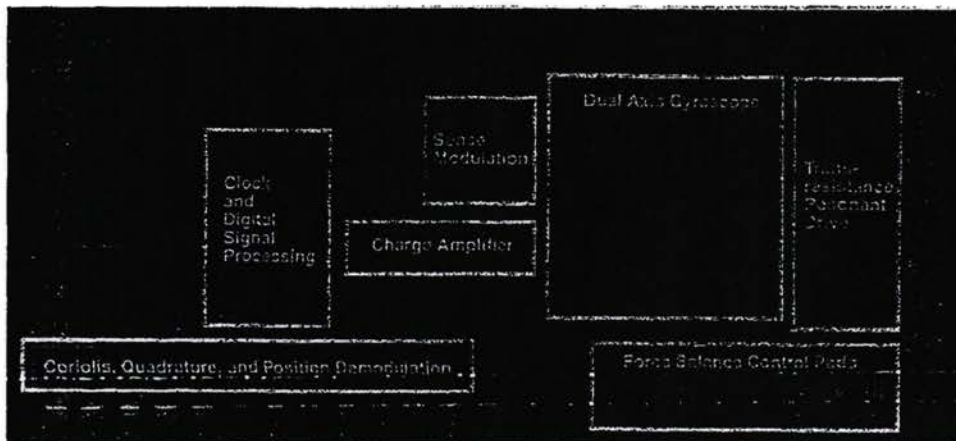


Figure 6-4. Micrograph showing the Sandia National Labs version of the dual-axis rate gyroscope. Note extensive interface and signal processing circuitry.

The lack of significant residual stress gradient in the SNL structural polysilicon allowed more design freedom than the ADI process. Hence, the rotor center is hollow with all drive combs located in the center for improved performance in a smaller structure (see Fig 6-4). The fact that polysilicon rather than n+ diffusions were used for interconnect increased interconnect density enough to allow the addition of feedback

electrodes and the drive signal bus in the rotor center. Finally, smaller minimum electronics feature size permitted extensive on-chip signal processing. In fact, only sources and passive components were needed off-chip.

6.2 VACUUM PACKAGING

As eluded to earlier, the viscosity of air seriously impedes inertial rotor resonance. To overcome this viscous resistance, the dual-axis gyroscope is operated in vacuum. A mini-vacuum chamber was used for all experiments because this allowed access to the micromachine in case of stiction to the substrate. However, prototype demonstration and actual product manufacture require a sealed vacuum package. Vacuum sealing die mounted by ADI in ceramic packages was developed with the help of James Bustillo. Concurrently, a method for measuring vacuum package pressure after sealing was also invented. This initial work produced medium vacuum packages with good long term stability. Lower pressures should be achievable by replacing the standard epoxy die attach with either vacuum epoxy or gold foil.

The first obstacle to vacuum package development is measuring the internal pressure of a sealed package. The fact that it is sealed makes access to the vacuum cavity impossible. Repeated measurements over time are imperative for stability verification, so destructive methods must be ruled out. The chosen solution used micromachined tuning forks inside the vacuum package. A method was found to measure tuning fork damping or quality factor inside the vacuum package. Then the relationship between quality factor and ambient pressure was experimentally determined using an unsealed die in a vacuum chamber. This allowed the estimate of sealed package pressure through first measuring tuning fork quality factor and then using the empirical relationship to estimate internal pressure.

The entire approach is founded upon accurately and consistently measuring tuning fork quality factor. This was achieved using a ring-down test. The tuning forks were driven into self resonance using an on-chip trans-resistance amplifier. The resonance feedback loop could be disconnected off-chip. When the feedback drive voltage was

disconnected from the drive combs, the tuning forks ring-down from resonance to no motion. A typical ring-down oscilloscope trace of the trans-resistance amplifier output is shown in Fig. 6-5. Fitting a damped, second order system model to the ring-down trace provides an estimate of tuning fork quality factor Q inside the vacuum package. A maximum likelihood exponential regression was used to fit the ring-down amplitude envelope.

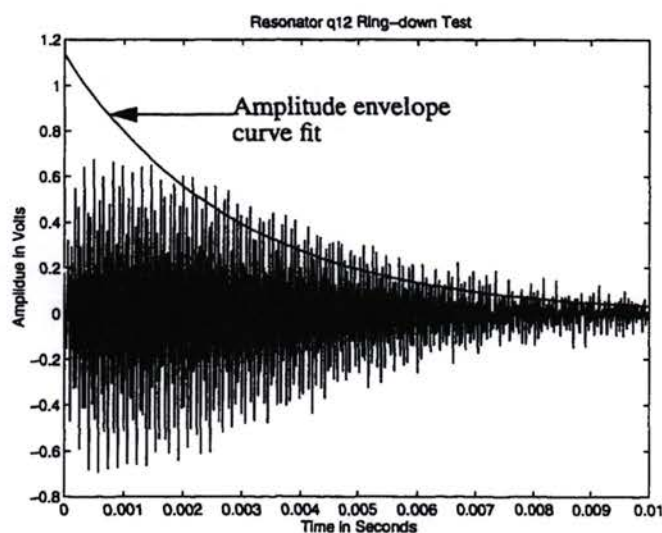


Figure 6-5. Oscilloscope trace from a typical tuning fork ring-down test as well as the amplitude envelope of the maximum likelihood regression.

With a consistent method of measuring tuning fork quality factor developed, the relationship between vacuum chamber pressure and tuning fork quality factor could be experimentally determined. Repeated ring-down tests were completed inside the controlled environment of an MMR vacuum chamber. The relationship between ambient pressure and tuning fork quality factor Q is summarized by Fig. 6-6. The relationship appears approximately logarithmic in nature. Estimate of internal package pressure was now accomplished by measuring Q via ring-down. Then the unknown pressure is estimated by applying the model to the measured Q to find pressure.

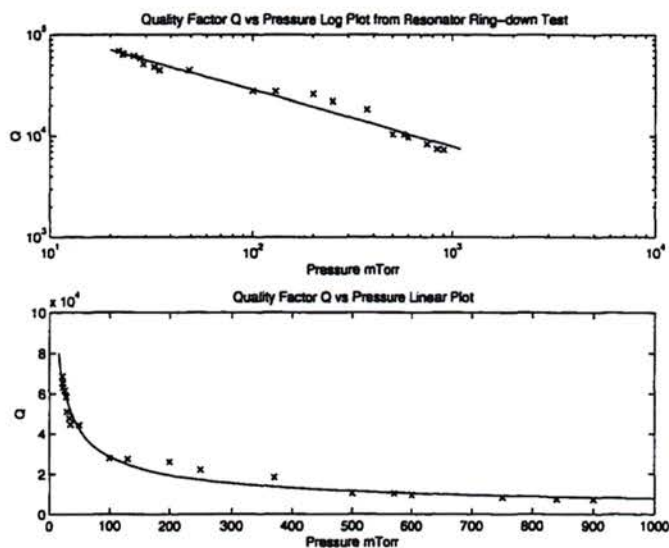


Figure 6-6. Relationship between tuning fork quality factor and vacuum chamber pressure. Note the nearly logarithmic dependence. All motion is in the lateral direction parallel to the substrate.

The following recipe was used to seal standard ADI ceramic DIP packages [75], [76] with epoxy die attach. The epoxy die attach presented a likely source for out-gassing, so extremely low pressures were not attained. The first step was to remove as much air, water films, and other volatile contaminants from the package die cavity as possible. The package was placed in a clear quartz vacuum chamber with the lid loosely placed in position. The chamber was repeatedly evacuated with dry nitrogen back fill to remove atmospheric air. During this process, a heat lamp raised the package temperature above the boiling point of water (to approximately 200 degrees C) to remove water molecules adhering to the package and die. With an inert environment inside the package die chamber, the lid was sealing by increasing heating temperature above the melting point of the indium lid solder to approximately 280 degrees C.

First attempts produced internal pressures of around 900 mTorr, while later recipes improved internal pressures to 300 mTorr. The time series data in Fig 6-7 is testimony to

the long term stability of the vacuum. No increase in pressure due to either out-gassing or leakage was detected after 140 days which bodes well for the much longer lifetimes required for commercial products. Switching die attach to vacuum epoxy or gold foil should allow much improved vacuum levels. However, this was not pursued as a sealed lid makes inspection and probing of the enclosed die impossible. For this reason, gyroscope testing inside sealed vacuum packages was not pursued.

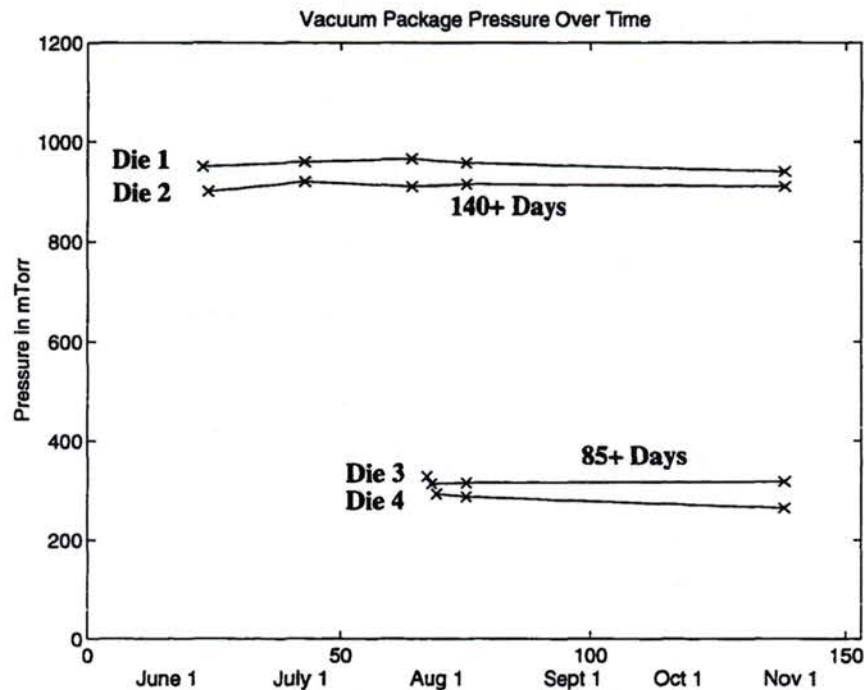


Figure 6-7. Endurance tests showing vacuum package integrity over time.

6.3 DESCRIPTION OF TEST SETUP

All dual-axis gyroscope testing was completed in the Berkeley Micromechanical Analysis and Design Labs. Accurate angular rate inputs were produced by a Contraves rate table in the basement of Etcheverry Hall. A mini-vacuum chamber attached to a roughing pump was used to produce the vacuum environment for gyroscope resonance. Vacuum pressures of between 60 and 100 mTorr were typical. All gyroscope support

circuitry and the mini-vacuum chamber were pinioned to a rate table mounting bracket allowing 180 degree of rotation orthogonal to the rate table spin axis. This arrangement is akin to a dividing head and allows cross-axis sensitivity measurements.

6.4 SYSTEM IDENTIFICATION AND PARAMETERIZATION

Experimental system identification and parameterization was completed to validate analytical models. Those parameters with greatest impact on performance, such as natural frequency, quality factor Q , and maximum sense voltage were experimentally identified. Both the devices fabricated by ADI and the devices fabricated by SNL were extensively tested.

Two versions of the dual-axis rate gyroscope were fabricated using the ADI surface micromachining processes. The two versions have identical inertial rotor sizes but different suspensions. The high frequency design had a straight single beam suspension with minimal stress relief (see Fig. 6-2) giving a drive natural frequency of 28.6kHz. The other low frequency design had serpentine springs with a drive frequency of 21kHz. Sense axis frequencies were 2-9% higher than the drive frequencies. This variation was apparently caused by variations in fabrication. The drive frequency was measured by exciting self resonant drive, while the sense axis frequencies were found by scanning sinewave frequencies on the sense electrodes and identifying resonance peaks. Both sensors were tested in 60 mTorr vacuum which gave the high frequency version a sense axis quality factor Q of 960 and the low frequency version a Q of 820. The ring-down test described in the vacuum packaging section was used in all cases. The pull down voltage of the high frequency sensor was 4.2 volts, whereas the low frequency sensor could withstand only 3.4 volts.

All of the above tests were also completed on the single SNL version. The sense natural frequency was found to be 13.7 kHz with drive frequency 40-50% higher. The z-axis translation frequency was 17.44 kHz resulting in a pull-down voltage of 2.5 volts. The low frequencies and extreme sense and drive mode mismatch were due to serious over etching of the suspension beams. A summary and comparison of all three version can be found in Table 5 below.

Table 5: Experimental Device Parameters

Parameters	ADI High Frequency	ADI Low Frequency	SNL
Drive Natural Frequency	28.6 kHz	21 kHz	7 kHz
Sense & Drive Frequency Mismatch	2-9%	5-9	13.7 kHz
Sense Mode Quality Factor Q_x	960	820	880
Pull-down Voltage	4.2	3.4	2.5
Z-axis Translational Frequency	18.2 kHz	13.5 kHz	17.4 kHz

6.5 PERFORMANCE WITHOUT FREQUENCY TUNING

There are many measures of gyroscope performance including minimum detectable signal, scale factor stability, and offset bias stability to name a few. The focus of this testing was upon minimum detectable signal. Because micromachined devices produce extremely tiny signals, noise level or minimum signal is of great interest. Furthermore, achieving stable offset and scale factor requires process control, compensation, and calibration more suitable to an industrial setting than academe.

The performance results for the ADI dual-axis gyroscope versions are summarized in Fig. 6-8. The results are for gyroscopes as fabricated without electrostatic tuning. Due to process variation, a distribution of nearly matched and unmatched rate sensors could be investigated. In all instances, a sinusoidal rate input was used to alleviate difficulties posed by the twisting of the wiring and vacuum tubing connecting the rate table mounted test bed to off table equipment. The dual-axis rate gyroscope having mode mismatch of 2% provided far better performance with 2mV per °/sec sensitivity and a 0.24 °/sec/√Hz noise floor limited by the electrical noise. Thus, a minimum detectable signal of approximately 1.2 °/sec over a 25 Hz bandwidth was achieved. Individual gyroscopes which possessed mode mismatch of 8-8.5% yielded a noise level nearly an order of magnitude larger. Interestingly, the low frequency version did not show proportional improvement over the high frequency versions because a lower pull-down voltage necessitated a low sense voltage.

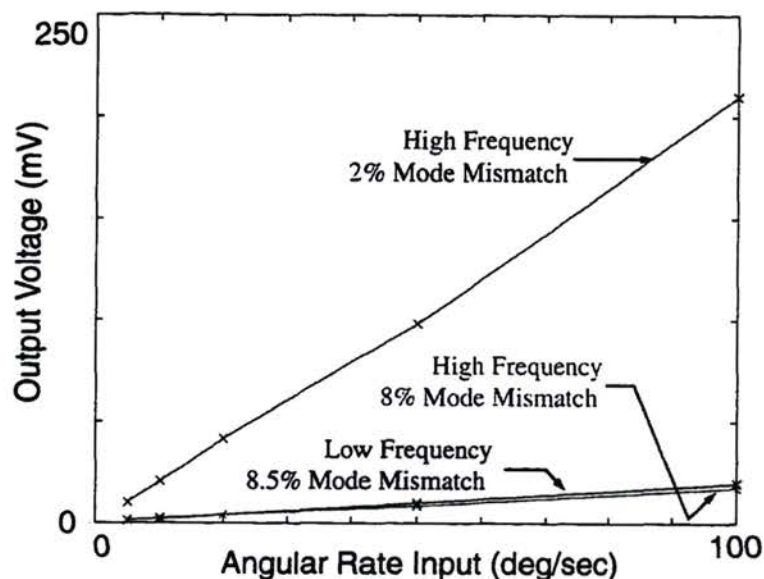


Figure 6-8. Plot showing dual-axis gyroscope voltage output versus angular rate amplitude input for a 1 Hz sinewave.

6.6 PERFORMANCE WITH FREQUENCY TUNING

Due to the high sense-mode quality factor Q_x , great performance improvement may be reaped using electrostatic tuning. As displayed in the upper graph of Fig. 6-9, increasing voltage between the quarter circle sense electrodes and the rotor tunes the sense frequency downward, while affecting the drive frequency only slightly. The bottom graph in Fig. 6-9 shows that sensitivity is maximized roughly at the point where the frequencies match with some offset due to experimental error. Tuning resulted in performance of 10 °/min over a 25 Hz bandwidth.

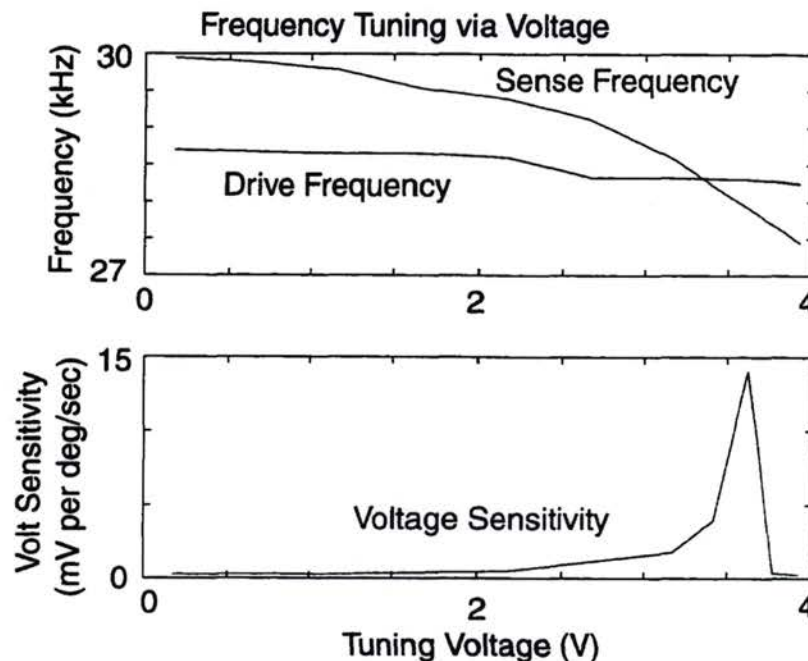


Figure 6-9. The upper plot shows sense frequency down-tuning. At the voltage where sense and drive frequencies are approximately equal, sensitivity is maximum.

6.7 CROSS-AXIS SENSITIVITY

A key parameter in all sensors, but especially dual-axis sensors, is cross-axis sensitivity. When there is an input rate on the one axis, then there should be no rate measurement from the other orthogonal axis, but in practice a small false measurement is unavoidable. Cross-axis sensitivity is here defined as the percentage of on-axis rate input which appears as a false off-axis rate measurement. For example, if a 100 °/sec rate input is applied to the on-axis and the orthogonal off-axis produces a 1 °/sec measurement, then the cross-axis sensitivity is 1%. Both die misalignment and intrinsic sources contribute to cross-axis sensitivity; however, misalignment was partially compensated by experimentally adjusting the die such that the measurement axis and the rate input axis

were approximately colinear. Essentially, a rate input was applied and the attitude was adjusted until the maximum output voltage was obtained. During open-loop operation the devices with better mode matching and hence better noise performance showed larger cross axis sensitivity. In fact, poorly matched modes exhibited 3% cross-sensitivity while closely matched modes exhibited over 16%. When modes were perfectly matched, cross-axis signals nearly as large as the rate input were witnessed. This is not unexpected as operating open-loop with mode matching can exacerbate cross coupling.

Matching drive and sense modes reduces the rotation rate noise floor, but cross axis sensitivity may consequently increase during *open-loop* operation. Part of this may be due to slight differences in the x- and y-axis natural frequencies, but the mechanical cross coupling is the primary source of cross axis sensitivity. Asymmetries and irregularities stemming from fabrication imperfections can result in inertial, viscous, and elastic cross coupling. Since the dual-axis rate gyroscope beams are sized at the minimum allowable process dimensions, elastic cross coupling is the dominant cross coupling term. Elastic cross coupling [54] simply implies that the suspension restoration torque is a function of both x- and y-axis angular displacements θ_x and θ_y as denoted in Eq. 6-1 and 6-2.

$$T_{X,Elastic} = K\theta_x + \delta K\theta_y \quad (6-1)$$

$$T_{Y,Elastic} = K\theta_y + \delta K\theta_x \quad (6-2)$$

Assuming for simplicity that the sense axes are identical, then K is the standard spring constant and δK is the elastic cross coupling term equal to the standard spring constant multiplied by a dimensionless coupling coefficient δ . Adding the dynamic effects of mode matching, the approximate function Eq. 6-3 for cross axis sensitivity S_{Cross} is derived where Q is the quality factor, ω_s is the sense mode frequency, and ω_D is the drive resonant frequency.

$$\|S_{Cross}\| \approx \left\| \frac{\delta \omega_s^2}{\omega_s^2 + (j\omega_D \omega_s)/Q - \omega_D^2} \right\| \quad (6-3)$$

This function is identical to a typical second order system with low damping and therefore a resonance peak. When mode matching is exact ($\omega_D = \omega_s$), cross axis

sensitivity due to elastic cross coupling is maximized. This was experimentally observed when electrostatic frequency tuning was used to match modes precisely. The white noise floor was reduced to $2^\circ/\text{minute}/\sqrt{\text{Hz}}$, but at the cost of excessive cross axis sensitivity. This undesirable increase in cross axis sensitivity is a consequence of open-loop dynamics which can be suppressed by closed-loop force balancing as discussed in the final chapter.

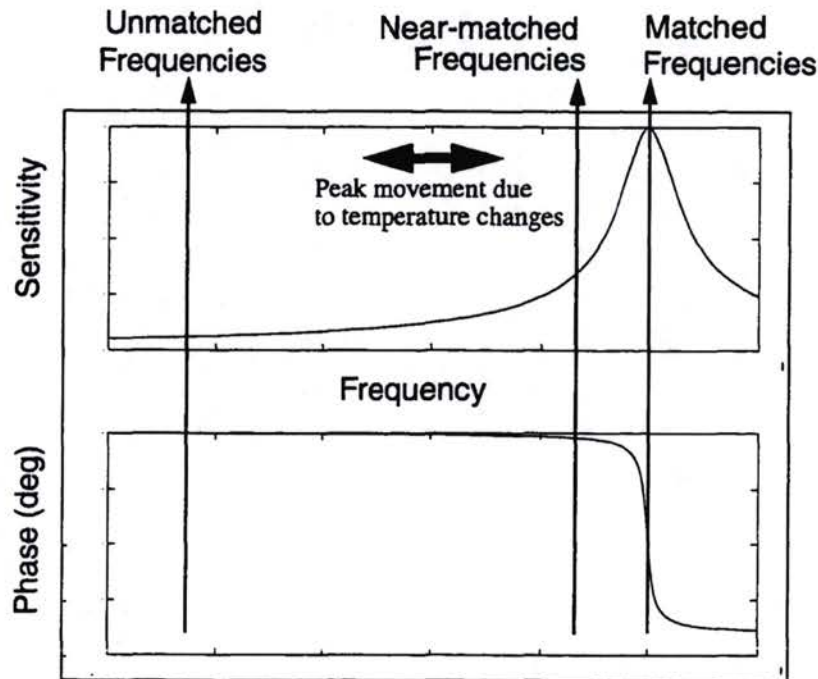


Figure 6-10. Bode plot showing sensitivity and output phase versus sense axis natural frequency.

6.8 PROBLEMS POSED BY MODE MATCHING

Under open-loop operation, matching natural frequencies closely forces the inertial instrument designer to confront the underlying trade off between noise performance and cross axis sensitivity. An additional consideration is that gain and phase both change dramatically at the resonance peak, so natural frequency drift can cause both substantial scale factor and output phase changes. Any lateral movement of the peak in Fig. 6-10 due to temperature changes would cause drastic changes in sensitivity and hence scale factor.

Yet another difficulty is that there is a tiny bandwidth over which scale factor is constant. In fact, with perfectly matched modes, the constant scale factor bandwidth is infinitesimal. These consequences limit practical mode matching for *open-loop* operation to no closer than 5-10%. However, future development of closed loop feedback control should alleviate many of these problems.

7. FUTURE DIRECTIONS AND CONCLUSIONS

7.1 PROJECT SUMMARY

A dual-axis rate gyroscope which measures rotation rate inputs about two input axes simultaneously has been designed, fabricated, and tested. All designs were surface micromachined with integrated circuitry on a single chip. The mechanical gyroscope itself could fit on the head of a pin, while the entire integrated chip was less than a few millimeters square. Dual-axis operation was successfully demonstrated and characterized. Automotive grade open-loop performance was achieved, but the peak performance via mode matching resulted in an undesirable increase of cross axis sensitivity and scale factor drift. Future directions include adding closed-loop feedback which should allow future designs to benefit from the performance advantages of mode matching without sacrificing cross axis sensitivity and scale factor stability. Other future directions include quadrature cancellation to reduce measurement offset drift and increased circuit integration. In the future, integration may come to apply to more than just circuitry. The mechanical sensors themselves may be integrated together to form entire monolithic microsystems.

7.2 WHY IS CLOSED LOOP CONTROL DESIRABLE?

Under open-loop operation, matching natural frequencies closely forces the inertial instrument designer to confront the underlying trade off between noise performance and cross axis sensitivity. Another consideration is that gain and phase both change dramatically at the resonance peak, so the scale factor is not constant over any appreciable bandwidth. Furthermore, natural frequency drift can cause both substantial scale factor and output phase changes. These consequences limit practical mode matching for *open-loop* operation to no closer than 5-10%.

However, closed-loop feedback is often used to enhance performance beyond the limitations of open-loop operation [77], [69]. Feedback typically improves scale factor stability, linearity, bandwidth, and operating range. A well-designed feedback loop will not significantly alter noise performance. Thus the noise advantages of mode matching

can be retained, while the dynamic advantages of closed-loop feedback can be added. In this case, null position moment balancing would be used. This technique balances all external moments thereby nulling rotor tilt position. All rotor tilt is zeroed and the rotor would remain parallel to the substrate despite Coriolis accelerations and external moments. The output voltage measurement is then the feedback voltage required to balance the Coriolis moment. It is projected that closed-loop moment balancing of the dual-axis rate gyroscope will alleviate many scale factor and phase difficulties while also reducing cross axis sensitivity significantly below that attained thus far.

Scale factor stability is important, but equally important is offset bias stability. Offset bias is the gyroscope output measurement when the gyroscope is not experiencing an angular rate input. Ideally the measurement should be zero, but in practice an offset always exists. Aside from electrical sources, rotor wobble and the ensuing quadrature error cause offset. As discussed in Chapter 4, the rotor wobble and Coriolis tilt motion are orthogonal in phase. Unfortunately, due to slight phase lag in the electronics, differentiating between these out of phase signals is difficult. Any phase lag will corrupt the Coriolis signal with quadrature error offset due to rotor wobble. Null position moment balancing feedback will improve phase performance whereby promoting better discrimination between Coriolis signals and quadrature error. However, an even better option is to cancel quadrature error at the source by negating rotor wobble. Improved offset stability may be achieved in the future by cancelling rotor wobble due to imbalance using a feedback loop driving special comb electrodes designed for levitation forcing.

Another possible closed-loop option is automatic frequency tuning of the sense mode to match the drive mode. This would ensure near constant noise performance in the face of temperature changes which alter natural frequencies. The possible future closed-loop feedback systems and the required moment generating electrodes are depicted in Fig. 7-1. Null position moment balancing and quadrature error cancellation will be discussed in the next two sections.

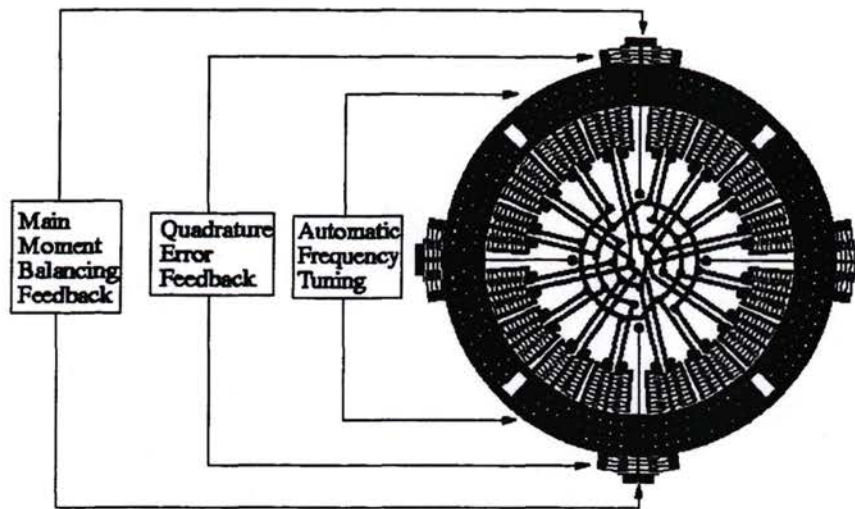


Figure 7-1. Top view of SNL structure showing forcing electrodes for both moment balancing and quadrature cancellation feedback. An additional frequency tuning loop using the sense electrodes is shown.

7.3 NULL POSITION MOMENT BALANCING

The main difficulty plaguing open-loop operation with closely matched sense and drive modes is poor scale factor stability. The resonant peak of the sense mode provides the gain needed to provide improved noise performance. On the other hand, the peak does not provide a flat or constant sensitivity over bandwidth and any movement of the peak due to temperature change results in radical changes in sensitivity. Accelerometers face less severe, but equivalent problems which have been solved using closed-loop or force balanced designs. The same advantages realized in accelerometer technology can be applied to the dual-axis micromachined gyroscope. The inherent limitations of the open-loop mechanical structure are surmounted by adding both moment generating electrodes and a feedback control law to keep the inertial rotor at the no tilt null position despite external rate inputs. In the SNL versions, the possibility of moment balancing was provided by four rectangular plate electrodes as part of the main moment balancing

feedback in Fig 7-1. A differential voltage applied to the electrodes creates two differential forces which in turn provide the restoring moment. The output signal is the differential voltage required to keep the rotor at this null position. Thus, the output is not an open-loop, indirect position measurement, but rather a direct measurement of the moment required to cancel Coriolis acceleration.

The underlying advantage of this approach is that gyroscope operation would be dominated by the electrical moment balancing feedback rather than the mechanical structure. However, the mechanical structure design is still important because the noise performance is determined by the original open-loop system. The virtues of null position moment balancing [77] can be readily seen in Fig. 7-2. The original open-loop system

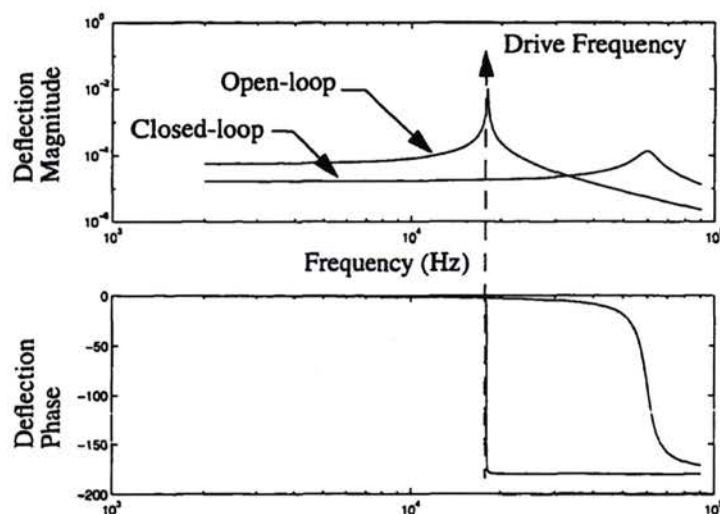


Figure 7-2. Bode frequency plot showing the enhancing effects of position nulling feedback. Notice closed-loop feedback provides a flat bandwidth with minimal phase change surrounding the drive frequency.

with modes matched exhibited a resonant peak at the drive frequency. The closed-loop system implementing null position moment balancing raises the system dominant pole providing a flat operational region surrounding the drive frequency. The flat region provides constant phase and constant sensitivity over the desired measurement bandwidth. Because there is no resonant peak near the drive frequency, scale factor changes due to

temperature changes are limited. Furthermore, the inertial rotor motion resulting from a Coriolis acceleration is attenuated by several orders of magnitude thereby reducing nonlinearity stemming from the suspension beams and sense electrodes.

However, it might appear that noise performance will be sacrificed because the sense and drive frequencies are intentionally mismatched. This is not true. Assuming the control electronics do not introduce a dominant noise source, then the closed-loop noise performance is equal to the original open-loop noise performance. Force balanced accelerometer manufacturers have taken advantage of this fact for years. Both the electrical Coriolis signal and the electrical noise are acted upon by the same linear feedback gain and loop gain. If the signal-to-noise ratio is considered, then the feedback gain and loop gain cancel out leaving the original open-loop signal-to-noise ratio. Therefore, null position moment balancing can significantly improve scale factor stability without limiting noise performance. This technique could allow future micromachined gyroscope designs to move towards tactical grade performance of conventional military and commercial gyroscopes.

7.4 QUADRATURE ERROR CANCELLATION

Rate gyroscopes are often used for attitude estimation, dead reckoning, or inertial navigation. In all these cases, the gyroscope measurement is integrated. The integration of a zero rate offset results in an angle ramp with error magnitude growing towards infinity over time. Hence, every effort must be made to cancel offset. As discussed in Chapter 4, rotor wobble can result in a quadrature error offset. The magnitude of the offset is proportional to higher order effects such as beam asymmetries, rotor imbalance, and electronic phase lag. These effects may not be stable over time and temperature thereby producing offset drift which will defy simple one time calibration.

The best way to combat quadrature error is to attack the underlying source, namely the rotor wobble. This is difficult because the rotor wobble is at the same frequency as the critical Coriolis tilting motion. In theory, the quadrature error will be 90° out of phase with the Coriolis signal. However, electrical lags make detecting quadrature error and

cancelling wobble difficult. Bill Clark [68] developed a novel method of cancelling quadrature error through carefully designed mechanical electrodes. A variation of this technique can be applied to the dual-axis gyroscope.

The key to this technique is the use of forcing electrode which provide a rotor tilting moment proportional to z-axis rotor drive angle. Ignoring forcing non-idealities, this produces a moment exactly in phase with rotor drive displacement and therefore in phase with rotor wobble. This allows more effective cancellation of rotor wobble despite some circuit lag. For the dual-axis rate gyroscope, restoring moments are needed about the sense axes (x- and y-axis). Therefore electrostatic forcing must be generated perpendicular to the substrate. This can be accomplished by taking advantage of comb finger levitation [49], [78]. If both the ground plane and a moving rotor mounted finger have the same voltage potential, then asymmetric electrical field lines are formed which provide a levitation force perpendicular to the substrate as shown in Fig. 7-3. This levitation force can be harnessed to provide the quadrature cancellation motion.

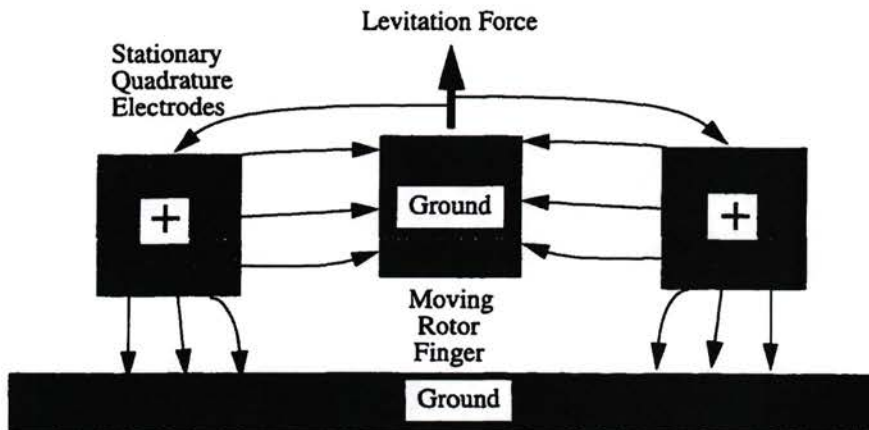


Figure 7-3. Cross-section of quadrature error cancellation electrodes showing electrostatic field lines. Changing stationary electrode voltages varies levitation force to cancel rotor wobble.

The schematic diagram depicting quadrature cancellation feedback for a single y-axis sensing mode is shown in Fig. 7-4. Conceptually, the rotor y-axis tilt signal is chopped with the rotor position signal generated by the a phase-lock-loop slaved to the structure

oscillation. This rotor position signal is in phase with the rotor wobble. After low pass filtering, this produces a differential signal proportional to rotor wobble. The procedure is imperfect due to electrical phase lag, but the imperfections should be small. The differential quadrature error signal is fed back to the stationary levitation electrodes as the wiring of Fig. 7-4 shows. The common mode or nominal bias voltage of the electrodes is well above the rotor bias voltage, so there is a steady levitation bias applied to the top and bottom of the rotor (looking down on the die) when the rotor is centered. Suppose that due to rotor wobble measurement, the stationary combs on the right are at a higher voltage than the stationary combs on the left. If the rotor rotates clockwise, the top rotor mounted finger moves away from the low voltage left comb and towards the high voltage right comb. This produces a net increase in levitation force proportional to angular displacement. The bottom rotor mounted fingers experiences opposite polarities, so a net decrease in levitation proportional to rotor angle is generated. This differential force produces a moment which is proportional to rotor angular drive displacement and can therefore cancel rotor wobble. At the time of publication, a BSAC patent was pending on this quadrature cancellation design.

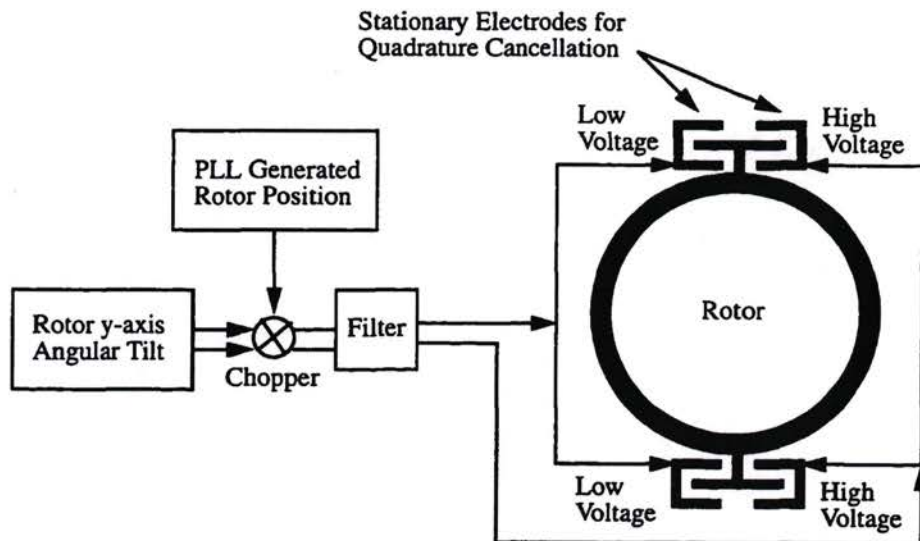


Figure 7-4. Schematic showing the quadrature error cancellation feedback electronics and accompanying electronics.

7.5 HIGHER LEVELS OF INTEGRATION

The dual-axis gyroscope fabricated by Sandia National Labs (SNL) had a high level of integration. All drive, sense, and signal processing electronics were integrated on chip. This included op-amps for filtering chopped signals used for the phase-lock-loop and automatic gain control. Hence, only voltages sources, passive components, and variable trim resistors were needed off-chip. Clearly future versions will incorporate circuitry to reduce voltages to only supply and ground. Further integration will reduce off-chip components and alleviate the need for external trimming.

However, the greatest challenge is designing circuitry which alleviates output offsets and exhibits stable scale factor over operational temperature range. Ensuring constant drive rate amplitude over time is critical for consistent scale factor. Equally important for open-loop operation are the amplitude of sense modulation voltages and the gain of the sensing signal path. Bandgap voltage references may be needed to ensure the stability of crucial voltages. An advantage of closed-loop operation involving null position moment balancing is that fewer voltages must be constant over temperature. In fact, the only truly critical voltage is the moment balancing bias voltage since all other voltages are included in the loop gain which cancels out of the final scale factor. Added functionality in the form of self-test or digital output may also be integrated on-chip. Furthermore, as the next section portends, the true future of integration may not be limited to a single gyroscope.

7.6 FUTURE OF MICROMACHINED GYROSCOPES

The field of micromachining is young and the field of micromachined gyroscopes is even younger. Future improvement in design and integration will allow performance and functionality far beyond what has been attained to date. Innovative technologies such as high-aspect-ratio MEMS and SOI-MEMS made possible by new deep trench etchers may allow significant improvements in performance. Regardless of the achieved performance, micromachined gyroscope size, weight, power consumption, and cost should be orders of magnitude below conventional technology. This will allow the consumer access to technology once reserved for aerospace and the military. Society as a whole will benefit

from improved automobile safety, personal navigation, virtual entertainment, and advanced manufacturing. More importantly, micromachined gyroscopes may be the enabling technology for previously undreamt of applications.

Yet the true micromachine revolution may not emanate from the individual sensors themselves, but the integration of many sensors into a microsystem. Orthogonal triads of accelerometers and gyroscopes could produce an inertial measurement unit on a chip with all signal processing included. This could revolutionize the navigation industry. Every portable computer and cell phone may some day have the navigation capability of present day aircraft. Total automobile safety and navigation modules may one day deploy multiple air-bags, operate anti-skid braking, optimize ride comfort, and display maps and best routing to the driver. A long term goal of the automotive industry is an affordable self-driving car like those being pursued by the California PATH project [79], [80].

Combining other chemical sensors, physical sensors, and functionality could drastically alter the economics and capabilities of many fields ranging from space exploration to health care. Fleets of micro-satellites smaller than a hockey puck may encircle the globe and explore the outer reaches of space [81]. Dreamers in the MEMS field even envision a future where tiny autonomous medical robots course through the blood stream searching for ailments. The possibilities are endless, and the revolution has only just begun!

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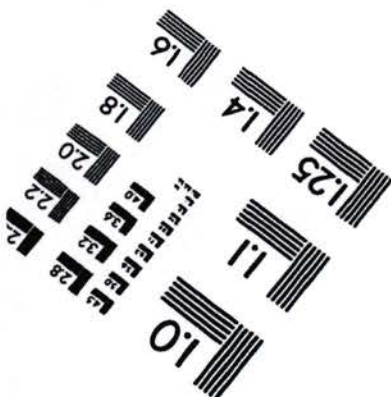
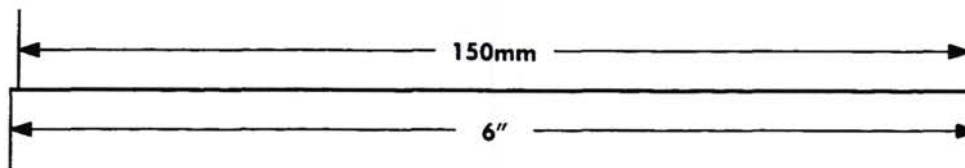
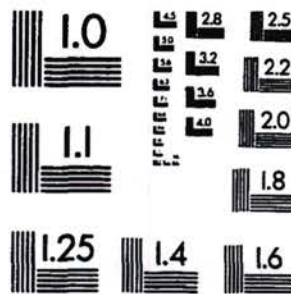
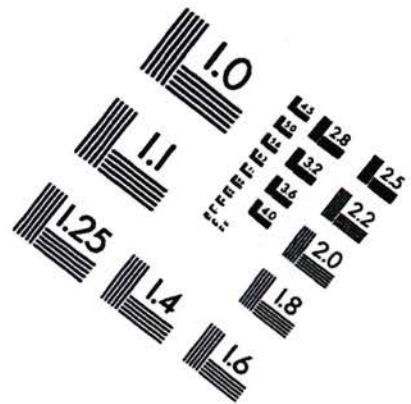
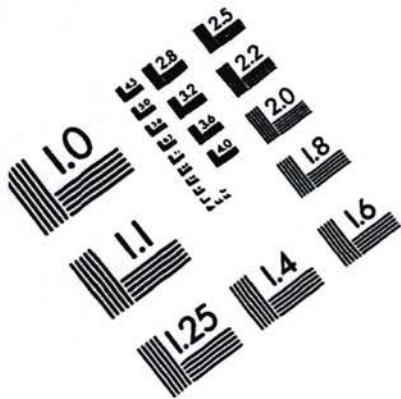
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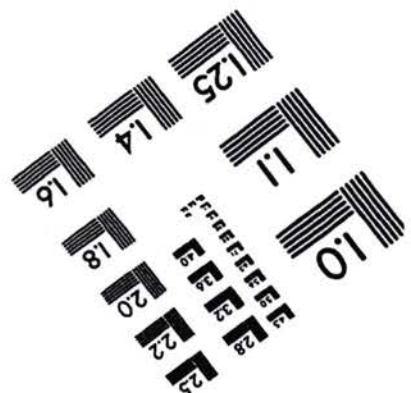
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IMAGE EVALUATION TEST TARGET (QA-3)



APPLIED IMAGE, Inc
1653 East Main Street
Rochester, NY 14609 USA
Phone: 716/482-0300
Fax: 716/288-5989

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