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Micro Accelerometer Design with Digital Feedback Control

by

Mark Alan Lemkin

**B.S. (Carnegie Mellon University) 1991
M.S. (University of California, Berkeley) 1993**

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I dedicate this thesis to my family.

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1. INTRODUCTION

Development of inertial instruments based on microelectromechanical systems (MEMS) has seen significant progress over the past decade. Commercially available accelerometers, and newly available rate gyroscopes, permit inertial measurement at a fraction of the cost and size of non-MEMS-based sensors. The availability of low-cost, low-power, miniature inertial measurement sensors is expected to generate new applications in automotive, consumer, and industrial markets [1, 2, 3]. Some applications which will be enabled by MEMS-based inertial sensors include package monitoring during shipping, building observation during natural disasters, automobile anti-skid control, attitude stabilization of miniature spacecraft, head-mounted virtual reality systems, activity monitoring for pacemakers, and global positioning system (GPS) augmentation.

Market surveys predict large future growth in the MEMS-based inertial sensor market [1, 2, 3]. According to these surveys, the estimated size of the MEMS-based inertial *system* market will be approximately 30 times the inertial *sensor* market. Given the large value added between sensor and system, it is clear that moving more system level functionality to the sensor will increase value of the assembled part. Recent research has demonstrated fabrication technologies capable of supporting high density electronics with an integrated mechanical sense element [4, 5, 6, 7]. These technologies will allow complex signal processing circuits with mechanical sensors to be fabricated on a single chip.

The objective of this research has been to investigate design of precision accelerometers with a resolution down to one-millionth of the acceleration due to earth's gravity (μG). To enable high-level integration, sensor electronics should be compatible with high density CMOS technologies. A sensor with digital output is also desirable because it eliminates the need for a separate analog-to-digital converter.

While this work concentrates on design of precision accelerometers, the solutions to

fundamental error sources and system topologies developed for micro accelerometers will find direct application in other microelectromechanical systems, including rate gyroscopes [8, 9, 10] and MEMS actuators [11]. The scope of this work extends from high-level system issues down to transistor level design. At each level of abstraction, important design issues are extracted, thereby enabling optimized sensor design.

In organizing this document, all efforts have been made to arrange topics in a manner that would be appropriate for a class on accelerometer design. Organization is from top level down, since it is important to understand system level design issues before delving into subsystem and circuit level design.

Chapter 2 presents the design of the mechanical sense-element. The problem is first broken up into mechanical and electrical components. This division fosters a clearer understanding of factors which affect sensor noise and sensitivity. After this analysis, the coupling of electrical and mechanical energy domains is explored, because this interaction is the means by which acceleration is sensed.

Chapter 3 focuses on modeling and design of a topology for enclosing the sense element in an oversampled force-feedback loop. In addition to force balancing the mechanical sense-element, this topology realizes analog-to-digital conversion of the analog input acceleration to a digital output. The derived model captures important system parameters affecting performance. In addition to enabling rapid simulations, this model is used to predict quantization noise, yielding insight into system level design trade-offs.

Chapter 4 presents an electrical interface for implementation of the system described in Chapters 2 and 3. The completely differential electronics enable use of switched-capacitor design techniques, relaxing requirements on circuit technology. Detailed analyses of interface noise and electrostatic feedback give accurate predictions of fabricated sensor performance. Correlated double sampling is introduced as a method for removal of offset and error arising from the electronics. A digital error cancellation technique permits in-package trimming of the sensor offset.

Chapter 5 describes testing methods and experimental results of two fabricated

accelerometers. Measured performance is compared to calculated performance to validate models developed in Chapters 2 through 4. By varying system parameters, such as clock rate and bias voltages, contributions from different error sources are isolated and compared with theory.

In the final chapter, Chapter 6, the major achievements of this work are summarized and future directions for investigation are outlined.

2. SENSING ELEMENT DESIGN

2.1 OVERVIEW

The heart of an accelerometer is the mechanical proof-mass, or sense-element. Under an applied acceleration, this passive mechanical device moves with respect to the substrate. By measuring the displacement of the proof-mass with an electrical interface, acceleration may be inferred. In this chapter, a mechanical model accounting for parameters relevant to accelerometer design is introduced. Mechanical suspension design is discussed for both in-plane and out-of-plane accelerometers. Results of gas damping analyses are presented, enabling estimation of both the Brownian noise floor and the mechanical Q .

A parallel-plate model is often used for hand calculation of air-gap capacitance, even though this model can result in significant errors. These errors arise from fringing fields around the plate edges. Results of computer simulations are compared with hand calculations for a typical comb-finger capacitor, permitting quantification of error for different aspect ratios. Complete electrical models including parasitic elements are necessary for the sensor analysis presented in Chapter 4. To enable this analysis, a lumped-parameter electrical model of the sense-element and interconnect is introduced and discussed.

The combination of both the electrical and the mechanical models completely characterizes the sense-element. Since it is this coupling between mechanical and electrical energy domains which enables electrical position measurement and electrostatic force feedback, electromechanical coupling mechanisms are discussed. The models obtained in this chapter are used throughout the rest of this document for design and performance analysis.

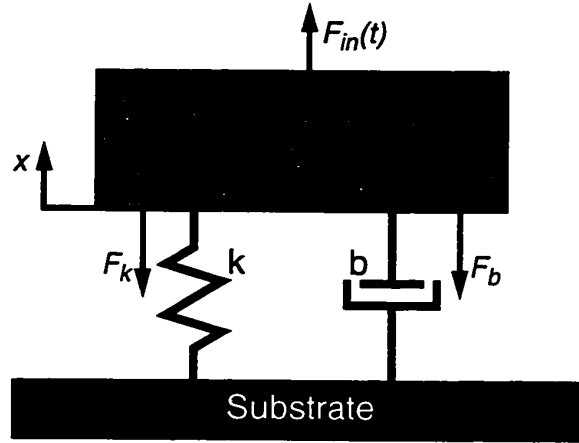


Figure 2-1. Mechanical schematic of proof-mass, spring, damper system used for acceleration measurement.

2.2 MECHANICAL DESIGN AND ANALYSIS

Accelerometry is achieved by transducing proof-mass displacement, resulting from acceleration input, into an electrical signal. The following analysis looks at mechanical design of the sense element, and how system parameters affect mechanical performance.

A schematic diagram of a lumped-parameter proof-mass system is shown in Figure 2-1. The proof-mass is suspended above the substrate by a compliant spring with a linear spring constant k . The spring exerts a restoring force equal to the spring constant multiplied by the displacement x : $F_k = -kx$. Viscous damping is represented by a linear dashpot with damping coefficient b . The dashpot applies a force opposite the direction of motion and proportional to the velocity: $F_b = -b\dot{x}$. Summation of the inertial force $m\ddot{x}$, spring restoring force, and damping force results in the differential equation:

$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = F_{in}(t). \quad (2-1)$$

Since the above equation is linear it may be transformed into the Laplace domain to obtain the following transfer function between substrate acceleration $A(s)$ and displacement $X(s)$:

$$\frac{X(s)}{A(s)} = \frac{1}{s^2 + \frac{b}{m}s + \frac{k}{m}} = \frac{1}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}. \quad (2-2)$$

In (2-2), the natural frequency ω_n has been defined as $\omega_n = \sqrt{k/m}$ and the mechanical Q has been defined as $(\omega_n m)/b$. For frequencies below ω_n , the change in position for an input acceleration a is approximately equal to:

$$\frac{x}{a} = \frac{1}{\omega_n^2}. \quad (2-3)$$

Equation (2-3) shows that to achieve high sensitivity, the natural frequency should be made small. This results in large displacement signals for the position detection circuitry.

In practice, the choice of ω_n calls for a trade-off between sensitivity and resistance to mechanical shock. Since the distance between the proof-mass and the substrate is typically only a few microns, mechanical shocks can cause the proof-mass to touch, or crash into, the substrate. Depending on surface characteristics and contact area, the proof-mass can stick to the substrate. If a voltage difference exists between the proof-mass and substrate, permanent welding of the two may occur.

An upper bound on the proof-mass deflection may be found by assuming all of the kinetic energy is converted into potential energy stored in the spring. The resulting deflection is found to equal $1/\omega_n$ times the velocity upon impact. If the assembled device is dropped on a hard surface from a height d , the velocity upon impact is given by $4.4\sqrt{d}$ m/s. As an example, for a proof-mass suspended $2\mu\text{m}$ above the substrate with a natural frequency of 10 kHz, the critical drop height above which crashing occurs is only 0.8mm.

In practice, the allowable drop height can be somewhat larger than the above calculated value due to package compliance and squeeze-film damping. Squeeze-film damping can be intentionally increased to provide protection from shocks. This is not a practical solution for surface micromachined devices, however, since Brownian noise increases with damping, as will be described in Section 4.5.1. An alternative method of shock protection uses limit stops to minimize contact area, preventing stiction upon contact. In this method, the limit stops are driven to the same potential as the proof-mass to ensure welding does not occur. Another method for obtaining both good sensitivity and good shock resistance uses electrostatic springs to lower the natural frequency during power-on conditions. Upon removing power, the natural frequency returns to a higher

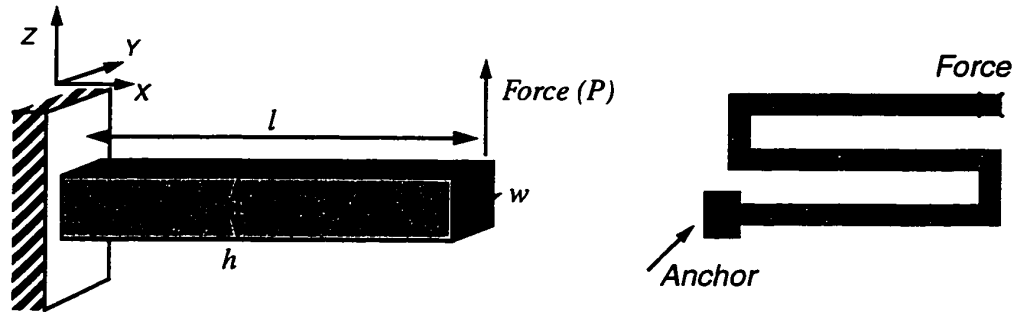


Figure 2-2. Definition of beam dimensions (a - left). Folded crab-leg suspension for realization of a low spring constant (b- right).

value to protect the sensor from mechanical shock.

2.2.1 SPRING DESIGN

In order to fabricate the micro accelerometer from one primary structural layer, mechanical springs are made of the same material as the proof-mass. To design for high accelerometer sensitivity, accurate predictions of the mechanical spring constant are needed. For simple suspensions the linear, or first order, mechanical spring constant may be derived from basic beam bending equations relating deflection to loading. Under typical mechanical deflections experienced by accelerometers, this approximation of linear bending is appropriate.

2.2.1.1 FOLDED CRAB-LEG SPRING

A crab-leg suspension is shown in Figure 2-2 (a). This type of suspension has been used successfully in several z-axis accelerometers [12, 13, 14]. Assuming a quasi-static response, when a force P is applied to the end of the beam in the z-direction, the tip deflects a distance [14]:

$$\delta_z \approx \frac{Pl^3}{12EI}, \quad (2-4)$$

where E is the modulus of elasticity for polysilicon equal to 160 GPa [15], and I is equal to the moment of inertia of the beam: $(wh^3/12)$. This deflection corresponds to a spring constant of:

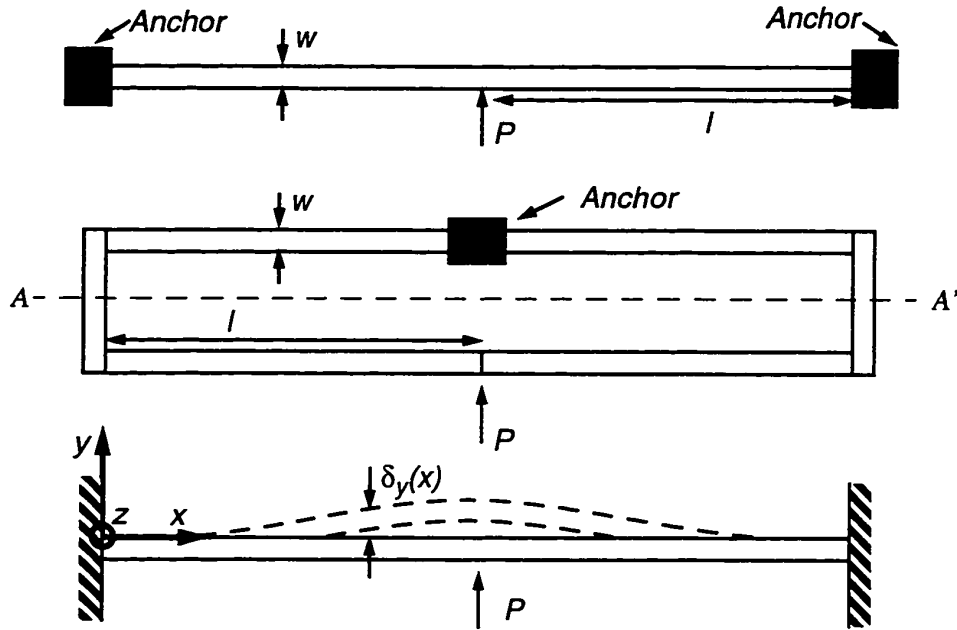


Figure 2-3. Schematic diagram of fixed-fixed suspension (a - top) and closed-end spring (b - middle). Diagram of half-spring used for analysis of spring constant, cut at line of symmetry A - A' (c - bottom).

$$k_z \equiv \frac{P}{\delta_z} = Ew \left(\frac{h}{l} \right)^3. \quad (2-5)$$

For lower compliance, several beams may be placed in series as shown in Figure 2-2 (b). To first order, the spring constant for a beam folded N times is N times smaller than the value predicted by (2-5).

2.2.1.2 CLOSED-END SPRING

A closed-end folded-beam suspension for a lateral accelerometer is shown in Figure 2-3 (b). An advantage of this type of folded-beam suspension over the fixed-fixed suspension of Figure 2-3 (a) is that residual stress from processing simply causes a slight change in the suspension dimensions, instead of gross changes in spring constant [16]. Higher compliance may also be achieved with this topology, resulting in a lower natural frequency, and thus higher sensitivity.

Since the suspension in Figure 2-3 (b) is symmetric about the line A - A', it may be

analyzed one-half at a time about this axis of symmetry. In this analysis, the compliance of the two stubby beams connecting the top and bottom suspensions is assumed to be negligible. For a fixed-fixed beam of length $2l$ with a point load P applied to the center of the beam, the generalized beam-bending equation [17] may be integrated and is equal to:

$$\delta_y(x) = \frac{1}{EI} \left(-\frac{Px^3}{2} + C_2 \frac{x^2}{2} + C_3 x + C_4 \right). \quad (2-6)$$

In this equation, I is the moment of inertia of the beam, equal to $(hw^3/12)$. The boundary conditions for (2-6) are zero angle at the beam ends ($\delta' = 0$), zero angle at the center of the beam (from symmetry), and zero deflection ($\delta = 0$) at $x = 0$. Applying the first three boundary conditions gives $C_3 = 0$ and $C_2 = Pl/4$. By applying the last boundary condition, C_4 is found to be equal to 0. Thus the deflection $\delta(x)$ along the beam is equal to:

$$\delta_y(x) = \frac{P}{EI} \left(-\frac{x^3}{12} + \frac{lx^2}{8} \right). \quad (2-7)$$

To obtain the deflection at the midpoint of the beam, x is set equal to l :

$$\delta_y(x=l) = \frac{Pl^3}{24EI}. \quad (2-8)$$

Since there are two beams in this suspension, the total deflection between the point of application of the force and the anchor is twice this value. Equation (2-8) may be solved for the spring constant in the y-direction:

$$k_y \equiv \frac{P}{2\delta_y(x=l)} = \frac{12EI}{l^3} = Eh \left(\frac{w}{l} \right)^3. \quad (2-9)$$

For z-axis deflections, the closed-end spring is equivalent to two double-fold crab-leg suspensions in parallel. The spring constant of two parallel springs is the sum of their individual spring constants. Therefore, the closed-end spring exhibits a z-axis spring constant of:

$$k_z = \left(\frac{1}{2} Eh \left(\frac{h}{l} \right)^3 \right) + \left(\frac{1}{2} Eh \left(\frac{h}{l} \right)^3 \right) = Eh \left(\frac{h}{l} \right)^3. \quad (2-10)$$

Note that for a square beam $w = h$, equation (2-9) equals (2-10). This result shows that

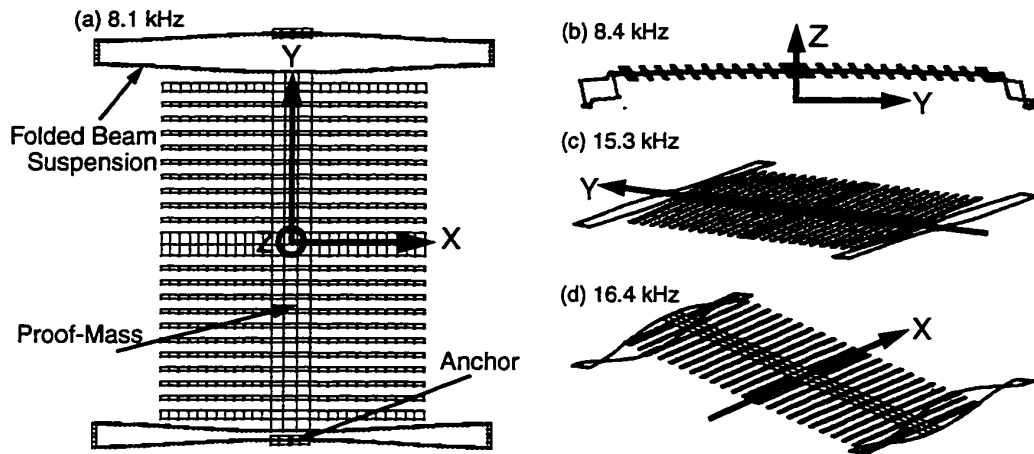


Figure 2-4. Finite element simulation results of the resonant modes for a lateral proof-mass (Y-axis is input axis). The resonant frequencies of the fundamental (a - left), second (b - top right), third (c - middle right) and fourth (d - bottom right) modes are 8.1 kHz, 8.4 kHz, 15.3 kHz, and 16.4 kHz respectively.

structures made from a thin-film will typically have multiple resonant modes spaced closely in frequency. For applications in which only one axis of motion is required, such as a single-axis accelerometer, this can be undesirable because off-axis accelerations cause proof-mass displacement. On the other hand, some rate gyroscopes [10] take advantage of the close spacing of in-plane and out-of-plane modes for measurement of Coriolis acceleration.

Figure 2-4 shows a finite element simulation of the proof-mass for a lateral accelerometer using two closed-end springs for the suspension [18]. The mass of this device is $0.17 \mu\text{gm}$ and the spring dimensions are $l = 180 \mu\text{m}$, $w = 1.7 \mu\text{m}$, $h = 2 \mu\text{m}$. Since the suspension has two identical springs, the total calculated spring constant is twice the value of (2-9). The above parameters yield a calculated resonant frequency of 8.9 kHz. Simulated resonant frequency in the lateral direction (Figure 2-4 (a)) is 8.1 kHz, which is within 10% of the calculated value.

Higher order modes are also shown in Figure 2-4. The z-axis, or trampoline mode, is shown in Figure 2-4 (b). The calculated resonant frequency of this mode using equation (2-10) is 10.5 kHz, which is within 25% of the simulated value of 8.4 kHz. Simulations

show that third- and fourth-order rotational modes occur at approximately twice the fundamental frequency. These modes are of little practical importance, however, because rotational accelerations are typically of lower magnitude than linear accelerations, and are canceled to first order through sense capacitor layout, as will be discussed in Section 2.3.1.

2.2.2 MECHANISMS OF VISCOUS DAMPING

The damping term in equation (2-1) arises from both structural damping and viscous flow of gas within the structural region. Since at typical operating pressures the structural damping component is orders of magnitude lower than gas damping, it will be ignored [15, 19]. The gas damping of MEMS accelerometers can be classified into a component due to Couette flow and a component due to squeeze-film damping. The next two sections present methods to quantify these two types of gas damping, enabling estimation of the Brownian noise floor and mechanical Q.

2.2.2.1 COUETTE-FLOW DAMPING

Couette flow is a viscous flow which occurs when two plates slide parallel to each other under zero pressure gradient. An example of Couette flow may be found between the proof-mass and the substrate during y-axis motion of the proof-mass shown in Figure 2-4. If the gas is assumed to be Newtonian, viscous forces may be described by a parameter μ which provides a linear correspondence between shear stress and velocity gradient. In this case, the viscous damping coefficient of Couette flow may be modeled by:

$$b \equiv \frac{\mu}{t} A, \quad (2-11)$$

where t is the film thickness and A is the plate area. This force is typically an order of magnitude smaller than squeeze-film damping forces, and in many designs may be neglected.

2.2.2.2 SQUEEZE-FILM DAMPING

Squeeze-film damping occurs when the gap between two closely spaced parallel surfaces changes, as shown in Figure 2-5. An example of squeeze film damping may be found between comb fingers during y-axis motion of the proof-mass shown in Figure 2-4.

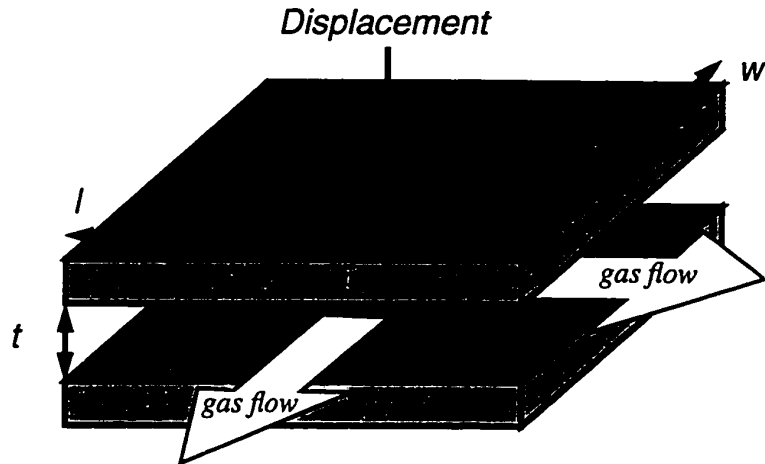


Figure 2-5. Parallel-plate model used for squeeze-film damping analysis.

Another example of squeeze film damping is found between the proof-mass and the substrate during z-axis motion of the accelerometer to be discussed in Section 2.3.2. As these gaps are reduced, gas is forced to flow out of the gap. During flow, viscous forces extract energy from the gas causing damping. Reference [20] models the damping caused by squeezing of thin films in response to perpendicular motion by using the compressible Reynolds gas-film equation [21]. In this model, a squeeze number σ , representing the compressibility of the squeeze film, is first defined:

$$\sigma \equiv \frac{12\mu l^2}{Pt^2} \omega, \quad (2-12)$$

where μ is the viscosity, l is the length of the rectangular plate, P is the atmospheric pressure, t is the separation distance and ω is the excitation frequency. For large plates and thin gaps (i.e., $\sigma \gg 0$), the gas acts as a compressible fluid since viscous effects at the plate edges trap air between the plates. Small plates and thick gaps (i.e., $\sigma \approx 0$) permit lateral motion of the gas, so the gas approximates incompressible viscous flow.

To estimate damping, a geometry-dependent parameter β is defined: $\beta \equiv w/l$. The non-dimensional damping force given in terms of the squeeze number and plate geometry is [20]:

$$f = \frac{64\sigma}{\pi^6} \left(\frac{1 + (1/\beta)^2}{[1 + (1/\beta)^2]^2 + \sigma^2/\pi^4} \right). \quad (2-13)$$

To determine the actual damping force, equation (2-13) must be multiplied by the plate area and atmospheric pressure. Since the linear damping coefficient in equation (2-1) is defined as the damping force normalized by angular frequency, the damping coefficient is given by:

$$b = \frac{l^2 \beta P 64 \sigma}{\omega t \pi^6} \left(\frac{1 + (1/\beta)^2}{[1 + (1/\beta)^2]^2 + \sigma^2/\pi^4} \right). \quad (2-14)$$

At low frequencies, the oscillation time is long enough such that the squeeze number is small and the gas may be modeled as incompressible [22]. In this case, equation (2-14) reduces to:

$$b = \frac{l^2 \beta P 64 \sigma}{\omega t \pi^6} \left(\frac{1 + (1/\beta)^2}{[1 + (1/\beta)^2]^2} \right). \quad (2-15)$$

For a square plate $w = l$, $\beta = l$, and equation (2-15) further reduces to:

$$b = \frac{l^2 \beta P 64 \sigma}{\omega t \pi^6} \frac{1}{2} = 0.4 \mu \frac{l^4}{t^3}. \quad (2-16)$$

Equations (2-15) and (2-16) accurately describe damping between parallel plates when the plate dimensions are much greater than the air gap. For situations where this condition is not satisfied, these results are inaccurate because equation (2-13) assumes gas pressure falls to ambient levels at the plate edges [22]. For narrow widths, edge effects represent a significant percentage of the total damping. In these cases, such as comb finger gaps, a more accurate estimate of damping may be found by assuming Hagen-Poiseuille flow [23]. The damping coefficient assuming Hagen-Poiseuille flow between a single comb finger gap is given by:

$$b \approx 7.2 \mu w \left(\frac{l}{t} \right)^3 \quad (2-17)$$

for a typical surface micromachined air-gap geometry. Reference [23] compares measured and calculated results of damping for the surface micromachined accelerometer described in [16]. The error between measured and calculated values was within 15% for this device,

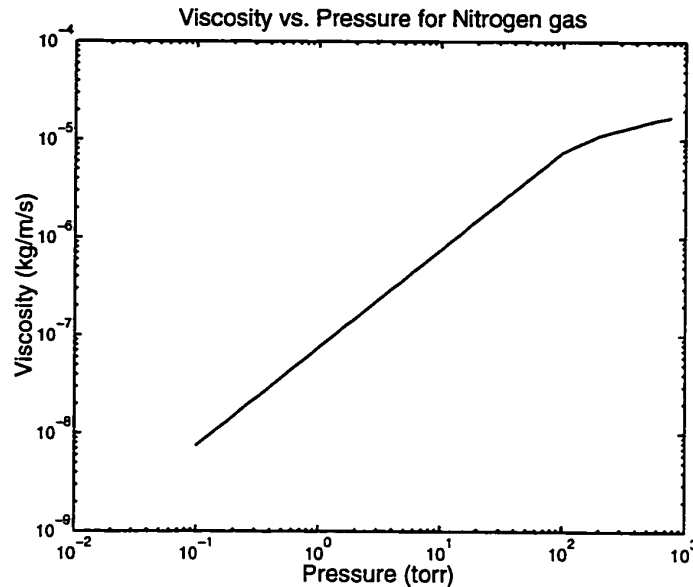


Figure 2-6. *Effective viscosity vs. pressure for a 2 μm -thick nitrogen film [29]. The effective viscosity of air is 1.56×10^{-5} kg/m/s at 760 torr and 20 C.*

verifying the accuracy of equation (2-17). Note that scale factor is the only difference between the damping predicted using the Reynolds gas-film equation and the damping predicted assuming Hagen-Poiseuille flow. Both equations (2-16) and (2-17) show that damping is linearly dependent on both gas viscosity and plate width, and that it scales with the cube of the ratio of the plate length to gap thickness.

Direct application of equations (2-15) and (2-16) to micromachined plates, such as z-axis accelerometers, will result in gross errors due to the presence of etch holes. In fact, etch holes can reduce damping by several orders of magnitude [24]. To obtain accurate predictions of damping with etch holes, the proof-mass is broken up into a collection of smaller plates acting in parallel, with the size of each plate equal to the distance between etch holes [25]. The total damping is the sum of damping from each of the individual plates, which may be calculated using equation (2-15) or (2-16).

2.2.2.3 VISCOSITY OF GASSES

Unlike in a liquid, where viscosity results from cohesion between adjacent molecules,

gas viscosity results from intermolecular collisions within the gas [26]. Since the molecular collision rate is dependent on temperature, gas viscosity increases as temperature is increased, but is unaffected by changes in pressure. Because micromachined sensors have system dimensions on the same order as the mean free path of gas molecules, intermolecular collisions rarely occur within the gas film at very low pressures [22]. In these cases, momentum is transferred between the gas molecules and the plates, yielding a pressure dependent *effective viscosity* μ_{eff} . This effective viscosity is used in place of the true viscosity μ for calculations. As an illustration, in Figure 2-6 the effective viscosity for a 2 μ m-thick nitrogen film is plotted against pressure. As this plot shows, the effective viscosity remains approximately constant near atmospheric pressure, but drops proportional to pressure at low pressures. For example, the effective viscosity drops from 7.4×10^{-6} kg/m/s to 0.74×10^{-6} kg/m/s as the pressure drops from 130 torr to 13 torr. Thus, while the true viscosity of a gas is independent of pressure, gas damping is not. Vacuum packaging may be used to take advantage of reduced damping to lower mechanical noise as will be described in Section 4.5.1.

2.3 ELECTRICAL DESIGN AND ANALYSIS

In this section, sensor layout of both lateral and z-axis accelerometers is described. To facilitate analysis of the electrical interface, discussed in Chapter 4, lumped-parameter models of parasitic capacitances and parasitic resistances are obtained. Basic electromechanical relationships are presented for electromechanical sensitivity, electrostatic forcing, and electrical spring softening, thus laying the groundwork for analysis of the complete sensor in Chapters 3 and 4.

2.3.1 LATERAL ACCELEROMETER

A schematic diagram of a proof-mass suitable for lateral acceleration measurement is shown in Figure 2-7 (a) [16, 18]. The proof-mass is suspended above the substrate by two closed-end springs, discussed in Section 2.2.1. A differential comb-finger array provides two nominally equal-sized capacitors, both of which have the proof-mass as a common node. Typical surface micromachined comb finger dimensions are on the order of

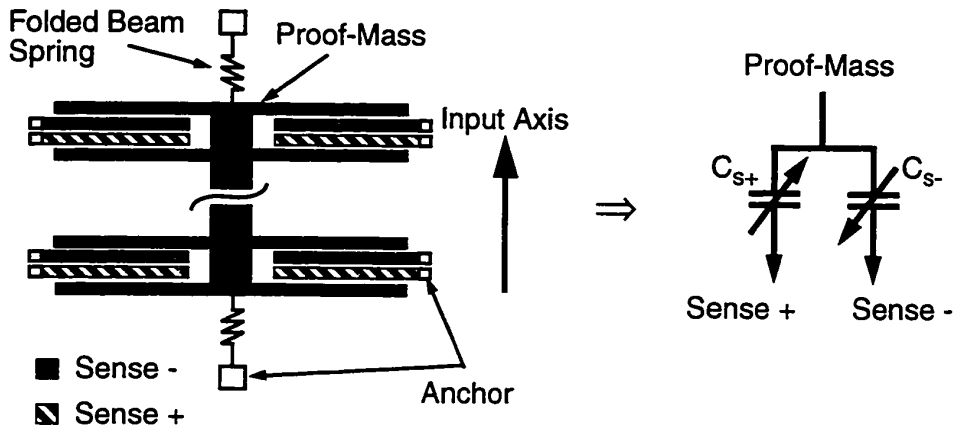


Figure 2-7. Lateral accelerometer capacitor layout (a - left) and equivalent schematic diagram (b - right). The proof-mass is a common node for the capacitive half-bridge formed between C_{s+} and C_{s-} .

$150\ \mu\text{m} \times 2\ \mu\text{m} \times 2\ \mu\text{m}$, with a $1\ \mu\text{m}$ air gap between fingers. An array of 100 fingers would result in a total air-gap capacitance of approximately 270 fF. Distributing the comb fingers around the proof-mass in four symmetric quadrants, creates a common-centroid layout. Common centroid layout achieves first order cancellation of processing gradients, reducing capacitor mismatch.

A simplified electrical equivalent of the proof-mass is shown in Figure 2-7 (b). Movement of the proof-mass in the positive direction causes air gaps to decrease between the comb fingers which make up capacitor C_{s+} , raising this capacitance. At the same time, air-gaps between comb fingers comprising the capacitor C_{s-} increase, lowering this capacitance.

The output of the sense element is taken as the imbalance, or capacitance difference, of the half-bridge formed by the series combination of C_{s-} and C_{s+} . Measurement of this capacitance difference will be discussed in detail in Section 4.2. Note that off-axis accelerations cause equal changes in both C_{s+} and C_{s-} due to the symmetry of the device. Since only the difference in capacitance is measured by the sense interface, off-axis accelerations are canceled to first order.

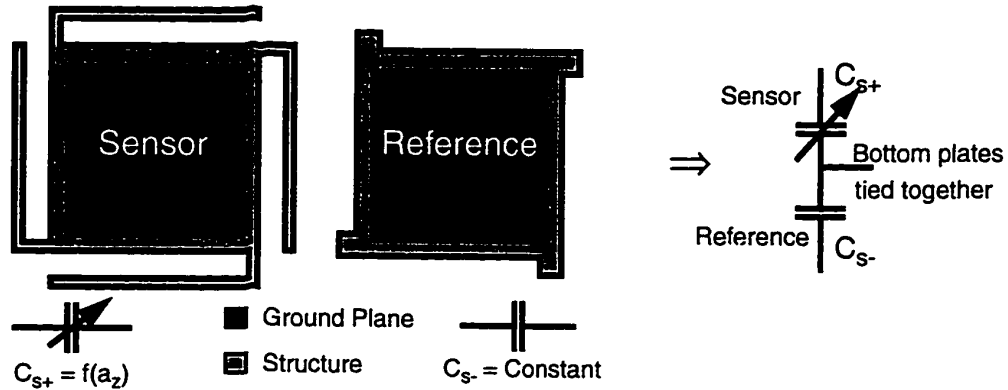


Figure 2-8. Z-axis (out-of-plane) accelerometer layout (a - left) and equivalent schematic diagram (b - right). A half-bridge is formed with a movable proof-mass and a stiffly anchored reference capacitor.

2.3.2 Z-AXIS ACCELEROMETER

A schematic diagram of a proof-mass suitable for out-of-plane, or z-axis, acceleration measurement is shown in Figure 2-8 (a) [12]. Suspension of the proof-mass above the substrate is accomplished using four folded-beam suspensions, discussed in Section 2.2.1. The sense capacitor is formed by the suspended proof-mass and a layer of ground-plane polysilicon located below the proof-mass. To enable use of a differential position measurement interface, a stiffly anchored reference capacitor is fabricated beside the proof-mass. Joining together the bottom plates of both capacitors creates a half-bridge configuration. The reference capacitor is made from the same material as the proof-mass to achieve good matching between the reference and sense capacitors, thereby minimizing systematic offset. A typical proof-mass measuring $200\ \mu\text{m}$ on a side and suspended $2\ \mu\text{m}$ above the substrate attains a sense capacitance of 175 fF.

When the substrate undergoes a z-axis acceleration, the air-gap between the proof-mass and substrate changes value. The difference in gap causes an imbalance in the half bridge. As is the case with the lateral accelerometer, the output of the sense element is taken as the imbalance, or capacitance difference, of the half-bridge.

An alternative configuration for measurement of z-axis accelerations in a single layer of structural polysilicon is discussed in [28]. This sensor measures acceleration using two

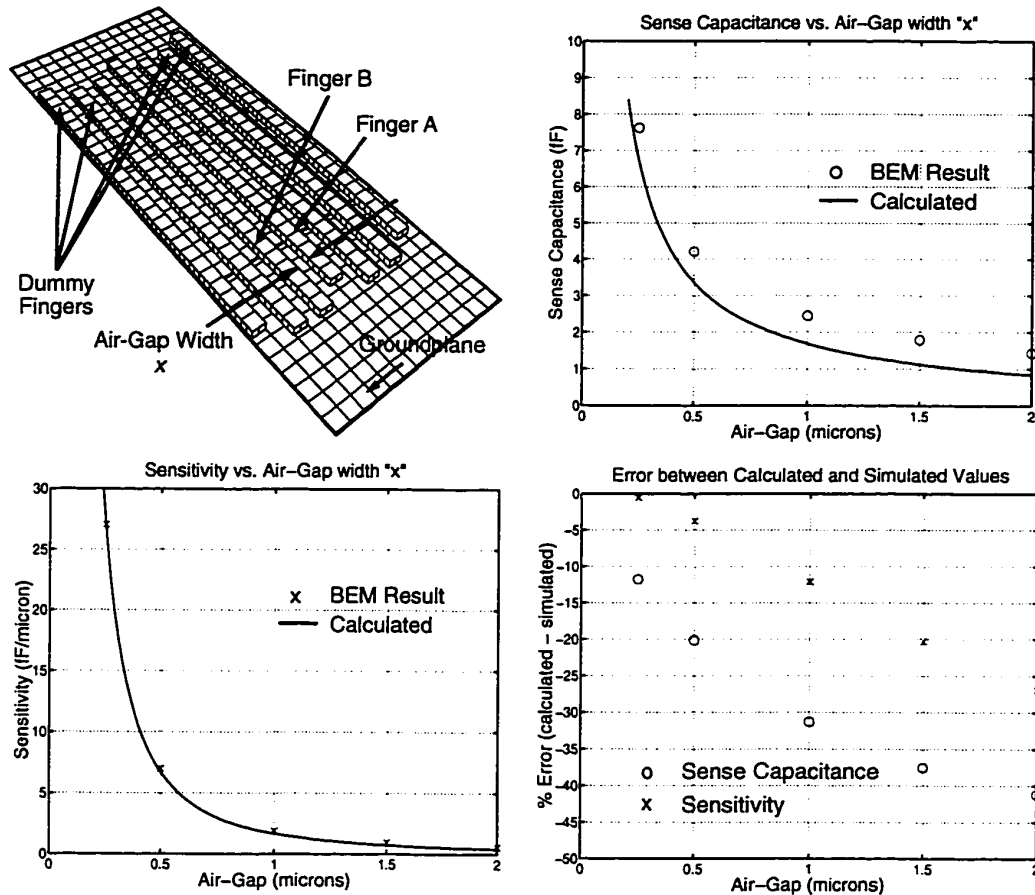


Figure 2-9. FASTCAP [27] mesh used for comb-finger simulations (a- upper right). Calculated and simulated sense capacitance (b - upper right) and sensitivity (c - lower left). Error vs. gap width assuming a parallel-plate model is plotted in (d - lower right).

torsional microstructures, similar to playground see-saws, with a different mass on each side. Capacitive pick-off using identically sized areas of ground-plane polysilicon under both sides of each microstructure cancel rotational acceleration inputs, and yield a truly differential output. Because this sense-element is entirely symmetric, it would be expected to result in significantly less systematic offset than the accelerometer shown in Figure 2-8.

2.3.3 AIR-GAP CAPACITORS

Since air-gap capacitors are the method by which mechanical displacements are detected and mechanical feedback forces applied, it is important to determine precisely their magnitude and their sensitivity to changes in gap spacing. For two large parallel

plates separated by a distance x with an overlap area A , the capacitance is given by:

$$C \approx \frac{\epsilon A}{x}, \quad (2-18)$$

where ϵ is the permittivity of the dielectric between the two plates. In air, ϵ is approximately equal to the permittivity of free space: $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$.

Unlike the oxide capacitors used in the electronic interface, the thickness of the air-gap between comb fingers used for sensing and force feedback can be comparable to the thickness of the thin film used to make the mechanical structures. In these cases, fringing of electrostatic field lines can cause substantial underestimation of the total sense capacitance when equation (2-18) is used.

To quantify this error, hand calculations of the air-gap capacitance between comb fingers A and B (shown in Figure 2-9 (a)) are compared with computer simulations in Figure 2-9 (b). The comb fingers measure $100\mu\text{m} \times 4\mu\text{m} \times 2\mu\text{m}$, and are suspended $2\mu\text{m}$ above the substrate. By including dummy fingers, shown in Figure 2-9 (a), the effect of fringing fields to adjacent fingers is simulated. As shown in Figure 2-9 (d), the error between hand calculated and simulated sense capacitance increases as the air-gap width approaches the mechanical film thickness. Error increases because a larger percentage of the electric field lines are composed of fringing fields.

The derivative of capacitance with respect to gap width is another important parameter in electrical interface design and performance estimation. The derivative determines sensitivity, to be discussed in Section 2.3.5, as well as feedback force, to be discussed in Section 2.3.6. Again ignoring fringing fields, the sensitivity to changes in gap width x at a gap separation of x_0 is estimated by the derivative of (2-18):

$$\left. \frac{dC}{dx} \right|_{x=x_0} = \left. \frac{d}{dx} \left(\frac{\epsilon A}{x} \right) \right|_{x=x_0} = -\frac{\epsilon A}{x_0^2} = -\frac{C_0}{x_0}. \quad (2-19)$$

The results of hand calculations of dC/dx for a single comb finger are compared with FASTCAP simulations in Figure 2-9 (c). Error between simulated results and hand-calculated values are plotted in Figure 2-9 (d). This plot shows that the calculated

sensitivity is more accurate than the calculated sense capacitance for all values of gap width. An intuitive explanation for this result is that while the external fringing field lines add significant capacitance, they have a longer path length than straight field lines between the fingers. Thus, the change in capacitance due to the change in path length of the fringing field lines adds significantly less to dC/dx than the change in capacitance from the straight field lines.

The plot of error vs. gap, Figure 2-9 (d), shows that hand calculated values give excellent estimates of both sense capacitance and sensitivity when the air-gap width is at least four times less than the finger height. As these dimensions become closer in magnitude both capacitance and sensitivity are underestimated, with errors approaching 45% and 30% as the gap width becomes equal to the film thickness. For fingers with such a small aspect ratio, computer simulations must be used for accurate determination of sense capacitance and sensitivity.

2.3.4 PARASITIC RESISTANCES AND CAPACITANCES

To design an electrical position sense interface and accurately predict performance, it is necessary to have a model which completely describes relevant electrical behavior. Figure 2-10 shows a lumped-parameter model of the lateral sense-element consisting of sense capacitors, parasitic resistors, and parasitic capacitors. The air-gap sense capacitors are represented by $C_{s+,-}$ and, as discussed in the previous section, are approximately given by:

$$C_{s+,-} = N \frac{\epsilon_0 l w}{t}, \quad (2-20)$$

where N is the number of comb fingers and ϵ_0 is the permittivity of free space equal to $8.85 \times 10^{-14} \text{ F/m}$.

Parasitic capacitances shown in Figure 2-10 increase both settling time and input-referred electrical noise of the position-sense circuitry, as will be discussed in Section 4.5.2. The capacitance C_{ab} represents parasitic capacitance from the positive stationary comb fingers to the negative stationary comb fingers, and is typically of the

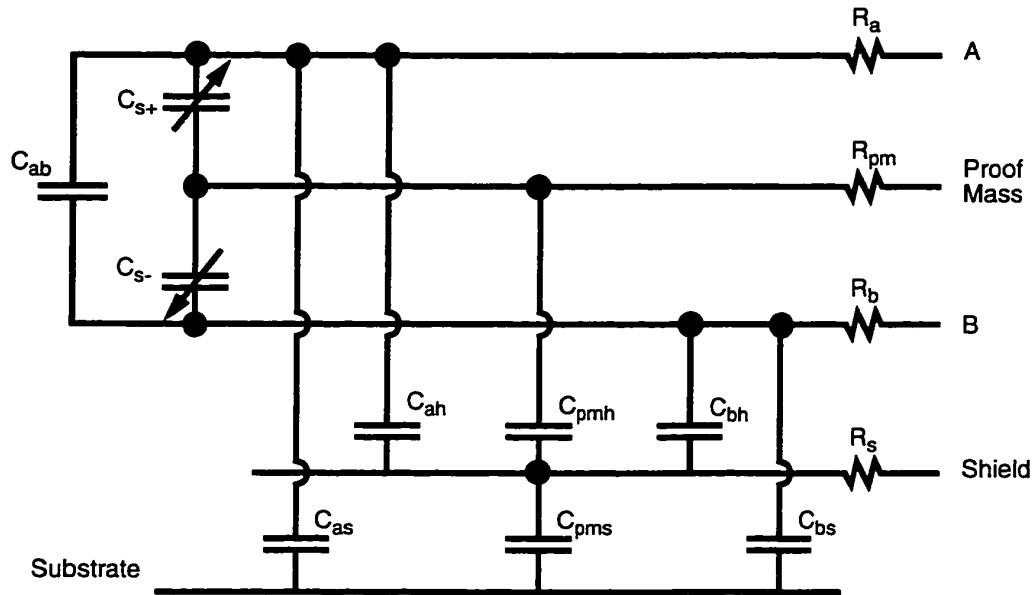


Figure 2-10. Lumped capacitance and resistance model of lateral sense-element.

same order as C_s . Capacitors C_{ah} , C_{bh} , and C_{pmh} are parasitic air-gap capacitors formed by the ground-plane shield and positive stationary comb fingers, negative stationary comb fingers, and the proof-mass respectively. The magnitude of all three of these capacitors is also typically of the same order as C_s . Capacitance between the substrate and ground-plane polysilicon, used for interconnect between the comb fingers, is represented by C_{as} and C_{bs} . These capacitances are typically on the order of 0.5 pF in an integrated technology.

The capacitor C_{pms} represents parasitic capacitance between the ground plane shielding and the substrate. Since the entire proof-mass must be shielded from the substrate to prevent electrostatic pull-down, this capacitance is typically on the order of 50 pF. While this capacitance does not directly increase noise, it does require the use of a low impedance voltage source in order to keep the shield voltage equal to the proof-mass voltage.

Parasitic resistors in series with the stationary comb-fingers, R_a and R_b , result from the finite conductivity of structural materials and ground-plane wiring. These resistors add noise to the position-sense circuitry and reduce speed of the electrical interface, as will be

discussed in Section 4.5.2.2. Typical values of these resistors in a polysilicon MEMS technology are about 1K. In-line resistors R_s and R_{pm} are common-mode noise generators, and thus do not contribute noise to the sense interface to first order. These resistors, in conjunction with C_{pms} , set the settling time of the center node of the capacitive half-bridge, limiting the speed of the position sense interface.

2.3.5 ELECTROMECHANICAL SENSITIVITY

Since the position sense interface detects changes in capacitance, it is desirable to refer electrical signals, in the form of capacitance changes, to an equivalent input acceleration. For signals below the natural frequency of the mechanical sense element, a differential input acceleration a results in a differential displacement x equal to $(1/\omega_n^2)a$. For small displacements, the change in capacitance may be found by combining this result with equation (2-19) to yield:

$$\frac{dC_s}{da} = \frac{1}{\omega_n^2} \frac{C_s}{x_0} \quad \frac{F}{m/s^2}, \quad (2-21)$$

where C_s is the nominal sense capacitance, and x_0 is the nominal air gap. When two capacitors are connected in a capacitive-half bridge configuration, the total difference in capacitance is twice this number. By multiplying equation (2-21) by 9.8, the relationship between change in capacitance and input acceleration in G may be found, where $1 G$ is defined as the acceleration due to earth's gravitational field and is equal to $9.8 m/s^2$. Chapter 4 uses this relationship to predict both sensitivity and noise floor.

2.3.6 ELECTROSTATIC FORCING

When an electrical potential is applied to two plates of a capacitor C separated by a distance x , a potential field is created. The energy stored in the capacitor C is given by:

$$E(x) = \frac{1}{2} QV = \frac{1}{2} C(x) V^2. \quad (2-22)$$

Mechanical work is performed when a force F is applied over a distance d :

$$W = Fd. \quad (2-23)$$

To determine the force of attraction of two parallel plates, the energy difference between two points along the potential field is first found:

$$\Delta E = \frac{1}{2} V^2 (C(x_2) - C(x_1)). \quad (2-24)$$

A force must be applied to move the plates from position 1 to position 2. The work done by this force must equal the change in energy of equation (2-24):

$$F = \frac{1}{2} V^2 \frac{C(x_2) - C(x_1)}{x_2 - x_1}. \quad (2-25)$$

By taking the limit as $x_2 - x_1$ approaches zero we find:

$$F = \frac{1}{2} \frac{dC}{dx} V^2. \quad (2-26)$$

For parallel plates, the derivative of the capacitance with respect to gap width is given by equation (2-19), and thus the force at a given position x and voltage V may be rewritten as:

$$F(x) = -\frac{1}{2} \frac{\epsilon A}{x^2} V^2 = -\frac{1}{2} \frac{C(x)}{x} V^2, \quad (2-27)$$

where the negative sign results from the force being attractive.

2.3.7 ELECTROSTATIC SPRING SOFTENING

2.3.7.1 SINGLE PARALLEL-PLATE CAPACITOR

As described in the previous section, application of a voltage V across the terminals of a single parallel-plate capacitor generates an attractive force. As the plates are moved closer together, the magnitude of this attractive force increases. This effect may be thought of as an electrostatic spring with a *negative* spring constant k_e . Since the resonant frequency is dependent on the sum of the electrostatic and mechanical spring constants, $k_{tot} = k_e + k_m$, the resonant frequency of the proof-mass is lowered under an applied electric field.

Because a linear spring constant k is defined as the negative of the derivative of force over distance, $k = -dF/dx$, the electrical spring constant may be found by differentiating

equation (2-27) with respect to position:

$$k_e = \frac{dF}{dx} = -\frac{d}{dx}\left(-\frac{1}{2}\frac{\epsilon A}{x^2}V^2\right) = -\frac{\epsilon A}{x^3}V^2 = -\frac{C(x)}{x^2}V^2. \quad (2-28)$$

When the electrical spring becomes comparable to the mechanical spring, the resonant frequency starts to drop rapidly with small increases in plate voltage. The snap-in voltage, or voltage at which the resonant frequency drops to zero, occurs when the electrical spring constant is equal to the mechanical spring constant. Note that this does not occur at the power off position, x_0 , since the gap is reduced as the plate voltage is increased. The position at which snap-in occurs may be found by equating (2-27) with the mechanical restoring force:

$$k_m(x_0 - x) = -\frac{1}{2}\frac{\epsilon A}{x^2}V^2. \quad (2-29)$$

From (2-29), the equilibrium voltage at a position x is given by:

$$V(x) = \sqrt{\frac{2k_m(x_0 - x)x^2}{\epsilon A}}. \quad (2-30)$$

Differentiating voltage with respect to position and setting the result to zero, equation (2-30) is found to have a maximum at $(2/3)x_0$:

$$V_{CritPlate} = \sqrt{\frac{8}{27}\frac{k_mx_0^3}{\epsilon A}} = \sqrt{\frac{8}{27}\frac{k_mx_0^2}{C_0}}, \quad (2-31)$$

where C_0 is the capacitance with zero deflection.

For gaps smaller than $(2/3)x_0$, the decreasing gap increases electrostatic force faster than the mechanical restoring force, and thus smaller plate voltages are needed for equilibrium. Substituting equation (2-31) in the equation for the electrostatic spring, k_e is found to equal $-k_m$, which implies instability.

2.3.7.2 DIFFERENTIAL COMB-FINGER CAPACITOR

A differential comb-finger array has a slightly different effect than a single parallel-plate capacitor. Since there are two sets of electrical springs, the total electrical

spring constant is the sum of these two springs:

$$k_e = -\left(\frac{\epsilon A_+}{x_+^3} V_+^2 + \frac{\epsilon A_-}{x_-^3} V_-^2\right), \quad (2-32)$$

where the + and - subscripts refer to the two different capacitors. Assuming the same voltage is applied across both sets of capacitors, the electrical spring constant is given by:

$$k_e = -2 \frac{\epsilon A}{x^3} V^2. \quad (2-33)$$

Since the capacitors are symmetric about the proof-mass, there will be no deflection under an applied bias voltage V . Therefore, the equilibrium gap spacing, x , is equal to the initial gap, x_0 , and equation (2-33) may be rewritten as:

$$k_e = -2 \frac{C_0}{x_0^2} V^2, \quad (2-34)$$

where C_0 is the capacitance of *each* sense capacitor under zero deflection. Setting the mechanical spring constant equal to the electrical spring constant, the critical voltage for a differential capacitor array is found to be:

$$V_{CritComb} = \sqrt{\frac{1}{2} \frac{k_m x_0^3}{\epsilon A}} = \sqrt{\frac{1}{2} \frac{k_m x_0^2}{C_0}}. \quad (2-35)$$

Since the proof-mass remains centered, equation (2-35) is 1.3 times higher than the result for the single parallel-plate capacitor.

Note the *total* capacitance of the differential comb-finger array is twice that of the single parallel-plate capacitor: two capacitors of equal value, C_0 , make up the pair. In conjunction with a higher V_{crit} , the larger capacitance of the differential array enables a sensitivity 2.6 times that of the single-plate capacitor.

2.4 SUMMARY

This section has presented methods of analysis suitable for designing an accelerometer sense-element. Mechanical design issues including spring compliance and estimation of

mechanical damping coefficient have been discussed. A combination of both simulated and experimental results have shown the error of these elementary hand calculations to be typically less than 25%. Mechanical topologies of sense-elements have been presented for sensing acceleration in the x-, y-, and z- axes.

Calculated values of capacitance and sensitivity using a parallel plate model agree well with actual capacitance for aspect ratios greater than four. Because parasitic capacitances and resistances have first order effects on performance, sources of parasitics have been described, and a lumped-parameter model presented. Finally, electro-mechanical coupling of the mass-spring-damper system with the sense capacitors has been characterized. It is this coupling which enables electrical measurement of position and electrostatic force feedback for position nulling.

3. FEEDBACK LOOP ANALYSIS AND DESIGN

3.1 INTRODUCTION

Force balancing of the sense-element is attained by enclosing the proof-mass in a negative feedback loop. The feedback loop measures deviations of the proof-mass, and applies a force in the opposite direction to keep the proof mass centered. Output from the accelerometer is taken as the force needed to null, or zero, the position. By maintaining small deflections, nonlinearities from the capacitive pick-off and mechanical springs are minimized. Because the output is dependent only on the feedback force, variations in the spring constant do not affect the output. In addition, a high-bandwidth feedback loop can extend the sensor bandwidth beyond the natural frequency of the proof-mass.

As discussed in Section 2.2, high sensitivity requires a low natural frequency. Since a low natural frequency yields large proof-mass displacements in response to input accelerations, the electronic interface can saturate for even small inputs. For example, suppose the natural frequency of the proof-mass shown in Figure 2-4 was reduced to 100 Hz. If the accelerometer were operated open-loop, the output voltage of the position sense interface would be approximately 20 Volts for a 1 G input. Thus, in a 5 Volt technology, the maximum input signal would be limited to only 0.25 G. Since an ideal feedback network does not affect noise, enclosure of the proof-mass in a feedback loop maintains the lower noise floor, while increasing the full scale of the device. For typical surface micromachined MEMS technologies, feedback can extend the full scale to several hundred Gs.

Although force balancing can significantly raise linearity, bandwidth, and dynamic range, there are some applications where feedback is impractical or undesirable. For instance, high-G accelerometers, used for shock and impact measurement, have input accelerations in the thousands of Gs. Because it is not possible to attain the forces needed to balance these accelerations, a feedback loop would saturate. Another example is low-cost applications. Since many applications are highly cost sensitive, the increased

system complexity of a force balance loop is often prohibitive.

While there are many ways of implementing a feedback loop, sigma-delta ($\Sigma\Delta$) modulation is particularly attractive because it is simple, provides a digital output, and can be easily implemented in high density CMOS technologies, enabling co-fabrication of digital signal processing circuitry [14, 30]. In this topology, the proof-mass is enclosed in a negative feedback loop, where it provides the filtering necessary for a $\Sigma\Delta$ modulator.

The sigma-delta loop may be operated as a sampled-data system by time-multiplexing position sensing and force feedback operations. Time multiplexion requires careful attention to timing since the proof-mass is governed by continuous-time dynamics. As will be shown in this chapter, the time from when the position sense circuitry is sampled to the center of the feedback pulse is critical in determining loop delay, and thus stability. Similarly, the feedback pulse duration has a first order effect on the accelerometer full scale.

Because the proof-mass is a second order system, compensation of the feedback loop is required. In $\Sigma\Delta$ voltage converters, compensation is typically implemented using multiple feedback or feedforward loops, which add zeros to the loop transfer function [31]. In a mechanical system, the only accessible input is force: there is no method for direct application of a velocity to a mass. Thus alternative compensation methods must be found. In this chapter, several possible methods of compensation are presented and discussed.

To prevent the proof-mass from crashing into the substrate, resonant frequencies are typically designed in the kHz range. The large bandwidth of the proof-mass reduces noise shaping at lower frequencies. The result is a larger in-band quantization noise than exists in typical electrical sigma-delta converters, where filter bandwidths can be on the order of a few Hz. In addition to reduced noise shaping, the high resonant frequency can cause a dead zone [31, 32]. This is not a problem in typical surface micromachined technologies, however, since electrical and mechanical broadband noise sources act as a dither source.

In Section 3.2, basic principles of $\Sigma\Delta$ converters are outlined and methods for

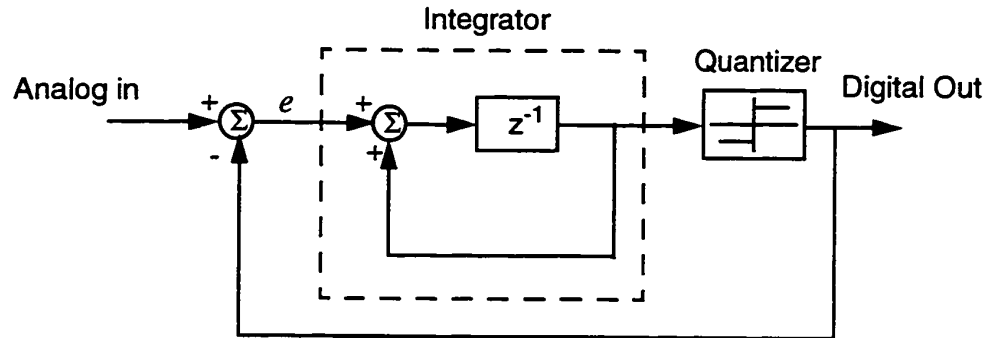


Figure 3-1. Block diagram of a first order $\Sigma\Delta$ A/D converter.

compensation of the feedback loop are introduced. Section 3.3 provides a detailed analysis of the sampled-data system, deriving an exact discrete-time model for simulation and analysis. The effect of compensation on quantization noise and stability are discussed in Section 3.4. With the results from Section 3.3, the correct amount of compensation for different amounts of loop delay are determined. Finally, in Section 3.5 analytical results are compared with simulations for verification of the derived models.

3.2 SIGMA-DELTA CONVERSION

A block diagram of a first order $\Sigma\Delta$ converter is shown in Figure 3-1. A one-bit quantizer is used for analog-to-digital (A/D) conversion of the integrator output. The one-bit signal is fed back to the summing node at the input. By taking the output of the modulator as this one-bit signal, analog-to-digital conversion is achieved. When an input signal is applied to the summing junction, the one-bit digital feedback is subtracted from this input and the resulting error integrated. Assuming the feedback loop is properly compensated, the negative feedback drives the average error e over many periods to zero, causing the average output to track the input. The feedback loop is operated at a sampling rate f_s much faster than the Nyquist rate of the input signal f_N , enabling a moving average of the output to be constructed at the Nyquist rate. By averaging the digital output, the one-bit data stream is converted to a multi-bit digital signal at a lower bandwidth. This ability to trade speed for accuracy is the basis by which $\Sigma\Delta$ converters operate.

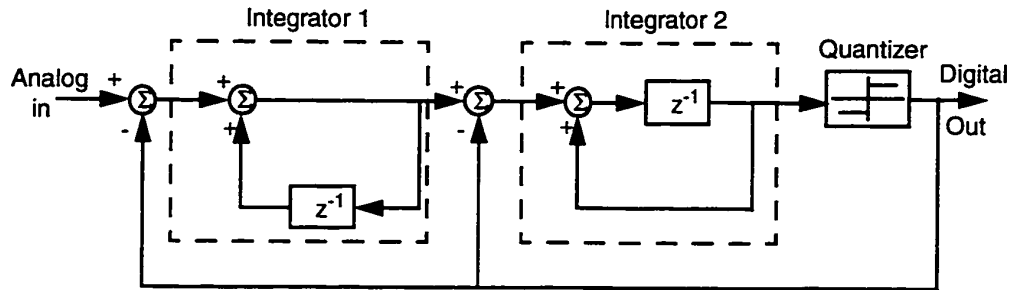


Figure 3-2. Block diagram of a second-order $\Sigma\Delta$ A/D converter.

Although the system is highly nonlinear, due to the one-bit quantizer, linear system analyses provide useful insights into factors affecting converter performance. For these analyses, quantization error is typically approximated a uniformly distributed, additive white noise source [33]. The variance of a uniformly distributed random variable over the interval $[-\Delta/2, \Delta/2]$ is given by:

$$q_n^2 = \frac{\Delta^2}{12}. \quad (3-1)$$

In equation (3-1), $\pm \Delta/2$ represents the full scale feedback value. Since this white noise is added after the integrator, it appears in the output shaped by the inverse of the forward path transfer function. By placing a low-pass filter in the forward path, the output noise is differentiated; thus low frequency noise is moved to higher frequencies. Filtering the output removes unwanted high frequency noise, permitting downsampling of the converter output to the Nyquist rate of the input signal.

A second order $\Sigma\Delta$ converter is shown in Figure 3-2. This topology uses two forward path integrators to achieve higher quantization noise shaping for a given oversampling ratio $M \equiv f_s/f_N$ [31]. Note the output is fed back to both first and second integrators to compensate the system.

A mechanical mass may be used for double integration in a $\Sigma\Delta$ converter. In this case, the input acceleration is integrated twice to position. Unfortunately it is impossible to directly obtain the stabilizing inner feedback loop shown in Figure 3-2 because there is no

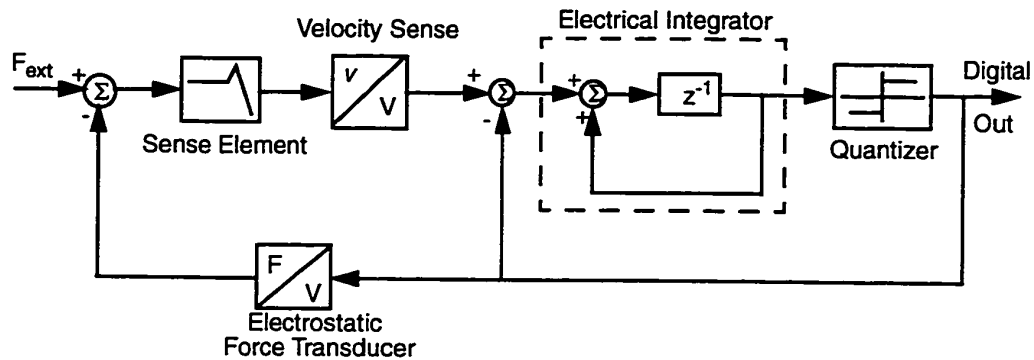


Figure 3-3. Stable second-order $\Sigma\Delta$ converter system using velocity sensing. Since position is unobservable, the proof-mass wanders from the zero position.

way to directly input a velocity to a mechanical system. Several possibilities exist for overcoming this problem, yielding a properly compensated $\Sigma\Delta$ loop.

Reference [34] uses an overdamped bulk-micromachined sense element to split the two mechanical poles to high and low frequencies. By operating the feedback loop below the high-frequency pole, first-order filtering is realized. This method is not applicable to surface-micromachined sensors due to the increase of Brownian noise in an overdamped sensor. Furthermore this method has limited usefulness for large oversampling ratios, since operation significantly above the second pole will result in instability.

Instead of measuring position, proof-mass velocity may be estimated by measuring the difference in position over a constant time period. By taking velocity as the output from the mechanical system a zero is added to the forward path resulting in a first order system. Addition of a purely electrical integrator after the mechanical sense element enables feedback to the inner loop as shown in Figure 3-3. This topology requires precise matching of the inner and outer feedback loop gains, which unfortunately are set by two different physical mechanisms. Furthermore, since position is not directly measured the proof-mass has a tendency to wander from its nominal position; thus offset in velocity sensing circuitry can cause the proof-mass to be driven out of range.

A third method of compensation, discussed in detail here, uses a discrete-time FIR lead filter to provide the phase lead necessary for stable operation. By using a lead filter

positive phase may be added only where needed, thereby keeping a majority of the second order noise shaping characteristic of the mechanical system. Since the loop through the system is linear from the quantizer output to the quantizer input, it ideally does not matter if the filter is put in the feedback or forward paths. In practice, however, placing the compensator in the feedback path reduces system linearity, since a multi-bit DAC is needed for conversion of the digital feedback signal to an analog voltage. In addition, under normal operation in a feedback-path compensated loop, the average feedback level is only one-third full scale. Thus, full scale range is reduced by a factor of approximately two-thirds for a given power supply voltage. The reduced full scale range has implications on system robustness, as will be discussed in Section 3.5.

3.3 SYSTEM FORMULATION

In order to analyze and simulate the $\Sigma\Delta$ converter, it is necessary to first model the system as a set of differential equations. Various methods of closed-loop simulation may be employed, including Runge-Kutta and Euler integration; however, an exact *discrete time* model of the *continuous time* differential equations may be found. Use of an exact discrete time model, with the quantizer outside the linear equations, permits quick numerical simulation and enables the effect of system parameters on performance to be understood.

The system model used for the following analysis is shown in Figure 3-4. Second-order integration, from acceleration to position, is performed by the proof-mass. Polysilicon springs suspend the proof-mass above the substrate, as described in Section 2.2. Since the feedback loop keeps the proof-mass displacement small, the springs are assumed to be operating in their linear region. Rearranging the continuous time dynamics (2-1) into state-space form, the linear system is given by:

$$\dot{X} = A_c X + B_c e \quad (3-2)$$

$$y = C_c X \quad (3-3)$$

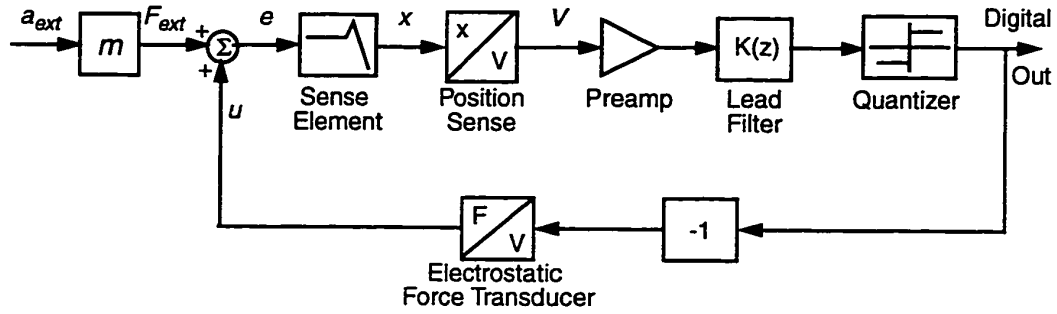


Figure 3-4. Block diagram of a second-order $\Sigma\Delta$ A/D converter with a lead filter in the forward path for compensation of the feedback loop.

$$A_c = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -\omega_n/Q \end{bmatrix}, B_c = \begin{bmatrix} 0 \\ 1/m \end{bmatrix} \quad (3-4)$$

$$C_c = \begin{bmatrix} 1 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ v \end{bmatrix}, \quad (3-5)$$

where m , ω_n , and Q are the mass, natural frequency and quality factor of the sensor, and e is the input force. States x and v represent the sensor position and velocity respectively. Note the input to the feedback loop is a force, while the desired measurand is acceleration. Input acceleration, a_{ext} , is converted to an input force, F_{ext} , by multiplying by the proof-mass, m , as shown in Figure 3-4.

Operation of the $\Sigma\Delta$ loop is time multiplexed into three basic phases: 1) *zero*, 2) *position sense* and *compare*, and 3) *force feedback*. During the *zero* phase, errors in the electronic interface are canceled. During the *position sense* phase, proof-mass displacement is detected and quantized to a one-bit output. During the *force feedback* phase, a force of constant magnitude, F_{fb} , is applied to the proof-mass. The sign of the feedback force, u , is dependent on the output of the quantizer during the previous phase. Figure 3-5 (a) shows a clocking diagram of the three phases in relation to each other. The individual length of each phase is set by settling time requirements of the electronic interface in conjunction with full scale range requirements. Since the mechanical system is linear, the exact discrete time model of the sense element over one complete period is

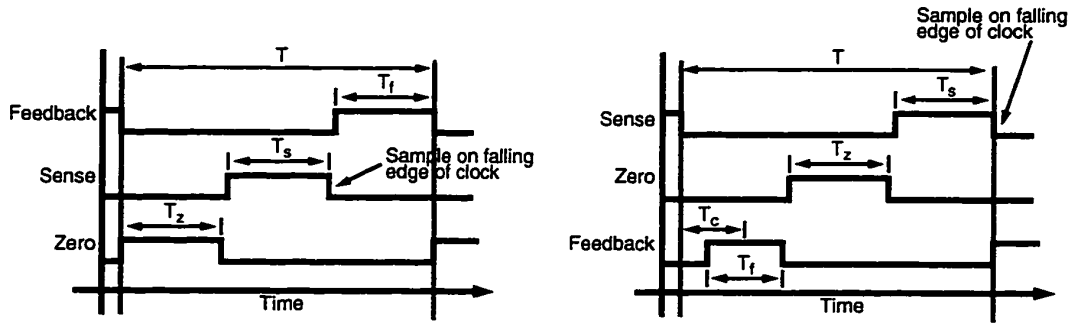


Figure 3-5. (a) Clocking diagram of different phases of the feedback loop operation. (b) Modified clocking diagram including delay between position sensing and feedback pulse application, T_c

given by:

$$X((k+1)T) = e^{A_c T} X(kT) + \int_{kT}^{(k+1)T} e^{A_c(T-\tau)} B_c \underbrace{(F_{ext}(\tau) + u(\tau))}_{e(\tau)} d\tau, \quad (3-6)$$

where T is the length of time between sampling instants. Because position quantization occurs in the middle of this cycle, it is not possible to find an exact solution in this form. Rewriting the clocking diagram as shown in Figure 3-5 (b) eliminates this problem by moving the quantization operation to the end of the cycle.

In Figure 3-5 (b), the feedback pulse is characterized with two parameters, T_f and T_c . T_c has two components. The first portion models delay between sampling, during the *position sense* phase, and the start of the feedback pulse. The second portion of T_c is due to the finite length of the feedback pulse: longer values of T_f delay the temporal centroid of the feedback pulse. The effect of both sources of delay on system behavior is identical.

Since feedback is constant over the feedback phase in a discrete time $\Sigma\Delta$ converter, equation (3-6) may be explicitly solved:

$$X(kT + T) = FX(kT) + Gu(kT) + G_i a_{ext}(kT), \quad (3-7)$$

where

$$F = e^{A_c T}, \quad G = \int_{T_f - \frac{1}{2}T_c}^{T_f + \frac{1}{2}T_c} e^{A_c(T-\tau)} B_c d\tau, \quad G_i = m \int_0^T e^{A_c(T-\tau)} B_c d\tau. \quad (3-8)$$

For the special case of $\omega_n = 0$, matrices G , G_i , and F reduce to the following discrete time transfer functions:

$$H(z) \equiv \frac{x(z)}{u(z)} = \frac{T_f(T - T_c) \left(z + \frac{T_c}{T - T_c} \right)}{m(z - 1)^2}, \quad (3-9)$$

$$H_i(z) \equiv \frac{x(z)}{a_{ext}(z)} = \frac{T^2(z + 1)}{2(z - 1)^2}. \quad (3-10)$$

Equations (3-9) and (3-10) provide an intuitive understanding of factors affecting performance. Since the transfer function $H_i(z)$, from input acceleration to position, is independent of T_f the ratio of $H(z)/H_i(z)$ is directly proportional to T_f . Thus the full scale value of the $\Sigma\Delta$ converter is directly proportional to T_f and equal to:

$$a_{fs} = \frac{a_{ext}(z) x(z)}{x(z) u(z)} F_{fb} = \frac{1}{H_i(z)} H(z) F_{fb} = \frac{T_f F_{fb}}{T m}, \quad (3-11)$$

where F_{fb} is the full scale feedback force. Note that the length of the feedback pulse, T_f affects only the magnitude of $H(z)$, not the phase. This result shows the accelerometer full scale is set by the fraction of the period the feedback pulse is applied, multiplied by the full scale feedback force.

The phase lag of the system, fixed by the ratio T_c/T , determines the amount of phase lead needed from the compensator for stability. Figure 3-6 shows the phase response of (3-9) for increasing values of T_c/T . The increase of phase lag with larger values of T_c/T makes intuitive sense. Since T_c is the temporal centroid of the feedback pulse, T_c/T represents the normalized delay between sense measurement and feedback application. Longer pulses and greater delays between position sampling and feedback pulse application are represented by a corresponding increase in T_c/T , and thus an increase in total phase lag.

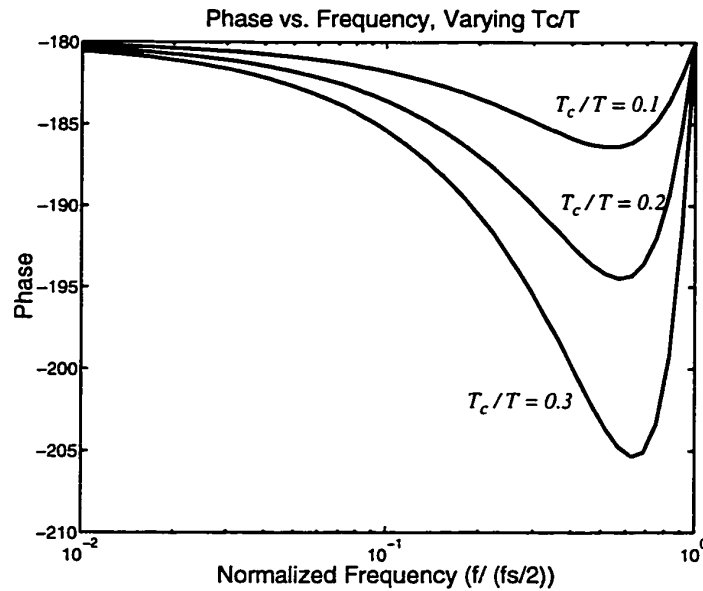


Figure 3-6. Phase lag vs. normalized frequency for varying values of T_c/T .

A linear compensator in either the forward or feedback paths may be included in the system model given above through direct multiplication in the z -domain. Section 3.4 uses equations (3-7) and (3-8) to analytically predict performance. These equations are also used in Section 3.5 for comparison of analytical and simulated results.

3.4 STABILITY

$\Sigma\Delta$ converters are inherently unstable by most conventional stability definitions since for any static input, their output does not converge to an equilibrium point but tends to form limit cycles. The existence of limit cycles is not harmful as long as they occur at very high frequencies and have a small domain of attraction [35]. If the system is not properly compensated, these limit cycles move to lower frequencies where they cause degradation of performance due to strong tones in the baseband, a decrease in noise shaping, and large dead zones. Additionally, low frequency oscillations can overload electrical circuitry due to their large magnitude. This section discusses modulator compensation for small input signals, with the goal of minimizing quantization noise floor. A good discussion on stability under large input signals, nearing full scale, may be found in reference [36].

In order to compensate the electro-mechanical $\Sigma\Delta$ modulator, a 2-tap finite impulse response (FIR) filter of the form:

$$K(z) = \frac{\alpha - z^{-1}}{\alpha + 1} = \frac{\alpha z - 1}{(\alpha + 1)z} \quad (3-12)$$

is proposed. Note this 2-tap lead filter provides compensation equivalent to the stable switched-capacitor implementation shown in Figure 3-2 when $\alpha = 2$ and $T_c/T = 0$. Physically, this corresponds to a feedback impulse at the start of the period. By varying the value of α , it is possible to adjust the phase lead introduced by the compensator: *lower values of α provide higher levels of compensation*. This is observed by noting the zero of $K(z)$ moves to lower frequencies as α is decreased, while the pole remains fixed at 0.

The quantization noise of equation (3-1) is shaped by the inverse of the loop transfer function, $H(z)K(z)$ [33]. Since the compensator effectively adds a zero to the loop transfer function, the second-order noise shaping supplied by the proof-mass is reduced to a first-order noise shaping function above the effective compensation-zero frequency. Second-order noise shaping resumes below the compensation zero down to the resonant frequency. Thus the quantization noise in a bandwidth $f_N/2$ is given by:

$$S_q^2 = \left(\frac{\Delta^2}{12}\right)\left(\frac{f_N}{f_s}\right)\left(\frac{f_r^4}{\left(\frac{f_z(\alpha)}{2}\right)^2\left(\frac{f_s}{2}\right)^2}\right) = \frac{4}{3}\Delta^2 f_N \left(\frac{f_r^4}{f_z^2(\alpha) f_s^3}\right) \quad , \quad f_N < 2f_r \quad , \quad (3-13)$$

where $f_z(\alpha)$ is the +3 dB frequency of the compensator zero. Although the initial reaction to this result would be to increase the compensator zero frequency, increasing the zero frequency has the undesired effect of moving limit cycles to lower frequencies.

Limit cycles are the cyclic movement of the proof-mass about the zero position. Since input accelerations also result in proof-mass displacement, these cyclic movements show up as an input referred acceleration. If the motion of the proof-mass is below the quantization noise floor, the output noise is described by equation (3-13). However if the system is undercompensated, limit cycles can dominate output noise. By analytically determining limit cycle dependence on system parameters, the total noise contribution

from both quantization noise and proof-mass motion may be minimized.

We can make the following observations about the possible existence of limit cycles, under zero applied input, from general knowledge about the system with this 2-tap FIR filter:

1) Limit cycles may only occur at even multiples of the sampling period. This constraint arises from the fact that if there was a limit cycle with an odd period, the proof-mass would receive feedback in one direction more often as the other direction. Unequal feedback would make the proof-mass move away from the zero position, preventing the limit cycle from sustaining itself.

2) Limit cycles may not exist for any sampling frequency less than $f_s/4$ in this second order system. By constraint 1, limit cycles of f_s and $f_s/3$ may not exist. If a limit cycle could exist at $f_s/2$, the output of the compensator would alternate between two values $(\alpha+1)$ and $-(\alpha+1)$. Thus, the output of the compensator effectively exhibits zero phase lead, as the feedback would not be changed if the compensator were replaced with a static gain of $K(z) = (\alpha+1)$. Since the system is unstable with no phase lead from the compensator, the limit cycle will move to a lower frequency. By adding large amounts of phase lead it is possible to create limit cycles at $f_s/2$; however, doing so reduces the system to a first order modulator having decreased noise shaping.

Describing functions may be used to predict the existence of limit cycles [37]. This method uses a Fourier series decomposition of system nonlinearities to determine an equivalent gain and phase of the nonlinearity at a given amplitude and frequency. When the loop gain, consisting of the plant and compensator dynamics multiplied by the quantizer describing function, equals +1, conditions for existence of a limit cycle are met. The describing function for a single-bit quantizer is found to equal [37]:

$$N(A) = \frac{4F_{fb}}{\pi A}, \quad (3-14)$$

where A represents the amplitude of the input sinusoid to the quantizer. Since equation (3-14) lies on the real axis, a limit cycle is predicted when the following condition is

satisfied:

$$\angle H(e^{j\omega_l})K(e^{j\omega_l}) = -180^\circ, \quad 0 \leq \omega_l \leq \pi. \quad (3-15)$$

At the limit cycle frequency, ω_l , the amplitude at the proof-mass output equals:

$$x_l = |H(e^{j\omega_l})| \frac{4F_{fb}}{\pi}. \quad (3-16)$$

In this analysis, the limit cycle is assumed to have a sinusoidal trajectory and the position error is assumed to be equally distributed across frequencies up to the limit cycle frequency. From equation (3-16), the total output acceleration noise power due to limit cycles is found to be:

$$S_l^2 = \left(2.8 \omega_n^2 F_{fb} \frac{f_N}{\omega_l f_s} |H(e^{j\omega_l})| \right)^2, \quad f_N < 2f_r. \quad (3-17)$$

Equation (3-17) shows the noise due to limit cycles is decreased by moving the limit cycle to higher frequencies - i.e. increasing ω_l . This is achieved by increasing compensation. Since increasing compensation raises quantization noise, it is expected that an optimal value of α exists: when the noise contribution from motion of the proof-mass is equal to quantization noise. The existence of this point is demonstrated in the next section.

3.5 RESULTS

The above method of calculating noise has been applied to an example system and compared with simulations. Results were simulated in the time domain using the equations derived in Section 3.3, with the 2-tap FIR compensator (3-12) in the forward path. All simulations were done with MATLAB on a Hewlett Packard 9000 workstation. Simulations were very fast due to the use of the exact discrete time equations derived in Section 3.3: each millisecond of simulated operation required approximately 0.5 seconds of computer time.

Comparison of simulations and hand calculated values of noise vs. α , with a mechanical Q of 1, is shown in Figure 3-7. At low values of α , the system is overcompensated and noise shaping is reduced to that of a first order system with an

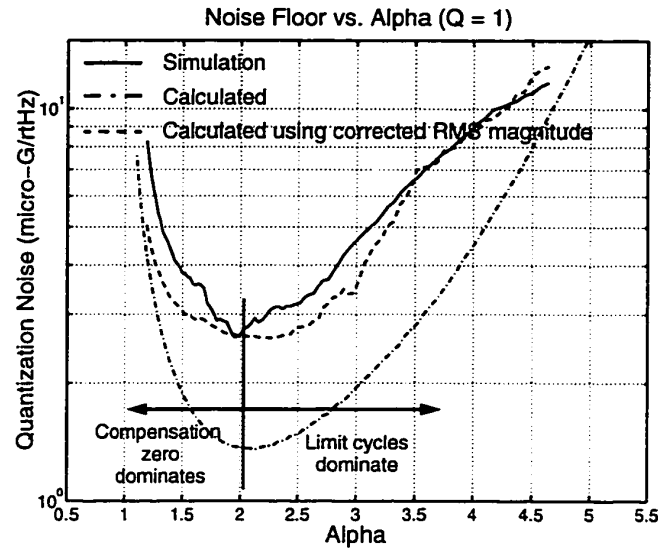


Figure 3-7. Simulated and calculated quantization noise power density vs. α for $Q = 1$, $T_c/T = 0.25$, $f_s = 1\text{MHz}$, $f_r = 2\text{kHz}$, and full-scale $= \pm 12.5G$.

increased noise floor. At high values of α , the system is undercompensated and becomes unstable, causing limit cycles to move to lower frequencies and degrade performance. Note the hand calculated values, using equation (3-17), under predicts noise floor (dash-dotted line in Figure 3-7). This error is because the describing function method, used to predict limit-cycle magnitude, is an approximate method assuming a single limit cycle trajectory. In fact, the position output is not composed of a limit cycle with a single trajectory, but can contain many different trajectories [38]. The RMS limit cycle magnitude predicted by (3-16) may be replaced with simulated RMS values. The result is a significant improvement in prediction of noise floor, as shown by the dashed line in Figure 3-7. Even though the describing function underestimates absolute RMS magnitude, the value of α minimizing noise is accurately predicted. Thus, equation (3-17) may be used as a design guideline, capturing the most important aspects of system performance.

Figure 3-8 shows simulated and calculated noise vs. α for a proof-mass with zero damping ($Q \Rightarrow \infty$). In this instance, the system is far more sensitive to choice of α . The increased sensitivity to compensation is due to the rapid phase drop to -180° immediately above resonance, requiring additional phase lead at lower frequencies. In the low Q case,

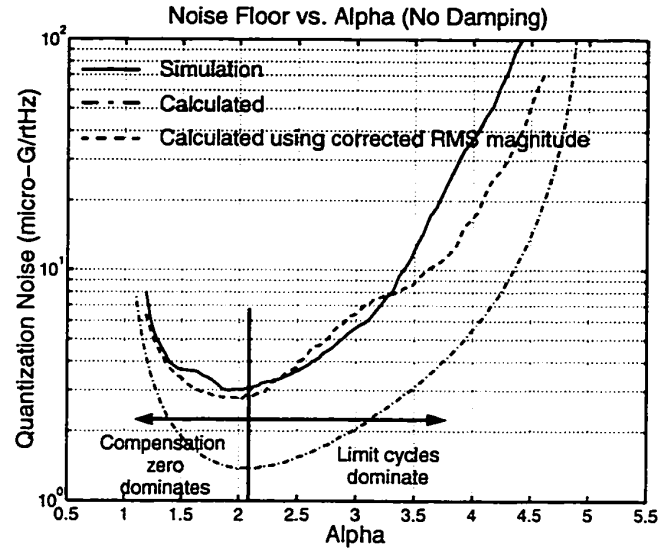


Figure 3-8. Simulated and calculated quantization noise power density vs. α for $Q = \infty$, $T_c/T = 0.25$, $f_s = 1\text{MHz}$, $f_r = 2\text{kHz}$, and full-scale = $\pm 12.5\text{G}$.

phase more gradually decreases, reducing the amount of phase lead necessary to keep limit cycles at high frequencies.

Higher sampling frequencies have an effect similar to a high mechanical Q : more phase lag is contributed where limit cycles are undesired. The effect on performance is identical to that of an increased mechanical Q , as shown in Figure 3-9. Although the system becomes less robust for both high Q and high sampling rates, the optimal value for compensation remains constant since when the system is optimally compensated, limit cycles occur at frequencies much greater than resonance. At these frequencies, the phase contribution of the proof-mass is approximately 180° , independent of Q or M .

The effect of delay, T_c/T , on robustness is shown in Figure 3-10. This graph shows the range of α which will provide a signal to quantization noise ratio (SQNR) within 3, 6, and 12dB of peak in a high Q system. The lower limit on α is approximately independent of delay, since it arises from overcompensation. The upper limit on α is decreased with increases in delay, since more lead is required from the compensator to prevent large limit cycles.

The component of delay from position measurement and quantization operations may

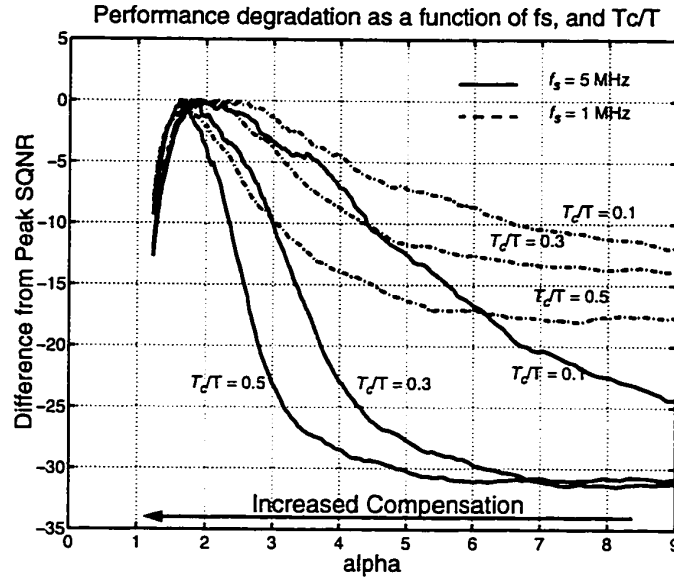


Figure 3-9. Deviations from optimal noise performance for $f_s = 1$ and 5 MHz and varying values of T_c/T .

be made small with a fast CMOS technology and careful circuit design. Even so, feedback pulses should be as short as possible, since the minimum value of T_c/T is set by half the length of the feedback pulse. Small feedback pulses may be used to achieve nominal full scale requirements by increasing the feedback pulse force F_{fb} . Since electrostatic forces are proportional to the square of the feedback voltage, full scale ranges on the order of ± 100 G, may be obtained in even a low voltage technology. As discussed in Section 3.2, locating the compensator in the forward path provides a full scale acceleration of approximately 3 times that of a feedback path compensated system for a given full scale feedback force. This permits shortening of the feedback pulse by a factor of approximately 2/3, increasing system robustness.

3.6 SUMMARY

Use of a mechanical sense element to provide second order noise shaping in a $\Sigma\Delta$ converter topology simultaneously realizes a high dynamic range, good linearity, and a digital output. Because $\Sigma\Delta$ converters include nonlinear feedback elements, system level simulations are a necessity to enable accurate performance estimates. An exact

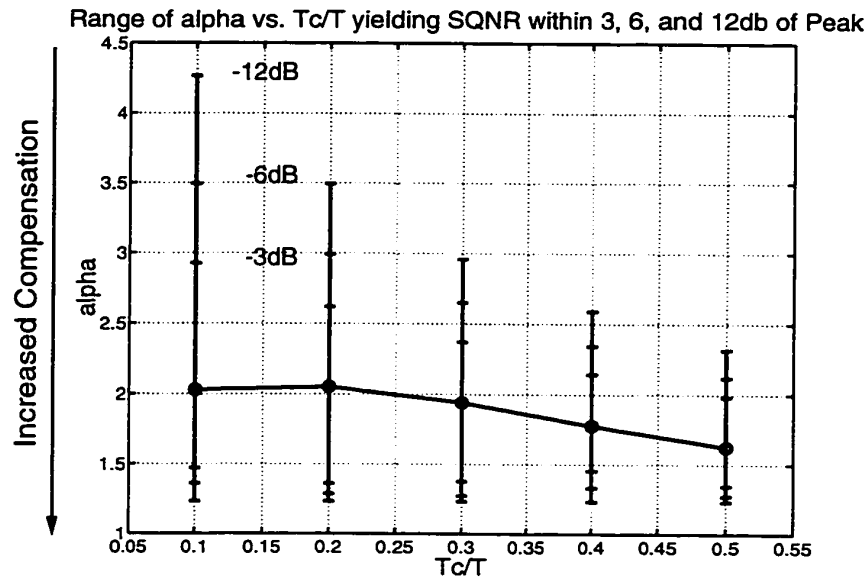


Figure 3-10. Range of alpha giving noise performance within 3, 6, and 12 dB of peak. As the delay T_c/T is increased system performance is more sensitive to variations in compensation.

discrete-time model characterized by a minimal number of parameters has been derived, permitting rapid simulation of the system.

Because the mechanical sense element has two poles, a compensator is necessary for stable operation. A 2-Tap FIR lead filter in the forward path can be used to provide stabilizing compensation. Placement of the compensator zero has been demonstrated to significantly affect modulator noise performance. If the zero is placed at too low a frequency the system suffers from poor noise shaping. If the zero is placed at too high a frequency limit cycles dominate output noise. An approximate method, based on describing functions, has been shown to yield qualitatively accurate estimates of noise performance, enabling optimal choice of compensator values.

4. ELECTRICAL INTERFACE

4.1 OVERVIEW

A displacement accelerometer infers acceleration by measuring displacement of a suspended proof-mass in response to an input acceleration. Reference [39] discusses various methods of transduction which may be used for deflection measurement. These methods include electron tunneling, piezoresistive sensing, piezoelectric sensing, and capacitive sensing. Capacitive position sensing has a low intrinsic temperature coefficient, can be highly sensitive, and is easily integrated with CMOS for monolithic sensor based systems [14, 40]. In addition, electrostatic force balancing of the proof-mass may be accomplished with the same capacitors that are used for sensing displacement. Capacitive interfaces are particularly sensitive to parasitic capacitance at the input to the electronic interface, however. For this reason, piezoresistive or other transduction mechanisms are often preferable in two-chip solutions, where interconnect capacitance can reduce sensitivity by several orders of magnitude.

As described in Sections 2.3.1 and 2.3.2, displacement of the proof-mass causes an imbalance in the capacitive half-bridge. This imbalance can be measured using either voltage buffering [14, 16, 41] or charge integration [13, 34, 42] in response to a voltage pulse applied to the sense capacitors. Charge integration is used here since it lends itself well to differential switched-capacitor techniques, which have a proven track record in high dynamic range analog-to-digital converters.

Besides dynamic range, another reason for choosing a switched-capacitor topology is ease of implementation in a CMOS technology. In order to maintain the potential cost advantage of integrated MEMS sensors, the number of processing steps must be kept small. Since patterning of the sense-element and interconnect can already require many processing steps, a simple circuit technology is desirable in order to maintain a high yield and a low cost. The topology described in this chapter may be implemented in a simple CMOS process. This enables large amounts of functionality to be integrated through

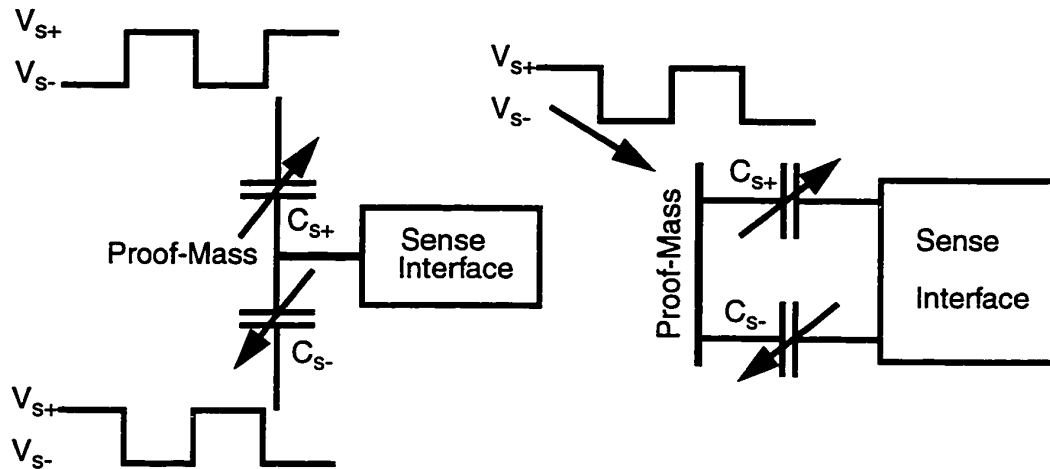


Figure 4-1. (a) Capacitive half-bridge configured for single-ended output (left) and (b) for differential output (right). Proof-mass is the center node of the half-bridge in both configurations.

on-chip digital logic, with minimal process complexity.

In light of the above discussion, this chapter explores electrical design of a precision $\Sigma\Delta$ accelerometer. A differential position-sense interface for a sense-element with capacitive half-bridge output is described in Section 4.2. Section 4.3 discusses cancellation of errors introduced by this electrical interface, which would otherwise appear in the output as an unwanted acceleration signal. Sections 4.4 and 4.5 analyze factors limiting performance, thereby permitting optimal sensor design. Sections 4.6 and 4.7 discuss design of a compensator and force-feedback. Since device mismatch causes undesirable offsets in the sensor output, a method for digitally trimming post-packaging is presented in Section 4.8. Finally, in Section 4.9, a 3-axis accelerometer using a single proof-mass demonstrates the flexibility of this topology for multi-axis servoing.

4.2 DIFFERENTIAL POSITION SENSE INTERFACE

Previous capacitive interfaces for accelerometers and gyroscopes have used a single-ended sense-element output to interface with the sense electronics [8, 16, 34, 42, 43, 44, 45]. In many of these applications, a pair of sense capacitors are configured in a

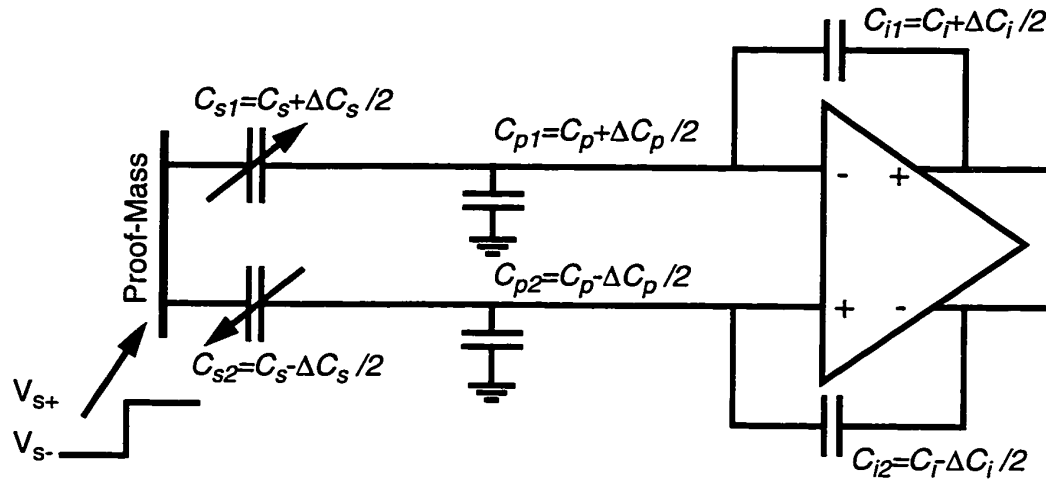


Figure 4-2. Differential sense interface. Mismatch between capacitors $C_{x1,2}$ is represented by a nominal capacitance, C_x with the difference in capacitance ΔC_x

capacitive half-bridge, similar to the bridge described in Section 2.3. Capacitance is measured by driving the ends of the capacitive bridge and taking the output as the center node, shown in Figure 4-1(a). As described in [46], high output stability requires precise generation of the two AC drive signals over temperature and power supply fluctuations. In fact, it is estimated the bias stability of these voltages should be on the order of 10^{-7}V for inertial grade accelerometers and gyroscopes. Reference [46] proposes a switched-capacitor (SC) topology which uses feedback to generate these precision signals.

An alternative sensing method, presented in this chapter, uses a differential sense interface. Differential sensor output may be achieved with the same sense-element by reversing the roles of the center and the end terminals, as shown in Figure 4-1(b). In this case, voltage pulses are applied to the center of the capacitive half bridge with the ends connected to the differential position sense interface. Variations in the magnitude of this single pulse cause only scale factor changes, which are removed by the quantizer in the sigma-delta topology.

In addition to relaxing requirements on sense pulse stability, driving the center node of the bridge enables use of differential circuitry, which results in better power supply rejection and first order cancellation of substrate noise. Due to the small size of the sense

capacitors, switch charge injection represents a potentially large source of error in SC topologies. A fully differential interface converts switch charge injection to a common-mode signal, providing first order cancellation of this error. The differential interface also allows ground-plane shielding, needed to prevent electrostatic pull-down of the proof-mass, to be connected directly to the proof-mass. Connecting the proof-mass to the shield eliminates the need for a buffer [16], with its associated increase in distortion and decrease in stability. By driving the proof-mass, multiple sets of sense capacitors in different axes may be simultaneously force-balanced. This allows multi-axis servo control as will be discussed in Section 4.9.

Figure 4-2 shows a diagram of the sense-element connected to a differential charge integrator for position measurement. Capacitors $C_{pl,2}$ are parasitic capacitors which include both the structural wiring capacitance and the input capacitance of the charge integrator. Position is sensed by applying a voltage pulse to the center node of the capacitive half-bridge. The voltage pulse, typically on the order of several volts, is made as large as possible to obtain a high signal-to-noise ratio (SNR) at the integrator output. A capacitive imbalance in the half bridge causes different amounts of charge to flow from the integrating capacitors onto the sense capacitors. The charge difference, ΔQ , which flows onto C_{s1} and C_{s2} in response to the voltage step $\Delta V_s = V_{s+} - V_{s-}$ is given by:

$$\Delta Q \equiv Q_{s1} - Q_{s2} = \Delta C_s (\Delta V_s - \Delta V_{icm}), \quad (4-1)$$

where ΔV_{icm} is the difference in input common mode voltage in response to the voltage step. Note the voltage $\Delta V_s - \Delta V_{icm}$ is the change in voltage across the sense capacitors due to application of the sense pulse. The input common mode shift, ΔV_{icm} , results from the fact that the feedback in Figure 4-2 is applied to only the differential voltage at the opamp input. Since the charge in (4-1) must flow from the integrating capacitors, and the voltage difference at the opamp input is driven to zero, the differential output voltage is given by:

$$\Delta V_{out} \equiv V_{out}^+ - V_{out}^- = \frac{\Delta C_s}{C_i} (\Delta V_s - \Delta V_{icm}) = \Delta V_s \frac{\Delta C_s}{C_i} \left(1 - \frac{C_s}{C_s + C_i + C_p} \right). \quad (4-2)$$

Mismatch in the parasitic capacitors, $C_{pl,2}$, and the integrating capacitors, $C_{il,2}$, cause different amounts of charge to flow onto the integrating capacitors in response to the input

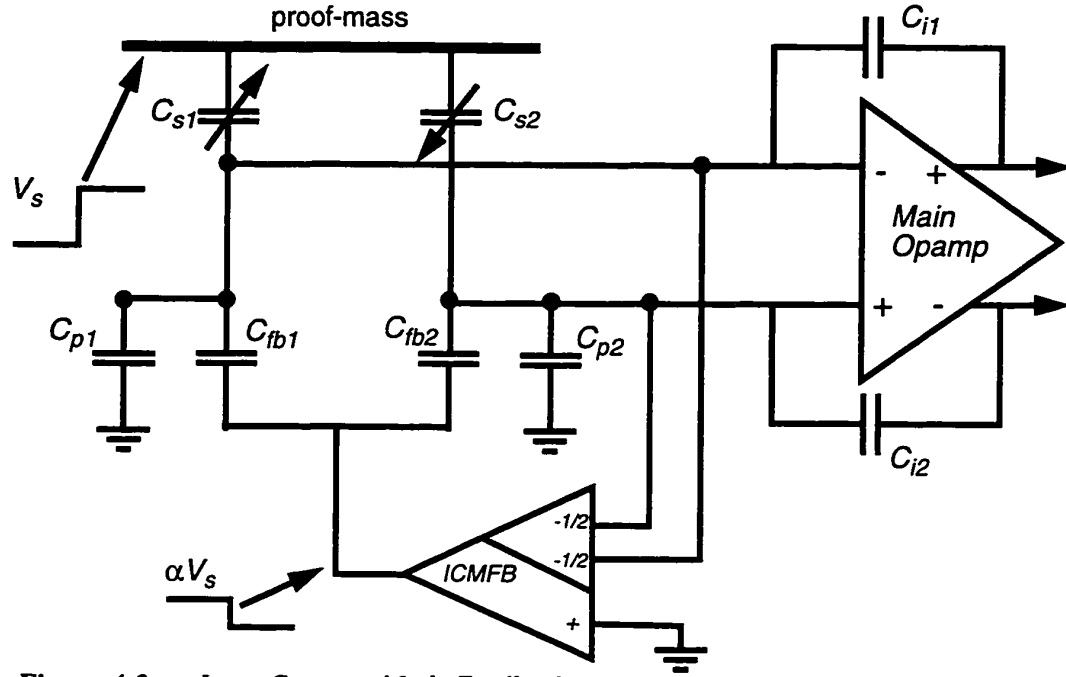


Figure 4-3. Input Common Mode Feedback (ICMFB) keeps the opamp input common mode at a constant value, eliminating the gain and offset errors in equation (4-3).

common mode shift. The result is a mismatch-dependent offset appearing in the output:

$$\Delta V_{out} = \Delta V_s \left(\underbrace{\frac{\Delta C_s}{C_i} \left(1 - \frac{C_s}{C_s + C_i + C_p} \right)}_{\text{Gain Error}} - \underbrace{\frac{\Delta C_i + \Delta C_p}{C_i} \frac{C_s}{C_s + C_i + C_p}}_{\text{Offset Error}} \right). \quad (4-3)$$

Because the offset error can be large, the dynamic range of the sensor can be compromised. Furthermore, small changes in parasitic mismatch over temperature or aging will show up in the output as drift.

In addition to causing an offset, the input common mode voltage shift places severe requirements on the opamp. The opamp must be designed to handle large input common mode swings, even at the expense of noise performance and bandwidth. The opamp must also have an excellent common-mode-rejection-ratio (CMRR) to prevent large output errors from the input common mode shift. In addition, sense-pulse voltages must be kept small to maintain the opamp within its input common mode range.

All of the above problems may be minimized through input common mode feedback

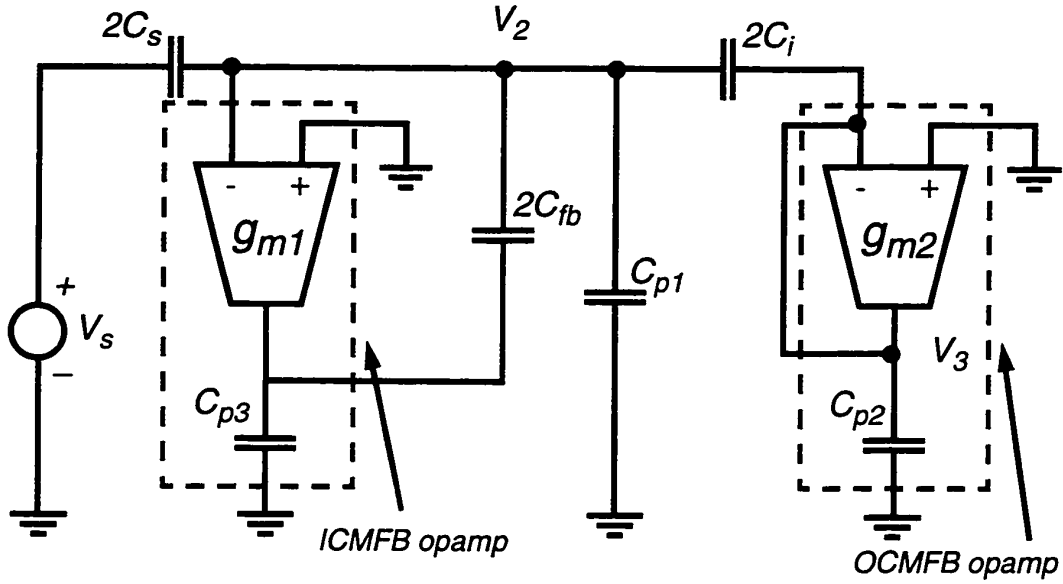


Figure 4-4. Common mode schematic diagram used for settling analysis. Voltage V_s represents the proof-mass voltage. Voltages V_2 and V_3 represent the common mode voltages at the main opamp input and output respectively.

(ICMFB), shown in Figure 4-3. ICMFB measures the input common mode of the opamp and applies voltage feedback through two feedback capacitors, C_{fb} . Negative feedback drives the voltage αV_s towards the value necessary to keep the input common mode constant. This operation may be thought of as driving both sides of the full bridge, formed by C_s and C_{fb} , with voltage pulses chosen for zero input common mode shift. The integrator output with ICMFB is now given by:

$$\Delta V_{out} = \Delta V_s \left(\frac{\Delta C_s}{C_i} - \underbrace{\frac{C_s}{C_{fb} C_i} \Delta C_{fb}}_{\text{Offset Error}} \right). \quad (4-4)$$

Note that ICMFB, in conjunction with differential feedback from C_i , keeps both terminals of the opamp input at a virtual ground, removing the effects of mismatch between $C_{p1,2}$ or $C_{il,2}$. Attenuation of the desired output signal, represented by the gain error in (4-3), is avoided since the full sense pulse voltage appears across the sense capacitors. Mismatch between $C_{fb1,2}$ does show up as an offset at the integrator output,

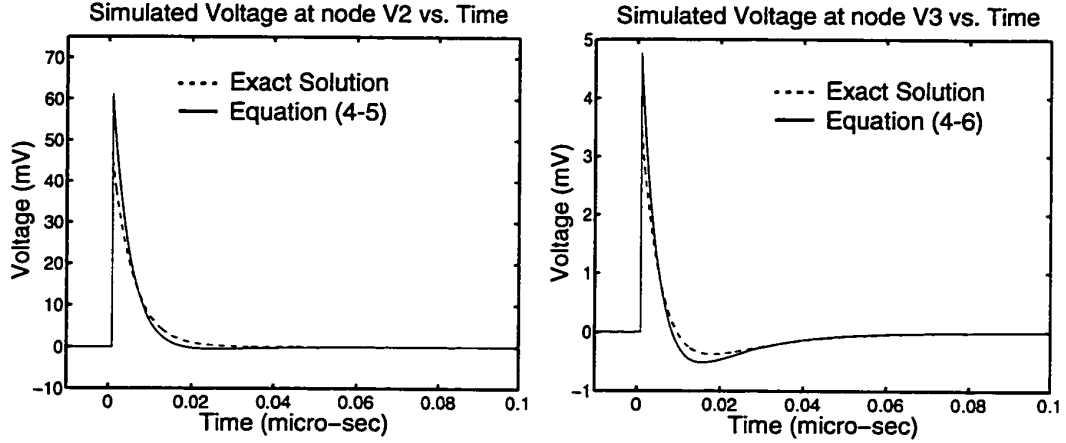


Figure 4-5. Simulated input and output common mode responses to a step change of 1 Volt in V_s using equations (4-5) and (4-6). For these simulations $C_s = 100\text{fF}$, $C_{fb} = 200\text{fF}$, $C_i = 200\text{fF}$, $C_{p1} = 3.5\text{pF}$, $C_{p2} = 4.75\text{pF}$, $C_{p3} = 1.5\text{pF}$, $g_{m1} = 4.1\text{mA/V}$, and $g_{m2} = 0.31\text{mA/V}$.

however. Note that this mismatch may be digitally trimmed out using a binary weighted capacitor array, as will be discussed in Section 4.8.

Because the opamp is differential, output common mode feedback (OCMFB) is needed to keep the opamp in its operating region [47]. The coupling of ICMFB and OCMFB through C_i results in second order settling of both input and output common mode voltages. Figure 4-4 shows a schematic diagram of the common mode which may be used for settling analysis. Capacitors C_{p1} , C_{p2} , and C_{p3} represent parasitic capacitances at the main opamp input, the main opamp output (including loading from the subsequent stage), and the ICMFB output node respectively. Analysis of the transfer function between V_s and $V_{2,3}$ yields a very complex characteristic equation, derived in Appendix A. By eliminating coefficients with a small magnitude, simplified transfer functions may be found. The reduced transfer function between V_s and V_2 is given by:

$$\frac{V_2(s)}{V_s(s)} \approx \frac{2C_s(2C_{fb} + C_{p3})((2C_i + C_{p2})s + g_{m2})s}{(C_{p2}C_{p1}(C_{p3} + 2C_{fb}))s^2 + (2C_{fb}C_{p2}g_{m1} + C_{p3}C_{p1}g_{m2})s + 2C_{fb}g_{m1}g_{m2}} \quad (4-5)$$

The reduced transfer function between V_s and V_3 is given by:

$$\frac{V_3(s)}{V_s(s)} \approx \frac{4C_s C_i (2C_{fb} + C_{p3}) s^2}{(C_{p2} C_{p1} (C_{p3} + 2C_{fb})) s^2 + (2C_{fb} C_{p2} g_{m1} + C_{p3} C_{p1} g_{m2}) s + 2C_{fb} g_{m1} g_{m2}} \quad (4-6)$$

Both of these transfer functions have a zero at 0. This zero arises from the AC coupling of the input voltage, V_s , through the sense capacitors, C_s .

Figure 4-5 shows simulated input and output common mode settling for the capacitor values listed in the caption. Note that for these values the output common mode, V_3 , has a slight amount of overshoot. A closer look at the characteristic equations of (4-5) and (4-6) reveals that both poles are real for reasonable values of capacitors, and the overshoot in Figure 4-5 (b) is caused by the two zeros in (4-6). Since the system is not underdamped, settling is governed by the dominant pole. For the parameters listed in Figure 4-5, the dominant pole is approximately 10 MHz and is set by the speed of the output common mode feedback loop.

4.3 ERROR CANCELLATION

In order to implement the differential position sense interface in a sigma-delta feedback loop, several issues need to be addressed:

- 1) In order to maximize dynamic range, all of the air-gap capacitors are used for both sensing and force feedback. Since feedback must be applied to the proof-mass with the same capacitors used for sensing, they must be disconnected from the charge integrator to prevent overdriving the main opamp.
- 2) The input nodes of opamp, shown in Figure 4-3, have no d.c. path to ground. Because these nodes are floating, the voltage on these nodes is not defined. Small amounts of charge leakage onto or off of these nodes cause the input common node voltage to move beyond the operating range of the main and ICMFB opamps.
- 3) Opamp flicker noise and d.c. offset couple directly into the integrator, causing a large error signal in the position sense output.

Continuous time topologies have used diode leakage [41], high valued resistors [16],

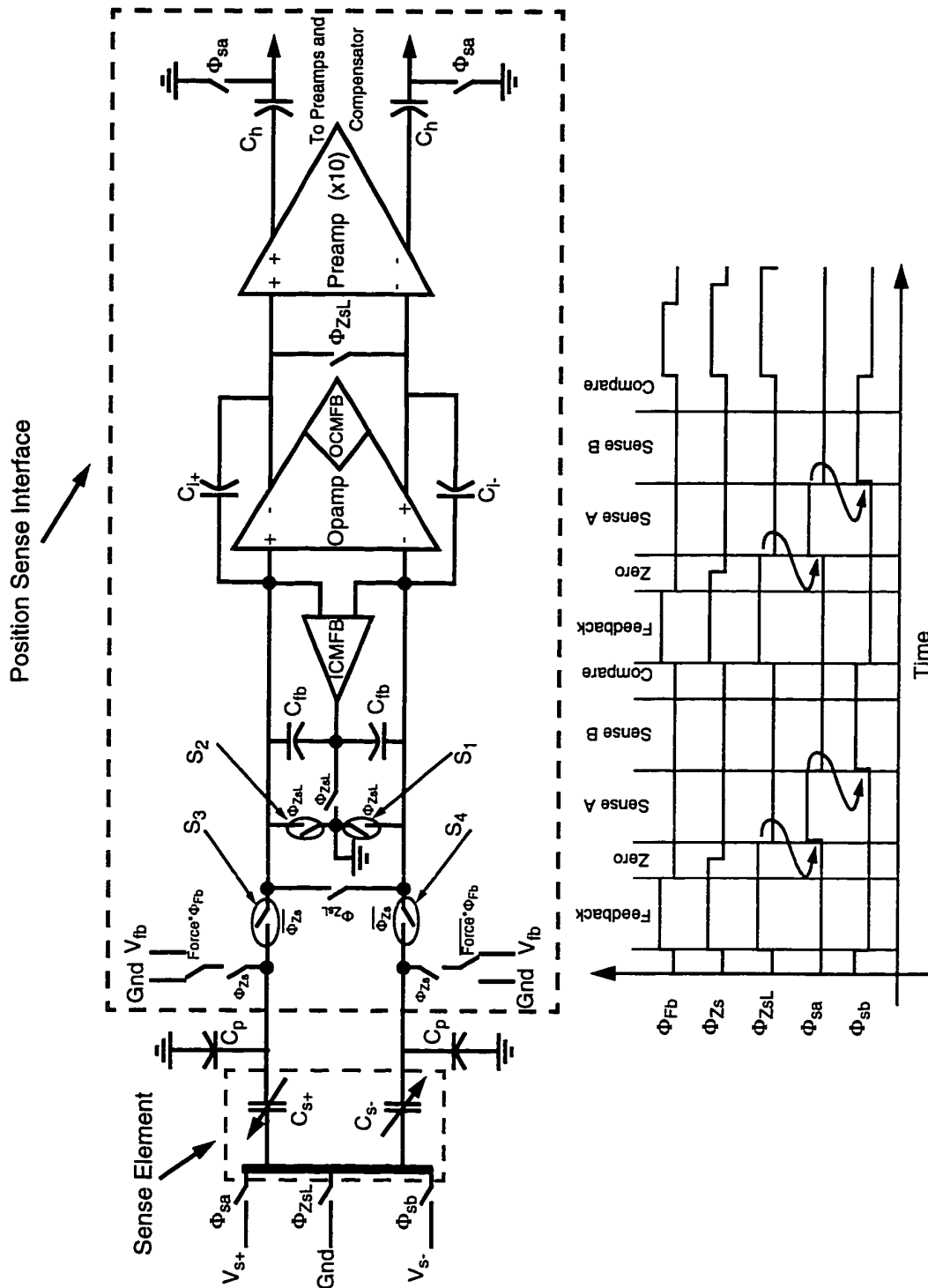


Figure 4-6. Schematic Diagram of the position sense interface with correlated double sampling. The timing diagram shows switch positions during different phases.

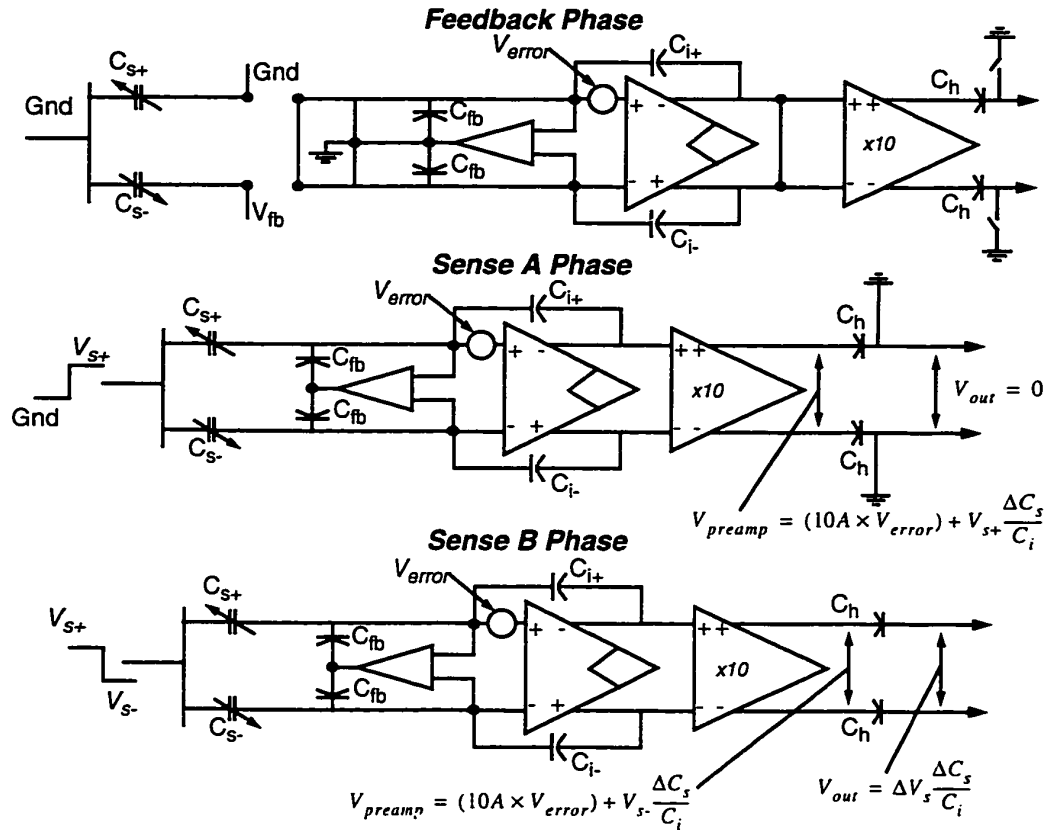


Figure 4-7. Position sense interface during Feedback (a - top), Sense A (b - middle), and Sense B (c - bottom) phases. Errors from the sense interface are referred to a voltage V_{error} at the opamp input. (Force feedback in the negative direction is shown).

and MOSFETS operating in the subthreshold region [8] to create a d.c. path to ground at the opamp input. These high impedance paths to ground can increase noise and result in poor control of the d.c. value at this node. Switched-capacitor topologies use switches to provide a low-impedance path to ground during a *zero* phase, while releasing the input node from ground during sense capacitor measurement [13, 34]. Although thermal noise is sampled onto the input node of the opamp when these switches are opened, this noise can be measured and subtracted using correlated double sampling, described below [48, 49].

A schematic diagram of the differential position sense interface is shown in Figure 4-6. Operation of the $\Sigma\Delta$ loop occurs in five phases: *feedback*, *zero*, *sense a*, *sense b*, and *compare*. Figure 4-6 shows a system timing diagram of these phases with respect to

each other. During the *feedback* phase (Figure 4-7 (a)), the opamp input is first disconnected from the sense element by switches s_3 and s_4 to prevent overdriving the main opamp. One-bit electrostatic force feedback is then applied by grounding the proof-mass and applying a reference voltage, V_{fb} , to the stationary terminal of either C_{s+} or C_{s-} , while holding the other capacitor at ground. During feedback, the input terminals of the main opamp are connected to ground while the output terminals of the main opamp are shorted together. This operation zeroes the integrating capacitors and prevents the main opamp from railing due to input-referred offset voltage. By grounding the output terminal of the ICMFB amplifier, railing of the input common mode feedback node due to mismatch in the ICMFB amplifier is prevented.

During the first half of the *zero* phase, Φ_{Zs} , the sense capacitors are grounded to remove charge from the feedback phase. During the second half of the *zero* phase the sense capacitors are connected to the opamp. Since the switches at the input and output of the main opamp remain closed, the voltages at the input terminals settle to ground. This sets the d.c. voltage at these nodes. At the end of the *zero* phase, the input of the main opamp and the ICMFB output are disconnected from ground, the main opamp output is unclamped, and the proof-mass is disconnected from ground. Unclamping the ICMFB amplifier output closes the feedback loop, activating input common mode feedback. When the input terminals are disconnected from ground, an error voltage, consisting of charge injection mismatch and $kT/(C_s+C_p)$ noise, is sampled by switches s_1 and s_2 . The sampling error, opamp flicker noise, and opamp offset may be lumped together and referred to the opamp input as a single error source, V_{error} , shown in Figure 4-7. Cancellation of these errors occurs over the next two clock phases, *sense a* and *sense b*.

In the first sense phase, *sense a*, a positive voltage pulse of magnitude V_{s+} is applied to the proof-mass. At this point, the integrator output includes both the desired position signal and an undesired component due to V_{error} sampled during the end of the *zero* phase. The error voltage, V_{error} , is multiplied by a gain A , dependent on the values of C_i , C_s , C_{fb} , and parasitic capacitances. The integrator output is amplified by a preamplifier and stored on capacitors C_h , as shown in Figure 4-7 (b).

During the second sense phase, *sense b*, the capacitors C_h are disconnected from ground, and a negative voltage of magnitude V_{s-} is applied to the proof-mass. By taking the output as the top plate voltage of C_h , only the *difference* in preamp output between the *sense a* and *sense b* phases is measured. Since $kT/(C_s+C_p)$, charge injection mismatch, and d.c. offset errors remain constant over Φ_{sa} and Φ_{sb} , they are canceled, while the position signal is doubled [48, 49] (Figure 4-7 (c)). Because opamp flicker noise varies slowly, it is not entirely canceled, but rather attenuated by $1-z^{-1/4}$. This differentiation lowers the flicker noise below opamp thermal noise at low frequencies. Since the sampling capacitors follow the preamp, preamp offset and flicker noise are also sampled and cancelled.

Because the integrator has a gain of only a few $\mu\text{V/mG}$, several stages of preamps are used to boost the signal before it reaches the compensator. Charge injection mismatch and kT/C_h noise from sampling the first preamp output may be canceled through delayed sampling of the second and third preamps. In this topology, the bottom plate of C_h is connected to the preamp output to minimize attenuation by the capacitive divider formed between C_h and the bottom plate parasitic of this capacitor. The holding capacitor is made large enough to minimize attenuation from the input capacitance of the subsequent stage, while not significantly slowing settling. This value is typically a few picoFarads.

The compensator, described in Section 4.6, samples the output of the third preamp after settling has occurred at the end of the *sense b* phase.

4.4 CIRCUIT TOPOLOGIES

The main operational amplifier used for implementing the charge integrator must be fully-differential in order to implement a differential charge integrator. Since the output swing requirements of the charge integrator are very small, on the order of several mV, a telescopic opamp configuration may be used, as shown in Figure 4-8. A telescopic topology is desirable because it is simple and because it has fewer noise-contributing transistors than other opamp topologies, such as the folded cascode.

The primary noise contributing transistors in the telescopic opamp shown in Figure 4-8 are designated by $M_1 \rightarrow M_4$. Because the cascode transistors $M_5 \rightarrow M_8$ are

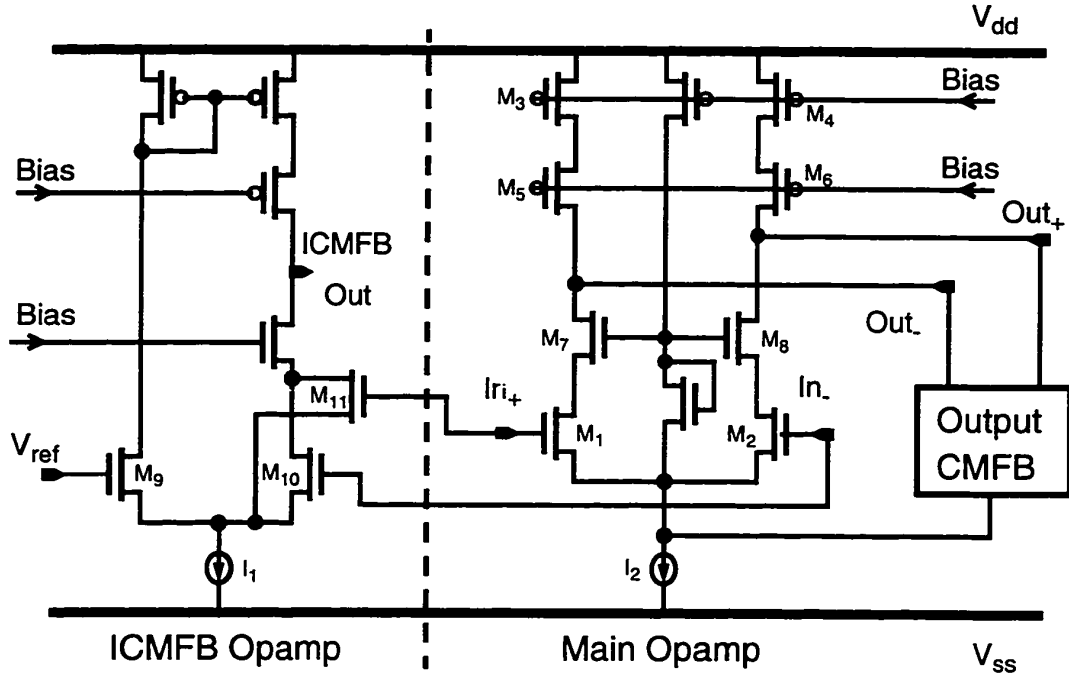


Figure 4-8. Simplified schematic of main opamp and input common mode feedback opamp.

degenerated by the output resistance of transistors $M_1 \rightarrow M_4$, their noise contributions are negligible. Analysis of the input-referred noise of this opamp yields:

$$\frac{V_n^2}{\Delta f} = \frac{16kT}{3} \left(\underbrace{\frac{1}{g_{m1,2}}}_{M_{1,2}} + \underbrace{\frac{g_{m3,4}}{(g_{m1,2})^2}}_{M_{3,4}} \right). \quad (4-7)$$

The noise contribution made from PMOS transistors M_3 and M_4 may be reduced by making the ratio $g_{m3,4}/g_{m1,2}$ less than one. In this case the dominant noise source becomes the input transistors M_1 and M_2 .

The optimal size of the input transistors depends on the parasitic loading of the input node. Figure 4-9 shows a single-ended schematic of the opamp which may be used to determine device parameters for optimal noise performance. Note that optimization of the opamp for use in a charge integrator is equivalent to designing the opamp for optimal input-referred current noise, I_n^2 . The input-referred current noise from Figure 4-9 is given

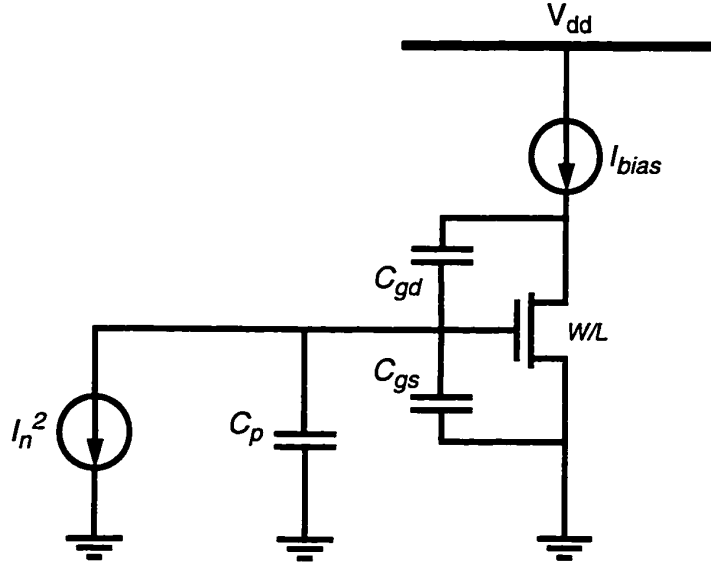


Figure 4-9. Circuit diagram for analysis of optimal input transistor size.

by:

$$\frac{I_n^2}{\Delta f} = \left(4kT \left(\frac{2}{3} \frac{1}{\mu C_{ox} V_{dsat}} \frac{L}{W} \right) \left(C_p + \frac{2}{3} W L C_{ox} + W L_d C_{ox} \right)^2 \right) \omega^2 = K_n \omega^2. \quad (4-8)$$

Equation (4-8) is minimized over all frequencies when K_n is minimized, since K_n is independent of frequency. Note that optimization of K_n is undertaken for constant V_{dsat} as opposed to bias current I_{bias} , since it is assumed the power supply voltage is fixed. Since the transistor gate length L is in the numerator, K_n is optimized when the gate length is chosen as the minimum possible for a given technology. The optimal gate width W may be found by differentiating K_n with respect to W and setting the derivative to zero:

$$\frac{\partial}{\partial W} K_n = \frac{2C_T C_T' - C_T^2}{W^2} = 0, \quad (4-9)$$

where $C_T \equiv C_{gs} + C_{gd} + C_p$, and $C_T' \equiv C_{gs} + C_{gd}$. Equation (4-9) is satisfied when the capacitance $C_T' = C_p$. This results in an optimal transistor width of:

$$W_{opt} = \frac{C_p}{C_{ox} \left(\frac{2}{3} L + L_d \right)}. \quad (4-10)$$

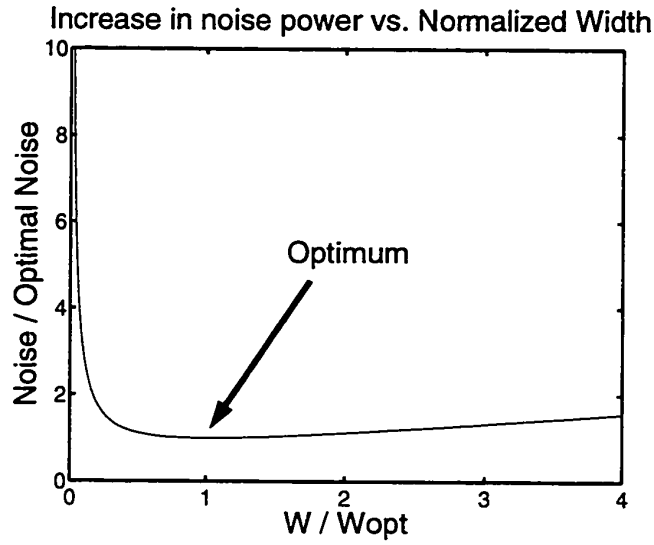


Figure 4-10. Normalized noise power vs. normalized input transistor width.

Intuitively, this result makes sense. For a very narrow gate, input-referred voltage noise from the transistor increases, causing increased current noise. For a very wide gate, the input capacitance increases faster than voltage noise decreases. An optimum gate width exists because the RMS voltage noise power is proportional to the inverse of the square root of width, while the input capacitance is directly proportional to width. Figure 4-10 plots the input-referred current noise versus normalized transistor gate width W/W_{opt} . This plot shows that noise performance is more sensitive to undersizing of input transistors than oversizing of input transistors.

The input common mode feedback amplifier uses a single-ended opamp with an active load to drive the capacitors C_{fb} shown in Figure 4-3. The main opamp input common mode voltage is calculated by two MOSFET transistors in parallel, M_{10} and M_{11} in Figure 4-8, each having a gate width one-half the size of the other transistor in the differential pair, M_9 . Since there are three transistor inputs to the differential pair, the sources of $M_9 \rightarrow M_{11}$ are not at a virtual ground. This causes an error in the calculation of the effective input common mode voltage when a differential voltage exists at the main opamp input. The output current error, as a function of $\Delta V_{in} \equiv V_{inp} - V_{inm}$, and the difference in input common mode voltage $\Delta V_{icm} \equiv (V_{inp} + V_{inm})/2 - V_{ref}$ is given by:

$$I_{error} = \frac{-\gamma \Delta V_{in}^2}{4} - \Delta V_{icm} \left(\sqrt{\frac{\gamma(4I_1 - \gamma \Delta V_{in}^2 - 2\gamma \Delta V_{icm}^2)}{2}} - \sqrt{\gamma(2I_1 - \gamma \Delta V_{icm}^2)} \right), \quad (4-11)$$

where $\gamma \equiv (k_n/2)(W/L)$. This may be expressed as an input-referred voltage error by dividing by the transconductance of M_9 . Since the input common mode feedback drives ΔV_{icm} toward zero, equation (4-11) reduces to:

$$V_{error} = \frac{-\Delta V_{in}^2}{8V_{dsat}}. \quad (4-12)$$

During the *sense* phases, differential feedback through the integrating capacitors drives ΔV_{in} towards the offset voltage of the main opamp. Since ΔV_{in} settles to within several mV, the error due to the effect modeled by equation (4-11) is negligible.

Several open-loop preamplifiers are cascaded after the integrator to increase gain before the compensator. The total delay product added by these preamps must be small enough to allow settling within the allotted period. Since the preamps are operating in the open loop configuration, they must have a high -3dB bandwidth. Given n identical preamps in series, each having a -3dB bandwidth of f_{3dB} , settling in response to a unit step is given by:

$$V_{out}(t) = 1 - e^{-at} \left(1 + at + \frac{(at)^2}{2!} + \dots + \frac{(at)^{n-1}}{(n-1)!} \right), \quad (4-13)$$

where $a \equiv 1/\tau_p \equiv 1/(2\pi f_{3dB})$. Equation (4-13) is a transcendental equation; therefore t may not be explicitly solved for as a function of desired settling accuracy. For the case of 3 preamps cascaded in series, numerically solving (4-13) for the time required to achieve 1%, 0.1% and 0.01% settling accuracy yields $8.8\tau_p$, $11.2\tau_p$, and $13.9\tau_p$ respectively. Since the charge integrator is a first order system, settling to the specified accuracies is achieved in $4.6\tau_i$, $6.9\tau_i$, and $9.2\tau_i$, where τ_i is the closed loop integrator time constant. To prevent slowing the position sense interface, the individual preamp settling time constant, τ_p , should be approximately twice as fast as τ_i .

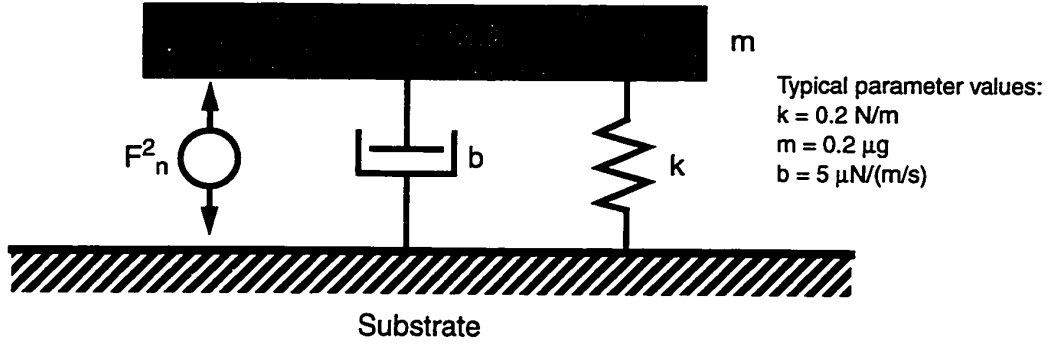


Figure 4-11. Schematic diagram of Brownian noise source in mechanical spring-mass-damper system.

4.5 NOISE ANALYSIS

Sensor output noise has 3 sources in addition to the quantization noise discussed in Chapter 3: mechanical or Brownian noise, electrical noise from the position interface and feedback circuits, and reflected noise from the electrical interface. The effects of system parameters on these various noise sources must be quantified to enable accurate predictions of accelerometer performance and optimized design of low-noise sensors.

4.5.1 BROWNIAN NOISE

Brownian noise enters the system at the proof-mass as a noisy force generator of magnitude:

$$F_n^2 = 4k_B T b \quad N^2/Hz, \quad (4-14)$$

[50] where k_B is Boltzman's constant ($1.38 \times 10^{-23} \text{ J/K}$), T is absolute temperature, and b is the mechanical damping coefficient described in Section 2.2.2. A schematic diagram of this force generator is shown in Figure 4-11. The noisy force generator may be transferred to an equivalent input acceleration by dividing by the sense-element mass, m :

$$a_n^2 = \frac{4k_B T b}{9.8^2 m^2} \quad G^2/Hz. \quad (4-15)$$

As an example, for the parameters shown in Figure 4-11, the equivalent noise force equals

$2.88 \times 10^{-13} \text{ N}/\text{Hz}$, corresponding to an effective input acceleration of $145 \mu\text{G}/\text{Hz}$. As equation (4-15) shows, both reducing air damping and increasing the sense-element mass have the desired effect of lowering the mechanical noise floor. Due to size limitations, mass is limited to only a fraction of a micro-gram for surface micromachined sensors. As described in Section 2.2.2.3, gas damping is strongly dependent on pressure within the cavity [7]; therefore, Brownian noise may be decreased by vacuum packaging. Since gas damping over low to medium vacuum is proportional to pressure, decreasing RMS Brownian noise by one order of magnitude necessitates lowering cavity pressure by two orders of magnitude.

4.5.2 ELECTRICAL NOISE SOURCES

In order to quantify the effects of electrical noise on the sensor output, it is useful to model the sensor with a noiseless electrical interface and all electrical noise generators referred to equivalent input acceleration noise generators. We do this by first referring all electrical noise generators to the output of the charge integrator. Electrical noise may then be converted to an equivalent acceleration by dividing by the transfer function between acceleration input and charge integrator output. Assuming that ICMFB keeps the voltage at the integrator input constant, this gain may be found using equations (2-21) and (4-4):

$$\frac{dV_{out}}{dA_{in}} = 9.8 \left(2 \frac{C_s}{x_0} \right) \frac{V_s}{C_i} \left(\frac{1}{2\pi f_r} \right)^2. \quad (4-16)$$

4.5.2.1 OPAMP NOISE

For clarity in the following analysis, a single-ended representation of the differential topology is shown in Figure 4-12. Use of correlated double sampling, described in Section 4.3, effectively eliminates the effects of flicker noise, and thus flicker noise will be ignored in this analysis. Opamp white noise may be referred to an equivalent input noise generator, as shown in Figure 4-12. The voltage fluctuations at the opamp input cause charge to flow onto the integrating capacitor from both the sense and parasitic capacitors, causing the opamp noise to be amplified by a factor of:

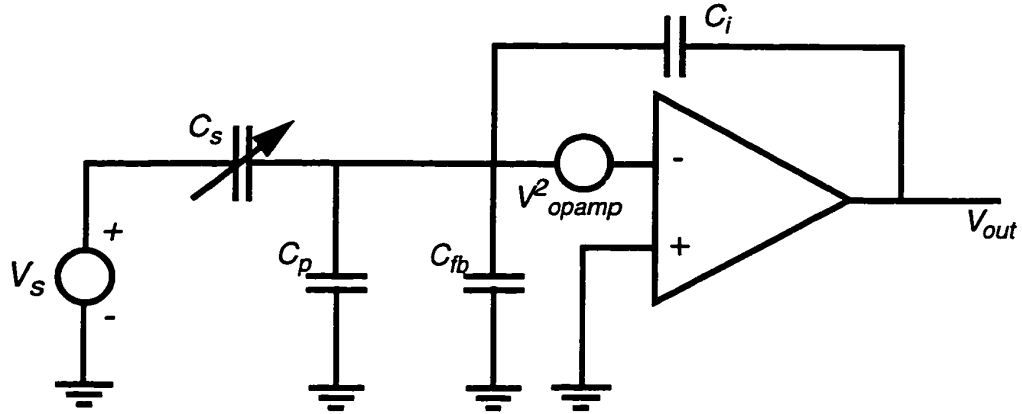


Figure 4-12. Single-ended representation of opamp noise source and capacitive feedback network for opamp thermal noise analysis. All opamp thermal noise sources are referred to the opamp input.

$$\frac{V_{out}^2}{V_{opamp}^2} = \left(\frac{C_s + C_p + C_i + C_{fb}}{C_i} \right)^2. \quad (4-17)$$

Since this noise is wideband and the output is sampled, the output noise is aliased into the inband frequency range. Furthermore, because the output is sampled twice, once at the end of the *sense a* phase and once at the end of the *sense b* phase, the noise power contributed by this source is doubled. The total effective in-band noise power may be written as:

$$\frac{V_{out}^2}{V_{opamp}^2} = \left(\frac{C_T}{C_i} \right) \left(\frac{2\pi f_u}{f_s} \right), \quad (4-18)$$

where $C_T \equiv C_s + C_p + C_i + C_{fb}$ and f_u is the unity-gain bandwidth of the amplifier. Assuming that all opamp noise comes from the input transistors $M_{I,2}$, equation (4-18) may be rewritten as:

$$\frac{V_{out}^2}{\Delta f} = \frac{16 C_T}{3 C_i C_{out}} \frac{kT}{f_s} \frac{1}{Hz}, \quad (4-19)$$

where C_{out} includes both the output capacitance and the load on the opamp.

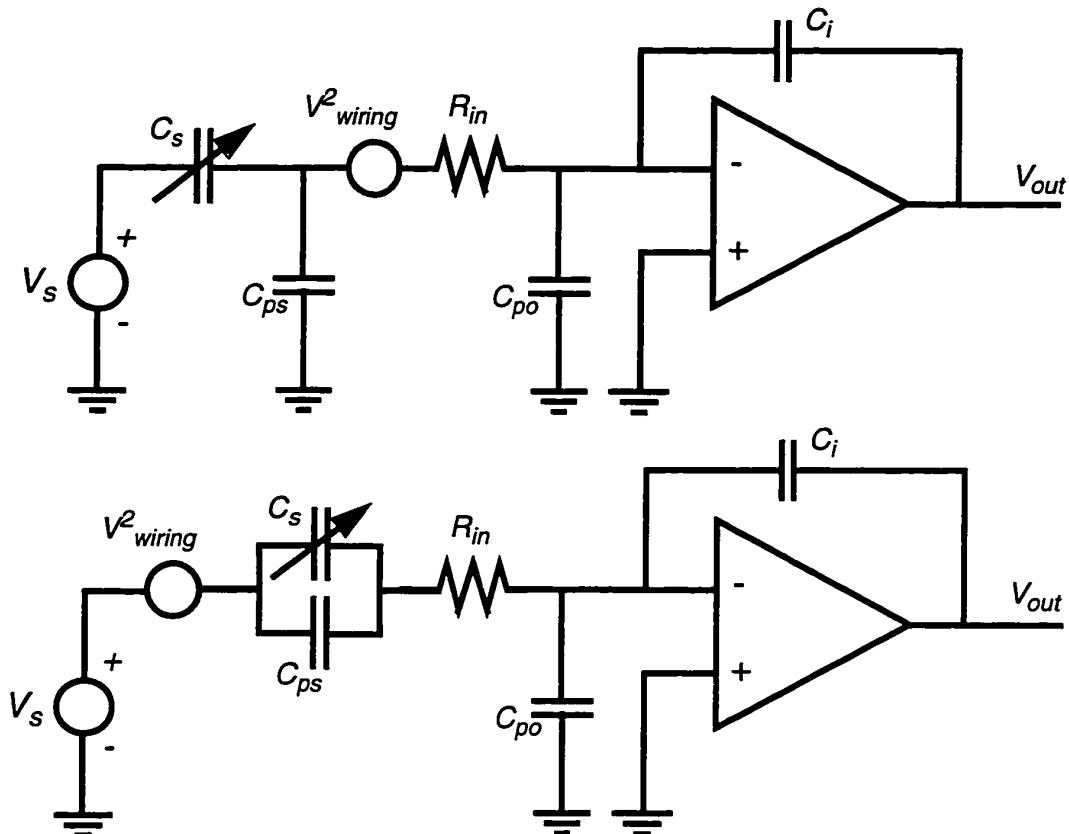


Figure 4-13. (a - Top) Wiring resistance thermal noise analysis. Parasitic capacitances are separated into capacitances located at the structure (C_{ps}) and at the opamp input (C_{po}). (b - Bottom) Schematic redrawn for easier analysis.

This result may be interpreted as follows. Settling requirements dictate a required minimum gain-bandwidth product: g_m/C_{out} . While increasing only g_m decreases opamp noise power density, bandwidth simultaneously increases, and thus total sampled noise is unchanged. By raising both g_m and C_{out} in equal proportions, the noise power density is lowered while bandwidth is kept constant. The result is reduced aliasing of high frequency noise, and hence a lowered noise floor.

4.5.2.2 WIRING RESISTANCE

Wiring resistance between the sense capacitors and the charge integrator also contributes white noise to the sensor output. Because thermal noise from the suspension resistance is a common mode input to the charge integrator, it will be ignored in this analysis. Noise from wiring of the stationary comb fingers and the switches $s_{3,4}$, shown in

Figure 4-6, may be lumped together into one equivalent resistance R_{in} as illustrated in Figure 4-13 (a). Rearranging this circuit into the configuration shown in Figure 4-13 (b) permits an easier analysis. Assuming the opamp has infinite bandwidth, the gain from V_{wiring}^2 to the integrator output is given by:

$$\frac{V_{out}^2}{V_{wiring}^2} = \left(\frac{C_s + C_{ps}}{C_i} \right)^2 \left(\frac{1}{(1 + j\omega R_{in}(C_s + C_{ps}))} \right)^2. \quad (4-20)$$

Note that equation (4-20) has a constant gain of $(C_s + C_{ps}/C_i)^2$ for frequencies below the -3dB point of the RC network:

$$f_c \equiv \frac{1}{2\pi R_{in}(C_s + C_{ps})}. \quad (4-21)$$

Integrating the output noise power from $f = 0$ to $f = \infty$, and dividing by the sampling bandwidth, the power spectral density with aliasing is found to be:

$$\frac{V_{out}^2}{\Delta f} = \frac{4}{f_s/2} \frac{C_s + C_{ps}}{C_i} \left(\frac{kT}{C_i} \right) \frac{V^2}{Hz}. \quad (4-22)$$

The first term includes a factor of 4 to account for both the double sampling of the integrator output and the presence of wiring resistance on both sides of the integrator. In a real implementation, the closed loop opamp bandwidth will be somewhat less than infinite. If the closed loop integrator bandwidth, f_{-3dB} , is less than f_c in (4-21), the noise due to wiring resistance will be reduced to:

$$\frac{V_{out}^2}{\Delta f} = 16kTR_{in} \left(\frac{\pi f_{-3dB}}{f_s} \right) \left(\frac{C_s + C_{ps}}{C_i} \right)^2 \frac{V^2}{Hz}. \quad (4-23)$$

Equation (4-23) shows that noise from wiring resistance may be attenuated by making R_{in} small. Wiring resistance should also be made small to ensure $f_c \gg f_{-3dB}$, so settling is limited by the charge integrator.

Using equations (4-23) and (4-18), the value of R_{in} at which noise from wiring resistance equals opamp thermal noise may be found:

$$R_{in} = \frac{2}{3g_m} \left(\frac{C_T^2}{(C_s + C_{ps})^2} \right), \quad (4-24)$$

where g_m is the transconductance of transistors $M_{1,2}$ in Figure 4-8. For typical capacitor values, (4-24) evaluates to approximately $2/g_m$, or 1-2K Ω in a MOS technology.

4.5.2.3 PREAMP NOISE

Noise from the first stage of the preamp again is double sampled and aliased into the sampling bandwidth, yielding a noise floor of:

$$\frac{V_{out}^2}{V_{preamp}^2} = 2 \left(\frac{f_u}{f_s/2} \right) \frac{\pi}{2} = 2\pi \left(\frac{f_u}{f_s} \right). \quad (4-25)$$

Later stages of preamps contribute negligible thermal noise to the sensor output since noise, when referred to the integrator output, is divided by the gain of both the first preamp and the integrator. As described in Section 4.3, kT/C_h noise from sampling of the hold capacitors, C_h , may be reduced by staggering sampling of the hold capacitors in subsequent stages.

4.5.3 REFLECTED NOISE

Low frequency opamp noise and opamp offset drive the input terminals of the integrator to different potentials during the *sense a* and *sense b* phases, when the opamp is operating closed loop. During the *sense a* and *sense b* phases, a common mode d.c. bias equal to $\pm V_s$ also appears across the capacitive half bridge. The voltage across each capacitor imparts an electrostatic force to the proof-mass. Since the opamp input terminals are at different potentials, the magnitudes of these forces are slightly unequal, as shown in Figure 4-14. Mismatch between these forces results in a net force on the proof-mass, indistinguishable from an input acceleration. The effective magnitude of this force is dependent not only on the error and sense pulse voltages, but also on the length of the two *sense* phases. Integrating the force error over one complete period allows the input-referred reflected *acceleration* noise to be found:

reflected signals through the position sense interface is negligible because the proof-mass acts as a second order filter, effectively filtering out high frequency forces. The beating of high-frequency opamp noise to low frequency forces is larger, since multiplication of high frequency noise by the sense pulse brings energy at multiples of the sense pulse frequency into the baseband. In practice, however, the energy contribution from high frequency opamp noise is smaller than that of low frequency components due to the $1/f$ noise spectrum seen in MOSFET electronics. To minimize reflected noise, the gain factor, λ , in equation (4-26) may be made small by setting $V_{s-} = -V_{s+}(T_{sa}/T_{sb})$.

Note that reflected noise is independent of the proof-mass natural frequency. This is because reflected noise appears as a *force* on the proof-mass. In contrast, electrical noise in the position sense interface is referred to an equivalent *position* noise at the proof-mass.

4.6 COMPENSATOR & QUANTIZER DESIGN

Chapter 3 provided an analysis of the effects of system level design parameters on stability and quantization noise. The result of this analysis was a specification for a stabilizing discrete-time lead compensator of the form shown in equation (3-12). In order to implement the compensator in the sampled data system there are two additional requirements:

- 1) The compensator must settle very quickly after sampling of the position sense output for two reasons. First, in order to minimize the delay T_c/T , the time between sampling the position sense circuitry and applying feedback should be small. Compensator settling time contributes directly to this time delay. Second, time allotted for compensator settling necessarily comes from the feedback phase, and thereby reduces the dynamic range of the sensor.
- 2) The compensator must be differential in order to maintain the benefits obtained by using a completely differential electrical interface.

A simple method of implementing the compensator to achieve these goals is shown in Figure 4-15. The compensator uses charge redistribution among three capacitors to obtain

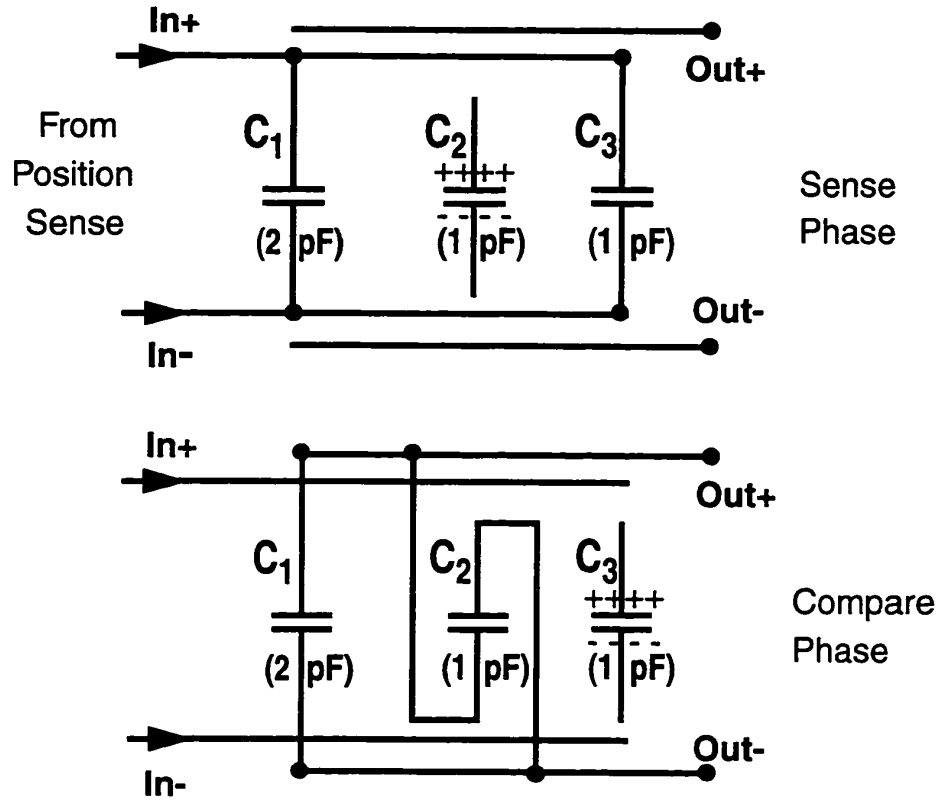


Figure 4-15. Simplified compensator schematic diagram. Capacitors C_2 and C_3 alternate storing the value of the position sense circuitry output.

the delay and coefficients comprising the 2-tap FIR filter. Capacitor C_1 realizes the first tap of the FIR. Capacitors C_2 and C_3 alternate storing the value of the delayed position, realizing both the coefficient and the delay operator z^{-1} of the second tap. Operation of this compensator occurs in two phases. During the end of the position sense phase, capacitors C_1 and C_3 sample the output of the final preamp. During the beginning of the compare phase, C_1 and C_3 are disconnected from the position sense circuitry. Capacitor C_3 is set aside for the next period, thereby implementing the delay operator. Capacitor C_2 , which has stored the output of the position sense circuitry from the previous period, is inverted and connected with C_1 . The charge stored on each capacitor is given by:

$$Q_1 = C_1(V_{in+}(k) - V_{in-}(k)) = C_1\Delta V_{in}(k), \quad (4-27)$$

$$Q_{2,3} = C_{2,3}(V_{in+}(k-1) - V_{in-}(k-1)) = C_{2,3}\Delta V_{in}(k-1). \quad (4-28)$$

When $C_{2,3}$ is inverted and connected to C_1 , the differential output voltage across the combined capacitors is given by:

$$\Delta V_{out}(k) = \frac{C_1 \Delta V_{in}(k) - C_{2,3} \Delta V_{in}(k-1)}{C_1 + C_{2,3}}. \quad (4-29)$$

As shown in equation (4-29), inverting $C_{2,3}$ produces a negative sign, scaling the capacitors $C_1/C_{2,3}$ sets the compensator zero, and alternating sampling between C_2 and C_3 achieves the delay operator. Equation (4-29) may be rewritten in the z-transform domain as:

$$\frac{\Delta V_{out}(z)}{\Delta V_{in}(z)} = \frac{C_1 - z^{-1} C_{2,3}}{C_1 + C_{2,3}} = \frac{C_{2,3}}{C_1 + C_{2,3}} (\alpha - z^{-1}) \quad \text{where } \alpha \equiv \frac{C_1}{C_{2,3}}. \quad (4-30)$$

Note the direct correspondence between the capacitor ratio $C_1/C_{2,3}$ and the location of the compensator zero in equation (3-12). Since the quantizer immediately follows the compensator, the value of the static coefficient $C_{2,3}/(C_1 + C_{2,3})$ is irrelevant.

Unfortunately, the simplified circuit of Figure 4-15 has two problems. First, the common mode output voltage is undefined since the output is taken as the difference in plate voltages. Second, the parasitic capacitor formed by the bottom plate and the substrate creates asymmetry between the two terminals. A schematic of a compensator which solves these problems is shown in Figure 4-16. By using two capacitors to store the differential voltage, both differential- and common-mode voltages are sampled. By connecting the bottom plates to ground, the parasitic capacitance to the substrate is shunted.

Settling of the compensator is governed by the RC time constant set by the switch on-resistance and the size of the sampling capacitors. Typical values of sampling capacitance, $C_1 = 2\text{pF}$, and switch on-resistance, $1\text{K}\Omega$, result in a settling time constant of 2ns . During the beginning of the compare phase, settling is even faster since the effective capacitance is the series combination of C_1 and $C_{2,3}$, yielding a time constant of 0.7 ns . As illustrated by this example, this topology satisfies the requirement for fast settling.

To obtain accurate coefficients for the lead compensator, parasitic capacitances from

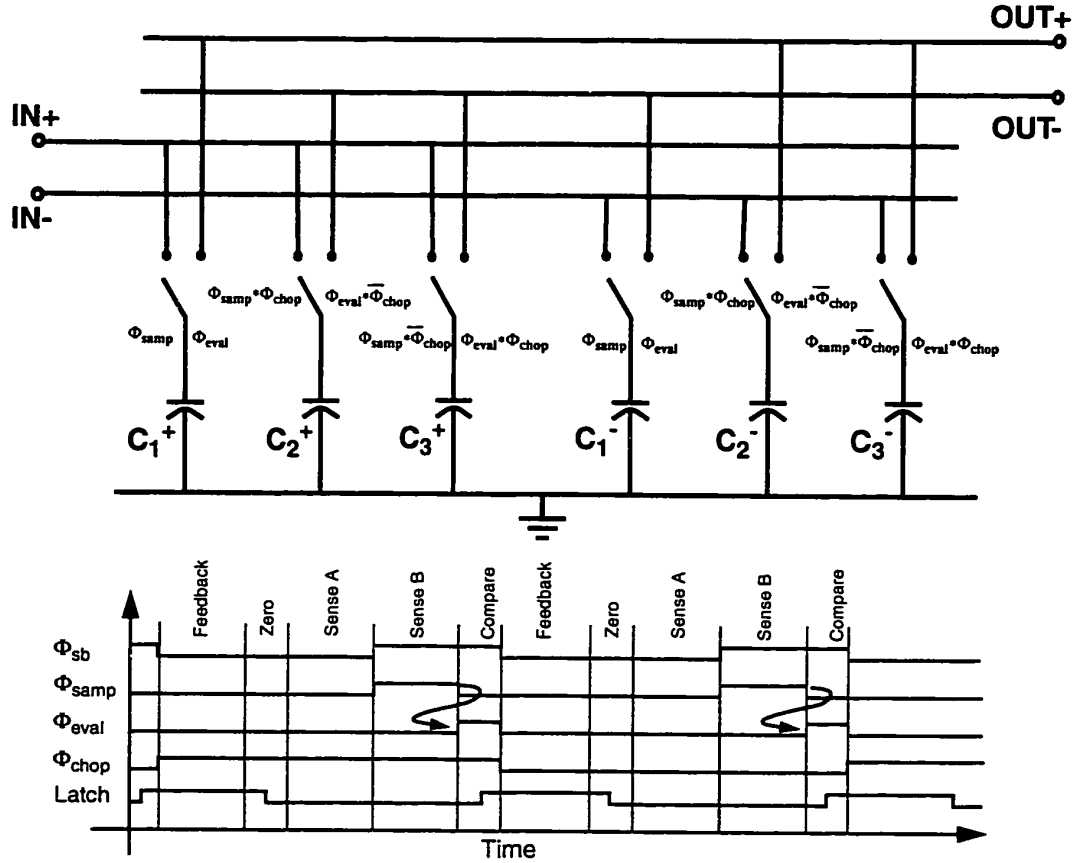


Figure 4-16. Actual implementation of compensator with timing diagram. Clock phase Φ_{sb} is shown as a reference to the sense interface clocking diagram in Figure 4-6.

both the source and drain diffusions of the MOSFET switches, as well as charge injection onto the holding capacitors must be modeled. Parasitic capacitors C_{lp} and $C_{2p,3p}$, in parallel with the sampling capacitors, increase the effective capacitance of both sampling capacitors, lowering α below the value predicted by equation (4-30).

Charge is injected onto the sampling capacitors when the position sense circuitry is sampled, introducing an error in the transfer function. Assuming fast rise/fall times on the clock edges, the error due to charge injection onto a capacitor C_h may be written as a gain and offset error [51]:

$$V_{hold} = V_{in}(1 + \epsilon) + V_{OSH}, \quad (4-31)$$

where

$$\varepsilon = \frac{1}{2} \frac{L_{eff} W C_{ox}}{C_h} \quad (4-32)$$

and V_{OSH} is a constant dependent on the switch sizes and the hold capacitor size. In (4-32), L_{eff} is the effective gate length of the switch, W is the gate width, C_{ox} is the oxide capacitance per unit area and C_h is the value of the capacitor being sampled. Since V_{OSH} is sampled onto each capacitor in the pair, the differential voltage error from this source is canceled. By including parasitic capacitances with the results of (4-31) and (4-30) we find:

$$\frac{\Delta V_{out}(z)}{\Delta V_{in}(z)} = \frac{C_1' - z^{-1} C_{2,3}'}{C_1' + C_{2,3}'} + \frac{L_{eff} W C_{ox} (1 - z^{-1})}{2(C_1' + C_{2,3}')}, \quad (4-33)$$

where $C_1' \equiv C_1 + C_{lp}$ and $C_{2,3}' \equiv C_{2,3} + C_{2p,3p}$. By placing (4-33) in the form of (4-30) we can solve for the value of α :

$$\frac{\Delta V_{out}(z)}{\Delta V_{in}(z)} = \frac{C_{2,3}' + 0.5 L_{eff} W C_{ox}}{C_1' + C_{2,3}'} (\alpha - z^{-1}) \quad \text{where } \alpha \equiv \frac{C_1' + 0.5 L_{eff} W C_{ox}}{C_{2,3}' + 0.5 L_{eff} W C_{ox}}. \quad (4-34)$$

Again, the static gain in (4-34) is unimportant since the quantizer immediately follows the compensator. The effect of parasitic capacitances and charge injection on the value of α can be seen via an example. For $C_1 = 2\text{pF}$, $C_{2,3} = 1\text{pF}$, $C_{lp,2p,3p} = 150\text{fF}$, $L_{eff} = 1.8\mu\text{m}$, $W = 10\mu\text{m}$, and $C_{ox} = 1.9\text{fF}/\mu\text{m}^2$ equation (4-30) predicts $\alpha = 2.0$ while (4-34) predicts $\alpha = 1.86$. Simulations using SPICE show an actual value of $\alpha = 1.87$. Thus, neglecting charge injection and parasitic capacitances would result in overcompensation.

Quantization of the compensator output is accomplished with a regenerative comparator after compensator settling. Because the quantizer follows several gain stages, requirements on comparator offset are relaxed, enabling use of the very fast and simple CMOS comparator described in [52]. A switch between the input terminals of the comparator is closed during the *zero* phase to clear charge stored on the input capacitance from the previous period.

4.7 ONE-BIT FEEDBACK

As described in Section 3.2, incorporating the compensator in the forward path

enables single-bit force feedback, simplifying the feedback topology greatly. When a voltage V_{fb} is applied across a parallel plate capacitor C_s , an attractive electrostatic force results, given by equation (2-26):

$$F_{fb} = \frac{1}{2} \frac{dC_s}{dx} V_{fb}^2. \quad (4-35)$$

One-bit forcing may be achieved by applying a feedback voltage, V_{fb} , across one sense capacitor while applying zero potential difference across the second sense capacitor. Feedback is applied during the *feedback* phase for a fraction of the period equal to T_f/T . Using the results of (3-11) and (4-35), the full scale feedback acceleration in G is given by:

$$a_{fs} = \frac{1}{9.8m} \frac{1}{T} \left(\frac{1}{2} \frac{dC_s}{dx} V_{fb}^2 \right). \quad (4-36)$$

For example, with a proof-mass of $0.25\mu\text{g}$, $T_f/T = 0.25$, $dC_s/dx = 50\text{fF}/\mu\text{m}$, and $V_{fb} = 5\text{V}$ the full scale feedback acceleration is $\pm 64\text{G}$. Note that this value is independent of the resonant frequency of the proof-mass.

A bulk micromachined $\Sigma\Delta$ accelerometer reported in [34] achieves one-bit feedback by applying a voltage $0.5V_{fb}$ to one stationary plate and a voltage $-0.5V_{fb}$ to the other stationary plate. Positive and negative feedback values are obtained by varying the proof-mass voltage between $\pm 0.5V_{fb}$. In this manner a voltage of V_{fb} is applied across one sense capacitor, while zero voltage appears across the second capacitor. Note the proof-mass must be charged to one of two different voltages, depending on feedback direction. Proof-mass settling to these two different voltages will be different, due to variations in switch on-resistance at the two different bias voltages. Differences in settling time create a systematic offset since feedback in one direction will be greater than in the other direction. Changes in switch on-resistance due to fluctuations in temperature cause this offset to shift.

A method which is entirely symmetrical, avoiding the above problem, is shown in Figure 4-17. Note the proof-mass is grounded to the same potential every period, regardless of feedback direction. A voltage of V_{fb} is applied across the capacitor in the

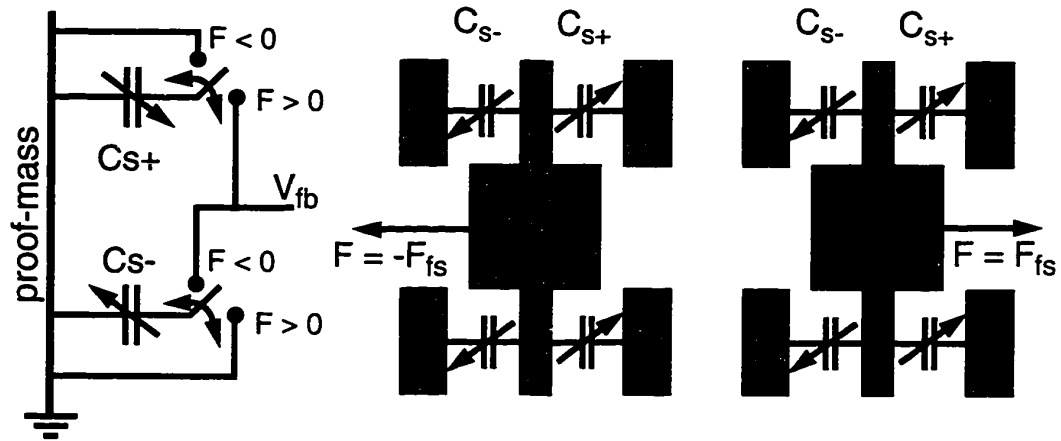


Figure 4-17. One-bit force feedback is applied by grounding the proof-mass and applying a voltage V_{fb} across one set of sense capacitors.

desired feedback direction while the other capacitor is grounded. Since both sides settle to the same potential under application of the feedback voltage, V_{fb} , settling times are equal.

Mismatch of dC_s/dx in the two sets of sense capacitors will cause an offset due to unequal forcing in the positive and negative directions. Mismatch causes more pulses in the direction with a lower dC_s/dx under zero input, thus shifting the output by an offset acceleration equal to:

$$\Delta a = \frac{1}{9.8m} \frac{1}{T} \frac{1}{2} V_{fb}^2 \left(\Delta \frac{dC_s}{dx} \right). \quad (4-37)$$

Note that mismatch in dC_s/dx is as small as a few hundredths of a percent, as will be discussed in Section 4.8, which corresponds to an offset on the order of 10mG. Compared to the other sources of mismatch, this source is negligible.

4.8 DIGITAL OFFSET TRIMMING

Offsets from many sources can appear in the sensor output if not canceled, reducing dynamic range and causing the d.c. output level to vary from die to die. Shift in offset over time or temperature causes output drift. System topologies described in this chapter make the output insensitive to some of these offsets. Other offsets cannot be canceled in this

manner. In order to manufacture accelerometers, these offsets must be easily trimmed to within specifications.

Use of fully differential position sense circuitry, described in Section 4.2, eliminated the need for good matching between two sense pulses. Section 4.2 also described using input common mode feedback to remove output offset from mismatch between both parasitic capacitances and the charge integrator capacitors. Section 4.3 discussed the use of correlated double sampling to sample opamp and preamp offsets and subtract them from the position sense output.

Two sources of offset which are indistinguishable from an applied input acceleration are sense capacitor mismatch and input common mode feedback capacitor mismatch. Mismatch in capacitors is caused both by processing gradients across the wafer, and by variations in edge location due to the limited resolution of the photolithographic process [53]. References [54, 55] show that for small oxide capacitors, i.e. those less than approximately $50\mu\text{m}$ on a side, edge roughness is the dominant source of mismatch. For capacitors larger than $50\mu\text{m}$, mismatch is dominated by thickness and permittivity variations in the oxide used as a dielectric. References [55, 56] show a 1σ mismatch in $20\mu\text{m}$ capacitors of approximately 0.2%. Capacitors with an edge size of $100\mu\text{m}$ had a measured mismatch an order of magnitude less than that of the $20\mu\text{m} \times 20\mu\text{m}$ capacitors.

Common-centroid layout of the air-gap sense capacitors, as described in Section 2.3, can decrease mismatch due to processing gradients. Since the sense element has a lateral dimension of order $1000\mu\text{m}$, edge roughness will also be averaged out yielding a very small amount of mismatch. The capacitors C_{fb} , on the other hand, will typically not be as well matched since the dimensions of these capacitors are several orders of magnitude smaller than the sense capacitors, and are more sensitive to variations in vertical film thickness and oxide permittivity.

Using equation (4-4) and the results of equation (2-21) in Section 2.3.5, these capacitor mismatches may be translated into an equivalent acceleration offset:

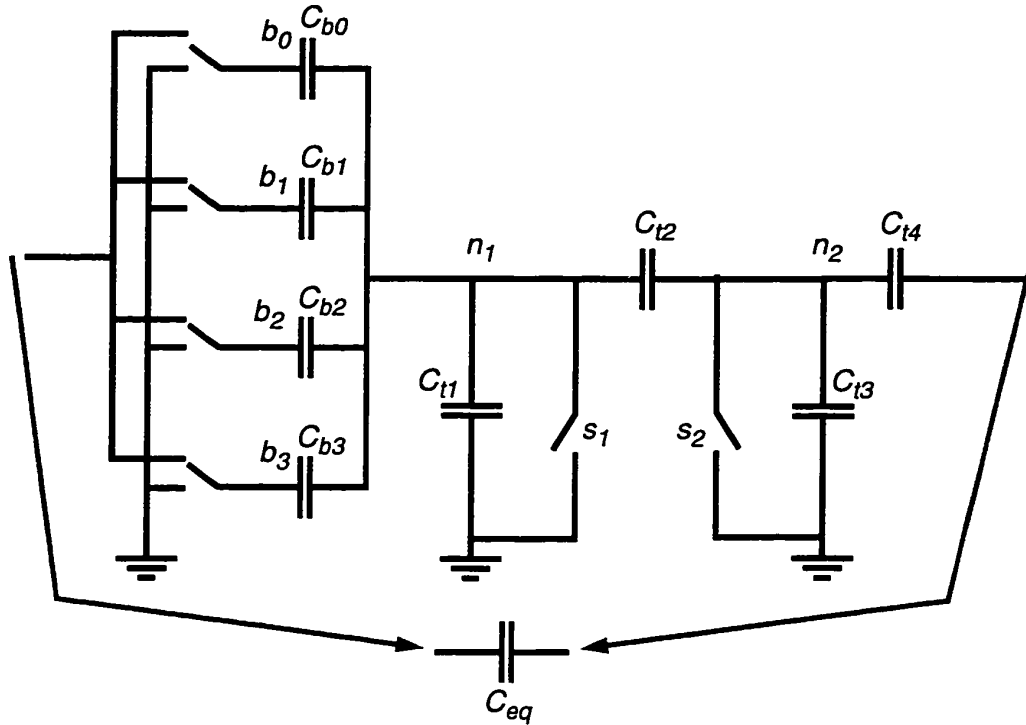


Figure 4-18. Binary weighted array for digital trimming. By switching in capacitors C_{bx} the value of C_{eq} may be varied.

$$\Delta a = \frac{x_0(2\pi f_r)^2}{9.8} \left(\frac{\Delta C_s}{C_s} + \frac{\Delta C_{fb}}{C_{fb}} \right). \quad (4-38)$$

For $C_{fb} = C_s = 200$ fF, $x_0 = 2\mu\text{m}$, $f_r = 5$ kHz, 0.2% mismatch of $C_{fb1,2}$ and 0.02% mismatch of $C_{s1,2}$ we find an expected output offset of 0.5G.

Since these offsets are caused by capacitive mismatch, they may be canceled in the position sense interface through addition of a small capacitor in parallel with the sense capacitors. Two binary weighted capacitor arrays, one in parallel with each sense capacitor, may be used to trim the output of the device to within one LSB, for both positive and negative values of $\Delta C_{s,fb}$. According to the above analysis, the total capacitance needed to cancel offset is on the order of ± 0.5 fF. It is impractical to build oxide capacitors of this size, however capacitors may be formed indirectly through the use of a capacitive divider [57]. A 4-bit binary weighted array using this topology is shown in Figure 4-18. Two stages of attenuation allow reasonable sized capacitors to be used in the binary

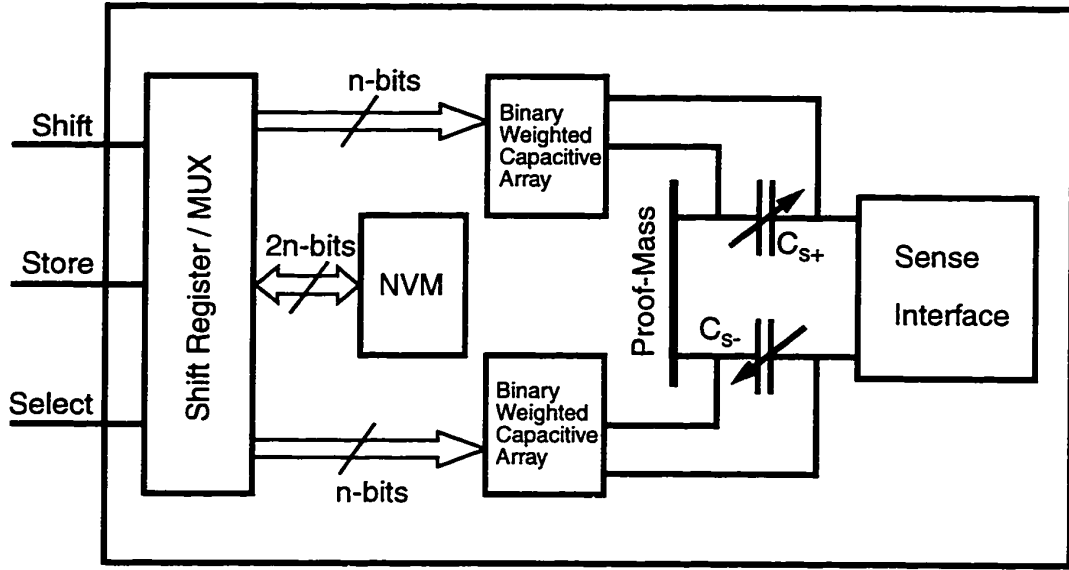


Figure 4-19. Complete digital offset trimming system. A one-bit serial input (*Shift*) selects proper coefficients after packaging. Non-volatile memory (*NVM*), which is multiplexed with the shift register (*Select*), stores calibration coefficients (*Store*).

weighted array. The effective value of capacitance from this network is:

$$C_{eq} \approx \frac{(b_0 C_{b0} + b_1 C_{b1} + b_2 C_{b2} + b_3 C_{b3})}{(C_{b0} + C_{b1} + C_{b2} + C_{b3}) + C_{t1}} \left(\frac{C_{t2}}{C_{t2} + C_{t3}} \right) C_{t4}. \quad (4-39)$$

In this equation, C_{t1} and C_{t3} are chosen to be much greater than C_{t2} and C_{t4} to realize a very small effective capacitance. The switches of the binary weighted array are set to cancel offset from mismatches in C_s and C_{fb} . A compact method for achieving a monotonic, capacitive array with a larger number of bits is outlined in [58].

The nodes marked n_1 and n_2 in Figure 4-18 are floating and must be zeroed. Zeroing is accomplished by closing switches s_1 and s_2 during the feedback phase of the sigma-delta loop. At the end of the *feedback* phase these switches are opened, injecting charge and kT/C noise onto the nodes. During the *zero* phase, both the sense capacitors and the binary weighted arrays are grounded. Since the nodes n_1 and n_2 are capacitively coupled, charge injection and kT/C noise sampled during the end of the feedback phase are canceled.

Trimming may be performed post-packaging using a serial input line attached to a shift register, as shown in Figure 4-19. Coefficients are stored in a non-volatile memory (NVM), such as polysilicon fuses [59, 60] or an on-chip EEPROM. In-package trimming permits cancellation of package-induced stress, in addition to random offset from capacitance mismatch. In keeping with the goals set out in Section 4.1, this method of offset trimming lends itself to standard CMOS technologies.

4.9 MULTI-AXIS SERVO CONTROL

Simultaneous multi-axis servoing of a proof-mass has applications beyond multi-axis acceleration measurement. When operating in vacuum, to achieve a lower noise floor for instance, force balancing may be used to stabilize undamped modes. In this case, feedback is used only to damp certain structural modes and not for direct measurement of these modes. This allows the use of smaller air-gap capacitors. Other sensors requiring multi-axis sensing, such as vibratory rate gyroscopes, may use this interface to both sense and drive the proof-mass around different axes. The digital output of the $\Sigma\Delta$ topology enables digital demodulation and quadrature correction of the gyro output signal.

Recent work has shown the feasibility of three axis accelerometry using a single proof-mass in a closed-loop configuration [61]. Reference [61] measures position and force balances the proof-mass with a method similar to [16], using multiple carrier frequencies to sense different axes. Multiple carrier frequencies require careful attention to nonlinearities in the analog signal processing circuitry, which can cause intermodulation distortion. Additionally, the feedback loops of [16, 61] have a relatively narrow bandwidth necessitated by their compensation methods. By using the switched capacitor interface described in this section for force balancing all three axes, these problems may be overcome.

Figure 4-20 shows a 2.3 μm -thick mechanical sense-element for 3-axis servoing, with FEM simulations of lateral (x- and y-axes) and out-of-plane (z-axis) resonant modes. Resonant frequencies along the x-, y-, and z- axes are designed to be approximately equal for comparable performance in all 3-axes, as shown in Table 5-2. Quad symmetry of the

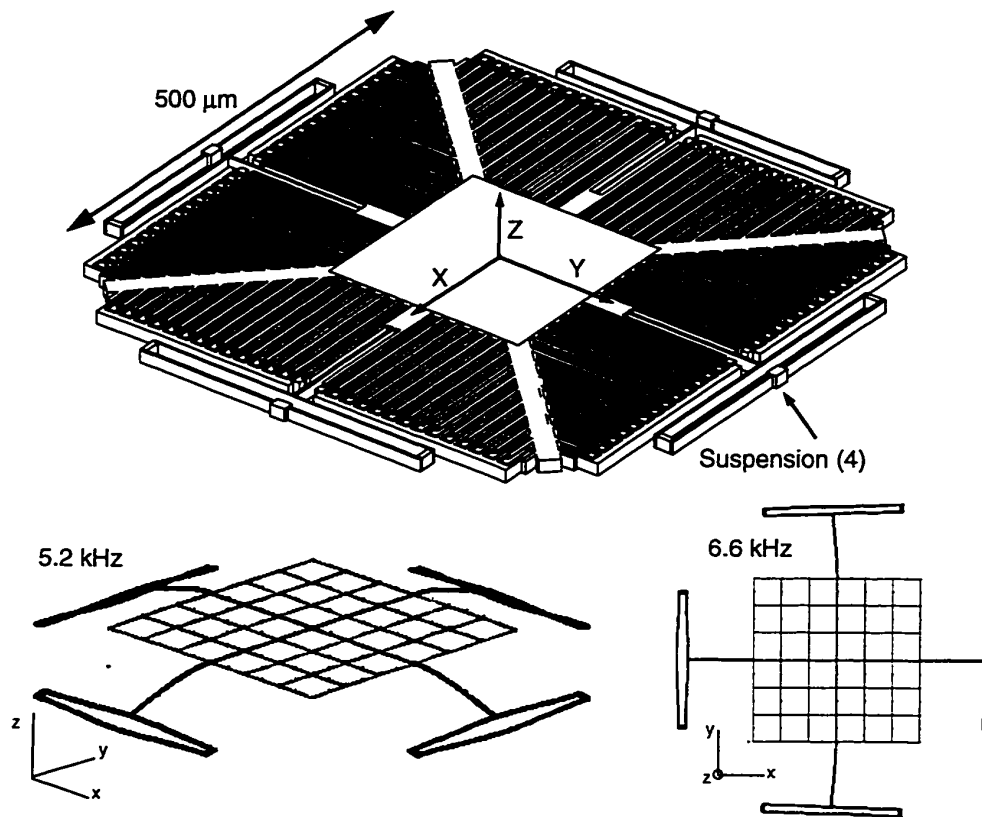


Figure 4-20. Undeformed 2.3 μm thick structure (top). PATRAN simulations of deformation due to applied z-axis (bottom left) and x-axis (bottom right) accelerations are shown.

proof-mass about the z-axis minimizes sensitivity to off-axis accelerations. When a lateral acceleration is applied to the substrate, comb finger gaps change from their nominal 2.2 μm value, causing an imbalance in the capacitive half bridge shown in Figure 4-21(a) [18]. By laying out comb fingers in a common centroid geometry, off-axis accelerations become a common-mode signal, resulting in first-order rejection of both translational and rotational off-axis accelerations [18].

Under an applied z-axis acceleration, the proof-mass moves out of plane. This motion causes a change in the parallel plate capacitance formed between the center of the proof-mass and the bottom plate, as shown in Figure 4-21(b). To realize a differential z-axis sense interface, a reference capacitor made from a separate mechanical structure is used in the half bridge.

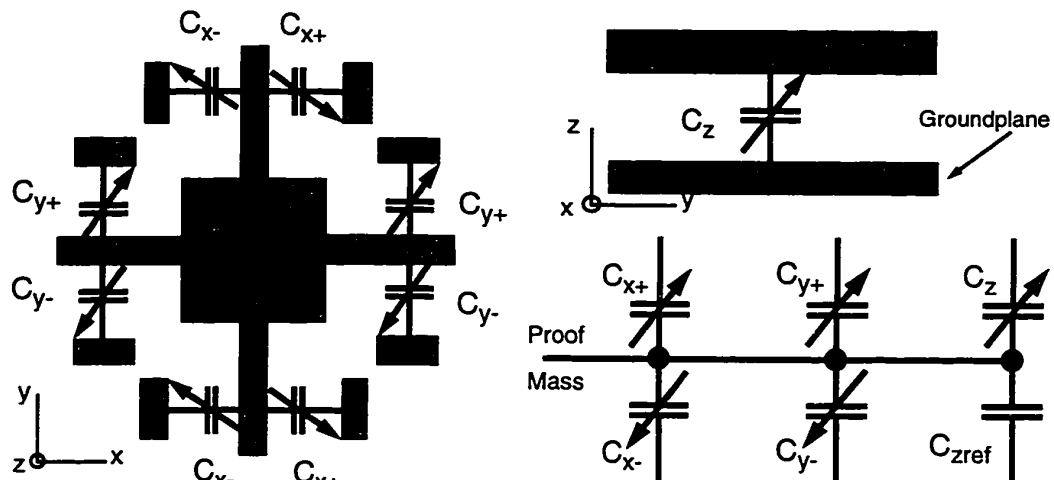


Figure 4-21. Proof-mass schematic for lateral acceleration measurement. (a - top left) schematic of x- and y-axis capacitive half bridges, (b - top right) z-axis sense capacitor, (c - bottom) complete sensor schematic. Proof-mass is a common node for all capacitors.

The mechanical sense element is designed so that the x-, y-, and z-axis capacitors are insensitive to off-axis deflections. Additionally, mechanical symmetry of the sense element provides good selectivity between different axes when force feedback is applied. Because of the small amount of electromechanical coupling between these orthogonal axes, feedback loop design may be undertaken one axis at a time, independent of other axes.

As discussed in Section 4.2, the center node voltage of the single-axis capacitive half bridge is driven by a low impedance voltage source during all phases of operation. Looking at Figure 4-21(c) it is clear this center node is common to all three capacitive half bridges. By connecting a set of differential position sense and feedback circuitry to both ends of each half bridge, shown in Figure 4-22, multi-axis servoing can be accomplished. Synchronization is required to ensure all three sets of position sense circuitry have been sampled before voltage pulses are applied to the proof-mass. Synchronization is attained by waiting for acknowledgment pulses from each set of circuitry before the proof-mass voltage is switched.

This same proof-mass may be converted to an x-, y-, z-axis *angular* accelerometer by

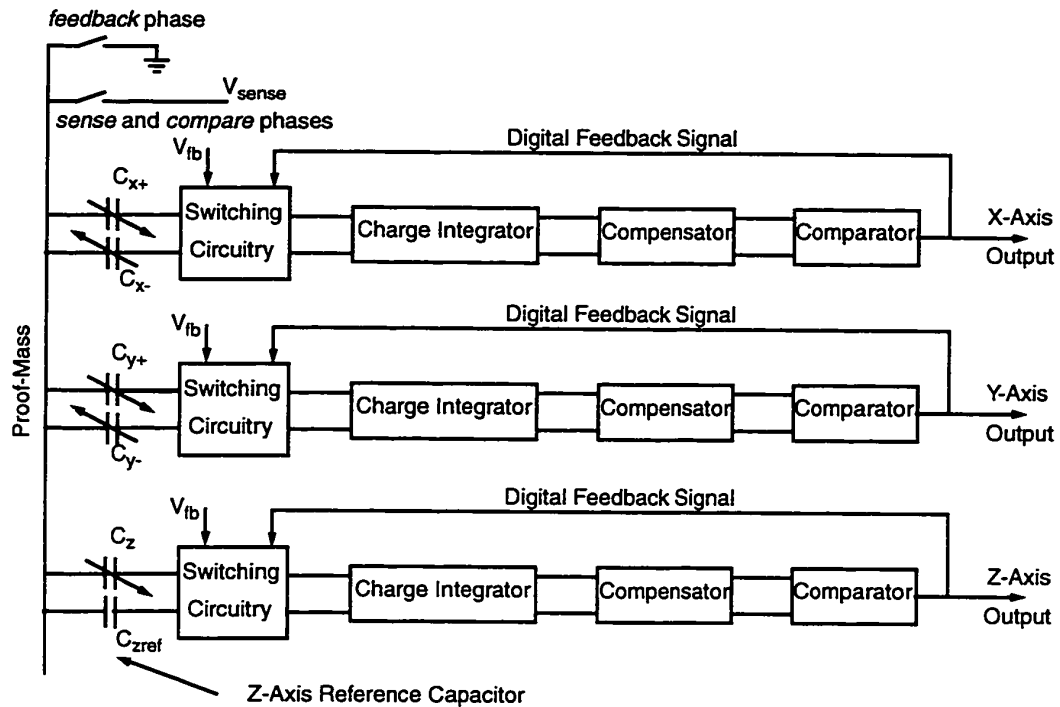


Figure 4-22. 3-Axis system diagram. Note the proof-mass is a common node for all capacitors, with the voltage at this node driven during both sense and feedback operations.

creating slightly different proof-mass/ground plane capacitors and rewiring the capacitors in the lateral axes as shown in Figure 4-23. Note the only difference between the configuration shown in Figure 4-21 and Figure 4-23 is in the wiring of the individual sense capacitor arrays. Thus the same mechanical sense element may be force balanced in six degrees-of-freedom (translation and rotation in x-, y-, z-axes) by interleaving two periods in the sigma-delta feedback loop, during which the capacitors are rewired with MOSFET switches.

4.10 SUMMARY

Following an overview of conventional methods of capacitive sensing techniques for microelectromechanical systems, this chapter explored a fully differential position sense interface. The interface is implemented in a digital CMOS technology, to enable co-fabrication of the sensor with signal processing circuitry. A fully differential position

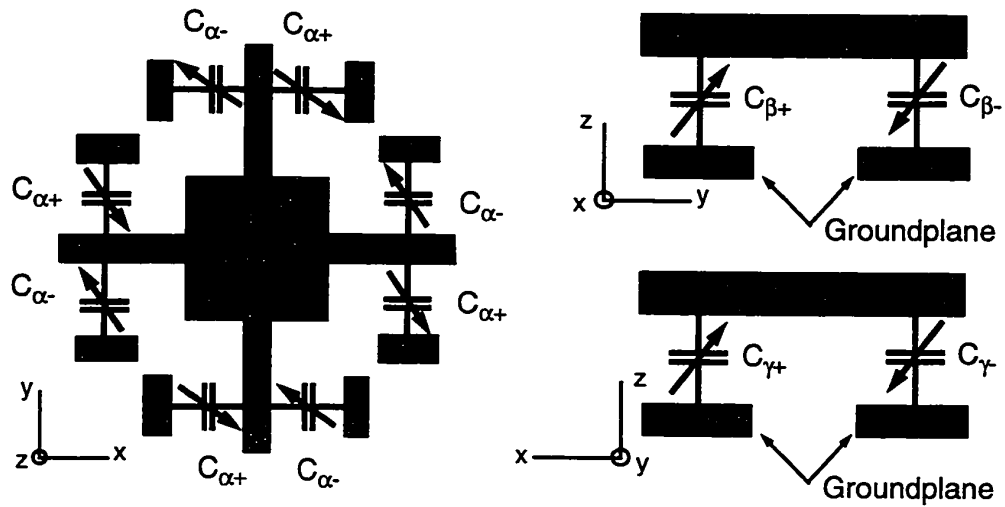


Figure 4-23. Capacitor configuration for rotational acceleration measurement about the z- (left), x- (top right) and y-axes (bottom right).

sense interface provides good power supply independence, allows first order cancellation of both substrate noise and charge injection, and permits ground-plane shielding to be connected to the proof-mass directly. Opamp flicker noise and offset are both cancelled using correlated double sampling. The d.c. levels of both the opamp input and the sense capacitors are maintained at a constant level through input common-mode feedback. Use of a binary weighted capacitive array controlled by a one-bit serial input allows trimming of offset errors after packaging, enabling cancellation of package-induced stresses.

In order to stabilize the feedback loop, as discussed in Chapter 3, compensation is necessary. Loop stability is achieved by locating a 2-tap finite impulse response filter in the forward path. A fully differential switched-capacitor compensator based on charge redistribution has been introduced. The compensator achieves fast settling and stray parasitic insensitivity in a simple implementation. A new method for realizing single-bit force feedback has been outlined. By maintaining symmetry about both the positive and negative directions, bias stability is increased over previously reported methods.

Finally, a 3-axis force-balanced accelerometer with a single proof-mass has been presented. By utilizing the differential sense interface discussed in this chapter, position sensing of multiple axes is easily achieved. Using MOS switches to rewire capacitors in

the three-axis accelerometer enables six degree-of-freedom force balancing of the same mechanical structure (x-, y-, and z- translational and rotational accelerations). Multi-axis force balancing damps parasitic vibrational modes when an accelerometer is operated in a vacuum to reduce noise floor. Multi-axis force balancing may also be used in applications such as gyroscopes, where one axis must be driven into oscillation while one or more separate axes are force balanced to zero position.

5. EXPERIMENTAL SYSTEM

5.1 DESCRIPTION OF FABRICATED DEVICE

The system topologies and design techniques discussed in Chapters 2 through 4 have been used to design two accelerometers, each measuring translational acceleration in the x-, y-, and z- axes. The first accelerometer, which will be referred to as the three-mass accelerometer, measures acceleration in three axes using three separate proof masses. The second accelerometer, which will be referred to as the single-mass accelerometer, force balances a single proof-mass in three axes, as described in Section 4.9.

Both of these designs were fabricated by Sandia National Laboratories in a surface micromachined technology with integrated CMOS. In this technology, the mechanical sense-elements are fabricated below the surface of the wafer in a recessed trench etched into bulk silicon. Polysilicon studs provide connectivity between the mechanical structures and integrated CMOS, as shown in Figure 5-1. After sense-element patterning, the trench region is filled with oxide, passivated, and the whole wafer planarized. At this point, the wafer is sent through a standard CMOS process for fabrication of the interface electronics [4, 62]. The CMOS used for the test accelerometers presented in this chapter has a minimum gate length of 2 μ m and uses a single layer of metal for interconnect. In this technology, capacitors are formed between gate polysilicon and a p+ substrate diffusion.

5.2 DESCRIPTION OF TEST SETUP

In order to test the performance of the sigma-delta accelerometers, it is necessary to apply test signals while simultaneously acquiring the one-bit output from the accelerometer. DC test accelerations may be applied to the accelerometer with a dividing head or a precision centrifuge, permitting calibration [63]. A dividing head provides a method for precisely orientating the accelerometer to the earth's gravitational field, enabling application of input accelerations between -1 G and +1 G. While a dividing head is useful for noise floor and linearity measurements over a ± 1 G field, it does not allow

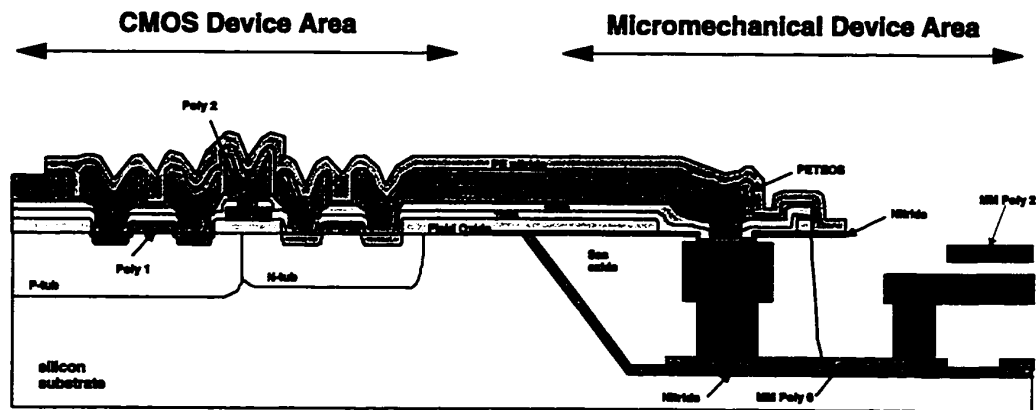


Figure 5-1. Cross section of the integrated MEMS with CMOS process used to fabricate test devices [4] (courtesy Jim Smith and Stephen Montague, Sandia National Laboratories).

measurement of these parameters near the upper end of the accelerometer's range. A precision centrifuge extends the upper limit of test accelerations, but does not allow testing of the accelerometer in response to a dynamic input.

An alternative test method uses a vibration exciter for measurement of dynamic properties in addition to the parameters mentioned above. In this testing arrangement, the device-under-test (DUT) and a reference accelerometer are fixed to the vibrator's face plate. A sinusoidal input acceleration is applied to the face plate and the DUT output is captured. Vibration exciters can apply large test accelerations to the DUT, enabling testing of the device over its full scale range. The tests described in this chapter were undertaken with a Brüel & Kjær vibration exciter (Model number 4808). The vibration exciter is powered by a power amplifier (Brüel & Kjær Model 2712) driven by an HP signal generator (Model 33120A). This setup has the capability of providing a maximum test acceleration of 20 G (14.1 G RMS) to a 0.9 lb load. A Brüel & Kjær piezoelectric reference accelerometer (Model 4381) mounted next to the DUT is used for calibration and measurement of the applied test accelerations.

The one-bit data stream from the DUT must be captured for analysis. This data must also be decimated to the bandwidth of interest to remove high frequency noise components from the output signal and lower the sample frequency to a useful rate.

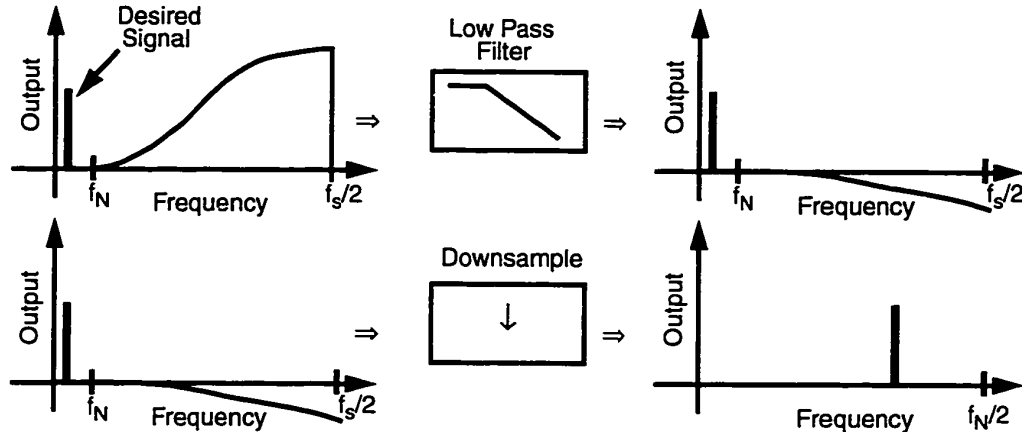


Figure 5-2. Schematic representation of decimation. High frequency noise is filtered from the output before downsampling to the new sample rate.

Decimation is a process in which a high bandwidth signal is first digitally filtered to remove high frequency noise, then downsampled to a lower rate. This procedure is graphically illustrated in Figure 5-2. Because high frequency signals are filtered out before the downsampling operation, there is minimal aliasing of high frequency noise components to baseband frequencies. Reference [64] describes a computationally efficient decimation algorithm which was used for testing.

The filtering and downsampling of the one-bit data stream in the test setup is computed by a digital signal processing board (Berkeley Camera Engineering part number S56X33) connected to a Sun Sparcstation 5. Data is acquired by a custom assembly language program. An interrupt service routine samples the one-bit output while simultaneously decimating to four times the desired final sample rate. Direct virtual memory access (DVMA) transfer of the decimated signal to the workstation permits continuous data transfer to the computer, where it is written to a file for analysis. The file is further decimated off-line using MATLAB. Performance specifications are extracted using the methods described in Section 5.3.

5.3 PERFORMANCE METRICS

The output response of the accelerometer to a sinusoidal input signal consists of a d.c.

level, output at the fundamental frequency, harmonic distortion components, and random components due to noise. In order to quantify accelerometer dynamic performance, it is necessary to separate the output signal into these various elements.

A computationally efficient method for decomposing the output into the above components, called the sinusoidal minimum error method, is described in [65]. This method uses least squares estimation to find the best fit between the accelerometer output signal and a template comprised of a d.c. offset, a fundamental component, and harmonic components. For an input signal:

$$u(t) = A \cos(2\pi f_u t), \quad (5-1)$$

with the output sampled at discrete time intervals kT , a template of the following form is used:

$$\hat{y}(kT) = \underbrace{a_0}_{\text{offset}} + \underbrace{a_1 \cos(2\pi f_u kT + \phi_1)}_{\text{signal}} + \underbrace{\sum_{n=2}^N a_n \cos(2\pi n f_u kT + \phi_n)}_{\text{harmonics}}. \quad (5-2)$$

Best fit in this context refers to minimization of the mean squared error between the output and the template:

$$\min_{a_n, \phi_n} \|e(kT)\|_2 = \min_{a_n, \phi_n} \|y(kT) - \hat{y}(kT)\|_2. \quad (5-3)$$

The minimization problem is linear in amplitude a_n but nonlinear in phase ϕ_n . Transforming the problem from the time domain into the frequency domain makes the minimization problem linear in both amplitude and phase. This transformation uses the unit circle in the complex plane formed by $e^{j\theta}$, with one revolution about the circle corresponding to one period:

$$\hat{y}(kT) = \sum_{n=-N}^N \frac{a_n}{2} e^{j(2\pi n f_u)kT + \phi} = \sum_{n=-N}^N \underbrace{\frac{a_n}{2} e^{j\phi_n}}_{\text{Magnitude \& Phase Coefficient}} e^{j(2\pi n f_u)kT}. \quad (5-4)$$

Equation (5-4) is in a form such that the complex variables $(a_n \exp(j\phi_n))/2$ may be optimally determined using a least squares projection.

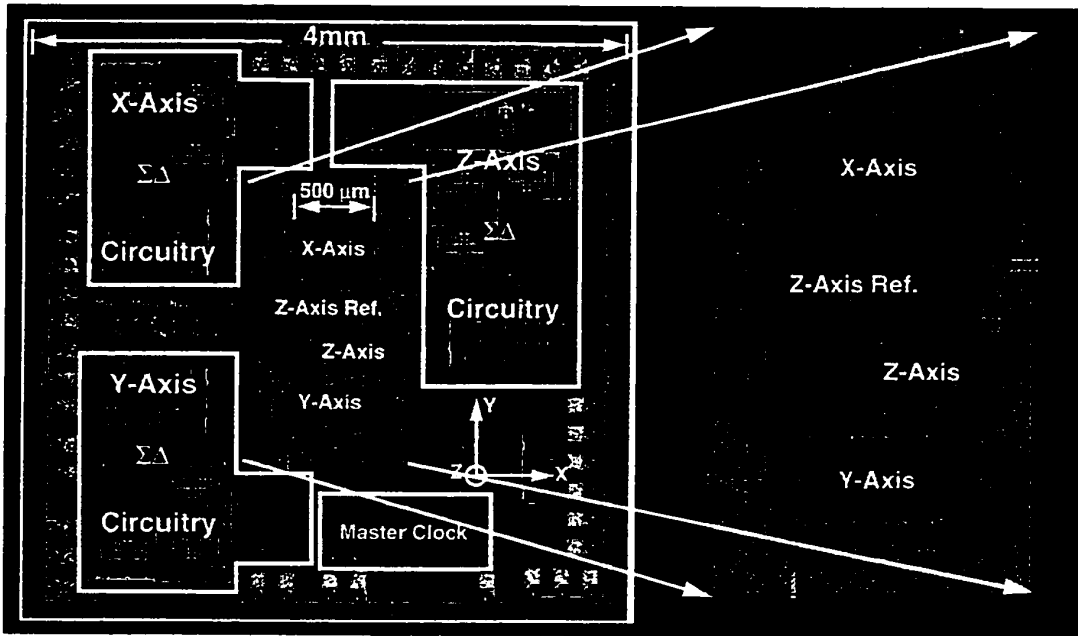


Figure 5-3. Die photograph of the three-mass accelerometer (left). A close-up of the structural area is shown on the right.

After the least squares estimate of the template has been fitted, the d.c. offset, the fundamental frequency, and the harmonic components of the output signal may be reconstructed and subtracted from the output. The remainder, $e(kT)$, is random noise. By comparing the noise power with the power of the fundamental in the output, the signal-to-noise ratio (*SNR*) may be found:

$$SNR \equiv \frac{\text{Power of Fundamental}}{\text{Noise Power}} = \frac{(a_1^2/2)}{\text{var}(e(kT))}. \quad (5-5)$$

Since the data set is finite, variance is approximated by the sample variance. The *SNR* includes noise from mechanical and electrical noise sources as well as quantization noise.

Another performance specification which can be measured from the template coefficients and the original output data set is the signal-to-noise plus distortion ratio (*SNDR*). The *SNDR* is defined as the power of the output signal to the power of the noise plus harmonic distortion caused by nonlinearities in the accelerometer. Both *SNR* and *SNDR* are useful in measuring the noise floor of the accelerometer and determining the dynamic range of the device. The dynamic range is defined as the maximum input power

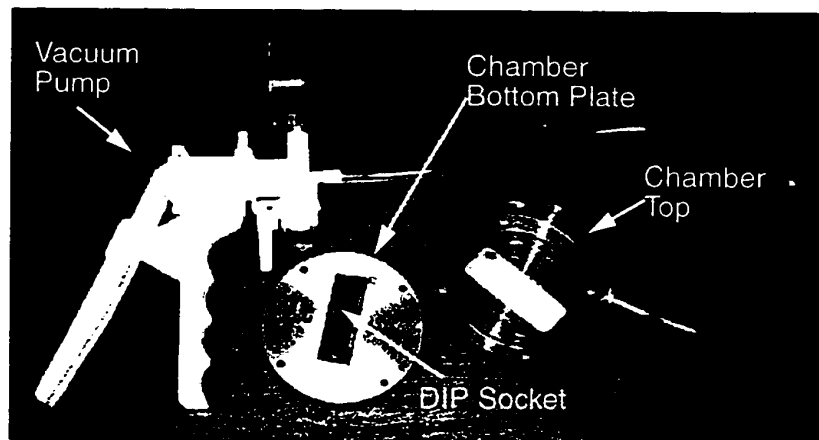


Figure 5-4. Photograph of the vacuum chamber used for measuring effects of Brownian noise. The DUT is placed in the DIP socket located in the bottom plate (courtesy William Clark).

measurable by the device divided by the minimum detectable signal. This parameter is a measure of the size of the operating range over which the accelerometer is useful.

5.4 THREE-MASS ACCELEROMETER

A die photograph of the three-mass accelerometer is shown in Figure 5-3. Detailed measurements from this accelerometer are presented in this section. The results from testing the single-mass accelerometer are presented in Section 5.5.

5.4.1 NOISE

The noise floor of an accelerometer is an important specification, quantifying the fundamental resolution of the device. Section 4.5 derived relations between electrical and mechanical noise sources and their appearance in the output as acceleration noise. In this section, noise sources will be calculated from the design parameters of the device, and detailed comparisons will be made with the results of measurements from the x-axis output of the three-mass accelerometer. For comparison purposes, it is often useful to make a noise source under investigation intentionally higher than other noise sources. By increasing a noise contributor above the nominal total noise, it is possible to separate and

Device Parameter	Source	x-axis	y-axis	z-axis
Proof-mass	H	0.294 μgm	0.210 μgm	0.391 μgm
Spring constant	H	0.177 N-m	0.191 N-m	1.03 N-m
Natural frequency	H	3.9 kHz	4.8 kHz	8.2 kHz
Mechanical Q	H	1.6	1.6	3.2
Sense capacitance - C_s	H	101 fF	78 fF	322 fF
Sense-capacitor gap	M	2.13 μm	2.13 μm	2.3 μm
Parasitic capacitance - C_{po}	H	0.84 pF	0.84 pF	0.84 pF
Parasitic capacitance - C_{ps}	H	0.86 pF	0.65 pF	0.1 pF
Integrating capacitors - C_i	D	200 fF	200 fF	200 fF
ICMFB capacitors - C_{fb}	D	200 fF	200 fF	200 fF
Wiring resistance from structure	H	13.8 k Ω	7.64 k Ω	1.71 k Ω
In-line switch on resistance	S&H	15.3 k Ω	15.3 k Ω	15.3 k Ω
Total wiring resistance - R_{in}	S&H	29.1 k Ω	17.8 k Ω	17.0 k Ω
Input-referred opamp noise - V_{opamp}^2	S	4.5 nV/ $\sqrt{\text{Hz}}$	4.5 nV/ $\sqrt{\text{Hz}}$	4.5 nV/ $\sqrt{\text{Hz}}$
Main opamp f_u (with loading)	S	91 MHz	91 MHz	91 MHz
Input-referred preamp noise - V_{preamp}^2	S	17 nV/ $\sqrt{\text{Hz}}$	17 nV/ $\sqrt{\text{Hz}}$	17 nV/ $\sqrt{\text{Hz}}$
Preamplifier f_u (with loading)	S	13.4 MHz	13.4 MHz	13.4 MHz
Feedback Fraction (T_f/T)	D	2/8	2/8	2/8
Sense Phase A Fraction (T_{sa}/T)	D	2/8	2/8	2/8
Sense Phase B Fraction (T_{sb}/T)	D	3/8	3/8	3/8

Table 5-1. Parameters of fabricated three-mass sensor. Key to source column: D - Designed value, S - Value obtained from SPICE simulation, H - Hand calculated, M - Measured.

look at the effect of parameters on a single noise source. This enables verification of the models derived in Chapters 2 through 4, even in cases where several noise sources are of similar magnitude.

5.4.1.1 BROWNIAN NOISE

Brownian noise may be directly calculated using a combination of designed and measured proof-mass dimensions. Since gap spacing is a critical dimension in calculation of the viscous damping, it was measured using a split-field microscope. Other dimensions were taken directly from the mechanical layout. As discussed in Chapter 2, viscous damping is comprised of both squeeze-film and Couette flow. At atmospheric pressure the

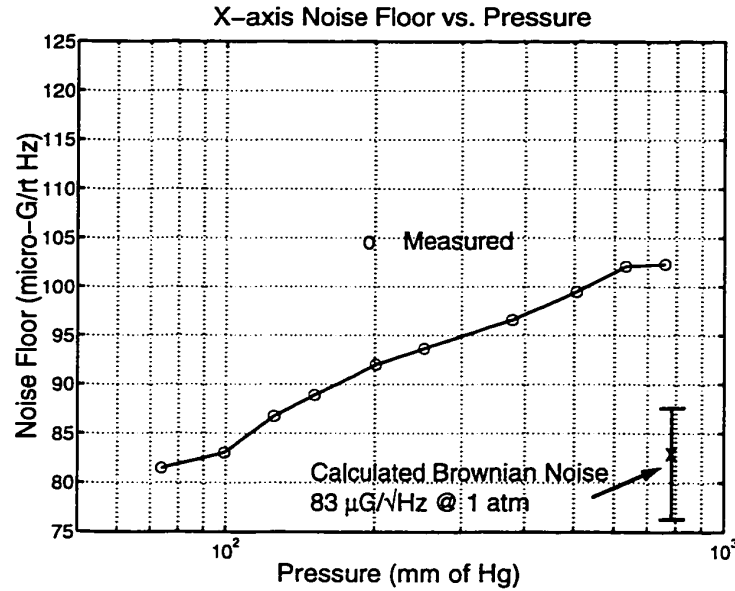


Figure 5-5. Output noise vs. pressure for the x-axis accelerometer. The decrease in noise as pressure is decreased is due to a reduction in air-damping.

squeeze-film damping coefficient, from equation (2-17), is:

$$b \approx 136(7.2) \left(1.56 \times 10^{-5} \frac{\text{kg}}{\text{m/s}} \right) (155 \times 10^{-6} \text{m}) \left(\frac{2.3 \mu\text{m}}{2.13 \mu\text{m}} \right)^3 = 2.98 \frac{\mu\text{N}}{\text{m/s}}, \quad (5-6)$$

where 136 represents the number of gaps between the proof-mass and stationary comb fingers. Damping from Couette flow is given by equation (2-11):

$$b = \frac{1.56 \times 10^{-5} \frac{\text{kg}}{\text{m/s}}}{2 \times 10^{-6} \text{m}} 5.48 \times 10^{-8} \text{m}^2 = 0.43 \frac{\mu\text{N}}{\text{m/s}}, \quad (5-7)$$

resulting in a total viscous damping coefficient of $3.41 \mu\text{N}/(\text{m/s})$. Using equation (4-15), the input-referred Brownian noise is calculated to be $83 \mu\text{G}/\text{rt Hz}$.

As discussed in Section 2.2.2, mechanical damping caused by air drops at lower pressures. Thus, Brownian noise is expected to fall if the ambient pressure surrounding the accelerometer decreases. This phenomenon may be observed by placing the accelerometer into a vacuum chamber, pictured in Figure 5-4, and reducing the pressure. Output noise is noted at different pressures. Figure 5-5 shows the measured noise floor vs. pressure as the

pressure varies from 74 torr to 760 torr (1 atm). Since the Brownian noise floor is comparable to the electrical noise floor, the decrease in noise is not as striking as it would be if the electrical noise floor were reduced. There is, however, a clear decrease in total output noise, as would be expected if Brownian noise were appearing in the output.

5.4.1.2 ELECTRICAL NOISE

Electrical noise sources may be calculated from equations (4-16) through (4-25) and the parameters listed in Table 5-1. The following analysis and comparison with experimental data are for a sample rate of $f_s = 500$ kHz. Opamp thermal noise is referred to an equivalent noise voltage at the integrator output by equation (4-18):

$$\frac{V_{out}^2}{\Delta f} = 1.26 \times 10^4 V_{opamp}^2 = 2.54 \times 10^{-13} \frac{V^2}{Hz} \Rightarrow \left[0.503 \frac{\mu V}{\sqrt{Hz}} \right]^2. \quad (5-8)$$

To calculate the noise contribution due to wiring resistance, it is first necessary to determine whether the corner frequency of the RC network shown in Figure 4-13 is above or below the closed loop opamp bandwidth. Equation (4-21) yields:

$$f_c = \frac{1}{2\pi(29.1k\Omega)(0.96pF)} = 5.7MHz. \quad (5-9)$$

This is less than the closed loop bandwidth of the opamp, and thus the noise contribution due to the wiring resistance is found from equation (4-22):

$$\frac{V_{out}^2}{\Delta f} = \left(\frac{4}{2.5 \times 10^5 Hz} \right) \left(\frac{0.96pF}{0.2pF} \right) \left(\frac{4.15 \times 10^{-21}}{0.2pF} \right) = \left[1.26 \frac{\mu V}{\sqrt{Hz}} \right]^2. \quad (5-10)$$

Thermal noise from the first preamplifier is found from equation (4-25):

$$\frac{V_{out}^2}{\Delta f} = 2\pi \left(\frac{13.4MHz}{5 \times 10^5 Hz} \right) V_{preamp}^2 = \left[0.221 \frac{\mu V}{\sqrt{Hz}} \right]^2. \quad (5-11)$$

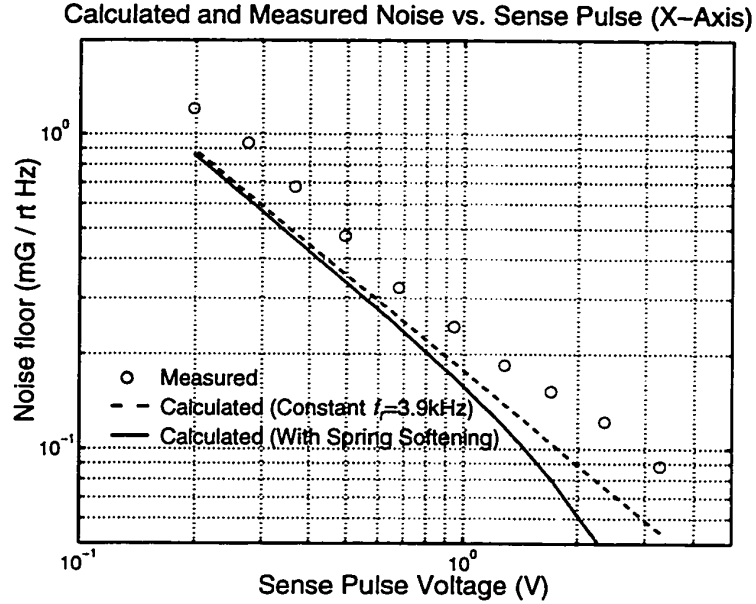


Figure 5-6. Measured and calculated output noise floor as a function of sense pulse voltage V_s . The solid line, including the effects of electrical spring softening, shows a lower noise floor at high V_s than if resonant frequency is assumed constant.

Since the noise sources from equations (5-8), (5-10), and (5-11) are independent, the total noise power referred to the integrator output may be obtained by adding the three powers, resulting in a total noise power of

$$\frac{V_{out}^2}{\Delta f} = 1.89 \times 10^{-12} \frac{V_s^2}{Hz} \Rightarrow \sqrt{\frac{V_{out}^2}{\Delta f}} = 1.37 \frac{\mu V}{\sqrt{Hz}}. \quad (5-12)$$

In order to refer this noise density to an equivalent input-referred acceleration noise, the relationship between input acceleration and integrator output (4-16) must be used:

$$\frac{dV_{out}}{dA_{in}} = 1.18 \times 10^5 \frac{V_s}{f_r^2} \text{ Volts/G}. \quad (5-13)$$

The calculated electronic noise floor using equations (5-12) and (5-13) is plotted in Figure 5-6 for varying values of V_s , along with measured values from the fabricated device. For larger values of V_s , the negative electrical spring constant causes a lowering of the natural frequency as described in Section 2.3.7. This effect is included in the solid line

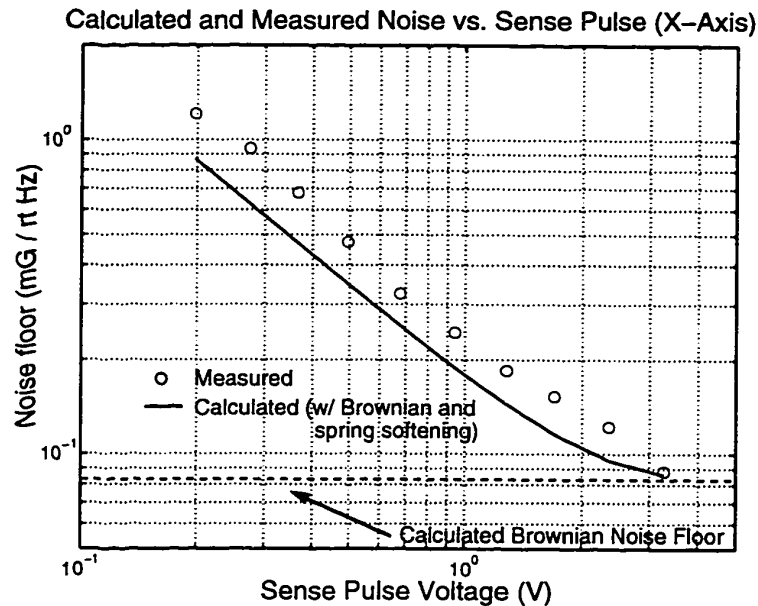


Figure 5-7. Measured and calculated output noise floor as a function of sense pulse voltage V_s , including effects due to electric spring softening and Brownian noise. Average error between calculated and measured noise is 2dB.

of Figure 5-6, and results in a decreased noise floor. For large values of V_s , the measured noise floor deviates from calculated values. This deviation occurs because Brownian noise becomes the dominant noise source as the electrical noise floor is lowered. If both Brownian and electrical noise are included, the discrepancy is greatly reduced, as shown in Figure 5-7. The average error between calculated and measured noise when both electrical and mechanical sources are included is 2dB.

Electrical noise in this device is particularly high due to two factors: sheet resistance of the structural polysilicon used for interconnect, and high threshold voltages of the in-line MOSFETS which are used to connect the electrical interface to the sense capacitors (switches s_3 and s_4 in Figure 4-6). The large sheet resistance of the polysilicon results from attack during release of the mechanical sense elements, and may be decreased through the use of alternative doping strategies. Switch on-resistance may be decreased through use of a clock-boosting circuit to increase switch conductance [66, 67, 68]. In a redesign which implements only these changes, the electrical noise floor would be dominated by opamp thermal noise and would drop by a factor of three to approximately

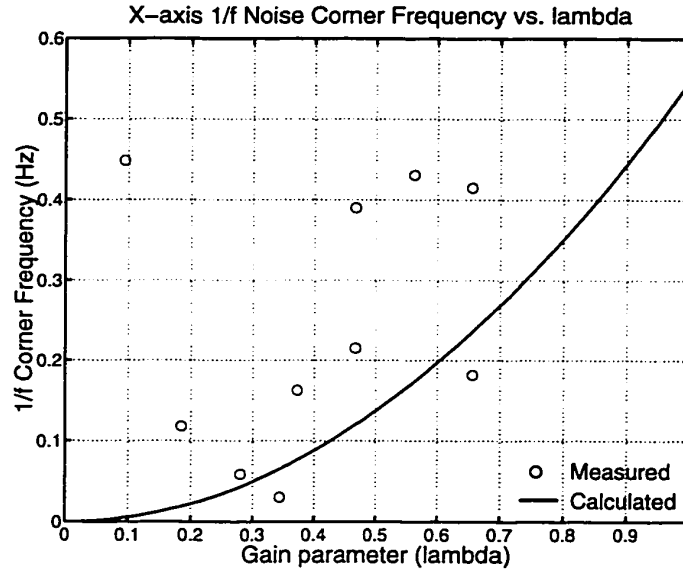


Figure 5-8. Measured and calculated flicker noise corner frequency due to reflected noise from the opamp. Calculated values assume thermal noise floor of $100 \mu\text{G}/\sqrt{\text{Hz}}$.

$30 \mu\text{G}/\sqrt{\text{Hz}}$. Electrical noise can be further lowered by decreasing the gap between comb fingers or reducing resonant frequency.

5.4.1.3 REFLECTED NOISE

The reflected noise for the x-axis device may be determined from equation (4-26). As noted in Section 4.5.3, the dominant reflected noise source is low frequency opamp flicker noise. Assuming a $1/f$ corner frequency of 1 MHz and using the parameters from Table 5-1, we find a noise power density at 1 Hz of:

$$\sqrt{\frac{a_n^2}{\Delta f}} = \left(\frac{1}{9.8} \frac{1}{2.94 \times 10^{-10}} \frac{0.1 \text{ pF}}{2.13 \mu\text{m}} \underbrace{\left(V_{s+\frac{2}{8}} + V_{s-\frac{3}{8}} \right)}_{\lambda} \right) 4.5 \frac{\mu\text{V}}{\sqrt{\text{Hz}}} = 7.3 \times 10^{-5} \lambda \frac{\text{G}}{\sqrt{\text{Hz}}} \quad (5-14)$$

The output $1/f$ noise corner may be determined for a given noise power density at 1 Hz by calculating the frequency at which the noise power density from equation (5-14) equals the noise power due to thermal noise:

$$1/f \text{ Corner} = \left(\frac{\sqrt{a_n^2}}{7.3 \times 10^{-5} \lambda} \right)^2 \text{ Hz.} \quad (5-15)$$

Figure 5-8 shows measured and calculated values of the flicker noise corner as the parameter λ in equation (5-15) is varied. The sense pulse magnitude, $\Delta V_s = \Delta V_{s+} - \Delta V_{s-}$, is kept at 1.75 V as λ is varied. Measured values of the flicker noise corner were obtained from power spectral density plots of 14 minute-long samples of data under zero input. Although the measured corner frequencies vary about calculated values, the trend predicted by equation (5-15) is still evident in the plot of Figure 5-8.

Some of the discrepancy between measured and calculated flicker noise corners may arise from other drift mechanisms. One such drift mechanism which has been identified is water vapor inside the package. Long-term measurements over 3 days have shown an output drift of up to 2.5 G when the devices are packaged in air. Drift was reduced to approximately 50 mG over an equivalent period when the sensor was placed in a chamber with desiccant. The nature of water molecules provides a possible explanation of this effect. Because water vapor is comprised of polar molecules, charge can build up on comb fingers in the presence of a d.c. potential field. Charge build-up imparts a force on the proof-mass indistinguishable from an input acceleration. Changes in the charge cause drift in this force. These preliminary results must be followed by further investigation to increase understanding of the relationship between water vapor and drift.

5.4.1.4 QUANTIZATION NOISE

Quantization noise from the sigma-delta loop topology is approximately an order of magnitude lower than both mechanical and electrical noise sources at the nominal sampling frequency of $f_s = 500$ kHz and the full scale value of ± 20 G. For a constant bandwidth, Brownian noise is independent of the sampling rate, while inband electrical noise increases by only 3 dB for every halving of the sampling rate. Quantization noise, on the other hand, is highly dependent on the oversampling ratio and thus may be raised

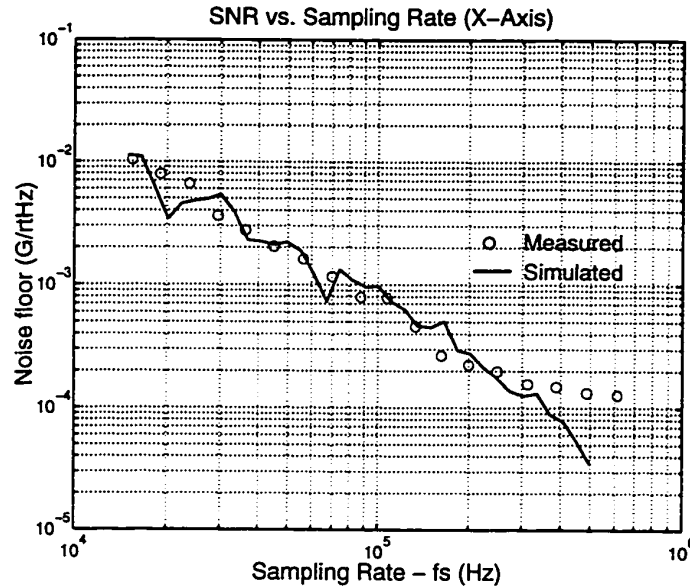


Figure 5-9. Measured and calculated output noise floor as a function of sampling rate, f_s . The deviation of measured results from simulations at high sample rates is due to Brownian noise.

above the other noise sources at low sampling rates. In order to determine whether the models derived in Section 3.3 are accurate, the sampling rate is varied to compare measured noise floor with results of simulations in a region of operation where quantization noise dominates.

Figure 5-9 shows measured and simulated values of output noise. Simulated output noise includes only the quantization noise. For both simulated and experimental measurements, the full scale of the x-axis device was set to ± 20 G. Simulated noise levels track experimental measurements very well up to approximately 300 kHz, at which point the measured noise floor flattens to a constant level. At this frequency the measured output noise is dominated by Brownian noise, and thus no further decrease for higher sampling rates is observed.

5.4.2 FULL SCALE VS. FEEDBACK VOLTAGE

Equation (4-36) predicts the full-scale value of the accelerometer as a function of feedback pulse magnitude and the fraction of the total period allotted to the feedback operation. For

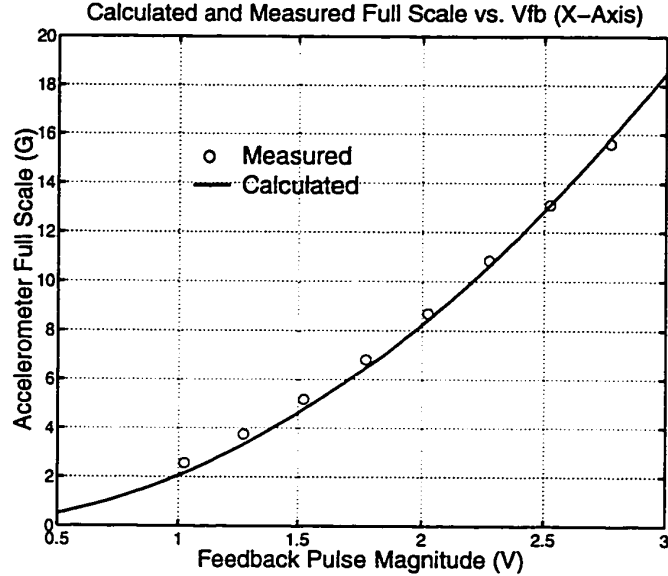


Figure 5-10. Measured and calculated accelerometer full scale vs. feedback pulse magnitude.

the fabricated device, one-quarter of the total period is allotted for feedback. Thus $T_f/T = 0.25$, and the full scale is calculated to be:

$$a_{fs} = \frac{0.25}{9.8(2.94 \times 10^{-12})} \left(\frac{1}{2} \frac{0.1 pF}{2.13 \times 10^{-6} m} V_{fb}^2 \right) = 2.05 V_{fb}^2 \text{ G.} \quad (5-16)$$

Measured full scale versus feedback pulse magnitude V_{fb} is plotted in Figure 5-10. The average error between calculated and measured full-scale values is 5%. From the above result, it is determined a full scale range of $\pm 50G$ may be achieved by raising the feedback pulse to 5 Volts in the current technology. Further increases in full scale may also be achieved through reduction of the sense capacitor gaps. For instance, decreasing the gaps from their current 2.13 micron size to a gap of 1 micron would result in a full scale of $\pm 225G$.

5.4.3 DYNAMIC RANGE

A plot of the measured SNR vs. input acceleration may be used to determine the noise floor, peak SNR , and dynamic range of the accelerometer. Figure 5-11 shows measured SNR vs. RMS input power in a 100 Hz bandwidth for the x-axis device. By extrapolating

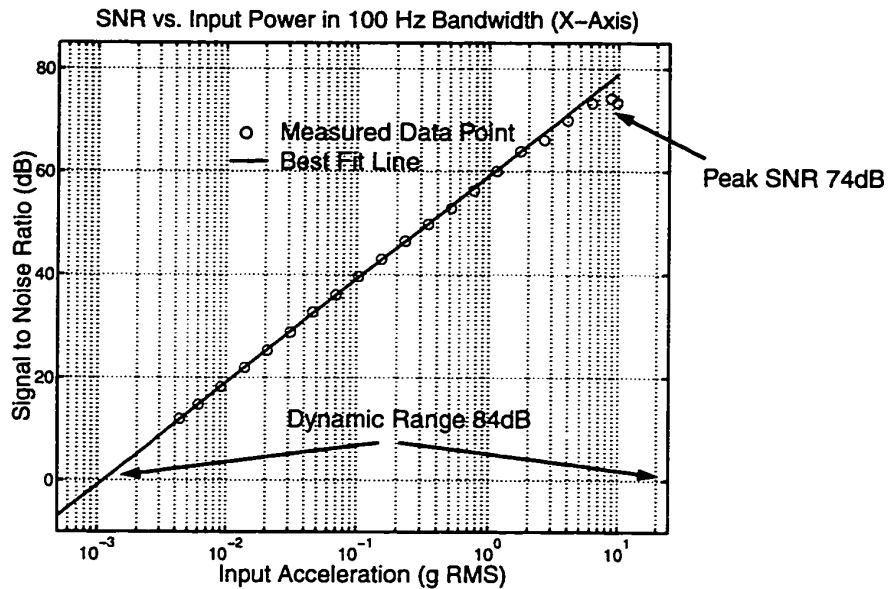


Figure 5-11. Measured signal to noise ratio (SNR) vs. input acceleration for a test signal at 40 Hz. The X-axis device has a measured peak SNR of 74 dB and a measured dynamic range of 84 dB in a 100 Hz bandwidth.

the data to find the input signal corresponding to a SNR of zero dB, the noise floor of the device may be determined. The line in Figure 5-11 crosses zero dB at approximately 1.1 mG, corresponding to a noise floor of $110 \mu\text{G}/\sqrt{\text{Hz}}$.

The peak SNR of 74 dB occurs for an input signal of approximately 12G (8.5G RMS), above which point the noise floor increases. Dynamic range of the device is found from Figure 5-11 to be 84 dB.

The noise floor for the y- and z-axes has been measured at $160 \mu\text{G}/\sqrt{\text{Hz}}$ and $990 \mu\text{G}/\sqrt{\text{Hz}}$ respectively. This corresponds to a dynamic range of 81 dB and 70 dB in a 100 Hz bandwidth. The differences in these values are due to different y- and z-axis proof-mass designs, which provide smaller sensitivity in response to input accelerations.

5.4.4 DIGITAL ERROR CANCELLATION

Digital offset trimming was implemented in a second fabrication run of the three-mass

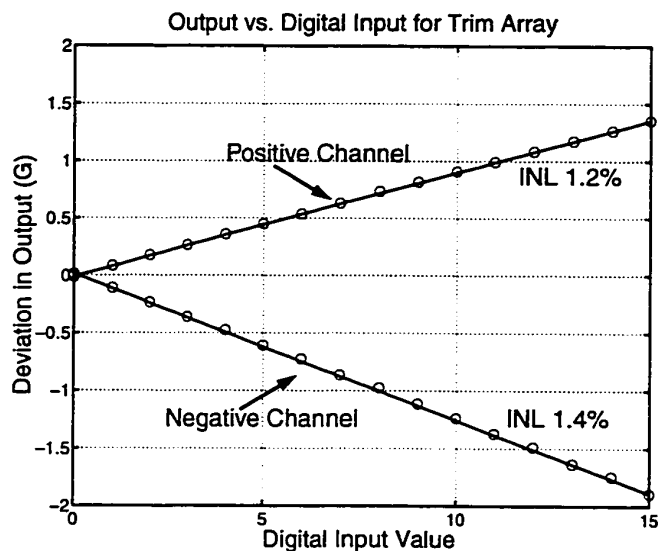


Figure 5-12. Measured offset vs. digital input to the 4-bit binary-weighted capacitive array. For this 4-bit array measured trimming range was found to be $[+1.4\text{ G}, -1.9\text{ G}]$. Calculated trim range was $\pm 1.9\text{ G}$

accelerometer. As described in Section 4.8, two sets of capacitor arrays were utilized to enable cancellation of both positive and negative offsets. In this implementation, each array was designed to have 4-bits of trimming capability as a proof of concept. The capacitor sizes were chosen such that offsets of approximately $\pm 2\text{ G}$ may be canceled out.

Figure 5-12 shows measured results of the digital trim network for the lateral accelerometer. Measured trimming range was approximately $+1.4\text{ G}$ to -1.9 G with a resolution of 91 and 127 mG/bit in the positive and negative directions respectively. Differences in resolution arise from mismatch in the attenuation ratios of the two different channels. Calculated resolution and full scale range values are 123 mG/bit and $\pm 1.9\text{ G}$ respectively. The integral nonlinearity (INL) of the trimming array was found to be 1.2% and 1.4% for the positive and negative directions.

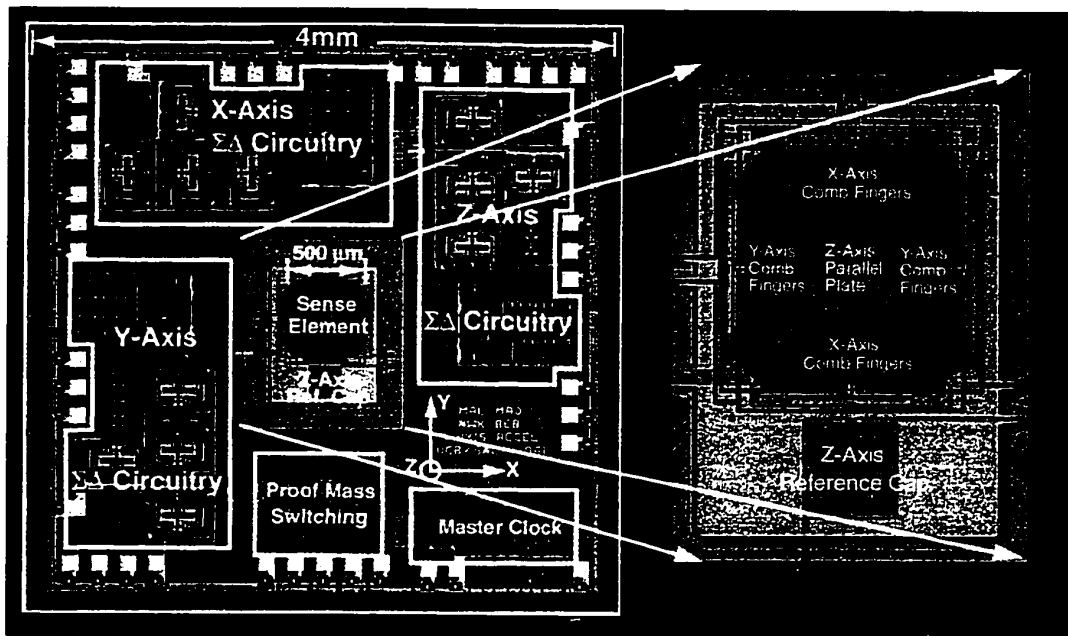


Figure 5-13. Die photograph of the single-mass accelerometer (left). A close-up of the structural area is shown on the right, including labels of sense capacitors. An air-gap reference capacitor allows the same differential circuitry to be used for the z-axis.

5.5 SINGLE-MASS ACCELEROMETER

A die photograph of the fabricated single-mass accelerometer, described in Section 4.9, is shown in Figure 5-13. System parameters and measured performance are listed in Table 5-2. The poorer performance of this device, compared to the three-mass device, is due to higher resonant frequencies and smaller sense capacitors.

In Section 4.9 it was hypothesized that the position sense and feedback circuitry are insensitive to off-axis deflections, and therefore permit simultaneous force balancing of three axes while maintaining good rejection of off-axis accelerations. Verification of this hypothesis may be achieved by measuring cross-axis sensitivity. Measurement of cross-axis sensitivity is accomplished by applying accelerations along one axis of the device while measuring the output signal from the other two axes. The sinusoidal minimum error method, described in Section 5.3, is used to precisely measure the output signal amplitude from the off-axis channels. Results of these measurements are shown in Table 5-3. Attenuation of transverse signals averages about 40 dB, corresponding to 1% cross-axis sensitivity. These numbers are conservative estimates of transverse sensitivity

Parameter	x-axis	y-axis	z-axis
Natural Frequency	6.6 kHz	6.6 kHz	5.2 kHz
Sense Capacitance - C_s	49 fF	49 fF	117
Sense-capacitor gap	2.13 μm	2.13 μm	2.3 μm
Measured noise	0.73 mG/ $\sqrt{\text{Hz}}$	0.73 mG/ $\sqrt{\text{Hz}}$	0.76 mG/ $\sqrt{\text{Hz}}$
Measured full scale	± 11 G	± 11 G	± 5.5 G

Table 5-2. *Single-mass three-axis accelerometer system parameters.*

Input Axis	X-Output	Y-Output	Z-Output
X	●	-36 dB	-39 dB
Y	-38 dB	●	-39 dB
Z	-47 dB	-56 dB	●

Table 5-3. *Measured cross-axis sensitivity of single-mass accelerometer.*

of the sensor, since die-to-package misalignment and cross-axis vibrations from the vibration exciter are also included in these values. Even so, 1% transverse sensitivity is better than many commercial grade accelerometers [69, 70, 71, 72].

5.6 DISCUSSIONS OF LIMITATIONS IN TESTING

Accelerometer testing in this chapter was undertaken using a vibration exciter with relatively crude test fixtures. While the results from these tests provide a large amount of useful data, these tests do not allow precise measurements of certain parameters.

Combined harmonic distortion in the power amplifier, vibration exciter, and test fixture of up to 10% (at full scale) prevent accurate measurement of accelerometer nonlinearity, since the distortion of the test acceleration is above the distortion of the accelerometer. A precision centrifuge has been proposed for measurement of accelerometer nonlinearity [63]; however, this test method only measures static nonlinearities.

Off-axis resonances in the vibration exciter and test fixture introduce cross-axis acceleration components to the input test signal, yielding low estimates of cross-axis

rejection. Due to input-axis misalignment is another source of error when measuring intrinsic transverse sensitivity. A more accurate measurement technique for cross-axis sensitivity uses a four point tumble test, in which static acceleration is applied in four directions using a precision dividing head [63]. Since transverse sensitivity has already been measured to within typical die-to-package alignment tolerances, this test would not be expected to yield better results.

Both sources of error listed above may be attenuated through the use of specialized test equipment. Since the cost of acquiring this equipment is prohibitive, testing may be contracted out to specialized calibration laboratories, such as the National Institute of Standards and Technology in Gaithersburg, Maryland.

5.7 SUMMARY

In this section, experimental measurements of fabricated accelerometers have been presented. Experimental values of mechanical, electrical, and quantization noise agree with calculated values obtained from the analysis presented in Chapters 2 through 4. Measured full scale and dynamic range are also in close agreement with expected values, verifying that the important issues in both capacitive position sense interface design and accelerometer design have been accurately modeled and understood. Experimental measurements of the digital offset trimming system have shown effective cancellation of residual accelerometer offset.

In addition to a working three-axis accelerometer, consisting of three single-axis accelerometers located on a single chip, a functional three-axis accelerometer using a single proof-mass was presented. Measurement of the three-axis single-mass accelerometer shows good rejection of transverse accelerations. This is due to the high degree of orthogonality achievable with photolithographic patterning of mechanical features. The single-mass accelerometer has proven the feasibility of force balancing multiple axes of a single mass simultaneously. Extensions of multi-axis servoing include damping vacuum-packaged accelerometers and force-balancing vibratory rate gyroscopes.

6. CONCLUSIONS

6.1 RESULTS

Integration of capacitive-based inertial sensors with VLSI technology enables increased system functionality. On-chip calibration, signal processing, and communication capability is desirable for many applications, including sensor arrays, embedded devices, and functional units such as air-bag deployment modules. Addition of these system features increases the value of the MEMS device at a small marginal cost.

This work has investigated design issues in precision accelerometry using an integrated CMOS electrical interface. For precise measurement, errors from the electrical interface must be canceled. Methods to attenuate errors including flicker noise, sampling noise, and offset have been proposed. In-depth analyses of noise contributions at the electrical, the mechanical, and the system level were derived, thereby permitting quantitative predictions of accelerometer performance. The accuracy of these analyses has been experimentally verified with a fabricated 3-axis surface-micromachined micro accelerometer.

Accelerometer performance is limited by both mechanical and electrical characteristics of the fabrication technology. Results presented in this dissertation suggest technology directions for higher performance accelerometers. Sensitivity to input accelerations can be made larger by reducing air-gap spacing and decreasing resonant frequency. Current air-gap width is set by both photolithographic and etching constraints. Better photolithography and etching chemistries are expected to enable sub-micron gaps in the near future. Two factors limiting resonant frequency are sensitivity to mechanical shock and stiction during release. Movement to a thick-film integrated technology [5, 73] in conjunction with mechanical “bumpers” will enable manufacture of viable sense-elements with a low resonant frequency.

While the electrical interface presented here attenuates many sources of error, it cannot cancel thermal noise from interconnect resistance. Interconnect resistance can easily

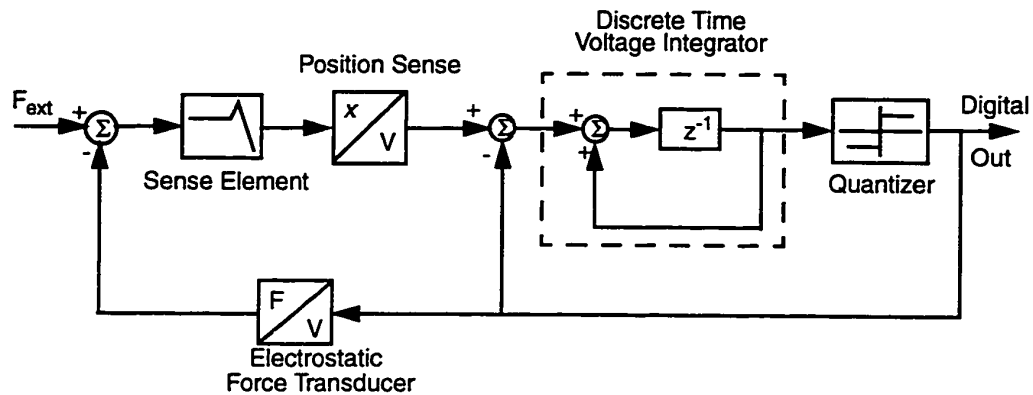


Figure 6-1. Third-order $\Sigma\Delta$ feedback loop. Inclusion of the electrical integrator simultaneously increases noise shaping and boosts low-frequency gain.

become the dominant noise source in a silicon-based technology due to the relatively high sheet resistance of doped silicon. The equivalent noise resistance of the active electronics is typically on the order of 1 K Ω . To prevent unnecessarily high output noise, total wiring resistance should be maintained below this number. Possible methods of achieving a low resistance include minimization of interconnect length, use of metal interconnect [6], and use of structural materials such as electroplated metal.

A technology which implements these improvements will result in increased resolution. In such a technology, the topology described in this dissertation could be expected to achieve a noise floor in the 100s of nG/ $\sqrt{\text{Hz}}$.

6.2 FUTURE WORK

In the current design, open-loop preamps realize voltage gain before the compensator. Because of settling requirements, this arrangement results in very high power consumption. An alternative method of providing gain must be found for many low-power applications, such as distributed sensor arrays and battery powered devices. Since an integrator provides high gain at low frequencies, the preamps may be replaced with a single discrete-time voltage integrator, as shown in Figure 6-1. An additional benefit of replacing the preamps with a discrete-time integrator is lower quantization noise. Because adding another integrator to the forward-path increases phase delay, alternative

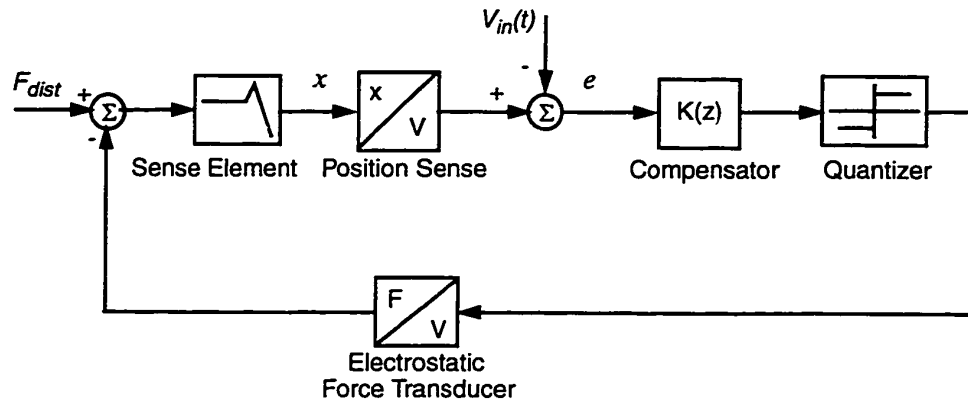


Figure 6-2. Feedback-loop configuration for position servoing. Proof-mass position x tracks the analog input voltage $V_{in}(t)$. Negative feedback attenuates the effect of acceleration disturbances (represented by F_{dist}) on position.

compensation methods must be examined.

While on-chip trimming of d.c. offset has been implemented, the problem of scale factor adjustment needs to be solved. Since scale factor is solely dependent on the feedback force, trimming should be easily accomplished. Feedback force magnitude may be varied using an on-chip digital-to-analog converter (DAC) to adjust the feedback pulse voltage. Programmable memory may be used to store the DAC coefficients, enabling scale factor calibration after packaging.

Instead of force-balancing the proof-mass to zero displacement, many applications require servoing the proof-mass to a reference position or trajectory. Examples of these applications include both micro actuated disk-drive heads and vibratory rate gyroscopes. The system presented in this dissertation may be modified for position servoing by subtracting an input voltage, $V_{in}(t)$, from the feedback loop shown in Figure 6-2. The inherently high bandwidth of the $\Sigma\Delta$ feedback loop is expected to result in both good tracking and good disturbance rejection. In addition to proof-mass servoing, a vibratory rate gyroscope requires simultaneous force balancing of an orthogonal axis from which Coriolis acceleration is detected. Although simultaneous multi-axis force balancing has already been demonstrated, significant work will be required to create functional gyroscopes.

Dicing and packaging of fabricated devices remains a major difficulty for commercial production. To enable sensing of accelerations on the order of $1\ \mu\text{G}$, the devices must be packaged in vacuum on the order of 10 milli-torr. Guaranteeing package hermeticity over the part lifetime is a difficult task and can increase cost dramatically. One possible solution, a die-level cap which facilitates dicing and packaging, has been presented in [74]. By encapsulating the proof-mass in a vacuum with a polysilicon cap, the device is protected from particles during assembly. Other back-end processing for protection of the sense-element will need to be explored in detail. Future MEMS capabilities and commercial market penetration will depend in large part on the success of research in packaging technology.

APPENDIX A - COMMON MODE SETTLING

In this appendix the common mode settling transfer functions (4-5) and (4-6) are derived. To enable algebraic manipulation of the nodal equations, the capacitors in Figure 4-4 are replaced with their equivalent impedances: $z = 1/(j\omega C)$. The voltages and currents are labeled as in Figure A-1. Kirchhoff's current law is applied to the node V_2 :

$$I_1 + I_2 + I_3 + I_4 = 0, \quad (\text{A-1})$$

where:

$$I_1 = \frac{V_s - V_2}{z_1}, I_2 = \frac{V_4 - V_2}{z_2}, I_3 = \frac{V_3 - V_2}{z_3}, \text{ and } I_4 = \frac{-V_2}{z_6}. \quad (\text{A-2})$$

The voltage V_4 is equal to the current through z_5 times z_5 :

$$V_4 = (-g_{m1} V_2 - I_2) z_5. \quad (\text{A-3})$$

A similar relation may be written for z_4 :

$$V_3 = (-g_{m2} V_3 - I_3) z_4. \quad (\text{A-4})$$

The above seven equations have seven unknown variables: $V_2 \rightarrow V_4$, and $I_1 \rightarrow I_4$. A symbolic math solver may be used to determine the voltages $V_2(s)$ and $V_3(s)$ as a function of $V_s(s)$. Using the symbolic math package *Mathematica*, the numerator of the transfer function $V_2(s)/V_s(s)$ is found to equal:

$$2C_s(2C_{fb} + C_{p3})((2C_i + C_{p2})s + g_{m2})s. \quad (\text{A-5})$$

The numerator of the transfer function $V_3(s)/V_s(s)$ is found to equal:

$$4C_s C_i (2C_{fb} + C_{p3}) s^2. \quad (\text{A-6})$$

The exact characteristic equation for both $V_2(s)/V_s(s)$ and $V_3(s)/V_s(s)$ is equal to:

$$a_2 s^2 + a_1 s + a_0 = 0, \quad (\text{A-7})$$

where:

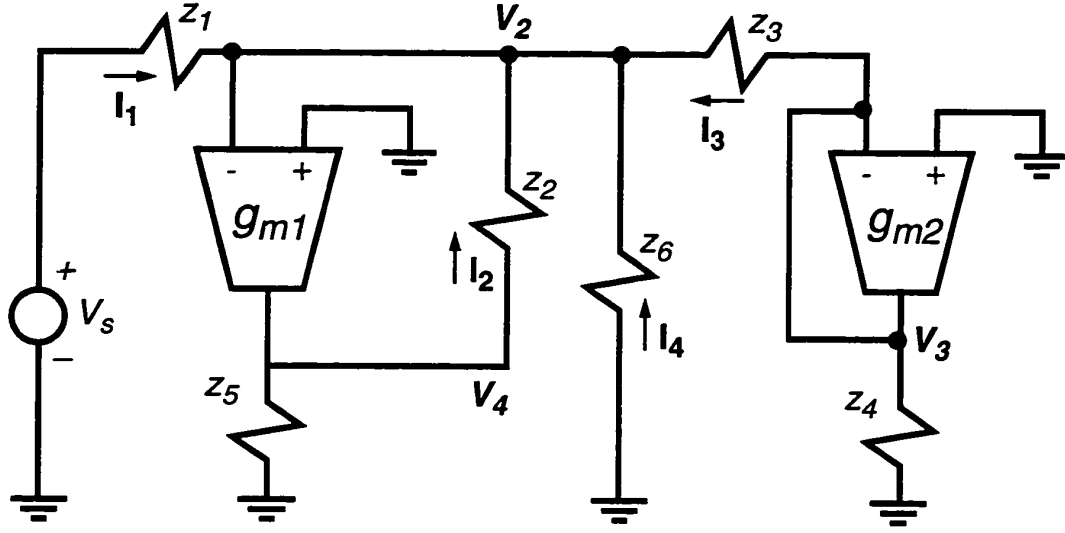


Figure A-1. Common mode schematic diagram used for settling analysis. Voltage V_s represents the proof-mass voltage. Voltages V_2 and V_3 represent the common mode voltages at the main opamp input and output respectively. The capacitors have been replaced with their equivalent impedances.

$$a_0 = 2C_{fb}g_{m1}g_{m2} \quad (\text{A-8})$$

$$a_1 = g_{m1}[2C_{fb}(2C_i + C_{p2})] + g_{m2}[2C_{fb}(2C_s + 2C_i + C_{p1} + C_{p3}) + C_{p3}(2C_s + 2C_i + C_{p1})] \quad (\text{A-9})$$

$$a_2 = 2C_{fb}(4C_sC_i + 2C_{p2}(C_s + C_i) + C_{p3}(2C_i + C_{p2}) + C_{p1}(2C_i + C_{p2})) + C_{p3}(2C_i(2C_s + C_{p1} + C_{p2}) + C_{p1}C_{p2}) \quad (\text{A-10})$$

Substituting parameter values for the variables in equations (A-8) through (A-10) allows small terms to be identified and removed. For the parameter values listed in the caption of Figure 4-5, the characteristic equation may be reduced to the following simplified form:

$$(C_{p2}C_{p1}(C_{p3} + 2C_{fb}))s^2 + (2C_{fb}C_{p2}g_{m1} + C_{p3}C_{p1}g_{m2})s + 2C_{fb}g_{m1}g_{m2} = 0, \quad (\text{A-11})$$

where terms contributing less than 10% of each coefficient have been eliminated. This is the denominator in equations (4-5) and (4-6).

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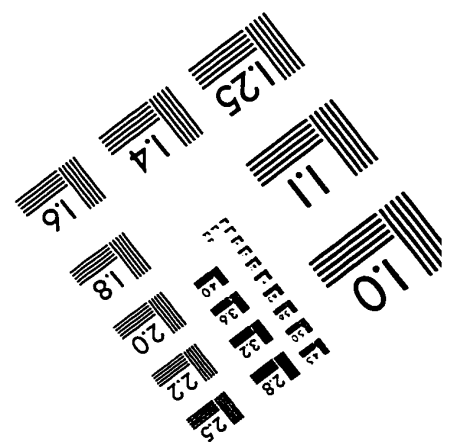
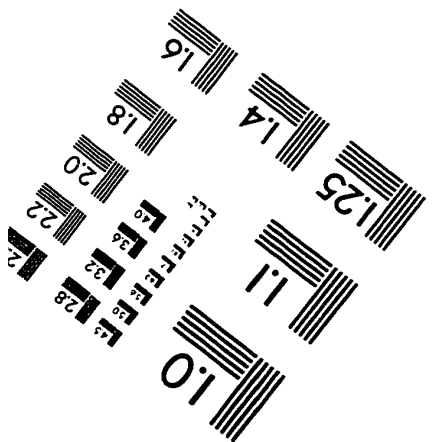
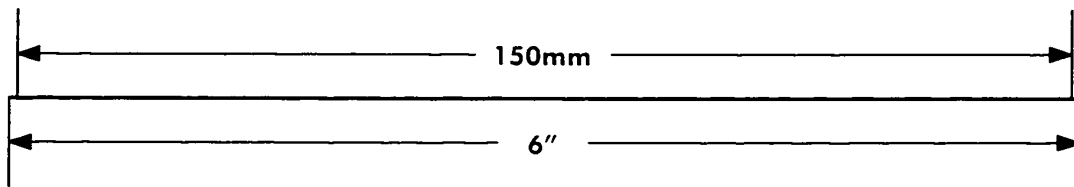
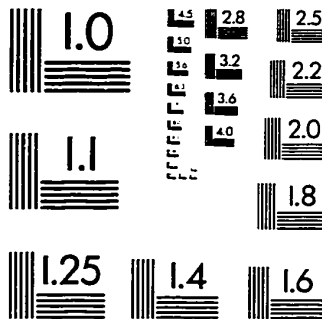
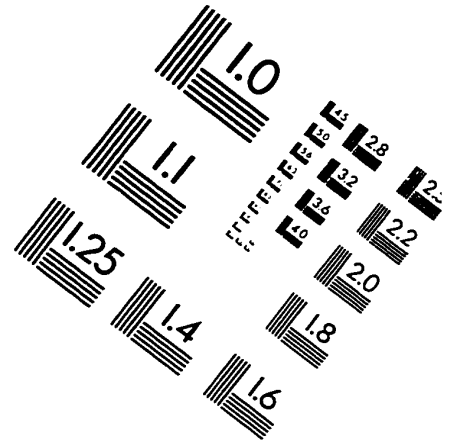
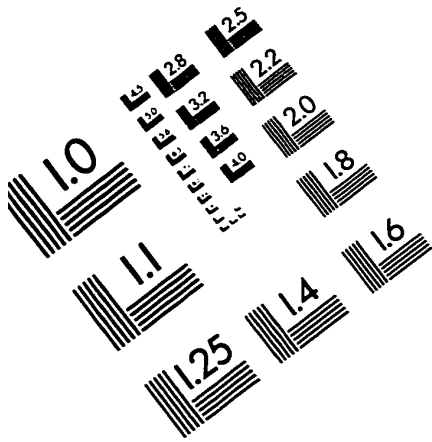
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IMAGE EVALUATION TEST TARGET (QA-3)



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