

Aerospace Sensor Systems and Applications

SHMUEL MERHAV



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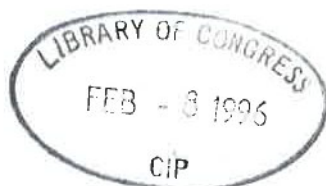
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Introduction and historical background

The purpose of aerospace sensors is to provide all the necessary information on the *variables and parameters* of the flight system for the execution of the mission at hand. A *mission* is generally defined in three dimensional space. Sometimes, on adding the fourth 'dimension', namely, time, a four-dimensional space is defined. In contrast to the three spatial coordinates which can increase or decrease, time always grows. The recent concept of 4-D guidance has special significance in coordinating flight paths such as in air traffic control or in space rendezvous operations, where the notions of 'here-now' or 'there-then' assume paramount importance.

As compared to the first generation aircraft at the turn of the century, which were equipped with a compass, a barometric altimeter, and a relative air speed sensor, a modern military aircraft or airliner is equipped with hundreds of sensors at all levels of the flight system. As such, they have become an integral part of it, not only in regular operation but also in preflight procedures for maintenance and safety. It is paradoxical that by introducing components and subsystems with more sophisticated technologies and a myriad of sensors for monitoring their performance and safety, a situation of diminishing returns may be created. This may be so, because in view of their large numbers, a failure in any of them easily causes a mission abort even if an imminent catastrophe is not necessarily indicated. It is therefore not surprising that along with the ever-increasing demands for accuracy and precision, research and development efforts continue to seek improved technologies for better reliability and maintainability. Currently, one already occasionally encounters the term 'unlimited' for the item MTBF (mean time between failures) in the data sheet of an aerospace sensor.

Modern aerospace sensors are direct descendants of the discoveries of physical laws in theoretical and experimental physics in the seventeenth and eighteenth centuries followed by the avalanche of revolutionary inventions in the nineteenth and early twentieth centuries.

At this point, it is enough to recall only a few of the most salient milestones in order to appreciate the direct link between the late twentieth century technology of aerospace sensors and systems and the discoveries in physics and the inventions that followed. Thus, for example, Torricelli's discovery of atmospheric pressure in 1643 was quickly followed by Pascal's invention of the barometric altimeter in 1647 and consequently by his discovery during 1647—1654 of the properties of our gaseous atmosphere. Newton's discovery of the law of gravitation in 1687, his first law on the conservation of momentum, and his second law, that force equals the rate of change of momentum, laid the foundation for the much later development of the accelerometer at the turn of this century following the invention of the seismometer by Milner in 1883. Other principal examples are the early invention of a steam-powered vertical

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8

The Ring Laser Gyro

8.0 Introduction

This chapter introduces the ring laser gyro, which is considered one of the most revolutionary twentieth century developments in the aerospace sensor field. It appeared on the scene in the early 1960s and since then has matured as a dominant product in the aerospace industry, having accumulated millions of successful operational flight hours in military and commercial aircraft.

The often quoted paper by Killpatrick [1], is a comprehensive tutorial that explains the principle of operation and the basic properties of this device. As in many later publications on the subject, the author explains that the RLG is really not a gyro in the sense of the conventional concept of a spinning mass. However, as pointed out in [1], the original meaning of the Greek term "*γυρος*" is *ring*, which, linguistically, may be more appropriate than the conventional connotation of a spinning mass. The outstanding property of ring laser gyros (RLG) is that there are no moving and sliding components. The sensitive element consists of a pair of colocated, closed path clockwise and counterclockwise light beams. Their interaction causes interference, which manifests itself as fringes resulting from the corresponding changes of the wavelengths and oscillating frequencies. These changes are proportional to the inertial angular rate. The intrinsic readout is by counting the pulses due to the fringes. This constitutes a natural interface with the digital processor and computer. This development has significantly contributed to the practical implementation of guidance and inertial navigation systems, especially of strapdown mechanization.

8.1 Operating principle

Basically, the RLG is a Sagnac interferometer. Thus, it has much in common with the IFOG and RFOG. The common denominator is the interferometry. The principal difference is that unlike the passive IFOG, it is an active device essentially operating like an electro-optical oscillator. The device is a laser that incorporates three or more reflectors, arranged so that the light beams form a polygon enclosing an area A . The triangular shape is lowest cost, but it encloses the least area A for a given size and is therefore the least sensitive. A comparative geometrical analysis of different polygonal shapes is given in [2]. The reflecting mirrors, together with the light-amplifying medium in the light path, constitute an oscillator. There are two such oscillators, each propagating

light energy along the same path but in opposite directions. The frequencies at which these oscillators operate are determined by the length of the optical path. For the oscillation to be sustained, as in any oscillator, two conditions must be fulfilled: 1) the overall gain around the loop must be unity at a power level determined by the amplifying medium, and 2) the phase shift around the loop must be zero. This condition implies that the number of light wavelengths must be an exact integer, namely, the wavelength must be an exact integer fraction of the path length. This condition is actually the one that determines the oscillation frequency of the laser. Further details can be found in [3] and [4]. If the ring laser is stationary in inertial space, the cw and ccw beams oscillate at exactly the same frequency. However, if put into rotation, a slight difference in the cw and the ccw path lengths occurs in the same manner as in the passive IFOG. Consequently, the corresponding wavelengths and thus the oscillating frequency will differ. This difference is proportional to the inertial rotation rate Ω of the RLG. The measurement of this difference, which is the essence of the operation of the RLG, is by monitoring this frequency difference.

Description

A schematic description of the basic operating elements of a triangular RLG is given in Figure 8.1. A closed cavity in a polygonally shaped Zerodur block contains the two counterrotating single-frequency light beams that are generated from the lasing action of the Helium-Neon gas discharge within the optical cavity. Zerodur is a ceramic glass that has an extremely low thermal expansion coefficient. Thus, the dimensions of the RLG are very firm constants which are vital to maintain a constant lasing frequency. The gas is typically a mixture of one part of neon to ten parts of helium at a total pressure of 1–10 mm of mercury. The reflecting surfaces are dielectric mirrors designed to selectively reflect the oscillation frequency of the Helium Neon gas mixture. The mirrors require high-quality optical seals to prevent any contaminants from leaking into the gas mixture, thus degrading performance and shortening life-time. Since the apparent path lengths of the cw and ccw waves differ slightly, and since the corresponding light wavelengths λ_{cw} and λ_{ccw} must be N integer fractions of the path lengths L_{cw} and L_{ccw} , it follows that the corresponding oscillating frequencies f_{cw} and f_{ccw} must differ too. N is typically in the range of 10^5 – 10^6 . The path length difference ΔL is related to Ω by the Sagnac effect, given by

$$\Delta L = \frac{4A}{c_m} \Omega \quad (8.1.1)$$

which is in close correspondence with (7.2.6). As before, A is the enclosed area and c_m is the velocity of light in the medium. With the RLG at rest, i.e., $\Omega = 0$, we have

$$\lambda = \frac{L}{N} \quad (8.1.2)$$

The increment in wavelength is

$$\Delta\lambda = \frac{\Delta L}{N} \quad (8.1.3)$$

and in view of the change $\Delta\lambda$ caused by ΔL , the corresponding relative change in frequency is

$$\frac{\Delta f}{f} = \frac{\Delta L}{L} \quad (8.1.4)$$

Substituting ΔL from (8.1.1), we have

$$\Delta f = \frac{\Delta L}{L} f = \frac{4A\Omega}{c_m L} f = \frac{4A}{\lambda L} \Omega \quad (8.1.5)$$

The cw and ccw beams at their corresponding frequencies f and $f + \Delta f$ are combined, as indicated in Figure 8.1 by the readout optics.

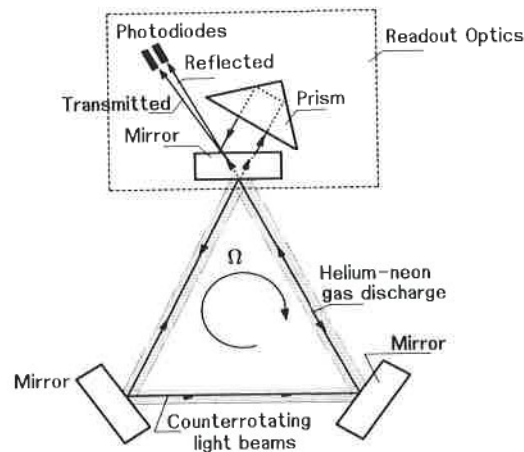


Figure 8.1. Schematic description of the ring laser gyro.

Δf is measured by allowing a small fraction of the light to escape through the mirror, which is part of the readout optics. The prism is designed to reflect one of the beams so that it partially overlaps the other by making it inclined by a very small angle, referred to as the *wedge angle*. Due to the finite width of the beams, the wedge angle permits partial merging of the two beams.

The interference fringe pattern

This merging, which is just a summation, creates a fringe pattern in the readout zone. The spatial light signals are $E_{cw} = E_1 \sin 2\pi f t$ and $E_{ccw} = E_2 \sin 2\pi (f + \Delta f) t$.

Assuming that $E_1 = E_2 = E$, and since $\Delta f \ll f$, their sum x is

$$\begin{aligned}
 x &= E[\sin 2\pi ft + \sin 2\pi(f + \Delta f)t] = 2E \sin \frac{1}{2} 2\pi(2f + \Delta f)t \cos \frac{1}{2} 2\pi \Delta f t \\
 &\cong 2E \sin(2\pi ft) \cos(\pi \Delta f t) = 2E \sin(2\pi ft) \cos\left\{\pi \frac{4A}{\lambda L} \Omega t\right\} \quad (8.1.6)
 \end{aligned}$$

which consists of the carrier signal at frequency f modulated by $\cos(\pi \Delta f t)$. It is required to express the combined readout in terms of its intensity, namely, $I = x^2$. We therefore have

$$\begin{aligned}
 I &= 4E^2 \sin^2(2\pi ft) \cos^2\left\{\pi \frac{4A}{\lambda L} \Omega t\right\} = 4E^2 \frac{1 - \cos 4\pi ft}{2} \frac{1 + \cos\left\{\frac{2\pi 4A}{\lambda L} \Omega t\right\}}{2} \\
 &= E^2 [1 - \cos(4\pi ft)] [1 + \cos\left(\frac{8\pi A}{\lambda L} \Omega t\right)] \\
 &= E^2 [1 - \cos(4\pi ft)] [1 + \cos(2\pi \gamma \theta)] \quad (8.1.7)
 \end{aligned}$$

where

$$\gamma = \frac{4A}{\lambda L} \quad (8.1.8)$$

is the spatial angular frequency and $\theta = \Omega t$ is the angle through which the gyro has rotated. γ is exactly the constant that translates Ω to Δf in accordance with (8.1.5). The so-called period is

$$\Pi = \gamma^{-1} = \frac{\lambda L}{4A} \quad (8.1.9)$$

If the RLG were annular in shape, (8.1.9) would be given by

$$\Pi = \frac{\lambda}{D} \quad (8.1.10)$$

which is exactly the expression for resolution of the circular IFOG, as given in (7.4.20). This result is indeed to be expected because the angular period Π , namely, the length of the standing wave, is exactly the natural unit of the resolution.

In (8.1.7), the first term in brackets is non-negative and represents the average light intensity at frequency f . The second non-negative term in brackets expresses the 100% modulation caused by the interference between the two combined counterrotating beams and thus represents the fringes. They appear only if $\Omega \neq 0$ and their angular frequency is proportional to Ω . This means that at a given point in the rotating RLG axis system, say the reflection point from the readout mirror seen by the photodiodes, the periodic light intensity variation behaves as if an inertially fixed annular standing wave outside the RLG is moving across an open window as the RLG rotating at the inertial angular rate Ω . This interpretation is depicted in Figure 8.2 as an inertially fixed ring on which a periodic light intensity wave having the angular

period Π is superimposed and that is sensed by the RLG which rotates at the angular rate Ω . Digital integrated angular rate increment pulses are generated by the pair of diodes depicted in Figure 8.1. The operation of the RLG from the point of view of the user is similar to a digital rotational encoder in which the optical bar-code is fixed in inertial space. The function of the electro-optical pickoff is essentially the same. Any drift of the RLG manifests itself as a slow rotation of the annular standing wave with respect to inertial space.

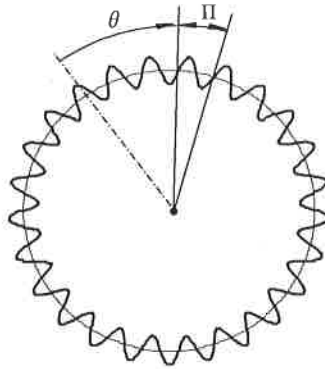


Figure 8.2. Annular standing wave interpretation of the RLG.

As the intensity fringes cross a diode, a corresponding sinusoidal signal is generated. Two diodes are required in order to determine whether Ω is clockwise or counter-clockwise. The diodes are placed at 90° with respect to each other in the coordinates of the fringe angle γ . With this arrangement, it is possible to determine which of the two diodes precedes the other. Digital pulse conditioning and logic convert the sinusoidal outputs into a train of pulses whose frequency is directly proportional to Ω and whose accumulated count represents the angle θ . The pulse size depends on the wavelength λ and the path length between mirrors. Thus, the RLG, in conjunction with its output logic, performs as a digital rate integrating gyro. As such, its null point is always dictated by the point at which the θ counter is initialized. This is precisely what is needed to implement a strapdown digital attitude reference system, commonly used in modern inertial navigation units.

EXAMPLE 8.1-1: Distance error propagation to limited resolution

The dimensions of a ring laser gyro are as follows: $L = 30$ cm; $A = 87$ cm²; $\lambda = 0.63$ μ . In accordance with (8.1.9), the period Π or the resolution is

$$\Pi = \frac{0.63 \times 10^{-6} \times 30 \times 10^{-1}}{4 \times 87 \times 10^{-4}} = 54.3 \text{ } \mu\text{rad} = 11.2 \text{ arc sec}$$

This book is about aerospace sensors and their essential roles in guidance and navigation systems. Performance characteristics, developed from basic physical laws, are consolidated in typical product-data lists, thus bridging between theory and practice. Sensor dynamics and random processes are discussed as the basis of statistical error modeling. Examples, problems, and qualitative discussions are used throughout to enhance the reader's intuitive understanding.

Intended as a textbook for advanced undergraduates, and for self-study by engineers, the book prepares readers to design aerospace guidance systems, to evaluate the feasibility of proposed concepts, and to evaluate the performance of existing systems.

The first two chapters introduce generic sensor models, random signal processing, and filtering and estimation, thus providing a common basis for students from diverse backgrounds. The seven chapters that follow cover three generations of sensor technology spanning fifty years, ranging from classical electromechanical sensors for rotation and force to modern fiber-optic gyros and silicon micromechanical devices. Sensors are presented as constituents of systems in which measurements are blended by Kalman filtering to achieve superior precision and reliability and lower cost. Effects of noise and interference, for example on automatic tracking, homing, and inertial navigation, are analyzed and discussed in detail.

The core of the book is based on courses developed and taught by the author at the Technion Department of Aerospace Engineering and on his experience at the Israel Ministry of Defense and NASA research institutes.

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