Fifth Edition

UNIVERSITY PHYSICS

Part II

Francis W. Sears
Professor Emeritus
Dartmouth College

Mark W. Zemansky
Professor Emeritus
The City College of
The City University of New York

Hugh D. Young
Associate Professor of Physics
Carnegie-Mellon University

ADDISON-WESLEY PUBLISHING COMPANY
Reading, Massachusetts · Menlo Park, California
London · Amsterdam · Don Mills, Ontario · Sydney

Delta 2010 - Kohler v. Delta - IPR2024-0700

This book is in the
ADDISON-WESLEY SERIES IN PHYSICS

Third printing, March 1977

Copyright © 1976, 1970, 1964, 1955, 1949 by Addison-Wesley Publishing Company, Inc. Philippines copyright 1976, 1970, 1964, 1955, 1949 by Addison-Wesley Publishing Company, Inc.
All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the publisher. Printed in the United States of America. Published simultaneously in Canada. Library of Congress Catalog Card No. 75–20989.

ISBN 0-201-06938-5
ABCDEFGHIJ-DO-7987

Preface

University Physics is intended for students of science and engineering who are taking an introductory calculus course concurrently. The complete text may be taught in an intensive two- or three-semester course and is also adaptable to a variety of shorter courses. Primary emphasis is on physical principles and problem solving; historical background and specialized practical applications have been given a place of secondary importance. Many worked-out examples and an extensive collection of problems are included with each chapter. *University Physics* is available as a single volume or as two separate parts. Part I includes mechanics, heat, and sound, and Part II includes electricity and magnetism, optics, and atomic and nuclear physics.

In this new edition, the basic philosophy and outline and the balance between depth of treatment and breadth of subject-matter coverage are unchanged from previous editions. We have tried to preserve those features that users of previous editions have found desirable, while incorporating a number of changes that should enhance the book's usefulness. Here are the most important changes.

1. The mks system of units, with the conventions and nomenclature of the Système Internationale, has become the principal unit system in the book. In this system the joule is the fundamental unit of energy of all forms, including heat. In the first half of the book, however, some examples and problems using English units have been retained.

2. The material on atomic and nuclear physics (Chapters 44 through 46) has been completely rewritten and expanded into three chapters, now including an elementary discussion of physics of solids, high-energy physics, and elementary particles.

3. A new chapter on relativistic mechanics has been added. Positioned somewhat arbitrarily at the end of the mechanics material, this chapter can be taken up earlier or later, or may be omitted completely if desired.

4. The chapter on electromagnetic waves has been completely rewritten, to exhibit more clearly and in simpler context the relation of wave propagation to the basic principles of electromagnetism.

5. Several sections have been added to broaden subject coverage. Among these are: 16–7 Examples (of calorimetric calculations), 19–25 (practical aspects of) Energy Conversion, 23–5 Musical Intervals and Scales, 23–7 Applications of Acoustic Phenomena, 28–10 Physiological Effects of Electric Currents, 38–7 Absorption (of light), 38–8 Illumination, 41–10 Defects of Vision, 42–1 Coherent Sources (of light), 42–13 Holography, 45–7 Semiconductors, 45–8 Semiconductor Devices, and 46–11 Radiation and the Life Sciences.

6. Some material has been reorganized. The material on surface tension has been shortened and incorporated into the hydrostatics chapter; the treatment of thermoelectricity has been shortened to a single section. The chapter on electromagnetic in-

29 Direct-current circuits and instruments 501

29–1 Resistors in series and in parallel 501
29–2 Kirchhoff's rules 503
29–3 Ammeters and voltmeters 505
29–4 The Wheatstone bridge 507
29–5 The ohmmeter 508
29–6 The potentiometer 508
29–7 The *R–C* series circuit 509
29–8 Displacement current 511

30 The magnetic field 519

30–1 Magnetism 519
30–2 The magnetic field 520
30–3 Lines of induction. Magnetic flux 522
30–4 Motion of charged particles in magnetic fields 523
30–5 Thomson's measurement of e/m 524
30–6 Isotopes 526
30–7 Mass spectroscopy 528
30–8 The cyclotron 529

31 Magnetic forces on current-carrying conductors 535

31–1 Force on a conductor 535
31–2 The Hall effect 536
31–3 Force and torque on a complete circuit 537
31–4 The galvanometer 539
31–5 The pivoted-coil galvanometer 540
31–6 The ballistic galvanometer 541
31–7 The direct-current motor 541
31–8 The electromagnetic pump 542

32 Magnetic field of a current 547

32–1 Field of a moving charge 547
32–2 Magnetic field of a current element. The Biot law 548
32–3 Magnetic field of a long straight conductor 549
32–4 Force between parallel conductors. The ampere and the coulomb 551
32–5 Magnetic field of a circular turn 552
32–6 Ampère's law 553
32–7 Applications of Ampère's law 554
32–8 Magnetic fields and displacement currents 557

33 Induced electromotive force 561

33–1 Motional electromotive force 561
33–2 Faraday's law 564
33–3 The search coil 566
33–4 Galvanometer damping 567
33–5 Induced electric fields 568
33–6 Lenz's law 569
33–7 The betatron 570
33–8 Eddy currents 571

34 Inductance 579

34–1 Mutual inductance 579
34–2 Self-inductance 580
34–3 Energy associated with an inductor 581
34–4 The *R–L* circuit 582
34–5 The *L–C* circuit 584
34–6 The *R–L–C* circuit 585

35 Magnetic properties of matter 589

35–1 Magnetic materials 589
35–2 Magnetic permeability 590
35–3 Molecular theory of magnetism 591
35–4 Magnetization and magnetic intensity 592
35–5 Magnetic susceptibility and permeability 594
35–6 Ferromagnetism 595
35–7 Magnetic domains 596
35–8 Hysteresis 598
35–9 Self-inductance 600
35–10 Permanent magnets 600
35–11 The magnetic field of the earth 602
35–12 Magnetic circuits 603

36 Alternating currents 609

36–1 Introduction 609
36–2 Circuits containing resistance, inductance, or capacitance 610
36–3 The *R–L–C* series circuit 613
36–4 Average and root-mean-square values. AC instruments 615
36–5 Power in ac circuits 617
36–6 Series resonance 618
36–7 Circuits in parallel 620
36–8 The transformer 621

37 Electromagnetic waves 627

37–1 Introduction 627
37–2 Speed of an electromagnetic wave 628
37–3 Energy in electromagnetic waves 630
37–4 Electromagnetic waves in matter 632
37–5 Sinusoidal waves 632
37–6 Radiation from an antenna 634
37–7 Waves on transmission lines 634

38 The nature and propagation of light 639

38–1 The nature of light 639
38–2 Sources of light 640
38–3 Waves, wave fronts, and rays 642
38–4 The speed of light 643
38–5 The laws of reflection and refraction 645
38–6 Index of refraction 647
38–7 Absorption 648
38–8 Illumination 649

39 Reflection and refraction at plane surfaces 653

39–1 Huygens' principle 653
39–2 Derivation of the law of reflection from Huygens' principle 654
39–3 Derivation of Snell's law from Huygens' principle 655
39–4 Total internal reflection 657
39–5 Refraction by a prism 658
39–6 Dispersion 660
39–7 The rainbow 661

40 Images formed by a single reflection or refraction 665

40–1 Introduction 665
40–2 Reflection at a plane mirror 665
40–3 Reflection at a spherical mirror 668
40–4 Focal point and focal length 672
40–5 Graphical methods 674
40–6 Refraction at a plane surface 675
40–7 Refraction at a spherical surface 677
40–8 Summary 679

41 Lenses and optical instruments 683

41–1 Images as objects 683
41–2 The thin lens 684
41–3 Diverging lenses 687
41–4 Graphical methods 688
41–5 Images as objects for lenses 689
41–6 The newtonian form of the lens equation 689
41–7 Thick lenses 691
41–8 Lens aberrations 692
41–9 The eye 693
41–10 Defects of vision 695
41–11 The magnifier 696
41–12 The camera 697
41–13 The projector 698
41–14 The compound microscope 699
41–15 Telescopes 700

42 Interference and diffraction 707

42–1 Interference and coherent sources 707
42–2 Young's experiment and Pohl's experiment 710
42–3 Intensity distribution in interference fringes 711
42–4 Phase changes in reflection 713
42–5 Interference in thin films. Newton's rings 713
42–6 Thin coatings on glass 715
42–7 The Michelson interferometer 716
42–8 Fresnel diffraction 719
42–9 Fraunhofer diffraction from a single slit 721
42–10 The plane diffraction grating 726
42–11 Diffraction of x-rays by a crystal 728
42–12 The resolving power of optical instruments 729
42–13 Holography 731

43 Polarization 737

43–1 Polarization 737
43–2 Polarization by reflection 738
43–3 Double refraction 739
43–4 Polarizers 741
43–5 Percentage polarization. Malus' law 743
43–6 The scattering of light 744
43–7 Circular and elliptic polarization 745
43–8 Production of colors by polarized light 747
43–9 Optical stress analysis 747
43–10 Study of crystals 748
43–11 Optical activity 748

44 Photons, electrons, and atoms 753

44–1 Emission and absorption of light 753
44–2 Thermionic emission 754
44–3 The photoelectric effect 756
44–4 Line spectra 757
44–5 The Bohr atom 759
44–6 Atomic spectra 763
44–7 The laser 764
44–8 X-ray production and scattering 766
44–9 Wave mechanics 769
44–10 The electron microscope 771

45 Atoms, molecules, and solids 775

45–1 The exclusion principle 775
45–2 Atomic structure 777
45–3 Diatomic molecules 779
45–4 Molecular spectra 780
45–5 Structure of solids 782
45–6 Properties of solids 783

while the capacitor is being charged, the rate at which energy flows into the space between the plates is equal to the rate at which the electrostatic energy increases.

37–11 A very long solenoid of n turns per unit length and radius a carries an increasing current i.

a) Calculate the induced electric field at a point at a distance r from the solenoid axis.

b) Compute the magnitude and direction of the Poynting vector at this point.

37–12 An FM radio station antenna radiates a power of 10 kW at a wavelength of 3 m. Assume for simplicity that the radiated power is confined to, and is uniform over, a hemisphere with the antenna at its center. What are the amplitudes of E and H in the radiation field at a distance of 10 km from the antenna?

37–13 In a TV picture, ghost images are formed when the signal from the transmitter travels directly to the receiver and also indirectly after reflection from a building or other large metallic mass. In a 25-inch set the ghost is about 1 cm to the right of the principal image if the reflected signal arrives 1 μs after the principal signal. In this case, what is the difference in path length for the two signals?

37–14 For a sinusoidal electromagnetic wave in vacuum, such as that described by Eqs. (37–19), show that the average density of energy in the electric field is the same as that in the magnetic field.

37–15 A plane electromagnetic wave has a wavelength of 3.0 cm and an $\boldsymbol{E}$-field amplitude of 30 V m^{-1}.

a) What is the frequency?

b) What is the $\boldsymbol{B}$-field amplitude?

c) What is the intensity?

d) What average force does the radiation pressure exert on a totally absorbing surface of area 0.5 m^2 perpendicular to the the direction of propagation?

37–16 If the intensity of direct sunlight is 1.4 kW m^{-2}, find the radiation pressure (in pascals) on (a) a totally absorbing surface; (b) a totally reflecting surface. Also express your results in atmospheres.

Chapter 38

The Nature and Propagation of Light

38–1 THE NATURE OF LIGHT

Until about the middle of the seventeenth century it was generally believed that light consists of a stream of particles or *corpuscles*. These corpuscles were supposedly emitted by light sources, such as the sun or a candle flame, and traveled outward from the source in straight lines. They could penetrate transparent materials and were reflected from the surfaces of opaque materials. When the corpuscles entered the eye, the sense of sight was stimulated.

By the middle of the seventeenth century, while most workers in the field of optics accepted the corpuscular theory, the idea had begun to develop that light might be a *wave* motion of some sort. Christian Huygens, in 1678, showed that the laws of reflection and refraction could be explained on the basis of a wave theory and that such a theory furnished a simple explanation of the recently discovered phenomenon of double refraction. The wave theory did not find immediate acceptance, however. For one thing, the objection was raised that if light were a wave motion, one should be able to see around corners, since waves can bend around obstacles in their path. We know now that the wavelengths of light waves are so short that the bending, while it *does* actually take place, is so small that it is not ordinarily observed. As a matter of fact, the bending of a light wave around the edges of an object, a phenomenon known as *diffraction*, was noted by Grimaldi in a book published in 1665, but the significance of his observations was not understood at the time.

In the first quarter of the nineteenth century the experiments of Thomas Young and Augustin Fresnel on interference, and the measurements of the velocity of light in liquids by Leon Foucault at a somewhat later date, conclusively demonstrated the existence of optical phenomena for whose explanation a corpuscular theory was inadequate. These phenomena, including interference and diffraction, can be understood on the basis of a wave theory and will be discussed further in Chapter 42. Young's experiments enabled him to measure the wavelength of the waves and Fresnel showed that the rectilinear propagation of light, as well as the diffraction effects observed by Grimaldi and others, could be accounted for by the behavior of waves of short wavelength.

The next great forward step in the theory of light was the work of Maxwell, discussed in the preceding chapter. In 1873, Maxwell showed that an oscillating electrical circuit should radiate electromagnetic waves. The speed of propagation of the waves could be computed from purely electrical and magnetic

Page 4 of 16

measurements, and it turned out to be very nearly 3×10^8m s^{-1}. Within the limits of experimental error, this was equal to the measured speed of propagation of light. The evidence seemed inescapable that light consisted of electromagnetic waves of extremely short wavelength. In 1887 Heinrich Hertz, using an oscillating circuit of small dimensions, succeeded in producing short-wavelength waves (we would speak of them today as *microwaves*) of undoubted electromagnetic origin and showed that they possessed all the properties of light waves. They could be reflected, refracted, focused by a lens, polarized, and so on, just as could waves of light. Maxwell's electromagnetic theory of light and its experimental justification by Hertz constituted one of the triumphs of physical science. By the end of the nineteenth century it was the general belief that little, if anything, would be added in the future to our knowledge of the nature of light. Such was not to be the case!

The classical electromagnetic theory failed to account for several phenomena associated with *emission* and *absorption* of light. One example is the phenomenon of photoelectric emission, that is, the ejection of electrons from a conductor by light incident on its surface. In 1905, Einstein extended an idea proposed 5 years earlier by Planck and postulated that the energy in a light beam was concentrated in packets or *photons*. A vestige of the wave picture was retained, in that a photon was still considered to have a frequency and the energy of a photon was proportional to its frequency. The mechanism of the photoelectric effect consisted in the transfer of a photon's energy to an electron. Experiments by Millikan showed that the kinetic energies of photoelectrons were in excellent agreement with the formula proposed by Einstein. These experiments will be discussed in more detail in Chapter 44.

Another striking confirmation of the photon nature of light is the Compton effect. A. H. Compton, in 1921, succeeded in determining the motion of a photon and a single electron, both before and after a "collision" between them, and found that they behaved like material bodies having kinetic energy and momentum, both of which were conserved in the collision. The photoelectric effect and the Compton effect, then, both seem to demand a return to a corpuscular theory of light.

The reconciliation of these apparently contradictory experiments has come only since 1930 with the development of quantum electrodynamics, a comprehensive theory which includes both wave and particle properties. The phenomena of light *propagation* may best be described by the electromagnetic *wave* theory, while the interaction of light with matter, in the processes of emission and absorption, is a *corpuscular* phenomenon.

38–2 SOURCES OF LIGHT

All bodies emit electromagnetic radiation as a result of thermal motion of their molecules; this radiation, called *thermal radiation*, is a mixture of different wavelengths. At a temperature of 300°C the most intense of these waves has a wavelength of 5000 $\times 10^{-9}$m or 5000 nm, which is in the *infrared* region. At a temperature of 800°C a body emits enough visible radiant energy to be self-luminous and appears "red hot." By far the larger part of the energy emitted, however, is still carried by infrared waves. At 3000°C, which is about the temperature of an incandescent lamp filament, the radiant energy contains enough of the "visible" wavelengths, between 400 nm and 700 nm, that the body appears nearly "white hot."

In modern incandescent lamps, the filament is a coil of fine tungsten wire. An inert gas such as argon is introduced to reduce evaporation of the filament. Incandescent lamps vary in size from one no larger than a grain of wheat to one with a power input of 5000 W, used for illuminating airfields.

One of the brightest sources of light is the *carbon arc*. Rods of carbon from 10 to 20 cm in length and about 1 cm in diameter are placed either horizontally, as shown in Fig. 38–1(a), or at an angle, as shown in Fig. 38–1(b). Sometimes the carbon rods are copper coated to improve electrical conductivity. To start a carbon arc, the two carbons are connected to a 110-V or 220-V dc source, are allowed to touch momentarily, and are then drawn apart. Intense

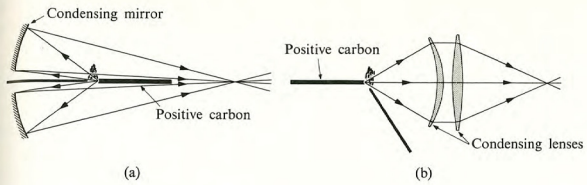


Fig. 38–1 Two types of carbon arcs. (a) With condensing mirror, for moderately sized motion picture theaters. (b) With condensing lenses for large motion picture theaters.

electron bombardment of the positive rod causes an extremely hot crater to form at its end. This end, at a temperature of about 4000°C, is the source of light. An electric motor or a clockwork mechanism is used to keep the carbons the correct distance apart as they burn away. Carbon arcs are used in all motion-picture theaters, where they operate on from 50 to several hundred amperes.

A common laboratory source of light is provided by a mercury arc. A glass or quartz tube has tungsten electrodes sealed in each end. A pool of mercury surrounds the negative electrode. A difference of potential is established across the electrodes and the tube is tilted until the mercury makes contact between the two electrodes. Some mercury is vaporized, and when the tube is restored to its vertical position an electric discharge is maintained by electrons and positive mercury ions. When the mercury is at low pressure, the mercury atoms emit a characteristic light consisting of only yellow, green, blue, and violet. A didymium filter may be used to absorb the yellow, and a yellow glass filter to absorb the blue and violet, leaving an intense green light consisting of a very small band of wavelengths whose average value is 546 nm. Low-pressure mercury arcs containing only one isotope of mercury, of atomic mass 198, are obtainable from the United States National Bureau of Standards. The green light from these lamps consists of an extremely narrow band of wavelengths, and is a close approach to *monochromatic* light.

If the current through a quartz mercury-arc lamp is allowed to increase, the temperature rises greatly and the vapor pressure of the mercury rises to between 50 and 100 atm. Such a lamp requires water cooling, and when operating is a very intense source of white light.

An intense source of yellow light of average wavelength 589.3 nm is provided by a sodium arc lamp. This is usually made of a special kind of glass that is not attacked by sodium and into which electrodes are sealed. Each electrode is a filament for providing electrons to maintain an electric discharge through an inert gas. After the inert gas discharge has taken place for a few minutes, the temperature rises to a value at which the vapor pressure of the sodium is great enough to provide sufficient sodium atoms to emit the characteristic yellow sodium light. Sodium lamps are sometimes used for street lighting because of their economy and because great visual acuity results from the absence of chromatic aberration of the eye when almost monochromatic light is used.

A very important development in illumination engineering is provided by the *fluorescent* lamp. This consists of a glass tube containing argon and a droplet of mercury. The electrodes consist of tungsten filaments. When an electric discharge takes place in the mercury–argon mixture, only a small amount of visible light is emitted by the mercury and argon atoms. There is, however, considerable *ultraviolet* light (light of wavelength shorter than that of visible violet). This ultraviolet light is absorbed in a thin layer of material, called a *phosphor*, with which the interior walls of the glass tube are coated. The phosphor has the property of *fluorescence*, which means that it emits visible light when illuminated by light of shorter wavelength. Lamps may be obtained which will fluoresce with any desired color, depend-

Page 5 of 16

ing on the nature of the phosphor. Phosphors commonly used are cadmium borate for pink, zinc silicate for green, calcium tungstate for blue, and a mixture for white.

The brightest source among those described in the preceding pages is the carbon arc, which emits a large amount of radiant power from a relatively small surface. The radiation, however, spreads out in a large solid angle and only a fraction of it may be collected by a lens or a mirror and focused on a piece of equipment. In Section 44–7, a source of light readily available, called a *laser*, will be described and explained. The laser produces a narrow beam of enormously intense radiation, *all* of which may be intercepted by a lens and focused on an object. The power of this beam can be so great that high-intensity lasers have been used to cut through steel, fuse high-melting-point materials, and bring about many other effects that are important in physics, chemistry, biology, and engineering.

38–3 WAVES, WAVE FRONTS, AND RAYS

It is convenient to represent a train of waves of any sort by means of *wave fronts. A wave front is defined as the locus of all points at which the phase of vibration of a physical quantity is the same.* Thus, in the case of sound waves spreading out in all directions from a point source, any spherical surface concentric with the source is a possible wave front. Some of these spherical surfaces are the loci of points at which the pressure is a *maximum*, others where it is a *minimum*, and so on; but the *phase* of the pressure variations is the same over any spherical surface. It is customary to draw only a few wave fronts, usually those which pass through the maxima and minima of the disturbance. Such wave fronts are separated from one another by one-half wavelength.

If the wave is a light wave, the quantity corresponding to the pressure in a sound wave is the electric or magnetic field. It is usually unnecessary to indicate in a diagram either the magnitude or direction of the field, but simply to show the *shape* of the wave by drawing the wave fronts of their intersections with some reference plane. For example, the

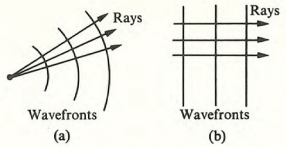


Fig. 38–2 *Wave fronts and rays.*

electromagnetic waves radiated by a small light source may be represented by spherical surfaces concentric with the source or, as in Fig. 38–2(a), by the intersections of these surfaces with the plane of the diagram. At a sufficiently great distance from the source, where the radii of the spheres have become very large, the spherical surfaces can be considered planes and we have a train of plane waves, as in Fig. 38–2(b).

A train of light waves may often be represented more simply by means of *rays* than by wave fronts. In a corpuscular theory, a ray is simply the path followed by a light corpuscle. From the wave viewpoint, *a ray is an imaginary line drawn in the direction in which the wave is traveling.* Thus, in Fig. 38–2(a) the rays are the radii of the spherical wave fronts and in Fig. 38–2(b) they are straight lines perpendicular to the wave fronts. In fact, in every case in which the waves are traveling in a homogeneous isotropic medium, the rays are straight lines normal to the wave fronts. At a boundary surface between two media, such as the surface between a glass plate and the air outside it, the direction of a ray may change suddenly, but it is a straight line both in the air and in the glass. If the medium is *not* homogeneous, for instance, if one is considering the passage of light through the earth's atmosphere, where the density and hence the velocity vary with elevation, the rays are curved but are still normal to the wave fronts. If the medium is anisotropic, as is the case in certain crystals, the direction of rays is not always normal to the wave fronts. The problem will be considered in more detail in Chapter 43.

A narrow cone of rays diverging from a common point is called a *pencil.* The entire group of

pencils orginating at all points of a surface of finite extent is called a *beam.*

The wavelength of visible electromagnetic waves (those capable of affecting the sense of sight) lies between 0.00004 cm and 0.00007 cm. Because these wavelengths are so small, it is convenient to express them in terms of a small unit of length.

Three such units are commonly used: the micrometer (1 μm), the nanometer (1 nm) (both accented on the *first* syllable), and the angstrom (1 Å):

$$1\ \mu\text{m} = 10^{-6}\text{m} = 10^{-4}\text{cm},$$
$$1\ \text{nm} = 10^{-9}\text{m} = 10^{-7}\text{cm},$$
$$1\ \text{Å} = 10^{-10}\text{m} = 10^{-8}\text{cm}.$$

In older literature the micrometer is sometimes called the *micron*, and the nanometer is sometimes called the *millimicron*; these terms are passing out of common usage. Most workers in the fields of optical instrument design, color, and physiological optics express wavelengths in *nanometers.* For example, the wavelength of the yellow light from a sodium flame is 589 nm. A spectroscopist, however, would specify the wavelengths present in this same yellow light as 5889.963 Å and 5895.930 Å.

Figure 38–3 is a chart of the electromagnetic spectrum. Although waves of different wavelengths must be produced by different methods, all are alike so far as their fundamental nature is concerned. Note the relatively small portion occupied by the visible spectrum.

Different parts of the visible spectrum evoke the sensations of different colors. Wavelengths for colors in the visible spectrum are (approximately) as follows:

From 400 to 450 nm	Violet
From 450 to 500 nm	Blue
From 500 to 550 nm	Green
From 550 to 600 nm	Yellow
From 600 to 650 nm	Orange
From 650 to 700 nm	Red

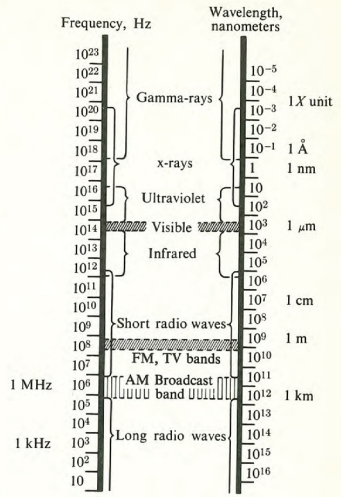


Fig. 38–3 *A chart of the electromagnetic spectrum.*

By the use of special sources or special filters, it is possible to limit the wavelength spread to a small band, say from 1 to 10 nm. Such light is called roughly *monochromatic light*, meaning light of a single color. Light consisting of only one wavelength is an idealization that is useful in theoretical calculations but represents an experimental impossibility. When the expression "monochromatic light of wavelength 550 nm" is used in theoretical discussions, it refers to one wavelength, but in descriptions of laboratory experiments it means a small band of wavelengths *around* 550 nm.

38–4 THE SPEED OF LIGHT

The speed of light in free space is one of the fundamental constants of nature. Its magnitude is so great (about 186,000 mi s^{-1} or 3×10^8m s^{-1}) that it

evaded experimental measurement until 1676. Up to that time it was generally believed that light traveled with an infinite speed.

The first attempt to measure the speed of light involved a method proposed by Galileo. Two experimenters were stationed on the tops of two hills about a mile apart. Each was provided with a lantern, the experiment being performed at night. One man was first to uncover his lantern and, observing the light from this lantern, the second was to uncover his. The velocity of light could then be computed from the known distance between the lanterns and the time elapsing between the instant when the first observer uncovered his lantern and when he observed the light from the second. While the experiment was entirely correct in principle, we know that the speed is too great for the time interval to be measured in this way with any degree of precision.

In 1676, the Danish astronomer Olaf Roemer, from astronomical observations made on one of the satellites of the planet Jupiter, obtained the first definite evidence that light is propagated with a finite speed. Jupiter has twelve small satellites or moons, four of which are sufficiently bright to be seen with a moderately good telescope or a pair of field glasses. The satellites appear as tiny bright points at one side or the other of the disk of the planet. These satellites revolve about Jupiter just as does our moon about the earth and, since the plane of their orbits is nearly the same as that in which the earth and Jupiter revolve, each is eclipsed by the planet during a part of every revolution.

Roemer was engaged in measuring the time of revolution of one of the satellites by taking the time interval between consecutive eclipses (about 42 hr). He found, by a comparison of results over a long period of time, that while the earth was receding from Jupiter the periodic times were all somewhat longer than the average, and that while it was approaching Jupiter the times were all somewhat shorter. He concluded rightly that the cause of these variations was the varying distance between Jupiter and the earth.

Figure 38–4, not to scale, illustrates the situation. Let observations be started when the earth and Jupiter are in the positions E_1 and J_1. Since Jupiter

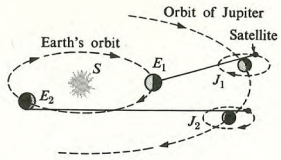


Fig. 38–4 *Roemer's method for deducing the velocity of light.*

requires about 12 years to make one revolution in its orbit, then by the time the earth has moved to E_2 (about 5 months later) Jupiter has moved only to J_2. During this interval the distance between the planets has been continually increasing. Hence at each eclipse, the light from the satellite must travel a slightly greater distance than at the preceding eclipse, and the observed time of revolution is slightly larger than the true time.

Roemer concluded from his observations that a time of about 22 min was required for light to travel a distance equal to the diameter of the earth's orbit. The best figure for this distance, in Roemer's time, was about 172,000,000 mi. Although there is no record that Roemer actually made the computation, if he *had* used the data above he would have found a speed of about 130,000 mi s^{-1} or 2.1×10^8 m s^{-1}.

The first successful determination of the speed of light from purely *terrestrial* measurements was made by the French scientist Fizeau in 1849. A schematic diagram of his apparatus is given in Fig. 38–5. Lens L_1 forms an image of the light source S at a point near the rim of a toothed wheel T, which can be set into rapid rotation. G is an inclined plate of clear glass. Suppose first that the wheel is stationary and the light passes through one of the openings between the teeth. Lenses L_2 and L_3, which are separated by about 8.6 km, form a second image on the mirror M. The light is reflected from M, retraces its path, and is in part reflected from the glass plate G through the lens L_4 into the eye of an observer at E.

If the wheel T is set in rotation, the light from S is "chopped up" into a succession of wave trains of

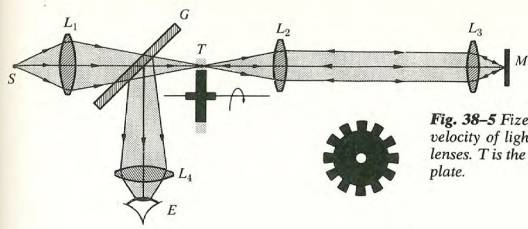


Fig. 38–5 *Fizeau's toothed wheel method for measuring the velocity of light. S is a light source, L_1, L_2, L_3, and L_4 are lenses. T is the toothed wheel, M is a mirror, and G is a glass plate.*

limited length. If the speed of rotation is such that by the time the front of one wave train has traveled to the mirror and returned, an opaque segment of the wheel has moved into the position formerly occupied by an open portion, no reflected light will reach the observer E. At twice this angular velocity, the light transmitted through any one opening will return through the next and an image of S will again be observed. From a knowledge of the angular velocity and radius of the wheel, the distance between openings, and the distance from wheel to mirror, the speed of light may be computed. Fizeau's measurements were not of high precision. He obtained a value of

$$3.15 \times 10^8 \text{ m s}^{-1}.$$

Fizeau's apparatus was modified by Foucault, who replaced the toothed wheel with a rotating mirror. In addition, Foucault introduced between the wheel and the mirror a tube filled with water and proved that the speed of light in water is less than in air. (The old corpuscular theory demanded that it should be greater, and at the time these measurements were made they were taken as conclusive proof that a corpuscular theory was untenable.)

The most precise measurements by the Foucault method were made by the American physicist Albert A. Michelson (1852-1931). His first experiments were performed in 1878 while he was on the staff of the Naval Academy at Annapolis. The last, under way at the time of his death, were completed in 1935 by Pease and Pearson.

From analysis of all measurements up to 1969, the most probable value is

$$c = 2.9979250 \times 10^8 \text{ m s}^{-1},$$

which is believed to be correct within ± 100 m s^{-1}.

Since we feel sure today that the speed of any electromagnetic wave in free space is given by

$$c = \sqrt{1/\epsilon_0\mu_0} = \sqrt{k/k'},$$

and since in the mks system k' is assigned a value of *exactly* 10^{-7} N s^2 C^{-2}, the preceding equation provides the most precise means for finding the value of the electrical constant k:

$$k = k'c^2,$$

since the speed of light can be measured with much higher precision than the force between two charged bodies.

38–5 THE LAWS OF REFLECTION AND REFRACTION

Most of the objects we see are visible because they *reflect* light into our eyes. In the most common class of reflection, called *diffuse reflection*, light is reflected in all directions. A book on a table in a room lighted only by a point source of light can be seen from any part of the room. This type of reflection occurs whenever the roughness of the reflecting body has "dimensions" large compared with the wavelength of the reflected wave. In the other class of reflection, called *regular* or *specular reflection*, a narrow beam or

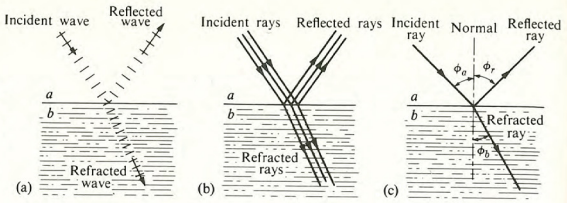


Fig. 38–6 *(a) A train of plane waves is in part reflected and in part refracted at the boundary between two media. (b) The waves in (a) are represented by rays. (c) For simplicity, only one example of incident, reflected, and refracted rays is drawn.*

pencil of light is reflected in one direction only. This type of reflection occurs from smooth surfaces whose irregularities are small compared with the wavelength of the reflected wave. For example, reflection from a blotter is diffuse, whereas reflection from a mirror is specular. Reflection from a polished automobile is intermediate, in that the reflected light is both diffuse and specular. Our main interest is in specular reflection, and in what follows, the word "reflection" will mean "specular reflection."

Figure 38–6(a) shows a narrow train of plane waves of light incident from the upper left on a plane surface separating two transparent substances, *a* and *b*, such as air and glass. A part of the incident light is *reflected* at the surface and a part passes into the lower medium, or is *refracted.*

The directions of the incident, reflected, and refracted beams of light are specified in terms of the angles they make with the normal to the surface at the point of incidence. For this purpose it is sufficient to indicate one ray, as in Fig. 38–6(c), although a single ray of light is a geometrical abstraction. A careful experimental study of the directions of the incident, reflected, and refracted beams leads to the following results:

1. *The incident, reflected, and refracted rays, and the normal to the surface, all lie in the same plane.* Thus, if the incident ray is in the plane of the diagram, and the surface of separation is perpendicular to this plane, the reflected and refracted rays are in the plane of the diagram.

2. *The angle of reflection ϕ_r is equal to the angle of incidence ϕ_a for all colors and any pair of substances.* Thus

$$\boxed{\phi_r = \phi_a.} \qquad (38\text{–}1)$$

The experimental result that $\phi_r = \phi_a$, and that the incident and reflected rays and the normal all lie in the same plane, is known as the *law of reflection.*

3. For monochromatic light and for a given pair of substances, *a* and *b*, on opposite sides of the surface of separation, *the ratio of the sine of the angle ϕ_a* (between the ray in substance *a* and the normal) *and the sine of the angle ϕ_b* (between the ray in substance *b* and the normal) *is a constant.* Thus

$$\frac{\sin \phi_a}{\sin \phi_b} = \text{constant}. \qquad (38\text{–}2)$$

This experimental result, together with the fact that the incident and refracted rays and the normal to the surface all lie in the same plane, is known as the *law of refraction.* The discovery of this law is usually credited to Willebrord Snell (1591–1626), although there appears to be some doubt that it was actually original with him. It is called *Snell's law.*

The laws of reflection and refraction relate only to the *directions* of the corresponding rays but say nothing about an equally important question, namely, the *intensities* of the reflected and refracted rays. These depend on the angle of incidence; for the present we simply state that the fraction reflected is smallest at *normal* incidence, where it is a few percent, and that it increases with increasing angle of incidence to almost 100% at grazing incidence or when $\phi_a = 90°$.

Table 38–1 INDEX OF REFRACTION FOR YELLOW SODIUM LIGHT (λ = 589 nm)

Substance	Index of refraction	Substance	Index of refraction
Solids		Liquids at 20°C	
Ice (H_2O)	1.309	Methyl alcohol (CH_3OH)	1.3290
Fluorite (CaF_2)	1.434	Water (H_2O)	1.3330
Rock salt (NaCl)	1.544	Ethyl alcohol (C_2H_5OH)	1.3618
Quartz (SiO_2)	1.544	Carbon tetrachloride (CCl_4)	1.4607
Zircon ($ZrO_2 \cdot SiO_2$)	1.923	Turpentine	1.4721
Diamond (C)	2.417	Glycerine	1.4730
Fabulite ($SrTiO_3$)	2.409	Benzene	1.5012
Rutile (TiO_2)	2.616 to 2.903	Carbon disulfide (CS_2)	1.6276

Suppose next that a ray of light is directed from *below* the surface in Fig. 38–6. Again, we find that reflected and refracted rays exist, and that these, together with the incident ray and the normal, all lie in the same plane. The same law of reflection applies as when the ray is originally traveling in air, and the same law of refraction, Eq. (38–2). *The passage of a ray of light in going from one medium to another is reversible.* It follows the same path in going from *b* to *a* as when going from *a* to *b*. Also, in going from *b* to *a*, when the incident ray is normal to the surface, the same fraction of the incident light is reflected as when normal to the surface from above. As the angle between the ray in water and the normal is increased, the fraction reflected increases, but according to a different law from that applying in the direction from *a* to *b*.

38–6 INDEX OF REFRACTION

Let a beam of monochromatic light traveling *in vacuum* make an angle of incidence ϕ_v with the normal to the surface of a substance *a*, and let ϕ_a be the angle of refraction in the substance. The constant in Snell's law is then called the *index of refraction* of substance *a* and is designated by n_a:

$$\boxed{\frac{\sin \phi_v}{\sin \phi_a} = n_a.} \qquad (38\text{–}3)$$

The index of refraction depends not only on the substance but on the wavelength of the light. If no wavelength is stated, the index is usually assumed to be that corresponding to the yellow light from a sodium lamp, of wavelength 589 nm.

The index of refraction of most of the common glasses used in optical instruments lies between 1.46 and 1.96. There are only a very few substances having indices larger than this value, diamond being one, with an index of 2.42, and rutile (crystalline titanium dioxide) another, with an index of 2.7. The values for a number of solids and liquids are given in Table 38–1.

The index of refraction of air at standard conditions, for violet light of wavelength 436 nm, is 1.0002957, while for red light of wavelength 656 nm, the index is 1.0002914. It follows that for most purposes the *index of refraction of air can be assumed to be unity.* The index of refraction of a gas increases uniformly as the density of the gas is increased.

It follows from Eq. (38–3) that the angle of refraction ϕ_a is always *less than* the angle of incidence ϕ_v for a ray passing from a vacuum into one of the materials listed in Table 38–1, where all indices are seen to be greater than unity. In such a case, the ray is bent *toward* the normal. If the light is traveling in the opposite direction, the reverse is true and the ray is bent *away from* the normal.

Consider two parallel-sided plates of substances *a* and *b* placed parallel to each other with an arbitrary space between them, as shown in Fig. 38–7(a). Let the medium surrounding both plates be vacuum, although the behavior of light would be

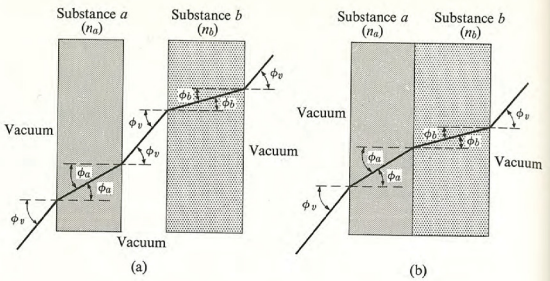


Fig. 38–7 *The transmission of light through parallel plates of different substances. The incident and emerging rays are parallel, regardless of direction and regardless of the thickness of the space between adjacent slabs.*

practically the same if the plates were surrounded by air. If a ray of monochromatic light starts at the lower left with an angle of incidence ϕ_v, the angle between ray and normal in substance a is ϕ_a, and the light emerges from substance a at an angle ϕ_v equal to its incident angle. The light ray therefore enters plate b with an angle of incidence ϕ_v, makes an angle ϕ_b in substance b, and emerges again at an angle ϕ_v. Exactly the same path would be traversed if the same light ray were to start at the upper right and enter substance b at an angle ϕ_v. Moreover, *the angles are independent of the thickness of the space between the two plates,* and are the same when the space shrinks to nothing, as in Fig. 38–7(b).

Applying Snell's law to the refractions that take place at the surface between vacuum and substance a, and also at the surface between vacuum and substance b, we have

$$\frac{\sin \phi_v}{\sin \phi_a} = n_a, \qquad \frac{\sin \phi_v}{\sin \phi_b} = n_b.$$

Dividing the second equation by the first, we get

$$\frac{\sin \phi_a}{\sin \phi_b} = \frac{n_b}{n_a}, \tag{38–4}$$

which shows that the *constant of Snell's law for the refraction between substances a and b is the ratio of the indices of refraction.* From Eq. (38–4) we see that the simplest and most symmetrical way of writing Snell's law for any two substances a and b, and for any direction, is

$$\boxed{n_a \sin \phi_a = n_b \sin \phi_b.} \tag{38–5}$$

38–7 ABSORPTION

No material is perfectly transparent; as light passes through any optical medium (except vacuum) its energy is partially absorbed, increasing the internal energy in the material, and the intensity (power per unit area) is correspondingly attenuated.

When a beam of light passes through a thin sheet of material, of thickness dx, the decrease dI in its intensity I is found to be proportional to the initial intensity I and to the thickness dx. Thus

$$dI = -\alpha I\, dx. \tag{38–6}$$

The proportionality constant α, which depends on the material, is called the *absorption coefficient.* The intensity after passage through a slab of finite thickness x can be obtained by integrating Eq. (38–6):

$$\boxed{I = I_0\, e^{-\alpha x},} \tag{38–7}$$

where I_0 is the intensity at $x = 0$. Equation (38–7) is called *Lambert's law.*

The absorption coefficient is often strongly wavelength-dependent. A clear optical glass for which $\alpha = 4\ \text{m}^{-1}$ (a typical value) in the middle of the visible spectrum may have $\alpha = 1000\ \text{m}^{-1}$ at $\lambda = 250$ nm (the near ultraviolet) and hence be essentially opaque to ultraviolet radiation. If α varies substantially *within* the visible spectrum, white light incident on the material appears colored when it emerges. For example, if α is larger at 500 nm than at 600 nm, proportionally more green and blue light is absorbed, and the emergent light appears red.

In some materials, absorption depends on the *polarization* of the incident light, that is, the plane in which the electric-field vector oscillates. A familiar example is the Polaroid filter, discussed in Section 43–4, used for sunglasses and many other applications. This material is transparent for light with one plane of polarization but absorbent for light in the perpendicular plane; the two absorption coefficients may differ by as much as a factor of 100.

A beam of light passing through an optical medium may also be attenuated by *scattering.* In contrast to absorption, in which the energy is ordinarily converted to internal energy, scattering simply redirects some of the radiation into directions other than that of the beam. The visibility from the side of a searchlight beam at night results from scattering of light out of the beam by dust particles and water droplets in the air and, to a small extent, by the air molecules themselves.

Scattering of visible light is usually wavelength-dependent, usually increasing with increasing frequency (decreasing wavelength). The sky is blue because the shorter-wavelength (blue) visible light from the sun is scattered more strongly in the earth's atmosphere than is the longer-wavelength (red) light. Similarly, at sunset the sun appears red because proportionally more of the blue light has been scattered out of the beam.

38–8 ILLUMINATION

We have defined the *intensity* of light and other electromagnetic radiation as power per unit area, measured in watts per square meter. Similarly, the *total* rate of radiation of energy from any of the sources of light discussed in Section 38–2 is called the *radiant power* or *radiant flux,* measured in watts. These quantities are not adequate to measure the visual sensation of *brightness,* however, for two reasons: First, not all the radiation from a source lies in the visible spectrum; an ordinary incandescent light bulb radiates more energy in the infrared than in the visible spectrum. Second, the eye is not equally sensitive to all wavelengths; a bulb emitting 1 watt of yellow light appears brighter than one emitting one watt of blue light.

The quantity analogous to radiant power, but compensated to include the above effects, is called *luminous flux,* denoted by F. The unit of luminous flux is the *lumen,* abbreviated lm, defined as that quantity of light emitted by $\frac{1}{60}$ cm^2 surface area of pure platinum at its melting temperature (about 1770°C), within a solid angle of 1 steradian (1 sr). As an example, the total light output (luminous flux) of a 40-watt incandescent light bulb is about 500 lm, while that of a 40-watt fluorescent tube is about 2300 lm.

When luminous flux strikes a surface, the surface is said to be *illuminated.* The intensity of illumination, analogous to the intensity of electromagnetic radiation (which is power per unit area) is the *luminous flux per unit area,* called the *illuminance,* denoted by E. The unit of illuminance is the lumen per square meter, also called the *lux*:

$$1\ \text{lux} = 1\ \text{lm m}^{-2}.$$

An older unit, the lumen per square foot, or footcandle, has become obsolete. If luminous flux F falls at normal incidence on an area A, the illuminance E is given by

$$E = \frac{F}{A}. \tag{38–8}$$

Most light sources do not radiate equally in all directions; it is useful to have a quantity that describes the intensity of a source in a specific direction, without using any specific distance from the source. We place the source at the center of an imaginary sphere of radius R. A small area A of the sphere subtends a solid angle Ω given by $\Omega = A/R^2$. If the luminous flux passing through this area is F, we define the *luminous*

intensity I in the direction of the area as

$$I = \frac{F}{\Omega}. \qquad (38\text{–}9)$$

The unit of luminous intensity is one lumen per steradian, also called one *candela*, abbreviated cd:

$$1\ \text{cd} = 1\ \text{lm sr}^{-1}.$$

The term "luminous intensity" is somewhat misleading. The usual usage of *intensity* connotes power per unit area, and the intensity of radiation from a point source decreases as the square of the distance. Luminous intensity, however, is flux per unit *solid angle*, not per unit *area*, and the luminous intensity of a source in a particular direction *does not* decrease with increasing distance.

Example A certain 100-watt light bulb emits a total luminous flux of 1200 lm, distributed uniformly over a hemisphere. Find the illuminance and the luminous intensity at a distance of 1 m, and at 5 m.

The area of a half-sphere of radius 1 m is

$$(2\pi)(1\ \text{m})^2 = 6.28\ \text{m}^2.$$

The illuminance at 1 m is

$$E = \frac{1200\ \text{lm}}{6.28\ \text{m}^2} = 191\ \text{lm m}^{-2} = 191\ \text{lux}.$$

Similarly, the area of a half-sphere of radius 5 m is

$$(2\pi)(5\ \text{m})^2 = 157\ \text{m}^2,$$

and the illuminance at 5 m is

$$E = \frac{1200\ \text{lm}}{157\ \text{m}^2} = 7.64\ \text{lm m}^{-2} = 7.64\ \text{lux}.$$

This is smaller by a factor of 5^2 than the illuminance at 1 m, and illustrates the inverse-square law for illuminance from a point source.

The solid angle subtended by a hemisphere is 2π sr. The luminous intensity is

$$I = \frac{1200\ \text{lm}}{2\pi\ \text{sr}} = 191\ \text{lm sr}^{-1} = 191\ \text{cd}.$$

The luminous intensity does not depend on distance.

PROBLEMS

38–1 What is the wavelength in meters, microns, nanometers, and angstrom units of (a) soft x-rays of frequency 2×10^{17} Hz? (b) green light of frequency 5.6×10^{14} Hz?

38–2 The visible spectrum includes a wavelength range from about 400 nm to about 700 nm. Express these wavelengths in inches.

38–3 Assuming the radius of the earth's orbit to be 92,900,000 mi, and taking the best value of the speed of light, compute the time required for light to travel a distance equal to the diameter of the earth's orbit. Compare with Roemer's value of 22 min.

38–4 Fizeau's measurements of the speed of light were continued by Cornu, using Fizeau's apparatus but with the distance between mirrors increased to 22.9 km. One of the toothed wheels used was 40 mm in diameter and had 180 teeth. Find the angular velocity at which it should rotate so that light transmitted through one opening will return through the next.

38–5 The speed of light in matter is less than in vacuum by a factor of the refractive index of the material. When orange light of wavelength 600 nm enters glass having refractive index 1.5, find (a) the wavelength, (b) the frequency, in the material.

38–6 An inside corner of a cube is lined with mirrors. A ray of light is reflected successively from each of three mutually perpendicular mirrors; show that its final direction is always exactly opposite to its initial direction. This principle is used in tail-light lenses and reflecting highway signs.

38–7 The density of the earth's atmosphere increases as the surface of the earth is approached.

a) Draw a diagram showing how the light from a star or planet is bent as it goes through the atmosphere.
b) Indicate the apparent position of the light source.
c) Explain how one can see the sun after it has set.
d) Explain why the setting sun appears flattened.

38–8

a) Why does the surface of a smooth highway under hot sun appear wet when it is seen at a glancing angle?
b) Account for a mirage in the desert.

38–9 A ray of light traveling with speed c leaves point 1 of Fig. 38–8 and is reflected to point 2. Show that the time

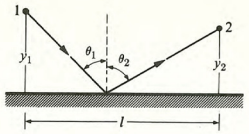


Fig. 38–8

required for the light to travel from 1 to 2 is $(y_1 \sec \theta_1 + y_2 \sec \theta_2)/c$.

38–10 Prove that a ray of light reflected from a plane mirror rotates through an angle 2θ when the mirror rotates through an angle θ about an axis perpendicular to the plane of incidence.

38–11 A parallel beam of light is incident on a prism, as shown in Fig. 38-9, part reflected from one face and part from another. Show that the angle θ between the two reflected beams is twice the angle A between the two reflecting surfaces.

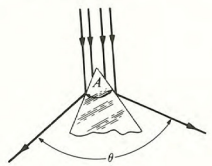


Fig. 38–9

38–12 A parallel beam of light makes an angle of 30° with the surface of a glass plate having a refractive index of 1.50. (a) What is the angle between the refracted beam and the surface of the glass? (b) What should be the angle of incidence ϕ with this plate for the angle of refraction to be $\phi/2$?

38–13 Light strikes a glass plate at an angle of incidence of 60°, part of the beam being reflected and part refracted. It is observed that the reflected and refracted portions make an angle of 90° with each other. What is the index of refraction of the glass?

38–14 A ray of light is incident on a plane surface separating two transparent substances of indices 1.60 and 1.40. The angle of incidence is 30° and the ray originates in the medium of higher index. Compute the angle of refraction.

38–15 A parallel-sided plate of glass having a refractive index of 1.60 is held on the surface of water in a tank. A ray coming from above makes an angle of incidence of 45° with the top surface of the glass. (a) What angle does the ray make with the normal in the water? (b) How does this angle vary with the refractive index of the glass?

38–16 A lens for sunglasses is to be made from a gray-tinted glass having an absorption coefficient of 500 m^{-1}. If the intensity of light passing through the lens is to be decreased by a factor of $\frac{1}{4}$, what should be the thickness of the lens?

38–17 If a swimming pool is 5m deep and the water has an absorption coefficient of 2 m^{-1}, by what factor is the intensity of light attenuated as it travels from the surface to the bottom of the pool?

38–18 If the absorption coefficient of sea water is 2 m^{-1}, and if the eye can perceive light of intensity smaller than that of sunlight by a factor of 10^{-18}, what is the greatest depth below the ocean surface at which light can be seen?

38–19 The illuminance of direct sunlight is about 10^5 lux. If a photoflash lamp has an intensity in a certain direction of 5×10^6 cd, at what distance from a surface should it be placed to produce illuminance equal to that of sunlight?

38–20 The *luminous efficiency* of a lamp is defined as the ratio of luminous flux to electric power input. A certain lamp mounted 3 m above a desk top has a luminous efficiency of 20 lm W^{-1}. What is the power input to the lamp if the illuminance on the desk is equal to that of sunlight, about 10^5 lux? Assume that the lamp radiates uniformly over its lower half-sphere.

38–21 A certain baseball field in the shape of a square 140 m on a side is to be illuminated for night games by six towers supporting banks of 1000-watt incandescent lamps with luminous efficiency (see Problem 38–20) of 30 lm W^{-1}. The illuminance required on the playing field is 200 lux. Assume that 50% of the luminous flux from the lamps reaches the field.

a) How many lamps are required in each tower?
b) What is the electric power input to each tower?
c) If power for all six towers is supplied by a generator driven by a gasoline engine, what must be the power capacity of the engine?

Page 10 of 16

Chapter 39

Reflection and Refraction at Plane Surfaces

39–1 HUYGENS' PRINCIPLE

In Chapters 37 and 38 we have considered electromagnetic waves for which the wavefronts are planes or spheres, the latter originating from a very small source which may be idealized as a point. In the absence of obstructions, such a wave can propagate indefinitely far; a relatively simple expression can be written down which describes the electric or magnetic field at any point and at any time. We have written such expressions for sinusoidal plane electromagnetic waves in Section 37–5.

The absence of obstructions is of course an idealization; in real situations waves almost always *are* obstructed. When light enters a telescope, only that portion of a wave front corresponding to the diameter of the objective lens is admitted. When light passes through a narrow slit, almost the entire wave front is absorbed by the screen and only a small portion goes through. The fundamental problem of wave propagation is to determine the amplitude, the phase, and the polarization of the light arriving at a point when only a portion of a wave front is exposed. This can be an extraordinarily difficult problem when the exposed portion of the wave front has an unusual shape, and when the electrical and magnetic characteristics of the obstructing screen have to be taken into account. If, however, the direction of the $\boldsymbol{E}$-vector is unimportant, a scalar theory may be used and the mathematical equations become somewhat simpler. The technique of finding the intensity and phase of the light at any point when only a portion of a wave front is exposed is called *Huygens' principle.* (The diphthong "uy" is pronounced like the French sound "euille," somewhat as the "oy" in "boy," and the "g" is guttural.)

The principle, as stated originally by Huygens, is a geometrical method for finding, from the known shape of a portion of a wave front at some instant, what the shape will be at some later instant. It is assumed that *every point of a wave front may be considered the source of secondary wavelets, which spread out in all directions with a speed equal to the speed of propagation of the waves.* The new wave front is then found by constructing a surface *tangent* to the secondary wavelets or, as it is called, the *envelope* of the wavelets.

Huygens' principle is illustrated in Fig. 39–1. The original wave front, AA', is traveling as indicated by the small arrows. We wish to find the shape of the wave front after a time interval t. Let v represent the speed of propagation. Construct a number of circles (traces of spherical wavelets) of radius $r = vt$, with centers along AA'. The trace of the envelope of these

Page 11 of 16

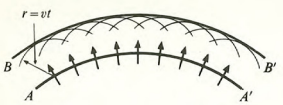


Fig. 39–1 *Huygens' principle.*

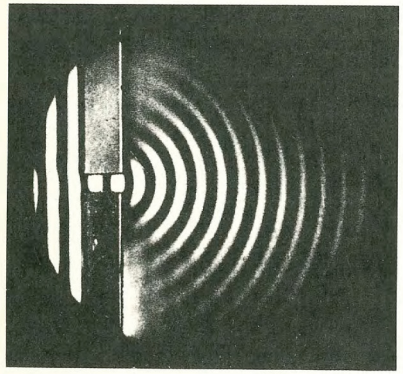


Fig. 39–2 *Demonstration of a Huygens wavelet with the aid of water waves in a shallow tank. (Reproduced from* Ripple Tank Studies of Wave Motion *with permission of W. Llowarch, the Clarendon Press, Oxford.)*

wavelets, which is the new wave front, is the curve BB'. The speed v has been assumed the same at all points and in all directions.

The fact that any point on a wave front may be regarded as a source of a secondary wavelet may be demonstrated convincingly with the aid of a ripple tank. If plane wave fronts are produced in the water in a shallow tank and these wave fronts come to a narrow slit, a complete circular wave is formed, as shown in Fig. 39–2, rather than a narrow parallel beam, as might have been expected.

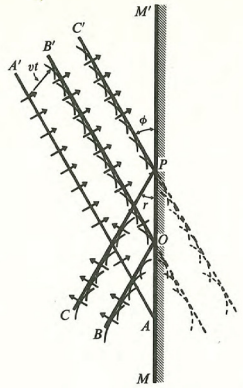


Fig. 39–3 *Successive positions of a plane wave AA' as it is reflected from a plane surface.*

39–2 DERIVATION OF THE LAW OF REFLECTION FROM HUYGENS' PRINCIPLE

In Fig. 39–3, consider the trace of a plane wave surface AA', called the incident *wave front*, which is just making contact with a reflecting surface MM' along a line through A perpendicular to the plane of the diagram. The planes of the wave front and the reflecting surface are also normal to the plane of the figure. The position of the wave front after a time interval t may be found by applying Huygens' principle. With points on AA' as centers, draw a number of secondary wavelets of radius vt, where v is the speed of propagation in the medium near the surface. Those wavelets originating near the upper end of AA' spread out unhindered, and their envelope gives that portion of the new wave surface OB'. The wavelets originating near the lower end of AA', however, strike the reflecting surface. If the latter had not been there, they would have occupied the positions shown by the dotted circular arcs. The effect of the reflecting surface is to reverse the direction of travel of

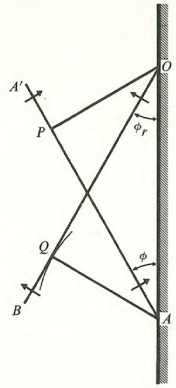


Fig. 39–4 *A portion of Fig. 39–3.*

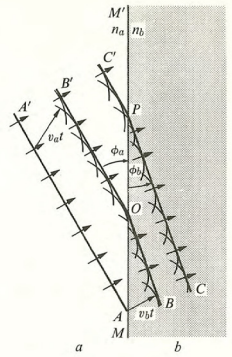


Fig. 39–5 *Successive positions of a plane wave front AA' as it is refracted by a plane surface.*

those wavelets which strike it, so that that part of a wavelet which would have penetrated the surface actually lies to the left of it, as shown by the full lines. The envelope of these reflected wavelets is then that portion of the wave front OB. The trace of the entire wave front at this instant is the broken line BOB'. A similar construction gives the line CPC' for the wave front after another interval t.

The angle ϕ between the incident *wave front* and the *surface* is the same as that between the incident *ray* and the *normal* to the surface, and is therefore the angle of incidence. Similarly, ϕ_r is the angle of reflection. To find the relation between these angles, consider Fig. 39–4, which is the same as a portion of Fig. 39–3. From O, draw $OP = vt$, perpendicular to AA'. Now OB, by construction, is tangent to a circle of radius vt with center at A. Hence, if AQ is drawn from A to the point of tangency, the triangles APO and AQO are equal (right triangles with the side AO in common and with $AQ = OP$). The angle ϕ therefore equals the angle ϕ_r, and we have the law of reflection.

39–3 DERIVATION OF SNELL'S LAW FROM HUYGENS' PRINCIPLE

Consider the trace of a plane wave front AA' (Fig. 39–5) which is just making contact with the surface MM' along a line through A perpendicular to the plane of the diagram. MM' represents a boundary surface between two transparent media a and b, of indices of refraction n_a and n_b. (The reflected waves are not shown in the figure; they proceed exactly as in Fig. 39–3.) Let us apply Huygens' principle to find the position of the refracted wave front after a time t.

With points on AA' as centers, draw a number of secondary wavelets. Those originating near the upper end of AA' travel with speed v_a and, after a time interval t, are spherical surfaces of radius v_at. The wavelet originating at point A, however, is traveling in the second medium b with speed v_b and at time t is a spherical surface of radius v_bt. The envelope of the wavelets from the original wave front is the plane whose trace is the broken line BOB'. A similar construction leads to the trace CPC' after a second interval t.

Page 12 of 16

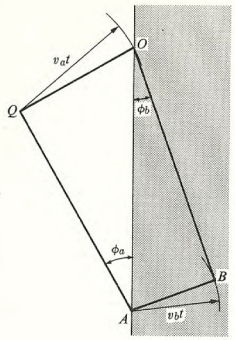


Fig. 39–6 A portion of Fig. 39–5.

The angles ϕ_a and ϕ_b between the surface and the incident and refracted wave fronts are, respectively, the angle of incidence and the angle of refraction. To find the relation between these angles, refer to Fig. 39–6 which is the same as a portion of Fig. 39–5. Draw $OQ = v_a t$, perpendicular to AQ, and draw $AB = v_b t$, perpendicular to BO. From the right triangle AOQ,

$$\sin \phi_a = \frac{v_a t}{AO},$$

and from the right triangle AOB,

$$\sin \phi_b = \frac{v_b t}{AO}.$$

Hence,

$$\frac{\sin \phi_a}{\sin \phi_b} = \frac{v_a}{v_b}. \qquad (39\text{–}1)$$

Since v_a/v_b is a constant, Eq. (39–1) expresses Snell's law, and we have derived Snell's law from a wave theory.

The most general form of Snell's law is given by Eq. (38–4), namely

$$\frac{\sin \phi_a}{\sin \phi_b} = \frac{n_b}{n_a},$$

whence

$$\frac{v_a}{v_b} = \frac{n_b}{n_a},$$

and

$$n_a v_a = n_b v_b.$$

When either medium is a vacuum, the index is 1 and the speed is c. Hence

$$\boxed{n_a = \frac{c}{v_a}, \qquad n_b = \frac{c}{v_b},} \qquad (39\text{–}2)$$

showing that the *index of refraction of any medium is the ratio of the speed of light in a vacuum to the speed in the medium.*

It was shown in Chapter 37 that the speed of an electromagnetic wave in vacuum is given by

$$c = \sqrt{1/\epsilon_0 \mu_0}.$$

For a light wave in a *material* medium of permittivity ϵ and permeability μ, the speed v is

$$v = \sqrt{1/\epsilon\mu} = \sqrt{1/K\epsilon_0 K_m \mu_0} = c\sqrt{1/KK_m}.$$

Most transparent substances show a magnetic behavior that differs only slightly from that of a vacuum. The quantity K_m is therefore very close to 1, and $v = c/\sqrt{K}$, or

$$\boxed{n = \sqrt{K}.} \qquad (39\text{–}3)$$

The values of K listed in Table 27–1 are *static* values, obtained by measurements with nonvarying electric fields. When the electric field varies rapidly, as it does in a light wave, K is found to depend on the frequency. The index of refraction, therefore, cannot be calculated from the values in Table 27–1.

In Fig. 39–5, the spacing of the incident wave fronts is arbitrary. If t is chosen to be the period T of the wave, the spacing is vT, which is the wavelength λ. Figure 39–5 shows that the wavelength in the second medium is smaller, since the wave speed is smaller. When a light wave proceeds from one medium to another, where the speed is different, the wavelength changes *but not the frequency*. Since

$$v_a = f\lambda_a \quad \text{and} \quad v_b = f\lambda_b,$$

$$\frac{\lambda_a}{v_a} = \frac{\lambda_b}{v_b} \quad \text{and} \quad \lambda_a \frac{c}{v_a} = \lambda_b \frac{c}{v_b}.$$

Therefore,

$$\lambda_a n_a = \lambda_b n_b.$$

If either medium is vacuum, the index is 1 and the wavelength in a vacuum is represented by λ_v. Hence

$$\boxed{\lambda_a = \frac{\lambda_v}{n_a}, \qquad \lambda_b = \frac{\lambda_v}{n_b},} \qquad (39\text{–}4)$$

showing that *the wavelength in any medium is the wavelength in a vacuum divided by the index of refraction of the medium.*

39–4 TOTAL INTERNAL REFLECTION

Figure 39–7 shows a number of rays diverging from a point source P in medium a of index n_a and striking the surface of a second medium b of index n_b, where $n_a > n_b$. From Snell's law,

$$\sin \phi_b = \frac{n_a}{n_b} \sin \phi_a.$$

Since n_a/n_b is greater than unity, $\sin \phi_b$ is larger than $\sin \phi_a$ and evidently equals unity (i.e., $\phi_b = 90°$) for some angle ϕ_a less than 90°. This is illustrated by ray 3 in the diagram, which emerges just grazing the surface at an angle of refraction of 90°. The angle of incidence for which the refracted ray emerges tangent to the surface is called the *critical angle* and is designated by ϕ_{crit} in the diagram. If the angle of incidence is greater than the critical angle, the sine of the angle of refraction, as computed by Snell's law, would have to be greater than unity. This may be interpreted to mean that beyond the critical angle the ray *does not* pass into the upper medium but is *totally internally reflected* at the boundary surface. Total internal reflection can occur only when a ray is incident on the surface of a medium whose index is *smaller* than that of the medium in which the ray is traveling.

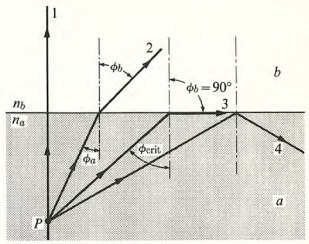


Fig. 39–7 Total internal reflection. The angle of incidence ϕ_a for which the angle of refraction is 90°, is called the critical angle.

The critical angle for two given substances may be found by setting $\phi_b = 90°$ or $\sin \phi_b = 1$ in Snell's law. We then have

$$\boxed{\sin \phi_{crit} = \frac{n_b}{n_a}.} \qquad (39\text{–}5)$$

The critical angle of a glass–air surface, taking 1.50 as a typical index of refraction of glass, is

$$\sin \phi_{crit} = \frac{1}{1.50} = 0.67, \qquad \phi_{crit} = 42°.$$

This angle, very conveniently, is slightly less than 45°, which makes possible the use in many optical instruments of prisms of angles 45°-45°-90° as totally reflecting surfaces. The advantages of totally reflecting prisms over metallic surfaces as reflectors are, first, that the light is *totally* reflected, while no metallic surface reflects 100% of the light incident on it, and second, the reflecting properties are permanent and not affected by tarnishing. Offsetting these is the fact that there is some loss of light by reflection at the surfaces where light enters and leaves the prism, although coating the surfaces with so-called "nonreflecting" films can reduce this loss considerably.

The simplest type of reflecting prism is shown in Fig. 39–8. Its angles are 45°-45°-90°. Light incident normally on one of the shorter faces strikes the inclined face at an angle of incidence of 45°. This is

Page 13 of 16

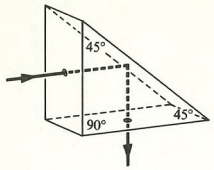


Fig. 39–8 *A totally reflecting prism.*

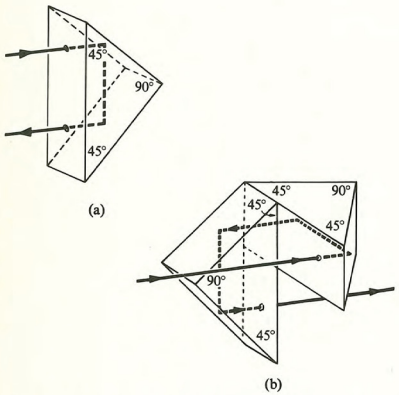


Fig. 39–9 *(a) A Porro prism. (b) A combination of two Porro prisms.*

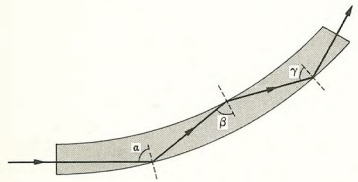


Fig. 39–10 *A light ray "trapped" by internal reflections.*

greater than the critical angle, so the light is totally internally reflected and emerges from the second of the shorter faces after undergoing a deviation of 90°.

A 45°-45°-90° prism, used as in Fig. 39–9(a), is called a *Porro* prism. Light enters and leaves at right angles to the hypotenuse and is reflected at each of the shorter faces.The deviation is 180°. Two Porro prisms are often combined, as in Fig. 39–9(b).

If a beam of light enters one end of a transparent rod, as in Fig. 39–10, the light is totally reflected internally and is "trapped" within the rod even if it is curved, provided the curvature is not too great. The rod is sometimes referred to as a *light pipe.* A bundle of fine fibers will behave in the same way and has the advantage of being flexible. The study of the properties of such a bundle is an active field of research known as *fiber optics.*

A bundle may consist of thousands of individual fibers, each the order of 0.002 to 0.01 mm in diameter. If the fibers can be assembled in the bundle so that the relative positions of the ends are the same at both ends, the bundle can transmit an image, as shown in Fig. 39–11. Bundles several feet in length have been made. Devices using fiber optics are finding a wide range of applications in medical science. The interior of lungs and other passages in the human body can be viewed by insertion of a fiber bundle. A bundle can be enclosed in a hypodermic needle for the study of tissues and blood vessels far beneath the skin. Despite the enormous technical difficulties of manufacturing fiber-optic components, such systems promise to become an extremely important class of optical systems.

39–5 REFRACTION BY A PRISM

The prism, in one or another of its many forms, is second only to the lens in its usefulness as a piece of optical apparatus. Totally reflecting prisms have been mentioned briefly. We consider now the *deviation* and the *dispersion* produced by a prism.

Consider a light ray incident at an angle ϕ on one face of a prism, as in Fig. 39–12(a). Let the index of the prism be n, the included angle at the apex be A, and let the medium on either side of the prism be air. It is desired to find the *angle of deviation,* δ. In

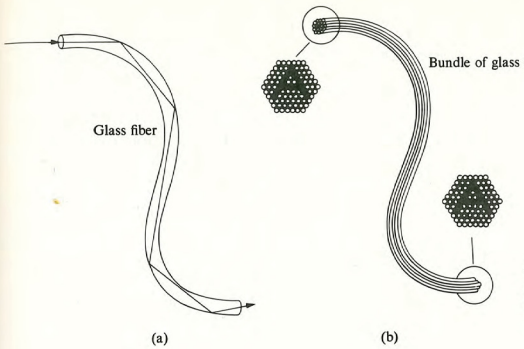


Fig. 39–11 *(a) Total internal reflection in a single fiber. (b) Image transmission by a bundle of fibers.*

principle, this is a straightforward problem in surveying. One has only to apply Snell's law at the first surface, compute the angle of refraction, then by geometry find the angle of incidence at the second surface, and from a second application of Snell's law find the angle of refraction at the second surface.The direction of the emergent ray is then known, and the angle of deviation may be found.

While the method is simple enough, the expression for the angle δ turns out in the general case to be rather complicated. However, as the angle of incidence is, say, decreased from a large value, the angle of deviation is found to decrease at first and then increase, and to be a *minimum* when the ray passes through the prism symmetrically, as in Fig. 39–12(b). The angle δ_m is then called the angle of *minimum deviation*; it is related to the angle of the prism and its index by the equation

$$n = \frac{\sin[(A + \delta_m)/2]}{\sin A/2}. \qquad (39\text{–}6)$$

To derive Eq. 39–6, we have, from Fig. 39–12(b),

$$\phi_1' = \frac{A}{2} \quad \text{(sides mutually perpendicular)},$$

$$\delta_1 = \frac{\delta_m}{2} \quad \text{(half the deviation takes place at each surface)},$$

$$\phi_1 = \phi_1' + \delta_1 = \frac{A}{2} + \frac{\delta_m}{2} = \frac{A + \delta_m}{2},$$

$$\sin\phi_1 = n \sin\phi_1', \quad \text{and} \quad \sin\frac{A + \delta_m}{2} = n \sin\frac{A}{2},$$

which is Eq. (39–6).

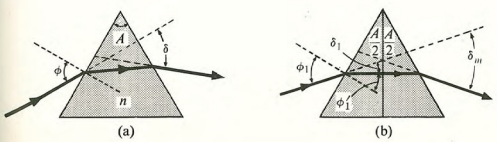


Fig. 39–12 *(a) Deviation by a prism. (b) The deviation is a minimum when the ray passes through the prism symmetrically.*

Page 14 of 16

The index of refraction of a transparent solid may be measured, making use of the equation derived above. The specimen whose index is desired is ground into the form of a prism. The angle of the prism A and the angle of minimum deviation δ_m are measured with the aid of a spectrometer. Since these angles may be determined with a high degree of precision, this method is an extremely accurate one; indices of refraction may be measured to six significant figures.

If the angle of the prism is *small*, the angle of minimum deviation is small also, and we may replace the sines of the angles by the angles themselves. One then obtains

$$n = \frac{A + \delta_m}{A},$$

or

$$\delta_m = (n - 1)A, \qquad (39\text{–}7)$$

a useful approximate relation.

39–6 DISPERSION

Most light beams are a mixture of waves whose wavelengths extend throughout the visible spectrum. While the speed of light waves in a vacuum is the same for all wavelengths, the speed in a material substance is different for different wavelengths. Hence the index of refraction of a substance is a function of wavelength. The dependence of wave speed on wavelength was first mentioned in connection with water waves, in Section 21–7. A substance in which the speed of a wave varies with wavelength is said to exhibit *dispersion*. Figure 39–13 is a diagram showing the variation of index of refraction with wavelength for a number of the more common optical materials.

Consider a ray of white light, a mixture of all visible wavelengths, incident on a prism, as in Fig. 39–14. Since the deviation produced by the prism increases with increasing index of refraction, violet light is deviated most and red least, with other colors occupying intermediate positions. On emerging from the prism, the light is spread out into a fan-shaped beam, as shown. The light is said to be *dispersed* into a spectrum.

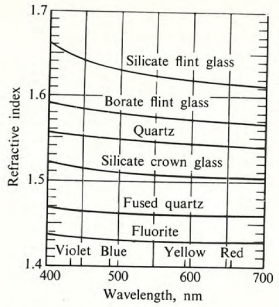


Fig. 39–13 *Variation of index with wavelength.*

When white light is dispersed by a prism, it can be seen from Fig. 39–14 that the whole fan-shaped beam is deviated from the incident direction. A convenient measure of this *deviation* is provided by the angle of deviation of yellow light, since yellow is roughly midway between red and violet. A simple measure of the *dispersion* is provided by the angular separation of the red and violet rays. Since deviation and index of refraction are related, the deviation of the entire spectrum is controlled by the index of refraction for yellow light, whereas the dispersion

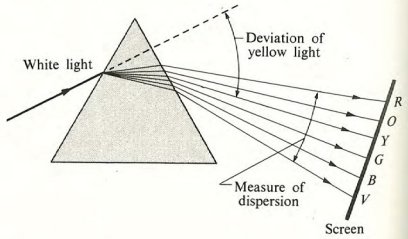


Fig. 39–14 *Dispersion by a prism. The bank of colors on the screen is called a spectrum.*

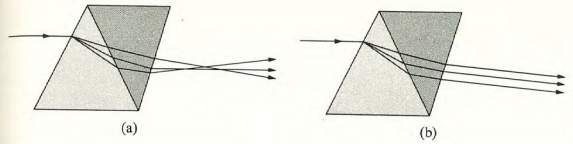


Fig. 39–15 *(a) A direct-vision prism. (b) An achromatic prism.*

depends on the *difference* between the index for violet light and that for red light. From Fig. 38–13 it can be seen that for a substance such as fluorite, whose index for yellow light is small, the difference between the indices for red and violet is also small. On the other hand, in the case of silicate flint glass, both the index for yellow light and the difference between extreme indices are large. In other words, for most transparent materials the greater the deviation, the greater the dispersion.

The brilliance of diamond is due in part to its large dispersion. In recent years synthetic crystals of titanium dioxide and of strontium titanate, with about eight times the dispersion of diamond, have been produced.

Because of the shape of the curves in Fig. 39–13, the dispersions produced by two prisms of equal angles are not exactly proportional to the mean deviations. It is therefore possible to combine two (or more) prisms of different materials in such a way that there is *no net deviation* of a ray of some chosen wavelength, while there remains an outstanding dispersion of the spectrum as a whole. Such a device, illustrated in Fig. 39–15(a), is known as a *direct-vision prism*. Two prisms may also be designed so that the dispersion of one is offset by the dispersion of the other, although the deviation is not. A compound prism of this sort, shown in Fig. 39–15(b), is called *achromatic* (without color).

39–7 THE RAINBOW

The rainbow is produced by the combined effects of refraction, dispersion, and internal reflection of sunlight by drops of rain. When conditions for its observation are favorable, two bows may be seen, the inner being called the primary bow and the outer the secondary bow. The inner bow, which is the brighter, is red on the outside and violet on the inside, while in the fainter outer bow the colors are reversed.

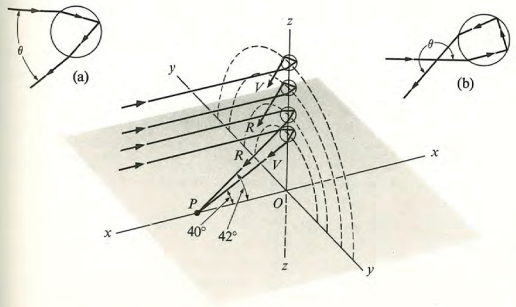


Fig. 39–16 *The rainbow.*

The primary bow is produced in the following manner. Assume that the sun's rays are horizontal, and consider a ray striking a raindrop, as in Fig. 39–16(a). This ray is refracted at the first surface and is in part reflected at the second surface, passing out again at the front surface as shown. The French scientist Descartes computed the paths of some thousands of rays incident at different points on the surface of a raindrop and showed that, if a ray of any given color were incident at such a point that its deviation was a *maximum*, all other rays of the same color which struck the surface of the drop in the immediate neighborhood of this point would be reflected in a direction very close to that of the first. Hence each color is strongly reflected in the direction of maximum deviation of that particular color. The angle of maximum deviation of red light is 138°, so the angle θ in Fig. 39–16(a) is $180° - 138° = 42°$. The corresponding angle for violet light is 40°, while that for other colors lies between these.

Consider now an observer at P in Fig. 39–16. The xy-plane is horizontal and sunlight is coming from the left parallel to the x-axis. All drops which lie on a circle subtending an angle of 42° at P, and with the center at O, will reflect red light strongly to P. All those on a circle subtending 40° at P will reflect violet light strongly, while those occupying intermediate positions will reflect the intermediate colors of the spectrum.

The point O, the center of the circular arc of the bow, may be considered the shadow of P on the yz-plane. As the sun rises above the horizon, the point O moves down, and hence with increasing elevation of the sun a smaller and smaller part of the bow is visible. Evidently an observer at ground level cannot see the primary bow when the sun is more than 42° above the horizon. If the observer is in an elevated position, however, the point O moves up and more and more of the bow may be seen. In fact, it is not uncommon for a complete *circular* rainbow to be seen from an airplane.

The secondary bow is produced by *two* internal reflections, as shown in Fig. 39–16(b). As before, the light which is reflected in any particular direction depends upon the angle of maximum deviation. Since the angle of deviation is here the angle θ, and since the violet is deviated more than the red, the violet rays in the secondary bow are deflected down at a steeper angle than the red and the secondary bow is red on the *inside* and violet on the outside edge. The corresponding angles are 50.5° for red and 54° for violet.

The preceding discussion applies when the drops producing a rainbow are relatively large. When the drops are small, diffraction plays just as important a role as dispersion and reflection, and red light, for example, is received in appreciable amounts from drops lying on circles other than those subtending an angle of 42°. The bow is then a complicated mixture of colors and its appearance depends on the size of the drops.

PROBLEMS

39–1

a) What is the speed of light of wavelength 500 nm (in vacuum), in glass whose index at this wavelength is 1.50?

b) What is the wavelength of these waves in the glass?

39–2 A glass plate 3 mm thick, of index 1.50, is placed between a point source of light of wavelength 600 nm (in vacuum) and a screen. The distance from source to screen is 3 cm. How many waves are there between source and screen?

39–3 The speed of light of wavelength 656 nm in heavy flint glass is 1.60×10^8 m s^{-1}. What is the index of refraction of this glass?

39–4 Light of a certain frequency has a wavelength in water of 442 nm. What is the wavelength of this light when it passes into carbon disulfide? (See Table 38–1.)

39–5 A ray of light goes from point A in a medium where the velocity of light is v_1 to point B in a medium where the velocity is v_2, as in Fig. 39–17. Show (a) that the time required for the light to go from A to B is

$$t = \frac{h_1 \sec \theta_1}{v_1} + \frac{h_2 \sec \theta_2}{v_2},$$

and (b) that this time is minimum when the relation between the angles is $n_1 \sin \theta_1 = n_2 \sin \theta_2$. (Note that $\ell = h_1 \tan \theta_1 + h_2 \tan \theta_2$.)

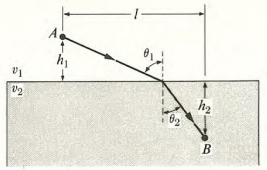


Fig. 39–17

39–6 A point light source is 5 cm below a water-air surface. Compute the angles of refraction of rays from the source making angles with the normal of 10°, 20°, 30°, and 40°, and show these rays in a carefully drawn full-size diagram.

39–7 A glass cube in air has a refractive index of 1.50. Parallel rays of light enter the top obliquely and then strike a side of the cube. Is it possible for the rays to emerge from this side?

39–8 A point source of light is 20 cm below the surface of a body of water. Find the diameter of the largest circle at the surface through which light can emerge from the water.

39–9 The index of refraction of the prism shown in Fig. 39–18 is 1.56. A ray of light enters the prism at point a and follows in the prism the path ab which is parallel to the line cd. (a) Sketch carefully the path of the ray from a point outside the prism at the left, through the glass, and out some distance into the air again. (b) Compute the angle between the original and final directions in air. (Dotted lines are construction lines only.)

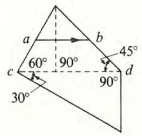


Fig. 39–18

39–10 Light is incident normally on the short face of a 30°-60°-90° prism, as in Fig. 39–19. A drop of liquid is placed on the hypotenuse of the prism. If the index of the prism is 1.50, find the maximum index the liquid may have if the light is to be totally reflected.

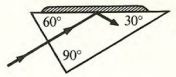


Fig. 39–19

39–11 A 45°-45°-90° prism is immersed in water. What is the minimum index of refraction the prism may have if it is to reflect totally a ray incident normally on one of its shorter faces?

39–12 The velocity of a sound wave is 330 m s^{-1} in air and 1320 m s^{-1} in water.

a) What is the critical angle for a sound wave incident on the surface between air and water?

b) Which medium has the higher "index of refraction" for sound?

39–13 Light is incident at an angle ϕ_1 (as in Fig. 39–20) on the upper surface of a transparent plate, the surfaces of the plate being plane and parallel to each other.

a) Prove that $\phi_1 = \phi_2'$.

b) Show that this is true for any number of different parallel plates.

c) Prove that the lateral displacement of the emergent beam is given by the relation

$$PQ = t \frac{\sin(\phi_1 - \phi_1')}{\cos \phi_1'},$$

where t is the thickness of the plate.

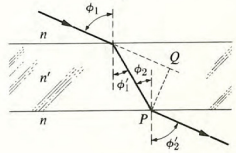


Fig. 39–20

39–14 A parallel beam of light containing wavelengths A and B is incident on the face of a triangular glass prism having a refracting angle of 60°. The indices of refraction are $n_A = 1.40$ and $n_B = 1.60$. If beam A goes through the prism at minimum deviation, find (a) the angle of emergence of each beam and (b) the angle of deviation of each.

39–15 A ray of light is incident at an angle of 60° on one surface of a glass plate 2 cm thick, of index 1.50. The