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The Penguin Dictionary of MATHEMATICS

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function (map, mapping) A rule that assigns to every element x of a *set X a unique element y of a set Y , written as $y = f(x)$ where f denotes the function. X is called the *domain* and Y the *codomain*. The set of all the elements $f(x)$ is called the *range* or *image* of f , and is denoted by R_f , $\text{Im } f$, or $f(X)$. It is a subset of the codomain. For example, the area of a circle, y , is a function of the radius, x , written as $y = f(x) = \pi x^2$. x is called the *independent variable* or *argument*, and y is called the *dependent variable* or the *image* of x . If a function can be expressed algebraically the value of y can be calculated for any particular value of x . For example, a circle of radius 2 has area $f(2) = 4\pi$. However, some functions cannot be expressed algebraically: for example, the function 'is the birthday of', which has domain the set of all individuals and range the set of all days in a year.

A function can also be defined as the set of all *ordered pairs (x, y) , with x belonging to the domain X and y belonging to the codomain Y ,

where there is a *many-to-one correspondence between the members of X and the members of Y .

A *multiple-valued function is not a function as defined above because for each value of x the corresponding y is not necessarily unique, but it can be considered as being made up of several branches, each of which is a (single-valued) function.

A function $y = f(x)$ can be graphically represented if (x, y) is plotted on rectangular coordinate axes for every x in X .

A function of two variables assigns to every element (x_1, x_2) of a set of ordered pairs a single element $y = f(x_1, x_2)$. Here, x_1 and x_2 are the independent variables and y is the dependent variable. For example, the volume of a right circular cylinder y is $\pi x_1^2 x_2$ where x_1 is the radius of

where $|A|$ denotes the value of A .

The main problem with many-valued logics has been to find an acceptable and interesting interpretation. *See also* three-valued logic.

mapping (map) *See* function.

marginal distribution *See* bivariate distribution; multivariate distribution.

Markov, Andrei Andreevich (1856 – 1922) Russian mathematician noted for his work in probability theory and his introduction in 1906 of what has since become known as a *Markov chain.

Markov chain A *stochastic process in which a discrete *random variable $X(t)$ may change state (i.e. value) at times $t_1, t_2, t_3, \dots$ (usually equally spaced) is called a Markov chain if the *conditional distribution of $X(t_{i+1})$ at t_{i+1} depends only on $X(t_i)$ and not on the value of X at any earlier time. This is often expressed by saying that the state of the system in the future is unaffected by its history. The simplest case is that in which X takes values 0 and 1 only (corresponding, for example, to a circuit in which current may be either off (0) or on (1)). At any transition time t_i there is a probability P_{rs} that the system, if in state r , will change to state s , where $r, s = 0, 1$. Thus p_{01} is the probability of a change from state 0 to state 1, and p_{11} is the probability of the system remaining in state 1 if it is already there (*see* diagram). The matrix

$$\begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix}$$

where $p_{00} + p_{01} = p_{10} + p_{11} = 1$ is called the *transition matrix*. If $p_{10} = p_{01} = 1$ this implies that $p_{00} = p_{11} = 0$, and the system oscillates repeatedly from state 0 to state 1. If $p_{00} = p_{11} = 1$ the system never changes state, remaining in one of them at all times, and that state is called an *absorbing state*. If all the elements in the transition matrix are nonzero then, for equally spaced t_i , the system approaches a condition in which at any time it has a probability $p_{10}/(p_{01} + p_{10})$ of being in state 0 and a probability $p_{01}/(p_{01} + p_{10})$ of being in state 1. This property is