SOME STANDARD SYMBOLS USED IN ELECTRONICS

Prefix	Symbol	Exponent	Multiplication Factor
exa	E	18	1,000,000,000,000,000,000
peta	P	15	1,000,000,000,000,000
tera	T	12	1,000,000,000,000
giga	G	9	1,000,000,000
mega	M	6	1,000,000
kilo	k	3	1,000
hecto	h	2	100
deca	da	1	10
(unit)		0	1
deci	da	-1	0.1
centi	c	-2	0.01
milli	m	-3	0.001
micro	u	-6	0.000001
nano	n	-9	0.000000001
pico	p	-12	0.000000000001
femto	f	-15	0.000000000000001
atto	a	-18	0.000000000000000001

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SOME FUNDAMENTAL DEFINITIONS

They start with the electron or the flow of electrons:

1 amp = flow of 1 coulomb/sec
where 1 coulomb = 6.25×10^{18} electrons

1 volt = (Electromotive) force (EMF) between two points for which a flow of 1 amp will generate 1 joule of energy (work)

1 watt = rate (power) at which work is done (joules/second)
= volt x amp (Note: 1 horsepower = 746 watts)

work = joule = power x time = watt x time
(At home we pay for work in kilowatt-hours)

1 ohm = resistance that allows 1 amp to flow at 1-volt EMF

1 farad = storage of 1 coulomb of charge that will generate 1-volt EMF

1 henry = inductance that will cause an induced voltage of 1-volt EMF when the inducing current is changing at 1 amp/second

IMPORTANT NAMES IN ELECTRONICS

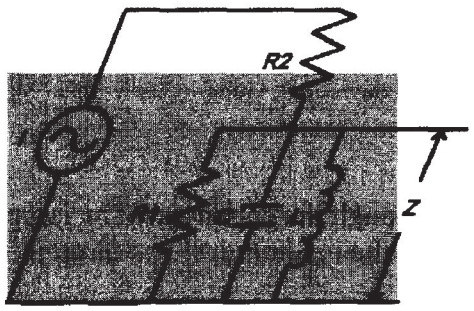
Name	Life Span	Symbol	Attribute
Charles Augustin de Coulomb	1736 – 1806	C	Charge
Andre Marie Ampere	1775 – 1836	A	Current
Alessandro Volta	1745 – 1827	V	Potential
James Prescott Joule	1818 – 1889	J	Energy, work
James Watt	1736 – 1819	W	Power
George Simon Ohm	1787 – 1854	W	Resistance
Michael Faraday	1791 – 1867	F	Capacitance
Joseph Henry	1797 – 1878	H	Inductance
Nikola Tesla	1857 – 1943	T	Magnetic flux density
Hans Christian Oersted	1777 – 1851	Oe	Magnetic field intensity
Wilhelm Eduard Weber	1804 – 1865	Wb	Magnetic flux

Others:

Carl Friedrich Gauss	1777 – 1855
Gustav Robert Kirchhoff	1824 – 1887
Heinrich Friedrich Emil Lenz	1804 – 1865
James Clerk Maxwell	1831 – 1879
E. L. Norton	1926*
M. Leon Thevenin	1857 – 1926

* Date Norton published his theorem

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2 PROPAGATION TIMES

PROPAGATION SPEED

*E*lectrical signals traveling along a wire or trace in the air travel at the speed of light, 186,280 miles per second (which, if you do the arithmetic, works out to 11.8 in/ns). That speed is equivalent to 7.5 trips around the Earth's equator in one second. Yet we need to worry about how fast these signals travel along traces on our boards! That is pretty amazing when you think about it.

Consider this experiment: String a copper wire in the air across a lake and measure the time it takes a signal to travel from one end of the wire to the other end. Now lower the wire into the lake and measure the signal propagation time again. What do you think happens to the propagation time?

When a current flows along a wire or trace, it generates an electromagnetic field around the conductor. There are two components to this electromagnetic field, an electrical field and a magnetic field. The electrical field originates with the charged electrons. It radiates radially out from the conductor with a strength that decreases with the square of the distance from the conductor. The magnetic field originates from the motion of the electrons along the conductor. There is no magnetic field if there is no motion (current), and the strength of the field changes with the magnitude of the current. The magnetic field radiates around the conductor, with a polarity that depends on the direction of the current, and with a strength that is inversely related to the square of the distance from the conductor.

The electromagnetic field and the current must move together. The current can't move out in front of the fields. The magnetic field can't move out in front of the electric field. It turns out the issue is not how fast the current (electrons) can

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flow through the conductor. The issue is how fast the electromagnetic field can flow through the medium it flows through, the medium *around* the wire.

All materials have a characteristic called the *relative dielectric coefficient* (ε_r). It is a measure of the material's ability to store charge. It is called relative because it is referenced to the dielectric coefficient of air (really that of a vacuum, but the difference between air and a vacuum is so small that only astronomers care). Its relevance here is that the speed with which an electromagnetic field can travel through a material is equal to the speed of light divided by the square root of the dielectric coefficient of the material. Therefore, the propagation speed (in in/ns) of a signal is given by the formula in Equation 2-1:

$$\text{Propagation Speed} = 11.8/\sqrt{\varepsilon_r} \quad \text{in/ns} \qquad [2\text{-}1]$$

So, back to our illustration of the wire dropped into the lake. The issue is not how fast the electrons can travel through the copper wire, the issue is how fast the electromagnetic field can travel through the water. The relative dielectric coefficient of water is approximately 80 (depending on its purity). The square root of 80 is approximately 9. So the signal will slow down when the wire is lowered into the water by about a factor of 9.

PROPAGATION TIMES

It is easy to confuse propagation *speed* and propagation *time*. Propagation speed is velocity. Its units are distance per unit time. Propagation time is, well, time. A related measure, wavelength (the distance a signal travels in one complete cycle), will be discussed later in this chapter. We express propagation time either in units of time (e.g., nanoseconds), or as units of time per unit length (e.g., nanoseconds/inch). Propagation time (expressed as time per unit length) is the inverse of propagation speed. That is:

Propagation Time (per unit length) = 1/Propagation Speed, or
Propagation Time = Length/Propagation Speed

For example, if an electrical signal in a copper wire in air travels at the speed of 11.8 in/ns, then it propagates down the wire at 1/11.8 or .085 ns/in.

TRACE CONFIGURATIONS AND SIGNAL PROPAGATION

Figure 2-1 illustrates various trace configurations designers usually work with when designing high-speed boards. Most boards in situations where signal integrity is an issue have internal planes (as will be seen in almost every chapter in Part 2, "Signal Integrity"). From a signal propagation standpoint, traces that are located between two reference planes are considered to be in a stripline environment, regardless of whether the actual environment is a simple, centered, dual, offset, or asymmetric stripline configuration. A trace with a reference plane on only one side is considered to be in a microstrip environment.

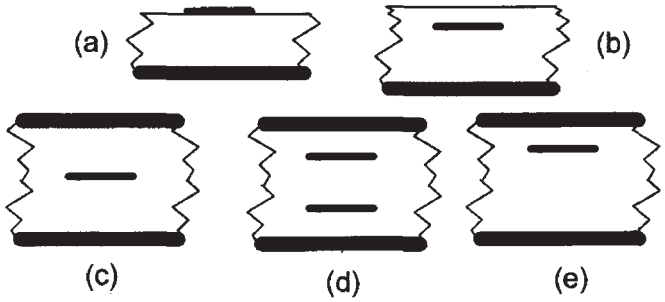


Figure 2-1 Common trace configurations on PCBs: microstrip (a), embedded microstrip (b), stripline (c), dual stripline (d), and asymmetric stripline (e).

A microstrip trace might simply be an external trace with air above it and board material between it and an underlying plane. An embedded microstrip trace has board material above the trace, also. This would particularly be the case if there were two external trace layers above the plane. At least the traces on layer 2 would be embedded microstrip traces. A coated microstrip trace would be one with board material between it and the reference plane, and with an additional material coat above the trace. The additional coat could be a variety of materials, including solder mask, conformal coating material protection, and so on.

We generally consider the material surrounding a trace in a stripline environment to be homogeneous (uniform). In fact, we usually specify it to be this way.

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Therefore, the propagation speed of a signal in a stripline environment is dependably expected to be as described in Equation 2-1:

$$\text{Propagation Speed} = 11.8/\sqrt{\varepsilon_r} \quad \text{in/ns}$$

Actual experience generally only varies from this calculation if the ε_r is not as expected or if the material is quite nonuniform. Since the relative dielectric material of FR4 is approximately 4, and since the square root of 4 is 2, the propagation speed of a signal in a stripline environment is often considered to be $11.8/2 \cong 6$ in/ns, a rule we are all pretty familiar with. If we want to be more precise in our calculations, then we have to know the relative dielectric coefficient of the material we are using more precisely and plug that value into Equation 2-1.

The microstrip environment is more problematic. The material surrounding the trace is not uniform. In the simplest case it is air above the trace and dielectric below the trace. In more complex cases the dividing line between the dielectric and the air might not be precisely uniform, and there may be more than one type of material involved.

Therefore, if you want to estimate the signal propagation speed in microstrip you need to estimate the effective dielectric coefficient of the material surrounding the trace. There is a generally accepted rule of thumb for estimating the effective dielectric coefficient that has been in use for over 30 years.[1] That estimate is given in Equation 2-2:

$$\varepsilon'_r = 0.475\varepsilon_r + .67 \qquad [2\text{-}2]$$

There are a couple of problems with this estimate. The most significant one is that it is a constant. People have discovered that propagation time in microstrip is a variable, and, all other things being equal, is a function of trace width and height above the plane.

As trace widths get wider, propagation speed slows down! The reason for this is that as traces widen, a larger percentage of the field lines between the trace and the plane is contained within the dielectric. In the limit, with infinite trace

[1]There are numerous places where this equation can be found. See for example, IPC-D-317A, *Design Guidelines for Electronic Packaging Utilizing High-Speed Techniques*, p. 18. The reference given for this formula is H.R. Kaupp, "Characteristics of Microstrip Transmission Lines," *IEEE Trans.*, Vol EC-16, No. 2, April 1967.

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width, virtually all the electromagnetic field is contained within the dielectric. In this case, the microstrip trace really looks very much like a stripline trace.

Microstrip propagation speed also slows down as the trace gets closer to the plane, for the same reason. A greater percentage of the field lines are contained within the dielectric than in the air. Brooks has shown that a much better estimate of propagation time in microstrip can be obtained by expressing propagation time in microstrip as a ratio of the propagation time for a signal in a stripline environment surrounded with the same dielectric material. He generates a formula for estimating this ratio.[2] The propagation *time* for a microstrip trace will never be longer than for a trace in a stripline environment surrounded by the same material. It will be shorter, depending on how the electromagnetic field splits between the air above the trace and the dielectric under the trace. His estimate for propagation time (in ns/in) is as follows:

$$\text{Propagation Time(Microstrip)} = \text{Br} \times \text{Propagation Time(Stripline)}$$

or,

$$\text{Propagation Time} = \text{Br}\sqrt{\varepsilon_r}/11.8 \qquad [2\text{-}3]$$

where:

$\text{Br} = .8566 + (.0294)\text{Ln}(\text{W}) - (.00239)\text{H} - (.0101)\varepsilon_r$
W= trace width (mils)
H = distance between the trace and the plane (mils)
ε_r = relative dielectric material between the trace and the underlying plane
Ln represents the natural logarithm, base e.

The fraction, Br, can never be greater than 1.0. A microstrip trace can never be slower than a stripline trace (surrounded by the same material). The formula was derived for simple microstrip traces with air above the trace and dielectric material underneath. Embedded and coated microstrip traces would have slightly higher fractions (and slightly slower propagation times) than estimated here, subject to an upper limit of 1.0.

[2]Brooks, Douglas G., "Microstrip Propagation Times: Slower Than We Think." Available for download at www.mentor.com/pcb/tech_papers.cfm.

CIRCUIT TIMING ISSUES

"Timing is everything" as the saying goes. In complex circuits we have signals on buses running everywhere. There are certain moments in time when those signals must align exactly right. Just a few of those times might include:

1. The three colors of a video display (red, green, and blue) must arrive at a receiver at the same time or the video picture will be distorted.
2. The two traces of a differential pair must be the same length so that the two opposite signals arrive at the receiver at the same time or common mode noise may be generated (see Chapter 14).
3. The data lines must be present and stable at their respective gates when the clock pulse triggers or there will be data errors.

Figure 2-2a illustrates what might happen if there is a timing problem. Three signal lines are shown, the top slightly misaligned from the others. A clock pulse samples the data state. The data signals don't line up quite right, but the clock signal still samples the data signals when they are all high or all low. In Figure 2-2b, however, the top signal is so badly misaligned that it transitions during the time the clock is sampling the data lines. In Figure 2-2b there will be data logic errors.

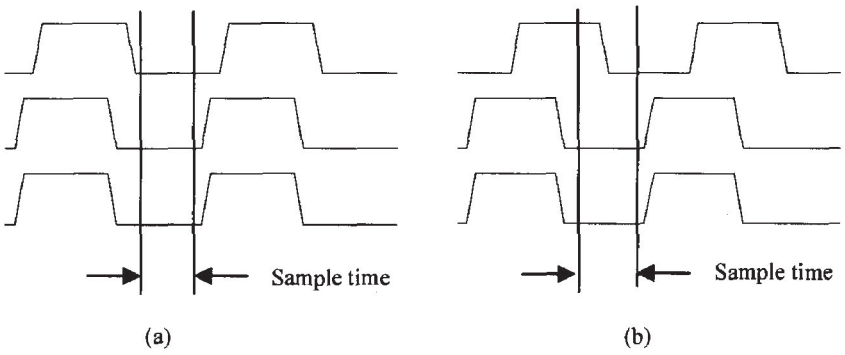


Figure 2-2 Circuits can tolerate slight differences in timing (a), but if the differences in timing are too great, sampling errors will occur (b).

Signals get misaligned for several reasons. One of them involves the devices themselves. There is a throughput time for a signal to travel through a device.

There are tolerances on output timing of devices, and circuits have different numbers of devices the signals propagate through. There are propagation delays on traces themselves. Yet, at certain well-defined points and times in a circuit and system, all the signals must align properly. The *specification* for this is the responsibility of the circuit design engineer. But *making it happen* can be the responsibility of the board designer.

Board designers control signal timing with trace length. Multiplying the propagation time (in units of ns/in) of a trace by its length gives the propagation time of the signal down the trace. If we need the trace to be a specific increment of time, then we adjust its length to make it so. This process is often called *tuning* the trace.

Adjusting timing this way requires two things, (a) a precise knowledge of the propagation speed of the signal along the trace we are interested in, and (b) the ability to adjust the trace length. We determine the propagation speed from the preceding discussion. Inherent in the determination is the knowledge of the relative dielectric coefficient (ε_r) of the material surrounding the trace. We cannot make the trace shorter than the distance between its connection points, but we can make it longer. The normal way to adjust trace length is to "snake" it.

Many of the higher end PCB design programs will automatically adjust trace length once you give it the appropriate parameters. Figure 2-3 illustrates the tuning editor from one of them. Note the variety of patterns the designer can specify. Figure 2-4 illustrates a portion of a board UltraCAD Design, Inc. has designed that required a considerable amount of tuning.

Some engineers worry about EMI radiation that might be caused by tuning patterns. In our experience we have seen no EMI issues from any type of tuning pattern as long as (a) every trace is referenced to a signal plane, and (b) the tuning is confined to stripline signal layers. Lots of traces are successfully tuned on microstrip layers (look at almost any computer motherboard), and we do so all the time. But if you or your engineers have significant concerns about tuning causing EMI problems, confining tuning to inner layers will almost always eliminate any possibility of radiation.

It should be noted that there are some articles that suggest crosstalk between segments of tuning loops can interact with the signal propagation speed and distort timing calculations. This is probably a concern at only the highest clock

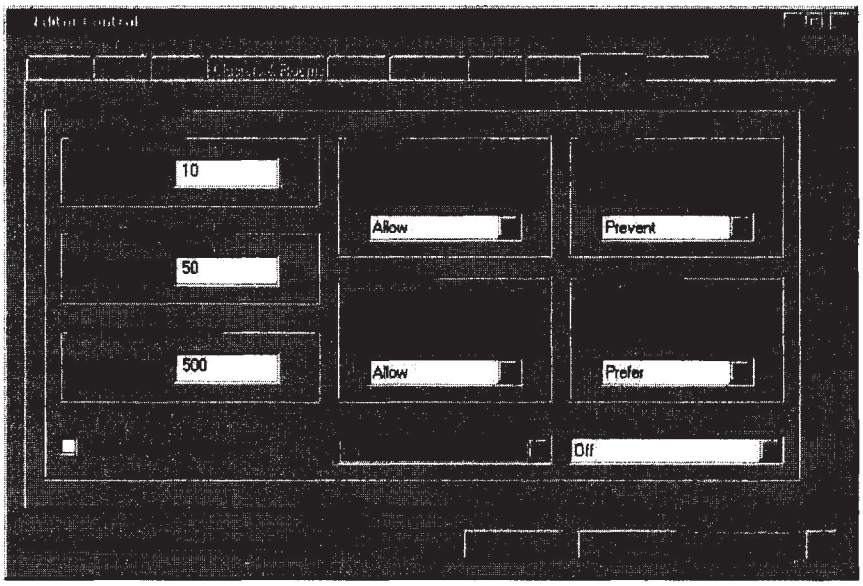


Figure 2-3 Tuning options from a PCB design software package.

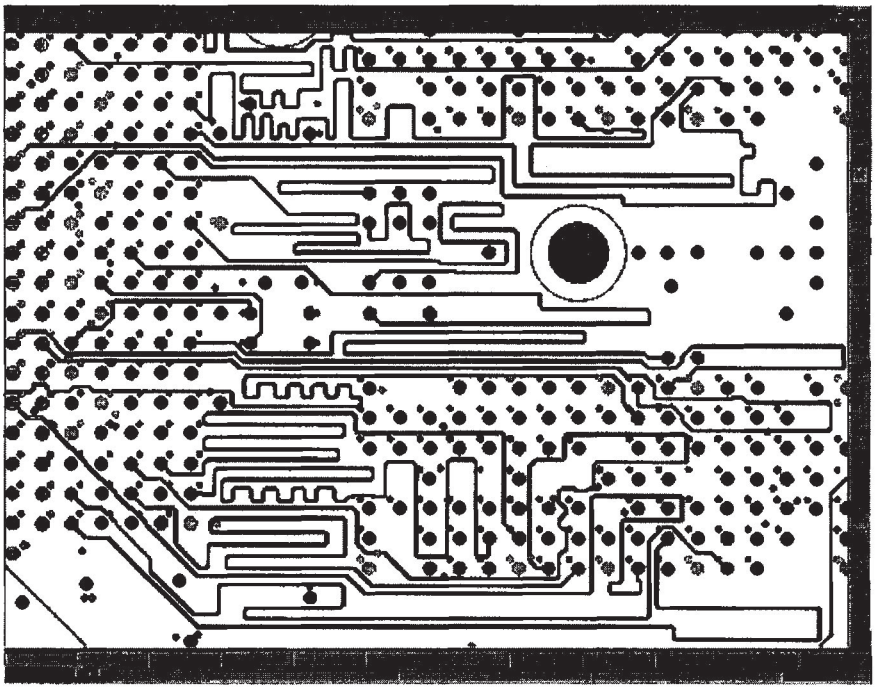


Figure 2-4 Tuned traces on a circuit board.

frequencies and is beyond the scope of this book. We have heard many people argue about whether or not via length should be taken into consideration while calculating propagation time. There does not seem to be a definitive conclusion in the industry about this.

WAVELENGTH

The wavelength of a signal, λ, is defined as the distance a signal travels during one complete cycle (see Figure 2-5). In air, this is simply the speed of light divided by the frequency in Hertz (Equation 2-4):

$$\lambda = c/f \qquad [2\text{-}4]$$

where λ = wavelength
c = speed of light in units of length per second
f = frequency in Hertz

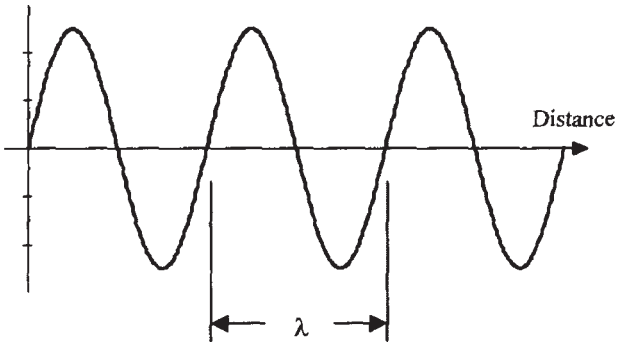


Figure 2-5 Wavelength, λ, is the distance the signal travels in one complete cycle.

On circuit boards, we need to know the propagation speed of the signal on the board before we can calculate wavelength. Then we would substitute that value

for c in Equation 2-4. For example, the wavelength of a 300-MHz (which is .3 cycles/ns) signal in an FR4 stripline environment ($\varepsilon_r = 4$) would be:

From Equation 2-1:

$$\text{Propagation speed} = 11.8/\sqrt{4} = 5.9 \text{ in/ns}$$

Then from Equation 2-4:

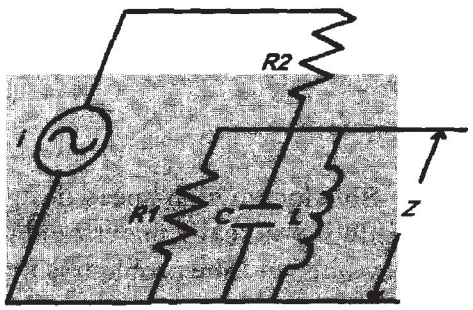
$$\lambda = 5.9/(.3) = 19.7 \text{ in.}$$

Wavelength is normally not an important parameter in board design. But it arises in at least two types of discussions. The first is if we are designing a transmitting antenna onto the board. (Note: Receiving antennas are not nearly as dependent on wavelength as transmitting antennas are.) This might be the case for a cell phone, pager, automotive door locking device, and so on. Transmitting antennas tend to perform best when they are some (fractional) multiple of a wavelength. Therefore, we need to know propagation speed in order to calculate wavelength and therefore an optimum length for the antenna we are putting on the board.

Note that if we are designing an antenna onto the board and are concerned about wavelength, then it is important that we *specify* the relative dielectric coefficient of the board material to be used in the fabrication of the board. In the case of microstrip traces we must also specify the type and extent of any coatings that might be used over the traces. Since propagation speed is dependent on relative dielectric coefficient, we cannot be certain what the speed will be, and therefore the wavelength of the signal, if we do not control the material. We control the material through a material specification that is part of the fabrication drawing for the board. More than one company has fallen into the trap of not controlling the material specification and then wondering why the effective transmitting length or performance of their boards changes from production lot to production lot.

Another context in which board designers may see wavelength mentioned relates to via spacing. In Chapter 9 we mention the concept of a Faraday shield in the context of controlling EMI emissions from an inner layer of a board. Some people advocate placing a row of vias along the edge of the board to trap EMI emissions within the board. If so, then they usually recommend that the vias be spaced some fraction of a wavelength (often 1/20).

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3 ELECTRICAL COMPONENTS

THE THREE BASICS

When we think about all the challenging complexities of high-speed designs, it is sometimes hard to believe how few components there really are in the world of electronics. Are you ready for this? There are only three passive components we will ever have to worry about in high-speed design issues. Only three! They are (a) resistors, (b) capacitors, and (c) inductors. Virtually everything we encounter in board designs can be described and analyzed in terms of these three components and their combination and effects.

To go with them, we may need an energy source. If so, we can consider it to be either a pure current source or a pure voltage source (an example of a voltage source might be a battery). And, as we will see later, the fundamental nature of inductance is electromagnetic energy, which relates to EMI and crosstalk. But *pure* energy sources are simple concepts, and electromagnetic energy is at the heart of inductance. So this does not negate the statement that we need understand only three basic passive components.

We usually consider semiconductor devices to be active components. But even these, in steady state, can be analyzed in terms of a simple energy source and just the three passive components, as we shall see. A clock driver or a logic gate at its "one" or "zero" output level, for example, can be considered to be at a steady state. (Note: A semiconductor device during switching between states, an amplifier with varying input, and circuits with feedback—such as control loops—often, but not always, are a little more difficult to analyze.)

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So why are high-speed design issues so difficult? Well, actually, they aren't so difficult if you really understand the behavior of the three basic components and how they interact. The problem is that many people, even many degreed engineers, do not really understand them. And that is a shame, because that single, simple truth is in large part the reason for so much confusion and mystery about high-speed design problems.

This chapter focuses on the three basic components and their fundamental natures. Hydraulic analogs are used as examples. The next chapters will cover their electrical effects (resistance and reactance) and what happens when we combine resistance and reactance to get impedance.

The discussions in this chapter relate to "ideal" components: ideal resistors, capacitors, and inductors. Ideal components are "pure" components. That is, an ideal resistor is a pure resistor with no (parasitic) capacitance or inductance associated with it. An ideal inductor has no resistance or capacitance associated with it, and so on.

In reality, of course, no component is ideal or pure. But once we learn to deal with these ideal components, and their combinations, we will then know how to deal with more realistic situations (e.g., capacitors with lead inductance).

RESISTANCE

Remember, current is the flow of electrons. Electrons (typically) flow through a wire or trace. A hydraulic analog would be water flowing though a pipe. In order to "push" the electrons along a wire we need an energy source. Figure 3-1a shows a battery as the energy source. Similarly, water needs a force to push it through the pipe. Figure 3-1b shows a pump as the source of the force pushing water through a pipe.

A resistor affects the flow of the electrons. (Of course, so do capacitors and inductors.) Its effect is to impede (related to impedance) the flow. The analog would be a crimp in the pipe that impedes the flow of water. A small crimp (resistance) offers a little impedance to the flow, whereas a bigger crimp offers a lot of impedance. A very large crimp (resistance) might almost stop the flow altogether.

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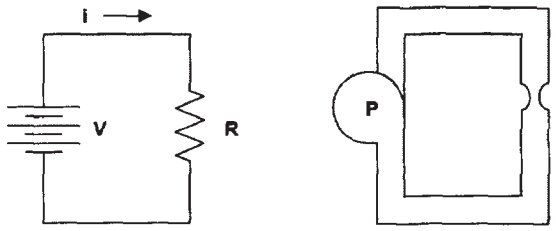


Figure 3-1 Resistance can be modeled by a crimp in a hydraulic pipe.

A small crimp might allow a certain flow of water. A larger crimp would reduce the flow and cause pressure to build up behind the crimp, effectively fighting the pump pressure. If we wanted to maintain the same flow with the tighter crimp (bigger resistance) we would have to provide more pump force (voltage) to overcome the pressure backing up behind the crimp. Thus, there is a relationship between the size of the crimp (resistance), the flow of the water (current), and the pump force (voltage).

OHM'S LAW

There is an extremely important formula that relates the flow of electrons through a resistor. It is called Ohm's Law (after George Simon Ohm):

$$V = i \times R \qquad [3\text{-}1]$$

where V is the voltage force (in volts)
i is the current (in amps), and
R is the resistance (in ohms, or Ω)

Other variations on Ohm's Law are:

$$i = V/R$$
$$R = V/i$$

Given any two values, the third can be calculated from Ohm's Law. For example, if V = 5 volts and R = 1K (1,000 Ω), then i will be 5 mA (.005 amps).

This is perhaps the most important formula in the book! If there is only one thing in this book you need to commit to memory, this is it. Absolutely anyone and everyone in our industry needs to be able to write down and work with Ohm's Law without having to think about it.

There are a couple of simple observations we can make by looking at Figure 3-1. For example, the flow of water (and the current) must be the same at every point in the pipe (or wire or trace). If this were not true, where would the electrons or the water go? You can't pump water out of a pump if you don't have water coming into the pump. And you can't force electrons out of a battery if they are not also flowing in.

Consider the potential impact if this were not true. If we could force electrons out of the battery without returning them in the other side, then somewhere in the circuit there would be a big buildup of electrons and somewhere else there would be a big shortage. These two areas would then have large negative and positive charges, respectively. Then the two areas would really be hard to handle, attracting or repelling lots of other things around them. We know intuitively that this doesn't happen. So, current flows in a loop, and when it does so it must be the same everywhere in that loop.

We will see later that current might divide into branches, and we will discuss the laws relating to that at that time. But current in any branch is the same at every point in the branch, and ultimately returns to its source.

A second observation relates to the forces around the loop. Consider the voltage at the battery in Figure 3-1 (or the pressure at the pump). It is the same as the voltage across the resistor (or the pressure across the crimp). If we were to go around the loop and add up the voltages (or pressures) they would sum to zero. For example, if we went clockwise around the loop the voltage across the resistor would be +V and the voltage across the battery would be measured as –V (in a clockwise rotation we would see the negative terminal of the battery first). This is a simple illustration, but nevertheless it is a fundamental truth that voltages around even a more complex loop must always sum to zero. Always. This is known as Kirchhoff's (Gustav Robert Kirchhoff) 2nd Law (see Chapter 5).

Let's now look at two resistors (crimps) in series (Figure 3-2). If one resistor (crimp) is very large, very little will flow, regardless of how small the other resistor is. For there to be a large flow, both resistors (crimps) need to be relatively

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small. We will see later that the effect of two resistors (crimps) in series is additive; that is, the equivalent resistance, Req, is given by Equation 3-2.

$$Req = R1 + R2 \qquad [3\text{-}2]$$

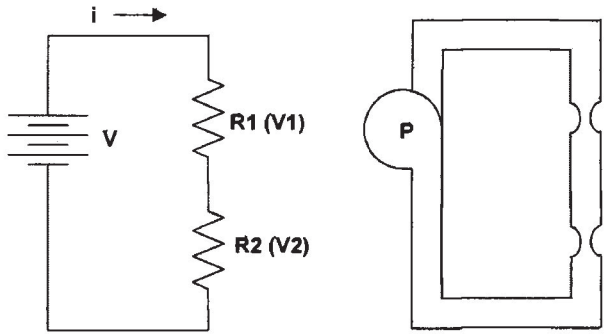


Figure 3-2 Hydraulic analog of two resistors connected in series.

Of particular note, if R2, for example, is very much larger than R1, then the equation for equivalent series resistance reduces to Equation 3-3.

$$Req = R1 + R2 \cong R2 \qquad [3\text{-}3]$$

If we put two resistors (crimps) in parallel, as in Figure 3-3, the situation is a little different. First, notice that the current out of the battery must equal the current into the battery, as we discussed before, just as the water flowing out of the pump must equal the water flowing into the pump. But the current (water) divides into two branches. Since we can't create current (or water) from nothing, the sum of the current (water) in the two branches must equal the current (water) through the battery (pump).

In these figures, there are two nodes: one at the top where the two branches break off from the main line, and the other at the bottom. The current (water) into a node must equal the current (water) out of the node. If this were not true, where would the electrons (or water) come from or go?

Also, since there can only be a single voltage (pressure) at a node, then the voltage across R1 must be the same as the voltage across R2.

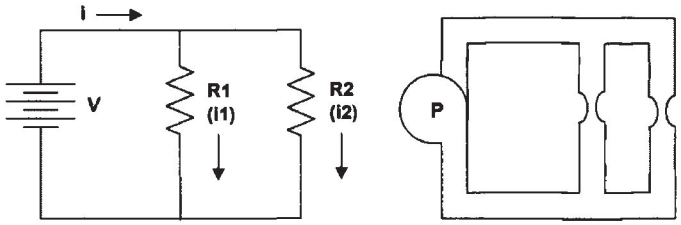


Figure 3-3 Hydraulic analog of two resistors connected in parallel.

Now consider what happens if R1 and R2 are significantly different, say R1 is very much larger than R2. In that case, most of the current will flow through R2. In fact, no matter how large R1 is, the effective resistance of their combination will never be greater than R2. The equivalent resistance of their combination will be smaller than R2 as R1 reduces in size. It is reasonably intuitive to see that if R1 = R2, then the current through R1 will equal the current through R2 (since the voltage across them is the same, and in fact equals V in this illustration.) If i1 = i2, then the equivalent (single) resistor that looks like the parallel pair of resistors R1 and R2 (where R1 = R2) would be R/2. The formula for the equivalent resistance, Req, of two parallel resistors is given by Equation 3-4:

$$Req = \frac{1}{\frac{1}{R1} + \frac{1}{R2}} = \frac{R1R2}{R1 + R2} \qquad [3\text{-}4]$$

This is called the *parallel combination* of two resistors. We will see later that a similar formula will apply to some combinations (not necessarily parallel ones) of inductance, capacitance, reactance, and impedance.

Of particular note, if R2 is very much larger than R1, then the equation for the equivalent parallel resistance reduces to Equation 3-5 (compare that with Equation 3-2).

$$Req = \frac{1}{\frac{1}{R1} + \frac{1}{R2}} = \frac{R1R2}{R1 + R2} \cong R1 \qquad [3\text{-}5]$$

CAPACITANCE

Capacitors are interesting devices. The hydraulic analog to a capacitor is a storage device with two separate, nonconnected, sealed chambers (Figure 3-4). As a pump forces the water around the pipe, water flows into the top chamber and is pumped out of the lower chamber. At first, this happens with almost no impedance to the flow. But as the chamber begins to fill, the pressure in it builds up, pushing against the pressure of the pump and causing the flow of water to slow. When the chamber fills, the pressure at the chamber equals the pressure at the pump and the flow stops.

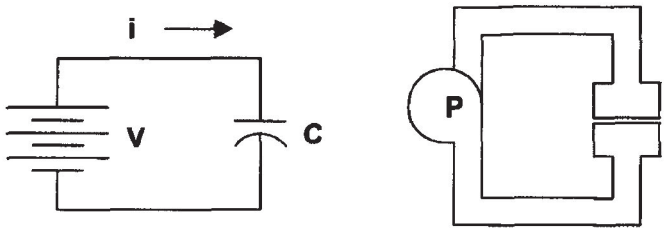


Figure 3-4 Hydraulic model of a capacitor.

A capacitor, in its simplest form, consists of two parallel plates placed closely together. Actual fabrication techniques are, of course, more complicated than this. But all capacitors, fundamentally, consist of two separate surfaces placed very close together with a thin material placed between them.

When a voltage is applied to a capacitor (perhaps by connecting a battery across its leads), electrons flow onto one of these plates. (Remember, the flow of electrons is current.) Electrons flow off the other plate and back around into the battery. This preserves the condition that current is the same everywhere in the loop. During the first instant, the flow is virtually unimpeded. But as the electrons begin to build up on the plate, they start building up a charge that begins to resist the continuing flow of more electrons. Thus the flow starts slowing down. At some point,

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the charge buildup is great enough to stop any further flow, and all current stops. The charge buildup is manifested by a voltage on the plate. The voltage increases as more charge builds, and current stops flowing when the voltage equals the initial voltage that was applied to the capacitor.

One key to understanding what happens with a capacitor is the recognition that what happens is *time dependent*. (This is *not* true with resistance.) That is, when current first starts flowing into a capacitor, there is no voltage across the plates (i.e., across the capacitor). There can be no voltage across the capacitor until charge builds up on the plates. But charge can't build up on the plates until charge (current) has actually flowed into the capacitor. Since current must flow before a voltage appears, we say that current leads voltage. (Why this concept is important will become more apparent when, later on, we see that the opposite is true with inductors; that is, current lags voltage in the case of an inductor.)

Now, consider the hydraulic analog and think about what happens if we turn the pump on in the forward direction for a moment and then reverse it, pumping in the opposite direction. Water will start to flow into the storage tank, but if we reverse the pump soon enough, negligible pressure will have built up. Then water will flow in the reverse direction. Now if we reverse the pump again, quickly enough, pressure will not build in the opposite direction either.

In this way, we can envision how it would be possible to have a pump that continually reverses direction fast enough that water pressure cannot build up in the tank. The flow of water in the pipes will *look* as though the storage tank were not there. Of course it is, but the flow reverses so quickly that its effects are not seen. We would see this same effect if the pump reversed more slowly, but the storage tank was larger. But if the pump reversal was relatively slow, and the storage tank was relatively small, then we would see the flow of water speed up and slow down in different parts of the pumping cycle as pressure built up and then decreased again.

So, the effect of the storage tank is a function of both the size of the tank and the frequency at which we alternate the pumping cycle. Water will flow as if the tank were not there if the tank size is very large or if the pump frequency is very high. Otherwise, water flow will slow down, and might even stop, during part of the cycle.

So it is with electrons and a capacitor. If a capacitor is very large, alternating current will flow virtually unimpeded, as if the capacitor were not there. If the

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frequency is very high, current will flow as if the capacitor were not there. It is not that *individual* electrons are flowing around the loop; they are not. Individual electrons cannot travel past the plate of the capacitor. But the flow of electrons past any particular point will be the same as it would be if the capacitor were not present. On the other hand, there will be significant impedance to current flow if (a) the frequency is relatively low, or (b) the capacitor is relatively small.

Another way to think about this is to recognize how many electrons actually flow during a half-cycle of an AC current. A 1-MHz waveform changes direction 10^6 times each second. Therefore, for the same magnitude of peak current (charge/unit time), only one-millionth (10^{-6}) as many electrons flow during the half-cycle of a 1-MHz (10^6) waveform as flow during the half-cycle of a 1-Hz waveform. Therefore, the voltage buildup on a capacitor will be substantially less for a half-waveform of a 1-MHz signal than for a 1-Hz signal. Table 3-1 summarizes the range of possibilities.

		Frequency		
		High (Fast)	Low (Slow)	DC
Cap	Big	Lots of flow	Some flow	No flow
	Small	Some flow	Very low flow	No flow

Table 3-1 Capacitor size and frequency both affect the flow of current.

Suppose we put two capacitors in parallel (Figure 3-5). If the value of one capacitor is very large, then it almost doesn't matter what the value of the other one is. Even if the other capacitor is very small, almost zero, the overall circuit will still behave as if there is a large capacitor in the flow loop. The flow will be determined, first, by the larger capacitor. The effective capacitance of the two will be at least as large as the larger of the two capacitors, and perhaps even more if the second capacitor is nearly the same size as the first.

In contrast, consider the case of two capacitors in series (Figure 3-6). Here, the flow will be restricted by the smaller capacitor. The smaller capacitor will charge up quickly (the smaller storage tank will fill up quickly) and flow will stop. No matter how large the larger capacitor is, the flow will never be greater than that allowed by the smaller capacitor. The total flow may be less if the other capacitor

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adds additional restriction (impedance to the flow), which would happen if it were nearly as small as the first one.

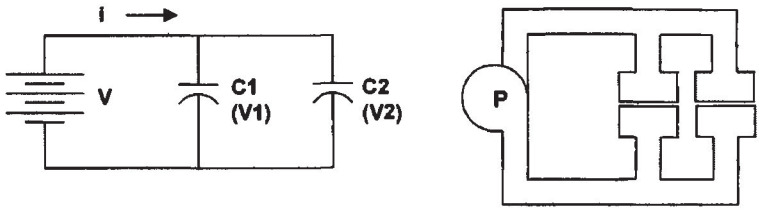


Figure 3-5 Hydraulic analog of two capacitors connected in parallel.

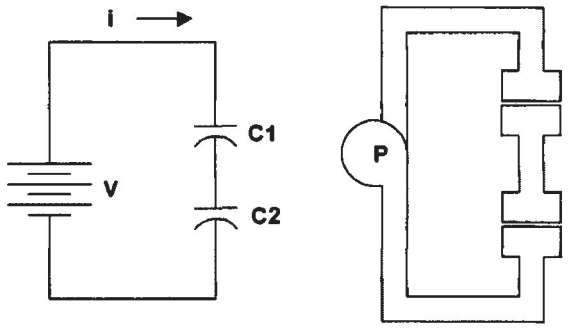


Figure 3-6 Hydraulic analog of two capacitors connected in series.

For the parallel combination of capacitors, Ceq is given by Equation 3-6:

$$Ceq = C1 + C2 \qquad [3\text{-}6]$$

Note that if, for example, C2 were very much larger than C1, the equivalent parallel combination would reduce to Equation 3-7:

$$Ceq = C1 + C2 \cong C2 \qquad [3\text{-}7]$$

For the series combination of capacitors, Ceq is given by Equation 3-8:

$$Ceq = \frac{1}{\frac{1}{C1} + \frac{1}{C2}} = \frac{C1C2}{C1 + C2} \qquad [3\text{-}8]$$

Note in Equation 3-8 that if C2, for example, were very large, the equation would reduce to Equation 3-9

$$Ceq = \frac{1}{\frac{1}{C1} + \frac{1}{C2}} = \frac{C1C2}{C1 + C2} \cong \frac{C1C2}{C2} = C1 \qquad [3\text{-}9]$$

This illustrates how the smaller capacitor becomes the limiting one in this case. Note that this is the opposite case from that of resistors (Equations 3-3 and 3-5).

The measure of capacitance is the farad (Michael Faraday). One farad of capacitance will have a force of one volt across its plates when the capacitor stores one coulomb of charge. (Remember one coulomb is 6.25×10^{18} electrons.) Recall also that one amp is the flow of one coulomb of charge in one second. So, if one amp of current flows for one second onto the plates of a one-farad capacitor, the voltage across the plates will rise one volt, or:

$$\Delta V = i \times \Delta t / C$$

Perhaps of more relevance, what if 1.0 mA flows onto the plate of a 10.0 uF capacitor for 1.0 ms? Well, 1.0 mA (10^{-3}) for 1.0 ms (10^{-3}) would move 10^{-6} coulombs. A capacitance of 10.0 uF is 10^{-5} farads. So the voltage buildup would be:

$$V = \text{charge/capacitance}$$
$$= 10^{-6}/10^{-5} = 0.1 \text{ volts}$$

CHARGE STORAGE

Suppose we charged a capacitor up to 10 volts and then disconnected it from the circuit. How long would the capacitor hold the charge? Conceptually, a perfect capacitor would hold the charge forever. In practice the charge would decay

through time, primarily because of what we call *leakage resistance*. Almost all capacitors will slowly leak charge internally from one plate to the other. The mechanism for how this happens is a function of the materials and the fabrication processes used. The speed with which this happens is determined by the magnitude of the leakage resistance, which is usually part of the specification for the capacitor or capacitor family. Leakage resistance is usually quite high and not an issue in high-speed designs except in very special cases.

If we had a charged, disconnected capacitor sitting on a workbench and wanted to see how fast it was discharging (how fast the charge was leaking off), this would be hard to measure. Most measurement tools (especially multimeters and scopes) absorb charge during measurement, therefore affecting the very effect you are trying to measure.

Certain metal-oxide-silicon (MOS) devices (e.g., some charge coupled devices) use little silicon capacitors as "charge buckets" to store digital information. Some devices can be designed to hold charge for a considerable time, allowing them to function as nonvolatile memories (i.e., memories that don't lose their information when the power supply is turned off).

A more serious consideration involves capacitors in high-voltage circuits normally associated with CRTs, laser printers, and copiers. These capacitors may be charged to many thousands of volts and may still hold this charge when the device is turned off. Good safety practice is to place a very high resistance "bleeder" resistor around such capacitors so that they will discharge safely within a few minutes. For various reasons, these resistors may be missing or ineffective in any particular circuit. Service personnel learn quickly (if they don't already know) to be very careful around such circuits when servicing them. An inadvertent shock from a highly charged capacitor is, at a minimum, uncomfortable, and can very easily be quite dangerous.

FORMULA FOR CAPACITANCE

The formula for capacitance is fairly straightforward (see Equation 3-10). It is purely a function of the geometry of the plates and the material between the plates. Some capacitors (e.g., ceramic ones) are made up of several layers of plates, and the $n - 1$ term accounts for this.

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$$C(pF) = .2248 \times \varepsilon_r \times A \times (n - 1)/d \qquad [3\text{-}10]$$

where:
ε_r = relative dielectric coefficient (air = 1, FR4 ≅ 4)
A = area of each plate (in^2)
d = distance between plates (in)
n = number of plates

Applying this formula to a PCB made from FR4 with $\varepsilon_r = 4$, 3-inches square, and with power and ground planes spaced 10 mils apart would lead to a capacitance of

$$C = .2248 \times 4 \times 9 \times 1/.01 = 809 \text{ pF}$$

CAPACITANCE FUNCTIONS AND EFFECTS

The capacitance between closely spaced power and ground planes on a good-sized PCB can be fairly significant. We will see later that this might be used to our advantage in high-speed designs. If the planes are spaced further apart, however, the available capacitance drops quickly. We cannot just blindly assume the capacitance formed by the planes will work to our advantage. Some conditions do apply.

Capacitors provide several different important functions in a circuit. Their ability to store DC charge and to offer relatively low impedance to alternating frequency signals make them useful in power supply circuits and bypass applications. We can think of them as functioning as little batteries here. Since they tend to block DC but allow AC to pass, they are used to "couple" circuit stages together. The signal can pass from one stage to the next, but DC bias voltages are isolated to their own stage. Since the amount of charge that accumulates on their storage plates is a function of time and current flow, they can be used to "time" various circuit functions, such as the length of time a digital camera shutter is open or a circuit gate is open to accept a signal. We will see later that capacitors can be combined in a unique way with inductors to control frequency. Thus they are an important part of tuning and filter circuits.

But sometimes capacitors show up where we don't want or expect them to. For example, consider two traces routed side by side. Their edges can form the

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plates of a capacitor, and a signal can be coupled from one trace to the other, creating unwanted noise on the other trace. This is an important cause of crosstalk. Wirewound resistors have so much capacitive coupling between their wire turns that they are virtually useless, and almost never used, in higher frequency circuits. Unwanted and undesirable capacitive effects are among the top three problems in high-speed PCB designs (the other two are inductance and EMI).

INDUCTANCE

Inductors seem to be very difficult components for many people to understand. Appendix B is devoted to explaining why inductors do what they do (induct). Here, I try to give an intuitive understanding of what happens when there is an inductor in a circuit.

The hydraulic analog for an inductor (Figure 3-7) is a paddle wheel with significant mass. When the pump applies pressure through the pipe, the paddle wheel's inertia resists the initial pressure, and pressure builds up behind it in the pipe. The pressure begins to overcome the inertia, and the paddle begins to turn. As long as there is pressure behind it, the paddle wheel will continue to speed up until it reaches an equilibrium with the flow.

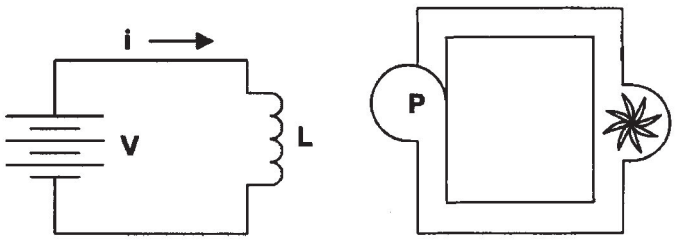


Figure 3-7 Hydraulic analog for an inductor.

Most of us probably don't see many paddle wheels, so let's consider a perhaps more familiar illustration of inertia first. When your car is sitting in the driveway and won't start, think of what happens when you try to push it. It is very hard to get started, and you have to push it very hard to get it started. But once it starts to

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roll, it can be equally hard to stop. Some of us have had the unfortunate experience of having our car roll into another car or into the side of our garage before we could stop it again.

I had an opportunity once to open and close a huge (5,000 lb.) bank vault door. It was beautifully balanced on its hinges, and swung easily after I got it started. It took considerable effort to stop it again before it swung so far that it closed. It was a beautiful example of almost pure inertia.

So now think what happens when we turn on the pump (voltage). Pressure builds instantly against the vanes of the paddle wheel (inductor) but no water (current) flows. The pressure must "work" against the inertia for a moment before the wheel begins to turn. After a period of time, the wheel begins to turn and allow water (current) to flow. How long this takes is a function of how much force is applied and how much inertia there is in the paddle wheel. The force will continue to speed up the wheel until the wheel reaches an equilibrium speed such that all the water will flow unimpeded around the loop and there will be no pressure at all across the wheel.

Now, suppose we turn off the pump. The paddle wheel's inertia will cause it to continue to turn, forcing water to continue to flow around the loop. It is as though the paddle wheel takes over the pump's function. Reverse pressure will build up across the pump as water is forced to its "back" side and drawn away from its front. This will continue until the force that is built up is finally able to counteract the inertia of the paddle wheel, and the paddle wheel slows to a stop.

So it is with electrons. When we place a battery across an inductor, there is force (voltage) across the inductor, but in the first instant there is no current flow. Current does begin to flow, however, and after a moment the current will reach an equilibrium and flow unimpeded around the loop. At that point the voltage across the inductor has dropped to zero. How quickly the current begins to flow and how quickly it reaches equilibrium depend on how large the inductor is. It takes longer for this to happen with a larger inductor than it does with a smaller inductor.

The impedance to the initial flow of current has its source in the buildup of the magnetic field around the conductor. For a more extended discussion of this characteristic see Appendix B, "Why Inductors Induct." After the current stabilizes there is a stable magnetic field around the wire. Now when we remove the source of current, the magnetic field around the conductor begins to collapse, generating the continuing force to keep current flowing in the same direction. Electrons will con-

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tinue to flow and build up on the far side of the inductor. As they build up, the charge (voltage) increases. When the voltage increases enough, it will repel the further flow of electrons. The force will finally balance against the "inertia" of the inductor. At that point, the flow will stop.

So, as with a capacitor, what happens to the relationship between voltage and current with an inductor is time dependent. At the instant when voltage is first applied, voltage appears across the inductor but no current flows. Thus, voltage *leads* current, or current *lags* voltage through an inductor. In this sense, an inductor behaves exactly the opposite as a capacitor.

Also, in contrast to a capacitor, consider what happens when we pump water in one direction, then reverse the pump and pump it in the other direction. If we reverse the direction of the pump quickly, before the "inertia" can be overcome, very little water flows at all. On the other hand, if we pump for a longer time in one direction, then reverse the pump and pump in the other direction for a while, a reasonable flow will exist. A large inertia (inductor) will resist the flow more and for a longer time, and a smaller inertia will let the flow start more quickly.

Thus, the effect of the inductor is a function of both the size of the inductor and the frequency. Flow is small if the inductor is large or if the frequency is high; flow is larger if the inductor is smaller or the frequency is lower (see Table 3-2).

		High (Fast)	Low (Slow)	DC
L	Big	Very low flow	Some flow	Full flow
	Small	Some flow	Lots of flow	Full flow

Table 3-2 Inductor size and frequency both affect the flow of current.

Suppose two inductors are connected in series (Figure 3-8). If L1 is much larger than L2 (i.e., the first paddle wheel is much larger than the other), the current (flow) cannot be larger than that allowed by the larger one. The second, smaller inductor will not have much effect, unless it is about the same size as the larger one.

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By the time the force can overcome the inertia of the larger one, the smaller one's inertia has already been overcome.

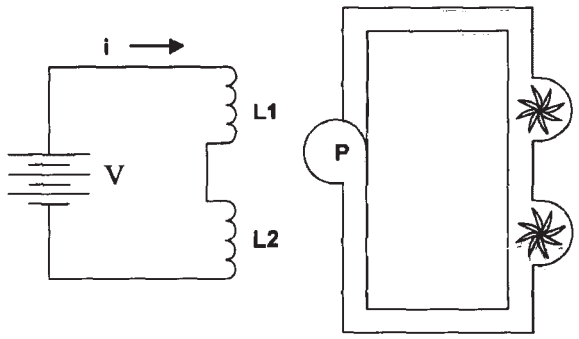


Figure 3-8 Hydraulic analog of two inductors connected in series.

As is the case with resistors, inductors in series add together. So the equivalent inductance, Leq, if two inductors are connected in series is given by Equation 3-11:

$$Leq = L1 + L2 \qquad [3\text{-}11]$$

Note that if L2 is very much larger than L1, then the equation for Leq reduces to Equation 3-12:

$$Leq = L1 + L2 \cong L2 \qquad [3\text{-}12]$$

If there are two inductors in parallel, however (Figure 3-9), the current will not be less than what the smaller one will allow. The smaller inductor's inertia is quickly overcome by the force, so current flows through it whether or not the inertia of the larger one has been overcome. If the other inductor is small enough, additional current can flow through it. Thus, parallel inductors combine into a single equivalent, Leq, in the same manner that parallel resistors do (see Equation 3-13):

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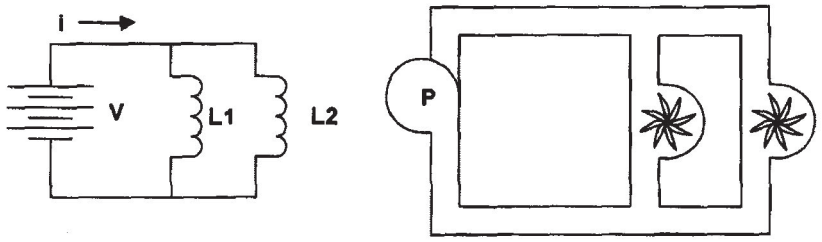


Figure 3-9 Hydraulic analog of two inductors connected in parallel.

$$Leq = \frac{1}{\frac{1}{L1} + \frac{1}{L2}} = \frac{L1L2}{L1 + L2} \qquad [3\text{-}13]$$

Note also that if one inductor, say L2, is very large, the equivalent parallel inductance simply reduces to Equation 3-14:

$$Leq = \frac{1}{\frac{1}{L1} + \frac{1}{L2}} = \frac{L1L2}{L1 + L2} \cong \frac{L1L2}{L2} = L1 \qquad [3\text{-}14]$$

Note that inductors combine exactly as resistors do. Inductors and capacitors are opposites in this characteristic also.

The measure of inductance is the henry (Joseph Henry). Remember the measure of capacitance is the farad, and the voltage across a capacitor will rise one volt when one amp flows onto the plates. A current flowing through an inductor may generate a voltage across it, also. But from the preceding it is clear that, in the case of an inductor, we are not talking about a DC current. A DC current does *not* generate voltage across an inductor just as a steady flow past the paddle wheel does *not* need a force to keep it going (once the paddle wheel is up to speed). Force builds up across an inductance when the current flow through it is changing, working against the inertia of the inductance. One henry of inductance will cause one volt to appear across it when the current through it is *changing* at the rate of one amp per second, or, more generally:

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$$V = L \times \Delta i / \Delta t$$

FORMULAS FOR INDUCTANCE

Formulas for inductance are moderately complicated and are also somewhat approximate. Equations 3-15 and 3-16 give generally accepted formulas for a wire in air and for a microstrip trace:

Wire in air:

$$L = .00508\, b\left\{\left[\mathrm{Ln}\left(\frac{2b}{a}\right)\right] - .75\right\} \qquad [3\text{-}15]$$

where L = inductance in μH
a = radius (in)
b = length (in)
Ln = natural log (base e)

For example, let's assume a typical through hole wire is .018 in in diameter. Then a 1.0 in length would have an inductance of:

$$L = .00508 \times 1 \times \left\{\left[\mathrm{Ln}\left(\frac{2 \times 1}{.009}\right)\right] - .75\right\} = .012\ \mu H = 12\ nH/in$$

Microstrip trace:

$$L \cong 5\mathrm{Ln}(2\pi h/w) \qquad [3\text{-}16]$$

Where: L = inductance in nH/in
w = trace width (mils)
h = height above the plane (mils)

For example, a 10-mil-wide trace 1.0 in long and 10 mils above the plane would have an approximate inductance of

$$L \cong 5\mathrm{Ln}(2\pi 10/10) \cong 9.2\ nH/in$$

Note that these component lead and trace inductances are pretty small, on the order of 9 to 12 nH per inch. But if the current is rising at, say 10 mA per ns, (which might be representative of a 1-ns rise time device) the voltages generated can be surprisingly large:

$$V = L\Delta i/\Delta t$$
$$V = (12)(10^{-9})(10^{-2}/10^{-9}) = .12 \text{ volts} = 120 \text{ mV}$$

This gives an idea of why stray inductance is a signal integrity problem in high-speed designs. Ordinarily 9 to 12 nH is an inductance that would be considered negligible (unless we were talking about RF or microwave circuits). But when we talk about nanosecond rise times, such inductances can be a problem, indeed.

High-speed digital circuits don't usually have very many inductors in them, but inductors are an integral part of electromechanical relays and transformers. They may be used in power supply filtering circuits because of their ability to pass DC power supply voltages while "blocking" (thereby filtering out) higher frequency noise. Sometimes a simple ferrite bead is used to provide localized inductance for this purpose. Inductors sometimes are used to set frequencies or as part of filtering circuits, but there are often better ways to accomplish these functions with capacitors and active circuits.

It is the stray inductance that causes problems in high-speed designs. There is enough inductance in a short length of component lead or PCB trace to cause really significant potential problems in a circuit. We will see later that (just like capacitive coupling) inductive coupling between traces must often be carefully controlled or minimized.

CHARGE AND DISCHARGE CURRENTS

There are a few waveforms and relationships that people in our industry should be able to recognize and sketch from memory. Figures 3-10 through 3-12 represent some of these. Figure 3-10 illustrates a simple resistor-capacitor (RC) circuit. When the switch is one position, current flows toward the capacitor. When the switch is in the other position, the capacitor discharges.

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When we first connect the battery, current flows toward the capacitor. In fact, from Ohm's Law, if R = 1.0 the flow will be 1.0V/1.0 Ω = 1.0 amp. Remembering that for capacitors, current leads voltage, and since the capacitor initially has no charge on it, even though the current flow is 1.0 amp, there will be 0.0 volts across the capacitor. Therefore, 1.0 volt must appear across the resistor (1.0 amp through 1.0 Ω).

As current flows onto the plate of the capacitor, the voltage across the capacitor must begin to rise. Since the battery voltage does not change, the voltage across the resistor must necessarily decrease (the sum of the voltages across the capacitor and across the resistor must be constant and, in this case, equal to the voltage of the battery, 1.0 volt). Since the value of the resistor does not change, then the current through it must begin to decrease. After some period of time, the current will decline to zero and the voltage across the capacitor will rise to equal that at the battery.

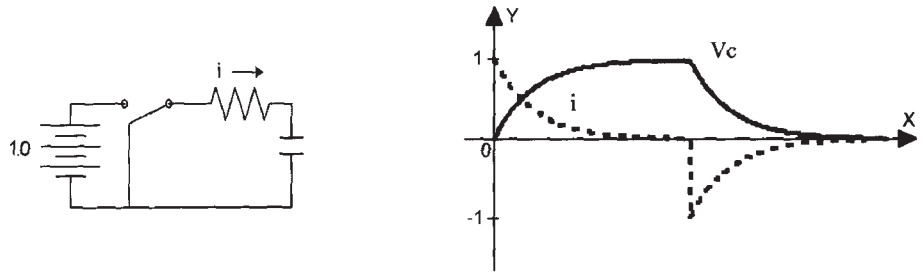


Figure 3-10 Typical charging and discharge voltage and currents for an RC circuit.

The graph in Figure 3-10 illustrates the relationship. As voltage across the capacitor increases, current decreases. All RC circuits follow a curve such as this, and differ only in terms of the height of the curves or how "stretched out" they are along the horizontal axis. But the shape of the curve is always that of an exponential, and we will define the equation for it in Chapter 4 in the section on time constants.

When we flip the switch to remove the battery from the circuit, the capacitor then discharges through the resistor. Note that the fully charged capacitor (1.0 volt) now causes a current of 1.0 amp (initially) to flow through the resistor in the

opposite direction. Thus, the voltage across the resistor is now of the opposite sign. As before, the curves follow a very well-defined pattern. In fact, in this type of situation (a fixed DC source, a simple discharge path around the battery, and time to reach equilibrium) the discharge curves must be identical mirror images of the charging curves.

Figure 3-11 shows the corresponding RL (resistor-inductor) circuit. The analysis is identical. When the battery is first connected, the voltage across the inductor jumps up to 1.0 volt and there is no current through the inductor. Then as current begins to flow through the inductor, the voltage across the inductor begins to decline. At equilibrium, there is no voltage across the inductor, and the current is defined by the battery and the resistor:

$$i = V/R = 1.0/1.0 = 1.0 \text{ amp}$$

After the switch is thrown, disconnecting the battery from the circuit, the inductor tries to maintain the same current flow. Since the current doesn't change instantaneously, the voltage across the resistor doesn't change. So the voltage remains at 1.0 volt across the resistor. The left side of the resistor is now the same node as the bottom of the inductor (since the switch has switched), so the voltage across the inductor is now the opposite of what it was when the switch was first connected to the battery.

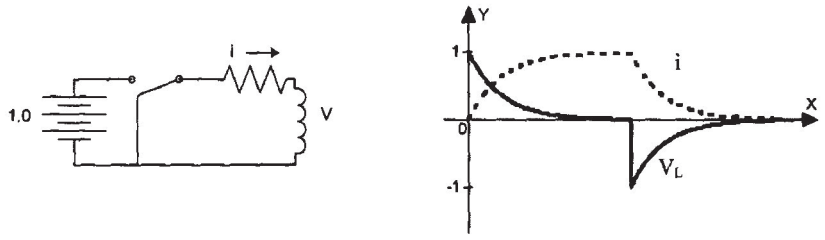


Figure 3-11 Typical charging and discharge voltage and currents for an RL circuit.

Again, these curves follow a well-defined exponential relationship, and the discharge curves are mirror images of the charging curves.

When a sine wave current source is applied to a capacitor or an inductor, the voltage across the device also follows a sine wave. For pure capacitors and in-

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ductors (i.e., no resistance component at all) the voltage waveform will be shifted (exactly) 90 degrees from the driving current waveform. (Remember, there are 360 degrees in one cycle.) This result can be proven mathematically, but the proof requires mathematical analyses well beyond the scope of this book. So you'll just have to take this fact on faith.

For capacitors, current will lead voltage by 90 degrees (Figure 3-12a) and for inductors it lags voltage by 90 degrees (Figure 3-12b). Conversely, for capacitors, voltage lags current by 90 degrees and for inductors voltage leads current by 90 degrees. If we connect a capacitor and an inductor in series, current must necessarily be the same through both of them (current is the same everywhere along a loop). Therefore, *the voltage across the capacitor would lag the voltage across the inductor by 180 degrees* (or exactly one-half cycle). Later on we will look at the total voltage across the inductor–capacitor (LC) series pair and note that since their voltages are 180 degrees out of phase, one subtracts from the other. A particularly interesting case is when the voltages across each component are equal, but 180 degrees opposite in phase. Even though there can be voltage across each component, the voltage across the pair will be equal and opposite at all times. Therefore, the combined voltage across the pair will be zero! This surprising result can have very important consequences for certain types of circuits.

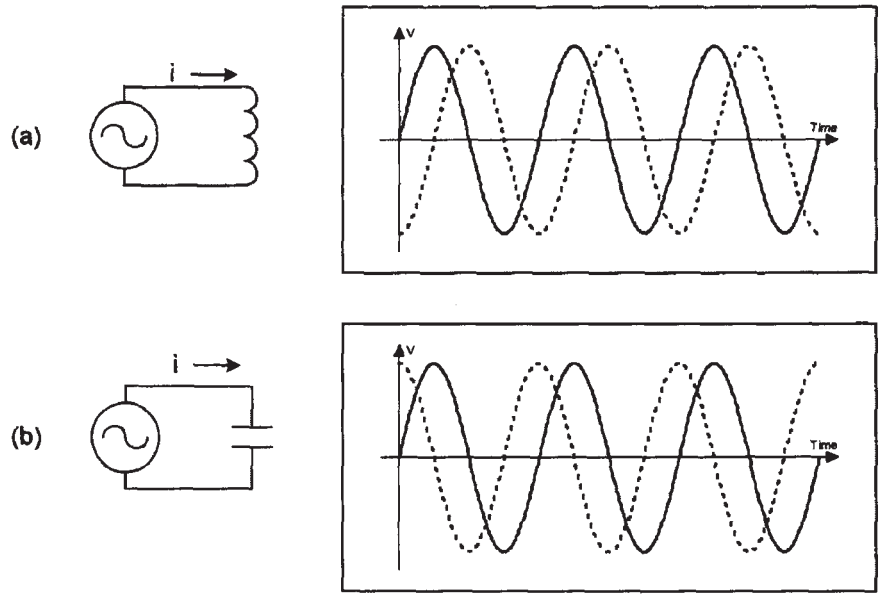


Figure 3-12 Voltage (solid line) leads current through an inductor (a) and voltage lags current through a capacitor (b).

RESONANCE

Before we leave our hydraulic analogies, let's look at one more special case. Suppose we have the storage tank (capacitor) and paddle wheel (inductor) in parallel as shown in Figure 3-13. Suppose we turn on the pump long enough to have the paddle wheel turning freely (no resistance to the flow). Then, let's turn off the pump.

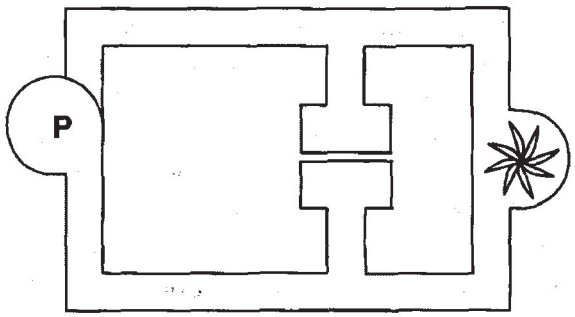


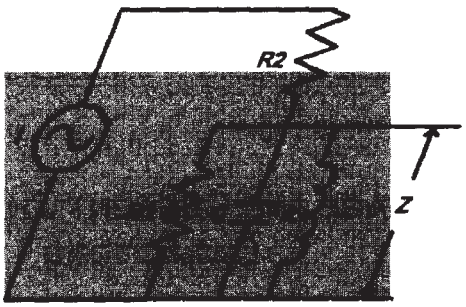
Figure 3-13 Hydraulic analogs for inductance and capacitance connected in parallel.

If the conditions are just right, inertia from the paddle wheel will cause water to continue flowing up into the lower storage tank. This will continue until the pressure in the tank gets strong enough to slow, and then stop, the paddle wheel. But when the paddle wheel finally stops, there will be considerable pressure in the tank. This will cause the paddle wheel to start turning in the opposite direction, moving water from the lower storage tank to the upper storage tank. This will continue until the pressure in the upper storage tank becomes strong enough to slow, and then stop, the paddle wheel. By that time, the pressure in the upper tank will be strong enough to start the paddle wheel turning in the opposite direction again, moving water from the upper storage tank to the lower one.

In the absence of friction (resistance), and if the tanks and paddle wheel are sized correctly, this can go on indefinitely! It requires no outside force or power to keep it going once it starts. (Of course, in the real world there is always friction, or resistance, but electronic circuits can be designed where these oscillations can be very large and strong.) If one component is very much larger or very much smaller than the other, then this oscillation will die out very quickly. But if the components

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happen to be sized just right, oscillations can continue for a very long time. This very special set of conditions is called *resonance*, and as we will see, it is a very important concept.



4 VOLTAGE AND CURRENT CHANGES AND TIME CONSTANTS

VOLTAGE AND CURRENT CHANGES THROUGH RESISTORS

If current through a resistor changes, voltage across the resistor changes immediately. There are no time lags for voltage or for current through resistors. Therefore, the *impedance* that a resistor offers to the flow of current is solely a function of the size of the resistor. Frequency does not play a role. This is one of the biggest distinctions between resistors and the other two types of components.

If we plot the relationship for either voltage or current for a resistor over time, it is a straight line. For example, Figure 4-1 plots current through time for a fixed voltage applied across three different resistors. I do that here simply as a comparison against plots that will follow for capacitors and inductors.

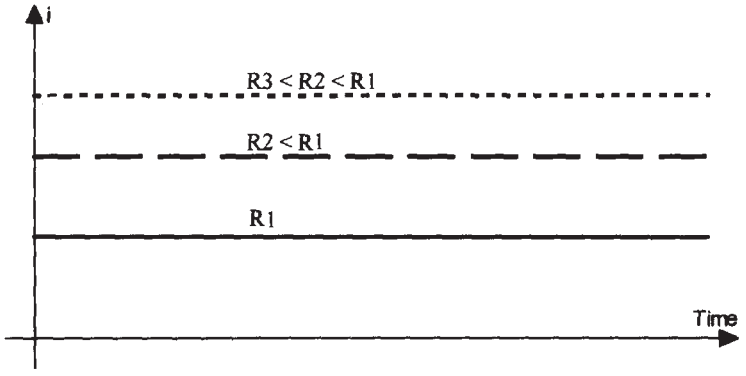


Figure 4-1 Current through a resistor through time.

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VOLTAGE AND CURRENT CHANGES THROUGH CAPACITORS

Earlier, we defined capacitance by saying that a one-farad capacitor with one coulomb of charge stored on its plates will have one volt across it. Therefore, the voltage across a one-farad capacitor would rise one volt for each additional coulomb of charge flowing onto its plates. It follows that the voltage across a 2.0-farad capacitor would rise only half as much (.5 volts) for the same flow of charge. So we are talking about a change of charge producing a change of voltage for any given value of capacitance. Using the symbol, Δ, for change, we can state that

$$\Delta V = \Delta Coulombs/C$$

Since current is the flow of electrons (coulombs) during some measure of time, we can state that:

$$i = \Delta Coulombs/\Delta t(ime), \text{ or}$$
$$\Delta Coulombs = i \times \Delta t$$

Therefore, the voltage across a capacitor is related to current and to capacitance according to the fundamental relationship shown in Equation 4-1:

$$\Delta V = i \times \Delta t/C, \text{ or} \qquad [4\text{-}1]$$
$$i = C \times \Delta V/\Delta t, \text{ or}$$
$$C = i \times \Delta t/\Delta V$$

Expressed in more conventional (calculus) terms, substituting d for Δ, the relationships are shown in Equation 4-2:

$$dV = i \times dt/C, \text{ or} \qquad [4\text{-}2]$$
$$i = C \times dV/dt, \text{ or}$$
$$C = i \times dt/dV$$

Thus, the voltage buildup across a capacitor depends (a) on the value of capacitance (larger capacitors will build up voltage more slowly) and (b) how large a current flows for (c) how long a time. This actually is fairly intuitive. Larger capacitors build up voltage more slowly. Larger currents charge capacitors faster. Even small currents will charge up a capacitor over time. Figure 4-2 illustrates how the voltage will build up on a capacitor as a function of time and current level. If we are

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