

pulling current out of a capacitor, the same relationship exists. The change (decrease) of voltage will be a function of how much current we drain off for how much time for any given value of capacitance.

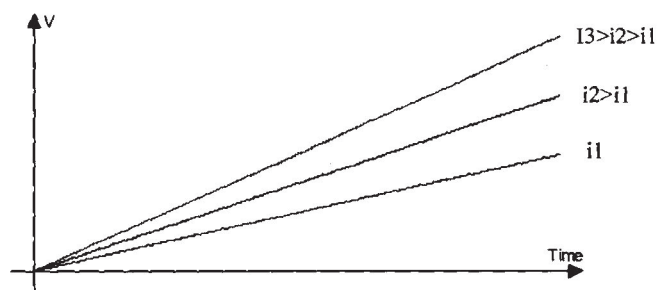


**Figure 4-2** Curves showing voltage across a capacitor as a function of time for three different current levels.

Note that for any given level of current,  $i$ , even very large levels of current, the change in voltage is small for *small* periods of time. This illustrates the fact that voltage lags current, or, alternatively, current leads voltage with capacitors.

As an example, suppose we could charge a capacitor with a constant 10 mA of current for 1.0 microsecond. Table 4-1 illustrates the results for three values of capacitance. There will be almost no voltage built up across a relatively large (1.0  $\mu\text{F}$ ) capacitor, but almost 10 volts will build up across a .001  $\mu\text{F}$  capacitor in the same time with the same current. This is one indication of why we want to use relatively larger capacitors as filter capacitors in power supplies.

| Suppose we charge a capacitor with 10 mA for 1 $\mu\text{s}$    |                                  |                   |      |
|-----------------------------------------------------------------|----------------------------------|-------------------|------|
| $\Delta V = i \times \Delta t / C$ $= 10(10^{-3})(10^{-6}) / C$ |                                  |                   |      |
| if                                                              | $C = 1 \mu\text{F} = 10^{-6}$    | $\Delta V = .01$  | volt |
|                                                                 | $C = .01 \mu\text{F} = 10^{-8}$  | $\Delta V = 1.0$  |      |
|                                                                 | $C = .001 \mu\text{F} = 10^{-9}$ | $\Delta V = 10.0$ |      |

**Table 4-1** Voltage buildup on a capacitor depends on current, time, and capacitance value.

In a typical bypass capacitor application, we are both charging and discharging the capacitor, depending on the switching cycle of the device being bypassed. The desire is to hold the voltage as constant as possible. It would follow, therefore, that larger capacitors would be better for bypass applications. Yet we often use smaller capacitor values in bypass applications. We will learn later that, whereas larger values appear to be preferable, lead inductance associated with bypass capacitors has some impact when selecting the appropriate values for bypass capacitors. It turns out that we sometimes compromise between capacitor value and the magnitude of lead (and connection) inductance.

## VOLTAGE AND CURRENT CHANGES THROUGH INDUCTORS

It is the very nature of inductance that a constant current flow through an ideal inductor has no effect (except that it does generate a constant magnetic field around the inductor). But if we *change* the flow of current, then the magnetic field around the inductor changes and, as a consequence, a voltage is generated across the inductor. So, voltage across an inductor is a function of the *change* ( $\Delta$ ) in current flow. For a given change in current, the voltage generated is higher for larger values of inductance. And, the voltage level is higher if the current flow changes quickly than it is if the current flow changes more slowly.

This relationship is captured in Equation 4-3:

$$\begin{aligned} V &= L \times \Delta i / \Delta t, \text{ or} \\ L &= V \times \Delta t / \Delta i, \text{ or} \\ \Delta i &= V \times \Delta t / L \end{aligned} \quad [4-3]$$

Expressed in more conventional (calculus) terms, substituting  $d$  for  $\Delta$ , these relationships become those shown in Equation 4-4:

$$\begin{aligned} V &= L \times di/dt, \text{ or} \\ L &= V \times dt/di, \text{ or} \\ di &= V \times dt/L \end{aligned} \quad [4-4]$$

If we apply a sudden voltage across an inductor, no current will flow during the first instant of time. (Refer to Appendix B for a thorough discussion of why in-

ductors do what they do, i.e., induct.) As time goes by, more current will flow (see Figure 4-3). A higher voltage will cause the magnitude of the current to increase more quickly, and a larger inductor will cause the magnitude of the current to increase more slowly. But we have to change voltage *first* before current changes. As a result, we say that voltage leads current for inductors, or, alternatively, current lags voltage.

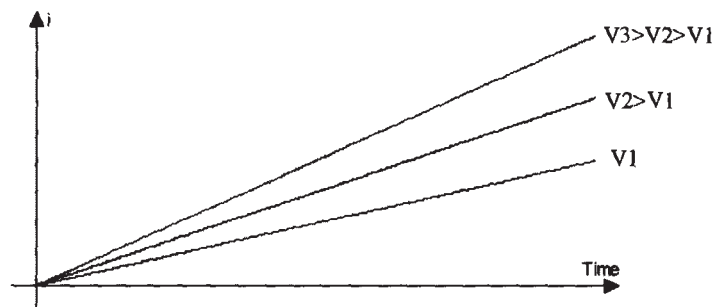


**Figure 4-3** Curves showing current through an inductor through time for three different voltage levels.

One thing that is sometimes difficult for students to grasp initially is that, for a given voltage, current will increase linearly (as a function of time) forever! Obviously, most voltage sources can't supply infinite current, so the relationship shown in Figure 4-3 works for current levels that a source *can* supply and still hold voltage constant. In real-world applications, there comes a point where the source can no longer (both) supply increasing current and hold the voltage constant at the same time. Typically, the voltage begins to fall. This causes a reduction in the increase of current flow, and pretty soon a level of stability is reached at some level of current with no voltage across the inductor.

Note again that the instantaneous voltage developed across an inductor is given by Equation 4-4:

$$V = L di/dt$$

Table 4-2 shows why the  $di/dt$  term can be so important. If we change current by one amp in one second, the rate is equal to 1 amp/second. But if we change the current by only one mA over just one ns, the instantaneous rate of change is

1,000,000 ( $10^6$ ) amp/seconds. It is this effect that is at the heart of so many high-speed design noise problems.

| If di =       | And dt =         | Then di/dt = | Units      |
|---------------|------------------|--------------|------------|
| 1 amp         | 1 second         | 1            | amp/second |
| $10^{-3}$ amp | $10^{-3}$ second | 1            | amp/second |
| $10^{-6}$ amp | $10^{-3}$ second | $10^{-3}$    | amp/second |
| $10^{-6}$ amp | $10^{-6}$ second | 1            | amp/second |
| $10^{-3}$ amp | $10^{-9}$ second | $10^6$       | amp/second |

**Table 4-2** The di/dt term can become significant when the dt term is very small.

For example, suppose we have a lead inductance of 10 nH ( $10^{-8}$ ) and we switch 10 mA ( $10^{-2}$ ) of current through it. If we switch the current over 1.0 usec ( $10^{-6}$ ), the instantaneous voltage generated across the lead inductance will be:

$$V = Ldi/dt = (10^{-8})(10^{-2})/(10^{-6}) = 10^{-4} = 0.1 \text{ mV}$$

But, under identical conditions, if we simply switch the current faster, at say 1.0 ns ( $10^{-9}$ ) then the induced voltage will be:

$$V = Ldi/dt = (10^{-8})(10^{-2})/(10^{-9}) = 10^{-1} = 100 \text{ mV}$$

This latter voltage, 1,000 times higher than in the former case, might cause a noise problem, whereas the former case probably did not. Yet the *only* difference in these two illustrations is the change in time (rise time).

Some people have been unexpectedly “burned” when their integrated circuit (IC) suppliers “improved” their devices so that they switched more rapidly. Switching current rise times (di/dt) that were not a problem before suddenly become a problem, causing noise problems to occur where they didn’t before. If you are not aware that your supplier has “improved” your circuits, or if you change suppliers of a particular IC and the new source switches faster, you might have a new problem and not have a clue how it happened.

## SOME INTERESTING INDUCTIVE CIRCUIT DYNAMICS

Figure 4-4 illustrates a circuit design characteristic that has caught more than a few engineers by surprise, much to their expense. The illustration shows a 10-volt source energizing an electromagnetic relay through a 100- $\Omega$  resistor. The transistor switch, Q, activates the relay. This is a very common type of circuit that is found, for example, in a variety of relay switching, scoreboard, and display applications. As shown, when the relay is activated, 100 mA ( $10\text{V}/100\ \Omega$ ) will flow through the inductive coil of the relay.

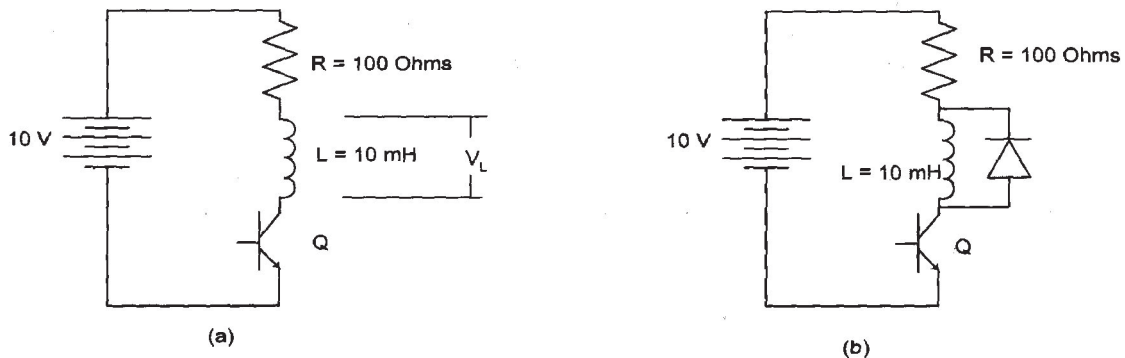


**Figure 4-4** Reverse voltage protection diode (b) can protect a switching transistor (Q) from damaging voltage transients.

Now, let's turn off the relay in Figure 4-4a by turning off the transistor switch. And let's say the switch turns off in one microsecond. The inductive action (recall the inertia analogy) of the inductive relay coil will try to keep the current flowing. What happens is the magnetic field around the coil starts to collapse. This will generate a voltage, which will appear across the transistor. The instantaneous magnitude of the voltage will be, from Equation 4-4:

$$V = L di/dt$$

In this illustration,  $L$  is 10 mH ( $10^{-2}$ ),  $di$  (the change in current) is 100 mA ( $10^{-1}$ ) and  $dt$  (the change in time) is one microsecond ( $10^{-6}$ ). So, the instantaneous voltage generated at the coil is

$$V = (10^{-2})(10^{-1})/(10^{-6}) = 1,000 \text{ V.}$$

The polarity of this voltage is positive at the junction of the collector of the transistor and the coil and negative at the other end of the coil. That is, the circuit is attempting to *maintain* the flow of current in the same direction as it was flowing before the transistor turned off.

This voltage exists only for a very short time (on the order of a microsecond), and if you saw it on a scope at all, it would appear as a very short voltage spike. But this voltage appears across the transistor. Transistors tend to be more tolerant of current spikes than voltage spikes (the current here is only 100 mA). After all, voltage spikes as small as tens of volts can puncture through the doped regions of a transistor, ruining it. More perversely, voltage spikes sometimes only weaken a transistor, so that the device may not fail until the voltage spike has occurred a large number of times (i.e., after the product has been in the field for a while).

This design flaw is, unfortunately, more common than you would think. If the circuit designer has forgotten (or maybe never learned) about inductive effects, it may go undiagnosed for a long time.

The solution is quite simple. Place a diode across the coil as shown in Figure 4-4b. This is called a *reverse voltage* diode in that it provides reverse voltage protection for the transistor. The diode provides a path for the current that the inductor tries to keep flowing, so that a voltage never has a chance to build up. The maximum current is only the 100 mA flowing through the coil, so the diode can be a fairly routine one (no special requirements).

This fairly routine circuit illustrates how destructive inductive effects ( $L \times di/dt$ ) can be in a design, and why an understanding of inductors and their effects is important for a circuit designer.

There is another fairly common circuit that depends on this same type of inductive effect for operation. It is the old automotive ignition system shown in Figure 4-5. When the points are closed, current flows from the 12-volt car battery through the coil. At the right moment, the points open and the current stops flowing, causing the magnetic field around the coil to collapse. The automotive ignition coil is also a transformer, so the collapsing field around the entire coil causes a very high voltage to appear across the spark plug. A high voltage also appears across the points, which can cause them to burn out. That is why older ignition systems had

“condensers” (really capacitors) around them to prevent the points from arcing and burning. Modern cars still have coils and they generate spark across plugs in much the same way. But the points have been replaced by improved electronic switching, which is (a) faster (shorter  $dt$ ), so higher spark voltages can be generated, (b) more reliable, and (c) requires less maintenance (no mechanical points to replace).

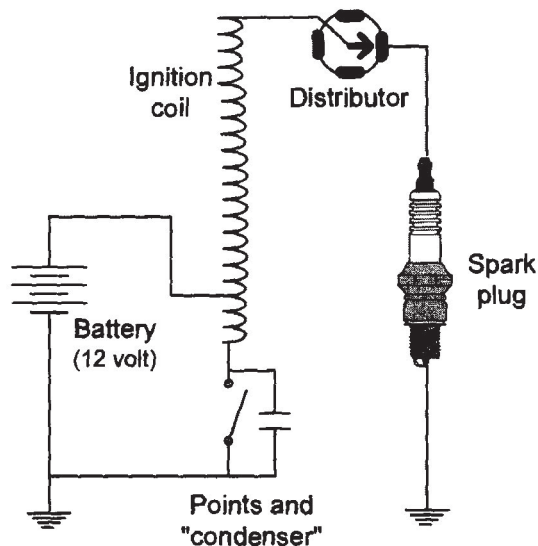


**Figure 4-5** Early automotive ignition systems used points in the spark generation system.

## TIME CONSTANTS

Let's look at Figure 4-6. Recall from earlier that when the switch connects the resistor to the battery, current will immediately begin to flow onto the plates of the capacitor. The initial magnitude of the current (assuming the capacitor is initially uncharged) is  $V/R$ . Voltage across the capacitor will begin to increase according to Equation 4-1:

$$\Delta V = i \times \Delta t / C$$

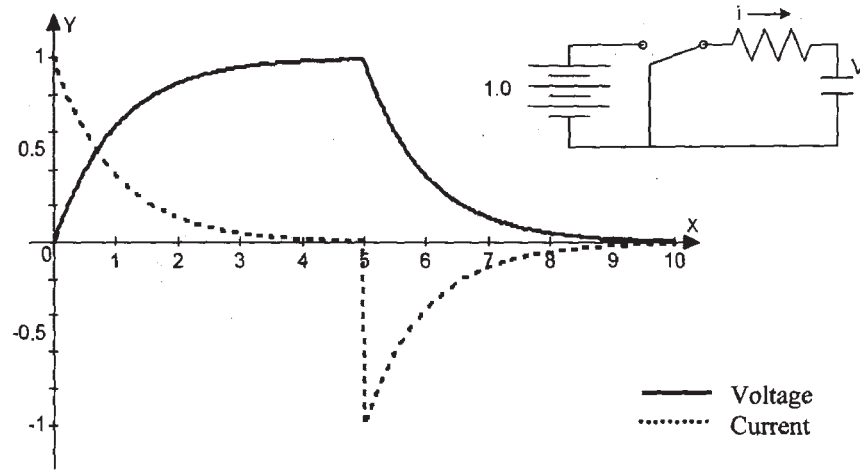


**Figure 4-6** Charging and discharge curves associated with RC circuits.

As the voltage across the capacitor increases, the voltage across the resistor must decrease, since the battery voltage remains unchanged. If the voltage across the resistor decreases, the current through the resistor must also decrease. Thus, the current starts out at the value  $V/R$  but decreases steadily through time until it reaches zero (when the capacitor is fully charged). Similarly, the voltage across the resistor starts out at  $V$ , but declines steadily through time until it reaches zero (when the current stops flowing because the capacitor is fully charged). Finally, the voltage across the capacitor starts out at zero, but grows steadily until it reaches the value  $V$ .

The curves these voltages and currents follow are shown in Figure 4-6. For basic RC circuits such as the one shown, they follow well-defined, predictable shapes. When the capacitor is charging, the equations for the curves are given by Equation 4-5 and 4-6.

Voltage across the capacitor:

$$VC(t) = V(1 - e^{-t/RC}) \quad [4-5]$$

Current through the capacitor:

$$i(t) = (V/R)e^{-t/RC} \quad [4-6]$$

The value  $e$  is a constant ( $e = 2.71828\dots$ ) and it is the base of the natural logarithm (see Appendix C). These types of expressions are called *exponential equations* and they are very important in electrical engineering. If  $t$  equals zero then the exponent of  $e$  also equals zero, regardless of the values of  $R$  and  $C$ . Any value raised to the zero power has a value of one (e.g.,  $e^0 = 1$ ). So the initial conditions of these equations are

$$VC(t = 0) = V(1 - e^{-0}) = 0$$

$$i(t = 0) = (V/R)e^{-0} = V/R$$

as we would expect. If any value is raised to a very large negative value (e.g.  $e^{-n}$ , where  $n \cong \text{infinity}$ ) its value becomes zero. So, for very large values of  $t$  (after the circuit stabilizes), the equations become:

$$VC(t) = V(1 - 0) = V$$

$$i(t) = (V/R)0 = 0$$

We could, if we wanted to, generalize these equations in units of  $RC$ . That is, we can look at the equations at the points where  $t = 1RC$ ,  $t = 2RC$ , ...,  $t = nRC$ . To do so, let's think of a unit of time called  $t_c$  where  $t_c = 1$  if  $t = 1RC$ ,  $t_c = 2$  if  $t = 2RC$ , and so on. If we do that, the preceding equations become generalized as Equations 4-7 and 4-8.

$$VC(t) = V(1 - e^{-t_c}) \quad [4-7]$$

$$i(t) = (V/R)e^{-t_c} \quad [4-8]$$

Interestingly enough, these equations now can apply to *any* circuit with a resistor and a capacitor of *any* arbitrary values, as long as we recognize that  $t_c = t/RC$ . The expressions  $e^{-t_c}$  and  $(1 - e^{-t_c})$  are graphed in Figure 4-7. They illustrate some interesting characteristics. For example, at a time equal to  $1t_c$ , the curves are 63% of the way toward their final value. At a time equal to  $3t_c$  they are 95% of the way to their final value. After a time of  $5t_c$  they are virtually at their final value (for all practical purposes).

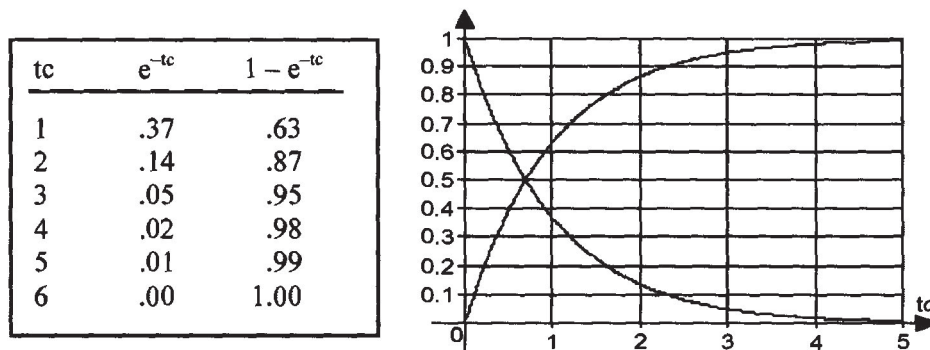


**Figure 4-7** Exponential curves expressed in “time constants.”

The value  $RC$  (the value of  $R$  times the value of  $C$ ) is called an *RC time constant* and is an important concept. The response of *any* circuit with a resistance and a capacitance will follow the shape of an exponential curve and will reach 63% of its final value in a time equal to  $R \times C$ . It will reach 95% of its final value within three  $RC$  time constants. For all practical purposes it has stabilized within 5 time constants.

This has both good and bad implications. For example, a stable resistor combined with a stable capacitor can be (and is routinely) used as a very accurate timing circuit. If we combine a 1.0-K resistor with a .001-uF capacitor in a timing circuit, the voltage across the capacitor will reach .95 V in  $3RC$  or  $3(10^3)(10^{-9}) = 3 \mu\text{sec}$ . If the resistor and capacitor are good quality and stable, this will *always* be true for the life of the product.  $RC$  timing circuits are routinely used in products ranging from home appliances to cameras to audio sound systems to industrial systems to spacecraft.

On the other hand, the relationships also show that a circuit requires some time to respond. AC circuits usually cannot respond instantaneously. If we have an IC that is switching an output voltage through a  $50\text{-}\Omega$  output resistance into another IC with 20-picofarads (pF) input capacitance, it will take  $3RC = 3(50)(20)(10^{-12}) = 3 \text{ ns}$  to reach 95% of its value at the input of the receiving IC. That delay may not be fast enough for certain high-speed circuits.

An inductive circuit works in a very similar way. Figure 4-8 shows a resistor and an inductor in series. When the switch first closes, no current flows because of the inductive inertia of the inductor. If there is no current, there is no voltage

across the resistor, so all the voltage appears across the inductor. As soon as some current flows, voltage is developed across the resistor, so the voltage across the inductor begins to decrease. The shapes of the curves for this circuit are identical to the shapes of the curves for the RC circuit, except:

1. The voltage across the inductor starts out high and declines, whereas the current through the inductor starts out low and increases,
2. The voltage across the capacitor starts out low and increases, whereas the current through the capacitor starts out high and decreases.

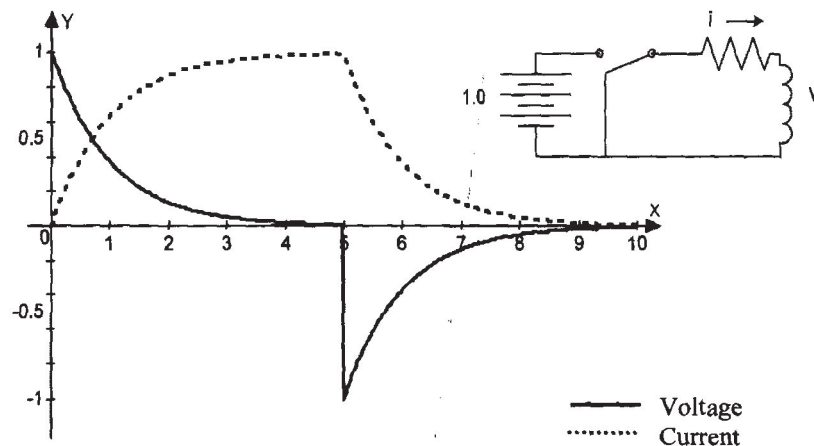


**Figure 4-8** Charging and discharge curves for RL circuits.

This is another example in which capacitors and inductors are nearly equal but opposite in their effects.

The formulas for the initial charging curves for the inductor are similar to those for an RC circuit and are given by Equations 4-9 and 4-10:

$$i(t) = (V/R)(1 - e^{-tR/L}) \quad [4-9]$$

$$V_L(t) = (V)e^{-tR/L} \quad [4-10]$$

Just as we developed an explanation for an RC time constant earlier, we can think about an  $L/R$  time constant here. The circuit will reach 63% of stability in a length of time equal to  $L/R$  and 95% of stability in a length of time equal to  $3L/R$ .

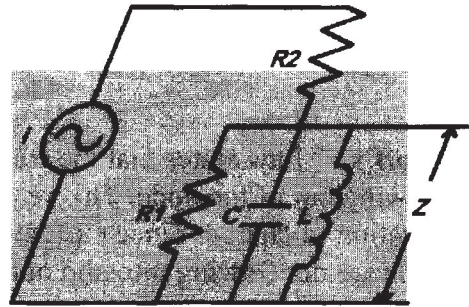
All of the strengths and weaknesses associated with an RC time constant are also true for an L/R time constant. We usually do not use inductors in timing circuits, however, because they tend to be large, bulky, and (often) not as stable as capacitors. In general, RC circuits perform better as timing circuits. (Note: We will leave the discussion about LC timing and frequency control for later.)

## A NOTE ON CHARGE AND DISCHARGE EQUATIONS

The discussion here has focused on charging circuits, when we switch the circuit to the battery. The description of what happens when we switch the circuit back to discharge the capacitor or inductor follows the same logic. The charging and discharge equations are summarized in Table 4-3. Note that the capacitor discharge current flows in a direction opposite to that of the charging current; hence the negative sign. When the magnetic field around the inductor begins to collapse, it tries to maintain the current flow around the loop. The voltage across the resistor (initially) still is  $V$ . So the voltage across the inductor reverses (hence the negative sign) to try to maintain the current flow. That also helps explain why the reverse voltage protection diode is oriented the way it is.

**Table 4-3** Summary of RC and RL charging and discharge equations.

| Circuit | Parameter | Charging                                   | Discharging                                 |
|---------|-----------|--------------------------------------------|---------------------------------------------|
| RC      | VC        | $V_c(t) = V(1 - e^{-t/RC})$                | $V_c(t) = Ve^{-t/RC}$                       |
|         | i         | $i(t) = \left(\frac{V}{R}\right)e^{-t/RC}$ | $i(t) = -\left(\frac{V}{R}\right)e^{-t/RC}$ |
| RL      | VL        | $V_L(t) = Ve^{-tR/L}$                      | $V_L(t) = -Ve^{-tR/L}$                      |
|         | i         | $i(t) = \frac{V}{R}(1 - e^{-tR/L})$        | $i(t) = \frac{V}{R}(e^{-tR/L})$             |



## 5 RESISTANCE

People who study electronic circuit analysis use a lot of calculus. Other than the basic  $di/dt$  term I've already introduced, I promise to stay as far away from calculus as I can in this book, but we can't stay away from equations. The whole point of selecting component values in circuits is to obtain a desired result. And the primary way you select component values is to perform calculations.

This book is not intended to show you how to make all the calculations. If you want to know that, you should go get your degree. But this book will cover some of the more fundamental calculations related to electronic circuits. And it will suggest what some of the more difficult calculations are, without actually trying to teach you how to do them. The objective is to give you enough familiarity with the topic that you can do some of the more basic calculations yourself, and can understand some of the more complex ones when someone does them for you.

### KIRCHHOFF'S LAWS

Gustav Robert Kirchhoff is credited with (among many other things) two laws related to how currents flow in circuits. These two laws are almost as important to the field of electronics as Ohm's Law is. Consider Figure 5-1a and Figure 5-1b.

Figure 5-1a illustrates the case of two components (they don't have to be resistors) connected in parallel and driven by a voltage source (which could be DC or AC). Current will flow from the source and divide between the two (or more) components. The point where the voltage source and the two components come together is called a *node*. (This is equivalent to a net in PCB design terms.)

Kirchhoff's 1<sup>st</sup> Law states that the current flowing into a node must equal the current flowing out of the node. This really makes intuitive sense; it is a statement of conservation of charge. Since current is the flow of electrons, if the same number of electrons are not flowing into and out of a node, where do the excess ones come from or go to?

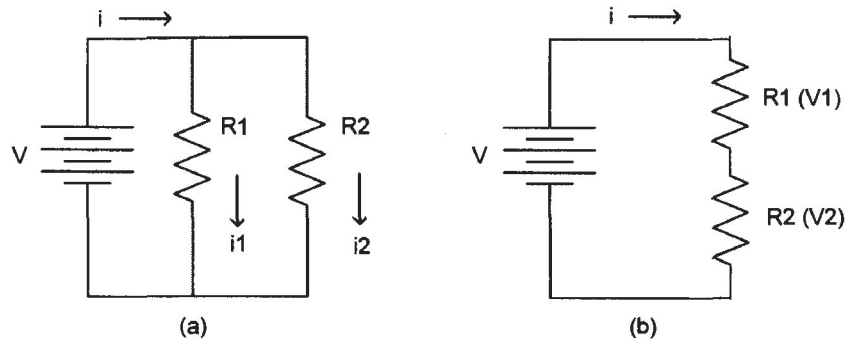


**Figure 5-1** Two resistors connected in parallel (a) and in series (b).

Figure 5-1b illustrates two components connected in series and driven by a voltage source (AC or DC). It should be intuitively clear that the voltage across the series components must equal the voltage source. If not, how is the extra voltage generated and where does it go? Kirchhoff's 2<sup>nd</sup> Law states this in a form that students sometimes find a little awkward at first. The law states that the sum of the voltages around a loop must be zero. This is a conservation of energy law. Consider how to explain the case where they might not sum to zero.

The way to interpret Kirchhoff's 2<sup>nd</sup> Law is to start at the top and move clockwise around the loop. The first component has a voltage drop across it of  $V1$ . The second component has a voltage drop of  $V2$ . So far, the sum is  $V1 + V2$ . Continuing around the loop we come to the voltage source. The voltage at the voltage source is *minus*  $V$ . It is minus because we are approaching it (measuring it) from the bottom to the top as we continue to travel around the loop. So now we have Equation 5-1:

$$V1 + V2 + (-V) = 0, \text{ or} \quad [5-1]$$

$$V1 + V2 = V$$

The trick to applying Kirchhoff's 2<sup>nd</sup> Law is to remember that the direction you are traveling around the loop determines the sign of the voltages you are considering.

## SERIES RESISTORS

There are numerous times in circuits where we want to replace two or more resistors with an equivalent single resistor, or, occasionally, replace a single resistor with two of them. Thus, we think in terms of circuit equivalents. In this example, the question is, "What single resistor is the equivalent of two resistors connected in series?"

Most of us already know the answer. But the point is not in knowing the answer; the point is knowing how to do the analysis to get the answer, because later on we will ask the same question about equivalent capacitors and equivalent inductors. Those analyses will be very similar to the analysis we go through here. Although many people can talk about equivalent resistive circuits, they really don't understand similar equivalent circuits when we get to the other types of components.

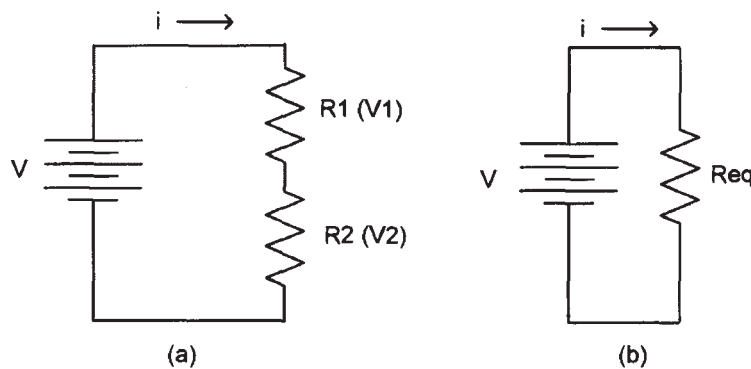


**Figure 5-2** Equivalent circuit (b) of two resistors connected in series (a).

Figure 5-2a illustrates the case in which we are going to determine the single resistance,  $R_{eq}$ , that is the equivalent of the two resistors,  $R1$  and  $R2$ , in series. First, recall from Kirchhoff's 2<sup>nd</sup> law that the sum of the voltages around a loop

must equal zero. If  $V_1$  is the voltage across  $R_1$  and  $V_2$  is the voltage across  $R_2$ , then if the sum of the voltages around the loop (from Equation 5-1) must equal zero:

$$V_1 + V_2 - V = 0, \text{ or}$$

$$V = V_1 + V_2$$

Also, if  $V_{eq}$  is the voltage across the equivalent resistor, then the same analysis leads to Equation 5-2:

$$V_{eq} - V = 0, \text{ or}$$

$$V = V_{eq} \quad [5-2]$$

Remember, as we go around the loop clockwise, we see the negative side of the battery first. So, going around the loop, the voltage across the battery is  $-V$ .

Remember, also, that the current around a loop is constant at all points in the loop. Let the current here be designated as  $i$ . The voltage across each resistor is (from Ohm's Law)  $V = iR$ . So we can write Equations 5-1 and 5-2, respectively, as Equation 5-3:

$$V = iR_1 + iR_2, \text{ and}$$

$$V = iR_{eq} \quad [5-3]$$

This leads to:

$$iR_{eq} = iR_1 + iR_2$$

and dividing through by  $i$ , to Equation 5-4:

$$R_{eq} = R_1 + R_2 \quad [5-4]$$

This might seem to be a lot of trouble to get to something that is somewhat obvious. But before we make that judgment, let's do a similar thing to parallel resistors. Then, wait until you see how this applies later to the combination of capacitors.

## PARALLEL RESISTORS

Figure 5-3 shows two resistors connected in parallel. Now the question is, “What value of resistor,  $R_{eq}$ , is equivalent to the two resistors,  $R_1$  and  $R_2$ , connected in parallel?” Already the situation is going to get a little more interesting.

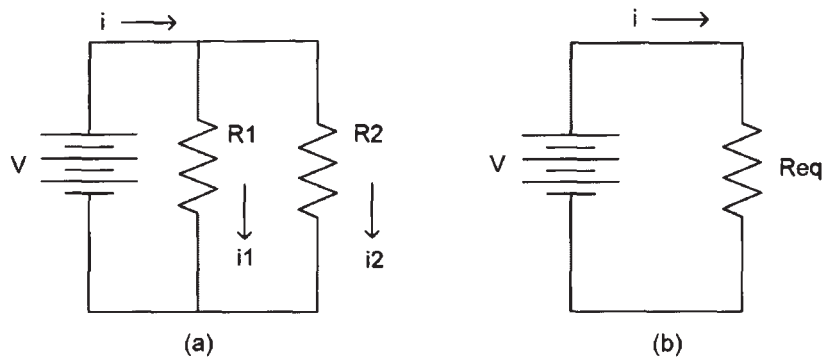


**Figure 5-3** Equivalent circuit (b) of two resistors connected in parallel (a).

Kirchhoff's 1<sup>st</sup> Law states that the current going into a node has to equal the current going out of a node. This example is actually pretty simple, with just a single node at the top and another at the bottom. The current going into the node at the top is  $i$ . The current through  $R_1$  is  $i_1$  and the current through  $R_2$  is  $i_2$ . By Kirchhoff's 1<sup>st</sup> Law we can write Equation 5-5:

$$i = i_1 + i_2 \quad [5-5]$$

In this example, there are three loops: (a) the loop from the battery through  $R_1$ , (b) the loop from the battery through  $R_2$ , and (c) the loop between the two resistors,  $R_1$  and  $R_2$ . By Kirchhoff's 2<sup>nd</sup> Law, the voltages around a loop must sum to zero. So we can write the following three equations to illustrate this:

$$i_1 R_1 - V = 0 \quad [5-6]$$

$$i_2 R_2 - V = 0 \quad [5-7]$$

$$i_2 R_2 - i_1 R_1 = 0$$

These are not really three different equations. Any one of these is a necessary result of the other two. We actually have only two independent equations here, and the third one is redundant.

We can write Equations 5-6, 5-7, and also 5-3 as follows:

$$\begin{aligned}i_1 &= V / R_1 \\i_2 &= V / R_2 \\i &= V / R_{eq}\end{aligned}$$

Then, substituting into Equation 5-5

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2}$$

And, dividing through by V we get Equation 5-8:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad [5-8]$$

which can alternatively be expressed as Equation 5-9:

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2} \quad [5-9]$$

The form of Equation 5-8 is important. By extension, it can be shown that if there are an arbitrary number of (n) resistors in parallel, the equivalent resistance is found from Equation 5-10:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} \quad [5-10]$$

which can alternatively be expressed as Equation 5-11:

$$R_{eq} = \frac{1}{\frac{1}{R1} + \frac{1}{R2} + \dots + \frac{1}{Rn}} \quad [5-11]$$

These are relatively easy forms to use if it is ever necessary to calculate the equivalent resistance of several resistors in parallel.

It is interesting to note what happens in a few special cases. Suppose R1 is zero. The equivalent series resistance is still R2, but the equivalent *parallel* resistance is zero. If R1 is infinite, the equivalent *series* resistance is infinite, but the equivalent parallel resistance is R2. Thus, it follows that:

1. In the case of resistors in series, the resistance is never less than the largest resistor.
2. In the case of parallel resistors, the resistance is never more than the smallest resistance.

Also, if there are n resistors in parallel, and they are all the same value (R), the equivalent resistance is

$$R_{eq} = R/n$$

This is very handy to remember when we have multiple loads, for example, at the end of a trace.

## VOLTAGE DIVIDERS

One of the more common resistive circuit configurations is the voltage divider (Figure 5-4). Designers should be able to recognize this particular configuration and the principle behind it.

A voltage source, Vout (from a previous stage in the circuit) is connected to two resistors in series. Recognize that the current is:

$$i = V_{out} / (R1 + R2)$$

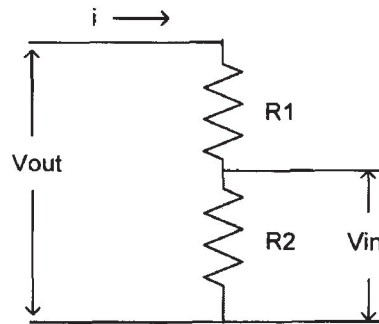


**Figure 5-4** Classic voltage divider circuit.

and therefore we obtain Equation 5-12:

$$V_{out} = i(R1 + R2) \quad [5-12]$$

The input (to the next stage) voltage,  $V_{in}$ , is given by Equation 5-13:

$$V_{in} = iR2 \quad [5-13]$$

So, the ratio of  $V_{in}$  to  $V_{out}$  is given by Equation 5-14:

$$V_{in}/V_{out} = R2/(R1 + R2) \quad [5-14]$$

Or, put more simply, the ratio of the input voltage (to the next stage) to the output voltage (from the previous stage) is the same as the ratio of the resistance below the input to the total resistance.

Figures 5-5a and 5-5b illustrate two common voltage divider configurations. In Figure 5-5a the voltage input to the operational amplifier follows equation 5-14. Figure 5-5b illustrates a typical simple volume control application. In this case,  $V_{in}$  is  $V_{out} \times R1/R$ , where  $R1$  is the resistance between the wiper of the control and the end connected to ground.

This type of configuration, where the result is proportional simply to one resistance divided by a total resistance, is so common it would be helpful if a designer could readily recognize it.

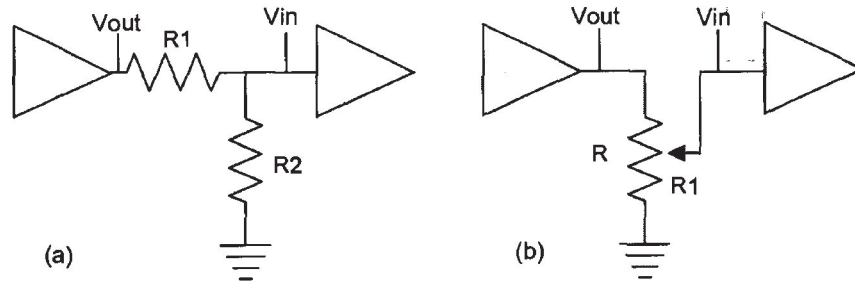


Figure 5-5 Two common voltage divider circuits.

## AMPLIFIER FEEDBACK AND GAIN

A more subtle type of circuit will be included here simply for thoroughness. It is a common circuit, but sometimes the configuration is subtle enough that it might not be immediately recognized.

In Figure 5-6,  $V_{1out}$  is a voltage signal being fed to the inverting input of an amplifier stage through resistor  $R1$ .  $V_{2out}$  is the output of the next amplifier stage.  $V_v$  is the voltage at the input of the amplifier, so  $V_{2out}$  is  $-V_v$  times the gain of the amplifier. Assuming the gain is very high, then it is true that

$$V_{2out} \gg -V_v$$

It is also true that with a high-gain amplifier, whose other input is tied to ground (zero volts),  $V_v$  will be very close to zero; so close in fact that we can assume it does equal zero for simplicity. Since there is typically no current into the high-impedance amplifier, and since the current into a node must equal the current out of it, the current through  $R1$  ( $i1$ ) must be equal to the current through  $R2$  ( $i2$ ). If  $V_v$  is (virtually) zero, then the following relationships must hold true:

$$\begin{aligned} i1 &= i2 \\ V_{1out}/R1 &= -V_{2out}/R2 \\ V_{2out}/V_{1out} &= -R2/R1 \\ V_{2out} &= -(R2/R1)V_{1out} \end{aligned} \quad [5-15]$$

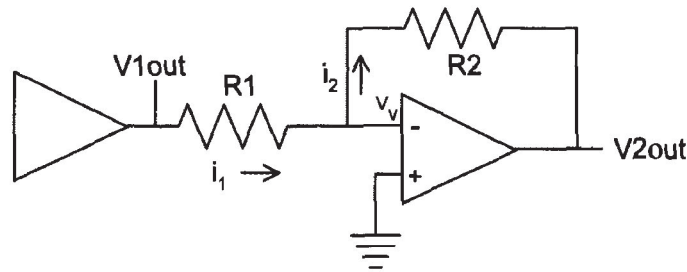


**Figure 5-6** The amplifier gain of the second stage is  $R2/R1$ .

This is a very common relationship in analog design, where the gain of a stage is simply the (negative) ratio of the feedback resistor ( $R2$ ) to the input resistor ( $R1$ ). Sometimes, by looking at a schematic, it is not immediately obvious what a circuit designer has done. And there are many variations on this design concept. But when a circuit designer points out to you that the gain of a stage is simply the ratio of two resistances, you can now visualize why that is so.

## POWER

We intuitively know that if we short-circuit the terminals of a typical 9-volt battery found in small electronic appliances, not much happens. But if you have ever seen an accidental short-circuit of the terminals of a 12-volt car battery, the result can be severe. (If you haven't seen it, don't try it!) The difference is that the car battery can supply a lot more power than the little 9-volt battery can.

Power is defined as voltage times current. A car battery that supplies 12 volts at 60 amps to a starter motor is supplying 720 watts of power (almost one horsepower: one hp = 746 watts). A 100-watt light bulb requires almost 1.0 amp at 110 volts. A 15-amp circuit breaker in your home will allow 1,650 watts (at 110 volts) before it trips. An alkaline D-cell (1.5-V) battery is typically rated for 100 hours at 150 mA., meaning it can supply  $1.5 \times .150$  or almost .25 watt of power for that 100 hours.

Looking at the basic circuit in Figure 5-7, and remembering Ohm's Law ( $V = iR$ ), we can express the formula for power these three ways:

$$\begin{aligned}\text{Power} &= Vi \\ &= V^2 / R \\ &= i^2 R\end{aligned}\tag{5-16}$$

These equations reflect things we may intuitively recognize. For example, if we increase the voltage across a resistor, the resistor begins to get warmer. That reflects the increased power being dissipated within it ( $V^2/R$ ). If we increase the amount of current through a PCB trace, the temperature of the trace increases ( $i^2R$ ). If we put too much current through it, the trace can get so hot it melts. CMOS circuits run cooler than ECL circuits because, for the same voltage levels, they require much less current, and therefore much less power.

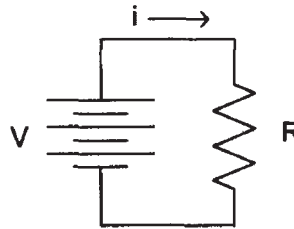


**Figure 5-7** Basic circuit for illustrating power.

It is important to note that in all these cases, the power (and therefore the temperature) goes up as the *square* of the voltage or current. Thus, a 100- $\Omega$  half-watt resistor can handle 7 volts at .07 amps. But a 1-watt (twice as large) 100- $\Omega$  resistor can only handle 10 volts at .1 amps.

Most of the time, PCB designers don't have to worry much about power. The exception can be in power supplies, and this can really be a problem if we are designing large motherboards that supply lots of power to numerous daughter boards. If we are concerned about power, it is usually a concern about the  $i^2R$  (pronounced "eye-squared-R") power dissipation in traces. A discussion about  $i^2R$  power capabilities of traces is outside the scope of this book, but the reader can consult several articles on the subject, including the IPC trace/temperature tables.<sup>1</sup> There is also a PCB trace temperature calculator available for download from UI-

<sup>1</sup>IPC-2221, *Generic Standard on Printed Board Design*, February 1998, Figure 6-4, p. 38.

traCAD's Web site at <http://www.ultracad.com/calc>. You are particularly encouraged to read the Help file that accompanies that calculator.

While the responsibility for power management usually belongs (first) to the circuit designer, board designers should understand the implications associated with  $i^2R$  losses and considerations on their boards and be alert to potential heating problems caused by high currents. The most common and effective ways to deal with heat range from (a) relocating parts (to avoid heat buildup at a localized area), (b) resizing parts (for example increasing the size of traces), and (c) improving cooling techniques (perhaps with fans, board orientation, heat sinks, or even metal "cores" within boards).

## EQUIVALENT CIRCUITS

So far, all the circuit examples we have looked at have been fairly simple, mostly involving a single power source and one or two resistors. What if we have a more complicated example like that illustrated in Figure 5-8? How would we go about analyzing the output voltage of that circuit?

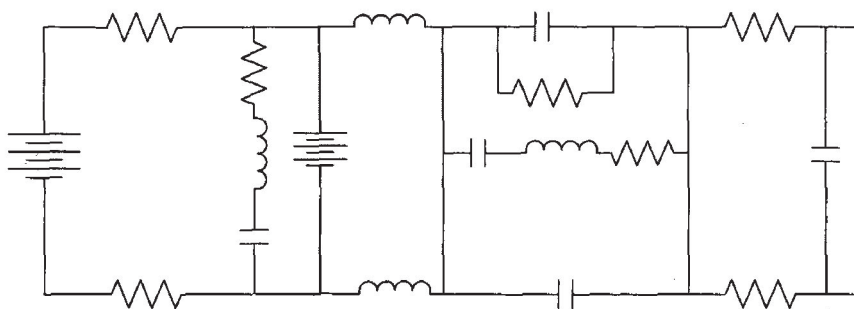


**Figure 5-8** This complex circuit would be difficult to analyze.

Well, it can be done. The process would be a sequential one of taking a couple of components at a time and finding their equivalents, and then reducing the circuit almost component by component until it becomes manageable.

There are two very interesting theorems in electronics that conceptually make this a very simple task, at least in an empirical (experimental) sense. They go like this:

Any circuit, such as the one shown in Figure 5-8, can be reduced to an equivalent circuit containing a *single* ideal power source and a *single* resistor, as shown in either Figure 5-9a or Figure 5-9b.

Figure 5-9a illustrates what is known as a Thevenin equivalent circuit. It consists of an ideal voltage source,  $V$ , with a resistor,  $R$ , in series with it. Figure 5-8 can be reduced to Figure 5-9a using the following strategy: The voltage,  $V$ , is the open circuit voltage that would be measured at the output (right) side of the network if there were no other connections made there. The resistor,  $R$ , is determined by dividing this voltage by the short-circuit current that would be measured if you shorted the terminals at the right of the network. Thus, the measured open-circuit voltage of Figure 5-9a is the same as that of Figure 5-8. Also, the measured short-circuit current of Figure 5-9a is the same as the short-circuit current of Figure 5-8. Now, if you connected any resistor to the output of the equivalent circuit represented in Figure 5-9a, the current through it, and the voltage across it, would be the same as if you connected that resistor to the output side of the circuit in Figure 5-8. This is the power and the beauty of Thevenin's Theorem.

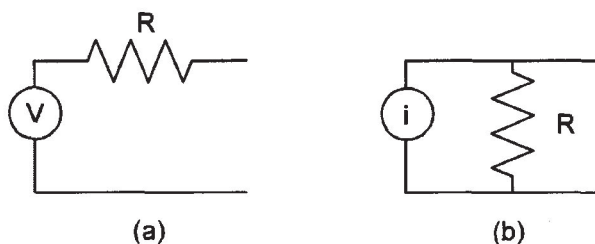


Figure 5-9 Thevenin (a) and Norton (b) equivalent circuits

Figure 5-9b illustrates a Norton equivalent circuit. It consists of an ideal current source,  $I$  (as opposed to the voltage source,  $V$ , in the Thevenin equivalent circuit), and a resistor,  $R$ , in parallel with the current source. In a manner similar to the preceding, the magnitude of the current source in Figure 5-9b is equal to the short-circuit current at the output in Figure 5-8. The resistor,  $R$ , is determined by dividing the open-circuit voltage at the output of Figure 5-8 by this short-circuit

current. As with the Thevenin circuit, if you attach any resistor at the output of the equivalent circuit of Figure 5-9b, the current through it, and the voltage across it, will be the same as if you added it to the output of the actual circuit of Figure 5-8.

So what? Does this have any practical value?

I picked up a National Semiconductor data sheet at random. It happened to be for a DS26F31C/DS26F31M Quad High Speed Differential Line Driver. It, like Figure 5-8, has a lot of stuff inside it. But it has a typical “high” voltage spec of 3.2 volts and an output short-circuit spec of 60 mA. If we divide the voltage by the current ( $3.2/.06$ ) we get  $53\ \Omega$ . If we want to see how this device will work in a circuit (at least while its voltage is high) we can simply substitute Figure 5-9a or Figure 5-9b in its place in the circuit, with  $R = 53.3$  and  $V = 3.2$  (Figure 5-9a) or  $i = .06$  (Figure 5-9b). We don’t *need* to know all the circuitry inside it. All we need to know is the open circuit voltage and the short-circuit current. Virtually all data sheets will provide this information. The data sheet will also provide other general guidelines, for example, for the output capability of a device. But using the implied output resistance is a way to determine circuit performance for, perhaps, configurations that are a little bit different than standard.

As an aside, the value, in this case, for  $R$ ,  $53\ \Omega$ , is for each side of the differential output for this particular device. The differential output pair of this device is designed to be terminated with a  $100\text{-}\Omega$  resistor. It is not an accident that this happens to be twice the value for  $R$  we just determined. The significance of this relates to transmission line considerations, which we will cover in great detail in Chapters 10 and 11. There is a very good reason why the implied value for  $R$  we just determined is  $53\ \Omega$  in this equivalent circuit, and not some other value.

The calculations we just made apply to the “high” output voltage. It might well be that the equivalent circuit will look different when the device is at its “low” state. That is one problem with analyzing the performance and effects of an active (semiconductor) circuit. Equivalent circuits are legitimate tools when ICs are in a “stable” state (at a “high” or a “low,” or at a “one” or a “zero”). They usually don’t apply during transient switching times between states. But we are usually concerned about different things then. At a stable state, we are concerned about voltage and current levels. During switching, we are usually concerned that the device can, in fact, *complete* the switching cycle in the appropriate time. We will see later that other device specifications (especially things like input capacitance and output rise times) are of interest during switching times.

## POWER CURVE

Linear ICs can often be looked at from the standpoint of their equivalent circuits. For example, Figure 5-9a could easily apply to the output stage of a transmitter or to an audio amplifier. When we add a load to such circuits (such as an antenna to a transmitter or speakers to an audio amplifier) it is interesting to see what happens to output power as a function of that load.

Referring to Figure 5-10, let  $V$  be 5 volts and  $R_s$  (source resistance) be  $50\ \Omega$ .  $R_L$  is the load resistance. The short-circuit output current, if we let  $R_L$  be zero, would be  $V/R_s = 5/50 = 100\text{ mA}$ . If we did short-circuit the outputs there would be 5 volts across, and 100 mA through the  $50\text{-}\Omega$  source resistor,  $R_s$ . Consequently, .5 watts would be dissipated in the resistor. It is important to note that this power is dissipated within the device. The power dissipated in the load outside the device would be  $i^2 R_L$ , but since  $R_L$  is zero (short circuit), there is no power dissipated in the load.

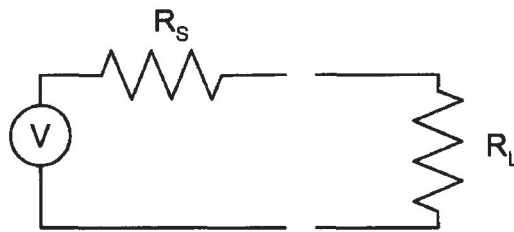


**Figure 5-10** Voltage source powering a load,  $R_L$ , through a source resistor,  $R_s$ .

If the load resistor,  $R_L$ , were  $50\ \Omega$ , then the output current would be:

$$i_{\text{out}} = V/(R_s + R_L) = 5/100 = 50\text{ mA}$$

There would be 2.5 volts across each resistor, and therefore .125 watt (2.5 volt x .05 amp) dissipated in each resistor. Now, the power dissipated within the device is only .125 watt, and the output power, the power in the output resistor, is also .125 watt. The total power has decreased, since the total resistance looking at the voltage source has increased, but the output power (the power dissipated in the load) has increased. Since power usually manifests itself as heat, the heat generated

within the device is much less, as we would expect. Short-circuits are usually hard on devices.

Now let  $R_L$  be  $100\ \Omega$ . The total current is now:

$$i_{\text{out}} = V/(R_s + R_L) = 5/150 = 33\ \text{mA}$$

There is now .055 watts dissipated within the device (in  $R_s$ ) and .11 watts in the load resistor, for a total power produced of .166 watts.

What has been illustrated here is a fundamental truth. Output power (the power dissipated in  $R_L$ ) is maximized when  $R_L$  equals  $R_s$  (Figure 5-11). *Total* power decreases as  $R_L$  increases, but the power in the *load* is a maximum when the load resistor is the same as the source resistor. This illustration has been with resistors for simplicity, but this relationship is true, in general, for any type of impedance. Output power is maximized when the load impedance equals the source impedance. When output power is maximized, an equal amount of power is dissipated within the device and in its load.

That is why we buy audio speakers to match the output impedance of our audio systems and why antennas are matched to transmitters. Even good CB radios have antenna-matching adjustments to help match the antenna load impedance to the output impedance of the transmitter. The reason is so that the available output power (the power dissipated in the load, i.e., in the speakers or the antenna) is maximized.

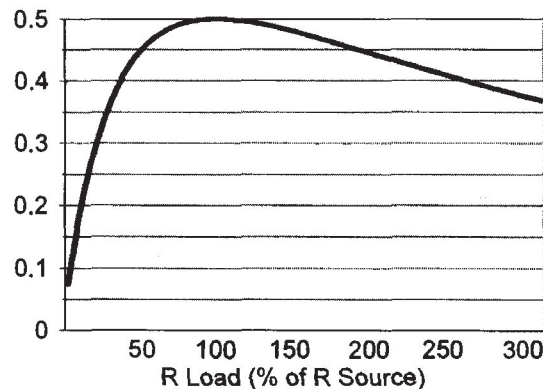


**Figure 5-11** Output power is a maximum when the load impedance equals the source impedance.

Do we always want to maximize power? Of course not. Sometimes we want to maximize signal sensitivity. Then we would design input stages with higher levels of impedance so they don't "load down" their source (but not too high, because this might increase noise sensitivity). Sometimes we have to design low-impedance outputs for a device so that can drive other devices with lower impedance inputs. With transmission lines, we want to match impedances in order to reduce reflections (not to maximize power). But, sometimes we do want to maximize power transfer between devices, and in this case doing so requires that we match the load impedance to the source impedance.

## POWER SOURCES

Up until now we have used pure, or ideal, energy sources in our examples. For example, Figure 5-12a shows a battery and a resistor. We introduced Ohm's Law showing that, for example,

$$i = V/RL$$

But what if  $R_L$  is zero? That would imply an infinite current, which we know intuitively is not possible. The equivalent circuit for a battery (or more generally for any energy source) is more properly shown in Figure 5-12b, an ideal energy source with a source resistor,  $R_s$ , in series with it.

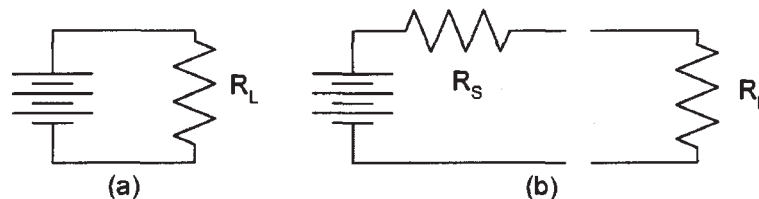


**Figure 5-12** Battery voltage source shown as an ideal source (a), and as the more realistic case with source resistance (b).

A 12-volt car battery, for example, might be able to deliver a short-circuit current of perhaps 80 amps. That would imply a value for  $R_s$  of around  $12/80 = .15 \Omega$ . Now, when driving the starter motor, we want to deliver maximum current to

the starter. If the cable between the battery and the starter motor had a resistance of just  $.15 \Omega$ , we would cut the maximum available current at the starter by 50%. That is why battery cables are so big. Their resistance must be small compared to the source resistance,  $R_s$ , of the battery.

Looking at this another way, what would the battery cable resistance need to be if we only wanted to drop a maximum of 1.0 volt across the cable when drawing 60 amps to the starter motor? A resistor of  $1.0\text{V}/60\text{A} = .017 \Omega$  will develop a voltage across it of 1.0 volt at 60 amps, so the total battery cable resistance, including the connections to the battery and the started motor, must be just  $.017 \Omega$  if we want to limit the voltage drop to 1.0 volt. (Keep those battery cable connections clean!)

A fresh alkaline D-cell battery can (perhaps surprisingly) deliver 10 amps of short-circuit current (at least for a few moments). So, the initial output resistance of a fresh battery is

$$R_s = 1.5 \text{ volts}/10 \text{ amps} = .15 \Omega$$

When the battery appears to be running down, the output resistance is going up. Therefore, it can deliver less and less current without pulling the output voltage down across the output resistance. A so-called “dead” battery has a large output resistance and drops virtually all the available voltage across that output resistance at almost any load.

## CONDUCTANCE

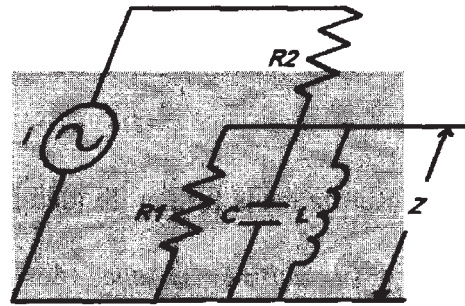
Conductance is a term we don’t very often hear used in general engineering discussions. It is defined as the inverse of resistance. If something offers high resistance to current flow, there will be very little conductance, and vice versa. Conductance is designated by the symbol  $G$ . Thus,

$$G = 1/R$$

The concept of conductance is primarily used to simplify certain kinds of mathematical analyses. For example, if we express Equation 5-11 in terms of conductances, it would be Equation 5-17:

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}} = \frac{1}{G_1 + G_2 + \dots + G_n} \quad [5-17]$$

We will not use conductance again in this book until we discuss lossy transmission lines in Chapter 17.



## 6 REACTANCE

As I have indicated, there are only three types of passive components: resistors, capacitors, and inductors. Each of these *impedes* the flow of current, but each does so in a different way. We categorize them these three ways:

- *Resistance* (designated by the symbol  $R$ ) is the impedance to current flow that is presented by a pure (ideal) resistor. Resistance is measured in ohms. Resistance is directly related to the size of the resistor, but it is not a function of frequency and there is no phase shift associated with this impedance to current flow.
- *Reactance* (designated by the symbol  $X$ ) is the impedance to current flow that is presented by an ideal inductor or capacitor. Reactance is measured in ohms. Reactance is related to the size of the component but is *also* related to the frequency of the signal. There is a 90-degree (only) phase shift associated with reactance.<sup>1</sup>
- *Impedance* (designated by the symbol  $Z$ ) is what we get when we *combine* resistance and reactance (see Chapter 7). Impedance is measured in ohms. It is a function of frequency and there is normally a phase shift associated with impedance. That phase shift can range between +90 degrees and -90 degrees.<sup>2</sup>

<sup>1</sup>Note that this statement says there is a 90-degree phase shift (plus or minus) associated with reactance. It is *exactly*  $\pm 90$  degrees and nothing else. It is important to understand that reactance *only* has a 90-degree phase shift, and no other value of phase shift. Reactance, for example, cannot have a 45-degree phase shift under any conditions. (It is only impedance, discussed in the next chapter, that can have *any* value of phase shift between  $\pm 90$  degrees.)

<sup>2</sup>It is not wrong to describe resistance as impedance, but more properly resistance is only the “real” part of impedance. It is not wrong to describe reactance as impedance, but more properly it is only the “imaginary” (shifted 90 degrees) component of impedance.

Reactance is further divided into two components:

- *Capacitive reactance* (designated by the symbol  $X_C$ ) is inversely related to the size of the capacitor and to frequency. That is, larger capacitors have lower reactance at the same frequency, and a given capacitor has a lower reactance as frequency increases. Voltage *lags* current (current leads voltage) through a capacitor, so there is a (negative) 90-degree phase shift associated with capacitive reactance.
- *Inductive reactance* (designated by the symbol  $X_L$ ) is directly related to the size of the inductor and to frequency. That is, larger inductors have higher reactance at the same frequency, and a given inductor has a higher reactance as frequency increases. Voltage *leads* current (current lags voltage) through an inductor, so there is a (positive) 90-degree phase shift associated with inductive reactance.

It is sometimes helpful to note that capacitive and inductive reactance are almost exactly opposite in their characteristics and effects. It's as if they were at extreme opposite ends of the same scale. Resistance, then, is a very special case exactly between these two extremes.

## CAPACITIVE REACTANCE

It would follow from the preceding that the measure of capacitive reactance, for example, might be given by the formula  $X_C = -1/(\text{frequency} \times C)$ . (The minus sign means a minus 90-degree phase shift. It is a consequence of the "imaginary operator," something we introduce in the next chapter.) To use this formula we need to know what measure of frequency we need to use. Recall from the discussion in Chapter 1 there are several ways we can measure, or describe, frequency. The measure used here is the angular frequency,  $\omega$  (which is  $2\pi f$ ; refer back to Equation 1-2). So the definition of capacitive reactance is as shown in Equation 6-1:

$$X_C = -1/\omega C \text{ ohms} \quad [6-1]$$

For example, suppose we have a .01  $\mu\text{F}$  ( $10^{-8}$ ) capacitor at an angular frequency of  $\omega = 10^6$ . (Remember an angular frequency of  $10^6$  would equate to a cyclical frequency of  $10^6/2\pi = 159,155$  kHz). The reactance associated with this capacitor at this frequency would be:

$$X = -1/(10^6 \times 10^{-8}) = -100 \, \Omega$$

(The minus sign means that the voltage phase shift is  $-90$  degrees).

This same capacitor at a frequency 10 times higher ( $\omega = 10^7$ ) would have a reactance of:

$$X = -1/(10^7 \times 10^{-8}) = -10 \, \Omega$$

demonstrating the importance of the frequency of the circuit.

A larger capacitor, say  $.1 \, \mu\text{F}$  ( $10^{-7}$ ) at a frequency  $\omega = 10^6$  would have a reactance of

$$X = -1/(10^6 \times 10^{-7}) = -10 \, \Omega$$

These examples illustrate the importance of both the value of the capacitor and the frequency in determining the reactance (in ohms) the device presents to the circuit.

The relationship between capacitive reactance and frequency is nonlinear. That is, a graph of the relationship would be a curved line (see Figure 6-1). Even though we calculate capacitive reactance as a negative number indicating phase shift, we graph it with a positive sign. The reason for that will become clearer in the next chapter, when we show that  $Z = \sqrt{X^2}$ . Since frequencies of possible interest cover a very wide range (from a few Hz to hundreds of MHz and more), we usually graph the reactive relationship on “log-log” axes. The scales for both the horizontal and vertical axes are logarithmic (common log, to the base 10.) See Appendix C for more information about logarithms. When graphed this way (Figure 6-2) the capacitive reactance relationship is a straight line. Notice in what direction the graphs move as capacitance changes.

## INDUCTIVE REACTANCE

Inductive reactance is given by the relationship shown in Equation 6-2:

$$X_L = +\omega L \quad [6-2]$$

where  $X_L$  is inductive reactance in ohms

$\omega$  is angular frequency and

$L$  is inductance in henrys

The + sign (usually understood and not used) means a +90-degree phase shift.

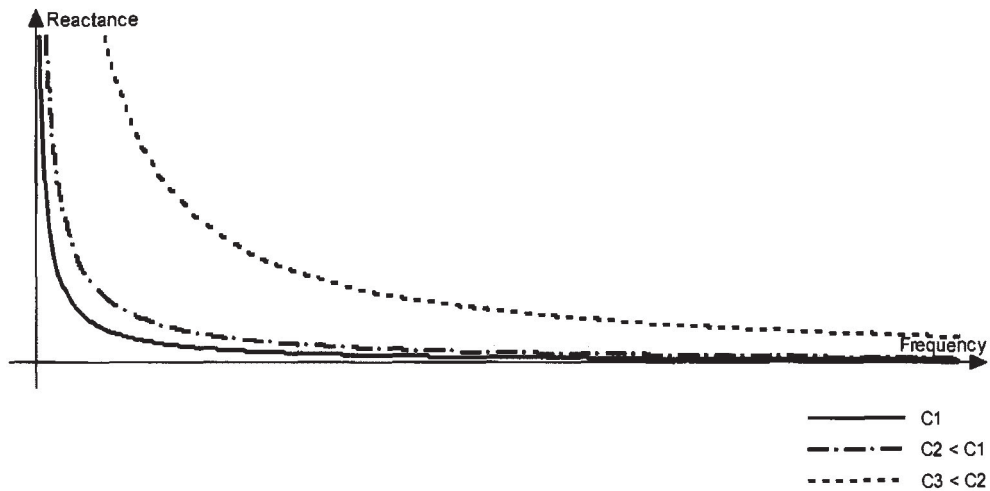


**Figure 6-1** Capacitive reactance graphed on linear scales.

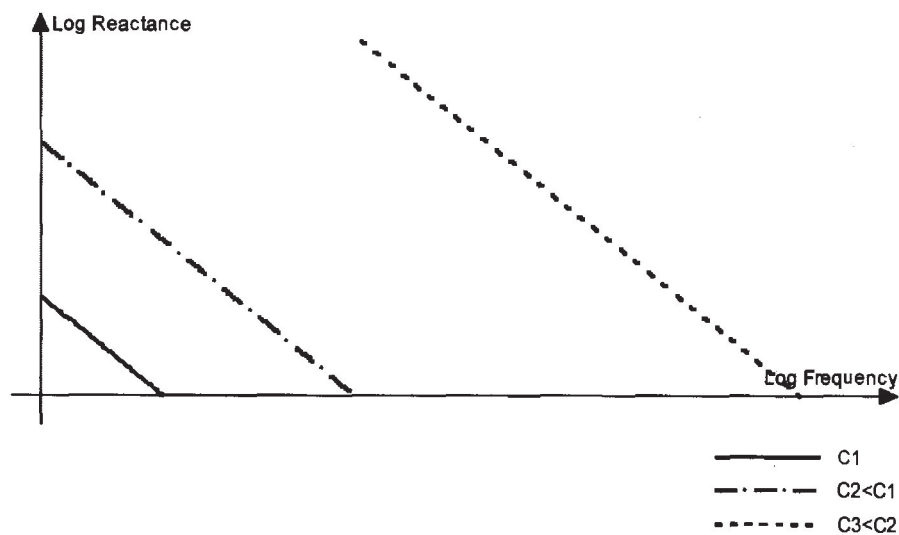


**Figure 6-2** Capacitive reactance graphed on logarithmic scales.

For example, consider a component lead with 10 nH ( $10^{-8}$ ) of inductance at a frequency  $\omega = 10^6$ .  $X_L$  would be  $(10^6)(10^{-8}) = 10 \text{ m}\Omega$ . But the same lead at a frequency of  $\omega = 10^8$  would present a reactance of  $X_L = (10^8)(10^{-8}) = 1.0 \Omega$ .

Inductive reactance is a linear relationship. That means that it would graph as a straight line whether linear or logarithmic scales were used (see Figure 6-3). We usually use logarithmic scales to graph inductive reactance because the range of frequencies is so large.

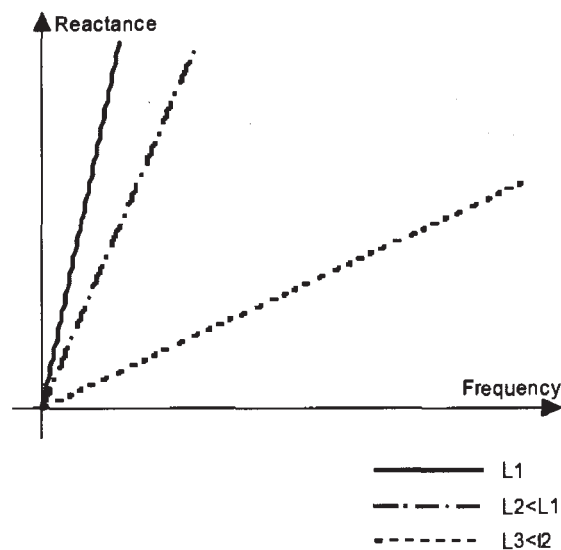


Figure 6-3 Inductive reactance graphed on logarithmic scales.

## OHM'S LAW FOR REACTANCE

Just as Ohm's Law applies to resistance, Ohm's Law also applies to reactance (see Equation 6-3):

$$V = iX \quad [6-3]$$

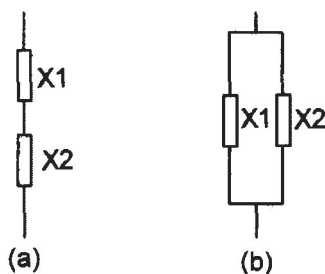


**Figure 6-4** Series (a) and parallel (b) combinations of reactance.

Reactances combine just as resistances do (Figure 6-4). The equivalent reactance,  $X_{eq}$ , for two reactances in series is shown in Equation 6-4:

$$X_{eq} = X1 + X2 \quad [6-4]$$

Reactances in parallel combine using the normal parallel combination formula as shown in Equation 6-5:

$$X_{eq} = \frac{1}{\frac{1}{X1} + \frac{1}{X2} + \dots + \frac{1}{Xn}} \quad [6-5]$$

From here we can see how the formula for an equivalent capacitance of two or more capacitors in series is derived (see Equation 6-6):

$$X_{eq} = X1 + X2 + \dots + Xn$$

$$\frac{1}{\omega C_{eq}} = \frac{1}{\omega C1} + \frac{1}{\omega C2} + \dots + \frac{1}{\omega Cn}$$

$$C_{eq} = \frac{1}{\frac{1}{C1} + \frac{1}{C2} + \dots + \frac{1}{Cn}} \quad [6-6]$$

When we place two or more capacitors in parallel the result is shown in Equation 6-7:

$$X_{eq} = \frac{1}{\frac{1}{X_1} + \frac{1}{X_2} + \dots + \frac{1}{X_n}}$$

$$\frac{1}{\omega C_{eq}} = \frac{1}{\omega C_1 + \omega C_2 + \dots + \omega C_n}$$

$$C_{eq} = C_1 + C_2 + \dots + C_n \quad [6-7]$$

Similarly, two or more series inductors combine as shown in Equation 6-8:

$$X_{eq} = X_1 + X_2 + \dots + X_n$$

$$\omega L_{eq} = \omega L_1 + \omega L_2 + \dots + \omega L_n$$

$$L_{eq} = L_1 + L_2 + \dots + L_n \quad [6-8]$$

Two or more inductors in parallel combine as shown in Equation 6-9:

$$X_{eq} = \frac{1}{\frac{1}{X_1} + \frac{1}{X_2} + \dots + \frac{1}{X_n}}$$

$$\omega L_{eq} = \frac{1}{\frac{1}{\omega L_1} + \frac{1}{\omega L_2} + \dots + \frac{1}{\omega L_n}}$$

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n}} \quad [6-9]$$

## SERIES LC COMBINATIONS

It gets more interesting if we combine inductive and capacitive reactances. Suppose we have a capacitor and an inductor connected in series (Figure 6-5). The equivalent reactance for this circuit is shown in Equation 6-10:

$$\begin{aligned} X_{\text{total}} &= X_1 + X_2, \text{ or} \\ &= X_L + X_C \\ &= \omega L - 1/\omega C \\ &= (\omega^2 LC - 1)/\omega C \end{aligned} \quad [6-10]$$

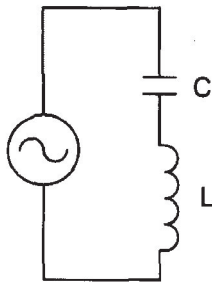


Figure 6-5 Series LC circuit.

Assume we have an AC current flowing through this circuit. Remember that current must be constant everywhere within a loop. This current (by Ohm's Law) generates a voltage across the capacitor, a voltage across the inductor, and a voltage across the pair. Ohm's Law applies for each case. Now the voltage across the capacitor *lags* the current by 90 degrees. The voltage across the inductor *leads* the current by 90 degrees. Therefore the voltage across the capacitor must *lag* the voltage across the inductor *by 180 degrees*.

*This is a very important point.* Since there are only 360 degrees in a cycle, that means the voltages across the capacitor and inductor are *exactly* opposite in phase. And they are *always* so. Always! Let's look at a couple of examples. Suppose we let:

$$\begin{aligned} C &= .1 \text{ uF} = 10^{-7} \\ L &= 4 \text{ uH} = 4 \times 10^{-6} \end{aligned}$$

Let current = 5 mA at an angular frequency of  $2\pi f = 10^6$

$$X_C = -1/\omega C = -1/(10^6)(10^{-7}) = -10 \Omega$$

$$X_L = \omega L = (10^6)(4 \times 10^{-6}) = 4 \Omega$$

$$X_{\text{total}} = X_C + X_L = -6 \Omega$$

(Remember, the minus sign means that the voltage lags current.)

We should get the same total reactance if we calculate it from Equation

6-10:

$$\begin{aligned} X_{\text{total}} &= [(10^6)(10^6)(10^{-7})(4 \times 10^{-6}) - 1]/(10^{-7})(10^6) \\ &= -.6/.1 = -6 \Omega \end{aligned}$$

The three voltage waveforms, then, would be:

$$V_C = iX_C = 5(10^{-3})(-10) \text{ with } -90\text{-degree phase shift}$$

$$V_L = iX_L = 5(10^{-3})(4) \text{ with } +90\text{-degree phase shift, and}$$

$$V_{\text{total}} = iX_{\text{total}} = 5(10^{-3})(-6) \text{ with } -90\text{-degree phase shift}$$

The three voltage waveforms are graphed in Figure 6-6. There are several things to note from this example. First, the voltages across the capacitor and the inductor are exactly opposite in phase. Second, the total voltage across the circuit is the algebraic sum of the two voltages. (It could be determined graphically by simply adding the two individual curves.) Third, since the reactance of the capacitor is larger than the reactance of the inductor, the voltage across the capacitor is (naturally) larger than the voltage across the inductor. Consequently, the voltage phase shift of the total voltage is the same as the voltage phase shift across the capacitor. That is, for the total circuit the phase shift is  $-90$  degrees, the voltage lags the current.