

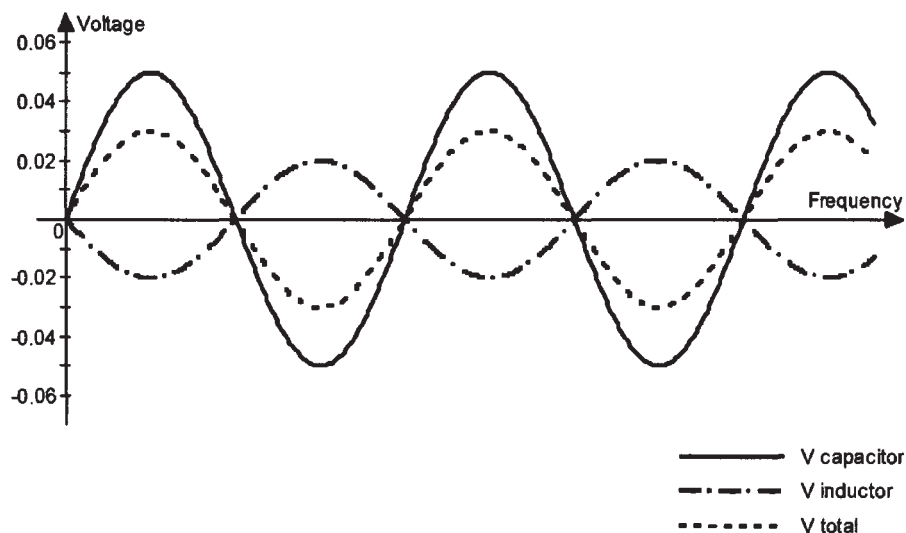


Figure 6-6 Plots of the three voltages at $\omega = 10^6$.

It is important to recognize that since the phase shift of the pair of components is -90 degrees, this circuit looks more like a capacitor than an inductor. This makes sense since the capacitive reactance is more than twice the magnitude of the inductive reactance. We call this circuit *capacitive* (at *this* frequency).

Now let's make just one change in this example. Let's change the frequency from $\omega = 10^6$ to $\omega = 2 \times 10^6$. Intuitively we expect that the capacitive reactance will go down and the inductive reactance will go up. We calculate:

$$X_C = -1/\omega C = -1/(2 \times 10^6)(10^{-7}) = -5 \Omega$$

$$X_L = \omega L = (2 \times 10^6)(4 \times 10^{-6}) = 8 \Omega$$

$$X_{\text{total}} = X_C + X_L = 3 \Omega$$

Now the total reactance phase shift is $+90$ degrees, which means the total voltage is exactly in phase with the voltage across the inductor. The voltage curves now are shown in Figure 6-7. This figure looks very similar to Figure 6-6, except the voltages across the two components are different (they are still 180 degrees different in phase from each other), and the total voltage is now in phase with the voltage across the inductor.

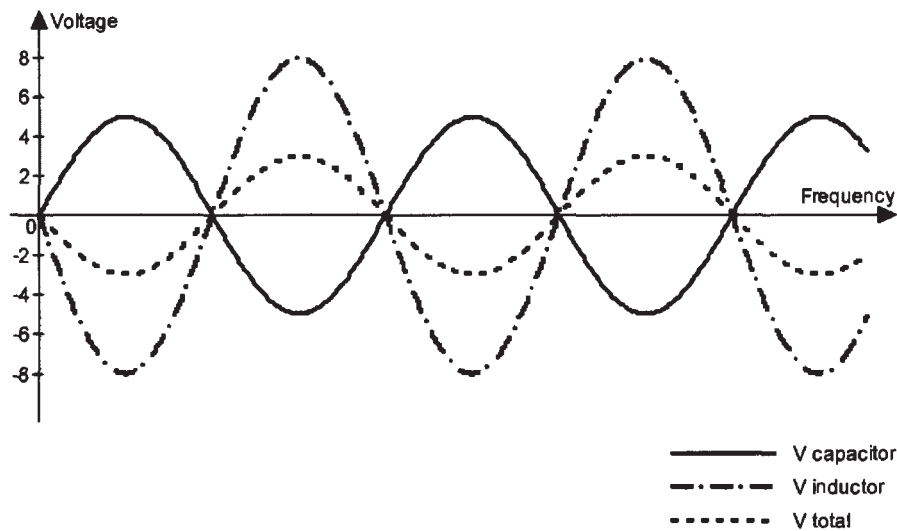


Figure 6-7 Plots of the three voltages at $\omega = 2 \times 10^6$.

It is important to recognize that since the phase shift of the pair of components is +90 degrees, this circuit looks more like an inductor than a capacitor. This makes sense since the inductive reactance is now larger than the capacitive reactance (totally because the frequency, and *nothing* else, has changed). We now call this circuit *inductive* (at *this* frequency).

Note that this circuit has gone from a capacitive circuit to an inductive circuit without changing any component values at all. The change is solely the result of a change in frequency. The change in frequency wasn't really very much, just from $\omega = 10^6$ to $\omega = 2 \times 10^6$. This example could be that of a normal bypass capacitor with lead inductance. Figure 6-5 could describe such a bypass capacitor exactly. (We introduce the impact of series resistance, ESR, in Chapter 15.) The bypass capacitor changes from a capacitive device to an inductive device simply as a result of the frequency increasing beyond a certain value.

PARALLEL LC CIRCUITS

Figure 6-8 illustrates a parallel LC circuit formed with the same components used in the series LC example. Now, however, the total reactance presented by the parallel combination is shown in Equation 6-11:

$$\begin{aligned}
 X_{\text{total}} &= 1 / [(1/XC) + (1/XL)] \\
 &= 1/[-\omega C + 1/\omega L] \\
 &= \omega L / (1 - \omega^2 LC)
 \end{aligned}
 \tag{6-11}$$

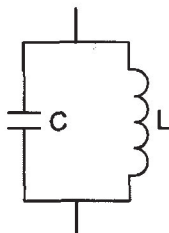


Figure 6-8 Parallel LC circuit.

At a frequency of $\omega = 10^6$ the values work out to be:

$$\begin{aligned}
 XC &= -1/\omega C = -1/(10^6)(10^{-7}) = -10 \, \Omega \\
 XL &= \omega L = (10^6)(4 \times 10^{-6}) = 4 \, \Omega \\
 X_{\text{total}} &= (10^6)(4 \times 10^{-6})/[1 - (10^6)(10^6)(4 \times 10^{-6})(10^{-7})] \\
 &= 4/.6 = 6.66 \, \Omega
 \end{aligned}$$

(Note: X_{total} does not equal $XC + XL$ in this case, as you might expect, because XC and XL are connected in parallel, not series.)

If the voltage across the parallel combination is held at 5 volts with zero phase shift, we can calculate that the current through the components will be:

$$\begin{aligned}
 i_C &= 5/(-10) = -.5 \text{ amps} \\
 i_L &= 5/4 = +1.25 \text{ amps} \\
 i_{\text{total}} &= 5/6.666 = +.75 \text{ amps.}
 \end{aligned}$$

These waveforms are graphed in Figure 6-9. The minus (–) signs mean that the current through the capacitor leads the voltage (the voltage lags the current) by 90 degrees. The current through the inductor lags the voltage (the voltage leads the current) by 90 degrees. Therefore, the current through the capacitor leads the current through the inductor by 180 degrees. This relationship (between the parallel

capacitor and inductor) cannot change; their currents are always out of phase by this 180 degrees.

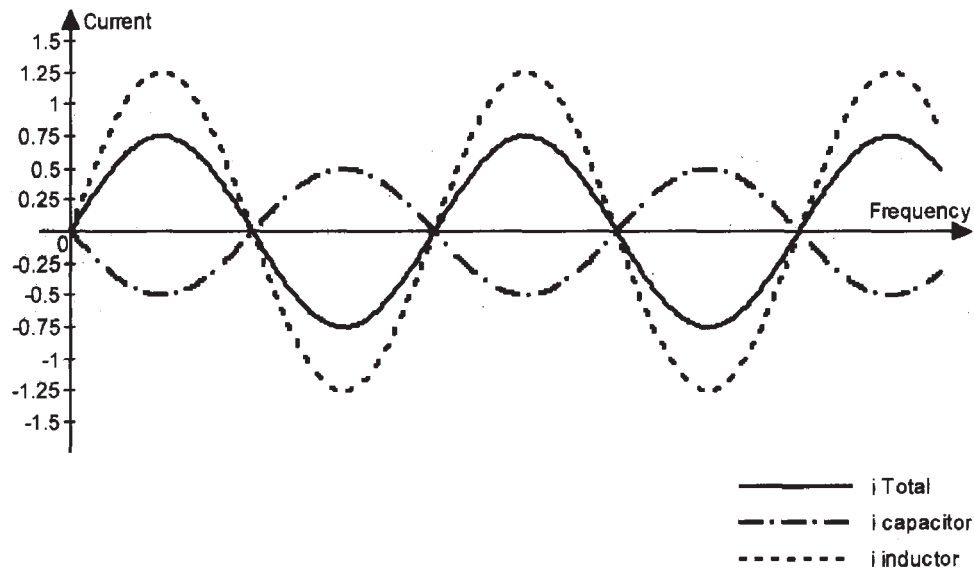


Figure 6-9 Parallel currents for $\omega = 10^6$.

The total current is the sum of the two currents, consistent with Kirchhoff's 1st Law (the current into a node must equal the current going out of a node). Since the sign of the total reactance is positive, we say this parallel combination is *inductive* at this frequency. That is, most of the current is flowing through the inductor, with only a small amount through the capacitor. If we extend this toward the limit at very low frequencies, the capacitor would have almost no effect on the circuit at all.

As we did before, let's just change one thing, frequency, to $\omega = 2 \times 10^6$. Now we find:

$$\begin{aligned}
 X_C &= -1/\omega C = -1/(2 \times 10^6)(10^{-7}) = -5 \, \Omega \\
 X_L &= \omega L = (2 \times 10^6)(4 \times 10^{-6}) = 8 \, \Omega \\
 X_{\text{total}} &= (2 \times 10^6)(4 \times 10^{-6})/[1 - (2 \times 10^6)(2 \times 10^6)(4 \times 10^{-6})(10^{-7})] \\
 &= 8/(-.6) = -13.3333 \, \Omega
 \end{aligned}$$

If the voltage across the parallel combination is held at 5 volts with zero phase shift, we can calculate that the current through the components will be the following:

$$\begin{aligned} i_C &= 5/(-5) = -1 \text{ amp} \\ i_L &= 5/8 = .625 \text{ amps} \\ i_{\text{total}} &= 5/(-13.333) = -.375 \text{ amps} \end{aligned}$$

These waveforms are graphed in Figure 6-10. As before, the minus (–) signs mean that the current through the capacitor leads the voltage (the voltage lags the current) by 90 degrees. The current through the inductor lags the voltage (the voltage leads the current) by 90 degrees. Therefore, the current through the capacitor leads the current through the inductor by 180 degrees. This relationship (between the parallel capacitor and inductor) cannot change; their currents are always out of phase by this 180 degrees.

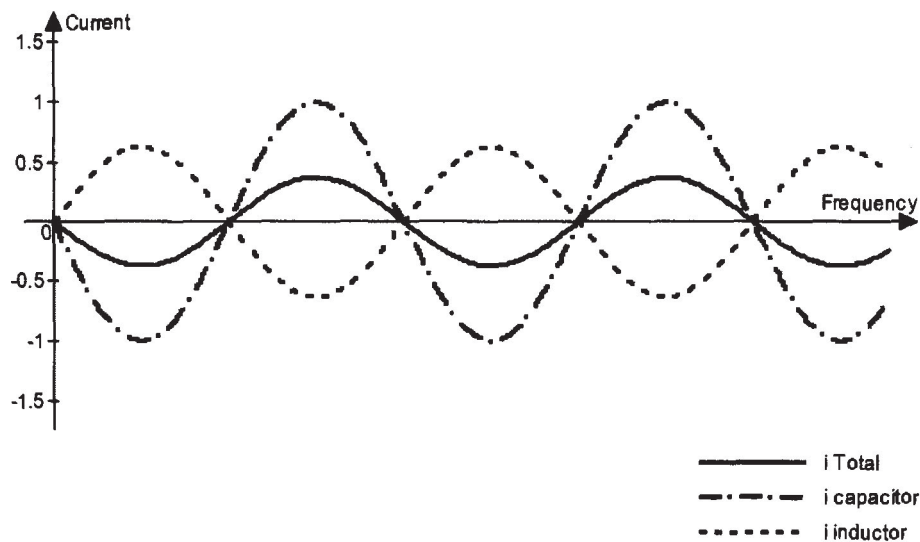


Figure 6-10 Parallel currents for $\omega = 2 \times 10^6$.

Note also as before, the total current is the sum of the two currents, consistent with Kirchhoff's 1st Law. But this time, since the sign of the total reactance is negative, we say this parallel combination is *capacitive* at this frequency. That is, most of the current is flowing through the capacitor, with only a small amount through the inductor. If we extend this toward the limit at very high frequencies, the

inductor would have almost no effect on the circuit at all, and all the current would flow through the capacitor.

RESONANCE

When we plot the impedance of a circuit against frequency, we often refer to the graph as an impedance plot. Such a graph is usually plotted against log-log axes. Figure 6-11 shows impedance plots of the serial and parallel LC circuits described in Figures 6-5 and 6-8, respectively. Even though the reactance of a capacitive circuit is negative, we plot the impedance as a positive number on the axes. More properly, we say we plot the *absolute value* (the value without regard to sign).³

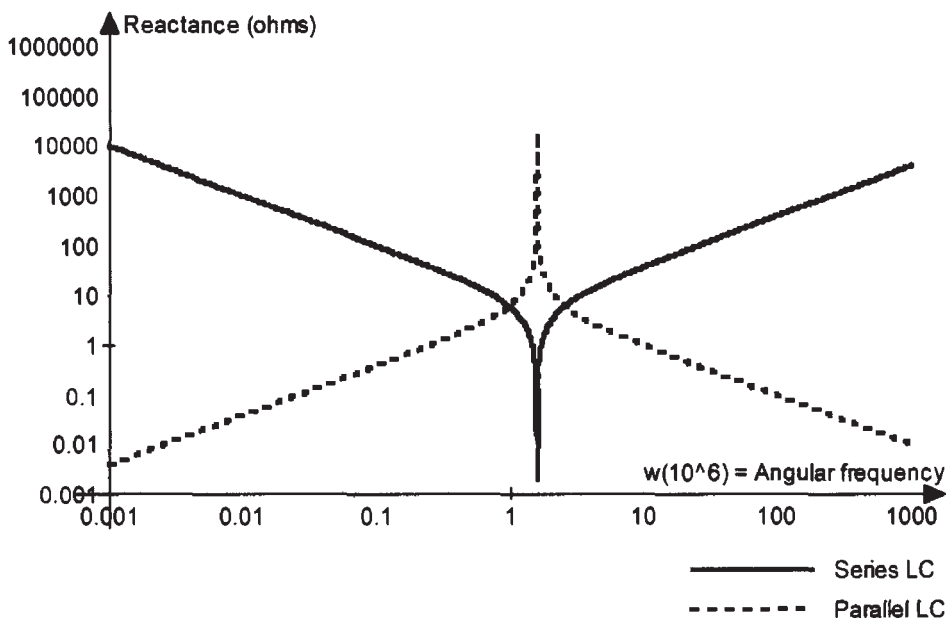


Figure 6-11 Impedance plots for the series and parallel circuits of Figures 6-5 and 6-8.

³When you stop to think about it, you can't have negative impedance. That would imply that a positive voltage, for example, would result in a negative current, a current flowing in the wrong direction. We will see in the next chapter how we get a positive impedance value from a negative reactance.

We often say that the capacitive portion of an impedance curve is “downward sloping to the right.” This is true for both of these curves. On any impedance curve, for any area in which the impedance is falling as the frequency is increasing, the circuit will look capacitive (the current will lead the voltage through the circuit at those frequencies).

Each of the curves in Figure 6-11 exhibits an interesting characteristic at a particular frequency. The frequency is the same in each curve, about 250 kHz, where $\omega = 1.58 \times 10^6$. To see what is special about this point, look at Equations 6-10 and 6-11. Consider the term $\omega^2 LC$. If this term exactly equals 1.0, then the numerator of the series reactance term (Equation 6-10) goes to exactly zero. That means there is zero reactance (impedance) through the series LC circuit. (Not just a small reactance; absolutely *zero* reactance.⁴) The frequency at which this term goes to zero is found by setting (in Equation 6-12):

$$\begin{aligned}\omega^2 LC &= 1 & [6-12] \\ \omega^2 &= 1/LC = 1/(4 \times 10^{-6})(10^{-7}) = 2.5 \times 10^{12} \\ \omega &= 1.58 \times 10^6 = 2\pi f \\ f &= 1.58 \times 10^6 / 2\pi \cong 251,646 \text{ kHz}\end{aligned}$$

This does *not* mean that there is no voltage across each component. There is current flowing, and by Kirchhoff's 2nd Law (see Chapter 5), the current is constant everywhere in the loop. The reactance through the capacitor is not zero, nor is the reactance through the inductor. Since there is reactance through each component, then the current will introduce a voltage across each component, even at this particular frequency. What is special about this particular frequency is that the reactance of each component is *exactly* equal at this frequency. That means a common current will create *exactly* equal voltages across each component. And since the voltages across a capacitor and an inductor are exactly 180 degrees out of phase, their *sum* is exactly zero and they exactly cancel. That is why there is no voltage that can exist across the series pair *at this particular frequency*.

At this same frequency, the denominator of the parallel LC reactance term (Equation 6-11) also goes to zero. At that point, the parallel reactance goes to infinity. (Not just very high; infinitely high.) It's as if there is an open circuit at that particular frequency.

⁴From a practical standpoint there must be some residual (parasitic) resistance in any circuit. In this chapter, however, we are considering “ideal” capacitors and inductors, those without any resistance.

That does not mean that there is no current flowing through the components. As can be seen from Figure 6-8, the voltage across the capacitor is exactly the same as the voltage across the inductor. At this particular frequency, the reactance of the capacitor is exactly the same as the reactance of the inductor. Therefore, the current through the capacitor must be exactly the same as the current through the inductor. But since the phase of the currents differs by exactly 180 degrees, the currents are equal and opposite. Therefore, the current into the node from one component is exactly the same as the current out of the node toward the other component.

By Kirchhoff's 1st Law the total current into the node must equal the total current out of the node. Under this condition that means there is zero current that can enter the node from the voltage source. Since there is zero current from the voltage source, the voltage source must be looking into an infinitely high impedance.

This very special frequency (defined by Equation 6-12), where

$$\begin{aligned}\omega^2 LC &= 1 \\ \omega^2 &= 1/LC \\ \omega &= 1/\sqrt{LC} = 2\pi f \text{ or} \\ f &= 1/(2\pi\sqrt{LC})\end{aligned}\tag{6-13}$$

is called the *resonant* frequency of the circuit (see Equation 6-13). Resonance is a very important concept. One place where its use might be obvious is in a tuning circuit. If we placed a series LC circuit between two stages of an amplifier, we can envision that the only frequency that would pass through from one stage to another is the resonant frequency. All other frequencies would (tend to be) blocked by the LC circuit. If we placed a parallel LC circuit to ground between two stages of an amplifier, the resonant frequency would be passed on to the next stage. All other frequencies would be shunted toward ground. Of course, in real life, circuits are a little more complicated than this, but this principle is the one on which most tuning circuits are based.

Consider the case of a typical bypass capacitor, say one with a value of .01 μ F and a lead inductance of 5 nH. This would look to the circuit like a series LC circuit whose reactance would be given by the expression:

$$\begin{aligned}
 X &= X_L + X_C \\
 &= \omega(5)(10^{-9}) - 1/\omega(10^{-8}) \\
 &= [5\omega^2(10^{-9})(10^{-8}) - 1]/\omega(10^{-8})
 \end{aligned}$$

Since this capacitor has some associated inductance, there must be a resonant frequency. The resonant frequency of this capacitor is

$$\begin{aligned}
 f &= 1/(2\pi\sqrt{LC}) \cong 22.5 \text{ MHz} \\
 &(22,507,926.92 \text{ Hz to be more exact!})
 \end{aligned}$$

Figure 6-12 is a plot of this impedance function for this bypass capacitor. Included also is a plot of an ideal .01 capacitor and an ideal 5 nH inductor. Note that the bypass capacitor plot is capacitive (what we refer to as downward sloping to the right) until the resonant frequency, then becomes upward sloping (inductive) past that point. Beyond 25.5 MHz the capacitor actually starts looking like an inductor. That is the point beyond which the inductive reactance begins to exceed the capacitive reactance.

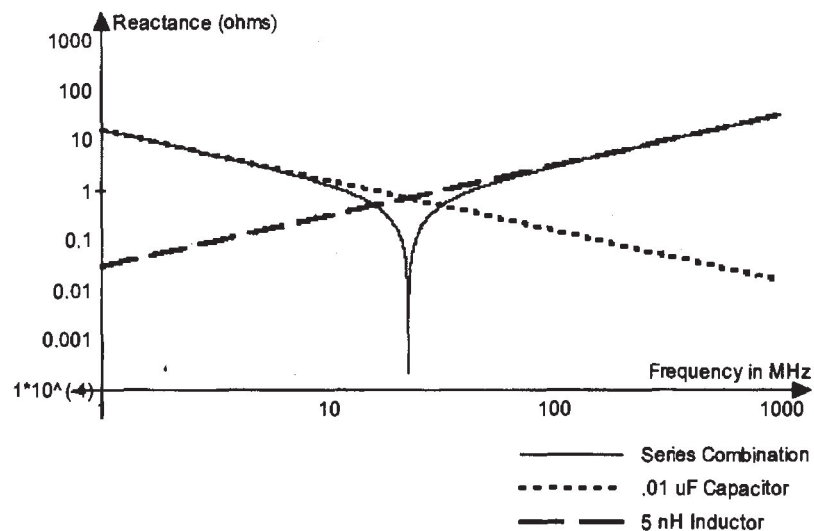


Figure 6-12 Impedance plot of the bypass capacitor and each component individually.

At frequencies well below the resonant frequency, the capacitive reactance dominates; the inductive reactance is so small that it is negligible. At frequencies

well above the resonant frequency the inductive reactance dominates and the capacitive reactance is so small that it is negligible. But at frequencies near resonance, the two interact in such a way that the combined effect is lower impedance than either component taken separately.

There are two important points to recognize in this example. The first is that the phase shift caused by this bypass capacitor is either -90 degrees (capacitive) or $+90$ degrees (inductive). It is never anything else (except that the phase shift is *exactly* zero at *exactly* the resonant frequency). We sometimes think of this as “shooting” rapidly from -90 degrees, through zero, to $+90$ degrees. In fact it does not progress somehow from one extreme to the other. There is a discontinuity at the resonant frequency. It is -90 degrees below the resonant frequency, no matter how close to the resonant frequency you want to get. It is $+90$ degrees above the resonant frequency, no matter how close to the resonant frequency you want to get. It is *exactly* zero at *exactly* the resonant frequency. It is *never* anything else. (Remember, we are assuming no resistance at all.)

There are those who might say that above the resonant frequency this bypass capacitor (a) looks like an inductor and therefore (b) is useless in the circuit. Comment (a) is true, but (b) does not follow logically from it. First note that even (for a little ways) above the resonant frequency the impedance of the real device is still *below* that of the ideal capacitor, because of the interaction with the inductance. This effect becomes more pronounced in circuits that have many capacitors, as we will see in Chapter 15. Second, it is not necessarily the bypass capacitor effect that is important in our circuits; it is often the net impedance offered by the device. Even in the inductive region the net impedance can be very small, and therefore the device is providing a “service” to the circuit. Be careful of people who make absolute statements about components either having or not having an effect at certain frequencies. It is not necessarily that simple.

POLES AND ZEROS

Figure 6-13 illustrates a case with two series LC circuits in parallel. We will look at them for now as just that; but later we will see that this is exactly the case that we have when $C1$ and $C2$ are bypass capacitors and $L1$ and $L2$ represent the stray inductance contributed by leads, pads, vias, and traces.

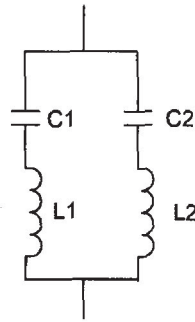


Figure 6-13 Schematic of two parallel capacitors, each with associated lead and mounting inductance.

Let these components have the following values:

$$L1 = 1.0 \text{ nH} = 10^{-9}$$

$$L2 = .1 \text{ nH} = 10^{-10}$$

$$C1 = .1 \text{ uF} = 10^{-7}$$

$$C2 = .01 \text{ uF} = 10^{-8}$$

Remember that the resonant frequency of a series LC pair is given by Equation 6-13:

$$f = 1/(2\pi \sqrt{LC})$$

Using this formula we can find that these pairs will resonate at about 16 MHz and 160 MHz, respectively. Figure 6-14 plots the impedance function of each of these LC circuits. At lower frequencies each circuit looks more like a capacitor than an inductor. But above each resonance, along the upward-sloping part of the reactance curve, each circuit looks more like an inductor than a capacitor. Over this rising portion of the curve the sign of the reactance would be positive.

Now here comes some interesting stuff. Look on the graph at about where the frequency would be 50 MHz. The left-hand curve (Pair 1) is inductive and its reactance has a positive sign here. The downward-sloping curve (Pair 2) is capacitive and its reactance here is negative. The curves are plotted for each one separately, as if the other one weren't there. But in the circuit, of course, they *are* both there and their effects combine. One hint about the way they are going to combine is the fact that at (approximately) 50 MHz their reactances are equal and opposite. This implies another resonance point. But these *pairs* are not in series, they are in

parallel. So, if there is another resonance point, it will be a parallel resonance point and the effect must be infinite reactance.

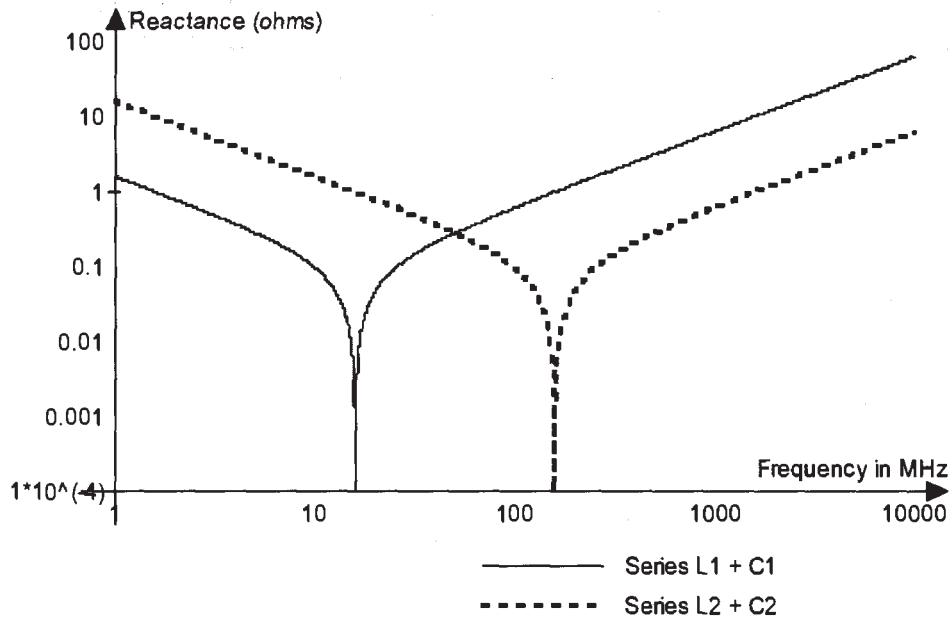


Figure 6-14 Impedance function of each capacitor separately.

Another way to view this is to recognize that the four components do form a parallel circuit of L s and C s. At some frequencies one side will look mostly like an inductor and the other side will look mostly like a capacitor. Therefore, it makes sense that there would be a parallel resonance formed by this set somewhere in that frequency range.

And indeed there is! Figure 6-15 plots the combined effect of all the components making up this circuit. The parallel resonance point between the two series resonance points is very clear. Not coincidentally, it is at the precise point where the two individual curves cross.

The series and parallel resonance points in Figure 6-15 are very sharp because we have included no resistance effects in the analysis. In real life there is always some resistance around, so results on a PCB would normally not be as sharp as is shown here, which is fortunate for all of us. The impact of resistance on LC circuits will be explored in the next chapter and in Chapter 15.

The series resonance points, where the impedance goes to zero, are called (appropriately enough) *zeros*. The parallel resonance points are called *poles*. It seems intuitive that they are called that because they stick up on the frequency plots. In a 3D reactance or impedance plot the poles and zeros can look just like they are holding up or tying down a sheet of canvas for a circus tent.

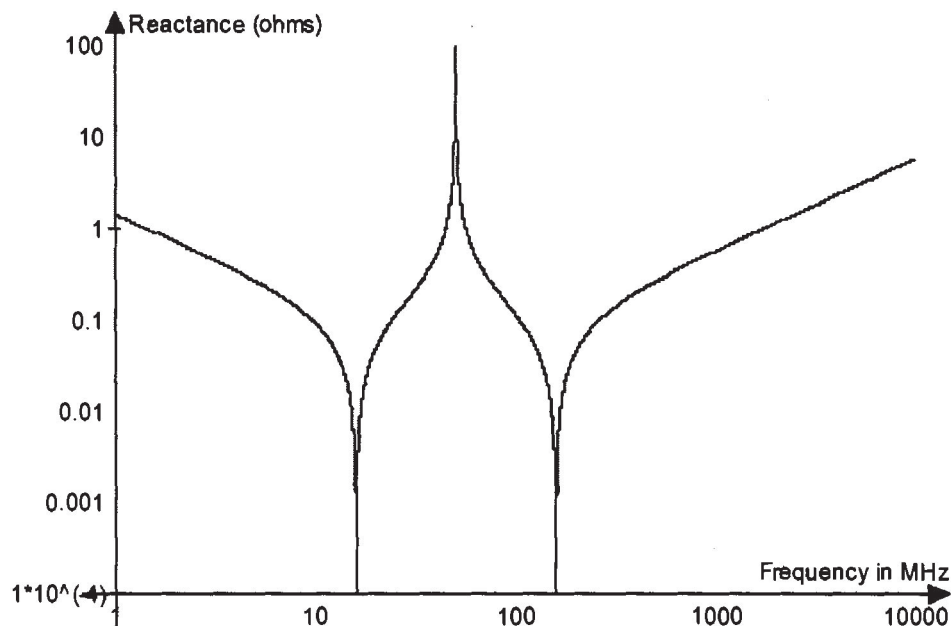


Figure 6-15 Combined impedance of L1 and C1 in Parallel with L2 and C2

There is another reason for looking at the concept of poles and zeros. It is true that:

1. If there are two zeros, there will be a pole in between.
2. If there are two poles, there will be a zero in between.
3. Anytime there is a pole, there will also be a zero.
4. Anytime there is a zero, there will also be a pole.
5. It is possible that some the poles and zeros occur at the same frequencies.

This can be helpful in understanding how circuits will react and where certain unexpected results might be coming from.

Now, perhaps you have already said, "Well, wait a minute. If I have a single capacitor with a zero at infinity, where is the pole?" The answer is at DC (zero frequency.) An inductor, with a pole at infinitely high frequencies, has a zero at DC. A single series combination LC circuit has a zero at its resonant frequency. It has poles at DC and at infinite frequency. A parallel LC circuit with a pole at its resonant frequency has zeros at both DC and at infinite frequency. Poles and zeros exist in pairs and one will always be accompanied by the other.

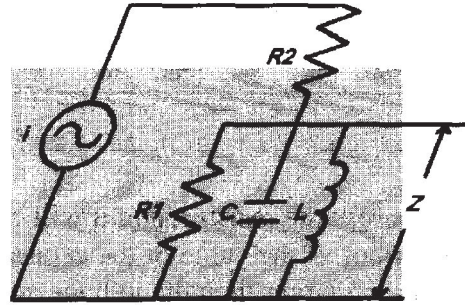
Later we will see that capacitors almost always have some associated inductance with them, so every capacitor probably has a series resonant point (a zero). One could wonder if we have 200 bypass capacitors on our board, and therefore 200 zeros, will we also have 200 parallel resonance points, or poles? The answer is yes, this is true. This can have a serious impact on bypass capacitor and power supply strategy, and can be an important design consideration. We will talk more about this later, particularly in Chapter 15. One clue why we don't have more problems than we already do, however, is because there are a lot of effective resistance paths that help attenuate the sharp peaks. The real world situation is not quite as bad, therefore, as is suggested by Figure 6-15.

SUSCEPTANCE

In the interest of thoroughness I introduce here the term *susceptance*. Susceptance is the inverse of reactance, just as conductance is the inverse of resistance (see Chapter 5). Susceptance is denoted by the symbol B and is defined as:

$$B = 1/X$$

The phase shift associated with susceptance is the same as the phase shift associated with reactance. That is, if the capacitive (voltage) phase shift is -90 degrees, then capacitive susceptance will also have a (voltage) phase shift of -90 degrees.



7 IMPEDANCE AND PHASE SHIFT

IMPEDANCE

Most people find that analyses regarding resistance are fairly easy. Analyses regarding capacitive reactance aren't too difficult, because capacitive effects seem to be somewhat intuitive to a lot of people. The effects of inductors are much less intuitive, and people tend to have more problems with them. And then there is impedance.

Table 7-1 Summary of the Effects of the Primary Passive Components

Component	"Impedance" to Current	Measured in	Voltage Phase	Frequency Dependent?	Impedance Is High if
R	Resistive (R)	ohms	0	No	$R \Rightarrow \text{high}$
L	Inductive reactance (X)	ohms	+90	Yes	$L \Rightarrow \text{high}$ $f \Rightarrow \text{low}$
C	Capacitive reactance (X)	ohms	-90	Yes	$C \Rightarrow \text{low}$ $f \Rightarrow \text{high}$

Consider the simple RC circuit shown in Figure 7-1, driven by an AC constant current source.¹ If the value of the capacitor is very large, there will be very little voltage across it. The voltage across the RC pair will be almost identical to the

¹A constant current source maintains a constant current and phase under all conditions. Constant current sources are harder to design than constant voltage sources, but they are not uncommon in circuits.

voltage across the resistor, and it will be in phase (i.e., zero phase shift) with respect to the current. If the value of the capacitor is very small, then the voltage across it will be much larger than the voltage across the resistor. The voltage across the RC pair will be almost equal to the voltage across the capacitor. Its voltage phase will be -90 degrees (90 degrees behind) the phase of the current.

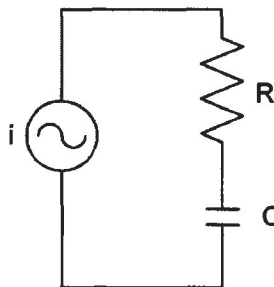


Figure 7-1 Simple RC circuit driven by a constant current source.

Therefore, the voltage across the RC pair ranges from 0 -degrees phase shift for large values of capacitance to -90 -degrees phase shift for small values of capacitance. There must, therefore, be a large number of values of capacitance for which the phase shift across the RC pair will be somewhere in between these values, say 45 degrees, 57 degrees, or 82 degrees, for example.

Resistors offer resistance to the flow of current. Resistance is really a very special case of impedance, that of zero phase shift. Capacitors and inductors resist the flow of current with reactance. Reactance is also a very special case of (exactly) 90 -degree phase shift (either plus or minus). The general case of resistance to current flow is called *impedance* and it covers all cases of phase shift from $+90$ degrees to -90 degrees (including $+90$ degrees, -90 degrees and zero-degree phase shift). Since impedance is the general case, we can say that pure resistance is always a measure of impedance, but only in a very special case would impedance be purely resistive.

Formally, impedance (Z) is the combination of resistance (R) and reactance (X) of a circuit according to the expression shown in Equation 7-1:

$$Z = R + jX \quad [7-1]$$

The symbol j stands for the square root of -1 , $\sqrt{-1}$, which, of course, doesn't exist (see the Chapter Endnote on imaginary numbers at the end of this chapter and also see Appendix E). The impedance expression has what we call a real part (R) and an imaginary part (X). The practical distinction between these parts is that they are *orthogonal*, or perpendicular, to each other. That is, they are shifted from each other by 90 degrees. This is, in fact, the 90-degree phase shift between reactance and resistance.

When we look at impedance we usually picture it as being graphed on axes as shown in Figure 7-2. R is usually graphed on the horizontal (zero phase) axis, and reactance is graphed on the vertical (90-degree phase) axis. So inductive reactance (+90 degree) would be plotted on the vertical axis above the horizontal axis, and capacitive reactance (-90 degree) would be plotted on the vertical axis below the horizontal axis.

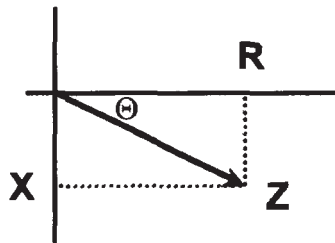


Figure 7-2 For any given R and X , impedance is the vector to their intersection and the phase shift is the angle between that vector and the horizontal axis.

Impedance is referred to as a vector, Z . A vector has magnitude and it has direction (angle from some reference, usually the origin, the intersection of the horizontal and vertical axes). Z can be found graphically by plotting the intersection of R and X (Figure 7-2). The magnitude of Z is the length of the vector from the origin to the intersection. The phase shift is the angle (Θ) formed between the vector and the horizontal axis. The magnitude of Z can also be solved for mathematically. It is the hypotenuse of the right triangle formed by R , X , and Z and is found by the Pythagorean theorem (Equation 7-2) we all learned in trigonometry.²

²Now we can see why we treat negative capacitive reactance as a positive impedance. If $R = 0$ and XC is negative, $Z = \sqrt{(-XC)^2} = XC$ (a positive value).

$$Z = \sqrt{R^2 + X^2} \quad [7-2]$$

Return now to Figure 7-1. If the capacitor is very large, X is very small, and Z is almost the same as R , with no phase shift. But if the capacitor is very small, then X is very large, Z is almost the same as X , and the phase shift is almost the same as that for a pure capacitor, -90 degrees.

Let's look now at a more normal case. Let's say $R = 100 \, \Omega$ and $C = .02 \, \mu\text{F}$. But wait a minute! We can't plot that yet because we haven't specified a frequency. We must know the frequency before we can calculate X . So let's let $\omega = 10^6$.

Now we can calculate X as follows:

$$-1/\omega C = -1/(10^6 \times .02 \times 10^{-6}) = -50 \, \Omega$$

(Remember, the minus sign means a capacitive, or negative, phase shift.) So Z , now, is given by Equation 7-3:

$$Z = \sqrt{100^2 + (-50)^2} = 111.8 \, \Omega \quad [7-3]$$

The angle, Θ , is the resulting phase shift of the RC pair. Clearly Θ will be negative because the circuit must look at least a little capacitive (there is no inductance). We can measure the angle Θ graphically, or we can calculate Θ from trigonometry. The tangent of an angle is the ratio of the side opposite (X) over the side adjacent (R). So the phase angle is found by finding the angle whose tangent is X/R , or by the expression in Equation 7-4:

$$\Theta = \tan^{-1}(X/R) \quad [7-4]$$

The expression $\tan^{-1}$ is called the arctangent, and almost all current spreadsheets and many handheld calculators calculate it (be careful to know whether your calculator or spreadsheet is working in degrees or radians). In our illustration, we use Equation 7-5 to find Θ :

$$\Theta = \tan^{-1}(-50/100) = -26.6 \text{ degrees.} \quad [7-5]$$

Figure 7-3 shows the waveforms for the circuit in Figure 7-1 for the following values:

$$\begin{aligned}
 i &= \sin(\Theta) \\
 \omega &= 10^6 \\
 R &= 100 \, \Omega \\
 C &= .02 \, \mu\text{F} \\
 XC &= -50 \, \Omega
 \end{aligned}$$

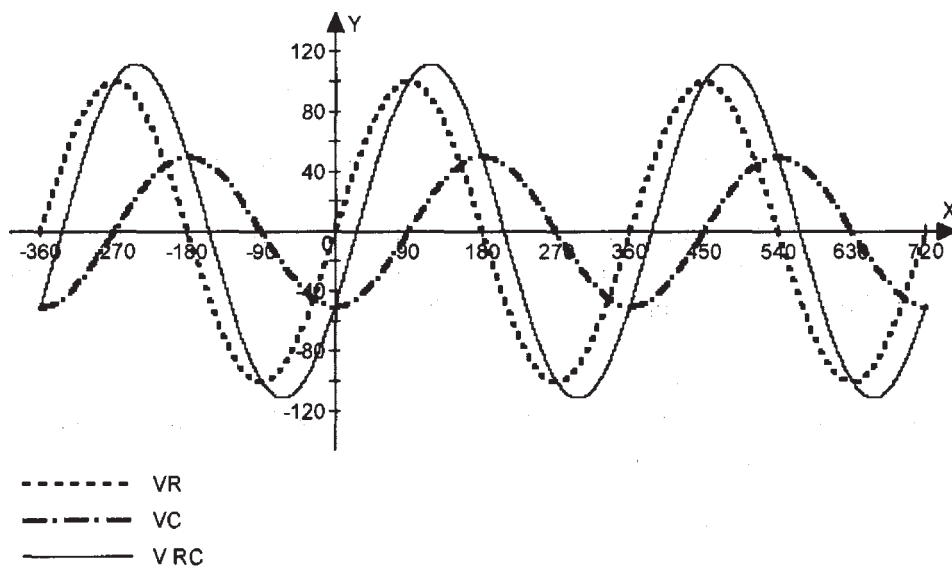


Figure 7-3 Voltage waveforms for the circuit in Figure 7-1.

The voltage across the resistor is:

$$VR = iR = 100\sin(\Theta)$$

It is in phase with the current and it is the curve in Figure 7-3 that goes through the origin (i.e., the value is 0 at 0 degrees).

The voltage across the capacitor is:

$$VC = iXC = -50\sin(\Theta) = 50\sin(\Theta - 90)$$

Here the minus sign means a phase shift of -90 degrees. So the voltage curve for VC is the curve whose value is 0 at 90 degrees (the 90-degree voltage lag). The voltage across the RC pair can be found two ways:

(a) We found from Equations 7-3 and 7-5 that $Z = 111.8 \Omega$ with a phase shift of -26.6 degrees. So we can use:

$$VRC = iZ = 111.8\sin(\Theta - 26.6)$$

This is the curve in Figure 7-3 that crosses the origin between the curves for VR and VC.

(b) We can also simply sum the two curves for VR and VC. That is:

$$VRC = VR + VC = 100\sin(\Theta) + 50\sin(\Theta - 90)$$

You can verify visually from Figure 7-3 that VRC is the arithmetic sum of the other two curves. If you go back to high school trigonometry, you can work this out through those formulas, but we won't do that here.

Now let's look at the same circuit, at the same frequency, but this time make both R and C a little smaller. Thus the resistance will drop but the capacitive reactance will increase. If we let:

$$\begin{aligned} R &= 50 \Omega \\ C &= .01 \mu\text{F} \\ \omega &= 10^6 \\ XC &= -1/\omega C = -1/(10^6)(10^{-8}) = -100 \Omega \end{aligned}$$

The voltage across the resistor is:

$$VR = Ri = 50i = 50\sin(\Theta)$$

The voltage across the capacitor is:

$$VC = XCi = -100i = 100\sin(\Theta - 90)$$

When we calculate the impedance we get:

$$\begin{aligned}
 Z &= \sqrt{50^2 + 100^2} \\
 &= 111.8 \, \Omega, \text{ and} \\
 \Theta &= \tan^{-1} (X/R) \\
 &= \tan^{-1} (-100/50) \\
 &= -63.4 \text{ degrees, and} \\
 VRC &= Zi = 111.8 \sin(\Theta - 63.4)
 \end{aligned}$$

These curves are drawn in Figure 7-4. Note that the relative phase of VR and VC *has not changed*, but their magnitudes have. The phase relationship between VR and VC cannot change. Voltage across a resistor is *always* in phase with the current and the voltage across a capacitor *always* lags current by exactly 90 degrees. As before, VR goes through the origin of the graph and VC crosses the horizontal axis at 90 degrees. Their magnitudes changed by such an amount that their combined impedance didn't change (I did that on purpose). Because the circuit is more capacitive now, the total voltage, VRC, has shifted further in phase. We can see this by comparing the impedance plot in Figure 7-5 with Figure 7-2.

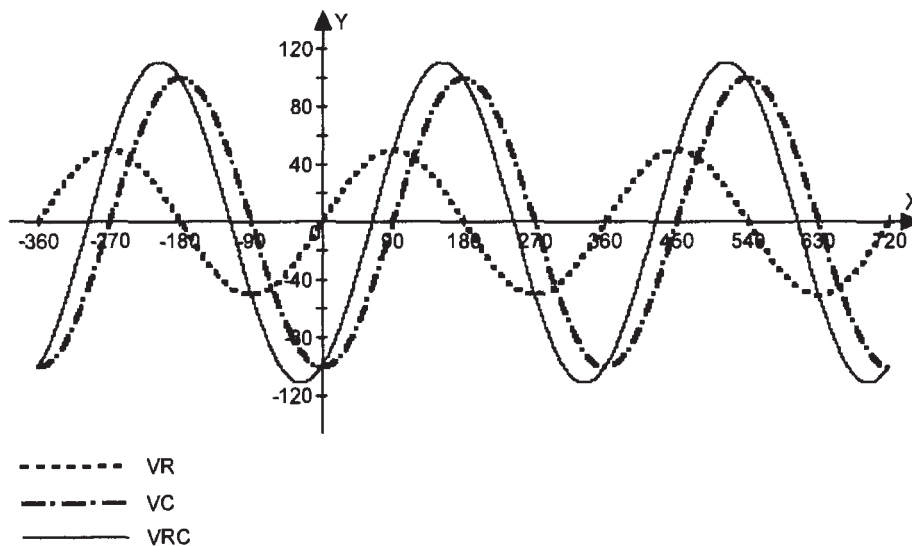


Figure 7-4 Voltage waveforms.

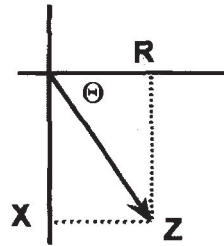


Figure 7-5 Impedance plot for $Z = 111.8 \Omega$ and $\Theta = -63.4$ degrees.

EFFECT OF FREQUENCY

Reviewing what we have just done with our circuit in Figure 7-1, if:

$$\begin{aligned} R &= 50 \Omega \\ C &= .01 \mu\text{F} \\ \omega &= 10^6 \\ XC &= -100 \Omega, \text{ then:} \\ Z &= 111.8 \Omega, \text{ and} \\ \Theta &= -64.3 \text{ degrees.} \end{aligned}$$

This is what is plotted in Figure 7-5. Now let's change nothing except to let $\omega = 10^7$. With this change, $XC = -1/\omega C = -10 \Omega$. We have changed no components, only frequency. Now:

$$\begin{aligned} Z &= \sqrt{50^2 + 10^2} \\ &= 51.0 \Omega, \text{ and} \\ \Theta &= \tan^{-1}(X/C) = \tan^{-1}(-10/50) = -11.3 \text{ degrees.} \end{aligned}$$

This impedance is shown graphically in Figure 7-6, illustrating the truly dramatic effect that frequency has on impedance. All other things being equal (i.e., the Rs and Cs don't change), impedance and phase shift will change dramatically with frequency.

Even if we have a relatively low clock frequency associated with our circuit, if that clock signal has a fast rise time there will be some high-frequency harmonic signals through our circuits. In fact, there will be several transient harmonics at several different frequencies. Each of these harmonics will see our circuit differently because the impedance that our circuits present to a signal, and the phase shift it causes with our signal, is dependent on the frequency of the *particular* harmonic. Thus, if we are not careful, the string of harmonics (say representing a square wave) we inject into a circuit may look quite different when they come out of the other end of the circuit. One name for this difference in signal shape is *distortion*. This is what can make high-speed signal analysis so difficult, and why we have to design our circuits so they present as little distortion to our signal waveforms as possible.

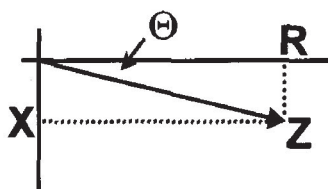


Figure 7-6 Impedance plot for $Z = 51 \Omega$ and $\Theta = -11.3$ degrees.

It is beyond the scope of this book to cover how to make impedance measurements and calculations. The purpose instead is to give you an appreciation of what good, experienced engineers are faced with when they design fast, complex circuits. The rest of this chapter covers some simple concepts, such as RC filters, how impedances combine in circuits, and voltage waveforms in RLC circuits.

ANOTHER RC EXAMPLE

The illustration in Figure 7-1 used a constant current source to drive the RC circuit. Let's do the same analysis with the same circuit, but this time with a constant voltage source as shown in Figure 7-7.

Let:

$$V = 100\sin(\Theta)$$

$$\omega = 10^6$$

$$R = 100 \Omega$$

$$C = .02 \mu\text{F}$$

$$XC = -50 \Omega$$

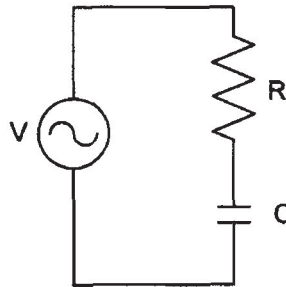


Figure 7-7 Simple RC circuit driven with a constant voltage source.

We have already calculated (Equations 7-3 and 7-5) Z as 111.8Ω with a voltage phase shift, Θ , equal to -26.6 degrees. So current, by Ohm's Law, is:

$$i = V/Z = 100/111.8 = .894 \text{ amps with a phase of } +26.6$$

(capacitive, current leading voltage), or

$$i = .894\sin(\Theta + 26.6)$$

The voltages across R and C , then, are:

$$VR = Ri = (100)(.894)\sin(\Theta + 26.6)$$

$$= 89.4\sin(\Theta + 26.6), \text{ and}$$

$$VC = XCi = (50)(.894)\sin(\Theta - 63.4)$$

$$= 44.7\sin(\Theta - 63.4)$$

(that is, VC lags both the current and the resistive voltage by 90 degrees).

It is tempting to look at this and say, therefore, that the total maximum voltage can be $89.4 + 44.7 = 134.1$ volts. Don't make this mistake. We can't just total these coefficients. We must first adjust them for the phase and *then* add them. The total voltage is graphed in Figure 7-8 and is found by:

$$V_{\text{total}} = 100\sin(\Theta) = 89.4\sin(\Theta + 26.6) + 44.7\sin(\Theta - 63.4)$$

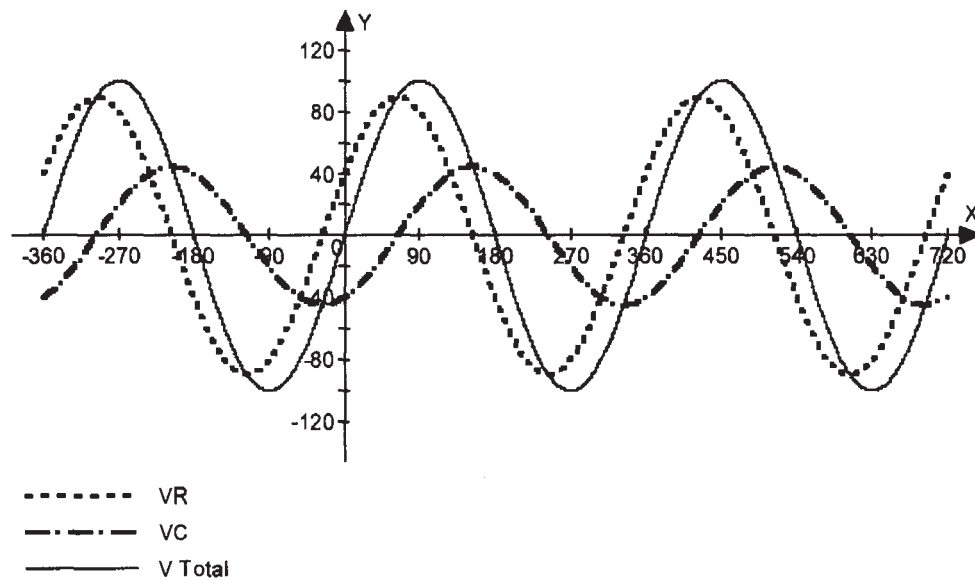


Figure 7-8 Voltage waveforms for this new example.

Table 7-2 in the notes at the end of this chapter shows the calculations for these terms for various angles between 0 and 90 degrees, showing that the calculations do, in fact, work.

CLASSIC RC FILTER

Remember our discussion about voltage dividers (Chapter 5)? Let's look at a classic RC filter circuit (Figure 7-9) from the standpoint of a voltage divider. This is called a *filter* circuit because we are going to filter out (shunt to ground) the higher frequencies and pass through only the lower frequencies. You have probably seen many of these circuits at the power input section of most PCBs.

Just as the voltage ratio of a resistive voltage divider is R_L/R_T , the voltage ratio here is:

$$V_{out}/V_{in} = Z_C/Z_{RC}$$

where Z_C is the impedance of the capacitor and Z_{RC} is the total impedance seen by V_{in} .

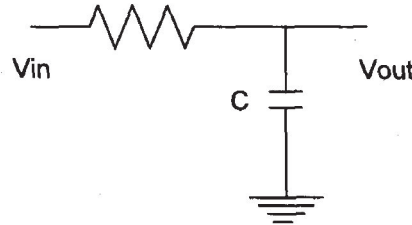


Figure 7-9 Classic RC filter commonly found in power supply circuits.

Since:

$$Z_C = 1/\omega C$$

$$Z_{total} = R + jX = R + 1/j\omega C$$

The voltage ratio, then, becomes

$$\frac{V_{out}}{V_{in}} = \frac{Z_C}{Z_{total}} = \frac{1/\omega C}{\sqrt{R^2 + \left(\frac{-1}{\omega C}\right)^2}} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

If we plot this equation against frequency (using a logarithmic scale), the curve looks like Figure 7-10. Lower frequencies are passed without attenuation, but higher frequencies get shunted to ground through the capacitor. The frequency where the voltage ratio falls to .707 is commonly called the *cutoff frequency* for the filter. That is the point at which $\omega = 1/RC$. We recognize $1/RC$ as the RC time constant we talked about in Chapter 4. Remember also that power is related to voltage squared. Noting that $(.707)^2$ is .5, we see that the RC filter cutoff point, $1/RC$, the RC time constant, also represents the point on the curve where *power* is reduced by 50%.

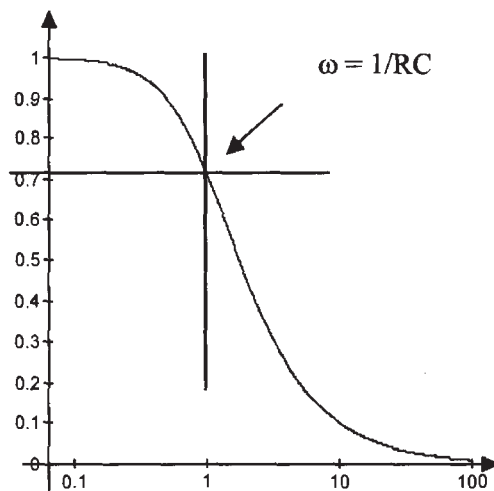


Figure 7-10 Frequency response of an RC filter.

COMBINING IMPEDANCES

We have already looked at how resistors, capacitors, inductors, and reactances combine in series and parallel. What about impedance? How do impedances combine in series and parallel?

Two impedances in series (Figure 7-11a) combine easily. If:

$$\begin{aligned} Z_1 &= R_1 + jX_1, \text{ and} \\ Z_2 &= R_2 + jX_2, \text{ then} \\ Z_{\text{total}} &= (R_1 + R_2) + j(X_1 + X_2) \end{aligned}$$

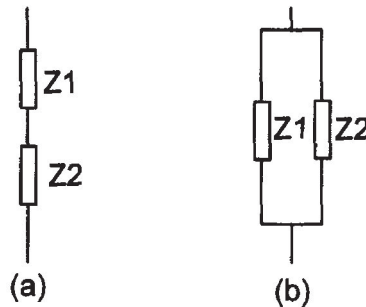


Figure 7-11 Combining impedances.

Parallel impedances pose a problem that is beyond the scope of this book. For practical purposes, the approach to this problem would be to reduce the impedances to their real and imaginary components, combine each of these components separately, and then recombine them into an impedance expression. The general solutions to complex impedance analyses can be very difficult. Since the reactive terms are functions of frequency, it is often easier to assume a *specific frequency*, and then work with the numerical values. Of course, if there are several frequencies of interest, this can be tedious, time consuming, and difficult to do without making errors. There comes a point where we stop doing this manually and start using higher order calculus and transforms and computerized programs to aid in the analyses.

RESONANCE AND Q

It is almost impossible to have an LC circuit without some resistance somewhere. Even if we don't add a resistor to achieve some effect, there is still parasitic resistance at other places in the circuit.

The impedance equation for a series RLC circuit can be developed as follows:

$$Z = \sqrt{R^2 + X^2} \text{ where}$$

$$R = \text{resistance and}$$

$$X = \text{reactance} = \omega L - 1/\omega C = (\omega^2 LC - 1)/\omega C$$

Figure 7-12b illustrates a series RLC circuit. We will let $L = .1 \mu\text{H}$ and $C = .1 \mu\text{F}$. The resonant frequency for this LC pair is:

$$f = \frac{1}{2\pi\sqrt{LC}} = 316 \text{ kHz}$$

Figure 7-12a shows the impedance curve for this circuit as a function of frequency. It is centered on the resonant frequency. Three curves are shown, each for progressively higher values of R_1 . (The resistor R_2 is there simply to establish a uniform maximum impedance for visual purposes.)

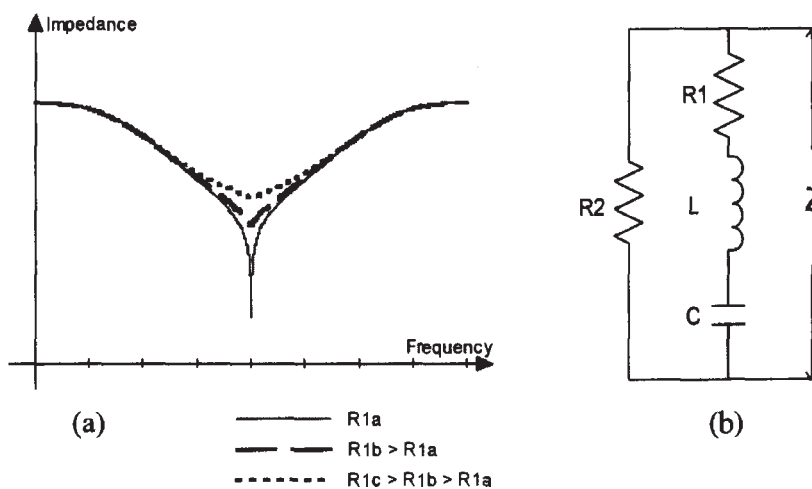


Figure 7-12 Series RLC circuit (a) and its impedance plot (b).

Recall from the discussion in Chapter 6 how sharp the resonance point could be at resonance. These curves illustrate how the frequency selectivity of an RLC series circuit depends heavily on the series resistance. Low values of series resistance (curve R_{1a} in Figure 7-12a) result in a very sharp resonance point, or a very selective filter, while higher values of R_1 (R_{1b} and R_{1c} in Figure 7-12a) create broader patterns as frequency varies. If we were tuning a radio, for example, we would likely want a very sharp selectivity. But we might want a broader pattern if we were building a band pass filter and wanted to pass a range of frequencies.

Figure 7-13b shows a parallel RLC circuit, this time driven by a current source. The plotted curves (Figure 7-13a) show the voltage across the RLC circuit as a function of frequency. The values of L and C are as before, with a resonant frequency of 316 kHz. If the parallel resistor, R_1 , is not there (or is a very high value), the voltage, at least in theory, can become almost infinitely high at resonance. This is because we are assuming a constant current driving an almost infinite parallel LC impedance at resonance. The resulting voltage, from Ohm's Law ($V = iX$) must also be almost infinite. But if there is some resistance in the circuit, the peak voltage will be limited by the value of that resistor times the current ($V_{\max} = iR_1$). Thus, a lower value of parallel resistance, R_1 , will result in a lower peak voltage across the circuit. (As before, R_2 is included in the circuit simply to improve the visual characteristics of the graph.)

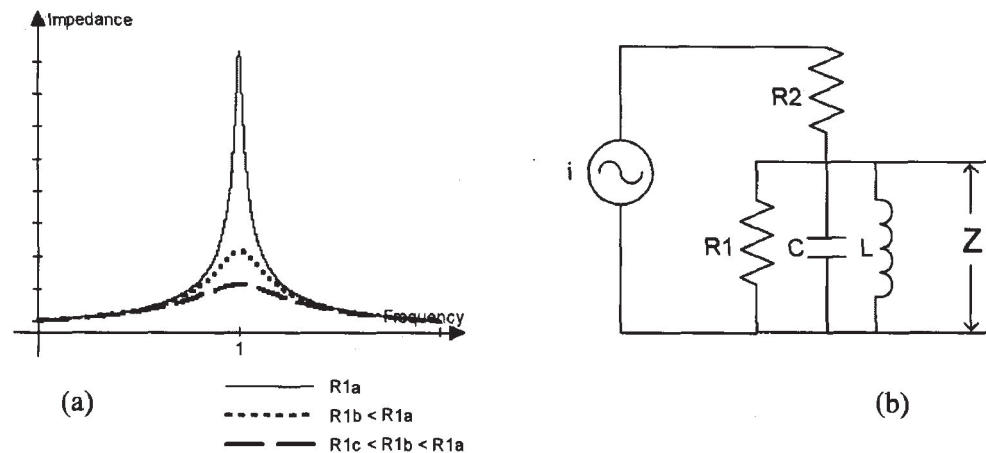


Figure 7-13 Impedance plot of a parallel RLC circuit.

The more sharply peaked curve in Figure 7-13 is for the highest resistance value for R_1 . The other curves are for progressively lower values of resistance. As before, the selectivity and effectiveness of this RLC filter depends heavily on the value of R_1 .

We sometimes talk about the Q of an RLC circuit. Q is called the quality factor of the circuit. For a series RLC circuit Q is defined as

$$Q = \omega_r L / R$$

where ω_r is the resonant frequency. As R gets very small, the bandwidth of the circuit gets very narrow and the Q of the circuit increases, sometimes to a quite high value.

The corresponding measure for a parallel RLC circuit is (perhaps not surprisingly)

$$Q = R/\omega_r L$$

again where ω_r is the resonant frequency. In the parallel circuit, as R increases, the bandwidth of the circuit gets narrower and the Q of the circuit increases.

On our circuit boards, bypass capacitors are really series RLC circuits. The C is, of course, the value of the capacitor. L is the inductance associated with the leads and terminals that are a part of the capacitor as well as any additional inductance associated with mounting the capacitor on the board. R is often referred to as equivalent series resistance (ESR) which is usually a part of the specification for a capacitor or capacitor family. Since bypass capacitors are really RLC circuits, they have a resonant frequency and a Q associated with them. If we connect two different capacitors together in parallel, there can be a *parallel* resonance between them, also characterized by a frequency and a Q . We will look more into these effects in Chapter 15.

Note: The design of frequency selective filter circuits (whether they are LC or RLC) can become very complicated very quickly. Filter design is a specialty by itself within the field of electronics, and is well beyond the scope of this book. All we can do here is give you a flavor for the most basic concepts behind filter design.

SERIES RLC CIRCUITS

A series RLC circuit can be really interesting to look at. Consider the circuit in Figure 7-14.

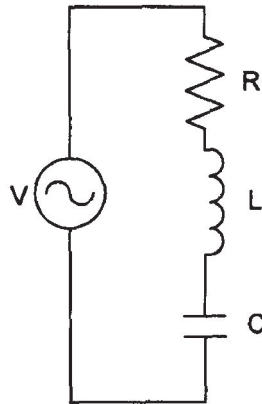


Figure 7-14 Series RLC circuit.

Let's let:

$$V_{in} = 10\sin(\Theta)$$

$$R = .1 \, \Omega$$

$$L = 5 \, \text{nH}$$

$$C = .1 \, \mu\text{F}$$

$$\omega = 50 \times 10^6$$

You might recognize that these are reasonable values for a bypass capacitor.

We pointed out earlier that the way to calculate the impedance of this RLC circuit is to first define its resistive and reactive parts. The resistive part is simply $.1 \, \Omega$. Its reactive part is the series combination of the capacitive and inductive reactances, or

$$\begin{aligned} X &= \omega L + (-1/\omega C) \\ &= 50(10^6)(5)(10^{-9}) + (-1/50(10^6)(10^{-7})) \\ &= .250 - .200 = (+).05 \, \Omega \end{aligned}$$

Since the sign of X is positive, this circuit is slightly inductive at this frequency. Based on this, we can now calculate:

$$\begin{aligned} Z &= \sqrt{.1^2 + .05^2} = .112 \, \Omega \\ \Theta &= \tan^{-1}(.05/.1) = 26.6 \, \text{degrees} \end{aligned}$$

Since $V = iZ$, then the current, i , is found by

$$i = (10/.112) \sin(\Theta - 26.6) = 83.3 \sin(\Theta - 26.6)$$

(Current lags voltage.)

The other voltages, then, are as follows:

$$\begin{aligned} V_R &= (.1)(83.3) \sin(\Theta - 26.6) \\ &= 8.33 \sin(\Theta - 26.6) \end{aligned}$$

$$\begin{aligned} V_L &= .250(83.3) \sin(\Theta - 26.6 + 90) \\ &= 20.83 \sin(\Theta + 63.4) \end{aligned}$$

$$\begin{aligned} V_C &= .200(83.3) \sin(\Theta - 26.6 - 90) \\ &= 16.66 \sin(\Theta - 116.6) \end{aligned}$$

Remember, if the circuit is slightly inductive, then the current lags the voltage by a little bit (in this case by 26.6 degrees). V_R is always in phase with the current, so it lags the overall voltage by the same 26.6 degrees. V_L and V_C will lead and lag the current by 90 degrees, respectively. These voltages are graphed in Figure 7-15.

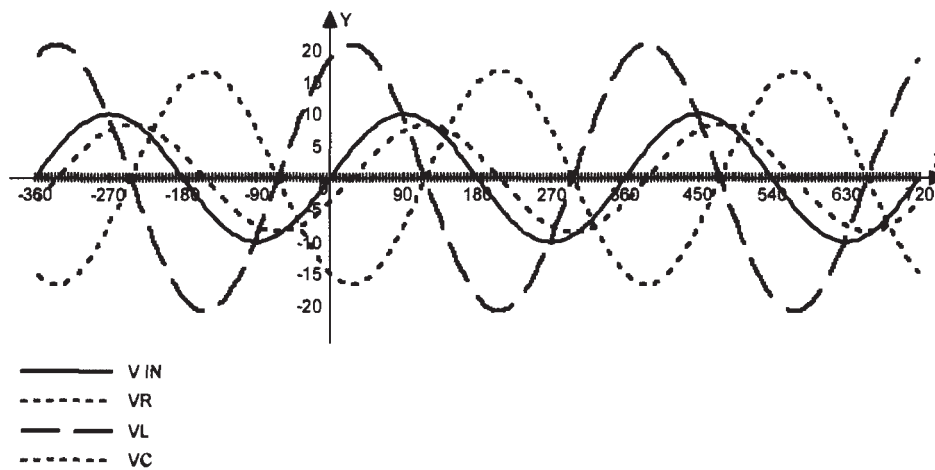


Figure 7-15 Voltages in the RLC circuit.

Looking at Figure 7-15, the overall voltage, $V_{in} = 10\sin(\Theta)$, is the curve that goes through the origin. V_R , in phase with the current, crosses the x-axis 26.6 degrees later (current is lagging voltage). The curves for V_L and V_C are each shifted 90 degrees from V_R and 180 degrees from each other. That is the first observation.

A second, really interesting observation is this. Consider the voltages at a point, say $\Theta = 0$ degrees. At this point:

$$\begin{aligned} V_{in} &= 10\sin(0) &= 0 \\ V_R &= 8.33\sin(-26.6) &= -3.7 \\ V_L &= 20.83\sin(+63.4) &= 18.6 \\ V_C &= 16.66\sin(-116.6) &= -14.9 \end{aligned}$$

The latter three voltages sum to the input voltage (zero), as they must.³ But note that the absolute value of both V_L and V_C are larger than V_{in} *ever* is. It is not intuitively obvious that this can happen, but it can. This case illustrates clearly that it can. The total voltage never exceeds the input voltage, because V_L and V_C tend to cancel each other at every point in time. In fact, the total voltage must always equal the input voltage. But there can be some interesting dynamics *within* the loop when both capacitive and inductive influences are present. And, in particular, the voltage across the inductive and capacitive devices can be considerably larger than the input voltage.

SERIES RLC AT RESONANCE

It gets even more interesting at the point of resonance. Remember that at the point of resonance X_L and X_C are equal and opposite. So let's look at this example at the resonance point:

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5(10^{-9})(10^{-7})}} = 44.721 \times 10^6$$

³If you try to replicate these calculations or use a different set of values there are almost certain to be minor round-off errors. Impedance calculations typically must be carried out to a large number of decimal places to eliminate these round-off errors.

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{5(10^{-9})(10^{-7})}} = 7.117 \text{ MHz}$$

$$R = .1 \Omega$$

$$XL = 5(10^{-9})(44.721)10^6 = .2236$$

$$XC = -1/(10^{-7})(44.721)10^6 = -.2236$$

$$X_{\text{total}} = XL + XC = .2236 - .2236 = 0$$

$$Z = \sqrt{.1^2 + 0^2} = .1 \text{ (same as R)}$$

$$\Theta = \tan^{-1}(0/.1) = 0 \text{ degrees}$$

So current, and then the individual voltages are:

$$i = (10/.1)\sin(\Theta) = 100\sin(\Theta)$$

$$VR = .1(100)\sin(\Theta) = 10\sin(\Theta)$$

$$VL = .2236(100)\sin(\Theta + 90) = 22.36\sin(\Theta + 90)$$

$$VC = .2236(100)\sin(\Theta - 90) = 22.36\sin(\Theta - 90)$$

$$V_{\text{total}} = VR + VL + VC = 10 \times \sin(\Theta)$$

These voltages are graphed in Figure 7-16.

VL and VC are equal and opposite (180-degrees difference, exactly opposite in phase) at every point in time, so their voltages sum to zero. There is *never* any voltage across the pair of them. But each one individually has a voltage waveform across it that has over twice the amplitude of the input voltage. VR is identical to the input voltage, as it must be, since the other voltages cancel.

You can work out for yourself what happens if R decreases. If R decreases, then current increases. Since there is no reactive component (i.e., $X = 0$), the current is totally determined by the value of R. So if R were, say, $.05 \Omega$, then the peak current would be 200 amps ($200\sin(\Theta)$), and the peak voltage across each reactive component would be about 44 volts. Some strange and exotic things can happen at resonance.

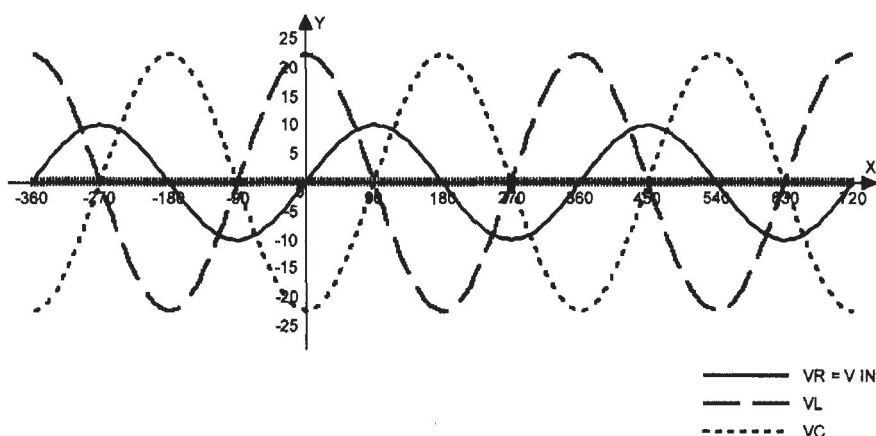


Figure 7-16 Voltages for the RLC circuit at resonance.

ADMITTANCE

In the interest of thoroughness, I add here the concept of admittance. If you are not an active circuit design engineer you probably will never use this term (and I fear even some engineers don't fully understand it). Admittance is simply the inverse of impedance (just as conductance is the inverse of resistance and susceptance is the inverse of reactance; see Chapters 5 and 6). Admittance is designated by the symbol Y and is defined as:

$$Y = 1/Z$$

Ohm's Law applies to admittance (indeed as it must). If $Z = V/i$, then it follows that $Y = i/V$.

Just as impedance is related to resistance and reactance by Equation 7-2,

$$Z = \sqrt{R^2 + X^2}$$

admittance is related to conductance and susceptance by the relationship shown in Equation 7-6:

$$Y = \sqrt{G^2 + B^2} \quad [7-6]$$

The phase shift associated with admittance must be the same as the phase shift associated with impedance. The phase shift is associated with the components and their values, not by how we manipulate the definitions of the terms. So, instead of defining the phase shift in terms of resistance and reactance (from Equation 7-4):

$$\Theta = \tan^{-1}(X/R)$$

we can define it in terms of conductance and susceptance (Equation 7-7):

$$\Theta = \tan^{-1}(B/G) \quad [7-7]$$

I do not use the term admittance again in this book!

Chapter Endnotes

DETAILED CALCULATIONS FOR FIGURE 7-8

Table 7-2 These Calculations Demonstrate that VR and VC for the Example in Figure 7-8 do Add up to Vtotal.

Angle = A	Vin = 100(sin(A))	VR	VC	Vtotal	Difference
0	0.00	40.00	-40.00	0.00	0.00
26.6	44.78	71.59	-26.81	44.78	0.00
45	70.71	84.85	-14.14	70.71	0.00
90	100.00	80.00	20.00	100.00	0.00
5	8.72	46.82	-38.10	8.72	0.00
10	17.36	53.28	-35.92	17.36	0.00
15	25.88	59.34	-33.46	25.88	0.00
20	34.20	64.95	-30.75	34.20	0.00
25	42.26	70.06	-27.80	42.26	0.00
30	50.00	74.64	-24.64	50.00	0.00
35	57.36	78.65	-21.29	57.36	0.00
40	64.28	82.06	-17.79	64.28	0.00
45	70.71	84.85	-14.14	70.71	0.00
50	76.60	87.00	-10.39	76.60	0.00
55	81.92	88.48	-6.56	81.92	0.00
60	86.60	89.28	-2.68	86.60	0.00
65	90.63	89.41	1.22	90.63	0.00
70	93.97	88.86	5.11	93.97	0.00
75	96.59	87.63	8.97	96.59	0.00
80	98.48	85.73	12.75	98.48	0.00
85	99.62	83.18	16.44	99.62	0.00
90	100.00	80.00	20.00	100.00	0.00

$$VR = (100)(.894)\sin(A + 26.6)$$

$$VC = (50)(.894)\sin(A - 63.4)$$

RLC SIMULATION

UltraCAD has developed a simulation program for use in exploring these relationships further. It is available for download from UltraCAD's Web site. The program will show the waveforms for VR, VL, VC, Vin and Iin for a serial RLC circuit (see Figure 7-17). The program is dynamic in that you can change any one component and watch the waveforms shift in real time. This will give you a perspective on how voltages in a circuit shift in phase as the values of the circuit components change. The instructions for obtaining and running the program can be found in Appendix D.

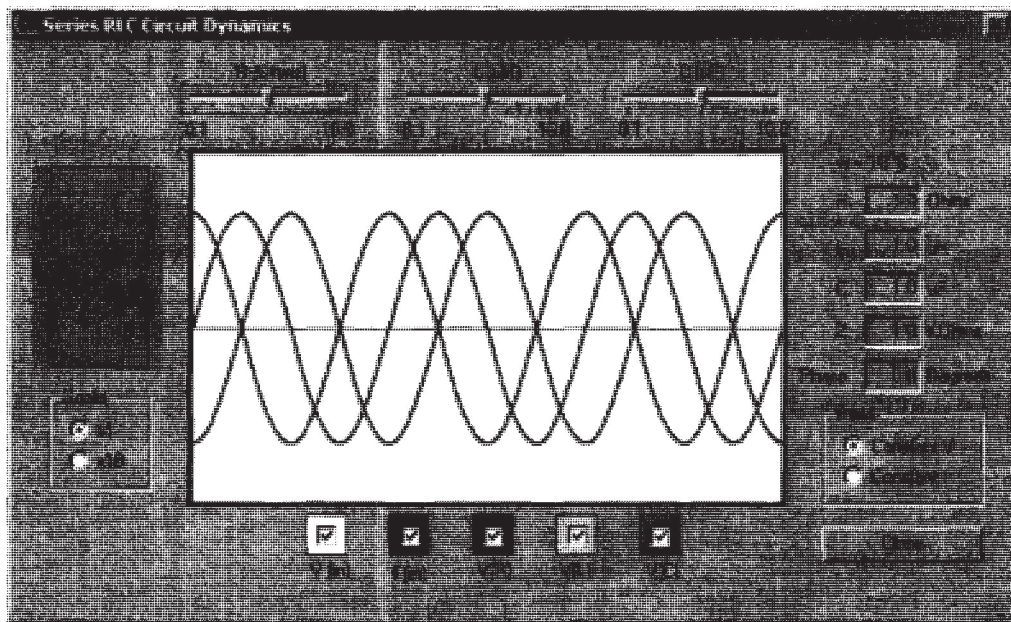


Figure 7-17 UltraCAD's RLC simulation tool.

Imaginary Numbers

In the calculus derivation of the expression $Z = R + jX$, it turns out that the value of j is $\sqrt{-1}$. You should recognize that the square root of -1 does not exist. That is, there is no number that, when multiplied by itself, equals -1 .

Electronics is not the only discipline in which this happens. Several other areas of study also have similar expressions, and the common symbol used for $\sqrt{-1}$ is i . In electronics we use j instead so as not to confuse this symbol with the symbol for current.

Expressions of the type $R + jX$ constitute an area of mathematics called complex algebra. Since the square root of -1 doesn't exist, we often call the jX term *imaginary* and we sometimes call complex algebra *imaginary arithmetic*. The two terms making up the expression are called the real part (R) and the imaginary part (X).

See Appendix E for a more in-depth discussion of complex algebra.

We can now look again at why capacitive reactance is given a negative sign. Suppose we have a single capacitor, so $X = X_C$. X_C is given by:

$$X_C = 1/j\omega C$$

If we multiply the numerator and denominator by j :

$$X_C = j/j^2 \omega C$$

Since:

$$j^2 = (\sqrt{-1})^2 = -1$$

this becomes

$$X_C = -j/\omega C = j(-1/\omega C)$$

This fits the format of Equation 7-1. This is where the minus sign for X_C comes from.