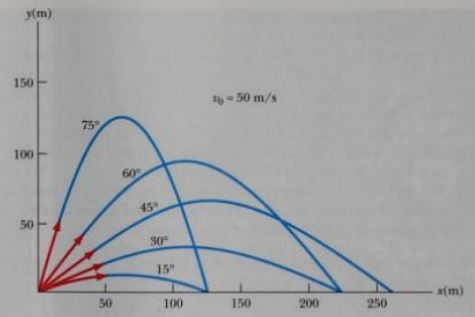


Figure 3.8 A projectile fired from the origin with an initial speed of 50 m/s at various angles of projection. Note that complementary values of θ_0 will result in the same value of x (range of the projectile).

is unity, which occurs when $2\theta_0 = 90°$. Therefore, we see that R is a maximum when $\theta_0 = 45°$, as you would expect if air resistance is neglected.

Figure 3.8 illustrates various trajectories for a projectile of a given initial speed. As you can see, the range is a maximum for $\theta_0 = 45°$. In addition, for any θ_0 other than 45°, a point with coordinates $(R, 0)$ can be reached by using either one of two complementary values of θ_0, such as 75° and 15°. Of course, the maximum height and the time of flight will be different for these two values of θ_0.

Before we look at some numerical examples dealing with projectile motion, let us pause to summarize what we have learned so far about this kind of motion:

- Provided air resistance is negligible, the horizontal component of velocity, v_x, remains constant because there is no horizontal component of acceleration.
- The vertical component of acceleration is equal to the free-fall acceleration, **g**.
- The vertical component of velocity, v_y, and the displacement in the y direction are identical to those of a freely falling body.
- Projectile motion can be described as a superposition of the two motions in the x and y directions.

PROBLEM-SOLVING STRATEGY • Projectile Motion

We suggest that you use the following approach to solving projectile motion problems:

1. Select a coordinate system.
2. Resolve the initial velocity vector into x and y components.
3. Treat the horizontal motion and the vertical motion independently.

4. Follow the techniques for solving problems with constant velocity to analyze the horizontal motion of the projectile.
5. Follow the techniques for solving problems with constant acceleration to analyze the vertical motion of the projectile.

Thinking Physics 2

A home run is hit in a baseball game. The ball is hit from home plate into the centerfield stands along a long, parabolic path. What is the acceleration of the ball (a) while it is rising, (b) at the highest point of the trajectory, and (c) while it is descending after reaching the highest point? Ignore air resistance.

Reasoning The answers to all three parts are the same—the acceleration is that due to gravity, 9.8 m/s^2, because the force of gravity is pulling downward on the ball during the entire motion. During the rising part of the trajectory, the downward acceleration results in the decreasing positive values of the vertical component of the velocity of the ball. During the falling part of the trajectory, the downward acceleration results in the increasing negative values of the vertical component of the velocity. Many people have trouble with the topmost point, mistakenly thinking that the acceleration of the ball at the highest point is zero. This interpretation arises from a confusion between zero vertical velocity and zero acceleration. Even though the ball has momentarily come to rest at the highest point (vertically; it is still moving horizontally), it is still accelerating, because the force of gravity is still acting. If the ball were to come to rest and experience zero acceleration, then the velocity would not change—the ball would remain suspended at the highest point! We don't see that happening, because the acceleration is *not* equal to zero.

CONCEPTUAL PROBLEM 3

Suppose that as you are running at constant speed, you wish to throw a ball such that you will catch it as it comes back down. How should you throw the ball?

CONCEPTUAL PROBLEM 4

As a projectile moves in its parabolic path, is there any point along its path at which the velocity and acceleration vectors are (a) perpendicular to each other? (b) parallel to each other?

Example 3.2 That's Quite an Arm

A stone is thrown from the top of a building upward at an angle of 30.0° to the horizontal and with an initial speed of 20.0 m/s, as in Figure 3.9. If the height of the building is 45.0 m, (a) how long is the stone "in flight"?

Solution The initial x and y components of the velocity are

$$v_{x0} = v_0 \cos\theta_0 = (20.0 \text{ m/s})(\cos 30.0°) = 17.3 \text{ m/s}$$

$$v_{y0} = v_0 \sin\theta_0 = (20.0 \text{ m/s})(\sin 30.0°) = 10.0 \text{ m/s}$$

To find t, we can use $y = v_{y0}t - \frac{1}{2}gt^2$ (Eq. 3.13) with $y = -45.0$ m and $v_{y0} = 10.0$ m/s (we have chosen the top of the building as the origin, as in Fig. 3.9):

$$-45.0 \text{ m} = (10.0 \text{ m/s})t - \tfrac{1}{2}(9.80 \text{ m/s}^2)t^2$$

Solving the quadratic equation for t gives, for the positive root, $t = 4.22$ s. Does the negative root have any physical

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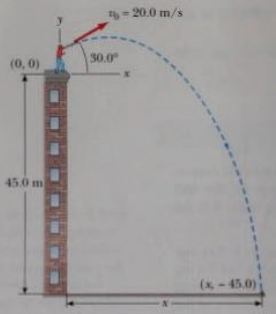


Figure 3.9 (Example 3.2)

meaning? (Can you think of another way of finding t from the information given?)

(b) What is the speed of the stone just before it strikes the ground?

Solution The y component of the velocity just before the stone strikes the ground can be obtained using the equation $v_y = v_{y0} - gt$ (Eq. 3.11) with $t = 4.22$ s:

$$v_y = 10.0 \text{ m/s} - (9.80 \text{ m/s}^2)(4.22 \text{ s}) = -31.4 \text{ m/s}$$

Because $v_x = v_{x0} = 17.3$ m/s, the required speed is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(17.3)^2 + (-31.4)^2} \text{ m/s} = 35.9 \text{ m/s}$$

EXERCISE 3 Where does the stone strike the ground? Answer 73.0 m from the base of the building.

Example 3.3 The Stranded Explorers

An Alaskan rescue plane drops a package of emergency rations to a stranded party of explorers, as shown in Figure 3.10. If the plane is traveling horizontally at 40.0 m/s at a height of 100 m above the ground, where does the package strike the ground relative to the point at which it is released?

Reasoning and Solution The coordinate system for this problem is selected as shown in Figure 3.10, with the positive x direction to the right and the positive y direction upward.

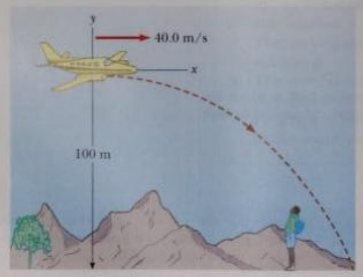


Figure 3.10 (Example 3.3) According to a ground observer, a package released from the rescue plane travels along the path shown. How does the path followed by the package appear to an observer on the plane (assumed to be moving at constant speed)?

Consider first the horizontal motion of the package. The only equation available to us is $x = v_{x0}t$ (Eq. 3.12).

The initial x component of the package velocity is the same as that of the plane when the package is released, 40.0 m/s. Thus, we have

$$x = (40.0 \text{ m/s})t$$

If we know t, the length of time the package is in the air, we can determine x, the distance traveled by the package in the horizontal direction. To find t, we move to the equations for the vertical motion of the package. We know that at the instant the package hits the ground, its y coordinate is -100 m. We also know that the initial component of velocity of the package in the vertical direction, v_{y0}, is zero because the package was released with only a horizontal component of velocity. From Equation 3.13, we have

$$y = -\tfrac{1}{2}gt^2$$
$$-100 \text{ m} = -\tfrac{1}{2}(9.80 \text{ m/s}^2)t^2$$
$$t^2 = 20.4 \text{ s}^2$$
$$t = 4.51 \text{ s}$$

The value for the time of flight substituted into the equation for the x coordinate gives

$$x = (40.0 \text{ m/s})(4.51 \text{ s}) = 180 \text{ m}$$

The package hits the ground not directly under the drop point but 180 m to the right of that point.

EXERCISE 4 What are the horizontal and vertical components of the velocity of the package just before it hits the ground? Answer $v_x = 40.0$ m/s; $v_y = -44.1$ m/s

EXERCISE 5 It has been said that in his youth George Washington threw a silver dollar across a river. Assuming that the river was 75 m wide, (a) what *minimum initial* speed was necessary to get the coin across the river, and (b) how long was the coin in flight? Answer (a) 27.1 m/s (b) 3.91 s

3.4 • UNIFORM CIRCULAR MOTION

Figure 3.11a shows an object moving in a circular path with *constant linear speed* v. Such motion is called **uniform circular motion.** It is often surprising to students to find that **even though the object moves at a constant speed, it still has an acceleration.** To see why, consider the defining equation for average acceleration, $\bar{\mathbf{a}} = \Delta\mathbf{v}/\Delta t$ (Eq. 3.4).

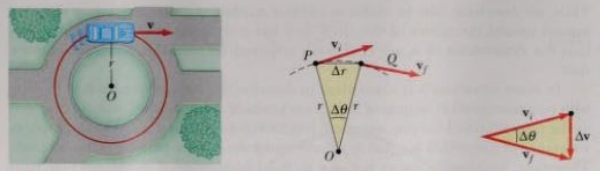
Note that the acceleration depends on *the change in the velocity vector.* Because velocity is a vector quantity, there are two ways in which an acceleration can be produced, as mentioned in Section 3.1: by a change in the *magnitude* of the velocity and by a change in the *direction* of the velocity. It is the latter situation that is occurring for an object moving with constant speed in a circular path. The velocity vector is always tangent to the path of the object and perpendicular to the radius r of the circular path. We now show that the acceleration vector in uniform circular motion is always perpendicular to the path and always points toward the center of the circle. An acceleration of this nature is called a *centripetal acceleration* (center seeking) and its magnitude is

$$a_r = \frac{v^2}{r} \qquad [3.19]$$

• *Magnitude of centripetal acceleration*

where r is the radius of the circle.

To derive Equation 3.19, consider Figure 3.11b. Here an object is seen first at point P and then at point Q. The particle is at P at time t_i, and its velocity at that time is $\mathbf{v}_i$; it is at Q at some later time t_f, and its velocity at that time is $\mathbf{v}_f$. Let us



(a) (b) (c)

Figure 3.11 (a) An object moving along a circular path at constant speed experiences uniform circular motion. (b) As the particle moves from P to Q, the direction of its velocity vector changes from $\mathbf{v}_i$ to $\mathbf{v}_f$. (c) The construction for determining the direction of the change in velocity $\Delta\mathbf{v}$ that is toward the center of the circle.

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also assume here that $\mathbf{v}_i$ and $\mathbf{v}_f$ differ only in direction; their magnitudes are the same (that is, $v_i = v_f = v$). In order to calculate the acceleration of the particle, let us begin with the defining equation for average acceleration (Eq. 3.4):

$$\bar{\mathbf{a}} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta \mathbf{v}}{\Delta t}$$

This equation indicates that we must vectorially subtract $\mathbf{v}_i$ from $\mathbf{v}_f$, where $\Delta \mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$ is the change in the velocity. Because $\mathbf{v}_i + \Delta \mathbf{v} = \mathbf{v}_f$, the vector $\Delta \mathbf{v}$ can be found using the vector triangle in Figure 3.11c. When Δt is very small, Δr and $\Delta \theta$ are also very small. In this case, $\mathbf{v}_f$ is almost parallel to $\mathbf{v}_i$ and the vector $\Delta \mathbf{v}$ is approximately perpendicular to them, pointing toward the center of the circle.

Now consider the triangle in Figure 3.11b, which has sides Δr and r. This triangle and the one in Figure 3.11c that has sides Δv and v are similar. (Two triangles are *similar* if the angle between any two sides is the same for both triangles and if the ratio of lengths of these sides is the same.) This enables us to write a relationship between the lengths of the sides:

$$\frac{\Delta v}{v} = \frac{\Delta r}{r}$$

This equation can be solved for Δv and the expression obtained from it can be substituted into $\bar{a} = \Delta v / \Delta t$ (Eq. 3.4) to give

$$\bar{a} = \frac{v \, \Delta r}{r \Delta t}$$

Now imagine that points P and Q in Figure 3.11b come extremely close together. In this case $\Delta \mathbf{v}$ would point toward the center of the circular path, and because the acceleration is in the direction of $\Delta \mathbf{v}$, it too is toward the center. Furthermore, as P and Q approach each other, Δt approaches zero, and the ratio $\Delta r / \Delta t$ approaches the speed v. Hence, in the limit $\Delta t \to 0$, the magnitude of the acceleration is

$$a_r = \frac{v^2}{r}$$

Thus, we conclude that in uniform circular motion the acceleration is directed inward toward the center of the circle and has magnitude v^2/r. You should show that the dimensions of a_r are $\mathrm{L/T^2}$—as required because this is a true acceleration.

In many situations it is convenient to describe the motion of a particle moving with constant speed in a circle of radius r in terms of the **period,** T, which is defined as the time required for one complete revolution. In the time T the particle moves a distance of $2\pi r$, which is equal to the circumference of the particle's circular path. Therefore, because its speed is equal to the circumference of the circular path divided by the period, or $v = 2\pi r / T$, it follows that

$$T = \frac{2\pi r}{v} \qquad [3.20]$$

Thinking Physics 3

An airplane travels from Los Angeles to Sydney, Australia. For 15 hours after cruising altitude is reached, the instruments on the plane indicate that the ground speed holds rock-steady at 700 km/hr and that the heading of the airplane does not change. Is the velocity of the airplane constant?

Reasoning The velocity is not constant, due to the curvature of the Earth. Even though the speed does not change and the heading is always toward Sydney (is this actually true?), the airplane is traveling around a significant portion of the circumference of the Earth. Thus, the direction of the velocity vector does change. We could extend this by imagining that the airplane passes over Sydney and continues (assuming it has enough fuel!) around the Earth until it arrives at Los Angeles again. It is impossible for an airplane to have a constant velocity (relative to the Universe, not to the Earth's surface) and return to its starting point.

EXERCISE 6 Find the acceleration of a particle moving with a constant speed of 8.0 m/s in a circle 2.0 m in radius. Answer 32 $\mathrm{m/s^2}$ toward the center of the circle

3.5 • TANGENTIAL AND RADIAL ACCELERATION

Let us consider the motion of a particle along a curved path where the velocity changes both in direction and in magnitude, as described in Figure 3.12. In this situation, the velocity vector is always tangent to the path; however, the acceleration vector $\mathbf{a}$ is at some angle to the path.

As the particle moves along the curved path in Figure 3.12, we see that the direction of the total acceleration vector, $\mathbf{a}$, changes from point to point. This vector can be resolved into two components: a radial component, a_r, and a tangential component, a_t. The *total* acceleration vector, $\mathbf{a}$, can be written as the vector sum of the component vectors:

$$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t \qquad [3.21]$$ • *Total acceleration*

The tangential acceleration arises from the change in the speed of the particle, and the projection of the tangential acceleration along the direction of the velocity is

$$a_t = \frac{d\,|\mathbf{v}|}{dt} \qquad [3.22]$$ • *Tangential acceleration*

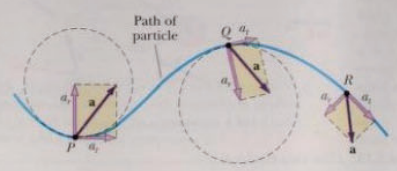


Figure 3.12 The motion of a particle along an arbitrary curved path lying in the xy plane. If the velocity vector $\mathbf{v}$ (always tangent to the path) changes in direction and magnitude, the acceleration $\mathbf{a}$ has a tangential component a_t and a radial component a_r.

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The radial acceleration is due to the change in direction of the velocity vector and has an absolute magnitude given by

Radial or centripetal acceleration

$$a_r = \frac{v^2}{r} \qquad [3.23]$$

where r is the radius of curvature of the path at the point in question. Because $\mathbf{a}_r$ and $\mathbf{a}_t$ are perpendicular component vectors of $\mathbf{a}$, it follows that $a = \sqrt{a_r^2 + a_t^2}$. As in the case of uniform circular motion, $\mathbf{a}_r$ always points toward the center of curvature, as shown in Figure 3.12. Also, at a given speed, a_r is large when the radius of curvature is small (as at points P and Q in Fig. 3.12) and small when r is large (such as at point R). The direction of $\mathbf{a}_t$ is either in the same direction as $\mathbf{v}$ (if v is increasing) or opposite $\mathbf{v}$ (if v is decreasing).

Note that in the case of uniform circular motion, where v is constant, $a_t = 0$ and the acceleration is always radial, as we described in Section 3.4. (*Note:* Eq. 3.23 is identical to Eq. 3.19.) In other words, uniform circular motion is a special case of motion along a curved path. Furthermore, if the direction of $\mathbf{v}$ does not change, then there is no radial acceleration and the motion is one-dimensional ($a_r = 0$, but a_t may not be zero).

Thinking Physics 4

In uniform circular motion, the acceleration vector is always perpendicular to the velocity vector. (a) Suppose a particle is moving at uniform speed around an *elliptical* path. What is the relationship between the acceleration and velocity vectors in this case? (b) Now, let the elliptical path be that of Halley's Comet in orbit around the Sun. This is *not* a case of uniform speed—the comet moves slowly when it is far from the Sun and rapidly when it is close. Are the acceleration and velocity vectors perpendicular in this case?

Reasoning (a) We consider first the case of the particle moving at constant speed around the elliptical path (Fig. 3.13a). Because the speed is constant, there must be no component of the acceleration tangent to the path. Thus, the acceleration must be

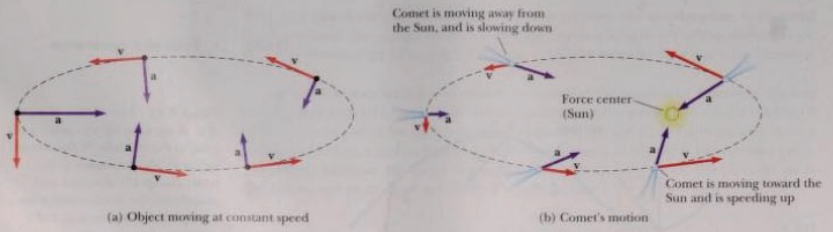


Figure 3.13 (Thinking Physics 4)

perpendicular to the path and, therefore, always perpendicular to the velocity vector. The magnitude of the acceleration vector is not constant—it will be largest at the ends of the ellipse, because the velocity is changing direction the most rapidly there. Figure 3.13a shows the velocity and acceleration vectors at several locations around the path.

(b) In the case of Halley's Comet, or any other object in an elliptical orbit around a gravitational-force center, the direction of the acceleration is the same as the direction of the gravitational force on the object, which is toward the *focus* of the ellipse, where the gravitational force center lies (Fig. 3.13b). Thus, the acceleration and velocity vectors are not perpendicular, except for the points at the ends of the ellipse. As the comet moves away from the Sun, the acceleration has a component opposite to the velocity. As it moves toward the Sun, the acceleration has a component parallel to the velocity. This results in the slowing down of the comet as it moves away from the Sun and its speeding up as it moves toward the Sun. Notice in Figure 3.13b that the velocity vectors have large magnitudes when the comet is near the Sun and small magnitudes when it is far away.

Example 3.4 The Swinging Ball

A ball tied to the end of a string 0.50 m in length swings in a vertical circle under the influence of gravity, as in Figure 3.14. When the string makes an angle of $\theta = 20°$ with the vertical, the ball has a speed of 1.5 m/s. (a) Find the magnitude of the radial component of acceleration at this instant.

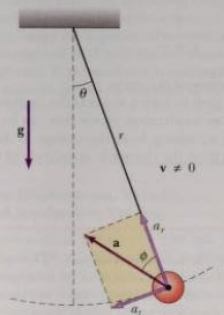


Figure 3.14 (Example 3.4) Motion of a ball suspended by a string of length r. The ball swings with nonuniform circular motion in a vertical plane, and its acceleration $\mathbf{a}$ has a radial component a_r and a tangential component a_t.

Solution Because $v = 1.5$ m/s and $r = 0.50$ m, we find that

$$a_r = \frac{v^2}{r} = \frac{(1.5 \text{ m/s})^2}{0.50 \text{ m}} = 4.5 \text{ m/s}^2$$

(b) When the ball is at an angle θ to the vertical, it has a tangential acceleration of magnitude $g \sin \theta$ (the component of $\mathbf{g}$ tangent to the circle). Therefore, at $\theta = 20°$, $a_t = g \sin 20° = 3.4$ m/s^2. Find the magnitude and direction of the *total* acceleration at $\theta = 20°$.

Solution Because $\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$, the magnitude of $\mathbf{a}$ at $\theta = 20°$ is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(4.5)^2 + (3.4)^2} \text{ m/s}^2 = 5.6 \text{ m/s}^2$$

If ϕ is the angle between $\mathbf{a}$ and the string, then

$$\phi = \tan^{-1}\frac{a_t}{a_r} = \tan^{-1}\left(\frac{3.4 \text{ m/s}^2}{4.5 \text{ m/s}^2}\right) = 36°$$

Note that all of the vectors—$\mathbf{a}$, $\mathbf{a}_t$, and $\mathbf{a}_r$—change in direction *and* magnitude as the ball swings through the circle. When the ball is at its lowest elevation ($\theta = 0$), $a_t = 0$, because there is no tangential component of $\mathbf{g}$ at this angle, and a_r is a *maximum* because v is a maximum. If the ball has enough speed to reach its highest position ($\theta = 180°$), a_t is again zero, but a_r is a minimum because v is now a minimum. Finally, in the two horizontal positions ($\theta = 90°$ and 270°), $|\mathbf{a}| = g$ and a_r has a value between its minimum and maximum values.

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SUMMARY

If a particle moves with *constant* acceleration $\mathbf{a}$ and has velocity $\mathbf{v}_0$ and position $\mathbf{r}_0$ at $t = 0$, its velocity and position vectors at some later time t are

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t \quad [3.8]$$

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{a}t^2 \quad [3.9]$$

For two-dimensional motion in the xy plane under constant acceleration, these vector expressions are equivalent to two component expressions, one for the motion along x and one for the motion along y.

Projectile motion is two-dimensional motion under constant acceleration, where $a_x = 0$ and $a_y = -g$. In this case, if $x_0 = y_0 = 0$, the components of Equations 3.8 and 3.9 reduce to

$$v_x = v_{x0} = \text{constant} \quad [3.10]$$

$$v_y = v_{y0} - gt \quad [3.11]$$

$$x = v_{x0}t \quad [3.12]$$

$$y = v_{y0}t - \tfrac{1}{2}gt^2 \quad [3.13]$$

where $v_{x0} = v_0 \cos\theta_0$, $v_{y0} = v_0 \sin\theta_0$, v_0 is the initial speed of the projectile, and θ_0 is the angle $\mathbf{v}_0$ makes with the positive x axis. Note that these expressions give the velocity components (and hence the velocity vector) and the coordinates (and hence the position vector) at *any* time t that the projectile is in motion.

It is useful to think of projectile motion as the superposition of two motions: (1) uniform motion in the x direction and (2) motion in the vertical direction subject to a constant downward acceleration of magnitude $g = 9.80\ \text{m/s}^2$.

A particle moving in a circle of radius r with constant speed v undergoes a centripetal (or radial) acceleration, $\mathbf{a}_r$, because the direction of $\mathbf{v}$ changes in time. The magnitude of $\mathbf{a}_r$ is

$$a_r = \frac{v^2}{r} \quad [3.19]$$

and its direction is always toward the center of the circle.

If a particle moves along a curved path in such a way that the magnitude and direction of $\mathbf{v}$ change in time, the particle has an acceleration vector that can be described by two components: (1) a radial component, a_r, arising from the change in direction of $\mathbf{v}$, and (2) a tangential component, a_t, arising from the change in magnitude of $\mathbf{v}$.

CONCEPTUAL QUESTIONS

1. If you know the position vectors of a particle at two points along its path and also know the time it took to get from one point to the other, can you determine the particle's instantaneous velocity? Its average velocity? Explain.
2. Explain whether or not the following particles have an acceleration: (a) a particle moving in a straight line with constant speed and (b) a particle moving around a curve with constant speed.
3. Correct the following statement: "The racing car rounds the turn at a constant velocity of 90 miles per hour."
4. A spacecraft drifts through space at a constant velocity. Suddenly a gas leak in the side of the spacecraft causes a constant acceleration of the spacecraft in a direction perpendicular to the initial velocity. The orientation of the spacecraft does not change, so that the acceleration remains perpendicular to the original direction of the velocity. What is the shape of the path followed by the spacecraft in this situation?
5. A ball is projected horizontally from the top of a building. One second later another ball is projected horizontally from the same point with the same velocity. At what point in the motion will the balls be closest to each other? Will the first ball always be traveling faster than the second ball? What will be the time difference between when the balls hit the ground? Can the horizontal projection velocity of the second ball be changed so that the balls arrive at the ground at the same time?
6. At the end of a pendulum's arc, its velocity is zero. Is its acceleration also zero at that point?
7. If a rock is dropped from the top of a sailboat's mast, will it hit the deck at the same point whether the boat is at rest or in motion at constant velocity?
8. Two projectiles are thrown with the same magnitude of initial velocity, one at an angle θ with respect to the level ground and the other at angle $90° - \theta$. Both projectiles will strike the ground at the same distance from the projection point. Will both projectiles be in the air for the same time interval?
9. A projectile is fired at some angle to the horizontal with some initial speed v_0, and air resistance is neglected. Is the projectile a freely falling body? What is its acceleration in the vertical direction? What is its acceleration in the horizontal direction?
10. State which of the following quantities, if any, remain constant as a projectile moves through its parabolic trajectory: (a) speed, (b) acceleration, (c) horizontal component of velocity, (d) vertical component of velocity.
11. The maximum range of a projectile occurs when it is launched at an angle of 45.0° with the horizontal, if air resistance is neglected. If air resistance is not neglected, will the optimum angle be greater or less than 45.0°? Explain.
12. An object moves in a circular path with constant speed v. (a) Is the velocity of the object constant? (b) Is its acceleration constant? Explain.
13. A projectile is fired on the Earth with some initial velocity. Another projectile is fired on the Moon with the *same* initial velocity. Neglecting air resistance, which projectile has the greater range? Which reaches the greater altitude? (Note that the free-fall acceleration on the Moon is about $1.6\ \text{m/s}^2$.)
14. A coin on a table is given an initial horizontal velocity such that it ultimately leaves the end of the table and hits the floor. At the instant the coin leaves the end of the table, a ball is released from the same height and falls to the floor. Explain why the two objects hit the floor simultaneously, even though the coin has an initial velocity.
15. Describe how a driver can steer a car traveling at constant speed so that (a) the acceleration is zero or (b) the magnitude of the acceleration remains constant.
16. An ice skater is executing a figure eight, consisting of two equal, tangent circular paths. Throughout the first loop she increases her speed uniformly, and during the second loop she moves at a constant speed. Make a sketch of her acceleration vector at several points along the path of motion.
17. Construct motion diagrams showing the velocity and acceleration of a projectile at several points along its path if (a) the projectile is fired horizontally and (a) the projectile is fired at an angle θ with the horizontal.
18. A baseball is thrown such that its initial x and y components of velocity are known. Neglecting air resistance, describe how you would calculate, at the instant the ball reaches the top of its trajectory, (a) its coordinates, (b) its velocity, and (c) its acceleration. How would these results change if air resistance were taken into account?

Boxed numbers indicate questions that have answers available in the Student Solutions Manual and Study Guide.

PROBLEMS

Section 3.1 The Displacement, Velocity, and Acceleration Vectors

1. A motorist drives south at 20.0 m/s for 3.00 min, then turns west and travels at 25.0 m/s for 2.00 min, and finally travels northwest at 30.0 m/s for 1.00 min. For this 6.00-min trip, find (a) the net vector displacement of the motorist, (b) the motorist's average speed, and (c) the average velocity of the motorist.
2. Suppose that the position-vector function for a particle is given as $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, with $x(t) = at + b$ and $y(t) = ct^2 + d$, where $a = 1.00\ \text{m/s}$, $b = 1.00\ \text{m}$, $c = 0.125\ \text{m/s}^2$, and $d = 1.00\ \text{m}$. (a) Calculate the average velocity during the time interval $t = 2.00\ \text{s}$ to $t = 4.00\ \text{s}$. (b) Determine the velocity and speed at $t = 2.00\ \text{s}$.
3. A golf ball is hit off a tee at the edge of a cliff. Its x and y coordinates versus time are given by the following expressions:

$$x = (18.0\ \text{m/s})t \qquad y = (4.00\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2$$

(a) Write a vector expression for the position $\mathbf{r}$ versus time t, using the unit vectors $\mathbf{i}$ and $\mathbf{j}$. By using derivatives, repeat

Boxed numbers indicate problems that have full solutions available in the Student Solutions Manual and Study Guide. Cyan numbers indicate intermediate level problems and magenta numbers indicate more challenging problems.

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for (b) the velocity vector $\mathbf{v}$ versus time and (c) the acceleration vector $\mathbf{a}$ versus time. (d) Find the x and y coordinates of the golf ball at $t = 3.00$ s. Using the unit vectors $\mathbf{i}$ and $\mathbf{j}$, write expressions for (e) the velocity $\mathbf{v}$ and (f) the acceleration $\mathbf{a}$ at the instant $t = 3.00$ s.

4. The coordinates of an object moving in the xy plane vary with time according to the equations $x = -(5.00\text{ m})\sin(t)$ and $y = (4.00\text{ m}) - (5.00\text{ m})\cos(t)$, where t is in seconds. (a) Determine the components of velocity and components of acceleration at $t = 0$ s. (b) Write expressions for the position vector, the velocity vector, and the acceleration vector at any time $t > 0$. (c) Describe the path of the object in an xy plot.

Section 3.2 Two-Dimensional Motion with Constant Acceleration

5. At $t = 0$, a particle moving in the xy plane with constant acceleration has a velocity of $\mathbf{v}_0 = (3.00\mathbf{i} - 2.00\mathbf{j})$ m/s at the origin. At $t = 3.00$ s, the particle's velocity is $\mathbf{v} = (9.00\mathbf{i} + 7.00\mathbf{j})$ m/s. Find (a) the acceleration of the particle and (b) its coordinates at any time t.

6. The vector position of a particle varies in time according to the expression $\mathbf{r} = (3.00\mathbf{i} - 6.00t^2\mathbf{j})$ m. (a) Find expressions for the velocity and acceleration as functions of time. (b) Determine the particle's position and velocity at $t = 1.00$ s.

7. A fish swimming in a horizontal plane has velocity $\mathbf{v}_0 = (4.00\mathbf{i} + 1.00\mathbf{j})$ m/s when its displacement from a certain rock is $\mathbf{r}_0 = (10.0\mathbf{i} - 4.00\mathbf{j})$ m. After the fish swims with constant acceleration for 20.0 s, its velocity is $\mathbf{v} = (20.0\mathbf{i} - 5.00\mathbf{j})$ m/s. (a) What are the components of the acceleration? (b) What is the direction of the acceleration with respect to unit vector $\mathbf{i}$? (c) Where is the fish at $t = 25.0$ s and in what direction is it moving?

8. A particle initially located at the origin has an acceleration of $\mathbf{a} = 3.00\mathbf{j}$ m/s^2 and an initial velocity of $\mathbf{v}_0 = 5.00\mathbf{i}$ m/s. Find (a) the vector position and velocity at any time t and (b) the coordinates and speed of the particle at $t = 2.00$ s.

Section 3.3 Projectile Motion

(Neglect air resistance in all problems. Use $g = 9.80$ m/s^2.)

9. In a local bar, a customer slides an empty beer mug on the counter for a refill. The bartender is momentarily distracted and does not see the mug, which slides off the counter and strikes the floor 1.40 m from the base of the counter. If the height of the counter is 0.860 m, (a) with what velocity did the mug leave the counter, and (b) what was the direction of the mug's velocity just before it hit the floor?

10. A student decides to measure the muzzle velocity of a pellet from his gun. He points the gun horizontally. He places a target on a vertical wall a distance x away from the gun. The pellet hits the target a vertical distance y below the gun. (a) Show that the position of the pellet when traveling through the air is given by $y = Ax^2$, where A is a constant. (b) Express the constant A in terms of the initial velocity and the free-fall acceleration. (c) If $x = 3.00$ m and $y = 0.210$ m, what is the initial speed of the pellet?

11. A golfer wants to drive a golf ball a distance of 310 yards (283 m). If the four-wood launches the ball at 15.0° above the horizontal, what must be the initial speed of the ball to achieve the required distance?

12. One strategy in a snowball fight is to throw a snowball at a high angle over level ground. While your opponent is watching the first one, you throw a second snowball at a low angle timed to arrive before or at the same time as the first one. Assume both snowballs are thrown with a speed of 25.0 m/s. The first one is thrown at an angle of 70.0° with respect to the horizontal. (a) At what angle should the second snowball be thrown to arrive at the same point as the first? (b) How many seconds later should the second snowball be thrown after the first to arrive at the same time?

13. A tennis player standing 12.6 m from the net hits the ball at 3.00° above the horizontal. To clear the net, the ball must rise at least 0.330 m. If the ball just clears the net at the apex of its trajectory, how fast was the ball moving when it left the racket?

14. An artillery shell is fired with an initial velocity of 300 m/s at 55.0° above the horizontal. It explodes on a mountainside 42.0 s after firing. What are the x and y coordinates of the shell where it explodes relative to its firing point?

15. An astronaut on a strange planet finds that she can jump a *maximum* horizontal distance of 15.0 m if her initial speed is 3.00 m/s. What is the free-fall acceleration on the planet?

16. A ball is tossed from an upper-story window. The ball is given an initial velocity of 8.00 m/s at an angle of 20.0° below the horizontal. It strikes the ground 3.00 s later. (a) How far horizontally from the base of the building does the ball strike the ground? (b) Find the height from which the ball was thrown. (c) How long does it take the ball to reach a point 10.0 m below the level of launching?

17. A ball is thrown horizontally from the top of a building 35.0 m high. The ball strikes the ground at a point 80.0 m from the base of the building. Find (a) the time the ball is in flight, (b) its initial velocity, and (c) the x and y components of velocity just before the ball strikes the ground.

18. A cannon with a muzzle speed of 1000 m/s is used to destroy a target on a mountain top. The target is 2000 m from the cannon horizontally and 800 m above the ground. At what angle, relative to the ground, should the cannon be fired?

19. A placekicker must kick a football from a point 36.0 m (about 40 yards) from the goal and the ball must clear the crossbar, which is 3.05 m high. The ball leaves the ground when kicked with a speed of 20.0 m/s at an angle of 53.0° to the horizontal. (a) By how much does the ball clear or fall short of clearing the crossbar? (b) Does the ball approach the crossbar while still rising or while falling?

20. A fireman, 50.0 m away from a burning building, directs a stream of water from a fire hose at an angle of 30.0° above the horizontal. If the speed of the stream is 40.0 m/s, at what height will the stream of water strike the building?

21. The speed of a projectile when it reaches its maximum height is one half of its speed when it is at half its maximum height. What is the initial projection angle of the projectile?

22. A soccer player kicks a rock horizontally off a 40.0-m–high cliff into a pool of water. If the player hears the sound of the splash 3.00 s later, what was the initial speed given to the rock? Assume the speed of sound in air to be 343 m/s.

23. During the 1968 Olympics in Mexico City, Bob Beamon executed a record long jump. The horizontal distance he achieved was 8.90 m. His center of gravity started at an elevation of 1.00 m, reached a maximum height of 1.90 m, and finished at 0.150 m. From these data determine (a) his time of flight, (b) his horizontal and vertical velocity components at the takeoff time, and (c) his takeoff angle.

Section 3.4 Uniform Circular Motion

24. The orbit of the Moon around the Earth is approximately circular, with a mean radius of 3.84×10^8 m. It takes 27.3 days for the Moon to complete one revolution around the Earth. Find (a) the mean orbital speed of the Moon and (b) its centripetal acceleration.

25. An athlete rotates a 1.00-kg discus along a circular path of radius 1.06 m (Figure P3.25). The maximum speed of the discus is 20.0 m/s. Determine the magnitude of the maximum radial acceleration of the discus.

26. A tire 0.500 m in radius rotates at a constant rate of 200 rev/min. Find the speed and magnitude of acceleration of a small stone lodged in the outer edge of the tread of the tire.

27. Young David who slew Goliath experimented with slings before tackling the giant. He found that he could revolve a sling of length 0.600 m at the rate of 8.00 rev/s. If he increased the length to 0.900 m, he could revolve the sling only 6.00 times per second. (a) Which rate of rotation gives the greater linear speed? (b) What is the centripetal acceleration at 8.00 rev/s? (c) What is the centripetal acceleration at 6.00 rev/s?

28. An astronaut standing on the Moon fires a gun so that the bullet leaves the barrel initially moving in a horizontal direction. (a) What must be the muzzle speed of the bullet so that it travels completely around the Moon and returns to its original location? (b) How long is the bullet in flight? Assume that the free-fall acceleration on the Moon is one sixth that on the Earth.

29. If the rotation of the Earth increased so that the centripetal acceleration was equal to the gravitational acceleration at the Equator, (a) what would be the tangential speed of a person standing at the Equator, and (b) how long would a day be?



Figure P3.25

Section 3.5 Tangential and Radial Acceleration

30. An automobile, the speed of which is increasing at a rate of 0.600 m/s^2, travels along a circular road of radius 20.0 m. When the instantaneous speed of the automobile is 4.00 m/s, find (a) the tangential acceleration component, (b) the centripetal acceleration component, and (c) the magnitude and direction of the total acceleration.

31. A train slows down as it rounds a sharp horizontal turn, slowing from 90.0 km/h to 50.0 km/h in the 15.0 s that it takes to round the bend. The radius of the curve is 150 m. Compute the acceleration at the moment the train speed reaches 50.0 km/h, assuming it continues to slow down at this time.

32. A point on a rotating turntable 20.0 cm from the center accelerates from rest to a final speed of 0.700 m/s in 1.75 s. At $t = 1.25$ s, find the magnitude and direction of

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(a) the centripetal acceleration, (b) the tangential acceleration, and (c) the total acceleration of the point.

33. Figure P3.33 represents the total acceleration of a particle moving clockwise in a circle of radius 2.50 m at a given instant of time. At this instant, find (a) the centripetal acceleration, (b) the speed of the particle, and (c) its tangential acceleration.

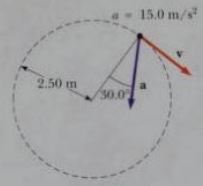


Figure P3.33

34. A ball swings in a vertical circle at the end of a 1.50-m long rope. When it is 36.9° past the lowest point on its way up, the ball's total acceleration is $(-22.5\mathbf{i} + 20.2\mathbf{j})$ m/s². At that instant, (a) sketch a vector diagram showing the components of its acceleration, (b) determine the magnitude of its centripetal acceleration, and (c) determine the speed and velocity of the ball.

Additional Problems

35. At $t = 0$ a particle leaves the origin with a velocity of 6.00 m/s in the positive y direction. Its acceleration is given by $\mathbf{a} = (2.00\mathbf{i} - 3.00\mathbf{j})$ m/s². When the particle reaches its *maximum* y coordinate, its y component of velocity is zero. At this instant, find (a) the velocity of the particle and (b) its x and y coordinates.

36. A ball on the end of a string is whirled around in a horizontal circle of radius 0.300 m. The plane of the circle is 1.20 m above the ground. The string breaks and the ball lands 2.00 m (horizontally) away from the point on the ground directly beneath the ball's location when the string breaks. Find the centripetal acceleration of the ball during its circular motion.

37. A home run is hit in such a way that the baseball just clears a wall 21.0 m high, located 130 m from home plate. The ball is hit at an angle of 35.0° to the horizontal, and air resistance is negligible. Find (a) the initial speed of the ball, (b) the time it takes the ball to reach the wall, and (c) the velocity components and the speed of the ball when it reaches the wall. (Assume the ball is hit at a height of 1.00 m above the ground.)

38. The astronaut orbiting the Earth in the photograph is preparing to dock with a spinning Westar VII satellite. The satellite is in a circular orbit 600 km above the Earth's surface, where the free-fall acceleration is 8.21 m/s². The radius of the Earth is 6400 km. Determine the speed of the satellite and the time required to complete one orbit around the Earth.



Figure P3.38 *(Courtesy of NASA)*

39. A particle has velocity components

$$v_x = +4.00 \text{ m/s} \qquad v_y = -(6.00 \text{ m/s}^2)t + 4.00 \text{ m/s}$$

Calculate the speed of the particle and the direction $\theta = \tan^{-1}(v_y/v_x)$ of the velocity vector at $t = 2.00$ s.

40. When baseball outfielders throw the ball in, they usually allow it to take one bounce on the theory that the ball arrives sooner this way. Suppose that after the bounce the ball rebounds at the same angle θ as before, as in Figure P3.40 but loses half its speed. (a) Assuming the ball is always thrown with the same initial speed, at what angle θ should the ball be thrown in order to go the same distance D with one bounce (blue path) as one thrown upward at 45.0° with no bounce (green path)? (b) Determine the ratio of the times for the one-bounce and no-bounce throws.

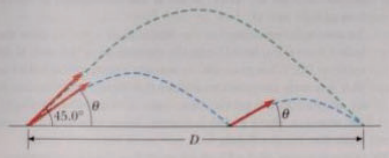


Figure P3.40

41. A girl can throw a ball a maximum horizontal distance of 40.0 m on a level field. How far can she throw the same ball vertically upward? Assume that her muscles give the ball the same speed in each case.

42. A girl can throw a ball a maximum horizontal distance of R on a level field. How far can she throw the same ball vertically upward? Assume that her muscles give the ball the same speed in each case.

43. A bomber is flown horizontally, with a ground speed of 275 m/s, at an altitude of 3000 m over level terrain. Neglect the effects of air resistance. (a) How far from the point that is vertically under the point of release will a bomb hit the ground? (b) If the plane maintains its original course and speed, where will it be when the bomb hits the ground? (c) For the preceding conditions, at what angle from the vertical at the point of release must the telescopic bomb sight be set so that the bomb will hit the target seen in the sight at the time of release?

44. A football is thrown toward a receiver with an initial speed of 20.0 m/s, at an angle of 30.0° above the horizontal. At that instant, the receiver is 20.0 m from the quarterback. In what direction and with what constant speed should the receiver run in order to catch the football at the level at which it was thrown?

45. A hawk is flying horizontally at 10.0 m/s in a straight line, 200 m above the ground. A mouse it has been carrying is released from its grasp. The hawk continues on its path at the same speed for 2.00 s before attempting to retrieve its prey. To accomplish the retrieval, it dives in a straight line at constant speed and recaptures the mouse 3.00 m above the ground. (a) Assuming no air resistance, find the diving speed of the hawk. (b) What angle did the hawk make with the horizontal during its descent? (c) For how long did the mouse "enjoy" free fall?

46. A rocket is launched at an angle of 53.0° to the horizontal with an initial speed of 100 m/s. For 3.00 s it moves along its initial line of motion with an acceleration of 30.0 m/s². Then its engines fail and the rocket proceeds to move in free fall. Find (a) the maximum altitude reached by the rocket, (b) its total time of flight, and (c) its horizontal range.

47. A car is parked on a steep incline overlooking the ocean, where the incline makes an angle of 37.0° with the horizontal. The negligent driver leaves the car in neutral, and the parking brakes are defective. The car rolls from rest down the incline with a constant acceleration of 4.00 m/s², traveling 50.0 m to the edge of a cliff. The cliff is 30.0 m above the ocean. Find (a) the speed of the car when it reaches the edge of the cliff and the time it takes to get there, (b) the velocity of the car when it lands in the ocean, (c) the total time the car is in motion, and (d) the position of the car when it lands in the ocean, relative to the base of the cliff.

48. The determined coyote is out once more to try to capture the elusive roadrunner. The coyote wears a pair of Acme jet-powered roller skates, which provide a constant horizontal acceleration of 15.0 m/s², as pictured in Figure P3.48. The coyote starts off at rest 70.0 m from the edge of a cliff at the instant the roadrunner zips past him in the direction of the cliff. (a) If the roadrunner moves with constant speed, determine the minimum speed he must have in order to reach the cliff before the coyote. (b) If the cliff is 100 m above the base of a canyon, determine where the coyote lands in the canyon (assume his skates are still in operation when he is in flight). (c) Determine the coyote's velocity components just before he lands in the canyon. (As usual, the roadrunner saves himself by making a sudden turn at the cliff.)

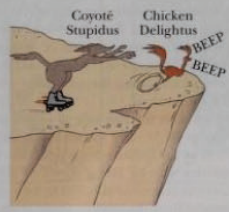


Figure P3.48

49. A skier leaves the ramp of a ski jump with a velocity of 10.0 m/s, 15.0° above the horizontal, as in Figure P3.49. The slope is inclined at 50.0°, and air resistance is negligi-

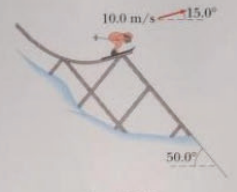


Figure P3.49

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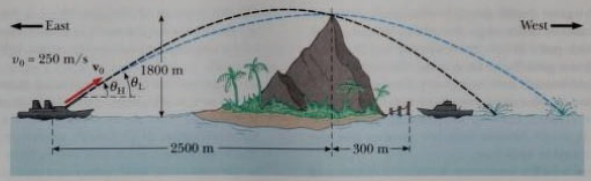


Figure P3.51

ble. Find (a) the distance from the ramp to where the jumper lands and (b) the velocity components just before the landing. (How do you think the results might be affected if air resistance were included? Note that jumpers lean forward in the shape of an airfoil, with their hands at their sides, to increase their distance. Why does this work?)

50. Describe what you might do to give your hand a large acceleration without hurting yourself or striking your hand against anything. Compute an order-of-magnitude estimate of this acceleration, stating the quantities you measure or estimate and their values.

51. An enemy ship is on the east side of a mountain island, as shown in Figure P3.51. The enemy ship can maneuver to within 2500 m of the 1800-m–high mountain peak and can shoot projectiles with an initial speed of 250 m/s. If the western shoreline is horizontally 300 m from the peak, what are the distances from the western shore at which a ship can be safe from the bombardment of the enemy ship?

Spreadsheet Problems

S1. The punter for the St. Louis Rams needs to kick the football 40 yd to 45 yd (36.6 m to 41.1 m) to keep the opposing team inside the 5-yd line. Use Spreadsheet 3.1 to estimate the angle at which he should kick the ball. *Note:* He must keep the ball in the air as long as possible in order to help prevent a runback (in other words, he wants a long "hang time"). The initial speed of the ball should be in the range of 20 m/s to 30 m/s.

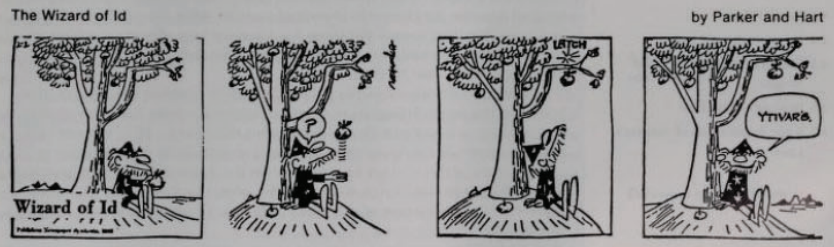
S2. Modify Spreadsheet 3.1 by adding two columns and then calculate the maximum height reached by the football in Problem S1 and the total time it is in the air.

S3. The x and y coordinates of a projectile are given by Equations 3.12 and 3.13. Using these equations, set up a spreadsheet to calculate the x and y coordinates of a projectile and the components of its velocity v_x and v_y as functions of time. The initial speed and initial angle of the projectile should be input parameters. Use the graph function of the spreadsheet to plot the x and y coordinates as functions of time. Also graph v_x and v_y as functions of time. Use this spreadsheet to solve the following problem.

The New York Giants are tied with the Chicago Bears with only a few seconds left in the game. The Giants have the football and call the placekicker into the game. He must kick the ball 52 yd (47.5 m) for a field goal. If the crossbar of the goal is 10 ft (3.05 m) high, at what angle and speed should he kick the ball to score the winning points? Is there only one solution to this problem? Solutions can be found by varying the parameters and studying the graph of y versus x.

ANSWERS TO CONCEPTUAL PROBLEMS

1. (a) Yes. Although its speed may be constant, the direction of its motion (that is, the direction of **v**) may change, causing an acceleration. For example, an object moving in a circle with constant speed has an acceleration directed toward the center of the circle. (b) No. An object that moves with constant velocity has zero acceleration. Constant velocity means that both the direction and magnitude of **v** remain constant.
2. A constant velocity is interpreted as constant in both magnitude and direction. Thus, a constant velocity clearly implies a nonvarying speed. On the other hand, a constant speed indicates that only the magnitude of the velocity vector is constant—the direction can change. Thus, as long as a particle covers the same distance in equal time intervals, the speed is constant, and its direction can change continuously, giving a varying velocity. A familiar example is a particle moving in a circular path at constant speed, such that the velocity is constantly changing direction.
3. You should simply throw it straight up in the air. Because the ball was moving along with you, it would follow a parabolic trajectory with a horizontal component of velocity that is the same as yours.
4. (a) At the top of its flight, **v** is horizontal and **a** is vertical. This is the only point at which the velocity and acceleration vectors are perpendicular. (b) If the object is thrown straight up or down, then **v** and **a** will be parallel throughout the downward motion. Otherwise, the velocity and acceleration vectors are never parallel.



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4

The Laws of Motion

CHAPTER OUTLINE

4.1 The Concept of Force
4.2 Newton's First Law and Inertial Frames
4.3 Inertial Mass
4.4 Newton's Second Law
4.5 The Gravitational Force and Weight
4.6 Newton's Third Law
4.7 Some Applications of Newton's Laws

In the preceding two chapters on kinematics, we described the motions of particles based on the definitions of displacement, velocity, and acceleration. However, we would like to be able to answer specific questions related to the causes of motion, such as, "What mechanism causes motion?" and, "Why do some objects accelerate at higher rates than others?" In this chapter we shall describe the change in motion of particles using the concepts of force and mass. We then discuss the three fundamental laws of motion, which are based on experimental observations and were formulated about three centuries ago by Sir Isaac Newton.

Classical mechanics describes the relationship between the motion of a body and the forces acting on it. Classical mechanics deals only with objects that are large compared with the dimensions of atoms ($\sim 10^{-10}$ m) and that move at speeds much slower than the speed of light (3.00×10^8 m/s).

We learn in this chapter how to describe the acceleration of an object in terms of the resultant force acting on the object and its mass. This force represents the interaction of the object with its environment. Mass is a measure of the object's tendency to resist an acceleration when a force acts on it.

We also discuss *force laws*, which describe how to



Calf-roping, as shown in this photograph taken in Steamboat, Colorado, is a standard rodeo event. The external forces exerted on the horse are the force of friction between the horse and ground, the downward force of gravity, the force exerted by the rope attached to the calf, the downward force exerted by the cowboy on the horse, and the upward force of the ground. Can you identify the forces acting on the calf? *(Fourbyfive, Inc.)*

calculate the force on an object if its environment is known. We shall see that, although the force laws are simple in form, they successfully explain a wide variety of phenomena and experimental observations. They, together with the laws of motion, are the foundations of classical mechanics.

4.1 • THE CONCEPT OF FORCE

Everyone has a basic understanding of the concept of force as a result of everyday experiences. When you push or pull an object, you exert a force on it. You exert a force when you throw or kick a ball. In these examples, the word *force* is associated with the result of muscular activity and with some change in the state of motion of an object. Forces do not always cause an object to move, however. For example, as you sit reading this book, the force of gravity acts on your body, and yet you remain stationary. Likewise, you can push on a block of stone and yet fail to move it.

What force (if any) causes a distant star to drift freely through space? Newton answered such questions by stating that the change in velocity of an object is caused by unbalanced forces. Therefore, if an object moves with uniform motion (constant velocity), no force is required to maintain the motion. Because only a force can cause a change in velocity, we can think of force as that which causes a body to accelerate.

• *A body accelerates due to an external force.*

Now consider a situation in which several forces act simultaneously on an object. In this case, the object accelerates only if the **net force** acting on it is not equal to zero. We shall often refer to the net force as the *resultant force* or as the *unbalanced force*. **If the net force is zero, the acceleration is zero and the velocity of the object remains constant.** That is, if the net force acting on the object is zero, either the object will be at rest or it will move with constant velocity. **When a body has constant velocity or is at rest, it is said to be in equilibrium.**

• *Definition of equilibrium*

This chapter is concerned with the relation between the force on an object and the acceleration of that object. If you pull on a spring, as in Figure 4.1a, the spring stretches. If the spring is calibrated, the distance it stretches can be used to measure the strength of the force. If a child pulls hard enough on a cart to overcome friction, as in Figure 4.1b, the cart moves. When a football is kicked, as in Figure 4.1c, it is both deformed and set in motion. These are all examples of a class of forces called *contact forces*. That is, they represent the result of physical contact between two objects.

Another class of forces, which do not involve physical contact between two objects but act through empty space, are known as *field forces*. The force of gravitational attraction between two objects is an example of this class of force, illustrated in Figure 4.1d. This gravitational force keeps objects bound to the Earth and gives rise to what we commonly call the *weight* of an object. The planets of our solar system are bound under the action of gravitational forces. Another common example of a field force is the electric force that one electric charge exerts on another electric charge, as in Figure 4.1e. These charges might be an electron and proton forming a hydrogen atom. A third example of a field force is the force that a bar magnet exerts on a piece of iron, as shown in Figure 4.1f. The forces holding an atomic nucleus together are also field forces but are usually very short-range. They are the dominating interaction for particle separations of the order of 10^{-15} m.

Early scientists, including Newton, were uneasy with the concept of a force acting between two disconnected objects. To overcome this conceptual problem,



A football is set in motion as a result of the contact force **F** on it due to the kicker's foot. The ball is distorted in the short time it is in contact with the foot. *(Ralph Cowan, Tony Stone Worldwide)*

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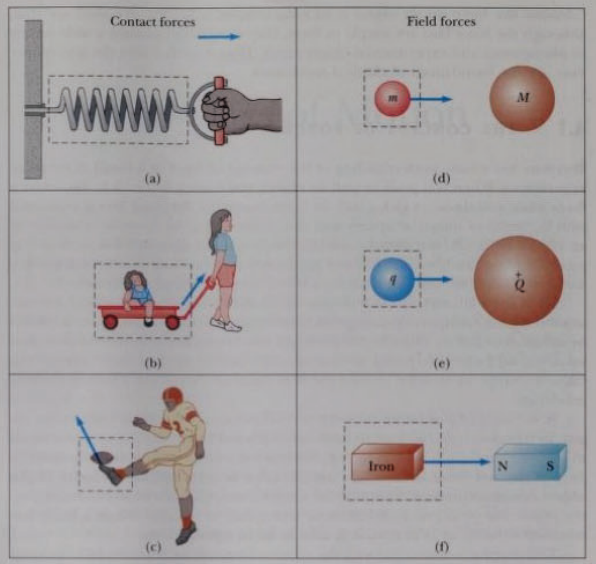


Figure 4.1 Some examples of forces applied to various objects. In each case a force is exerted on the particle or object within the boxed area. The environment external to the boxed area provides this force.

Michael Faraday (1791–1867) introduced the concept of a *field*. According to this approach, when a mass m_1 is placed at some point P near a mass m_2, one can say that m_1 interacts with m_2 by virtue of the gravitational field that exists at P. The field at P is created by mass m_2. Likewise, a field exists at the position of m_2 created by m_1. In fact, all objects create a gravitational field in the space around them.

The distinction between contact forces and field forces is not as sharp, however, as the previous section might lead you to believe. At the atomic level, all the forces we classify as contact forces turn out to be due to repulsive electrical (field) forces of the type illustrated in Figure 4.1e. Nevertheless, in developing models for macroscopic phenomena, it is convenient to use both classifications of forces. However, the only known *fundamental* forces in nature are all field forces: (1) gravitational attractions between objects, (2) electromagnetic forces between charges, (3) strong nuclear forces between subatomic particles, and (4) weak nuclear forces that arise in certain radioactive decay processes. In classical physics we are concerned only with gravitational and electromagnetic forces.

Fundamental forces in nature •

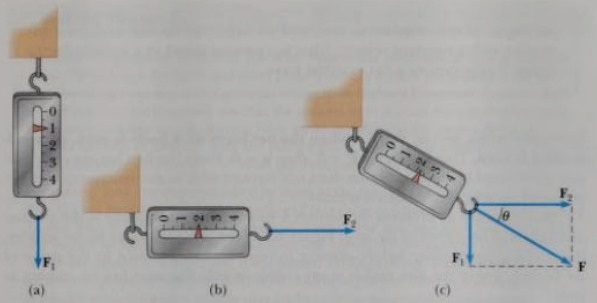


Figure 4.2 The vector nature of a force is tested with a spring scale. (a) The downward vertical force $\mathbf{F}_1$ elongates the spring 1 unit. (b) The horizontal force $\mathbf{F}_2$ elongates the spring 2 units. (c) The combination of $\mathbf{F}_1$ and $\mathbf{F}_2$ elongates the spring $\sqrt{1^2 + 2^2} = \sqrt{5}$ units.

It is sometimes convenient to use the deformation of a spring to measure force. Suppose a force is applied vertically to a spring that has a fixed upper end, as in Figure 4.2a. We can calibrate the spring by defining the unit force, $\mathbf{F}_1$, as the force that produces an elongation of 1.00 cm. If a force $\mathbf{F}_2$, applied horizontally as in Figure 4.2b, produces an elongation of 2.00 cm, the magnitude of $\mathbf{F}_2$ is 2 units. If the two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are applied simultaneously, as in Figure 4.2c, the elongation of the spring is $\sqrt{5} = 2.24$ cm. The single force $\mathbf{F}$ that would produce this same elongation is the vector sum of $\mathbf{F}_1$ and $\mathbf{F}_2$, as described in Figure 4.2c. That is, $|\mathbf{F}| = \sqrt{F_1^2 + F_2^2} = \sqrt{5}$ units, and its direction is $\theta = \arctan(-0.500) = -26.6°$. **Because forces are vectors, you must use the rules of vector addition to get the resultant force on a body.**

Springs that elongate in proportion to an applied force are said to obey **Hooke's law.** Such springs can be constructed and calibrated to measure unknown forces.

4.2 • NEWTON'S FIRST LAW AND INERTIAL FRAMES

Before about 1600, scientists believed that the natural state of matter was the state of rest. Galileo was the first to take a different approach to motion and the natural state of matter. He devised thought experiments, such as an object moving on a frictionless surface, and concluded that it is not the nature of an object to stop once it is set in motion; rather, it is an object's nature to resist changes in its motion. In his words, "Any velocity, once imparted to a moving body, will be rigidly maintained as long as the external causes of retardation are removed."

This new approach to motion was later formalized by Newton in a statement that has come to be known as **Newton's first law of motion:**

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A statement of Newton's first law

An object at rest remains at rest, and an object in motion continues in motion with a constant velocity (that is, constant speed in a straight line), unless it experiences a net external force.

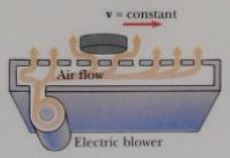


Figure 4.3 A disk moving on a layer of air is an example of uniform motion—that is, motion in which the acceleration is zero.

In simpler terms, we can say that **when the net force on a body is zero, its acceleration is zero.** That is, when $\Sigma \mathbf{F} = 0$, then $\mathbf{a} = 0$. From the first law, we conclude that an isolated body (a body that does not interact with its environment) is either at rest or moving with constant velocity.

One example of uniform motion on a nearly frictionless plane is the motion of a light disk on a layer of air, as in Figure 4.3. If the disk is given an initial velocity, it coasts a great distance before coming to rest. This idea is used in the game of air hockey, where the disk makes many collisions with the walls before coming to rest.

As a second example, consider a spaceship traveling in space, far removed from any planets or other matter. The spaceship requires some propulsion system to change its velocity. However, if the propulsion system is turned off when the spaceship reaches a velocity $\mathbf{v}$, the spaceship "coasts" in space with that velocity, and the astronauts get a free ride (that is, no propulsion system is required to keep them moving at the velocity $\mathbf{v}$).

Inertial Frames

Inertial frame

Newton's first law, sometimes called the *law of inertia*, defines a special set of reference frames called inertial frames. **An inertial frame of reference is one in which Newton's first law is valid.** Any reference frame that moves with constant velocity with respect to an inertial frame is itself an inertial frame. A reference frame that moves with constant velocity relative to the distant stars is the best approximation of an inertial frame. The Earth is not an inertial frame because of its orbital motion around the Sun and rotational motion around its own axis. As the Earth travels in its nearly circular orbit around the Sun, it experiences a centripetal acceleration of about $4.4 \times 10^{-3}\ \mathrm{m/s^2}$ toward the Sun. In addition, because the Earth rotates around its own axis once every 24 h, a point on the Equator experiences an additional centripetal acceleration of $3.37 \times 10^{-2}\ \mathrm{m/s^2}$ toward the center of the Earth. However, these accelerations are small compared with g and can often be neglected. In most situations **we shall assume that a frame on or near the Earth's surface is an inertial frame.**

Thus, if an object is moving with constant velocity, an observer in one inertial frame (say, one at rest with respect to the object) will claim that the acceleration and the resultant force on the object are zero. An observer in *any other* inertial frame will also find that $\mathbf{a} = 0$ and $\mathbf{F} = 0$ for the object. According to the first law, a body at rest and one moving with constant velocity are equivalent. Unless stated otherwise, we shall usually write the laws of motion with respect to an observer "at rest" in an inertial frame.

Thinking Physics 1

When the Copernican Theory of the Solar System was proposed, a natural question arose: "What keeps the Earth and other planets moving in their paths around the Sun?" An interesting response to this question comes from Richard Feynman[1]: "In those days, one of the theories proposed was that the planets went around because behind them there were invisible angels, beating their wings and driving the planets forward. . . . It turns out that in order to keep the planets going around, the invisible angels must fly in a different direction. . . ." What did Feynman mean by this?

Reasoning First, the question asked by those at the time of Copernicus indicates that they did not have a proper understanding of inertia, as described by Newton's first law. At that time in history, before Galileo and Newton, the interpretation was that *motion* was caused by force. This is different from our current understanding that *changes in motion* are caused by force. Thus, it was natural for Copernicus' contemporaries to ask what force propelled a planet in its orbit. According to our current understanding, it is equally natural for us to realize that no force is necessary—the motion simply continues due to inertia. Thus, in Feynman's imagery, we do not need the angels pushing the planet *from behind*. We do need the angels to push *inward*, however, to provide the centripetal acceleration needed to change its direction into a circle. Of course, the angels are not real, from a scientific point of view—they are a metaphor for the *gravitational force*.

CONCEPTUAL PROBLEM 1

Is it possible to have motion in the absence of a force?

CONCEPTUAL PROBLEM 2

If a single force acts on an object, is there an acceleration? If an object experiences an acceleration, is a force acting on it? If an object experiences no acceleration, is there no force acting?

4.3 • INERTIAL MASS

Inertia

If you attempt to change the velocity of an object, the object resists this change. **Inertia** is solely a property of an individual object; it is a measure of the response of an object to an external force. For instance, consider two large, solid cylinders of equal size, one balsa wood and the other steel. If you were to push the cylinders along a horizontal, rough surface, the force required to give the steel cylinder some acceleration would be larger than the force needed to give the balsa wood cylinder the same acceleration. Therefore, we say that the steel cylinder has more inertia than the balsa wood cylinder.

Mass is used to measure inertia, and the SI unit of mass is the kilogram. The greater the mass of a body, the less it accelerates (changes its state of motion) under the action of an applied force.

A quantitative measurement of mass can be made by comparing the accelera-

[1] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Vol. 1, Reading, Massachusetts, 1963, Addison-Wesley Publishing Co., p. 7-2.

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tions that a given force produces on different bodies. Suppose a force acting on a body of mass m_1 produces an acceleration $\mathbf{a}_1$, and the *same force* acting on a body of mass m_2 produces an acceleration $\mathbf{a}_2$. The ratio of the two masses is defined as the *inverse* ratio of the magnitudes of the accelerations produced by the same force:

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} \qquad [4.1]$$

If one mass is standard and known—say, 1 kg—the mass of an unknown object can be obtained from acceleration measurements. For example, if the standard 1-kg mass undergoes an acceleration of 3 m/s² under the influence of some force, a 2-kg mass will undergo an acceleration of 1.5 m/s² under the action of the same force.

Mass is an inherent property of a body, independent of the body's surroundings and of the method used to measure it. It is an experimental fact that **mass is a scalar quantity.** Finally, **mass is a quantity that obeys the rules of ordinary arithmetic.** That is, several masses can be combined in simple numerical fashion. For example, if you combine a 3-kg mass with a 5-kg mass, their total mass is 8 kg. This can be verified experimentally by comparing the acceleration of each object produced by a known force with the acceleration of the combined system using the same force.

Mass should not be confused with weight. **Mass and weight are two different quantities.** The weight of a body is equal to the magnitude of the force exerted by the Earth on the body and varies with location. For example, a person who weighs 180 lb on Earth weighs only about 30 lb on the Moon. However, the mass of a body is the same everywhere. An object having a mass of 2 kg on Earth also has a mass of 2 kg on the Moon.



Isaac Newton (1642–1727)

An English physicist and mathematician, Newton was one of the most brilliant scientists in history. Before the age of 30, he formulated the basic concepts and laws of mechanics, discovered the law of universal gravitation, and invented the mathematical methods of calculus. As a consequence of his theories, Newton was able to explain the motions of the planets, the ebb and flow of the tides, and many special features of the motions of the Moon and the Earth. He also interpreted many fundamental observations concerning the nature of light. His contributions to physical theories dominated scientific thought for two centuries and remain important today. *(Giraudon/Art Resource)*

4.4 · NEWTON'S SECOND LAW

Newton's first law explains what happens to an object when the net external force on it is zero: It either remains at rest or moves in a straight line with constant speed. Newton's second law answers the question of what happens to an object that has a nonzero net force acting on it.

Imagine you are pushing a block of ice across a frictionless horizontal surface. When you exert some horizontal force $\mathbf{F}$, the block moves with some acceleration $\mathbf{a}$. If you apply a force twice as large, the acceleration doubles. If you increase the applied force to $3\mathbf{F}$, the original acceleration is tripled, and so on. From such observations, we conclude that **the acceleration of an object is directly proportional to the net force acting on it.**

As stated in the preceding section, the acceleration of an object also depends on its mass. This can be understood by considering the following set of experiments. If you apply a force $\mathbf{F}$ to a block of ice on a frictionless surface, the block will undergo some acceleration $\mathbf{a}$. If the mass of the block is doubled, the same applied force will produce an acceleration $\mathbf{a}/2$. If the mass is tripled, the same applied force will produce an acceleration $\mathbf{a}/3$, and so on. We conclude that **the acceleration of an object is inversely proportional to its mass.**

These observations are summarized in **Newton's second law:**

> The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

Thus, we can relate mass and force through the following mathematical statement of Newton's second law:[2]

$$\Sigma\mathbf{F} = m\mathbf{a} \qquad [4.2]$$

• *Newton's second law*

You should note that Equation 4.2 is a *vector* expression and hence is equivalent to the following three component equations:

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y \qquad \Sigma F_z = ma_z \qquad [4.3]$$

• *Newton's second law—component form*

Units of Force and Mass

The SI unit of force is the **newton,** which is defined as the force that, when acting on a 1-kg mass, produces an acceleration of 1 m/s². From this definition and Newton's second law, we see that the newton can be expressed in terms of the fundamental units of mass, length, and time:

$$1\ \text{N} \equiv 1\ \text{kg}\cdot\text{m/s}^2 \qquad [4.4]$$

• *Definition of a newton*

The units of force and mass are summarized in Table 4.1. Most of the calculations we shall make in our study of mechanics will be in SI units. Conversion factors between the three systems are given in Appendix A.

Thinking Physics 2

You have most likely had the experience of standing in an elevator that accelerates upward as it leaves to move toward a higher floor. In this case, you *feel* heavier. If you are standing on a bathroom scale at the time, it will *measure* a force larger than your weight. Thus, you have tactile and measured evidence that lead you to believe that you are heavier in this situation. *Are* you heavier?

Reasoning No, you are not—your weight is unchanged. In order to provide the acceleration upward, the floor or the bathroom scale must apply an upward force larger than your weight. It is this larger force that you feel, which you interpret as feeling heavier. A bathroom scale reads this upward force, not your weight, so its reading also increases.

TABLE 4.1 Units of Force, Mass, and Acceleration[a]

Systems of Units	Mass	Acceleration	Force
SI	kg	m/s²	N = kg·m/s²
cgs	g	cm/s²	dyne = g·cm/s²
British engineering	slug	ft/s²	lb = slug·ft/s²

[a] 1 N = 10^5 dyne = 0.225 lb.

[2] Equation 4.2 is valid only when the speed of the object is much less than the speed of light. We will treat the relativistic situation in Chapter 9.

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Thinking Physics 3

In a train, the cars are connected by *couplers*. The couplers between the cars are under tension as the train is pulled by the locomotive in the front. As you move from the locomotive to the caboose, does the tension force in the couplers *increase, decrease,* or *stay the same* as the train speeds up? What if the engineer applies the brakes, so that the couplers are under compression? How does the compression force vary from locomotive to caboose in this case?

Reasoning The tension force *decreases* from the front of the train to the back. The coupler between the locomotive and the first car must apply enough force to accelerate all of the remaining cars. As we move back along the train, each coupler is accelerating less mass behind it. The last coupler only has to accelerate the caboose, so it is under the least tension. If the brakes are applied, the couplers are under compression. The force of compression decreases from the front to the back of the train in this case also. The first coupler, at the back of the locomotive, must apply a large force to slow down all of the remaining cars. The final coupler must only apply a force large enough to slow down the mass of the caboose.

Example 4.1 An Accelerating Hockey Puck

A hockey puck with a mass of 0.30 kg slides on the horizontal frictionless surface of an ice rink. Two forces act on the puck as shown in Figure 4.4. The force $\mathbf{F}_1$ has a magnitude of 5.0 N, and $\mathbf{F}_2$ has a magnitude of 8.0 N. Determine the acceleration of the puck.

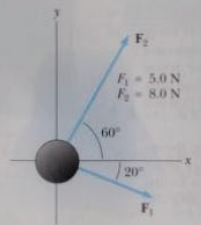


Figure 4.4 (Example 4.1) An object moving on a frictionless surface accelerates in the direction of the *resultant* force, $\mathbf{F}_1 + \mathbf{F}_2$.

Solution The resultant force in the x direction is

$$\sum F_x = F_{1x} + F_{2x} = F_1 \cos 20^\circ + F_2 \cos 60^\circ$$
$$= (5.0\text{ N})(0.940) + (8.0\text{ N})(0.500) = 8.7\text{ N}$$

The resultant force in the y direction is

$$\sum F_y = F_{1y} + F_{2y} = -F_1 \sin 20^\circ + F_2 \sin 60^\circ$$
$$= -(5.0\text{ N})(0.342) + (8.0\text{ N})(0.866) = 5.2\text{ N}$$

Now we can use Newton's second law in component form to find the x and y components of acceleration:

$$a_x = \frac{\Sigma F_x}{m} = \frac{8.7\text{ N}}{0.30\text{ kg}} = 29\text{ m/s}^2$$

$$a_y = \frac{\Sigma F_y}{m} = \frac{5.2\text{ N}}{0.30\text{ kg}} = 17\text{ m/s}^2$$

The acceleration has a magnitude of

$$a = \sqrt{(29)^2 + (17)^2}\text{ m/s}^2 = 34\text{ m/s}^2$$

and its direction is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 31^\circ$$

relative to the positive x axis.

EXERCISE 1 Determine the components of a third force that, when applied to the puck, will cause it to be in equilibrium. Answer $F_x = -8.7$ N, $F_y = -5.2$ N

EXERCISE 2 A 6.0-kg object undergoes an acceleration of 2.0 m/s². (a) What is the magnitude of the resultant force acting on the object? (b) If this same force is applied to a 4.0-kg object, what acceleration does it produce? Answer (a) 12 N (b) 3.0 m/s²

EXERCISE 3 An 1800-kg car is traveling in a straight line with a speed of 25.0 m/s. What is the magnitude of the constant horizontal force that is needed to bring the car to rest in a distance of 80.0 m? Answer 7030 N

4.5 • THE GRAVITATIONAL FORCE AND WEIGHT

We are well aware of the fact that all objects are attracted to the Earth. The force exerted by the Earth on an object is the gravitational force $\mathbf{F}_g$ (Fig. 4.5). This force is directed toward the center of the Earth.[3] The magnitude of the gravitational force is called the **weight** of the object, w.

We have seen that a freely falling object experiences an acceleration $\mathbf{g}$ directed toward the center of the Earth. Applying Newton's second law to a freely falling object of mass m, we have $\mathbf{F} = m\mathbf{a}$. Because $\mathbf{F}_g = m\mathbf{g}$, it follows that $\mathbf{a} = \mathbf{g}$ and

$$w = mg \qquad [4.5]$$

Because it depends on g, weight varies with geographic location. (You should not confuse the italicized symbol g that we use for gravitational acceleration with the symbol g that is used for grams.) Bodies weigh less at higher altitudes than at sea level because g decreases with increasing distance from the center of the Earth. Hence, weight, unlike mass, is not an inherent property of a body. For example, if a body has a mass of 70 kg, then its weight in a location where $g = 9.80$ m/s² is $mg = 686$ N (about 154 lb). At the top of a mountain where $g = 9.76$ m/s², the



Astronaut Edwin E. Aldrin, Jr., walking on the Moon after the Apollo 11 lunar landing. The weight of the astronaut on the Moon is less than it is on Earth but his mass remains the same. *(Courtesy of NASA)*



Figure 4.5 The only force acting on an object in free fall is the gravitational force, $\mathbf{F}_g$. The magnitude of this force is the weight of the object, mg.

[3]This statement is a simplification in that it ignores the fact that the mass distribution of the Earth is not perfectly spherical.

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body's weight would be 683 N. Thus, if you want to lose weight without going on a diet, climb a mountain or weigh yourself at 30 000 ft during an airplane flight.

Because $w = mg$, we can compare the masses of two bodies by measuring their weights with a spring scale or balance. At a given location the ratio of the weights of two bodies equals the ratio of their masses.

Thinking Physics 4

In the absence of air friction, it is claimed that all objects fall with the same acceleration. A heavier object is pulled to the Earth with more force than a light object. Why does the heavier object not fall faster?

Reasoning It is indeed true that the heavier object is pulled with a larger force. The *strength* of the force is determined by the gravitational mass of the object. But the *resistance* to the force and, therefore, to the change in motion of the object, is represented by the object's inertial mass. Inertial mass and gravitational mass are chosen to be equal in Newtonian mechanics. Thus, if an object has twice as much mass as another, it is pulled to the Earth with twice the force, but it also exhibits twice the resistance to having its motion changed. These factors cancel, so that the change in motion, the acceleration, is the same for all objects, regardless of mass.

CONCEPTUAL PROBLEM 3

Suppose you are talking by interplanetary telephone to your friend, who lives on the Moon. He tells you that he has just won a newton of gold in a contest. You tell him that you entered the Earth-version of the same contest and also won a newton of gold! Who is richer?

CONCEPTUAL PROBLEM 4

A baseball of mass m is thrown upward with some initial speed. If air resistance is neglected, what is the force on the ball (a) when it reaches half its maximum height and (b) when it reaches its peak?

4.6 • NEWTON'S THIRD LAW

A statement of Newton's third law •

> Newton's third law states that if two bodies interact, the force exerted on body 1 by body 2 is equal in magnitude but opposite in direction to the force exerted on body 2 by body 1.

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \qquad [4.6]$$

This law, which is illustrated in Figure 4.6a, is equivalent to stating that **forces always occur in pairs,** or that **a single isolated force cannot exist.** The force that body 1 exerts on body 2 is sometimes called the *action force,* and the force of body 2 on body 1 is called the *reaction force.* In reality, either force can be labeled the action or reaction force. **The action force is equal in magnitude to the reaction force and opposite in direction. In all cases, the action and reaction forces act on different objects and must be of the same type.** For example, the force acting on a freely falling projectile is the force of the Earth on the projectile, $\mathbf{F}_g$, and the

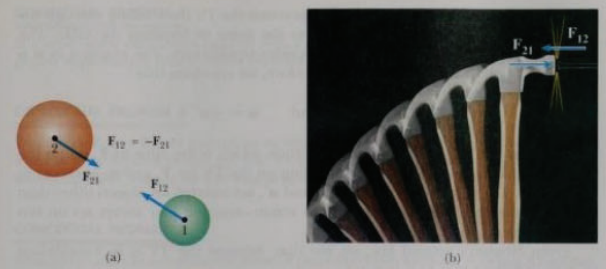


Figure 4.6 Newton's third law. (a) The force exerted by object 1 on object 2 is equal in magnitude and opposite the force exerted by object 2 on object 1. (b) The force exerted by the hammer on the nail is equal in magnitude and opposite the force exerted by the nail on the hammer. *(John Gillmoure, The Stock Market)*

magnitude of this force is mg. The reaction to this force is the force of the projectile on the Earth, $\mathbf{F}'_g = -\mathbf{F}_g$. The reaction force, $\mathbf{F}'_g$, must accelerate the Earth toward the projectile just as the action force, $\mathbf{F}_g$, accelerates the projectile toward the Earth. However, because the Earth has such a large mass, its acceleration due to this reaction force is negligibly small.

Another example of Newton's third law in action is shown in Figure 4.6b. The force exerted by the hammer on the nail (the action) is equal to and opposite the force exerted by the nail on the hammer (the reaction). You directly experience the law if you slam your fist against a wall or kick a football with your bare foot. You should be able to identify the action and reaction forces in these cases.

As we mentioned earlier, the Earth exerts a force $\mathbf{F}_g$ on any object. If the object is a TV at rest on a table, as in Figure 4.7a, the reaction force to $\mathbf{F}_g$ is the force the TV exerts on the Earth, $\mathbf{F}'_g$. The TV does not accelerate, because it is held up by the table. The table, therefore, exerts on the TV an upward action force, $\mathbf{n}$, called

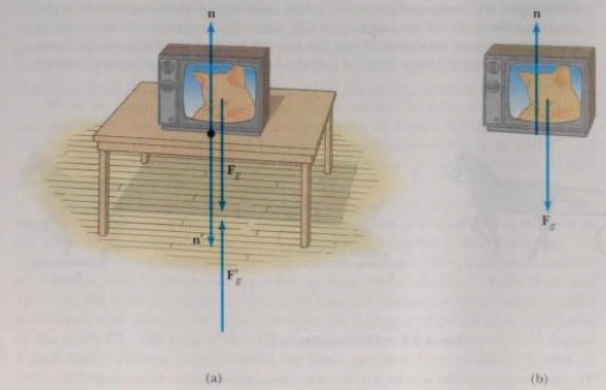


Figure 4.7 (a) When a TV set is sitting on a table, the forces acting on the set are the normal force, $\mathbf{n}$, and the force of gravity, $\mathbf{F}_g$. The reaction to $\mathbf{n}$ is the force of the TV set on the table, $\mathbf{n}'$. The reaction to $\mathbf{F}_g$ is the force of the TV set on the Earth, $\mathbf{F}'_g$. (b) The free-body diagram for the TV set.

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Normal force •

the **normal force.**[4] This is the force that prevents the TV from falling through the table; it can have any value needed, up to the point of breaking the table. The normal force balances the weight and provides equilibrium. The reaction to **n** is the force of the TV on the table, **n′**. Therefore, we conclude that

$$\mathbf{F}_g = -\mathbf{F}'_g \quad \text{and} \quad \mathbf{n} = -\mathbf{n}'$$

The forces **n** and **n′** have the same magnitude, which is the same as $\mathbf{F}_g$ unless the table has broken. Note that the forces acting on the TV are $\mathbf{F}_g$ and **n**, as shown in Figure 4.7b. The two reaction forces, $\mathbf{F}'_g$ and **n′**, are exerted on objects other than the TV. Remember, the two forces in an action–reaction pair always act on two different objects.

From Newton's second law, we see that, because the TV is in equilibrium ($\mathbf{a} = 0$), it follows that $F_g = n = mg$.

Thinking Physics 5

A horse pulls a sled with a horizontal force, causing it to accelerate as in Figure 4.8a. Newton's third law says that the sled exerts an equal and opposite force on the horse. In view of this, how can the sled accelerate? Under what condition does the system (horse plus sled) move with constant velocity?

Reasoning When applying Newton's third law, it is important to remember that the forces involved act on different objects. When you are determining the motion of an object, you must add the forces on that object alone. The force that accelerates the system (horse and sled) is the force exerted by the Earth on the horse's feet. Newton's third law tells us that the horse exerts an equal and opposite force on the Earth. The horizontal forces exerted on the sled are the forward force exerted by the horse and the backward force of friction between sled and surface (Fig. 4.8b). When the forward force exerted by the horse on the sled exceeds the backward force, the sled accelerates to the right. The horizontal forces exerted on the horse are the forward force of the Earth and the backward force of the sled (Fig. 4.8c). The resultant of these two forces causes the horse to accelerate. When the forward force of the Earth on the horse balances the force of friction between sled and surface, the system moves with constant velocity.

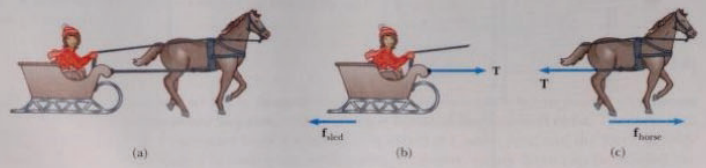


Figure 4.8 (Thinking Physics 5)

[4]The word *normal* is used because the direction of **n** is always *perpendicular* to the surface.

CONCEPTUAL PROBLEM 5

If a small sports car collides head-on with a massive truck, which vehicle experiences the greater impact force? Which vehicle experiences the greater acceleration?

CONCEPTUAL PROBLEM 6

Two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, are exerted on an object of mass m. $\mathbf{F}_1$ is directed toward the right, whereas $\mathbf{F}_2$ is directed toward the left. Under what condition does the object accelerate toward the right? Toward the left? Can its acceleration ever be zero?

CONCEPTUAL PROBLEM 7

What force causes an automobile to move? A propeller-driven airplane? A rocket? A rowboat?

CONCEPTUAL PROBLEM 8

A large man and a small boy stand facing each other on frictionless ice. They put their hands together and push toward each other, so that they move away from each other. Who exerts the larger force? Who experiences the larger acceleration? Who moves away with the higher velocity? Who moves over a longer distance while their hands are in contact?

4.7 • SOME APPLICATIONS OF NEWTON'S LAWS

In this section we present some simple applications of Newton's laws to bodies that are either in equilibrium ($\mathbf{a} = 0$) or moving linearly under the action of constant external forces. For our model, we shall assume that the bodies behave as particles so that we need not worry about rotational motions. In this section we also neglect the effects of friction for those problems involving motion. This is equivalent to stating that the surfaces are *frictionless*. Finally, we usually neglect the masses of any ropes involved. In this approximation, the magnitude of the force exerted at any point along the rope is the same at all points along the rope. In problem statements, the terms *light* and *of negligible mass* are used to indicate that a mass is to be ignored when you work the problem. These two terms are synonymous in this context.

When we apply Newton's laws to an object, we shall be interested only in those external forces that act *on the object*. For example, in Figure 4.7 the only external forces acting on the TV are **n** and $\mathbf{F}_g$. The reactions to these forces, **n′** and $\mathbf{F}'_g$, act on the table and on the Earth, respectively, and do not appear in Newton's second law as applied to the TV.

When an object such as a block is being pulled by a rope attached to it, the rope exerts a force on the object. In general, **tension** is a scalar and is defined as the magnitude of the force that the rope exerts on whatever is attached to it. • *Tension*

Consider a crate being pulled to the right on the frictionless, horizontal surface, as in Figure 4.9a. Suppose you are asked to find the acceleration of the crate and the force the floor exerts on it. First, note that the horizontal force being applied to the crate acts through the rope. The force the rope exerts on the crate is denoted by the symbol **T**. The magnitude of **T** is equal to the tension in the rope. In Figure 4.9a, a dotted circle is drawn around the block to remind you to isolate it from its surroundings.

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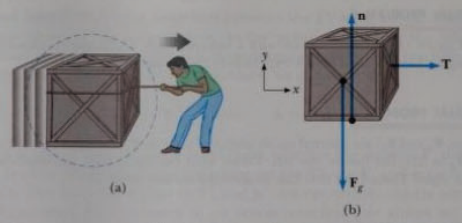


Figure 4.9 (a) A crate being pulled to the right on a frictionless surface. (b) The free-body diagram that represents the external forces on the crate.

Free-body diagrams are important when applying Newton's laws.

Because we are interested only in the motion of the block, we must be able to identify all external forces acting on it. These are illustrated in Figure 4.9b. In addition to the force **T**, the force diagram for the block includes the force of gravity, $\mathbf{F}_g$, and the normal force, **n**, exerted by the floor on the crate. Such a force diagram is referred to as a **free-body diagram.** The construction of a correct free-body diagram is an essential step in applying Newton's laws. The *reactions* to the forces we have listed—namely, the force exerted by the rope on the hand, the force exerted by the crate on the Earth, and the force exerted by the crate on the floor—are not included in the free-body diagram, because they act on *other* bodies and not on the crate.

Now let us apply Newton's second law to the block. First we must choose an appropriate coordinate system. In this case it is convenient to use the coordinate system shown in Figure 4.9b, with the x axis horizontal and the y axis vertical. We can apply Newton's second law in the x direction, y direction, or both, depending on what we are asked to find in the problem. In addition, we may be able to use the equations of motion for constant acceleration that are found in Chapter 2. However, you should use these equations only when the acceleration is constant. For example, if the force **T** in Figure 4.9 is constant, then it follows that the acceleration in the x direction is also constant, because $\mathbf{a}_x = \mathbf{T}/m$. Hence, if we need to find the displacement or the velocity of the block at some instant of time, we can use the equations of motion with constant acceleration.

Objects in Equilibrium and Newton's First Law

Objects that are either at rest or moving with constant velocity are said to be in equilibrium, and Newton's first law is a statement of one condition that must be true for equilibrium conditions to prevail. In equation form, this condition of equilibrium can be expressed as

First condition of equilibrium

$$\sum \mathbf{F} = 0 \qquad [4.7]$$

This statement signifies that the *vector* sum of all the forces (the net force) acting on an object in equilibrium is zero.[5]

[5]This is only one condition of equilibrium. A second condition of equilibrium is a statement of rotational equilibrium. This condition will be discussed in Chapter 10.

Usually, the problems we encounter in our study of equilibrium are easier to solve if we work with Equation 4.7 in terms of the components of the external forces acting on an object. By this we mean that, in a two-dimensional problem, the sum of all the external forces in the x and y directions must separately equal zero; that is,

$$\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0 \qquad [4.8]$$

This set of equations is often referred to as the **first condition for equilibrium.** We shall not consider three-dimensional problems in this book, but the extension of Equations 4.8 to a three-dimensional situation can be made by adding a third equation, $\Sigma F_z = 0$.

PROBLEM-SOLVING STRATEGY • Objects in Equilibrium

The following procedure is recommended for problems involving objects in equilibrium:

1. Make a sketch of the object under consideration.
2. Draw a free-body diagram for the *isolated* object, and label all external forces acting on the object. Assume a direction for each force. If you select a direction that leads to a negative sign in your solution for a force, do not be alarmed; this merely means that the direction of the force is the opposite of what you assumed.
3. Resolve all forces into x and y components, choosing a convenient coordinate system.
4. Use the equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Remember to keep track of the signs of the various force components.
5. Application of the last step leads to a set of equations with several unknowns. Solve the simultaneous equations for the unknowns in terms of the known quantities.

Example 4.2 A Traffic Light at Rest

A traffic light weighing 125 N hangs from a cable tied to two other cables fastened to a support, as in Figure 4.10a. The upper cables make angles of 37.0° and 53.0° with the horizontal. Find the tension in the three cables.

Reasoning We must construct two free-body diagrams in order to work this problem. The first of these is for the traffic light, shown in Figure 4.10b; the second is for the knot that holds the three cables together, as in Figure 4.10c. This knot is a convenient point to choose because all the forces we are interested in act through this point. Because the acceleration of the system is zero, we can use the condition that the net force on the light is zero, and the net force on the knot is zero.

Solution First we construct a free-body diagram for the traffic light, as in Figure 4.10b. The force exerted by the vertical cable, $\mathbf{T}_3$, supports the light, and so $T_3 = w = 125$ N. Next, we choose the coordinate axes as shown in Figure 4.10c and resolve the forces into their x and y components:

Force	x component	y component
$\mathbf{T}_1$	$-T_1 \cos 37.0°$	$T_1 \sin 37.0°$
$\mathbf{T}_2$	$T_2 \cos 53.0°$	$T_2 \sin 53.0°$
$\mathbf{T}_3$	0	-125 N

The first condition for equilibrium gives us the equations

(1) $\sum F_x = T_2 \cos 53.0° - T_1 \cos 37.0° = 0$

(2) $\sum F_y = T_1 \sin 37.0° + T_2 \sin 53.0° - 125 \text{ N} = 0$

From (1) we see that the horizontal components of $\mathbf{T}_1$ and $\mathbf{T}_2$ must be equal in magnitude, and from (2) we see that the sum of the vertical components of $\mathbf{T}_1$ and $\mathbf{T}_2$ must balance the weight of the light. We can solve (1) for T_2 in terms of T_1 to give

$$T_2 = T_1 \left(\frac{\cos 37.0°}{\cos 53.0°} \right) = 1.33 T_1$$

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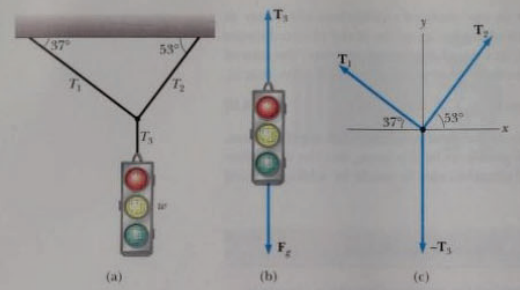


Figure 4.10 (Example 4.2) (a) A traffic light suspended by cables. (b) Free-body diagram for the traffic light. (c) Free-body diagram for the knot in the cable.

This value for T_2 can be substituted into (2) to give

$$T_1 \sin 37.0° + (1.33T_1)(\sin 53.0°) - 125 \text{ N} = 0$$

$$T_1 = 75.1 \text{ N}$$

$$T_2 = 1.33T_1 = 99.9 \text{ N}$$

EXERCISE 4 In what situation will $T_1 = T_2$?
Answer The angles must be equal.



Figure 4.11 (Exercise 5) A child holding a sled on a frictionless hill.

EXERCISE 5 A child holds a sled weighing 77.0 N at rest on a frictionless snow-covered incline as in Figure 4.11. Find (a) the magnitude of the force the child must exert on the rope, and (b) the magnitude of the force the incline exerts on the sled. (c) What happens to the normal force as the angle of incline increases? (d) Under what conditions would the normal force equal the weight of the sled?
Answer (a) 38.5 N (b) 66.7 N (c) It decreases. (d) If the sled were on a horizontal surface and the applied force were either zero or along the horizontal

Accelerating Objects and Newton's Second Law

In a situation in which a net force is acting on an object, the object is accelerated, and we use Newton's second law in order to determine the features of the motion. The representative problems and suggestions that follow should help you to solve problems of this kind.

PROBLEM-SOLVING STRATEGY • Newton's Second Law

The following procedure is recommended when dealing with problems involving the application of Newton's second law:

1. Draw a diagram of the system.
2. Isolate the object the motion of which is being analyzed. Draw a free-body diagram for this object, showing *all external forces acting on it.* For systems containing more than one object, draw a *separate* diagram for each object.
3. Establish convenient coordinate axes for each object and find the components of the forces along those axes. Apply Newton's second law, $\Sigma \mathbf{F} = m\mathbf{a}$, in the x and y directions for each object.
4. Solve the component equations for the unknowns. Remember that in order to obtain a complete solution, you must have as many independent equations as you have unknowns.
5. If necessary, use the equations of kinematics (motion with constant acceleration) from Chapter 2 to find all the unknowns.

Example 4.3 A Crate on a Frictionless Incline

A crate of mass m is placed on a frictionless, inclined plane of angle θ, as in Figure 4.12a. (a) Determine the acceleration of the crate after it is released.

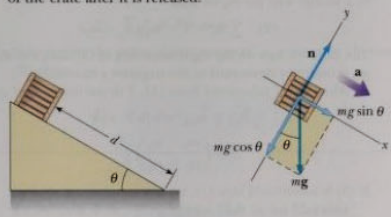


Figure 4.12 (Example 4.3) (a) A crate sliding down a frictionless incline. (b) Free-body diagram for the crate. Note that the magnitude of its acceleration along the incline is $g \sin \theta$.

Reasoning Because the forces acting on the crate are known, Newton's second law can be used to determine its acceleration. First, we construct the free-body diagram for the crate as in Figure 4.12b. The only forces on the crate are the normal force $\mathbf{n}$ acting perpendicular to the plane and the force of gravity $\mathbf{F}_g$ acting vertically downward. For problems of this type involving inclined planes, **it is convenient to choose the coordinate axes with x along the incline and y perpendicular to it.** Then, we replace $\mathbf{F}_g$ by a component of magnitude $mg \sin \theta$ along the *positive* x axis and one of magnitude $mg \cos \theta$ in the *negative* y direction.

Solution Applying Newton's second law to the crate in component form while noting that $a_y = 0$ gives

$$(1) \quad \sum F_x = mg \sin \theta = ma_x$$

$$(2) \quad \sum F_y = n - mg \cos \theta = 0$$

From (1) we see that the acceleration along the incline is provided by the component of the force of gravity down the incline:

$$(3) \quad a_x = g \sin \theta$$

From (2) we conclude that the component of the force of gravity perpendicular to the incline is *balanced* by the normal force; that is, $n = mg \cos \theta$. Note that the acceleration given by (3) is *independent* of the mass of the crate—it depends only on the angle of inclination and on g.

Special Cases When $\theta = 90°$, $a = g$ and $n = 0$. This case corresponds to the crate in free fall. When $\theta = 0$, $a_x = 0$ and $n = mg$ (its maximum value).

(b) Suppose the crate is released from rest at the top, and the distance from the front edge of the crate to the bottom is d. How long does it take the front edge of the crate to reach the bottom, and what is its speed just as it gets there?

Solution Because a_x = constant, we can apply the Equation 2.10, $x - x_0 = v_{x0}t + \frac{1}{2}a_x t^2$ to the crate. Because the displacement $x - x_0 = d$ and $v_{x0} = 0$, we get $d = \frac{1}{2}a_x t^2$, or

$$(4) \quad t = \sqrt{\frac{2d}{a_x}} = \sqrt{\frac{2d}{g \sin \theta}}$$

Using Equation 2.11, $v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$, with $v_{x0} = 0$, we find that $v_x^2 = 2a_x d$, or

$$(5) \quad v_x = \sqrt{2a_x d} = \sqrt{2gd \sin \theta}$$

Again, t and v_x are *independent* of the mass of the crate. This fact suggests a simple method of measuring g using an inclined air track or some other frictionless incline. Simply measure the angle of inclination, the distance traveled by the crate, and the time it takes to reach the bottom. The value of g can then be calculated from (4) and (5).

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Example 4.4 Atwood's Machine

When two unequal masses are hung vertically over a light, frictionless pulley as in Figure 4.13a, the arrangement is called *Atwood's machine*. The device is sometimes used in the laboratory to measure the free-fall acceleration. Calculate the magnitude of the acceleration of the two masses and the tension in the string.

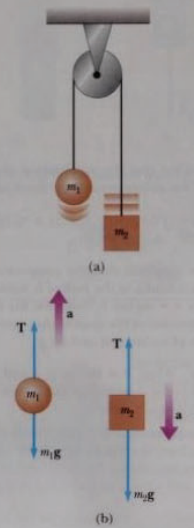


Figure 4.13 (Example 4.4) Atwood's machine. (a) Two masses connected by a light string over a frictionless pulley. (b) Free-body diagrams for m_1 and m_2.

Reasoning The free-body diagrams for the two masses are shown in Figure 4.13b. Two forces act on each block: the upward force exerted by the string, **T**, and the downward force of gravity. Thus, the magnitude of the net force exerted on m_1 is $T - m_1g$, whereas the magnitude of the net force exerted on m_2 is $T - m_2g$. Because the blocks are connected by a string, their accelerations must be equal in magnitude. If we assume that $m_2 > m_1$, then m_1 must accelerate upward, whereas m_2 must accelerate downward.

Solution When Newton's second law is applied to m_1, with **a** upward for this mass (because $m_2 > m_1$), we find (taking upward to be the positive y direction)

$$(1) \quad \sum F_y = T - m_1g = m_1a$$

In a similar way, for m_2 we find

$$(2) \quad \sum F_y = T - m_2g = -m_2a$$

The negative sign on the right-hand side of (2) indicates that m_2 accelerates downward in the negative y direction.

When (2) is subtracted from (1), T drops out and we get

$$-m_1g + m_2g = m_1a + m_2a$$

$$(3) \quad a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

If (3) is substituted into (1), we get

$$(4) \quad T = \left(\frac{2m_1m_2}{m_1 + m_2}\right) g$$

Special Cases When $m_1 = m_2$, $a = 0$ and $T = m_1g = m_2g$, as we would expect for the balanced case. Also, if $m_2 \gg m_1$, $a \approx g$ (a freely falling body) and $T \approx 2m_1g$.

EXERCISE 6 Find the acceleration and tension of an Atwood's machine in which $m_1 = 2.00$ kg and $m_2 = 4.00$ kg. Answer $a = 3.27$ m/s^2; $T = 26.1$ N

Example 4.5 One Block Pushes Another

Two blocks of masses m_1 and m_2 are placed in contact with each other on a frictionless, horizontal surface, as in Figure 4.14a. A constant horizontal force **F** is applied to m_1 as shown. (a) Find the magnitude of the acceleration of the system.

Reasoning and Solution Both blocks must experience the *same* acceleration because they are in contact with each other. Because **F** is the only horizontal force exerted on the system (the two blocks), we have

$$\sum F_x(\text{system}) = F = (m_1 + m_2)a$$

$$(1) \quad a = \frac{F}{m_1 + m_2}$$

(b) Determine the magnitude of the contact force between the two blocks.

Reasoning and Solution To solve this part of the problem, first construct a free-body diagram for each block, as shown in Figures 4.14b and 4.14c, where the contact force is denoted by **P**. From Figure 4.14c we see that the only horizontal force acting on m_2 is the contact force **P** (the force exerted by m_1 on m_2), which is directed to the right. Applying Newton's second law to m_2 gives

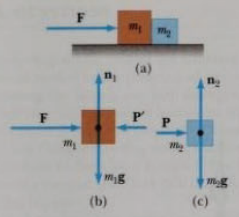


Figure 4.14 (Example 4.5)

$$(2) \quad \sum F_x = P = m_2a$$

Substituting the value of the acceleration a given by (1) into (2) gives

$$(3) \quad P = m_2a = \left(\frac{m_2}{m_1 + m_2}\right) F$$

From this result, we see that the contact force **P** is *less* than the applied force **F**. This is consistent with the fact that the force required to accelerate m_2 alone must be less than the force required to produce the same acceleration for the system of two blocks.

It is instructive to check this expression for P by considering the forces acting on m_1, shown in Figure 4.14b. The horizontal forces acting on m_1 are the applied force **F** to the right and the contact force **P′** to the left (the force exerted by m_2 on m_1). From Newton's third law, **P′** is the reaction to **P**, so $|\mathbf{P}'| = |\mathbf{P}|$. Applying Newton's second law to m_2 gives

$$(4) \quad \sum F_x = F - P' = F - P = m_2a$$

Substituting the value of a from (2) into (4) gives

$$P = F - m_2a = F - \frac{m_1F}{m_1 + m_2} = \left(\frac{m_2}{m_1 + m_2}\right) F$$

This agrees with (3), as it must.

EXERCISE 7 If $m_1 = 4.00$ kg, $m_2 = 3.00$ kg, and $F = 9.00$ N, find the magnitude of the acceleration of the system and that of the contact force. Answer $a = 1.29$ m/s^2; $P = 3.86$ N

Example 4.6 Weighing a Fish in an Elevator

A person weighs a fish on a spring scale attached to the ceiling of an elevator, as shown in Figure 4.15. Show that if the elevator accelerates or decelerates, the spring scale reads a value different from the weight of the fish.

Reasoning The external forces acting on the fish are the downward force of gravity m**g** and the upward force **T** exerted on it by the scale. By Newton's third law, the tension T is also the reading of the spring scale. If the elevator is either at rest or moving at constant velocity, then the fish is not accelerating and $T = mg$. However, if the elevator accelerates in either direction, the tension is no longer equal to the weight of the fish.

Solution If the elevator accelerates with an acceleration **a** relative to an observer outside the elevator in an inertial frame, then the second law applied to the fish gives the total force on the fish:

$$\sum F = T - mg = ma_y$$

which leads to

$$(1) \quad T = mg + ma_y$$

Thus, we conclude from (1) that the scale reading, T, is greater than the weight, mg, if **a** is upward, as in Figure 4.15a. Furthermore, we see that T is less than mg if **a** is downward, as in Figure 4.15b.

For example, if the weight of the fish is 40.0 N, and a_y is 2.00 m/s^2 upward, then the scale reading is

$$T = mg + ma = mg\left(\frac{a}{g} + 1\right)$$

$$= w\left(\frac{a}{g} + 1\right) = (40.0 \text{ N})\left(\frac{2.00 \text{ m/s}^2}{9.80 \text{ m/s}^2} + 1\right)$$

$$= 48.2 \text{ N}$$

If a_y is 2.00 m/s^2 downward, then

$$T = mg - ma = mg\left(1 - \frac{a}{g}\right) = 31.8 \text{ N}$$

Hence, if you buy a fish in an elevator, make sure the fish is weighed while the elevator is at rest or accelerating down-

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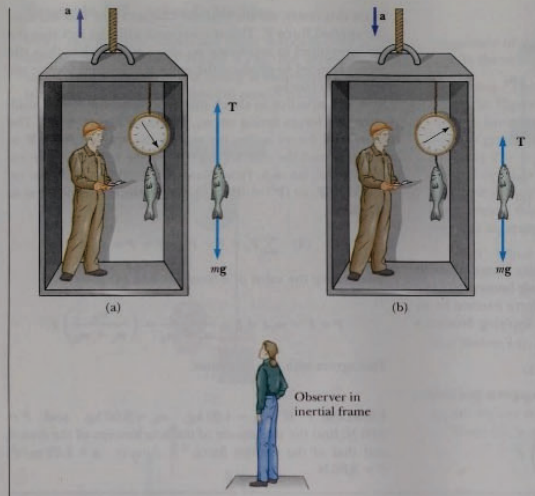


Figure 4.15 (Example 4.6). (a) When the elevator accelerates *upward*, the spring scale reads a value *greater* than the weight. (b) When the elevator accelerates *downward*, the spring scale reads a value *less* than the weight.

ward! Furthermore, note that one cannot determine the *direction* of motion of the elevator from the information given here.

Special Cases If the cable breaks, then the elevator falls freely and $a_y = -g$, and from (1) we see that the tension, T, is zero; that is, the fish appears to be weightless. If the elevator accelerates *downward* with an acceleration *greater* than g, the fish (along with the person in the elevator) will eventually hit the ceiling, because the acceleration of the fish will still be that of a freely falling body relative to an outside observer.

SUMMARY

Newton's first law states that a body at rest will remain at rest, and a body in uniform motion in a straight line will maintain that motion, unless an external resultant force acts on the body.

Newton's second law states that the acceleration of an object is directly proportional to the resultant force acting on the object and inversely proportional to the object's mass. If the mass of the body is constant, the net force equals the product of the mass and its acceleration, or $\Sigma \mathbf{F} = m\mathbf{a}$.

Newton's first and second laws are valid in an inertial frame of reference. An **inertial frame** is one in which Newton's first law is valid.

The **weight** of a body is equal to the product of its mass (a scalar quantity) and the magnitude of the free-fall acceleration, or $w = mg$.

Newton's third law states that if two bodies interact, the force exerted on body 1 by body 2 is equal in magnitude but opposite in direction to the force exerted on body 2 by body 1. Thus, an isolated force cannot exist in nature.

CONCEPTUAL QUESTIONS

1. Draw a free-body diagram for each of the following objects: (a) a projectile in motion in the presence of air resistance, (b) a rocket leaving the launch pad with its engines operating, (c) an athlete running along a horizontal track.
2. In the motion picture *It Happened One Night* (Columbia Pictures, 1934), Clark Gable is standing inside a stationary bus in front of Claudette Colbert, who is seated. The bus suddenly starts moving forward and Clark falls into Claudette's lap. Why did this happen?
3. As you sit in a chair, the chair pushes up on you with a normal force. This force is equal to your weight and in the opposite direction. Is it the Newton's third law reaction force to your weight?
4. The observer in the elevator of Example 4.6 would claim that the "weight" of the fish is T, the scale reading. This is obviously wrong. Why does this observation differ from that of a person outside the elevator, at rest with respect to the Earth?
5. Identify the action–reaction pairs in the following situations: a man takes a step; a snowball hits a girl in the back; a baseball player catches a ball; a gust of wind strikes a window.
6. While a football is in flight, what forces act on it? What are the action–reaction pairs while the football is being kicked and while it is in flight?
7. A rubber ball is dropped onto the floor. What force causes the ball to bounce back into the air?
8. What is wrong with the statement "Because the car is at rest, there are no forces acting on it"? How would you correct this sentence?
9. A weightlifter stands on a bathroom scale. He pumps a barbell up and down. What happens to the reading on the bathroom scale as this is done? Suppose he is strong enough to actually *throw* the barbell upward. How does the reading on the scale vary now?
10. The mayor of a city decides to fire some city employees because they will not remove the sags from the cables that support the city traffic lights. If you were a lawyer, what defense would you give on behalf of the employees? Who do you think would win the case in court?
11. In a tug-of-war between two athletes, each athlete pulls on the rope with a force of 200 N. What is the tension in the rope? If the rope does not move what force does each athlete exert against the ground?
12. Suppose a truck loaded with sand accelerates at 0.5 m/s^2 on a highway. If the driving force on the truck remains constant, what happens to the truck's acceleration if its trailer leaks sand at a constant rate through a hole in its bottom?

PROBLEMS

Sections 4.1 Through 4.6

1. A force, $\mathbf{F}$, applied to an object of mass m_1 produces an acceleration of 3.00 m/s^2. The same force applied to a second object of mass m_2 produces an acceleration of 1.00 m/s^2. (a) What is the value of the ratio m_1/m_2? (b) If m_1 and m_2 are combined, find their acceleration under the action of the force $\mathbf{F}$.
2. A force of 10.0 N acts on a body of mass 2.00 kg. What are (a) the body's acceleration, (b) its weight in newtons, and (c) its acceleration if the force is doubled?
3. A 3.00-kg mass undergoes an acceleration given by $\mathbf{a} = (2.00\mathbf{i} + 5.00\mathbf{j})$ m/s^2. Find the resultant force, $\mathbf{F}$, and its magnitude.
4. A heavy freight train has a mass of 15 000 metric tons. If the locomotives can exert a pulling force of 750 000 N, how long does it take to increase the speed from 0 to 80.0 km/h?
5. A pitcher throws a baseball of weight 1.40 N with velocity (32.0$\mathbf{i}$) m/s by uniformly accelerating her arm for 0.0900 s. If the ball starts from rest, (a) through what distance does the ball accelerate before its release? (b) What vector force does she exert on it?
6. A pitcher throws a baseball of weight $-w\mathbf{j}$ with velocity $v\mathbf{i}$ by uniformly accelerating her arm for time t. If the ball starts from rest, (a) through what distance does the ball accelerate before its release? (b) What vector force does she exert on it?
7. A 4.00-kg object has a velocity of 3.00$\mathbf{i}$ m/s at one instant. Eight seconds later, its velocity is (8.00$\mathbf{i}$ + 10.0$\mathbf{j}$) m/s. Assuming the object was subject to a constant net force, find (a) the components of the force and (b) its magnitude.
8. The average speed of a nitrogen molecule in air is about 6.70 $\times$ 10^2 m/s, and its mass is about 4.68 $\times$ 10^{-26} kg. (a) If it takes 3.00 $\times$ 10^{-13} s for a nitrogen molecule to hit a wall and rebound with the same speed but in an opposite direction, what is the average acceleration of the molecule during this time interval? (b) What average force does the molecule exert on the wall?
9. An electron of mass 9.11 $\times$ 10^{-31} kg has an initial speed of 3.00 $\times$ 10^5 m/s. It travels in a straight line, and its speed

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increases to 7.00×10^5 m/s in a distance of 5.00 cm. Assuming its acceleration is constant, (a) determine the force on the electron and (b) compare this force with the weight of the electron, which we neglected.

10. (a) A car with a mass of 850 kg is moving to the right with a constant speed of 1.44 m/s. What is the total force on the car? (b) What is the total force on the car if it is moving to the left?
11. A woman weighs 120 lb. Determine (a) her weight in newtons and (b) her mass in kilograms.
12. If a man weighs 900 N on Earth, what would he weigh on Jupiter, where the acceleration due to gravity is 25.9 m/s²?
13. Two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, act on a 5.00-kg mass. If $F_1 = 20.0$ N and $F_2 = 15.0$ N, find the accelerations in (a) and (b) of Figure P4.13.

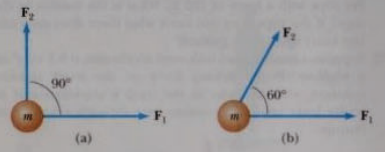


Figure P4.13

14. Besides its weight, a 2.80-kg object is subjected to one other constant force. The object starts from rest and in 1.20 s experiences a displacement of $(4.20\ \text{m})\mathbf{i} - (3.30\ \text{m})\mathbf{j}$, where the direction of $\mathbf{j}$ is the upward vertical direction. Determine the other force.
15. You stand on the seat of a chair and then hop off. (a) During the time you are in flight down to the floor, estimate the acceleration of the Earth as it lurches up toward you. (b) Estimate the upward distance through which the Earth moves as you fall to the floor. Visualize the Earth as a perfectly solid object.
16. Forces of 10.0 N north, 20.0 N east, and 15.0 N south are simultaneously applied to a 4.00-kg mass. Obtain the object's acceleration.
17. A 1000-kg boat moves through the water with two forces acting on it. One is a 2000-N forward push by the propeller; the other is an 1800-N resistive force due to the water. (a) What is the acceleration of the boat? (b) If it starts from rest, how far will it move in 10.0 s? (c) What will be its speed at the end of this time?
18. Three forces, $\mathbf{F}_1 = (-2.00\mathbf{i} + 2.00\mathbf{j})$ N, $\mathbf{F}_2 = (5.00\mathbf{i} - 3.00\mathbf{j})$ N, and $\mathbf{F}_3 = (-45.0\mathbf{i})$ N, act on an object to give it an acceleration of magnitude 3.75 m/s². (a) What is the direction of the acceleration? (b) What is the mass of the object? (c) If the object is initially at rest, what is its speed after 10.0 s? (d) What are the velocity components of the object after 10.0 s?
19. A 15.0-lb block rests on the floor. (a) What force does the floor exert on the block? (b) If a rope is tied to the block and run vertically over a pulley and the other end is attached to a free-hanging 10.0-lb weight, what is the force of the floor on the 15.0-lb block? (c) If we replace the 10.0-lb weight in part (b) with a 20.0-lb weight, what is the force of the floor on the 15.0-lb block?

Section 4.7 Some Applications of Newton's Laws

20. A 3.00-kg mass is moving in a plane, with its x and y coordinates given by $x = 5t^2 - 1$ and $y = 3t^3 + 2$ (x and y are in meters and t is in seconds). Find the magnitude of the net force acting on this mass at $t = 2.00$ s.
21. Find the tension in each cord of the systems described in Figure P4.21. (Neglect the masses of the cords.)

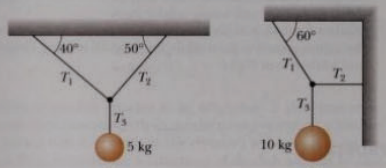


Figure P4.21

22. The distance between two telephone poles is 50.0 m. When a 1.00-kg bird lands on the telephone wire midway between the poles, the wire sags 0.200 m. Draw a free-body diagram of the bird. How much tension does the bird produce in the wire? Ignore the weight of the wire.
23. A bag of cement hangs from three wires as shown in Figure P4.23. Two of the wires make angles θ_1 and θ_2 with the horizontal. If the system is in equilibrium, (a) show that

$$T_1 = \frac{w \cos \theta_2}{\sin(\theta_1 + \theta_2)}$$

(b) Given that $w = 325$ N, $\theta_1 = 10.0°$ and $\theta_2 = 25.0°$, find the tensions T_1, T_2, and T_3 in the wires.

24. The systems shown in Figure P4.24 are in equilibrium. If the spring scales are calibrated in newtons, what do they read? (Neglect the masses of the pulleys and strings, and assume the incline is frictionless.)

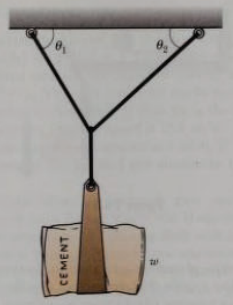


Figure P4.23

25. A fire helicopter carries a 620-kg bucket of water at the end of a 20.0-m long cable. Flying back from a fire at a constant speed of 40.0 m/s, the cable makes an angle of 40.0° with respect to the vertical. (a) Determine the force of air resistance on the bucket. (b) After filling the bucket with sea water, the helicopter returns to the fire at the same speed with the bucket now making an angle of 7.00° with the vertical. What is the mass of the water in the bucket?
26. A simple accelerometer is constructed by suspending a mass m from a string of length L that is tied to the top of a cart. As the cart is accelerated the string-mass system makes an angle of θ with the vertical. (a) Assuming that the string mass is negligible compared to m, derive an expression for the cart's acceleration in terms of θ, and show that it is independent of the mass m and the length L. (b) Determine the acceleration of the cart when $\theta = 23.0°$.
27. A 1.00-kg mass is observed to accelerate at 10.0 m/s² in a direction 30.0° north of east (Fig. P4.27). The force $\mathbf{F}_2$ acting on the mass has a magnitude of 5.00 N and is directed north. Determine the magnitude and direction of the force $\mathbf{F}_1$ acting on the mass.

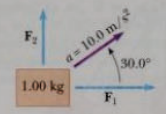


Figure P4.27

28. Draw a free-body diagram of a block that slides down a frictionless plane having an inclination of $\theta = 15.0°$ (Fig. P4.28). If the block starts from rest at the top and the length of the incline is 2.00 m, find (a) the acceleration of the block and (b) its speed when it reaches the bottom of the incline.

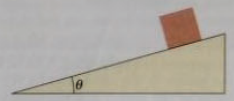


Figure P4.28

29. A block is given an initial velocity of 5.00 m/s up a frictionless 20.0° incline (Fig. P4.28). How far up the incline does the block slide before coming to rest?

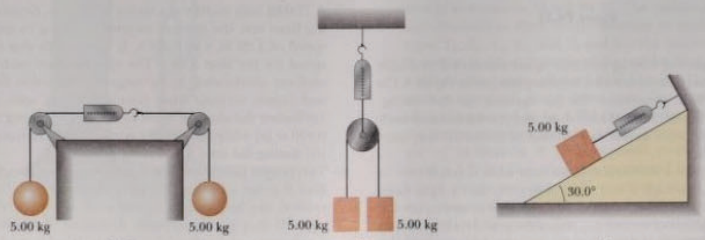


Figure P4.24

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30. Two masses are connected by a light string that passes over a frictionless pulley, as in Figure P4.30. The incline is frictionless, $m_1 = 2.00$ kg, $m_2 = 6.00$ kg, and $\theta = 55.0°$. (a) Draw free-body diagrams of both masses. Find (b) the accelerations of the masses, (c) the tension in the string, and (d) the speed of each mass 2.00 s after being released from rest.

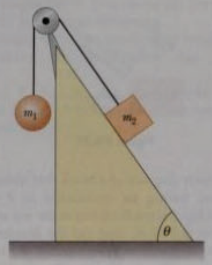


Figure P4.30

31. Two masses, m_1 and m_2, situated on a frictionless, horizontal surface, are connected by a light string. A force, $\mathbf{F}$, is exerted on one of the masses to the right (Fig. P4.31). Determine the acceleration of the system and the tension, T, in the string.

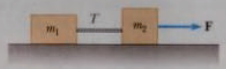


Figure P4.31

32. Two masses of 3.00 kg and 5.00 kg are connected by a light string that passes over a frictionless pulley, as in Figure 4.13a (page 98). Determine (a) the tension in the string, (b) the acceleration of each mass, and (c) the distance each mass will move in the first second of motion if they start from rest.

33. Mass m_1 on a frictionless horizontal table is connected to mass m_2 through a very light pulley, P_1, and a light fixed pulley, P_2, as shown in Figure P4.33. (a) If a_1 and a_2 are the accelerations of m_1 and m_2, respectively, what is the relation between these accelerations? Express (b) the tensions in the strings and (c) the accelerations a_1 and a_2 in terms of the masses m_1, and m_2, and g.

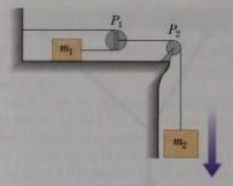


Figure P4.33

34. A 5.00-kg mass placed on a frictionless, horizontal table is connected to a cable that passes over a pulley and then is fastened to a hanging 10.0-kg mass, as in Figure P4.34. Draw free-body diagrams of both masses. Find the acceleration of the two objects and the tension in the string.

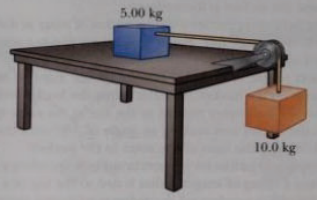


Figure P4.34

35. A 72.0-kg man stands on a spring scale in an elevator. Starting from rest, the elevator ascends, attaining its maximum speed of 1.20 m/s in 0.800 s. It travels with this constant speed for the next 5.00 s. The elevator then undergoes a uniform acceleration in the negative y direction for 1.50 s and comes to rest. What does the spring scale register (a) before the elevator starts to move? (b) during the first 0.800 s? (c) while the elevator is traveling at constant speed? (d) during the time it is slowing down?

36. Two people pull as hard as they can on ropes attached to a boat that has a mass of 200 kg. If they pull in the same direction, the boat has an acceleration of 1.52 m/s^2 to the right. If they pull in opposite directions, the boat has an acceleration of 0.518 m/s^2 to the left. What is the force exerted by each person on the boat? (Disregard any other forces on the boat.)

Additional Problems

37. A time-dependent force, $\mathbf{F} = (8.00\mathbf{i} - 4.00t\mathbf{j})$ N (where t is in seconds) is applied to a 2.00-kg object initially at rest. (a) At what time will the object be moving with a speed of 15.0 m/s? (b) Through what total displacement has the object traveled at this time? (c) How far is the object from its initial position when its speed is 15.0 m/s?

38. An elevator accelerates upward at 1.50 m/s^2. If the elevator has a mass of 200 kg, find the tension in the supporting cable.

39. The largest-caliber anti-aircraft gun operated by the German air force during World War II was the 12.8-cm Flak 40. This weapon fired a 25.8-kg shell with a muzzle speed of 880 m/s. What propulsive force was necessary to attain the muzzle speed within the 6.00-m barrel? (Assume constant acceleration and neglect the Earth's gravitational effect.)

40. One of the great dangers to mountain climbers is the *avalanche*, in which a mass of snow and ice breaks loose and goes on an essentially frictionless "ride" down the mountain on a cushion of compressed air. If you were on a 30.0° mountain slope and an avalanche started 400 m up the slope, how much time would you have to get out of the way?

41. An inventive child named Brian wants to reach an apple in a tree without climbing the tree. Sitting in a chair connected to a rope that passes over a frictionless pulley (Fig. P4.41), Brian pulls on the loose end of the rope with such a force that the spring scale reads 250 N. Brian's weight is 320 N, and the chair weighs 160 N. (a) Draw free-body diagrams for Brian and the chair considered as separate systems and another diagram for Brian and the chair considered as one system. (b) Show that the acceleration of the system is *upward* and find its magnitude. (c) Find the force Brian exerts on the chair.

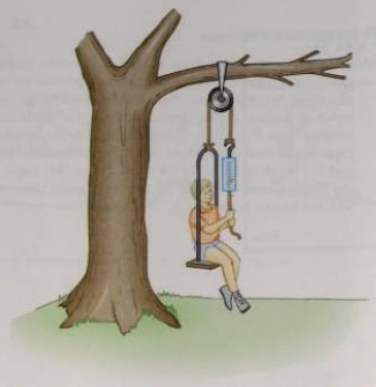


Figure P4.41

42. Three blocks are in contact with each other on a frictionless, horizontal surface, as in Figure P4.42. A horizontal force $\mathbf{F}$ is applied to m_1. If $m_1 = 2.00$ kg, $m_2 = 3.00$ kg, $m_3 = 4.00$ kg, and $F = 18.0$ N, draw free-body diagrams of each block and find (a) the acceleration of the blocks, (b) the *resultant* force on each block, and (c) the magnitudes of the contact forces between the blocks.

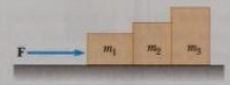


Figure P4.42

43. A high diver of mass 70.0 kg jumps off a board 10.0 m above the water. If her downward motion is stopped 2.00 s after she enters the water, what average upward force did the water exert on the diver?

44. Two forces, $\mathbf{F}_1 = (-6\mathbf{i} - 4\mathbf{j})$ N and $\mathbf{F}_2 = (-3\mathbf{i} + 7\mathbf{j})$ N, act on a particle of mass 2.00 kg that is initially at rest at coordinates (−2.00 m, +4.00 m). (a) What are the components of the particle's velocity at $t = 10.0$ s? (b) In what direction is the particle moving at $t = 10.0$ s? (c) What displacement does the particle undergo during the first 10.0 s? (d) What are the coordinates of the particle at $t = 10.0$ s?

45. A mass M is held in place by an applied force $\mathbf{F}$ and a pulley system as shown in Figure P4.45. The pulleys are massless and frictionless. Find (a) the tension in each section of rope, T_1, T_2, T_3, T_4, and T_5 and (b) the magnitude of $\mathbf{F}$.

46. A student is asked to measure the acceleration of a cart on a "frictionless" inclined plane, as in Figure 4.12, using an air track, a stopwatch, and a meter stick. The height of the incline is measured to be 1.774 cm, and the total length of the incline is measured to be $d = 127.1$ cm. Hence, the angle of inclination θ is determined from the relation $\sin\theta = 1.774/127.1$. The cart is released from rest at the top of the incline, and its displacement along the incline, x, is measured versus time, where $x = 0$ refers to the initial position of the cart. For x values of 10.0 cm, 20.0 cm, 35.0 cm, 50.0 cm, 75.0 cm, and 100 cm, the measured times to undergo these displacements (averaged over five runs) are 1.02 s, 1.53 s, 2.01 s, 2.64 s, 3.30 s, and 3.75 s, respec-

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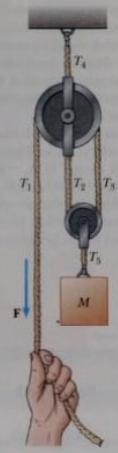


Figure P4.45

tively. Construct a graph of x versus t^2, and perform a linear least-squares fit to the data. Determine the acceleration of the cart from the slope of this graph and compare it with the value you would get using $a' = g \sin \theta$, where $g = 9.80\ \text{m/s}^2$.

47. What horizontal force must be applied to the cart shown in Figure P4.47 in order that the blocks remain stationary relative to the cart? Assume all surfaces, wheels, and pulley are frictionless. (*Hint:* Note that the force exerted by the string accelerates m_1 and that m_2 is in contact with the cart.)

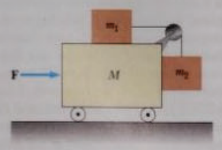


Figure P4.47

48. A 2000-kg car is slowed down uniformly from a speed of 20.0 m/s to 5.00 m/s in a time of 4.00 s. What force did the road exert on the car during this time, and how far did the car travel during the deceleration?

49. A van accelerates down a hill (Fig. P4.49), going from rest to 30.0 m/s in 6.00 s. During the acceleration, a toy ($m = 0.100$ kg) hangs by a string from the van's ceiling. The acceleration is such that the string remains perpendicular to the ceiling. Determine (a) the angle θ and (b) the tension in the string.

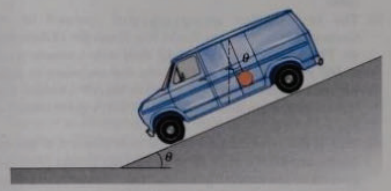


Figure P4.49

50. A car is at rest at the top of a driveway that has a slope of 20.0°. If the brake of the car is released with the car in neutral gear, find (a) the acceleration of the car down the drive and (b) the time it takes for the car to reach the street 10.0 m away.

Spreadsheet Problems

S1. An 8.4-kg mass slides down a fixed, frictionless inclined plane. Design and write a spreadsheet to determine the normal force exerted on the mass and its acceleration for a series of incline angles (measured from the horizontal) ranging from 0° to 90° in 5° increments. Use the graphing capability of your spreadsheet data to plot the normal force and the acceleration as functions of the incline angle. In the limiting cases of 0° and 90°, are your results consistent with the known behavior?

ANSWERS TO CONCEPTUAL PROBLEMS

1. Motion requires no force. Newton's first law says an object in motion continues to move by itself in the absence of external forces.
2. If a single force acts on an object, it must accelerate. (From Newton's second law, $a = F/m$.) If an object accelerates, at least one force must act on it. However, if an object has no acceleration, you cannot conclude that no forces act on it. In this case, you can only say that the *net* force on the object is zero.
3. Because the gravitational field g_M is smaller on the Moon than on the Earth, more mass of gold would be required to represent 1 newton of weight. Thus, your friend on the Moon is richer, by about a factor of 6!
4. The only force acting on the ball during its motion is the downward force of gravity. The magnitude of this force is mg.
5. The car and truck experience forces that are equal in magnitude but in opposite directions. A calibrated spring scale placed between the colliding vehicles reads the same whichever way it faces. Because the car has the smaller mass, it experiences greater acceleration.
6. The *net* force on the object is $\mathbf{F}_1 + \mathbf{F}_2$, thus from Newton's second law, its acceleration is $(\mathbf{F}_1 + \mathbf{F}_2)/m$. The object accelerates toward the right when $|\mathbf{F}_1| > |\mathbf{F}_2|$, and toward the left when $|\mathbf{F}_2| > |\mathbf{F}_1|$. The acceleration of the object is zero when the two forces have equal magnitudes, that is, when $|\mathbf{F}_1| = |\mathbf{F}_2|$.
7. The force causing an automobile to move is the friction between the tires and the roadway as the automobile attempts to push the roadway backward. Notice that forces from the engine are not directly what accelerate the automobile. The force driving a propeller-driven airplane forward is the force of the air on the propeller as the rotating propeller pushes the air backward. The force propelling a rocket is the force of the exhausted gases from the back of the rocket on the rocket, as the rocket pushes the gases backward. In a rowboat, the rower pushes the water backward with the oars. As a result, the water pushes forward on the boat, and the force is applied at the oarlocks. Notice that all of these situations involve the propulsion of an object by Newton's third law.
8. According to Newton's third law, the force of the man on the boy and the force of the boy on the man are third law reaction forces, so they must be equal in magnitude. Both individuals experience the same force, so the boy, with the smaller mass, experiences the larger acceleration. Both individuals experience an acceleration over the same time interval. The larger acceleration of the boy will result in his moving away from the interaction with a larger velocity. Again, because of the larger acceleration, the boy moves through a larger distance during the interval while the hands are in contact.



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5

More Applications of Newton's Laws

CHAPTER OUTLINE

5.1 Forces of Friction
5.2 Newton's Second Law Applied to Uniform Circular Motion
5.3 Nonuniform Circular Motion
5.4 Motion in the Presence of Velocity-Dependent Resistive Forces
5.5 Numerical Modeling in Particle Dynamics
5.6 The Fundamental Forces of Nature
5.7 The Gravitational Field

In Chapter 4 we introduced Newton's laws of motion and applied them to some linear motion situations in which we were able to neglect frictional forces. In this chapter we shall expand our investigation to systems moving in the presence of friction forces. These systems include objects moving on rough surfaces and objects moving through viscous media such as liquids and air. In one section we discuss how numerical methods can be used to solve such real-world problems as motion in which the resistive force is velocity-dependent. We also apply Newton's laws to the dynamics of circular motion. Finally, we conclude this chapter with a discussion of the fundamental forces in nature.

5.1 · FORCES OF FRICTION

When a body is in motion either on a surface or through a viscous medium such as air or water, there is resistance to the motion because the body interacts with its surroundings. We call such resistance a **force of friction.** Forces of friction are very important in our everyday lives. They allow us to walk or run and are necessary for the motion of wheeled vehicles.



The passengers on this corkscrew rollercoaster experience the thrill of various forces as they travel along the curved track. The forces on one of the passenger cars include the force exerted by the track, the force of gravity, and the force of air resistance. *(Robin Smith/ Tony Stone Images)*

Consider a block on a horizontal table, as in Figure 5.1a. If we apply an external horizontal force **F** to the block, acting to the right, the block remains stationary if **F** is not too large. The force that counteracts **F** and keeps the block from moving acts to the left and is called the force of static friction, $\mathbf{f}_s$. As long as the block is not moving, $f_s = F$. Thus, if **F** is increased, $\mathbf{f}_s$ also increases. Likewise, if **F** decreases, $\mathbf{f}_s$ also decreases. Experiments show that the frictional force arises from the nature of the two surfaces: Because of their roughness, contact is made only at a few points, as shown in the magnified view of the surface in Figure 5.1a. Actually, the frictional force is much more complicated than presented here, because it ultimately involves the electrostatic force between atoms or molecules.

If we increase the magnitude of **F**, as in Figure 5.1b, the block eventually slips. When the block is on the verge of slipping, f_s is a maximum as shown in Figure 5.1c. When F exceeds $f_{s,\max}$, the block moves and accelerates to the right. When

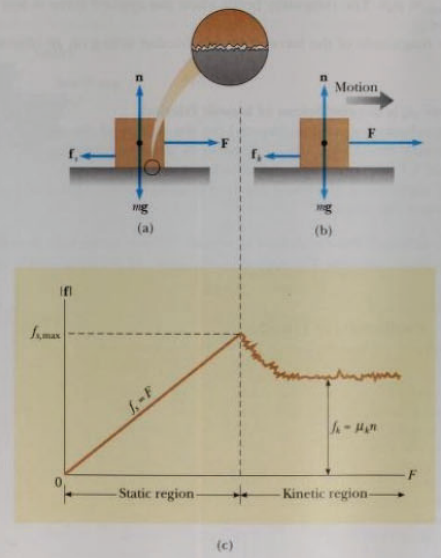


Figure 5.1 The force of friction, **f**, between a block and a rough surface is opposite the applied force, **F**. (a) The force of static friction equals the applied force. (b) When the applied force exceeds the force of kinetic friction, the block accelerates to the right. (c) A graph of the magnitude of the frictional force versus the applied force. Note that $f_{s,\max} > f_k$.

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the block is in motion, the frictional force becomes less than $f_{s\ max}$ (Fig. 5.1c). We call the frictional force for an object in motion the **force of kinetic friction, f_k.** The unbalanced force in the x direction, $F - f_k$, produces an acceleration to the right. If $F = f_k$, the block moves to the right with constant speed. If the applied force is removed, then the frictional force acting to the left accelerates the block in the negative x direction and eventually brings it to rest.

Experimentally, one finds that, to a good approximation, both $f_{s\ max}$ and f_k are proportional to the normal force acting on the block. The experimental observations can be summarized as follows:

- The magnitude of the force of static friction between any two surfaces in contact can have the values

Force of static friction

$$f_s \leq \mu_s n \qquad [5.1]$$

where the dimensionless constant μ_s is called the **coefficient of static friction** and n is the magnitude of the normal force. The equality in Equation 5.1 holds when the block is on the verge of slipping—that is, when $f_s = f_{s\ max} \equiv \mu_s n$. The inequality holds when the applied force is less than this value.

- The magnitude of the force of kinetic friction acting on an object is

Force of kinetic friction

$$f_k = \mu_k n \qquad [5.2]$$

where μ_k is the **coefficient of kinetic friction.**

- The values of μ_k and μ_s depend on the nature of the surfaces, but μ_k is generally less than μ_s. Typical values of μ range from around 0.05 to 1.5. Table 5.1 lists some reported values.
- The coefficients of friction are nearly independent of the area of contact between the surfaces.

Finally, although the coefficient of kinetic friction varies with speed, we shall neglect any such variations. The approximate nature of the equations is easily demonstrated by trying to get a block to slip down an incline at constant speed. Especially at low speeds, the motion is likely to be characterized by alternate *stick* and *slip* episodes.

TABLE 5.1 Coefficients of Friction[a]

	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.0	0.8
Wood on wood	0.25–0.5	0.2
Glass on glass	0.94	0.4
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Ice on ice	0.1	0.03
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.01	0.003

[a] All values are approximate.

Thinking Physics 1

In the motion picture *The Abyss* (Twentieth Century Fox, 1990), an underwater oil exploration rig sits at the bottom of very deep water. It is connected to a ship on the ocean surface by an "umbilical cord," as suggested in Figure 5.2a. On the ship, the umbilical cord is attached to a gantry. During a hurricane, the gantry structure breaks loose from the ship, falls into the water, and sinks to the bottom, passing over the edge of an extremely deep abyss. As a result, the rig is dragged by the umbilical cord along the ocean bottom, as described in Figure 5.2b. As the rig approaches the edge of the abyss, however, it is not pulled over the edge but stops just short of the edge, as shown in Figure 5.2c. Is this purely a cinematic edge-of-the-seat scenario, or is there a reason within the principles of physics for why the rig does not go over the edge?

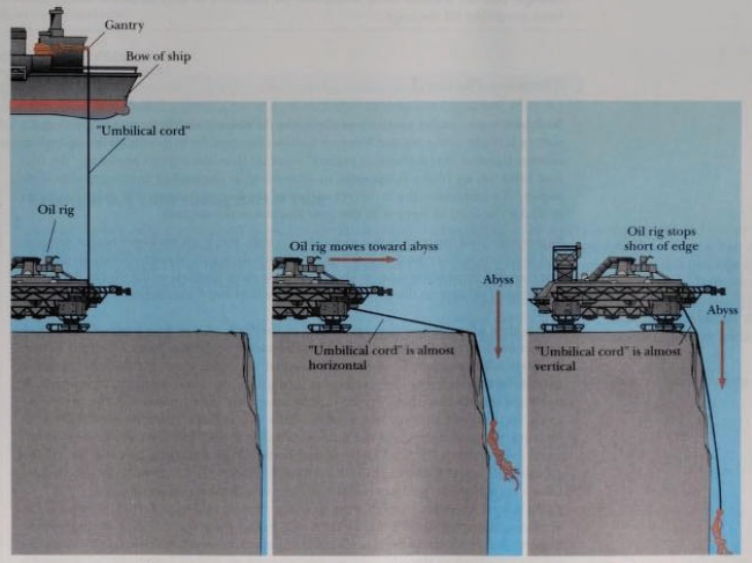


Figure 5.2 (Thinking Physics 1)

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Reasoning There is a physics reason for this phenomenon. While the rig is being pulled across the ocean floor, it is pulled by the section of the umbilical cord that is almost horizontal—almost parallel to the ocean floor. Thus, the rig is subject to two horizontal forces—the tension in the umbilical cord pulling it forward and friction with the ocean floor pulling back. Let us assume that these forces are equal, so that the rig moves with constant velocity. As the rig nears the edge of the abyss, the angle that the umbilical cord makes with the horizontal increases. As a result, the component of force parallel to the ocean floor decreases and the downward vertical component increases. As a result of the increased vertical force, the rig is pulled downward more strongly into the ocean floor, increasing the normal force on it and, in turn, increasing the friction force between the rig and the ocean floor. Thus, with less force pulling it forward (from the umbilical cord) and more force pulling it back (due to friction), the rig slows down. By the time that the rig reaches the edge of the abyss, the force from the umbilical cord is almost *straight down*, resulting in little forward force. This downward force pulls the rig hard into the ocean floor, so that the friction force is large, and the rig stops.

This is not to say that this would happen in any situation. The characters on the rig had some luck in their favor. For example, if the umbilical cord had been attached at a lower point on the rig or if the coefficient of friction between the rig feet and the ocean floor were smaller, then the forward velocity of the rig could have been large enough that the friction force would not be sufficient to stop it, and it would have been propelled off the edge.

Thinking Physics 2

Sometimes, an incorrect statement about the frictional force between an object and a surface is made—"the friction force on an object is opposite to its motion or impending motion (motion that is about to occur)"—rather than the correct phrasing, "the friction force on an object is opposite to its motion or impending motion *relative to the surface*." To emphasize the incorrect nature of the first statement, give some examples in which the friction force is in the *same direction* as the motion.

Reasoning Walking is a familiar example. To walk, you push backward on the ground, and the friction force pushes you forward, in the direction of your walking motion. In this case, the impending motion (because this is static friction) of the object, your foot, is backward relative to the ground, and the friction force exerted on your foot is forward.

Another example is a package sitting in the back of a pickup truck. As the truck accelerates forward, the natural inertia of the package tends to "leave it behind." Thus, the impending motion relative to the floor of the truck is toward the rear, and the friction force exerted on the package is forward, in the same direction as it moves. If the truck accelerates rapidly enough, the package will slide backward in the bed of the truck. In this case, we have real motion of the object rather than impending motion (relative to the surface), and the motion is backward. The friction force, then, is forward. Even though the package is sliding backward in the truck, an observer on the ground will see the package moving forward relative to the ground, so the friction force exerted on the package is again in the same direction as its motion.

As a final example, consider the popular dish-and-tablecloth trick, in which the demonstrator quickly pulls a tablecloth out from under some dishes. It may appear that the dishes do not move. Careful measurement, however, will show that the dishes are slightly closer to the edge of the table after the trick than before. As the tablecloth is pulled, the motion of the dishes relative to the tablecloth is away from the puller. Thus, the friction force on the dishes is toward the puller. This force causes the dishes to move toward the puller. Once the tablecloth is completely out from under the dishes, the tabletop becomes the new surface. The motion of the dishes relative to the tabletop is toward the puller. Thus, the friction force on the dishes is now away from the puller and opposite to the motion of the dishes, causing them to come to rest. The success of the trick depends on a relatively small friction force between the dishes and the tablecloth, so that the dishes do not achieve a very large velocity toward the puller, and a relatively large friction force between the dishes and the tabletop, so that they stop quickly.

CONCEPTUAL PROBLEM 1

You are playing with your daughter in the snow. She is sitting on a sled and asking you to slide her across a flat, horizontal field. You have a choice of pushing her from behind, by applying a force at 30° below the horizontal, or attaching a rope to the front of the sled and pulling with a force at 30° above the horizontal. Which would be easier for you and why?

CONCEPTUAL PROBLEM 2

A book is given a push at the bottom of a nonfrictionless ramp so that it slides up to a point and then slides back down to the starting point. Does it take the same time to go up as it does to come down?

CONCEPTUAL PROBLEM 3

The driver of an empty speeding truck slams on the brakes and skids to a stop through a distance d. (a) If the truck carried a heavy load such that its mass were doubled, what would be its skidding distance? (b) If the initial speed of the truck is halved, what would be its skidding distance?

Example 5.1 The Sliding Hockey Puck

A hockey puck on a frozen pond is hit and given an initial speed of 20.0 m/s, as in Figure 5.3. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and the ice.

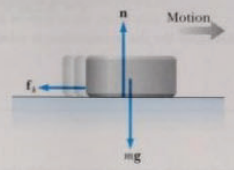


Figure 5.3 (Example 5.1) *After* the puck is given an initial velocity, the external forces acting on it are the weight, $m\mathbf{g}$, the normal force, $\mathbf{n}$, and the force of kinetic friction, $\mathbf{f}_k$.

Reasoning The forces acting on the puck after it is in motion are shown in Figure 5.3. If we assume that the force of friction, f_k, remains constant, then this force produces a uniform negative acceleration of the puck. First, we find the acceleration using Newton's second law. Knowing the acceleration of the puck and the distance it travels, we can then use kinematics to find the coefficient of kinetic friction.

Solution Applying Newton's second law in component form to the puck gives

$$(1)\quad \sum F_x = -f_k = ma$$

$$(2)\quad \sum F_y = n - mg = 0 \qquad (a_y = 0)$$

But $f_k = \mu_k n$, and from (2) we see that $n = mg$. Therefore, (1) becomes

$$-\mu_k n = -\mu_k mg = ma$$

$$a = -\mu_k g$$

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The negative sign means that the acceleration is to the left, corresponding to a negative acceleration of the puck. The acceleration is independent of the mass of the puck and is constant, because we assume that μ_k remains constant.

Because the acceleration is constant, we can use Equation 2.12, $v^2 = v_0^2 + 2ax$, with the final speed $v = 0$:

$$v_0^2 + 2ax = v_0^2 - 2\mu_k g x = 0$$

$$\mu_k = \frac{v_0^2}{2gx}$$

$$\mu_k = \frac{(20.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(115 \text{ m})} = 0.177$$

Note that μ_k has no dimensions.

Example 5.2 Connected Objects

A ball and a cube are connected by a light string that passes over a frictionless pulley, as in Figure 5.4a. The coefficient of kinetic friction between the cube and the surface is 0.30. Find the acceleration of the two objects and the tension in the string.

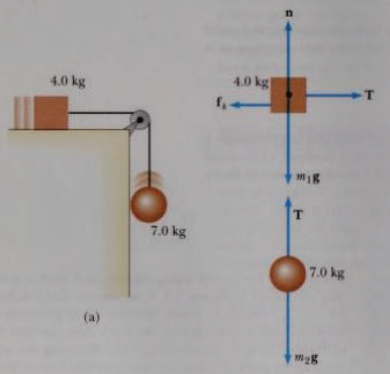


Figure 5.4 (Example 5.2) (a) Two objects connected by a light string that passes over a frictionless pulley. (b) Free-body diagrams for the objects.

Reasoning First, we draw the free-body diagrams for the two objects as in Figure 5.4b. Next, we apply Newton's second law in component form to each object and make use of the fact that the magnitude of the force of kinetic friction acting on the block is proportional to the normal force according to $f_k = \mu_k n$. Finally, we solve for the acceleration in terms of the parameters given.

Solution Newton's second law applied to the cube in component form, with the positive x direction to the right, gives

$$\sum F_x = T - f_k = (4.0 \text{ kg})a$$

$$\sum F_y = n - (4.0 \text{ kg})g = 0$$

Because $n = mg = (4.0 \text{ kg})(9.80 \text{ m/s}^2) = 39.2 \text{ N}$ and $f_k = \mu_k n$, we have $f_k = \mu_k n = (0.30)(39.2 \text{ N}) = 11.8 \text{ N}$. Therefore,

$$(1) \quad T = f_k + (4.0 \text{ kg})a = 11.8 \text{ N} + (4.0 \text{ kg})a$$

Now we apply Newton's second law to the ball moving in the vertical direction, where the downward direction is selected as positive:

$$\sum F_y = (7.0 \text{ kg})g - T = (7.0 \text{ kg})a$$

or

$$(2) \quad T = 68.6 \text{ N} - (7.0 \text{ kg})a$$

Subtracting (1) from (2) eliminates T:

$$56.8 \text{ N} - (11 \text{ kg})a = 0$$

$$a = 5.2 \text{ m/s}^2$$

When this value for the acceleration is substituted into (1), we get

$$T = 33 \text{ N}$$

EXERCISE 1 A block moves up a 45° incline with constant speed under the action of a force of 15 N applied *parallel* to the incline. If the coefficient of kinetic friction is 0.30, determine (a) the weight of the block and (b) the minimum force required to allow it to move *down* the incline at constant speed. Answer (a) 16.3 N (b) 8.07 N up the incline

EXERCISE 2 A block is placed on a plane that is inclined at 60° with respect to the horizontal. If the block slides down the plane with an acceleration of $g/2$, determine the coefficient of kinetic friction between the block and the plane. Answer 0.732

5.2 • NEWTON'S SECOND LAW APPLIED TO UNIFORM CIRCULAR MOTION

Solving problems involving friction is just one of many applications of Newton's second law. Let us now apply Newton's second law to another common situation: uniform circular motion. In Chapter 3, we found that a particle moving in a circular path of radius r with uniform speed v experiences an acceleration of magnitude

$$a_r = \frac{v^2}{r}$$

• *Centripetal acceleration*

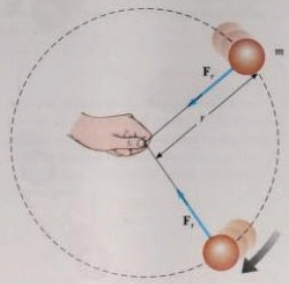
Because the velocity vector **v** changes its direction continuously during the motion, the acceleration vector **a**, is directed toward the center of the circle and hence is called **centripetal acceleration.** Furthermore, **a**, is *always* perpendicular to **v**.

Consider a ball of mass m tied to a string of length r and being whirled in a horizontal circular path on a table top as in Figure 5.5. Let us assume that the ball moves with constant speed. The ball tends to maintain the motion in a straight-line path; however, the string prevents this motion along a straight line by exerting a force on the ball to make it follow its circular path. This force is directed along the length of the string toward the center of the circle, as shown in Figure 5.5. This force can be any one of our familiar forces playing the role of causing an object to follow a circular path.

If we apply Newton's second law along the radial direction, we find that the value of the force causing the centripetal acceleration can be evaluated:

$$F_r = ma_r = m\frac{v^2}{r} \qquad [5.3]$$

Figure 5.5 An overhead view of a ball moving in a circular path in a horizontal plane. A force **F**, directed toward the center of the circle keeps the ball moving in the circle with constant speed.



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An athlete in the process of throwing the hammer. The force exerted by the chain on the hammer provides the centripetal acceleration. Only when the athlete releases the hammer will it move along a straight-line path tangent to the circle. *(Focus on Sports)*

A force causing a centripetal acceleration acts toward the center of the circular path and causes a change in the direction of the velocity vector. In the case of a ball rotating at the end of a string, the force exerted by the string on the ball causes the centripetal acceleration of the ball. For a satellite in a circular orbit around the Earth, the force of gravity causes the centripetal acceleration of the satellite. The force acting on a car rounding a curve on a flat road to cause the centripetal acceleration of the car is the force of static friction between the tires and pavement, and so forth. In general, a body can move in a circular path under the influence of various types of forces or a combination of forces.

Regardless of the type of force causing the centripetal acceleration, if the force acting on an object should vanish, the object would no longer move in its circular path; instead it would move along a straight-line path tangent to the circle. This idea is illustrated in Figure 5.6 for the case of the ball whirling in a circle at the end of a string. If the string breaks at some instant, the ball will move along the straight-line path tangent to the circle at the point at which the string broke.

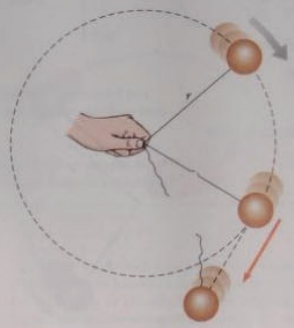


Figure 5.6 When the string breaks, the ball moves in the direction tangent to the circular path.

Thinking Physics 3

The force causing centripetal acceleration is often called a *centripetal force.* We are familiar with a variety of forces in nature—friction, gravity, normal forces, electrical forces, tension, and so on. Should we add *centripetal* force to this list?

Reasoning Centripetal force should *not* be added to this list. This is a pitfall for many students. Giving the force causing circular motion a name—centripetal force—leads many students to consider this as a new *kind* of force rather than a new *role* for force. A common mistake in force diagrams is to draw in all of the usual forces and then add another vector for the centripetal force. But it is not a separate force—it is simply one of our familiar forces *acting in the role of causing a circular motion.* Consider some examples. For the motion of the Earth around the Sun, the centripetal force is *gravity.* For an object sitting on a rotating turntable, the centripetal force is *friction.* For a rock whirled on the end of a string, the centripetal force is the *tension* in the string. For an amusement park patron pressed against the inner wall of a rapidly rotating circular room, the centripetal force is the *normal force* from the wall. In addition, the centripetal force could be a combination of two or more forces. For example, as a Ferris-wheel rider passes through the lowest point of the ride, the centripetal force on the rider is the difference between the normal force from the seat and the force of gravity.

Thinking Physics 4

High-speed, curved roadways are *banked*—that is, the roadway is tilted toward the inside of tight curves. Why is this?

Reasoning If an automobile is rounding a curve, there must be a force providing the centripetal acceleration. Normally, this is the static friction force between the tires and the roadway, parallel to the roadway surface and perpendicular to the centerline of the car. This is the only horizontal force—the weight and normal force are both vertical. If the curve is tight or the speed is high, the required friction force may be larger than the maximum possible friction force. In this case, Newton's first law takes over and the car skids along a straight line, which is off the side of the curved road. This situation is even more likely if the road is icy, which reduces the friction coefficient. Banking the roadway tilts the normal force away from the vertical, so that it has a horizontal component toward the center of the circular roadway. Thus, the normal force and friction work together to provide the needed centripetal acceleration. The larger the acceleration, the stronger the banking, in order to provide a larger component of the normal force in the horizontal direction.



As these cyclists in the Tour de France negotiate a curve on a flat racing track, the centripetal acceleration is provided by the force of static friction between the tires and the track surface. *(Michel Gouverneur, Photo News, Gamma Sport)*

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CONCEPTUAL PROBLEM 4

An object executes circular motion with a constant speed whenever a net force of constant magnitude acts perpendicular to the velocity. What happens to the speed if the force is not perpendicular to the velocity?

Example 5.3 How Fast Can It Spin?

A ball of mass 0.500 kg is attached to the end of a cord, the length of which is 1.50 m. The ball is whirled in a horizontal circle as in Figure 5.6. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed the ball can have before the cord breaks?

Solution Because the force that provides the centripetal acceleration in this case is the force **T** exerted by the cord on the ball, Equation 5.3 gives

$$T = m\frac{v^2}{r}$$

Solving for v, we have

$$v = \sqrt{\frac{Tr}{m}}$$

The maximum speed that the ball can have corresponds to the maximum value of the tension. Hence, we find

$$v_{max} = \sqrt{\frac{T_{max}r}{m}} = \sqrt{\frac{(50.0\text{ N})(1.50\text{ m})}{0.500\text{ kg}}} = 12.2\text{ m/s}$$

EXERCISE 3 Calculate the tension in the cord if the speed of the ball is 5.00 m/s. Answer 8.33 N

Example 5.4 The Conical Pendulum

A small body of mass m is suspended from a string of length L. The body revolves in a horizontal circle of radius r with constant speed v, as in Figure 5.7. (Because the string sweeps out the surface of a cone, the system is known as a *conical pendulum*.) Find the speed of the body and the period of revolution, T_P, defined as the time needed to complete one revolution.

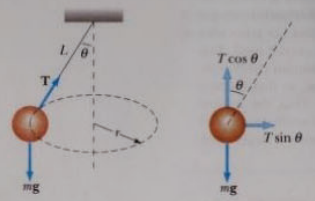


Figure 5.7 (Example 5.4) The conical pendulum and its free-body diagram.

Solution The free-body diagram for the mass m is shown in Figure 5.7, where the force exerted by the string, **T**, has been resolved into a vertical component, $T\cos\theta$, and a component $T\sin\theta$ acting toward the center of rotation. Because the body does not accelerate in the vertical direction, the vertical component of **T** must balance the weight. Therefore,

$$(1)\quad T\cos\theta = mg$$

Because the force that provides the centripetal acceleration in this example is the component $T\sin\theta$, from Newton's second law we get

$$(2)\quad T\sin\theta = ma_r = \frac{mv^2}{r}$$

By dividing (2) by (1), we eliminate T and find that

$$\tan\theta = \frac{v^2}{rg}$$

But from the geometry we note that $r = L\sin\theta$; therefore,

$$v = \sqrt{rg\tan\theta} = \sqrt{Lg\sin\theta\tan\theta}$$

Because the ball travels a distance of $2\pi r$ (the circumference of the circular path) in a time equal to the period of revolution, T_P (not to be confused with the force **T**), we find

$$(3)\quad T_P = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{rg\tan\theta}} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$

The intermediate algebraic steps used in obtaining (3) are left to the reader. Note that T_P is independent of m. If we take $L = 1.00$ m and $\theta = 20.0°$, using (3) we find that

$$T_P = 2\pi\sqrt{\frac{(1.00\text{ m})(\cos 20.0°)}{9.80\text{ m/s}^2}} = 1.95\text{ s}$$

Is it physically possible to have a conical pendulum with $\theta = 90°$?

Example 5.5 What Is the Maximum Speed of the Car?

A 1500-kg car moving on a flat, horizontal road negotiates a curve, the radius of which is 35.0 m, as in Figure 5.8. If the coefficient of static friction between the tires and the dry pavement is 0.500, find the maximum speed the car can have in order to make the turn successfully.

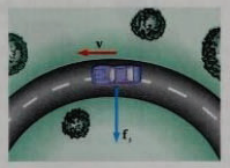


Figure 5.8 (Example 5.5) The force of static friction directed toward the center of the arc keeps the car moving in a circle.

Solution In this case, the force that provides the centripetal acceleration and enables the car to remain in its circular path is the force of static friction. Hence, from Equation 5.3 we have

$$(1)\quad f_s = m\frac{v^2}{r}$$

The maximum speed that the car can have around the curve corresponds to the speed at which it is on the verge of skidding outward. At this point, the friction force has its maximum value

$$f_{s,\,max} = \mu_s n$$

Because the magnitude of the normal force equals the weight in this case, we find

$$f_{s,\,max} = \mu_s mg = (0.500)(1500\text{ kg})(9.80\text{ m/s}^2) = 7350\text{ N}$$

Substituting this value into (1), we find that the maximum speed is

$$v_{max} = \sqrt{\frac{f_{s,\,max}r}{m}} = \sqrt{\frac{(7350\text{ N})(35.0\text{ m})}{1500\text{ kg}}} = 13.1\text{ m/s}$$

EXERCISE 4 On a wet day, the car described in this example begins to skid on the curve when its speed reaches 8.00 m/s. What is the coefficient of static friction in this case? Answer 0.187

Example 5.6 Let's Go Loop-the-loop

A pilot of mass m in a jet aircraft executes a loop-the-loop maneuver, as illustrated in Figure 5.9a. In this flying pattern, the aircraft moves in a vertical circle of radius 2.70 km at a *constant speed* of 225 m/s. Determine the force exerted by the seat on the pilot at (a) the bottom of the loop and (b) the top of the loop. Express the answers in terms of the weight of the pilot, mg.

Solution (a) The free-body diagram for the pilot at the bottom of the loop is shown in Figure 5.9b. The only forces acting on the pilot are the downward force of gravity, $m\mathbf{g}$, and the upward force $\mathbf{n}_{bot}$ exerted by the seat on the pilot. Because the net upward force that provides the centripetal acceleration has a magnitude $n_{bot} - mg$, Newton's second law for the radial direction gives

$$n_{bot} - mg = m\frac{v^2}{r}$$

$$n_{bot} = mg + m\frac{v^2}{r} = mg\left[1 + \frac{v^2}{rg}\right]$$

Substituting the values given for the speed and radius gives

$$n_{bot} = mg\left[1 + \frac{(225\text{ m/s})^2}{(2.70\times10^3\text{ m})(9.80\text{ m/s}^2)}\right] = 2.91mg$$

Hence, the force exerted by the seat on the pilot is *greater* than his weight by a factor of 2.91. The pilot experiences an apparent weight that is greater than his weight by the factor 2.91. This is discussed further in Section 5.3.

(b) The free-body diagram for the pilot at the top of the loop is shown in Figure 5.9c. At this point, both the force of gravity and the force exerted by the seat on the pilot, $\mathbf{n}_{top}$, act *downward*, so the net force downward that provides the centripetal acceleration has a magnitude $n_{top} + mg$. Applying Newton's second law gives

$$n_{top} + mg = m\frac{v^2}{r}$$

$$n_{top} = m\frac{v^2}{r} - mg = mg\left[\frac{v^2}{rg} - 1\right]$$

$$n_{top} = mg\left[\frac{(225\text{ m/s})^2}{(2.70\times10^3\text{ m})(9.80\text{ m/s}^2)} - 1\right]$$

$$= 0.911mg$$

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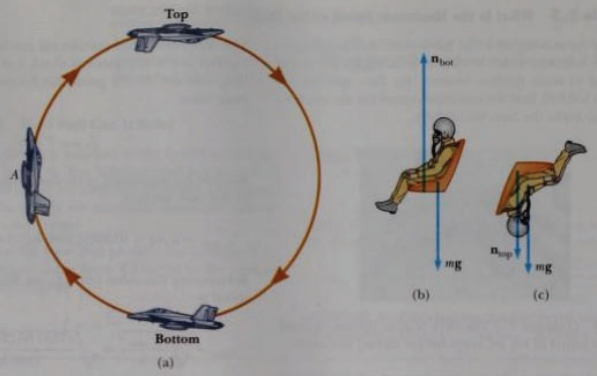


Figure 5.9 (Example 5.6)

In this case, the force exerted by the seat on the pilot is *less* than the weight by a factor of 0.911. Hence, the pilot will feel lighter at the top of the loop.

EXERCISE 5 Calculate the force on the pilot causing the centripetal acceleration if the aircraft is at point *A* in Figure 5.9a, midway up the loop. Answer 1.911*mg* directed to the right

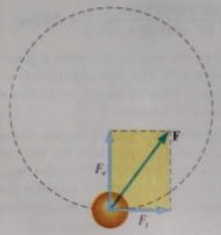


Figure 5.10 When the force acting on a particle moving in a circular path has a tangential component F_t, its speed changes. The total force on the particle also has a component F_r directed toward the center of the circular path. Thus, total force is $\mathbf{F} = \mathbf{F}_t + \mathbf{F}_r$.

EXERCISE 6 An automobile moves at constant speed over the crest of a hill. The driver moves in a vertical circle of radius 18.0 m. At the top of the hill, she notices that she barely remains in contact with the seat. Find the speed of the vehicle. Answer 13.3 m/s

EXERCISE 7 A 1500-kg car rounds an unbanked curve with a radius of 52 m at a speed of 12 m/s. What minimum coefficient of friction must exist between the road and tires to prevent the car from slipping? Answer 0.283

5.3 · NONUNIFORM CIRCULAR MOTION

In Chapter 3 we found that if a particle moves with varying speed in a circular path, there is, in addition to the centripetal component of acceleration, a tangential component of magnitude dv/dt. Therefore, the force acting on the particle must also have a tangential and a radial component, as shown in Figure 5.10. That is, because the total acceleration is $\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$, the total force exerted on the particle is $\mathbf{F} = \mathbf{F}_r + \mathbf{F}_t$. The component F_r is directed toward the center of the circle and is responsible for the centripetal acceleration. The component F_t tangent to the circle is responsible for the tangential acceleration, which causes the speed of the particle to change with time. The following examples demonstrate this type of motion.

Thinking Physics 5

Suppose you are the only rider on a Ferris wheel. The drive mechanism breaks after the ride starts, so that you continue to rotate freely, without friction. As you pass through the lowest point (let us call it the 6-o'clock point), you will feel the heaviest—that is, the seat will press upward on you with the largest force. As you pass over the highest point (the 12-o'clock point), you will feel the lightest. At the 3-o'clock and 9-o'clock points, will you feel as if you have the "correct" weight? If not, where *do* you feel as if your weight is correct?

Reasoning If the wheel were rotating at a constant speed, the answer to the question would be *yes*. If you are the only rider, however, the wheel is *unbalanced* and would not rotate with a constant speed. As you come down from the top point, your velocity increases. After you pass through the lowest point, your velocity decreases as you move back to the top. Thus, you will experience tangential acceleration as well as centripetal acceleration. At the 3-o'clock position, at which you are moving downward, you have a downward tangential acceleration. Thus, just as in a downward accelerating elevator, you will feel lighter. At the 9-o'clock position, when you are moving upward, you are accelerating downward tangentially. Again, you will feel lighter. In order to counteract the *downward* component of tangential acceleration near both positions, you need some *upward* component from the centripetal acceleration. This will be present if you are at positions *below* 3 o'clock or 9 o'clock, so that the centripetal acceleration has an upward component. The exact positions will depend on your mass and the details of the Ferris wheel.

Example 5.7 Follow the Rotating Ball

A small sphere of mass m is attached to the end of a cord of length R, which rotates in a *vertical* circle about a fixed point O, as in Figure 5.11a. Let us determine the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.

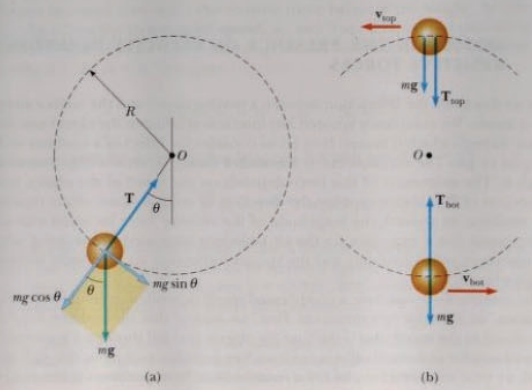


Figure 5.11 (Example 5.7) (a) Forces acting on a mass m connected to a string of length R and rotating in a vertical circle centered at O. (b) Forces acting on m when it is at the top and bottom of the circle. Note that the tension at the bottom is a maximum and the tension at the top is a minimum.

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Solution First we note that the speed is *not* uniform, because there is a tangential component of acceleration arising from the weight of the sphere. From the free-body diagram in Figure 5.11a, we see that the only forces acting on the sphere are the force of gravity, $m\mathbf{g}$, and the force exerted by the cord, $\mathbf{T}$. Now we resolve $m\mathbf{g}$ into a tangential component, $mg \sin\theta$, and a radial component, $mg\cos\theta$. Applying Newton's second law to the forces in the tangential direction gives

$$\sum F_t = mg \sin\theta = ma_t$$

$$a_t = g \sin\theta$$

This component causes v to change in time, because $a_t = dv/dt$.

Applying Newton's second law to the forces in the radial direction and noting that both $\mathbf{T}$ and $\mathbf{a}_r$ are directed toward O, we get

$$\sum F_r = T - mg\cos\theta = \frac{mv^2}{R}$$

$$T = m\left(\frac{v^2}{R} + g\cos\theta\right)$$

Limiting Cases At the top of the path, where $\theta = 180°$, we have $\cos 180° = -1$, and the tension equation becomes

$$T_{top} = m\left(\frac{v_{top}^2}{R} - g\right)$$

This is the *minimum* value of T. Note that at this point $a_t = 0$ and therefore the acceleration is radial and directed downward, as in Figure 5.11b.

At the bottom of the path, where $\theta = 0$, we see that because $\cos 0 = 1$,

$$T_{bot} = m\left(\frac{v_{bot}^2}{R} + g\right)$$

This is the *maximum* value of T. Again, at this point $a_t = 0$ and the acceleration is radial and directed upward.

EXERCISE 8 At what orientation of the system would the cord most likely break if the average speed increased?
Answer At the bottom of the path, where T has its maximum value.

EXERCISE 9 A 0.40-kg object is swung in a vertical circular path on a string 0.50 m long. If a constant speed of 4.0 m/s is maintained, what is the tension in the string when the object is at the top of the circle? Answer 8.88 N

EXERCISE 10 A car traveling on a straight road at 9.00 m/s goes over a hump in the road. The hump may be regarded as an arc of a circle of radius 11.0 m. What must be the speed of the car over the hump if a 600-N passenger is to experience weightlessness?
Answer 10.4 m/s

5.4 · MOTION IN THE PRESENCE OF VELOCITY-DEPENDENT RESISTIVE FORCES

Earlier we described the interaction between a moving object and the surface along which it moves. We completely ignored any interaction between the object and the medium through which it moves. Now let us consider the effect of a medium such as a liquid or gas. The medium exerts a **resistive force, R,** on the object moving through it. The magnitude of this force depends on the speed of the object, and the direction of $\mathbf{R}$ is always opposite the direction of motion of the object relative to the medium. In general, the magnitude of the resistive force increases with increasing speed. Some examples are the air resistance associated with moving vehicles (sometimes called air drag) and the viscous forces that act on objects moving through a liquid.

The resistive force can have a complicated speed dependence. In the following discussions, we consider two situations. First, we assume that the resistive force is proportional to the speed; this is the case for objects that fall through a liquid with low speed and for very small objects, such as dust particles, that move through air. Second, we treat situations for which the resistive force is proportional to the square of the speed of the object; large objects, such as a skydiver moving through air in free fall, experience such a force.

Resistive Force Proportional to Object Speed

If we assume that the resistive force acting on an object that is moving through a viscous medium is proportional to the object's velocity, then the resistive force can be expressed as

$$\mathbf{R} = -b\mathbf{v} \qquad [5.4]$$

where $\mathbf{v}$ is the velocity of the object and b is a constant that depends on the properties of the medium and on the shape and dimensions of the object. If the object is a sphere of radius r, then b is proportional to r.

Consider a sphere of mass m released from rest in a liquid, as in Figure 5.12a. Assuming the only forces acting on the sphere are the resistive force, $-b\mathbf{v}$, and the gravitational force, $m\mathbf{g}$, let us describe its motion.[1]

Applying Newton's second law to the vertical motion, choosing the downward direction to be positive, and noting that $\Sigma F_y = mg - bv$, we get

$$mg - bv = m\frac{dv}{dt}$$

where the acceleration is downward. Simplifying this expression gives

$$\frac{dv}{dt} = g - \frac{b}{m}v \qquad [5.5]$$

Equation 5.5 is called a *differential equation*, and the methods of solving such an equation may not be familiar to you as yet. However, note that initially, when $v = 0$, the resistive force is zero and the acceleration, dv/dt, is simply g. As t increases, the resistive force increases and the acceleration decreases. Eventually, the acceleration becomes zero when the resistive force balances the weight. At this point, the object reaches its **terminal speed,** v_t, and from then on it continues to move with zero acceleration. The terminal speed can be obtained from Equation 5.5 by setting $a = dv/dt = 0$. This gives

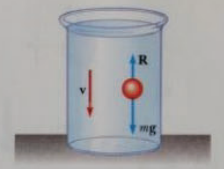
$$mg - bv_t = 0 \quad \text{or} \quad v_t = \frac{mg}{b}$$

The expression for v that satisfies Equation 5.5 with $v = 0$ at $t = 0$ is

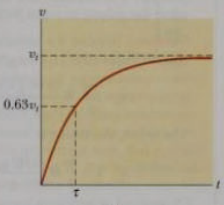
$$v = \frac{mg}{b}(1 - e^{-bt/m}) = v_t(1 - e^{-t/\tau}) \qquad [5.6]$$

This function is plotted in Figure 5.12b. The time constant $\tau = m/b$ is the time it takes the object to reach 63.2% of its terminal speed. This can be seen by noting that when $t = \tau$, Equation 5.6 gives $v = 0.632v_t$. We can check that Equation 5.6 is a solution to Equation 5.5 by direct differentiation:

[1]There is also a *buoyant* force acting on the submerged object, and this force is constant and equal to the weight of the displaced fluid, as will be discussed in Chapter 12. This force will only change the apparent weight of the sphere by a constant factor, so we can ignore it here.



(a)



(b)

Figure 5.12 (a) A small sphere falling through a viscous fluid. (b) The speed–time graph for an object falling through a viscous medium. The object reaches a maximum, or terminal speed v_t, and τ is the time it takes to reach $0.63v_t$.

• ***Terminal speed***

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$$\frac{dv}{dt} = \frac{d}{dt}\left(\frac{mg}{b} - \frac{mg}{b}e^{-bt/m}\right) = -\frac{mg}{b}\frac{d}{dt}e^{-bt/m} = ge^{-bt/m}$$

Substituting this expression and Equation 5.6 into Equation 5.5 shows that our solution satisfies the differential equation.

Example 5.8 A Sphere Falling in Oil

A small sphere of mass 2.00 g is released from rest in a large vessel filled with oil. The sphere reaches a terminal speed of 5.00 cm/s. Determine the time constant τ and the time it takes the sphere to reach 90% of its terminal speed.

Solution Because the terminal speed is given by $v_t = mg/b$, the coefficient b is

$$b = \frac{mg}{v_t} = \frac{(2.00\text{ g})(980\text{ cm/s}^2)}{5.00\text{ cm/s}} = 392\text{ g/s}$$

Therefore, the time τ is

$$\tau = \frac{m}{b} = \frac{2.00\text{ g}}{392\text{ g/s}} = 5.10 \times 10^{-3}\text{ s}$$

The speed of the sphere as a function of time is given by Equation 5.6. To find the time t it takes the sphere to reach a speed of $0.900v_t$, we set $v = 0.900v_t$ into the expression and solve for t:

$$0.900v_t = v_t(1 - e^{-t/\tau})$$
$$1 - e^{-t/\tau} = 0.900$$
$$e^{-t/\tau} = 0.100$$
$$-\frac{t}{\tau} = -2.30$$
$$t = 2.30\tau = 2.30(5.10 \times 10^{-3}\text{ s})$$
$$= 11.7 \times 10^{-3}\text{ s} = 11.7\text{ ms}$$

EXERCISE 11 What is the sphere's speed through the oil at $t = 11.7$ ms? Compare this value with the speed the sphere would have if it were falling in a vacuum and so influenced only by gravity. Answer 4.50 cm/s in oil versus 11.5 cm/s in free fall

Air Drag at High Speeds

For large objects moving at high speeds through air, such as airplanes, skydivers, and baseballs, the resistive force is approximately proportional to the square of the speed. In these situations, the magnitude of the resistive force can be expressed as

$$R = \tfrac{1}{2}D\rho Av^2 \qquad [5.7]$$

where ρ is the density of air, A is the cross-sectional area of the falling object measured in a plane perpendicular to its motion, and D is a dimensionless empirical quantity called the *drag coefficient*. The drag coefficient has a value of about 0.5 for spherical objects but can be as high as 2.0 for irregularly shaped objects.



Aerodynamic car. Streamlined bodies are used for sports cars and other vehicles to reduce air drag and increase fuel efficiency. *(© 1992 Dick Kelley)*

Consider an airplane in flight that experiences such a resistive force. Equation 5.7 shows that the force is proportional to the density of air and hence decreases with decreasing air density. Because air density decreases with increasing altitude, the resistive force on a jet airplane flying at a given speed must also decrease with increasing altitude. However, if at given altitude the plane's speed is doubled, the resistive force increases by a factor of four. In order to maintain this increased speed, the propulsive force must also increase by a factor of four.

Now let us analyze the motion of a mass in free fall subject to an upward air resistive force, the magnitude of which is $R = \frac{1}{2}D\rho Av^2$. Suppose a mass m is released from rest from the position $y = 0$, as in Figure 5.13. The mass experiences two external forces: the downward force of gravity, $m\mathbf{g}$, and the resistive force, $\mathbf{R}$, upward. (There is also an upward buoyant force that we neglect.) Hence, the magnitude of the net force is

$$F_{net} = mg - \tfrac{1}{2}D\rho Av^2 \qquad [5.8]$$

Substituting $F_{net} = ma$ into Equation 5.8, we find that the mass has a downward acceleration of magnitude

$$a = g - \left(\frac{D\rho A}{2m}\right)v^2 \qquad [5.9]$$

Again, we can calculate the terminal speed, v_t, using the fact that when the force of gravity is balanced by the resistive force, the net force is zero and therefore the acceleration is zero. Setting $a = 0$ in Equation 5.9 gives

$$g - \left(\frac{D\rho A}{2m}\right)v_t^2 = 0$$
$$v_t = \sqrt{\frac{2mg}{D\rho A}} \qquad [5.10]$$

Using this expression, we can determine how the terminal speed depends on the dimensions of the object. Suppose the object is a sphere of radius r. In this case, $A \propto r^2$ and $m \propto r^3$ (because the mass is proportional to the volume). Therefore, $v_t \propto \sqrt{r}$.

Table 5.2 lists the terminal speeds for several objects falling through air, all computed on the assumption that the drag coefficient is 0.5

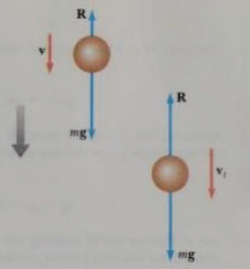


Figure 5.13 An object falling through air experiences a resistive drag force $\mathbf{R}$ and a gravitational force $m\mathbf{g}$. The object reaches terminal speed (on the right) when the net force is zero, that is, when $\mathbf{R} = -m\mathbf{g}$, or $R = mg$. Before this occurs, the acceleration varies with speed according to Equation 5.9.

TABLE 5.2 Terminal Speeds for Various Objects Falling Through Air

Object	Mass (kg)	Cross-sectional Area (m^2)	v_t(m/s)[a]
Skydiver	75.0	0.70	60
Baseball (radius 3.7 cm)	0.145	4.2×10^{-3}	33
Golf ball (radius 2.1 cm)	0.046	1.4×10^{-3}	32
Hailstone (radius 0.50 cm)	4.8×10^{-4}	7.9×10^{-5}	14
Raindrop (radius 0.20 cm)	3.4×10^{-5}	1.3×10^{-3}	9

[a] The drag coefficient, D, is assumed to be 0.5 in each case.

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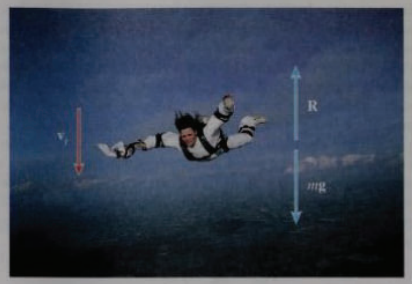


Figure 5.14 (Conceptual Problem 5) By spreading their arms and legs and by keeping their bodies parallel to the ground, skydivers experience maximum air resistance resulting in a minimum terminal speed. *(Guy Sauvage, Photo Researchers, Inc.)*

CONCEPTUAL PROBLEM 5

Consider a skydiver falling through air before reaching her terminal speed, as in Figure 5.14. As the speed of the skydiver increases, what happens to her acceleration? What is her acceleration once she reaches terminal speed?

5.5 • NUMERICAL MODELING IN PARTICLE DYNAMICS[2]

As we have seen in this and the preceding chapter, the study of dynamics of a particle focuses on describing the position, velocity, and acceleration as functions of time. Cause-and-effect relationships exist between these quantities: A velocity causes position to change, an acceleration causes velocity to change, and an acceleration is the direct result of applied forces. Therefore, the study of motion usually begins with an evaluation of the net force on the particle.

In this section, we confine our discussion to one-dimensional motion, so boldface notation will not be used for vector quantities. If a particle of mass m moves under the influence of a net force F, Newton's second law tells us that the acceleration of the particle is given by $a = F/m$. In general, we can then obtain the solution to a dynamics problem by using the following procedure:

1. Sum all the forces on the particle to get the net force F.
2. Use this force to determine the acceleration, using $a = F/m$.
3. Use this acceleration to determine the velocity from $dv/dt = a$.
4. Use this velocity to determine the position from $dx/dt = v$.

[2]The author is most grateful to Colonel James Head of the U.S. Air Force Academy for preparing this section.

Example 5.9 An Object Falling in a Vacuum

We can illustrate the procedure just described by considering a particle falling in a vacuum under the influence of the force of gravity, as in Figure 5.15.

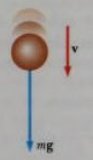


Figure 5.15 (Example 5.9) An object falling in vacuum under the influence of gravity.

Reasoning and Solution Applying Newton's second law, we set the sum of the external forces equal to the mass of the particle times its acceleration (taking upward to be the positive direction):

$$F = ma = -mg$$

Thus, $a = -g$, a constant. Because $dv/dt = a$, the resulting differential equation for velocity is $dv/dt = -g$, which may be integrated to give

$$v(t) = v_0 - gt$$

Then, because $dx/dt = v$, the position of the particle is obtained from another integration, which yields the well-known result

$$x(t) = x_0 + v_0 t - \tfrac{1}{2}gt^2$$

In this expression, x_0 and v_0 represent the position and speed of the particle at $t = 0$.

The procedure just described is straightforward for some physical situations, such as the one described in the previous example. In the real world, however, complications often arise that make analytical solutions for many practical situations difficult and perhaps beyond the mathematical abilities of most students taking introductory physics. For example, the force may depend on the position of the particle, as in cases in which the variation in gravitational acceleration with height must be taken into account. The force may also vary with velocity, as we have seen in cases of resistive forces caused by motion through a liquid or gas. The force may depend on both position and velocity, as in the case of an object falling through air where the resistive force depends on velocity and on height (air density). In rocket motion, the mass changes with time, so even if the force is constant, the acceleration is not.

Another complication arises because the equations relating acceleration, velocity, position, and time are differential equations, not algebraic equations. Differential equations are usually solved using integral calculus and other special techniques that introductory students may not have mastered.

So how does one proceed to solve real-world problems without advanced mathematics? One answer is to solve such problems on personal computers, using elementary numerical methods. The simplest of these is the Euler method, named after the Swiss mathematician Leonhard Euler (1707–1783).

The Euler Method

In the **Euler method** of solving differential equations, derivatives are approximated as finite differences. Considering a small increment of time, Δt, the relationship between speed and acceleration may be approximated as

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$$a(t) = \frac{\Delta v}{\Delta t} = \frac{v(t + \Delta t) - v(t)}{\Delta t}$$

Then the speed of the particle at the end of the period Δt is approximately equal to the speed at the beginning of the period, plus the acceleration during the interval multiplied by Δt:

$$v(t + \Delta t) = v(t) + a(t)\,\Delta t \qquad [5.11]$$

Because the acceleration is a function of time, this estimate of $v(t + \Delta t)$ will be accurate only if the time interval Δt is short enough that the change in acceleration during it is very small (as will be discussed later).

The position can be found in the same manner:

$$v(t) = \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

$$x(t + \Delta t) = x(t) + v(t)\,\Delta t \qquad [5.12]$$

It may be tempting to add the term $\frac{1}{2}a(\Delta t)^2$ to this result to make it look like the familiar kinematics equation, but this term is not included in the Euler method of integration because Δt is assumed to be so small that $(\Delta t)^2$ is nearly zero.

If the acceleration at any instant t is known, the particle's velocity and position at $(t + \Delta t)$ can be calculated from Equations 5.11 and 5.12. The calculation can then proceed in a series of finite steps to determine the velocity and position at any later time. The acceleration is determined by the net force acting on the object,

$$a(x, v, t) = \frac{F(x, v, t)}{m} \qquad [5.13]$$

which may depend explicitly on the position, velocity, or time.

It is convenient to set up the numerical solution to this kind of problem by numbering the steps and entering the calculations in a table. Table 5.3 illustrates how to do this in an orderly way. The equations provided in the table can be entered into a spreadsheet and the calculations performed row by row to determine the velocity, position, and acceleration as functions of time. The calculations can also be carried out using a program written in BASIC, PASCAL, or FORTRAN, or with commercially available mathematics packages for personal computers. Many small increments can be taken, and accurate results can usually be obtained with the help of a computer. Graphs of velocity versus time or position versus time can be displayed to help you visualize the motion.

TABLE 5.3 The Euler Method for Solving Dynamics Problems

Step	Time	Position	Velocity	Acceleration
0	t_0	x_0	v_0	$a_0 = F(x_0, v_0, t_0)/m$
1	$t_1 = t_0 + \Delta t$	$x_1 = x_0 + v_0\,\Delta t$	$v_1 = v_0 + a_0\,\Delta t$	$a_1 = F(x_1, v_1, t_1)/m$
2	$t_2 = t_1 + \Delta t$	$x_2 = x_1 + v_1\,\Delta t$	$v_2 = v_1 + a_1\,\Delta t$	$a_2 = F(x_2, v_2, t_2)/m$
3	$t_3 = t_2 + \Delta t$	$x_3 = x_2 + v_2\,\Delta t$	$v_3 = v_2 + a_2\,\Delta t$	$a_3 = F(x_3, v_3, t_3)/m$
	.	.	.	.
	.	.	.	.
	.	.	.	.
n	t_n	x_n	v_n	a_n

The Euler method has the advantage that the dynamics are not obscured—the fundamental relationships of acceleration to force, velocity to acceleration, and position to velocity are clearly evident. Indeed, these relationships form the heart of the calculations. There is no need to use advanced mathematics, and the basic physics governs the dynamics.

The Euler method is completely reliable for infinitesimally small time increments, but for practical reasons a finite increment size must be chosen. In order for the finite difference approximation of Equation 5.11 to be valid, the time increment must be small enough that the acceleration can be approximated as being constant during the increment. We can determine an appropriate size for the time increment by examining the particular problem that is being investigated. The criterion for the size of the time increment may need to be changed during the course of the motion. In practice, however, we usually choose a time increment appropriate to the initial conditions and use the same value throughout the calculations.

The size of the time increment influences the accuracy of the result, but unfortunately it is not easy to determine the accuracy of a solution by the Euler method without a knowledge of the correct analytical solution. One method of determining the accuracy of the numerical solution is to repeat the calculations with a smaller time increment and compare results. If the two calculations agree to a certain number of significant figures, you can assume that the results are correct to that precision.

5.6 · THE FUNDAMENTAL FORCES OF NATURE

We have described a variety of forces that are experienced in our everyday activities, such as the force of gravity that acts on all objects at or near the Earth's surface and the force of friction as one surface slides over another. Forces also act in the atomic and subatomic world. For example, atomic forces within the atom are responsible for holding its constituents together, and nuclear forces act on different parts of the nucleus to keep its parts from separating.

Until recently, physicists believed that there were four fundamental forces in nature: the gravitational force, the electromagnetic force, the strong nuclear force, and the weak nuclear force.

The Gravitational Force

The gravitational force is the mutual force of attraction between any two objects in the Universe. It is interesting and rather curious that, although the gravitational force can be very strong between macroscopic objects, it is the weakest of all the fundamental forces. For example, the gravitational force between the electron and proton in the hydrogen atom is only about 10^{-47} N, whereas the electromagnetic force between these same two particles is about 10^{-7} N.

Newton's law of gravitation states that every particle in the Universe attracts every other particle with a force that is directly proportional to the product of the two masses and inversely proportional to the square of the distance between them.

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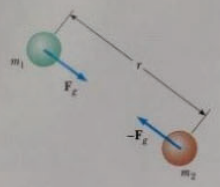


Figure 5.16 Two particles of masses m_1 and m_2 attract each other with a force of magnitude Gm_1m_2/r^2.

If the particles have masses m_1 and m_2 and are separated by a distance r, as in Figure 5.16, the magnitude of the gravitational force is

$$F_g = G\frac{m_1 m_2}{r^2} \qquad [5.14]$$

where $G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$ is the universal gravitational constant.

The Electromagnetic Force

The electromagnetic force is the force that binds atoms and molecules in compounds to form ordinary matter. It is much stronger than the gravitational force. The force that causes a rubbed comb to attract bits of paper and the force that a magnet exerts on an iron nail are electromagnetic forces. Essentially all forces at work in our macroscopic world (apart from the gravitational force) are manifestations of the electromagnetic force. For example, friction forces, contact forces, tension forces, and forces in elongated springs are consequences of electromagnetic forces between charged particles in proximity.

The electromagnetic force involves two types of particles: those with positive charge and those with negative charge. (The properties of these two types of charges will be discussed shortly.) Unlike the gravitational force, which is always an attractive interaction, the electromagnetic force can be either attractive or repulsive, depending on the charges on the particles. In general, the electromagnetic force acts between two charged particles that are in relative motion.

When the charged particles are at rest relative to each other, we refer to the electromagnetic force between them as the electrostatic force. Coulomb's law expresses the magnitude of the electrostatic force F_e between two charged particles separated by a distance r:

$$F_e = k_e\frac{q_1 q_2}{r^2} \qquad [5.15]$$

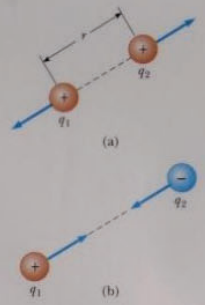


Figure 5.17 Two point charges separated by a distance r exert an electrostatic force on each other given by Coulomb's law. (a) When the charges are of the same sign, the charges repel each other. (b) When the charges are of opposite sign, the charges attract each other.

where q_1 and q_2 are the charges on the two particles, measured in coulombs (C), and k_e is the Coulomb constant, with the value $k_e = 8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$. Note that the electrostatic force has the same mathematical form as Newton's law of gravity (Eq. 5.14); however, charge replaces mass, and the constants are different. The electrostatic force is attractive if the two charges have opposite signs and repulsive if the two charges have the same sign, as indicated in Figure 5.17. In other words, opposite charges attract each other, and like charges repel each other.

The smallest amount of isolated charge found (so far) in nature is the charge on an electron or proton. This fundamental unit of charge has the magnitude $e = 1.60 \times 10^{-19}$ C. Theories developed in the 1970s and 1980s propose that protons and neutrons are made up of smaller particles called quarks, which have charges of either $2e/3$ or $-e/3$ (discussed further in Chapter 31). Although experimental evidence has been found for such particles inside nuclear matter, free quarks have never been detected.

The most important properties of electric charges can be summarized as follows:

- Charges are scalar quantities; hence, their values are additive.
- The two kinds of charges that exist in nature are labeled positive and negative. Charges of like sign repel each other, and charges of opposite sign attract each other.
- The net charge on any object has discrete values; in other words, charge is said to be quantized. Any isolated elementary charged particle has a charge of e. For example, electrons have a charge $-e$ and protons have a charge $+e$. (Neutrons have no charge.) Because e is the fundamental unit of charge, the net charge of an object is Ne, where N is an integer. For macroscopic objects, N is very large and the quantization of charge is not noticed.
- Electric charge is always conserved. This means that in any kind of process—such as a collision event, a chemical reaction, or nuclear decay—the total charge of an isolated system remains constant.

• *Properties of electric charge*

The Strong Force

An atom, as we currently understand it, consists of an extremely dense, positively charged nucleus surrounded by a cloud of negatively charged electrons, with the electrons attracted to the nucleus by the Coulomb force. Because all nuclei except those of hydrogen are combinations of positively charged protons and neutral neutrons (collectively called nucleons), why does the repulsive electrostatic force between the protons not cause nuclei to break apart? Clearly, there must be an attractive force that overcomes the strong electrostatic repulsive force and is responsible for the stability of nuclei. This great force that binds the nucleons to form a nucleus is called the *strong force*. It should be noted that electrons and certain other particles are immune to the strong force. Unlike the gravitational and electromagnetic forces, which depend on distance in an inverse-square fashion, the strong force is extremely short-range; its strength decreases very rapidly outside the nucleus and is negligible for separations greater than approximately 10^{-14} m. For separations of approximately 10^{-15} m (a typical nuclear dimension), the strong force is about two orders of magnitude stronger than the electromagnetic force.

The Weak Force

The *weak force* is a short-range force that tends to produce instability in certain nuclei. It was first observed in naturally occurring radioactive substances and was later found to play a key role in most radioactive decay reactions. The weak force is about 10^{25} times stronger than the gravitational force and about 10^{12} times weaker than the electromagnetic force.

The Current View of Fundamental Forces

For years physicists have searched for a simplification scheme that would reduce the number of fundamental forces in nature. In 1967 physicists predicted that the electromagnetic force and the weak force, originally thought to be independent of each other and both fundamental, are in fact manifestations of one force, now called the *electroweak force*. The prediction was confirmed experimentally in 1984. We also now know that protons and neutrons are not fundamental particles but are composed of simpler particles called quarks. For example, the proton consists of

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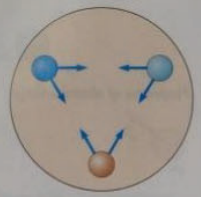


Figure 5.18 A proton is composed of three quarks that are bound together by a very strong force called the color force.

three quarks as shown in Figure 5.18. Quarks are bound together by a strong force called the **color force.**

Scientists believe that the fundamental forces of nature are closely related to the origin of the Universe. The Big Bang theory holds that the Universe began with a cataclysmic explosion 15 billion to 20 billion years ago. According to this theory, the first moments after the Big Bang saw such extremes of energy that all the fundamental forces were unified into one force. Physicists are continuing their search for connections among the known fundamental forces—connections that could eventually prove that the forces are all merely different forms of a single superforce. The recent success linking the electromagnetic and weak nuclear forces has spurred greater efforts at a unification scheme (yet to be proved) called the Grand Unified Theory. This fascinating subject continues to be at the forefront of physics.

5.7 • THE GRAVITATIONAL FIELD

When Newton first published his theory of gravitation, his contemporaries found it difficult to accept the concept of a force that one object could exert on another without anything happening in the space between them. They asked how it was possible for two masses to interact even though they were not in contact with each other. Although Newton himself could not answer this question, his theory was considered a success because it satisfactorily explained the motions of the planets.

An alternative approach is to think of the gravitational interaction as a two-step process. First, one object creates a gravitational field $\mathbf{g}$ throughout the space around it. Then a second object of mass m experiences a force $\mathbf{F}_g = m\mathbf{g}$. In other words, the field $\mathbf{g}$ exerts a force on the particle. Hence, the gravitational field is defined by

Definition of gravitational field •

$$\mathbf{g} \equiv \frac{\mathbf{F}_g}{m} \qquad [5.16]$$

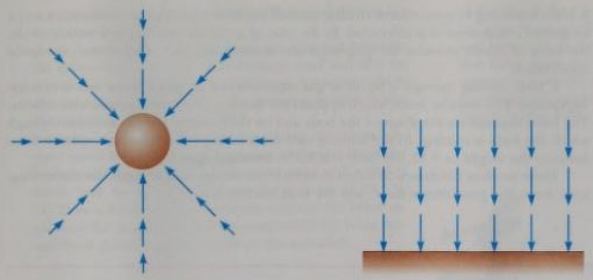
That is, the gravitational field at a point in space equals the gravitational force that a test mass experiences at that point divided by the mass. As a consequence, if $\mathbf{g}$ is known at some point in space, a test particle of mass m experiences a gravitational force $m\mathbf{g}$ when placed at that point.

As an example, consider an object of mass m near the Earth's surface. The gravitational force on the object is directed toward the center of the Earth and has a magnitude mg. Thus we see that the gravitational field experienced by the object at some point has a magnitude equal to the acceleration of gravity at that point. Because the gravitational force on the object has a magnitude $GM_E m/r^2$ (where M_E is the mass of the Earth), the field $\mathbf{g}$ at a distance r from the center of the Earth is given by

Gravitational field of the Earth •

$$\mathbf{g} = \frac{\mathbf{F}_g}{m} = -\frac{GM_E}{r^2}\hat{\mathbf{r}} \qquad [5.17]$$

where $\hat{\mathbf{r}}$ is a unit vector pointing radially outward from the Earth, and the minus sign indicates that the field points toward the center of the Earth, as shown in Figure



(a) (b)

Figure 5.19 (a) The gravitational field vectors in the vicinity of a uniform spherical mass vary in both direction and magnitude. (b) The gravitational field vectors in a small region near the Earth's surface are uniform—that is, they match in both direction and magnitude.

5.19a. Note that the field vectors at different points surrounding the spherical mass vary in both direction and magnitude. In a small region near the Earth's surface, $\mathbf{g}$ is approximately constant and the downward field is uniform, as indicated in Figure 5.19b. This expression is valid at all points outside the Earth's surface, assuming that the Earth is spherical and that rotation can be neglected. At the Earth's surface, where $r = R_E$, $\mathbf{g}$ has a magnitude of 9.80 m/s^2.

The field concept is used in many other areas of physics. In fact, it was introduced by Michael Faraday (1791–1867) in his study of electromagnetism. In spite of its abstract nature, the field concept is particularly useful for describing electric and magnetic interactions.

Gravitational, electric, and magnetic fields are vector fields, because a vector is associated with each point in space. In the same way, a scalar field is one in which a scalar quantity is used to describe each point in space. For example, the variation in temperature over a given region can be described by a scalar temperature field.

SUMMARY

The maximum force of static friction, $f_{s,\max}$, between a body and a surface is proportional to the normal force acting on the body. This maximum force occurs when the body is on the verge of slipping. In general, $f_s \leq \mu_s n$, where μ_s is the *coefficient of static friction* and $\mathbf{n}$ is the normal force. When a body slides over a rough surface, the *force of kinetic friction*, $\mathbf{f}_k$, is opposite the motion and is also proportional to the normal force. The magnitude of this force is given by $f_k = \mu_k n$, where μ_k is the *coefficient of kinetic friction*. Usually, $\mu_k < \mu_s$.

Newton's second law, applied to a particle moving in **uniform circular motion,** states that the net force in the radial direction must equal the product of the mass and the centripetal acceleration:

$$F_r = ma_r = \frac{mv^2}{r} \qquad [5.3]$$

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A particle moving in nonuniform circular motion has both a centripetal acceleration and a tangential component of acceleration. In the case of a particle rotating in a vertical circle, the force of gravity provides the tangential acceleration and part or all of the centripetal acceleration.

A body moving through a liquid or gas experiences a **resistive force** that is velocity-dependent. This resistive force, which opposes the motion, generally increases with velocity. The force depends on the shape of the body and on the properties of the medium through which the body is moving. In the limiting case for a falling body, when the resistive force balances the weight ($a = 0$), the body reaches its **terminal speed**.

There are four fundamental forces in nature: the strong nuclear force, the electromagnetic force, the gravitational force, and the weak nuclear force.

CONCEPTUAL QUESTIONS

1. If you push on a heavy box that is at rest, it requires some force **F** to start its motion. However, once it is sliding, it requires a smaller force to maintain that motion. Why?
2. Suppose you are driving a car along a highway at a high speed. Why should you avoid slamming on your brakes if you want to stop in the shortest distance? That is, why should you keep the wheels turning as you brake?
3. Twenty people are playing tug-of-war, with ten people on a team. The teams are so evenly matched that neither team wins. After they give up the game, they notice that one of the team member's cars is mired in mud. They attach the tug-of-war rope to the bumper of the car and the 20 people try to pull the car from the mud. The friction force between the car and the mud is huge, the car does not move, and the rope breaks. Why did the rope break in this situation when it did not break when 20 people pulled on it in the tug-of-war?
4. What causes a rotary lawn sprinkler to turn?
5. It has been suggested that rotating cylinders about 10 mi in length and 5 mi in diameter be placed in space and used as colonies. The purpose of the rotation is to simulate gravity for the inhabitants. Explain this concept for producing an effective gravity.
6. Consider a rotating space station, spinning with just the right speed such that the centripetal acceleration on the inner surface is g. Thus, astronauts standing on this inner surface would feel pressed to the surface as if they were pressed into the floor because of the Earth's gravitational force. Suppose an astronaut in this station holds a ball above her head and "drops" it to the floor. Will the ball fall just like it would on the Earth?
7. A pail of water can be whirled in a vertical path such that none is spilled. Why does the water stay in, even when the pail is above your head?
8. How would you explain the force that pushes a rider toward the side of a car as the car rounds a corner?
9. Why does a pilot tend to black out when pulling out of a steep dive?
10. Suppose a race track is built on the Moon. For a given speed of a car around the track, would the roadway have to be banked at a steeper angle or a shallower angle compared to the same roadway on the Earth?
11. A skydiver in free fall reaches terminal speed. After the parachute is opened, what parameters change to decrease this terminal speed?
12. Consider a small raindrop and a large raindrop falling through the atmosphere. Compare their terminal speeds. What are their accelerations when they reach terminal speed?
13. On long journeys, jet aircraft usually fly at high altitudes of about 30 000 ft. What is the main advantage of flying at these altitudes from an economic viewpoint?
14. If someone told you that astronauts are weightless in orbit because they are beyond the pull of gravity, would you accept the statement? Explain.

PROBLEMS

Section 5.1 Forces of Friction

1. A 25.0-kg block is initially at rest on a horizontal surface. A horizontal force of 75.0 N is required to set the block in motion. After it is in motion, a horizontal force of 60.0 N is required to keep the block moving with constant speed. Find the coefficients of static and kinetic friction from this information.
2. A racing car accelerates uniformly from 0 to 80.0 mi/h in 8.00 s. The external force that accelerates the car is the frictional force between the tires and the road. If the tires do not spin, determine the *minimum* coefficient of friction between the tires and the road.
3. A car is traveling at 50.0 mi/h on a horizontal highway. (a) If the coefficient of friction between road and tires on a rainy day is 0.100, what is the minimum distance in which the car will stop? (b) What is the stopping distance when the surface is dry and $\mu_s = 0.600$?
4. A woman at an airport is towing her 20.0-kg suitcase at constant speed by pulling on a strap at an angle of θ above the horizontal (Fig. P5.4). She pulls on the strap with a 35.0-N force, and the friction force on the suitcase is 20.0 N. (a) Draw a free-body diagram of the suitcase. (b) What angle does the strap make with the horizontal? (c) What normal force does the ground exert on the suitcase?



Figure P5.4

5. A 3.00-kg block is released from rest at the top of a 30.0° incline and slides a distance of 2.00 m down the incline in 1.50 s. Find (a) the magnitude of the acceleration of the block, (b) the coefficient of kinetic friction between block and plane, (c) the frictional force acting on the block, and (d) the speed of the block after it has slid 2.00 m.
6. In order to determine the coefficients of friction between rubber and various surfaces, a student uses a rubber eraser and an incline. In one experiment the eraser slips down the incline when the angle of inclination is 36.0° and then moves down the incline with constant speed when the angle is reduced to 30.0°. From these data, determine the coefficients of static and kinetic friction for this experiment.
7. A boy drags his 60.0-N sled at constant speed up a 15.0° hill. He does so by pulling with a 25.0-N force on a rope attached to the sled. If the rope is inclined at 35.0° to the horizontal, (a) what is the coefficient of kinetic friction between sled and snow? (b) At the top of the hill, he jumps on the sled and slides down the hill. What is the magnitude of his acceleration down the slope?
8. Determine the stopping distance for a skier with a speed of 20.0 m/s (Fig. P5.8). Assume $\mu_k = 0.180$ and $\theta = 5.00°$.

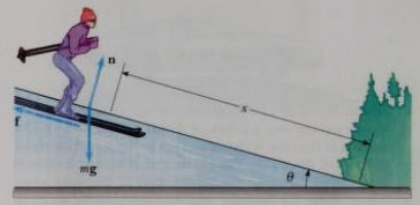


Figure P5.8

9. A 9.00-kg hanging weight is connected by a string over a pulley to a 5.00-kg block that is sliding on a flat table (Fig. P5.9). If the coefficient of kinetic friction is 0.200, find the tension in the string.

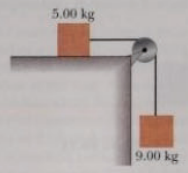


Figure P5.9

10. Three masses are connected on the table as shown in Figure P5.10. The table is rough and has a coefficient of sliding friction of 0.350. The three masses are $m_1 = 4.00$ kg, $m_2 = 1.00$ kg, and $m_3 = 2.00$ kg, respectively, and the pulleys are frictionless. (a) Draw free-body diagrams for each of the masses. (b) Determine the acceleration of each block and their directions. (c) Determine the tensions in the two cords.

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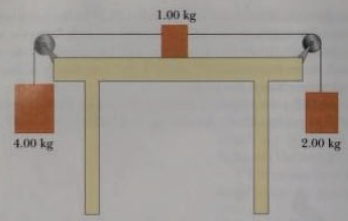


Figure P5.10

11. A mass $M = 2.20$ kg is accelerated across a rough surface by a rope passing over a pulley, as shown in Figure P5.11. The tension in the rope is 10.0 N and the pulley is 10.0 cm above the top of the block. The coefficient of kinetic friction is 0.400. (a) Determine the acceleration of the block when $x = 40.0$ cm. (b) Find the value of x at which the acceleration becomes zero.

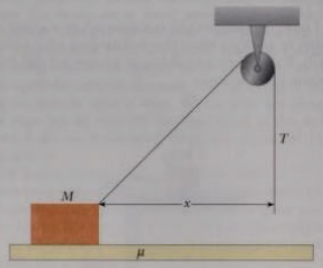


Figure P5.11

Section 5.2 Newton's Second Law Applied to Uniform Circular Motion

12. A toy car moving at constant speed completes one lap around a circular track (a distance of 200 m) in 25.0 s. (a) What is the average speed? (b) If the mass of the car is 1.50 kg, what is the magnitude of the force that keeps it moving in a circle?

13. A 3.00-kg mass attached to a light string rotates on a horizontal, frictionless table. The radius of the circle is 0.800 m, and the string can support a mass of 25.0 kg before breaking. What range of speeds can the mass have before the string breaks?

14. A 55.0-kg ice skater is moving at 4.00 m/s when she grabs the loose end of a rope the opposite end of which is tied to a pole. She then moves in a circle of radius 0.800 m around the pole. (a) Determine the force exerted by the rope on her arms. (b) Compare this force with her weight.

15. In the Bohr model of the hydrogen atom, the speed of the electron is approximately 2.20×10^6 m/s. Find (a) the centripetal acceleration of the electron and (b) the force acting on the electron as it revolves in a circular orbit of radius 0.530×10^{-10} m.

16. A crate of eggs is located in the middle of the flat bed of a pickup truck as the truck negotiates an unbanked curve in the road. The curve may be regarded as an arc of a circle of radius 35.0 m. If the coefficient of static friction between crate and truck is 0.600, what must be the maximum speed of the truck if the crate is not to slide?

17. A coin placed 30.0 cm from the center of a rotating, horizontal turntable slips when its speed is 50.0 cm/s. (a) What force provides the centripetal acceleration when the coin is stationary relative to the turntable? (b) What is the coefficient of static friction between coin and turntable?

18. A small turtle, appropriately named Dizzy, is placed on a horizontal, rotating turntable 20.0 cm from its center. Dizzy's mass is 50.0 g, and the coefficient of static friction between his feet and the turntable is 0.300. Find (a) the maximum number of revolutions per second the turntable can have if Dizzy is to remain stationary relative to the turntable and (b) Dizzy's speed and centripetal acceleration when he is on the verge of slipping.

19. Consider a conical pendulum with an 80.0-kg bob on a 10.0-m wire making an angle of 5.00° with the vertical (Fig. P5.19). Determine (a) the horizontal and vertical components of the force exerted by the wire on the pendulum, and (b) the centripetal acceleration of the bob.

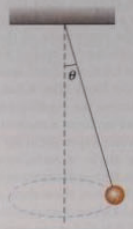


Figure P5.19

Section 5.3 Nonuniform Circular Motion

20. A 40.0-kg child sits in a swing supported by two chains, each 3.00 m long. If the tension in each chain at the lowest point is 350 N, find (a) the child's speed at the lowest point and (b) the force of the seat on the child at the lowest point. (Neglect the mass of the seat.)

21. Tarzan ($m = 85.0$ kg) tries to cross a river by swinging from a vine. The vine is 10.0 m long, and his speed at the bottom of the swing (as he just clears the water) is 8.00 m/s. Tarzan doesn't know that the vine has a breaking strength of 1000 N. Does he make it safely across the river?

22. At the Six Flags Great America amusement park in Gurnee, Illinois, there is a roller coaster that incorporates some of the latest design technology and some basic physics. Each vertical loop, instead of being circular, is shaped like a teardrop (Fig. P5.22). The cars ride on the inside of the loop at the top, and the speeds are high enough to ensure that the cars remain on the track. The biggest loop is 40.0 m high, with a maximum speed of 31.0 m/s (nearly 70 mph) at the bottom. Suppose the speed at the top is 13.0 m/s and the corresponding centripetal acceleration is $2g$. (a) What is the radius of the arc of the teardrop at the top? (b) If the total mass of the cars plus people is M, what force does the rail exert on it at the top? (c) Suppose the roller coaster had a loop of radius 20.0 m. If the cars have the same speed, 13.0 m/s at the top, what is the centripetal acceleration at the top? Comment on the normal force at the top in this situation.



Figure P5.22 *(Frank Cezus, FPG International)*

23. An 1800-kg car passes over a hump in a road that follows the arc of a circle of radius 42.0 m, as in Figure P5.23. (a) What force does the road exert on the car as the car passes the highest point of the hump if the car travels at 16.0 m/s? (b) What is the maximum speed the car can have as it passes this highest point before losing contact with the road?



Figure P5.23

24. A car of mass m passes over a hump in a road that follows the arc of a circle of radius R, as in Figure P5.23. (a) What force does the road exert on the car as the car passes the highest point of the hump if the car travels at a speed v? (b) What is the maximum speed the car can have as it passes this highest point before losing contact with the road?

25. A pail of water is rotated in a vertical circle of radius 1.00 m. What is the minimum speed of the pail at the top of the circle if no water is to spill out?

Section 5.4 Motion in the Presence of Velocity-Dependent Resistive Forces

26. A sky diver of mass 80.0 kg jumps from a slow-moving aircraft and reaches a terminal speed of 50.0 m/s. (a) What is the acceleration of the sky diver when her speed is 30.0 m/s? What is the drag force on the diver when her speed is (b) 50.0 m/s? (c) 30.0 m/s?

27. A small piece of Styrofoam packing material is dropped from a height of 2.00 m above the ground. Until the terminal speed is reached, the magnitude of the acceleration is given by $a = g - cv$. After falling 0.500 m, it reaches terminal speed, and then the Styrofoam takes an extra 5.00 s to reach the ground. (a) What is the value of the constant c? (b) What is the acceleration at $t = 0$? (c) What is the acceleration when the speed is 0.150 m/s?

28. (a) Estimate the terminal speed of a wooden sphere (density 0.830 g/cm^3) moving in air if its radius is 8.00 cm. (b) From what height would a freely falling object reach this speed in the absence of air resistance? Note that the density of air is 1.20 kg/m^3.

29. A motorboat cuts its engine when its speed is 10.0 m/s and coasts to rest. The equation governing the motion of the motorboat during this period is $v = v_0 e^{-ct}$, where v is the

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speed at time t, v_0 is the initial speed, and c is a constant. At $t = 20.0$ s, the speed is 5.00 m/s. (a) Find the constant c. (b) What is the speed at $t = 40.0$ s? (c) Differentiate the expression for $v(t)$ and thus show that the acceleration of the boat is proportional to the speed at any time.

30. Assume that the resistive force exerted on a speed skater is $f = -kmv^2$, where k is a constant and m is the skater's mass. The skater crosses the finish line of a straight-line race with speed v_f and then slows down by coasting without skating. Show that the skater's speed at any time t after crossing the finish line is $v(t) = v_f/(1 + ktv_f)$.

Section 5.5 Numerical Modeling in Particle Dynamics

31. A hailstone of mass 4.80×10^{-4} kg falls through the air and experiences a net force given by

$$F = -mg + Dv^2$$

where $D = 2.50 \times 10^{-5}$ kg/m. (a) Calculate the terminal speed of the hailstone. (b) Use Euler's method of numerical analysis to find the speed and position of the hailstone at 0.2-s intervals, taking the initial speed to be zero. Continue the calculation until the hailstone reaches 99% of terminal speed.

32. A 3.00-g leaf is dropped from a height of 2.00 m above the ground. Assume the net downward force on the leaf is $F = mg - bv$, where the drag factor is $b = 0.0300$ kg/s. (a) Calculate the terminal speed of the leaf. (b) Use Euler's method of numerical analysis to find the speed and position of the leaf, as functions of time, from the instant it is released until 99% of terminal speed is reached. (*Hint:* Try $\Delta t = 0.005$ s.)

33. A 50.0-kg parachutist jumps from an airplane and falls to Earth with a drag force proportional to the square of the speed, $R = Dv^2$. Take $D = 0.200$ kg/m (with the parachute closed) and $D = 20.0$ kg/m (with the chute open). (a) Determine the terminal speed of the parachutist in both configurations, before and after the chute is opened. (b) Set up a numerical analysis of the motion, and compute the speed and position as functions of time, assuming the jumper begins the descent at 1000 m above the ground and is in free fall for 10.0 s before opening the parachute. (*Hint:* When the parachute opens, a sudden large acceleration takes place; a smaller time step may be necessary in this region.)

34. Consider a 10.0-kg projectile launched with an initial speed of 100 m/s, at an angle of 35.0° elevation. The resistive force is $\mathbf{R} = -b\mathbf{v}$, where $b = 10.0$ kg/s. (a) Use a numerical method to determine the horizontal and vertical positions of the projectile as functions of time. (b) What is the range of this projectile? (c) Determine the elevation angle that gives the maximum range for the projectile. (*Hint:* Adjust the elevation angle by trial and error to find the greatest range.)

Section 5.6 The Fundamental Forces of Nature

35. Two identical isolated particles, each of mass 2.00 kg, are separated by a distance of 30.0 cm. What is the magnitude of the gravitational force of one particle on the other?

36. In a thundercloud there may be electric charges of +40.0 C near the top of the cloud and −40.0 C near the bottom of the cloud. These charges are separated by 2.00 km. What is the electric force on the top charge?

Section 5.7 The Gravitational Field

37. When a falling meteor is at a distance above the Earth's surface of 3.00 times the Earth's radius, what is its free-fall acceleration due to the gravitational force exerted on it?

Additional Problems

38. A crate of weight w is pushed by a force $\mathbf{F}$ on a horizontal floor. (a) If the coefficient of static friction is μ_s and $\mathbf{F}$ is directed at angle ϕ below the horizontal, show that the minimum value of F that will move the crate is given by

$$F = \frac{\mu_s w \sec \phi}{1 - \mu_s \tan \phi}$$

(b) Find the minimum value of F that can produce motion when $\mu_s = 0.400$, $w = 100$ N, and $\phi = 0°$, 15.0°, 30.0°, 45.0°, and 60.0°.

39. A 1.30-kg toaster is not plugged in. The coefficient of static friction between the toaster and a horizontal countertop is 0.350. To make the toaster start moving, you carelessly pull on its electric cord. (a) In order for the cord tension to be as small as possible, you should pull at what angle above the horizontal? (b) Then how large must the tension be?

40. A student builds and calibrates an accelerometer, which she uses to determine the speed of her car around a certain unbanked highway curve. The accelerometer is a plumb bob with a protractor that she attaches to the roof of her car. A friend riding in the car with her observes that the plumb bob hangs at an angle of 15.0° from the vertical when the car has a speed of 23.0 m/s. (a) What is the centripetal acceleration of the car rounding the curve? (b) What is the radius of the curve? (c) What is speed of the car if the plumb bob deflection is 9.00° while rounding the same curve?

41. A space station, in the form of a large wheel 120 m in diameter, rotates to provide an artificial gravity of 3.00 m/s² for persons situated at the outer rim. Find the rotational frequency of the wheel (in revolutions per minute) that will produce this effect.

42. An amusement park ride consists of a rotating circular platform 8.00 m in diameter from which 10.0-kg seats are suspended at the end of 2.50-m massless chains (Fig. P5.42). When the system rotates, the chains make an angle $\theta = 28.0°$ with the vertical. (a) What is the speed of each seat?

(b) Draw a free-body diagram of a 40.0-kg child riding in a seat and find the tension in the chain.

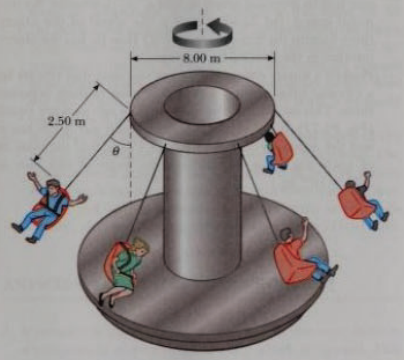


Figure P5.42

43. Because the Earth rotates about its axis, a point on the Equator experiences a centripetal acceleration of 0.0337 m/s², while a point at the poles experiences no centripetal acceleration. (a) Show that at the Equator the magnitude of the gravitational force on an object (its weight) must exceed the object's apparent weight. (b) What is the apparent weight at the Equator and at the poles of a person having a mass of 75.0 kg? (Assume the Earth is a uniform sphere and take $g = 9.800$ N/kg.)

44. The Earth rotates around its axis with a period of 24.0 h. Imagine that the rotational speed can be increased. If an object at the Equator is to have zero apparent weight, (a) what must the new period be? (b) By what factor would the speed of the object be increased when the planet is rotating at the higher speed? (*Hint:* See Problem 43 and note that the apparent weight of the object becomes zero when the normal force exerted on it is zero. Also, the distance traveled during one period is $2\pi R$, where R is the Earth's radius.)

45. An engineer wishes to design a curved exit ramp for a toll road in such a way that a car will not have to rely on friction to round the curve without skidding. Suppose that a typical car rounds the curve with a speed of 30 mi/h (13.4 m/s) and that the radius of the curve is 50.0 m. At what angle should the curve be banked? (See Fig. P5.45.)

46. A car rounds a banked curve as in Figure P5.45. The radius of curvature of the road is R, the banking angle is θ, and the coefficient of static friction is μ_s. (a) Determine the range of speeds the car can have without slipping up or down the road. (b) Find the minimum value for μ_s such that the minimum speed is zero. (c) What is the range of speeds possible if $R = 100$ m, $\theta = 10.0°$, and $\mu_s = 0.100$ (slippery conditions)?

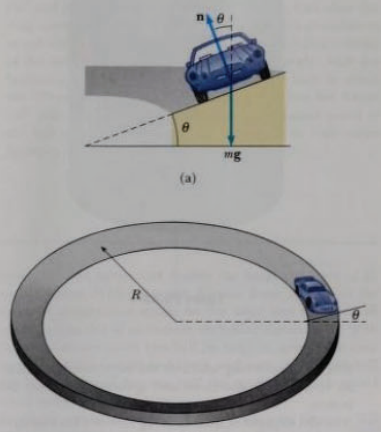


Figure P5.45

47. An amusement park ride consists of a large vertical cylinder that spins around its axis fast enough that any person inside is held up against the wall when the floor drops away (Fig. P5.47). The coefficient of static friction between person and wall is μ_s, and the radius of the cylinder is R. (a) Show that the maximum period of revolution necessary to keep the person from falling is $T = (4\pi^2 R\mu_s/g)^{1/2}$. (b) Obtain a numerical value for T if $R = 4.00$ m and $\mu_s = 0.400$. How many revolutions per minute does the cylinder make?

48. The expression $F = arv + br^2v^2$ gives the magnitude of the resistive force (in newtons) on a sphere of radius r (in meters) exerted by a stream of air with speed v (in meters per second), where a and b are constants with appropriate SI units. Their numerical values are $a = 3.10 \times 10^{-4}$ and $b = 0.870$. Using this formula, find the terminal speed for water droplets falling under their own weight in air, taking the following values for the drop radii: (a) 10.0 μm, (b) 100 μm, (c) 1.00 mm. Note that for (a) and (c) you can obtain accurate answers without solving a quadratic equa-

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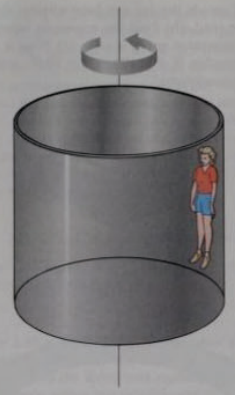


Figure P5.47

tion, by considering which of the two contributions to the air resistance is dominant and ignoring the lesser contribution.

49. A model airplane of mass 0.750 kg flies in a horizontal circle at the end of a 60.0-m control wire, with a speed of 35.0 m/s. Compute the tension in the wire if it makes a constant angle of 20.0° with the horizontal. The forces exerted on the airplane are the pull of the control wire, the force of gravity, and aerodynamic lift, which acts at 20.0° inward from the vertical as shown in Figure P5.49.

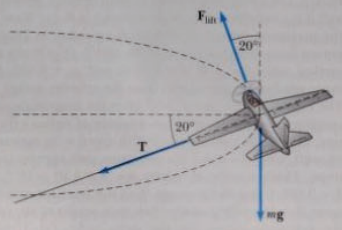


Figure P5.49

50. A 9.00-kg object starting from rest falls through a viscous medium and experiences a resistive force $\mathbf{R} = -b\mathbf{v}$, where $\mathbf{v}$ is the velocity of the object. If the object's speed reaches one half its terminal speed in 5.54 s, (a) determine the terminal speed. (b) At what time is the speed of the object three fourths the terminal speed? (c) How far has the object traveled in the first 5.54 s of motion?

51. Plaskett's binary system consists of two stars that revolve in a circular orbit around a center of mass midway between them. This means that the masses of the two stars are equal (Fig. P5.51). If the orbital velocity of each star is 220 km/s and the orbital period of each is 14.4 days, find the mass M of each star. (For comparison, the mass of our Sun is 1.99×10^{30} kg.)

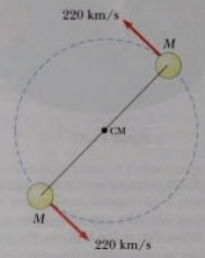


Figure P5.51

52. One block of mass 5.00 kg sits on top of a second rectangular block of mass 15.0 kg, which in turn is on a horizontal table. The coefficients of friction between the two blocks are $\mu_s = 0.300$ and $\mu_k = 0.100$. The coefficients of friction between the lower block and the rough table are $\mu_s = 0.500$ and $\mu_k = 0.400$. You apply a constant horizontal force to the lower block, just large enough to make this block start sliding out from between the upper block and the table. (a) Draw a free-body diagram of each block, naming the forces on each. (b) Determine the magnitude of each force on each block at the instant at which you have started pushing but motion has not yet started. In particular, what force must you apply? (c) Determine the acceleration you measure for each block.

53. Determine the order of magnitude of the gravitational force that you exert on another person 2 m away. In your solution state the quantities you estimate and the values you estimate for them.

Spreadsheet Problems

S1. A 0.142-kg baseball has a terminal speed of 42.5 m/s (95 mph). (a) If a baseball experiences a drag force of magnitude $R = Dv^2$, what is the value of D? (b) What is the magnitude of the drag force when the speed of the baseball is 36 m/s? (c) Set up a spreadsheet to determine the motion of a baseball thrown vertically upward at an initial speed of 36 m/s. What maximum height does the ball reach? How long is it in the air? What is its speed just before it hits the ground?

S2. A 10.0-kg projectile is launched with an initial speed of 150 m/s at an elevation angle of 35.0° with the horizontal. The resistive force acting on the projectile is $\mathbf{R} = -b\mathbf{v}$, where $b = 15$ kg/s. (a) Set up a spreadsheet to determine the horizontal and vertical positions of the projectile as functions of time. (b) Find the range of this projectile. (c) Determine the elevation angle that gives the maximum range for this projectile. (*Hint:* Adjust the elevation angle by trial and error to find the greatest range.)

S3. A professional golfer hits her 5-iron 155 m (170 yd). A 46-g golf ball experiences a drag force of magnitude $R = Dv^2$ and has a terminal speed of 44 m/s. (a) Calculate the drag coefficient D for the golf ball. (b) Set up a spreadsheet to calculate the trajectory of this shot. If the initial velocity of the ball makes an angle of 31° (the loft angle) with the horizontal, what initial speed must the ball have to reach the 155-m distance? (c) If this same golfer hits her 9-iron (47° loft) a distance of 119 m, what is the initial speed of the ball in this case? Discuss the differences in trajectories between the two shots.

ANSWERS TO CONCEPTUAL PROBLEMS

1. It is easier to attach the rope and pull. In this case, there is a component of your applied force which is upward. This reduces the normal force between the sled and the snow. In turn, this reduces the friction force between the sled and the snow, making it easier to move. If you push from behind, with a force with a downward component, the normal force is larger, the friction force is larger, and the sled is harder to move.
2. On the way up the ramp, the friction force and the component of the gravitational force on the book are both directed downhill. When the book returns back down the ramp, the component of the gravitational force is still downhill, but the friction force is uphill. Thus, there is a smaller net force when the book is sliding downhill, and according to Newton's second law, a smaller magnitude of acceleration. With a smaller acceleration, the velocity of the book at any point on the ramp is smaller when coming downhill than it was when going uphill. As a result, it takes longer to come down than to go up. The extreme case would be if the angle of the hill is such that once the book stops at the top of its motion, the maximum static friction force is larger than the component of the gravitational force, and the book "sticks" to the ramp. Then it takes *forever* to come down!
3. (a) Doubling the truck's mass would double the normal force the road exerts and double the backward force of kinetic friction. With twice the friction force acting on the mass, its acceleration would be the same and so would all other parameters of its motion, including distance. (b) With the same acceleration but half its original speed, the time required to stop would be halved, and moving with one-half the average speed for one-half the time, the truck would travel one-quarter the distance.
4. An object can move in a circle even if the total force on it is not perpendicular to its velocity, but then its speed will change. Resolve the total force into an inward radial component and a perpendicular tangential component. If the tangential force is forward, the object will speed up, and if the tangential force acts backward, it will slow down.
5. The forces exerted on the sky diver are the downward force of gravity, $m\mathbf{g}$, and an upward force of air resistance, $\mathbf{R}$, the magnitude of which is less than her weight before she reaches terminal speed. As her downward speed increases, the force of air resistance increases. The vector sum of the force of gravity and the force of air resistance gives a total force that decreases with time, so her acceleration decreases. Once she reaches terminal speed, the two forces balance each other, the net force is zero, and her acceleration is zero.

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6

Work and Energy

CHAPTER OUTLINE

6.1 Work Done by a Constant Force
6.2 The Scalar Product of Two Vectors
6.3 Work Done by a Varying Force
6.4 Kinetic Energy and the Work-Kinetic Energy Theorem
6.5 Power

Energy is present in the Universe in various forms, including mechanical energy, electromagnetic energy, chemical energy, thermal energy, and nuclear energy. When energy is changed from one form to another, its total amount remains the same. Conservation of energy says that if an isolated system loses energy in some form, it will gain an equal amount of energy in other forms.

In this chapter we shall concern ourselves only with the mechanical form of energy. We shall see that the concepts of work and energy can be applied to the dynamics of a mechanical system without resorting to Newton's laws. (However, it is important to note that the work–energy concepts are based on Newton's laws and therefore do not involve any new physical principles.) This energy approach to describing motion is especially useful when the force acting on a particle is not constant. Because the acceleration is not constant, we cannot apply the kinematic equations we developed in Chapter 2. Particles in nature are often subject to forces that vary with the particles' positions. These forces include gravitational forces and the force exerted on a body attached to a spring. We shall describe techniques for treating such systems with the help of an extremely important concept called the *work-kinetic energy theorem*, which is the central topic of this chapter.

We begin by defining work, a concept that provides a link between the concepts of force and energy.



These cyclists are working hard and expending energy as they pedal uphill in Marin County, California. *(David Madison, Tony Stone Images)*

6.1 · WORK DONE BY A CONSTANT FORCE

Almost all of the terms we have used thus far—*velocity*, *acceleration*, *force*, and so on—have conveyed the same meaning in physics as they do in everyday life. Now, however, we encounter a term the meaning of which in physics is distinctly different from its everyday meaning. This new term is **work**. Consider a particle that undergoes a displacement **d** along a straight line while acted on by a constant force **F**, which makes an angle θ with **d**, as in Figure 6.1.

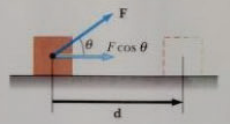


Figure 6.1 If an object undergoes a displacement **d**, the work done by the force **F** is $(F \cos \theta)d$.

The work W done by an agent exerting a constant force is the product of the component of the force along the direction of the displacement of the point of application of the force and the magnitude of the displacement:

$$W \equiv Fd \cos \theta \qquad [6.1]$$

From this definition, we see that a force does no work on a particle if the particle does not move: If $d = 0$, Equation 6.1 gives $W = 0$. Also note from Equation 6.1 that the work done by a force is zero when the force is perpendicular to the displacement. That is, if $\theta = 90°$, then $\cos 90° = 0$, and $W = 0$. For example, in Figure 6.2, the work done by the normal force and the work done by the force of gravity during the horizontal displacement are zero for the same reason. In general, the particle may be moving with a constant or varying velocity under the influence of several forces. In that case, because work is a scalar quantity, the total work done as the particle undergoes some displacement is the algebraic sum of the work done by each of the forces.

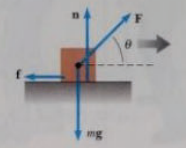


Figure 6.2 When an object is displaced horizontally, the normal force **n** and the force of gravity m**g** do no work.

The sign of the work also depends on the direction of **F** relative to **d**. The work done by the applied force is positive when the vector associated with the component $F \cos \theta$ is in the *same direction* as the displacement. For example, when an object is lifted, the work done by the applied force is positive because the lifting force is upward—that is, in the same direction as the displacement.

When the vector associated with the component $F \cos \theta$ is in the direction *opposite* the displacement, W is *negative*. In the case of the object being lifted, for instance, the work done by the gravitational force is negative. It is important to note that work is an energy transfer; if energy is transferred *to* the system, W is positive; if energy is transferred *from* the system, W is negative.

If an applied force **F** acts along the direction of the displacement, then $\theta = 0$ and $\cos 0 = 1$. In this case, Equation 6.1 gives

$$W = Fd \qquad [6.2]$$

Work is a scalar quantity, and its units are force multiplied by length. Therefore, the SI unit of work is the **newton · meter** (N · m). The newton · meter, when it refers to work or energy, is called the **joule** (J). The unit of work in the cgs system is the **dyne · cm**, also called the **erg**; the unit in the British engineering system is the **ft · lb**. These are summarized in Table 6.1. Note that $1 \text{ J} = 10^7$ ergs.

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TABLE 6.1 Units of Work in the Three Common Systems of Measurement

System	Unit of Work	Name of Combined Unit
SI	newton·meter (N·m)	joule (J)
cgs	dyne·centimeter (dyne·cm)	erg
British engineering (conventional)	foot·pound (ft·lb)	foot·pound (ft·lb)



Figure 6.3 (Thinking Physics 1) A person lifts a cement block of mass m a vertical distance h and then walks horizontally a distance d.

Thinking Physics 1

A person slowly lifts a cement block of mass m a vertical height h and then walks horizontally a distance d while holding the block, as in Figure 6.3. Determine the work done by the person and by the force of gravity in this process.

Reasoning Assuming that the person lifts the block with a force of magnitude equal to the weight of the block, mg, the work done by the person during the vertical displacement is mgh, because the force in this case is in the direction of the displacement. The work done by the person during the horizontal displacement of the block is zero, because the applied force in this process is perpendicular to the displacement. Thus, the net work done by the person is mgh. The work done by the force of gravity during the vertical displacement of the block is $-mgh$, because this force is opposite the displacement. The work done by the force of gravity is zero during the horizontal displacement because this force is also perpendicular to the displacement. Hence, the net work done by the force of gravity is $-mgh$. The net work on the block is zero $(+mgh - mgh = 0)$.

Thinking Physics 2

A planet moves in a perfectly circular orbit around the Sun. What can be said about the work done by the gravitational force on the planet? Suppose the orbit is elliptical; what can be said now about the work done by the gravitational force?

Reasoning In the case of the circular orbit, the gravitational force is always perpendicular to the displacement of the planet as it moves around the orbit. Thus, there is no work done over any portion of the orbit, and no net work done in one complete orbit. In the case of the elliptical orbit, the Sun is at one focus of the ellipse. Thus, the force vector is not, in general, perpendicular to the displacement. While the planet is moving away from the Sun toward its farthest distance, called the aphelion, the force vector has a component opposite to the displacement. Thus, negative work is done on the planet and it slows down. As the planet passes aphelion and moves back toward its closest distance to the Sun, called the perihelion, there is a component of the gravitational force that is parallel to the displacement. The planet thus has positive work done on it and speeds up. The total negative work done on the outbound trip is canceled by the positive work done on the return trip. Thus, in the elliptical orbit, there is work done over any given portion of the orbit (except for portions symmetric about the aphelion or perihelion), but no net work is done over one complete orbit.

CONCEPTUAL PROBLEM 1

Consider a tug-of-war, in which the two teams pulling on the rope are evenly matched, so that no motion takes place. Is work done on the rope? On the pullers? On the ground? Is work done on anything?

CONCEPTUAL PROBLEM 2

A team of furniture movers wishes to load a truck using a ramp from the ground to the rear of the truck. One of the movers claims that less work would be required to load the truck if the length of the ramp were increased, reducing the angle of the ramp with respect to the horizontal. Is his claim valid? Explain.

CONCEPTUAL PROBLEM 3

Roads going up mountains are formed into *switchbacks*, with the road weaving back and forth along the face of the slope, such that there is only a gentle rise on any portion of the roadway. Does this require any less work to be done by an automobile climbing the mountain, compared to driving on a roadway that runs straight up the slope? Why are the switchbacks used?

Example 6.1 Mr. Clean

A man cleaning his apartment pulls a vacuum cleaner with a force of magnitude $F = 50$ N. The force makes an angle of 30° with the horizontal as shown in Figure 6.4. The vacuum cleaner is displaced 3.0 m to the right. Calculate the work done by the 50-N force.

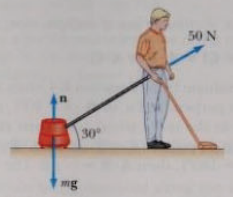


Figure 6.4 (Example 6.1) A vacuum cleaner being pulled at an angle of 30° with the horizontal.

Solution Using the definition of work (Equation 6.1), we have

$$W_F = (F\cos\theta)d = (50\text{ N})(\cos 30°)(3.0\text{ m})$$
$$= 130\text{ N}\cdot\text{m} = 130\text{ J}$$

Note that the normal force, $\mathbf{n}$, the force of gravity, $m\mathbf{g}$, and the upward component of the applied force, (50 N) sin 30°, do *no* work because they are perpendicular to the displacement.

EXERCISE 1 Find the work done by the man on the vacuum cleaner if he pulls it 3.0 m with a horizontal force of 32 N. Answer 96 J

EXERCISE 2 If a person lifts a 20.0-kg bucket from a well and does 6.00 kJ of work, how deep is the well? Assume the speed of the bucket remains constant as it is lifted. Answer 30.6 m

EXERCISE 3 A 65-kg woman climbs a flight of 20 stairs, each 23 cm high. How much work is done against the force of gravity in the process? Answer 2.93 kJ

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6.2 · THE SCALAR PRODUCT OF TWO VECTORS

We have defined work as a *scalar* quantity given by the product of the magnitude of the displacement and the component of a force in the direction of the displacement. It is convenient to express Equation 6.1 in terms of a **scalar product** of the two vectors **F** and **d**. We write this scalar product **F·d.** Because of the dot symbol, the scalar product is often called the *dot product.* Thus, we can express Equation 6.1 as a scalar product:

Work expressed as a scalar product

$$W = \mathbf{F}\cdot\mathbf{d} = Fd\cos\theta \qquad [6.3]$$

In other words, **F·d** (read "F dot d") is a shorthand notation for $Fd\cos\theta$.

In general, the scalar product of any two vectors **A** and **B** is a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle θ between them:

Scalar product of any two vectors **A** *and* **B**

$$\mathbf{A}\cdot\mathbf{B} \equiv AB\cos\theta \qquad [6.4]$$

where θ is the smaller angle between **A** and **B**, as in Figure 6.5. Note that **A** and **B** need not have the same units.

In Figure 6.5, $B\cos\theta$ is the projection of **B** onto **A**. Therefore, Equation 6.4 says that **A·B** is the product of the magnitude of **A** and the projection of **B** onto **A**.[1]

From Equation 6.4 we also see that the scalar product is *commutative.* That is,

The order of the scalar product can be reversed.

$$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A} \qquad [6.5]$$

Finally, the scalar product obeys the *distributive law of multiplication,* so that

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{C}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{C} \qquad [6.6]$$

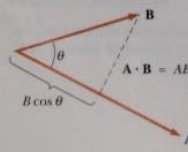


Figure 6.5 The scalar product **A·B** equals the magnitude of **A** multiplied by the projection of **B** onto **A**.

The dot product is simple to evaluate from Equation 6.4 when **A** is either perpendicular or parallel to **B**. If **A** is perpendicular to **B** ($\theta = 90°$), then $\mathbf{A}\cdot\mathbf{B} = 0$. (The equality $\mathbf{A}\cdot\mathbf{B} = 0$ also holds in the more trivial case when either **A** or **B** is zero.) If **A** and **B** point in the same direction ($\theta = 0°$), then $\mathbf{A}\cdot\mathbf{B} = AB$. If **A** and **B** point in opposite directions ($\theta = 180°$), then $\mathbf{A}\cdot\mathbf{B} = -AB$. The scalar product is negative when $90° < \theta < 180°$.

The unit vectors **i**, **j**, and **k**, which were defined in Chapter 1, lie in the positive *x*, *y*, and *z* directions, respectively, of a right-handed coordinate system. Therefore, it follows from the definition of **A·B** that the scalar products of these unit vectors are given by

Scalar products of unit vectors

$$\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = \mathbf{k}\cdot\mathbf{k} = 1 \qquad [6.7]$$

$$\mathbf{i}\cdot\mathbf{j} = \mathbf{i}\cdot\mathbf{k} = \mathbf{j}\cdot\mathbf{k} = 0 \qquad [6.8]$$

Two vectors **A** and **B** can be expressed in component form as

[1]This is equivalent to stating that **A·B** equals the product of the magnitude of **B** and the projection of **A** onto **B** or, vice versa, the product of the magnitude of **A** and the projection of **B** onto **A.**

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

Therefore, Equations 6.7 and 6.8 reduce the scalar product of **A** and **B** to

$$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z \qquad [6.9]$$

In the special case where **A** = **B**, we see that

$$\mathbf{A}\cdot\mathbf{A} = A_x^2 + A_y^2 + A_z^2 = A^2$$

Example 6.2 The Scalar Product

The vectors **A** and **B** are given by $\mathbf{A} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{B} = -\mathbf{i} + 2\mathbf{j}$.

(a) Determine the scalar product **A·B**.

Solution $\mathbf{A}\cdot\mathbf{B} = (2\mathbf{i} + 3\mathbf{j})\cdot(-\mathbf{i} + 2\mathbf{j})$

$= -2\mathbf{i}\cdot\mathbf{i} + 2\mathbf{i}\cdot2\mathbf{j} - 3\mathbf{j}\cdot\mathbf{i} + 3\mathbf{j}\cdot2\mathbf{j}$

$= -2 + 6 = 4$

where we have used the facts that $\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = 1$ and $\mathbf{i}\cdot\mathbf{j} = \mathbf{j}\cdot\mathbf{i} = 0$. The same result is obtained using Equation 6.9 directly, where $A_x = 2$, $A_y = 3$, $B_x = -1$, and $B_y = 2$.

(b) Find the angle θ between **A** and **B**.

Solution The magnitudes of **A** and **B** are given by

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

Using Equation 6.4 and the result from (a) gives

$$\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{8.06} = 60.2°$$

EXERCISE 4 For $\mathbf{A} = 4\mathbf{i} + 3\mathbf{j}$ and $\mathbf{B} = -\mathbf{i} + 3\mathbf{j}$, find (a) **A·B** and (b) the angle between **A** and **B**. Answer (a) 5.00 (b) 71.6°

EXERCISE 5 As a particle moves from the origin to $(3\mathbf{i} - 4\mathbf{j})$ m, it is acted on by a force given by $(4\mathbf{i} - 5\mathbf{j})$ N. Calculate the work done by this force as the particle moves through the given displacement. Answer 32 J

6.3 · WORK DONE BY A VARYING FORCE

Consider a particle being displaced along the *x* axis under the action of a varying force, as in Figure 6.6. The particle is displaced in the direction of increasing *x* from $x = x_i$ to $x = x_f$. In such a situation, we cannot use $W = (F\cos\theta)d$ to calculate the work done by the force, because this relationship applies only when **F** is constant in magnitude and direction. However, if we imagine that the particle undergoes a small displacement, Δx, shown in Figure 6.6a, then the *x* component of the force, F_x, is approximately constant over this interval and we can express the work done by the force for this small displacement as

$$W_i = F_x\,\Delta x \qquad [6.10]$$

This quantity is just the area of the shaded rectangle in Figure 6.6a. If we imagine that the F_x versus *x* curve is divided into a large number of such intervals, then the

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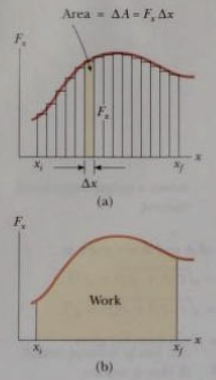


Figure 6.6 (a) The work done by the force F_x for the small displacement Δx is $F_x \Delta x$, which equals the area of the shaded rectangle. The total work done for the displacement from x_i to x_f is approximately equal to the sum of the areas of all the rectangles. (b) The work done by the variable force F_x as the particle moves from x_i to x_f is *exactly* equal to the area under this curve.

total work done for the displacement from x_i to x_f is approximately equal to the sum of a large number of such terms:

$$W \equiv \sum_{x_i}^{x_f} F_x \, \Delta x$$

If the displacements Δx are allowed to approach zero, then the number of terms in the sum increases without limit, but the value of the sum approaches a definite value equal to the area under the curve bounded by F_x and the x axis as in Figure 6.6b. As you probably have learned in calculus, this limit of the sum is called an *integral* and is represented by

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \, \Delta x = \int_{x_i}^{x_f} F_x \, dx$$

The limits on the integral, $x = x_i$ to $x = x_f$, define what is called a **definite integral.** (An *indefinite integral* is the limit of a sum over an unspecified interval. Appendix B.7 gives a brief description of integration.) This definite integral is numerically equal to the area under the F_x versus x curve between x_i and x_f. Therefore, we can express the work done by F_x for the displacement of the particle from x_i to x_f as

$$W = \int_{x_i}^{x_f} F_x \, dx \qquad [6.11]$$

This equation reduces to Equation 6.1 when $F_x = F \cos \theta$ is constant.

If more than one force acts on a particle, the total work done is just the work done by the resultant force. If we express the resultant force in the x direction as ΣF_x, then the *net work* done as the particle moves from x_i to x_f is

$$W_{\text{net}} = \int_{x_i}^{x_f} \left(\sum F_x \right) dx \qquad [6.12]$$

Thinking Physics 3

Consider a single force that causes an object to follow a path. If the force is constant, under what conditions can the work done over the entire path be zero? If the force is varying, under what conditions can the work done over the entire path be zero?

Reasoning In the case of the constant force, there are several possibilities that will give a result of zero work. One possibility is that the constant value of the force is zero, in which case we have a trivial result of nothing pushing. The second possibility for zero work is that a nonzero force does act, but it does not succeed in moving the object, although this does not fit with the question, because it is stated that the object *does* move over a path. The third possibility is that the force is always perpendicular to the path, as in the case of a planet moving in a circular orbit around a gravitational-force center.

For the varying force, only one of the above possibilities is consistent with the question. The zero-force possibility is not available, because a force whose value is *always* zero is not a *varying* force. The second possibility, that of zero displacement, as is the case above, does not fit with the statement that the object does move through a path. The third possibility, in which the force is perpendicular to the displacement, will also give zero work in the case of a varying force.



Does the weight lifter do any work as he holds the weight on his shoulders? Does he do any work as he raises the weight?

We have additional possibilities in the case of the varying force that were not available in the constant force case. One possibility is that the varying force (or a constant force applied opposite the velocity) carries the object around a path such that the object ends up back where it started. In this case the net work is zero, because there was no net displacement. This is the case for a comet in an elliptical orbit around the Sun—no net work is done by the gravitational force during one complete orbit. Another possibility is that the relative direction of the displacement and the force reverse during the path in such a way that the area above the axis in the force-displacement graph exactly equals the area below the axis. This is the case for an automobile accelerating to some speed and then braking to a stop. The positive work done while the automobile is speeding up exactly equals the negative work done as the automobile slows back down (the equality of the positive and negative work is probably not obvious at this point, but will be after Section 6.4).

Example 6.3 Calculating Total Work Done from a Graph

A force acting on a particle varies with x, as shown in Figure 6.7. Calculate the work done by the force as the particle moves from $x = 0$ to $x = 6.0$ m.

Solution The work done by the force is equal to the area under the curve from $x = 0$ to $x = 6.0$ m. This area is equal to the area of the rectangular section from $x = 0$ to $x = 4.0$ m plus the area of the triangular section that extends from $x = 4.0$ m to $x = 6.0$ m. The area of the rectangle is $(4.0)(5.0)$ N·m = 20 J, and the area of the triangle is $\frac{1}{2}(2.0)(5.0)$ N·m = 5.0 J. Therefore, the total work done is 25 J.

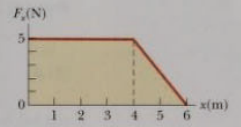


Figure 6.7 (Example 6.3) The force acting on a particle is constant for the first 4.0 m of motion and then decreases linearly with x from $x = 4.0$ m to $x = 6.0$ m. The net work done by this force is the area under this curve.

Work Done by a Spring

A common physical system for which the force varies with position is shown in Figure 6.8. A block on a horizontal, frictionless surface is connected to a spring. If the spring is stretched or compressed a small distance from its unstretched, or equilibrium, configuration, the spring will exert a force on the block given by

$$F_x = -kx \qquad [6.13]$$

• *Spring force*

where x is the displacement of the block from its unstretched ($x = 0$) position and k is a positive constant called the *force constant* of the spring. As mentioned in Chapter 4, Section 4.2, this force law for springs is known as **Hooke's law.** For many materials, Hooke's law can be very accurate, provided the displacement is not too large. The value of k is a measure of the stiffness of the spring. Stiff springs have large k values, and soft springs have small k values.

The negative sign in Equation 6.13 signifies that the force exerted by the spring is always directed *opposite* the displacement. For example, when $x > 0$, as in Figure 6.8a, the spring force is to the left, or negative. When $x < 0$, as in Figure 6.8c, the

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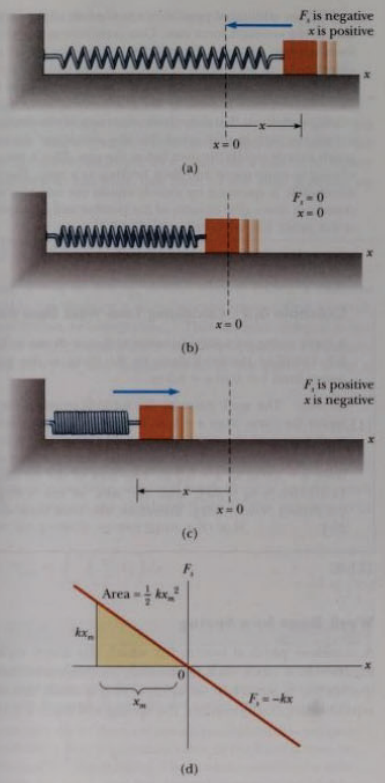


Figure 6.8 The force exerted by a spring on a block varies with the block's displacement from the equilibrium position $x = 0$. (a) When x is positive (stretched spring), the spring force is to the left. (b) When x is zero, the spring force is zero (natural length of the spring). (c) When x is negative (compressed spring), the spring force is to the right. (d) Graph of F_s versus x for the mass–spring system. The work done by the spring force as the block moves from $-x_m$ to 0 is the area of the shaded triangle, $\frac{1}{2}kx_m^2$.

spring force is to the right, or positive. Of course, when $x = 0$, as in Figure 6.8b, the spring is unstretched and $F_s = 0$. Because the spring force always acts toward the equilibrium position, it is sometimes called a *restoring force*. Once the block is displaced some distance x_m from equilibrium and then released, it moves from $-x_m$ through zero to $+x_m$. The details of the ensuing oscillating motion will be given in Chapter 12.

Suppose that the block is pushed to the left a distance x_m from equilibrium, as in Figure 6.8c, and then released. Let us calculate the *work done by the spring force* as

the block moves from $x_i = -x_m$ to $x_f = 0$. Applying Equation 6.11 assuming the block may be treated as a particle, we get

$$W_s = \int_{x_i}^{x_f} F_s\,dx = \int_{-x_m}^{0} (-kx)\,dx = \tfrac{1}{2}kx_m^2 \qquad [6.14]$$

• *Work done by a spring*

where we have used the indefinite integral $\int x\,dx = x^2/2$. That is, the work done by the spring force is positive because the spring force is in the same direction as the displacement caused by the spring force (both are to the right). However, if we consider the work done by the spring force as the block moves from $x_i = 0$ to $x_f = x_m$, we find that $W_s = -\frac{1}{2}kx_m^2$, because for this part of the motion the displacement caused by the spring force is to the right and the spring force is to the left. Therefore, the *net* work done by the spring force as the block moves from $x_i = -x_m$ to $x_f = x_m$ is *zero*.

If we plot F_s versus x, as in Figure 6.8d, we arrive at the same results. Note that the work calculated in Equation 6.14 is equal to the area of the shaded triangle in Figure 6.8d, with base x_m and height kx_m. This area is $\frac{1}{2}kx_m^2$.

If the block undergoes an *arbitrary* displacement from $x = x_i$ to $x = x_f$, the work done by the spring force is

$$W_s = \int_{x_i}^{x_f} (-kx)\,dx = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad [6.15]$$

From this equation we see that the work done by the spring force is zero for any motion that ends where it began ($x_i = x_f$). We shall make use of this important result in Chapter 7, where we describe the motion of this system in more detail.

Equations 6.14 and 6.15 describe the work done by the spring force exerted on the block. Now let us consider the work done on the spring by an *external agent* as the spring is stretched *very slowly* from $x_i = 0$ to $x_f = x_m$, as in Figure 6.9. This work can be easily calculated by noting that the *applied force*, $\mathbf{F}_{app}$, is equal to and opposite the spring force, $\mathbf{F}_s$, at any value of the displacement, so that $F_{app} = -(-kx) = kx$. Therefore, the work done on the spring by this applied force (the external agent) is

$$W_{F_{app}} = \int_0^{x_m} F_{app}\,dx = \int_0^{x_m} kx\,dx = \tfrac{1}{2}kx_m^2$$

You should note that this work is equal to the negative of the work done by the spring force for this displacement.

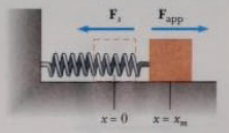


Figure 6.9 A block being pulled from $x = 0$ to $x = x_m$ on a frictionless surface by a force $\mathbf{F}_{app}$. If the process is carried out very slowly, the applied force is equal to and opposite the spring force at all times.

CONCEPTUAL PROBLEM 4

A spring is stretched by a distance x_m, resulting in work W being done by the external agent. The spring is then relaxed, cut in half, and one of the halves stretched by the same distance. How much work is done in this case?

Example 6.4 Work Required to Stretch a Spring

One end of a horizontal Hooke's-law spring ($k = 80$ N/m) is held fixed while an external force is applied to the free end, stretching it from $x_0 = 0$ to $x_1 = 4.0$ cm. (a) Find the work done by the external force.

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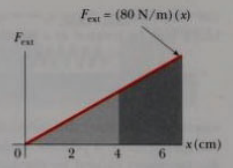


Figure 6.10 (Example 6.4) A graph of the external force required to stretch a spring that obeys Hooke's law versus the elongation of the spring.

Solution We place the zero reference of the coordinate axis at the free end of the unstretched spring (Fig. 6.10). The external force is $F_{ext} = (80\ N/m)(x)$. The work done by F_{ext} is the area of the triangle from 0 to 4.0 cm:

$$W = \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}(80\ N/m)(0.040\ m)^2 = 64\ mJ$$

(b) Find the additional work done in stretching the spring from $x_1 = 4.0$ cm to $x_2 = 7.0$ cm.

Solution The work done in stretching the spring the additional amount, from $x_1 = 0.040$ m to $x_2 = 0.070$ m, is the darker-shaded area between these limits. Geometrically, it is

$$W = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2$$
$$= \tfrac{1}{2}(80\ N/m)\,[(0.070\ m)^2 - (0.040\ m)^2] = 130\ mJ$$

Using calculus, we find that

$$W = \int_{x_1}^{x_2} F_{ext}\,dx = \int_{0.04\,m}^{0.07\,m} (80\ N/m)\,dx$$
$$= \tfrac{1}{2}(80\ N/m)(x^2)\Big|_{0.04\,m}^{0.07\,m}$$
$$W = \tfrac{1}{2}(80\ N/m)\,[(0.070\ m)^2 - (0.040\ m)^2] = 130\ mJ$$

EXERCISE 6 If an applied force varies with position according to $F_x = 3x^2 - 5$, where x is in meters, how much work is done by this force on an object that moves from $x = 4$ m to $x = 7$ m? Answer 264 J

6.4 · KINETIC ENERGY AND THE WORK-KINETIC ENERGY THEOREM

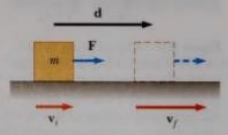


Figure 6.11 A particle undergoing a displacement and change in velocity under the action of a constant net force **F**.

Solutions using Newton's second law can be difficult if the forces in the problem are complex. An alternative approach that enables us to understand and solve such motion problems is to relate the speed of the particle to its displacement under the influence of some net force. As we shall see in this section, if the work done by the net force on a particle can be calculated for a given displacement, the change in the particle's speed will be easy to evaluate.

Figure 6.11 shows a particle of mass m moving to the right under the action of a constant net force **F**. Because the force is constant, we know from Newton's second law that the particle will move with a constant acceleration **a**. If the particle is displaced a distance d, the work done by the force **F** is

$$W_{net} = Fd = (ma)d \qquad [6.16]$$

In Chapter 2 (Eqs. 2.4 and 2.9) we found that the following relationships are valid when a particle undergoes constant acceleration:

$$d = \tfrac{1}{2}(v_i + v_f)t \qquad a = \frac{v_f - v_i}{t}$$

where v_i is the speed at $t = 0$ and v_f is the speed at time t. Substituting these expressions into Equation 6.16 gives

$$W_{net} = m\left(\frac{v_f - v_i}{t}\right)\tfrac{1}{2}(v_i + v_f)t$$

$$W_{net} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \qquad [6.17]$$

The quantity $\frac{1}{2}mv^2$ represents the energy associated with the motion of a particle. It is so important that it has been given a special name—**kinetic energy.** The kinetic energy, K, of a particle of mass, m, moving with a speed, v, is defined as

$$K \equiv \tfrac{1}{2}mv^2 \qquad [6.18]$$

- *Kinetic energy is energy associated with the motion of a particle.*

Kinetic energy is a scalar quantity and has the same units as work. For example, a 2.0-kg mass moving with a speed of 4.0 m/s has a kinetic energy of 32 J. It is often convenient to write Equation 6.17 in the form

$$W_{net} = K_f - K_i = \Delta K \qquad [6.19]$$

- *The work-kinetic energy theorem states that the work done on a particle equals the change in its kinetic energy.*

That is,

> the work done by the constant net force $\mathbf{F}_{net}$ in displacing a particle equals the change in kinetic energy of the particle.

Equation 6.19 is an important result known as the **work-kinetic energy theorem.** For convenience, it was derived under the assumption that the net force acting on the particle was constant. A more general derivation would show that this equation is also valid for a variable force.

The work-kinetic energy theorem also says that the speed of the particle will increase if the net work done on it is positive, because the final kinetic energy will be greater than the initial kinetic energy. The speed will decrease if the net work is negative, because the final kinetic energy will be less than the initial kinetic energy. The speed and kinetic energy of a particle change only if work is done on the particle by some external force.

Consider the relationship between the work done on a particle and the change in its kinetic energy as expressed by Equation 6.19. Because of this connection, we can also think of kinetic energy as the work the particle can do in coming to rest. For example, suppose a hammer is on the verge of striking a nail, as in Figure 6.12. The moving hammer has kinetic energy and can do work on the nail. The work done on the nail appears as the product Fd, where F is the average force exerted on the nail by the hammer and d is the distance the nail is driven into the wall. However, the hammer and the nail are not particles, so part of the kinetic energy of the hammer goes into warming the hammer and nail, and all of the work done on the nail goes into warming the nail and wall and locally deforming the wall.



Figure 6.12 The hammer has kinetic energy associated with its motion and can do work on the nail, driving it into the wall.

CONCEPTUAL PROBLEM 5

You are working in a library, reshelving books. You lift a book from the floor to the top shelf. The kinetic energy of the book on the floor was zero, and the kinetic energy of the book on the top shelf is zero, so there is no change in kinetic energy. Yet you did some work in lifting the book. Is the work-kinetic energy theorem violated?

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Situations Involving Kinetic Friction

When dealing with a force acting on an extended object, one must be careful in calculating the work done by that force because the displacement of the object is generally not equal to the displacement of the point of application of that force. Suppose that an object of mass m sliding on a horizontal surface is pulled with a constant horizontal external force $\mathbf{F}$ to the right and a kinetic frictional force $\mathbf{f}$ acts to the left, where $\mathbf{F} > \mathbf{f}$. In this case, the net force is to the right as in Figure 6.11, and the net work done on the object as it undergoes a displacement $\mathbf{d}$ to the right is

$$W_{\text{net}} = (\mathbf{F} - \mathbf{f}) \cdot \mathbf{d} = Fd - fd \qquad [6.20]$$

The quantity Fd is the work done on the object by the constant force $\mathbf{F}$. The quantity $-fd$ is negative because the force of kinetic friction is opposite the displacement. The work done by kinetic friction depends on both the displacement of the object and on the details of the motion between the initial and final positions. In fact, the work done by kinetic friction on an extended object cannot be explicitly evaluated because friction forces and their individual displacements are complex.

Now suppose that a block moving on a horizontal surface and given an initial horizontal velocity $\mathbf{v}_i$ slides a distance d before reaching a final velocity $\mathbf{v}_f$ as in Figure 6.13. The external force that causes the block to undergo an acceleration in the negative x direction is the force of kinetic friction $\mathbf{f}$ acting to the left, opposite the motion. The initial kinetic energy of the block is $\frac{1}{2}mv_i^2$ and its final kinetic energy is $\frac{1}{2}mv_f^2$. The change in kinetic energy of the block is equal to $-fd$. This can be shown by applying Newton's second law to the block. (Newton's second law gives the acceleration of the center of mass of any object regardless of how or where the forces act.) Because the net force on the block in the x direction is the friction force, Newton's second law gives $-f = ma$. Multiplying both sides of this expression by d and using the expression $v_f^2 - v_i^2 = 2ad$ for motion under constant acceleration gives $-fd = (ma)d = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ or

$$\Delta K = -fd \qquad [6.21]$$

This result says that the change in kinetic energy of the block is equal to $-fd$, which corresponds to the energy dissipated by the force of kinetic friction. Part of this energy is transferred to internal thermal energy of the block, and part is transferred from the block to the surface.[2] In effect, the loss in kinetic energy of the block results in an increase in thermal energy of both the block and surface. For example, if the loss in kinetic energy of the block is 300 J, and 100 J appears as an increase in thermal energy of the block, then the remaining 200 J must have been transferred from the block to the surface.

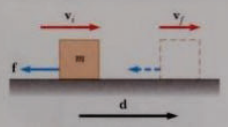


Figure 6.13 A block sliding to the right on a horizontal surface slows down in the presence of a force of kinetic friction acting to the left. The initial velocity of the block is $\mathbf{v}_i$, and its final velocity is $\mathbf{v}_f$. The normal force and force of gravity are not included in the diagram because they are perpendicular to the direction of motion and therefore do not influence the change in velocity of the block.

Thinking Physics 4

A car traveling at a speed v skids a distance d after its brakes lock. Estimate how far it will skid if its brakes lock when its initial speed is $2v$. What happens to the car's kinetic energy as it stops?

[2]For more details on energy transfer situations involving forces of kinetic friction, see B. A. Sherwood and W. H. Bernard, *American Journal of Physics*, 52:1001, 1984, and R. P. Bauman, *The Physics Teacher*, 30:264, 1992.

Reasoning Let us assume that the force of kinetic friction between car and road surface is constant and the same in both cases. The net force times the displacement of the car is equal to its initial kinetic energy. If the speed is doubled as in this example, the kinetic energy of the car is quadrupled. For a given applied force (in this case, the frictional force), the distance traveled is four times as great when the initial speed is doubled, so the estimated distance it skids is $4d$. The kinetic energy of the car is changed into internal energy associated with the tires, brake pads, and road as they increase in temperature.

Thinking Physics 5

In most situations we have encountered in this chapter, frictional forces tend to reduce the kinetic energy of an object. However, frictional forces can sometimes increase an object's kinetic energy. Describe a few situations in which friction causes an increase in kinetic energy.

Reasoning If a crate is located on the bed of a truck and the truck accelerates to the east, the static friction force exerted on the crate by the truck acts to the east to give the crate the same acceleration as the truck (assuming the crate doesn't slip). Another example is a car that accelerates because of the frictional forces exerted on the car's tires by the road. These forces act in the direction of the car's motion, and the sum of these forces causes an increase in the car's kinetic energy.

CONCEPTUAL PROBLEM 6

It is a known fact that more energy is expended by walking downstairs than by walking horizontally at the same speed. Why do you think this is so?

Example 6.5 A Block Pulled on a Frictionless Surface

A 6.0-kg block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant, horizontal force of 12 N, as in Figure 6.14. Find the speed of the block after it has moved 3.0 m.

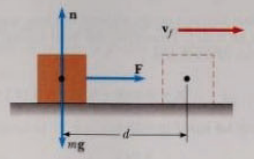


Figure 6.14 (Example 6.5)

Solution The gravitational force acting on the block is balanced by the normal force, and neither of these forces does work because the displacement is horizontal. Because there is no friction, the resultant external force is the 12-N force. The work done by this force is

$$W = Fd = (12\ \text{N})(3.0\ \text{m}) = 36\ \text{N}\cdot\text{m} = 36\ \text{J}$$

Using the work-kinetic energy theorem and noting that the initial kinetic energy is zero, we get

$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - 0$$

$$v_f^2 = \frac{2W}{m} = \frac{2(36\ \text{J})}{6.0\ \text{kg}} = 12\ \text{m}^2/\text{s}^2$$

$$v_f = 3.5\ \text{m/s}$$

EXERCISE 7 Find the acceleration of the block using the kinematic equation $v_f^2 = v_i^2 + 2ax$.
Answer $a = 2.0\ \text{m/s}^2$

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Example 6.6 A Block Pulled on a Rough Surface

Find the final speed of the block described in Example 6.5 if the surface is rough and the coefficient of kinetic friction is 0.15.

Reasoning In this case, we must use Equation 6.21 to calculate the change in kinetic energy, ΔK. The net force exerted on the block is the sum of the applied 12-N force and the frictional force, as in Figure 6.15. Because the frictional force is in the direction opposite the displacement, it must be subtracted.

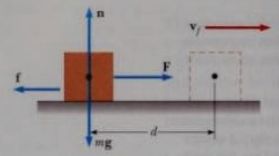


Figure 6.15 (Example 6.6)

Solution The magnitude of the frictional force is $f = \mu_k n = \mu_k mg$. Therefore the net force acting on the block is

$$F_{net} = F - \mu_k mg = 12\text{ N} - (0.15)(6.0\text{ kg})(9.80\text{ m/s}^2)$$
$$= 12\text{ N} - 8.82\text{ N} = 3.18\text{ N}$$

Multiplying this constant force by the displacement and using Equation 6.21 gives

$$\Delta K = F_{net}d = (3.18\text{ N})(3.0\text{ m}) = 9.54\text{ J} = \tfrac{1}{2}mv_f^2$$

using the information that $v_i = 0$. Therefore,

$$v_f^2 = \frac{2(9.54\text{ J})}{6.0\text{ kg}} = 3.18\text{ m}^2/\text{s}^2$$

$$v_f = 1.8\text{ m/s}$$

EXERCISE 8 Find the acceleration of the block from Newton's second law. Answer $a = 0.53\text{ m/s}^2$

Example 6.7 A Mass-Spring System

A block of mass 1.6 kg is attached to a spring that has a force constant of 1.0×10^3 N/m as in Figure 6.16. The spring is compressed a distance of 2.0 cm, and the block is released from rest. (a) Calculate the speed of the block as it passes through the equilibrium position $x = 0$ if the surface is frictionless.

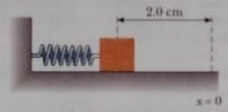


Figure 6.16 (Example 6.7)

Solution We use Equation 6.14 to find the work done by the spring with $x_m = -2.0\text{ cm} = -2.0 \times 10^{-2}\text{ m}$:

$$W_s = \tfrac{1}{2}kx_m^2 = \tfrac{1}{2}(1.0 \times 10^3\text{ N/m})(-2.0 \times 10^{-2}\text{ m})^2 = 0.20\text{ J}$$

Using the work-kinetic energy theorem with $v_i = 0$ gives

$$W_s = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$0.20\text{ J} = \tfrac{1}{2}(1.6\text{ kg})v_f^2 - 0$$

$$v_f^2 = \frac{0.40\text{ J}}{1.6\text{ kg}} = 0.25\text{ m}^2/\text{s}^2$$

$$v_f = 0.50\text{ m/s}$$

(b) Calculate the speed of the block as it passes through the equilibrium position if a constant frictional force of 4.0 N retards its motion.

Solution We use Equation 6.21 to calculate the kinetic energy lost due to friction and add this to the kinetic energy found in the absence of friction. Considering only the frictional force, the kinetic energy lost due to friction is

$$-fd = -(4.0\text{ N})(2.0 \times 10^{-2}\text{ m}) = -0.08\text{ J}$$

The final kinetic energy, without this loss, was found in part (a) to be 0.20 J. Therefore, the final kinetic energy in the presence of friction is

$$K_f = 0.20\text{ J} - 0.08\text{ J} = 0.12\text{ J} = \tfrac{1}{2}mv_f^2$$

$$\tfrac{1}{2}(1.6\text{ kg})v_f^2 = 0.12\text{ J}$$

$$v_f^2 = \frac{0.24\text{ J}}{1.6\text{ kg}} = 0.15\text{ m}^2/\text{s}^2$$

$$v_f = 0.39\text{ m/s}$$

Note that this value for v_f is less than that obtained in the frictionless case. Does this result surprise you?

EXERCISE 9 A 100-kg sled is dragged by a team of dogs a distance of 2.00 km over a horizontal surface at a constant velocity. If the coefficient of friction between the sled and snow is 0.150, find (a) the work done by the team of dogs and (b) the energy lost due to friction. Answer (a) 2.94×10^5 J (b) All of this energy is dissipated due to friction.

EXERCISE 10 A 15-kg block is dragged over a horizontal surface by a constant force of 70 N acting at an angle of 20° above the horizontal. The block is displaced 5.0 m, and the coefficient of kinetic friction between the block and surface is 0.30. Find the work done by (a) the 70-N force, (b) the normal force, and (c) the force of gravity. (d) Calculate the energy lost due to friction. Answer (a) 330 J (b) 0 (c) 0 (d) −185 J

6.5 · POWER

From a practical viewpoint, it is interesting to know not only the work done on an object but also the rate at which the work is being done. The time rate at which work is done, or the time rate of energy transfer, is called **power**.

If an external force is applied to an object (which we assume acts as a particle), and if the work done by this force is W in the time interval Δt, then the **average power** during this interval is defined as

$$\bar{P} \equiv \frac{W}{\Delta t} \qquad [6.22]$$

• *Average power*

The work done on the object contributes to increasing the energy of the object. A more general definition of power is the *time rate of energy transfer*. The **instantaneous power,** P, is the limiting value of the average power as Δt approaches zero:

$$P \equiv \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt} \qquad [6.23]$$

• *Instantaneous power*

where we have represented the infinitesimal value of the work done by dW (even though it is not a change and therefore not a differential). We know from Equation 6.3 that $dW = \mathbf{F} \cdot d\mathbf{x}$. Therefore, the instantaneous power can be written

$$P = \frac{dW}{dt} = \mathbf{F} \cdot \frac{d\mathbf{x}}{dt} = \mathbf{F} \cdot \mathbf{v} \qquad [6.24]$$

• *The watt*

where we have used the fact that $\mathbf{v} = d\mathbf{x}/dt$.

The SI unit of power is joules per second (J/s), also called a *watt* (W) (after James Watt):

$$1\text{ W} = 1\text{ J/s} = 1\text{ kg} \cdot \text{m}^2/\text{s}^3$$

The upright symbol W for watt should not be confused with the italic symbol W for work.

The unit of power in the British engineering system is the horsepower (hp):

$$1\text{ hp} = 550\text{ ft} \cdot \text{lb/s} = 746\text{ W}$$

A new unit of energy (or work) can now be defined in terms of the unit of power. One kilowatt hour (kWh) is the energy converted or consumed in 1 h at the constant rate of 1 kW. The numerical value of 1 kWh is

$$1\text{ kWh} = (10^3\text{ W})(3600\text{ s}) = 3.60 \times 10^6\text{ J}$$

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It is important to realize that a kilowatt hour is a unit of energy, not power. When you pay your electric bill, you are buying energy, and the amount of electricity used by an appliance is usually expressed in kilowatt hours. For example, an electric bulb rated at 100 W would "consume" 3.60×10^5 J of energy in 1 h.

Thinking Physics 6

A light bulb is described by some individuals as "having 60 watts." What's wrong with this phrase?

Reasoning The number given represents the power of the light bulb, which is the rate at which energy passes through it, entering as electrical energy from the power source. It is not a value that the light bulb "possesses," however. It is only the approximate power rating of the bulb when it is connected to a normal 120-volt household supply of electricity. If the bulb were connected to a 12-volt supply, energy would enter the bulb at a rate other than 60 watts. Thus, the 60 watts is not an intrinsic property of the bulb, such as density is for a given solid material.

CONCEPTUAL PROBLEM 7

An older model car accelerates from 0 to speed v in 10 seconds. A newer, more powerful sports car accelerates from 0 to $2v$ in the same time period. What is the ratio of powers expended by the two cars? Consider the energy coming from the engine to appear only as kinetic energy of the car.

Example 6.8 Power Delivered by an Elevator Motor

An elevator has a mass of 1000 kg and carries a maximum load of 800 kg. A constant frictional force of 4000 N retards its motion upward, as in Figure 6.17. (a) What must be the minimum power delivered by the motor to lift the elevator at a constant speed of 3.00 m/s?

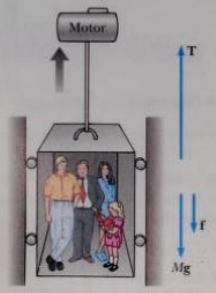


Figure 6.17 (Example 6.8) A motor exerts an upward force, equal in magnitude to the tension, T, on the elevator. A frictional force **f** and the force of gravity $M\mathbf{g}$ act downward.

Solution The motor must supply the force **T** that pulls the elevator upward. From Newton's second law and from the fact that $a = 0$ because v is constant, we get

$$T - f - Mg = 0$$

where M is the *total* mass (elevator plus load), equal to 1800 kg. Therefore,

$$\begin{aligned} T &= f + Mg \\ &= 4.00 \times 10^3 \text{ N} + (1.80 \times 10^3 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 2.16 \times 10^4 \text{ N} \end{aligned}$$

Using Equation 6.24 and the fact that **T** is in the same direction as **v** gives

$$\begin{aligned} P = \mathbf{T}\cdot\mathbf{v} &= Tv \\ &= (2.16 \times 10^4 \text{ N})(3.00 \text{ m/s}) = 6.48 \times 10^4 \text{ W} \\ &= 64.8 \text{ kW} \end{aligned}$$

(b) What power must the motor deliver at any instant if it is designed to provide an upward acceleration of 1.00 m/s²?

Solution Applying Newton's second law to the elevator gives

$$\begin{aligned} T - f - Mg &= Ma \\ T &= M(a + g) + f \\ &= (1.80 \times 10^3 \text{ kg})(1.00 + 9.80) \text{ m/s}^2 \\ &\quad + 4.00 \times 10^3 \text{ N} \\ &= 2.34 \times 10^4 \text{ N} \end{aligned}$$

Therefore, using Equation 6.24, for the required power we get

$$P = Tv = (2.34 \times 10^4\, v) \text{ W}$$

where v is the instantaneous speed of the elevator in meters per second. Hence, the power required increases with increasing speed.

EXERCISE 11 A 65-kg athlete runs a distance of 600 m up a mountainside that is inclined at 20° to the horizontal. He performs this feat in 80 s. Assuming that air resistance is negligible, (a) how much work is done against gravity, and (b) what is his average power output during the run? Answer (a) 1.3×10^5 J (b) 1.6 kW

SUMMARY

The **work** done by a *constant* force **F** acting on a particle is defined as the product of the component of the force in the direction of the particle's displacement and the magnitude of the displacement. If **F** makes an angle θ with the displacement **d**, the work done by **F** is

$$W \equiv Fd\cos\theta \qquad [6.1]$$

The **scalar**, or dot **product** of any two vectors **A** and **B** is defined by the relationship

$$\mathbf{A}\cdot\mathbf{B} \equiv AB\cos\theta \qquad [6.5]$$

where the result is a scalar quantity and θ is the angle between the directions of the two vectors. The scalar product obeys the commutative and distributive laws.

The **work** done by a *varying* force acting on a particle moving along the x axis from x_i to x_f is

$$W = \int_{x_i}^{x_f} F_x\, dx \qquad [6.11]$$

where F_x is the component of force in the x direction. If there are several forces acting on the particle, the net work done by all forces is the sum of the individual amounts of work done by each force.

The **kinetic energy** of a particle of mass m moving with a speed v (where v is small compared with the speed of light) is

$$K \equiv \tfrac{1}{2}mv^2 \qquad [6.18]$$

The **work-kinetic energy theorem** states that the net work done on a particle by external forces equals the change in kinetic energy of the particle:

$$W_{\text{net}} = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \qquad [6.19]$$

Average power is the time rate of doing work:

$$\overline{P} \equiv \frac{W}{\Delta t} \qquad [6.22]$$

If an agent applies a force **F** to an object moving with a velocity **v**, the **instantaneous power** delivered by that agent is

$$P \equiv \frac{dW}{dt} = \mathbf{F}\cdot\mathbf{v} \qquad [6.24]$$

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CONCEPTUAL QUESTIONS

1. Explain why the work done by the force of sliding friction is negative when an object undergoes a displacement on a rough surface.
2. When a particle rotates in a circle, a force acts on it directed toward the center of rotation. Why is it that this force does no work on the particle?
3. When a punter kicks a football, is he doing any work on the ball while his toe is in contact with it? Is he doing any work on the ball after it loses contact with his toe? Are any forces doing work on the ball while it is in flight?
4. Cite two examples in which a force is exerted on an object without doing any work on the object.
5. As a simple pendulum swings back and forth, the forces acting on the suspended mass are the force of gravity, the tension in the supporting cord, and air resistance. (a) Which of these forces, if any, does no work on the pendulum? (b) Which of these forces does negative work at all times during its motion? (c) Describe the work done by the force of gravity while the pendulum is swinging.
6. If the dot product of two vectors is positive, does this imply that the vectors must have positive rectangular components?
7. A hockey player pushes a puck with his hockey stick over frictionless ice. If the puck starts from rest and ends up moving at speed v, has the hockey player performed any work in his reference frame? An ant is on the puck and hangs on tightly while the puck is accelerated. In the reference frame of the ant, has any work been done? Is the work-kinetic energy theorem satisfied for each observer?
8. Can kinetic energy be negative? Explain.
9. One bullet has twice the mass of a second bullet. If both are fired so that they have the same speed, which has more kinetic energy? What is the ratio of the kinetic energies of the two bullets?
10. If the speed of a particle is doubled, what happens to its kinetic energy?
11. What can be said about the speed of a particle if the net work done on it is zero?
12. Can the average power ever equal the instantaneous power? Explain.
13. In Example 6.8, does the required power increase or decrease as the force of friction is reduced?
14. Sometimes physics words are used in popular literature in interesting ways. For example, consider a description of a rock falling from the top of a cliff as "gathering force as it falls to the beach below." What does the phrase "gathering force" mean and can you repair this phrase?

PROBLEMS

Section 6.1 Work Done by a Constant Force

1. A tugboat exerts a constant force of 5000 N on a ship moving at constant speed through a harbor. How much work does the tugboat do on the ship in a distance of 3.00 km?
2. A shopper in a supermarket pushes a cart with a force of 35.0 N directed at an angle of 25.0° downward from the horizontal. Find the work done by the shopper as he moves down a 50.0-m length of aisle.
3. A raindrop ($m = 3.35 \times 10^{-5}$ kg) falls vertically at constant speed under the influence of gravity and air resistance. After the drop has fallen 100 m, what is the work done (a) by gravity and (b) by air resistance?
4. A block of mass 2.50 kg is pushed 2.20 m along a frictionless horizontal table by a constant 16.0-N force directed 25.0° below the horizontal. Determine the work done by (a) the applied force, (b) the normal force exerted by the table, (c) the force of gravity, and (d) the net force on the block.
5. Batman, whose mass is 80.0 kg, is holding on to the free end of a 12.0-m rope, the other end of which is fixed to a tree limb directly above. He is able to get the rope in motion as only Batman knows how, eventually getting it to swing enough that he can reach a ledge when the rope makes a 60.0° angle with the vertical. How much work was done by him against the force of gravity in this maneuver?

Section 6.2 The Scalar Product of Two Vectors

6. Vector $\mathbf{A}$ has a magnitude of 5.00 units, and $\mathbf{B}$ has a magnitude of 9.00 units. The two vectors make an angle of 50.0° with each other. Find $\mathbf{A} \cdot \mathbf{B}$.
7. Vector $\mathbf{A}$ extends from the origin to a point having polar coordinates (7, 70°), and vector $\mathbf{B}$ extends from the origin to a point having polar coordinates (4, 130°). Find $\mathbf{A} \cdot \mathbf{B}$.
8. Given two arbitrary vectors $\mathbf{A}$ and $\mathbf{B}$, show that $\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$. (*Hint:* Write $\mathbf{A}$ and $\mathbf{B}$ in unit vector form and use Equations 6.7 and 6.8.)
9. A force $\mathbf{F} = (6\mathbf{i} - 2\mathbf{j})$ N acts on a particle that undergoes a displacement $\mathbf{d} = (3\mathbf{i} + \mathbf{j})$ m. Find (a) the work done by the force on the particle and (b) the angle between $\mathbf{F}$ and $\mathbf{d}$.
10. For $\mathbf{A} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{B} = -\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$, and $\mathbf{C} = 2\mathbf{j} - 3\mathbf{k}$, find $\mathbf{C} \cdot (\mathbf{A} - \mathbf{B})$.
11. Using the definition of the scalar product, find the angles between (a) $\mathbf{A} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{B} = 4\mathbf{i} - 4\mathbf{j}$; (b) $\mathbf{A} = -2\mathbf{i} + 4\mathbf{j}$ and $\mathbf{B} = 3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$; (c) $\mathbf{A} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{B} = 3\mathbf{j} + 4\mathbf{k}$.

Section 6.3 Work Done by a Varying Force

12. The force acting on a particle varies as in Figure P6.12. Find the work done by the force as the particle moves (a) from $x = 0$ to $x = 8.00$ m, (b) from $x = 8.00$ m to $x = 10.0$ m, and (c) from $x = 0$ to $x = 10.0$ m.

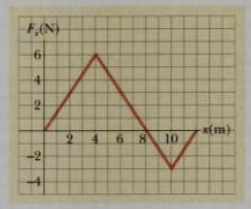


Figure P6.12

13. A particle is subject to a force $\mathbf{F}_x$ that varies with position as in Figure P6.13. Find the work done by the force on the body as it moves (a) from $x = 0$ to $x = 5.00$ m, (b) from $x = 5.00$ m to $x = 10.0$ m, and (c) from $x = 10.0$ m to $x = 15.0$ m. (d) What is the total work done by the force over the distance $x = 0$ to $x = 15.0$ m?

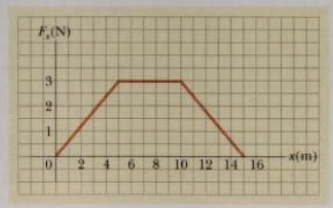


Figure P6.13

14. The force acting on a particle is $F_x = (8x - 16)$ N, where x is in meters. (a) Make a plot of this force versus x from $x = 0$ to $x = 3.00$ m. (b) From your graph, find the net work done by this force as the particle moves from $x = 0$ to $x = 3.00$ m.
15. A force $\mathbf{F} = (4.0\mathbf{i} + 3.0\mathbf{j})$ N acts on an object as the object moves in the x direction from the origin to $x = 5.00$ m. Find the work done on the object by the force.
16. When a 4.00-kg mass is hung vertically on a certain light spring that obeys Hooke's law, the spring stretches 2.50 cm. If the 4.00-kg mass is removed, (a) how far will the spring stretch if a 1.50-kg mass is hung on it, and (b) how much work must an external agent do to stretch the same spring 4.00 cm from its unstretched position?
17. An archer pulls her bow string back 0.400 m by exerting a force that increases uniformly from 0 to 230 N. (a) What is the equivalent spring constant of the bow? (b) How much work is done in pulling the bow?
18. A 100-g bullet is fired from a rifle having a barrel 0.600 m long. Assuming the origin is placed where the bullet begins to move, the force (in newtons) exerted on the bullet by the expanding gas is $15\,000 + 10\,000x - 25\,000x^2$, where x is in meters. (a) Determine the work done by the gas on the bullet as the bullet travels the length of the barrel. (b) If the barrel is 1.00 m long, how much work is done, and how does this value compare to the work calculated in (a)?
19. If it takes 4.00 J of work to stretch a Hooke's-law spring 10.0 cm from its unstressed length, determine the extra work required to stretch it an additional 10.0 cm.
20. If it takes work W to stretch a Hooke's-law spring a distance d from its unstressed length, determine the extra work required to stretch it an additional distance d.

Section 6.4 Kinetic Energy and the Work-Kinetic Energy Theorem

21. A 0.600-kg particle has a speed of 2.00 m/s at point A and kinetic energy of 7.50 J at point B. What is (a) its kinetic energy at A? (b) its speed at B? (c) the total work done on the particle as it moves from A to B?
22. A 0.300-kg ball has a speed of 15.0 m/s. (a) What is its kinetic energy? (b) If its speed is doubled, what is its kinetic energy?
23. A 3.00-kg mass has an initial velocity $\mathbf{v}_0 = (6.00\mathbf{i} - 2.00\mathbf{j})$ m/s. (a) What is its kinetic energy at this time? (b) Find the total work done on the object if its velocity changes to $(8.00\mathbf{i} + 4.00\mathbf{j})$ m/s. (*Hint:* Remember that $v^2 = \mathbf{v} \cdot \mathbf{v}$.)
24. You can think of the work-kinetic energy theorem as a second theory of motion, parallel to Newton's laws in describing how outside influences affect the motion of an object. In this problem work out parts (a) and (b) separately from parts (c) and (d) to compare the predictions of the two theories. In a rifle barrel, a 15.0-g bullet is accelerated from rest to a speed of 780 m/s. (a) Find the work done on it. (b) If the rifle barrel is 72.0 cm long, find the magnitude of the average total force that acted on it, as $F = W/(d \cos \theta)$. (c) Find the constant acceleration of a bullet that starts from rest and gains speed 780 m/s over a distance

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of 72.0 cm. (d) If the bullet has mass 15.0 g, find the total force that acted on it as $\Sigma F = ma$.

25. A cart loaded with bricks has a total mass of 18.0 kg and is pulled at constant speed by a rope. The rope is inclined at 20.0° above the horizontal, and the cart moves a distance of 20.0 m on a horizontal surface. The coefficient of kinetic friction between the cart and surface is 0.500. (a) What is the tension of the rope? (b) How much work is done on the cart by the rope? (c) What is the energy lost due to friction?

26. A 40.0-kg box initially at rest is pushed 5.00 m along a rough, horizontal floor with a constant applied horizontal force of 130 N. If the coefficient of friction between box and floor is 0.300, find (a) the work done by the applied force, (b) the energy lost due to friction, (c) the change in kinetic energy of the box, and (d) the final speed of the box.

27. A sled of mass m is given a kick on a frozen pond, imparting to it an initial speed $v_i = 2.00$ m/s. The coefficient of kinetic friction between sled and ice is $\mu_k = 0.100$. Use energy considerations to find the distance the sled moves before stopping.

28. A block of mass 12.0 kg slides from rest down a frictionless 35.0° incline and is stopped by a strong spring with $k = 3.00 \times 10^4$ N/m. The block slides 3.00 m from the point of release to the point where it comes to rest against the spring. When the block comes to rest, how far has the spring been compressed?

29. A crate of mass 10.0 kg is pulled up a rough incline with an initial speed of 1.50 m/s. The pulling force is 100 N parallel to the incline, which makes an angle of 20.0° with the horizontal. The coefficient of kinetic friction is 0.400, and the crate is pulled 5.00 m. (a) How much work is done by gravity? (b) How much energy is lost due to friction? (c) How much work is done by the 100-N force? (d) What is the change in kinetic energy of the crate? (e) What is the speed of the crate after being pulled 5.00 m?

30. A picture tube in a certain television set is 36.0 cm long. The electrical force accelerates an electron in the tube from rest to 1.00% of the speed of light over this distance. Determine (a) the kinetic energy of the electron as it strikes the screen at the end of the tube, (b) the magnitude of the average electrical force acting on the electron over this distance, (c) the magnitude of the average acceleration of the electron over this distance, and (d) the time of flight.

31. A time-varying net force acting on a 4.00-kg object causes the object to have a displacement given by $x = 2.0t - 3.0t^2 + 1.0t^3$, where x is in meters and t is in seconds. Find the work done on the object in the first 3.00 s of motion.

32. A 5.00-kg steel ball is dropped onto a copper plate from a height of 10.0 m. If the ball leaves a dent 0.320 cm deep, what is the average force exerted on the ball by the plate during the impact?

Section 6.5 Power

33. A 700-N Marine in basic training climbs a 10.0-m vertical rope at a constant speed in 8.00 s. What is her power output?

34. Make an order-of-magnitude estimate of the power your car engine puts into speeding the car up to highway speed. In your solution state the physical quantities you take as data and the values you measure or estimate for them. The mass of the vehicle is given in the owner's manual. If not a car, consider a bus or truck that you specify.

35. If a certain horse can maintain 1.00 hp of output for 2.00 h, how many 70.0-kg bundles of shingles can that horse hoist (via some pulley arrangement) to the roof of a house 8.00 m tall, assuming 70.0% efficiency?

36. A certain automobile engine delivers 30.0 hp (2.24×10^4 W) to its wheels when moving at a constant speed of 27.0 m/s (≈60 mi/h). What is the resistive force acting on the automobile at that speed?

37. A skier of mass 70.0 kg is pulled up a slope by a motor-driven cable. (a) How much work is required to pull him a distance of 60.0 m up a 30.0° slope (assumed frictionless) at a constant speed of 2.00 m/s? (b) A motor of what power is required to perform this task?

38. A 650-kg elevator starts from rest. It moves upward for 3.00 s with constant acceleration until it reaches its cruising speed of 1.75 m/s. (a) What is the average power of the elevator motor during this period? (b) How does this power compare with its power when it moves at its cruising speed?

Additional Problems

39. A baseball outfielder throws a 0.150-kg baseball at a speed of 40.0 m/s and an initial angle of 30.0°. What is the kinetic energy of the baseball at the highest point of the trajectory?

40. While running, a person dissipates about 0.600 J of mechanical energy per step per kilogram of body mass. If a 60.0-kg runner dissipates a power of 70.0 W during a race, how fast is the person running? Assume a running step is 1.50 m long.

41. A particle of mass m moves with a constant acceleration $\mathbf{a}$. If the initial position vector and velocity of the particle are $\mathbf{r}_0$ and $\mathbf{v}_0$, respectively, show that its speed v at any time satisfies the equation

$$v^2 = v_0^2 + 2\mathbf{a}\cdot(\mathbf{r} - \mathbf{r}_0)$$

where $\mathbf{r}$ is the position vector of the particle at that same time.

42. The direction of an arbitrary vector $\mathbf{A}$ can be completely specified with the angles α, β, and γ that the vector makes with the x, y, and z axes, respectively. If $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$, (a) find expressions for cos α, cos β, and cos γ (these are known as *direction cosines*), and (b) show that these angles satisfy the relation $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. (*Hint:* Take the scalar product of $\mathbf{A}$ with $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ separately.)

43. A 4.00-kg particle moves along the x axis. Its position varies with time according to $x = t + 2.0t^3$, where x is in meters and t is in seconds. Find (a) the kinetic energy at any time t, (b) the acceleration of the particle and the force acting on it at time t, (c) the power being delivered to the particle at time t, and (d) the work done on the particle in the interval $t = 0$ to $t = 2.00$ s.

44. When a spring is stretched beyond its elastic limit, the restoring force satisfies the equation $F = -kx + \beta x^3$. If $k = 10.0$ N/m and $\beta = 100$ N/m^3, calculate the work done by this force when the spring is stretched 0.100 m.

45. A 2100-kg pile driver is used to drive a steel I-beam into the ground. The pile driver falls 5.00 m before contacting the beam, and it drives the beam 12.0 cm into the ground before coming to rest. Using energy considerations, calculate the average force the beam exerts on the pile driver while the pile driver is brought to rest after each hit.

46. A cyclist and her bicycle have a combined mass of 75.0 kg. She coasts down a road inclined at 2.00° with the horizontal at 4.00 m/s and down a road inclined at 4.00° at 8.00 m/s. She then holds on to a moving vehicle and coasts on a level road. What power must the vehicle expend to maintain her speed at 3.00 m/s? Assume that the force of air resistance is proportional to her speed and assume that other frictional forces remain constant.

47. A 200-g block is pressed against a spring of force constant 1.40 kN/m until the block compresses the spring 10.0 cm. The spring rests at the bottom of a ramp inclined at 60.0° to the horizontal. Use energy considerations to determine how far up the incline the block moves before it stops if (a) there is no friction between block and ramp and if (b) the coefficient of kinetic friction is 0.400.

48. A 0.400-kg particle slides on a horizontal circular track 1.50 m in radius. It is given an initial speed of 8.00 m/s. After one revolution, its speed drops to 6.00 m/s because of friction. (a) Find the energy lost due to friction in one revolution. (b) Calculate the coefficient of kinetic friction. (c) What is the total number of revolutions the particle makes before stopping?

49. The ball launcher in a pinball machine has a spring that has a force constant of 1.20 N/cm (Fig. P6.49). The surface on which the ball moves is inclined 10.0° with respect to the horizontal. If the spring is initially compressed 5.00 cm, find the launching speed of a 100-g ball when the plunger is released. Friction and the mass of the plunger are negligible.

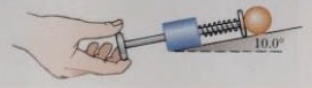


Figure P6.49

50. In diatomic molecules, the constituent atoms exert attractive forces on each other at large distances and repulsive forces at short distances. For many molecules, the Lennard-Jones law is a good approximation to the magnitude of these forces:

$$F = F_0\left[2\left(\frac{\sigma}{r}\right)^{13} - \left(\frac{\sigma}{r}\right)^{7}\right]$$

where r is the center-to-center distance between the atoms in the molecule, σ is a length parameter, and F_0 is the force when $r = \sigma$. For an oxygen molecule, the values of F_0 and σ are $F_0 = 9.60 \times 10^{-11}$ N and $\sigma = 3.50 \times 10^{-10}$ m. Determine the work done by this force from $r = 4.00 \times 10^{-10}$ m to $r = 9.00 \times 10^{-10}$ m.

51. Suppose a car is modeled as a cylinder with cross-sectional area A moving with a speed v, as in Figure P6.51. In a time Δt, a column of air of mass Δm must be moved a distance $v\,\Delta t$ and given a kinetic energy $\frac{1}{2}(\Delta m)v^2$. Using this model, show that the power loss due to air resistance is $\frac{1}{2}\rho A v^3$ and the resistive force is $\frac{1}{2}\rho A v^2$, where ρ is the density of air.

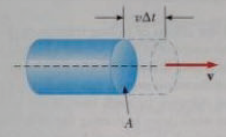


Figure P6.51

Spreadsheet Problems

S1. When different weights are hung on a spring, the spring stretches to different lengths as shown in the table that follows. (a) Use a spreadsheet to plot the length of the spring versus applied force. Use the spreadsheet's least-squares fitting features to determine the straight line that best fits the data. (You may not want to use all the data points.) (b) From the slope of the least-squares fit line, find the spring constant k. (c) If the spring is extended to 105 mm, what force does it exert on the suspended weight?

F(N)	L(mm)	F(N)	L(mm)
2.0	15	14	112
4.0	32	16	126
6.0	49	18	149
8.0	64	20	175
10	79	22	190
12	98		

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