of the wave pulse doesn't change with time, we can represent the displacement y for all later times, measured in a stationary frame with the origin at O, as

Pulse traveling to the right •

$$y = f(x - vt) \qquad [13.1]$$

In the same way, if the wave pulse travels to the left, the displacement of the string is

Pulse traveling to the left •

$$y = f(x + vt) \qquad [13.2]$$

The displacement y, sometimes called the **wave function,** depends on the two variables x and t. For this reason, it is often written $y(x, t)$, which is read "y as a function of x and t." It is important to understand the meaning of y.

Consider a point, P, on the string, identified by a particular value of its coordinates. As the wave pulse passes through P, the y coordinate of this point increases, reaches a maximum, and then decreases to zero. **The wave function $y(x, t)$ represents the y coordinate of any point P at any time t.** Furthermore, if t is fixed, then the wave function y as a function of x, sometimes called the *waveform*, defines a curve representing the actual shape of the pulse at that time. This is equivalent to a snapshot of the pulse at time t.

For a pulse that moves without changing its shape, the velocity of the pulse is the same as the velocity of any point along the pulse, such as the crest. To find the velocity of the pulse, we can calculate how far the crest moves horizontally in a short time and then divide that distance by the time interval. In order to follow the motion of the crest, some particular value, say x_0, must be substituted for $x - vt$ in Equation 13.1. (The value x_0 is called the *argument* of the function y.) Regardless of how x and t change individually, we must require that $x - vt = x_0$ in order to stay with the crest. This equation, therefore, represents the motion of the crest. At $t = 0$, the crest is at $x = x_0$; after an interval of dt, the crest is at $x = x_0 + v\,dt$. Therefore, in the time dt the crest has moved a distance of $dx = (x_0 + v\,dt) - x_0 = v\,dt$. Clearly, the wave speed, often called the **phase speed,** is

Phase speed •

$$v = \frac{dx}{dt} \qquad [13.3]$$

The *wave velocity*, or *phase velocity*, must not be confused with the *transverse velocity* of a particle in the medium (which is always perpendicular to the wave velocity). The phase velocity is the same at any point along the pulse.

The following example illustrates how a specific wave function is used to describe the motion of a traveling wave pulse.

Example 13.1 A Pulse Moving to the Right

A wave pulse moving to the right along the x axis is represented by the wave function

$$y(x, t) = \frac{2}{(x - 3.0t)^2 + 1}$$

where x and y are measured in centimeters and t is in seconds. Let us plot the waveform at $t = 0$, $t = 1.0$ s, and $t = 2.0$ s.

Solution First, note that this function is of the form $y = f(x - vt)$. By inspection, we see that the speed of the wave is $v = 3.0$ cm/s. Furthermore, the wave amplitude (the maximum value of y) is given by $A = 2.0$ cm. The location of the peak of the pulse occurs at the value of x for which the denominator is a minimum, that is, where $(x - 3.0t) = 0$. At times $t = 0$, $t = 1.0$ s, and $t = 2.0$ s, the wave function expressions are

$$y(x, 0) = \frac{2}{x^2 + 1} \quad \text{at } t = 0$$

$$y(x, 1.0) = \frac{2}{(x - 3.0)^2 + 1} \quad \text{at } t = 1.0 \text{ s}$$

$$y(x, 2.0) = \frac{2}{(x - 6.0)^2 + 1} \quad \text{at } t = 2.0 \text{ s}$$

We can now use these expressions to plot the wave function versus x at these times. For example, let us evaluate $y(x, 0)$ at $x = 0.50$ cm:

$$y(0.50, 0) = \frac{2}{(0.50)^2 + 1} = 1.6 \text{ cm}$$

Likewise, $y(1.0, 0) = 1.0$ cm, $y(2.0, 0) = 0.40$ cm, and so on. A continuation of this procedure for other values of x yields the waveform shown in Figure 13.7a. In a similar manner, one obtains the graphs of $y(x, 1.0)$ and $y(x, 2.0)$, shown in Figures 13.7b and 13.7c, respectively. These snapshots show that the wave pulse moves to the right without changing its shape and has a constant speed of 3.0 cm/s.

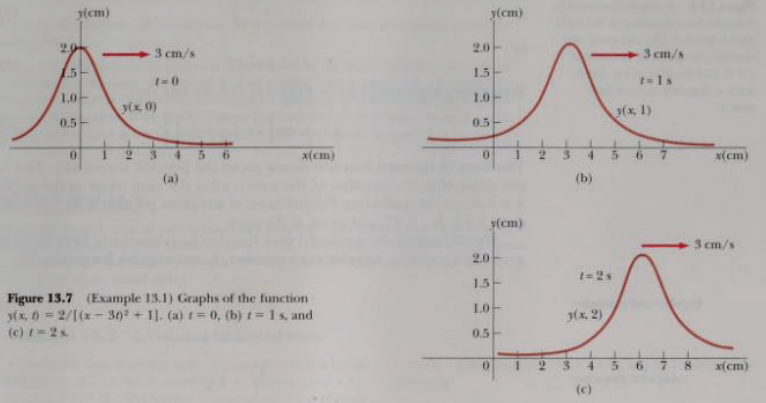


Figure 13.7 (Example 13.1) Graphs of the function $y(x, t) = 2/[(x - 3t)^2 + 1]$. (a) $t = 0$, (b) $t = 1$ s, and (c) $t = 2$ s.

13.4 • SINUSOIDAL TRAVELING WAVES

In this section we introduce an important periodic waveform known as a **sinusoidal wave** (Fig. 13.8). The red curve represents a snapshot of a sinusoidal traveling wave at $t = 0$, and the blue curve represents a snapshot of the wave at some later t. At $t = 0$, the displacement of the curve can be written

$$y = A \sin\left(\frac{2\pi}{\lambda} x\right) \qquad [13.4]$$

where the amplitude, A, as usual, represents the maximum value of the displacement, and λ is the wavelength as defined in Figure 13.1a. Thus, we see that the displacement repeats itself when x is increased by an integral multiple of λ. If the wave moves to the right with a phase speed of v, the wave function at some later t is

INTEL-1013
U.S. Patent 7,151,430

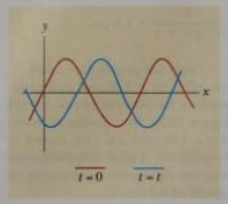


Figure 13.8 A one-dimensional sinusoidal wave traveling to the right with a speed v. The red curve represents a snapshot of the wave at $t = 0$, and the blue curve represents a snapshot at some later time t.

$$y = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right] \qquad [13.5]$$

That is, the sinusoidal wave moves to the right a distance of vt in the time t, as in Figure 13.8. Note that the wave function has the form $f(x - vt)$ and represents a wave traveling to the right. If the wave were traveling to the left, the quantity $x - vt$ would be replaced by $x + vt$, just as in the case of the traveling pulse described by Equations 13.1 and 13.2.

Because the period T is the time it takes the wave to travel a distance of one wavelength, the phase speed, wavelength, and period are related by

$$v = \frac{\lambda}{T} \qquad [13.6]$$

or

$$\lambda = vT \qquad [13.7]$$

Substituting Equation 13.6 into Equation 13.5, we find that

$$y = A \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] \qquad [13.8]$$

This form of the wave function clearly shows the periodic nature of y. That is, at any given time t (a snapshot of the wave), y has the same value at the positions x, $x + \lambda$, $x + 2\lambda$, and so on. Furthermore, at any given position x, the value of y at times t, $t + T$, $t + 2T$, and so on, is the same.

We can express the sinusoidal wave function in a convenient form by defining two other quantities: **angular wave number,** k, and **angular frequency,** ω:

Angular wave number •

$$k \equiv \frac{2\pi}{\lambda} \qquad [13.9]$$

Angular frequency •

$$\omega \equiv \frac{2\pi}{T} = 2\pi f \qquad [13.10]$$

Note that in Equation 13.10, we used the definition of frequency, $f = 1/T$. Using these definitions, we see that Equation 13.8 can be written in the more compact form

$$y = A \sin(kx - \omega t) \qquad [13.11]$$

We shall use this form most frequently.

Using Equations 13.9 and 13.10, we can express the phase speed, v, in the alternative forms

$$v = \frac{\omega}{k} \qquad [13.12]$$

Speed of a traveling sinusoidal wave •

$$v = f\lambda \qquad [13.13]$$

The wave function given by Equation 13.11 assumes that the displacement, y, is zero at $x = 0$ and $t = 0$. This need not be the case. If the transverse displacement is not zero at $x = 0$ and $t = 0$, we generally express the wave function in the form

$$y = A \sin(kx - \omega t - \phi) \qquad [13.14]$$

Wave function for a traveling sinusoidal wave •

where ϕ is called the **phase constant** and can be determined from the initial conditions.

Thinking Physics 2

The quantity that we call the angular wave number k is sometimes called *spatial frequency*. Why?

Reasoning The angular frequency ω is 2π divided by the *time* interval for one cycle of oscillation for the wave, the period, T. The angular wave number k is 2π divided by the *space* interval for one cycle of oscillation for the wave, the wavelength, λ (the spatial interval of one cycle of the wave). Due to the similarity in these parameters, it is reasonable to consider ω the *temporal* frequency and k the *spatial* frequency.

CONCEPTUAL PROBLEM 2

Sound waves from a musical performance are encoded onto radio waves for transmission from the radio studio to your home receiver. A radio wave with a wavelength of 3 meters carries a sound wave that has a wavelength of 3 meters in air. Which has the higher frequency, the radio wave or the sound wave?

Example 13.2 A Traveling Sinusoidal Wave

A sinusoidal wave traveling in the positive x direction has an amplitude of 15.0 cm, a wavelength of 40.0 cm, and a frequency of 8.00 Hz. The vertical displacement of the medium at $t = 0$ and $x = 0$ is also 15.0 cm, as shown in Figure 13.9. (a) Find the angular wave number, period, angular frequency, and speed of the wave.

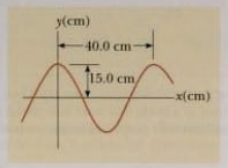


Figure 13.9 (Example 13.2) A sinusoidal wave of wavelength $\lambda = 40.0$ cm and amplitude $A = 15.0$ cm. The wave function can be written in the form $y = A\cos(kx - \omega t)$.

Solution Using Equations 13.9, 13.10, and 13.13, we find the following:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{40.0 \text{ cm}} = 0.157 \text{ rad/cm}$$

$$T = \frac{1}{f} = \frac{1}{8.00 \text{ s}^{-1}} = 0.125 \text{ s}$$

$$\omega = 2\pi f = 2\pi(8.00 \text{ s}^{-1}) = 50.3 \text{ rad/s}$$

$$v = f\lambda = (8.00 \text{ s}^{-1})(40.0 \text{ cm}) = 320 \text{ cm/s}$$

(b) Determine the phase constant ϕ, and write a general expression for the wave function.

Solution Because $A = 15.0$ cm and because it is given that $y = 15.0$ cm at $x = 0$ and $t = 0$, substitution into Equation 13.14 gives

$$15 = 15 \sin(-\phi) \quad \text{or} \quad \sin(-\phi) = 1$$

INTEL-1013
U.S. Patent 7,151,430

Because $\sin(-\phi) = -\sin\phi$, we see that $\phi = -\pi/2$ rad (or $-90°$). Hence, the wave function is of the form

$$y = A\sin\left(kx - \omega t + \frac{\pi}{2}\right) = A\cos(kx - \omega t)$$

That the wave function must have this form can be seen by inspection, noting that the cosine argument is displaced by 90° from the sine function. Substituting the values for A, k, and ω into this expression gives

$$y = (15.0\text{ cm})\cos(0.157x - 50.3t)$$

Sinusoidal Waves on Strings

One method of producing a traveling sinusoidal wave on a very long string is shown in Figure 13.10. One end of the string is connected to a blade that is set in vibration. As the blade oscillates vertically with simple harmonic motion, a traveling wave moving to the right is set up on the string. Figure 13.10 represents snapshots of the wave at intervals of one quarter of a period. Note that **each particle of the string, such as *P*, oscillates vertically in the *y* direction with simple harmonic motion.** This must be the case because each particle follows the simple harmonic motion of the blade. Therefore, every segment of the string can be treated as a simple harmonic oscillator vibrating with a frequency equal to the frequency of vibration of the blade that drives the string.[3] Note that although each segment oscillates in the *y* direction, the wave (or disturbance) travels in the *x* direction with speed v.

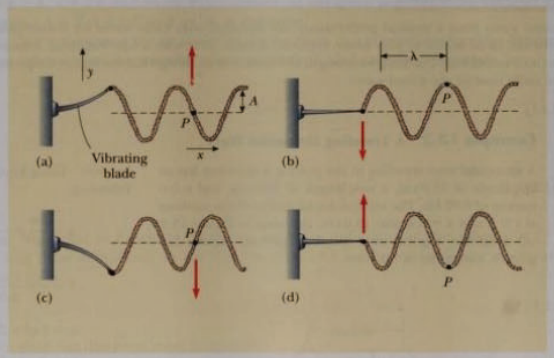


Figure 13.10 One method for producing a train of sinusoidal wave pulses on a continuous string. The left end of the string is connected to a blade that is set into vibration. Every segment of the string, such as the point *P*, oscillates with simple harmonic motion in the vertical direction.

[3]In this arrangement, we are assuming that the segment always oscillates in a vertical line. The tension in the string would vary if the segment were allowed to move sideways. Such a motion would make the analysis very complex.

Of course, this is the definition of a transverse wave. In this case, the energy carried by the traveling wave is supplied by the vibrating blade.

If the waveform at $t = 0$ is as described in Figure 13.10b, then the wave function can be written

$$y = A\sin(kx - \omega t)$$

We can use this expression to describe the motion of any point on the string. The point P (or any other point on the string) moves vertically, and so its x coordinate *remains constant.* Therefore, the **transverse speed,** v_y, of the point P (not to be confused with the wave speed, v) and its **transverse acceleration,** a_y, are

$$v_y = \left.\frac{dy}{dt}\right]_{x=\text{constant}} = \frac{\partial y}{\partial t} = -\omega A\cos(kx - \omega t) \qquad [13.15]$$

$$a_y = \left.\frac{dv_y}{dt}\right]_{x=\text{constant}} = \frac{\partial v_y}{\partial t} = -\omega^2 A\sin(kx - \omega t) \qquad [13.16]$$

The maximum values of these quantities are simply the absolute values of the coefficients of the cosine and sine functions:

$$(v_y)_{\max} = \omega A \qquad [13.17]$$

$$(a_y)_{\max} = \omega^2 A \qquad [13.18]$$

You should recognize that the transverse speed and transverse acceleration of any point on the string do not reach their maximum values simultaneously. In fact, the transverse speed reaches its maximum value (ωA) when the displacement $y = 0$, whereas the transverse acceleration reaches its maximum value ($\omega^2 A$) when $y = -A$. Finally, Equations 13.17 and 13.18 are identical to the corresponding equations for simple harmonic motion.

Example 13.3 A Sinusoidally Driven String

The string shown in Figure 13.10 is driven at a frequency of 5.00 Hz. The amplitude of the motion is 12.0 cm, and the wave speed is 20.0 m/s. Furthermore, the wave is such that $y = 0$ at $x = 0$ and $t = 0$. Determine the angular frequency and wave number for this wave, and write an expression for the wave function.

Solution Using Equations 13.10 and 13.12 gives

$$\omega = \frac{2\pi}{T} = 2\pi f = 2\pi(5.00\text{ Hz}) = 31.4\text{ rad/s}$$

$$k = \frac{\omega}{v} = \frac{31.4\text{ rad/s}}{20.0\text{ m/s}} = 1.57\text{ rad/m}$$

Because $A = 12.0\text{ cm} = 0.120\text{ m}$, we have

$$y = A\sin(kx - \omega t) = (0.120\text{ m})\sin(1.57x - 31.4t)$$

EXERCISE 1 Calculate the maximum values for the transverse speed and transverse acceleration of any point on the string. Answer 3.77 m/s; 118 m/s²

EXERCISE 2 When a particular wire is vibrating with a frequency of 4.00 Hz, a transverse wave of wavelength 60.0 cm is produced. Determine the speed of wave pulses along the wire. Answer 2.40 m/s

EXERCISE 3 For a certain transverse wave, the distance between two successive maxima is 1.2 m and eight maxima pass a given point along the direction of travel every 12 s. Calculate the wave speed. Answer 0.80 m/s

INTEL-1013
U.S. Patent 7,151,430

13.5 · SUPERPOSITION AND INTERFERENCE OF WAVES

Many interesting wave phenomena in nature cannot be described by a single moving pulse. Instead, one must analyze complex waveforms in terms of a combination of many traveling waves. To analyze such wave combinations, one can make use of the **superposition principle,** which states that

Linear waves obey the superposition principle.

> if two or more traveling waves are moving through a medium, the resultant wave function at any point is the sum of the wave functions of the individual waves.

This rather striking property is exhibited by many waves in nature. Waves that obey this principle are called *linear waves,* and they are generally characterized by small amplitudes. Waves that violate the superposition principle are called *nonlinear waves* and are often characterized by large amplitudes. In this book, we shall deal only with linear waves.

One consequence of the superposition principle is that **two traveling waves can pass through each other without being destroyed or even altered.** For instance, when two pebbles are thrown into a pond, the expanding circular surface waves do not destroy each other. In fact, the ripples pass through each other. The complex pattern that is observed can be viewed as two independent sets of expanding circles. Likewise, when sound waves from two sources move through air, they also can pass through each other. The sound one hears at a given point is the resultant of both disturbances.

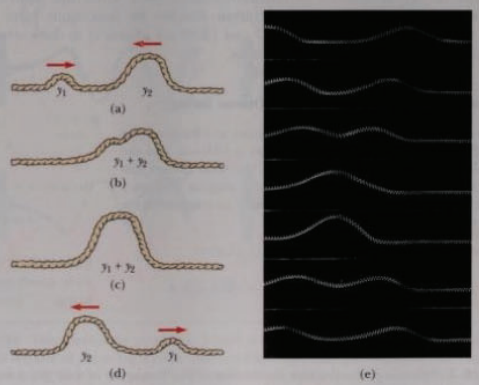


Figure 13.11 *(Left)* Two wave pulses traveling on a stretched string in opposite directions pass through each other. When the pulses overlap, as in (b) and (c), the net displacement of the string equals the sum of the displacements produced by each pulse. Because each pulse produces positive displacements of the string, we refer to their superposition as *constructive interference.* *(Right)* Photograph of superposition of two equal and symmetric pulses traveling in opposite directions on a stretched spring. *(Photo, Education Development Center, Newton, Mass.)*

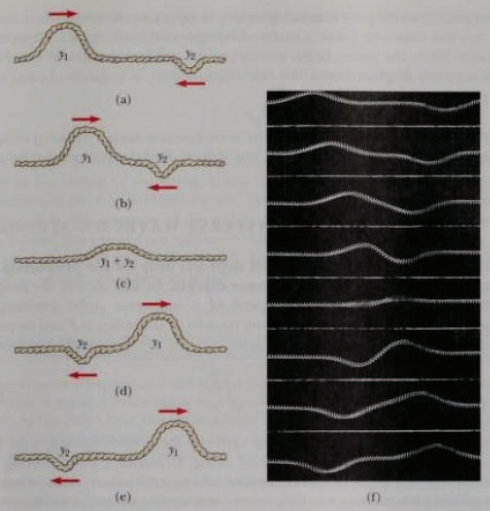


Figure 13.12 *(Left)* Two wave pulses traveling in opposite directions with displacements that are inverted relative to each other. When the two overlap as in (c), their displacements subtract from each other. *(Right)* Photograph of superposition of two symmetric pulses traveling in opposite directions, where one is inverted relative to the other. *(Photo, Education Development Center, Newton, Mass.)*

A simple pictorial representation of the superposition principle is obtained by considering two pulses traveling in opposite directions on a stretched string, as in Figure 13.11. The wave function for the pulse moving to the right is y_1, and the wave function for the pulse moving to the left is y_2. The pulses have the same speed but different shapes. Each pulse is assumed to be symmetric (although this is not a necessary condition), and in both cases the vertical displacements of the string are taken to be positive. When the waves begin to overlap (Fig. 13.11b), the resulting complex waveform is given by $y_1 + y_2$. When the crests of the pulses exactly coincide (Fig. 13.11c), the resulting waveform, $y_1 + y_2$, is symmetric. The two pulses finally separate and continue moving in their original directions (Fig. 13.11d). Note that the final waveforms remain unchanged, as if the two pulses had never met! The combination of separate waves in the same region of space to produce a resultant wave is called **interference.**

For the two pulses shown in Figure 13.11, the vertical displacements caused by the pulses are in the same direction, and so the resultant waveform (when the pulses overlap) exhibits a displacement greater than those of the individual pulses. Now



Interference patterns produced by outward spreading waves from several drops of water falling into a pond. *(Martin Dohrn/SPL/Photo Researchers)*

INTEL-1013
U.S. Patent 7,151,430

consider two identical pulses, again traveling in opposite directions on a stretched string, but this time one pulse is inverted relative to the other, as in Figure 13.12. In this case, when the pulses begin to overlap, the resultant waveform is the sum of the two separate displacements, but one displacement is negative. Again, the two pulses pass through each other.

EXERCISE 4 Two waves are traveling in the same direction along a stretched string. Each has an amplitude of 4.0 cm and they are 90° out of phase. Find the amplitude of the resultant wave. Answer 5.7 cm

13.6 · THE SPEED OF TRANSVERSE WAVES ON STRINGS

For linear mechanical waves, **the speed depends only on the properties of the medium through which the disturbance travels.** In this section we shall focus our attention on determining the speed of a transverse pulse traveling on a stretched string. We shall show that if the tension in the string is F and its mass per unit length is μ, then the wave speed, v, is

Speed of a wave on a stretched string

$$v = \sqrt{\frac{F}{\mu}} \qquad [13.19]$$

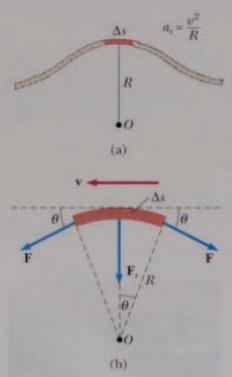


Figure 13.13 (a) To obtain the speed v of a wave on a stretched string, it is convenient to describe the motion of a small segment of the string in a moving frame of reference. (b) The net force on a small segment of length Δs is in the radial direction. The horizontal components of the tension force cancel.

Now let us use a mechanical analysis to derive the preceding expression for the speed of a pulse traveling on a stretched string. Consider a pulse moving to the right with a uniform speed of v, measured relative to a stationary frame of reference. Instead of using a stationary frame, however, it is more convenient to choose as our reference frame one that moves along with the pulse at the same speed, so that the pulse appears to be at rest in the frame, as in Figure 13.13a. This is permitted because Newton's laws are valid in either a stationary frame or one that moves with constant velocity. A small segment of the string, of length Δs, forms the approximate arc of a circle of radius R, as shown in Figure 13.13a and magnified in Figure 13.13b. In the pulse's frame of reference, the segment moves to the left with the speed v. This segment has a centripetal acceleration of v^2/R, which is supplied by the tension, F, in the string. The force **F** acts on each side of the segment, tangent to the arc, as in Figure 13.13b. The horizontal components of **F** cancel, and each vertical component $F \sin \theta$ acts radially inward toward the center of the arc. Hence, the total radial force is $2F \sin \theta$. Because the segment is small, θ is small, and we can use the small-angle approximation $\sin \theta \approx \theta$. Therefore, the total radial force can be expressed as

$$F_r = 2F \sin \theta \approx 2F\theta$$

The segment has the mass $m = \mu \, \Delta s$, where μ is the mass per unit length of the string. Because the segment forms part of a circle and subtends an angle of 2θ at the center, $\Delta s = R(2\theta)$, and hence

$$m = \mu \, \Delta s = 2\mu R\theta$$

If we apply Newton's second law to this segment, the radial component of motion gives

$$F_r = \frac{mv^2}{R} \quad \text{or} \quad 2F\theta = \frac{2\mu R\theta v^2}{R}$$

where F_r is the force that supplies the centripetal acceleration of the segment and maintains the curvature of this segment.

Solving for v gives

$$v = \sqrt{\frac{F}{\mu}}$$

Notice that this derivation is based on the assumption that the pulse height is small relative to the length of the string. Using this assumption, we were able to use the approximation $\sin \theta \approx \theta$. Furthermore, the model assumes that the tension, F, is not affected by the presence of the pulse, so that F is the same at all points on the string. Finally, this proof does *not* assume any particular shape for the pulse. Therefore, we conclude that a pulse of *any shape* will travel on the string with speed $v = \sqrt{F/\mu}$, without changing its shape.

Thinking Physics 3

A secret agent is trapped in a building on top of the elevator car at a lower floor. She attempts to signal a fellow agent on the roof by tapping a message on the elevator cable. As the pulses from this tapping move up the cable to the accomplice, does the speed with which they move stay the same, increase, or decrease? If the pulses are sent one second apart, are they received one second apart by her partner?

Reasoning The elevator cable can be modeled as a heavy string. The speed of waves on the cable is a function of the tension in the cable. As the waves move higher on the cable, they encounter increased tension, because each higher point on the cable must support the weight of all the cable below it (and the elevator). Thus, the speed of the pulses increases as they move higher on the cable. The frequency of the pulses will not be affected, because each pulse takes the same total time to reach the top—they will still arrive at the top of the cable at intervals of one second.

CONCEPTUAL PROBLEM 3

In mechanics, massless strings are often assumed. Why is this not a good assumption when discussing waves on strings?

Example 13.4 The Speed of a Pulse on a Cord

A uniform cord has a mass of 0.300 kg and a total length of 6.00 m. Tension is maintained in the cord by suspending a 2.00-kg mass from one end (Fig. 13.14). Find the speed of a pulse on this cord. Assume the tension is not affected by the mass of the cord.

Solution The tension F in the cord is equal to the weight of the suspended 2.00-kg mass:

$$F = mg = (2.00 \text{ kg})(9.80 \text{ m/s}^2) = 19.6 \text{ N}$$

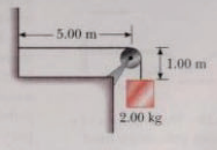


Figure 13.14 (Example 13.4) The tension F in the cord is maintained by the suspended mass. The wave speed is given by the expression $v = \sqrt{F/\mu}$.

INTEL-1013
U.S. Patent 7,151,430

(This calculation of the tension neglects the small mass of the cord. Strictly speaking, the cord can never be exactly horizontal, and therefore the tension is not uniform.)

The mass per unit length μ is

$$\mu = \frac{m}{\ell} = \frac{0.300 \text{ kg}}{6.00 \text{ m}} = 0.0500 \text{ kg/m}$$

Therefore, the wave speed is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{19.6 \text{ N}}{0.0500 \text{ kg/m}}} = 19.8 \text{ m/s}$$

EXERCISE 5 Show that Equation 13.19 is dimensionally correct using the known dimensions of F and μ.

EXERCISE 6 Transverse waves travel with a speed of 20.0 m/s in a string under a tension of 6.00 N. What tension is required for a wave speed of 30.0 m/s in the same string?
Answer 13.5 N

13.7 · REFLECTION AND TRANSMISSION OF WAVES

Whenever a traveling wave pulse reaches a boundary, part or all of the pulse is *reflected*. Any part not reflected is said to be *transmitted* through the boundary. Consider a pulse traveling on a string that is fixed at one end (Fig. 13.15). When the pulse reaches the fixed boundary, it is reflected. Because the support attaching the string to the wall is rigid, none of the pulse is transmitted through the wall.

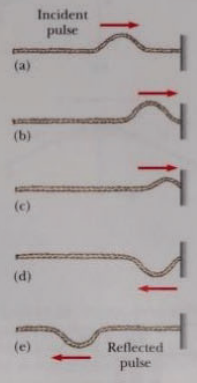


Figure 13.15 The reflection of a traveling wave pulse at the fixed end of a stretched string. The reflected pulse is inverted, but its shape remains the same.

Note that the reflected pulse has exactly the same amplitude as the incoming pulse but is inverted. The inversion can be explained as follows. When the pulse meets the end of the string that is fixed at the support, the string produces an upward force on the support. By Newton's third law, the support must exert an equal and opposite reaction force on the string. This downward force causes the pulse to invert on reflection.

Now consider another situation in which there is total reflection and zero transmission. In this case the pulse arrives at the end of a string that is free to move vertically, as in Figure 13.16. The tension at the free end is maintained by tying the string to a ring of negligible mass that is free to slide vertically on a frictionless post. Again, the pulse is reflected, but this time it is not inverted. As the pulse reaches the post, it exerts a force on the free end, causing the ring to accelerate upward. In the process, the ring has upward momentum as it reaches the top of its motion and is then returned to its original position by the downward component of the tension force. This produces a reflected pulse that is not inverted, the amplitude of which is the same as that of the incoming pulse.

Finally, we may have a situation in which the boundary is intermediate between these two extreme cases; that is, it is neither completely rigid nor completely free. In this case, part of the wave is transmitted and part is reflected. For instance, suppose a light string is attached to a heavier string, as in Figure 13.17. When a pulse traveling on the light string reaches the boundary between the two strings, part of the pulse is reflected and inverted and part is transmitted to the heavier string. As one would expect, both the reflected pulse and the transmitted one have a smaller amplitude than the incident pulse. The inversion in the reflected wave is similar to the behavior of a pulse meeting a rigid boundary.

When a pulse traveling on a heavy string strikes the boundary of a lighter string, as in Figure 13.18, again part is reflected and part transmitted. This time, however, the reflected pulse is not inverted. In both the lighter-to-heavier case and the heavier-to-lighter case, the relative heights of the reflected and transmitted pulses depend on the relative densities of the two strings. In an extreme case, if the second string were infinitely more dense, it would behave like a rigid wall.

In the preceding section, we found that the speed of a wave on a string increases as the mass per unit length of the string decreases. In other words, a pulse travels more slowly on a heavy string than on a light string if both are under the same tension. The following general rules apply to reflected waves:

> When a wave pulse travels from medium A to medium B and $v_A > v_B$ (that is, when B is more massive than A), the reflected part of the pulse is inverted on reflection. When a wave pulse travels from medium A to medium B and $v_A < v_B$ (A is more massive than B), the reflected pulse is not inverted.

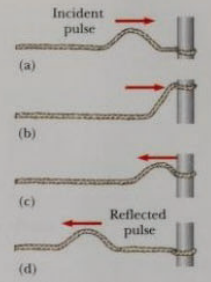


Figure 13.16 The reflection of a traveling wave pulse at the free end of a stretched string. In this case, the reflected pulse is not inverted.

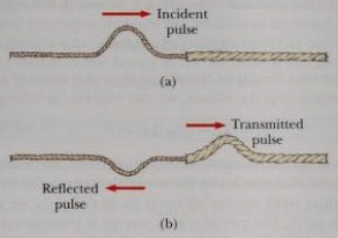


Figure 13.17 (a) A pulse traveling to the right on a light string attached to a heavier string. (b) Part of the incident pulse is reflected (and inverted), and part is transmitted to the heavier string. (Note that the change in pulse width is not shown.)

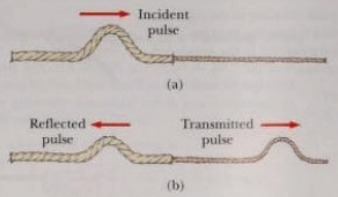


Figure 13.18 (a) A pulse traveling to the right on a heavy string attached to a lighter string. (b) The incident pulse is partially reflected and partially transmitted. In this case, the reflected pulse is not inverted. (Note that the change in pulse width is not shown.)

INTEL-1013
U.S. Patent 7,151,430

13.8 • ENERGY TRANSMITTED BY SINUSOIDAL WAVES ON STRINGS

As waves propagate through a medium, they transport energy. This is easily demonstrated by hanging a mass on a stretched string and then sending a pulse down the string, as in Figure 13.19. When the pulse meets the suspended mass, the mass is momentarily displaced, as in Figure 13.19b. In the process, energy is transferred to the mass, because work must be done in moving it upward.

This section examines the rate at which energy is transported along a string. We shall assume that this one-dimensional wave is sinusoidal when we calculate the power transferred. Later we shall extend the same ideas to three-dimensional waves.

Consider a sinusoidal wave traveling on a string (Fig. 13.20). The source of the energy is some external agent at the left end of the string, which does work in producing the oscillations. Let us focus our attention on an element of the string of length Δx and mass Δm. Each such segment moves vertically with simple harmonic motion. Furthermore, each segment has the same angular frequency, ω, and the same amplitude, A. As we found in Chapter 12, the total energy, E, associated with a particle moving with simple harmonic motion is $E = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2A^2$ (Eq. 12.22), where k is the effective force constant of the restoring force. If we apply this equation to the element of length Δx, we see that the total energy of this element is

$$\Delta E = \tfrac{1}{2}(\Delta m)\omega^2A^2$$

If μ is the mass per unit length of the string, then the element of length Δx has a mass, Δm, that is equal to $\mu\,\Delta x$. Hence, we can express the energy as

$$\Delta E = \tfrac{1}{2}(\mu\,\Delta x)\omega^2A^2 \qquad [13.20]$$

If the wave travels from left to right, as in Figure 13.20, the energy ΔE arises from the work done on the element Δm by the string element to the left of Δm. Similarly, the element Δm does work on the element to its right, so we see that energy is transmitted to the right. The rate at which energy is transmitted along the string, or the power (P), is dE/dt. If we let Δx approach 0, Equation 13.20 gives

$$P = \frac{dE}{dt} = \tfrac{1}{2}\left(\mu\,\frac{dx}{dt}\right)\omega^2A^2$$

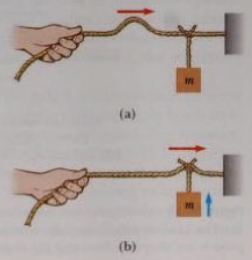


Figure 13.19 (a) A pulse traveling to the right on a stretched string on which a mass has been suspended. (b) Energy is transmitted to the suspended mass when the pulse arrives.

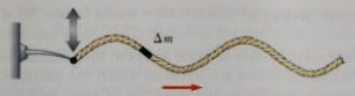


Figure 13.20 A sinusoidal wave traveling along the x axis on a stretched string. Every segment moves vertically, and each segment has the same total energy. The power transmitted by the wave equals the energy contained in one wavelength divided by the period of the wave.

Because dx/dt is equal to the wave speed, v, we have

$$P = \tfrac{1}{2}\mu\omega^2A^2v \qquad [13.21]$$ • *Power*

This shows that the power transmitted by a sinusoidal wave on a string is proportional to (1) the wave speed, (2) the square of the frequency, and (3) the square of the amplitude. In fact, *all* sinusoidal waves have the following general property: **The power transmitted by any sinusoidal wave is proportional to the square of the angular frequency and to the square of the amplitude.**

Thus, we see that a wave traveling through a medium corresponds to energy transport through the medium, with no net transfer of matter. An oscillating source provides the energy and produces a harmonic disturbance of the medium. The disturbance is able to propagate through the medium as the result of the interaction between adjacent particles. In order to verify Equation 13.21 by direct experiment, one would have to design some device at the far end of the string to extract the energy of the wave without producing any reflections.

Thinking Physics 4

We have shown that waves carry energy—not only waves on strings, but all waves. Devise a demonstration that shows that waves transfer momentum as well as energy.

Reasoning Imagine that we cover the ends of a cardboard tube with rubber sheets, secured with rubber bands, to form a crude drum with two heads. We now lay the tube horizontally on the table and suspend a Ping-Pong ball from a string so that the Ping-Pong ball is just touching one of the rubber sheets. If we now thump the other rubber sheet with a finger, the sound will travel down the tube, strike the first rubber sheet, and the Ping-Pong ball will bounce away from the rubber sheet. This demonstrates that the sound wave is carrying momentum to the Ping-Pong ball.

Example 13.5 Power Supplied to a Vibrating String

A string of mass per unit length $\mu = 5.00 \times 10^{-2}$ kg/m is under a tension of 80.0 N. How much power must be supplied to the rope to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.00 cm?

Solution The wave speed on the string is

$$v = \sqrt{\frac{F}{\mu}} = \left(\frac{80.0\text{ N}}{5.00 \times 10^{-2}\text{ kg/m}}\right)^{1/2} = 40.0\text{ m/s}$$

Because $f = 60.0$ Hz, the angular frequency ω of the sinusoidal waves on the string has the value

$$\omega = 2\pi f = 2\pi(60.0\text{ Hz}) = 377\text{ s}^{-1}$$

Using these values in Equation 13.21 for the power, with $A = 6.00 \times 10^{-2}$ m, gives

$$\begin{aligned}\text{Power} &= \tfrac{1}{2}\mu\omega^2A^2v \\ &= \tfrac{1}{2}(5.00 \times 10^{-2}\text{ kg/m})(377\text{ s}^{-1})^2 \\ &\quad \times (6.00 \times 10^{-2}\text{ m})^2(40.0\text{ m/s}) \\ &= 512\text{ W}\end{aligned}$$

INTEL-1013
U.S. Patent 7,151,430

13.9 • SOUND WAVES

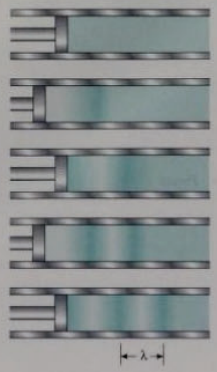


Figure 13.21 A sinusoidal longitudinal wave propagating down a tube filled with a compressible gas. The source of the wave is a vibrating piston at the left. The high- and low-pressure regions are dark and light, respectively.

Let us turn our attention from transverse waves to longitudinal ones. As stated in Section 13.2, longitudinal waves are waves in which the particles of the medium undergo displacements parallel to the direction of wave motion. Sound waves in air are the most important example of longitudinal waves. They can travel through any material medium, their speed depending on the properties of that medium.

The displacements accompanying a sound wave in air are longitudinal displacements of individual molecules from their equilibrium positions. Such displacements result if the source of the waves, such as the diaphragm of a loudspeaker, oscillates with simple harmonic motion. For instance, you can produce a one-dimensional harmonic sound wave in a long, narrow tube containing a gas by means of a vibrating piston at one end, as in Figure 13.21. The darker regions in the figure represent regions in which the gas is compressed and, consequently, the density and pressure are *above* their equilibrium values. Such a compressed layer of gas, called a **condensation,** is formed when the piston is being pushed into the tube. The condensation moves down the tube as a pulse, continuously compressing the layers in front of it. When the piston is withdrawn from the tube, the gas in front of it expands and consequently the pressure and density in this region fall below their equilibrium values. These low-pressure regions, called **rarefactions,** are represented by the lighter areas in Figure 13.21. The rarefactions also propagate along the tube, following the condensations. Both regions move with a speed equal to the speed of sound in that medium. The speed of sound waves in air at 20°C is about 343 m/s.

As the piston oscillates back and forth in a sinusoidal fashion, regions of condensation and rarefaction are continuously set up. The distance between two successive condensations (or two successive rarefactions) equals the wavelength, λ. As these regions travel down the tube, any small volume of the medium moves with simple harmonic motion parallel to the direction of the wave (in other words, longitudinally). If $s(x, t)$ is the displacement of a small volume element measured from its equilibrium position, we can express this displacement function as

$$s(x, t) = s_{max} \cos(kx - \omega t) \qquad [13.22]$$

where s_{max} is the **maximum displacement from equilibrium** (the displacement amplitude), k is the angular wave number, and ω is the angular frequency of the piston. Note that the displacement of the volume element is along x, the direction of motion of the sound wave, which of course means we are describing a longitudinal wave. The variation in the pressure of the gas, ΔP, measured from its equilibrium value is also sinusoidal; it is given by

$$\Delta P = \Delta P_{max} \sin(kx - \omega t) \qquad [13.23]$$

The **pressure amplitude,** ΔP_{max}, is the **maximum change in pressure from the equilibrium value.** It is proportional to the displacement amplitude, s_{max}:

Pressure amplitude •

$$\Delta P_{max} = \rho v \omega s_{max} \qquad [13.24]$$

where ρ is the density of the medium, v is the wave speed, and ωs_{max} is the maximum longitudinal speed of the medium in front of the piston.

Thus, we see that a sound wave may be considered as either a displacement wave or a pressure wave. A comparison of Equations 13.22 and 13.23 shows that **the pressure wave is 90° out of phase with the displacement wave.** Graphs of these functions are shown in Figure 13.22. Note that the pressure variation is a maximum when the displacement is zero, whereas the displacement is a maximum when the pressure variation is zero. Because the pressure is proportional to the density of the medium, the expression that describes how that density varies from its equilibrium value is similar to Equation 13.23.

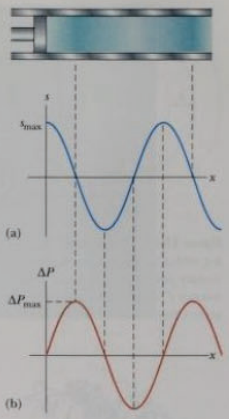


Figure 13.22 (a) Displacement amplitude versus position and (b) pressure amplitude versus position for a sinusoidal longitudinal wave. The displacement wave is 90° out of phase with the pressure wave.

Thinking Physics 5

Garth Brooks had a hit song with "The Thunder Rolls" (Caged Panther Music, Inc., 1990). Why *does* thunder produce an extended "rolling" sound? And how does lightning produce thunder in the first place?

Reasoning Let us assume we are at ground level and neglect ground reflections. When lightning strikes, a channel of ionized air carries a very large electric current from a cloud to the ground. This results in a very rapid temperature increase of this channel of air as the current flows through it. The temperature increase causes a sudden expansion of the air. This expansion is so sudden and so intense that a tremendous disturbance is produced in the air—thunder. The thunder rolls due to the fact that the lightning channel is a long extended source—the entire length of the channel produces the sound at essentially the same instant of time. Sound produced at the bottom of the channel reaches you first, if you are on the ground, because that is the point closest to you, and then sounds from progressively higher portions of the channel reach you. If the lightning channel were a perfectly straight line, the resulting sound might be a steady roar, but the zig-zagged shape of the path results in the rolling variation in loudness, with sound from some portions of the channel arriving at your location simultaneously. This results in periods of loud sounds, interspersed with other instants when the net sound reaching you is low in intensity.

13.10 • THE DOPPLER EFFECT

When a vehicle sounds its horn as it travels along a highway, the frequency of the sound you hear is higher as the vehicle approaches you than it is as the vehicle moves away from you. This is one example of the **Doppler effect,** named after the Austrian physicist Christian Johann Doppler (1803–1853).

> In general, a Doppler effect is experienced whenever there is relative motion between the source of sound and the observer. When the source and observer are moving toward each other, the observer hears a frequency that is higher than the true frequency of the source. When the source and observer are moving away from each other, the observer hears a frequency that is lower than the true frequency of the source.[4]

[4]Although the Doppler effect is most commonly experienced with sound waves, it is a phenomenon common to all harmonic waves. For example, a shift in frequencies of light waves (electromagnetic waves) is produced by the relative motion of source and observer. The relativistic Doppler effect demands that there can be no distinction between observer moving toward source and source moving toward observer. In 1842 Doppler first reported the frequency shift in connection with light emitted by stars revolving about each other in double-star systems.

INTEL-1013
U.S. Patent 7,151,430

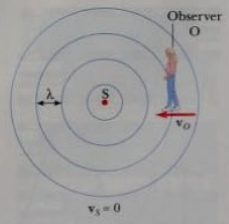


Figure 13.23 An observer O moving with a speed v_O toward a stationary point source S hears a frequency f' that is greater than the source frequency.



"I love hearing that lonesome wail of the train whistle as the magnitude of the frequency of the wave changes due to the Doppler effect."

The Doppler effect for electromagnetic waves is used in police radar systems to measure the speeds of motor vehicles. Likewise, astronomers use the effect to determine the relative motions of stars, galaxies, and other celestial objects.

First, let us consider the case where the observer O is moving with a speed of v_O and the sound source S is stationary, as in Figure 13.23. For simplicity, we shall assume that the air is stationary and that the observer moves directly toward the source (considered as a point source). We shall take the frequency of the source to be f, the wavelength to be λ, and the speed of sound to be v. A stationary observer would detect f vibrations each second, where $f = v/\lambda$. (That is, when the source and observer are both at rest, the observed frequency must equal the true frequency of the source.) However, the observer moving toward the source with the speed v_O receives an additional number of vibrations per second, equal to v_O/λ. Hence, the frequency the observer hears, f', is *increased:* $f' = v/\lambda + v_O/\lambda$. Because $\lambda = v/f$, we can express f' as

$$f' = f\left(\frac{v + v_O}{v}\right) \qquad \text{(Observer moving toward source)} \qquad [13.25]$$

Now consider the situation in which the source moves with a speed of v_S relative to the medium, and the observer is at rest. If the source moves directly toward observer A in Figure 13.24a, the wave fronts seen by the observer are closer to each other than they would be if the source were at rest. As a result, the wavelength λ' measured by observer A is shorter than the true wavelength λ of the source. During each vibration, with a duration of T (the period), the source moves a distance of $v_S T = v_S/f$. Therefore, the wavelength is *shortened* by this amount, and the observed wavelength has the shorter value $\lambda' = \lambda - v_S/f$. Because $\lambda = v/f$, the frequency heard by observer A is

$$f' = \frac{v}{\lambda'} = f\left(\frac{v}{v - v_S}\right) \qquad \text{(source moving toward observer)} \qquad [13.26]$$

That is, the frequency is *increased* when the source moves toward the observer. In a similar manner, if the source moves away from observer B at rest, the sign of v_S is reversed in Equation 13.26.

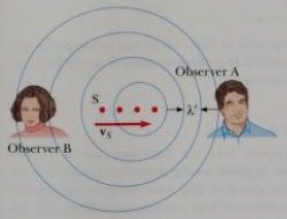
Finally, if both the source and the observer are in motion, one finds the following general formula for the observed frequency:

Frequency heard with observer and source in motion

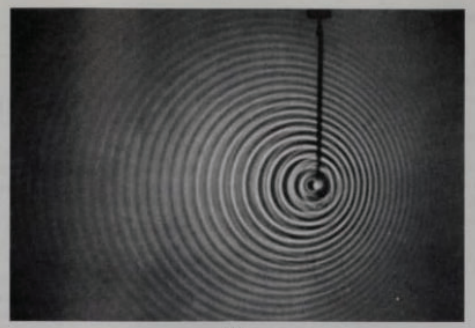
$$f' = f\left(\frac{v \pm v_O}{v \mp v_S}\right) \qquad [13.27]$$

In this expression, the upper signs ($+ v_O$ and $- v_S$) refer to motion of the one *toward* the other, and the lower signs ($- v_O$ and $+ v_S$) refer to motion of one *away from* the other.

When working with any Doppler effect problem, a convenient rule to remember concerning signs is the following: The word *toward* is associated with an *increase* in the observed frequency. The words *away from* are associated with a *decrease* in the observed frequency.



(a)



(b)

Figure 13.24 (a) A source S moving with a speed v_S toward a stationary observer A and away from a stationary observer B. Observer A hears an increased frequency, and observer B hears a decreased frequency. (b) The Doppler effect in water observed in a ripple tank. *(Courtesy Educational Development Center, Newton, Mass.)*

Thinking Physics 6

Suppose you place your stereo speakers far apart and run past them from right to left, or left to right. If you run rapidly enough and have excellent pitch discrimination, you may notice that the music that is playing seems to be out of tune when you are between the speakers. Why?

Reasoning When you are between the speakers, you are running away from one of them and toward the other. Thus, there is a Doppler shift downward for the sound from the speaker behind you and a Doppler shift upward for the sound from the speaker ahead of you. As a result, the sound from the two speakers will not be in tune. A calculation shows that a world-class sprint runner could run fast enough to generate about a semitone difference in the sound from the two speakers.

CONCEPTUAL PROBLEM 4

Suppose the wind blows. Does this cause a Doppler effect for an observer listening to a sound moving through the air? Is it like a moving source or a moving observer?

CONCEPTUAL PROBLEM 5

You are driving toward a cliff and you honk your horn. Is there a Doppler shift of the sound when you hear the echo? Is it like a moving source or moving observer? What if the reflection occurs not from a cliff but from the forward edge of a huge alien spacecraft which is moving toward you as you drive?

INTEL-1013
U.S. Patent 7,151,430

Example 13.6 **The Noisy Siren**

An ambulance travels down a highway at a speed of 33.5 m/s (75 mi/h). Its siren emits sound at a frequency of 400 Hz. What is the frequency heard by a passenger in a car traveling at 24.6 m/s (55 mi/h) in the opposite direction as the car approaches the ambulance and as the car moves away from the ambulance?

Solution Let us take the velocity of sound in air to be $v = 343$ m/s. We can use Equation 13.27 in both cases. As the ambulance and car approach each other, the observed apparent frequency is

$$f' = f\left(\frac{v + v_O}{v - v_S}\right) = (400 \text{ Hz})\left(\frac{343 \text{ m/s} + 24.6 \text{ m/s}}{343 \text{ m/s} - 33.5 \text{ m/s}}\right)$$

$$= 475 \text{ Hz}$$

Likewise, as they recede from each other, a passenger in the car hears a frequency

$$f' = f\left(\frac{v - v_O}{v + v_S}\right) = (400 \text{ Hz})\left(\frac{343 \text{ m/s} - 24.6 \text{ m/s}}{343 \text{ m/s} + 33.5 \text{ m/s}}\right)$$

$$= 338 \text{ Hz}$$

The *change* in frequency as detected by the passenger in the car is 475 − 338 = 137 Hz, which is more than 30% of the actual frequency emitted.

EXERCISE 7 Suppose that the passenger car is parked on the side of the highway as the ambulance travels down the highway at the speed of 33.5 m/s. What frequency will the passenger in the car hear as the ambulance (a) approaches the parked car and (b) recedes from the parked car? Answer (a) 443 Hz (b) 364 Hz

EXERCISE 8 A band is playing on a moving truck. The band strikes the note middle C (262 Hz), but it is heard by spectators ahead of the truck as C# (277 Hz). How fast is the truck moving? Answer 18.6 m/s

SUMMARY

A **transverse wave** is a wave in which the particles of the medium move in a direction perpendicular to the direction of the wave velocity. An example is a wave on a stretched string.

Longitudinal waves are waves in which the particles of the medium move parallel to the direction of the wave speed. Sound waves in air are longitudinal.

Any one-dimensional wave traveling with a speed of v in the positive x direction can be represented by a wave function of the form $y = f(x - vt)$. Likewise, the wave function for a wave traveling in the negative x direction has the form $y = f(x + vt)$. The shape of the wave at any instant (a snapshot of the wave) is obtained by holding t constant.

The **superposition principle** says that when two or more waves move through a medium, the resultant wave function equals the algebraic sum of the individual wave functions. Waves that obey this principle are said to be *linear*. When two waves combine in space, they interfere to produce a resultant wave.

The wave function for a one-dimensional harmonic wave traveling to the right can be expressed as

$$y = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right] = A \sin(kx - \omega t) \qquad [13.6, 13.12]$$

where A is the amplitude, λ is the wavelength, k is the angular wave number, and ω is the angular frequency. If T is the period and f is the frequency, then v, k, and ω can be written

$$v = \frac{\lambda}{T} = f\lambda \qquad [13.7, 13.14]$$

$$k = \frac{2\pi}{\lambda} \qquad [13.10]$$

$$\omega = \frac{2\pi}{T} = 2\pi f \qquad [13.11]$$

The speed of a wave traveling on a stretched string of mass per unit length μ and tension F is

$$v = \sqrt{\frac{F}{\mu}} \qquad [13.19]$$

When a pulse traveling on a string meets a fixed end, the pulse is reflected and inverted. If the pulse reaches a free end, it is reflected but not inverted.

The **power** transmitted by a harmonic wave on a stretched string is

$$P = \tfrac{1}{2}\mu\omega^2 A^2 v \qquad [13.21]$$

The change in frequency heard by an observer whenever there is relative motion between a wave source and the observer is called the **Doppler effect.** When the source and observer are moving toward each other, the observer hears a frequency that is higher than the true frequency of the source. When the source and observer are moving away from each other, the observer hears a frequency that is lower than the true frequency of the source.

CONCEPTUAL QUESTIONS

1. How would you set up a longitudinal wave in a stretched spring? Would it be possible to set up a transverse wave in a spring?
2. By what factor would you have to increase the tension in a taut string in order to double the wave speed?
3. When traveling on a taut string, does a wave pulse always invert on reflection? Explain.
4. Does the vertical speed of a segment of a horizontal taut string through which a wave is traveling depend on the wave speed?
5. A vibrating source generates a sinusoidal wave on a string under constant tension. If the power delivered to the string is doubled, by what factor does the amplitude change? Does the wave speed change under these circumstances?
6. Consider a wave traveling on a taut rope. What is the difference, if any, between the speed of the wave and the speed of a small section of the rope?
7. What happens to the wavelength of a wave on a string when the frequency is doubled? What happens to its speed? Assume the tension in the string remains the same.
8. When all the strings on a guitar are stretched to the same tension, will the speed of a wave along the more massive bass strings be faster or slower than the speed of a wave on the lighter strings?
9. If you stretch a rubber hose and pluck it, you can observe a pulse traveling up and down the hose. What happens to the speed if you stretch the hose tighter? If you fill the hose with water?
10. In a longitudinal wave in a spring, the coils move back and forth in the direction of wave motion. Does the speed of the wave depend on the maximum speed of each coil?
11. When two waves interfere, can the amplitude of the resultant wave be larger than either of the two original waves? Under what conditions?
12. A solid can transport both longitudinal waves and transverse waves, but a fluid can transport only longitudinal waves. Why?
13. In an earthquake both *S* (transverse) and *P* (longitudinal) waves are sent out. The *S* waves travel through the Earth more slowly than the *P* waves (4.5 km/s versus 7.8 km/s). By detecting the time of arrival of the waves, how can one determine how far away the epicenter of the quake was? How many detection centers are necessary to pinpoint the location of the epicenter?

INTEL-1013
U.S. Patent 7,151,430

PROBLEMS

Section 13.3 One-Dimensional Transverse Traveling Waves

1. At $t = 0$, a transverse wave pulse in a wire is described by the function

$$y = \frac{6}{x^2 + 3}$$

where x and y are in meters. Write the function $y(x, t)$ that describes this wave if it is traveling in the positive x direction with a speed of 4.50 m/s.

2. Ocean waves with a crest-to-crest distance of 10.0 m can be described by

$$y(x, t) = (0.800\ \text{m})\sin\,[0.628(x - vt)]$$

where $v = 1.20$ m/s. (a) Sketch $y(x, t)$ at $t = 0$. (b) Sketch $y(x, t)$ at $t = 2.00$ s. Note how the entire waveform has shifted 2.40 m in the positive x direction in this time interval.

3. A wave moving along the x axis is described by

$$y(x, t) = 5.0e^{-(x+5.0t)^2}$$

where x is in meters and t is in seconds. Determine (a) the direction of the wave motion and (b) the speed of the wave.

4. Two wave pulses A and B are moving in opposite directions along a taut string with a speed of 2.00 cm/s. The amplitude of A is twice the amplitude of B. The pulses are shown in Figure P13.4 at $t = 0$. Sketch the shape of the string at $t = 1$, 1.5, 2, 2.5, and 3 s.

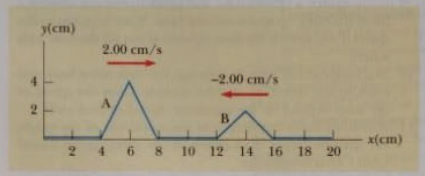


Figure P13.4

Section 13.4 Sinusoidal Traveling Waves

5. A sinusoidal wave is traveling along a rope. The oscillator that generates the wave completes 40.0 vibrations in 30.0 s. Also, a given peak travels 425 cm along the rope in 10.0 s. What is the wavelength?

6. A sinusoidal wave of wavelength 2.00 m and amplitude 0.100 m travels with a speed of 1.00 m/s on a string. Initially, the left end of the string is at the origin and the wave moves from left to right. Find (a) the frequency and angular frequency, (b) the angular wave number, and (c) the wave function for this wave. Determine the equation of motion for (d) the left end of the string, and (e) the point on the string at $x = 1.50$ m to the right of the left end. (f) What is the maximum speed of any point on the string?

7. (a) Write the expression for the function $y(x, t)$ for a sinusoidal wave traveling along a rope in the *negative* x direction with the following characteristics: $A = 8.00$ cm, $\lambda = 80.0$ cm, $f = 3.00$ Hz, and $y(0, 0) = 0$. (b) Write the expression for y as a function of x and t for the wave in part (a) assuming that $y(10\ \text{cm}, 0) = 0$.

8. A transverse wave on a string is described by

$$y = (0.120\ \text{m})\sin\,\pi(x/8 + 4t)$$

(a) Determine the transverse speed and acceleration of the string at $t = 0.200$ s for the point on the string located at $x = 1.60$ m. (b) What are the wavelength, period, and speed of propagation of this wave?

9. A wave is described by $y = (2.00\ \text{cm})\sin(kx - \omega t)$, where $k = 2.11$ rad/m, $\omega = 3.62$ rad/s, x is in meters, and t is in seconds. Determine the amplitude, wavelength, frequency, and speed of the wave.

10. A sinusoidal wave traveling in the $-x$ direction (to the left) has an amplitude of 20.0 cm, a wavelength of 35.0 cm, and a frequency of 12.0 Hz. The displacement of the wave at $t = 0$, $x = 0$ is $y = -3.00$ cm and has a positive velocity here. (a) Sketch the wave at $t = 0$. (b) Find the angular wave number, period, angular frequency, and phase velocity of the wave. (c) Write an expression for the wave function $y(x, t)$.

11. A transverse sinusoidal wave on a string has a period $T = 25.0$ ms and travels in the negative x direction with a speed of 30.0 m/s. At $t = 0$, a particle on the string at $x = 0$ has a displacement of 2.00 cm and is moving in the negative y direction with a speed of 2.00 m/s. (a) What is the amplitude of the wave? (b) What is the initial phase angle? (c) What is the maximum transverse speed of the string? (d) Write the wave function for the wave.

Section 13.5 Superposition and Interference of Waves

12. Two pulses traveling on the same string are described by

$$y_1 = \frac{5}{(3x - 4t)^2 + 2} \quad \text{and} \quad y_2 = \frac{-5}{(3x + 4t - 6)^2 + 2}$$

(a) In which direction does each pulse travel? (b) At what time do the two cancel? (c) At what point do the two waves always cancel?

13. Two sinusoidal waves in a string are defined by the functions

$$y_1 = (2.00\ \text{cm})\sin(20.0x - 30.0t)$$

$$y_2 = (2.00\ \text{cm})\sin(25.0x - 40.0t)$$

where y and x are in centimeters and t is in seconds. (a) What is the phase difference between these two waves at the point $x = 5.00$ cm at $t = 2.00$ s? (b) What is the positive x value closest to the origin for which the two phases differ by π at $t = 2.00$ s? (This is where the two waves add to zero.)

Section 13.6 The Speed of Transverse Waves on Strings

14. A piano string of mass-per-length 5.00×10^{-3} kg/m is under a tension of 1350 N. Find the speed with which a wave travels on this string.

15. A phone cord is 4.00 m long. The cord has a mass of 0.200 kg. If a transverse wave pulse travels from the receiver to the phone box in 0.100 s, what is the tension in the cord?

16. A 30.0-m steel wire and a 20.0-m copper wire, both with 1.00-mm diameters, are connected end to end and stretched to a tension of 150 N. How long does it take a transverse wave to travel the entire length of the two wires?

17. Transverse pulses travel with a speed of 200 m/s along a taut copper wire the diameter of which is 1.50 mm. What is the tension in the wire? (The density of copper is 8.92 g/cm^3.)

18. A light string of mass-per-unit length 8.00 g/m has its ends tied to two walls separated by a distance equal to three fourths the length of the string (Fig. P13.18). A mass m is suspended from the center of the string, putting a tension in the string. (a) Find an expression for the transverse wave speed in the string as a function of the hanging mass.

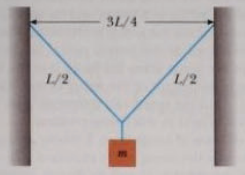


Figure P13.18

(b) How much mass should be suspended from the string to have a wave speed of 60.0 m/s?

19. An astronaut on the Moon wishes to measure the local value of the free-fall acceleration g_{Moon} by timing pulses traveling down a wire that has a large mass suspended from it. Assume a wire of mass 4.00 g is 1.60 m long and has a 3.00 kg mass suspended from it. A pulse requires 36.1 ms to traverse the length of the wire. Calculate g_{Moon} from these data. (You may neglect the mass of the wire when calculating the tension in it.)

Section 13.8 Energy Transmitted by Sinusoidal Waves on Strings

20. A taut rope has a mass of 0.180 kg and a length of 3.60 m. What power must be supplied to the rope in order to generate sinusoidal waves having an amplitude of 0.100 m and a wavelength of 0.500 m and traveling with a speed of 30.0 m/s?

21. Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string of linear density 4.00×10^{-2} kg/m. If the maximum power delivered by the source is 300 W and the string is under a tension of 100 N, what is the highest vibrational frequency at which the source can operate?

22. It is found that a 6.00-m segment of a long string contains four complete waves and has a mass of 180 g. The string is vibrating sinusoidally with a frequency of 50.0 Hz and a peak-to-valley displacement of 15.0 cm. (*Peak-to-valley* means the vertical distance from the farthest positive displacement to the farthest negative displacement.) (a) Write down the function that describes this wave traveling in the positive x direction. (b) Determine the power being supplied to the string.

23. A horizontal string can transmit a maximum power of P_0 (without breaking) if a wave with amplitude A and angular frequency ω is traveling along it. In order to increase this maximum power, a student folds the string and uses this double string as a transmitter. Determine the maximum power that can be transmitted along the double string, assuming that the tension remains the same.

24. A two-dimensional water wave spreads in circular wave fronts. Show that the amplitude A at a distance r from the initial disturbance is proportional to $1/\sqrt{r}$. (*Hint:* Consider the energy concentrated in the outward-moving ripple.)

Section 13.9 Sound Waves

(In this section, use the following values as needed unless otherwise specified: the equilibrium density of air, $\rho = 1.20$ kg/m^3; the speed of sound in air, $v = 343$ m/s. Also, pressure variations, ΔP, are measured relative to atmospheric pressure.)

INTEL-1013
U.S. Patent 7,151,430

25. A flowerpot is knocked off a balcony 20.0 m above the sidewalk heading for a man of height 1.75 m standing below. How high from the ground can the flowerpot be after which it would be too late for a shouted warning to reach the man in time? Assume that the man below requires 0.300 s to respond to the warning.
26. A rescue plane flies horizontally at a constant speed searching for a disabled boat. When the plane is directly above the boat, the boat's crew blows a loud horn. By the time the plane's sound detector perceives the horn's sound, the plane has traveled a distance equal to one-half its altitude above the ocean. If it takes the sound 2.00 s to reach the plane, determine (a) the speed of the plane, and (b) its altitude. Take the speed of sound to be 343 m/s.
27. A sound wave in a cylinder is described by Equations 13.22 through 13.24. Show that $\Delta P = \pm \rho v \omega \sqrt{s_{max}^2 - s^2}$.
28. Calculate the pressure amplitude of a 2.00-kHz sound wave in air if the displacement amplitude is 2.00×10^{-8} m.
29. An experimenter wishes to generate in air a sound wave that has a displacement amplitude of 5.50×10^{-6} m. The pressure amplitude is to be limited to 8.40×10^{-1} N/m^2. What is the minimum wavelength the sound wave can have?
30. A sound wave in air has a pressure amplitude of 4.00 N/m^2 and a frequency of 5.00 kHz. Take $\Delta P = 0$ at the point $x = 0$ when $t = 0$. (a) What is ΔP at $x = 0$ when $t = 2.00 \times 10^{-4}$ s, and (b) what is ΔP at $x = 0.0200$ m when $t = 0$?

Section 13.10 The Doppler Effect

31. A commuter train passes a passenger platform at a constant speed of 40.0 m/s. The train horn is sounded at its characteristic frequency of 320 Hz. (a) What total change in frequency is detected by a person on the platform as the train goes from approaching to receding? (b) What wavelength is detected by a person on the platform as the train approaches?
32. A driver traveling northbound on a highway is driving at a speed of 25.0 m/s. A police car driving southbound at a speed of 40.0 m/s approaches with its siren sounding at a base frequency of 2500 Hz. (a) What frequency is observed by the driver as the police car approaches? (b) What frequency is detected by the driver after the police car passes him? (c) Repeat parts (a) and (b) for the case when the police car is traveling northbound.
33. Standing at a crosswalk, you hear a frequency of 560 Hz from the siren of an approaching police car. After the police car passes, the observed frequency of the siren is 480 Hz. Determine the car's speed from these observations.
34. A tuning fork vibrating at 512 Hz falls from rest and accelerates at 9.80 m/s^2. How far below the point of release is the tuning fork when waves of frequency 485 Hz reach the release point? Take the speed of sound in air to be 340 m/s.
35. A block with a speaker bolted to it is connected to a spring having spring constant $k = 20.0$ N/m as in Figure P13.35. The total mass of the block and speaker is 5.00 kg, and the amplitude of this unit's motion is 0.500 m. If the speaker emits sound waves of frequency 440 Hz, determine the range in frequencies heard by the person to the right of the speaker.

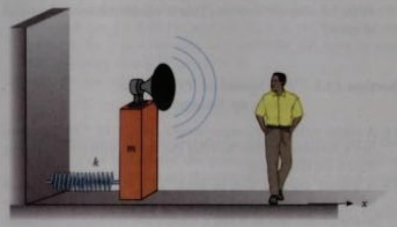


Figure P13.35

Additional Problems

36. Unoccupied by spectators, a large set of football bleachers has solid seats and risers. You stand on the field in front of it and sharply clap two wooden boards together once. You hear from the bleachers a sound with definite pitch, which may remind you of a short toot on a trumpet. Account for this sound. Compute order-of-magnitude estimates for its frequency, wavelength, and duration on the basis of data you specify.
37. A 2.00-kg block hangs from a rubber string, being supported so that the string is not stretched. The unstretched length of the string is 0.500 m and its mass is 5.00 g. The spring constant for the string is 100 N/m. The block is released and stops at the lowest point. (a) Determine the tension in the string when the block is at this lowest point. (b) What is the length of the string in this "stretched" position? (c) Find the speed of a transverse wave in the string if the block is held in this lowest position.
38. A block of mass M hangs from a rubber string, being supported so that the string is not stretched. The unstretched length of the string is L_0 and its mass is m, much less than M. The spring constant for the string is k. The block is released and stops at the lowest point. (a) Determine the tension in the string when the block is at this lowest point. (b) What is the length of the string in this "stretched" position? (c) Find the speed of a transverse wave in the string if the block is held in this lowest position.
39. The wave function for a traveling wave on a taut string is (in SI units)

$$y(x, t) = (0.350 \text{ m})\sin(10\pi t - 3\pi x + \pi/4)$$

(a) What is the speed and direction of travel of the wave? (b) What is the vertical displacement of the string at $t = 0$, $x = 0.100$ m? (c) What are the wavelength and frequency of the wave? (d) What is the maximum magnitude of the transverse speed of the string?
40. A stone is dropped into a deep canyon and is heard to strike the bottom 10.2 s after release. The speed of sound waves in air is 343 m/s. How deep is the canyon? What would be the percentage error in the depth if the time required for the sound to reach the canyon rim were ignored?
41. The ocean floor is underlain by a layer of basalt that constitutes the crust, or uppermost layer of the Earth, in this region. Below this crust is found denser peridotite rock, which forms the Earth's mantle. The boundary between these two layers is called the *Mohorovičić discontinuity* ("Moho" for short). If an explosive charge is set off at the surface of the basalt, it generates a seismic wave that is reflected back at the Moho. If the speed of this wave in basalt is 6.50 km/s, and the two-way travel time is 1.85 s, what is the thickness of this oceanic crust?
42. A worker strikes a steel pipeline with a hammer, generating both longitudinal and transverse waves. Reflected waves return 2.40 s apart. How far away is the reflection point? (For steel, $v_{long} = 6.20$ km/s and $v_{trans} = 3.20$ km/s.)
43. A rope of total mass m and length L is suspended vertically. Show that a transverse wave pulse will travel the length of the rope in a time $t = 2\sqrt{L/g}$. (*Hint:* First find an expression for the wave speed at any point a distance x from the lower end by considering the tension in the rope as resulting from the weight of the segment below that point.)
44. A train whistle ($f = 400$ Hz) sounds higher or lower in pitch depending on whether it approaches or recedes. (a) Prove that the difference in frequency between the approaching and receding train whistle is

$$\Delta f = \frac{2f\left(\frac{u}{v}\right)}{1 - \frac{u^2}{v^2}} \qquad \begin{aligned} u &= \text{speed of train} \\ v &= \text{speed of sound} \end{aligned}$$

(b) Calculate this difference for a train moving at a speed of 130 km/h. Take the speed of sound in air to be 340 m/s.

45. In order to be able to determine her speed, a skydiver carries a tone generator. A friend on the ground at the landing site has equipment for receiving and analyzing sound waves. While the skydiver is falling at terminal speed, her tone generator emits a steady tone of 1800 Hz. (Assume that the air is calm and that the sound speed is 343 m/s, independent of altitude.) (a) If her friend on the ground (directly beneath the skydiver) receives waves of frequency 2150 Hz, what is the skydiver's speed of descent? (b) If the skydiver were also carrying sound-receiving equipment sensitive enough to detect waves reflected from the ground, what frequency would she receive?
46. A wave pulse traveling along a string of linear mass density is described by the relationship

$$y = [A_0 e^{-bx}]\sin(kx - \omega t)$$

where the factors in brackets before the sine are said to be the amplitude. (a) What is the power $P(x)$ carried by this wave at a point x? (b) What is the power $P(0)$ carried by this wave at the origin? (c) Compute the ratio $P(x)/P(0)$.
47. A bat, moving at 5.00 m/s, is chasing a flying insect. If the bat emits a 40.0-kHz chirp and receives back an echo at 40.4 kHz, at what speed is the insect moving toward or away from the bat? (Take the speed of sound in air to be $v = 340$ m/s.)
48. An earthquake on the ocean floor in the Gulf of Alaska induces a *tsunami* (sometimes called a *tidal wave*) that reaches Hilo, Hawaii, 4450 km distant, in a time of 9 h 30 min. Tsunamis have enormous wavelengths (100 km to 200 km), and for such waves the propagation speed is $v \approx \sqrt{gd}$, where d is the average depth of the water. From the information given, find the average wave speed and the average ocean depth between Alaska and Hawaii. (This method was used in 1856 to estimate the average depth of the Pacific Ocean long before soundings were made to give a direct determination.)

Spreadsheet Problem

S1. Two transverse wave pulses traveling in opposite directions along the x axis are represented by the following wave functions:

$$y_1(x, t) = \frac{6}{(x - 3t)^2} \qquad y_2(x, t) = -\frac{3}{(x + 3t)^2}$$

where x and y are measured in centimeters and t is in seconds. Write a spreadsheet or program to add the two pulses and obtain the shape of the composite waveform $y_{tot} = y_1 + y_2$ as a function of time. Plot y_{tot} versus x. Make separate plots for $t = 0$, 0.5, 1, 1.5, 2, 2.5, and 3.0 s.

INTEL-1013
U.S. Patent 7,151,430

ANSWERS TO CONCEPTUAL PROBLEMS

1. A pulse in a long line of people is longitudinal, since the movement of people is parallel to the direction of propagation of the pulse. The speed is determined by the reaction time of the people and the speed with which they can move, once a space opens up. There is also a psychological factor, in that people will not want to fill a space that opens up in front of them too quickly, so as not to intimidate the person in front of them. The "wave" at a stadium is transverse, since the fans stand up vertically, as the wave sweeps past them horizontally. The speed of this pulse depends on the limits of the fans' abilities to rise and sit rapidly, and on psychological factors associated with the anticipation of seeing the pulse approach the observer's location.
2. Both the radio wave and the sound wave have the same wavelength. The speed of propagation for electromagnetic waves, however, is much higher than the speed of sound, so the frequency of the radio wave is much higher than that of the sound wave. Audible sound frequencies range from about 15 Hz to 15 000 Hz, while radio frequencies are in kilohertz for AM radio and megahertz for FM.
3. A wave on a massless string would have an infinite speed of propagation because its linear mass density is zero.
4. Wind can change a Doppler shift. Both v_O and v_S in our equation must be interpreted as speeds of observer and source relative to air. If source and observer are moving relative to each other, the observer will hear one shifted frequency in still air and a different shifted frequency if wind is blowing. However, if source and observer are at rest relative to each other, there is no shift when wind blows past.
5. The echo *is* Doppler shifted, and the shift is like both a moving source and a moving observer. The sound which leaves your horn in the forward direction is Doppler shifted to a higher frequency, because it is coming from a moving source. As the sound reflects back and comes toward you, you are a moving observer, so there is a second Doppler shift to an even higher frequency. If the sound reflects from the spacecraft moving toward you, there is another moving source shift to an even higher frequency. The reflecting surface of the spacecraft acts as a moving source.

14

Superposition and Standing Waves

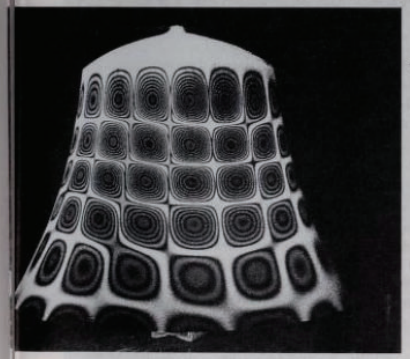
An important aspect of waves is their combined effect when two or more travel in the same medium. For instance, what happens to a string when a wave traveling toward a fixed end is reflected back on itself?

In a linear medium—that is, one in which the restoring force of the medium is proportional to the displacement of the medium—one can apply the **principle of superposition** to obtain the resultant disturbance. This principle can be applied to many types of waves under certain conditions, including waves on strings, sound waves, surface water waves, and electromagnetic waves. The superposition principle states that the net displacement of any part of the disturbed medium equals the vector sum of the displacements caused by the individual waves. In Chapter 13, the term **interference** was used to describe the effect produced by combining two waves that are moving simultaneously through a medium.

This chapter is concerned with the superposition principle as it applies to sinusoidal waves. If the sinusoidal waves that com-

CHAPTER OUTLINE

14.1 Superposition and Interference of Sinusoidal Waves
14.2 Standing Waves
14.3 Natural Frequencies in a Stretched String
14.4 Standing Waves in Air Columns
14.5 Beats: Interference in Time (Optional)
14.6 Complex Waves (Optional)



◀ **Photograph of standing waves on a vibrating handbell. The photograph was taken using a technique called *time-average holographic interferometry*. Such a hologram consists of millions of superimposed holograms, but this hologram emphasizes the two positions of maximum deflection of the vibrating bell. The pattern recorded here occurred at a frequency of 2684 Hz, but other patterns are observed at lower and higher frequencies.** *(Courtesy of Professor Thomas D. Rossing, Northern Illinois University)*

INTEL-1013
U.S. Patent 7,151,430

bine in a given medium have the same frequency and wavelength, one finds that a stationary pattern, called a **standing wave,** can be produced at certain frequencies under certain circumstances. For example, a stretched string fixed at both ends has a discrete set of standing wave patterns, called **modes of vibration,** that depend on the tension and the mass-per-unit-length of the string. These modes of vibration are found in the strings of stringed musical instruments. Other musical instruments, such as the organ and flute, make use of the natural frequencies of sound waves in hollow pipes. Such frequencies depend on the length and shape of the pipe and on whether the ends are open or closed.

In this chapter we also consider the superposition and interference of waves with different frequencies and wavelengths. When two sound waves with nearly the same frequency interfere, one hears variations in loudness called *beats.* The beat frequency corresponds to the rate of alternation between constructive and destructive interference. Finally, we describe how any complex periodic waveform can, in general, be described by a sum of sine and cosine functions.

14.1 · SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

Let us apply the superposition principle to two sinusoidal waves traveling in the same direction in a medium. If the two waves are traveling to the right and have the same frequency, wavelength, and amplitude but differ in phase, we can express their individual wave functions as

$$y_1 = A\sin(kx - \omega t) \qquad \text{and} \qquad y_2 = A\sin(kx - \omega t - \phi)$$

Hence, the resultant wave function, y, is

$$y = y_1 + y_2 = A[\sin(kx - \omega t) + \sin(kx - \omega t - \phi)]$$

In order to simplify this expression, it is convenient to use the trigonometric identity

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

If we let $a = kx - \omega t$ and $b = kx - \omega t - \phi$, the resultant wave, y, reduces to

Resultant of two traveling sinusoidal waves •

$$y = \left(2A\cos\frac{\phi}{2}\right)\sin\left(kx - \omega t - \frac{\phi}{2}\right) \qquad [14.1]$$

This result has several important features. The resultant wave function, y, is also harmonic and has the *same* frequency and wavelength as the individual waves. The amplitude of the resultant wave is $2A\cos(\phi/2)$ and the phase is $\phi/2$. If the phase constant, ϕ, equals 0, then $\cos(\phi/2) = \cos 0 = 1$ and the amplitude of the resultant wave is $2A$. In other words, the amplitude of the resultant wave is twice the amplitude of either individual wave. In this case, the waves are said to be everywhere *in phase* and to thus **interfere constructively.** That is, the crests and troughs of the individual waves occur at the same positions, as is shown by the blue lines in Figure 14.1a. In general, constructive interference occurs when $\cos(\phi/2) = \pm 1$, or when $\phi = 0, 2\pi, 4\pi, \ldots$. However, if ϕ is equal to π radians or to any *odd* multiple of π then $\cos(\phi/2) = \cos(\pi/2) = 0$ and the resultant wave has *zero* amplitude everywhere. In this case, the two waves **interfere destructively.** That is, the crest of one

Constructive interference •

Destructive interference •

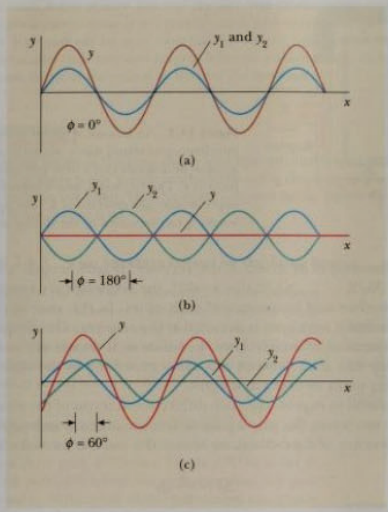


Figure 14.1 The superposition of two waves with amplitudes y_1 and y_2, where $y_1 = y_2$. (a) When the two waves are in phase, the result is constructive interference. (b) When the two waves are 180° out of phase, the result is destructive interference. (c) When the phase angle lies in the range $0 < \phi < 180°$, the resultant y falls somewhere between that shown in (a) and that shown in (b).

wave coincides with the trough of the second (Fig. 14.1b) and their displacements cancel at every point. Finally, when the phase constant has an arbitrary value between 0 and π, as in Figure 14.1c, the resultant wave has an amplitude the value of which is somewhere between 0 and $2A$.

Interference of Waves

One simple device for demonstrating interference of sound waves is illustrated in Figure 14.2. Sound from speaker S is sent into a tube at P, at which point there is a T-shaped junction. Half the sound power travels in one direction, and half in the opposite direction. Thus, the sound waves that reach receiver R at the other side can travel along either of two paths. The total distance from speaker to receiver is called the **path length,** r. The length of the lower path is fixed at r_1. The upper path length, r_2, can be varied by sliding the U-shaped tube (similar to that on a slide trombone). When the difference in the path lengths, $\Delta r = |r_2 - r_1|$, is either zero or some integral multiple of the wavelength λ, the two waves reaching the receiver are in phase and interfere constructively, as in Figure 14.1a. In this case, a maximum

INTEL-1013
U.S. Patent 7,151,430

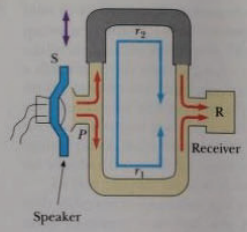


Figure 14.2 An acoustical system for demonstrating interference of sound waves. Sound from the speaker propagates into a tube and splits into two parts at *P*. The two waves, which superimpose at the opposite side, are detected at R. The upper path length r_2 can be varied by the sliding section.

in the sound intensity is detected at the receiver. If path length r_2 is adjusted so that Δr is $\lambda/2$, $3\lambda/2$, ... $n\lambda/2$ (for n odd), the two waves are exactly 180° out of phase at the receiver and hence cancel each other. In this case of completely destructive interference, no sound is detected at the receiver. This simple experiment is a striking illustration of interference. In addition, it demonstrates the fact that a phase difference may arise between two waves generated by the same source when they travel along paths of unequal lengths.

It is often useful to express the path difference in terms of the phase difference, ϕ, between the two waves. Because a path difference of one wavelength corresponds to a phase difference of 2π radians, we obtain the ratio $\lambda/2\pi = \Delta r/\phi$, or

Relationship between path difference and phase angle •

$$\Delta r = \frac{\lambda}{2\pi}\,\phi \qquad [14.2]$$

Nature provides many other examples of interference phenomena. Later in the book we shall describe several interesting interference effects involving light waves.

Thinking Physics 1

If stereo speakers are connected to the amplifier "out of phase," one speaker is moving outward when the other is moving inward. This results in a weakness in the bass notes, which can be corrected by reversing the wires on one of the speaker connections. Why is there only an effect on the bass notes in this case and not the treble notes?

Reasoning Imagine that you are sitting in front of the speakers, midway between them. Then, the sound from each speaker travels the same distance to you, and there is no phase difference in the sound that is due to a path difference. Because the speakers are connected out of phase, the sound waves are half a wavelength out of phase on leaving the speaker and, consequently, on arriving at your ear. As a result, there will be cancellation of the sound for all frequencies in the ideal case of a zero-size head located exactly on the midpoint between the speakers. If the ideal head were moved off the centerline, there will be an additional phase difference introduced by the path-length difference for the sound from the two speakers. In the case of low frequency, long wavelength bass notes, the path-length differences will be a small fraction of a wavelength, so there will still be significant cancellation. For the high-frequency, short-wavelength treble notes, a small movement of the ideal head will result in a much larger fraction of a wavelength in path length difference or even multiple wavelengths. Thus, the treble notes could be in phase with this head movement. If we now add the fact that the head is not of zero size and the fact that there are two ears, we can see that the complete cancellation is not possible, and, with even small movements of the head, one or both ears will be at or near maxima for the treble notes. The size of the head is much smaller than bass wavelengths, however, so there is significant weakening of the bass over much of the region in front of the speakers.

CONCEPTUAL PROBLEM 1

As oppositely moving pulses of the same shape (one upward, one downward) on a string pass through each other, there is one instant at which the string shows no displacement from the equilibrium position at any point. Has the energy carried by the pulses disappeared at this instant in time? If not, where is it?

Example 14.1 Two Speakers Driven by the Same Source

A pair of speakers placed 3.00 m apart are driven by the same oscillator (Fig. 14.3). A listener is originally at point *O*, which is located 8.00 m from the center of the line connecting the two speakers. The listener then walks to point *P*, which is a perpendicular distance 0.350 m from *O* before reaching the *first minimum* in sound intensity. What is the frequency of the oscillator?

Solution The first minimum occurs when the two waves reaching the listener at *P* are 180° out of phase—in other words, when their path difference equals $\lambda/2$. In order to calculate the path difference, we must first find the path lengths r_1 and r_2. Making use of the two shaded triangles in Figure 14.3, we find the path lengths to be

$$r_1 = \sqrt{(8.00\text{ m})^2 + (1.15\text{ m})^2} = 8.08\text{ m}$$

$$r_2 = \sqrt{(8.00\text{ m})^2 + (1.85\text{ m})^2} = 8.21\text{ m}$$

Hence, the path difference is $r_2 - r_1 = 0.13$ m. Because we require that this path difference be equal to $\lambda/2$ for the first minimum, we find that $\lambda = 0.26$ m.

To obtain the oscillator frequency, we can use $v = \lambda f$, where v is the speed of sound in air, 343 m/s:

$$f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{0.26\text{ m}} = 1.3\text{ kHz}$$

EXERCISE 1 If the oscillator frequency is adjusted such that the listener hears the first minimum at a distance of 0.75 m from *O*, what is the new frequency? Answer 0.63 kHz

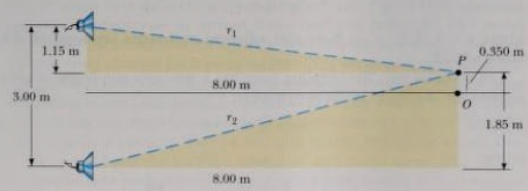


Figure 14.3 (Example 14.1)

14.2 • STANDING WAVES

If a stretched string is clamped at both ends, waves traveling in both directions are reflected from the ends. The incident and reflected waves combine according to the superposition principle.

INTEL-1013
U.S. Patent 7,151,430

Consider two sinusoidal waves, in the same medium, that have the same amplitude, frequency, and wavelength but are traveling in *opposite* directions. If we neglect any phase angle ϕ for simplicity, their wave functions can be written

$$y_1 = A\sin(kx - \omega t) \quad \text{and} \quad y_2 = A\sin(kx + \omega t)$$

where y_1 represents a wave traveling to the right and y_2 represents a wave traveling to the left. Adding these two functions gives the resultant wave function, y:

$$y = y_1 + y_2 = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

where $k = 2\pi/\lambda$ and $\omega = 2\pi f$, as usual. Using the trigonometric identity $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$, this reduces to

Wave function for a standing wave

$$y = (2A\sin kx)\cos \omega t \qquad [14.3]$$

This expression represents the wave function of a **standing wave.** From this result, we see that a standing wave has an angular frequency of ω and an amplitude of $2A\sin kx$ (the quantity in the parentheses of Eq. 14.3). That is, every particle of the string vibrates in simple harmonic motion with the same frequency. However, the amplitude of motion of a given particle depends on x. This is in contrast to the situation involving a traveling sinusoidal wave, in which all particles oscillate with the same amplitude as well as the same frequency.

Because the amplitude of the standing wave at any value of x is equal to $2A\sin kx$, we see that the *maximum* amplitude has the value $2A$. This occurs when the coordinate x satisfies the condition $\sin kx = 1$, or when

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

Because $k = 2\pi/\lambda$, the positions of maximum amplitude, called **antinodes,** are

Position of antinodes

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots = \frac{n\lambda}{4} \qquad [14.4]$$

where $n = 1, 3, 5, \ldots$. Note that **adjacent antinodes are separated by a distance of $\lambda/2$.**

In a similar way, the standing wave has a *minimum* amplitude of zero when x satisfies the condition $\sin kx = 0$ or when

$$kx = \pi, 2\pi, 3\pi, \ldots$$

giving

Position of nodes

$$x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots = \frac{n\lambda}{2} \qquad [14.5]$$

where $n = 1, 2, 3, \ldots$. **These points of zero amplitude, called nodes, are also spaced by $\lambda/2$.** The distance between a node and an adjacent antinode is $\lambda/4$.

The standing wave patterns produced at various times by two waves traveling in opposite directions are depicted graphically in Figure 14.4. The upper part of each figure represents the individual traveling waves, and the lower part represents the standing wave patterns. The nodes of the standing wave are labeled *N*, and the antinodes are labeled *A*. At $t = 0$ (Fig. 14.4a), the two waves are spatially identical,

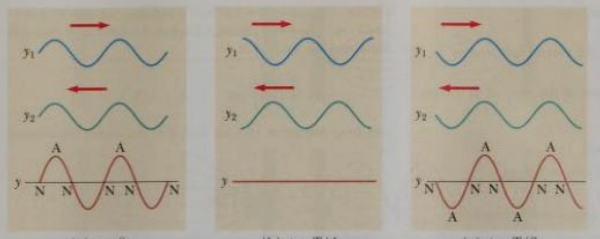


Figure 14.4 Standing wave patterns at various times produced by two waves of equal amplitude traveling in opposite directions. For the resultant wave y, the nodes (N) are points of zero displacement and the antinodes (A) are points of maximum displacement.

giving a standing wave of maximum amplitude, $2A$. One quarter of a period later, at $t = T/4$ (Fig. 14.4b), the individual waves have moved one quarter of a wavelength (one to the right and the other to the left). At this time, the individual displacements are equal and opposite for all values of x, and hence the resultant wave has zero displacement everywhere. At $t = T/2$ (Fig. 14.4c), the individual waves are again identical spatially, producing a standing wave pattern that is inverted relative to the $t = 0$ pattern.

It is instructive to describe the energy associated with the motion of a standing wave. To illustrate this point, consider a standing wave formed on a stretched string that is fixed at each end, as in Figure 14.5. Except for the nodes, which are stationary, all points on the string oscillate vertically with the same frequency. Furthermore, different points have different amplitudes of motion. Figure 14.5 represents snapshots of the standing wave at various times over one half-cycle. The nodes represent points on the string that are always at rest, and the locations of the maxima and minima never change. Each point on the string executes simple harmonic motion in the vertical direction. That is, one can view the standing wave as a large number of oscillators vibrating parallel to each other. The energy of the vibrating string continually alternates between elastic potential energy, at which time the string is momentarily stationary (Fig. 14.5a), and kinetic energy, at which time the string is horizontal and the particles have their maximum speed (Fig. 14.5c). The string particles have both potential energy and kinetic energy at intermediate times (Figs. 14.5b and 14.5d).

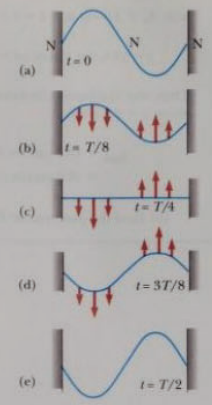


Figure 14.5 A standing wave pattern in a taut string showing snapshots during one half-cycle. (a) At $t = 0$, the string is momentarily at rest, and so $K = 0$ and all of the energy is potential energy U associated with the vertical displacements of the string segments. (b) At $t = T/8$, the string is in motion, and the energy is half kinetic and half potential. (c) At $t = T/4$, the string is horizontal (undeformed) and, therefore, $U = 0$; all of the energy is kinetic. The motion continues as indicated in (d) and (e), and ultimately the initial configuration in part (a) is repeated.

Example 14.2 Formation of a Standing Wave

Two waves traveling in opposite directions produce a standing wave. The individual wave functions are

$$y_1 = (4.0\text{ cm})\sin(3.0x - 2.0t)$$
$$y_2 = (4.0\text{ cm})\sin(3.0x + 2.0t)$$

where x and y are in centimeters. (a) Find the maximum displacement of the motion at $x = 2.3$ cm.

Solution When the two waves are summed, the result is a standing wave the function of which is given by Equation 14.3,

INTEL-1013
U.S. Patent 7,151,430

with $A_0 = 4.0$ cm and $k = 3.0$ rad/cm:

$$y = (2A_0 \sin kx)\cos \omega t = [(8.0 \text{ cm})\sin 3.0x]\cos \omega t$$

Thus, the maximum displacement of the motion at the position $x = 2.3$ cm is

$$y_{max} = (8.0 \text{ cm})\sin 3.0x]_{x=2.3}$$
$$= (8.0 \text{ cm})\sin(6.9 \text{ rad}) = 4.6 \text{ cm}$$

(b) Find the positions of the nodes and antinodes.

Solution Because $k = 2\pi/\lambda = 3$ rad/cm, we see that $\lambda = 2\pi/3$ cm. Therefore, from Equation 14.4 we find that the antinodes are located at

$$x = n\left(\frac{\pi}{6}\right) \text{ cm} \qquad (n = 1, 3, 5, \ldots)$$

and from Equation 14.5 we find that the nodes are located at

$$x = n\frac{\lambda}{2} = n\left(\frac{\pi}{3}\right) \text{ cm} \qquad (n = 1, 2, 3, \ldots)$$

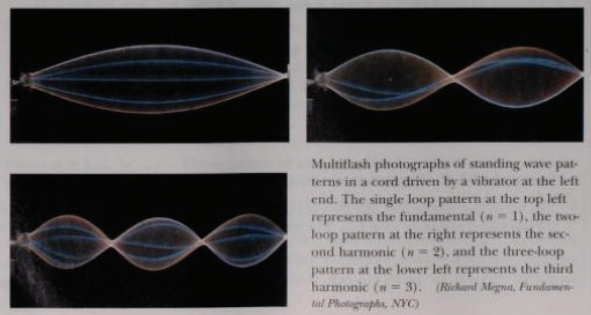
14.3 · NATURAL FREQUENCIES IN A STRETCHED STRING

Consider a string that has a length of L and is fixed at both ends, as in Figure 14.6. Standing waves are set up in the string by a continuous superposition of waves incident on and reflected from the ends. The string has a number of natural patterns of vibration, called **normal modes.** Each of these modes has a characteristic frequency; the frequencies are easily calculated.

First, note that the ends of the string must be nodes, because these points are fixed. If the string is displaced at its midpoint and released, all modes that do not have a node at the center point are excited. Consider the normal mode in which the center of the string is an antinode (Fig. 14.6b). For this normal mode, the length of the string equals $\lambda/2$ (the distance between nodes):

$$L = \lambda_1/2 \qquad \text{or} \qquad \lambda_1 = 2L$$

The next normal mode, of wavelength λ_2 (Fig. 14.6c), occurs when the length of the string equals one wavelength, that is, when $\lambda_2 = L$. The third normal mode (Fig. 14.6d) corresponds to the case in which the length equals $3\lambda/2$; therefore, $\lambda_3 = 2L/3$. In general, the wavelengths of the various normal modes can be conveniently expressed as



Multiflash photographs of standing wave patterns in a cord driven by a vibrator at the left end. The single loop pattern at the top left represents the fundamental ($n = 1$), the two-loop pattern at the right represents the second harmonic ($n = 2$), and the three-loop pattern at the lower left represents the third harmonic ($n = 3$). *(Richard Megna, Fundamental Photographs, NYC)*

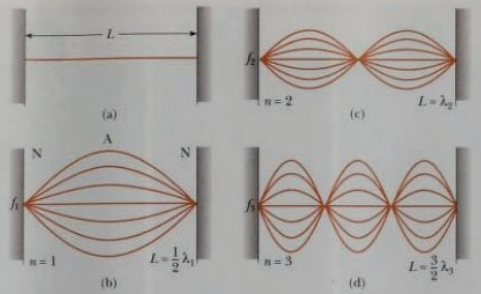


Figure 14.6 (a) A string of length L fixed at both ends. The normal modes of vibration form a harmonic series: (b) the fundamental frequency, or first harmonic; (c) the second harmonic; and (d) the third harmonic.

$$\lambda_n = \frac{2L}{n} \qquad (n = 1, 2, 3, \ldots) \qquad [14.6]$$

• *Wavelengths of normal modes*

where the index n refers to the nth mode of vibration. The natural frequencies associated with these modes are obtained from the relationship $f = v/\lambda$, where **the wave speed, v, is the same for all frequencies.** Using Equation 14.6, we find that the frequencies of the normal modes are

$$f_n = \frac{v}{\lambda_n} = \frac{n}{2L}v \qquad (n = 1, 2, 3, \ldots) \qquad [14.7]$$

• *Frequencies of normal modes as functions of wave speed and length of string*

Because $v = \sqrt{F/\mu}$ (Eq. 13.19), where F is the tension in the string and μ is its mass per unit length, we can express the natural frequencies of a stretched string (sometimes called *harmonics*) as

$$f_n = \frac{n}{2L}\sqrt{\frac{F}{\mu}} \qquad (n = 1, 2, 3, \ldots) \qquad [14.8]$$

• *Frequencies of normal modes as functions of string tension and linear mass density*

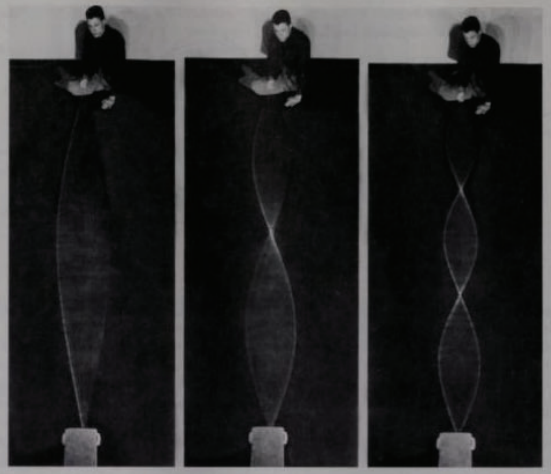
The lowest frequency, corresponding to $n = 1$, is called the **fundamental frequency,** f_1, and is

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}} \qquad [14.9]$$

• *Fundamental frequency of a taut string*

It is clear that the frequencies of the remaining modes are integral multiples of the fundamental frequency—that is, $2f_1$, $3f_1$, $4f_1$, and so on. These higher natural frequencies, together with the fundamental frequency, form a **harmonic series.** The fundamental, f_1, is the first harmonic; the frequency $f_2 = 2f_1$ is the second harmonic; the frequency f_n is the nth harmonic.

INTEL-1013
U.S. Patent 7,151,430



Photographs of standing waves. As one end of the rope is moved from side to side with increasing frequency, patterns with more and more loops are formed; only certain definite frequencies will produce fixed patterns. *(Photos, Education Development Center, Newton, Mass.)*

We can obtain the foregoing results in an alternative manner. Because we require that the string be fixed at $x = 0$ and $x = L$, the wave function $y(x, t)$ given by Equation 14.3 must be *zero* at these points for *all* times. That is, the boundary conditions require that $y(0, t) = 0$ and $y(L, t) = 0$ for all values of t. Because $y = (2A \sin kx)\cos \omega t$, the first condition, $y(0, t) = 0$, is automatically satisfied because $\sin kx = 0$ at $x = 0$. To meet the second condition, $y(L, t) = 0$, we require that $\sin kL = 0$. This condition is satisfied when the angle kL equals an integral multiple of π (180°). Therefore, the allowed values of k are[1]

$$k_n L = n\pi \qquad (n = 1, 2, 3, \ldots) \qquad \textbf{[14.10]}$$

Because $k_n = 2\pi/\lambda_n$, we find that

$$\left(\frac{2\pi}{\lambda_n}\right) L = n\pi \qquad \text{or} \qquad \lambda_n = \frac{2L}{n}$$

which is identical to Equation 14.6.

When a stretched string is distorted to a shape that corresponds to any one of its harmonics, after being released it will vibrate at the frequency of that harmonic. However, if the string is plucked or bowed, the resulting vibration will include frequencies of various harmonics, including the fundamental. In effect, the string

[1]We exclude $n = 0$, because this corresponds to the trivial case in which no wave exists ($k = 0$).

"selects" the normal-mode frequencies when disturbed by a nonharmonic disturbance (for example, a finger plucking it).

Figure 14.7 shows a stretched string vibrating with its first and second harmonics simultaneously. In this figure, the combined vibration is the superposition of the two vibrations shown in Figures 14.6b and 14.6c. The larger loop corresponds to the fundamental frequency of vibration, f_1, and the smaller loops correspond to the second harmonic, f_2. In general, the resulting motion, or displacement, of the string can be described by a superposition of the various harmonic wave functions, with different frequencies and amplitudes. Hence, the sound that one hears corresponds to a complex waveform associated with these modes of vibration. (We shall return to this point in Section 14.6.)

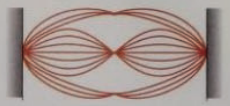


Figure 14.7 Multiple exposures of a string vibrating simultaneously in its first and second harmonics.

The frequency of a stringed instrument can be changed either by varying the strings' tension, F, or by changing their length, L. For example, the tension in the strings of guitars and violins is adjusted by a screw mechanism or by turning pegs on the neck of the instrument. As the tension increases, the frequencies of the normal modes increase according to Equation 14.8. Once the instrument is "tuned," the player varies the frequency by moving his or her fingers along the neck, thereby changing the length of the vibrating portion of the string. As this length is reduced, the frequency increases, because the normal-mode frequencies are inversely proportional to (vibrating) string length.

CONCEPTUAL PROBLEM 2

Guitarists sometimes play a "harmonic," by lightly touching a string at the exact center and plucking the string. The result is a clear note one octave higher than the fundamental of the string, even though the string is not pressed to the fingerboard. Why does this happen?

CONCEPTUAL PROBLEM 3

An archer shoots an arrow from a bow. Does the string of the bow exhibit standing waves after the arrow leaves? If so, and if the bow is perfectly symmetric so that the arrow leaves from the center of the string, what harmonics are excited?

Example 14.3 Give Me a C Note

A middle C string of the C-major scale on a piano has a fundamental frequency of 262 Hz, and the A note has a fundamental frequency of 440 Hz. (a) Calculate the frequencies of the next two harmonics of the C string.

Solution Because $f_1 = 262$ Hz, we can use Equations 14.8 and 14.9 to find the frequencies f_2 and f_3:

$$f_2 = 2f_1 = 524 \text{ Hz}$$

$$f_3 = 3f_1 = 786 \text{ Hz}$$

(b) If the strings for the A and C notes are assumed to have the same mass per unit length and the same length, determine the ratio of tensions in the two strings.

Solution Using Equation 14.8 for the two strings vibrating at their fundamental frequencies gives

$$f_{1A} = \frac{1}{2L}\sqrt{F_A/\mu} \qquad \text{and} \qquad f_{1C} = \frac{1}{2L}\sqrt{F_C/\mu}$$

$$f_{1A}/f_{1C} = \sqrt{F_A/F_C}$$

$$F_A/F_C = (f_{1A}/f_{1C})^2 = (440/262)^2 = 2.82$$

(c) In a real piano, the assumption we made in (b) is only half true. The string densities are equal, but the A string is 64% as long as the C string. What is the ratio of their tensions?

$$f_{1A}/f_{1C} = (L_C/L_A)\sqrt{F_A/F_C} = (100/64)\sqrt{F_A/F_C}$$

$$F_A/F_C = (0.64)^2(440/262)^2 = 1.16$$

INTEL-1013
U.S. Patent 7,151,430

EXERCISE 2 A stretched string is 160 cm long and has a linear density of 0.0150 g/cm. What tension in the string will result in a second harmonic of 460 Hz? Answer 813 N

EXERCISE 3 A string of linear density 1.0×10^{-3} kg/m and length 3.0 m is stretched between two points. One end is vibrated transversely at 200 Hz. What tension in the string will establish a standing-wave pattern with three loops along the string's length?
Answer 160 N

14.4 • STANDING WAVES IN AIR COLUMNS

Standing longitudinal waves can be set up in a tube of air, such as an organ pipe, as the result of interference between longitudinal waves traveling in opposite directions. The phase relationship between the incident wave and the wave reflected from one end depends on whether that end is open or closed. This is analogous to the phase relationships between incident and reflected transverse waves at the ends of a string. **The closed end of an air column is a displacement node,** just as the fixed end of a vibrating string is a displacement node. As a result, the reflected wave at a closed end of a tube of air is 180° out of phase with the incident wave. Furthermore, because the pressure wave is 90° out of phase with the displacement wave (Section 13.9), **the closed end of an air column corresponds to a pressure antinode** (that is, a point of maximum pressure variation).

If the end of an air column is open to the atmosphere, the air molecules have complete freedom of motion. The wave reflected from an open end is nearly in phase with the incident wave when the tube's diameter is small relative to the wavelength of the sound. As a consequence, **the open end of an air column is approximately a displacement antinode and a pressure node.**

Strictly speaking, the open end of an air column is not exactly an antinode. A condensation does not reach full expansion until it passes somewhat beyond an open end. For a thin-walled tube of circular cross-section, this correction that must be added for each open end is about $0.6R$, where R is the tube's radius. Hence, the effective length of the tube is somewhat greater than the true length, L.

The first three modes of vibration of a pipe that is open at both ends are shown in Figure 14.8a. Note that in this figure, the standing longitudinal waves are drawn as transverse waves because it is difficult to draw longitudinal displacements. When air is directed into the pipe from the left, longitudinal standing waves are formed and the pipe resonates at its natural frequencies. All modes of vibration are excited simultaneously (although not with the same amplitude). Note that the ends are displacement antinodes (approximately). In the fundamental mode, the wavelength is twice the length of the pipe, and hence the frequency of the fundamental, f_1, is $v/2L$. In a similar way, the frequencies of the higher harmonics are $2f_1$, $3f_1$, ... Thus, in a pipe that is open at both ends, the natural frequencies of vibration form a harmonic series; that is, the higher harmonics are integral multiples of the fundamental frequency. Because all harmonics are present, we can express the natural frequencies of vibration as

Natural frequencies of a pipe open at both ends •

$$f_n = n\frac{v}{2L} \qquad (n = 1, 2, 3, \ldots) \qquad [14.11]$$

where v is the speed of sound in air.

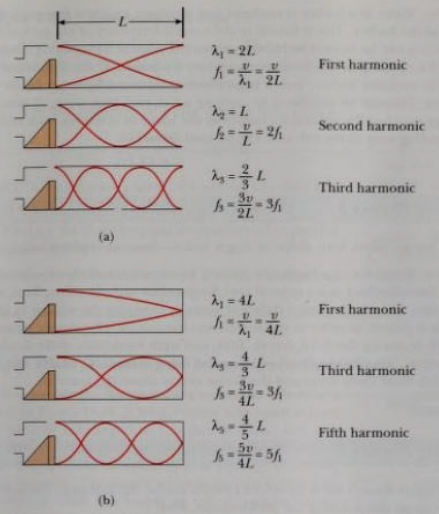


Figure 14.8 (a) Standing longitudinal waves in an organ pipe open at both ends. The natural frequencies that form a harmonic series are f_1, $2f_1$, $3f_1$, ... (b) Standing longitudinal waves in an organ pipe closed at one end. Only the *odd* harmonics are present, and so the natural frequencies are f_1, $3f_1$, $5f_1$, ...

If a pipe is closed at one end and open at the other, the closed end is a displacement node (Fig. 14.8b). In this case, the wavelength for the fundamental mode is four times the length of the tube. Hence, the fundamental, f_1, is equal to $v/4L$, and the frequencies of the higher harmonics are equal to $3f_1$, $5f_1$, ... That is, **in a pipe that is closed at one end, only odd harmonics are present,** and these are

$$f_n = n\frac{v}{4L} \qquad (n = 1, 3, 5, \ldots) \qquad [14.12]$$

Natural frequencies of a pipe closed at one end and open at the other •

Standing waves in air columns are the primary sources of the sounds produced by wind instruments.

Thinking Physics 2

Passing ocean waves sometimes cause the water in a harbor to undergo very large oscillations, called *seiches*. Why would this happen?

INTEL-1013
U.S. Patent 7,151,430

Reasoning Water in a harbor is enclosed and possesses a natural frequency based on the size of the harbor. This is similar to the natural frequency of the enclosed air in a bottle, which can be excited by blowing across the edge of the opening. Ocean waves pass by the opening of the harbor at a certain frequency. If this frequency matches that of the enclosed harbor, then a large standing wave can be set up in the water by resonance. This can be simulated by carrying a fish tank with water. If your walking frequency matches the natural frequency of the water as it sloshes back and forth, a large standing wave in the fish tank can be established.

Thinking Physics 3

A bugle has no valves, keys, slides, or finger holes—how can it play a song?

Reasoning Songs for the bugle are limited to harmonics of the fundamental frequency, because there is no control over frequencies without valves, keys, slides, or finger holes. The player obtains different notes by changing the tension in the lips as the bugle is played, in order to excite different harmonics. The normal playing range of a bugle is among the third, fourth, fifth, and sixth harmonics of the fundamental. For example, "Reveille" is played with just the three notes G, C, and E, and "Taps" is played with these three notes and the G one octave above the lower G.

Thinking Physics 4

If an orchestra doesn't warm up before a performance, the strings go flat and the wind instruments go sharp during the performance. Why?

Reasoning Without warming up, all the instruments will be at room temperature at the beginning of the concert. As the wind instruments are played, they fill with warm air from the player's exhalation. The increase in temperature of the air in the instrument causes an increase in the speed of sound, which raises the resonance frequencies of the air columns. As a result, the instruments go sharp. The strings on the stringed instruments also increase in temperature due to the friction of rubbing with the bow. This results in thermal expansion, which causes a decrease in the tension in the strings. With a decrease in tension, the wave speed on the strings drops, and the fundamental frequencies decrease. Thus, the stringed instruments go flat.

CONCEPTUAL PROBLEM 4

In Balboa Park in San Diego, California, there is an outdoor organ. Does the fundamental frequency of a particular pipe on this organ change on hot and cold days? How about on days with high and low atmospheric pressure?

CONCEPTUAL PROBLEM 5

If you have a series of identical glass bottles, with varying amounts of water in them, you can play musical notes by either striking the bottles with a spoon, or blowing across the open tops of the bottles. When hitting the bottles, the frequency of the note decreases as the water level rises. When blowing on the bottles, the frequency of the note increases as the water level rises. Why is the behavior of the frequency different in these two cases?

Example 14.4 Resonance in a Pipe

A pipe has a length of 1.23 m. (a) Determine the frequencies of the first three harmonics if the pipe is open at each end. Take $v = 343$ m/s as the speed of sound in air.

Solution The first harmonic of a pipe open at both ends is

$$f_1 = \frac{v}{2L} = \frac{343 \text{ m/s}}{2(1.23 \text{ m})} = 139 \text{ Hz}$$

Because all harmonics are present, the second and third harmonics are $f_2 = 2f_1 = 278$ Hz and $f_3 = 3f_1 = 417$ Hz.

(b) What are the three frequencies determined in part (a) if the pipe is closed at one end?

Solution The fundamental frequency of a pipe closed at one end is

$$f_1 = \frac{v}{4L} = \frac{343 \text{ m/s}}{4(1.23 \text{ m})} = 69.7 \text{ Hz}$$

In this case, only odd harmonics are present, and so the next two resonances have frequencies $f_3 = 3f_1 = 209$ Hz and $f_5 = 5f_1 = 349$ Hz.

(c) For the pipe open at both ends, how many harmonics are present in the normal human hearing range (20 to 20 000 Hz)?

Solution Because all harmonics are present, $f_n = nf_1$. For $f_n = 20\,000$ Hz, we have $n = 20\,000/139 = 144$, so that 144 harmonics are present in the audible range. Actually, only the first few harmonics have sufficient amplitude to be heard.

Example 14.5 Measuring the Frequency of a Tuning Fork

A simple apparatus for demonstrating resonance in a tube is described in Figure 14.9a. A long, vertical tube open at both ends is partially submerged in a beaker of water, and a vibrating tuning fork of unknown frequency is placed near the top. The length of the air column, L, is adjusted by moving the tube vertically. The sound waves generated by the fork are reinforced when the length of the air column corresponds to one of the resonant frequencies of the tube.

For a certain tube, the smallest value of L for which a peak occurs in the sound intensity is 9.00 cm. From this measurement, determine the frequency of the tuning fork and the value of L for the next two resonant modes.

Reasoning Although the tube is open at both ends to allow the water in, the water surface acts like a wall at one end of the length L. Therefore this setup represents a pipe closed at one end and the fundamental has a frequency of $v/4L$ (Fig. 14.9b).

Solution Taking $v = 343$ m/s for the speed of sound in air and $L = 0.0900$ m, we get

$$f_1 = \frac{v}{4L} = \frac{343 \text{ m/s}}{4(0.0900 \text{ m})} = 953 \text{ Hz}$$

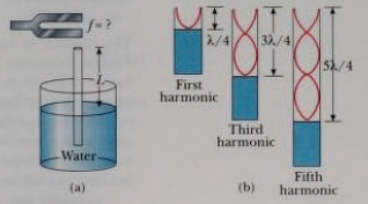


Figure 14.9 (Example 14.5) (a) Apparatus for demonstrating the resonance of sound waves in a tube closed at one end. The length L of the air column is varied by moving the tube vertically while it is partially submerged in water. (b) The first three normal modes of the system shown in part (a).

From this information about the fundamental mode, we see that the wavelength is $\lambda = 4L = 0.360$ m. Because the frequency of the source is constant, the next two resonance modes (Fig. 14.9b) correspond to lengths of $3\lambda/4 = 0.270$ m and $5\lambda/4 = 0.450$ m.

EXERCISE 4 The longest pipe on a certain organ is 4.88 m. What is the fundamental frequency (at 0.0°C) if the nondriven end of the pipe is (a) closed and (b) open?
Answer (a) 17.0 Hz (b) 34.0 Hz

EXERCISE 5 An air column 2.00 m in length is open at both ends. The frequency of a certain harmonic is 410 Hz, and the frequency of the next higher harmonic is 492 Hz. Determine the speed of sound in the air column. Answer 328 m/s

INTEL-1013
U.S. Patent 7,151,430

OPTIONAL

14.5 • BEATS: INTERFERENCE IN TIME

The interference phenomena with which we have been dealing so far involve the superposition of two or more waves with the same frequency, traveling in opposite directions. Because the resultant waveform in this case depends on the coordinates of the disturbed medium, we can refer to the phenomenon as *spatial interference.* Standing waves in strings and pipes are common examples of spatial interference.

We now consider another type of interference effect, one that results from the superposition of two waves with slightly *different frequencies.* In this case, when the two waves are observed at a given point, they are periodically in and out of phase. That is, there occurs a temporal alternation between constructive and destructive interference. We refer to this phenomenon as **interference in time** or **temporal interference.**

For example, if two tuning forks of slightly different frequencies are struck, one hears a sound of pulsating intensity, called a **beat.**

Definition of beats •

> A **beat** is the periodic variation in intensity at a given point due to the superposition of two waves with slightly different frequencies.

The number of beats one hears per second, or **beat frequency,** equals the difference in frequency between the two sources. The maximum beat frequency that the human ear can detect is about 20 beats/s. When the beat frequency exceeds this value, it blends indistinguishably with the compound sounds producing the beats.

One can use beats to tune a stringed instrument, such as a piano, by beating a note against a reference tone of known frequency. The string can then be adjusted to equal the frequency of the reference by tightening or loosening it until the beats become too infrequent to notice.

Consider two waves with equal amplitudes, traveling through a medium with slightly different frequencies, f_1 and f_2. We can represent the displacement that each wave would produce at a point as

$$y_1 = A \cos 2\pi f_1 t \qquad \text{and} \qquad y_2 = A \cos 2\pi f_2 t$$

Using the superposition principle, we find that the resultant displacement at that point is given by

$$y = y_1 + y_2 = A(\cos 2\pi f_1 t + \cos 2\pi f_2 t)$$

It is convenient to write this in a form that uses the trigonometric identity

$$\cos a + \cos b = 2 \cos\left(\frac{a-b}{2}\right) \cos\left(\frac{a+b}{2}\right)$$

Letting $a = 2\pi f_1 t$ and $b = 2\pi f_2 t$, we find that

Resultant of two waves of different frequencies but equal amplitude •

$$y = 2A \cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t \cos 2\pi \left(\frac{f_1 + f_2}{2}\right) t \qquad [14.13]$$

Graphs demonstrating the individual waveforms as well as the resultant wave are shown in Figure 14.10. From the factors in Equation 14.13, we see that the resultant

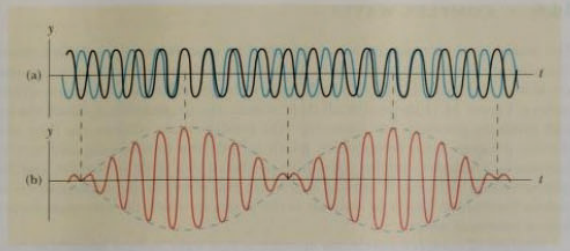


Figure 14.10 Beats are formed by the combination of two waves of slightly different frequencies traveling in the same direction. (a) The individual waves. (b) The combined wave has an amplitude (broken line) that oscillates in time.

vibration at a point has an effective frequency equal to the average frequency, $(f_1 + f_2)/2$, and an amplitude of

$$A = 2A \cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t \qquad [14.14]$$

That is, the *amplitude varies in time* with a frequency of $(f_1 - f_2)/2$. When f_1 is close to f_2, this amplitude variation is slow, as illustrated by the envelope (broken line) of the resultant waveform in Figure 14.10b.

Note that a beat, or a maximum in amplitude, will be detected whenever

$$\cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t = \pm 1$$

That is, there are *two* maxima in each cycle. Because the amplitude varies with frequency as $(f_1 - f_2)/2$, the number of beats per second, or the beat frequency, f_b, is twice this value:

$$f_b = f_1 - f_2 \qquad [14.15]$$

Beat frequency •

For instance, if two tuning forks vibrate individually at frequencies of 438 Hz and 442 Hz, respectively, the resultant sound wave of the combination has a frequency of 440 Hz (the musical note A) and a beat frequency of 4 Hz. That is, the listener hears the 440-Hz sound wave go through an intensity maximum four times every second.

CONCEPTUAL PROBLEM 6

You have a standard tuning fork whose frequency you know and a second tuning fork whose frequency you don't know. When you play them together, you hear a beat frequency of 4 Hz. You know that the frequency of the mystery tuning fork is different from that of the standard fork by 4 Hz, but you can't tell whether it is higher or lower. What could you do to determine this? (*Hint:* You are chewing gum at the time that you perform these measurements.)

INTEL-1013
U.S. Patent 7,151,430

OPTIONAL

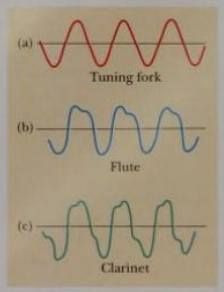


Figure 14.11 Waveform produced by (a) a tuning fork, (b) a flute, and (c) a clarinet, each at approximately the same frequency. *(Adapted from C. A. Culver, Musical Acoustics, 4th ed., New York, McGraw-Hill, 1956, p. 128.)*

14.6 · COMPLEX WAVES

The sound wave patterns produced by most instruments are very complex. Some characteristic waveforms produced by a tuning fork, a harmonic flute, and a clarinet are shown in Figure 14.11. Although each instrument has its own characteristic pattern, Figure 14.11 shows that all three waveforms are periodic. A struck tuning fork produces primarily one harmonic (the fundamental), whereas the flute and clarinet produce many frequencies, which include the fundamental and various harmonics. Thus, the complex waveforms produced by a violin or clarinet, and the corresponding richness of musical tones, are the result of the superposition of various harmonics. This is in contrast to the drum, in which the overtones do not form a harmonic series.

Analysis of complex waveforms appears at first sight to be a formidable task. However, if the waveform is periodic, it can be represented with arbitrary precision by the combination of a sufficiently large number of sinusoidal waves that form a harmonic series. In fact, one can represent any periodic function or any function over a finite interval as a series of sine and cosine terms by using a mathematical technique based on *Fourier's theorem.* The corresponding sum of terms that represents the periodic waveform is called a **Fourier series.**

Let $y(t)$ be any function that is periodic in time, with a period of T, so that $y(t + T) = y(t)$. **Fourier's theorem** states that this function can be written

$$y(t) = \sum_n (A_n \sin 2\pi f_n t + B_n \cos 2\pi f_n t) \qquad [14.16]$$

where the lowest frequency is $f_1 = 1/T$.

The higher frequencies are integral multiples of the fundamental, and so $f_n = nf_1$. The coefficients A_n and B_n represent the amplitudes of the various waves. The amplitude of the nth harmonic is proportional to $\sqrt{A_n^2 + B_n^2}$, and its intensity is proportional to $A_n^2 + B_n^2$.

Figure 14.12 represents a harmonic analysis of the waveforms shown in Figure 14.11. Note the variation of relative intensity with harmonic content for the flute and clarinet. In general, any musical sound contains components that are members of a harmonic set with varying relative intensities.

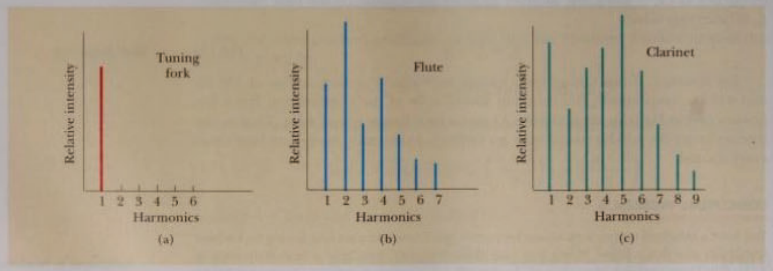


Figure 14.12 Harmonics of the waveforms shown in Figure 14.11. Note the variations in intensity of the various harmonics. *(Adapted from C. A. Culver, Musical Acoustics, 4th ed., New York, McGraw-Hill, 1956.)*

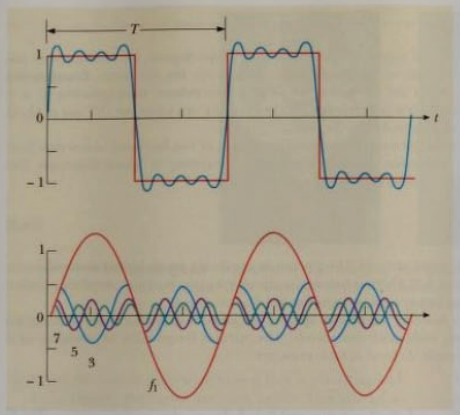


Figure 14.13 Harmonic synthesis of a square wave, which can be represented by the sum of odd harmonics of the fundamental. *(From M. L. Warren, Introductory Physics, New York, W. H. Freeman, 1979, p. 178; by permission of the publisher.)*

As an example of Fourier synthesis, consider the periodic square wave shown in Figure 14.13. The square wave is synthesized by a series of *odd* harmonics of the fundamental. The series contains only sine functions (that is, $B_n = 0$ for all n). Only the first four odd harmonics and their respective amplitudes are shown. One obtains a better fit to the true waveform by adding more harmonics.

Using modern technology, one can produce a variety of musical tones by mixing harmonics with varying amplitudes. Electronic music synthesizers that do this are now widely used.

Thinking Physics 5

A professor performs a demonstration in which he breathes helium and then speaks with a comical voice. One student explains, "The velocity of sound in helium is higher than in air, so the fundamental frequency of the standing waves in the mouth is increased." Another student says, "No, the fundamental frequency is determined by the vocal folds and cannot be changed. Only the *quality* of the voice has changed." Which student is correct?

Reasoning The second student is correct. The fundamental frequency of the complex tone from the voice is determined by the vibration of the vocal folds and is not changed by substituting a different gas in the mouth. The introduction of the helium into the mouth results in higher harmonics being excited more than in the normal voice, but the fundamental frequency of the voice is the same—only the quality has changed. The unusual inclusion of the higher frequency harmonics results in a common description of this effect as a "high-pitched" voice, but this is incorrect.

INTEL-1013
U.S. Patent 7,151,430

SUMMARY

When two waves with equal amplitudes and frequencies superimpose, the resultant wave has an amplitude that depends on the phase angle, ϕ, between the two waves. **Constructive interference** occurs when the two waves are *in phase* everywhere, corresponding to $\phi = 0, 2\pi, 4\pi, \ldots$. **Destructive interference** occurs when the two waves are 180° out of phase everywhere, corresponding to $\phi = \pi, 3\pi, 5\pi, \ldots$.

Standing waves are formed from the superposition of two harmonic waves that have the same frequency, amplitude, and wavelength but are traveling in *opposite* directions. The resultant standing wave is described by the wave function

$$y = (2A \sin kx)\cos \omega t \qquad [14.3]$$

Hence, its amplitude varies as $\sin kx$. The maximum amplitude points (called **antinodes**) are separated by a distance $\lambda/2$. Halfway between antinodes are points of zero amplitude (called **nodes**). The distance between adjacent nodes is $\lambda/2$.

One can set up standing waves with specific frequencies in such systems as stretched strings, hollow pipes, rods, and drumheads. The natural frequencies of vibration of a stretched string of length, L, fixed at both ends, are

$$f_n = \frac{n}{2L}\sqrt{\frac{F}{\mu}} \qquad (n = 1, 2, 3, \ldots) \qquad [14.8]$$

where F is the tension in the string and μ is its mass-per-unit length. The natural frequencies of vibration form a **harmonic series**—that is, $f_1, 2f_1, 3f_1, \ldots$.

The standing wave patterns for longitudinal waves in a hollow pipe depend on whether the ends of the pipe are open or closed. If the pipe is open at both ends, the natural frequencies of vibration form a harmonic series. If one end is closed, only odd harmonics of the fundamental are present.

The phenomenon of **beats** occurs as a result of the superposition of two traveling waves of slightly different frequencies. For sound waves at a given point, one hears an alternation in sound intensity with time. Thus, beats correspond to *interference as time passes*.

Any periodic waveform can be represented by the combination of the sinusoidal waves that form a harmonic series. The process is called *Fourier synthesis* and is based on *Fourier's theorem*.

CONCEPTUAL QUESTIONS

1. When two waves interfere constructively or destructively, is there any gain or loss in energy? Explain.
2. Does the phenomenon of wave interference apply only to sinusoidal waves?
3. Some singers claim to be able to shatter a wine glass by maintaining a certain vocal pitch over a period of several seconds (Fig. Q14.3). What mechanism causes the glass to break? (The glass must be very clean in order to break.)
4. What limits the amplitude of motion of a real vibrating system that is driven at one of its natural frequencies?
5. Explain why all harmonics are present in an organ pipe open at both ends, but only the odd harmonics are present in a pipe closed at one end.
6. An airplane mechanic notices that the sound from a twin-engine aircraft rapidly varies in loudness when both engines are running. What could be causing this variation from loud to soft?
7. Why does a vibrating guitar string sound louder when placed on the instrument than it would if allowed to vibrate in the air while off the instrument?



Figure Q14.3 (Question 3) *(Left)* Standing wave pattern in a vibrating wine glass. The wine glass will shatter if the amplitude of vibration becomes too large. *(Courtesy of Professor Thomas D. Rossing, Northern Illinois University)* *(Right)* A wine glass shattered by the amplified sound of a human voice. *(© Ben Rose 1992/The IMAGE Bank)*

8. When the base of a vibrating tuning fork is placed against a chalkboard, the sound becomes louder. How does this affect the length of time for which the fork vibrates? Does this agree with conservation of energy?
9. To keep animals away from their cars, some people mount short, thin pipes on the fenders. The pipes give out a high-pitched wail when the cars are moving. How do they create the sound?
10. If you wet your fingers and lightly run them around the rim of a fine wine glass, a high-pitched sound is heard. Why? How could you produce various musical notes with a set of wine glasses?
11. When a bell is rung, standing waves are set up around the bell's circumference. What boundary conditions must be satisfied by the resonant wavelengths? How does a crack in the bell, such as in the Liberty Bell, affect the satisfying of the boundary conditions and the sound emanating from the bell?
12. Explain why your voice seems to sound better than usual when you sing in the shower.
13. What is the purpose of the slide on a trombone or the valves on a trumpet?
14. Despite a reasonably steady hand, a certain person often spills his coffee when carrying it to his seat. Discuss resonance as a possible cause of this difficulty and devise a means for solving the problem.

PROBLEMS

Section 14.1 Superposition and Interference of Sinusoidal Waves

1. Two harmonic waves are described by

$$y_1 = (5.00\ \text{m})\sin[\pi(4.00x - 1200t)]$$

$$y_2 = (5.00\ \text{m})\sin[\pi(4.00x - 1200t - 0.250)]$$

where x, y_1, and y_2 are in meters and t is in seconds. (a) What is the amplitude of the resultant wave? (b) What is the frequency of the resultant wave?

2. A sinusoidal wave is described by

$$y_1 = (0.0800\ \text{m})\sin[2\pi(0.100x - 80.0t)]$$

where y_1 and x are in meters and t is in seconds. Write an expression for a wave that has the same frequency, amplitude, and wavelength as y_1 but when added to y_1 gives a resultant with an amplitude of $8\sqrt{3}$ cm.

3. Two waves are traveling in the same direction along a stretched string. Each has an amplitude of 4.00 cm and they are 90.0° out of phase. Find the amplitude of the resultant wave.
4. Two identical sinusoidal waves with wavelengths of 3.00 m travel in the same direction at a speed of 2.00 m/s. The second wave originates from the same point as the first but at a later time. Determine the minimum possible time interval between the starting moments of the two waves if the amplitude of the resultant wave is the same as that of the two initial waves.
5. Two speakers are driven by a common oscillator at 800 Hz and face each other at a distance of 1.25 m. Locate the points along a line joining the two speakers at which relative minima would be expected. (Use $v = 343$ m/s.)
6. A tuning fork generates sound waves with a frequency of 246 Hz. The waves travel in opposite directions along a hall-

INTEL-1013
U.S. Patent 7,151,430

way, are reflected by walls, and return. What is the phase difference between the reflected waves when they meet? The corridor is 47.0 m long and the tuning fork is located 14.0 m from one end. The speed of sound in air is 343 m/s.

Section 14.2 Standing Waves

7. Use the trigonometric identity

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

to show that the resultant of two wave functions each of amplitude A, angular frequency ω, and propagation number k and traveling in opposite directions can be written

$$y = (2A \sin kx) \cos \omega t$$

For simplicity, set all phase angles ϕ to zero.

8. Two sinusoidal waves traveling in opposite directions interfere to produce a standing wave described by

$$y = (1.50 \text{ m}) \sin(0.400x) \cos(200t)$$

where x is in meters and t is in seconds. Determine the wavelength, frequency, and speed of the interfering waves.

9. Two waves in a long string are given by

$$y_1 = (0.0150 \text{ m}) \cos\left(\frac{x}{2} - 40t\right)$$

$$y_2 = (0.0150 \text{ m}) \cos\left(\frac{x}{2} + 40t\right)$$

where the y's and x are in meters and t is in seconds. (a) Determine the positions of the nodes of the resulting standing wave. (b) What is the maximum displacement at the position $x = 0.400$ m?

10. Two waves that set up a standing wave in a long string are given by

$$y_1 = A \sin(kx - \omega t + \phi)$$

$$y_2 = A \sin(kx + \omega t)$$

Show (a) that the addition of the arbitrary phase angle will change only the position of the nodes and (b) that the distance between nodes remains constant.

Section 14.3 Natural Frequencies in a Stretched String

11. A student wants to establish a standing wave on a wire that is 1.80 m long and clamped at both ends. The wave speed is 540 m/s. What is the minimum frequency the student should apply to set up standing waves?

12. A standing wave is established in a 120–cm-long string fixed at both ends. The string vibrates in four segments when driven at 120 Hz. (a) Determine the wavelength. (b) What is the fundamental frequency?

13. A cello A-string vibrates in its fundamental mode with a frequency of 220 vibrations/s. The vibrating segment is 70.0 cm long and has a mass of 1.20 g. (a) Find the tension in the string. (b) Determine the frequency of the harmonic that causes the string to vibrate in three segments.

14. A string of length L, mass-per-unit length μ, and tension F is vibrating at its fundamental frequency. What effect will the following have on the fundamental frequency? (a) The length of the string is doubled, with all other factors held constant. (b) The mass per unit length is doubled, with all other factors held constant. (c) The tension is doubled, with all other factors held constant.

15. A 60.0-cm guitar string under a tension of 50.0 N has a mass per unit length of 0.100 g/cm. What is the highest resonant frequency that can be heard by a person capable of hearing frequencies up to 20 000 Hz?

16. A stretched wire vibrates in its fundamental mode at a frequency of 400 vibrations/s. What would be the fundamental frequency if the wire were half as long, with twice the diameter and with four times the tension?

17. A 2.00–m-long wire having a mass of 0.100 kg is fixed at both ends. The tension in the wire is maintained at 20.0 N. What are the frequencies of the first three allowed modes of vibration? If a node is observed at a point 0.400 m from one end, in what mode and with what frequency is it vibrating?

18. A violin string has a length of 0.350 m and is tuned to concert G, having a frequency $f_G = 392$ Hz. Where must the violinist place her finger to play concert A, having a frequency $f_A = 440$ Hz? If this position is to remain correct to one half the width of a finger (i.e., to within 0.600 cm), the string tension cannot be allowed to slip by more than what fraction?

19. Find the fundamental frequency and the next three frequencies that could cause a standing wave pattern on a string that is 30.0 m long, has a mass per unit length 9.00×10^{-3} kg/m, and is stretched to a tension of 20.0 N.

20. Standing-wave vibrations are set up in a crystal goblet with two nodes and two antinodes equally spaced around the 20.0-cm circumference of its rim. If transverse waves move around the glass at 900 m/s, an opera singer would have to produce a high harmonic with what frequency in order to shatter the glass with a resonant vibration?

Section 14.4 Standing Waves in Air Columns

(In this section, unless otherwise indicated, assume that the speed of sound in air is 343 m/s.)

21. Calculate the length for a pipe that has a fundamental frequency of 240 Hz if the pipe is (a) closed at one end and (b) open at both ends.

22. A glass tube (open at both ends) of length L is positioned near an audio speaker of frequency $f = 0.680$ kHz. For what values of L will the tube resonate with the speaker?

23. The overall length of a piccolo is 32.0 cm. The resonating air column vibrates as a pipe open at both ends. (a) Find the frequency of the lowest note a piccolo can play, assuming the speed of sound in air is 340 m/s. (b) Opening holes in the side effectively shortens the length of the resonant column. If the highest note a piccolo can sound is 4000 Hz, find the distance between adjacent nodes for this mode of vibration.

24. The fundamental frequency of an open organ pipe corresponds to middle C (261.6 Hz on the chromatic musical scale). The third resonance of a closed organ pipe has the same frequency. What are the lengths of the two pipes?

25. A tuning fork the frequency of which is f is used to set up a resonance condition in a pipe. Write an expression for the length that will cause the pipe to resonate in its nth mode if it is (a) open at both ends; (b) closed at one end. (Assume that the speed of sound is v.)

26. Do not stick anything into your ear! Estimate the length of your ear canal, from its opening at the external ear in to the eardrum. If you regard the canal as a tube that is open at one end and closed at the other, at approximately what fundamental frequency would you expect your hearing to be most sensitive? Explain why you can hear especially soft sounds just around this frequency.

27. A shower stall measures 86.0 cm × 86.0 cm × 210 cm. When you sing in the shower, which frequencies will sound the richest (resonate), assuming the shower acts as a pipe closed at both ends (nodes at opposite sides)? Assume also that the human voice ranges from 130 Hz to 2000 Hz (not necessarily one person's voice, however). Let the speed of sound in the hot shower stall be 355 m/s.

28. An open pipe 0.400 m in length is placed vertically in a cylindrical bucket, nearly touching the bottom of the bucket, which has area 0.100 m^2. Water is slowly poured into the bucket until a sounding tuning fork of frequency 440 Hz, held over the pipe, produces resonance. Find the mass of water in the bucket at this moment.

29. Water is pumped into a long cylinder at a rate of 18.0 cm^3/s. The radius of the cylinder is 4.00 cm, and at the open top of the cylinder there is a tuning fork vibrating with a frequency of 200 Hz. As the water rises, how much time elapses between successive resonances?

30. Water is pumped into a long cylinder at a volume flow rate R. The radius of the cylinder is r (cm), and at the open top of the cylinder there is a tuning fork vibrating with a frequency f. As the water rises, how much time elapses between successive resonances?

31. A piece of metal pipe is just the right length so that when it is cut into two pieces, their lowest resonance frequencies are 256 Hz for one and 440 Hz for the other. (a) What resonant frequency would have been produced by the original length of pipe, and (b) how long was the original piece?

32. A piece of cardboard tubing, closed at one end, is just the right length so that when it is cut into two (unequal) pieces, their lowest resonant frequencies are 256 Hz for the piece with the closed end and 440 Hz for the piece with both ends open. (a) What resonant frequency would have been produced by the original cardboard tubing, and (b) how long was the original piece?

Section 14.5 Beats: Interference in Time (Optional)

33. In certain ranges of a piano keyboard, more than one string is tuned to the same note to provide extra loudness. For example, the note at 110 Hz has two strings at this pitch. If one string slips from its normal tension of 600 N to 540 N, what beat frequency will be heard when the two strings are struck simultaneously?

34. A student holds a tuning fork oscillating at 256 Hz. He walks toward a wall at a constant speed of 1.33 m/s. (a) What beat frequency does he observe between the tuning fork and its echo? (b) How fast must he walk away from the wall to observe a beat frequency of 5.00 Hz?

35. A flute is designed so that it plays a frequency of 261.6 Hz, middle C, when all the holes are covered and the temperature is 20.0°C. (a) Consider the flute as a pipe that is open at both ends; find the length of the flute, assuming that middle C is the fundamental. (b) A second player, nearby in a colder room, also attempts to play middle C on an identical flute. A beat frequency of 3.00 Hz is heard. What is the temperature of the room? The speed of sound in air is described by

$$v = (331 \text{ m/s}) \sqrt{1 + \frac{T}{273^\circ}}$$

where T is the Celsius temperature.

Additional Problems

36. Two loudspeakers are placed on a wall, 2.00 m apart. A listener stands directly in front of one of the speakers, 3.00 m from the wall. The speakers are being driven by a single oscillator at a frequency of 300 Hz. (a) What is the phase difference between the two waves when they reach the observer? (b) What is the frequency closest to 300 Hz to which the oscillator may be adjusted such that the observer will hear minimal sound?

37. On a marimba, the wooden bar that sounds a tone when struck vibrates as a transverse standing wave with three antinodes and two nodes. The lowest frequency note is 87.0 Hz, produced by a bar 40.0 cm long. (a) Find the speed of transverse waves on the bar. (b) The loudness and duration of the emitted sound are enhanced by a resonant pipe suspended vertically below the center of the bar. If the pipe is open at the top end only and the speed of sound in air is 340 m/s, what is the length of the pipe required to resonate with the bar in part (a)?

INTEL-1013
U.S. Patent 7,151,430



(Problem 37) Marimba players in Mexico City. *(Murray Greenberg)*

38. A speaker at the front of a room and an identical speaker at the rear of the room are being driven by the same oscillator at 456 Hz. A student walks at a uniform rate of 1.50 m/s along the length of the room. How many beats does the student hear per second?
39. Two train whistles have identical frequencies of 180 Hz. When one train is at rest in the station sounding its whistle, a beat frequency of 2.00 Hz is heard from a moving train. What two possible speeds and directions can the moving train have?
40. Jane waits on a railroad platform, while two trains approach from the same direction at equal speeds of 8.00 m/s. Both trains are blowing their whistles (which have the same frequency), and one train is some distance behind the other. After the first train passes Jane, but before the second train passes her, she hears beats of frequency 4.00 Hz. What is the frequency of the train whistles?
41. A string (mass = 4.80 g, length = 2.00 m, and tension = 48.0 N), fixed at both ends, vibrates in its second ($n = 2$) natural mode. What is the wavelength in air of the sound generated by this vibrating string?
42. A pipe that is open at both ends has a fundamental frequency of 300 Hz when the speed of sound in air is 333 m/s. (a) What is the length of the pipe? (b) What is the frequency of the second harmonic when the temperature of the air is increased so that the speed of sound in the pipe is 344 m/s?
43. A string with a mass of 8.00 g and a length of 5.00 m has one end attached to a wall; the other end is draped over a pulley and attached to a hanging mass of 4.00 kg. If the string is plucked, what is the fundamental frequency of vibration?
44. In a major chord on the physical-pitch musical scale, the frequencies are in the ratios 4:5:6:8. A set of pipes, closed at one end, are to be cut so that when sounded in their fundamental mode, they will sound out a major chord. (a) What is the ratio of the lengths of the pipes? (b) What length pipes are needed if the lowest frequency of the chord is 256 Hz? (c) What are the frequencies of this chord?
45. Two wires are welded together. The wires are of the same material, but one is twice the diameter of the other one. They are subjected to a tension of 4.60 N. The thin wire has a length of 40.0 cm and a linear mass density of 2.00 g/m. The combination is fixed at both ends and vibrated in such a way that two antinodes are present with the node between them being right at the weld. (a) What is the frequency of vibration? (b) How long is the thick wire?
46. Two identical strings, each fixed at both ends, are arranged near each other. If string A starts oscillating in its fundamental mode, it is observed that string B will begin vibrating in its third ($n = 3$) natural mode. Determine the ratio of the tension of string B to the tension of string A.
47. A standing wave is set up in a string of variable length and tension by a vibrator of variable frequency. When the vibrator has a frequency f, using a string of length L and tension F, there are n antinodes set up in the string. (a) If the length of the string is doubled, by what factor should the frequency be changed to get the same number of antinodes? (b) If the frequency and length are held constant, what tension will produce $n + 1$ antinodes? (c) If the frequency is tripled and the length halved, by what factor should the tension be changed to get twice as many antinodes?
48. Radar detects the speed of a car, using the Doppler shift of microwaves that are reflected off the moving car, by beating the received wave with the transmitted wave and measuring the difference. The Doppler shift for microwaves is

$$f = f_0\sqrt{\frac{c+v}{c-v}}$$

where f_0 is the transmitted frequency, c is the speed of light (3×10^8 m/s), and v is the relative speed of the two objects. (a) Show that the wave that reflects back to the source has a frequency

$$f = f_0\frac{(c+v)}{(c-v)}$$

(b) Show that the expression for the beat frequency of the microwaves may be written as $f_b = 2v/\lambda$. (Because the beat frequency is much smaller than the transmitted frequency, use the approximation $f + f_0 = 2f_0$.) (c) What beat frequency is measured for a speed of 30.0 m/s (67 mph) if the microwaves have a frequency of 10.0 GHz? (1 GHz = 10^9 Hz.) (d) If the beat frequency measurement is accurate to $\pm$ 5 Hz, how accurate is the velocity measurement?
49. If two adjacent natural frequencies of an organ pipe are determined to be 0.550 kHz and 0.650 kHz, calculate the fundamental frequency and length of this pipe. (Use v = 340 m/s.)

50. A 0.0100 kg and 2.00-m-long wire is fixed at both ends and vibrates in its simplest mode under tension 200 N. When a tuning fork is placed near the wire, a beat frequency of 5.00 Hz is heard. (a) What is the frequency (frequencies) of the tuning fork? (b) What should the tension in the wire be to make the beats disappear?
51. The wave function for a standing wave is given in Equation 14.3 as $y = 2A \sin kx \cos \omega t$. (a) Rewrite this wave function in terms of the wavelength λ and wavespeed v of the wave. (b) Write the wave function of the simplest standing-wave vibration of a stretched string of length L. (c) Write the wave function for the second harmonic. (d) Generalize these results and write the wave function for the nth resonance vibration.

Spreadsheet Problems

S1. Spreadsheet 14.1 adds two traveling waves at some fixed time t. The resultant wave function is

$$y = y_1 + y_2 = A_1 \sin(k_1 x - \omega_1 t + \phi_1) + A_2 \sin(k_2 x - \omega_2 t + \phi_2)$$

Add two waves traveling in opposite directions having the same wavelengths, the same phases, and the same speeds. (a) Use $A_1 = A_2 = 0.10$ m, $\omega_1 = -\omega_2 = 3.0$ rad/s, $\phi_1 = \phi_2 = 0$, and $k_1 = k_2 = 2.0$ rad/m. View the associated graph. Choose different values of t to see the time evolution of the resultant wave function. Do you get standing waves? (b) Repeat part (a) using $A_1 = 0.10$ m, $A_2 = 0.20$ m.

S2. Use Spreadsheet 14.1 to add two traveling waves that differ only in phase. (a) Choose $A_1 = A_2 = 0.10$ m, $\omega_1 = \omega_2 = 2.5$ rad/s, $k_1 = k_2 = 1.0$ rad/m, $\phi_1 = 0$, and $\phi_2 = 0$, $\pi/8$, $\pi/4$, $\pi/2$, and π. View the associated graphs. For which values of ϕ_2 do you get constructive interference? Destructive interference? (b) Repeat part (a) using $A_2 = 0.20$ m.

S3. Use Spreadsheet 14.1 to add two traveling waves with different wavelengths. (a) Choose $A_1 = A_2 = 0.10$ m, $\omega_1 = \omega_2 = 3.0$ rad/s, $k_1 = 2.0$ rad/m, $k_2 = 1.0$ rad/m, and $\phi_1 = \phi_2 = 0$. View the associated graph. (b) Repeat part (a) with $k_2 = 3, 4, 5, 6, 7, 8, 9$, and 10. Examine the graph for each case. Explain the appearance of the graphs.

S4. Three waves with the same frequency, wavelength, and amplitude are traveling in the same direction. Each differs in phase such that $(\phi_2 - \phi_1) = (\phi_3 - \phi_2) = \Delta\phi$. Modify Spreadsheet 14.1 to add the three waves at some fixed time t. Calculate the resultant wave function at t = 0.0, 1.0 s, and 2.0 s. Use $A_1 = A_2 = A_3 = 0.050$ m, $\omega_1 = \omega_2 = \omega_3 = 2.5$ rad/s, and $k_1 = k_2 = k_3 = 1.0$ rad/m. Choose $\Delta\phi = \pi/6$. View the associated graph. Repeat for different values of $\Delta\phi$.

S5. Write a spreadsheet or computer program to add two traveling waves of different frequencies:

$$y_1 = y_0 \cos 2\pi f_1 t \qquad y_2 = y_0 \cos 2\pi f_2 t$$

If the two frequencies are close to each other, beats will be produced. Does your program show the beats? From your numerical results, how does the beat frequency relate to the frequencies of the two waves?

S6. The Fourier theorem states that any periodic wave of frequency f, no matter how complicated, can be expressed as a sum of even or odd harmonic functions. That is,

$$y(t) = \sum_{n=0}^{\infty} A_n \sin(n\omega t + \phi_n)$$

where $\omega = 2\pi f$ is the fundamental angular frequency. (a) Use $A_1 = 1$, $A_2 = 1/2$, $A_3 = 1/3$, and so on. All the ϕ_n's are zero. (b) Use $A_1 = 1$, $A_3 = 1/3$, $A_5 = 1/5$, and so on. All the even A_n's and all the ϕ_n's are zero. Write a spreadsheet or computer program that will calculate this sum. You should design your spreadsheet or program so that the number of terms in the sum can be easily specified. Start with just the first term ($n = 1$). Then rerun the calculation with more terms in the sum (n = 2, 3, 4, . . . 20). Plot the sum so that one or two periods are displayed. Note how the waveform builds up as more terms are added.

ANSWERS TO CONCEPTUAL PROBLEMS

1. At the instant at which there is no displacement of the string, the string is still moving. Thus, the energy is stored at that instant completely as kinetic energy of the string.
2. At the center of the string, there is a node for the second harmonic, as well as for every even-numbered harmonic. By placing the finger at the center and plucking, the guitarist is eliminating any harmonic that does not have a node at that point, which is all of the odd harmonics. The even harmonics can vibrate relatively freely with the finger at the center because they exhibit no displacement at that point. The result is a sound with a mixture of frequencies that are integer multiples of the second harmonic, which is one octave higher than the fundamental.
3. The bow string is pulled away from equilibrium and released, similar to the way that a guitar string is pulled and released when it is plucked. Thus, standing waves will be excited in the bow string. If the arrow leaves from the exact center of the string, then a series of odd harmonics will be excited. Even harmonics will not be excited because they have a node at the point where the string exhibits its maximum displacement.
4. A change in temperature will result in a change in the speed

INTEL-1013
U.S. Patent 7,151,430

of sound and, therefore, in the fundamental frequency. The speed of sound for audible frequencies in the open atmosphere is only a function of temperature, and does not depend on the pressure. Thus, there will be no effect on the fundamental frequency of variations in atmospheric pressure.

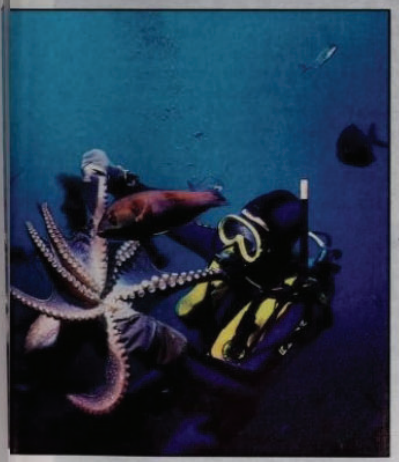
5. When the bottles are struck, standing wave vibrations are established in the glass material of the bottles. The frequencies of these vibrations are determined by the tension in the glass and the mass loading of the glass material. As the water level rises, there is more mass loading, since the glass is in contact with the water. This increased mass loading decreases the frequency. On the other hand, blowing into a bottle establishes a standing wave vibration in the air cavity above the water. As the water level rises, the length of this cavity decreases, and the frequency rises.
6. If you take the gum out of your mouth and stick it to one of the tines of the mystery tuning fork, it will lower its frequency, due to the mass loading of the gum. Now, when the two forks are played together, the beat frequency will be different. If the beat frequency is lower, then the forks are closer together in frequency, so the mystery fork must be higher in frequency than the standard. If the beat frequency is higher, the frequencies are farther apart, and the mystery fork must have had a lower frequency. This argument assumes that the piece of gum is small enough that the frequency shift for the fork is less than 4 Hz.

15

Fluid Mechanics

Matter is normally classified as being in one of three states: solid, liquid, or gaseous. Everyday experience tells us that a solid has a definite volume and shape. A brick maintains its familiar shape and size day in and day out. We also know that a liquid has a definite volume but no definite shape. Finally, an unconfined gas has neither definite volume nor definite shape. These definitions help us to picture the states of matter, but they are somewhat artificial. For example, asphalt and plastics are normally considered solids, but over long periods of time they tend to flow like liquids. Likewise, most substances can be a solid, liquid, or gas (or combinations of these), depending on the temperature and pressure. In general, the time it takes a particular substance to change its shape in response to an external force determines whether we treat the substance as a solid, liquid, or gas.

A **fluid** is a collection of molecules that are randomly arranged and held together by weak cohesive forces and forces exerted by the walls of a container. Both liquids and gases are fluids. In our treatment of the mechanics of fluids, we shall see that no new physical principles are needed to



◀ **A scuba diver plays with an octopus in Poor Knights Island, New Zealand. As the diver descends to greater depths, the water pressure increases above atmospheric pressure, and the internal pressure of the body also increases accordingly to maintain equilibrium. Scuba divers have been able to swim at depths of more than 1000 ft.** *(Darryl Torckler/Tony Stone Images)*

CHAPTER OUTLINE

15.1 Pressure
15.2 Variation of Pressure with Depth
15.3 Pressure Measurements
15.4 Buoyant Forces and Archimedes' Principle
15.5 Fluid Dynamics
15.6 Streamlines and the Equation of Continuity
15.7 Bernoulli's Principle
15.8 Other Applications of Bernoulli's Principle (Optional)

INTEL-1013
U.S. Patent 7,151,430

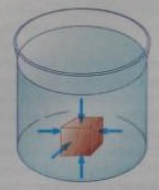


Figure 15.1 The force of the fluid on a submerged object at any point is perpendicular to the surface of the object. The force of the fluid on the walls of the container is perpendicular to the walls at all points.

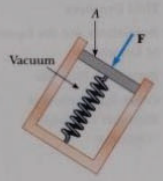


Figure 15.2 A simple device for measuring pressure in a fluid.

explain such effects as the buoyant force on a submerged object and the dynamic lift on an airplane wing. First, we consider a fluid at rest and derive an expression for the pressure exerted by the fluid as a function of its density and depth. We then treat fluids in motion, an area of study called fluid dynamics. A fluid in motion can be described by a model in which certain simplifying assumptions are made. We use this model to analyze some situations of practical importance. An underlying principle known as the **Bernoulli principle** enables us to determine relationships between the pressure, density, and velocity at every point in a fluid.

15.1 • PRESSURE

The study of fluid mechanics involves the density of a substance, defined as its mass per unit volume. For this reason, Table 15.1 lists the densities of various substances. These values vary slightly with temperature, because the volume of a substance is temperature dependent (as we shall see in Chapter 16). Note that under standard conditions (0°C and atmospheric pressure) the densities of gases are about 1/1000 the densities of solids and liquids. This difference implies that the average molecular spacing in a gas under these conditions is about ten times greater in each dimension than in a solid or liquid.

Fluids do not sustain shearing stresses, and thus the only stress that can exist on an object submerged in a fluid is one that tends to compress the object. The force exerted by the fluid on the object is always perpendicular to the surfaces of the object, as shown in Figure 15.1.

The pressure at a specific point in a fluid can be measured with the device pictured in Figure 15.2. The device consists of an evacuated cylinder enclosing a light piston connected to a spring. As the device is submerged in a fluid, the fluid presses down on the top of the piston and compresses the spring until the inward force of the fluid is balanced by the outward force of the spring. The fluid pressure can be measured directly if the spring is calibrated in advance. This is accomplished by applying a known force to the spring to compress it a given distance.

TABLE 15.1 Densities of Some Common Substances

Substance	ρ (kg/m^3)[a]	Substance	ρ (kg/m^3)[a]
Ice	0.917×10^3	Water	1.00×10^3
Aluminum	2.70×10^3	Sea water	1.03×10^3
Iron	7.86×10^3	Ethyl alcohol	0.806×10^3
Copper	8.92×10^3	Benzene	0.879×10^3
Silver	10.5×10^3	Mercury	13.6×10^3
Lead	11.3×10^3	Air	1.29
Gold	19.3×10^3	Oxygen	1.43
Platinum	21.4×10^3	Hydrogen	8.99×10^{-2}
Glycerine	1.26×10^3	Helium	1.79×10^{-1}

[a] All values are at standard atmospheric pressure and temperature (STP)—that is, atmospheric pressure and 0°C. To convert to grams per cubic centimeter, multiply by 10^{-3}.

If F is the magnitude of the normal force on the piston and A is the surface area of the piston, then the pressure, P, of the fluid at the level to which the device has been submerged is defined as the ratio of force to area:

$$P \equiv \frac{F}{A} \qquad [15.1]$$

• *Definition of pressure*

To define the pressure at a specific point, consider a fluid acting on the device shown in Figure 15.2. If the normal force exerted by the fluid is F over a surface element of area δA that contains the point in question, then the pressure at that point is

$$P = \lim_{\delta A \to 0} \frac{F}{\delta A} = \frac{dF}{dA} \qquad [15.2]$$

As we see in the next section, the pressure in a fluid varies with depth. Therefore, to get the total force on a flat wall of a container, we have to integrate Equation 15.2 over the surface.

Because pressure is force per unit area, it has units of N/m^2 in the SI system. Another name for the SI unit of pressure is **pascal** (Pa).

$$1 \text{ Pa} \equiv 1 \text{ N/m}^2 \qquad [15.3]$$



Snowshoes prevent the person from sinking into the soft snow because the person's weight is spread over a larger area, which reduces the pressure on the snow's surface. *(Earl Young/FPG)*

Thinking Physics 1

The daring physics professor, after a long lecture, stretches out for a nap on a bed of nails, as in the photograph. How is this possible?

Reasoning If you try to support your entire weight on a single nail, the pressure on your body is your weight divided by the very small area of the nail. This pressure is sufficiently large to penetrate the skin. However, if you distribute your weight over several hundred nails, as the professor is doing, the pressure is considerably reduced because the area that supports your weight is the total area of all nails in contact with your body. (Note that lying on a bed of nails is much more comfortable than sitting on a bed of nails. Standing on the bed of nails without shoes is not recommended.)



(Thinking Physics 1) *(Jim Lehman)*

INTEL-1013
U.S. Patent 7,151,430

Thinking Physics 2

Suction cups can be used to hold objects onto surfaces. Why don't astronauts use suction cups to hold onto the outside surface of the space shuttle?

Reasoning The action of a suction cup depends on the fact that air is pushed out from under the cup when it is pressed against a surface. When released, it tends to spring back a bit, causing the trapped gas under the cup to expand and have a reduced pressure. Thus, the difference between the atmospheric pressure on the outside of the cup and the reduced pressure inside provides a net force pushing the cup against the surface. For astronauts in orbit around the Earth, there is no air outside the surface of the spacecraft. Thus, if a suction cup were to be pressed against the outside surface of the spacecraft, the pressure differential could not occur.

CONCEPTUAL PROBLEM 1

A woman wearing high-heeled shoes is invited into a home in which the kitchen has vinyl floor covering. Why should the homeowner be concerned?

EXERCISE 1 Estimate the density of the *nucleus* of an atom. What does this result suggest concerning the structure of matter? (Use the fact that the mass of a proton is equal to 1.67×10^{-27} kg and its radius is approximately 10^{-15} m.) Answer 4.0×10^{17} kg/m^3; matter is mostly empty space.

15.2 • VARIATION OF PRESSURE WITH DEPTH

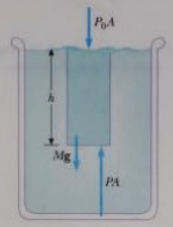


Figure 15.3 The variation of pressure with depth in a fluid. The net force on the volume of water within the darker region must be zero.

As divers know well, the pressure in the sea or a lake increases as they dive to greater depths. Likewise, atmospheric pressure decreases with increasing altitude. For this reason, aircraft flying at high altitudes must have pressurized cabins.

We now show how the pressure in a liquid increases linearly with depth. Consider a liquid of density ρ at rest and open to the atmosphere, as in Figure 15.3. Let us select a sample of the liquid contained within an imaginary cylinder of cross-sectional area A extending from the surface of the liquid to a depth h. The pressure exerted by the fluid on the bottom face is P, and the pressure on the top face of the cylinder is atmospheric pressure, P_0. Therefore, the upward force exerted by the liquid on the bottom of the cylinder is PA, and the downward force exerted by the atmosphere on the top is P_0A. Because the mass of liquid in the cylinder is $\rho V = \rho Ah$, the weight of the fluid in the cylinder is $w = \rho gV = \rho gAh$. Because the cylinder is in equilibrium, the upward force at the bottom must be greater than the downward force at the top of the sample to support its weight:

$$PA - P_0A = \rho gAh$$

or

$$P = P_0 + \rho gh \qquad [15.4]$$

Variation of pressure with depth •

That is, the absolute pressure P at a depth h below the surface of a liquid open to the atmosphere is *greater* than atmospheric pressure by an amount ρgh.

In our calculations and end-of-chapter problems, we usually take atmospheric pressure to be

$$P_0 = 1.00 \text{ atm} \approx 1.013 \times 10^5 \text{ Pa}$$

Equation 15.4 verifies that the pressure is the same at all points having the same depth, independent of the shape of the container.

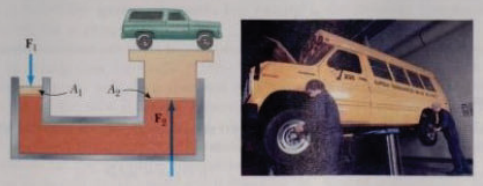
In view of the fact that the pressure in a liquid depends only on depth, any increase in pressure at the surface must be transmitted to every point in the fluid. This was first recognized by the French scientist Blaise Pascal (1623–1662) and is called **Pascal's law: A change in the pressure applied to an enclosed liquid is transmitted undiminished to every point of the liquid and to the walls of the container.** • *Pascal's law*

An important application of Pascal's law is the hydraulic press, illustrated by Figure 15.4. A force F_1 is applied to a small piston of area A_1. The pressure is transmitted through a liquid to a larger piston of area A_2. Because the pressure is the same on both sides, we see that $P = F_1/A_1 = F_2/A_2$. Therefore, the force F_2 is larger than F_1 by the multiplying factor A_2/A_1. Hydraulic brakes, car lifts, hydraulic jacks, and forklifts all make use of this principle.

Thinking Physics 3

Blood pressure is normally measured with the cuff of the sphygmomanometer around the arm. Suppose that the blood pressure were measured with the cuff around the calf of the leg of a standing person. Would the reading of the blood pressure be the same here as it was for the arm?

Reasoning The blood pressure measured at the calf would be larger than that measured at the arm. If we imagine the vascular system of the body to be a vessel containing a liquid (the blood), the pressure in the liquid will increase with depth. The blood at the calf is deeper in the liquid than that at the arm and is at a higher pressure.



(a) (b)

Figure 15.4 (a) Diagram of a hydraulic press. Because the increase in pressure is the same at the left and right sides, a small force $\mathbf{F}_1$ at the left produces a much larger force $\mathbf{F}_2$ at the right. (b) A bus under repair is supported by a hydraulic lift in a garage. *(Superstock)*

INTEL-1013
U.S. Patent 7,151,430



(Conceptual Problem 2) *(Henry Leap)*

CONCEPTUAL PROBLEM 2

A typical silo on a farm has many bands wrapped around its perimeter as shown in the photograph. Why is the spacing between successive bands smaller at the lower portions of the silo?

Example 15.1 The Car Lift

In a car lift used in a service station, compressed air exerts a force on a small piston of circular cross-section having a radius of 5.00 cm. This pressure is transmitted by a liquid to a second piston of radius 15.0 cm. What force must the compressed air exert in order to lift a car weighing 13 300 N? What air pressure will produce this force?

Solution Because the pressure exerted by the compressed air is transmitted undiminished throughout the fluid, we have

$$F_1 = \left(\frac{A_1}{A_2}\right)F_2 = \frac{\pi(5.00 \times 10^{-2}\ \text{m})^2}{\pi(15.0 \times 10^{-2}\ \text{m})^2}(1.33 \times 10^4\ \text{N})$$
$$= 1.48 \times 10^3\ \text{N}$$

The air pressure that will produce this force is

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3\ \text{N}}{\pi(5.00 \times 10^{-2}\ \text{m})^2} = 1.88 \times 10^5\ \text{Pa}$$

This pressure is approximately twice atmospheric pressure.

The input work (the work done by $\mathbf{F}_1$) is equal to the output work (the work done by $\mathbf{F}_2$), so that energy is conserved.

Example 15.2 The Force on a Dam

Water is filled to a height H behind a dam of width w (Fig. 15.5). Determine the resultant force on the dam.

Reasoning We cannot calculate the force on the dam by simply multiplying the area times the pressure, because the pressure varies with depth. The problem can be solved by finding the force dF on a narrow horizontal strip at depth h and then integrating the expression to find the total force on the dam.

Solution The pressure at the depth h beneath the surface at the shaded portion is

$$P = \rho gh = \rho g(H - y)$$

(We have left out atmospheric pressure because it acts on both sides of the dam.) Using Equation 15.2, we find the force on the shaded strip of area $dA = w\,dy$ to be

$$dF = P\,dA = \rho g(H - y)w\,dy$$

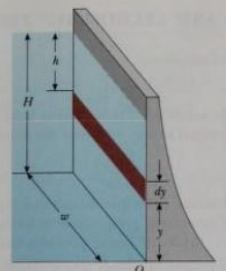


Figure 15.5 (Example 15.2) The total force on a dam must be obtained from the expression $F = \int P\,dA$, where dA is the area of the dark strip.

Therefore, the total force on the dam is

$$F = \int P\,dA = \int_0^H \rho g(H - y)w\,dy = \tfrac{1}{2}\rho g w H^2$$

Note that because the pressure increases with depth, the dam is designed so that its thickness increases with depth, as in Figure 15.5.

EXERCISE 2 Use the fact that the pressure increases linearly with depth to find the average pressure on the dam.
Answer $\frac{1}{2}\rho gH$

EXERCISE 3 What is the hydrostatic force on the back of Grand Coulee Dam if the water in the reservoir is 150 m deep and the width of the dam is 1200 m?
Answer 1.32×10^{11} N

EXERCISE 4 In some places, the Greenland ice sheet is 1.0 km thick. Estimate the pressure on the ground underneath the ice ($\rho_{ice} = 920\ \text{kg/m}^3$). Answer 9×10^6 Pa

15.3 · PRESSURE MEASUREMENTS

One simple device for measuring pressure is the open-tube manometer illustrated in Figure 15.6a. One end of a U-shaped tube containing a liquid is open to the atmosphere, and the other end is connected to a system of unknown pressure P. The difference in pressure $P - P_0$ is equal to ρgh. Therefore, we see that $P = P_0 + \rho gh$. The pressure P is called the **absolute pressure,** and the difference $P - P_0$ is called the **gauge pressure.** For example, the pressure you measure in your bicycle tire is gauge pressure.

Another instrument used to measure pressure is the common barometer, invented by Evangelista Torricelli (1608–1647). A long tube closed at one end is filled with mercury and then inverted into a dish of mercury (Fig. 15.6b). The closed end of the tube is nearly a vacuum, so its pressure can be taken as zero. Therefore, it follows that $P_0 = \rho_{Hg}gh$, where ρ_{Hg} is the density of the mercury and h is the height of the mercury column. One atmosphere ($P_0 = 1$ atm) of pressure is defined to be the pressure equivalent of a column of mercury that is exactly 0.7600 m in height at 0°C, with $g = 9.80665\ \text{m/s}^2$. At this temperature, mercury has a density of $13.595 \times 10^3\ \text{kg/s}^3$; therefore

$$P_0 = \rho_{Hg}gh = (13.595 \times 10^3\ \text{kg/m}^3)(9.80665\ \text{m/s}^2)(0.7600\ \text{m})$$
$$= 1.013 \times 10^5\ \text{Pa}$$

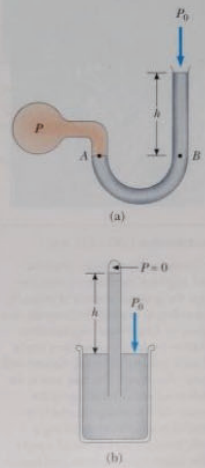


Figure 15.6 Two devices for measuring pressure: (a) an open-tube manometer and (b) a mercury barometer.

INTEL-1013
U.S. Patent 7,151,430

15.4 · BUOYANT FORCES AND ARCHIMEDES' PRINCIPLE

Archimedes' principle •

Archimedes' principle can be stated as follows:

> Any body completely or partially submerged in a fluid is buoyed up by a force the magnitude of which is equal to the weight of the fluid displaced by the body.



Figure 15.7 The external forces on the cube of water are the force of gravity **w** and the buoyant force **B**. Under equilibrium conditions, $B = w = mg$.



Archimedes (287–212 B.C.)

A Greek mathematician, physicist, and engineer, Archimedes was perhaps the greatest scientist of antiquity. According to legend, Archimedes was asked by King Hieron to determine whether the king's crown was made of pure gold or had been alloyed with some other metal. The task was to be performed without damaging the crown. Archimedes presumably arrived at a solution while taking a bath, noting a partial loss of weight after submerging his arms and legs in the water. As the story goes, he was so excited about his great discovery that he ran through the streets of Syracuse naked shouting, "Eureka!" which is Greek for "I have found it."

Everyone has experienced Archimedes' principle. Recall that it is relatively easy to lift someone if the person is in a swimming pool, whereas lifting that same individual on dry land is much harder. Evidently, water provides partial support to any object placed in it. The upward force that the fluid exerts on an object submerged in it is called the **buoyant force.** According to Archimedes' principle, **the magnitude of the buoyant force always equals the weight of the fluid displaced by the object.**

Archimedes' principle can be verified in the following manner. Suppose we focus our attention on the indicated cube of fluid in the container of Figure 15.7. This cube of fluid is in equilibrium under the action of the forces on it. One of these forces is the force of gravity. What cancels this downward force? Apparently, the rest of the fluid inside the container is holding it in equilibrium. Thus, the magnitude of the buoyant force is exactly equal to the weight of the fluid inside the cube:

$$B = w$$

Now, imagine that the cube of fluid is replaced by a cube of steel of the same dimensions. What is the buoyant force on the steel? The fluid surrounding a cube behaves in the same way whether a cube of fluid or a cube of steel is being buoyed up; therefore, **the buoyant force acting on the steel is the same as the buoyant force acting on a cube of fluid of the same dimensions.** This result applies for a submerged object of any shape, size, or density.

Let us show explicitly that the magnitude of the buoyant force is equal to the weight of the displaced fluid. The pressure at the bottom of the cube in Figure 15.7 is greater than the pressure at the top by an amount $\rho_f gh$, where ρ_f is the density of the fluid and h is the height of the cube. The pressure difference, ΔP, is equal to the buoyant force per unit area—that is, $\Delta P = B/A$. Thus, we see that $B = (\Delta P)A = (\rho_f gh)A = \rho_f gV$, where V is the volume of the cube. Because the mass of the fluid in the cube is $M = \rho_f V$, we see that

$$B = w = \rho_f Vg = Mg \qquad [15.5]$$

where w is the weight of the displaced fluid.

Before proceeding with a few examples, it is instructive to compare the forces acting on a totally submerged object with those acting on a floating object.

Case I: A Totally Submerged Object When an object is totally submerged in a fluid of density ρ_f, the magnitude of the upward buoyant force is $B = \rho_f V_0 g$, where V_0 is the volume of the object. If the object has a density ρ_0, its weight is $w = Mg = \rho_0 V_0 g$, and the net force on it is $B - w = (\rho_f - \rho_0)V_0 g$. Hence, if the density of the object is less than the density of the fluid, as in Figure 15.8a, the unsupported object will accelerate upward. If the density of the object is greater than the density of the fluid as in Figure 15.8b, the unsupported object will sink.

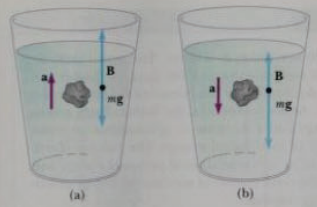


Figure 15.8 (a) A totally submerged object that is less dense than the fluid in which it is submerged will experience a net upward force. (b) A totally submerged object that is denser than the fluid sinks.

Case II: A Floating Object Now consider an object in static equilibrium floating on a fluid—that is, an object that is only partially submerged. In this case, the upward buoyant force is balanced by the downward force of gravity exerted on the object. If V is the volume of the fluid displaced by the object (which corresponds to that volume of the object beneath the fluid level), then the buoyant force has a magnitude $B = \rho_f Vg$. Because the weight of the object is $w = Mg = \rho_0 V_0 g$, and $w = B$, we see that $\rho_f Vg = \rho_0 V_0 g$, or

$$\frac{\rho_0}{\rho_f} = \frac{V}{V_0} \qquad [15.6]$$

Under normal conditions, the average density of a fish is slightly greater than the density of water. This being the case, a fish would sink if it did not have some mechanism for adjusting its density. The fish accomplishes this by internally regulating the size of its swim bladder. In this manner, fish are able to swim to various depths.



Hot-air balloons over Albuquerque, New Mexico. Because hot air is less dense than cold air, there is a net upward buoyant force on the ballons. *(William Moriarity/Rainbow)*

Thinking Physics 4

Suppose an office party is taking place on the top floor of a tall building. Carrying an iced soft drink, you step on the elevator, which begins to accelerate downward. What happens to the ice in the drink? Does it rise farther out of the liquid? Sink deeper into the liquid? Or is it unaffected by the motion?

Reasoning The level of the ice in the liquid is unaffected by the motion. The acceleration of the elevator is equivalent to a change in the gravitational field, according to the principle of equivalence. If the elevator accelerates downward, one might be tempted to say that the effect is the same as if gravity decreases—the weight of the ice cube decreases, causing it to float higher in the liquid. Recall, however, that the magnitude of the buoyant force is equal to the weight of the liquid displaced by the ice cube. The weight of the liquid also decreases with the effectively decreased gravity. With both the weight of the ice cube and the buoyant force decreasing by the same factor, the level of the ice cube in the liquid is unaffected.

INTEL-1013
U.S. Patent 7,151,430

Thinking Physics 5

A florist delivery person is delivering a flower basket to a home. The basket includes an attached helium-filled balloon, which suddenly comes loose from the basket and begins to accelerate upward toward the sky. Startled by the release of the balloon, the delivery person drops the flower basket. As the basket falls, the basket–Earth system experiences an increase in kinetic energy and a decrease in gravitational potential energy, consistent with the conservation of mechanical energy. The balloon–Earth system, however, experiences an increase in *both* gravitational potential energy and kinetic energy. Is this inconsistent with the conservation of mechanical energy principle? If not, from where is the extra energy coming?

Reasoning In the case of the system of the flower basket and the Earth, a good approximation to the motion of the basket can be made by ignoring the effects of the air. Thus, the conservation of mechanical energy principle is obeyed, with the exception of a small amount of transformation to internal energy of the air and the basket due to air friction. For the balloon–Earth system, we cannot ignore the effects of the air—it is the buoyant force of the air that causes the balloon to rise.

CONCEPTUAL PROBLEM 3

Atmospheric pressure varies from day to day. Does a ship float higher in the water on a high pressure day compared to a low pressure day?

CONCEPTUAL PROBLEM 4

Suppose a damaged ship just barely floats in the ocean after a hole in its hull has been plugged. It is pulled toward shore and into a river, heading toward a dry dock for repair. As it is pulled up the river, it sinks. Why?

CONCEPTUAL PROBLEM 5

A pound of styrofoam and a pound of lead have the same weight. If they are placed on an equal arm balance, will it balance?

CONCEPTUAL PROBLEM 6

A person in a boat floating in a small pond throws an anchor overboard. Does the level of the pond rise, fall, or remain the same?

Example 15.3 A Submerged Object

A piece of aluminum is suspended from a string and then completely immersed in a container of water (Fig. 15.9). The mass of the aluminum is 1.0 kg, and its density is $2.7 \times 10^3\ \text{kg/m}^3$. Calculate the tension in the string before and after the aluminum is immersed.

Solution When the aluminum is suspended in air, as in Figure 15.9a, the tension in the string, T_1 (the reading on the scale), is equal to the weight, Mg, of the aluminum, assuming that the buoyant force of air can be neglected:

$$T_1 = Mg = (1.0\ \text{kg})(9.80\ \text{m/s}^2) = 9.8\ \text{N}$$

When immersed in water, the aluminum experiences an upward buoyant force **B**, as in Figure 15.9b, which reduces the tension in the string. Because the system is in equilibrium,

$$T_2 + B - Mg = 0$$

$$T_2 = Mg - B = 9.8\ \text{N} - B$$

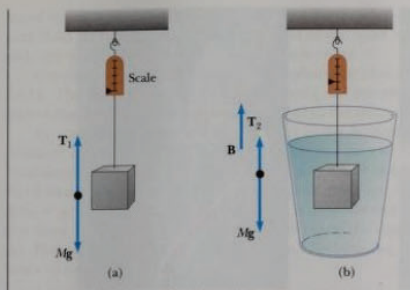


Figure 15.9 (Example 15.3) (a) When the aluminum is suspended in air, the scale reads the weight, Mg (neglecting the buoyancy of air). (b) When the aluminum is immersed in water, the buoyant force **B** reduces the scale reading to $T_2 = Mg - B$.

In order to calculate B, we must first calculate the volume of the aluminum:

$$V_{\text{Al}} = \frac{M}{\rho_{\text{Al}}} = \frac{1.0\ \text{kg}}{2.7 \times 10^3\ \text{kg/m}^3} = 3.7 \times 10^{-4}\ \text{m}^3$$

Because the buoyant force equals the weight of the water displaced, we have

$$\begin{aligned} B &= M_w g = \rho_w V_{\text{Al}} g \\ &= (1.0 \times 10^3\ \text{kg/m}^3)(3.7 \times 10^{-4}\ \text{m}^3)(9.80\ \text{m/s}^2) \\ &= 3.6\ \text{N} \end{aligned}$$

Therefore,

$$T_2 = 9.8\ \text{N} - B = 9.8\ \text{N} - 3.6\ \text{N} = 6.2\ \text{N}$$

EXERCISE 5 What is the true weight of one cubic meter of a light plastic that has a specific gravity of 0.15? Note the true weight of an object is its weight in vacuum.
Answer 1.47 kN

15.5 · FLUID DYNAMICS

Thus far, our study of fluids has been restricted to fluids at rest. We now turn our attention to fluid dynamics—that is, fluids in motion. Instead of trying to study the motion of each particle of the fluid as a function of time, we describe the properties of the fluid at each point as a function of time.

Flow Characteristics

When fluid is in motion, its flow can be characterized as being one of two main types. The flow is said to be **steady** or **laminar** if each particle of the fluid follows a smooth path, so that the paths of different particles never cross each other, as in Figure 15.10. Thus, in steady flow, the velocity of the fluid at any point remains constant in time.

Above a certain critical speed, fluid flow becomes **turbulent.** Turbulent flow is an irregular flow characterized by small whirlpool-like regions, as in Figure 15.11. As an example, the flow of water in a stream becomes turbulent in regions in which rocks and other obstructions are encountered, often forming white-water rapids.

The term **viscosity** is commonly used in fluid flow to characterize the degree of internal friction in the fluid. This internal friction or viscous force is associated with the resistance of two adjacent layers of the fluid against moving relative to each other. Because of viscosity, part of the kinetic energy of a fluid is converted to thermal energy. This is similar to the mechanism by which an object sliding on a rough horizontal surface loses kinetic energy.

INTEL-1013
U.S. Patent 7,151,430



Figure 15.10 An illustration of streamline flow around an automobile in a test wind tunnel. The streamlines in the airflow are made visible by smoke particles. *(Andy Sacks/Tony Stone Images)*



Figure 15.11 Turbulent flow: The tip of a rotating blade (the dark region at the top) forms a vortex in air that is being heated by an alcohol lamp (the wick is at the bottom). Note the air turbulence on both sides of the rotating blade. *(Kim Vandiver and Harold Edgerton, Palm Press, Inc.)*

Because the motion of a real fluid is complex and not yet fully understood, we make some simplifying assumptions in our approach. As we shall see, many features of real fluids in motion can be understood by considering the behavior of an ideal fluid. In our model, we make the following four assumptions:

Properties of an ideal fluid •

1. **Nonviscous fluid.** In a nonviscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.
2. **Incompressible fluid.** The density of an incompressible fluid is assumed to remain constant because it does not change due to a change in pressure on the fluid.
3. **Steady flow.** In steady flow, we assume that the velocity of the fluid at each point remains constant in time.
4. **Irrotational flow.** Fluid flow is irrotational if there is no angular momentum of the fluid about any point. If a small wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, the flow is irrotational. (If the wheel were to rotate, as it would if turbulence were present, the flow would be rotational.)

The first two assumptions are properties of our ideal fluid. The last two are descriptions of the way that the fluid flows.



Figure 15.12 This diagram represents a set of streamlines (blue lines). A particle at P follows one of these streamlines, and its velocity is tangent to the streamline at each point along its path.

15.6 • STREAMLINES AND THE EQUATION OF CONTINUITY

The path taken by a fluid particle under steady flow is called a **streamline.** The velocity of the fluid particle is always tangent to the streamline, as shown in Figure 15.12. No two streamlines can cross each other, for if they did, a fluid particle could move either way at the crossover point and then the flow would not be steady. A set of streamlines, as shown in Figure 15.12, forms what is called a *tube of flow.* Note that fluid particles cannot flow into or out of the sides of this tube, because if they did, the streamlines would be crossing each other.

Consider an ideal fluid flowing through a pipe of nonuniform size, as in Figure 15.13. The particles in the fluid move along the streamlines in steady flow. At all points the velocity of any particle is tangent to the streamline along which it moves.

In a small time interval, Δt, the fluid at the bottom end of the pipe moves a distance $\Delta x_1 = v_1 \Delta t$. If A_1 is the cross-sectional area in this region, then the mass contained in the shaded region is $\Delta m_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 \Delta t$. In a similar way, the fluid that moves through the upper end of the pipe in the time Δt has a mass $\Delta m_2 = \rho A_2 v_2 \Delta t$. However, because *mass is conserved* and because the flow is steady, the mass that crosses A_1 in a time Δt must equal the mass that crosses A_2 in a time Δt. That is, $\Delta m_1 = \Delta m_2$, or $\rho A_1 v_1 = \rho A_2 v_2$. Because the density is common to both sides of this expression, we get

$$A_1 v_1 = A_2 v_2 = \text{constant} \qquad [15.7]$$

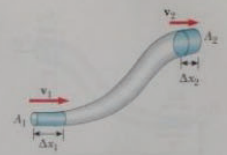


Figure 15.13 A fluid moving with streamline flow through a pipe of varying cross-sectional area. The volume of fluid flowing through A_1 in a time interval Δt must equal the volume flowing through A_2 in the same time interval. Therefore, $A_1 v_1 = A_2 v_2$.

• *Equation of continuity*

This expression, called the **equation of continuity,** says that **the product of the area and the fluid speed at all points along the pipe is a constant.** Therefore, the speed is high where the tube is constricted and low where the tube is wide. The product Av, which has the dimensions of volume/time, is called the **volume flow rate.** The condition Av = constant is equivalent to the fact that **the amount of fluid that enters one end of the tube in a given time interval equals the amount leaving in the same time interval, assuming no leaks.**

Example 15.4 Filling a Water Bucket

A water hose 2.00 cm in diameter is used to fill a 20.0-liter bucket. If it takes 1.00 min to fill the bucket, what is the speed v at which the water leaves the hose? (1 L = 10^3 cm^3)

Solution The cross-sectional area of the hose is

$$A = \pi r^2 = \pi \frac{d^2}{4} = \pi\left(\frac{2.00^2}{4}\right)\text{cm}^2 = \pi\ \text{cm}^2$$

According to the data given, the flow rate is equal to 20.0 liters/min. Equating this to the product Av gives

$$Av = 20.0\ \frac{\text{L}}{\text{min}} = \frac{20.0 \times 10^3\ \text{cm}^3}{60.0\ \text{s}}$$

$$v = \frac{20.0 \times 10^3\ \text{cm}^3}{(\pi\ \text{cm}^2)(60.0\ \text{s})} = 106\ \text{cm/s}$$

EXERCISE 6 If the diameter of the hose is reduced to 1.00 cm, what will the speed of the water be as it leaves the hose, assuming the same flow rate? Answer 424 cm/s

15.7 • BERNOULLI'S PRINCIPLE

As a fluid moves through a pipe of varying cross-section and elevation, the pressure changes along the pipe. In 1738 the Swiss physicist Daniel Bernoulli first derived an expression that relates the pressure to fluid speed and elevation.

Consider the flow of an ideal fluid through a nonuniform pipe in a time, Δt, as illustrated in Figure 15.14. The force on the lower end of the fluid is $P_1 A_1$, where P_1 is the pressure in section 1. The work done by this force is $W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1 = P_1 \Delta V$, where ΔV is the volume of section 1. In a similar manner, the

INTEL-1013
U.S. Patent 7,151,430

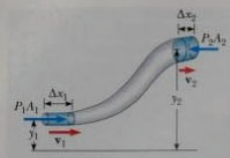


Figure 15.14 A fluid flowing through a constricted pipe with streamline flow. The fluid in the section of length Δx_1 moves to the section of length Δx_2. The volumes of fluid in the two sections are equal.

work done on the fluid at the upper end in the time Δt is $W_2 = -P_2A_2\,\Delta x_2 = -P_2\,\Delta V$. (The volume that passes through section 1 in a time Δt equals the volume that passes through section 2 in the same time interval.) This work is negative because the fluid force opposes the displacement. Thus the net work done by these forces in the time Δt is

$$W_F = (P_1 - P_2)\Delta V$$

If a fluid element of mass Δm enters the tube with speed v_1 and leaves with speed v_2, then the change in its kinetic energy is

$$\Delta K = \tfrac{1}{2}(\Delta m)v_2^2 - \tfrac{1}{2}(\Delta m)v_1^2$$

As the fluid element rises a vertical distance $y_2 - y_1$, the work done by gravity is negative and given by

$$W_g = -\Delta mg(y_2 - y_1)$$

Because the net work done on the fluid is $W_F + W_g$, we can apply the work-kinetic energy theorem to this volume of fluid to give

$$(P_1 - P_2)\Delta V - \Delta mg(y_2 - y_1) = \tfrac{1}{2}(\Delta m)v_2^2 - \tfrac{1}{2}(\Delta m)v_1^2$$

If we divide each term by ΔV, noting that $\rho = \Delta m/\Delta V$, and rearrange terms, the expression reduces to

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \qquad [15.8]$$

This is **Bernoulli's equation** as applied to an ideal fluid. It is often expressed as

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant} \qquad [15.9]$$

Bernoulli's equation says that the sum of the pressure, (P), the kinetic energy per unit volume ($\frac{1}{2}\rho v^2$), and gravitational potential energy per unit volume ($\rho g y$) has the same value at all points along a streamline.

Note the Bernoulli's equation is *not* a sum of energy density terms because P is pressure, not energy density. When the fluid is at rest, $v_1 = v_2 = 0$ and Equation 15.8 becomes

$$P_1 - P_2 = \rho g(y_2 - y_1) = \rho g h$$

which agrees with Equation 15.4.



Daniel Bernoulli (1700–1782)

Bernoulli was a Swiss physicist and mathematician who made important discoveries in hydrodynamics. His most famous work, *Hydrodynamica*, published in 1738, is both a theoretical and a practical study of equilibrium, pressure, and velocity of fluids. In this publication, Bernoulli also attempted the first explanation of the behavior of gases with changing pressure and temperature; this was the beginning of the kinetic theory of gases. *(Corbis-Bettmann)*

Thinking Physics 6

In tornadoes, it sometimes occurs that the windows of a house explode outward. Sometimes, the roof of a house is lifted off the house. What causes these events to happen? How does opening the windows in a tornado (which may seem counterintuitive!) help in theory?

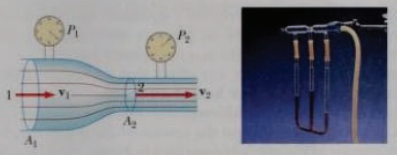
Reasoning The wind speed in a tornado is very high. According to Bernoulli's principle, such high wind speeds will result in low pressures. When these winds pass by a window, the pressure outside the window is reduced well below that of the pressure on the inside surface, which is due to the still air inside the house. As a result, the net force outward can burst the window. When these high-speed winds pass over the roof of a house, the pressure above the roof is much lower than the pressure below the roof, again from the still air inside the house. The pressure difference can be enough to lift the roof off the house. By opening the windows, the pressure differential between the outside and inside air can be reduced by allowing a flow of air through the open windows. This will reduce the possibility of exploding windows or a flying roof.

CONCEPTUAL PROBLEM 7

When you are driving a small car on the freeway and a truck passes you at a high speed, you feel pulled *toward* the truck. Why?

Example 15.5 The Venturi Tube

The horizontal constricted pipe illustrated in Figure 15.15, known as a *Venturi tube*, can be used to measure flow speeds such as air speed in flight. Let us determine the flow speed at point 2 if the pressure difference $P_1 - P_2$ is known.



(a) (b)

Figure 15.15 (a) (Example 15.5) The pressure P_1 is greater than the pressure P_2, because $v_1 < v_2$. This device can be used to measure the speed of fluid flow. (b) A Venturi tube. *(Courtesy of Central Scientific Co.)*

Solution Because the pipe is horizontal, $y_1 = y_2$ and Equation 15.8 applied to points 1 and 2 gives

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

From the equation of continuity (Eq. 15.7), we see that $A_1v_1 = A_2v_2$, or

$$v_1 = \frac{A_2}{A_1}v_2$$

Substituting this expression into the previous equation gives

$$P_1 + \tfrac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2 v_2^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

$$v_2 = A_1\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

We can also obtain an expression for v_1 using this result and the continuity equation. Note that because $A_2 < A_1$, it follows that $P_1 > P_2$. In other words, the pressure is reduced in the constricted part of the pipe. This result is somewhat analogous to the following situation: Consider a very crowded room in which people are squeezed together. As soon as a door is opened and people begin to exit, the squeezing (pressure) is least near the door where the motion (flow) is greatest.

EXERCISE 7 The Garfield Thomas water tunnel at Pennsylvania State University has a circular cross-section that constricts from a diameter of 3.6 m to the test section, which is 1.2 m in diameter. If the speed of flow is 3.0 m/s in the larger-diameter pipe, determine the speed of flow in the test section. Answer 27 m/s

15.8 · OTHER APPLICATIONS OF BERNOULLI'S PRINCIPLE OPTIONAL

Consider the streamlines that flow around an airplane wing as shown in Figure 15.16. Let us assume that the airstream approaches the wing horizontally from the right with a velocity $\mathbf{v}_1$. The tilt of the wing causes the airstream to be deflected downward with a velocity $\mathbf{v}_2$. Because the airstream is deflected by the wing, the wing must exert a force on the airstream. According to Newton's third law, the airstream must exert an equal and opposite force $\mathbf{F}$ on the wing. This force has a

INTEL-1013
U.S. Patent 7,151,430

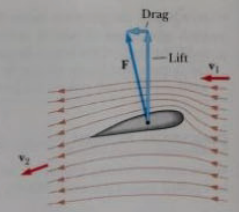


Figure 15.16 Streamline flow around a moving airplane wing. The air approaching from the right with a velocity $\mathbf{v}_1$ is deflected downward by the wing, leaving the trailing edge of the wing with a velocity $\mathbf{v}_2$. Because the airstream is deflected, it exerts a force $\mathbf{F}$ on the wing that has a vertical and horizontal component.

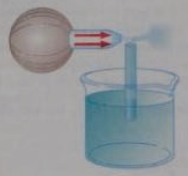


Figure 15.17 A stream of air passing over a tube dipped into a liquid will cause the liquid to rise in the tube as shown.

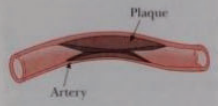


Figure 15.18 Blood must travel faster than normal through a constricted artery.

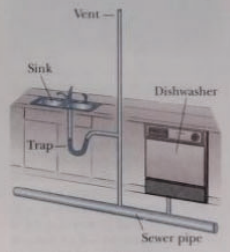


Figure 15.19 (Thinking Physics 7)

vertical component called the **lift** (or aerodynamic lift) and a horizontal component called **drag.** The lift depends on several factors, such as the speed of the airplane, the area of the wing, its curvature, and the angle between the wing and the horizontal. As this angle increases, turbulent flow can set in above the wing to reduce the lift.

The lift on the wing is consistent with Bernoulli's equation. The speed of the airstream is greater above the wing, hence the air pressure above the wing is less than the pressure below the wing, which results in a net upward force.

In general, an object experiences lift by any effect that causes the fluid to change its direction as it flows past the object. Some factors that influence the lift are the shape of the object, its orientation with respect to the fluid flow, spinning motion (for example, a spinning baseball), and the texture of the object's surface.

A number of devices operate in the manner described in Figure 15.17. A stream of air passing over an open tube reduces the pressure above the tube. This reduction in pressure causes the liquid to rise into the air stream. The liquid is then dispersed into a fine spray of droplets. You might recognize that this so-called atomizer is used in perfume bottles and paint sprayers.

Bernoulli's equation explains one symptom called vascular flutter in a person with advanced arteriosclerosis. The artery is constricted as a result of an accumulation of plaque on its inner walls (Fig. 15.18). If the blood speed is sufficiently high in the constricted region, the artery may collapse under external pressure, causing a momentary interruption in blood flow. At this point, Bernoulli's equation does not apply, and the vessel reopens under arterial pressure. As the blood rushes through the constricted artery, the internal pressure drops and again the artery closes. Such variations in blood flow can be heard with a stethoscope. If the plaque becomes dislodged and ends up in a smaller vessel that delivers blood to the heart, the person can suffer a heart attack.

Thinking Physics 7

Consider the portion of a home plumbing system shown in Figure 15.19. The water trap in the pipe below the sink captures a plug of water that prevents sewer gas from finding its way from the sewer pipe, up the sink drain, and into the home. Suppose the dishwasher is draining, so that water is moving to the left in the sewer pipe. What is the purpose of the vent, which is open to the air above the roof of the house? In which direction is air moving at the opening of the vent, upward or downward?

Reasoning Let us imagine that the vent is not present, so that the drain pipe for the sink is simply connected through the trap to the sewer pipe. As water from the dishwasher moves to the left in the sewer pipe, the pressure in the sewer pipe is reduced below atmospheric pressure, according to Bernoulli's principle. The pressure at the drain in the sink is still at atmospheric pressure. Thus, this pressure differential can push the plug of water in the water trap of the sink down the drain pipe into the sewer pipe, removing it as a barrier to sewer gas. With the addition of the vent to the roof, the reduced pressure of the dishwasher water will result in air entering the vent pipe at the roof. This will keep the pressure in the vent pipe and the right-hand side of the sink drain pipe at a pressure close to atmospheric, so that the plug of water in the water trap will remain.

SUMMARY

The **pressure,** P, in a fluid is the force per unit area that the fluid exerts on any surface:

$$P \equiv \frac{F}{A} \qquad [15.1]$$

In the SI system, pressure has units of N/m^2, and $1\ N/m^2 = 1$ pascal (Pa).

The pressure in a fluid varies with depth h according to the expression

$$P = P_0 + \rho g h \qquad [15.4]$$

where P_0 is atmospheric pressure ($= 1.013 \times 10^5\ N/m^2$) and ρ is the density of the fluid, assumed uniform.

Pascal's law states that when pressure is applied to an enclosed fluid, the pressure is transmitted undiminished to every point in the fluid and to every point on the walls of the container.

When an object is partially or fully submerged in a fluid, the fluid exerts an upward force on the object called the **buoyant force.** According to **Archimedes' principle,** the buoyant force is equal to the weight of the fluid displaced by the body.

Various aspects of fluid dynamics can be understood by assuming that the fluid is nonviscous and incompressible and that the fluid motion is a steady flow with no turbulence.

Using these assumptions, two important results regarding fluid flowing through a pipe of nonuniform size can be obtained:

- The flow rate through the pipe is a constant, which is equivalent to stating that the product of the cross-sectional area, A, and the speed, v, at any point is a constant. This gives the **equation of continuity:**

$$A_1 v_1 = A_2 v_2 = \text{constant} \qquad [15.7]$$

- The sum of the pressure, kinetic energy per unit volume, and gravitational potential energy per unit volume has the same value at all points along a streamline. This corresponds to **Bernoulli's equation:**

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant} \qquad [15.9]$$

INTEL-1013
U.S. Patent 7,151,430

CONCEPTUAL QUESTIONS

1. Figure Q15.1 shows aerial views from directly above two dams. Both dams are equally long (the vertical dimension in the diagram) and equally deep (into the page in the diagram). The dam on the left holds back a very large lake, and the dam on the right holds back a narrow river. Which dam has to be built more strongly?

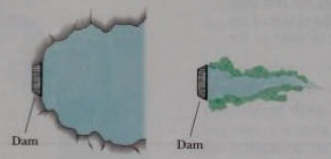


Figure Q15.1 (Conceptual Question 1)

2. Some physics students attach a long tube to the opening of a balloon. Leaving the balloon on the ground, the other end of the tube is hoisted to the roof of a multistory campus building. Students at the top of the building start pouring water into the tube. The students on the ground watch the balloon fill with water. On the roof, the students are surprised to see that the tube never seems to fill up—more and more water can continue to be poured into the tube. On the ground, the balloon bursts and drenches the students there. Why did the balloon-tube system never fill up, and why did the balloon burst?
3. A barge is carrying a load of gravel along a river. It approaches a low bridge, and the captain realizes that the top of the pile of gravel is not going to make it under the bridge. The captain orders the crew to quickly shovel gravel from the pile into the water. Is this a good decision?
4. Two drinking glasses having equal weights but different shapes and different cross-sectional areas are filled to the same level with water. According to the expression $P = P_0 + \rho gh$, the pressure is the same at the bottom of both glasses. In view of this, why does one weigh more than the other?
5. Pascal used a barometer with water as the working fluid. Why is it impractical to use water for a typical barometer?
6. A helium-filled balloon rises until its density becomes the same as that of the air. If a sealed submarine begins to sink, will it go all the way to the bottom of the ocean or will it stop when its density becomes the same as that of the surrounding water?
7. A fish rests on the bottom of a bucket of water while the bucket is being weighed. When the fish begins to swim around, does the weight change?
8. Lead has a greater density than iron, and both are denser than water. Is the buoyant force on a lead object greater than, less than, or equal to the buoyant force on an iron object of the same volume?
9. An ice cube is placed in a glass of water. What happens to the level of the water as the ice melts?
10. The water supply for a city is often provided from reservoirs built on high ground. Water flows from the reservoir, through pipes, and into your home when you turn the tap on your faucet. Why is the water flow more rapid out of a faucet on the first floor of a building than in an apartment on a higher floor?
11. If air from a hair dryer is blown over the top of a Ping-Pong ball, the ball can be suspended in air. Explain.
12. When ski-jumpers are airborne, why do they bend their bodies forward and keep their hands at their sides?
13. Explain why a sealed bottle partially filled with a liquid can float.
14. When is the buoyant force on a swimmer greater: after exhaling or after inhaling?
15. A piece of unpainted wood is partially submerged in a container filled with water. If the container is sealed and pressurized above atmospheric pressure, does the wood rise, fall, or remain at the same level? (*Hint:* Wood is porous.)
16. Because atmospheric pressure is about 10^5 N/m^2 and the area of a person's chest is about 0.13 m^2, the force of the atmosphere on one's chest is approximately 13 000 N. In view of this enormous force, why don't our bodies collapse?
17. If you release a ball while inside a freely falling elevator, the ball remains in front of you rather than falling to the floor, because the ball, the elevator, and you all experience the same downward acceleration, **g**. What happens if you repeat this experiment with a helium-filled balloon? (This one is tricky.)
18. Two identical ships set out to sea. One is loaded with a cargo of Styrofoam, and the other is empty. Which ship is more submerged?
19. A small piece of steel is tied to a block of wood. When the wood is placed in a tub of water with the steel on top, half of the block is submerged. If the block is inverted so that the steel is under water, does the amount of the block submerged increase, decrease, or remain the same? What happens to the water level in the tub when the block is inverted?
20. Prairie dogs ventilate their burrows by building a mound over one entrance, which is open to a stream of air. A second entrance at ground level is open to almost stagnant air. How does this construction create an air flow through the burrow?
21. An unopened can of diet cola floats when placed in a tank of water, whereas a can of regular cola of the same brand sinks in the tank. What do you suppose could explain this behavior?

PROBLEMS

Section 15.1 Pressure

1. A 50.0-kg woman balances on one heel of a high-heel shoe. If the heel is circular with radius 0.500 cm, what pressure does she exert on the floor?
2. A king orders a gold crown having a mass of 0.500 kg. When it arrives from the metalsmith, the volume of the crown is found to be 185 cm^3. Is the crown made of solid gold?
3. What is the total mass of the earth's atmosphere? (The radius of the Earth is 6.37×10^6 m, and atmospheric pressure at the surface is 1.013×10^5 N/m^2.)

Section 15.2 Variation of Pressure with Depth

4. Determine the absolute pressure at the bottom of a lake that is 30.0 m deep.
5. The spring of the pressure gauge shown in Figure 15.2 has a force constant of 1000 N/m, and the piston has a diameter of 2.00 cm. Find the depth in water for which the spring compresses by 0.500 cm.
6. The small piston of a hydraulic lift has a cross-sectional area of 3.00 cm^2, and the large piston has an area of 200 cm^2 (Fig. 15.4). What force must be applied to the small piston to raise a load of 15.0 kN? (In service stations this is usually accomplished with compressed air.)
7. What must be the contact area between a suction cup (completely exhausted) and a ceiling in order to support the weight of an 80.0-kg student?
8. A swimming pool has dimensions 30.0 m × 10.0 m and a flat bottom. When the pool is filled to a depth of 2.00 m with fresh water, what is the total force due to the water on the bottom? On each end? On each side?
9. Normal atmospheric pressure is 1.013×10^5 Pa. The approach of a storm causes the height of a mercury barometer to drop by 20.0 mm from the normal height. What is the atmospheric pressure? (The density of mercury is 13.59 g/cm^3.)
10. Mercury is poured into a U-shaped tube, as in Figure P15.10a. The left arm of the tube has a cross-sectional area of $A_1 = 10.0$ cm^2, and the right arm has a cross-sectional area of $A_2 = 5.00$ cm^2. One hundred grams of water are then poured into the right arm, as in Figure P15.10b. (a) Determine the length of the water column in the right arm of the U-shaped tube. (b) Given that the density of mercury is 13.6 g/cm^3, what distance, h, does the mercury rise in the left arm?
11. Blaise Pascal duplicated Torricelli's barometer using (as a Frenchman would) a red Bordeaux wine as the working liquid (Fig. P15.11). The density of the wine he used was 984 kg/m^3. What was the height h of the wine column for normal atmospheric pressure? Would you expect the vacuum above the column to be as good as for mercury?

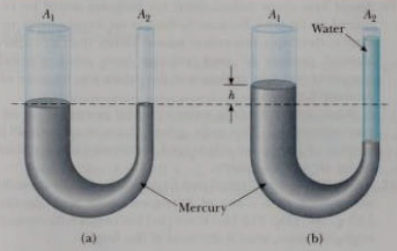


Figure P15.10

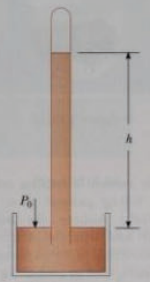


Figure P15.11

Section 15.4 Buoyant Forces and Archimedes' Principle

12. A Styrofoam slab has a thickness of 10.0 cm and a density of 300 kg/m^3. What is the area of the slab if it floats just awash in fresh water when a 75.0-kg swimmer is aboard?
13. A Styrofoam slab has thickness h and density ρ_s. What is the area of the slab if it floats with its top surface just awash in fresh water when a swimmer of mass m is on top?
14. A balloon is filled with 400 m^3 of helium. How big a payload can the balloon lift? (The density of air is 1.29 kg/m^3; the density of helium is 0.180 kg/m^3.)
15. A cube of wood 20.0 cm on a side and having a density of 650 kg/m^3 floats on water. (a) What is the distance from the horizontal top of the cube to the water level? (b) How

INTEL-1013
U.S. Patent 7,151,430

much lead weight has to be placed on top of the cube so that its top is just level with the water?

16. About how many helium-filled toy balloons would be required to lift you? Because helium is an irreplaceable resource, develop a theoretical answer rather than an experimental answer. In your solution state what physical quantities you take as data and the values you measure or estimate for them.

17. A plastic sphere floats in water with 50.0 percent of its volume submerged. This same sphere floats in oil with 40.0 percent of its volume submerged. Determine the densities of the oil and the sphere.

18. A frog in a hemispherical pod finds that he just floats without sinking into a sea of blue-green ooze with density 1.35 g/cm^3 (Fig. P15.18). If the pod has radius 6.00 cm and negligible mass, what is the mass of the frog?



Figure P15.18

19. How many cubic meters of helium are required to lift a balloon with a 400-kg payload to a height of 8000 m? ($\rho_{He} = 0.18$ kg/m^3.) Assume the balloon maintains a constant volume and that the density of air decreases with altitude z according to the expression $\rho_{air} = \rho_0 e^{-z/8000}$, where z is in meters, and ρ_0 (= 1.25 kg/m^3) is the density of air at sea level.

20. A 10.0-kg block of metal measuring 15.0 cm × 10.0 cm × 10.0 cm is suspended from a scale and immersed in water, as in Figure 15.9b. The 15.0-cm dimension is vertical, and the top of the block is 5.00 cm from the surface of the water. (a) What are the forces on the top and bottom of the block? (Take $P_0 = 1.0130 \times 10^5$ N/m^2.) (b) What is the reading of the spring scale? (c) Show that the buoyant force equals the difference between the forces at the top and bottom of the block.

Section 15.5 Fluid Dynamics

Section 15.6 Streamlines and the Equation of Continuity

Section 15.7 Bernoulli's Principle

21. The volume flow rate of water through a horizontal pipe is 2.00 m^3/min. Determine the speed of flow at a point at which the diameter of the pipe is (a) 10.0 cm, (b) 5.00 cm.

22. The legendary Dutch boy who saved Holland by placing his finger in the hole of a dike had a finger 1.20 cm in diameter. Assuming the hole was 2.00-m below the surface of the sea (density 1030 kg/m^3), (a) what was the force on his finger? (b) If he removed his finger from the hole, how long would it take the released water to fill 1 acre of land to a depth of 1 foot, assuming the hole remained constant in size? (A typical U.S. family of four uses 1 acre-foot of water, 1234 m^3, in 1 year.)

23. A large storage tank, open at the top and filled with water, develops a small hole in its side at a point 16.0 m below the water level. If the rate of flow from the leak is 2.50×10^{-3} m^3/min, determine (a) the speed at which the water leaves the hole and (b) the diameter of the hole.

24. A horizontal pipe 10.0 cm in diameter has a smooth reduction to a pipe 5.00 cm in diameter. If the pressure of the water in the larger pipe is 8.00×10^4 Pa and the pressure in the smaller pipe is 6.00×10^4 Pa, at what rate does water flow through the pipes?

25. Water is pumped from the Colorado River up to Grand Canyon Village through a 15.0-cm diameter pipe. The river is at 564 m elevation and the village is at 2096 m. (a) What is the minimum pressure the water must be pumped at to arrive at the village? (b) If 4500 m^3 are pumped per day, what is the speed of the water in the pipe? (c) What additional pressure is necessary to deliver this flow? *Note:* You may assume that the acceleration due to gravity and the density of air are constant over this range of elevations.

26. Old Faithful geyser in Yellowstone Park erupts at approximately 1-hour intervals, and the height of the fountain reaches 40.0 m. (a) With what speed does the water leave the ground? (b) What is the pressure (above atmospheric) in the heated underground chamber?

Section 15.8 Other Applications of Bernoulli's Principle (Optional)

27. An airplane has a mass of 1.60×10^4 kg, and each wing has an area of 40.0 m^2. During level flight, the pressure on the lower wing surface is 7.00×10^4 Pa. Determine the pressure on the upper wing surface.

28. A Venturi tube may be used as a fluid flow meter (Fig. 15.15). If the difference in pressure $P_1 - P_2 = 21.0$ kPa, find the fluid flow rate in m^3/s given that the radius of the outlet tube is 1.00 cm, the radius of the inlet tube is 2.00 cm, and the fluid is gasoline ($\rho = 700$ kg/m^3).

29. A Pitot tube can be used to determine the velocity of air flow by measuring the difference between the total pressure and the static pressure (Fig. P15.29). If the fluid in the tube is mercury, density $\rho_{Hg} = 13\,600$ kg/m^3, and $\Delta h = 5.00$ cm, find the speed of air flow. (Assume that the air is stagnant at point A and take $\rho_{air} = 1.25$ kg/m^3.)

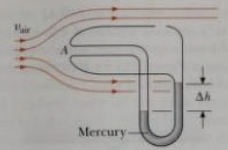


Figure P15.29

30. A siphon is used to drain water from a tank, as indicated in Figure P15.30. The siphon has a uniform diameter. Assume steady flow. (a) If the distance $h = 1.00$ m, find the speed of outflow at the end of the siphon. (b) What is the limitation on the height of the top of the siphon above the water surface? (In order to have a continuous flow of liquid, the pressure in the liquid cannot go below the vapor pressure of the liquid.)

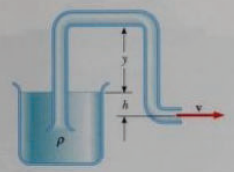


Figure P15.30

31. A large storage tank is filled to a height h_0. The tank is punctured at a height h from the bottom of the tank (Fig. P15.31). (a) Prove that the speed at which the water comes out is $\sqrt{2g(h_0 - h)}$ if the flow is steady and frictionless. (b) How far from the tank will the stream land?

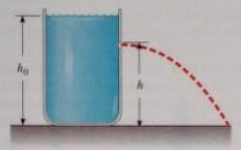


Figure P15.31

32. A hole is punched at height h in the side of a container of height h_0, full of water as in Figure P15.31. If the water is to shoot as far as possible horizontally, (a) how far from the bottom of the container should the hole be punched? (b) Neglecting friction losses, how far (initially) from the side of the container will the water land?

Additional Problems

33. A Ping-Pong ball has a diameter of 3.80 cm and average density of 0.0840 g/cm^3. What force would be required to hold it completely submerged under water?

34. Figure P15.34 shows a water tank with a valve at the bottom. If this valve is opened, what is the maximum height attained by the water stream coming out of the right side of the tank? Assume that $h = 10.0$ m, $L = 2.00$ m, and $\theta = 30.0°$ and that the cross-sectional area at A is very large compared with that at B.

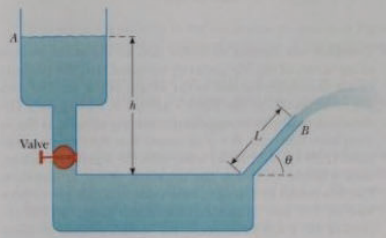


Figure P15.34

35. A helium-filled balloon is tied to a 2.00–m-long, 0.0500-kg uniform string. The balloon is spherical with a radius of 0.400 m. When released, it lifts a length, h, of string and then remains in equilibrium, as in Figure P15.35. Determine the value of h. The envelope of the balloon has mass 0.250 kg.

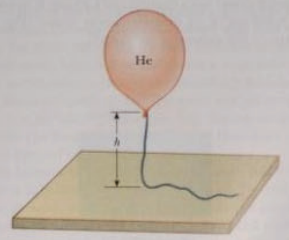


Figure P15.35

INTEL-1013
U.S. Patent 7,151,430

36. Water is forced out of a fire extinguisher by air pressure, as shown in Figure P15.36. How much gauge air pressure (above atmospheric) is required for a water jet to have a speed of 30.0 m/s when the water level is 0.500 m below the nozzle?

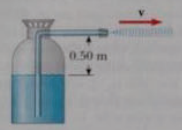


Figure P15.36

37. Torricelli was the first to realize that we live at the bottom of an ocean of air. He correctly surmised that the pressure of our atmosphere is due to the weight of the air. The density of air at 0°C at the Earth's surface is 1.29 kg/m³. The density actually decreases with increasing altitude as the atmosphere thins out. But *if we assume* that the density is constant (1.29 kg/m³) up to some altitude h and zero above, then h would represent the thickness of our atmosphere. Use this model to determine the value of h that gives a pressure of 1.00 atm at the surface of the Earth. Would the peak of Mt. Everest rise above the surface of such an atmosphere?

38. The true weight of a body is its weight when measured in a vacuum in which there are no buoyant forces. A body of volume V is weighed in air on a balance using weights of density ρ. If the density of air is ρ_a and the balance reads w', show that the true weight w is

$$w = w' + \left(V - \frac{w'}{\rho g}\right)\rho_a g$$

39. A wooden dowel has a diameter of 1.20 cm. It floats in water with 0.400 cm of its diameter above water (Fig. P15.39). Determine the density of the dowel.

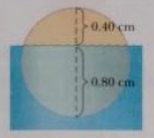


Figure P15.39

40. A 1.00-kg beaker containing 2.00 kg of oil (density = 916.0 kg/m³) rests on a scale. A 2.00-kg block of iron is suspended from a spring scale and completely submerged in the oil, as in Figure P15.40. Determine the equilibrium readings of both scales.

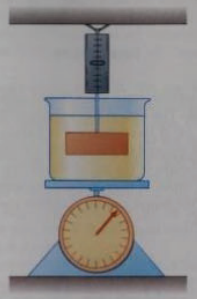


Figure P15.40

41. If a 1-megaton nuclear weapon is exploded at ground level, the peak overpressure (that is, the pressure increase above normal atmospheric pressure) will be 0.200 atm at a distance of 6.00 km. What force due to such an explosion will be exerted on the side of a house with dimensions 4.50 m × 22.0 m?

42. A light spring of constant k = 90.0 N/m rests vertically on a table (Fig. P15.42a). A 2.00-g balloon is filled with helium (density = 0.180 kg/m³) to a volume of 5.00 m³ and connected to the spring, causing it to expand as in Figure P15.42b. Determine the expansion length L when the balloon is in equilibrium.

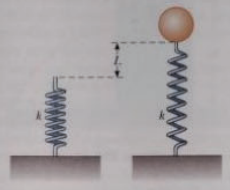


Figure P15.42

43. With reference to Figure 15.5, show that the total torque exerted by the water behind the dam about an axis through O is $\frac{1}{6}\rho g w H^3$. Show that the effective line of action of the total force exerted by the water is at a distance $\frac{1}{3}H$ above O.

44. About 1654 Otto von Guericke, inventor of the air pump, evacuated a sphere made of two brass hemispheres. Two teams of eight horses each could pull the hemispheres apart only on some trials, and then "with greatest difficulty," making a sound like a cannon firing (Fig. P15.44). (a) Show that the force F required to pull the evacuated hemispheres apart is $\pi R^2(P_0 - P)$, where R is the radius of the hemispheres and P is the pressure inside the hemispheres, which is much less than P_0. (b) Determine the force if $P = 0.100P_0$ and R = 0.300 m.

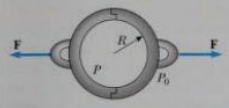




Figure P15.44 *(Henry Leap and Jim Lehman)*

45. In 1983, the United States began coining the cent piece out of copper-clad zinc rather than pure copper. If the mass of the old copper cent is 3.083 g and that of the new cent is 2.517 g, calculate the percent of zinc (by volume) in the new cent. The density of copper is 8.960 g/cm³ and that of zinc is 7.133 g/cm³. The new and old coins have the same volume.

46. A thin spherical shell of mass 4.00 kg and diameter 0.200 m is filled with helium (density = 0.180 kg/m³). It is then released from rest on the bottom of a pool of water that is 4.00 m deep. (a) Neglecting frictional effects, show that the shell rises with constant acceleration, and determine the value of that acceleration. (b) How long will it take for the top of the shell to reach the water surface?

47. An incompressible, nonviscous fluid initially rests in the vertical portion of the pipe shown in Figure P15.47a, where L = 2.00 m. When the valve is opened, the fluid flows into the horizontal section of the pipe. What is the speed of the fluid when it is entirely in the horizontal section as in Figure P15.47b? Assume the cross-sectional area of the entire pipe is constant.

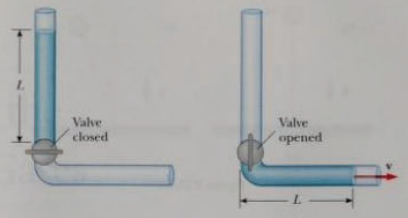


Figure P15.47

48. Water falls over a dam of height h meters at a rate of R kg/s. (a) Show that the power available from the water is

$$P = Rgh$$

where g is the acceleration of gravity. (b) Each hydroelectric unit at the Grand Coulee Dam discharges water at a rate of 8.50 × 10⁵ kg/s from a height of 87.0 m. The power developed by the falling water is converted to electric power with an efficiency of 85.0%. How much electric power is produced by each hydroelectric unit?

49. Show that the variation of atmospheric pressure with altitude is given by $P = P_0 e^{-\alpha h}$, where $\alpha = \rho_0 g/P_0$. P_0 is atmospheric pressure at some reference level, and ρ_0 is the atmospheric density at this level. Assume that the decrease in atmospheric pressure with increasing altitude is given by Equation 15.4, so that $dP/dh = -\rho g$, and that the density of air is proportional to the pressure.

50. A cube of ice the edge of which is 20.0 mm is floating in a glass of ice-cold water with one of its faces parallel to the water surface. (a) How far below the water surface is the bottom face of the block? (b) Ice-cold ethyl alcohol is gently poured onto the water surface to form a layer 5.00 mm thick above the water. When the ice cube attains hydrostatic equilibrium again, what will be the distance from the top of the water to the bottom face of the block? (c) Additional cold ethyl alcohol is poured onto the water surface until the top surface of the alcohol coincides with the top surface of the ice cube (in hydrostatic equilibrium). How thick is the required layer of ethyl alcohol?

51. A light balloon filled with helium of density 0.180 kg/m³ is tied to a light string of length L = 3.00 m. The string is tied to the ground, forming an "inverted" simple pendulum as in Figure P15.51a. If the balloon is displaced slightly from equilibrium, as in Figure P15.51b, (a) show that the motion

INTEL-1013
U.S. Patent 7,151,430

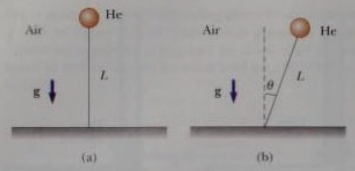


Figure P15.51

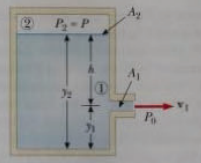


Figure P15.52

is simple harmonic, and (b) determine the period of the motion. Take the density of air to be 1.29 kg/m^3, and ignore any energy lost due to air friction.

52. *Torricelli's Law:* A tank, with a cover, containing a liquid of density ρ has a hole in its side at a distance y_1 from the bottom (Fig. P15.52). The diameter of the hole is small compared to the diameter of the tank. The air above the liquid is maintained at a pressure P. Assume steady, frictionless flow. Show that the speed at which the fluid leaves the hole when the liquid level is a distance h above the hole is

$$v_1 = \sqrt{\frac{2(P - P_0)}{\rho} + 2gh}$$

ANSWERS TO CONCEPTUAL PROBLEMS

1. She can exert enough pressure on the floor to dent or puncture the floor covering. The large pressure is caused by the fact that her weight is distributed over the very small cross-sectional area of her high heels. If you are the homeowner, you might want to suggest that she remove her high heels and put on some slippers.
2. If you think of the grain stored in the silo as a fluid, the pressure the grain exerts on the walls of the silo increases with increasing depth just as water pressure in a lake increases with increasing depth. Thus, the spacing between bands is made smaller at the lower portions to overcome the larger outward forces on the walls in these regions.
3. The level of floating of a ship in the water is unaffected by the atmospheric pressure. Air exerts negligible buoyant force compared to water. The buoyant force of the water results from the pressure *differential* in the fluid. On a high-pressure day, the pressure at all points in the water is higher than on a low-pressure day. Since water is almost incompressible, however, the rate of change of pressure with depth is the same, resulting in no change in the buoyant force.
4. In the ocean, the ship floats due to the buoyant force from *salt water*. Salt water is denser than fresh water. As the ship is pulled up the river, the buoyant force from the fresh water in the river is not sufficient to support the weight of the ship, and it sinks.
5. The balance will not be in equilibrium—the lead side will be lower. Despite the fact that the weights on both sides of the balance are the same, the styrofoam, due to its larger volume, will experience a larger buoyant force from the surrounding air. Thus, the net force of the weight and buoyant force is larger, in the downward direction, for the lead than for the styrofoam.
6. The level of the pond falls. This is because the anchor displaces more water while in the boat. A floating object displaces a volume of water whose weight is equal to the weight of the object. A submerged object displaces a volume of water equal to the volume of the object. Because the density of the anchor is greater than that of water, a volume of water that weighs the same as the anchor will be greater than the volume of the anchor.
7. As the truck passes, the air between your car and the truck is compressed into the channel between you and the truck and moves at a higher speed than when your car is in the open. According to Bernoulli's principle, this high-speed air has a lower pressure than the air on the outer side of your car. The difference in pressure provides a net force on your car toward the truck.

16

Temperature and the Kinetic Theory of Gases

Our study thus far has focused mainly on Newtonian mechanics and relativity, which explain a wide range of phenomena, such as the motion of baseballs, rockets, and planets. We now turn to the study of thermodynamics, which is concerned with the concepts of heat and temperature. As we shall see, thermodynamics is very successful in explaining the bulk properties of matter and the correlation between these properties and the mechanics of atoms and molecules.

Have you ever wondered how a refrigerator cools, or what types of transformations occur in an automobile engine, or what happens to the kinetic energy of an object once the object comes to rest? The laws of thermodynamics and the concepts of heat and temperature enable us to answer such practical questions.

Many things can happen to an object when its temperature is raised. Its size changes slightly, but it may also melt, boil, ignite,



◀ **The glass vessel contains dry ice (solid carbon dioxide). The white cloud is carbon dioxide vapor, which is denser than air and hence falls from the vessel.** *(R. Falwell/Science Photo Library/Photo Researchers)*

CHAPTER OUTLINE

16.1 Temperature and the Zeroth Law of Thermodynamics
16.2 Thermometers and Temperature Scales
16.3 Thermal Expansion of Solids and Liquids
16.4 Macroscopic Description of an Ideal Gas
16.5 The Kinetic Theory of Gases

INTEL-1013
U.S. Patent 7,151,430

or even explode. The outcome depends on the composition of the object, the degree to which its temperature is raised, and its environment. In general, thermodynamics deals with the physical and chemical transformations of matter in all of its forms: solid, liquid, gas, and plasma.

This chapter concludes with a study of ideal gases. We shall approach this study on two levels. The first will examine ideal gases on the macroscopic scale. Here we shall be concerned with the relationships among such quantities as pressure, volume, and temperature. On the second level, we shall examine gases on a microscopic scale, using a model that pictures the components of a gas as small particles. The latter approach, called the *kinetic theory of gases*, will help us to understand what happens on the atomic level to affect such macroscopic properties as pressure and temperature.

16.1 • TEMPERATURE AND THE ZEROTH LAW OF THERMODYNAMICS

We often associate the concept of temperature with how hot or cold an object feels when we touch it. Thus, our senses provide us with a qualitative indication of temperature. However, our senses are unreliable and often misleading. For example, if we remove a metal ice tray and a cardboard package of frozen vegetables from the freezer, the ice tray feels colder to the hand than the vegetables even though the two are at the same temperature. This is because metal is a better thermal conductor than cardboard, and so the ice tray transfers energy from our hand more efficiently than does the cardboard package. What we need is a reliable and reproducible method for establishing the relative "hotness" or "coldness" of objects. Scientists have developed a variety of thermometers for making such quantitative measurements.

We are all probably familiar with the fact that two objects at different initial temperatures may eventually reach some intermediate temperature when placed in contact with each other. For example, if two soft drinks, one hot and the other cold, are placed in an insulated container, the two eventually reach an equilibrium temperature, with the cold one warming up and the hot one cooling off. Likewise, if a cup of hot coffee is cooled with an ice cube, the ice eventually melts and the coffee's temperature decreases.

In order to understand the concept of temperature, it is useful to first define two often used phrases, **thermal contact** and **thermal equilibrium.** To grasp the meaning of thermal contact, imagine two objects placed in an insulated container so that they interact with each other but not with the rest of the world. If the objects are at different temperatures, energy is exchanged between them. **The energy exchanged between objects because of a temperature difference is called heat.** We shall examine the concept of heat in more detail in Chapter 17. For purposes of the current discussion, we shall assume that two objects are in **thermal contact** with each other if heat can be exchanged between them. **Thermal equilibrium** is the situation in which two objects in thermal contact with each other cease to have any exchange of heat.

Now consider two objects, A and B, that are not in thermal contact with each other, and a third object, C, that will be our thermometer. We wish to determine whether or not A and B would be in thermal equilibrium with each other once placed in thermal contact. The thermometer (object C) is first placed in thermal contact with A until thermal equilibrium is reached. At that point, the thermometer's reading remains constant, and we record it. The thermometer is then placed in thermal contact with B, and its reading is recorded after thermal equilibrium is reached. If the two readings are the same, then A and B are in thermal equilibrium with each other.

We can summarize these results in a statement known as the **zeroth law of thermodynamics** (the law of equilibrium):

• *Zeroth law of thermodynamics*

> If objects A and B are separately in thermal equilibrium with a third object, C, then A and B are in thermal equilibrium with each other.

This statement, insignificant and obvious as it may seem, is easily proved experimentally and is very important because it can be used to define temperature. We can think of temperature as the property that determines whether or not an object is in thermal equilibrium with other objects. **Two objects in thermal equilibrium with each other are at the same temperature.**

16.2 • THERMOMETERS AND TEMPERATURE SCALES

Thermometers are devices used to measure the temperature of a system. All thermometers make use of a change in some physical property with temperature. Some of the physical properties used are (1) the change in volume of a liquid, (2) the change in length of a solid, (3) the change in pressure of a gas held at constant volume, (4) the change in volume of a gas held at constant pressure, (5) the change in electric resistance of a conductor, and (6) the change in color of a very hot object. For a given substance, a temperature scale can be established based on any one of these physical quantities.

The most common thermometer in everyday use consists of a mass of liquid—usually mercury or alcohol—that expands into a glass capillary tube when its temperature rises (Fig. 16.1). In this case the physical property is the change in volume of a liquid. Note that the glass tube also expands as its temperature rises. One can define any temperature change to be proportional to the change in length of the liquid column. The thermometer can be calibrated by placing it in thermal contact with some natural systems that remain at constant temperature. One such system is a mixture of water and ice in thermal equilibrium at atmospheric pressure, which is defined to have a temperature of zero degrees Celsius, written 0°C; this temperature is called the *ice point of water.* Another commonly used system is a mixture of water and steam in thermal equilibrium at atmospheric pressure; its temperature is 100°C, the *steam point of water.* Once the liquid levels in the thermometer have been established at these two points, the column is divided into 100 equal segments, each denoting a change in temperature of one Celsius degree.

Thermometers calibrated in this way present problems when extremely accurate readings are needed. For instance, an alcohol thermometer calibrated at the ice and steam points of water might agree with a mercury thermometer only at the calibration points. Because mercury and alcohol have different thermal expansion properties, when one indicates a temperature of 50°C, say, the other may indicate a slightly different value. The discrepancies between different types of thermome-

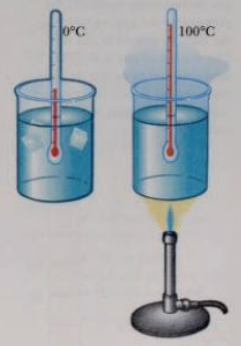


Figure 16.1 Schematic diagram of a mercury thermometer. As a result of thermal expansion, the level of the mercury rises as the mercury is heated from 0°C (the ice point) to 100°C (the steam point).

INTEL-1013
U.S. Patent 7,151,430

ters are especially large when the temperatures to be measured are far from the calibration points.

An additional practical problem of any thermometer is its limited temperature range. A mercury thermometer, for example, cannot be used below the freezing point of mercury, which is −39°C. To surmount these problems, we need a universal thermometer the readings of which are independent of the substances used. The gas thermometer approaches this requirement.

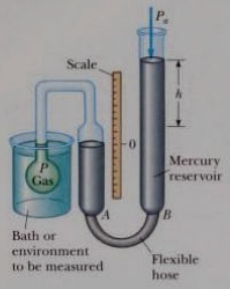


Figure 16.2 A constant-volume gas thermometer measures the pressure of the gas contained in the flask immersed in the bath. The volume of gas in the flask is kept constant by raising or lowering reservoir B such that the mercury level in column A remains constant.

The Constant-Volume Gas Thermometer and the Kelvin Scale

In a **gas thermometer**, the temperature readings are nearly independent of the substance used in the thermometer. One type of gas thermometer is the constant-volume unit, shown in Figure 16.2. The physical property used in this device is the pressure variation with temperature of a fixed volume of gas. When the constant-volume gas thermometer was developed, it was calibrated using the ice and steam points of water, as follows. (A different calibration procedure, to be discussed shortly, is now used.) The gas flask was inserted into an ice bath, and mercury reservoir B was raised or lowered until the volume of the confined gas was at some value, indicated by the zero point on the scale. The height h, the difference between the levels in the reservoir and column A, indicated the pressure in the flask at 0°C. The flask was inserted into water at the steam point, and reservoir B was readjusted until the height in column A was again brought to zero on the scale, ensuring that the gas volume was the same as it had been in the ice bath (hence the designation "constant-volume"). This gave a value for the pressure at 100°C. These pressure and temperature values were then plotted on a graph, as in Figure 16.3. The line connecting the two points serves as a calibration curve for measuring unknown temperatures. If we wanted to measure the temperature of a substance, we would place the gas flask in thermal contact with the substance and adjust the column of mercury until the level in column A again returns to zero. The height of the mercury column would tell us the pressure of the gas, and we could then find the temperature of the substance from the graph.

Now suppose that temperatures are measured with various gas thermometers containing different gases. Experiments show that the thermometer readings are nearly independent of the type of gas used, so long as the gas pressure is low and the temperature is well above the point at which the gas liquefies (Fig. 16.4). The agreement among thermometers using different gases improves as the pressure is reduced.

If you extend the curves in Figure 16.4 back toward negative temperatures, you find, in every case, that the pressure is zero when the temperature is −273.15°C. This significant temperature is used as the basis for the Kelvin temperature scale, which sets −273.15°C as its zero point (0 K). The size of a degree on the Kelvin scale is identical to the size of a degree on the Celsius scale. Thus, the relationship that enables one to convert between these temperatures is

$$T_C = T - 273.15 \qquad [16.1]$$

where T_C is the **Celsius temperature** and T is the **Kelvin temperature** (sometimes called the **absolute temperature**).

Early gas thermometers made use of ice and steam points according to the procedure just described. However, these points are experimentally difficult to duplicate. For this reason, a new procedure based on a single fixed point was adopted

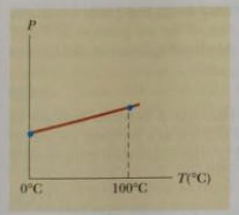


Figure 16.3 A typical graph of pressure versus temperature taken with a constant-volume gas thermometer. The dots represent known reference temperatures (the ice point and the steam point).

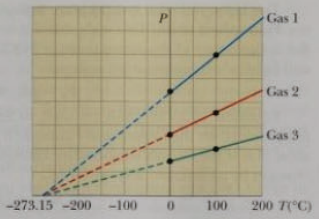


Figure 16.4 Pressure versus temperature for dilute gases. Note that, for all gases, the pressure extrapolates to zero at the unique temperature of −273.15°C.

in 1954 by the International Committee on Weights and Measures. The **triple point of water, which corresponds to the single temperature and pressure at which water, water vapor, and ice can co-exist in equilibrium,** was chosen as a convenient and reproducible reference temperature for the Kelvin scale. It occurs at a temperature of 0.01°C and a pressure of 4.58 mm of mercury. The temperature at the triple point of water on the Kelvin scale has been assigned a value of 273.16 kelvin (K). Thus, the SI unit of temperature, **the kelvin, is defined as 1/273.16 of the temperature of the triple point of water.**

Figure 16.5 shows the Kelvin temperature for various physical processes and structures. The temperature 0 K is often referred to as **absolute zero,** and as Figure 16.5 shows, this temperature has never been achieved, although laboratory experiments have come close.

What would happen to a substance if its temperature could reach 0 K? As Figure 16.4 indicates, the pressure it exerted on the walls of its container would be zero. In Section 16.5 we shall show that the pressure of a gas is proportional to the kinetic energy of the molecules of that gas. Thus, according to classical physics, the kinetic energy of the gas would go to zero, and there would be no motion at all of the individual components of the gas; hence, the molecules would settle out on the bottom of the container. Quantum theory, to be discussed in Chapter 28, modifies this statement to indicate that there would be some residual energy, called the *zero-point energy,* at this low temperature.

The Celsius, Fahrenheit, and Kelvin Temperature Scales

Equation 16.1 shows that the Celsius temperature, T_C, is shifted from the absolute (Kelvin) temperature, T, by 273.15°. Because the size of a degree is the same on the two scales, a temperature difference of 5°C is equal to a temperature difference of 5 K. The two scales differ only in the choice of the zero point. Thus, the ice point (273.15 K) corresponds to 0.00°C, and the steam point (373.15 K) is equivalent to 100.00°C.

• *The kelvin*

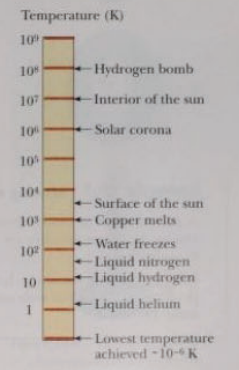


Figure 16.5 Absolute temperatures at which various selected physical processes occur. Note that the scale is logarithmic.

INTEL-1013
U.S. Patent 7,151,430

The most common temperature scale in everyday use in the United States is the **Fahrenheit scale.** This scale sets the temperature of the ice point at 32°F and the temperature of the steam point at 212°F. The relationship between the Celsius and Fahrenheit temperature scales is

$$T_F = \tfrac{9}{5}T_C + 32°\text{F} \qquad [16.2]$$

Equation 16.2 can easily be used to find a relationship between changes in temperature on the Celsius and Fahrenheit scales. It is left as a problem for you to show that if the Celsius temperature changes by ΔT_C, the Fahrenheit temperature changes by an amount ΔT_F, given by

$$\Delta T_F = \tfrac{9}{5}\Delta T_C \qquad [16.3]$$

Thinking Physics 1

A group of future astronauts lands on an inhabited planet. They strike up a conversation with the life forms about temperature scales. It turns out that the inhabitants of this planet have a temperature scale based on the freezing and boiling points of water separated by 100 of the inhabitants' degrees. Would these two temperatures on this planet be the same as those on Earth? Would the size of the planet inhabitants' degrees be the same as ours? Suppose that the inhabitants have also devised a scale similar to the Kelvin scale. Would their absolute zero be the same as ours?

Reasoning The values of 0°C and 100°C for the freezing and boiling points of water are defined at atmospheric pressure. On another planet, it is unlikely that atmospheric pressure would be exactly the same as that on Earth. Thus, water would freeze and boil at different temperatures on the planet. The inhabitants may *call* these temperatures 0° and 100°, but they would not be the same temperatures as our 0° and 100°. If the inhabitants did assign values of 0° and 100° for these temperatures, then their degrees would not be the same size as our Celsius degrees (unless their atmospheric pressure was the same as ours). For a version of the Kelvin scale from this other planet, the absolute zero would be the same as ours, because it is based on a natural, universal definition, rather than being associated with a particular substance or a given atmospheric pressure.

CONCEPTUAL PROBLEM 1

In an astronomy class, the temperature at the core of a star is given by the professor as 1.5×10^7 degrees. A student asks if this is Kelvin or Celsius. How would you respond?

Example 16.1 Heating a Pan of Water

A pan of water is heated from 25°C to 80°C. What is the change in its temperature on the Kelvin scale and on the Fahrenheit scale?

Solution From Equation 16.1, we see that the change in temperature on the Celsius scale equals the change on the Kelvin scale. Therefore,

$$\Delta T = \Delta T_C = 80 - 25 = 55°\text{C} = 55\ \text{K}$$

From Equation 16.3, we find

$$\Delta T_F = \tfrac{9}{5}\Delta T_C = \tfrac{9}{5}(80 - 25) = 99°\text{F}$$

EXERCISE 1 The melting point of gold is 1064°C, and the boiling point is 2660°C. (a) Express these temperatures in kelvin. (b) Compute the difference between these temperatures in Celsius degrees and kelvin.

Answer (a) 1337 K; 2933 K (b) 1596°C; 1596 K

16.3 · THERMAL EXPANSION OF SOLIDS AND LIQUIDS

Our discussion of the liquid thermometer made use of one of the best-known changes that occurs in a substance: As its temperature increases, its volume increases. (As we shall see shortly, in some materials the volume decreases when the temperature increases.) This phenomenon, known as **thermal expansion,** plays an important role in numerous applications. For example, thermal expansion joints must be included in buildings, concrete highways, and bridges to compensate for changes in dimensions with temperature variations.

The overall thermal expansion of an object is a consequence of the change in the average separation between its constituent atoms or molecules. Consider how the atoms in a solid substance behave. At ordinary temperatures, the atoms vibrate about their equilibrium positions with an amplitude of about 10^{-11} m, and the average spacing between the atoms is about 10^{-10} m. As the temperature of the solid increases, the average separation between atoms increases. The increase in average separation with increasing temperature (and subsequent thermal expansion) is due to the asymmetry of the potential energy between atoms. If the thermal expansion of an object is sufficiently small compared with the object's initial dimensions, then the change in any dimension is, to a good approximation, dependent on the first power of the temperature change.

Suppose an object has an initial length of L_0 along some direction at some temperature. The length increases by ΔL for the change in temperature ΔT. Experiments show that, when ΔT is small enough, ΔL is proportional to ΔT and to L_0:

$$\Delta L = \alpha L_0\, \Delta T \qquad [16.4]$$

or

$$L - L_0 = \alpha L_0 (T - T_0) \qquad [16.5]$$

where L is the final length, T is the final temperature, and the proportionality constant α is called the **average coefficient of linear expansion** for a given material and has units of $(°\text{C})^{-1}$.

It may be helpful to think of the process of thermal expansion as a magnification or a photographic enlargement. For example, as a metal washer is heated (Fig. 16.6), all dimensions, including the radius of the hole, increase according to Equation 16.4. Table 16.1 lists the average coefficient of linear expansion for various materials. Note that for these materials α is positive, indicating an increase in length with increasing temperature. This is not always the case. For example, some substances, such as calcite ($CaCO_3$), expand along one dimension (positive α) and contract along another (negative α) with increasing temperature.

Because the linear dimensions of an object change with temperature, it follows that volume and surface area also change with temperature. Consider a cube having an initial edge length, L_0, therefore an initial volume of $V_0 = L_0^3$. As the temperature is increased, the length of each side increases to

$$L = L_0 + \alpha L_0\, \Delta T$$



Thermal expansion joints are used to separate sections of roadways on bridges. Without these joints, the surfaces would buckle due to thermal expansion on very hot days or crack due to contraction on very cold days. *(Frank Siteman, Stock/Boston)*

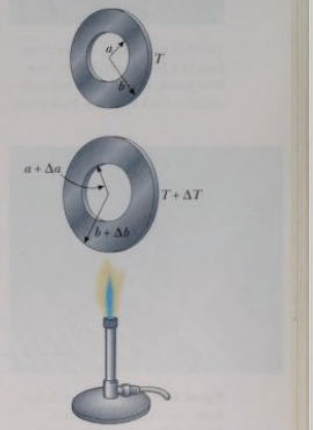


Figure 16.6 Thermal expansion of a homogeneous metal washer. Note that as the washer is heated, all dimensions increase. (The expansion is exaggerated in this figure.)

INTEL-1013
U.S. Patent 7,151,430



Thermal expansion: The extreme heat of a July day in Asbury Park, New Jersey, caused these railroad tracks to buckle. *(Wide World Photos)*

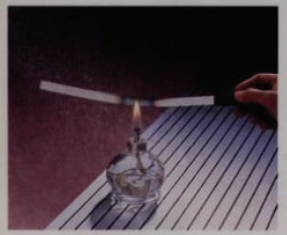


Figure 16.7 The two metals that form this bimetallic strip are bonded along their longest dimension. The strip in this photograph was straight before being heated and bends when heated. Which way would it bend if it were cooled? *(Courtesy of Central Scientific Company)*

TABLE 16.1 Expansion Coefficients for Some Materials Near Room Temperature

Material	Linear Expansion Coefficient $\alpha(^\circ\text{C})^{-1}$	Material	Volume Expansion Coefficient $\beta(^\circ\text{C})^{-1}$
Aluminum	24×10^{-6}	Alcohol, ethyl	1.12×10^{-4}
Brass and bronze	19×10^{-6}	Benzene	1.24×10^{-4}
Copper	17×10^{-6}	Acetone	1.5×10^{-4}
Glass (ordinary)	9×10^{-6}	Glycerin	4.85×10^{-4}
Glass (Pyrex)	3.2×10^{-6}	Mercury	1.82×10^{-4}
Lead	29×10^{-6}	Turpentine	9.0×10^{-4}
Steel	11×10^{-6}	Gasoline	9.6×10^{-4}
Invar (Ni–Fe alloy)	0.9×10^{-6}	Air at 0°C	3.67×10^{-3}
Concrete	12×10^{-6}	Helium at 0°C	3.665×10^{-3}

The new volume, $V = L^3$, is

$$L^3 = (L_0 + \alpha L_0 \Delta T)^3 = L_0^3 + 3\alpha L_0^3 \Delta T + 3\alpha^2 L_0^3 (\Delta T)^2 + \alpha^3 L_0^3 (\Delta T)^3$$

The last two terms in this expression contain the quantity $\alpha \Delta T$ raised to the second and third powers. Because $\alpha \Delta T$ is much less than one, squaring it makes it even smaller. Therefore, we can neglect these two terms to get a simpler expression:

$$V = L^3 = L_0^3 + 3\alpha L_0^3 \Delta T = V_0 + 3\alpha V_0 \Delta T$$

or

$$\Delta V = V - V_0 = \beta V_0 \Delta T \qquad [16.6]$$

where $\beta = 3\alpha$. The quantity β is called the **average coefficient of volume expansion.** We considered a solid cube in deriving it, but Equation 16.6 describes a sample of any shape of solid, liquid, or gas at constant pressure.

By a similar procedure, we can show that the *increase in area* of an object accompanying an increase in temperature is

$$\Delta A = \gamma A_0 \Delta T \qquad [16.7]$$

where γ, the **average coefficient of area expansion,** is given by $\gamma = 2\alpha$.

As Table 16.1 indicates, each substance has its own characteristic coefficients of expansion. For example, when the temperatures of a brass rod and a steel rod of equal length are raised by the same amount from some common initial value, the brass rod expands more than the steel rod because brass has a larger coefficient of expansion than steel. A simple device called a bimetallic strip that uses this principle is found in practical devices such as thermostats. The strip is made by securely bonding two different metals together. As the temperature of the strip increases, the two metals expand by different amounts, and the strip bends as in Figure 16.7.

Thinking Physics 2

Suppose a solid object slightly denser than water is sitting on the bottom of a container of water. Can the temperature of this system be changed in any way to make the object float to the surface?

Reasoning Let us imagine lowering the temperature. Liquids tend to have larger coefficients of linear expansion than solids, so the density of the water will increase more rapidly with the dropping temperature than that of the solid. If the density of the solid were initially only slightly larger than that of water, it would in principle be possible to make it float by lowering the temperature, although one faces the possible problem of reaching the freezing point of the water before this happens.

Thinking Physics 3

A homeowner is painting the ceiling, and a drop of paint falls from the brush onto an operating incandescent light bulb. The bulb breaks. Why?

Reasoning The glass envelope of an incandescent light bulb receives energy on the inside surface by radiation from the very hot filament and movement of the gas filling the bulb. Thus, the glass can become very hot. If a drop of paint falls onto the glass, that portion of the glass envelope will suddenly become cold, and the contraction of this region could cause thermal stresses that would break the glass.

CONCEPTUAL PROBLEM 2

Common thermometers are made of a mercury column in a glass tube. Based on the operation of these common thermometers, which has the larger coefficient of linear expansion—glass or mercury? (Don't answer this by looking in a table!)

CONCEPTUAL PROBLEM 3

Two spheres are made of the same metal and have the same radius, but one is hollow and the other is solid. The spheres are taken through the same temperature increase. Which sphere expands more?

Example 16.2 Does the Hole Get Bigger or Smaller?

A hole of cross-sectional area 100 cm² is cut in a piece of steel at 20°C. What is the change in area of the hole if the steel is heated from 20°C to 100°C?

Solution A hole in a substance expands in exactly the same way as would a piece of the substance having the same shape as the hole. The change in the area of the hole can be found by using Equation 16.7.

$$\begin{aligned}\Delta A &= \gamma A_0 \Delta T \\ &= [22 \times 10^{-6}(^\circ\text{C})^{-1}](100\ \text{cm}^2)(80^\circ\text{C}) \\ &= 0.18\ \text{cm}^2\end{aligned}$$

The Unusual Behavior of Water

Liquids generally increase in volume with increasing temperature and have volume expansion coefficients about ten times greater than those of solids. Water is an exception to this rule, as we can see from its density-versus-temperature curve in Figure 16.8. As the temperature increases from 0°C to 4°C, water contracts and thus its density increases. Above 4°C, water expands with increasing temperature. In other words, the density of water reaches a maximum value of 1000 kg/m³ at 4°C.

INTEL-1013
U.S. Patent 7,151,430

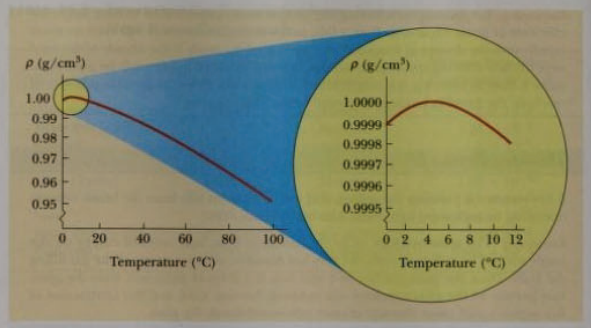


Figure 16.8 The variation of density with temperature for water at atmospheric pressure. The inset at the right shows that the maximum density of water occurs at 4°C.

We can use this unusual thermal expansion behavior of water to explain why a pond freezes at the surface. When the atmospheric temperature drops from, say, 7°C to 6°C, the water at the surface of the pond also cools and consequently decreases in volume. This means that the surface water is denser than the water below it, which has not cooled and decreased in volume. As a result, the surface water sinks and warmer water from below is forced to the surface to be cooled. When the atmospheric temperature is between 4°C and 0°C, however, the surface water expands as it cools, becoming less dense than the water below it. The mixing process stops, and eventually the surface water freezes. As the water freezes, the ice remains on the surface because ice is less dense than water. The ice continues to build up on the surface, and water near the bottom of the pool remains at 4°C. If this did not happen, fish and other forms of marine life would not survive.

EXERCISE 2 An automobile fuel tank is filled to the brim with 45 L of gasoline at 10°C. Immediately afterward, the vehicle is parked in a location where the temperature is 35°C. How much gasoline overflows from the tank as a result of expansion? (Neglect the expansion of the tank.) Answer 1.08 L



Figure 16.9 An ideal gas confined to a cylinder the volume of which can be varied with a movable piston. The state of the gas is defined by any two properties, such as pressure, volume, and temperature.

16.4 • MACROSCOPIC DESCRIPTION OF AN IDEAL GAS

In this section we shall be concerned with the properties of a gas of mass m confined to a container of volume V, pressure P, and temperature T. It is useful to know how these quantities are related. In general, the equation that interrelates these quantities, called the *equation of state*, is very complicated. However, if the gas is maintained at a very low pressure (or low density), the equation of state is found experimentally to be quite simple. Such a low-density gas is commonly referred to as an **ideal gas.** Most gases at room temperature and atmospheric pressure behave approximately as ideal gases.

It is convenient to express the amount of gas in a given volume in terms of the number of moles, n. As we learned in Section 1.2, **one mole** of any substance is that mass of the substance that contains Avogadro's number, $N_A = 6.022 \times 10^{23}$, of molecules. The number of moles, n, of a substance is related to its mass, m, through the expression

$$n = \frac{m}{M} \qquad [16.8]$$

where M is the **molar mass** of the substance, usually expressed in grams per mole. For example, the molar mass of molecular oxygen, O_2, is 32.0 g/mol. Therefore, the mass of one mole of oxygen is 32.0 g.

Now suppose an ideal gas is confined to a cylindrical container the volume of which can be varied by means of a movable piston, as in Figure 16.9. We shall assume that the cylinder does not leak, and so the mass (or the number of moles) remains constant. For such a system, experiments provide the following information. First, when the gas is kept at a constant temperature, its pressure is inversely proportional to the volume (Boyle's law). Second, when the pressure of the gas is kept constant, the volume is directly proportional to the temperature (the law of Charles and Gay-Lussac). These observations can be summarized by the following **equation of state for an ideal gas:**

$$PV = nRT \qquad [16.9]$$

In this expression, called the **ideal gas law,** R is a constant for a specific gas that can be determined from experiments, and T is the absolute temperature in kelvin. Experiments on several gases show that as the pressure approaches zero, the quantity PV/nT approaches the same value of R for all gases. For this reason, R is called the **universal gas constant.** In SI units, where pressure is expressed in pascals and volume in cubic meters, the product PV has units of newton-meters, or joules, and R has the value

$$R = 8.31 \text{ J/mol} \cdot \text{K} \qquad [16.10]$$

• *The universal gas constant*

If the pressure is expressed in atmospheres and the volume in liters (1 L = 10^3 cm^3 = 10^{-3} m^3), then R has the value

$$R = 0.0821 \text{ L} \cdot \text{atm/mol} \cdot \text{K}$$

Using this value of R and Equation 16.9, one finds that the volume occupied by 1 mol of any gas at atmospheric pressure and 0°C (273 K) is 22.4 L.

The ideal gas law is often expressed in terms of the total number of molecules, N. Because the total number of molecules equals the product of the number of moles and Avogadro's number, N_A, we can write Equation 16.9 as

$$PV = nRT = \frac{N}{N_A} RT$$

$$PV = Nk_B T \qquad [16.11]$$

where k_B is called **Boltzmann's constant** and has the value

$$k_B = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ J/K} \qquad [16.12]$$

• *Boltzmann's constant*



These colorful balloons rise as a gas burner heats the air inside them. Because warm air is less dense than cool air, the buoyant force upward can exceed the total force downward, causing the balloon to rise. The vertical motion can also be controlled by venting hot air at the top of the balloon. *(Richard Megna, Fundamental Photographs, NYC)*

INTEL-1013
U.S. Patent 7,151,430

CONCEPTUAL PROBLEM 4

Some picnickers stop at a convenience store to buy food, including bags of snack chips. They drive up into the mountains to their picnic site. When they unload the food, they notice that the chip bags are puffed up like balloons. Why did this happen?

CONCEPTUAL PROBLEM 5

A common material for cushioning objects in packages is made by trapping small bubbles of air between sheets of plastic. Many individuals find satisfaction in popping the bubbles after receiving a package (other individuals who are nearby and listening to the sound may not be so satisfied). Is this material more effective at cushioning the package contents on a hot day or a cold day?

Example 16.3 Squeezing a Tank of Gas

Pure helium gas is admitted into a tank containing a movable piston. The initial volume, pressure, and temperature of the gas are $15 \times 10^{-3}\ \text{m}^3$, 200 kPa, and 300 K. If the volume is decreased to $12 \times 10^{-3}\ \text{m}^3$ and the pressure increased to 350 kPA, find the final temperature of the gas. (Assume that helium behaves like an ideal gas.)

Solution If no gas escapes from the tank, the number of moles remains constant; therefore, using $PV = nRT$ at the initial and final points gives

$$\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f}$$

where i and f refer to the initial and final values. Solving for T_f, we get

$$T_f = \left(\frac{P_f V_f}{P_i V_i}\right) T_i = \frac{(350\ \text{kPa})(12 \times 10^{-3}\ \text{m}^3)}{(200\ \text{kPa})(15 \times 10^{-3}\ \text{m}^3)} (300\ \text{K})$$
$$= 420\ \text{K}$$

Example 16.4 Heating a Bottle of Air

A sealed glass bottle containing air at atmospheric pressure (101 kPa) and having a volume of 30 cm^3 is at 27°C. It is then tossed into an open fire. When the temperature of the air in the bottle reaches 200°C, what is the pressure inside the bottle? Assume any volume changes of the bottle are negligible.

Solution This example is approached in the same fashion as that used in Example 16.3. We start with the expression

$$\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f}$$

Because the initial and final volumes of the gas are assumed equal, this expression reduces to

$$\frac{P_i}{T_i} = \frac{P_f}{T_f}$$

This gives

$$P_f = \left(\frac{T_f}{T_i}\right) P_i = \left(\frac{473\ \text{K}}{300\ \text{K}}\right) (101\ \text{kPa}) = 159\ \text{kPa}$$

It is obvious that the higher the temperature, the higher the pressure exerted by the trapped air. Of course, if the pressure rises high enough, the bottle will shatter.

EXERCISE 3 In this example we neglected the change in volume of the bottle. If the coefficient of volume expansion for glass is $27 \times 10^{-6}\ (\text{°C})^{-1}$, find the magnitude of this volume change. Answer 0.14 cm^3

EXERCISE 4 An ideal gas is held in a container at constant volume. Initially, its temperature is 10.0°C and its pressure is 2.50 atm. What is its pressure when its temperature is 80.0°C? Answer 3.12 atm

EXERCISE 5 A full tank of oxygen (O_2) contains 12.0 kg of oxygen under a gauge pressure of 40.0 atm. Determine the mass of oxygen that has been withdrawn from the tank when the pressure reading is 25.0 atm. Assume the temperature of the tank remains constant. Answer 4.39 kg

16.5 · THE KINETIC THEORY OF GASES

In the preceding section we discussed the properties of an ideal gas, using such quantities as pressure, volume, number of moles, and temperature. In this section we shall show that these macroscopic properties can be understood on the basis of what is happening on the atomic (microscopic) scale. In addition, we shall reexamine the ideal gas law in terms of the behavior of the individual molecules that make up the gas.

Because the molecular interactions in a gas are much weaker than those in solids and liquids, our present discussion will be restricted to the molecular behavior of gases.

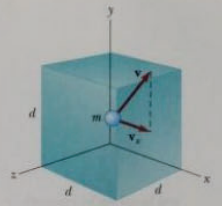


Figure 16.10 A cubical box with sides of length d containing an ideal gas. The molecule shown moves with velocity $\mathbf{v}$.

Molecular Model for the Pressure of an Ideal Gas

From the kinetic theory of gases, we shall find that the pressure a gas exerts on the walls of its container is a consequence of the collisions of the gas molecules with the walls. We make the following assumptions:

- The number of molecules is large, and the average separation between them is large compared with their dimensions. This means that the molecules occupy a negligible volume in the container.
- The molecules obey Newton's laws of motion, but as a whole they move randomly. By "randomly" we mean that any molecule can move in any direction with any speed. We also assume that the distribution of speeds does not change in time, despite the collisions between molecules. That is, at any given moment, a certain percentage of molecules move at high speeds, a certain constant percentage move at low speeds, and so on.
- We ignore the collisions of molecules with each other in our model.
- The forces between molecules are negligible except during a collision. The forces between molecules are short-range, so the molecules interact with each other only during collisions.
- The gas under consideration is a pure substance; that is, all molecules are identical.

* ***Assumptions of kinetic theory for an ideal gas***

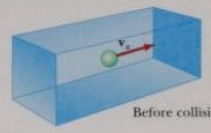
Although we often picture an ideal gas as consisting of single atoms, molecular gases exhibit equally good approximations to ideal behavior at low pressures. Effects of molecular rotations or vibrations have no effect, on the average, on the motions considered here.

Now let us derive an expression for the pressure of N molecules of an ideal gas in a container of volume V. The container is a cube with edges of length d (Fig. 16.10). We shall focus our attention on one of these molecules, of mass m and assumed to be moving so that its component of velocity in the x direction is v_x (Fig. 16.11). As the molecule collides elastically with any wall, its velocity is reversed. Because the x-component of the momentum, p_x, of the molecule is mv_x before the collision and $-mv_x$ after the collision, the **change in momentum of the molecule is**

$$\Delta p_x = mv_f - mv_i = -mv_x - mv_x = -2mv_x$$

Applying the impulse-momentum theorem (Eq. 8.12) to the molecule gives

$$F_1\,\Delta t = \Delta p_x = -2mv_x$$



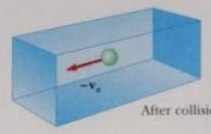


Figure 16.11 A molecule makes an elastic collision with the wall of the container. Its x component of momentum is reversed, thereby imparting momentum to the wall, and its y component remains unchanged. In this construction, we assume that the molecule moves in the xy plane.

INTEL-1013
U.S. Patent 7,151,430

In order for the molecule to make two collisions with the same wall, it must travel a distance of $2d$ in the x direction. Therefore, the time interval between two collisions with the same wall is

$$\Delta t = \frac{2d}{v_x}$$

The substitution of this result into the impulse-momentum equation enables us to express the force of the wall on the molecule:

$$F_1 = \frac{-2mv_x}{\Delta t} = \frac{-2mv_x^2}{2d} = \frac{-mv_x^2}{d}$$

Now, by Newton's third law, the force of the molecule on the wall is equal in magnitude and opposite in direction:

$$F_{1,\text{on wall}} = -F_1 = -\left(\frac{-mv_x^2}{d}\right) = \frac{mv_x^2}{d}$$

The total force exerted on the wall by all the molecules is found by adding the forces exerted by the individual molecules:

$$F = \frac{m}{d}\left(v_{x1}^2 + v_{x2}^2 + \cdots + v_{xN}^2\right)$$

In this equation, v_{x1} is the x component of velocity of molecule 1, v_{x2} is the x component of velocity of molecule 2, and so on. The summation terminates when we reach molecule N, because there are N molecules in the container.

To proceed further, note that the average value of the square of the velocity in the x direction for the N molecules is

$$\overline{v_x^2} = \frac{v_{x1}^2 + v_{x2}^2 + \cdots + v_{xN}^2}{N}$$

Thus, the total force on the wall can be written

$$F = \frac{m}{d}N\overline{v_x^2}$$

Now let us focus on one molecule in the container and say that this molecule has velocity components v_x, v_y, and v_z. The Pythagorean theorem relates the square of the velocity to the squares of these components:

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

Hence, the average value of v^2 for all the molecules in the container is related to the average values of v_x^2, v_y^2, and v_z^2 according to the expression

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$$

However, the average of the square of the velocity component is the same in any direction because the motion is completely random. Therefore,

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$$

and we have

$$\overline{v^2} = 3\overline{v_x^2}$$

Thus, the total force on the wall is

$$F = \frac{N}{3}\left(\frac{\overline{mv^2}}{d}\right)$$

This expression allows us to find the pressure exerted on the wall:

$$P = \frac{F}{A} = \frac{F}{d^2} = \frac{1}{3}\left(\frac{N}{d^3}\,\overline{mv^2}\right) = \frac{1}{3}\left(\frac{N}{V}\right)(\overline{mv^2})$$

$$P = \frac{2}{3}\left(\frac{N}{V}\right)\left(\tfrac{1}{2}\overline{mv^2}\right) \qquad [16.13]$$

• *Pressure of an ideal gas*

This result shows that **the pressure is proportional to the number of molecules per unit volume and to the average translational kinetic energy of the molecules, $\frac{1}{2}\overline{mv^2}$.** With this simplified model of an ideal gas, we have arrived at an important result that relates the large-scale quantity of pressure to an atomic quantity, the average value of the square of the molecular speed. Thus, we have a key link between the atomic world and the large-scale world.

You should note that Equation 16.13 verifies some features of pressure that are probably familiar to you. One way to increase the pressure inside a container is to increase the number of molecules per unit volume in the container. You do this when you add air to a tire. The pressure in the tire can also be increased by increasing the average translational kinetic energy of the molecules in the tire. As we shall see shortly, this can be accomplished by increasing the temperature of the gas inside the tire. That is why the pressure inside a tire increases as the tire heats up during long trips. The continuous flexing of the tires as they move along the road surface results in work done as parts of the tire distort and an increase in internal energy of the rubber. The increased temperature of the rubber results in transfer of energy into the air by conduction, increasing the air's temperature, which in turn produces an increase in pressure.

Molecular Interpretation of Temperature

We can obtain some insight into the meaning of temperature by first writing Equation 16.13 in the form

$$PV = \frac{2}{3}N\left(\tfrac{1}{2}\overline{mv^2}\right)$$

Let us now compare this with the equation of state for an ideal gas:

$$PV = Nk_B T$$

Recall that the equation of state is based on experimental facts concerning the macroscopic behavior of gases. Equating the right sides of these expressions, we find that

$$T = \frac{2}{3k_B}\left(\tfrac{1}{2}\overline{mv^2}\right) \qquad [16.14]$$

• *Temperature is proportional to average kinetic energy*

That is, **temperature is a direct measure of average translational molecular kinetic energy.**

INTEL-1013
U.S. Patent 7,151,430

By rearranging Equation 16.14, we can relate the average translational molecular kinetic energy to the temperature:

Average kinetic energy per molecule

$$\tfrac{1}{2}m\overline{v^2} = \tfrac{3}{2}k_B T \qquad [16.15]$$

That is, average translational kinetic energy per molecule is $\frac{3}{2}k_B T$. Because $\overline{v_x^2} = \frac{1}{3}\overline{v^2}$, it follows that

$$\tfrac{1}{2}m\overline{v_x^2} = \tfrac{1}{2}k_B T \qquad [16.16]$$

In a similar manner, for the y and z motions we find

$$\tfrac{1}{2}m\overline{v_y^2} = \tfrac{1}{2}k_B T \quad \text{and} \quad \tfrac{1}{2}m\overline{v_z^2} = \tfrac{1}{2}k_B T$$

Thus, each translational degree of freedom contributes an equal amount of energy to the gas, namely, $\frac{1}{2}k_B T$. (In general, the phrase "degrees of freedom" refers to the number of independent means by which a molecule can possess energy.) A generalization of this result, known as the **theorem of equipartition of energy,** says that **the energy of a system in thermal equilibrium is equally divided among all degrees of freedom.**

Theorem of equipartition of energy

The total translational kinetic energy of N molecules of gas is simply N times the average energy per molecule, which is given by Equation 16.15:

Total kinetic energy of N molecules

$$E = N\left(\tfrac{1}{2}m\overline{v^2}\right) = \tfrac{3}{2}Nk_B T = \tfrac{3}{2}nRT \qquad [16.17]$$

where we have used $k_B = R/N_A$ for Boltzmann's constant and $n = N/N_A$ for the number of moles of gas. From this result, we see that **the total kinetic energy of a system of molecules is proportional to the absolute temperature of the system.**

The square root of $\overline{v^2}$ is called the *root-mean-square* (rms) *speed* of the molecules. From Equation 16.15 we get, for the rms speed,

Root-mean-square speed

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3RT}{M}} \qquad [16.18]$$

where M is the molar mass in kg/mol. This expression shows that, at a given temperature, lighter molecules move faster, on the average, than heavier molecules. For example, hydrogen, with a molar mass of 2×10^{-3} kg/mol, moves four times as fast as oxygen, the molar mass of which is 32×10^{-3} kg/mol. Table 16.2 lists the rms speeds for various molecules at 20°C.

TABLE 16.2 Some rms Speeds

Gas	Molar Mass (g/mol)	v_{rms} at 20°C (m/s)
H_2	2.02	1902
He	4.0	1352
H_2O	18	637
Ne	20.1	603
N_2 or CO	28	511
NO	30	494
CO_2	44	408
SO_2	64	338

Thinking Physics 4

Imagine a gas in an insulated cylinder with a movable piston. The piston is pushed inward, compressing the gas, and then released. As the molecules of the gas strike the piston, they move it outward. From the points of view of (a) energy principles, and (b) kinetic theory, explain how the expansion of this gas causes its temperature to drop.

Reasoning (a) From the point of view of energy principles, the molecules strike the piston and move it through a distance. Thus, the molecules do work on the piston, which represents a transfer of energy out of the gas. As a result, the internal energy of the gas drops. Because temperature is related to internal energy, the temperature of the gas drops.

(b) From the point of view of kinetic theory, a molecule colliding with the piston causes the piston to rebound with some velocity. According to conservation of momentum, then, the molecule must rebound with less velocity than it had before the collision. Thus, as these collisions occur, the average velocity of the collection of molecules is reduced. Because temperature is related to the average velocity of molecules, the temperature of the gas drops.

CONCEPTUAL PROBLEM 6

Small planets tend to have little or no atmosphere. Why is this?

Example 16.5 A Tank of Helium

A tank of volume 0.300 m^3 contains 2.00 mol of helium gas at 20.0°C. Assuming the helium behaves like an ideal gas, (a) find the total thermal energy of the system.

Solution Using Equation 16.17 with $n = 2.00$ and $T = 293$ K, we get

$$E = \tfrac{3}{2}nRT = \tfrac{3}{2}(2.00\text{ mol})(8.31\text{ J/mol·K})(293\text{ K})$$
$$= 7.30 \times 10^3\text{ J}$$

(b) What is the average kinetic energy per molecule?

Solution From Equation 16.15, we see that the average kinetic energy per molecule is

$$\tfrac{1}{2}m\overline{v^2} = \tfrac{3}{2}k_B T = \tfrac{3}{2}(1.38 \times 10^{-23}\text{ J/K})(293\text{ K})$$
$$= 6.07 \times 10^{-21}\text{ J}$$

EXERCISE 6 Using the fact that the molar mass of helium is 4.00×10^{-3} kg/mol, determine the rms speed of the atoms at 20.0°C. Answer 1.35×10^3 m/s

SUMMARY

The **zeroth law of thermodynamics** states that if two objects, A and B, are separately in thermal equilibrium with a third object, then A and B are in thermal equilibrium with each other.

The relationship between T_C, the **Celsius temperature,** and T, the **Kelvin (absolute) temperature,** is

$$T_C = T - 273.15 \qquad [16.1]$$

The relationship between the **Fahrenheit and Celsius** temperatures is

$$T_F = \tfrac{9}{5}T_C + 32°\text{F} \qquad [16.2]$$

When a substance is heated, it generally expands. If an object has an initial length of L_0 at some temperature and undergoes a change in temperature of ΔT, its length changes by the amount ΔL, which is proportional to the object's initial length and the temperature change:

$$\Delta L = \alpha L_0 \Delta T \qquad [16.4]$$

The parameter α is called the **average coefficient of linear expansion.**

The change in area of a substance is given by

$$\Delta A = \gamma A_0 \Delta T \qquad [16.7]$$

where γ is the **average coefficient of area expansion** and is equal to 2α.

INTEL-1013
U.S. Patent 7,151,430

The change in volume of most substances is proportional to the initial volume, V_0, and the temperature change, ΔT:

$$\Delta V = \beta V_0 \, \Delta T$$

where β is the **average coefficient of volume expansion** and is equal to 3α.

An **ideal gas** is one that obeys the equation

$$PV = nRT \qquad [16.9]$$

where P is the pressure of the gas, V is its volume, n is the number of moles of gas, R is the universal gas constant (8.31 J/mol·K), and T is the absolute temperature in kelvin. A real gas at very low pressures behaves approximately as an ideal gas.

The **pressure** of N molecules of an ideal gas contained in a volume V is given by

$$P = \tfrac{2}{3}\left(\frac{N}{V}\right)\left(\tfrac{1}{2}m\overline{v^2}\right) \qquad [16.13]$$

where $\frac{1}{2}m\overline{v^2}$ is the **average kinetic energy per molecule.**

The average kinetic energy of the molecules of a gas is directly proportional to the absolute temperature of the gas:

$$\tfrac{1}{2}m\overline{v^2} = \tfrac{3}{2}k_B T \qquad [16.15]$$

where k_B is **Boltzmann's constant** (1.38×10^{-23} J/K).

The **root-mean-square** (rms) speed of the molecules of gas is

$$v_{\text{rms}} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3RT}{M}} \qquad [16.18]$$

CONCEPTUAL QUESTIONS

1. A piece of copper is dropped into a beaker of water. If the water's temperature rises, what happens to the temperature of the copper? Under what conditions are the water and copper in thermal equilibrium?
2. In the movie *Apollo 13* (Universal, 1995), the character who depicts astronaut Jim Lovell says, in describing his upcoming trip to the Moon, "I'll be walking in a place where there's 400° difference between sunlight and shadow." What exactly is it that is hot in sunlight and cold in shadow? Suppose an astronaut standing on the Moon holds a thermometer in his or her gloved hand. Is it reading the temperature of the vacuum at the Moon's surface? Does it read a temperature? If so, what object or substance has that temperature?
3. When the metal ring and metal sphere in Figure Q16.3 are both at room temperature, the sphere does not fit through the ring. After the ring is heated, the sphere can be passed through the ring. Explain.
4. Explain why a column of mercury in a thermometer first descends slightly and then rises when placed in hot water.
5. Markings to indicate length are placed on a steel tape in a room that has a temperature of 22°C. Are measurements made with the tape on a day when the temperature is 27°C too long, too short, or accurate? Defend your answer.



Figure Q16.3 *(Courtesy of Central Scientific Company)*

6. If a helium-filled balloon initially at room temperature is placed in a freezer, will its volume increase, decrease, or remain the same?
7. What does the ideal gas law predict about the volume of a gas at absolute zero? Why is this prediction incorrect?
8. What happens to a helium-filled balloon released into the air? Will it expand or contract? Will it stop rising at some height?

9. An inflated rubber balloon filled with air is immersed in a flask of liquid nitrogen that is at 77 K. Describe what happens to the balloon, assuming that it remains flexible while being cooled.
10. Two cylinders at the same temperature each contain the same kind and quantity of gas. If the volume of cylinder A is three times greater than the volume of cylinder B, what can you say about the relative pressures in the cylinders?
11. After food is cooked in a pressure cooker, why is it very important to cool the container with cold water before attempting to remove the lid?
12. The shore at a particular point of the ocean is very rocky. At one location, the rock formation is such that a cave is located with an opening that is underwater, as shown in Figure Q16.12a. (a) Inside the cave, there is a pocket of trapped air. As the level of the ocean rises and falls due to the tides, will the level of the water in the cave rise and fall? If so, will it have the same amplitude as that of the ocean? (b) Suppose the cave is deeper in the water, so that it is completely submerged and filled with water at high tide, as shown in Figure Q16.12b. At low tide, will the level of the water in the cave be the same as that of the ocean?

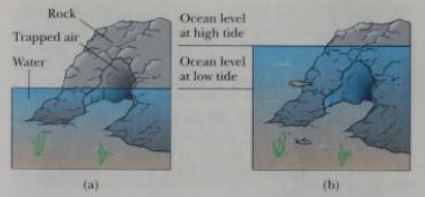


Figure Q16.12

13. An alcohol rub is sometimes used to help lower the temperature of a sick patient. Why does this help?
14. Ideal gas is contained in a vessel at 300 K. If the temperature is increased to 900 K, by what factor does each one of the following change? (a) The average kinetic energy of one molecule; (b) the rms molecular speed; (c) the average momentum change in one collision of a molecule with a wall; (d) the rate of collisions of molecules with walls; (e) the pressure of the gas.

PROBLEMS

Section 16.2 Thermometers and Temperature Scales

1. Convert the following temperatures to Celsius and kelvin: (a) the normal human body temperature, 98.6°F; (b) the air temperature on a cold day, −5.00°F.
2. In a constant-volume gas thermometer, the pressure at 20.0°C is 0.980 atm. (a) What is the pressure at 45.0°C? (b) What is the temperature if the pressure is 0.500 atm?
3. A constant-volume gas thermometer is calibrated in dry ice (which is carbon dioxide in the solid state and has a temperature of −80.0°C) and in boiling ethyl alcohol (78.0°C). The two pressures are 0.900 atm and 1.635 atm. (a) What value of absolute zero does the calibration yield? What is the pressure at (b) the freezing point of water and (c) the boiling point of water?
4. Show that the temperature −40.0° is unique in that it has the same numerical value on the Celsius and Fahrenheit scales.
5. Liquid nitrogen has a boiling point of −195.81°C at atmospheric pressure. Express this temperature in (a) degrees Fahrenheit and (b) kelvin.
6. On a Strange temperature scale, the freezing point of water is −15.0°S and the boiling point is +60.0°S. Develop a *linear* conversion equation between this temperature scale and the Celsius scale.
7. Show that if the temperature on the Celsius scale changes by ΔT_C, the Fahrenheit temperature changes by ΔT_F, which equals $\frac{9}{5}\Delta T_C$.
8. The temperature difference between the inside and the outside of an automobile engine is 450°C. Express this temperature difference on the (a) Fahrenheit scale, (b) Kelvin scale.
9. The temperature of one northeastern state varies from 105°F in the summer to −25.0°F in winter. Express this range of temperature in degrees Celsius.

Section 16.3 Thermal Expansion of Solids and Liquids

(Use Table 16.1.)

10. The concrete sections of a certain superhighway are designed to have a length of 25.0 m. The sections are poured and cured at 10.0°C. What minimum spacing should the engineer leave between the sections to eliminate buckling if the concrete is to reach a temperature of 50.0°C?
11. A copper telephone wire has essentially no sag between poles 35.0 m apart on a winter day when the temperature is −20.0°C. How much longer is the wire on a summer day when T_C = 35.0°C?
12. A brass ring of diameter 10.00 cm at 20.0°C is heated and slipped over an aluminum rod of diameter 10.01 cm at 20.0°C. Assuming the average coefficients of linear expansion are constant, (a) to what temperature must this combination be cooled to separate them? Is this attainable? (b) What if the aluminum rod were 10.02 cm in diameter?

INTEL-1013
U.S. Patent 7,151,430

13. A pair of eyeglass frames is made of epoxy plastic. At room temperature (assume 20.0°C), the frames have circular lens holes 2.20 cm in radius. To what temperature must the frames be heated in order to insert lenses 2.21 cm in radius? The average coefficient of linear expansion for epoxy is $1.30 \times 10^{-4}\ (°C)^{-1}$.
14. A copper rod and steel rod are heated. At 0°C the copper rod has a length of L_C, the steel one has a length L_S. When the rods are being heated or cooled, a difference of 5.00 cm is maintained between their lengths. Determine the values of L_C and L_S.
15. A square hole 8.00 cm along each side is cut in a sheet of copper. Calculate the change in the area of this hole if the temperature of the sheet is increased by 50.0 K.
16. The average volume coefficient of expansion for carbon tetrachloride is $5.81 \times 10^{-4}(°C)^{-1}$. If a 50.0-gal steel container is filled completely with carbon tetrachloride when the temperature is 10.0°C, how much will spill over when the temperature rises to 30.0°C?
17. The active element of a certain laser is made of a glass rod 30.0 cm long by 1.50 cm in diameter. If the temperature of the rod increases by 65.0°C, find the increase in (a) its length, (b) its diameter, and (c) its volume. (Take $\alpha = 9.00 \times 10^{-6}(°C)^{-1}$.)
18. A volumetric glass flask made of Pyrex is calibrated at 20.0°C. It is filled to the 100-mL mark with 35.0°C acetone. (a) What is the volume of the acetone when it cools to 20.0°C? (b) How significant is the change in volume of the flask?

Section 16.4 Macroscopic Description of an Ideal Gas

19. Gas is contained in an 8.00-liter vessel at a temperature of 20.0°C and a pressure of 9.00 atm. (a) Determine the number of moles of gas in the vessel. (b) How many molecules are there in the vessel?
20. A tank having a volume of 0.100 m^3 contains helium gas at 150 atm. How many balloons can the tank blow up if each filled balloon is a sphere 0.300 m in diameter at an absolute pressure of 1.20 atm?
21. An auditorium at 20.0°C and 101 kPa pressure has dimensions 10.0 m × 20.0 m × 30.0 m. How many molecules of air fill the auditorium?
22. Nine grams of water are placed in a 2.00-L pressure cooker and heated to 500°C. What is the pressure inside the container?
23. The mass of a hot-air balloon and its cargo (not including the air inside) is 200 kg. The air outside is at 10.0°C and 101 kPa. The volume of the balloon is 400 m^3. To what temperature must the air in the balloon be heated before the balloon will lift off? (Air density at 10.0°C is 1.25 kg/m^3.)
24. One mole of oxygen gas is at a pressure of 6.00 atm and a temperature of 27.0°C. (a) If the gas is heated at constant volume until the pressure triples, what is the final temperature? (b) If the gas is heated until both the pressure and volume are doubled, what is the final temperature?
25. An automobile tire is inflated using air originally at 10.0°C and normal atmospheric pressure. During the process, the air is compressed to 28.0 percent of its original volume and the temperature is increased to 40.0°C. (a) What is the tire pressure? (b) After the car is driven at high speed, the tire air temperature rises to 85.0°C and the interior volume of the tire increases by 2.00%. What is the new tire pressure in pascals (absolute)?
26. A weather balloon is designed to expand to a maximum radius of 20.0 m when in flight at its working altitude, where the air pressure is 0.0300 atm and the temperature is 200 K. If the balloon is filled at atmospheric pressure and 300 K, what is its radius at lift-off?
27. Estimate the mass of the air in your bedroom. State the quantities you take as data and the value you measure or estimate for each.
28. A room of volume 80.0 m^3 contains air having an average molar mass of 29.0 g/mol. If the temperature of the room is raised from 18.0°C to 25.0°C, what mass of air (in kg) will leave the room? Assume that the air pressure in the room is maintained at 101 kPa.
29. A room of volume V contains air having an average molar mass of M g/mol. If the temperature of the room is raised from T_1 to T_2, what mass of air will leave the room? Assume that the air pressure in the room is maintained at P_0.
30. At 25.0 m below the surface of the sea (density = 1025 kg/m^3), at which point the temperature is 5.00°C, a diver exhales an air bubble having a volume of 1.00 cm^3. If the surface temperature of the sea is 20.0°C, what is the volume of the bubble right before it breaks the surface?

Section 16.5 The Kinetic Theory of Gases

31. Use the definition of Avogadro's number to find the mass of a helium atom.
32. A sealed cubical container 20.0 cm on a side contains three times Avogadro's number of molecules at a temperature of 20.0°C. Find the force exerted by the gas on one of the walls of the container.
33. A cylinder contains a mixture of helium and argon gas in equilibrium at 150°C. (a) What is the average kinetic energy of each gas molecule? (b) What is the root-mean-square speed of each type of molecule?
34. At what temperature would the rms speed of helium atoms (mass = 6.66×10^{-27} kg) equal (a) the escape speed from Earth, 1.12×10^4 m/s, and (b) the escape speed from the Moon, 2.37×10^3 m/s?
35. In a period of 1.00 s, 5.00×10^{23} nitrogen molecules strike a wall of area 8.00 cm^2. If the molecules move with a speed of 300 m/s and strike the wall head-on in perfectly elastic collisions, find the pressure exerted on the wall. (The mass of one N_2 molecule is 4.68×10^{-26} kg.)
36. Suppose that the temperature at the center of the Sun is 2.00×10^7 K. Find (a) the average translational kinetic energy of a proton in the Sun's interior and (b) the proton's root-mean-square speed.
37. A 5.00-liter vessel contains 0.125 moles of an ideal gas at a pressure of 1.50 atm. What is the average translational kinetic energy of a single molecule?
38. A mixture of two gases will diffuse through a filter at rates proportional to their rms speeds. (a) Find the ratio of speeds for the two isotopes of chlorine, ^{35}Cl and ^{37}Cl, as they diffuse through the air. (b) Which isotope moves faster?

Additional Problems

39. At what Fahrenheit temperature are the Kelvin and Fahrenheit temperatures numerically equal?
40. A student measures the length of a brass rod with a steel tape at 20.0°C. The reading is 95.00 cm. What will the tape indicate for the length of the rod when the rod and the tape are at (a) −15.0°C, (b) 55.0°C?
41. The density of gasoline is 730 kg/m^3 at 0°C. Its volume expansion coefficient is $9.60 \times 10^{-4}/C°$. If 1.00 gallon of gasoline occupies 0.00380 m^3, how many extra kilograms of gasoline would you get if you bought 10.0 gallons of gasoline at 0°C rather than at 20.0°C from a pump not temperature-compensated?
42. A steel ball bearing is 4.000 cm in diameter at 20.0°C. A bronze plate has a hole in it that is 3.994 cm in diameter at 20.0°C. What common temperature must they have in order that the ball just squeeze through the hole?
43. A mercury thermometer is constructed as in Figure P16.43. The capillary tube has a diameter of 0.00400 cm, and the bulb has a diameter of 0.250 cm. Neglecting the expansion of the glass, find the change in height of the mercury column for a temperature change of 30.0°C.

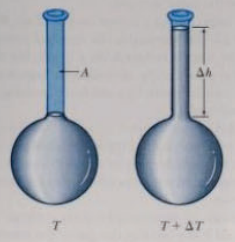


Figure P16.43

44. A liquid with a coefficient of volume expansion β just fills a spherical shell of volume V at a temperature of T (Fig. P16.43). The shell is made of a material that has a coefficient of linear expansion of α. The liquid is free to expand into a capillary of cross-sectional area A at the top. (a) If the temperature increases by ΔT, show that the liquid rises in the capillary by the amount Δh given by $\Delta h = (V/A)(\beta - 3\alpha)\Delta T$. (b) For a typical system, such as a mercury thermometer, why is it a good approximation to neglect the expansion of the shell?
45. The rectangular plate shown in Figure P16.45 has an area A equal to ℓw. If the temperature increases by ΔT, show that the increase in area is $\Delta A = 2\alpha A\,\Delta T$, where α is the average coefficient of linear expansion. What approximation does this expression assume? (*Hint:* Note that each dimension increases according to $\Delta \ell = \alpha \ell\,\Delta T$.)

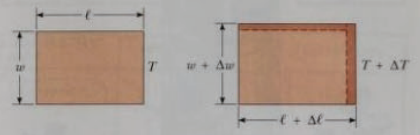


Figure P16.45

46. Show that the equation of state of an ideal gas can be written as $PM = \rho RT$, where M is its molar mass.
47. A liquid has a density ρ. (a) Show that the fractional change in density for a change in temperature ΔT is $\Delta\rho/\rho = -\beta\,\Delta T$. What does the negative sign signify? (b) Fresh water has a maximum density of 1.000 g/cm^3 at 4.0°C. At 10.0°C, its density is 0.9997 g/cm^3. What is β for water over this temperature interval?
48. An expandable cylinder has its top connected to a spring of constant 2.00×10^3 N/m (see Fig. P16.48). The cylinder is filled with 5.00 L of gas with the spring relaxed at a pressure

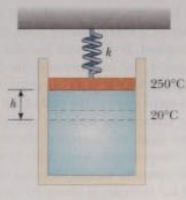


Figure P16.48

INTEL-1013
U.S. Patent 7,151,430

of 1.00 atm and a temperature of 20.0°C. (a) If the lid has a cross-sectional area of 0.0100 m^2 and negligible mass, how high will the lid rise when the temperature is raised to 250°C? (b) What is the pressure of the gas at 250°C?

49. A vertical cylinder of cross-sectional area A is fitted with a tight-fitting, frictionless piston of mass m (Fig. P16.49). (a) If there are n mol of an ideal gas in the cylinder at a temperature of T, determine the height h at which the piston is in equilibrium under its own weight. (b) What is the value for h if $n = 0.200$ mol, $T = 400$ K, $A = 0.00800$ m^2, and $m = 20.0$ kg?

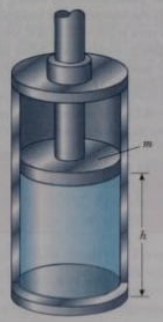


Figure P16.49

50. A bimetallic bar is made of two thin strips of dissimilar metals bonded together. As they are heated, the one with the larger average coefficient of expansion expands more than the other, forcing the bar into an arc, with the outer radius having a larger circumference (see Fig. P16.50). (a) Derive an expression for the angle of bending θ as a function of the initial length of the strips, their average coefficients of linear expansion, the change in temperature, and the separation of the centers of the strips ($\Delta r = r_2 - r_1$). (b) Show that the angle of bending goes to zero when ΔT goes to zero or when the two coefficients of expansion become equal. (c) What happens if the bar is cooled?

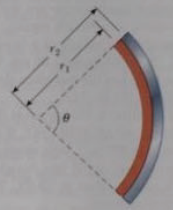


Figure P16.50

ANSWERS TO CONCEPTUAL PROBLEMS

1. The accurate response would be that it does not matter! Temperatures on the Kelvin and Celsius scales differ by only 273 degrees. This difference is insignificant for temperatures on the order of 10^7. If we imagine that the temperature given is in kelvin, and ignore any problems with significant figures, then the Celsius temperature is 1.4999727×10^7 °C.
2. Mercury must have the larger coefficient of expansion. As the temperature of a thermometer rises, both the mercury and the glass expand. If they both had the same coefficient of linear expansion, the mercury and the cavity in the glass would both expand by the same amount, and there would be no apparent movement of the end of the mercury column relative to the calibration scale on the glass. If the glass expanded more than the mercury, the reading would go down as the temperature went up! Now that we have argued this conceptually, we can look in a table and find that the coefficient for mercury is about 20 times as large as for glass, so that the expansion of the glass can effectively be ignored.
3. A cavity in a material expands in exactly the same way as if the cavity were filled with material. Thus, both spheres will expand by the same amount.
4. The chip bags contain a sealed sample of air. When the bags are taken up the mountain, the external atmospheric pressure on the bags is reduced. As a result, the difference between the pressure of the air inside the bags and the reduced pressure outside results in a net force pushing the plastic of the bag outward.
5. On a cold day, the trapped air in the bubbles would be reduced in pressure, according to the ideal gas law. Thus, the volume of the bubbles may be smaller than on a hot day, and the material would not be as effective in cushioning the package contents.
6. The retention of an atmosphere on a planet is due to the gravitational force holding the gas of the atmosphere to the planet. On a small planet, the gravitational force is very small, and the escape speed is correspondingly small. If a small planet starts its existence with an atmosphere, the molecules of the gas will have a distribution of speeds, according to kinetic theory. Some of these molecules will have speeds higher than the escape speed of the planet and will leave the atmosphere. As the remaining atmosphere is warmed by radiation from the Sun, more molecules will attain speeds high enough to escape. As a result, the atmosphere bleeds off into space.



By permission of John Hart and Field Enterprises, Inc.

INTEL-1013
U.S. Patent 7,151,430

17

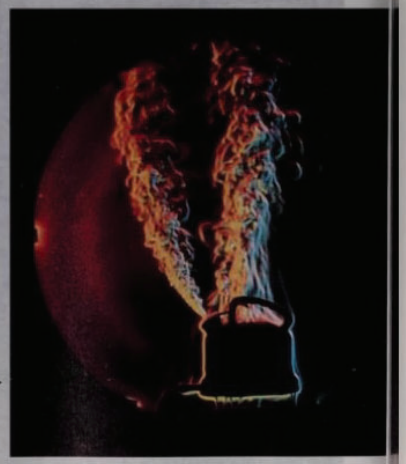
Heat and the First Law of Thermodynamics

CHAPTER OUTLINE

17.1 Heat, Thermal Energy, and Internal Energy
17.2 Specific Heat
17.3 Latent Heat and Phase Changes
17.4 Work and Thermal Energy in Thermodynamic Processes
17.5 The First Law of Thermodynamics
17.6 Some Applications of the First Law of Thermodynamics
17.7 Heat Transfer (Optional)

Until about 1850, the fields of heat and mechanics were considered to be two distinct branches of science, and the law of conservation of energy seemed to describe only certain kinds of mechanical systems. Mid–nineteenth-century experiments performed by the Englishman James Joule (1818–1889) and others showed that energy may be added to (or removed from) a system either as heat or as work done on (or by) the system. Now thermal energy is treated as a form of energy that can be transformed into mechanical energy. Once the concept of energy was broadened to include thermal energy, the law of conservation of energy emerged as a universal law of nature.

This chapter focuses on the concept of heat, the first law of thermodynamics, and some important applications. The first law of thermodynamics is merely the law of conservation of energy. It tells us only that an increase in one form of energy must be accompanied by a decrease in some other form of energy. The first law places no restrictions on the types of energy conversions that can occur. Furthermore, the first law makes no distinction between the results of heat



Schlieren photograph of a teakettle showing steam and turbulent convection currents. *(Gary Settles/Science Source/Photo Researchers)*

and work—a change in internal energy. According to the first law, a system's internal energy can be increased either by transfer of thermal energy to the system or by work done on the system. An important difference between thermal energy and mechanical energy is not evident from the first law: Energy transferred to a system as work can be stored completely as internal energy, but it is impossible for internal energy to completely leave a system only by the mechanism of work.

17.1 • HEAT, THERMAL ENERGY, AND INTERNAL ENERGY

A major distinction must be made between internal energy and heat. **Internal energy** is all of the energy belonging to a system while it is stationary (neither translating nor rotating), including nuclear energy, chemical energy, and strain energy (as in a compressed or stretched spring), as well as thermal energy associated with the random motion of atoms or molecules. **Heat is the energy that is transferred between the system and its surroundings because of a temperature difference between them.**

In the previous chapter we showed that the thermal energy of a monatomic ideal gas is associated with the internal motion of its atoms. In this special case, the thermal energy is simply the kinetic energy on a microscopic scale; the higher the temperature of the gas, the greater the kinetic energy of the atoms and the greater the thermal energy of the gas. More generally, however, thermal energy includes other forms of molecular energy, such as rotational energy and vibrational kinetic and potential energy.

As an analogy, consider the distinction between work and energy that we discussed in Chapter 7. The work done on (or by) a system is a measure of the energy transferred between the system and its surroundings, whereas the mechanical energy of the system (kinetic or potential) is a consequence of its motion and coordinates. Thus, when a person does work on a system, energy is transferred from the person to the system. It makes no sense to talk about the work *of* a system—one should instead refer only to the **work done on or by a system** when some process has occurred in which energy has been transferred to or from the system. Likewise, it makes no sense to use the term **heat** unless energy has been transferred as a result of a temperature difference.

It is also important to recognize that energy can be transferred between two systems even when no thermal energy transfer occurs. For example, when a gas is compressed by a piston, the gas is warmed and its thermal energy increases, but there is no transfer of thermal energy from the surroundings; if the gas then expands rapidly, it cools and its thermal energy decreases, but there is no transfer of thermal energy to the surroundings. In each case, energy is transferred to or from the system as work but appears within the system as an increase or decrease of thermal energy. The changes in internal energy in these examples are equal to the changes in thermal energy and are measured by corresponding changes in temperature.

Units of Heat

Early in the development of thermodynamics, before scientists realized that heat is transferred energy, heat was defined in terms of the temperature changes it produced in a body. Hence, the **calorie** (cal) was defined as **the amount of heat**



James Prescott Joule
(1818–1889)

A British physicist, Joule received some formal education in mathematics, philosophy, and chemistry from John Dalton but was in large part self-educated. Joule's most active research period, from 1837 through 1847, led to the establishment of the principle of conservation of energy and the equivalence of heat and other forms of energy. His study of the quantitative relationship among electrical, mechanical, and chemical effects of heat culminated in his announcement in 1843 of the amount of work required to produce a unit of heat, called the mechanical equivalent of heat. *(By kind permission of the President and Council of the Royal Society)*

• *Definition of the calorie*

INTEL-1013
U.S. Patent 7,151,430