necessary to raise the temperature of 1 g of water from 14.5°C to 15.5°C.[1] (Note that the "Calorie," with a capital C, used in describing the chemical energy content of foods, is actually a kilocalorie.) Likewise, the unit of heat in the British system, the **British thermal unit** (Btu), was defined as **the heat required to raise the temperature of 1 lb of water from 63°F to 64°F.**

In 1948, scientists decided that because heat (like work) is a measure of the transfer of energy, its SI unit should be the joule. The calorie is now defined to be exactly 4.186 J:

Mechanical equivalent of heat •

$$1 \text{ cal} \equiv 4.186 \text{ J} \qquad [17.1]$$

Note that this definition makes no reference to the heating of water. This relation is known, for purely historical reasons, as the **mechanical equivalent of heat.**

Example 17.1 Losing Weight the Hard Way

A student eats a dinner rated at 2000 (food) Calories. He wishes to do an equivalent amount of work in the gymnasium by lifting a 50.0-kg mass. How many times must he raise the mass to expend this much energy? Assume that he raises it a distance of 2.00 m each time and that he gains no work energy when it is dropped to the floor.

Solution Because 1 Calorie = 1.00×10^3 cal, the work required is 2.00×10^6 cal. Converting this to J, we have for the total work required

$$W = (2.00 \times 10^6 \text{ cal})(4.186 \text{ J/cal}) = 8.37 \times 10^6 \text{ J}$$

The work done in lifting the mass a distance h is equal to mgh, and the work done in lifting it n times is $nmgh$. We equate this to the total work required:

$$W = nmgh = 8.37 \times 10^6 \text{ J}$$

$$n = \frac{8.37 \times 10^6 \text{ J}}{(50.0 \text{ kg})(9.80 \text{ m/s}^2)(2.00 \text{ m})} = 8.54 \times 10^3 \text{ times}$$

If the student is in good shape and lifts the weight once every 5 s, it will take him about 12 h to perform this feat.

17.2 • SPECIFIC HEAT

The quantity of heat required to raise the temperature of a given mass of a substance by some amount varies with the substance. For example, the heat required to raise the temperature of 1 kg of water by 1.0°C is 4186 J, but the heat required to raise the temperature of 1 kg of copper by 1.0°C is only 387 J. Every substance requires a unique amount of heat to change the temperature of 1 kg of it by 1.0°C, and this number is a measure of the **specific heat** of the substance. Table 17.1 lists specific heats for several substances.

Suppose that a quantity Q of energy is transferred to mass m of a substance, thereby changing its temperature by ΔT. The **specific heat,** c, of the substance is defined as

Specific heat •

$$c \equiv \frac{Q}{m\,\Delta T} \qquad [17.2]$$

From this definition, we can express the energy transferred, Q, between a system of mass m and its surroundings for the temperature change ΔT as

[1]Originally, the calorie was defined as the heat necessary to raise the temperature of 1 g of water by 1°C. However, careful measurements showed that energy depends somewhat on temperature; hence, a more precise definition evolved.

TABLE 17.1 Specific Heats of Some Substances at 25.0°C and Atmospheric Pressure

Substance	Specific heat, c J/kg·°C	Specific heat, c cal/g·°C
Elemental Solids		
Aluminum	900	0.215
Beryllium	1830	0.436
Cadmium	230	0.055
Copper	387	0.0924
Germanium	322	0.077
Gold	129	0.0308
Iron	448	0.107
Lead	128	0.0305
Silicon	703	0.168
Silver	234	0.056
Other Solids		
Brass	380	0.092
Wood	1700	0.41
Glass	837	0.200
Ice (−5.0°C)	2090	0.50
Marble	860	0.21
Liquids		
Alcohol (ethyl)	2400	0.58
Mercury	140	0.033
Water (15.0°C)	4186	1.00

$$Q = mc\,\Delta T \qquad [17.3]$$

For example, the energy required to raise the temperature of 0.5 kg of water by 3.0°C is (0.5 kg)(4186 J/kg·°C)(3.0°C) = 6280 J. Note that when the temperature increases, ΔT and Q are *positive*, corresponding to energy flowing *into* the system. When the temperature decreases, ΔT and Q are *negative* and energy flows *out* of the system.

When specific heats are measured, the values obtained are also found to depend on the conditions of the experiment. In general, measurements made at constant pressure are different from those made at constant volume. For solids and liquids, the difference between the two values is usually no more than a few percent and is often neglected. For gases, the difference between the two values is significant.

Note from Table 17.1 that water has the highest specific heat of any substance with which we are likely to come in routine contact. The high specific heat of water is responsible for the moderate temperatures found in regions near large bodies of water. As the temperature of a body of water decreases during the winter, the

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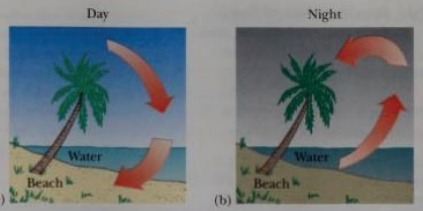


Figure 17.1 Circulation of air at the beach. (a) On a hot day, the air above the warm sand warms faster than the air above the cooler water. The cooler air over the water moving toward the beach displaces the rising warmer air. (b) At night, the sand cools more rapidly than the water and hence the air currents reverse their directions.

water transfers energy to the air, which carries the energy landward when prevailing winds are favorable. For example, the prevailing winds off the western coast of the United States are toward the land, and the energy liberated by the Pacific Ocean as it cools keeps coastal areas much warmer than they would be otherwise. This explains why the western coastal states generally have more favorable winter weather than the eastern coastal states, where the winds do not transfer energy toward land.

The fact that the specific heat of water is higher than that of sand is responsible for the pattern of air flow at a beach. During the day, the Sun adds roughly equal amounts of energy to beach and water, but the lower specific heat of sand causes the beach to reach a higher temperature than the water. The air above the land reaches a higher temperature than that above the water and the cold air pushes the hot air upward (due to Archimedes' principle). This results in a breeze from ocean to land during the day. Because the hot air gradually cools as it rises, it subsequently sinks, setting up the circulating pattern shown in Figure 17.1a. During the night, the sand cools more quickly than the water, and the circulating pattern reverses itself because the hotter air is now over the water (Fig. 17.1b). (The offshore and onshore breezes are certainly well known to sailors.)

Conservation of Energy: Calorimetry

Situations in which mechanical energy is converted to thermal energy occur frequently. We shall look at some in the examples following this section and in the problems at the end of the chapter, but most of our attention will be directed toward a particular kind of conservation-of-energy situation. In problems using the procedure we shall describe, called **calorimetry** problems, only the thermal energy transfer between the system and its surroundings is considered.

One technique for measuring the specific heat of a solid or liquid is simply to heat the substance to some known temperature, place it in a vessel containing water of known mass and temperature, and measure the temperature of the water after equilibrium is reached. Because a negligible amount of mechanical work is done in the process, the law of conservation of energy requires that the energy that leaves the warmer substance (of unknown specific heat) equals the energy that enters the water.[2] Devices in which this thermal energy transfer occurs are called **calorimeters.**

[2]For precise measurements, the container holding the water should be included in our calculations, because it also transfers thermal energy. This would require a knowledge of its mass and composition. However, if the mass of the water is large compared with that of the container, we can neglect the thermal energy gained by the container. Furthermore, precautions must be taken in such measurements to minimize thermal energy transfer between the system and the surroundings.

For example, suppose that m_x is the mass of a substance the specific heat of which we wish to determine, c_x its specific heat, and T_x its initial temperature. Let m_w, c_w, and T_w represent corresponding values for the water. If T is the final equilibrium temperature after everything is mixed, then from Equation 17.3 we find that the thermal energy gained by the water is $m_w c_w(T - T_w)$ and the thermal energy lost by the substance of unknown c is $-m_x c_x(T - T_x)$. Assuming that the combined system (water + unknown) does not lose or gain any thermal energy, it follows that the thermal energy gained by the water must equal the thermal energy lost by the unknown:

$$m_w c_w(T - T_w) = -m_x c_x(T - T_x)$$

Solving for c_x gives

$$c_x = \frac{m_w c_w(T - T_w)}{m_x(T_x - T)} \qquad [17.4]$$

Thinking Physics 1

The equation $Q = mc\,\Delta T$ indicates the relationship between energy, Q, transferred to an object of mass, m, and specific heat, c, by means of heat and the resulting temperature change, ΔT. In reality, the energy on the left-hand side of the equation could be transferred by any method, not just heat. Give a few examples in which this equation could be used to calculate a temperature change of an object but in which there is no heat involved.

Reasoning There are a number of examples, a few of which follow:

1. During the first few seconds after turning on a toaster, the temperature of the electrical coils rises. The transfer mechanism here is *electrical transmission* of energy through the power cord.
2. The temperature of a potato in a microwave oven increases due to the absorption of microwaves. In this case, the energy transfer mechanism is by *electromagnetic radiation*—the microwaves.
3. A bag of lead pellets is dropped to the floor many times, resulting in an increase in temperature of the pellets. The transfer mechanism here is *work* done during the deformation of the pellets as they strike the floor.
4. A carpenter attempts to use a dull drill bit to form a hole in a piece of wood. The bit fails to make much headway but becomes very warm. The increase in temperature in this case is due to *work* done on the bit by the wood.
5. The temperature of the fuel–air mixture in an automobile engine cylinder increases when the spark plug fires. In this case, the energy is already in the fuel–air mixture as electrical potential energy and is transformed to thermal energy in the mixture by the *chemical reaction* of combustion.

In each of these cases, as well as many others that could be generated, the Q on the left of the equation of interest is not a measure of heat but is replaced with the energy transferred or transformed by other means. Regardless of the fact that heat is not involved, the equation can still be used to calculate the temperature change.

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Example 17.2 **Cooling a Hot Ingot**

A 0.0500-kg ingot of metal is heated to 200.0°C and then dropped into a beaker containing 0.400 kg of water initially at 20.0°C. If the final equilibrium temperature of the mixed system is 22.4°C, find the specific heat of the metal.

Solution Because the thermal energy lost by the ingot equals the thermal energy gained by the water, we can write

$$m_w c_w (T_{wf} - T_{wi}) = -m_x c_x (T_{xf} - T_{xi})$$

$$(0.400\ \text{kg})(4186\ \text{J/kg}\cdot^\circ\text{C})(22.4^\circ\text{C} - 20.0^\circ\text{C}) = (0.0500\ \text{kg})(c_x)(200.0^\circ\text{C} - 22.4^\circ\text{C})$$

from which we find that

$$c_x = 453\ \text{J/kg}\cdot^\circ\text{C}$$

The ingot is most likely iron, as can be seen by comparing this result with the data in Table 17.1.

EXERCISE 1 What is the total thermal energy transferred to the water as the ingot is cooled? Answer 4020 J

EXERCISE 2 A cowboy fires a silver bullet of mass 2.00 g with a muzzle speed of 200 m/s into the pine wall of a saloon. Assume that all the internal energy generated by the impact remains with the bullet. What is the temperature change of the bullet? Answer 85.5°C

EXERCISE 3 How many calories of heat are required to raise the temperature of 3.0 kg of aluminum from 20°C to 50°C? Answer 19.4 kcal

17.3 · LATENT HEAT AND PHASE CHANGES

A substance usually undergoes a change in temperature when energy is transferred between the substance and its surroundings. There are situations, however, in which the transfer of energy does not result in a change in temperature. This is the case whenever the physical characteristics of the substance change from one form to another, commonly referred to as a **phase change.** Some common phase changes are solid to liquid (melting), liquid to gas (boiling), and a change in crystalline structure of a solid. All such phase changes involve a change in internal energy.

The thermal energy transfer required to change the phase of a given mass, m, of a pure substance is

$$Q = mL \qquad [17.5]$$

Latent heat •

where L is called the **latent heat** ("hidden" heat) of the substance and depends on the nature of the phase change as well as on the properties of the substance. **Latent heat of fusion,** L_f, is the term used when the phase change occurs during melting or freezing, and **latent heat of vaporization,** L_v, is the term used when the phase change occurs during boiling or condensing.[3] For example, the latent heat of fusion for water at atmospheric pressure is 3.33×10^5 J/kg, and the latent heat of vaporization of water is 2.26×10^6 J/kg. The latent heats of different substances vary considerably, as Table 17.2 illustrates.

[3]When a gas cools, it eventually returns to the liquid phase, or *condenses.* The thermal energy transfer per unit mass during the process is called the **latent heat of condensation,** and it equals in magnitude the latent heat of vaporization. When a liquid cools, it eventually solidifies, and the thermal energy transfer per unit mass during the process is called the **latent heat of fusion.**

TABLE 17.2 Latent Heats of Fusion and Vaporization

Substance	Melting Point (°C)	Latent Heat of Fusion (J/kg)	Boiling Point (°C)	Latent Heat of Vaporization (J/kg)
Helium	−269.65	5.23×10^3	−268.93	2.09×10^4
Nitrogen	−209.97	2.55×10^4	−195.81	2.01×10^5
Oxygen	−218.79	1.38×10^4	−182.97	2.13×10^5
Ethyl alcohol	−114	1.04×10^5	78	8.54×10^5
Water	0.00	3.33×10^5	100.00	2.26×10^6
Sulfur	119	3.81×10^4	444.60	3.26×10^5
Lead	327.3	2.45×10^4	1750	8.70×10^5
Aluminum	660	3.97×10^5	2450	1.14×10^7
Silver	960.80	8.82×10^4	2193	2.33×10^6
Gold	1063.00	6.44×10^4	2660	1.58×10^6
Copper	1083	1.34×10^5	1187	5.06×10^6

Consider, for example, the thermal energy required to convert a 1-g block of ice at −30.0°C to steam (water vapor) at 120.0°C. Figure 17.2 indicates the experimental results obtained when energy is gradually added to the ice. Let us examine each portion of the curve separately.

Part A. During this portion of the curve, we are changing the temperature of the ice from −30.0°C to 0.0°C. Because the specific heat of ice is 2090 J/kg·°C, we can calculate the amount of energy added from Equation 17.3:

$$Q = m_i c_i \Delta T = (1.00 \times 10^{-3}\ \text{kg})(2090\ \text{J/kg}\cdot^\circ\text{C})(30.0^\circ\text{C}) = 62.7\ \text{J}$$

Part B. When the ice reaches 0.0°C, the ice–water mixture remains at this temperature—even though energy is being added—until all the ice melts to become water at 0.0°C. The energy required to melt 1.00 g of ice at 0.0°C is, from Equation 17.5,

$$Q = mL_f = (1.00 \times 10^{-3}\ \text{kg})(3.33 \times 10^5\ \text{J/kg}) = 333\ \text{J}$$

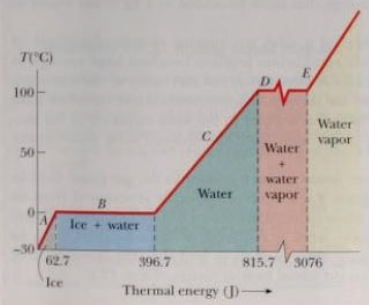


Figure 17.2 A plot of temperature versus thermal energy added when 1 g of ice initially at −30.0°C is converted to steam.

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Part C. Between 0.0°C and 100.0°C, nothing surprising happens. No phase change occurs in this region. The energy added to the water is being used to increase its temperature. The amount of energy necessary to increase the temperature from 0.0°C to 100.0°C is

$$Q = m_w c_w \Delta T = (1.00 \times 10^{-3}\ \text{kg})(4.19 \times 10^3\ \text{J/kg}\cdot\text{°C})(100.0\text{°C}) = 4.19 \times 10^2\ \text{J}$$

Part D. At 100.0°C, another phase change occurs as the water changes from water at 100.0°C to steam at 100.0°C. Just as in Part B, the water–steam mixture remains at 100.0°C—even though energy is being added—until all the liquid has been converted to steam. The energy required to convert 1.00 g of water to steam at 100.0°C is

$$Q = mL_v = (1.00 \times 10^{-3}\ \text{kg})(2.26 \times 10^6\ \text{J/kg}) = 2.26 \times 10^3\ \text{J}$$

Part E. On this portion of the curve, energy is being added to the steam with no phase change occurring. The energy that must be added to raise the temperature of the steam to 120.0°C is

$$Q = m_s c_s \Delta T = (1.00 \times 10^{-3}\ \text{kg})(2.01 \times 10^3\ \text{J/kg}\cdot\text{°C})(20.0\text{°C}) = 40.2\ \text{J}$$

The total amount of energy that must be added to change 1 gram of ice at −30.0°C to steam at 120.0°C is therefore about 3.11×10^3 J. Conversely, to cool 1 gram of steam at 120.0°C down to the point at which we have ice at −30.0°C, we must remove 3.11×10^3 J of energy.

Phase changes can be described in terms of a rearrangement of molecules when energy is added to or removed from a substance. Consider first the liquid-to-gas phase change. The molecules in a liquid are close together, and the forces between them are stronger than those between the molecules of a gas, which are far apart. Therefore, work must be done on the liquid, at the molecular level, against these attractive molecular forces in order to separate the molecules. The latent heat of vaporization is the amount of energy that must be added to 1 kg of the liquid to accomplish this.

Similarly, at the melting point of a solid, we imagine that the amplitude of vibration of the atoms about their equilibrium position becomes large enough to allow the atoms to pass the barriers of adjacent atoms and move to their new positions. The new locations are, on the average, less symmetrical and therefore have higher energy. The latent heat of fusion is equal to the work required, at the molecular level, to transform 1 kg of the mass from the ordered solid phase to the disordered liquid phase.

The average distance between atoms is much greater in the gas phase than in either the liquid or the solid phase. Each atom or molecule is removed from its neighbors, without the compensation of attractive forces to new neighbors. Therefore, it is not surprising that more work is required, at the molecular level, to vaporize a given mass of substance than to melt it, and that the latent heat of vaporization is much greater than the latent heat of fusion (Table 17.2).

Thinking Physics 2

The intuitive result of adding energy to a substance is to observe an increase in temperature. But if energy is added to ice at 0°C, its temperature does not increase. Where does the energy go?

Reasoning When energy is added to ice at, say, −10°C, much of the energy goes into increased molecular vibration, which has a physical manifestation of an increased temperature. When energy is added to ice at 0°C, the temperature of the ice remains constant and the ice melts. Much of the energy goes into increasing the electrical potential energy and breaking the bonds between molecules. This is similar to adding energy to a rock–Earth system by throwing the rock so fast that it escapes the Earth—the gravitational bond between the rock and the Earth has been broken. Breaking the bonds between the water molecules in the ice allows the molecules to move with respect to each other, which we recognize as the behavior of the liquid that results from the melting.

PROBLEM-SOLVING STRATEGY • Calorimetry Problems

If you are having difficulty with calorimetry problems, consider the following factors.

1. Be sure your units are consistent throughout. For instance, if you are using specific heats in cal/g·°C, be sure that masses are in grams and temperatures are Celsius throughout.
2. Losses and gains in energy are found by using $Q = mc\,\Delta T$ only for those intervals in which no phase changes are occurring. The equations $Q = mL_f$ and $Q = mL_v$ are to be used only when phase changes *are* taking place.
3. Often sign errors occur in heat loss = heat gain equations. One way to check your equation is to examine the signs of all ΔT's that appear in it.

Example 17.3 Boiling Liquid Helium

Liquid helium has a very low boiling point, 4.2 K, and a very low latent heat of vaporization, 2.09×10^4 J/kg (Table 17.2). A constant power of 10.0 W is transferred to a container of liquid helium from an immersed electric heater. At this rate, how long does it take to boil away 1.00 kg of liquid helium at 4.2 K?

Reasoning and Solution Because $L_v = 2.09 \times 10^4$ J/kg for liquid helium, we must supply 2.09×10^4 J of energy to boil away 1.00 kg. The power supplied to the helium is 10.0 W = 10.0 J/s. That is, in 1.00 s, 10.0 J of energy is transferred to the helium. Therefore, the time it takes to transfer an energy of 2.09×10^4 J is

$$t = \frac{2.09 \times 10^4\ \text{J}}{10.0\ \text{J/s}} = 2.09 \times 10^3\ \text{s} \approx 35\ \text{min}$$

EXERCISE 4 If 10.0 W of power is supplied to 1.00 kg of water at 100.0°C, how long will it take for the water to completely boil away? Answer 62.8 h

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17.4 · WORK AND THERMAL ENERGY IN THERMODYNAMIC PROCESSES

In the macroscopic approach to thermodynamics, we describe the *state* of a system with such variables as pressure, volume, temperature, and internal energy. The number of macroscopic variables needed to characterize a system depends on the system's nature. For a homogeneous system, such as a gas containing only one type of molecule, usually only two variables are needed. However, it is important to note that a *macroscopic state* of an isolated system can be specified only if the system is in thermal equilibrium internally. In the case of a gas in a container, internal thermal equilibrium requires that every part of the container be at the same pressure and temperature.

Consider gas contained in a cylinder fitted with a movable piston (Fig. 17.3). In equilibrium, the gas occupies a volume V and exerts a uniform pressure P on the cylinder walls and piston. If the piston has a cross-sectional area of A, the force exerted by the gas on the piston is $F = PA$. Now let us assume that the gas expands **quasi-statically**—that is, slowly enough to allow the system to remain essentially in thermodynamic equilibrium at all times. As the piston moves up a distance of dy, the work done by the gas on the piston is

$$dW = F\,dy = PA\,dy$$

Because $A\,dy$ is the increase in volume of the gas dV, we can express the work done as

$$dW = P\,dV \qquad [17.6]$$

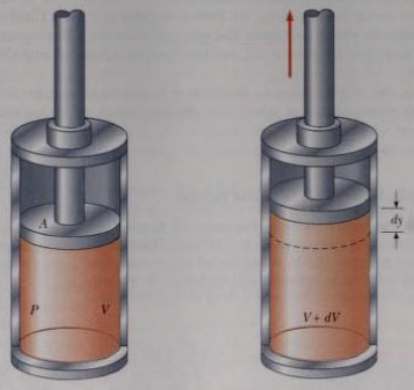


Figure 17.3 Gas contained in a cylinder at a pressure P does work on a moving piston as the system expands from a volume V to a volume $V + dV$.

Because the gas expands, dV is positive and the work done by the gas is positive, whereas if the gas is compressed, dV is negative, indicating that the work done by the gas is negative. (In the latter case, negative work can be interpreted as being work done *on* the gas.) Clearly, the work done by the gas is zero when the volume remains constant. The total work done by the gas as its volume changes from V_i to V_f is given by the integral of Equation 17.6:

$$W = \int_{V_i}^{V_f} P\,dV \qquad [17.7]$$

To evaluate this integral, one must know how the pressure varies during the expansion process. (Note that a *process* is *not* specified merely by the initial and final states. Rather, a process is a *fully specified* change in state of a system.) In general, the pressure is not constant throughout the expansion process but depends on the volume and temperature. If the pressure and volume are known at each step of the process, the states of the gas can be represented as a curve on a PV diagram, as in Figure 17.4. The work done in the expansion from the initial state to the final state is the area under the curve.

As Figure 17.4 shows, the work done in the expansion from the initial state to the final state depends on the path taken between the two states. To illustrate this important point, consider several different paths connecting i and f (Fig. 17.5). In the process depicted in Figure 17.5a, the pressure of the gas is first reduced from P_i to P_f by cooling at constant volume, V_i, and the gas then expands from V_i to V_f at constant pressure, P_f. The work done along this path is $P_f(V_f - V_i)$. In Figure 17.5b, the gas first expands from V_i to V_f at constant pressure, P_i, and then its pressure is reduced to P_f at constant volume, V_f. The work done along this path is $P_i(V_f - V_i)$, which is greater than that for the process described in Figure 17.5a. Finally, for the process described in Figure 17.5c, where both P and V change continuously, the work done has some value intermediate between the values obtained in the first two processes. To evaluate the work in this case, the shape of the PV curve must be known. Therefore, we see that the work done by a system depends on the process by which the system goes from the initial to the final state. In other words, the work done depends on the initial, final, and intermediate states of the system.

In a similar manner, the thermal energy transferred into or out of the system is also found to depend on the process. This can be demonstrated by the situations

- ***Work equals area under the curve in a PV diagram***

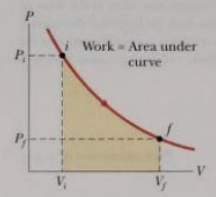


Figure 17.4 A gas expands reversibly (slowly) from state i to state f. The work done by the gas equals the area under the PV curve.

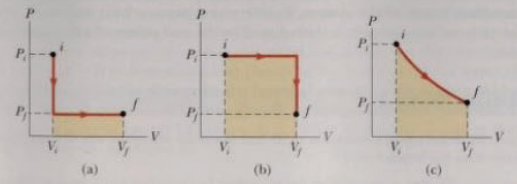


Figure 17.5 The work done by a gas as it is taken from an initial state to a final state depends on the path between these states.

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This device, called Hero's engine, was invented around 150 B.C. by Hero in Alexandria. When water is boiled in the flask, which is suspended by a cord, steam exits through two tubes at the sides of the flask (in opposite directions), creating a torque that rotates the flask. *(Courtesy of Central Scientific Co.)*

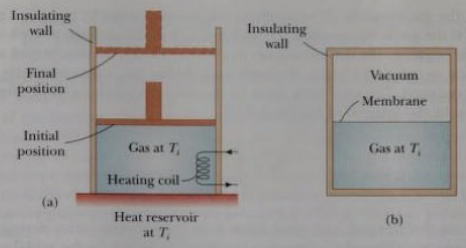


Figure 17.6 (a) A gas at temperature T_i expands slowly by absorbing thermal energy from a reservoir at the same temperature. (b) A gas expands rapidly into an evacuated region after a membrane is broken.

Free expansion of a gas

depicted in Figure 17.6. In each case, the gas has the same initial volume, temperature, and pressure and is assumed to be ideal. In Figure 17.6a, the gas is in thermal contact with a heat reservoir, and a small heating coil allows energy to be transferred to the gas by heat from the surface of the coil. As an infinitesimal amount of energy enters the gas from the coil, the pressure of the gas becomes infinitesimally greater than atmospheric pressure, and the gas expands, causing the piston to rise. During this expansion to some final volume, V_f, and final pressure P_f, sufficient thermal energy to maintain a constant temperature of T_i is transferred from the coil to the gas.

Now consider the thermally insulated system shown in Figure 17.6b. When the membrane is broken, the gas expands rapidly into the vacuum until it occupies a volume of V_f and is at a pressure of P_f. In this case, the gas does no work, because there is no movable piston. Furthermore, no energy is transferred through the thermally insulated wall, which we call an *adiabatic wall*, and so the temperature remains at T_i. This process is often referred to as **adiabatic free expansion** or simply **free expansion.** In general, **an adiabatic process is one in which no thermal energy is transferred between the system and its surroundings.**

Heat depends on the path between the initial and final states

The initial and final states of the ideal gas in Figure 17.6a are identical to the initial and final states in Figure 17.6b, but the paths are different. In the first case, thermal energy is transferred slowly to the gas, and the gas does work on the piston. In the second case, no thermal energy is transferred and the work done is zero. Therefore, we conclude that **heat, like work, depends on the initial, final, and intermediate states of the system.** Furthermore, because heat and work depend on the path, neither quantity is determined by the end points of a thermodynamic process.

Thinking Physics 3

Why is air at the top of a mountain cold? And why is the windward side of a mountain rainier than the leeward side?

Reasoning Imagine a parcel of air being pushed by prevailing winds from a valley to the top of a mountain. Air is a very poor thermal conductor, so the thermodynamic process that occurs in this parcel of air is close to adiabatic. As the parcel rises, it moves into regions of lower atmospheric pressure. As a result, the parcel expands. By expanding, it is doing work on the surrounding air. This represents a transfer of energy out of the system, so that the internal energy drops. Thus, the parcel of air becomes cold as it rises up the mountain. If the parcel of air contains water vapor, the drop in temperature as it rises up the mountain can result in the condensation into raindrops. Thus, the rain tends to fall more on the windward side as the temperature drops. On the leeward side, much of the water vapor has already condensed out as rain, and the air parcel is warming up again as it descends the mountain. Thus, there is little rain on the leeward side of the mountain.

An interesting example of this occurs in Hawaii. The prevailing winds are often from the northeast. The island of Kauai has a very tall mountain (Mount Waialeale, 1548 m), and it is one of the rainiest places on Earth, with an average annual rainfall of 1170 cm. Just to the southwest of Kauai is the island of Niihau, which is very dry and requires a system of irrigation for its farming. The leeward position of Niihau relative to Kauai is the reason for this.

17.5 • THE FIRST LAW OF THERMODYNAMICS

When the law of conservation of energy was first introduced in Chapter 7, it was stated that the mechanical energy of a system is constant in the absence of nonconservative forces, such as friction. That is, the changes in the internal energy of the system were not included in this mechanical model. The first law of thermodynamics is a generalization of the law of conservation of energy and encompasses possible changes in internal energy. It is a universally valid law that can be applied to all kinds of processes. Furthermore, it provides us with a connection between the microscopic and macroscopic worlds.

We have seen that energy can be transferred between a system and its surroundings in two ways. One is work done by (or on) the system that requires that there be a macroscopic displacement of the point of application of a force (or pressure). The other mode is heat, which occurs through random molecular collisions. Each of these represents a change of energy of the system, and therefore usually results in measurable changes in the macroscopic variables of the system, such as pressure, temperature, and volume of a gas.

To put these ideas on a more quantitative basis, suppose a system undergoes a change from an initial state to a final state. During this change, positive Q is the heat transferred *to* the system, and positive W is the work done *by* the system. As an example, suppose the system is a gas the pressure and volume of which change from P_i, V_i to P_f, V_f. If the quantity $Q - W$ is measured for various paths connecting the initial and final equilibrium states (that is, for various *processes*), one finds that it is the same for *all* paths connecting the initial and final states. We conclude that the quantity $Q - W$ is determined completely by the initial and final states of the system, and we call it the **change in the internal energy of the system.** Although Q and W both depend on the path, $\boldsymbol{Q - W}$ **is independent of the path.** If we represent the internal energy function with the letter U, then the *change* in internal energy, ΔU, can be expressed as

$$\Delta U = Q - W \qquad [17.8]$$

• *First-law equation*

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where all quantities must have the same energy units. Equation 17.8 is known as the **first-law equation** and is a key equation to many applications.

When a system undergoes an infinitesimal change in state, in which a small amount of heat dQ is transferred and a small amount of work dW is done, the energy also changes by a small amount dU. Thus, for infinitesimal processes we can express the first-law equation as[4]

First-law equation for infinitesimal changes

$$dU = dQ - dW \qquad [17.9]$$

In thermodynamics, we do not concern ourselves with the specific form of the internal energy. An analogy can be made between the internal energy of a system and the potential energy function associated with a body moving under the influence of gravity without friction. The potential energy function is independent of the path, and it is only the changes in it that are of concern. Likewise, the change in internal energy of a thermodynamic system is what matters, because only differences are defined. Because absolute values are not defined, any reference state can be chosen for the internal energy.

Let us look at some special cases in which the only changes in energy will be changes in internal energy. First consider an **isolated system**—that is, one that does not interact with its surroundings. In this case, no heat is added or removed from the system and the work done is zero; hence, the internal energy remains constant. That is, because $Q = W = 0$, then $\Delta U = 0$, and we conclude that **the internal energy of an isolated system remains constant.**

For isolated systems, U remains constant

Next consider the case in which a system (one not isolated from its surroundings) is taken through a **cyclic process**—that is, one that originates and ends at the same state. In this case, the change in the internal energy must again be *zero* and therefore the heat added to the system must equal the work done during the cycle. That is, in a cyclic process,

Cyclic process

$$\Delta U = 0 \quad \text{and} \quad Q = W$$

Note that **the net work done per cycle equals the area enclosed by the path representing the process on a *PV* diagram.** As we shall see in Chapter 18, cyclic processes are very important in describing the thermodynamics of **heat engines**—devices in which some part of the heat added to the system is extracted as mechanical work.

If a process occurs in which the work done is zero, then the change in internal energy equals the heat entering or leaving the system. If heat enters the system, Q is positive and the internal energy increases. For a gas, we can associate this increase in internal energy with an increase in the kinetic energy of the molecules. However, if a process occurs in which the heat transferred is zero and work is done by the system, then the magnitude of the change in internal energy equals the negative of the work done by the system. That is, the internal energy of the system decreases. For example, if a gas is compressed (by a moving piston, say), with no transfer of heat, the work done by the gas is negative and the internal energy again increases.

[4]It should be noted that dQ and dW are not true differential quantities, although dU is a true differential. In fact, dQ and dW are not differentials of any definable quantity. For further details on this point, see R. P. Bauman, *Modern Thermodynamics and Statistical Mechanics*, New York, Macmillan, 1992.

No practical distinction exists between heat and work on a microscopic scale. Each can produce a change in the internal energy of a system. Although the macroscopic quantities Q and W are *not* properties of a system, they are related to changes of the internal energy of a stationary system through the first-law equation. Once a process, or path, is defined, Q and W can be either calculated or measured, and the change in internal energy can be found from the first-law equation. One of the important consequences of the first law is the existence of a quantity we called internal energy, the value of which is determined by the state of the system. The internal energy function is therefore called a **state function.**

Thinking Physics 4

If an ideal gas undergoes a free expansion (an expansion into a vacuum) in an insulated container, it does no work, because it does not have to push anything out of the way. There is also no heat exchanged with its surroundings, because the container is insulated. Thus, the first law tells us that the internal energy and the temperature remain constant. What about a gas of positively charged ions? If it undergoes a free expansion in an insulated container, what happens to the temperature of this gas?

Reasoning In an ideal gas, it is assumed that there are no interactions between molecules (except during collisions), so that the energy of the gas does not depend on the intermolecular separation. For a gas of charged particles, however, the energy does depend on intermolecular separation, because there is electrical potential energy associated with the electrical forces between the ions. If the charged gas undergoes a free expansion, no work is done (by pushing anything out of the way), and there is no heat exchange. The overall electrical potential energy of the gas has decreased, because the particles have moved farther apart. As a result, the average kinetic energy of the particles must increase, and, in turn, the temperature of the gas will rise.

Thinking Physics 5

In the late 1970s, casino gambling was approved in Atlantic City, New Jersey, which can become quite cold in the winter. Energy projections that were performed for the design of the casinos showed that the air conditioning would need to operate in the casino even in the middle of a very cold January. Why?

Reasoning If we consider the air in the casino to be the gas to which we apply the first law, imagine that there is no air conditioning and no ventilation, so that this gas simply stays in the room. There is no work being done by the gas, so we focus on the heat input. A casino contains a large number of people, many of whom are active (throwing dice, cheering, etc.), and many of whom are in excited states (celebration, frustration, panic, etc.). As a result, these people have large rates of energy flow by heat from their bodies. This energy goes into the air of the casino, resulting in an increase in internal energy of the air. With the large number of excited people in a casino (along with the large number of incandescent lights and machines), the temperature of the gas can rise quickly and to a very high value. In order to keep the temperature at a reasonable value, there must be an energy transfer out of the air. Calculations show that conduction through the walls even in a 10°F January is not sufficient to maintain the required energy transfer, so the air conditioning system must be in almost continuous use throughout the year.

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17.6 • SOME APPLICATIONS OF THE FIRST LAW OF THERMODYNAMICS

In order to apply the first law of thermodynamics to specific systems, it is useful to first define some common thermodynamic processes. As we learned in Section 17.4, an **adiabatic process** is defined as one in which no heat enters or leaves the system—that is, $Q = 0$. An adiabatic process can be achieved either by thermally insulating the system from its surroundings (as in Fig. 17.6b) or by performing the process rapidly. Applying the first-law equation in this case, we see that

Adiabatic process

First-law equation applied to an adiabatic process

$$\Delta U = -W \qquad [17.10]$$

From this result, we see that when a gas expands adiabatically, W is positive and ΔU is negative. Conversely, when the gas is compressed adiabatically, ΔU is positive.

Adiabatic processes are very important in engineering practice. Common applications include the expansion of hot gases in an internal combustion engine, the liquefaction of gases in a cooling system, and the compression stroke in a diesel engine.

The **free expansion** depicted in Figure 17.6b is an adiabatic process in which no work is done on or by the gas. Because $Q = 0$ and $W = 0$, we see from the first law that $\Delta U = 0$ for this process. That is, **the initial and final internal energies of a gas are equal in an adiabatic free expansion.** As we shall see in Chapter 18, the internal energy of an ideal gas depends only on its temperature. Thus, we would expect no change in temperature during an adiabatic free expansion. This is in accord with experiments performed at low pressures. Careful experiments with real gases at high pressures show a slight increase or decrease in temperature after the expansion.

Isobaric process (constant pressure)

A process that occurs at constant pressure is called an **isobaric process.** When such a process occurs, energy is transferred by heat and the work done is simply the pressure multiplied by the change in volume, or $P(V_f - V_i)$.

Isovolumetric process (constant volume)

A process that takes place at constant volume is called an **isovolumetric process.** In such a process, the work done is clearly zero. Hence, from the first-law equation we see that, in an isovolumetric process,

First-law equation applied to an isovolumetric process

$$\Delta U = Q \qquad [17.11]$$

This equation tells us that **if heat is added to a system kept at constant volume, all of the heat goes into increasing the internal energy of the system.** For example, when a can of deodorant is thrown into a fire, heat enters the system (the gas in the can) by conduction through the metal walls of the can. As a consequence, the temperature and pressure in the can rise until the can possibly explodes.

Isothermal process (constant temperature)

A process that occurs at constant temperature is called an **isothermal process.** The internal energy of an ideal gas is a function of temperature only. Hence, in an isothermal process of a stationary ideal gas, $\Delta U = 0$.

Isothermal Expansion of an Ideal Gas

Suppose an ideal gas is allowed to expand quasi-statically at constant temperature as described by the PV diagram in Figure 17.7. (Isothermal expansion can be achieved by placing the gas in good thermal contact with a reservoir at the same temperature and carrying out the expansion very slowly, as in Figure 17.6a.) The curve is a hyperbola with the equation PV = constant. Let us calculate the work done by the gas in the expansion from state i to state f.

This work is given by Equation 17.7. Because the gas is ideal and the process is quasi-static, we can apply $PV = nRT$ for each point on the path. Therefore, we have

$$W = \int_{V_i}^{V_f} P\,dV = \int_{V_i}^{V_f} \frac{nRT}{V}\,dV$$

Because T is constant in this case, it can be removed from the integral, along with n and R:

$$W = nRT \int_{V_i}^{V_f} \frac{dV}{V}$$

To evaluate the integral, we use $\int \frac{dx}{x} = \ln x$. Thus, we find

$$W = nRT \ln\left(\frac{V_f}{V_i}\right) \qquad [17.12]$$

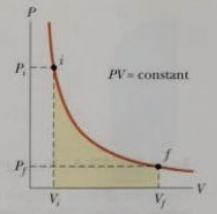


Figure 17.7 The PV diagram for an isothermal expansion of an ideal gas from an initial state to a final state. The curve is a hyperbola.

Work done in an isothermal process

Numerically, this work equals the shaded area under the PV curve in Figure 17.7. Because the gas expands, $V_f > V_i$, and the work done by the gas is positive, as we would expect. If the gas is compressed, then $V_f < V_i$, and the work done by the gas is negative. In the next chapter we shall find that the internal energy of an ideal gas depends only on temperature. Hence, for an isothermal process $\Delta U = 0$, and from the first law we conclude that the heat transferred from the reservoir to the gas equals the work done by the gas, or $Q = W$.

The Boiling Process

Suppose that a liquid of mass m vaporizes at constant pressure P. Its volume in the liquid state is V_ℓ, and its volume in the vapor state is V_v. Let us find the work done in the expansion and the change in internal energy of the system.

Because the expansion takes place at constant pressure, the work done by the system is

$$W = \int_{V_\ell}^{V_v} P\,dV = P\int_{V_\ell}^{V_v} dV = P(V_v - V_\ell)$$

The heat that must be transferred to the liquid to vaporize all of it is equal to $Q = mL_v$, where L_v is the heat of vaporization of the liquid. Using the first-law equation and the preceding result, we get

$$\Delta U = Q - W = mL_v - P(V_v - V_\ell) \qquad [17.13]$$

CONCEPTUAL PROBLEM 1

Suppose an isothermal process occurs for a dilute (or low density) gas, during which work W is done. How much energy was transferred by heat?

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CONCEPTUAL PROBLEM 2

Consider the human body performing a strenuous exercise, such as lifting weights or riding a bicycle. Work is being done by the body, and energy is leaving by conduction from the skin into the surrounding air. According to the first law of thermodynamics, the temperature of the body should be decreasing steadily during the exercise. This is not what happens, however. Is the first law invalid in this situation? Explain.

Example 17.4 Boiling Water

One gram of water occupies a volume of 1.00 cm^3 at atmospheric pressure. When this amount of water is boiled, it becomes 1671 cm^3 of steam. Calculate the change in internal energy for this process.

Solution Because the latent heat of vaporization of water is 2.26×10^6 J/kg at atmospheric pressure, the heat required to boil 1.00 g is

$$Q = mL_v = (1.00 \times 10^{-3}\ \text{kg})(2.26 \times 10^6\ \text{J/kg}) = 2260\ \text{J}$$

The work done by the system is positive and equal to

$$W = P(V_v - V_i)$$
$$= (1.013 \times 10^5\ \text{N/m}^2)[(1671 - 1.00) \times 10^{-6}\ \text{m}^3]$$
$$= 169\ \text{J}$$

Hence, the change in internal energy is

$$\Delta U = Q - W = 2260\ \text{J} - 169\ \text{J} = 2.09\ \text{kJ}$$

The internal energy of the system increases because ΔU is positive. We see that most (93%) of the thermal energy transferred to the liquid goes into increasing the internal energy. Only 7% goes into external work.

OPTIONAL

17.7 · HEAT TRANSFER



Melted snow patterns on a parking lot indicate the presence of underground hot water pipes used to aid snow removal. Heat from the water is conducted to the pavement from the pipes, causing the snow to melt. *(Courtesy of Dr. Albert A. Bartlett, University of Colorado, Boulder)*

In practice, it is important to understand the rate at which energy is transferred between a system and its surroundings and the mechanisms responsible for the transfer. You may have used a Thermos bottle or some other thermally insulated vessel to store hot coffee for a length of time. The vessel reduces energy transfer between the outside air and the hot coffee. Ultimately, of course, the liquid will reach air temperature because the vessel is not a perfect insulator. There is no energy transfer between a system and its surroundings when they are at the same temperature.

Heat Conduction

The easiest heat transfer process to describe quantitatively is called **conduction.** In this process, the transfer mechanism can be viewed on an atomic scale as an exchange of kinetic energy between molecules, where the less energetic particles gain energy by colliding with the more energetic particles. For example, if you insert a metallic bar into a flame while holding one end, you will find that the temperature of the metal in your hand increases. The energy reaches your hand through conduction. The manner in which energy is transferred from the flame, through the bar, and to your hand can be understood by examining what is happening to the atoms and electrons of the metal. Initially, before the rod is inserted into the flame, the metal atoms and electrons are vibrating about their equilibrium positions. At the surface of the rod, rapidly moving gas molecules in the flame collide with atoms at the surface of the metal, transferring energy into the metal by conduction. Those metal atoms and electrons near the flame begin to vibrate with larger and larger amplitudes. These, in turn, collide with their neighbors and transfer some of their

energy in the collisions. Slowly, metal atoms and electrons farther down the rod increase their amplitude of vibration, until the large-amplitude vibrations arrive at the end being held. The effect of this increased vibration is an increase in temperature of the metal and possibly a burned hand.

Although the transfer of energy through a metal can be partially explained by atomic vibrations and electron motion, the rate of conduction also depends on the properties of the substance being heated. For example, it is possible to hold a piece of asbestos in a flame indefinitely. This implies that very little energy is being conducted through the asbestos. In general, metals are good thermal conductors, and materials such as asbestos, cork, paper, and fiberglass are poor conductors. Gases also are poor thermal conductors because of their dilute nature. Metals are good thermal conductors because they contain large numbers of electrons that are relatively free to move through the metal and can transport energy from one region to another. Thus, in a good conductor, such as copper, conduction takes place via the vibration of atoms and via the motion of free electrons.

Conduction occurs only if there is a difference in temperature between two parts of the conducting medium. Consider a slab of material of thickness Δx and cross-sectional area A with its opposite faces at different temperatures T_1 and T_2, where $T_2 > T_1$ (Fig. 17.8). It is found from experiment that the energy Q transferred in a time Δt flows from the hotter end to the colder end. The rate of heat transfer, $Q/\Delta t$, is found to be proportional to the cross-sectional area and the temperature difference, and inversely proportional to the thickness:

$$\frac{Q}{\Delta t} \propto A \frac{\Delta T}{\Delta x}$$

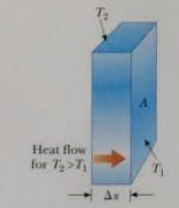


Figure 17.8 Heat transfer through a conducting slab of cross-sectional area A and thickness Δx. The opposite faces are at different temperatures T_1 and T_2.

It is convenient to use the symbol H to represent the rate of heat transfer. That is, we take $H = Q/\Delta t$. Note that H has units of watts when Q is in joules and Δt is in seconds. For a slab of infinitesimal thickness dx and temperature difference dT, we can write the **law of heat conduction**

$$H = -kA \frac{dT}{dx} \qquad [17.14]$$

where the proportionality constant k is called the **thermal conductivity** of the material, and dT/dx is the **temperature gradient** (the variation of temperature with position). The minus sign in Equation 17.14 denotes the fact that energy transfers as heat in the direction of decreasing temperature.

Suppose a substance is in the shape of a long uniform rod of length L, as in Figure 17.9, and is insulated so that heat cannot escape from its surface except at the ends, which are in thermal contact with reservoirs having temperatures T_1 and T_2. When a steady state has been reached, the temperature at each point along the rod is constant in time. In this case, the temperature gradient is the same everywhere along the rod and is

$$\frac{dT}{dx} = \frac{T_1 - T_2}{L}$$

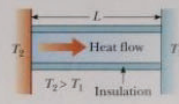


Figure 17.9 Conduction of heat through a uniform, insulated rod of length L. The opposite ends are in thermal contact with reservoirs at different temperatures.

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Thus the rate of heat transfer is

$$H = kA\frac{(T_2 - T_1)}{L} \qquad [17.15]$$

Substances that are good thermal conductors have large thermal conductivity values, whereas good thermal insulators have low thermal conductivity values. Table 17.3 lists thermal conductivities for various substances. We see that metals are generally better thermal conductors than nonmetals.

For a compound slab containing several materials of thicknesses L_1, L_2, . . . and thermal conductivities k_1, k_2, . . . , the rate of heat transfer through the slab at steady state is

$$H = \frac{A(T_2 - T_1)}{\Sigma_i(L_i/k_i)} \qquad [17.16]$$

where T_1 and T_2 are the temperatures of the outer extremities of the slab (which are held constant) and the summation is over all slabs.

TABLE 17.3 Thermal Conductivities of Some Substances

Substance	Thermal Conductivity (W/m·°C)
Metals (at 25°C)	
Aluminum	238
Copper	397
Gold	314
Iron	79.5
Lead	34.7
Silver	427
Gases (at 20°C)	
Air	0.0234
Helium	0.138
Hydrogen	0.172
Nitrogen	0.0234
Oxygen	0.0238
Nonmetals (approximate values)	
Asbestos	0.08
Concrete	0.8
Diamond	2300
Glass	0.8
Ice	2
Water	0.6
Wood	0.08

Convection

At one time or another you probably have warmed your hands by holding them over an open flame. In this situation, the air directly above the flame is heated and expands. As a result, the density of the air decreases and the air rises. This warmed mass of air heats your hands as it flows by. **Thermal energy transferred by the movement of a heated substance is said to have been transferred by convection.** When the movement results from differences in density, as in the example of air around a fire, it is referred to as **natural convection.** When the heated substance is forced to move by a fan or pump, as in some hot-air and hot-water heating systems, the process is called **forced convection.**

The circulating pattern of air flow at a beach is an example of convection. Likewise, the mixing that occurs as water is cooled and eventually freezes at its surface (Chapter 16) is an example of convection in nature. Recall that the mixing by convection currents ceases when the water temperature reaches 4°C. Because the water in a lake cannot be cooled by convection below 4°C, and because water is a relatively poor thermal conductor, the water near the bottom remains near 4°C for a long time. As a result, fish have a comfortable temperature in which to live even in periods of prolonged cold weather.

If it were not for convection currents, it would be difficult to boil water. As water is heated in a tea kettle, the lower layers are warmed first. These heated regions expand and rise to the top because their density is lowered. At the same time, the denser cool water falls to the bottom of the kettle so that it can be heated.

The same process occurs when a room is heated by a stove. The stove warms the air in the lower regions of the room. The warm air expands and rises to the ceiling because of its lower density. The denser regions of cooler air from above replace the warm air, setting up the continuous air current pattern shown in Figure 17.10.



Figure 17.10 Convection currents are set up in a room heated by a wood-burning stove.

Radiation

The third way of transferring energy is through **radiation.** All objects radiate energy continuously in the form of electromagnetic waves. Through electromagnetic radiation, approximately 1340 J of energy from the Sun strikes 1 m^2 of the top of the Earth's atmosphere every second. Some of this energy is reflected back into space and some is absorbed by the atmosphere, but enough arrives at the surface of the Earth each day to supply all of our energy needs on this planet hundreds of times over—if it could be captured and used efficiently. The growth in the number of solar houses in this country is one example of an attempt to make use of this free energy.

Radiant energy from the Sun affects our day-to-day existence in a number of ways. It influences the Earth's average temperature, ocean currents, agriculture, rain patterns, and so on. For example, consider what happens to the atmospheric temperature at night. If there is a cloud cover above the Earth, the water vapor in the clouds reflects back a part of the infrared radiation emitted by the Earth and consequently the temperature remains at moderate levels. In the absence of this cloud cover, however, there is nothing to prevent this radiation from escaping into space, and thus the temperature drops more on a clear night than when it is cloudy.

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The rate at which an object emits radiant energy is proportional to the fourth power of its absolute temperature. This is known as **Stefan's law** and is expressed in equation form as

$$P = \sigma A e T^4 \qquad [17.17]$$

where P is the power radiated by the body in watts, σ is a constant equal to $5.6696 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$, A is the surface area of the object in square meters, e is a constant called the **emissivity,** and T is temperature in kelvin. The value of e can vary between zero and unity, depending on the properties of the surface.

An object radiates energy at a rate given by Equation 17.17. At the same time, the object also absorbs electromagnetic radiation. If the latter process did not occur, an object would eventually radiate all of its energy and its temperature would reach absolute zero. The energy that a body absorbs comes from its surroundings, which consists of other objects that radiate energy. If an object is at a temperature T and its surroundings are at a temperature T_0, the net energy gained or lost each second by the object as a result of radiation is

$$P_{\text{net}} = \sigma A e (T^4 - T_0^4) \qquad [17.18]$$

When an object is in equilibrium with its surroundings, it radiates and absorbs energy at the same rate, and so its temperature remains constant. When an object is hotter than its surroundings, it radiates more energy than it absorbs and so it cools. An **ideal absorber** is defined as an object that absorbs all of the energy incident on it. The emissivity of an ideal absorber is equal to unity. Such an object is often referred to as a **black body**. An ideal absorber is also an ideal radiator of energy. In contrast, an object with an emissivity equal to zero absorbs none of the energy incident on it. Such an object reflects all the incident energy and so is a perfect reflector.

The Dewar Flask

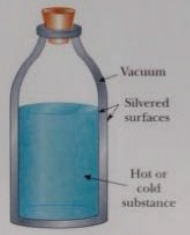


Figure 17.11 A cross-sectional view of a Dewar vessel, used to store hot or cold liquids or other substances.

The Thermos bottle, called a **Dewar flask** in the scientific community, is a practical example of a container designed to minimize energy losses by conduction, convection, and radiation. Such a container is used to store either cold or hot liquids for long periods of time. The standard construction (Fig. 17.11) consists of a double-walled Pyrex vessel with silvered inner walls. The space between the walls is evacuated to minimize heat transfer by conduction and convection. The silvered surfaces minimize heat transfer by radiation by reflecting most of the radiated energy. Very little thermal energy is lost over the neck of the flask, because glass is not a very good thermal conductor, and the cross-section of the glass in the direction of conduction is small. A further reduction in thermal energy loss is obtained by reducing the size of the neck. Dewar flasks are commonly used to store liquid nitrogen (boiling point 77 K) and liquid oxygen (boiling point 90 K).

In order to confine liquid helium, which has a very low heat of vaporization (boiling point 4.2 K), it is often necessary to use a double Dewar system in which the Dewar flask containing the liquid is surrounded by a second Dewar flask. The space between the two flasks is filled with liquid nitrogen.

Thinking Physics 6

If you sit in front of a fire in a fireplace with your eyes closed, you can feel significant warmth in your eyelids. If you now put on a pair of eyeglasses and repeat this activity, your eyelids will not feel nearly so warm. Why?

Reasoning Much of the warm feeling that one has while sitting in front of a fireplace is due to radiation from the fire. A large fraction of this radiation is in the infrared part of the spectrum. Your eyelids are particularly sensitive to infrared radiation. However, glass is opaque to infrared radiation. Thus, when you put on the glasses, you block much of the radiation from reaching your eyelids, and thus they feel cooler.

Thinking Physics 7

If you inspect an old or burned-out light bulb, you will notice a dark region on the inner surface. What's more, this region will be located on that part of the glass envelope that was the highest when the bulb was operating. What is the origin of this dark region, and why is it located at the high point?

Reasoning The dark region is tungsten, which has vaporized from the filament of the light bulb and collected on the inner surface of the glass. Many light bulbs contain gas, which allows convection to occur within the bulb. The gas near the filament is at a very high temperature, causing it to expand and float upward, due to Archimedes' principle. As it floats upward, it carries the vaporized tungsten with it, so that the tungsten collects on the surface at the top of the light bulb.

CONCEPTUAL PROBLEM 3

Rub the palm of your hand on a metal surface for 30 to 45 seconds. Place the palm of your other hand on an unrubbed portion of the surface and then the rubbed portion. The rubbed portion will feel warmer. Now repeat this process on a wooden surface. You should notice that the difference in temperature between the rubbed and unrubbed portions of the wood surface seems larger than that of the metal surface. Why is this?

CONCEPTUAL PROBLEM 4

In usually warm climates that experience an occasional hard freeze, fruit growers will spray the fruit trees with water, hoping that a layer of ice will form on the fruit. Why is this advantageous?

CONCEPTUAL PROBLEM 5

On a clear, cold night, why does frost tend to form on the tops of mailboxes and cars rather than the sides?

CONCEPTUAL PROBLEM 6

Cups of water for coffee or tea can be warmed with an immersion coil, which is immersed in the water and raised to a high temperature by means of electricity. The instructions warn the user not to operate the coils when they are out of water. Why? Should the immersion coil be used to warm up a cup of stew?

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SUMMARY

Thermal energy transfer is a form of energy transfer that takes place as a consequence of a temperature difference. The **internal energy** of a substance is a function of the state of the substance and generally increases with increasing temperature.

The **calorie** is the amount of heat necessary to raise the temperature of 1 g of water by 1.0°C. The calorie is related to the joule by experiment, yielding 4.186 J/cal. This relationship was historically called the mechanical equivalent of heat.

The thermal energy required to change the temperature of a substance by ΔT is

$$Q = mc\,\Delta T \qquad [17.3]$$

where m is the mass of the substance and c is its **specific heat.**

The thermal energy required to change the phase of a pure substance of mass m is

$$Q = mL \qquad [17.5]$$

The parameter L is called the **latent heat** of the substance and depends on the nature of the phase change and the properties of the substance.

A **quasi-static process** is one that proceeds slowly enough to allow the system to always be in a state of equilibrium.

The **work done** by a gas as its volume changes from some initial value, V_i, to some final value, V_f, is

$$W = \int_{V_i}^{V_f} P\,dV \qquad [17.7]$$

where P is the pressure, which may vary during the process. In order to evaluate W, the nature of the process must be specified—that is, P and V must be known during each step of the process. The work done depends on the initial, final, and intermediate states. In other words, W depends on the path taken between the initial and final states.

The **first law of thermodynamics** is the law of conservation of energy. It includes all forms of energy, including thermal energy.

When a stationary system undergoes a change from one state to another, the change in its internal energy, ΔU, is

$$\Delta U = Q - W \qquad [17.8]$$

where Q is the thermal energy transferred into (or out of) the system and W is the work done by (or on) the system. Although Q and W both depend on the path taken from the initial state to the final state, the quantity ΔU is independent of the path.

An isolated system is one that does not interact with its surroundings. The internal energy of an isolated system remains constant. In a **cyclic process** (one that originates and terminates at the same state), $\Delta U = 0$, and therefore $Q = W$. An **adiabatic process** is one in which no thermal energy is transferred between the system and its surroundings ($Q = 0$). In this case, the first-law equation gives $\Delta U = -W$. In the **adiabatic free expansion** of a gas, $Q = 0$ and $W = 0$, and so $\Delta U = 0$. An **isobaric process** is one that occurs at constant pressure. The work done in such a process is $P\Delta V$. An **isovolumetric process** is one that occurs at constant volume. No work (of expansion) is done in such a process. An **isothermal process** is one that occurs at constant temperature. The work done by an ideal gas during an isothermal process is

$$W = nRT \ln\left(\frac{V_f}{V_i}\right) \qquad [17.12]$$

CONCEPTUAL QUESTIONS

1. Clearly distinguish among temperature, thermal energy, heat, and internal energy.
2. When a sealed Thermos bottle full of hot coffee is shaken, what are the changes, if any, in (a) the temperature of the coffee and (b) the internal energy of the coffee?
3. Using the first law of thermodynamics, explain why the *total* energy of an isolated system is always constant.
4. Ethyl alcohol has about one half the specific heat of water. If equal masses of alcohol and water in separate beakers are supplied with the same amount of heat, compare the temperature increases of the two liquids.
5. A small crucible is taken from a 200°C oven and immersed in a tub full of water at room temperature (this process is often referred to as *quenching*). What is the approximate final equilibrium temperature?
6. The U.S. penny is now made of copper-coated zinc. Can a calorimetric experiment be devised to test for the metal content in a collection of pennies? If so, describe the procedure you would use.
7. What is wrong with the statement: "Given any two bodies, the one with the higher temperature contains more heat"?
8. What is a major problem that arises in measuring specific heats if a sample with a temperature above 100°C is placed in water?
9. The air temperature above coastal areas is profoundly influenced by the large specific heat of water. One reason is that the heat released when 1 cubic meter of water cools by 1.0°C will raise the temperature of an enormously larger volume of air by 1.0°C. Estimate this volume of air. The specific heat of air is approximately 1.0 kJ/kg·°C. Take the density of air to be 1.3 kg/m^3.
10. You need to pick up a very hot cooking pot in your kitchen. You have a pair of hot pads. Should you soak them in cold water or keep them dry in order to pick up the pot most comfortably?
11. Pioneers stored fruits and vegetables in underground cellars. Discuss fully this choice for a storage site.
12. In winter, the pioneers mentioned in Question 11 stored an open barrel of water alongside their produce. Why?

PROBLEMS

Section 17.1 Heat, Thermal Energy, and Internal Energy

Section 17.2 Specific Heat

1. Consider Joule's apparatus described in Figure P17.1. The two masses are 1.50 kg each, and the tank is filled with 200 g of water. What is the increase in the temperature of the water after the masses fall through a distance of 3.00 m?
2. A 50.0-g sample of copper is at 25.0°C. If 1200 J of thermal energy is added to it, what is the final temperature of the copper?
3. The temperature of a silver bar rises by 10.0°C when it absorbs 1.23 kJ of heat. The mass of the bar is 525 g. Determine the specific heat of silver.
4. What is the order of magnitude of the heat that a water heater puts into the water for a hot bath? How does this energy compare with your daily food–energy intake? In your solution state the quantities you estimate and the values you estimate for them.
5. A 1.50-kg iron horseshoe initially at 600°C is dropped into a bucket containing 20.0 kg of water at 25.0°C. What is the final temperature of the water and horseshoe? (Neglect the heat capacity of the container and assume that a negligible amount of water boils away.)
6. An aluminum cup of mass 200 g contains 800 g of water in thermal equilibrium at 80.0°C. The combination of cup and water is cooled uniformly so that the temperature decreases by 1.50 °C per minute. At what rate is thermal energy being removed? Express your answer in watts.
7. An aluminum calorimeter of mass 100 g contains 250 g of

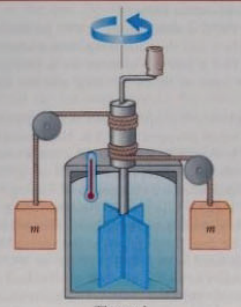


Figure P17.1 The falling weights rotate the paddles, causing the temperature of the water to increase.

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water. The calorimeter and water are in thermal equilibrium at 10.0°C. Two metallic blocks are placed in the water. One is a 50.0-g piece of copper at 80.0°C; the other has a mass of 70.0 g and is originally at a temperature of 100°C. The entire system stabilizes at a final temperature of 20.0°C. (a) Determine the specific heat of the unknown sample. (b) Guess the material of the unknown using Table 17.1.

8. Lake Erie contains roughly 4.00×10^{11} m^3 of water. (a) How much heat is required to raise the temperature of that volume of water from 11.0°C to 12.0°C? (b) Approximately how many years would it take to supply this amount of heat by using the full output of a 1000-MW electric power plant?

9. A 3.00-g copper penny at 25.0°C drops 50.0 m to the ground. (a) If 60.0% of its initial potential energy goes into increasing the internal energy, determine its final temperature. (b) Does the result depend on the mass of the penny? Explain.

Section 17.3 Latent Heat and Phase Changes

10. How much thermal energy is required to change a 40.0-g ice cube from a solid at −10.0°C to steam at 110°C?
11. If 90.0 g of molten lead at 327.3°C is poured into a 300-g casting form made of iron and initially at 20.0°C, what is the final temperature of the system? (Assume no energy loss to the environment occurs.)
12. Steam at 100°C is added to ice at 0°C. (a) Find the amount of ice melted and the final temperature when the mass of steam is 10.0 g and the mass of ice is 50.0 g. (b) Repeat when the mass of steam is 1.00 g and the mass of ice is 50.0 g.
13. A 1.00-kg block of copper at 20.0°C is dropped into a large vessel of liquid nitrogen at 77.3 K. How many kilograms of nitrogen boil away by the time the copper reaches 77.3 K? (The specific heat of copper is 0.0920 cal/g·°C. The latent heat of vaporization of nitrogen is 48.0 cal/g.)
14. A 50.0-g copper calorimeter contains 250 g of water at 20.0°C. How much steam must be condensed into the water to make the final temperature of the system 50.0°C?
15. In an insulated vessel, 250 g of ice at 0°C is added to 600 g of water at 18.0°C. (a) What is the final temperature of the system? (b) How much ice remains when the system reaches equilibrium?
16. A 3.00-g lead bullet at 30.0°C is fired at a speed of 240 m/s into a large block of ice at 0°C, in which it embeds itself. What quantity of ice melts?
17. Two speeding lead bullets, each of mass 5.00 g and at temperature 20.0°C, collide head-on at speeds of 500 m/s each. Assuming a perfectly inelastic collision and no loss of heat to the atmosphere, describe the final state of the two-bullet system.

Section 17.4 Work and Thermal Energy in Thermodynamic Processes

18. Gas in a container is at a pressure of 1.50 atm and a volume of 4.00 m^3. What is the work done by the gas if (a) it expands at constant pressure to twice its initial volume? (b) It is compressed at constant pressure to one quarter of its initial volume?
19. A sample of ideal gas is expanded to twice its original volume of 1.00 m^3 in a quasi-static process for which $P = \alpha V^2$, with $\alpha = 5.00$ atm/m^6, as shown in Figure P17.19. How much work was done by the expanding gas?

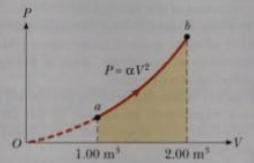


Figure P17.19

20. (a) Determine the work done by a fluid that expands from *i* to *f* as indicated in Figure P17.20. (b) How much work is performed by the fluid if it is compressed from *f* to *i* along the same path?

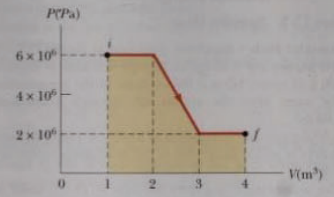


Figure P17.20

21. One mole of an ideal gas is heated slowly so that it goes from the *PV* state (P_0, V_0) to $(3P_0, 3V_0)$ in such a way that the pressure of the gas is directly proportional to the volume. (a) How much work is done in the process? (b) How is the temperature of the gas related to its volume during this process?
22. A sample of helium behaves as an ideal gas as it is heated at constant pressure from 273 K to 373 K. If the gas does 20.0 J of work, what is the mass of helium present?

Section 17.5 The First Law of Thermodynamics

23. A gas is compressed at a constant pressure of 0.800 atm from 9.00 L to 2.00 L. In the process, 400 J of thermal energy leaves the gas. (a) What is the work done by the gas? (b) What is the change in its internal energy?
24. A thermodynamic system undergoes a process in which its internal energy decreases by 500 J. If at the same time, 220 J of work is done on the system, find the thermal energy transferred to or from it.
25. A gas is taken through the cyclic process described in Figure P17.25. (a) Find the net thermal energy transferred to the system during one complete cycle. (b) If the cycle is reversed—that is, the process goes along *ACBA*—what is the net thermal energy transferred per cycle?

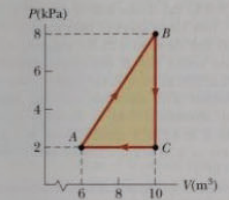


Figure P17.25

26. Consider the cyclic process depicted in Figure P17.25. If *Q* is negative for the process *BC*, and ΔU is negative for the process *CA*, determine the signs of *Q*, *W*, and ΔU that are associated with each process.

Section 17.6 Some Applications of the First Law of Thermodynamics

27. An ideal gas initially at 300 K undergoes an isobaric expansion at 2.50 kPa. If the volume increases from 1.00 m^3 to 3.00 m^3 and 12.5 kJ of thermal energy is transferred to the gas, find (a) the change in its internal energy and (b) its final temperature.
28. One mole of an ideal gas does 3000 J of work on its surroundings as it expands isothermally to a final pressure of 1.00 atm and volume of 25.0 L. Determine (a) the initial volume and (b) the temperature of the gas.
29. How much work is done by the steam when 1.00 mol of water at 100°C boils and becomes 1.00 mol of steam at 100°C at 1.00 atm pressure? Determine the change in internal energy of the steam as it vaporizes. Consider the steam to be an ideal gas.
30. One mole of gas initially at a pressure of 2.00 atm and a volume of 0.300 L has an internal energy that may be set equal to 91.0 J. In its final state, the pressure is 1.50 atm, the volume is 0.800 L, and the internal energy equals 182 J. For the three paths *IAF*, *IBF*, and *IF* in Figure P17.30, calculate (a) the work done by the gas and (b) the net thermal energy transferred in the process.

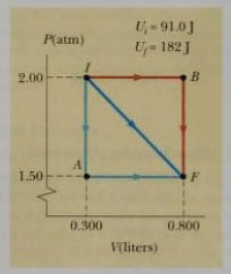


Figure P17.30

31. Two moles of helium gas initially at 300 K and 0.400 atm are compressed isothermally to 1.20 atm. Find (a) the final volume of the gas, (b) the work done by the gas, and (c) the thermal energy transferred. Consider the helium to behave as an ideal gas.
32. A 1.00-kg block of aluminum is heated at atmospheric pressure such that its temperature increases from 22.0°C to 40.0°C. Find (a) the work done by the aluminum, (b) the thermal energy added to it, and (c) the change in its internal energy.
33. One mole of water vapor at a temperature of 373 K cools down to 283 K. The heat given off by the cooling liquid is absorbed by 10.0 mol of an ideal gas, causing it to expand at a constant temperature of 273 K. If the final volume of the ideal gas is 20.0 L, determine the initial volume of the ideal gas.
34. One mole of steam at temperature T_h cools down to liquid water at T_c. The heat it gives off is absorbed by *n* mol of an ideal gas, causing it to expand at a constant temperature of T_0. If the final volume of the ideal gas is V_f, determine the initial volume of the ideal gas.
35. An ideal gas is carried through a thermodynamic cycle consisting of two isobaric and two isothermal processes as shown in Figure P17.35. Show that the net work done in the entire cycle is given by

$$W_{net} = P_1(V_2 - V_1)\ln\left(\frac{P_2}{P_1}\right)$$

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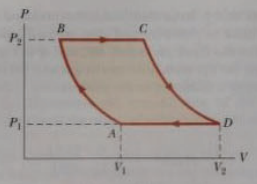


Figure P17.35

Section 17.7 Heat Transfer (Optional)

36. A steam pipe is covered with 1.50–cm-thick insulating material of heat conductivity 0.200 cal/cm·°C·s. How much heat is lost every second when the steam is at 200°C and the surrounding air is at 20.0°C? The pipe has a circumference of 20.0 cm and a length of 50.0 m. Neglect losses through the ends of the pipe.

37. A bar of gold is in thermal contact with a bar of silver of the same length and area (Fig. P17.37). One end of the compound bar is maintained at 80.0°C, and the opposite end is at 30.0°C. When the heat flow reaches steady state, find the temperature at the junction.

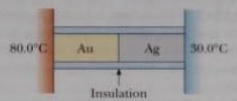


Figure P17.37

38. A box with a total surface area of 1.20 m² and a wall thickness of 4.00 cm is made of an insulating material. A 10.0-W electric heater inside the box maintains the inside temperature at 15.0°C above the outside temperature. Find the thermal conductivity k of the insulating material.

39. A Thermopane window of area 6.00 m² is constructed of two layers of glass, each 4.00 mm thick, separated by an air space of 5.00 mm. If the inside is at 20.0°C and the outside is at −30.0°C, what is the rate of heat loss through the window?

40. The surface of the Sun has a temperature of about 5800 K. Taking the radius of the Sun to be 6.96×10^8 m, calculate the total energy radiated by the sun each day. (Assume emissivity $e = 1.00$.)

Additional Problems

41. One hundred grams of liquid nitrogen at 77.3 K is stirred into a beaker containing 200 g of 5.00°C water. If the nitrogen leaves the solution as soon as it turns to gas, how much water freezes? (The heat of vaporization of nitrogen is 48.0 cal/g, and the heat of fusion of water is 80.0 cal/g.)

42. A 75.0-kg cross-country skier moves across snow so that the coefficient of friction between skis and snow is 0.200. Assume that all the snow beneath his skis is at 0°C and that all the thermal energy generated by friction is added to the snow, which sticks to his skis until melted. How far would he have to ski to melt 1.00 kg of snow?

43. An aluminum rod, 0.500 m in length and of cross-sectional area 2.50 cm², is inserted into a thermally insulated vessel containing liquid helium at 4.22 K. The rod is initially at 300 K. (a) If one half of the rod is inserted into the helium, how many liters of helium boil off by the time the inserted half cools to 4.22 K? (Assume the upper half does not yet cool.) (b) If the upper end of the rod is maintained at 300 K, what is the approximate boil-off rate of liquid helium after the lower half has reached 4.22 K? (Note that aluminum has a thermal conductivity of 31.0 J/s·cm·K at 4.2 K, a specific heat of 0.210 cal/g·°C, and a density of 2.70 g/cm³. The density of liquid helium is 0.125 g/cm³.)

44. A *flow calorimeter* is an apparatus used to measure the specific heat of a liquid. The technique is to measure the temperature difference between the input and output points of a flowing stream of the liquid while adding heat at a known rate. In one particular experiment, a liquid of density 0.780 g/cm³ flows through the calorimeter at the rate of 4.00 cm³/s. At steady state, a temperature difference of 4.80°C is established between the input and output points when heat is supplied at the rate of 30.0 J/s. What is the specific heat of the liquid?

45. One mole of an ideal gas is contained in a cylinder with a movable piston. The initial pressure, volume, and temperature are P_0, V_0, and T_0, respectively. Find the work done by the gas for the following processes and show each process on a PV diagram: (a) an isobaric compression in which the final volume is one half the initial volume, (b) an isothermal compression in which the final pressure is four times the initial pressure, (c) an isovolumetric process in which the final pressure is triple the initial pressure.

46. One mole of an ideal gas, initially at 300 K, is cooled at constant volume so that the final pressure is one fourth the initial pressure. Then the gas expands at constant pressure until it reaches the initial temperature. Determine the work done by the gas.

47. An ideal gas initially at P_0, V_0, and T_0 is taken through a cycle as in Figure P17.47. (a) Find the net work done by the gas per cycle. (b) What is the net heat added to the system per cycle? (c) Obtain a numerical value for the net work done per cycle for 1.00 mol of gas initially at 0°C.

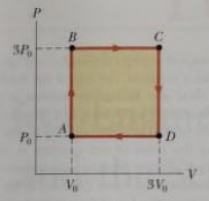


Figure P17.47

48. An iron plate is held against an iron wheel so that there is a sliding frictional force of 50.0 N acting between the two pieces of metal. The relative speed at which the two surfaces slide over each other is 40.0 m/s. (a) Calculate the rate at which mechanical energy is converted to thermal energy. (b) The plate and the wheel have a mass of 5.00 kg each, and each receives 50.0% of the thermal energy. If the system is run as described for 10.0 s and each object is then allowed to reach a uniform internal temperature, what is the resultant temperature increase?

49. A "solar cooker" consists of a curved reflecting mirror that focuses sunlight onto the object to be heated (Fig. P17.49). The solar power-per-unit area reaching the Earth at the location is 600 W/m², and the cooker has a diameter of 0.600 m. Assuming that 40.0% of the incident energy is transferred to the water, how long would it take to completely boil off 0.500 liter of water initially at 20.0°C? (Neglect the heat capacity of the container.)



Figure P17.49

50. An electric teakettle is boiling. The electric power consumed by the water is 1.00 kW. Assuming that the pressure of vapor in the kettle equals atmospheric pressure, determine the speed of effusion of vapor from the kettle's spout if the spout has a cross-sectional area of 2.00 cm².

51. An ideal gas is enclosed in a cylinder with a movable piston on top of it. The piston has a mass of 8000 g and an area of 5.00 cm² and is free to slide up and down, keeping the pressure of the gas constant. How much work is done as the temperature of 0.200 mol of the gas is raised from 20.0°C to 300°C?

52. A chamber contains 1.00 kg of water at 0°C under a piston, just touching the water surface. The piston is then raised quickly, so that part of the water is vaporized and the other part is frozen (no liquid left). Assuming the temperature remains constant at 0°C, determine the mass of ice formed in the chamber.

Spreadsheet Problems

S1. The rate at which an object with an initial temperature T_i cools when the surrounding temperature is T_0 is given by Newton's law of cooling:

$$\frac{dQ}{dt} = hA(T - T_0)$$

where A is the surface area of the object, and h is a constant that characterizes the rate of cooling, called the surface coefficient of heat transfer. The object's temperature T at any time t is

$$T = T_0 + (T_i - T_0)e^{-hAt/mc}$$

where m is the object's mass and c is its specific heat.

Spreadsheet 17.1 evaluates and plots the temperature of an object as a function of time. Consider the cooling of a cup of coffee the initial temperature of which is 75°C. After 3 min, it cools to a drinkable 45°C. Room temperature is 22°C, the effective area of the coffee is 125 cm², and its mass is 200 g. Take the specific heat c to be that of water. Use Spreadsheet 17.1 to find h for this cup of coffee. That is, find the value of h for which $T = 45$°C at $t = 180$ s.

S2. In the Einstein model of a crystalline solid the molar specific heat at constant volume is

$$C_V = 3R\left(\frac{T_E}{T}\right)^2 \frac{e^{T_E/T}}{(e^{T_E/T} - 1)^2}$$

where T_E is a characteristic temperature called the Einstein temperature and T is the temperature in Kelvin. (a) Verify that $C_V \approx 3R$, the Dulong-Petit law, for $T \gg T_E$, by evaluating $C_V - 3R$. (b) For diamond, T_E is approximately 1060 K. If 1 mol of diamond is heated from 300 K to 600 K, numerically integrate

$$\Delta U = \int C_V \, dT$$

to find the increase in the internal energy of the material. Use your spreadsheet to carry out a simple rectangular numeric integration. (You may also want to try to use the trapezoid and Simpson's methods to carry out the integration.)

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ANSWERS TO CONCEPTUAL PROBLEMS

1. If the process is isothermal, there is no change in internal energy. According to the first law, then, the heat is equal to the work done by the gas, $Q = W$.
2. The energy that is leaving the body by work and heat is replaced by means of biological processes that transform the potential energy in the food that the individual ate into internal energy. Thus, the temperature of the body can be maintained.
3. When you rub the surface, you are doing work on the surface, and the internal energy of the surface increases. The increase in internal energy is detected as an increase in temperature. With the metal surface, some of this energy is transferred away from the rubbing site by *conduction*. Thus, the internal energy remaining in the rubbed area is not as high for the metal as it is for the wood, and it feels relatively cooler than the wood.
4. If the fruit were open to the cold air, it is possible that the temperature of the fruit could drop to a point several degrees below freezing, causing damage. By spraying with water, a film of ice can form on the fruit. This provides some insulation to the fruit, so that its temperature will remain somewhat higher than that of the air.
5. One of the ways that objects transfer energy is by radiation. If we consider a mailbox, the top of the mailbox is oriented toward the clear sky. Radiation emitted by the top of the mailbox goes upward and into space. There is little radiation coming down from space into the top of the mailbox. Radiation leaving the sides of the mailbox is absorbed by the environment. Radiation *from* the environment (trees, houses, cars, etc.), however, can enter the sides of the mailbox, keeping it warmer than the top. As a result, the top is the coldest portion and frost forms there first.
6. The operation of the immersion coil depends on the convection of water to maintain a safe temperature. As the water near the coils warms up, it floats to the top due to Archimedes' principle. The temperature of the coils cannot go higher than the boiling temperature of water, 100°C. If the coils are operated in air, the convection process is reduced, and the upper limit of 100°C is removed. As a result, the coils can become hot enough to be damaged. If the coils are used in an attempt to warm a thick liquid like stew, the convection process cannot occur fast enough to carry energy away from the coils, so that they again become hot enough to be damaged.

18

Heat Engines, Entropy, and the Second Law of Thermodynamics

The first law of thermodynamics, studied in the previous chapter, is a statement of conservation of energy, generalized to include heat as a form of energy transfer. This law places no restrictions on the types of energy conversions that can occur. Furthermore, it makes no fundamental distinction between the results of energy input to a system by heat and work: According to the first law, the internal (thermal) energy of a body can be increased either by adding heat to it or by doing work on it. An important difference exists between heat and work, however, that is not evident from the first law. One manifestation of this difference is the fact that it is impossible to convert thermal energy into mechanical energy in an isothermal process.

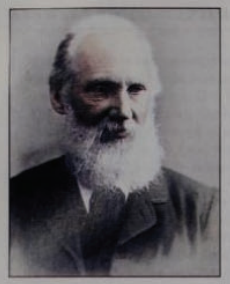


◀ This steam-driven locomotive runs from Durango to Silverton, Colorado. Early steam-driven locomotives obtained their energy by burning wood or coal. The generated heat produces the steam, which powers the locomotive. Modern trains use electricity or diesel fuel to power their locomotives. All heat engines extract heat from a burning fuel and convert only a fraction of this energy to mechanical energy. *(Lois Moulton, Tony Stone Images)*

CHAPTER OUTLINE

18.1 Heat Engines and the Second Law of Thermodynamics
18.2 Reversible and Irreversible Processes
18.3 The Carnot Engine
18.4 Heat Pumps and Refrigerators
18.5 An Alternative Statement of the Second Law
18.6 Entropy
18.7 Entropy Changes in Irreversible Processes
18.8 Entropy on a Microscopic Scale
18.9 Entropy and Disorder (Optional)

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Lord Kelvin, British physicist and mathematician (1824–1907)

Born William Thomson in Belfast, Kelvin was the first to propose the use of an absolute scale of temperature. The Kelvin scale, named in his honor, is discussed in Section 16.2. Kelvin's study of Carnot's theory led to the idea that heat cannot pass spontaneously from a colder body to a hotter body. *(J. L. Charmet/SPL/Photo Researchers)*

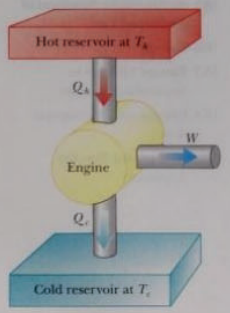


Figure 18.1 Schematic representation of a heat engine. The engine absorbs thermal energy, Q_h, from the hot reservoir, expels thermal energy Q_c to the cold reservoir, and does work W.

Contrary to what the first law implies, only certain types of energy conversions can take place. The second law of thermodynamics establishes which natural processes can and which cannot occur. The following are examples of processes that are consistent with the first law of thermodynamics in either direction, but proceed only in a particular direction governed by the second law.

1. When two objects at different temperatures are placed in thermal contact with each other, thermal energy always flows from the warmer to the cooler object—never from the cooler to the warmer.
2. A rubber ball dropped to the ground bounces several times and eventually comes to rest, but a ball lying on the ground never begins bouncing on its own.
3. An oscillating pendulum eventually comes to rest because of collisions with air molecules and friction at the point of suspension, and the initial mechanical energy is converted to thermal energy; the reverse conversion of energy never occurs.

These are all *irreversible processes*—that is, processes that occur naturally in only one direction. No irreversible process ever runs backward, because if it did, it would violate the second law of thermodynamics.[1]

From an engineering viewpoint, perhaps the most important application of the second law of thermodynamics is the limited efficiency of heat engines. The second law says that a machine capable of continuously converting thermal energy in a cyclic process *completely* to other forms of energy cannot be constructed.

18.1 • HEAT ENGINES AND THE SECOND LAW OF THERMODYNAMICS

A **heat engine** is a device that converts thermal energy to other useful forms, such as mechanical energy. In a typical process for producing electricity in a power plant, for instance, coal or some other fuel is burned, and the thermal energy produced is used to convert water to steam. This steam is directed at the blades of a turbine, setting it into rotation. Finally, the mechanical energy associated with this rotation is used to drive an electric generator. In another heat engine, the internal combustion engine in an automobile, energy enters the engine by mass transfer as the fuel is injected into the cylinder and a fraction of this energy is converted to mechanical energy.

A heat engine carries some working substance through a cyclic process during which (1) thermal energy is absorbed from a source at a high temperature, (2) work is done by the engine, and (3) thermal energy is expelled by the engine to a source at a lower temperature. As an example, consider the operation of a steam engine in which the working substance is water. The water is carried through a cycle in which it first evaporates into steam in a boiler and then expands against a piston. After the steam is condensed with cooling water, it is returned to the boiler, and the process is repeated.

[1]To be more precise, we should say that the set of events in the time-reversed sense is highly improbable. From this viewpoint, events in one direction are vastly more probable than those in the opposite direction.

It is useful to represent a heat engine schematically, as in Figure 18.1. The engine absorbs a quantity of energy that we will model as heat, Q_h, from the hot reservoir, does work W, and then gives up heat, Q_c, to the cold reservoir. Because the working substance goes through a cycle, its initial and final internal energies are equal, so $\Delta U = 0$. Hence, from the first-law equation we see that **the net work, W, done by a heat engine equals the net thermal energy flowing into it.** As we can see from Figure 18.1, $Q_{net} = Q_h - Q_c$; therefore,

$$W = Q_h - Q_c \qquad [18.1]$$

where Q_h and Q_c are taken to be positive quantities.

If the working substance is a gas, **the net work done for a cyclic process is the area enclosed by the curve representing the process on a *PV* diagram.** This is shown for an arbitrary cyclic process in Figure 18.2.

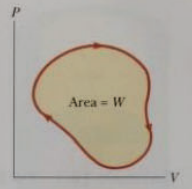


Figure 18.2 The PV diagram for an arbitrary cyclic process. The net work done equals the area enclosed by the curve.

The **thermal efficiency,** e, of a heat engine is defined as the ratio of the net work done to the thermal energy absorbed at the higher temperature during one cycle:

• *Thermal efficiency*

$$e \equiv \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h} \qquad [18.2]$$

We can think of the efficiency as the ratio of what you get (mechanical energy) to what you give (thermal energy at the higher temperature). Equation 18.2 shows that a heat engine has 100% efficiency ($e = 1$) only if $Q_c = 0$—that is, if no thermal energy is expelled to the cold reservoir. In other words, a heat engine with perfect efficiency would have to expel all the input energy by mechanical work. One of the consequences of the second law of thermodynamics is that this is impossible.

The **second law of thermodynamics** can be stated as follows: **It is impossible to construct a heat engine that, operating in a cycle, produces no other effect than the absorption of thermal energy from a reservoir and the performance of an equal amount of work.**

• *Second law of thermodynamics*

This form of the second law is useful in understanding the operation of heat engines. With reference to Equation 18.2, the second law says that, during the operation of a heat engine, W can never be equal to Q_h or, alternatively, that some heat, Q_c, must be rejected to the environment. As a result, it is theoretically impossible to construct an engine that works with 100% efficiency.

Our assessment of the first two laws of thermodynamics can be summed up as follows: the first law says **we cannot get more energy out of a cyclic process than the amount of thermal energy we put in, and the second law says we cannot break even because we must put more thermal energy in, at the higher temperature, than the net amount of work output.**

Thinking Physics 1

Does a heat engine have to take in and expel energy *by heat*?

Reasoning The typical schematic model of a heat engine shows the energy entering the engine by heat and leaving by heat and work. This is only a schematic, however. The energy could enter and leave by other means. In an automobile engine, for ex-

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ample, the energy enters the cylinder by *mass transfer*—the energy is carried in with the fuel, and then transformed to thermal energy by the ignition process. The exhaust is also by mass transfer—of the expended fuel–air mixture out the tailpipe. The atmosphere can be considered to be a heat engine—radiation from the ground is absorbed in the atmosphere. About 0.5% of this energy does work by driving the zonal wind systems in the general atmospheric circulation. The remainder is radiated out into space. Here, the input and output energy is transferred by *electromagnetic radiation.*

Example 18.1 The Efficiency of an Engine

Find the efficiency of an engine that introduces 2000 J of heat during the combustion phase and loses 1500 J at exhaust.

Solution The efficiency of the engine is given by Equation 18.2 as

$$e = 1 - \frac{Q_c}{Q_h} = 1 - \frac{1500\ \text{J}}{2000\ \text{J}} = 0.25, \text{ or } 25\%$$

EXERCISE 1 If an engine has an efficiency of 20% and loses 3000 J to the cooling water, how much work is done by the engine? Answer 750 J

EXERCISE 2 The heat absorbed by an engine is three times greater than the work it performs. (a) What is its thermal efficiency? (b) What fraction of the heat absorbed is expelled to the cold reservoir? Answer (a) 0.333 (b) 2/3

18.2 · REVERSIBLE AND IRREVERSIBLE PROCESSES

In the next section we shall discuss a theoretical heat engine that is the most efficient engine possible. In order to understand its nature, we must first examine the meaning of reversible and irreversible processes. A **reversible** process is one in which the system can be returned to its initial conditions along the same path and in which every point along the path is an equilibrium state. A process that does not satisfy these requirements is **irreversible.**

All natural processes are known to be irreversible. From the endless number of examples that could be selected, let us examine the free expansion of a gas, already discussed in Section 17.4, and show that it cannot be reversible. The gas is contained in an insulated container, as shown in Figure 18.3, with a membrane separating the gas from a vacuum. If the membrane is punctured, the gas expands freely into the vacuum. Because the gas does not exert a force through a distance on the surroundings, it does no work as it expands. In addition, no thermal energy is transferred to or from the gas, because the container is insulated from its surroundings. Thus, in this process, the system has changed but the surroundings have not. Now imagine that we try to reverse the process by first compressing the gas to its original volume. Let's say an engine is being used to force the piston inward. Note, however, that this action is changing both the system and surroundings. The surroundings are changing because work is being done by an outside agent on the system, and the system is changing because the compression is increasing the temperature of the gas. We can lower the temperature of the gas by allowing it to come into contact with an external heat reservoir. Although this second procedure returns the gas to its original state, the surroundings are again affected because thermal energy is added to the surroundings. If this thermal energy could somehow be used to drive the engine and compress the gas, the system and its surroundings could be returned to their initial states. However, our statement of the second law says that this extracted thermal energy cannot be completely converted into mechanical energy isothermally. We must conclude that a reversible process has not occurred.

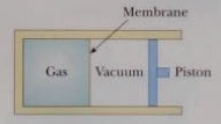


Figure 18.3 Free expansion of a gas.

Although real processes are always irreversible, some are *almost* reversible. If a real process occurs very slowly so that the system is virtually always in equilibrium, the process can be considered reversible. For example, imagine compressing a gas very slowly by dropping some grains of sand onto a frictionless piston, as in Figure 18.4. The compression process can be reversed by the placement of the gas in thermal contact with a heat reservoir. The pressure, volume, and temperature of the gas are well defined during this isothermal compression. Each added grain of sand represents a change to a new equilibrium state. The process can be reversed by the slow removal of grains of sand from the piston.

A general characteristic of a reversible process is that there can be no dissipative effects present, such as turbulence or friction, that convert mechanical energy into thermal energy. In reality, such effects are impossible to eliminate completely, and hence it is not surprising that real processes in nature are irreversible.

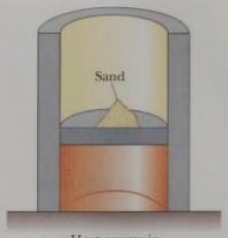


Figure 18.4 A gas in thermal contact with a heat reservoir is compressed slowly by dropping grains of sand onto the piston. The compression is isothermal and reversible.

18.3 · THE CARNOT ENGINE

In 1824 a French engineer named Sadi Carnot (1796–1832) described a theoretical engine, now called a **Carnot engine,** that is of great importance from both practical and theoretical viewpoints. He showed that a heat engine operating in an ideal, reversible cycle—called a **Carnot cycle**—between two heat reservoirs is the most efficient engine possible. Such an ideal engine establishes an upper limit on the efficiencies of all engines. That is, **the net work done by a working substance taken through the Carnot cycle is the greatest amount of work possible for a given amount of thermal energy supplied to the substance at the upper temperature.**

To describe the Carnot cycle, we shall assume that the substance working between temperatures T_c and T_h is an ideal gas contained in a cylinder with a movable piston at one end. The cylinder walls and the piston are thermally nonconducting. Four stages of the Carnot cycle are shown in Figure 18.5.

- The process $A \rightarrow B$ is an isothermal expansion at temperature T_h, in which the gas is placed in thermal contact with a heat reservoir at temperature T_h (Fig. 18.5a). During the process, the gas absorbs heat Q_h from the reservoir and does work W_{AB} in raising the piston.
- In the process $B \rightarrow C$, the base of the cylinder is replaced by a thermally nonconducting wall and the gas expands adiabatically—that is, no thermal energy enters or leaves the system (Fig. 18.5b). During the process, the temperature falls from T_h to T_c, and the gas does work W_{BC} in raising the piston.
- In the process $C \rightarrow D$, the gas is placed in thermal contact with a heat reservoir at temperature T_c (Fig. 18.5c) and is compressed isothermally at temperature T_c. During this time, the gas expels heat Q_c to the reservoir, and the work done on the gas is W_{CD}.



Sadi Carnot (1796–1832)

A French physicist, Carnot is considered to be the founder of the science of thermodynamics. Some of his notes found after his death indicate that he was the first to recognize the relationship between work and heat.

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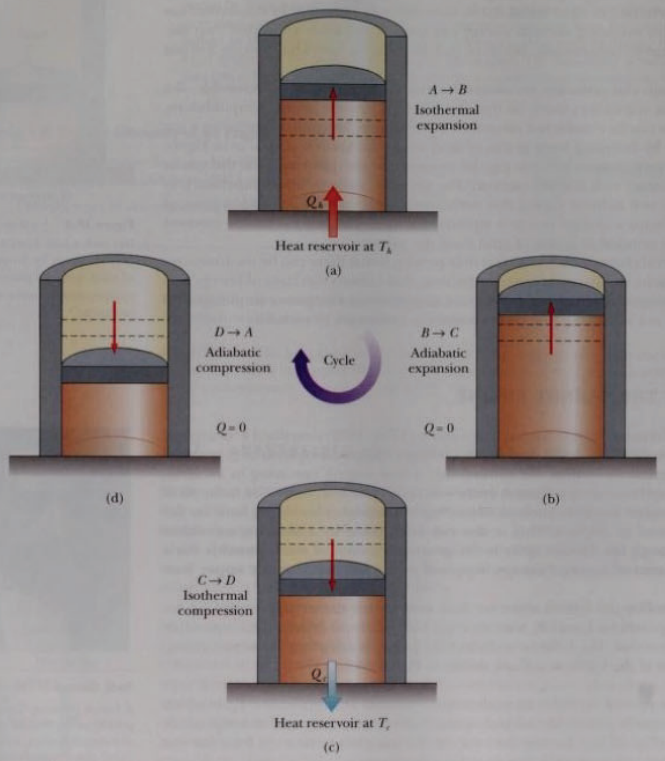


Figure 18.5 The Carnot cycle. (a) In process $A \rightarrow B$, the gas expands isothermally while in contact with a reservoir at T_h. (b) In process $B \rightarrow C$, the gas expands adiabatically ($Q = 0$). (c) In process $C \rightarrow D$, the gas is compressed isothermally while in contact with a reservoir at $T_c < T_h$. (d) In process $D \rightarrow A$, the gas is compressed adiabatically. The upward arrows on the piston in (a) and (b) indicate weights being removed during the expansions, and the downward arrows in (c) and (d) indicate the addition of weights during the compressions.

- In the final stage, $D \rightarrow A$, the base of the cylinder is again replaced by a thermally nonconducting wall (Fig. 18.5d) and the gas is compressed adiabatically. The temperature of the gas increases to T_h, and the work done on the gas is W_{DA}.

Figure 18.6 is the PV diagram for the cycle. The cycle consists of two adiabatic and two isothermal processes, all reversible.

Carnot showed that the thermal efficiency of a Carnot engine is

$$e_c = \frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h} \qquad [18.3]$$

- *Carnot efficiency*

It is important to note that in the equation, T must be in kelvin. From this result, we see that all Carnot engines operating between the same two temperatures have the same efficiency. Furthermore, the efficiency of a Carnot engine operating between two temperatures is greater than the efficiency of any real engine operating between the same two temperatures.

Equation 18.3 can be applied to any working substance operating in a Carnot cycle between two heat reservoirs. According to this result, the efficiency is zero if $T_c = T_h$, as one would expect. The efficiency increases as T_c is lowered and as T_h is increased. However, the efficiency can be unity (100%) only if $T_c = 0$ K. Such reservoirs are not available, and so the maximum efficiency is always less than unity. In most practical cases, the cold reservoir is near room temperature, about 300 K. Therefore, one usually strives to increase the efficiency by raising the temperature of the hot reservoir. **All real engines are less efficient than the Carnot engine because they are all subject to practical difficulties, including friction, but especially the need to operate irreversibly to complete a cycle in a brief time period.**

- *An alternative form of the second law of thermodynamics*

CONCEPTUAL PROBLEM 1

Which is more effective in increasing the efficiency of a Carnot engine—increasing T_h by ΔT, or decreasing T_c by ΔT?

CONCEPTUAL PROBLEM 2

An automobile manufacturer makes a claim that their cars obtain the same gasoline mileage with or without the air conditioner running. Do you believe this? Does playing the radio change the gas mileage? Turning on the headlights? Operating the windshield washer system? Rolling down a mechanical window?

CONCEPTUAL PROBLEM 3

If friction could be totally removed from a Carnot engine, would it then be 100% efficient?

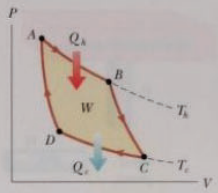
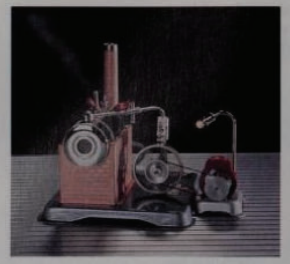


Figure 18.6 The PV diagram for the Carnot cycle. The net work done, W, equals the net heat received in one cycle, $Q_h - Q_c$. Note that $\Delta U = 0$ for the cycle.

Example 18.2 The Steam Engine

A steam engine has a boiler that operates at 500 K. The heat changes water to steam, and this steam then drives the piston. The exhaust temperature is that of the outside air, approximately 300 K. What is the maximum thermal efficiency of this steam engine?

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A model steam engine equipped with a built-in horizontal boiler. The water is heated electrically, which generates steam that is used to power the electric generator at the right. *(Courtesy of Central Scientific Co.)*

Solution From the expression for the efficiency of a Carnot engine, we find the maximum thermal efficiency for any engine operating between these temperatures:

$$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{300\ \text{K}}{500\ \text{K}} = 0.4, \text{ or } 40\%$$

You should note that this is the highest theoretical efficiency of the engine. In practice, the efficiency is considerably lower.

EXERCISE 3 Determine the maximum work the engine can perform in each cycle of operation if it absorbs 200 J of thermal energy from the hot reservoir during each cycle. Answer 80 J

EXERCISE 4 A heat engine operates between two reservoirs at 20°C and 300°C. What is the maximum efficiency possible for this engine? Answer 49%

18.4 • HEAT PUMPS AND REFRIGERATORS

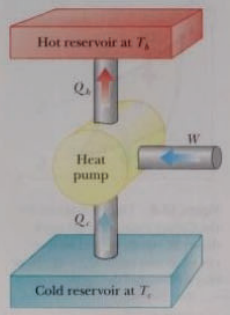


Figure 18.7 Schematic diagram of a heat pump, which absorbs thermal energy, Q_c, from the cold reservoir and expels thermal energy, Q_h, to the hot reservoir. The work done on the heat pump is W.

A heat pump is a mechanical device that moves thermal energy from a region at lower temperature to a region at higher temperature. Heat pumps have long been popular for cooling and are now becoming increasingly popular for heating purposes as well. In the heating mode, a circulating coolant fluid absorbs thermal energy from the outside and releases energy to the interior of the structure. The fluid is usually in the form of a low-pressure vapor when in the coils of the exterior part of the unit, where it absorbs thermal energy from either the air or the ground. This gas is then compressed into a hot, high-pressure vapor and enters the interior part of the unit where it condenses to a liquid and releases its stored thermal energy. An air conditioner is simply a heat pump installed backward, with "exterior" and "interior" interchanged.

Figure 18.7 is a schematic representation of a heat pump. The outside temperature is T_c, the inside temperature is T_h, and the thermal energy absorbed by the circulating fluid is Q_c. The compressor does work W on the fluid, and the thermal energy transferred from the pump into the structure is Q_h.

The effectiveness of a heat pump, in its heating mode, is described in terms of a number called the **coefficient of performance,** COP. This is defined as the ratio of the energy transferred as heat into the hot reservoir and the work required to transfer that heat:

$$\text{COP (heat pump)} = \frac{\text{energy transferred as heat}}{\text{work done by pump}} = \frac{Q_h}{W} \qquad [18.4]$$

If the outside temperature is 25°F or higher, the COP for a heat pump is about 4. That is, the energy transferred as heat into the house is about four times greater than the work done by the compressor in the heat pump. However, as the outside temperature decreases, it becomes more difficult for the heat pump to extract sufficient thermal energy from the air, and the COP drops. In fact, the COP can fall below unity for temperatures below the mid-teens.

A Carnot-cycle heat engine run in reverse constitutes a heat pump—in fact, the heat pump with the highest possible coefficient of performance for the temperatures between which it operates. The maximum coefficient of performance is

$$\text{COP}_c \text{ (heat pump)} = \frac{T_h}{T_h - T_c}$$

Although heat pumps are relatively new products in heating, the refrigerator has been a standard appliance in homes for years. The refrigerator works much like a heat pump, except that it cools its interior by pumping thermal energy from the food storage compartments into the warmer air outside. During its operation, a refrigerator removes thermal energy, Q_c, from the interior of the refrigerator, and in the process its motor does work W. The coefficient of performance of a refrigerator or of a heat pump used in its cooling cycle is

$$\text{COP (refrigerator)} = \frac{Q_c}{W} \qquad [18.5]$$

An efficient refrigerator is one that removes the greatest amount of thermal energy from the cold reservoir with the least amount of work. Thus, a good refrigerator should have a high coefficient of performance, typically 5 or 6.

The highest possible coefficient of performance is again that of a refrigerator the working substance of which is carried through the Carnot heat-engine cycle in reverse:

$$\text{COP}_c \text{ (refrigerator)} = \frac{T_c}{T_h - T_c}$$

As the difference between temperatures of the two reservoirs approaches zero, the theoretical coefficient of performance of a Carnot heat pump approaches infinity. In practice, the low temperature of the cooling coils and the high temperature at the compressor limit the COP to values below 10.

Thinking Physics 2

Suppose the temperature inside a house is held constant at 72°F. Assume that it is 92°F outside in the summer, and the air conditioner (heat pump) is operated to keep the house at 72°F. Later, during the winter, it is 52°F outside and the heat pump is used to keep the house at 72°F again. In both cases, there was a 20°F difference between the inside and outside temperatures. The electric bill for the heat pump will be higher in the summer than in the winter. Why?

Reasoning In the summer, the interior of the house serves as the low temperature reservoir for the heat engine, and the outside air is the high temperature reservoir. Energy drawn from the interior air combines with energy entering the heat pump by electrical transmission, and the combination is transferred to the external air. Thus,

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the only advantage of the energy entering by electrical transmission is to pump the energy from the interior to the exterior. In the winter, energy drawn from the exterior air combines with energy entering the heat pump by electrical transmission, and the combination is transferred to the internal air. Thus, the energy entering by electricity does two things—it operates the heat pump *and* it adds to the internal energy of the interior air. Thus, less energy must be drawn from the external air in winter than from the internal air in the summer, and it costs less to run the heat pump in the winter.

Thinking Physics 3

It is a sweltering summer day, and your air conditioning system is not operating. In your kitchen, you have a working refrigerator and an ice chest full of ice. Which should you open, and leave open, to cool the room more efficiently?

Reasoning The high temperature reservoir for your kitchen refrigerator is the air in the kitchen. If the refrigerator door were left open, energy would be drawn from the air in the kitchen, passed through the refrigeration system and transferred right back into the air. The result would be that the kitchen would become *warmer* due to the addition of the energy coming in by electricity to run the refrigeration system. If the ice chest were opened, energy in the air would enter the ice, raising its temperature and causing it to melt. The transfer of energy from the air would cause its temperature to drop. Thus, it would be more effective to open the ice chest.

CONCEPTUAL PROBLEM 4

Why are room air conditioners placed in windows or through holes in walls?

CONCEPTUAL PROBLEM 5

Can a heat pump deliver more energy than it consumes?

18.5 · AN ALTERNATIVE STATEMENT OF THE SECOND LAW

The first law of thermodynamics says that energy is conserved in every process. There is more to the story than that, however, and the second law adds the finishing touches. To illustrate what we mean, suppose you wish to cool off a hot piece of pizza by placing it on a block of ice. You will certainly be successful, because in every like situation that has ever been encountered, heat transfer has always taken place from a hot object to a cooler one. If you wait a short while, some amount of thermal energy—500 J, say—is transferred from the pizza to the ice. Energy is conserved, and the first law is satisfied. Yet, nothing in the first law says that this thermal energy transfer could not proceed in the opposite direction. (Imagine your astonishment if someday you place a piece of hot pizza on ice and 500 J of thermal energy moves from the ice to the pizza!) It is the second law that determines the directions of such natural phenomena.

An analogy can be made with the impossible sequence of events seen in a movie film running backward—such as a person diving out of a swimming pool, an apple rising from the ground and latching onto the branch of a tree, or a pot of hot water becoming colder as it rests over an open flame. Such events occurring backward in time are impossible because they violate the second law of thermodynamics. **Real processes have a preferred direction of time, often called the arrow of time.**

The second law has been stated in many different ways, but all the statements can be shown to be equivalent. Which form you use depends on the application you have in mind. For example, if you were concerned about the thermal energy transfer between pizza and ice, you might choose to concentrate on the second law in this form: **Thermal energy will not flow spontaneously from a cold object to a hot object.** At first glance, this statement of the second law seems to be radically different from that in Section 18.1. The two are, in fact, equivalent in all respects. Although we shall not prove it here, it can be shown that if either statement is false, so is the other.

18.6 · ENTROPY

The zeroth law of thermodynamics involves the concept of temperature, and the first law involves the concept of internal (thermal) energy. Temperature and internal energy are both state functions; that is, they can be used to describe the thermodynamic state of a system. Another state function, this one related to the second law of thermodynamics, is the **entropy,** S. In this section we define entropy on a macroscopic scale as it was first expressed by the German physicist Rudolf Clausius (1822–1888) in 1865.

Consider any infinitesimal process for a system between two equilibrium states. If dQ_r is the amount of thermal energy that would be transferred if the system had followed a reversible path, then the change in entropy, regardless of the actual path followed, is equal to this amount of thermal energy transferred along the reversible path divided by the absolute temperature of the system:

$$dS = \frac{dQ_r}{T} \qquad [18.6]$$

• *Clausius's definition of change in entropy*

The subscript r on the term dQ_r is a reminder that the thermal energy transfer is to be measured along a reversible path, even though the system may actually have followed some irreversible path. When thermal energy is absorbed by the system, dQ_r is positive and hence the entropy increases. When thermal energy is expelled by the system, dQ_r is negative and the entropy decreases. Note that Equation 18.6 defines not entropy but rather the *change* in entropy. Hence, the meaningful quantity in a description of a process is the *change* in entropy.

Entropy originally found its place in thermodynamics, but its importance grew tremendously as the field of physics called statistical mechanics developed, because this method of analysis provided an alternative way of interpreting entropy. In statistical mechanics, a substance's behavior is described in terms of the statistical behavior of its atoms and molecules. One of the main outcomes of this treatment is the principle that **isolated systems tend toward disorder, and entropy is a measure of that disorder.**

For example, consider the molecules of a gas in the air in your room. If all the gas molecules moved together like soldiers marching in step, they would be in a very ordered state. It is also an unlikely state. If you could see the molecules, you would see that they actually move haphazardly in all directions, bumping into one another, changing speed on collision, some going fast, some slowly. This is a highly



Rudolf Clausius (1822–1888)

"I propose . . . to call S the entropy of a body, after the Greek word 'transformation.' I have designedly coined the word 'entropy' to be similar to energy, for these two quantities are so analogous in their physical significance, that an analogy of denominations seems to be helpful." *(AIP Niels Bohr Library, Lande Collection)*

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disordered state. It is also highly possible that the actual state of the system is such a highly disordered state.

The cause of this perpetual drive toward disorder is easily seen. For any given energy of the system, only certain states are possible or accessible. Among those states, it is assumed that all are equally probable. However, when such possible states are examined, it is found that far more of them are disordered states than ordered states. Because each of the states is equally probable, it is highly probable that the actual state will be one of the highly disordered states, or more precisely, that the actual state will be one in which the system moves between states of equivalent amounts of disorder. In the following discussion, therefore, the term *state* will indicate a collection of states of equivalent amounts of disorder.

The second law again •

All physical processes tend toward more probable states for the system and its surroundings. The more probable state is always one of higher disorder. Because entropy is a measure of disorder, an alternative way of saying this is that **the entropy of the Universe increases in all natural processes.** This statement is yet another way of stating the second law of thermodynamics.

To calculate the change in entropy for a finite process, we must recognize that T is generally not constant. If dQ_r is the thermal energy transferred reversibly when the system is at a temperature of T, then the change in entropy in an arbitrary reversible process between an initial state and a final state is

Changes in entropy for a finite process •

$$\Delta S = \int_i^f dS = \int_i^f \frac{dQ_r}{T} \qquad \text{(reversible path)} \qquad [18.7]$$

Although we do not prove it here, the change in entropy of a system that goes from one state to another has the same value for *all* paths connecting the two states. That is, the change in entropy of a system depends only on the properties of the initial and final equilibrium states.

In the case of a **reversible, adiabatic process,** no thermal energy is transferred between the system and its surroundings, and therefore $\Delta S = 0$. Because no change in entropy occurs, such a process is often referred to as an **isentropic process.**

Consider the changes in entropy that occur in a Carnot heat engine operating between the temperatures T_c and T_h. In one cycle, the engine absorbs thermal energy Q_h from the hot reservoir and rejects thermal energy Q_c to the cold reservoir. Thus, the total change in entropy for one cycle is

$$\Delta S = \frac{Q_h}{T_h} - \frac{Q_c}{T_c}$$

where the negative sign in the second term represents the fact that thermal energy Q_c is expelled by the system. For a Carnot cycle,

$$\frac{Q_c}{Q_h} = \frac{T_c}{T_h}$$

Change in entropy for a Carnot cycle is zero. •

Using this result in the preceding expression for ΔS, we find that the total change in entropy for a Carnot engine operating in a cycle is *zero.*

Now consider a system taken through an arbitrary reversible cycle. Because the entropy function is a state function and hence depends only on the properties of a given equilibrium state, we conclude that $\Delta S = 0$ for *any* cycle. In general, we can write this condition in the mathematical form

$$\oint \frac{dQ_r}{T} = 0 \qquad [18.8]$$

• $\Delta S = 0$ *for any reversible cycle*

where the symbol $\oint$ indicates that the integration is over a *closed* path.

Another important property of entropy is the fact that the entropy of the Universe is not changed by a reversible process. This can be understood by noting that two bodies, A and B, that interact with each other reversibly must always be in thermal equilibrium with each other. That is, their temperatures must always be equal. Therefore, when a small amount of thermal energy, dQ, is transferred from A to B, the increase in entropy of B is dQ/T, whereas the corresponding change in entropy of A is $-dQ/T$. Thus, the total change in entropy of the system $(A + B)$ is zero, and the entropy of the Universe is unaffected by the reversible process.

As a special case, the following example shows how to calculate the change in entropy of a substance that undergoes a phase change.

Example 18.3 Change in Entropy: Melting Process

A solid substance that has a latent heat of fusion L_f melts at a temperature T_m. Calculate the change in entropy that occurs when m grams of this substance is melted.

Solution Let us assume that the melting occurs so slowly that it can be considered a reversible process. In that case the temperature can be regarded as constant and equal to T_m. Making use of Equation 18.7 and the fact that the latent heat of fusion $Q = mL_f$ (Eq. 17.5), we find that

$$\Delta S = \int \frac{dQ_r}{T} = \frac{1}{T_m}\int dQ = \frac{Q}{T_m} = \frac{mL_f}{T_m}$$

Note that we were able to remove T_m from the integral because the process is isothermal.

EXERCISE 5 Calculate the change in entropy when 0.30 kg of lead melts at 327°C. Lead has a latent heat of fusion equal to 24.5 kJ/kg. Answer $\Delta S = 12.3$ J/K

EXERCISE 6 What is the entropy decrease in 1 mol of helium gas that is cooled at 1 atm from room temperature (20°C) to 4 K? (C of helium = 21 J/mol K.)
Answer −90.2 J/K

18.7 • ENTROPY CHANGES IN IRREVERSIBLE PROCESSES

By definition, calculation of the change in entropy requires information about a reversible path connecting the initial and final equilibrium states. In order to calculate changes in entropy for real (irreversible) processes, we must first recognize that the entropy function (like internal energy) depends only on the *state* of the system. That is, entropy is a state function. Hence, the change in entropy when a system moves between any two equilibrium states depends only on the initial and final states. Experimentally one finds that the entropy change is the same for all processes that can occur between a given set of initial and final states. It is possible to show that if this were not the case, the second law of thermodynamics would be violated.

In view of the fact that the entropy of a system depends only on the state of the system, we can now calculate entropy changes for irreversible processes between two equilibrium states. We can do this by devising a reversible process (or series of reversible processes) between the same two equilibrium states and computing

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$\int dQ_r/T$ for the reversible process. The entropy change for the irreversible process is the same as that for the reversible process between the same two equilibrium states. In irreversible processes, it is critically important to distinguish between Q, the actual thermal energy transfer in the process, and Q_r, the thermal energy that would have been transferred along a reversible path. It is only the latter that gives the correct value for entropy change. For example, as we will see, if an ideal gas expands adiabatically into a vacuum, $Q = 0$, but $\Delta S \neq 0$ because $Q_r \neq 0$. The reversible path between the same two states is the reversible, isothermal expansion that gives $\Delta S > 0$.

As we shall see in the following examples, change in entropy for the system plus its surroundings is always positive for an irreversible process. In general, the total entropy (and disorder) always increases in irreversible processes. From these considerations, the second law of thermodynamics can be stated as follows: **The total entropy of an isolated system that undergoes a change cannot decrease.** Furthermore, if the process is *irreversible*, the total entropy of an isolated system always *increases*. However, in a reversible process, the total entropy of an isolated system remains constant.

When dealing with interacting bodies that are not isolated from the environment, remember that the increase of entropy applies for the system *and* its surroundings. When two bodies interact in an irreversible process, the increase in entropy of one part of the system is greater than the decrease in entropy of the other part. Hence, we conclude that the change in entropy of the Universe must be greater than zero for an irreversible process and equal to zero for a reversible process. Ultimately, the entropy of the Universe should reach a maximum value. At this point the Universe will be in a state of uniform temperature and density. All physical, chemical, and biological processes will cease, because a state of perfect disorder implies that no energy is available for doing work. This gloomy state of affairs is sometimes referred to as the *heat death of the Universe.*

Heat Conduction

Consider the reversible transfer of thermal energy Q from a hot reservoir at temperature T_h to a cold reservoir at temperature T_c. Because the cold reservoir absorbs thermal energy Q, its entropy increases by Q/T_c. At the same time, the hot reservoir loses thermal energy Q and its entropy decreases by Q/T_h. The increase in entropy of the cold reservoir is greater than the decrease in entropy of the hot reservoir, because T_c is less than T_h. Therefore, the total change in entropy of the system (and of the Universe) is greater than zero:

$$\Delta S_u = \frac{Q}{T_c} - \frac{Q}{T_h} > 0$$

Thinking Physics 4

The entropy statement of the second law states that the entropy of the Universe increases in irreversible processes. This sounds very different from the previous two forms of the second law: "1. It is impossible to construct a heat engine that, operating in a cycle, produces no other effect than the absorption of thermal energy from a reservoir and the performance of an equal amount of work. 2. Energy will not flow by heat spontaneously from a cold object to a hot object." Can these two statements be made consistent with the entropy interpretation of the second law?

Reasoning There is no problem with consistency. In the first statement, the thermal energy in the reservoir is disordered energy—the random motion of molecules. Performing work results in ordered energy—pushing a piston through a displacement, for example. In this case, the motion of all molecules of the piston is in the same direction. If a heat engine absorbs thermal energy and performs an equal amount of work, then we have converted disorder into order, in violation of the entropy statement. In the second statement, we start with an ordered system—higher temperature in the hot object, lower in the cold object. This separation of temperatures is an example of order. If energy were to flow from the cold object to the hot object, so that the temperatures became even farther apart, this would be an increase in order, in violation of the entropy statement.

Example 18.4 Which Way Does the Heat Flow?

A large cold object is at 273 K, and a large hot object is at 373 K. Show that it is impossible for a small amount of thermal energy, say 8.00 J, to be transferred from the cold object to the hot one without decreasing the entropy of the Universe.

Reasoning We assume that, during the thermal energy transfer, the two objects do not undergo a temperature change. This is not a necessary assumption; it is used to avoid reliance on the techniques of integral calculus. The process as described is irreversible, so we must find an equivalent reversible process. It is sufficient to assume that the hot and cold objects are connected by a poor thermal conductor the temperature of which spans the range from 273 K to 373 K, which transfers thermal energy but the state of which does not change during the process. Then, the thermal energy transfer to or from each object is reversible, and we may set $Q = Q_r$.

Solution The entropy change of the hot object is

$$\Delta S_h = \frac{Q}{T_h} = \frac{8.00\ \text{J}}{373\ \text{K}} = 0.0214\ \text{J/K}$$

The cold object loses thermal energy, and its entropy change is

$$\Delta S_c = \frac{Q}{T_c} = \frac{-8.00\ \text{J}}{273\ \text{K}} = -0.0293\ \text{J/K}$$

The net entropy change of the Universe is

$$\Delta S_u = \Delta S_c + \Delta S_h = -0.0079\ \text{J/K}$$

This is in violation of the concept that the entropy of the Universe always increases in natural processes. That is, *the spontaneous transfer of thermal energy from a cold to a hot object cannot occur.*

EXERCISE 7 In the preceding example, suppose that 8.00 J of thermal energy is transferred from the hot to the cold object. What is the net entropy change of the Universe?
Answer $+0.0079$ J/K

Free Expansion

An ideal gas in an insulated container initially occupies a volume of V_i (Fig. 18.8). A partition separating the gas from an evacuated region is suddenly broken so that the gas expands (irreversibly) to the volume V_f. Let us find the change in entropy of the gas and the Universe.

The process is clearly neither reversible nor quasi-static. The work done by the gas against the vacuum is zero, and because the walls are insulating, no thermal energy is transferred during the expansion. That is, $W = 0$ and $Q = 0$. Using the

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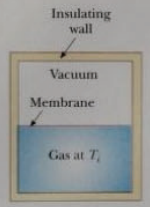


Figure 18.8 Free expansion of a gas. When the partition separating the gas from the evacuated region is ruptured, the gas expands freely and irreversibly so that it occupies a greater final volume. The container is thermally insulated from its surroundings, so $Q = 0$.

first law, we see that the change in internal energy is zero; therefore, $U_i = U_f$. Because the gas is ideal, U depends on temperature only, so we conclude that $T_i = T_f$.

To apply Equation 18.7, we must find Q_r—that is, we must find an equivalent reversible path that shares the same initial and final states. A simple choice is an isothermal, reversible expansion in which the gas pushes slowly against a piston. Because T is constant in this process, Equation 18.7 gives

$$\Delta S = \int \frac{dQ_r}{T} = \frac{1}{T}\int_i^f dQ_r$$

But $\int dQ_r$ is simply the work done by the gas during the reversible isothermal expansion from V_i to V_f, which is given by Equation 17.12. Because this is an isothermal process, $\Delta U = 0$, so the first law tells us that the energy input by heat is equal to the work done by the gas. Using this result, we find that

$$\Delta S = nR \ln\left(\frac{V_f}{V_i}\right) \qquad [18.9]$$

Because $V_f > V_i$, we conclude that ΔS is positive, and so both the entropy and the disorder of the gas (and the Universe) increase as a result of the irreversible, adiabatic expansion.

Irreversible Heat Transfer

A substance of mass m_1, specific heat c_1, and initial temperature T_1 is placed in thermal contact with a second substance of mass m_2, specific heat c_2, and initial temperature T_2, where $T_2 > T_1$. The two substances are contained in an insulated box so that no thermal energy is lost to the surroundings. The system is allowed to reach thermal equilibrium. What is the total entropy change for the system?

First, let us calculate the final equilibrium temperature, T_f. Energy conservation requires that the thermal energy lost by one substance equal the thermal energy gained by the other. Because, by definition, $Q = mc\,\Delta T$ for each substance, we get $Q_1 = -Q_2$, or

$$m_1c_1\,\Delta T_1 = -m_2c_2\,\Delta T_2$$
$$m_1c_1(T_f - T_1) = -m_2c_2(T_f - T_2)$$

Solving for T_f gives

$$T_f = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2} \qquad [18.10]$$

Note that $T_1 < T_f < T_2$, as expected.

The process is irreversible, because the system goes through a series of nonequilibrium states. During such a transformation, the temperature at any time is not well defined. However, we can imagine that the hot body, initially at temperature T_i, is slowly cooled to temperature T_f by contact with a series of reservoirs differing infinitesimally in temperature, the first reservoir being at T_i and the last at T_f. Such a series of very small changes in temperature would approximate a reversible process. Applying Equation 18.7 and noting that $dQ = mc\,dT$ for an infinitesimal change, we get

$$\Delta S = \int_1 \frac{dQ_1}{T} + \int_2 \frac{dQ_2}{T} = m_1c_1\int_{T_1}^{T_f}\frac{dT}{T} + m_2c_2\int_{T_2}^{T_f}\frac{dT}{T}$$

where we have assumed that the specific heats remain constant. Integrating, we find

$$\Delta S = m_1c_1 \ln\left(\frac{T_f}{T_1}\right) + m_2c_2 \ln\left(\frac{T_f}{T_2}\right) \qquad [18.11]$$

• *Change in entropy for a heat transfer process*

where T_f is given by Equation 18.10. If Equation 18.10 is substituted into Equation 18.11, it can be shown that one of the terms in Equation 18.11 will always be positive and the other negative. (You may want to verify this for yourself.) However, the positive term will always be larger than the negative term, resulting in a positive value for ΔS. Thus, we conclude that the entropy of the system and of the Universe increases in this irreversible process.

Finally, you should note that Equation 18.11 is valid only when no mixing occurs between the two substances in thermal contact. If the substances are liquids or gases and mixing occurs, Equation 18.11 applies only if the two fluids are identical.

18.8 • ENTROPY ON A MICROSCOPIC SCALE[2]

As we have seen, entropy can be approached via macroscopic concepts, using parameters such as pressure and temperature. Entropy can also be treated from a microscopic viewpoint through statistical analysis of molecular motions. We shall use a microscopic model to investigate the free expansion of an ideal gas discussed in the preceding section.

In the kinetic theory of gases, gas molecules are taken to be particles moving in a random fashion. Suppose all the gas is confined to the volume V_i (Fig. 18.9a). When the partition separating V_i from the larger container is removed, the molecules eventually are distributed in some fashion throughout the greater volume, V_f (Fig. 18.9b). The exact nature of the distribution is a matter of probability.

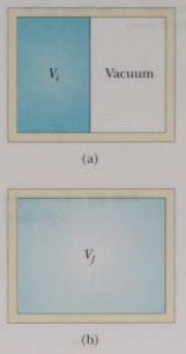


Figure 18.9 In a free expansion, the gas is allowed to expand into a larger volume that was previously a vacuum.

Such a probability can be determined by first finding the probabilities for the variety of molecular locations involved in the free expansion process. The instant after the partition is removed (and before the molecules have had a chance to rush into the other half of the container), all the molecules are in the initial volume. Now let us estimate the probability of the molecules' arriving at a particular configuration through natural random motions into some larger volume V_f. Assume each molecule occupies some microscopic volume, V_m. Then the total number of possible locations of that molecule, in a macroscopic initial volume, V_i, is the ratio, $W_i = V_i/V_m$, which is a huge number. We assume each of these locations is equally probable.

As more molecules are added to the system, the number of possible states multiply together. Neglecting the small probability of two molecules trying to occupy the same location, each molecule may go into any of the (V_i/V_m) locations, so the number of possibilities is $W_i^N = (V_i/V_m)^N$. Although a large number of the possible states would be considered unusual, the total number of states is so enormous that these unusual states have a negligible probability of occurring. Hence,

[2]This section was adapted from A. Hudson and R. Nelson, *University Physics*, Philadelphia, Saunders College Publishing, 1990, used with permission of the publisher.

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A full house (a pair and three of a kind) is a good hand in the game of poker. Can you calculate the probability of being dealt this hand in a standard deck of 52 cards assuming that 5 cards are dealt at one time? *(Tom Mareschel, The IMAGE Bank)*

the number of equivalent states is proportional to $(V_i/V_m)^N$. In a similar way, if the volume is increased to V_f, the number of equivalent states increases to $W_f^N = (V_f/V_m)^N$. Hence, the probabilities are $P_i = cW_i^N$ and $P_f = cW_f^N$, where the constant, c, has been left undetermined. The ratio of these probabilities is

$$\frac{P_f}{P_i} = \frac{cW_f^N}{cW_i^N} = \frac{\left(\frac{V_f}{V_m}\right)^N}{\left(\frac{V_i}{V_m}\right)^N} = \left(\frac{V_f}{V_i}\right)^N$$

If we now take the natural logarithm of this equation and multiply by Boltzmann's constant, k_B, we find

$$Nk_B \ln\left(\frac{W_f}{W_i}\right) = nN_A k_B \ln\left(\frac{V_f}{V_i}\right)$$

where we write the number of molecules, N, as nN_A, the number of moles times Avogadro's number. From Section 13.7 we know that $N_A k_B$ is the universal gas constant, R, so this equation may be written as

$$Nk_B \ln W_f - Nk_B \ln W_i = nR \ln\left(\frac{V_f}{V_i}\right) \qquad [18.12]$$

From thermodynamic considerations (specifically, Eq. 18.9) we know that when n mol of a gas undergoes a free expansion from V_i to V_f, the change in entropy, ΔS, is

$$S_f - S_i = nR \ln\left(\frac{V_f}{V_i}\right) \qquad [18.13]$$

Note that the right-hand sides of Equations 18.12 and 18.13 are identical. We thus make the following connection between entropy and the number of (microscopic) states available to the system in its (macroscopic) state:

Entropy (microscopic definition)

$$S = Nk_B \ln W \qquad [18.14]$$

Although our discussion used the specific example of the free expansion of an ideal gas, a more rigorous development of the statistical interpretation of entropy leads to the same conclusion: **Entropy is a measure of microscopic disorder.**

Imagine that a container of gas has molecules with speeds above the mean value on the left side and molecules with speeds lower than the mean value on the right side (an *ordered* arrangement), as in Figure 18.10a. Compare with an even mixture of fast- and slow-moving molecules like that in Figure 18.10b (a *disordered* arrangement). You might expect the ordered arrangement to be unlikely because random motions tend to mix the slow- and fast-moving molecules uniformly. Yet each of these arrangements *individually* is equally probable. However, there are far more disordered arrangements than ordered arrangements, so *collectively* the ordered arrangements are much less probable.

Let us estimate this probability for 100 molecules. The chance of any one molecule being in the left part of the container as a result of random motion is 1/2. If the molecules move independently, the probability of 50 faster molecules being found in the left part at any instant is $(1/2)^{50}$. Likewise, the probability of the remaining 50 slower molecules being found in the right part at any instant is $(1/2)^{50}$. Therefore, the probability of finding this fast–slow separation through

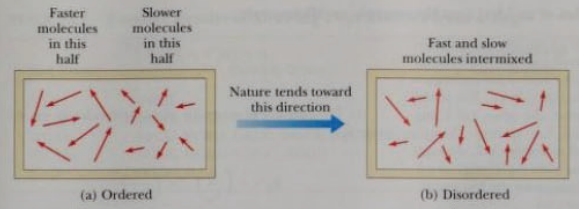


Figure 18.10 A box of gas in two equally probable states of molecular motion that may exist. (a) An *ordered* arrangement; one of a *few*, and therefore *collectively unlikely*, set. (b) A *disordered* arrangement; one of *many*, and therefore a *collectively likely set.*

random motion is the product $(1/2)^{50}(1/2)^{50} = (1/2)^{100}$, which corresponds to about 1 chance in 10^{30}. When this calculation is extrapolated from 100 molecules to a mole of gas (about 10^{23} molecules), the ordered arrangement is found to be *extremely* improbable!

Thinking Physics 5

A box contains 5 gas molecules, spread throughout the box. At some point in time, all 5 are in one half of the box. Does this violate the second law of thermodynamics? Is the second law valid for this system?

Reasoning Strictly speaking, this situation does violate the second law of thermodynamics. In response to the second question, however, the second law is not valid for small numbers of particles. The second law is based on collections of huge numbers of particles, so that disordered states have astronomically higher probabilities than ordered states. Because the macroscopic world is built from these huge numbers of particles, we see the second law as valid, as processes proceed from order to disorder. In the 5-molecule system, the general idea of the second law is valid in that there are more disordered states than ordered states, but the relatively high probability of the ordered states results in their existence from time to time.

CONCEPTUAL PROBLEM 6

A variety of flowers grow in a garden. The flowers are picked and arranged into a bouquet. After the flowers wilt, they are thrown into a fireplace and burned. Arrange these three states of the flowers (growing in the garden, arranged in a bouquet, and burned in a fireplace) in order of increasing entropy.

CONCEPTUAL PROBLEM 7

Imagine an adiabatic expansion of a gas. According to the definition of entropy, there is no entropy change, because there was no energy transferred by heat. But the molecules of the gas now have more volume in which to move around, so there seems to be an increase in disorder. An increase in disorder means an increase in entropy. How can we reconcile these notions—one that suggests no entropy change and the other that suggests an increase in entropy?

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Example 18.5 **Free Expansion of an Ideal Gas: Macroscopic vs. Microscopic**

Again consider the free expansion of an ideal gas. Let us verify that the macroscopic and microscopic approaches lead to the same conclusion. Suppose that 1 mol of gas undergoes a free expansion to four times its initial volume. The initial and final temperatures are, of course, the same. (a) Using a macroscopic approach, calculate the entropy change. (b) Find the probability, P_i, that all molecules will, through random motions, be found simultaneously in the original volume. (c) Calculate the change in entropy for the free expansion and show that it agrees with part (a).

Solution (a) From Equation 18.9 we obtain

$$\Delta S = nR \ln\left(\frac{V_f}{V_i}\right) = (1)R \ln\left(\frac{4V_i}{V_i}\right) = R \ln 4$$

(b) The number of states available to a single molecule in the initial volume V_i is $W_i = (V_i/V_m)$. For one mole (N_A molecules), the number of available states is $W_i^{N_A}$. Thus, the probability that all molecules are found in the original volume is

$$P_i = cW_i^{N_A} = c\left(\frac{V_i}{V_m}\right)^{N_A}$$

(c) The number of states for all N_A molecules in the volume $V_f = 4V_i$ is

$$W_f^{N_A} = \left(\frac{V_f}{V_m}\right)^{N_A} = \left(\frac{4V_i}{V_m}\right)^{N_A}$$

From Equation 18.14 we obtain

$$\Delta S = k_B \ln W_f^{N_A} - k_B \ln W_i^{N_A} = k_B \ln\left(\frac{W_f^{N_A}}{W_i^{N_A}}\right) = k_B \ln (4^{N_A})$$
$$= N_A k_B \ln 4 = R \ln 4$$

The answer is the same as that in part (a), which dealt with macroscopic parameters.

OPTIONAL

18.9 · ENTROPY AND DISORDER

As you look around at the beauty in nature, it is easy to recognize that the events taking place within natural processes involve a large element of chance. For example, the spacing between trees in a natural forest is quite random. If you were to discover a forest in which all the trees were equally spaced, you would conclude that the forest had probably been planted by humans. We can express the results of such an observation by saying that **a disorderly arrangement is much more probable than an orderly one if the laws of nature are allowed to act without interference.**

We already found that entropy is a measure of the disorder in a system through the relation $S = k_B \ln W$, where W is the number of available states over which the molecules can be distributed and is therefore a measure of the disorder, or randomness, of the system. We can further explore the meaning of this relation by considering another specific example. Imagine that you have a bag of 100 marbles, of which 50 are red and 50 are green. You are allowed to draw four marbles from the bag according to the following rules. Draw one marble, record its color, return it to the bag, and draw another. Continue this process until four marbles have been drawn. Because each marble is returned to the bag before another is drawn, the probability of drawing a red marble is always the same as that of drawing a green one. All the possible outcomes are shown in Table 18.1. For example, the result RRGR means that you drew a red marble on the first draw, a red one on the second, a green one on the third, and a red one on the fourth. As this table indicates, there is only one way to draw four red marbles. However, there are four possible sequences that could produce one green and three red marbles, six sequences that could produce two green and two red, four sequences that could produce three green and one red, and one sequence that could produce all green. The most likely outcome—two red and two green marbles—corresponds to the most disordered state. This is considered disorder simply because we cannot distinguish between individual molecules of the same color or between individual molecules. The probability that you would draw four red or four green marbles, these being the most ordered states, is much lower. From Equation 18.14 we see that the state with the greatest disorder has the highest entropy because it is the most probable. However, the most ordered states (all red or all green) are the least likely to occur and are the states of lowest entropy. The outcome of the draw can range from a highly ordered state, which has the lowest entropy, to a highly disordered state, which has the highest entropy.

TABLE 18.1 **Possible Results of Drawing Four Marbles from a Bag**

End Result	Possible Draws	Total Number of Same Results
All R	RRRR	1
1G, 3R	RRRG, RRGR, RGRR, GRRR	4
2G, 2R	RRGG, RGRG, GRRG, RGGR, GRGR, GGRR	6
3G, 1R	GGGR, GGRG, GRGG, RGGG	4
All G	GGGG	1

Thinking Physics 6

Touch a rubber band to your forehead and note the sensation of temperature. Remove the rubber band from your forehead, stretch it and touch it to your forehead again. It should now feel warmer. Remove the rubber band again from your forehead, let it return to its original length and touch it to your forehead again. It now feels cooler.

(a) Why do these temperature changes occur?
(b) Someone proposes that the answer to part (a) is *internal friction*, as portions of the rubber band rub against each other during the size changes. How would you respond to this proposal?

Reasoning (a) The answer lies in the second law of thermodynamics. Rubber consists of long, coiled molecules, the axes of which lie in random directions relative to the length of the rubber band. The force of the stretch tends to cause the coiled molecules to line up parallel with the rubber band. This represents an increase in the degree of *order* of these molecules, which represents a *decrease* in entropy. If the rubber band is stretched quickly, however, then the stretching process is very close to adiabatic, and the net entropy change is close to zero. Thus, there must be some change that occurs to compensate for the decrease in entropy due to the coils lining up. We can compensate for the decrease if the molecules increase their vibrational motion, so that their positions in the rubber represent a lower degree of order. Increased vibrational motion is represented by more internal energy that, in turn, means that the temperature increases—this is the warmth that is felt on the forehead.

When the rubber band is allowed to return to its original length, the coiled molecules can become more disordered, resulting in an increase in entropy. The increase in entropy of the directions of the molecules is accompanied by a decrease in entropy of the vibrations of the molecules. This decreased vibration represents a lower internal energy, and you feel a cooler temperature in this situation.

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(b) The internal friction argument could be claimed to be consistent with the increase in temperature associated with stretching the rubber band. When the rubber band is returned to its original length, however, there would be *additional rubbing* of portions of the rubber band, and the internal friction would result in a further temperature *increase*. This is inconsistent with the observation that the temperature decreases on returning the rubber band to its original length. Internal friction can be argued as the reason that a wire coat hanger increases in temperature when it is bent back and forth, but the rubber band phenomenon has the second law of thermodynamics as its basis.

SUMMARY

Real processes proceed in directions governed by the second law of thermodynamics. A **heat engine** is a device that converts thermal energy to other useful forms of energy. The net work done by a heat engine in carrying a substance through a cyclic process ($\Delta U = 0$) is

$$W = Q_h - Q_c \qquad [18.1]$$

where Q_h is the thermal energy absorbed from a hot reservoir and Q_c is the thermal energy rejected to a cold reservoir.

The **thermal efficiency,** e, of a heat engine is defined as the ratio of the net work done to the thermal energy absorbed per cycle from the higher temperature reservoir:

$$e \equiv \frac{W}{Q_h} = 1 - \frac{Q_c}{Q_h} \qquad [18.2]$$

The **second law of thermodynamics** can be stated as follows: **It is impossible to construct a heat engine that, operating in a cycle, produces no effect other than the absorption of thermal energy from a reservoir and the performance of an equal amount of work.**

A **reversible process** is one in which the system can be returned to its initial conditions along the same path and in which every point along the path is an equilibrium state. A process that does not satisfy these requirements is **irreversible.**

The **efficiency of a heat engine** operating in the **Carnot cycle** is given by

$$e_c = 1 - \frac{T_c}{T_h} \qquad [18.3]$$

where T_c is the absolute temperature of the cold reservoir and T_h is the absolute temperature of the hot reservoir.

No real heat engine operating between the temperatures T_c and T_h can be more efficient than an engine operating reversibly in a Carnot cycle between the same two temperatures.

The second law of thermodynamics states that when real (irreversible) processes occur, the degree of disorder in the system plus the surroundings increases. When a process occurs in an isolated system, ordered energy is converted into disordered energy. The measure of disorder in a system is called **entropy,** S.

The **change in entropy,** dS, of a system moving through an infinitesimal process between two equilibrium states is

$$dS = \frac{dQ_r}{T} \qquad [18.6]$$

The change in entropy of a system moving between two equilibrium states is

$$\Delta S = \int_i^f \frac{dQ_r}{T} \qquad [18.7]$$

The value of ΔS is the same for all reversible paths connecting the initial and final states.

The change in entropy for any reversible, cyclic process is zero, and when such a process occurs, the entropy of the Universe remains constant. Entropy is a state function—that is, it depends on the state of the system. The change in entropy for a system undergoing a real (irreversible) process between two equilibrium states is the same as that for a reversible process between the same states. In an irreversible process, the total entropy of an isolated system always increases. In general, the total entropy (and disorder) always increases in any irreversible process. Furthermore, the change in entropy of the Universe is greater than zero for an irreversible process.

From a microscopic viewpoint, **entropy,** S, is defined as

$$S = k_B \ln W \qquad [18.14]$$

where k_B is Boltzmann's constant and W is the number of (microscopic) states available to the system in its (macroscopic) state. Because of the statistical tendency of systems to proceed toward states of greater probability and greater disorder, all natural processes are irreversible and increase in entropy. Thus, **entropy is a measure of microscopic disorder.**

CONCEPTUAL QUESTIONS

1. What are some factors that affect the efficiency of automobile engines?
2. A steam-driven turbine is one major component of an electric power plant. Why is it advantageous to have the temperature of the steam as high as possible?
3. Is it possible to construct a heat engine that creates no thermal pollution?
4. Discuss the change in entropy of a gas that expands (a) at constant temperature and (b) adiabatically.
5. In solar ponds constructed in Israel, the Sun's energy is concentrated near the bottom of a salty pond. With the proper layering of salt in the water, convection is prevented, and temperatures of 100°C may be reached. Can you guess the maximum efficiency with which useful energy can be extracted from the pond?
6. Why does your automobile burn more gas in winter than in summer?
7. Can a heat pump have a coefficient of performance less than unity? Explain.
8. The Carnot cycle was illustrated in the chapter on a PV diagram. What would the Carnot cycle look like on a TS diagram?
9. Give some examples of irreversible processes that occur in nature. Give an example of a process in nature that is nearly reversible.
10. A thermodynamic process occurs in which the entropy of a system changes by -8.0 J/K. According to the second law of thermodynamics, what can you conclude about the entropy change of the environment?
11. If a supersaturated sugar solution is allowed to slowly evaporate, sugar crystals form in the container. Hence, sugar molecules go from a disordered form (in solution) to a highly ordered, crystalline form. Does this process violate the second law of thermodynamics? Explain.
12. How could you increase the entropy of 1 mol of a metal that is at room temperature? How could you decrease its entropy?
13. A heat pump is to be installed in a region where the average outdoor temperature in the winter months is -20°C. In view of this, why would it be advisable to place the outdoor compressor unit deep in the ground? Why are heat pumps not commonly used for heating in cold climates?
14. Suppose your roommate is "Mr. Clean" and tidies up your messy room after a big party. Because more order is being created by your roommate, does this represent a violation of the second law of thermodynamics?
15. Discuss the entropy changes that occur when you (a) bake a loaf of bread and (b) consume the bread.
16. A diesel engine operates with a compression ratio of about 15. A gasoline engine has a compression ratio of about 6. Which engine runs hotter? Which engine (at least theoretically) can operate more efficiently?
17. If you shake a jar full of jelly beans of different sizes, the larger beans tend to appear near the top, and the

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smaller ones tend to fall to the bottom. Why does this occur? Does this process violate the second law of thermodynamics?

18. The device in Figure Q18.18, called a thermoelectric converter, uses a series of semiconductor cells to convert thermal energy to electrical energy. In the photograph at the left, both legs of the device are at the same temperature, and no electrical energy is produced. However, when one leg is at a higher temperature than the other, as in the photograph on the right, electrical energy is produced as the device extracts energy from the hot reservoir and drives a small electric motor. (a) Why does the temperature differential produce electrical energy in this demonstration? (b) In what sense does this intriguing experiment demonstrate the second law of thermodynamics?

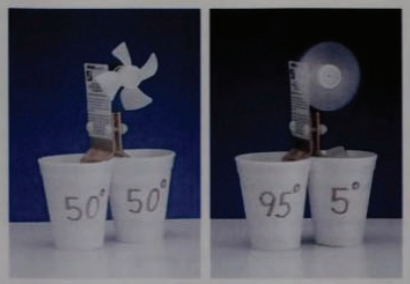


Figure Q18.18 (*Courtesy of PASCO Scientific Company*)

PROBLEMS

Section 18.1 Heat Engines and the Second Law of Thermodynamics

1. A heat engine absorbs 360 J of thermal energy and performs 25.0 J of work in each cycle. Find (a) the efficiency of the engine and (b) the thermal energy expelled in each cycle.
2. A heat engine performs 200 J of work in each cycle and has an efficiency of 30.0%. For each cycle, how much thermal energy is (a) absorbed and (b) expelled?
3. A particular engine has a power output of 5.00 kW and an efficiency of 25.0%. If the engine expels 8000 J of thermal energy in each cycle, find (a) the heat absorbed in each cycle and (b) the time for each cycle.

Section 18.3 The Carnot Engine

4. A heat engine operating between 200°C and 80.0°C achieves 20.0% of the maximum possible efficiency. What energy input will enable the engine to perform 10.0 kJ of work?
5. One of the most efficient engines ever built (its actual efficiency is 42.0%) operates between 430°C and 1870°C. (a) What is its maximum theoretical efficiency? (b) How much power does the engine deliver if it absorbs 1.40×10^5 J of thermal energy each second?
6. A Carnot engine has a power output of 150 kW. The engine operates between two reservoirs at 20.0°C and 500°C. (a) How much thermal energy is absorbed per hour? (b) How much thermal energy is lost per hour?
7. A steam engine is operated in a cold climate in which the exhaust temperature is 0°C. (a) Calculate the theoretical maximum efficiency of the engine using an intake steam temperature of 100°C. (b) If, instead, superheated steam at 200°C is used, find the maximum possible efficiency.
8. The exhaust temperature of a Carnot heat engine is 300°C. What is the intake temperature if the efficiency of the engine is 30.0%?
9. An ideal gas is taken through a Carnot cycle. The isothermal expansion occurs at 250°C, and the isothermal compression takes place at 50.0°C. If the gas absorbs 1200 J of heat during the isothermal expansion, find (a) the heat expelled to the cold reservoir in each cycle and (b) the net work done by the gas in each cycle.
10. A power plant operates at a 32.0% efficiency during the summer when the sea water for cooling is at 20.0°C. The plant uses 350°C steam to drive turbines. Assuming that the plant's efficiency changes in the same proportion as the ideal efficiency, what would be the plant's efficiency in the winter when the sea water is at 10.0°C?
11. A power plant has been proposed that would make use of the temperature gradient in the ocean. The system is to operate between 20.0°C (surface water temperature) and 5.00°C (water temperature at a depth of about 1 km). (a) What is the maximum efficiency of such a system? (b) If the power output of the plant is 75.0 MW, how much thermal energy is absorbed per hour? (c) In view of your answer to (a), do you think such a system is worthwhile (considering that there is no charge for fuel)?
12. Suppose a heat engine is connected to two heat reservoirs, one a pool of molten aluminum (660°C) and the other a block of solid mercury (−38.9°C). The engine runs by freezing 1.00 g of aluminum and melting 15.0 g of mercury during each cycle. The heat of fusion of aluminum is 3.97×10^5 J/kg; the heat of fusion of mercury is 1.18×10^4 J/kg. What is the efficiency of this engine?

13. A 20.0%-efficient real engine is used to speed up a train from rest to 5.00 m/s. It is known that an ideal (Carnot) engine using the same cold and hot reservoirs would accelerate the same train from rest to a speed of 6.50 m/s using the same amount of fuel. If the engines use air at 300 K as a cold reservoir, find the temperature of the steam serving as the hot reservoir.

Section 18.4 Heat Pumps and Refrigerators

14. What is the coefficient of performance of a refrigerator that operates with Carnot efficiency between temperatures −3.00°C and +27.0°C?
15. An ideal refrigerator or ideal heat pump is equivalent to a Carnot engine running in reverse. That is, heat Q_c is absorbed from a cold reservoir and heat Q_h is rejected to a hot reservoir. (a) Show that the work that must be supplied to run the refrigerator or pump is

$$W = \frac{T_h - T_c}{T_c} Q_c$$

(b) Show that the coefficient of performance of the ideal refrigerator is

$$\text{COP} = \frac{T_c}{T_h - T_c}$$

16. How much work is required, using an ideal Carnot refrigerator, to remove 1.00 J of thermal energy from helium gas at 4.00 K and reject this thermal energy to a room-temperature (293 K) environment?
17. A refrigerator has a coefficient of performance equal to 5.00. If the refrigerator absorbs 120 J of thermal energy from a cold reservoir in each cycle, find (a) the work done in each cycle and (b) the thermal energy expelled to the hot reservoir.
18. A heat pump, shown in Figure P18.18, is essentially a heat engine run backward. It extracts thermal energy from colder air outside and deposits it in a warmer room. Suppose that the ratio of the actual thermal energy entering the room to the work done by the device's motor is 10.0% of the theoretical maximum. Determine the thermal energy entering the room per joule of work done by the motor if the inside temperature is 20.0°C and the outside temperature is −5.00°C.

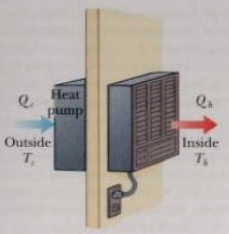


Figure P18.18

Section 18.6 Entropy

19. An ice tray contains 500 g of water at 0°C. Calculate the change in entropy of the water as it freezes completely and slowly at 0°C.
20. At a pressure of 1 atm, liquid helium boils at 4.20 K. The latent heat of vaporization is 20.5 kJ/kg. Determine the entropy change (per kilogram) resulting from vaporization.
21. Calculate the change in entropy of 250 g of water heated slowly from 20.0°C to 80.0°C. (*Hint:* Note that $dQ = mc\,dT$.)

Section 18.7 Entropy Changes in Irreversible Processes

22. The surface of the Sun is approximately at 5700 K, and the temperature of the Earth's surface is approximately 290 K. What entropy change occurs when 1000 J of thermal energy is transferred from the Sun to the Earth?
23. A 1500-kg car is moving at 20.0 m/s. The driver brakes to a stop. The brakes cool off to the temperature of the surrounding air, which is nearly constant at 20.0°C. What is the total entropy change?
24. An avalanche of ice and snow, of mass 1000 kg, slides downhill a vertical distance of 200 m. What is the total entropy change if the mountain air temperature is nearly constant at −3.00°C?
25. A 1.00-kg iron horseshoe is taken from a furnace at 900°C and dropped into 4.00 kg of water at 10.0°C. If no heat is lost to the surroundings, determine the total entropy change.
26. How fast are you personally making the entropy of the Universe increase right now? Compute an order-of-magnitude estimate, stating what quantities you take as data and the values you measure or estimate for them.
27. One mole of H_2 gas is contained in the left-hand side of the container shown in Figure P18.27, which has equal volumes

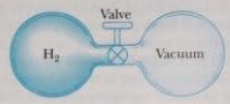


Figure P18.27

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left and right. The right-hand side is evacuated. When the valve is opened, the gas streams into the right side. What is the final entropy change? Does the temperature of the gas change?

28. A 2.00-liter container has a center partition that divides it into two equal parts, as shown in Figure P18.28. The left side contains H_2 gas, and the right side contains O_2 gas. Both gases are at room temperature and at atmospheric pressure. The partition is removed and the gases are allowed to mix. What is the entropy increase?

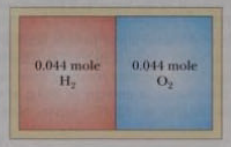


Figure P18.28

18.8 Entropy on a Microscopic Scale

18.9 Entropy and Disorder (Optional)

29. If you toss two dice, what is the total number of ways in which you can obtain (a) a 12 and (b) a 7?
30. Prepare a table like Table 18.1 for the following occurrence. You toss four coins into the air simultaneously. Record the results of your tosses in terms of the numbers of heads and tails that result. For example, HHTH and HTHH are two possible ways in which three heads and one tail can be achieved. (a) On the basis of your table, what is the most probable result of a toss? In terms of entropy, (b) what is the most ordered state, and (c) what is the most disordered?
31. Repeat the procedure used to construct Table 18.1 (a) for the case in which you draw three marbles from your bag rather than four and (b) for the case in which you draw five rather than four.

Additional Problems

32. On a hot day, a Florida home is kept cool by an air conditioner. The outside temperature is 36.0°C and the interior air is at 18.0°C. If 50 000 kJ/h of thermal energy is removed from the house, what is the minimum power that must be provided to the air conditioner?
33. Every second at Niagara Falls, some 5000 m^3 of water falls a distance of 50.0 m. What is the increase in entropy per second due to the falling water? (Assume that the mass of the surroundings is so large that its temperature and that of the water stays nearly constant at 20.0°C. Suppose that negligible water evaporates.)
34. If a 35.0%-efficient heat engine (Fig. 18.1) is run in reverse so as to form a refrigerator (Fig. 18.7), what would be this refrigerator's coefficient of performance?
35. How much work is required, using an ideal Carnot refrigerator, to change 0.500 kg of tap water at 10.0°C into ice at −20.0°C? Assume the freezer compartment is held at −20.0°C and the refrigerator exhausts heat into a room at 20.0°C.
36. A heat engine operates between two reservoirs at T_2 = 600 K and T_1 = 350 K. It absorbs 1000 J of thermal energy from the higher temperature reservoir and performs 250 J of work. Find (a) the entropy change of the Universe, ΔS_u, for this process and (b) the work, W, that could have been done by an ideal Carnot engine operating between these two reservoirs. (c) Show that the difference between the work done in parts (a) and (b) is $T_1 \Delta S_u$.
37. A house loses heat through the exterior walls and roof at a rate of 5000 J/s = 5.00 kW when the interior temperature is 22.0°C and the outside temperature is −5.00°C. Calculate the electric power required to maintain the interior temperature at 22.0°C for the following two cases. (a) The electric power is used in electric resistance heaters (which convert all of the electricity supplied into heat). (b) The electric power is used to drive an electric motor that operates the compressor of a heat pump (which has a coefficient of performance equal to 60.0% of the Carnot cycle value).
38. An athlete whose mass is 70.0 kg drinks 16 ounces (453.6 g) of refrigerated water. The water is at a temperature of 35.0°F. (a) Neglecting the temperature change of the body that results from the water intake (so that the body is regarded as a reservoir always at 98.6°F), find the entropy increase of the entire system. (b) Assume that the entire body is cooled by the drink and that the average specific heat of a human is equal to the specific heat of liquid water. Neglecting any other heat transfers and any metabolic heat release, find the athlete's temperature after she drinks the cold water, given an initial body temperature of 98.6°F. Under *these* assumptions, what is the entropy increase of the entire system? Compare this result with (a).
39. A refrigerator has a coefficient of performance of 3.00. The ice tray compartment is at −20.0°C, and the room temperature is 22.0°C. The refrigerator can convert 30.0 g of water at 22.0°C to 30.0 g of ice at −20.0°C each minute. What input power is required? Give your answer in watts.
40. An ideal (Carnot) freezer in a kitchen has a constant temperature of 260 K, whereas the air in the kitchen has a constant temperature of 300 K. Suppose the insulation for the freezer is not perfect so that some heat flows into the freezer at a rate of 0.150 W. Determine the average power of the freezer's motor which is needed to maintain the constant temperature in the freezer.
41. A power plant, having a Carnot efficiency, produces 1000 MW of electrical power from turbines that take in steam at 500 K and reject water at 300 K into a flowing river. If the water downstream is 6.00 K warmer due to the output of the power plant, determine the flow rate of the river.
42. A power plant, having a Carnot efficiency, produces electric power P from turbines that take in heat from steam at temperature T_h and discharge heat at temperature T_c through a heat exchanger into a flowing river. If the water downstream is warmer by ΔT due to the output of the power plant, determine the flow rate of the river.
43. What is the minimum amount of work that must be done to extract 400 J of heat from a massive object at 0°C while rejecting heat to a hot reservoir at a temperature of 20.0°C?
44. A gas follows the path 123 on the PV diagram in Figure P18.44 and 418 J of heat flows into the system. Also, 167 J of work is done. (a) What is the internal energy change of the system? (b) How much thermal energy flows into the system if the process follows the path 143? The work done by the gas along this path is 63.0 J. What net work would be done on or by the system if the system followed (c) the path 12341? (d) the path 14321? (e) What is the change in internal energy of the system in the processes described in parts (c) and (d)?

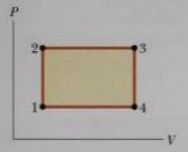


Figure P18.44

45. Figure P18.45 represents n mol of an ideal monatomic gas being taken through a cycle that consists of two isothermal processes at temperatures $3T_0$ and T_0 and two constant-volume processes. For each cycle, determine, in terms of n, R, and T_0, (a) the net heat transferred to the gas and (b) the efficiency of an engine operating in this cycle.

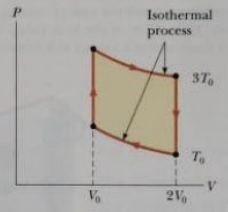


Figure P18.45

46. An electric power plant has an overall efficiency of 15.0%. The plant is to deliver 150 MW of power to a city, and its turbines use coal as the fuel. The burning coal produces steam that drives the turbines. This steam is then condensed to water at 25.0°C by passing it through cooling coils in contact with river water. (a) How many metric tons of coal does the plant consume each day (1 metric ton = 10^3 kg)? (b) What is the total cost of the fuel per year if the delivered price is $8.00/metric ton? (c) If the river water is delivered at 20.0°C, at what minimum rate must it flow over the cooling coils in order that its temperature not exceed 25.0°C? (*Note:* The heat of combustion of coal is 33.0 kJ/g.)

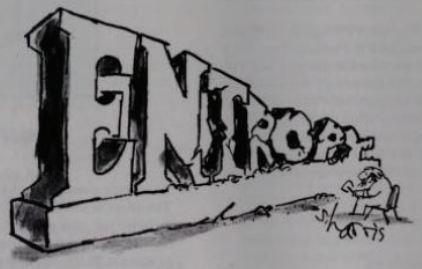
ANSWERS TO CONCEPTUAL PROBLEMS

1. The efficiency of a Carnot engine is given by the ratio of the difference in operating temperatures ($T_h - T_c$) to the temperature of the hot reservoir (T_h). Increasing T_h by ΔT or decreasing T_c by ΔT both have the same effect on the difference in operating temperatures. But only increasing T_h has an effect on the denominator of the fraction, increasing the denominator, and, thus, reducing the value of the fraction. Thus, it would be more effective to decrease T_c by ΔT.
2. An automobile engine has a certain efficiency, which determines how much energy is available to operate the car. With no accessories operating, this energy is completely available for changing the kinetic energy of the car (during acceleration) and overcoming friction as the car moves. If accessories are operating, the energy for their operation must come from the output energy of the engine, leaving less available for moving the car, and thus decreasing the gas mileage. Therefore, the air conditioner, the radio, the headlights, and the windshield washer system will all reduce the gas mileage when they are operating. Rolling down a manual window does not—the energy for this comes from the driver's body.
3. If friction were removed, the engine would be as efficient as possible, but even a Carnot engine would not be 100% efficient—this would violate the second law. The maximum efficiency depends on the operating temperatures.
4. The role of an air conditioner is that of a heat pump—it pumps energy from a cold reservoir (the interior air) to a warm reservoir (the exterior air). Thus, the "exhaust" of the air conditioner must be to the outside air, which is the case

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if the air conditioner is installed in a window or through a hole in the wall. If an air conditioner were operated in the middle of a room, air would enter the unit, have energy added to it (the energy entering the unit by electricity) and be expelled back into the room, warming the room up.

5. Yes, a heat pump can deliver significantly more energy than it consumes. In the winter, the energy delivered to the interior of the home is a combination of the energy drawn from the exterior air and the energy coming into the unit by electricity. If the COP of the heat pump is 5, then it is delivering 5 times as much energy as it consumes.
6. In order of increasing entropy: arranged in a bouquet, growing in the garden, burned in a fireplace.
7. There is no increase in entropy from either point of view. In an adiabatic expansion, the temperature must drop. Thus, although the molecules have more volume in which to move around, they move with lower velocities. The decrease in velocity represents more order, because the random jostling of the molecules is reduced. A calculation will show that the decrease in entropy due to the drop in jostling exactly compensates for the increase due to the increased volume. Thus, the net entropy change is zero.



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Electric Forces and Electric Fields

This chapter begins with a review of some of the basic properties of the electrostatic force, as well as some properties of the electric field associated with stationary charged particles. Our study of electrostatics then continues with the concept of an electric field that is associated with a continuous charge distribution and the effect of this field on other charged particles.

Two methods for calculating the electric fields associated with continuous charge distributions are discussed. The first method uses Coulomb's law and integral calculus, and the second uses Gauss's law. An outgrowth of Coulomb's law, Gauss's law is much more convenient for calculating electric fields of highly symmetric charge distributions. Furthermore, Gauss's law serves as a guide for understanding more complicated problems.

CHAPTER OUTLINE

19.1 Historical Overview
19.2 Properties of Electric Charges
19.3 Insulators and Conductors
19.4 Coulomb's Law
19.5 Electric Fields
19.6 Electric Field Lines
19.7 Electric Flux
19.8 Gauss's Law
19.9 Application of Gauss's Law to Charged Insulators
19.10 Conductors in Electrostatic Equilibrium



◀ **This metallic sphere is charged by a generator to a high voltage. The high concentration of charge formed on the sphere creates a strong electric field around the sphere. The charges then leak through the surrounding gas, producing the pink glow.** *(E. R. Degginger, H. Armstrong Roberts)*

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19.1 · HISTORICAL OVERVIEW

In this chapter we continue our study of electric and magnetic phenomena that we introduced in Chapter 5. The laws of electricity and magnetism play a central role in the operation of devices such as radios, televisions, electric motors, computers, high-energy particle accelerators, and a host of electronic devices used in medicine. More fundamental, however, is the fact that the interatomic and intermolecular forces that are responsible for the formation of solids and liquids are electric in origin. Furthermore, such forces as the pushes and pulls between objects and the elastic force in a spring arise from electric forces at the atomic level.

Chinese documents suggest that magnetism was recognized as early as about 2000 B.C. The ancient Greeks observed electric and magnetic phenomena possibly as early as 700 B.C. They found that a piece of amber, when rubbed, became electrified and attracted pieces of straw or feathers. The existence of magnetic forces was known from observations that pieces of a naturally occurring stone called *magnetite* (Fe_3O_4) were attracted to iron. (The word *electric* comes from the Greek word for amber, *elektron*. The word *magnetic* comes from *Magnesia*, a city on the coast of Turkey where magnetite was found.)

In 1600, the English scientist William Gilbert discovered that electrification was not limited to amber but was a general phenomenon. Scientists went on to electrify a variety of objects, including chickens and people! Experiments by Charles Coulomb in 1785 confirmed the inverse-square force law for electricity.

It was not until the early part of the 19th century that scientists established that electricity and magnetism are related phenomena. In 1820, the Danish scientist Hans Oersted discovered that a compass needle, which is magnetic, is deflected when placed near a circuit carrying an electric current. In 1831, Michael Faraday in England and, almost simultaneously, Joseph Henry in the United States, showed that when a wire is moved near a magnet (or when a magnet is moved near a wire), an electric current is observed in the wire. In 1873, James Clerk Maxwell used these observations and other experimental facts as a basis for formulating the laws of electromagnetism as we know them today. Shortly thereafter (around 1888), Heinrich Hertz verified Maxwell's predictions by producing electromagnetic waves in the laboratory. This achievement was followed by such practical developments as radio and television.

Maxwell's contributions to the science of electromagnetism were especially significant because the laws he formulated are basic to *all* forms of electromagnetic phenomena. His work is comparable in importance to Newton's discovery of the laws of motion and the theory of gravitation.

19.2 · PROPERTIES OF ELECTRIC CHARGES

A number of simple experiments demonstrate the existence of electrostatic forces. For example, after running a comb through your hair, you will find that the comb attracts bits of paper. The attractive electrostatic force is often strong enough to suspend the bits. The same effect occurs with other rubbed materials, such as glass or rubber.

Another simple experiment is to rub an inflated balloon with wool (or across your hair). On a dry day, the rubbed balloon will stick to the wall of a room, often

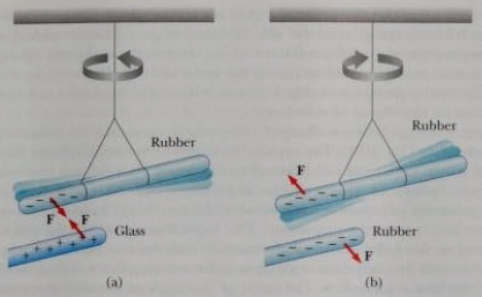


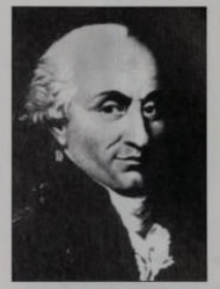
Figure 19.1 (a) A negatively charged rubber rod, suspended by a thread, is attracted to a positively charged glass rod. (b) A negatively charged rubber rod is repelled by another negatively charged rubber rod.

for hours. When materials behave this way, they are said to have become **electrically charged.** You can give your body an electric charge by walking across a wool rug or by sliding across a car seat. You can then feel, and remove, the charge on your body by lightly touching another person. Under the right conditions, a visible spark is seen when you touch, and a slight tingle is felt by both parties. (Such an experiment works best on a dry day because excessive moisture can provide a pathway for charge to leak off a charged object.)

Experiments also demonstrate that there are two kinds of electric charge, which were given the names **positive** and **negative** by Benjamin Franklin (1706–1790). Figure 19.1 illustrates the interactions of the two charges. A hard rubber (or plastic) rod that has been rubbed with fur (or an acrylic material) is suspended by a piece of string. When a glass rod that has been rubbed with silk is brought near the rubber rod, the rubber rod is attracted toward the glass rod (Fig. 19.1a). If two charged rubber rods (or two charged glass rods) are brought near each other, as in Figure 19.1b, the force between them is repulsive. This observation demonstrates that the rubber and glass have different kinds of charge. We use the convention suggested by Franklin, wherein the electric charge on the glass rod is called positive and that on the rubber rod is called negative. On the basis of observations such as these, we conclude that **like charges repel one another, and unlike charges attract one another.**

A natural tendency exists for charge to be transferred between unlike materials. Rubbing the two materials together only increases the area of contact and thus enhances the charge-transfer process.

Another important characteristic of electric charge is that **it is always conserved.** That is, when two initially neutral objects are charged by being rubbed together, charge is not created in the process. The objects become charged because *negative charge is transferred* from one object to the other. One object gains some amount of negative charge while the other loses an equal amount of negative charge and hence is left with a positive charge. For example, when a glass rod is rubbed with silk, as in Figure 19.2, the silk obtains a negative charge that is equal in mag-



Charles Coulomb (1736–1806), the great French physicist for whom the unit of electric charge called the *coulomb* was named.



Figure 19.2 When a glass rod is rubbed with silk, electrons are transferred from the glass to the silk. Because of conservation of charge, each electron adds negative charge to the silk, and an equal positive charge is left behind on the rod. Also, because the charges are transferred in discrete bundles, the charges on the two objects are $\pm e$, $\pm 2e$, or $\pm 3e$, and so on.

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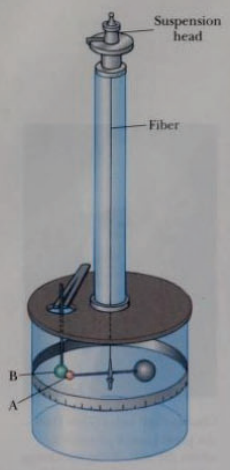


Figure 19.3 Coulomb's torsion balance, which was used to establish the inverse-square law for the electrostatic force between two charges.

nitude to the positive charge on the glass rod as negatively charged electrons are transferred from the glass to the silk. Likewise, when rubber is rubbed with fur, electrons are transferred from the fur to the rubber. An *uncharged object* contains an enormous number of electrons (on the order of 10^{23}). However, for every negative electron, a positively charged proton is also present; hence, an uncharged object has no net charge of either sign.

Electric forces between charged objects were measured by Coulomb with a torsion balance (Fig. 19.3). The apparatus consists of two small spheres fixed to the ends of a light horizontal rod made of an insulating material and suspended by a silk thread. Sphere A is given a charge, and charged object B is brought near sphere A. The attractive or repulsive force between the two charged objects causes the rod to rotate and to twist the suspension. The angle through which the rod rotates is measured via the deflection of a light beam by a mirror attached to the suspension. The rod rotates through some angle against the restoring force of the twisted thread before reaching equilibrium. The value of the angle of rotation increases as the charge on the objects increases. Thus, the angle of rotation provides a quantitative measure of the electric force of attraction or repulsion.

From our discussion thus far, and from what we learned in Chapter 5, we conclude that electric charge has the following important properties:

- Two kinds of charges exist in nature, with the property that unlike charges attract one another and like charges repel one another.
- The force between charges varies as the inverse square of their separation.
- Charge is conserved.
- Charge is quantized.

CONCEPTUAL PROBLEM 1

Two insulated rods are oppositely charged on the ends. They are mounted at their centers so that they are free to rotate, and then held in the position shown in a view from above (Fig. 19.4). The plane of the rotation of the rods is in the plane of the paper. Will the rods stay in these positions when released? If not, into what position(s) will they move? Will the final configuration(s) be stable?



Figure 19.4 (Conceptual Problem 1)

19.3 · INSULATORS AND CONDUCTORS

It is convenient to classify substances in terms of their ability to conduct electrical charge.

Conductors are materials in which electric charges move freely, and **insulators** are materials in which electric charges do not move freely.

Materials such as glass, rubber, and lucite are insulators. When such materials are charged by rubbing, only the rubbed area becomes charged, and there is no tendency for charge to move to other regions of the material. In contrast, materials such as copper, aluminum, and silver are good conductors. When such materials are charged in some small region, the charge readily distributes itself over the entire surface of the material. If you hold a copper rod in your hand and rub it with wool or fur, it will not attract a small piece of paper. This might suggest that a metal cannot be charged. However, if you hold the copper rod by an insulating handle and then rub, the rod will remain charged and attract the piece of paper. In the first case, the electric charges produced by rubbing readily move from the copper through your body and finally to Earth. In the second case, the insulating handle prevents the flow of charge to Earth.

Semiconductors are a third class of materials, and their electrical properties are somewhere between those of insulators and those of conductors. Silicon and germanium are well-known examples of semiconductors that are widely used to fabricate a variety of electronic devices. The electrical properties of semiconductors can be changed over many orders of magnitude by adding controlled amounts of certain foreign atoms to the materials.

- ***Metals are good conductors.***

Charging by Induction

When a conductor is connected to Earth by means of a conducting wire or pipe, it is said to be **grounded.** For our purposes, the Earth can be considered an infinite reservoir for electrons; this means that it can accept or supply an unlimited number of electrons. With this in mind, we can understand how to charge a conductor by a process known as **induction.**

Consider a negatively charged rubber rod brought near a neutral (uncharged) conducting sphere that is insulated so that there is no conducting path to ground (Fig. 19.5). The repulsive force between the electrons in the rod and those in the sphere causes a redistribution of charge on the sphere so that some electrons move to the side of the sphere farthest away from the rod. The region of the sphere nearest the rod has an excess of positive charge because of the migration of electrons away from this location. If a grounded conducting wire is then connected to the sphere, as in Figure 19.5b, some of the electrons leave the sphere and travel to the Earth. If the wire to ground is then removed (Fig. 19.5c), the conducting sphere is left with an excess of induced positive charge. Finally, when the rubber rod is removed from the vicinity of the sphere (Fig. 19.5d), the induced positive charge remains on the ungrounded sphere. This excess positive charge becomes uniformly distributed over the surface of the ungrounded sphere because of the repulsive forces among the like charges and the high mobility of charge carriers in a metal.

In the process of inducing a charge on the sphere, the charged rubber rod loses none of its negative charge, because it never came in contact with the sphere. **Charging an object by induction requires no contact with the object inducing the charge.** This is in contrast to charging an object by rubbing, which does require contact between the two objects.

A process very similar to charging by induction in conductors also takes place in insulators. In most neutral atoms and molecules, the center of positive charge

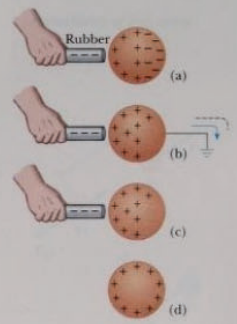


Figure 19.5 Charging a metallic object by induction. (a) The charge on a neutral metallic sphere is redistributed when a charged rubber rod is placed near the sphere. (b) The sphere is grounded, and some of the electrons leave. (c) The ground connection is removed, and the sphere has a net nonuniform positive charge. (d) When the rubber rod is removed, the excess positive charge becomes uniformly distributed over the surface of the sphere.

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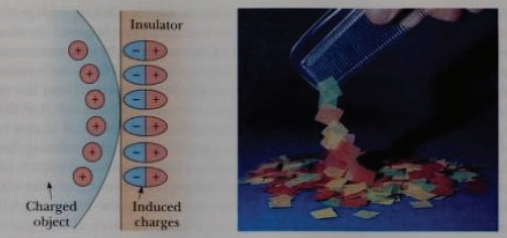


Figure 19.6 (a) A charged object induces charges on the surface of an insulator. (b) A charged comb attracts bits of paper because charges are displaced in the paper. The paper is neutral but polarized. *(Fundamental Photographs, NYC)*

coincides with the center of negative charge. However, in the presence of a charged object, these centers may shift slightly, resulting in more positive charge on one side of the molecule than on the other. This effect is known as **polarization.** The realignment of charge within individual molecules produces an induced charge on the surface of the insulator, as shown in Figure 19.6a. With these concepts in mind, you should be able to explain why a comb that has been rubbed through hair attracts bits of neutral paper or why a balloon that has been rubbed against your clothing can stick to a neutral wall.

Thinking Physics 1

A positively charged ball, hanging from a string, is brought near a nonconducting object. Based on the behavior of the ball–string combination, the ball is seen to be attracted to the object. From this experiment, it is not possible to determine whether the object is negatively charged or neutral. Why not? What additional experiment would help you decide between these two possibilities?

Reasoning The attraction between the ball and the object could be an attraction of unlike charges, *or* it could be an attraction between a charged object and a neutral object due to polarization of the molecules of the neutral object. Two possible additional experiments would help to determine if the object is charged. First, a known neutral ball could be brought near the object—if there is an attraction, the object is negatively charged. Another possibility is to bring a known negatively charged ball near the object—if there is a repulsion, then the object is negatively charged. If there is attraction, then the object is neutral.

CONCEPTUAL PROBLEM 2

If a suspended object A is attracted to object B, which is charged, can we conclude that object A is charged?

19.4 · COULOMB'S LAW

In Chapter 5 we introduced Coulomb's law, which describes the electrostatic force between two stationary charged particles. When the charges are of the same sign, the force on either charge is repulsive, and when the charges are of opposite sign, the force is attractive (Fig. 19.7). The magnitude of the electrostatic force between two charges separated by a distance of r is

$$F = k_e \frac{|q_1|\,|q_2|}{r^2} \qquad [19.1]$$

· *Coulomb's law*

where $k_e (= 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})$ is the Coulomb constant. The constant k_e is also written

$$k_e = \frac{1}{4\pi\epsilon_0}$$

where the constant ϵ_0 is known as the *permittivity of free space* and has the value

$$\epsilon_0 = 8.8542 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

Permittivity of free space

The charge of an electron or proton has the magnitude $|e| = 1.60 \times 10^{-19}$ C. Therefore, 1 C of charge is equal to the charge of 6.25×10^{18} electrons (that is, $1/e$). This can be compared with the number of free electrons in 1 $\mathrm{cm^3}$ of copper, which is on the order of 10^{23}. Note that 1 C is a substantial amount of charge. In typical electrostatic experiments, where a rubber or glass rod is charged by friction, a net charge on the order of 10^{-6} C ($= 1\ \mu$C) is obtained. In other words, only a very small fraction of the total available charge is transferred between the rod and the rubbing material. The charges and masses of the electron, proton, and neutron are given in Table 19.1.

When dealing with Coulomb's force law, you must remember that force is a *vector* quantity and must be treated accordingly. Furthermore, note that **Coulomb's law applies exactly only to point charges or particles.** The electrostatic force on q_2 due to q_1, written $\mathbf{F}_{21}$, can be expressed in vector form as

$$\mathbf{F}_{21} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}} \qquad [19.2]$$

where $\hat{\mathbf{r}}$ is a unit vector directed from q_1 to q_2 as in Figure 19.7a. From Newton's third law, we see that the electric force on q_1 due to q_2 is equal in magnitude to the

TABLE 19.1 Charge and Mass of the Electron, Proton, and Neutron

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.6021917 \times 10^{-19}$	9.1095×10^{-31}
Proton (p)	$+1.6021917 \times 10^{-19}$	1.67261×10^{-27}
Neutron (n)	0	1.67492×10^{-27}

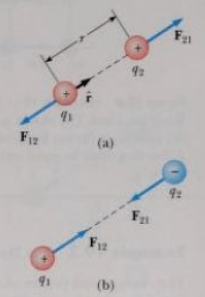


Figure 19.7 Two point charges separated by a distance r exert a force on each other given by Coulomb's law. Note that the force on q_1 is equal to and opposite the force on q_2. (a) When the charges are of the same sign, the force is repulsive. (b) When the charges are of the opposite sign, the force is attractive.

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force on q_2 due to q_1 and in the opposite direction; that is, $\mathbf{F}_{12} = -\mathbf{F}_{21}$. Finally, from Equation 19.2 we see that if q_1 and q_2 have the same sign, the product q_1q_2 is positive and the force is repulsive, as in Figure 19.7a. If q_1 and q_2 are of opposite sign, as in Figure 19.7b, the product q_1q_2 is negative and the force is attractive.

When more than two charges are present, the force between any pair is given by Equation 19.2. Therefore, **the resultant force on any one charge equals the *vector* sum of the forces due to the various individual charges.** This **principle of superposition** as applied to electrostatic forces is an experimentally observed fact. For example, if there are four charges, then the resultant force on particle 1 due to particles 2, 3, and 4 is given by the vector sum

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14}$$

Example 19.1 Where Is the Resultant Force Zero?

Three charges lie along the x axis as in Figure 19.8. The positive charge $q_1 = 15.0\ \mu\text{C}$ is at $x = 2.00$ m, and the positive charge $q_2 = 6.00\ \mu\text{C}$ is at the origin. Where must a negative charge q_3 be placed on the x axis such that the resultant force on it is zero?

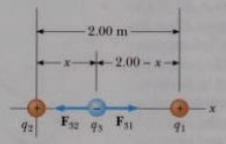


Figure 19.8 (Example 19.1) Three point charges are placed along the x axis. The charge q_3 is negative, whereas q_1 and q_2 are positive. If the net force on q_3 is zero, then the force on q_3 due to q_1 must be equal and opposite to the force on q_3 due to q_2.

Solution Because q_3 is negative and both q_1 and q_2 are positive, the forces $\mathbf{F}_{31}$ and $\mathbf{F}_{32}$ are both attractive, as indicated in Figure 19.8. If we let x be the coordinate of q_3, then the forces $\mathbf{F}_{31}$ and $\mathbf{F}_{32}$ have magnitudes

$$F_{31} = k_e \frac{|q_3|\,|q_1|}{(2.00 - x)^2} \quad \text{and} \quad F_{32} = k_e \frac{|q_3|\,|q_2|}{x^2}$$

In order for the resultant force on q_3 to be zero, $\mathbf{F}_{32}$ must be equal to and opposite $\mathbf{F}_{31}$, or

$$k_e \frac{|q_3|\,|q_2|}{x^2} = k_e \frac{|q_3|\,|q_1|}{(2.00 - x)^2}$$

Because k_e and q_3 are common to both sides, we solve for x and find that

$$(2.00 - x)^2|q_2| = x^2|q_1|$$

$$(4.00 - 4.00x + x^2)(6.00 \times 10^{-6}\ \text{C}) = x^2(15.0 \times 10^{-6}\ \text{C})$$

Solving this quadratic equation for x, we find that

$x = 0.775$ m. Why is the negative root not acceptable?

Example 19.2 The Hydrogen Atom

The electron and proton of a hydrogen atom are separated (on the average) by a distance of 5.3×10^{-11} m. Find the magnitude of the electrostatic force and the gravitational force between the two particles.

Solution From Coulomb's law, we find that the attractive electrical force has the magnitude

$$F_e = k_e \frac{|e|^2}{r^2} = 8.99 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\ \frac{(1.60 \times 10^{-19}\ \text{C})^2}{(5.3 \times 10^{-11}\ \text{m})^2}$$

$$= 8.2 \times 10^{-8}\ \text{N}$$

Using Newton's law of gravity and Table 19.1 for the particle masses, we find that the gravitational force has the magnitude

$$F_g = G \frac{m_e m_p}{r^2} =$$

$$\left(6.7 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) \frac{(9.11 \times 10^{-31}\ \text{kg})(1.67 \times 10^{-27}\ \text{kg})}{(5.3 \times 10^{-11}\ \text{m})^2}$$

$$= 3.6 \times 10^{-47}\ \text{N}$$

The ratio $F_e/F_g \approx 2 \times 10^{39}$. Thus the gravitational force between charged atomic particles is negligible compared with the electric force.

EXERCISE 1 Two point charges of magnitudes 3.0 nC and 6.0 nC are separated by a distance of 0.30 m. Find the electric force of repulsion between them.
Answer 1.8×10^{-6} N

19.5 · ELECTRIC FIELDS

The gravitational field $\mathbf{g}$ at a point in space was defined in Chapter 5 to be equal to the gravitational force $\mathbf{F}$ acting on a test mass m_0 divided by the test mass: $\mathbf{g} = \mathbf{F}/m_0$. In similar manner, an electric field at a point in space can be defined in terms of the electric force acting on a test charge q_0 placed at that point. To be more precise, **the electric field vector E at a point in space is defined as the electric force F acting on a positive test charge placed at that point divided by the magnitude of the test charge q_0:**

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} \qquad [19.3]$$

Note that $\mathbf{E}$ is the field produced by some charge *external* to the test charge—not the field produced by the test charge. This is analogous to the gravitational field set up by some body such as the Earth. The vector $\mathbf{E}$ has the SI units of newtons per coulomb (N/C). The direction of $\mathbf{E}$ is in the direction of $\mathbf{F}$ because we have assumed that $\mathbf{F}$ acts on a positive test charge. Thus, we can say that **an electric field exists at a point if a test charge at rest placed at that point experiences an electric force.** Once the electric field is known at some point, the force on *any* charged particle placed at that point can be calculated from Equation 19.3. Furthermore, the electric field is said to exist at some point (even empty space) regardless of whether or not a test charge is located at that point.

Consider a point charge q located a distance r from a small test charge q_0. According to Coulomb's law, the force exerted on the test charge by q is

$$\mathbf{F} = k_e \frac{qq_0}{r^2}\hat{\mathbf{r}}$$

Because the electric field at the position of the test charge is defined by $\mathbf{E} = \mathbf{F}/q_0$, we find that, at the position of q_0, the electric field created by q is

$$\mathbf{E} = k_e \frac{q}{r^2}\hat{\mathbf{r}} \qquad [19.4]$$

where $\hat{\mathbf{r}}$ is a unit vector directed from q toward q_0 (Fig. 19.9). If q is positive, as in Figure 19.9a, the electric field is directed radially outward from it. If q is negative, as in Figure 19.9b, the field is directed toward it.

In order to calculate the electric field at a point P due to a group of point charges, we first calculate the electric field vectors at P individually using Equation 19.4 and then add them vectorially. In other words, **the total electric field due to a group of charges equals the vector sum of the electric fields of all the charges.** This superposition principle applied to fields follows directly from the superposi-

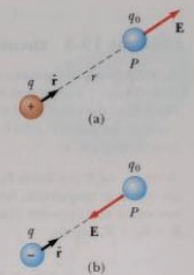


Figure 19.9 A test charge q_0 at point P is a distance r from a point charge q. (a) If q is positive, the electric field at P points radially *outward* from q. (b) If q is negative, the electric field at P points radially *inward* toward q.

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tion property of electric forces. Thus, the electric field of a group of charges (excluding the test charge q_0) can be expressed as

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i \qquad [19.5]$$

where r_i is the distance from the ith charge, q_i, to the point P (the location of the test charge) and $\hat{\mathbf{r}}_i$ is a unit vector directed from q_i toward P.

Thinking Physics 2

As an electron moving horizontally passes through deflection plates in an oscilloscope, a uniform electric field in the vertical direction exerts an electric force on the electron. What is the shape of the path followed by the electron while it is between the plates?

Reasoning The magnitude of the electrical force on the electron having charge e due to a uniform electric field of magnitude E is $F_e = eE$. Thus, the force is constant. Compare this to the force on a projectile of mass m moving in two dimensions in a uniform gravitational field g near the surface of the Earth. The magnitude of the gravitational force is $F_g = mg$. In both cases, the particle is subject to a constant force in the vertical direction and has an initial velocity in the horizontal direction. Thus, the path will be the same—the electron will follow a *parabolic* path, just like the projectile. Once the electron leaves the region between the plates, the electric field disappears and the electron continues moving in a straight line according to Newton's first law.

CONCEPTUAL PROBLEM 3

Why do TV screens and computer monitors become so dusty?

Example 19.3 Electric Field of a Dipole

An **electric dipole** consists of a positive charge q and a negative charge $-q$ separated by a distance of $2a$, as in Figure 19.10. Find the electric field $\mathbf{E}$ due to these charges along the y axis at the point P, which is a distance y from the origin. Assume that $y \gg a$.

Solution At P, the fields $\mathbf{E}_1$ and $\mathbf{E}_2$ due to the two charges are equal in magnitude, because P is equidistant from the two equal and opposite charges. The total field is given by $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$, where

$$E_1 = E_2 = k_e \frac{q}{r^2} = k_e \frac{q}{y^2 + a^2}$$

The y components of $\mathbf{E}_1$ and $\mathbf{E}_2$ cancel each other. The x components are equal, because they are both along the x axis. Therefore, $\mathbf{E}$ is parallel to the x axis and has a magnitude equal to $2E_1 \cos\theta$. From Figure 19.10 we see that $\cos\theta = a/r = a/(y^2 + a^2)^{1/2}$. Therefore,

$$E = 2E_1 \cos\theta = 2k_e \frac{q}{(y^2 + a^2)} \frac{a}{(y^2 + a^2)^{1/2}}$$

$$= k_e \frac{2qa}{(y^2 + a^2)^{3/2}}$$

Using the approximation $y \gg a$, we can neglect a^2 in the denominator and write

$$E \approx k_e \frac{2qa}{y^3}$$

Thus, we see that along the y axis the field of a dipole at a distant point varies as $1/r^3$, whereas the more slowly varying field of a point charge goes as $1/r^2$. (Note that in the geometry of this example, $r = y$.) This is because at distant points, the fields of the two equal and opposite charges almost cancel each other. The $1/r^3$ variation in E for the dipole is also obtained for a distant point along the x axis (Problem 8) and for a general distant point. The dipole is a good model of many molecules, such as HCl.

As we shall see in later chapters, neutral atoms and molecules behave as dipoles when placed in an external electric field. Furthermore, many molecules, such as HCl, are permanent dipoles. (HCl is partially described as an H^+ ion combined with a Cl^- ion.) The effect of such dipoles on the behavior of materials subjected to electric fields is discussed in Chapter 20.

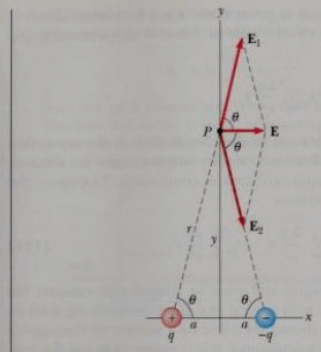


Figure 19.10 (Example 19.3) The total electric field $\mathbf{E}$ at P due to two equal and opposite charges (an electric dipole) equals the vector sum $\mathbf{E}_1 + \mathbf{E}_2$. The field $\mathbf{E}_1$ is due to the positive charge q, and $\mathbf{E}_2$ is the field due to the negative charge $-q$.

EXERCISE 2 A piece of aluminum foil of mass 5.0×10^{-2} kg is suspended by a string in an electric field directed vertically upward. If the charge on the foil is 3.0 μC, find the strength of the field that will reduce the tension in the string to zero. Answer 1.6×10^5 N/C

EXERCISE 3 The nucleus of a hydrogen atom, a proton, sets up an electric field. The average distance between the proton and the electron of a hydrogen atom is approximately 5.3×10^{-11} m. What is the magnitude of the electric field at this distance from the proton? Answer 5.1×10^{11} N/C

Electric Field Due to Continuous Charge Distributions

In most practical situations (such as an object charged by rubbing), the average separation between charges is small compared with their distances from the field point. In such cases, the system of charges can be considered *continuous*. That is, we imagine that the system of closely spaced charges is equivalent to a total charge that is continuously distributed through some volume or over some surface.

To evaluate the electric field of a continuous charge distribution, the following procedure is used. First, we divide the charge distribution into small elements, each of which contains a small charge Δq as in Figure 19.11. Next, we use Coulomb's law to calculate the electric field due to one of these elements at a point P. Finally, we evaluate the total field at P due to the charge distribution by summing the contributions of all the charge elements (that is, by applying the superposition principle).

The electric field at P due to one element of charge Δq is given by

$$\Delta\mathbf{E} = k_e \frac{\Delta q}{r^2} \hat{\mathbf{r}}$$

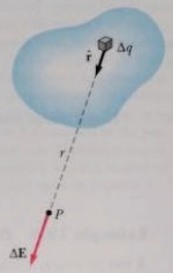


Figure 19.11 The electric field at P due to a continuous charge distribution is the vector sum of the fields due to all the elements Δq of the charge distribution.

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where r is the distance from the element to point P and $\hat{\mathbf{r}}$ is a unit vector directed from the element toward P. The total electric field at P due to all elements in the charge distribution is approximately

$$\mathbf{E} \approx k_e \sum_i \frac{\Delta q_i}{r_i^2} \hat{\mathbf{r}}_i$$

where the index i refers to the ith element in the distribution. If the separation between elements in the charge distribution is small compared with the distance to P, the charge distribution can be approximated as continuous. Therefore, the total field at P in the limit $\Delta q_i \to 0$ becomes

Electric field of a continuous charge distribution

$$\mathbf{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2} \hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}} \qquad [19.6]$$

where the integration is a *vector* operation and must be treated with caution. We shall illustrate this type of calculation with several examples in which we shall assume that the charge is *uniformly* distributed on a line or a surface or throughout some volume. When performing such calculations, it is convenient to use the concept of a charge density along with the following notations.

If a charge Q is uniformly distributed throughout a volume V the **charge per unit volume,** ρ, is defined by

Volume charge density

$$\rho \equiv \frac{Q}{V} \qquad [19.7]$$

where ρ has units of coulombs per cubic meter.

If a charge Q is uniformly distributed on a surface of area A, the **surface charge density,** σ, is defined by

Surface charge density

$$\sigma \equiv \frac{Q}{A} \qquad [19.8]$$

where σ has units of coulombs per square meter.

Finally, if a charge Q is uniformly distributed along a line of length ℓ, the **linear charge density,** λ, is defined by

Linear charge density

$$\lambda \equiv \frac{Q}{\ell} \qquad [19.9]$$

where λ has units of coulombs per meter.

Example 19.4 The Electric Field Due to a Charged Rod

A rod of length ℓ has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P along the axis of the rod, a distance d from one end (Fig. 19.12).

Reasoning and Solution For this calculation, the rod is taken to be along the x axis. Let us use Δx to represent the length of one small segment of the rod and let Δq be the charge on the segment. The ratio of Δq to Δx is equal to the ratio of the total charge to the total length of the rod. That is, $\Delta q/\Delta x = Q/\ell = \lambda$. Therefore, the charge Δq on the small segment is $\Delta q = \lambda\, \Delta x$.

The field $\Delta\mathbf{E}$ due to this segment at the point P is in the negative x direction, and its magnitude is[1]

$$\Delta E = k_e \frac{\Delta q}{x^2} = k_e \frac{\lambda\, \Delta x}{x^2}$$

Note that each element produces a field in the negative x direction, and so the problem of summing their contributions is particularly simple in this case. The total field at P due to all segments of the rod, which are at different distances from P, is given by Equation 19.6, which in this case becomes

$$E = \int_d^{\ell+d} k_e \lambda \frac{dx}{x^2}$$

where the limits on the integral extend from one end of the rod ($x = d$) to the other ($x = \ell + d$). Because k_e and λ are constants, they can be removed from the integral. Thus, we find that

$$E = k_e \lambda \int_d^{\ell+d} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x} \right]_d^{\ell+d}$$

$$= k_e \lambda \left(\frac{1}{d} - \frac{1}{\ell + d} \right) = \frac{k_e Q}{d(\ell + d)}$$

where we have used the fact that the total charge $Q = \lambda\ell$.

From this result we see that if the point P is far from the rod ($d \gg \ell$), then ℓ in the denominator can be neglected, and $E \approx k_e Q/d^2$. This is just the form you would expect for a point charge. Therefore, at large values of d/ℓ the charge distribution appears to be a point charge of magnitude Q. The use of the limiting technique (in this case, $d/\ell \to \infty$) is often a good method for checking a theoretical formula.

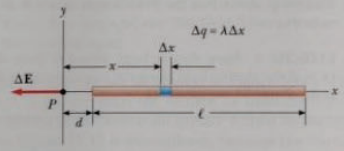


Figure 19.12 (Example 19.4) The electric field at P due to a uniformly charged rod lying along the x axis. The field at P due to the segment of charge Δq is $k_e\, \Delta q/x^2$. The total field at P is the vector sum over all segments of the rod.

Example 19.5 The Electric Field of a Uniform Ring of Charge

A ring of radius a has a uniform positive charge per unit length, with a total charge Q. Calculate the electric field along the axis of the ring at a point P lying a distance x from the center of the ring (Fig. 19.13a).

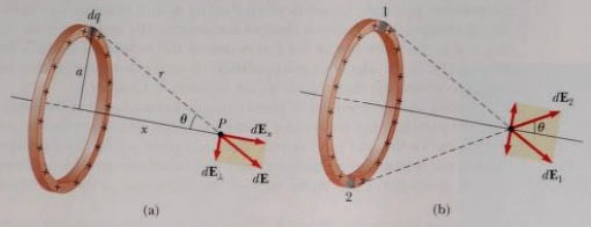


Figure 19.13 (Example 19.5) A uniformly charged ring of radius a. (a) The field at P on the x axis due to an element of charge dq. (b) The total electric field at P is along the x axis. Note that the perpendicular component of the electric field at P due to segment 1 is canceled by the perpendicular component due to segment 2, which is located on the ring opposite segment 1.

[1] It is important that you understand the procedure being used to carry out integrations such as this. First, choose an element the parts of which are all equidistant from the point at which the field is being calculated. Next, express the charge element Δq in terms of the other variables within the integral (in this example, there is one variable, x). The integral must be over scalar quantities and therefore must be expressed in terms of components. Then reduce the form to an integral over a single variable (or multiple integrals, each over a single variable). In examples that have spherical or cylindrical symmetry, the variables will be radial coordinates.

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Reasoning and Solution The magnitude of the electric field at P due to the segment of charge dq is

$$dE = k_e \frac{dq}{r^2}$$

This field has an x component $dE_x = dE \cos\theta$ along the axis of the ring and a component $dE_\perp$ perpendicular to the axis. But as we see in Figure 19.13b, the resultant field at P must lie along the x axis because the perpendicular components sum to zero. That is, the perpendicular component of any element is canceled by the perpendicular component of an element on the opposite side of the ring. Because $r = (x^2 + a^2)^{1/2}$ and $\cos\theta = x/r$, we find that

$$dE_x = dE\cos\theta = \left(k_e \frac{dq}{r^2}\right)\frac{x}{r} = \frac{k_e x}{(x^2 + a^2)^{3/2}}\,dq$$

In this case, all segments of the ring give the same contribution to the field at P because they are all equidistant from this point. Thus, we can integrate the expression above to get the total field at P:

$$E_x = \int \frac{k_e x}{(x^2 + a^2)^{3/2}}\,dq = \frac{k_e x}{(x^2 + a^2)^{3/2}}\int dq$$

$$= \frac{k_e x}{(x^2 + a^2)^{3/2}}\,Q$$

This result shows that the field is zero at $x = 0$. Does this surprise you?

EXERCISE 4 Show that at large distances from the ring ($x \gg a$) the electric field along the axis approaches that of a point charge of magnitude Q.

EXERCISE 5 A sphere of radius 4 cm has a net charge of $+39\ \mu C$. (a) If this charge is uniformly distributed throughout the volume of the sphere, what is the volume charge density? (b) If this charge is uniformly distributed on the sphere's surface, what is the surface charge density? Answer (a) $0.15\ C/m^3$ (b) $1.9 \times 10^{-3}\ C/m^2$

PROBLEM-SOLVING STRATEGY AND HINTS

1. Units: When performing calculations that involve the Coulomb constant, $k_e (= 1/4\pi\epsilon_0)$, charges must be in coulombs and distances in meters. If they appear in other units, you must convert them.
2. Applying Coulomb's law to point charges: It is important to use the superposition principle properly when dealing with a collection of interacting charges. When several charges are present, the resultant force on any one is the *vector sum* of the forces due to the individual charges. Be very careful in the algebraic manipulation of vector quantities. It may be useful to review the material on vector addition in Chapter 1.
3. Calculating the electric field of point charges: Remember that the superposition principle can be applied to electric fields, which are also vector quantities. To find the total electric field at a given point, first calculate the electric field at that point due to each individual charge. The resultant field at the point is the vector sum of the fields due to the individual charges.
4. Continuous charge distributions: When you are confronted with problems that involve a continuous distribution of charge, the vector sums for evaluating the total electric field at some point must be replaced by vector integrals. The charge distribution is divided into infinitesimal pieces, and the vector sum is carried out by integrating over the entire charge distribution. Examples 19.4 and 19.5 demonstrate such procedures.
5. Symmetry: Whenever dealing with either a distribution of point charges or a continuous charge distribution, take advantage of any symmetry in the system to simplify your calculations.

19.6 · ELECTRIC FIELD LINES

A convenient aid for visualizing electric field patterns is to draw lines pointing in the same direction as the electric field vector at any point. These lines, called **electric field lines,** are related to the electric field in any region of space in the following manner:

- The electric field vector **E** is *tangent* to the electric field line at each point.
- The number of lines per unit area through a surface that is perpendicular to the lines is proportional to the strength of the electric field in that region. Thus, **E** is large where the field lines are close together and small where they are far apart.

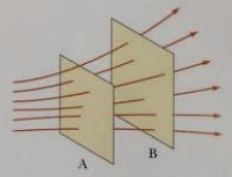


Figure 19.14 Electric field lines penetrating two surfaces. The magnitude of the field is greater on surface A than on surface B.

These properties are illustrated in Figure 19.14. The density of lines through surface A is greater than the density of lines through surface B. Therefore, the electric field is more intense on surface A than on surface B. Furthermore, the field drawn in Figure 19.14 is nonuniform, because the lines at different locations point in different directions.

Some representative electric field lines for a single positive point charge are shown in Figure 19.15a. Note that in this two-dimensional drawing we show only the field lines that lie in the plane containing the point charge. The lines are actually directed radially outward in *all* directions from the charge, somewhat like the needles of a porcupine. Because a positive test charge placed in this field would be repelled by the charge q, the lines are directed radially away from q. In a similar way, the electric field lines for a single negative point charge are directed toward the charge (Fig. 19.15b). In either case, the lines are radial and extend to infinity. Note that the lines are closer together as they get nearer the charge, indicating that the strength of the field is increasing.

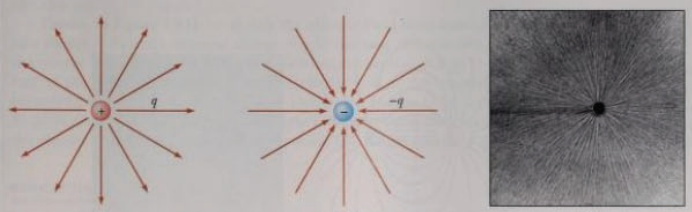


Figure 19.15 The electric field lines for a point charge. (a) For a positive point charge, the lines are radially outward. (b) For a negative point charge, the lines are radially inward. Note that the figures show only those field lines that lie in the plane containing the charge. (c) The dark areas are small pieces of thread suspended in oil, which align with the electric field produced by a small charged conductor at the center. *(Photo courtesy of Harold M. Waage, Princeton University)*

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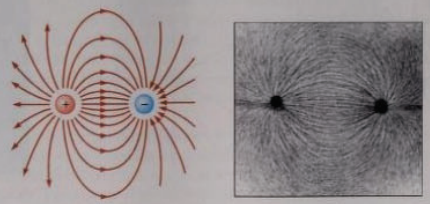
The rules for drawing electric field lines for any charge distribution follow:

- The lines must begin on positive charges (or at infinity) and must terminate either on negative charges or, in the case of an excess of positive charge, at infinity.
- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- No two field lines can cross each other.

Is this visualization of the electric field in terms of field lines consistent with Coulomb's law? To answer this question, consider an imaginary spherical surface of radius r, concentric with the charge. Using the principle of symmetry, we see that the magnitude of the electric field is the same everywhere on the surface of the sphere. The number of lines, N, that emerge from the charge is equal to the number that penetrate the spherical surface. Hence, the number of lines per unit area on the sphere is $N/4\pi r^2$ (where the surface area of the sphere is $4\pi r^2$). Because E is proportional to the number of lines per unit area, we see that E varies as $1/r^2$. This is consistent with the result obtained from Coulomb's law—that is, $E = k_e q/r^2$.

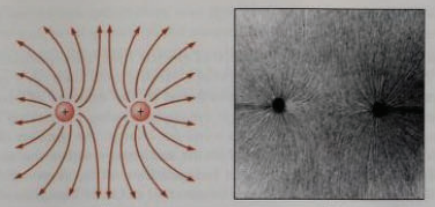
It is important to note that electric field lines are not material objects. They are used only to provide a qualitative description of the electric field. One problem with this model is the fact that one always draws a finite number of lines from each charge, which makes it appear as if the field were quantized and acted only in a certain direction. The field, in fact, is continuous—existing at every point. Another problem with this model is the danger of getting the wrong impression from a two-dimensional drawing of field lines used to describe a three-dimensional situation.

Because charge is quantized, the number of lines leaving any material object must be $0, \pm C'e, \pm 2C'e, \ldots$, where C' is an arbitrary (but fixed) proportionality constant. Once C' is chosen, the number of lines is not arbitrary. For example, if object 1 has charge Q_1 and object 2 has charge Q_2, then the ratio of numbers of lines is $N_2/N_1 = Q_2/Q_1$.



(a) (b)

Figure 19.16 (a) The electric field lines for two equal and opposite point charges (an electric dipole). Note that the number of lines leaving the positive charge equals the number terminating at the negative charge. (b) The photograph was taken using small pieces of thread suspended in oil, which align with the electric field. *(Photo courtesy of Harold M. Waage, Princeton University)*



(a) (b)

Figure 19.17 (a) The electric field lines for two positive point charges. (b) The photograph was taken using small pieces of thread suspended in oil, which align with the electric field. *(Photo courtesy of Harold M. Waage, Princeton University)*

The electric field lines for two point charges of equal magnitude but opposite signs (the electric dipole) are shown in Figure 19.16. In this case, the number of lines that begin at the positive charge must equal the number that terminate at the negative charge. At points very near the charges, the lines are nearly radial. The high density of lines between the charges indicates a region of strong electric field. The attractive nature of the force between the charges can also be seen from Figure 19.16.

Figure 19.17 shows the electric field lines in the vicinity of two equal positive point charges. Again, close to either charge the lines are nearly radial. The same number of lines emerges from each charge because the charges are equal in magnitude. At great distances from the charges, the field is approximately equal to that of a single point charge of magnitude $2q$. The bulging out of the electric field lines between the charges indicates the repulsive nature of the electric force between like charges.

Finally, in Figure 19.18 we sketch the electric field lines associated with a positive charge $+2q$ and a negative charge $-q$. In this case, the number of lines leaving the charge $+2q$ is twice the number terminating on the charge $-q$. Hence, only half of the lines that leave the positive charge end at the negative charge. The remaining half terminate on a negative charge we assume to be located at infinity. At great distances from the charges (great compared with the charge separation), the electric field lines are equivalent to those of a single charge $+q$.

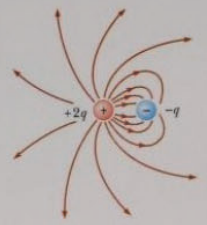


Figure 19.18 The electric field lines for a point charge $+2q$ and a second point charge $-q$. Note that two lines leave the charge $+2q$ for every one that terminates on $-q$.

CONCEPTUAL PROBLEM 4

In fair weather, there is an electric field at the surface of the Earth, pointing down into the ground. What is the sign of the electric charge on the ground in fair weather?

CONCEPTUAL PROBLEM 5

An uncharged, metallic-coated styrofoam ball is suspended in the region between two vertical metal plates. If the two plates are charged, one positive and one negative, describe the motion of the ball after it is brought into contact with one of the plates.

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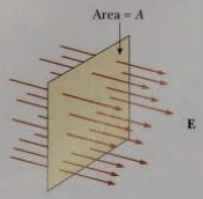


Figure 19.19 Field lines of a uniform electric field penetrating a plane of area A perpendicular to the field. The electric flux Φ through this area is equal to EA.

19.7 · ELECTRIC FLUX

Now that we have described the concept of electric field lines qualitatively, let us use a new concept, electric flux, to approach electric field lines on a quantitative basis. **Electric flux is a quantity proportional to the number of electric field lines penetrating some surface.** When the surface being penetrated is a closed surface (such as a sphere) that encloses some net charge, the number of lines that go through the surface is proportional to the net charge within the surface. The number of lines through the surface is independent of the shape of the surface enclosing the charge. This is essentially a statement of Gauss's law, described in the next section.

First consider an electric field that is uniform in both magnitude and direction, as in Figure 19.19. The field lines penetrate a plane rectangular surface of area A, which is perpendicular to the field. Recall that the number of lines per unit area is proportional to the magnitude of the electric field. Therefore, the number of lines penetrating the surface of area A is proportional to the product EA. The product of the electric field strength, E, and a surface area, A, perpendicular to the field is called the **electric flux,** Φ:

$$\Phi = EA \qquad [19.10]$$

From the SI units of E and A, we see that electric flux has the units $N \cdot m^2/C$.

If the surface under consideration is not perpendicular to the field, the number of lines (the flux) through it must be less than that given by Equation 19.10. This can be easily understood by considering Figure 19.20 where the normal to the surface of area A is at an angle of θ to the uniform electric field. Note that the number of lines that cross this area is equal to the number that cross the projected area A', which is perpendicular to the field. From Figure 19.20 we see that the two areas are related by $A' = A \cos \theta$. Because the flux through area A equals the flux through A', we conclude that the desired flux is

$$\Phi = EA \cos \theta \qquad [19.11]$$

From this result, we see that the flux through a surface of fixed area has the maximum value EA when the surface is perpendicular to the field (in other words, when the *normal* to the surface is parallel to the field—that is, $\theta = 0°$); the flux is zero when the surface is parallel to the field (when the normal to the surface is perpendicular to the field—that is, $\theta = 90°$).

In more general situations, the electric field may vary over the surface in question. Therefore, our definition of flux given by Equation 19.11 has meaning only over a small element of area. Consider a general surface divided up into a large

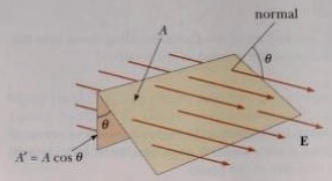


Figure 19.20 Field lines for a uniform electric field through an area A that is at an angle θ to the field. Because the number of lines that go through the shaded area A' is the same as the number that go through A, we conclude that the flux through A' is equal to the flux through A and is given by the expression $\Phi = EA \cos \theta$.

number of small elements, each of area ΔA. The variation in the electric field over the element can be neglected if the element is small enough. It is convenient to define a vector $\Delta \mathbf{A}_i$ the magnitude of which represents the area of the ith element and the direction of which is *defined to be perpendicular* to the surface, as in Figure 19.21. The electric flux $\Delta \Phi_i$ through this small element is

$$\Delta \Phi_i = E_i \, \Delta A_i \cos \theta = \mathbf{E}_i \cdot \Delta \mathbf{A}_i$$

where we have used the definition of the scalar product of two vectors ($\mathbf{A} \cdot \mathbf{B} = AB \cos \theta$). By summing the contributions of all elements, we obtain the total flux through the surface.[2] If we let the area of each element approach zero, then the number of elements approaches infinity and the sum is replaced by an integral. Therefore, the general definition of electric flux is

$$\Phi \equiv \lim_{\Delta A_i \to 0} \sum \mathbf{E}_i \cdot \Delta \mathbf{A}_i = \int_{\text{surface}} \mathbf{E} \cdot d\mathbf{A} \qquad [19.12]$$

Equation 19.12 is a surface integral, which must be evaluated over the surface in question. In general, the value of Φ depends both on the field pattern and on the specified surface.

We shall usually be interested in evaluating the flux through a *closed surface.* (A closed surface is defined as one that divides space into an inside region and an outside region, so that movement cannot take place from one region to the other without penetrating the surface. The surface of a sphere, for example, is a closed surface.) Consider the closed surface in Figure 19.22. Note that the vectors $\Delta \mathbf{A}_i$ point in different directions for the various surface elements. At each point, these vectors are *perpendicular* to the surface and, by convention, always point *outward.* At the elements labeled ① and ②, $\mathbf{E}$ is outward and $\theta < 90°$; hence, the flux $\Delta \Phi = \mathbf{E} \cdot \Delta \mathbf{A}$ through these elements is positive. For elements such as ③, where the field lines are directed into the surface, $\theta > 90°$ and the flux becomes negative because of the cos θ factor. (Element ③ is on the side of the surface we cannot see; hence the dashed lines representing the vectors $\mathbf{E}_3$ and $\Delta \mathbf{A}_3$.) The net flux through the surface is proportional to the net number of lines penetrating the surface, where

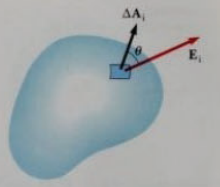


Figure 19.21 A small element of a surface of area ΔA_i. The electric field makes an angle θ with the normal to the surface (the direction of ΔA_i), and the flux through the element is equal to $E_i \, \Delta A_i \cos \theta$.

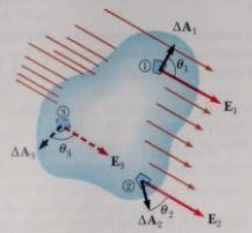


Figure 19.22 A closed surface in an electric field. The area vectors $\Delta \mathbf{A}_i$ are, by convention, normal to the surface and point outward. The flux through an area element can be positive (elements ① and ②) or negative (element ③). Element ③ is on the back side of the surface.

[2] It is important to note that drawings with field lines can have inaccuracies, because a small area (depending on its location) may happen to have too many or too few penetrating lines. At any rate, it is stressed that the basic definition of electric flux is $\int \mathbf{E} \cdot d\mathbf{A}$. The use of lines is only an aid for visualizing the concept.

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the *net number* means **the number leaving the volume surrounded by the surface minus the number entering the volume.** If more lines are leaving the surface than entering, the net flux is positive. If more lines enter than leave the surface, the net flux is negative. (The net flux can be nonzero only when some net charge is contained within the closed surface.) Using the symbol $\oint$ to represent an integral over a closed surface, we can write the net flux, Φ_e, through a closed surface

$$\Phi_e = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E_n \, dA \qquad [19.13]$$

where E_n represents the component of the electric field perpendicular to the surface and the subscript e denotes a closed surface.

Evaluating the net flux through a closed surface can be very cumbersome. However, if the field is perpendicular to the surface at each point and constant in magnitude, the calculation is straightforward. The following example illustrates this point.

Example 19.6 Flux Through a Cube

Consider a uniform electric field **E** oriented in the x direction. Find the net electric flux through the surface of a cube of edges ℓ oriented as shown in Figure 19.23.

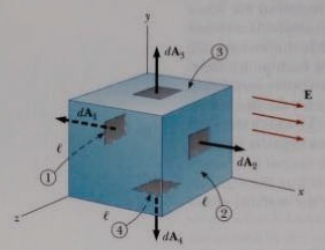


Figure 19.23 (Example 19.6) A hypothetical surface in the shape of a cube in a uniform electric field parallel to the x axis. The net flux through the surface is zero.

Solution The net flux can be evaluated by summing up the fluxes through each face of the cube. First, note that the flux through four of the faces is zero, because **E** is perpendicular to $d\mathbf{A}$ on these faces. In particular, the orientation of $d\mathbf{A}$ is perpendicular to **E** for the faces labeled ③ and ④ in Figure 19.23. Therefore, $\theta = 90°$, so that $\mathbf{E} \cdot d\mathbf{A} = E \, dA \cos 90° = 0$. The flux through each of the planes parallel to the yx plane is also zero for the same reason.

Now consider the faces labeled ① and ②. The net flux through these faces is

$$\Phi_e = \int_1 \mathbf{E} \cdot d\mathbf{A} + \int_2 \mathbf{E} \cdot d\mathbf{A}$$

For face ①, **E** is constant and inward and $d\mathbf{A}$ is outward ($\theta = 180°$), so that we find the flux through this face is

$$\int_1 \mathbf{E} \cdot d\mathbf{A} = \int_1 E \, dA \cos 180° = -E \int_1 dA = -EA = -E\ell^2$$

because the area of each face is $A = \ell^2$.

Likewise, for ②, **E** is constant and outward and in the same direction as $d\mathbf{A}$ ($\theta = 0°$), so that the flux through this face is

$$\int_2 \mathbf{E} \cdot d\mathbf{A} = \int_2 E \, dA \cos 0° = E \int_2 dA = +EA = E\ell^2$$

Hence, the net flux over all faces is zero, because

$$\Phi_e = -E\ell^2 + E\ell^2 = 0$$

EXERCISE 6 A flat surface having an area of 3.2 m^2 can be oriented in any direction in a uniform electric field of magnitude $E = 6.2 \times 10^5$ N/C. Calculate the electric flux through this area when the electric field (a) is perpendicular to the surface; (b) is parallel to the surface; (c) makes an angle of 75° with the plane of the surface.
Answer (a) 1.98×10^6 N·m^2/C (b) zero (c) 1.92×10^6 N·m^2/C

19.8 · GAUSS'S LAW

In this section we describe a general relation between the net electric flux through a closed surface (often called a *gaussian surface*) and the charge *enclosed* by the surface. This relation, known as **Gauss's law,** is of fundamental importance in the study of electrostatic fields.

First, let us consider a positive point charge, q, located at the center of a sphere of radius, r, as in Figure 19.24. From Equation 19.4, we know that the magnitude of the electric field everywhere on the surface of the sphere is $E = k_e q/r^2$. Furthermore, the field lines radiate outward and hence are perpendicular (or normal) to the surface at each point. That is, at each point on the surface, **E** is parallel to the vector $\Delta\mathbf{A}_i$ representing the local element of area ΔA_i. Therefore,

$$\mathbf{E} \cdot \Delta\mathbf{A}_i = E_n \, \Delta A_i = E \, \Delta A_i$$

and from Equation 19.13 we find that the net flux through the gaussian surface is

$$\Phi_e = \oint E_n \, dA = \oint E \, dA = E \oint dA$$

because E is constant over the surface and is given by $E = k_e q/r^2$. Furthermore, for a spherical gaussian surface, $\oint dA = A = 4\pi r^2$ (the surface area of a sphere). Hence, the net flux through the gaussian surface is

$$\Phi_e = \frac{k_e q}{r^2}(4\pi r^2) = 4\pi k_e q$$

Recalling that $k_e = 1/4\pi\epsilon_0$, we can write this in the form

$$\Phi_e = \frac{q}{\epsilon_0} \qquad [19.14]$$

Note that this result, which is independent of r, says that the net flux through a spherical gaussian surface is proportional to the charge q *inside* the surface. The fact that the flux is independent of the radius is a consequence of the inverse-square dependence of the electric field given by Equation 19.4. That is, E varies as $1/r^2$, but the area of the sphere varies as r^2. Their combined effect produces a flux that is independent of r.

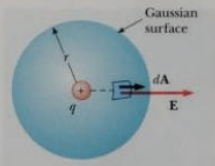


Figure 19.24 A spherical gaussian surface of radius r surrounding a point charge q. When the charge is at the center of the sphere, the electric field is normal to the surface and constant in magnitude everywhere on the surface.

Now consider several closed surfaces surrounding a charge, q, as in Figure 19.25. Surface S_1 is spherical, whereas surfaces S_2 and S_3 are nonspherical. The flux that passes through surface S_1 has the value q/ϵ_0. As we discussed in the preceding section, the flux is proportional to the number of electric field lines passing through that surface. The construction in Figure 19.25 shows that the number of electric field lines through the spherical surface S_1 is equal to the number of electric field lines through the nonspherical surfaces S_2 and S_3. Therefore, it is reasonable to conclude that the net flux through any closed surface is independent of the shape of that surface. (One can prove that this is the case if $E \propto 1/r^2$.) In fact, **the net flux through any closed surface surrounding the point charge q is q/ϵ_0.**

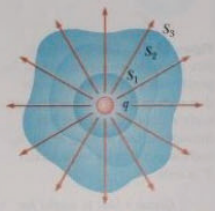


Figure 19.25 Closed surfaces of various shapes surrounding a charge q. Note that the net electric flux through each surface is the same.

Now consider a point charge located *outside* a closed surface of arbitrary shape, as in Figure 19.26. As you can see from this construction, electric field lines enter the surface, and field lines leave it. However, **the number of electric field lines entering the surface equals the number leaving the surface.** Therefore, we conclude that **the net electric flux through a closed surface that surrounds no**

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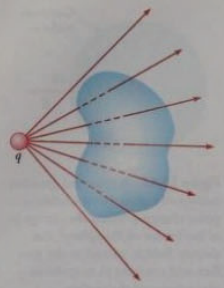


Figure 19.26 A point charge located *outside* a closed surface. In this case, note that the number of lines entering the surface equals the number leaving the surface.

charge is zero. If we apply this result to Example 19.6, we can easily see that the net flux through the cube is zero, because it was assumed there was no charge inside the cube.

Let us extend these arguments to the generalized case of many point charges, which we consider to be a continuous distribution of charge. We shall again make use of the superposition principle. That is, we can express the flux through any closed surface as

$$\oint \mathbf{E} \cdot d\mathbf{A} = \oint (\mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3) \cdot d\mathbf{A}$$

where $\mathbf{E}$ is the total electric field at any point on the surface and $\mathbf{E}_1$, $\mathbf{E}_2$, and $\mathbf{E}_3$ are the fields produced by the individual charges at that point. Consider the system of charges shown in Figure 19.27. The surface S surrounds only one charge, q_1; hence, the net flux through S is q_1/ϵ_0. The flux through S due to the charges outside it is zero, because each electric field line that enters S at one point leaves it at another. The surface S' surrounds charges q_2 and q_3; hence, the net flux through S' is $(q_2 + q_3)/\epsilon_0$. Finally, the net flux through surface S'' is zero, because there is no charge inside this surface. That is, *all* lines that enter S'' at one point leave S'' at another.

Gauss's law, which is a generalization of the foregoing discussion, states that the net flux through *any* closed surface is

$$\Phi_e = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0} \qquad [19.15]$$

Gauss's law



Figure 19.27 The net electric flux through any closed surface depends only on the charge *inside* that surface. The net flux through surface S is q_1/ϵ_0, the net flux through surface S' is $(q_2 + q_3)/\epsilon_0$, and the net flux through surface S'' is zero.

Gauss's law is useful for evaluating **E** *when the charge distribution has symmetry.*

where q_{in} represents the *net charge inside the gaussian surface* and $\mathbf{E}$ represents the electric field at any point on the gaussian surface. In other words,

> **Gauss's law** states that the net electric flux through any closed surface is equal to the net charge inside the surface divided by ϵ_0.

When using Equation 19.15, note that, although the charge q_{in} is the net charge inside the gaussian surface, the $\mathbf{E}$ that appears in Gauss's law represents the *total electric field*, which includes contributions from charges both inside and outside the gaussian surface. This point is often neglected or misunderstood.

In principle, Gauss's law can always be used to calculate the electric field of a system of charges or a continuous distribution of charge. In practice, however, **the technique is useful only for calculating the electric field in situations in which there is a high degree of symmetry.** As we shall see in the next section, **Gauss's law can be used to evaluate the electric field for charge distributions that have spherical, cylindrical, or plane symmetry.** If the gaussian surface surrounding the charge distribution is chosen carefully, the integral in Equation 19.15 will be easy to evaluate. Also note that a gaussian surface is a mathematical surface and need not coincide with any real physical surface.

Thinking Physics 3

Imagine a cubical conducting shell with a net charge placed in it. A cube has a high degree of symmetry. Why can't we use Gauss's law to evaluate the electric field around the cube?

Reasoning Although Gauss's law certainly *applies* to the electric field of the cube, it cannot be used to *evaluate* the field. Using Gauss's law to evaluate the electric field requires bringing the electric field outside of the integral in the equation for Gauss's law. The only way that the electric field can be brought outside the integral is if the electric field is a constant. There is no surface around a cube over which one can use symmetry arguments to convince oneself that the electric field is constant. Thus, the attempt is futile—the electric field must remain within the integral, so we need to know the field to do the integral.

However, the other geometries to which Gauss's law is applied in an effort to evaluate the field—spheres, cylinders, flat sheets—have sufficient symmetry that we can find a surface over which a valid argument can be made that the field is constant.



This beautiful closed surface generated by a computer is one example of a mathematical construct in hyperbolic space. *(Courtesy of Wolfram Research, Inc.)*

Thinking Physics 4

If the net flux through a gaussian surface is zero, which of the following statements are true? (a) There are no charges inside the surface. (b) The net charge inside the surface is zero. (c) The electric field is zero everywhere on the surface. (d) The number of electric field lines entering the surface equals the number leaving the surface.

Reasoning Statements (b) and (d) are true and follow from Gauss's law. Statement (a) is not necessarily true because Gauss's law says that the net flux through any closed surface equals the net charge inside the surface divided by ϵ_0. For example, an electric dipole (whose net charge is zero) might be inside the surface. Statement (c) is not necessarily true. Although the net flux through the surface is zero, the electric field in that region may not be zero (Fig. 19.8).

Thinking Physics 5

A spherical gaussian surface surrounds a point charge q. Describe what happens to the total flux through the surface if (a) the charge is tripled, (b) the volume of the sphere is doubled, (c) the surface is changed to a cube, and (d) the charge is moved to another location *inside* the surface.

Reasoning (a) If the charge is tripled, the flux through the surface is also tripled, because the net flux is proportional to the charge inside the surface. (b) The flux remains constant when the volume changes, because the surface surrounds the same amount of charge, regardless of its volume. (c) The total flux does not change when the shape of the closed surface changes. (d) The total flux through the closed surface remains unchanged as the charge inside the surface is moved to another location inside that surface. All of these conclusions are arrived at through an understanding of Gauss's law.

CONCEPTUAL PROBLEM 6

Explain why Gauss's law cannot be used to calculate the electric field of (a) an electric dipole, (b) a charged disk, and (c) three point charges at the corners of a triangle.

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EXERCISE 7 A point charge of $+5.0\ \mu C$ is located at the center of a sphere with a radius of 12 cm. What is the electric flux through the surface of this sphere?
Answer $5.7 \times 10^5\ N \cdot m^2/C$

EXERCISE 8 The electric field in the Earth's atmosphere is $E = 100\ N/C$, pointing downward. Determine the electric charge on the Earth. Answer $-4.6 \times 10^5\ C$

19.9 · APPLICATION OF GAUSS'S LAW TO CHARGED INSULATORS

In this section we give some examples of how to use Gauss's law to calculate **E** for a given charge distribution. It is important to recognize that **Gauss's law is useful when there is a high degree of symmetry in the charge distribution, as in the cases of uniformly charged spheres, long cylinders, and plane sheets.** In such cases, it is possible to find a simple gaussian surface over which the surface integral given by Equation 19.15 is easily evaluated.

> A surface should always be chosen to take advantage of the symmetry of the charge distribution.

Example 19.7 The Electric Field Due to a Point Charge

Starting with Gauss's law, calculate the electric field due to an isolated point charge q and show that Coulomb's law follows from this result.

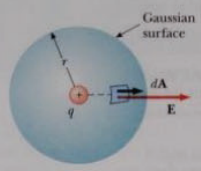


Figure 19.28 (Example 19.7) The point charge q is at the center of the spherical gaussian surface, and **E** is parallel to $d\mathbf{A}$ at every point on the surface.

Solution For this situation we choose a spherical gaussian surface of radius r and centered on the point charge, as in Figure 19.28. The electric field of a positive point charge is radial outward by symmetry and is therefore normal to the surface at every point. That is, **E** is parallel to $d\mathbf{A}$ at each point on the surface, and so $\mathbf{E} \cdot d\mathbf{A} = E\,dA$ and Gauss's law gives

$$\Phi_e = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E\,dA = \frac{q}{\epsilon_0}$$

By symmetry, E is constant everywhere on the surface, and so it can be removed from the integral. Therefore,

$$\oint E\,dA = E \int dA = E(4\pi r^2) = \frac{q}{\epsilon_0}$$

where we have used the fact that the surface area of a sphere is $4\pi r^2$. Hence, the magnitude of the field a distance r from q is

$$E = \frac{q}{4\pi\epsilon_0 r^2} = k_e \frac{q}{r^2}$$

If a second point charge q_0 is placed at a point where the field is E, the electric force on this charge has a magnitude

$$F = q_0 E = k_e \frac{q q_0}{r^2}$$

Previously we obtained Gauss's law from Coulomb's law. Here we show that Coulomb's law follows from Gauss's law: They are equivalent.

Example 19.8 A Spherically Symmetric Charge Distribution

An insulating sphere of radius a has a uniform charge density ρ and a total positive charge Q (Fig. 19.29). (a) Calculate the magnitude of the electric field at a point outside the sphere.

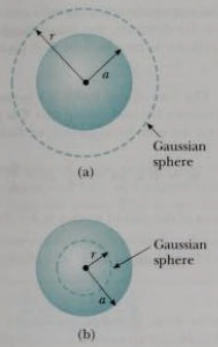


Figure 19.29 (Example 19.8) A uniformly charged insulating sphere of radius a and total charge Q. (a) The field at a point exterior to the sphere is $k_e Q/r^2$. (b) The field inside the sphere is due only to the charge *within* the gaussian surface and is given by $(k_e Q/a^3)r$.

Solution Because the charge distribution is spherically symmetric, we again select a spherical gaussian surface of radius r, concentric with the sphere, as in Figure 19.29a. Following the line of reasoning given in Example 19.7, we find that

$$E = k_e \frac{Q}{r^2} \qquad (\text{for } r > a)$$

Note that this result is identical to that obtained for a point charge. Therefore, we conclude that, for a uniformly charged sphere, the field in the region external to the sphere is *equivalent* to that of a point charge located at the center of the sphere.

(b) Find the magnitude of the electric field at a point inside the sphere.

Reasoning and Solution In this case we select a spherical gaussian surface with radius $r < a$, concentric with the charge distribution (Fig. 19.29b). Let us denote the volume of this smaller sphere by V'. To apply Gauss's law in this situation, it is important to recognize that the charge q_{in} within the gaussian surface of volume V' is a quantity less than the total charge Q. To calculate the charge q_{in}, we use the fact that $q_{in} = \rho V'$, where ρ is the charge per unit volume and V' is the volume enclosed by the gaussian surface, given by $V' = \frac{4}{3}\pi r^3$ for a sphere. Therefore,

$$q_{in} = \rho V' = \rho\left(\tfrac{4}{3}\pi r^3\right)$$

As in Example 19.7, the magnitude of the electric field is constant everywhere on the spherical gaussian surface and is normal to the surface at each point. Therefore, Gauss's law in the region $r < a$ gives

$$\oint E\,dA = E \oint dA = E(4\pi r^2) = \frac{q_{in}}{\epsilon_0}$$

Solving for E gives

$$E = \frac{q_{in}}{4\pi\epsilon_0 r^2} = \frac{\rho\frac{4}{3}\pi r^3}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0} r$$

Because by definition $\rho = Q/\frac{4}{3}\pi a^3$, this can be written

$$E = \frac{Qr}{4\pi\epsilon_0 a^3} = \frac{k_e Q}{a^3} r \qquad (\text{for } r < a)$$

Note that this result for E differs from that obtained in part (a). It shows that $E \to 0$ as $r \to 0$, as you might have guessed based on the spherical symmetry of the charge distribution. Therefore, the result fortunately eliminates the singularity that would exist at $r = 0$ if E varied as $1/r^2$ inside the sphere. That is, if $E \propto 1/r^2$, the field would be infinite at $r = 0$, which is clearly a physically impossible situation. A plot of E versus r is shown in Figure 19.30.

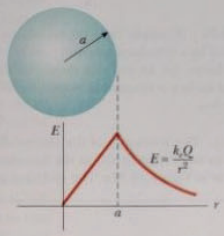
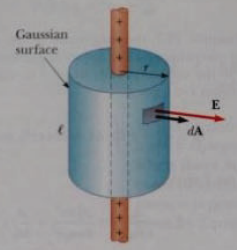


Figure 19.30 (Example 19.8) A plot of E versus r for a uniformly charged insulating sphere. The field inside the sphere ($r < a$) varies linearly with r. The field outside the sphere ($r > a$) is the same as that of a point charge Q located at the origin.

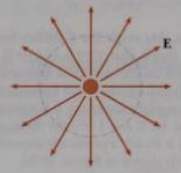
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Example 19.9 A Cylindrically Symmetric Charge Distribution

Find the electric field a distance r from a uniform positive line charge of infinite length the charge per unit length of which is λ = constant (Fig. 19.31).



(a)



(b)

Figure 19.31 (Example 19.9) (a) An infinite line of charge surrounded by a cylindrical gaussian surface concentric with the line charge. (b) An end view shows that the field on the cylindrical surface is constant in magnitude and perpendicular to the surface.

Reasoning The symmetry of the charge distribution shows that E must be perpendicular to the line charge and directed outward as in Figure 19.31a. The end view of the line charge shown in Figure 19.31b should help visualize the directions of the electric field lines. In this situation, we select a cylindrical gaussian surface of radius r and length ℓ that is coaxial with the line charge. For the curved part of this surface, $\mathbf{E}$ is constant in magnitude and perpendicular to the surface at each point. Furthermore, the flux through the ends of the gaussian cylinder is zero because $\mathbf{E}$ is parallel to these surfaces.

Solution The total charge inside our gaussian surface is $\lambda\ell$. Applying Gauss's law and noting that $\mathbf{E}$ is parallel to $d\mathbf{A}$ everywhere on the cylindrical surface, we find that

$$\Phi_e = \oint \mathbf{E}\cdot d\mathbf{A} = E\oint dA = \frac{q_{in}}{\epsilon_0} = \frac{\lambda\ell}{\epsilon_0}$$

But the area of the curved surface is $A = 2\pi r\ell$; therefore,

$$E(2\pi r\ell) = \frac{\lambda\ell}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = 2k_e\frac{\lambda}{r} \qquad [19.16]$$

Thus, we see that the field of a cylindrically symmetric charge distribution varies as $1/r$, whereas the field external to a spherically symmetric charge distribution varies as $1/r^2$. Equation 19.16 can also be obtained using Coulomb's law and integration; however, the mathematical techniques necessary for this calculation are more cumbersome.

If the line charge has a finite length, the result for E is not that given by Equation 19.16. For points close to the line charge and far from the ends, Equation 19.16 gives a good approximation for the value of the field. It turns out that Gauss's law is not useful for calculating E for a finite line charge. This is because the magnitude of the electric field is no longer constant over the surface of the gaussian cylinder. Furthermore, $\mathbf{E}$ is not perpendicular to the cylindrical surface at all points. When there is little symmetry in the charge distribution, as in this situation, it is necessary to calculate $\mathbf{E}$ using Coulomb's law.

It is left as a problem (Problem 29) to show that the $\mathbf{E}$ field inside a long uniformly charged rod of finite thickness is proportional to r.

Example 19.10 A Nonconducting Plane Sheet of Charge

Find the electric field due to a nonconducting, infinite plane with uniform charge per unit area σ.

Reasoning and Solution The symmetry of the situation shows that $\mathbf{E}$ must be perpendicular to the plane and that the direction of $\mathbf{E}$ on one side of the plane must be opposite its direction on the other side, as in Figure 19.32. It is convenient to choose for our gaussian surface a small cylinder the axis of which is perpendicular to the plane and the ends of which each have an area A and are equidistant from the plane. Here we see that because $\mathbf{E}$ is parallel to the cylindrical surface, there is no flux through this surface. The flux out of each end of the cylinder is EA (because $\mathbf{E}$ is perpendicular to the ends); hence, the total flux through our gaussian surface is $2EA$.

Solution Noting that the total charge inside the surface is σA, we use Gauss's law to get

$$\Phi_e = 2EA = \frac{q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0} \qquad [19.17]$$

Because the distance of the surfaces from the plane does not appear in Equation 19.17, we conclude that $E = \sigma/2\epsilon_0$ at any distance from the plane. That is, the field is uniform everywhere.

An important configuration related to this example is the case of two parallel planes of charge, with charge densities σ and $-\sigma$, respectively (Problem 46). In this situation, the electric field is σ/ϵ_0 between the planes and approximately zero elsewhere.

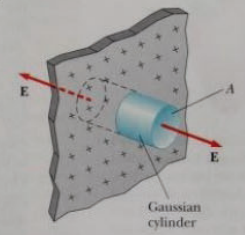


Figure 19.32 (Example 19.10) A cylindrical gaussian surface penetrating an infinite sheet of charge. The flux through each end of the gaussian surface is EA. There is no flux through the cylinder's curved surface.

EXERCISE 9 A conducting spherical shell of radius 15 cm carries a net charge of $-6.4\ \mu$C uniformly distributed on its surface. Find the electric field at points (a) just outside the shell and (b) inside the shell. Answer (a) 2.6×10^6 N/C (b) 0

PROBLEM-SOLVING STRATEGY AND HINTS • Applying Gauss's Law

Gauss's law may seem mysterious to you, and it is usually one of the most challenging concepts in introductory physics. However, as we have seen, Gauss's law is very powerful in solving problems with a high degree of symmetry. In this section, we have used Gauss's law to solve problems with three kinds of symmetry: plane, cylindrical, and spherical. It is important to review Examples 19.7 through 19.10 and to use the following procedure:

1. Select a gaussian surface that *has a symmetry appropriate for the charge distribution*. For point charges or spherically symmetric charge distributions, the gaussian surface should be a sphere centered on the charge, as in Examples 19.7 and 19.8. For uniform line charges or uniformly charged cylinders, the gaussian surface should be a cylindrical surface that is coaxial with the line charge or cylinder, as in Example 19.9. For sheets of charge having plane symmetry, the gaussian surface should be a small cylinder that straddles the sheet, as in Example 19.10. Note that in all cases, the gaussian surface is selected so that the electric field has the same magnitude everywhere on the surface and is directed perpendicular to the surface or is parallel to the surface. This enables you to easily evaluate the surface integral that appears on the left side of Gauss's law, and this integral represents the total electric flux through that surface.
2. Evaluate the right side of Gauss's law, which amounts to calculating the total electric charge, q_{in}, *inside* the gaussian surface. If the charge density

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is uniform, as is usually the case (that is, if λ, σ, or ρ is constant), simply multiply that charge density by the length, area, or volume enclosed by the gaussian surface. However, if the charge distribution is *nonuniform*, you must integrate the charge density over the region enclosed by the gaussian surface. For example, if the charge were distributed along a line, you would integrate the expression $dq = \lambda\, dx$, where dq is the charge on an infinitesimal element dx and λ is the charge per unit length. For a plane of charge, you would integrate $dq = \sigma\, dA$, where σ is the charge per unit area and dA is an infinitesimal element of area. For a volume of charge, you would integrate $dq = \rho\, dV$, where ρ is the charge per unit volume and dV is an infinitesimal element of volume.

3. Once the left and right sides of Gauss's law have been evaluated, you can calculate the electric field on the gaussian surface, assuming the charge distribution is given in the problem. Conversely, if the electric field is known, you can calculate the charge distribution that produces the field.

19.10 · CONDUCTORS IN ELECTROSTATIC EQUILIBRIUM

A good electrical conductor, such as copper, contains charges (electrons) that are not bound to any atom and are free to move about within the material. When no *net* motion of charge occurs within the conductor, the conductor is in **electrostatic equilibrium.** As we shall see, an isolated conductor (one that is insulated from ground) in electrostatic equilibrium has the following properties:

- The electric field is zero everywhere inside the conductor.
- Any excess charge on the conductor must reside entirely on its surface.
- The electric field just outside the charged conductor is perpendicular to the conductor's surface and has the magnitude σ/ϵ_0, where σ is the charge per unit area at that point.
- On an irregularly shaped conductor, charge tends to accumulate at locations where the radius of curvature of the surface is the smallest—that is, at sharp points. This will be discussed further in Chapter 20.

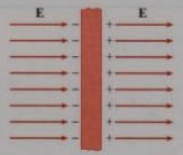


Figure 19.33 A conducting slab in an external electric field **E**. The charges induced on the surfaces of the slab produce an electric field that opposes the external field, giving a resultant field of zero inside the conductor.

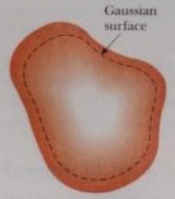


Figure 19.34 An insulated conductor of arbitrary shape. The broken line represents a gaussian surface just inside the conductor.

The first property can be understood by considering a conducting slab placed in an external field **E** (Fig. 19.33). In electrostatic equilibrium, the electric field *inside* the conductor must be zero. If this were not the case, the free charges would accelerate under the action of an electric field. Before the external field is applied, the electrons are uniformly distributed throughout the conductor. When the external field is applied, the free electrons accelerate to the left, causing a buildup of negative charge on the left surface (excess electrons) and of positive charge on the right (where electrons have been removed). These charges create their own electric field, which *opposes* the external field. The surface-charge density increases until the magnitude of the electric field set up by these charges equals that of the external field, giving a net field of zero *inside* the conductor. In a good conductor, the time it takes the conductor to reach equilibrium is on the order of 10^{-16} s, which for most purposes can be considered instantaneous.

We can use Gauss's law to verify the second and third properties. Figure 19.34 shows an arbitrarily shaped insulated conductor. A gaussian surface is drawn inside the conductor as close to the surface as we wish. As we have just shown, the electric field everywhere inside the conductor is zero when it is in electrostatic equilibrium. Because the electric field is therefore *zero* at every point on the gaussian surface, we see that the net flux through this surface is zero. From this result and Gauss's law, we conclude that the net charge inside the gaussian surface is zero. Because there can be no net charge inside the gaussian surface (which is arbitrarily close to the conductor's surface), **any net charge on the conductor must reside on its surface.** Gauss's law does *not* tell us how this excess charge is distributed on the surface.

Examining the third property, we can use Gauss's law to relate the electric field just outside the surface of a charged conductor in equilibrium to the charge distribution on the conductor. To do this, it is convenient to draw a gaussian surface in the shape of a small cylinder with end faces parallel to the surface (Fig. 19.35). Part of the cylinder is just outside the conductor and part is inside. There is no flux through the face on the inside of the cylinder, because $\mathbf{E} = 0$ inside the conductor. Furthermore, the field outside the conductor is normal to the surface. If **E** had a tangential component, the free charges would move along the surface, creating surface currents, and the conductor would not be in equilibrium. There is no flux through the cylindrical face of the gaussian surface, because **E** is tangent to this surface. Hence, the net flux through the gaussian surface is $E_n A$, where E_n is the electric field just outside the conductor. Applying Gauss's law to this surface gives

$$\Phi_e = \oint E_n\, dA = E_n A = \frac{q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

We have used the fact that the charge inside the gaussian surface is $q_{in} = \sigma A$, where A is the area of the cylinder's face and σ is the (local) charge per unit area. Solving for E_n gives

$$E_n = \frac{\sigma}{\epsilon_0} \qquad [19.18]$$

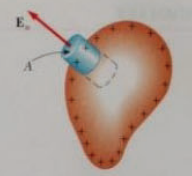


Figure 19.35 A gaussian surface in the shape of a small cylinder is used to calculate the electric field just outside a charged conductor. The flux through the gaussian surface is $E_n A$. Note that **E** is zero inside the conductor.

- ***Electric field just outside a charged conductor***

Thinking Physics 6

Suppose a point charge $+Q$ is in empty space. We sneak up and surround the charge with a spherical conducting shell. What effect does this have on the field lines from the charge?

Reasoning When the spherical shell is placed around the charge, the charges in the shell will adjust to satisfy the rules for a conductor in equilibrium and Gauss's law. A net charge of $-Q$ will move to the interior surface of the conductor, so that the electric field within the conductor will be zero (a spherical gaussian surface totally within the shell will enclose no net charge). A net charge of $+Q$ will reside on the outer surface. Thus, a gaussian surface outside the sphere will enclose a net charge of $+Q$, just as if the shell were not there. Thus, the only change in the field lines from the initial situation will be the absence of field lines within the conductor.

CONCEPTUAL PROBLEM 7

There are great similarities between electric and gravitational fields. A room can be electrically shielded so that there are no electric fields in the room by surrounding it with a conductor. Can a room be *gravitationally* shielded? Why or why not?



Electric field pattern of a charged conducting plate near an oppositely charged conducting cylinder. Small pieces of thread suspended in oil align with the electric field lines. Note that (1) the electric field lines are perpendicular to the conductors and (2) there are no lines inside the cylinder ($E = 0$). *(Courtesy of Harold M. Waage, Princeton University)*

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SUMMARY

Electric charges have the following important properties:

- Unlike charges attract one another, and like charges repel one another.
- Electric charge is always conserved.
- Charge is quantized; that is, the charge on any object is an integral multiple of the electronic charge.
- The force between charged particles varies as the inverse square of their separation.

Conductors are materials in which charges move freely. **Insulators** are materials that do not readily transport charge.

Coulomb's law states that the electrostatic force between two stationary, charged particles separated by a distance of r has the magnitude

$$F = k_e \frac{|q_1|\,|q_2|}{r^2} \qquad [19.1]$$

where the Coulomb constant $k_e = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$.

The *electric field* due to the point charge q at a distance of r from the charge is

$$\mathbf{E} = k_e \frac{q}{r^2}\hat{\mathbf{r}} \qquad [19.4]$$

where $\hat{\mathbf{r}}$ is a unit vector directed from the charge to the point in question. The electric field is directed radially outward from a positive charge and is directed *toward* a negative charge.

The electric field due to a group of charges can be obtained using the superposition principle. That is, the total electric field equals the *vector sum* of the electric fields of all the charges at some point:

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2}\hat{\mathbf{r}}_i \qquad [19.5]$$

In a similar way, the electric field of a continuous charge distribution at some point is

$$\mathbf{E} = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}} \qquad [19.6]$$

where dq is the charge on one element of the charge distribution and r is the distance from the element to the point in question.

Electric field lines are useful for describing the electric field in any region of space. The electric field vector $\mathbf{E}$ is always tangent to the electric field lines at every point. Furthermore, the number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of $\mathbf{E}$ in that region.

Electric flux is proportional to the number of electric field lines that penetrate a surface. If the electric field is uniform and makes an angle of θ with the normal to the surface, the electric flux through the surface is

$$\Phi = EA\cos\theta \qquad [19.11]$$

In general, the electric flux through a surface is defined by the expression

$$\Phi = \int_{\text{surface}} \mathbf{E}\cdot d\mathbf{A} \qquad [19.12]$$

Gauss's law says that the net electric flux Φ_e through any closed gaussian surface is equal to the *net* charge *inside* the surface divided by ϵ_0:

$$\Phi_e = \oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{in}}{\epsilon_0} \qquad [19.15]$$

Using Gauss's law, one can calculate the electric field due to various symmetric charge distributions.

A **conductor in electrostatic equilibrium** has the following properties:

- The electric field is zero everywhere inside the conductor.
- Any excess charge on an isolated conductor must reside entirely on its surface.
- The electric field just outside the conductor is perpendicular to its surface and has the magnitude σ/ϵ_0, where σ is the charge per unit area at that point.
- On an irregularly shaped conductor, charge tends to accumulate where the radius of curvature of the surface is the smallest—that is, at sharp points.

CONCEPTUAL QUESTIONS

1. Explain from an atomic viewpoint why charge is usually transferred by electrons.
2. A balloon is negatively charged by rubbing and then clings to a wall. Does this mean that the wall is positively charged? Why does the balloon eventually fall?
3. A charged comb often attracts small bits of dry paper that then fly away when they touch the comb. Explain.
4. Operating-room personnel must wear special conducting shoes while working around oxygen. Why? Contrast this procedure with what might happen if personnel wore rubber shoes.
5. Would life be different if the electron were positively charged and the proton were negatively charged? Does the choice of signs have any bearing on physical and chemical interactions? Explain.
6. When defining the electric field, why is it necessary to specify that the magnitude of the test charge be very small (that is, take the limit of $\mathbf{F}/q$ as $q \to 0$)?
7. Consider two equal point charges separated by some distance d. At what point (other than ∞) would a third test charge experience no net force?
8. A negative point charge $-q$ is placed at the point P near the positively charged ring shown in Figure 19.13 (Example 19.5). If $x \ll a$, describe the motion of the point charge if it is released from rest.
9. A very large, thin, flat plate of aluminum of area A has a total charge of Q uniformly distributed over its surfaces. The same charge is spread uniformly over only the *upper* surface of an otherwise identical glass plate. Compare the electric fields just above the centers of the plates' upper surfaces.
10. If there are more electric field lines leaving a gaussian surface than entering, what can you conclude about the net charge enclosed by that surface?
11. A uniform electric field exists in a region of space in which there are no charges. What can you conclude about the *net* electric flux through a gaussian surface placed in this region of space?
12. A common demonstration involves charging a balloon, which is an insulator, by rubbing it on your head, and touching the balloon to a ceiling or wall, which is also an insulator. The electrical attraction between the charged balloon and the neutral wall results in the balloon sticking to the wall. Imagine now that we have two infinitely large sheets of insulating material. One is charged and the other is neutral. If these are brought into contact, will there be an attractive force between them, as there was for the balloon and the wall?
13. If the total charge inside a closed surface is known but the distribution of the charge is unspecified, can you use Gauss's law to find the electric field? Explain.
14. Explain why excess charge on an isolated conductor must reside on its surface, using the repulsive nature of the force between like charges and the freedom of motion of charge within the conductor.
15. A person is placed in a large, hollow, metallic sphere that is insulated from ground. If a large charge is placed on the sphere, will the person be harmed on touching the inside of the sphere? Explain what will happen if the person also has an initial charge the sign of which is opposite that of the charge on the sphere.
16. Imagine two electric dipoles in empty space. Each dipole has zero net charge. Is there an electric force between the dipoles—that is, can two objects with *zero net charge* exert electric forces on each other? If so, is the force attractive or repulsive?

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PROBLEMS

Section 19.4 Coulomb's Law

1. Two identical conducting small spheres are placed with their centers 0.300 m apart. One is given a charge of 12.0 nC and the other a charge of −18.0 nC. (a) Find the electrostatic force exerted on one sphere by the other. (b) The spheres are connected by a conducting wire. After equilibrium has occurred, find the electrostatic force between the two.
2. (a) Two protons in a molecule are separated by 3.80×10^{-10} m. Find the electrostatic force exerted by one proton on the other. (b) How does the magnitude of this force compare to the magnitude of the gravitational force between the two protons? (c) What must be the charge-to-mass ratio of a particle if the magnitude of the gravitational force between two of these particles equals the magnitude of electrostatic force between them?
3. Richard Feynman once said that if two persons stood at arm's length from each other and each person had 1% more electrons than protons, the force of repulsion between them would be enough to lift a "weight" equal to that of the entire Earth. Carry out an order-of-magnitude calculation to substantiate this assertion.
4. In the Bohr theory of the hydrogen atom, an electron moves in a circular orbit about a proton, where the radius of the orbit is 0.529×10^{-10} m. (a) Find the electrostatic force between the two. (b) If this force provides the centripetal acceleration of the electron, what is the speed of the electron?
5. Two small silver spheres, each with a mass of 10.0 g, are separated by 1.00 m. Calculate the fraction of the electrons in one sphere that must be transferred to the other in order to produce an attractive force of 1.00×10^4 N (about 1 ton) between the spheres. (The number of electrons per atom of silver is 47, and the number of atoms per gram is Avogadro's number divided by the molar mass of silver, 107.87.)

Section 19.5 Electric Fields

6. The electrons in a particle beam each have a kinetic energy of 1.60×10^{-17} J. What are the magnitude and direction of the electric field that will stop these electrons in a distance of 10.0 cm?
7. The electrons in a particle beam each have a kinetic energy K. What are the magnitude and direction of the electric field that will stop these electrons in a distance d?
8. Consider the electric dipole shown in Figure P19.8. Show that the electric field at a *distant* point along the x axis is $E_x \equiv 4k_e qa/x^3$.
9. In Figure P19.9, determine the point (other than infinity) at which the electric field is zero.
10. A positively charged bead having a mass of 1.00 g falls from rest in a vacuum at a height of 5.00 m in a uniform vertical electric field of magnitude 1.00×10^4 N/C. The bead hits the ground at a speed of 21.0 m/s. Determine (a) the direction of the electric field (up or down), and (b) the charge on the bead.

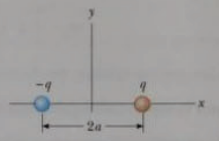


Figure P19.8

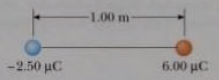


Figure P19.9

11. A continuous line of charge lies along the x axis, extending from $x = +x_0$ to positive infinity. The line carries a uniform linear charge density λ_0. What are the magnitude and direction of the electric field at the origin?
12. Consider n equal positive point charges each of magnitude q/n placed symmetrically around a circle of radius R. (a) Calculate the magnitude of the electric field E at a point a distance x on the line passing through the center of the circle and perpendicular to the plane of the circle. (b) Explain why this result is identical to the calculation done in Example 19.5.
13. Three equal positive charges q are at the corners of an equilateral triangle of sides a, as in Figure P19.13. (a) At what point in the plane of the charges (other than infinity) is the electric field zero? (b) What are the magnitude and direction of the electric field at P due to the two charges at the base?

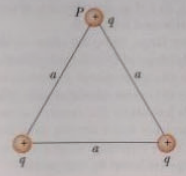


Figure P19.13

14. Three point charges, q, $2q$, and $3q$, are arranged on the vertices of an equilateral triangle with sides of length a. Determine the magnitude of electric field at the geometric center of the triangle.
15. A rod 14.0 cm long is uniformly charged and has a total charge of −22.0 μC. Determine the magnitude and direction of the electric field along the axis of the rod at a point 36.0 cm from its center.
16. A uniformly charged ring of radius 10.0 cm has a total charge of 75.0 μC. Find the electric field on the axis of the ring at (a) 1.00 cm, (b) 5.00 cm, (c) 30.0 cm, and (d) 100 cm from the center of the ring.
17. A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle as in Figure P19.17. If the rod has a total charge of −7.50 μC, find the magnitude and direction of the electric field at O, the center of the semicircle.

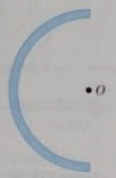


Figure P19.17

Section 19.7 Electric Flux

18. An electric field of intensity 3.50 kN/C is applied along the x axis. Calculate the electric flux through a rectangular plane 0.350 m wide and 0.700 m long if (a) the plane is parallel to the yz plane; (b) the plane is parallel to the xy plane; (c) the plane contains the y axis, and its normal makes an angle of 40.0° with the x axis.
19. A 40.0-cm-diameter loop is rotated in a uniform electric field until the position of maximum electric flux is found. The flux in this position is 5.20×10^5 N·m²/C. What is the electric field strength?
20. An electric field of magnitude 2.00×10^4 N/C exists perpendicular to the Earth's surface on a day when a thunderstorm is brewing. A car of approximately rectangular size 6.00 m by 3.00 m is traveling along a roadway sloping down at 10.0°. Determine the electric flux through the bottom of the car.

Section 19.8 Gauss's Law

21. A charge of 12.0 μC is at the geometric center of a cube. What is the electric flux through one of the faces?
22. The following charges are located inside a submarine: 5.00 μC, −9.00 μC, 27.0 μC, and −84.0 μC. Calculate the net electric flux through the submarine. Compare the number of electric field lines leaving the submarine with the number entering it.
23. A point charge Q is located just above the center of the flat face of a hemisphere of radius R as in Figure P19.23. What is the electric flux (a) through the curved surface and (b) through the flat face?

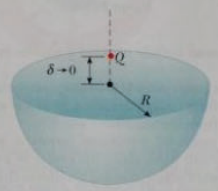


Figure P19.23

24. The electric field everywhere on the surface of a thin spherical shell of radius 0.750 m is measured to be equal to 890 N/C and points radially toward the center of the sphere. (a) What is the net charge within the sphere's surface? (b) What can you conclude about the nature and distribution of the charge inside the spherical shell?

Section 19.9 Application of Gauss's Law to Charged Insulators

25. Determine the magnitude of the electric field at the surface of a lead-208 nucleus, which contains 82 protons and 126 neutrons. Assume the lead nucleus has a volume 208 times that of 1 proton, and consider a proton to be a sphere of radius 1.20×10^{-15} m.
26. A solid sphere of radius 40.0 cm has a total positive charge of 26.0 μC uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) 0 cm, (b) 10.0 cm, (c) 40.0 cm, and (d) 60.0 cm from the center of the sphere.
27. A very thin, spherical shell of radius a has a total charge of Q distributed uniformly over its surface (Fig. P19.27). Find the electric field at points inside and outside the shell.
28. A cylindrical shell of radius 7.00 cm and length 240 cm has its charge uniformly distributed on its surface. The electric field at a point 19.0 cm radially outward from its axis (measured from the midpoint of the shell) is 36.0 kN/C. Use approximate relations to find (a) the net charge on the shell and (b) the electric field at a point 4.00 cm from the

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Figure P19.27

axis, measured radially outward from the midpoint of the shell.

29. Consider a long cylindrical charge distribution of radius R with a uniform charge density ρ. Find the electric field at distance r from the axis where $r < R$.

30. A 10.0-g piece of Styrofoam carries a net charge of $-0.700\ \mu C$ and floats above the center of a very large horizontal sheet of plastic that has a uniform charge density on its surface. What is the charge per unit area on the plastic sheet?

Section 19.10 Conductors in Electrostatic Equilibrium

31. A long, straight metal rod has a radius of 5.00 cm and a charge per unit length of 30.0 nC/m. Find the electric field (a) 3.00 cm, (b) 10.0 cm, and (c) 100 cm from the axis of the rod, where distances are measured perpendicular to the rod.

32. A charge q is placed on a conducting sheet that has a finite but very small thickness. What is the approximate magnitude of the electric field at an external point very near the surface of the sheet at its center if the area of the sheet is A?

33. A square plate of copper with 50.0-cm sides has no net charge and is placed in a region of uniform electric field of 80.0 kN/C directed *perpendicularly* to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

34. A solid conducting sphere of radius 2.00 cm has a charge 8.00 μC. A conducting spherical shell of inner radius 4.00 cm and outer radius 5.00 cm is concentric with the solid sphere and has a charge $-4.00\ \mu C$. Find the electric field at (a) $r = 1.00$ cm, (b) $r = 3.00$ cm, (c) $r = 4.50$ cm, and (d) $r = 7.00$ cm from the center of this charge configuration.

35. Two identical conducting spheres each having a radius of 0.500 cm are connected by a light 2.00-m-long conducting wire. Determine the tension in the wire if 60.0 μC is placed on one of the conductors. (*Hint:* Assume that the surface distribution of charge on each sphere is uniform.)

36. A long, straight wire is surrounded by a hollow metal cylinder the axis of which coincides with that of the wire. The wire has a charge per unit length of λ, and the cylinder has a net charge per unit length of 2λ. From this information, use Gauss's law to find (a) the charge per unit length on the inner and outer surfaces of the cylinder and (b) the electric field outside the cylinder a distance r from the axis.

37. The electric field on the surface of an irregularly shaped conductor varies from 56.0 kN/C to 28.0 kN/C. Calculate the local surface charge density at the point on the surface at which the radius of curvature of the surface is (a) smallest and (b) greatest.

Additional Problems

38. Three point charges are aligned along the x axis as shown in Figure P19.38. Find the electric field at (a) the position (2.00, 0) and (b) the position (0, 2.00).

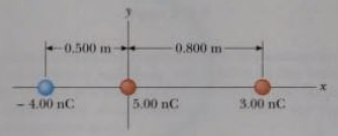


Figure P19.38

39. A small, 2.00-g plastic ball is suspended by a 20.0-cm-long string in a uniform electric field, as in Figure P19.39. If the ball is in equilibrium when the string makes a 15.0° angle with the vertical, what is the net charge on the ball?

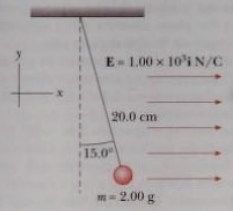


Figure P19.39

40. A uniform electric field of magnitude 640 N/C exists between two parallel plates that are 4.00 cm apart. A proton is released from the positive plate at the same instant that an electron is released from the negative plate. (a) Determine the distance from the positive plate that the two pass each other. (Ignore the electrostatic attraction between the proton and electron.) (b) Repeat part (a) for a sodium ion (Na^+) and a chlorine ion (Cl^-).

41. A charged cork ball of mass 1.00 g is suspended on a light string in the presence of a uniform electric field, as in Figure P19.41. When $\mathbf{E} = (3.00\mathbf{i} + 5.00\mathbf{j}) \times 10^5$ N/C, the ball is in equilibrium at $\theta = 37.0°$. Find (a) the charge on the ball and (b) the tension in the string.

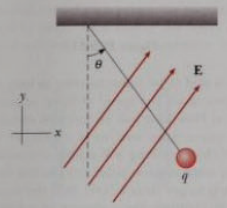


Figure P19.41

42. Four identical point charges ($q = +10.0\ \mu C$) are located on the corners of a rectangle, as shown in Figure P19.42. The dimensions of the rectangle are $L = 60.0$ cm and $W = 15.0$ cm. Calculate the magnitude and direction of the net electric force exerted on the charge at the lower left corner by the other three charges.

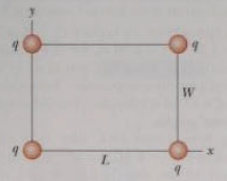


Figure P19.42

43. A solid, *insulating* sphere of radius a has a uniform charge density of ρ and a total charge of Q. Concentric with this sphere is an *uncharged, conducting* hollow sphere, the inner and outer radii of which are b and c, as in Figure P19.43. (a) Find the electric field intensity in the regions $r < a$, $a < r < b$, $b < r < c$, and $r > c$. (b) Determine the induced charge per unit area on the inner and outer surfaces of the hollow sphere.

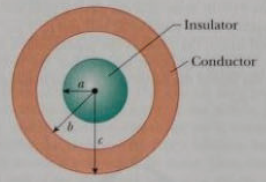


Figure P19.43

44. In nuclear fission, a nucleus of uranium-238, which contains 92 protons, divides into two smaller spheres, each having 46 protons and a radius of 5.90×10^{-15} m. What is the magnitude of the repulsive electric force pushing the two spheres apart?

45. An infinitely long cylindrical insulating shell of inner radius a and outer radius b has a uniform volume charge density ρ. A line of charge density λ is placed along the axis of the shell. Determine the electric field intensity everywhere.

46. Two infinite, nonconducting sheets of charge are parallel to each other as in Figure P19.46. The sheet on the left has a uniform surface charge density σ, and the one on the right has a uniform charge density $-\sigma$. Calculate the value of the electric field at points (a) to the left of, (b) in between, and (c) to the right of the two sheets. (*Hint:* See Example 19.10.)

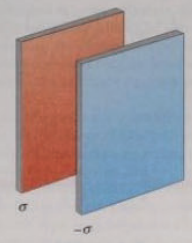


Figure P19.46

47. Repeat the calculations for Problem 46 when both sheets have *positive* uniform charge densities of value σ.

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48. A sphere of radius $2a$ is made of a nonconducting material that has a uniform volume charge density ρ. (Assume that the material does not affect the electric field.) A spherical cavity of radius a is now removed from the sphere, as shown in Figure P19.48. Show that the electric field within the cavity is uniform and is given by $E_x = 0$ and $E_y = \rho a/3\epsilon_0$. (*Hint:* The field within the cavity is the superposition of the field due to the original uncut sphere plus the field due to a sphere the size of the cavity with a uniform negative charge density $-\rho$.)

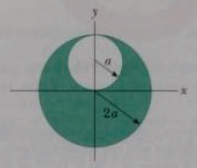


Figure P19.48

49. Three identical small Styrofoam balls ($m = 2.00$ g) are suspended from a fixed point by three nonconducting threads, each with a length of 50.0 cm and with negligible mass. At equilibrium the three balls form an equilateral triangle with sides of 30.0 cm. What is the common charge q carried by each ball?

50. Identical thin rods of length $2a$ carry equal charges, $+Q$, uniformly distributed along their lengths. The rods lie along the x axis with their centers separated by a distance of $b > 2a$ (Fig. P19.50). Show that the force exerted on the right rod is given by

$$F = \frac{k_e Q^2}{4a^2} \ln\left(\frac{b^2}{b^2 - 4a^2}\right)$$

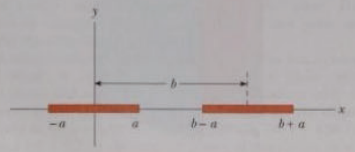


Figure P19.50

51. A line of positive charge is formed into a semicircle of radius $R = 60.0$ cm, as shown in Figure P19.51. The charge per unit length along the semicircle is described by the expression $\lambda = \lambda_0 \cos\theta$. The total charge on the semicircle is 12.0 μC. Calculate the total force on a charge of 3.00 μC placed at the center of curvature.

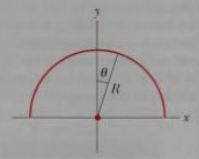


Figure P19.51

52. A particle is said to be nonrelativistic as long as its speed is less than one tenth the speed of light, or less than 3.00×10^7 m/s. (a) How long will an electron remain nonrelativistic if it starts from rest in a region of electric field 1.00 N/C? (b) How long will a proton remain nonrelativistic in the same electric field? (c) Electric fields are commonly much larger than 1 N/C. Will the charged particle remain nonrelativistic for a shorter or a longer time in a much larger electric field?

Spreadsheet Problems

S1. Spreadsheet 19.1 calculates the x and y components of the electric field **E** along the x axis due to any two point charges Q_1 and Q_2. Enter the values of the charges and their x and y coordinates. Choose $Q_1 = 4$ μC at the origin, and set $Q_2 = 0$. Plot the electric field along the x axis. *Note:* You must choose a step size and adjust the scaling for the graph because the electric field at the position of the point charge is infinite.

S2. (a) Using Spreadsheet 19.1, place $Q_1 = 4$ μC at the origin and $Q_2 = 4$ μC at $x = 0.08$ m, $y = 0$. Plot the components of the electric field **E** as a function of x. (b) Change Q_2 to -4 μC, and plot the components of the electric field as a function of x. (This is a dipole.) You will need to adjust the scales of your graphs.

S3. (a) Using Spreadsheet 19.1, take $Q_1 = 6$ nC at $x = 0$, $y = 0.03$ m, and $Q_2 = 6$ nC at $x = 0$, $y = -0.03$ m. Plot the components of the electric field **E** as a function of x. (b) Change Q_2 to -6 nC, and repeat part (a).

S4. Consider an electric dipole in which a charge $+q$ is on the x axis at $x = a$ and a charge $-q$ is on the x axis at $x = -a$. Take $q = 6$ nC and $a = 0.02$ m. Use Spreadsheet 19.1 to plot the components of the electric field along the x axis for values of x greater than 4 cm. Modify Spreadsheet 19.1 to add a plot $2k_e p/x^3$ (the **E** field in the approximation $x \gg a$), where $p = 2aq$ is the dipole moment of the charge distribution. For what range of values of x is the dipole approximation within 20% of the actual value? Within 5%?

ANSWERS TO CONCEPTUAL PROBLEMS

1. The configuration shown is inherently unstable. The negative charges are repelling each other. If there is any slight rotation of one of the rods, the repulsion can result in further rotation away from this configuration. There are three possible final configurations shown in Figure CP19.1: Configuration (a) is stable—if the positive upper ends are pushed toward each other, they will repel and move the system back to the original configuration. Configuration (b) is an equilibrium configuration, but is unstable—if the upper ends are moved toward each other, the attraction of the upper ends will be larger than that of the lower ends and the configuration will shift to (c). Configuration (c) is stable.

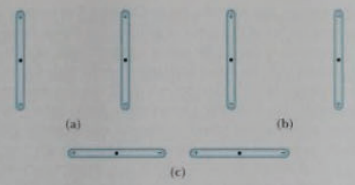


Figure CP19.1

2. No. Object A might have a charge opposite in sign to that of B, but it also might be a neutral conductor. In this latter case, object B causes A to be polarized, pulling charge of one sign to the near face of A and pushing an equal amount of charge of the opposite sign to the far face, as in Figure CP19.2. Then the force of attraction exerted on B by the induced charge on the near side of A is slightly larger than the force of repulsion exerted on B by the induced charge on the far side of A. Therefore, the net force on A is toward B.

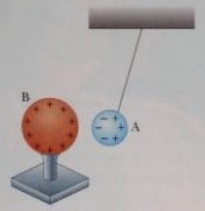


Figure CP19.2

3. A television screen is constantly bombarded with electrons from the electron gun. As a result, it becomes negatively charged, and there is an electric field established in the region in front of the screen. This electric field will polarize dust particles in the air in front of the screen, just like a charged object polarizes molecules in a neutral object. What's more, the electric field is not uniform, because the television screen is not an infinite sheet. Thus, the positively charged end of the dust particles will be nearer the screen and in a stronger electric field than the negative end. As a result, there is a net attractive force, pulling the dust particle to the screen, where it adheres.

4. Electric field lines start on positive charges and end on negative charges. Thus, if the fair weather field is directed into the ground, the ground must have a negative charge.

5. The two charged plates create a region of uniform electric field between them, directed from the positive toward the negative plate. Once the ball is disturbed so as to touch one plate, say the negative one, some negative charge will be transferred to the ball and it will experience an electric force that will accelerate it to the positive plate. Once the charge touches the positive plate, it will release its negative charge, acquire a positive charge, and accelerate back to the negative plate. The ball will continue to move back and forth between the plates until it has transferred all their net charge, thereby making both plates neutral.

6. The electric field patterns of each of these three configurations do not have sufficient symmetry to make the calculations practical. (Gauss's law is only useful for calculating the electric field of highly symmetric charge distributions, such as uniformly charged spheres, cylinders, and sheets.) In order to apply Gauss's law, you must be able to find a closed surface surrounding the charge distribution, which can be subdivided so that the field over the separate regions of the surface is constant. Such a surface cannot be found for these cases.

7. The electric shielding effect of conductors depends on the fact that there are two kinds of charge—positive and negative. As a result, charges can move within the conductor so that the combination of positive and negative charges establishes an electric field that exactly cancels the external field within the conductor and any cavities inside the conductor. There is only one type of gravitational charge, however—there is no "negative mass." As a result, gravitational shielding is not possible.

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20

Electric Potential and Capacitance

CHAPTER OUTLINE

20.1 Potential Difference and Electric Potential
20.2 Potential Differences in a Uniform Electric Field
20.3 Electric Potential and Electric Potential Energy Due to Point Charges
20.4 Obtaining **E** from the Electric Potential
20.5 Electric Potential Due to Continuous Charge Distributions
20.6 Electric Potential of a Charged Conductor
20.7 Capacitance
20.8 Combinations of Capacitors
20.9 Energy Stored in a Charged Capacitor
20.10 Capacitors with Dielectrics

The concept of potential energy was introduced in Chapter 7 in connection with such conservative forces as the force of gravity, the elastic force of a spring, and the electrostatic force. By using the principle of conservation of energy, we were often able to avoid working directly with forces when solving mechanical problems. In this chapter we shall use the energy concept in our study of electricity. Because the electrostatic force (given by Coulomb's law) is conservative, electrostatic phenomena can conveniently be described in terms of an *electric* potential energy function. This concept enables us to define a quantity called *electric potential*, which is a scalar function of position and thus leads to a simpler method of describing some electrostatic phenomena than the electric field method. Although the potential is clearly an easier path for many problems, one often requires **E**. In some situations, a knowledge of **E** provides the simplest path to calculation of the potential. As we shall see in subsequent chapters,



Jennifer is holding on to a charged sphere that reaches a potential of about 100 000 volts. The device that generates this high potential is called a Van de Graaff generator. Why do you suppose Jennifer's hair stands on end like the needles of a porcupine? Why is it important that she stand on a pedestal insulated from ground? *(Courtesy of Henry Leap and Jim Lehman)*

the concept of electric potential is of great practical value. For example, the measured voltage between any two points in an electrical circuit is simply the difference in electric potential between the points.

This chapter is also concerned with the properties of capacitors, devices that store charge. Capacitors are commonly used in a variety of electrical circuits. For instance, they are used to tune the frequency of radio receivers, as filters in power supplies, to eliminate unwanted sparking in automobile ignition systems, and as energy-storing devices in electronic flash units.

A capacitor basically consists of two conductors separated by an insulator. We shall see that the capacitance of a given device depends on its geometry and on the material called a *dielectric*, separating the charged conductors.

20.1 • POTENTIAL DIFFERENCE AND ELECTRIC POTENTIAL

When a test charge q_0 is placed in an electrostatic field **E**, the electric force on the charge is $q_0\mathbf{E}$. This force is the vector sum of the individual forces exerted on q_0 by the various charges producing the field **E**. It follows that the force $q_0\mathbf{E}$ is conservative because the individual forces governed by Coulomb's law are conservative. When a charge is moved within an electric field at constant velocity, the work done on q_0 by the electric field is equal to the negative of the work done by the external agent causing the displacement. For an infinitesimal displacement $d\mathbf{s}$, the work done by the electric field is $\mathbf{F} \cdot d\mathbf{s} = q_0\mathbf{E} \cdot d\mathbf{s}$. This decreases the potential energy of the charge–field system by an amount $dU = -q_0\mathbf{E} \cdot d\mathbf{s}$. For a finite displacement of the test charge between points A and B, the **change in potential energy** is

$$\Delta U = U_B - U_A = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad [20.1]$$

• *Change in potential energy*

The integral in Equation 20.1 is performed along the path by which q_0 moves from A to B and is called either a **path integral** or a **line integral.** Because the force $q_0\mathbf{E}$ is conservative, **this integral does not depend on the path taken between *A* and *B*.**

The potential energy per unit charge, U/q_0, is independent of the value of q_0 and has a unique value at every point in an electric field. The quantity U/q_0 is called the **electric potential** (or simply the **potential**), V. Thus, the electric potential at any point in an electric field is

$$V = \frac{U}{q_0} \qquad [20.2]$$

Because potential energy is a scalar, electric potential is also a scalar quantity.

The **potential difference,** $\Delta V = V_B - V_A$, between the points A and B is defined as the change in potential energy divided by the test charge q_0:

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad [20.3]$$

• *Potential difference between two points*

Potential difference should not be confused with potential energy. The potential difference is *proportional* to the potential energy difference, and we see from

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Equation 20.3 that the two are related by $\Delta U = q_0 \Delta V$. The potential is characteristic of the field, independent of the charges that may be placed in the field. However, when we speak of potential energy, we are referring to the charge–field system.

Because, as already noted, the change in the potential energy is the negative of the work done by the electric force, the potential difference ΔV equals the work per unit charge that an external agent must perform to move a test charge from A to B without a change in kinetic energy.

Equation 20.3 defines potential difference only. That is, only *differences* in V are meaningful. The electric potential function is often taken to be zero at some convenient point. We shall usually set the potential at zero for a point at infinity (that is, a point infinitely remote from the charges producing the electric field). With this choice, we can say that **the electric potential at an arbitrary point equals the work required per unit charge to bring a positive test charge from infinity to that point.** Thus, if we take $V_A = 0$ at infinity in Equation 20.3, then the potential at any point P is

$$V_P = -\int_{\infty}^{P} \mathbf{E} \cdot d\mathbf{s} \qquad [20.4]$$

In reality, V_P represents the potential difference between the point P and a point at infinity. (Equation 20.4 is a special case of Equation 20.3.)

Because potential is a measure of energy per unit charge, the SI units of potential are joules per coulomb, called the **volt** (V):

Definition of a volt •

$$1\ \text{V} = 1\ \text{J/C}$$

That is, 1 J of work must be done to take a 1-C charge through a potential difference of 1 V. Equation 20.3 shows that the potential difference also has the same units as electric field times distance. From this, it follows that the SI units of electric field, newtons per coulomb, can also be expressed as volts per meter.

$$1\ \text{N/C} = 1\ \text{V/m}$$

As we learned in Chapter 9, Section 9.7, a unit of energy commonly used in atomic and nuclear physics is the **electron volt** (eV):

The electron volt •

$$1\ \text{eV} = 1.60 \times 10^{-19}\ \text{C·V} = 1.60 \times 10^{-19}\ \text{J} \qquad [20.5]$$

For instance, an electron in the beam of a typical TV picture tube has a speed of 5.0×10^7 m/s. This corresponds to a kinetic energy of 1.1×10^{-15} J, which is equivalent to 7.1×10^3 eV. Such an electron has to be accelerated from rest through a potential difference of 7.1 kV to reach this speed.

Thinking Physics 1

Suppose scientists had chosen to measure small energies in *proton volts* rather than electron volts. What difference would this make?

Reasoning There would be no change at all. An electron volt is the kinetic energy gained by an electron in being accelerated through a potential difference of one volt. A proton accelerated through one volt would have the same kinetic energy, because it carries the same charge as the electron (except for the sign). The proton would be moving more slowly after accelerating through one volt, due to its larger mass, but it would still gain one electron volt, or one proton volt, of kinetic energy.

20.2 • POTENTIAL DIFFERENCES IN A UNIFORM ELECTRIC FIELD

In this section we shall describe the potential difference between any two points in a *uniform* electric field. The potential difference is independent of the path between the two points; that is, the work done in taking a test charge from point A to point B is the same along all paths. This confirms that a static, uniform electric field is conservative.

First, consider a uniform electric field directed along the negative y axis, as in Figure 20.1a. Let us calculate the potential difference between two points, A and B, separated by the distance d, where d is measured parallel to the field lines. If we apply Equation 20.3 to this situation, we get

$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\int_A^B E \cos 0^\circ\, d\mathbf{s} = -\int_A^B E\, ds$$

Because E is constant, it can be removed from the integral sign, giving

$$\Delta V = -E \int_A^B ds = -Ed \qquad [20.6]$$

• *Potential difference between two points in a uniform* **E** *field*

The minus sign results from the fact that point B is at a lower potential than point A; that is, $V_B < V_A$. In general, **electric field lines always point in the direction of decreasing electric potential,** as shown in Figure 20.1a.

Now suppose that a test charge q_0 moves from A to B. The change in its electric potential energy can be found from Equations 20.3 and 20.6:

$$\Delta U = q_0 \Delta V = -q_0 Ed \qquad [20.7]$$

From this result, we see that if q_0 is positive, ΔU is negative. This means that **when a positive charge moves in the direction of the electric field, its electric potential energy decreases.** (This is analogous to the decrease in gravitational potential energy of a mass falling in a gravitational field as suggested in Fig. 20.1b.) If a positive test charge is released from rest in the electric field, it experiences an electric force $q_0\mathbf{E}$ in the direction of **E** (downward in Fig. 20.1a). Therefore, it accelerates downward, gaining kinetic energy. **As the charged particle gains kinetic energy, it loses an equal amount of potential energy.**

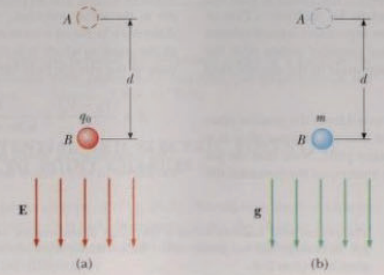


Figure 20.1 (a) When the electric field **E** is directed downward, point B is at a lower electric potential than point A. A positive test charge that moves from A to B loses electric potential energy. (b) A mass m moving downward in the direction of the gravitational field **g** loses gravitational potential energy.

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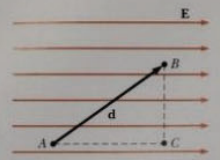


Figure 20.2 A uniform electric field directed along the positive x axis. Point B is at a lower potential than point A. Points B and C are at the *same* potential.

If q_0 is negative, then ΔU is positive and the situation is reversed. **A negative charge gains electric potential energy when it moves in the direction of the electric field.** If a negative charge is released from rest in the field **E**, it accelerates in a direction opposite the electric field.

Now consider the more general case of a charged particle moving between any two points in a uniform electric field directed along the x axis, as in Figure 20.2. If **d** represents the displacement vector between points A and B, Equation 20.3 gives

$$\Delta V = -\int_A^B \mathbf{E}\cdot d\mathbf{s} = -\mathbf{E}\cdot\int_A^B d\mathbf{s} = -\mathbf{E}\cdot\mathbf{d} \qquad [20.8]$$

where again we are able to remove **E** from the integral, because it is constant. Furthermore, the change in electric potential energy of the charge is

$$\Delta U = q_0\,\Delta V = -q_0\mathbf{E}\cdot\mathbf{d} \qquad [20.9]$$

Finally, our results show that all points in a plane *perpendicular* to a uniform electric field are at the same potential. This can be seen in Figure 20.2, where the potential difference $V_B - V_A$ is equal to $V_C - V_A$. Therefore, $V_B = V_C$. **The name *equipotential surface* is given to any surface consisting of a continuous distribution of points having the same electrical potential.** Note that because $\Delta U = q_0\,\Delta V$, no work is done in moving a test charge between any two points on an equipotential surface. The equipotential surfaces of a uniform electric field consist of a family of planes, all perpendicular to the field. Equipotential surfaces for fields with other symmetries will be described in later sections.

An equipotential surface

CONCEPTUAL PROBLEM 1

If a proton is released from rest in a uniform electric field, does its electric potential energy increase or decrease?

Example 20.1 The Electric Field Between Two Parallel Plates of Opposite Charge

A 12-V battery is connected between two parallel plates, as in Figure 20.3. The separation between the plates is 0.30 cm, and the electric field is assumed to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate size and if we do not consider points near the edges of the plates.) Find the magnitude of the electric field between the plates.

Solution The electric field is directed from the positive plate toward the negative plate. We see that the positive plate is at a higher potential than the negative plate. Note that the potential difference between plates must equal the potential difference between the battery terminals. This can be understood by noting that all points on a conductor in equilibrium are at the same potential,[1] and hence there is no potential difference between a terminal of the battery and any portion of the plate to which it is connected. Therefore, the magnitude of the electric field between the plates is

$$E = \frac{|V_B - V_A|}{d} = \frac{12\ \text{V}}{0.30\times10^{-2}\ \text{m}} = 4.0\times10^{3}\ \text{V/m}$$

This configuration, which is called a *parallel-plate capacitor*, is examined in more detail later in this chapter.

[1]The electric field vanishes within a conductor in electrostatic equilibrium, and so the path integral $\int \mathbf{E}\cdot d\mathbf{s}$ between any two points within the conductor must be zero. A fuller discussion of this point is given in Section 20.6.

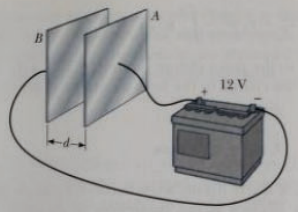


Figure 20.3 (Example 20.1) A 12-V battery connected to two parallel plates. The electric field between the plates has a magnitude given by the potential difference divided by the plate separation d.

Example 20.2 Motion of a Proton in a Uniform Electric Field

A proton is released from rest in a uniform electric field of magnitude 8.0×10^4 V/m directed along the positive x axis (Fig. 20.4). The proton undergoes a displacement of 0.50 m in the direction of **E**. (a) Find the change in the electric potential between the points A and B.

Solution From Equation 20.6 we have

$$\Delta V = -Ed = -\left(8.0\times10^4\ \frac{\text{V}}{\text{m}}\right)(0.50\ \text{m}) = -4.0\times10^4\ \text{V}$$

This negative result tells us that the electric potential decreases as the proton moves from A to B.

(b) Find the change in potential energy for this displacement.

Solution

$$\begin{aligned}\Delta U &= q_0\,\Delta V = e\,\Delta V\\ &= (1.6\times10^{-19}\ \text{C})(-4.0\times10^4\ \text{V})\\ &= -6.4\times10^{-15}\ \text{J}\end{aligned}$$

The negative sign here means that the potential energy decreases as the proton moves in the direction of the electric field. This makes sense, because as the proton accelerates in the direction of the field, it gains kinetic energy and at the same time (the proton–field system) loses electrical potential energy (energy is conserved).

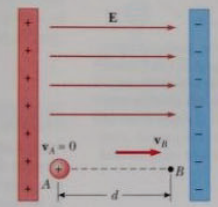


Figure 20.4 (Example 20.2) A proton accelerates from A to B in the direction of the electric field.

EXERCISE 1 Apply the principle of energy conservation to find the speed of the proton after it has moved 0.50 m, starting from rest. Answer 2.77×10^6 m/s

EXERCISE 2 The difference in potential between the deflection plates of a cathode ray tube is 300 V. If the distance between these plates is 2.0 cm, find the magnitude of the uniform electric field in this region. Answer 1.5×10^4 N/C

20.3 · ELECTRIC POTENTIAL AND ELECTRIC POTENTIAL ENERGY DUE TO POINT CHARGES

Consider an isolated positive point charge, q (Fig. 20.5). Recall that such a charge produces an electric field that is directed radially outward from the charge. In order to find the electric potential at a distance of r from the charge, we begin with the general expression for potential difference:

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