

Figure 20.5 The potential difference between points A and B due to a point charge q depends *only* on the initial and final radial coordinates, r_A and r_B, respectively. The two dashed circles represent cross-sections of spherical equipotential surfaces.

$$V_B - V_A = -\int_A^B \mathbf{E}\cdot d\mathbf{s}$$

Because the electric field due to the point charge is given by $\mathbf{E} = kq\hat{\mathbf{r}}/r^2$, where $\hat{\mathbf{r}}$ is a unit vector directed from the charge to the field point, the quantity $\mathbf{E}\cdot d\mathbf{s}$ can be expressed as

$$\mathbf{E}\cdot d\mathbf{s} = k_e\frac{q}{r^2}\hat{\mathbf{r}}\cdot d\mathbf{s}$$

The dot product $\hat{\mathbf{r}}\cdot d\mathbf{s} = ds\cos\theta$, where θ is the angle between $\hat{\mathbf{r}}$ and $d\mathbf{s}$, as in Figure 20.5. Furthermore, note that $ds\cos\theta$ is the projection of $d\mathbf{s}$ onto r, so that $ds\cos\theta = dr$. That is, any displacement $d\mathbf{s}$ produces the change dr in the magnitude of $\mathbf{r}$. With these substitutions, we find that $\mathbf{E}\cdot d\mathbf{s} = (k_e q/r^2)dr$, so that the expression for the potential difference becomes

$$V_B - V_A = -\int E_r\,dr = -k_e q\int_{r_A}^{r_B}\frac{dr}{r^2} = \frac{k_e q}{r}\Big]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \qquad [20.10]$$

The line integral of $\mathbf{E}\cdot d\mathbf{s}$ is *independent* of the path between A and B—as it must be, because the electric field of a point charge is conservative. Furthermore, Equation 20.10 expresses the important result that the potential difference between any two points A and B depends *only* on the *radial* coordinates r_A and r_B. As we learned in Section 20.1, it is customary to choose the reference of potential to be zero at $r_A = \infty$. With this choice, the electric potential due to a point charge at any distance r from the charge is

Potential of a point charge •

$$V = k_e\frac{q}{r} \qquad [20.11]$$

From this we see that V is constant on a spherical surface of radius r. Hence, we conclude that **the equipotential surfaces for an isolated point charge consist of a family of spheres concentric with the charge,** as shown in Figure 20.5. Note that the equipotential surfaces are perpendicular to the lines of electric force, as was the case for a uniform electric field.

The electric potential of two or more point charges is obtained by applying the superposition principle. That is, the total potential at some point P due to several point charges is the sum of the potentials due to the individual charges. For a group of charges, we can write the total potential at P in the form

The potential of several point charges •

$$V = k_e\sum_i\frac{q_i}{r_i} \qquad [20.12]$$

where the potential is again taken to be zero at infinity and r_i is the distance from the point P to the charge q_i. Note that the sum in Equation 20.12 is an *algebraic sum* of scalars rather than a vector sum (which is used to calculate the electric field of a group of charges). Thus, it is much easier to evaluate V than to evaluate $\mathbf{E}$.

We now consider the electric potential energy of interaction of a system of charged particles. If V_1 is the electric potential due to charge q_1 at point P, then

the work required to bring a second charge q_2 from infinity to point P without acceleration is q_2V_1. By definition, this work equals the potential energy U of the two-particle system when the particles are separated by a distance of r_{12} (Fig. 20.6). We can therefore express the potential energy as

$$U = q_2V_1 = k_e\frac{q_1q_2}{r_{12}} \qquad [20.13]$$

Electric potential energy of two charges •

Note that if the charges are of the same sign, U is positive. This is consistent with the fact that like charges repel, and so positive work must be done *on* the system to bring the two charges near one another. If the charges are of opposite sign, the force is attractive and U is negative. This means that negative work must be done to bring the unlike charges near one another.

If the system consists of more than two charged particles, the total electric potential energy can be obtained by calculating U for every pair of charges and summing the terms algebraically. The total electric potential energy of a system of point charges is equal to the work required to bring the charges, one at a time, from an infinite separation to their final positions.

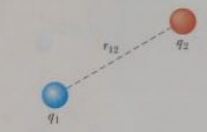


Figure 20.6 If two point charges are separated by a distance r_{12}, the potential energy of the pair of charges is given by $k_eq_1q_2/r_{12}$.

Thinking Physics 2

A spherical rubber balloon contains a charged object, which is mounted so that it always stays at the center of the balloon. As the balloon is inflated to a larger volume, what happens to the potential at the surface of the balloon? To the electric field at the surface of the balloon? To the total electric flux through the balloon? Answer the same questions for a *metallic* balloon being inflated.

Reasoning The potential at the surface of the balloon will decrease in inverse proportion to the radius. The electric field will decrease as the reciprocal of the squared radius of the balloon. The electric flux will remain constant, because all electric field lines must pass through the balloon, regardless of the size of the balloon. The answers will be exactly the same for the metallic balloon—the metal makes no difference, except for the fact that there will be no electric field within the skin of the balloon.

CONCEPTUAL PROBLEM 2

If the electric potential at some point is zero, can you conclude that there are no charges in the vicinity of that point?

Example 20.3 The Potential Due to Two Point Charges

A 2.00-μC point charge is located at the origin, and a second point charge of -6.00 μC is located on the y axis at the position (0, 3.00) m, as in Figure 20.7a. (a) Find the total electric potential due to these charges at the point P, the coordinates of which are (4.00, 0) m.

Solution For two charges, the sum in Equation 20.12 gives

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

In this example, $q_1 = 2.00$ μC, $r_1 = 4.00$ m, $q_2 = -6.00$ μC, and $r_2 = 5.00$ m. Therefore, V_P reduces to

$$V_P = 8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\left(\frac{2.00\times10^{-6}\ \text{C}}{4.00\ \text{m}} - \frac{6.00\times10^{-6}\ \text{C}}{5.00\ \text{m}}\right)$$

$$= -6.29\times10^3\ \text{V}$$

(b) How much work is required to bring a 3.00-μC point charge from infinity to the point P?

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Solution

$$W = q_3 V_P = (3.00 \times 10^{-6}\ \text{C})(-6.29 \times 10^{3}\ \text{V})$$
$$= -18.9 \times 10^{-3}\ \text{J}$$

The negative sign means that work is done by the field on the charge as it is displaced from infinity to P. Therefore, positive work would have to be done by an external agent to remove the charge from P back to infinity.

EXERCISE 3 Find the total potential energy of the system of three charges in the configuration shown in Figure 20.7b. Answer $-5.48 \times 10^{-2}\ \text{J}$

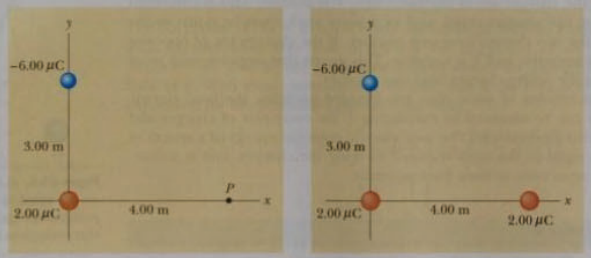


Figure 20.7 (Example 20.3) (a) The electric potential at the point P due to the two point charges q_1 and q_2 is the algebraic sum of the potentials due to the individual charges. (b) What is the potential energy of the system of three charges?

EXERCISE 4 (a) Find the potential at a distance of 1.0 cm from a proton. (b) What is the potential difference between two points that are 1.0 cm and 2.0 cm from a proton? (Take $V = 0$ at $r = \infty$) Answer (a) $1.4 \times 10^{-7}\ \text{V}$ (b) $-7.2 \times 10^{-8}\ \text{V}$

20.4 · OBTAINING E FROM THE ELECTRIC POTENTIAL

The electric field **E** and the electric potential V are related by Equation 20.3. Both quantities are determined by a specific charge distribution. We now show how to calculate the electric field if the electric potential is known in a certain region.

From Equation 20.3 we can express the potential difference dV between two points a distance ds apart as

$$dV = -\mathbf{E} \cdot d\mathbf{s} \qquad [20.14]$$

If the electric field has only *one* component, E_x, then $\mathbf{E} \cdot d\mathbf{s} = E_x\, dx$. Therefore, Equation 20.14 becomes $dV = -E_x\, dx$, or

$$E_x = -\frac{dV}{dx} \qquad [20.15]$$

That is, the electric field is equal to the negative of the derivative of the electric potential with respect to some coordinate. Note that the potential change is zero

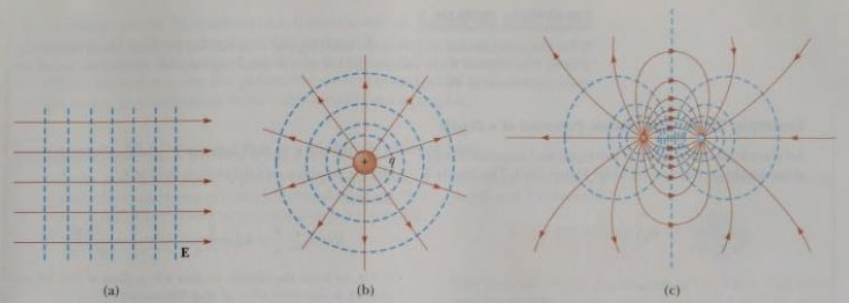


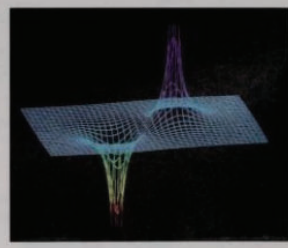
Figure 20.8 Equipotential surfaces (dashed blue lines) and electric field lines (red lines) for (a) a uniform electric field produced by an infinite sheet of charge, (b) a point charge, and (c) an electric dipole. In all cases, the equipotential surfaces are *perpendicular* to the electric field lines at every point.

for any displacement perpendicular to the electric field. This is consistent with the notion of equipotential surfaces being perpendicular to the field, as in Figure 20.8a.

If the charge distribution has spherical symmetry, where the charge density depends only on the radial distance, r, then the electric field is radial. In this case, $\mathbf{E} \cdot d\mathbf{s} = E_r\, dr$, and so we can express dV in the form $dV = -E_r\, dr$. Therefore,

$$E_r = -\frac{dV}{dr} \qquad [20.16]$$

For example, the potential of a point charge is $V = k_e q / r$. Because V is a function of r only, the potential function has spherical symmetry. Applying Equation 20.16, we find that the electric field due to the point charge is $E_r = k_e q / r^2$, a familiar result. Note that the potential changes only in the radial direction, not in a direction perpendicular to r. Thus, V (like E_r) is a function only of r. Again, this is consistent with the idea that **equipotential surfaces are perpendicular to field lines.** In this case the equipotential surfaces are a family of spheres concentric with the spherically symmetric charge distribution (Fig. 20.8b). The equipotential surfaces for the electric dipole are sketched in Figure 20.8c.



Computer-generated plot of the electric potential associated with an electric dipole. The charges lie in the horizontal plane, at the centers of the potential spikes. The contour lines help visualize the size of the potential the values of which are plotted vertically. *(Richard Megna, Fundamental Photographs, NYC)*

CONCEPTUAL PROBLEM 3

Can electric field lines ever cross? Why or why not? Can equipotential surfaces ever cross? Why or why not?

CONCEPTUAL PROBLEM 4

Suppose you know the value of the electric potential at one point. Can you find the electric field at that point from this information?

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CONCEPTUAL PROBLEM 5

If the electric potential is constant in some region, what can you conclude about the electric field in that region? If the electric field is zero in some region, what can you say about the electric potential in that region?

Example 20.4 The Electric Potential of a Dipole

An electric dipole consists of two equal and opposite charges separated by a distance $2a$, as in Figure 20.9. The dipole is along the x axis and is centered at the origin. Calculate the electric potential and the electric field at P.

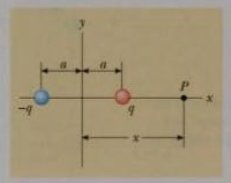


Figure 20.9 (Example 20.4) An electric dipole located on the x axis.

Solution

$$V = k_e \sum \frac{q_i}{r_i} = k_e\left(\frac{q}{x-a} - \frac{q}{x+a}\right) = \frac{2k_e qa}{x^2 - a^2}$$

If P is far from the dipole, so that $x \gg a$, then a^2 can be neglected in the term $x^2 - a^2$ and V becomes

$$V \approx \frac{2k_e qa}{x^2} \qquad (x \gg a)$$

Using Equation 20.15 and this result, we calculate the electric field at P:

$$E = -\frac{dV}{dx} = \frac{4k_e qa}{x^3} \qquad \text{for } x \gg a$$

20.5 · ELECTRIC POTENTIAL DUE TO CONTINUOUS CHARGE DISTRIBUTIONS

The electric potential due to a continuous charge distribution can be calculated in two ways. If the charge distribution is known, we can start with Equation 20.11 for the potential of a point charge. We then consider the potential due to a small charge element dq, treating this element as a point charge (Fig. 20.10). The potential dV at some point P due to the charge element dq is

$$dV = k_e \frac{dq}{r} \qquad [20.17]$$

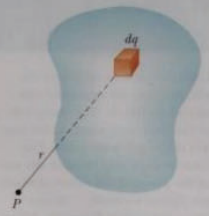


Figure 20.10 The electric potential at the point P due to a continuous charge distribution can be calculated by dividing the charged body into segments of charge dq and summing the potential contributions over all segments.

where r is the distance from the charge element to P. To get the total potential at P, we integrate Equation 20.17 to include contributions from all elements of the charge distribution. Because each element is, in general, at a different distance from P and because k_e is a constant, we can express V as

$$V = k_e \int \frac{dq}{r} \qquad [20.18]$$

In effect, we have replaced the sum in Equation 20.12 with an integral. Note that this expression for V uses a particular reference: The potential is taken to be zero when P is infinitely far from the charge distribution.

The second method for calculating the potential of a continuous charge distribution makes use of Equation 20.3. This procedure is useful when the electric field is already known from other considerations, such as Gauss's law. If the charge distribution is highly symmetric, we first evaluate $\mathbf{E}$ at any point using Gauss's law and then substitute the obtained value into Equation 20.3 to determine the potential difference between any two points. We then choose V to be zero at some convenient point. Let us illustrate both methods with two examples.

Example 20.5 Potential Due to a Uniformly Charged Ring

Find the electric potential at a point P located on the axis of a uniformly charged ring of radius a and total charge Q. The plane of the ring is chosen perpendicular to the x axis (Fig. 20.11).

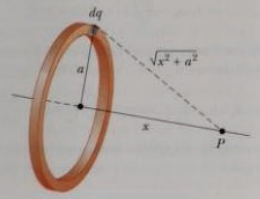


Figure 20.11 (Example 20.5) A uniformly charged ring of radius a, the plane of which is perpendicular to the x axis. All segments of the ring are at the same distance from any axial point P.

Reasoning and Solution Let us take P to be at a distance x from the center of the ring, as in Figure 20.11. The charge element dq is at a distance equal to $\sqrt{x^2 + a^2}$ from the point P. Hence, we can express V as

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{x^2 + a^2}}$$

In this case, each element dq is at the same distance from P. Therefore, the term $\sqrt{x^2 + a^2}$ can be removed from the integral, and V reduces to

$$V = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq = \frac{k_e Q}{\sqrt{x^2 + a^2}}$$

The only variable in this expression for V is x. This is not surprising, because our calculation is valid only for points along the x axis, where y and z are both zero. From the symmetry, we see that along the x axis $\mathbf{E}$ can have only an x component. Therefore, we can use Equation 20.15 to find the electric field at P:

$$\begin{aligned} E_x &= -\frac{dV}{dx} = -k_e Q \frac{d}{dx}(x^2 + a^2)^{-1/2} \\ &= -k_e Q\left(-\frac{1}{2}\right)(x^2 + a^2)^{-3/2}(2x) \\ &= \frac{k_e Q x}{(x^2 + a^2)^{3/2}} \end{aligned}$$

This result agrees with that obtained by direct integration (see Example 19.5). Note that $E_x = 0$ at $x = 0$ (the center of the ring). Could you have guessed this from Coulomb's law?

EXERCISE 5 What is the electric potential at the center of the uniformly charged ring? What does the field at the center imply about this result? **Answer** $V = k_e Q/a$ at $x = 0$. Because $E = 0$, V must have a maximum or minimum value; it is in fact a maximum.

Example 20.6 Potential of a Uniformly Charged Sphere

An insulating solid sphere of radius R has a uniform positive charge density with a total charge of Q (Fig. 20.12a). (a) Find the electric potential at a point outside the sphere—that is, for $r > R$. Take the potential to be zero at $r = \infty$.

Solution In Example 19.8 we found from Gauss's law that the magnitude of the electric field outside a uniformly charged sphere is

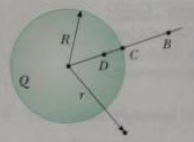
$$E_r = k_e \frac{Q}{r^2} \qquad (\text{for } r > R)$$

where the field is directed radially outward when Q is positive. To obtain the potential at an exterior point, such as B in Figure 20.12a, we substitute this expression for E into Equation 20.4. Because $\mathbf{E} \cdot d\mathbf{s} = E_r\, dr$ in this case, we get

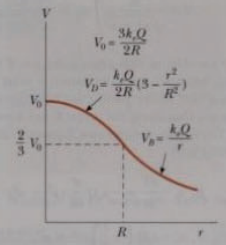
$$V_B = -\int_\infty^r E_r\, dr = -k_e Q \int_\infty^r \frac{dr}{r^2}$$

$$V_B = k_e \frac{Q}{r} \qquad (\text{for } r > R)$$

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(a)



(b)

Figure 20.12 (Example 20.6) (a) A uniformly charged insulating sphere of radius R and total charge Q. The electric potential at points B and C is equivalent to that of a point charge Q located at the center of the sphere. (b) A plot of the electric potential V versus the distance r from the center of a uniformly charged, insulating sphere of radius R. The curve for V_D inside the sphere is parabolic and joins smoothly with the curve for V_B outside the sphere, which is a hyperbola. The potential has a maximum value V_0 at the center of the sphere.

Note that the result is identical to that for the electric potential due to a point charge. Because the potential must be continuous at $r = R$, we can use this expression to obtain the potential at the surface of the sphere. That is, the potential at a point such as C in Figure 20.12a is

$$V_C = k_e \frac{Q}{R} \qquad (\text{for } r = R)$$

(b) Find the potential at a point inside the charged sphere—that is, for $r < R$.

Solution In Example 19.8 we found that the electric field inside a uniformly charged sphere is

$$E_r = \frac{k_eQ}{R^3} r \qquad (\text{for } r < R)$$

We can use this result and Equation 20.3 to evaluate the potential difference $V_D - V_C$, where D is an interior point:

$$V_D - V_C = -\int_R^r E_r\, dr = -\frac{k_eQ}{R^3}\int_R^r r\, dr = \frac{k_eQ}{2R^3}(R^2 - r^2)$$

Substituting $V_C = k_eQ/R$ into this expression and solving for V_D, we get

$$V_D = \frac{k_eQ}{2R}\left(3 - \frac{r^2}{R^2}\right) \qquad (\text{for } r < R)$$

At $r = R$, this expression gives a result for the potential that agrees with that for the potential at the surface—that is, V_C. A plot of V versus r for this charge distribution is given in Figure 20.12b.

EXERCISE 6 What are the electric field and electric potential at the center of a uniformly charged sphere?
Answer: $E = 0$ and $V_0 = 3k_eQ/2R$

PROBLEM-SOLVING STRATEGY AND HINTS

1. When working problems involving electric potential, remember that it is a *scalar quantity*, and so there are no components to worry about. Therefore, when using the superposition principle to evaluate the electric potential at a point due to a system of point charges, simply take the algebraic sum of the potentials due to each charge. You must keep track of signs, however. The potential for each positive charge ($V = k_eq/r$) is positive, whereas the potential for each negative charge is negative.
2. Just as with potential energy in mechanics, only *changes* in electric potential are significant; hence, the point at which the potential is set at zero is arbitrary. When dealing with point charges or a finite-sized charge distribution, we usually define $V = 0$ to be at a point infinitely far from the charges. However, if the charge distribution itself extends to infinity, some other nearby point must be selected as the reference point.
3. The electric potential at some point P due to a continuous distribution of charge can be evaluated by dividing the charge distribution into infinitesimal elements of charge dq located at a distance of r from the point P. This element is then treated as a point charge, and so the potential at P due to the element is $dV = k_e\, dq/r$. The total potential at P is obtained by integrating dV over the entire charge distribution. For most problems, it is necessary in performing the integration to express dq and r in terms of a single variable. In order to simplify the integration, it is important to give careful consideration to the geometry involved in the problem. Review Example 20.5 as a guide for using this method.
4. Another method that can be used to obtain the potential due to a finite continuous charge distribution is to start with the definition of the potential difference given by Equation 20.3. If $\mathbf{E}$ is known or can be obtained easily (say, from Gauss's law), then the line integral of $\mathbf{E} \cdot d\mathbf{s}$ can be evaluated. Example 20.6 uses this method.
5. Once you know the electric potential at a point, it is possible to obtain the electric field at that point by remembering that **the electric field is equal to the negative of the derivative of the potential with respect to some coordinate.** Example 20.5 illustrates how to use this procedure.

20.6 · ELECTRIC POTENTIAL OF A CHARGED CONDUCTOR

In Chapter 19 we found that when a solid conductor in electrostatic equilibrium carries a net charge, the charge resides on the outer surface of the conductor. Furthermore, we showed that the electric field just outside the surface of a conductor in equilibrium is perpendicular to the surface, whereas the field *inside* the conductor is zero. If the electric field had a component parallel to the surface, this would cause surface charges to move, creating a current and nonequilibrium.

We shall now show that **every point on the surface of a charged conductor in electrostatic equilibrium is at the same electric potential.** Consider two points A and B on the surface of a charged conductor, as in Figure 20.13. Along a surface path connecting these points, $\mathbf{E}$ is always perpendicular to the displacement, $d\mathbf{s}$; therefore, $\mathbf{E} \cdot d\mathbf{s} = 0$. Using this result and Equation 20.3, we conclude that the potential difference between A and B is necessarily zero. That is,

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0$$

This result applies to *any* two points on the surface. Therefore, V is constant everywhere on the surface of a charged conductor in equilibrium, and so such a surface is an equipotential surface. Furthermore, because the electric field is zero inside the conductor, we conclude that the potential is constant everywhere inside the conductor and equal to its value at the surface. It follows that no work is required

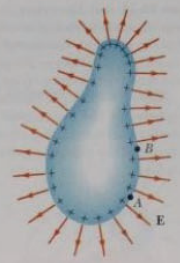


Figure 20.13 An arbitrarily shaped conductor with an excess positive charge. When the conductor is in electrostatic equilibrium, all of the charge resides at the surface, $\mathbf{E} = 0$ inside the conductor, and the electric field just outside the conductor is perpendicular to the surface. The potential is constant inside the conductor and is equal to the potential at the surface. The surface charge density is nonuniform.

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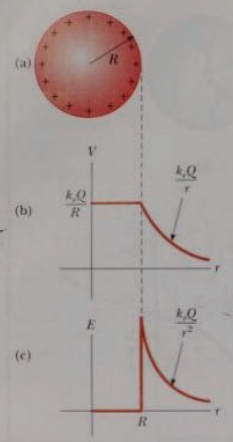


Figure 20.14 (a) The excess charge on a conducting sphere of radius R is uniformly distributed on its surface. (b) The electric potential versus the distance r from the center of the charged conducting sphere. (c) The electric field intensity versus the distance r from the center of the charged conducting sphere.

to move a test charge from the interior of a charged conductor to its surface. (Note that the potential is not zero inside the conductor, even though the electric field is zero.)

For example, consider a solid metal sphere of radius R and total positive charge Q, as shown in Figure 20.14a. The electric field outside the sphere is $k_e Q / r^2$ and points radially outward. Following Example 20.6, we see that the potential at the interior and surface of the sphere must be $k_e Q / R$ relative to infinity. The potential outside the sphere is $k_e Q / r$. Figure 20.14b is a plot of the potential as a function of r, and Figure 20.14c shows the variations of the electric field with r.

When a net charge is placed on a spherical conductor, the surface charge density is uniform, as indicated in Figure 20.14a. However, if the conductor is nonspherical, as in Figure 20.13, the surface charge density is high where the radius of curvature is small and convex and low where the radius of curvature is large and concave. Because the electric field just outside a charged conductor is proportional to the surface charge density, σ, we see that **the electric field is large near points having small convex radii of curvature and reaches very high values at sharp points.**

A Cavity Within a Conductor

Now consider a conductor of arbitrary shape containing a cavity, as in Figure 20.15. Let us assume there are no charges inside the cavity. We shall show that **the electric field inside the cavity must be zero,** regardless of the charge distribution on the outside surface of the conductor. Furthermore, the field in the cavity is zero even if an electric field exists outside the conductor.

In order to prove this point, we shall use the fact that every point on the conductor is at the same potential, and therefore any two points A and B on the surface of the cavity must be at the same potential. Now imagine that a field $\mathbf{E}$ exists in the cavity, and evaluate the potential difference $V_B - V_A$, defined by the expression

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

If $\mathbf{E}$ is nonzero, we can invariably find a path between A and B for which $\mathbf{E} \cdot d\mathbf{s}$ is always a positive number (a path along the direction of $\mathbf{E}$), and so the integral must be positive. However, because $V_B - V_A = 0$, the integral must also be zero. This contradiction can be reconciled only if $\mathbf{E} = 0$ inside the cavity. Thus, we conclude that a cavity surrounded by conducting walls is a field-free region as long as there are no charges inside the cavity.

This result has some interesting applications. For example, it is possible to shield an electronic circuit or even an entire laboratory from external fields by surrounding it with conducting walls. Shielding is often necessary during highly sensitive electrical measurements.

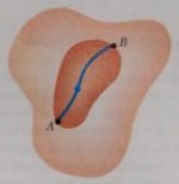
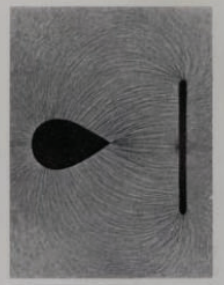


Figure 20.15 A conductor in electrostatic equilibrium containing an empty cavity. The electric field in the cavity is *zero* regardless of the charge of the conductor.

Thinking Physics 3

Why is the end of a lightning rod pointed?

Reasoning The role of a lightning rod is to serve as a point at which the lightning strikes, so that the charge delivered by the lightning will pass safely to the ground. A lightning strike begins when a channel of ionized air, called a *stepped leader,* starts to drop from the cloud. As it nears the ground, an upward-moving channel of ionized air, called a *return stroke,* jumps up to meet the stepped leader. The desire is for the return stroke to begin on a lightning rod, rather than on the rooftop of a house. If the lightning rod is pointed, then the electric field is very strong near the point, because the radius of curvature of the conductor is very small. This large electric field will cause large accelerations of stray electrons in the air and greatly increase the likelihood that ionization of the air to begin the return stroke will occur near the tip of the lightning rod rather than elsewhere.



The electric field pattern of a charged conducting plate near an oppositely charged pointed conductor. Small pieces of thread suspended in oil align with the electric field lines. Note that the electric field is most intense near the pointed part of the conductor and at other points at which the radius of curvature is small. *(Courtesy of Harold M. Waage, Princeton University)*

CONCEPTUAL PROBLEM 6

Suppose you are sitting in a car and a 20 000-volt power line drops across the car. Should you stay in the car or get out? The power line potential is 20 000 volts compared to the potential of the ground.

20.7 · CAPACITANCE

Consider two conductors having a potential difference of ΔV between them. Let us assume that the conductors have equal and opposite charges, as in Figure 20.16. This can be accomplished by connecting two uncharged conductors to the terminals of a battery. As we learned in Example 20.1, such a combination of two conductors is called a **capacitor.** The potential difference ΔV is proportional to the magnitude of the charge Q on the capacitor. The **capacitance,** C, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between them:

$$C \equiv \frac{Q}{\Delta V} \qquad [20.19]$$

• *Definition of capacitance*

Note that by definition **capacitance is always a positive quantity.** Furthermore, because the potential difference increases as the stored charge increases (see Eq. 20.7), the ratio $Q/\Delta V$ is constant for a given capacitor. Therefore, the capacitance of a device is a measure of its ability to store charge and electrical potential energy.

From Equation 20.19, we see that capacitance has the SI units coulombs per volt, called the **farad** (F) in honor of Michael Faraday. That is,

$$[\text{Capacitance}] = 1\ \text{F} = 1\ \text{C/V}$$

The farad is a very large unit of capacitance. In practice, typical devices have capacitances ranging from microfarads to picofarads. Capacitors are often labeled "mF" for microfarads and "mmF" for micromicrofarads (picofarads).

As you will soon see, the capacitance of a device depends on the geometric arrangement of the conductors. To illustrate this point, let us calculate the capacitance of an isolated spherical conductor of radius R and charge Q. (The second conductor can be taken as a concentric hollow, conducting sphere of infinite radius.) Because the sphere's potential is simply $k_e Q/R$ (where $V = 0$ at infinity), its capacitance is

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q/R} = \frac{R}{k_e} = 4\pi\epsilon_0 R \qquad [20.20]$$

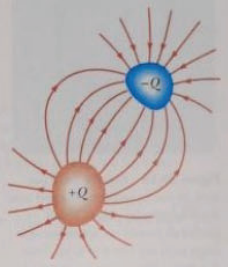


Figure 20.16 A capacitor consists of two conductors electrically isolated from each other and their surroundings. Once the capacitor is charged, the two conductors carry equal but opposite charges.

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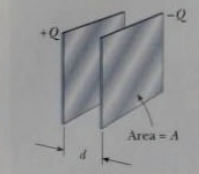


Figure 20.17 A parallel-plate capacitor consists of two parallel plates each of area A, separated by a distance d. When the capacitor is charged, the plates carry equal charges of opposite sign.

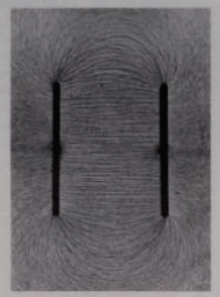


Figure 20.18 Electric field pattern of two oppositely charged conducting parallel plates. Small pieces of thread on an oil surface align with the electric field. Note the nonuniform nature of the electric field at the ends of the plates. Such end effects can be neglected if the plate separation is small compared to the length of the plates. *(Courtesy of Harold M. Waage, Princeton University)*

(Remember from Chapter 19, Section 19.4, that the Coulomb constant $k_e = 1/4\pi\epsilon_0$.) Equation 20.20 shows that the capacitance of an isolated charged sphere is proportional to the sphere's radius and is independent of both the charge and the potential difference.

Calculation of Capacitance

The capacitance of a pair of oppositely charged conductors can be calculated in the following manner. A convenient charge of magnitude Q is assumed, and the potential difference is calculated using the techniques described in Section 20.5. One then simply uses $C = Q/\Delta V$ to evaluate the capacitance. As you might expect, the calculation is relatively easy if the geometry of the capacitor is simple.

Let us illustrate this with two geometries with which we are all familiar: parallel plates and concentric cylinders. In these examples, we shall assume that the charged conductors are separated by a vacuum. (The effect of a dielectric material between the conductors will be treated in Section 20.10.)

The Parallel-Plate Capacitor

Two parallel plates of equal area A are separated by a distance of d, as in Figure 20.17. One plate has charge Q; the other, charge $-Q$. The charge per unit area on either plate is $\sigma = Q/A$. If the plates are very close together (compared with their length and width), we assume that the electric field is uniform between the plates and zero elsewhere. According to Example 19.10, the electric field between the plates is

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

The potential difference between the plates equals Ed; therefore,

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

Substituting this result into Equation 20.19, we find that the capacitance is

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{d} \qquad [20.21]$$

That is **the capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation.**

As you can see from the definition of capacitance, $C = Q/\Delta V$, the amount of charge a given capacitor can store for a given potential difference across its plates increases as the capacitance increases. Therefore, it seems reasonable that a capacitor constructed from plates having large areas should be able to store a large charge. The amount of charge needed to produce a given potential difference increases with decreasing plate separation.

A careful inspection of the electric field lines for a parallel-plate capacitor reveals that the field is uniform in the central region between the plates. However, the field is nonuniform at the edges of the plates. Figure 20.18 is a photograph of the electric field pattern of a parallel-plate capacitor showing the nonuniform field lines at the plates' edges.

PROBLEM-SOLVING STRATEGY AND HINTS

In practice, a variety of phrases are used to describe the potential difference between two points, the most common being *voltage*. A voltage *applied* to a device or *across* a device has the same meaning as the potential difference across the device. For example, if we say that the voltage across a certain capacitor is 12 volts, we mean that the potential difference between the capacitor's plates is 12 volts.

CONCEPTUAL PROBLEM 7

Why is it dangerous to touch the terminals of a high-voltage capacitor even after the voltage source that charged the battery is disconnected from the capacitor? What can be done to make the capacitor safe to handle after the voltage source has been removed?

Example 20.7 Parallel-Plate Capacitor

A parallel-plate capacitor has an area $A = 2.00 \times 10^{-4}\ \text{m}^2$ and a plate separation $d = 1.00$ mm. Find its capacitance.

Solution From Equation 20.21, we find

$$C = \epsilon_0 \frac{A}{d} = \left(8.85 \times 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(\frac{2.00 \times 10^{-4}\ \text{m}^2}{1.00 \times 10^{-3}\ \text{m}}\right)$$

$$= 1.77 \times 10^{-12}\ \text{F} = 1.77\ \text{pF}$$

EXERCISE 7 If the plate separation is increased to 3.00 mm, find the capacitance. **Answer** 0.590 pF

Example 20.8 The Cylindrical Capacitor

A cylindrical conductor of radius a and charge Q is coaxial with a larger cylindrical shell of radius b and charge $-Q$ (Fig. 20.19a). Find the capacitance of this cylindrical capacitor if its length is ℓ.

Reasoning and Solution If we assume that ℓ is long compared with a and b, we can neglect end effects. In this case, the field is perpendicular to the axis of the cylinders and is confined to the region between them (Fig. 20.19b). We must first calculate the potential difference between the two cylinders, which is given in general by

$$V_b - V_a = -\int_a^b \mathbf{E}\cdot d\mathbf{s}$$

where $\mathbf{E}$ is the electric field in the region $a < r < b$. In Chapter 19, using Gauss's law, we showed that the electric field of a cylinder of charge per unit length λ is $E = 2k_e\lambda/r$. The same result applies here, because the outer cylinder does not contribute to the electric field inside it. Using this result and noting that $\mathbf{E}$ is along r in Figure 20.19b, we find that

$$V_b - V_a = -\int_a^b E_r\, dr = -2k_e\lambda \int_a^b \frac{dr}{r} = -2k_e\lambda \ln\left(\frac{b}{a}\right)$$

Substituting this into Equation 20.19 and using the fact that $\lambda = Q/\ell$, we get

$$C = \frac{Q}{|\Delta V|} = \frac{Q}{\frac{2k_eQ}{\ell}\ln\left(\frac{b}{a}\right)} = \frac{\ell}{2k_e \ln\left(\frac{b}{a}\right)} \qquad [20.22]$$

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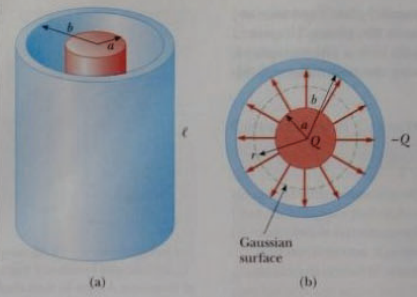


Figure 20.19 (Example 20.8) (a) A cylindrical capacitor consists of a cylindrical conductor of radius a and length ℓ surrounded by a coaxial cylindrical shell of radius b. (b) The end view of a cylindrical capacitor. The dashed line represents the end of the cylindrical gaussian surface of radius r and length ℓ.

where ΔV is the magnitude of the potential difference, given by $2k_e\lambda \ln (b/a)$, a positive quantity. That is, $\Delta V = V_a - V_b$ is positive because the inner cylinder is at the higher potential. Our result for C makes sense because it shows that the capacitance is proportional to the length of the cylinders. As you might expect, the capacitance also depends on the radii of the two cylindrical conductors. As an example, a coaxial cable consists of two concentric cylindrical conductors of radii a and b separated by an insulator. The cable carries currents in opposite directions in the inner and outer conductors. Such a geometry is especially useful for shielding an electrical signal from external influences. From Equation 20.22, we see that the capacitance per unit length of a coaxial cable is

$$\frac{C}{\ell} = \frac{1}{2k_e \ln\left(\frac{b}{a}\right)}$$

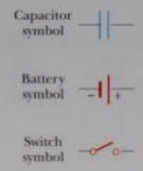


Figure 20.20 Circuit symbols for capacitors, batteries, and switches. Note that capacitors are in blue, and batteries and switches are in red.

20.8 • COMBINATIONS OF CAPACITORS

Two or more capacitors are often combined in circuits in several ways. The equivalent capacitance of certain combinations can be calculated using methods described in this section. Figure 20.20 shows the circuit symbols for capacitors and batteries, together with their color codes. The positive terminal of the battery is at the higher potential and is represented by the longer vertical line in the battery symbol.

Parallel Combination

Two capacitors connected as shown in Figure 20.21a are known as a **parallel combination** of capacitors. The left plates of the capacitors are connected by a conducting wire to the positive terminal of the battery, and therefore both are at the same potential. Likewise, the right plates are connected to the negative terminal of the battery. When the capacitors are first connected in the circuit, electrons are transferred through the battery from the left plates to the right plates, leaving the left plates positively charged and the right plates negatively charged. The energy source for this charge transfer is the internal chemical energy stored in the battery, which is converted to electric potential energy. The flow of charge ceases when the voltage across the capacitors is equal to that of the battery. The capacitors reach their maximum charge when the flow of charge ceases. Let us call the maximum charges on the two capacitors Q_1 and Q_2. Then the *total charge*, Q, stored by the two capacitors is

$$Q = Q_1 + Q_2 \qquad [20.23]$$

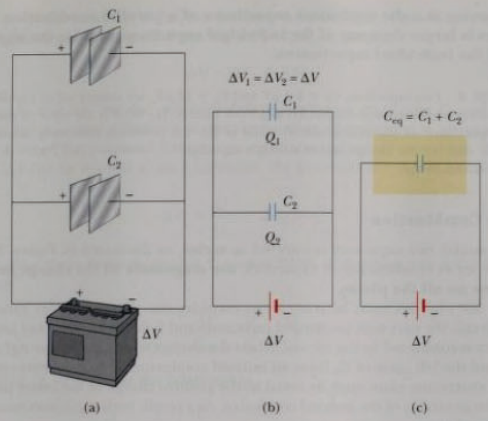


Figure 20.21 (a) A parallel combination of two capacitors. (b) The circuit diagram for the parallel combination. (c) The potential difference is the same across each capacitor, and the equivalent capacitance is $C_{eq} = C_1 + C_2$.

Suppose we wish to replace these two capacitors with one equivalent capacitor having the capacitance C_{eq}. This equivalent capacitor must have exactly the same external effect on the circuit as the original two. That is, it must store Q units of charge. We see from Figure 20.21b that **in the parallel circuit the potential differences across the two capacitors are the same,** and each is equal to the voltage of the battery, ΔV. From Figure 20.21c, we see that the voltage across the equivalent capacitor is also ΔV. Thus, we have

$$Q_1 = C_1 \Delta V \qquad Q_2 = C_2 \Delta V$$

and, for the equivalent capacitor,

$$Q = C_{eq} \Delta V$$

Substitution of these relations into Equation 20.23 gives

$$C_{eq} \Delta V = C_1 \Delta V + C_2 \Delta V$$

or

$$C_{eq} = C_1 + C_2 \quad \text{(parallel combination)} \qquad [20.24]$$

If we extend this treatment to three or more capacitors connected in parallel, the equivalent capacitance is

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \quad \text{(parallel combination)} \qquad [20.25]$$

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Thus, we see that **the equivalent capacitance of a parallel combination of capacitors is larger than any of the individual capacitances and is the algebraic sum of the individual capacitances.**

EXERCISE 8 Two capacitors, $C_1 = 5.0\ \mu F$ and $C_2 = 12\ \mu F$, are connected in parallel, and the resulting combination is connected to a 9.0-V battery. (a) What is the value of the equivalent capacitance of the combination? What is (b) the potential difference across each capacitor and (c) the charge stored on each capacitor? Answer (a)17 μF (b) 9.0 V (c) 45 μC and 108 μC

Series Combination

Now consider two capacitors connected in **series,** as illustrated in Figure 20.22a. For this series combination of capacitors, **the magnitude of the charge must be the same on all the plates.**

To see why this must be true, let us consider the charge transfer process in some detail. We start with uncharged capacitors and follow what happens just after a battery is connected to the circuit. When the connection is made, the right plate of C_1 and the left plate of C_2 form an isolated conductor. Thus, whatever negative charge enters one plate must be equal to the positive charge of the other plate, to maintain neutrality of the isolated conductor. As a result, both capacitors must have the same charge.

Suppose an equivalent capacitor performs the same function as the series combination. After it is fully charged, **the equivalent capacitor must end up with a charge of $-Q$ on its right plate and $+Q$ on its left plate.** By applying the definition of capacitance to the circuit shown in Figure 20.22b, we have

$$\Delta V = \frac{Q}{C_{eq}}$$

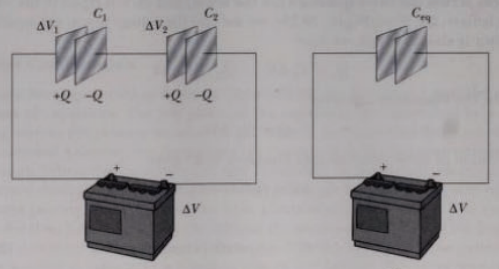


Figure 20.22 A series combination of two capacitors. The charge on each capacitor is the same, and the equivalent capacitance can be calculated from the relationship $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$.

where ΔV is the potential difference between the terminals of the battery and C_{eq} is the equivalent capacitance. From Figure 20.22a, we see that

$$\Delta V = \Delta V_1 + \Delta V_2 \qquad [20.26]$$

where ΔV_1 and ΔV_2 are the potential differences across capacitors C_1 and C_2. In general the potential difference across any number of capacitors in series is equal to the sum of the potential differences across the individual capacitors. Because $Q = C\,\Delta V$ can be applied to each capacitor, the potential difference across each is

$$\Delta V_1 = \frac{Q}{C_1} \qquad \Delta V_2 = \frac{Q}{C_2}$$

Substituting these expressions into Equation 20.26, and noting that $\Delta V = Q/C_{eq}$, we have

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

Canceling Q, we arrive at the relationship

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad \text{(series combination)} \qquad [20.27]$$

If this analysis is applied to three or more capacitors connected in series, the equivalent capacitance is found to be

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \qquad \text{(series combination)} \qquad [20.28]$$

This shows that **the equivalent capacitance of a series combination is always less than any individual capacitance in the combination and the inverse of the equivalent capacitance is the algebraic sum of the inverses of the individual capacitances.**

Example 20.9 Equivalent Capacitance

Find the equivalent capacitance between *a* and *b* for the combination of capacitors shown in Figure 20.23a. All capacitances are in microfarads.

Solution Using Equations 20.25 and 20.28, we reduce the combination step by step as indicated in the figure. The 1.0-μF and 3.0-μF capacitors are in parallel and combine according to $C_{eq} = C_1 + C_2$. Their equivalent capacitance is 4.0 μF. Likewise, the 2.0-μF and 6.0-μF capacitors are also in parallel and have an equivalent capacitance of 8.0 μF. The upper branch in Figure 20.23b now consists of two 4.0-μF capacitors in series, which combine according to

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0\ \mu F} + \frac{1}{4.0\ \mu F} = \frac{1}{2.0\ \mu F}$$

$$C_{eq} = 2.0\ \mu F$$

Likewise, the lower branch in Figure 20.23b consists of two 8.0-μF capacitors in *series,* which give an equivalent of 4.0 μF. Finally, the 2.0-μF and 4.0-μF capacitors in Figure 20.23c are in parallel and have an equivalent capacitance of 6.0 μF. Hence, the equivalent capacitance of the circuit is 6.0 μF, as shown in Figure 20.23d.

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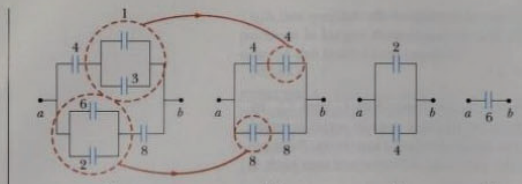


Figure 20.23 (Example 20.9) To find the equivalent combination of the capacitors in (a), the various combinations are reduced in steps as indicated in (b), (c), and (d), using the series and parallel rules described in the text.

EXERCISE 9 Consider three capacitors having capacitance of 3.0 μF, 6.0 μF, and 12 μF. Find their equivalent capacitance if they are connected (a) in parallel and (b) in series. Answer (a) 21 μF (b) 1.7 μF

20.9 • ENERGY STORED IN A CHARGED CAPACITOR

Almost everyone who works with electronic equipment has at some time verified that a capacitor can store energy. If the plates of a charged capacitor are connected by a conductor, such as wire, charge transfers from one plate to the other until the two are uncharged. The discharge can often be observed as a visible spark. If you accidentally touched the opposite plates of a charged capacitor, your fingers would act as pathways by which the capacitor could discharge, resulting in an electric shock. The degree of shock would depend on the capacitance and voltage applied to the capacitor. Where high voltages are present, such as in the power supply of a television set, the shock could be fatal.

Consider a parallel-plate capacitor that is initially uncharged, so that the initial potential difference across the plates is zero. Now imagine that the capacitor is connected to a battery and develops a maximum charge of Q. We shall assume that the capacitor is charged *slowly* so that the problem can be treated as an electrostatic system. The final potential difference across the capacitor is $\Delta V = Q/C$. Because the initial potential difference is zero, the *average* potential difference during the charging process is $\Delta V/2 = Q/2C$. From this we might conclude that the work needed to charge the capacitor is $W = Q\Delta V/2 = Q^2/2C$. Although this result is correct, a more detailed proof is desirable.

Suppose that q is the charge on the capacitor at some instant during the charging process. At the same instant, the potential difference across the capacitor is $\Delta V = q/C$. The work that is necessary[2] to transfer an increment of charge dq from the plate of charge $-q$ to the plate of charge q (which is at the higher potential) is

$$dW = \Delta V\, dq = \frac{q}{C}\, dq$$

Thus, the total work required to charge the capacitor from $q = 0$ to some final charge $q = Q$ is

[2]One mechanical analog of this process is the work required to stretch or compress a spring through some small distance.

$$W = \int_0^Q \frac{q}{C}\, dq = \frac{Q^2}{2C}$$

But the work done in charging the capacitor can be considered as potential energy U stored in the capacitor. Using $Q = C\ \Delta V$, we can express the electrostatic energy stored in a charged capacitor in the following alternative forms:

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2 \qquad [20.29]$$

• *Energy stored in a charged capacitor*

This result applies to *any* capacitor, regardless of its geometry. We see that the stored energy increases as C increases and as the potential difference increases. In practice, there is a limit to the maximum energy (or charge) that can be stored. This is because electrical discharge ultimately occurs between the plates of the capacitor at a sufficiently large value of ΔV. For this reason, capacitors are usually labeled with a maximum operating voltage.

The energy stored in a capacitor can be thought of as stored in the electric field created between the plates as the capacitor is charged. This is reasonable in view of the fact that the magnitude of the electric field is proportional to the charge on the capacitor. For a parallel-plate capacitor, the potential difference is related to the electric field through the relationship $\Delta V = Ed$. Furthermore, the capacitance is $C = \epsilon_0 A/d$. Substituting these expressions into Equation 20.29 gives

$$U = \tfrac{1}{2}\frac{\epsilon_0 A}{d}(E^2 d^2) = \tfrac{1}{2}(\epsilon_0 Ad)E^2 \qquad [20.30]$$

• *Energy stored in a parallel-plate capacitor*

Because the volume of a parallel-plate capacitor that is occupied by the electric field is Ad, the energy per unit volume $u = U/Ad$, called the **energy density,** is

$$u = \tfrac{1}{2}\epsilon_0 E^2 \qquad [20.31]$$

• *Energy density in an electric field*

Although Equation 20.31 was derived for a parallel-plate capacitor, the expression is generally valid. That is, **the energy density in any electrostatic field is proportional to the square of the electric field intensity at a given point.**

Thinking Physics 4

You have three capacitors and two batteries. How should you connect the capacitors and batteries so that the capacitors will store the maximum possible energy?

Reasoning The energy stored in a capacitor is proportional to the capacitance and the square of the potential difference. Thus, we would like to maximize each of these quantities. We can do this by connecting the three capacitors in *parallel* (so that the capacitances add) across the two batteries in *series* (so that the potential differences add).

Thinking Physics 5

You charge a capacitor and then remove it from the battery. The capacitor consists of large movable plates with air between them. You pull the plates a bit farther apart. What happens to the charge on the capacitor? To the potential difference? To the

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energy? To the capacitance? To the electric field between the plates? Was work done in pulling the plates apart?

Reasoning Because the capacitor is removed from the battery, charges on the plates have nowhere to go. Thus, the charge on the capacitor remains the same as the plates are pulled apart. Because the electric field of large plates is independent of distance, the electric field remains constant. Because the electric field is a measure of the rate of change of potential with distance, the potential difference between the plates must increase as the separation distance increases. Because the same charge is stored at a higher potential difference, the capacitance has decreased. Because energy stored is proportional to both charge and potential difference, the energy stored in the capacitor must increase. This energy must have been transferred from somewhere, so work was done. This is consistent with the fact that the plates attract one another, so work must be done to pull them apart.

Example 20.10 Rewiring Two Charged Capacitors

Two capacitors C_1 and C_2 (where $C_1 > C_2$) are charged to the same potential difference ΔV_0 but with opposite polarity. The charged capacitors are removed from the battery, and their plates are connected as shown in Figure 20.24a. The switches S_1 and S_2 are then closed, as in Figure 20.24b. (a) Find the final potential difference between a and b after the switches are closed.

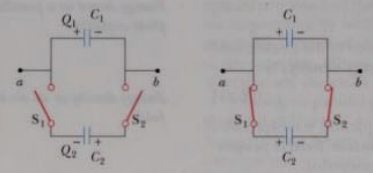


Figure 20.24 (Example 20.10)

Solution The charges on the left-hand plates of the capacitors before the switches are closed are

$$Q_1 = C_1 \Delta V_0 \quad \text{and} \quad Q_2 = -C_2 \Delta V_0$$

The negative sign for Q_2 is necessary, because this capacitor's polarity is opposite that of capacitor C_1. After the switches are closed, the charges on the plates redistribute until the total charge Q shared by the capacitors is

$$Q = Q_1 + Q_2 = (C_1 - C_2)\Delta V_0$$

The two capacitors are now in parallel, and so the final potential differences across each is the same:

$$\Delta V = \frac{Q}{C_1 + C_2} = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)\Delta V_0$$

(b) Find the total energy stored in the capacitors before and after the switches are closed.

Solution Before the switches are closed, the total energy stored in the capacitors is

$$U_i = \tfrac{1}{2}C_1(\Delta V_0)^2 + \tfrac{1}{2}C_2(\Delta V_0)^2 = \tfrac{1}{2}(C_1 + C_2)(\Delta V_0)^2$$

After the switches are closed and the capacitors have reached an equilibrium charge, the total energy stored in them is

$$U_f = \tfrac{1}{2}C_1(\Delta V)^2 + \tfrac{1}{2}C_2(\Delta V)^2 = \tfrac{1}{2}(C_1 + C_2)(\Delta V)^2$$
$$= \tfrac{1}{2}(C_1 + C_2)\left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2(\Delta V_0)^2 = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2 U_i$$

Therefore, the ratio of the final to the initial energy stored is

$$\frac{U_f}{U_i} = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2$$

This shows that the final energy is less than the initial energy. At first, you might think that energy conservation has been violated, but that is not the case. Part of the missing energy appears as thermal energy in the connecting wires, and part is radiated away in the form of electromagnetic waves (see Chapter 24).

EXERCISE 10 A 3.0-μF capacitor is connected to a 12-V battery. How much energy is stored in the capacitor? Answer 2.2×10^{-4} J

EXERCISE 11 A parallel-plate capacitor is charged and then disconnected from a battery. By what fraction does the stored energy change (increase or decrease) when the plate separation is doubled? Answer The stored energy doubles.

20.10 • CAPACITORS WITH DIELECTRICS

A **dielectric** is a nonconducting material such as rubber, glass, or waxed paper. When a dielectric material is inserted between the plates of a capacitor, the capacitance increases. If the dielectric completely fills the space between the plates, the capacitance increases by the dimensionless factor κ, called the **dielectric constant.**

The following experiment can be performed to illustrate the effect of a dielectric in a capacitor. Consider a parallel-plate capacitor of charge Q_0 and capacitance C_0 in the absence of a dielectric. The potential difference across the capacitor as measured by a voltmeter is $\Delta V_0 = Q_0/C_0$ (Fig. 20.25a). Notice that the capacitor circuit is *open;* that is, the plates of the capacitor are *not* connected to a battery and charge cannot flow through an ideal voltmeter. Hence, there is *no* path by which charge can flow and alter the charge on the capacitor. If a dielectric is now inserted between the plates as in Figure 20.25b, it is found that the voltmeter reading *decreases* by a factor of κ to the value ΔV, where

$$\Delta V = \frac{\Delta V_0}{\kappa}$$

Because $\Delta V < \Delta V_0$, we see that $\kappa > 1$.

Because the charge Q_0 on the capacitor *does not change,* we conclude that the capacitance must change to the value

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa\frac{Q_0}{\Delta V_0}$$

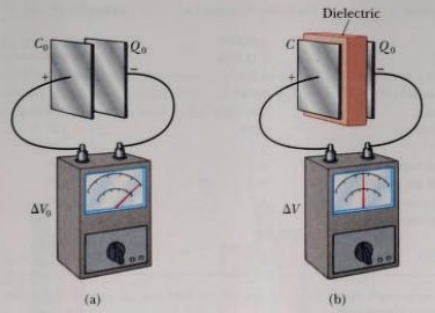


Figure 20.25 When a dielectric is inserted between the plates of a charged capacitor, the charge on the plates remains unchanged, but the potential difference as recorded by an electrostatic voltmeter is reduced from ΔV_0 to $\Delta V = \Delta V_0/\kappa$. Thus, the capacitance *increases* in the process by the factor κ.

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The capacitance of a filled capacitor is greater than that of an empty one by the factor κ.

$$C = \kappa C_0 \qquad [20.32]$$

where C_0 is the capacitance in the absence of the dielectric. That is, the capacitance *increases* by the factor κ when the dielectric completely fills the region between the plates.[3] For a parallel-plate capacitor, where $C_0 = \epsilon_0 A/d$, we can express the capacitance when the capacitor is filled with a dielectric as

$$C = \kappa \frac{\epsilon_0 A}{d} \qquad [20.33]$$

From this result, it would appear that the capacitance could be made very large by decreasing d, the distance between the plates. In practice, the lowest value of d is limited by the electrical discharge that could occur through the dielectric medium separating the plates. For any given separation d, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the **dielectric strength** (maximum electric field intensity) of the dielectric, which for air is equal to 3×10^6 V/m. If the field strength in the medium exceeds the dielectric strength, the insulating properties break down and the medium begins to conduct. Most insulating materials have dielectric strengths and dielectric constants greater than those of air, as Table 20.1 indicates. Thus, we see that a dielectric provides the following advantages:

- It increases the capacitance of a capacitor.
- It increases the maximum operating voltage of a capacitor.
- It may provide mechanical support between the conducting plates.

TABLE 20.1 Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (V/m)
Vacuum	1.00000	—
Air (dry)	1.00059	3×10^6
Bakelite	4.9	24×10^6
Fused quartz	3.78	8×10^6
Pyrex glass	5.6	14×10^6
Polystyrene	2.56	24×10^6
Teflon	2.1	60×10^6
Neoprene rubber	6.7	12×10^6
Nylon	3.4	14×10^6
Paper	3.7	16×10^6
Strontium titanate	233	8×10^6
Water	80	—
Silicone oil	2.5	15×10^6

[a] The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown.

[3] If another experiment is performed in which the dielectric is introduced while the potential difference is held constant by means of a battery, the charge increases to the value $Q = \kappa Q_0$. The additional charge is supplied by the battery, and the capacitance still increases by the factor κ.

Types of Capacitors

Commerical capacitors are often made using metal foil interlaced with a dielectric such as thin sheets of paraffin-impregnated paper. These alternate layers of metal foil and dielectric are then rolled into the shape of a cylinder to form a small package. High-voltage capacitors commonly consist of interwoven metal plates immersed in silicone oil. Small capacitors are often constructed from ceramic materials. Variable capacitors (typically 10 pF to 500 pF) usually consist of two interwoven sets of metal plates, one fixed and the other movable, with air as the dielectric.

An *electrolytic capacitor* is often used to store large amounts of charge at relatively low voltages. This device consists of a metal foil in contact with an electrolyte—a solution that conducts electricity by virtue of the motion of ions contained in the solution. When a voltage is applied between the foil and the electrolyte, a thin layer of metal oxide (an insulator) is formed on the foil, and this layer serves as the dielectric. Very large capacitance values can be attained because the dielectric layer is very thin.

When electrolytic capacitors are used in circuits, the polarity must be installed properly. If the polarity of the applied voltage is opposite what is intended, the oxide layer will be removed and the capacitor will not be able to store charge.

Thinking Physics 6

Consider a parallel-plate capacitor with a dielectric material between the plates. Is the capacitance higher on a cold day or a hot day?

Reasoning It is the polarization of the molecules in the dielectric that increases the capacitance when the dielectric is added. As the temperature increases, there is more vibrational motion of the polarized molecules. This disturbs the orderly arrangement of the polarized molecules, and the net polarization decreases. Thus, the capacitance must decrease as the temperature increases.

CONCEPTUAL PROBLEM 8

An electric stud finder will locate the studs in a wall in a home. It consists of a parallel-plate capacitor, with the plates next to each other, as shown in Figure 20.26. How does this device detect the presence of a wooden stud in the wall?

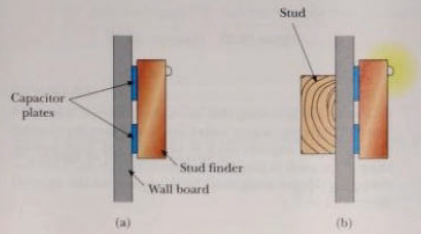


Figure 20.26 (Conceptual Problem 8) An electric stud finder positioned (a) on a piece of wall board and (b) over a wooden stud.

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Example 20.11 A Paper-Filled Capacitor

A parallel-plate capacitor has plates of dimensions 2.0 cm × 3.0 cm separated by a 1.0-mm thickness of paper. (a) Find the capacitance of this device.

Solution Because $\kappa = 3.7$ for paper (Table 20.1), we get

$$C = \kappa \frac{\epsilon_0 A}{d} = 3.7\left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(\frac{6.0 \times 10^{-4}\,\text{m}^2}{1.0 \times 10^{-3}\,\text{m}}\right)$$
$$= 20 \times 10^{-12}\,\text{F} = 20\,\text{pF}$$

(b) What is the maximum charge that can be placed on the capacitor?

Solution From Table 20.1 we see that the dielectric strength of paper is 16×10^6 V/m. Because the thickness of the paper is 1.0 mm, the maximum voltage that can be applied before breakdown is

$$\Delta V_{\max} = E_{\max} d = \left(16 \times 10^6\,\frac{\text{V}}{\text{m}}\right)(1.0 \times 10^{-3}\,\text{m})$$
$$= 16 \times 10^3\,\text{V}$$

Hence, the maximum charge is

$$Q_{\max} = C\,\Delta V_{\max} = (20 \times 10^{-12}\,\text{F})(16 \times 10^3\,\text{V})$$
$$= 0.32\,\mu\text{C}$$

EXERCISE 12 What is the maximum energy that can be stored in the capacitor? Answer 2.5×10^{-3} J

Example 20.12 Energy Stored Before and After Inserting a Dielectric

A parallel-plate capacitor is charged with a battery to a charge, Q_0, as in Figure 20.27a. The battery is then removed, and a slab of material that has a dielectric constant κ is inserted between the plates, as in Figure 20.27b. Find the energy stored in the capacitor before and after the dielectric is inserted.

Solution The energy stored in the capacitor in the absence of the dielectric is

$$U_0 = \tfrac{1}{2} C_0 (\Delta V_0)^2$$

Because $\Delta V_0 = Q_0/C_0$, this can be expressed as

$$U_0 = \frac{Q_0^2}{2C_0}$$

After the battery is removed and the dielectric is inserted between the plates, the *charge on the capacitor remains the same.* Hence, the energy stored in the presence of the dielectric is

$$U = \frac{Q_0^2}{2C}$$

But the capacitance in the presence of the dielectric is $C = \kappa C_0$, and so U becomes

$$U = \frac{Q_0^2}{2\kappa C_0} = \frac{U_0}{\kappa}$$

Because $\kappa > 1$, we see that the final energy is less than the initial energy by the factor $1/\kappa$. This missing energy can be accounted for by noting that when the dielectric is inserted into the capacitor, it gets pulled into the device. An external agent must do negative work to keep the slab from accelerating. This work is simply the difference $U - U_0$. (Alternatively, the positive work done by the system on the external agent is $U_0 - U$.)

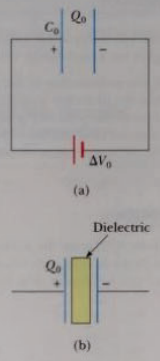


Figure 20.27 (Example 20.12)

EXERCISE 13 Suppose that the capacitance in the absence of a dielectric is 8.50 pF, and the capacitor is charged to a potential difference of 12.0 V. If the battery is disconnected and a slab of polystyrene ($\kappa = 2.56$) is inserted between the plates, calculate the energy difference $U - U_0$. Answer 373 pJ

EXERCISE 14 (a) How much charge can be placed on a capacitor with air between the plates before it breaks down if the area of each of the plates is 5.00 cm²? (b) Find the maximum charge if polystyrene is used between the plates instead of air. Answer (a) 13.3 nC (b) 272 nC

PROBLEM-SOLVING STRATEGY AND HINTS

1. Be careful with your choice of units. To calculate capacitance in farads, make sure that distances are in meters and use the SI value of ϵ_0. When checking consistency of units, remember that the units for electric fields can be either newtons per coulomb or volts per meter.
2. When two or more unequal capacitors are connected in series, they carry the same charge, but their potential differences are not the same. The capacitances add as reciprocals, and the equivalent capacitance of the combination is always less than the smallest individual capacitor.
3. When two or more capacitors are connected in parallel, the potential differences across them are the same. The charge on each capacitor is proportional to its capacitance; hence, the capacitances add directly to give the equivalent capacitance of the parallel combination.
4. A dielectric increases capacitance by the factor κ (the dielectric constant) because induced surface charges on the dielectric reduce the electric field inside the material from E_0 to E_0/κ.
5. Be careful about problems in which you may be connecting or disconnecting a battery to a capacitor. It is important to note whether modifications to the capacitor are being made while the capacitor is connected to the battery or after it is disconnected. If the capacitor remains connected to the battery, the voltage across the capacitor necessarily remains the same (equal to the battery voltage), and the charge is proportional to the capacitance, *regardless of how it may be modified* (say, by insertion of a dielectric). However, if you disconnect the capacitor from the battery before making any modifications to the capacitor, then its charge remains the same. In this case, as you vary the capacitance, the voltage across the plates changes in inverse proportion to capacitance, according to $\Delta V = Q/C$.



This photograph illustrates dielectric breakdown in air. Sparks are produced when a large alternating voltage is applied across the electrodes by way of a high-voltage induction coil power supply. *(Courtesy of Central Scientific Co.)*

SUMMARY

When a positive test charge q_0 is moved between points A and B in an electrostatic field **E**, the **change in potential energy** is

$$\Delta U = -q_0 \int_A^B \mathbf{E}\cdot d\mathbf{s} \qquad [20.1]$$

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The **potential difference,** ΔV, between points A and B in an electrostatic field $\mathbf{E}$ is defined as the change in potential energy divided by the test charge q_0.

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E}\cdot d\mathbf{s} \qquad [20.3]$$

where the electric potential V is a scalar and has the units joules per coulomb, defined as 1 volt (V).

The potential difference between two points, A and B, in a uniform electric field, $\mathbf{E}$, is

$$\Delta V = -\mathbf{E}\cdot\mathbf{d} \qquad [20.8]$$

where $\mathbf{d}$ is the displacement vector between A and B.

Equipotential surfaces are surfaces on which the electric potential remains constant. Equipotential surfaces are *perpendicular* to the electric field lines.

The electric potential due to a point charge q at any distance r from the charge is

$$V = k_e\frac{q}{r} \qquad [20.11]$$

The electric potential due to a group of point charges is obtained by summing the potentials due to the individual charges. Because V is a scalar, the sum is a simple algebraic operation.

The **electric potential energy of a pair of point charges** separated by a distance r is

$$U = k_e\frac{q_1q_2}{r} \qquad [20.13]$$

This represents the work required to bring the charges from an infinite separation to the separation r. The potential energy of a distribution of point charges is obtained by summing terms like Equation 20.13 over *all* pairs of particles.

If the electric potential is known as a function of coordinates x, y, z, the components of the electric field can be obtained by taking the negative derivative of the potential with respect to the coordinates. For example, the x component of the electric field is

$$E_x = -\frac{dV}{dx} \qquad [20.15]$$

The **electric potential due to a continuous charge distribution** is

$$V = k_e\int\frac{dq}{r} \qquad [20.18]$$

Every point on the surface of a charged conductor in electrostatic equilibrium is at the same potential. Furthermore, the potential is constant everywhere inside the conductor and equal to its value at the surface.

A *capacitor* is a charge storage device. It consists of two equal and oppositely charged conductors spaced very close together compared to their size, with a potential difference of ΔV between them. The **capacitance, C,** of any capacitor is defined to be the ratio of the magnitude of the charge Q on either conductor to the magnitude of the potential difference, ΔV:

$$C \equiv \frac{Q}{\Delta V} \qquad [20.19]$$

The SI units of capacitance are coulombs per volt, or the farad (F), and 1 F = 1 C/V.

If two or more capacitors are connected in parallel, the potential differences across them must be the same. The equivalent capacitance of a parallel combination of capacitors is

$$C_{eq} = C_1 + C_2 + C_3 + \ldots \qquad [20.25]$$

If two or more capacitors are connected in series, the charges on them are the same, and the equivalent capacitance of the series combination is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \qquad [20.28]$$

Work is required to charge a capacitor, because the charging process consists of transferring charges from one conductor at a lower potential to another conductor at a higher potential. The work done in charging the capacitor to the charge Q equals the electrostatic potential energy U stored in the capacitor, where

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2 \qquad [20.29]$$

When a dielectric material is inserted between the plates of a capacitor, the capacitance generally increases by the dimensionless factor κ, called the **dielectric constant.** That is,

$$C = \kappa C_0 \qquad [20.32]$$

where C_0 is the capacitance in the absence of the dielectric.

CONCEPTUAL QUESTIONS

1. Distinguish between electric potential and electrical potential energy.
2. Give a physical explanation of the fact that the potential energy of a pair of like charges is positive whereas the potential energy of a pair of unlike charges is negative.
3. Explain why, under static conditions, all points in a conductor must be at the same electric potential.
4. Why is it important to avoid sharp edges or points on conductors used in high-voltage equipment?
5. In what type of weather would a car battery be more likely to discharge and why?
6. How would you shield an electronic circuit or laboratory from stray electric fields? Why does this work?
7. If you are given three different capacitors, C_1, C_2, C_3, how many different combinations of capacitance can you produce?
8. The plates of a capacitor are connected to a battery. What happens to the charge on the plates if the connecting wires are removed from the battery? What happens to the charge if the wires are removed from the battery and connected to each other?
9. If you want to increase the maximum operating voltage of a parallel-plate capacitor, describe how you can do this for a fixed plate separation.
10. If the potential difference across a capacitor is doubled, by what factor does the energy stored change?
11. Because the charges on the plates of a parallel-plate capacitor are equal and opposite, they attract each other. Hence, it takes positive work to increase the plate separation. What happens to the external work done in this process? (Assume the capacitor plates are removed from the charging battery.)
12. If you were asked to design a capacitor where small size and large capacitance were required, what factors would be important in your design?
13. Explain why a dielectric increases the maximum operating voltage of a capacitor although the physical size of the capacitor does not change.
14. A pair of capacitors are connected in parallel and an identical pair are connected in series. Which pair would be more dangerous to handle after being connected to the same voltage source? Explain.
15. The energy stored in a particular capacitor is increased fourfold. What is the accompanying change in (a) the charge and (b) the potential difference across the capacitor?

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PROBLEMS

Section 20.1 Potential Difference and Electric Potential

1. A uniform electric field of magnitude 250 V/m is directed in the positive x direction. A $+12.0$-μC charge moves from the origin to the point $(x, y) = (20.0\text{ cm}, 50.0\text{ cm})$. (a) What was the change in the potential energy of this charge? (b) Through what potential difference did the charge move?
2. How much work is done (by a battery, generator, or some other source of electrical energy) in moving Avogadro's number of electrons from an initial point where the electric potential is 9.00 V to a point where the potential is -5.00 V? (The potential in each case is measured relative to a common reference point.)
3. (a) Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V. (b) Calculate the speed of an electron that is accelerated through the same potential difference.
4. Through what potential difference would an electron need to be accelerated for it to achieve a speed of 40.0% of the speed of light ($c = 3.00 \times 10^8$ m/s), starting from rest? Note that this problem must be solved using a relativistic expression for the energy.
5. An electron starts from rest and is accelerated through a potential difference of 20.0 kV. (a) Find its actual relativistic speed. (b) Find the speed that classical physics would predict for it. By how much is this in error?

Section 20.2 Potential Differences in a Uniform Electric Field

6. The difference in potential between the accelerating plates of a TV set is about 25 000 V. If the distance between these plates is 1.50 cm, find the magnitude of the uniform electric field in this region.
7. Suppose an electron is released from rest in a uniform electric field the strength of which is 5.90×10^3 V/m. (a) Through what potential difference will it have passed after moving 1.00 cm? (b) How fast will the electron be moving after it has traveled 1.00 cm?
8. An electron moving parallel to the x axis has an initial speed of 3.70×10^6 m/s at the origin. Its speed is reduced to 1.40×10^5 m/s at the point $x = 2.00$ cm. Calculate the potential difference between the origin and the point. Which point is at the higher potential?
9. A uniform electric field of magnitude 325 V/m is directed in the *negative* y direction in Figure P20.9. The coordinates of point A are $(-0.200, -0.300)$ m, and those of point B are (0.400, 0.500) m. Calculate the potential $V_B - V_A$ using the path shown in blue.

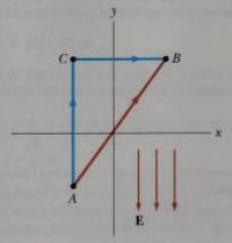


Figure P20.9

10. On planet Tehar, the acceleration of gravity is the same as that on Earth, but there is also a strong downward electric field with the field being uniform close to the planet's surface. A 2.00-kg ball having a charge of 5.00 μC is thrown upward at a speed of 20.1 m/s and it hits the ground after an interval of 4.10 s. What is the potential difference between the starting point and the top point of the trajectory?

Section 20.3 Electric Potential and Electric Potential Energy Due to Point Charges

(*Note:* Assume a reference level for potential as $V = 0$ at $r = \infty$ unless the statement of the problem requires otherwise.)

11. (a) Find the potential at a distance of 1.00 cm from a proton. (b) What is the potential difference between two points that are 1.00 cm and 2.00 cm from a proton? (c) Repeat parts (a) and (b) for an electron.
12. Given two 2.00-μC charges, as in Figure P20.12, and a positive test charge $q = 1.28 \times 10^{-18}$ C at the origin, (a) what is the net force exerted on q by the two 2.00-μC charges? (b) What is the electric field at the origin due to the two 2.00-μC charges? (c) What is the electrical potential at the origin due to the two 2.00-μC charges?

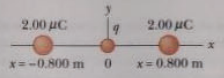


Figure P20.12

13. The three charges in Figure P20.13 are at the vertices of an isosceles triangle. Calculate the electric potential at the midpoint of the base, taking $q = 7.00$ μC.

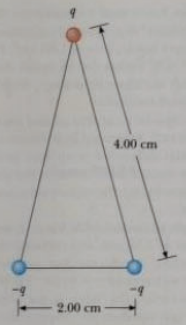


Figure P20.13

14. A charge $+q$ is at the origin. A charge $-2q$ is at $x = 2.00$ m on the x axis. For what finite value(s) of x is (a) the electric field zero? (b) the electric potential zero?
15. The Bohr model of the hydrogen atom holds that the electron can exist only in certain allowed orbits. The radius of each Bohr orbit is $r = n^2(0.0529\text{ nm})$ where $n = 1, 2, 3, \ldots$. Calculate the electric potential energy of a hydrogen atom when the electron is in the (a) first allowed orbit, $n = 1$; (b) second allowed orbit, $n = 2$; and (c) when the electron has escaped from the atom, $r = \infty$. Express your answers in electron volts.
16. Two point charges, $Q_1 = +5.00$ nC and $Q_2 = -3.00$ nC, are separated by 35.0 cm. (a) What is the potential energy of the pair? What is the significance of the algebraic sign of your answer? (b) What is the electric potential at a point midway between the charges?
17. Show that the amount of work required to assemble four identical point charges of magnitude Q at the corners of a square of side s is $5.41k_eQ^2/s$.
18. Compare this problem with Problem 42 in Chapter 19. Four identical point charges ($q = +10.0$ μC) are located on the corners of a rectangle, as shown in Figure P19.42. The dimensions of the rectangle are $L = 60.0$ cm and $W = 15.0$ cm. Calculate the electric potential energy of the charge at the lower left corner due to the other three charges.
19. Two insulating spheres having radii 0.300 cm and 0.500 cm, masses 0.100 kg and 0.700 kg, and charges -2.00 μC and 3.00 μC, are released from rest when their centers are separated by 1.00 m. (a) How fast will each be moving when they collide? (*Hint:* Consider conservation of energy and linear momentum.) (b) If the spheres were conductors, would the speeds be larger or smaller than those calculated in part (a)? Explain.

Section 20.4 Obtaining E from the Electric Potential

20. The potential in a region between $x = 0$ and $x = 6.00$ m is $V = a + bx$ where $a = 10.0$ V and $b = -7.00$ V/m. Determine (a) the potential at $x = 0$, 3.00 m, and 6.00 m, and (b) the magnitude and direction of the electric field at $x = 0$, 3.00 m and $x = 6.00$ m.
21. Over a certain region of space, the electric potential is $V = 5x - 3x^2y + 2yz^2$. Find the expressions for the x, y, and z components of the electric field over this region. What is the magnitude of the field at the point P, which has coordinates (1.00, 0, -2.00) m?
22. The electric potential inside a uniformly charged spherical insulator of radius R is

$$V = \frac{k_eQ}{2R}\left(3 - \frac{r^2}{R^2}\right)$$

Outside it is

$$V = \frac{k_eQ}{r}$$

Use $E_r = -\dfrac{dV}{dr}$ to derive the electric field both (a) inside ($r < R$) and (b) outside ($r > R$) this charge distribution.

Section 20.5 Electric Potential Due to Continuous Charge Distributions

23. Consider a ring of radius R with the total charge Q spread uniformly over its perimeter. What is the potential difference between the point at the center of the ring and a point on its axis a distance $2R$ from the center?
24. Compare this problem with Problem 17 in Chapter 19. A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle, as in Figure P19.17. If the rod has a total charge of -7.50 μC, find the electric potential at O, the center of the semicircle.
25. A rod of length L (Fig. P20.25) lies along the x axis with its left end at the origin and has a nonuniform charge density

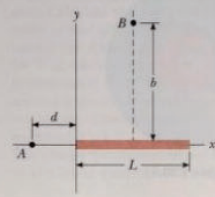


Figure P20.25

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$\lambda = \alpha x$ (where α is a positive constant). (a) What are the units of α? (b) Calculate the electric potential at A.

26. For the arrangement described in the previous problem, calculate the electric potential at point B that lies on the perpendicular bisector of the rod a distance b above the x axis.

Section 20.6 Electric Potential of a Charged Conductor

27. Two charged spherical conductors are connected by a long conducting wire, and a charge of 20.0 μC is placed on the combination. (a) If one sphere has a radius of 4.00 cm and the other has a radius of 6.00 cm, what is the electric field near the surface of each sphere? (b) What is the electrical potential of each sphere?

Section 20.7 Capacitance

28. (a) If a drop of liquid has capacitance 1.00 pF, what is its radius? (b) If another drop has radius 2.00 mm, what is its capacitance? (c) What is the charge on the smaller drop if its potential is 100 V?
29. Two conductors having net charges of +10.0 μC and −10.0 μC have a potential difference of 10.0 V. Determine (a) the capacitance of the system and (b) the potential difference between the two conductors if the charges on each are increased to +100 μC and −100 μC.
30. (a) How much charge is on each plate of a 4.00-μF capacitor when it is connected to a 12.0-V battery? (b) If this same capacitor is connected to a 1.50-V battery, what charge is stored?
31. A **spherical capacitor** consists of a spherical conducting shell of radius b and charge $-Q$ that is concentric with a smaller conducting sphere of radius a and charge $+Q$ (Fig. P20.31). (a) Show that its capacitance is

$$C = \frac{ab}{k_e(b-a)}$$

(b) Show that as b approaches infinity, the capacitance approaches the value $a/k_e = 4\pi\epsilon_0 a$.

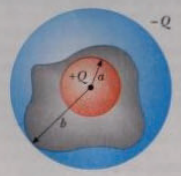


Figure P20.31

32. Regarding the Earth and a cloud layer 800 m above the Earth as the "plates" of a capacitor, calculate the capacitance if the cloud layer has an area of 1.00 km^2. If an electric field of 3.00×10^6 N/C makes the air break down and conduct electricity (that is, causes lightning), what is the maximum charge the cloud can hold?
33. An air-filled capacitor consists of two parallel plates, each with area 7.60 cm^2, separated by a distance of 1.80 mm. If a 20.0-V potential difference is applied to these plates, calculate (a) the electric field between the plates, (b) the surface charge density, (c) the capacitance, and (d) the charge on each plate.
34. A 50.0-m length of coaxial cable has an inner conductor that has a diameter of 2.58 mm and carries a charge of 8.10 μC. The surrounding conductor has an inner diameter of 7.27 mm and a charge of −8.10 μC. (a) What is the capacitance of this cable? (b) What is the potential difference between the two conductors? Assume the region between the conductors is air.

Section 20.8 Combinations of Capacitors

35. Two capacitors, $C_1 = 5.00$ μF and $C_2 = 12.0$ μF, are connected in parallel, and the resulting combination is connected to a 9.00-V battery. (a) What is the value of the equivalent capacitance of the combination? What is (b) the potential difference across each capacitor and (c) the charge stored on each capacitor?
36. The two capacitors of Problem 35 are now connected in series and to a 9.00-V battery. Find (a) the value of the equivalent capacitance of the combination, (b) the voltage across each capacitor, and (c) the charge on each capacitor.
37. Two capacitors when connected in parallel give an equivalent capacitance of 9.00 pF and an equivalent capacitance of 2.00 pF when connected in series. What is the capacitance of each capacitor?
38. Two capacitors when connected in parallel give an equivalent capacitance of C_p and an equivalent capacitance of C_s when connected in series. What is the capacitance of each capacitor?
39. Four capacitors are connected as shown in Figure P20.39. (a) Find the equivalent capacitance between points a and b. (b) Calculate the charge on each capacitor if $\Delta V_{ab} = 15.0$ V.
40. Evaluate the equivalent capacitance of the configuration shown in Figure P20.40. All the capacitors are identical and each has capacitance C.
41. Consider the circuit shown in Figure P20.41, where $C_1 = 6.00$ μF, $C_2 = 3.00$ μF, and $\Delta V = 20.0$ V. Capacitor C_1 is first charged by the closing of switch S_1. Switch S_1 is then opened, and the charged capacitor is connected to the uncharged capacitor by the closing of S_2. Calculate the initial charge acquired by C_1 and the final charge on each.

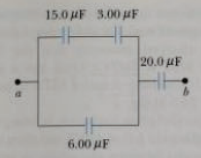


Figure P20.39

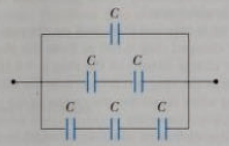


Figure P20.40

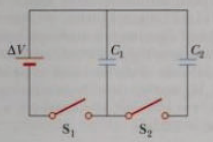


Figure P20.41

42. How should four 2.00-μF capacitors be connected to have a total capacitance of (a) 8.00 μF, (b) 2.00 μF, (c) 1.50 μF, and (d) 0.500 μF?

Section 20.9 Energy Stored in a Charged Capacitor

43. (a) A 3.00-μF capacitor is connected to a 12.0-V battery. How much energy is stored in the capacitor? (b) If the capacitor had been connected to a 6.00-V battery, how much energy would have been stored?
44. A parallel-plate capacitor is charged and then disconnected from a battery. By what fraction does the stored energy change (increase or decrease) when the plate separation is doubled?
45. Two capacitors, $C_1 = 25.0$ μF and $C_2 = 5.00$ μF, are connected in parallel and charged with a 100-V power supply. (a) Draw a circuit diagram and calculate the total energy stored in the two capacitors. (b) What potential difference would be required across the same two capacitors connected in series in order that the combination store the same energy as in (a)? Draw a circuit diagram of this circuit.
46. A uniform electric field $E = 3000$ V/m exists within a certain region. What volume of space contains an energy equal to 1.00×10^{-7} J? Express your answer in cubic meters and in liters.
47. A parallel-plate capacitor has a charge Q and plates of area A. Show that the force exerted on each plate by the other is $F = Q^2/2\epsilon_0 A$. (*Hint:* Let $C = \epsilon_0 A/x$ for an arbitrary plate separation x; then require that the work done in separating the two charged plates be $W = \int F\,dx$.)

Section 20.10 Capacitors with Dielectrics

48. Find the capacitance of a parallel-plate capacitor that uses Bakelite as a dielectric if each of the plates has an area of 5.00 cm^2 and the plate separation is 2.00 mm. (Refer to Table 20.1.)
49. A capacitor that has air between its plates is connected across a potential difference of 12.0 V and stores 48.0 μC of charge. It is then disconnected from the source while still charged. (a) Find the capacitance of the capacitor. (b) A piece of Teflon is inserted between the plates. Find the new capacitance. (c) Find the voltage and charge now on the capacitor.
50. Determine (a) the capacitance and (b) the maximum voltage that can be applied to a Teflon-filled parallel-plate capacitor having a plate area of 1.75 cm^2 and insulation thickness of 0.0400 mm.
51. The supermarket sells rolls of aluminum foil, of plastic wrap, and of waxed paper. Describe a capacitor made from supermarket materials. Compute order-of-magnitude estimates for its capacitance and its breakdown voltage.
52. (a) How much charge can be placed on a capacitor with air between the plates before it breaks down if the area of each of the plates is 5.00 cm^2? (b) Find the maximum charge if polystyrene is used between the plates instead of air.

Additional Problems

53. The liquid-drop model of the nucleus suggests that high-energy oscillations of cetain nuclei can split the nucleus into two unequal fragments plus a few neutrons. The fragments acquire kinetic energy from their mutual Coulomb repulsion. Calculate the electric potential energy (in electron volts) of two spherical fragments from a uranium nucleus having the following charges and radii: 38e and 5.50×10^{-15} m; 54e and 6.20×10^{-15} m. Assume that the charge is distributed uniformly throughout the volume of each sperical fragment and that their surfaces are initially in contact at rest. (The electrons surrounding the nucleus can be neglected.)
54. The charge distribution shown in Figure P20.54 is referred to as a linear quadrupole. (a) Show that the potential at a

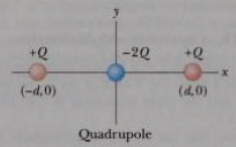
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point on the x axis where $x > d$ is

$$V = \frac{2k_e Qd^2}{x^3 - xd^2}$$

(b) Show that the expression obtained in (a) when $x \gg d$ reduces to

$$V = \frac{2k_e Qd^2}{x^3}$$



Quadrupole

Figure P20.54

55. Two parallel plates having equal but opposite charge are separated by 12.0 cm. Each plate has a surface-charge density of 36.0 nC/m². A proton is released from rest at the positive plate. Determine (a) the potential difference between the plates, (b) the energy of the proton when it reaches the negative plate, (c) the speed of the proton just before it strikes the negative plate, (d) the acceleration of the proton, and (e) the force on the proton. (f) From the force, find the electric field intensity and show that it is equal to the electric field intensity found from the charge densities on the plates.

56. For the system of capacitors shown in Figure P20.56, find (a) the equivalent capacitance of the system, (b) the potential across each capacitor, (c) the charge on each capacitor, and (d) the total energy stored by the group.

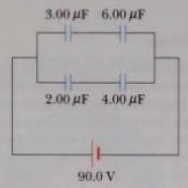


Figure P20.56

57. A certain spherical object can be raised to a maximum potential of 600 kV; then additional charge leaks off in sparks by producing dielectric breakdown of the surrounding dry air. Determine (a) the charge on the object and (b) the radius of the object.

58. The energy stored in a 52.0-μF capacitor is used to melt a 6.00-mg sample of lead. To what voltage must the capacitor be initially charged assuming that the initial temperature of the lead is 20.0°C? Lead has a specific heat of 128 J/kg·°C, a melting point of 327.3°C, and a latent heat of fusion of 24.5 kJ/kg.

59. A 2.00-nF parallel-plate capacitor is charged to an initial potential difference $\Delta V_i = 100$ V and then isolated. The dielectric material between the plates is mica ($\kappa = 5.00$). (a) How much work is required to withdraw the mica sheet? (b) What is the potential difference of the capacitor after the mica is withdrawn?

60. An electron is released from rest on the axis of a uniform positively charged ring 0.100 m from the ring's center. If the linear charge density of the ring is +0.100 μC/m and the radius of the ring is 0.200 m, how fast will the electron be moving when it reaches the center of the ring?

61. A parallel-plate capacitor is constructed using a dielectric material the dielectric constant of which is 3.00 and the dielectric strength of which is 2.00×10^8 V/m. The desired capacitance is 0.250 μF, and the capacitor must withstand a maximum potential difference of 4000 V. Find the minimum area of the capacitor plates.

62. A parallel-plate capacitor is constructed using three dielectric materials, as in Figure P20.62. (a) Find an expression for the capacitance of the device in terms of the plate area A and d, κ_1, κ_2, and κ_3. (b) Calculate the capacitance using the values $A = 1.00$ cm², $d = 2.00$ mm, $\kappa_1 = 4.90$, $\kappa_2 = 5.60$, and $\kappa_3 = 2.10$.

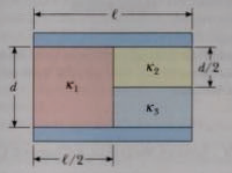


Figure P20.62

63. A conducting slab of thickness d and area A is inserted into the space between the plates of a parallel-plate capacitor with spacing s and surface area A, as in Figure P20.63. What is the capacitance of the system?

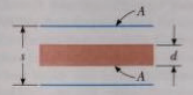


Figure P20.63

64. (a) Two spheres have radii a and b and their centers are a distance d apart. Show that the capacitance of this system is

$$C = \frac{4\pi\epsilon_0}{\frac{1}{a} + \frac{1}{b} - \frac{2}{d}}$$

provided that d is large compared with a and b. (*Hint:* Because the spheres are far apart, assume that the potential of each equals the sum of the potentials due to each sphere, and when calculating those potentials assume that $V = k_e Q/r$ applies.) (b) Show that as d approaches infinity the above result reduces to that of two spherical capacitors in series.

Spreadsheet Problems

S1. Spreadsheet 20.1 calculates the electric potential in the two-dimensional region around two charges. The region is divided into a 10 × 10 grid. The upper left corner of the grid corresponds to $x = 1.0$ m, $y = 1.0$ m, and the lower-right corner corresponds to $x = 10$ m, $y = 10$ m. The two charges can be placed anywhere in this grid. The first two tables calculate the distances from each charge to each grid point. The spreadsheet then calculates the electric potential at each grid point. Place charges of +2.0 μC at $x = 3.0$ m, $y = 5.0$ m and $x = 7.0$ m, $y = 5.0$ m. Print out the electric potential matrix. The location of the point charges will appear on the grid in the cells denoted in *ERR* (in Lotus1-2-3) or #*DIV/0* (in Excel) because the potential is infinite at the location of the point charges. Sketch by hand the equipotential lines for $V = 20$ kV, 15 kV, and 10 kV. Sketch in a representative set of electric field lines. (*Hint:* By adjusting the cell width on the spreadsheet and the printer line spacing, you can get the grid to print out approximately as a square.)

S2. Repeat Problem S1 with one charge replaced by −2.0 μC. Print out the electric potential matrix. Sketch the equipotential lines for $V = \pm 20$ kV, ± 15 kV, and ± 10 kV. Sketch in a representative set of electric field lines.

S3. Place two charges anywhere in the grid in Problem S1. Choose any values for the charges. Print out the electric potential matrix. Sketch representative equipotentials and electric field lines.

S4. Modify Spreadsheet 20.1 to include additional charges. Choose locations and values for each charge. Sketch the equipotentials and the electric field lines. Place a row of charges together. Are the field lines for this case uniform? If not, why not?

S5. Spreadsheet 11.1 calculates the gravitational potential energy of a mass moving along the x axis in the gravitational field of four fixed-point masses. Rework this spreadsheet to calculate the electrical potential along the x axis for four fixed-point charges at the same locations as the four-point masses. Find the electric field on the x axis as well. (*Hint:* $GM_i m$ must be replaced by $k_e Q_i$, and so forth.) Investigate various choices for the charges and their locations.

ANSWERS TO CONCEPTUAL PROBLEMS

1. The proton is displaced in the direction of the electric field, so the electric potential and electric potential energy will *decrease*. The decrease in potential energy is accompanied by an equal increase in kinetic energy, as required by the law of conservation of energy. An electron would be displaced opposite to the electric field so the electric potential would increase but the electric potential energy would decrease.
2. No. Suppose there are several charges in the vicinity of the point in question. If some charges are positive and some negative, their contributions to the potential at the point may cancel. For example, the electric potential at the midpoint of two equal but opposite charges is zero.
3. Field lines represent the direction of the electric force on a positive test charge. If electric field lines were to cross, then, at the point of crossing, there would be an ambiguity regarding the direction of the force on the test charge, because there would be two possible forces. Thus, electric field lines cannot cross. It is possible for equipotential surfaces to cross. For example, suppose two identical positive charges are at diagonally opposite corners of a square, and two negative charges of equal magnitude are at the other two corners. Then the planes perpendicular to the sides of the square at their midpoints are equipotential surfaces. These two planes cross each other, at the line perpendicular to the square at its center.
4. The value of the electric potential at one point is not sufficient to determine the electric field. The electric field is the negative of the gradient of the potential, so the nature in which the potential varies in space must be known.
5. If V is constant in a certain region, the electric field in that region must be zero because the electric field is equal to the negative gradient of E. (In one dimension, $E_x = -dV/dx$, so if V = constant, then $E = 0$.) Likewise, if $E = 0$ in some region, one can only conclude that V is a constant in that region.
6. The power line, if it makes electrical contact with the metal of the car, will raise the potential of the car to 20 000 volts.

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It will also raise the potential of your body to 20 000 volts, because you are in contact with the car. This is not a problem. If you step out of the car, your body at 20 000 volts will make contact with the ground, which is at ground potential of zero volts. As a result, a current will flow through your body, and you will likely be injured. Thus, it is best to stay in the car until help arrives.

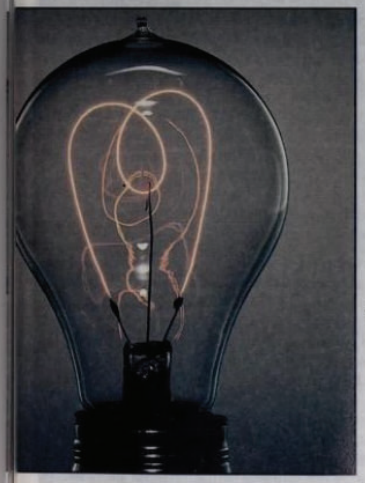
7. The capacitor often remains charged long after the voltage source is disconnected. This residual charge can be lethal. The capacitor can be safely handled after discharging the plates by short circuiting the device with a conductor, such as a screwdriver with an insulating handle.
8. The capacitance of the device is measured by the electronics in the stud finder. When a (dielectric) wooden stud is brought into the electric field region, it has the same effect as introducing a dielectric between the plates of a parallel-plate capacitor—the capacitance changes. The stud finder detects this change in capacitance and signals the presence of the stud.

21

Current and Direct Current Circuits

Thus far, our discussion of electrical phenomena has been confined to charges at rest, or electrostatics. We shall now consider situations involving electric charges in motion. The term *electric current*, or simply *current*, is used to describe the rate of flow of charge through some region of space. Most practical applications of electricity involve electric currents. For example, the battery of a flashlight supplies current to the filament of the bulb when the switch is turned on. In these common situations, the flow of charge takes place in a conductor, such as a copper wire. It is also possible for currents to exist outside a conductor. For instance, a beam of electrons in a television picture tube constitutes a current.

In this chapter we shall first define current and current density. A microscopic description of current will be given, and some of the factors that contribute to resistance to the flow of charge in conductors will be discussed. Mechanisms responsible for the electrical resistances of various



◀ **Photograph of a carbon filament incandescent lamp. The resistance of such a lamp is typically 10 Ω, but its value changes with temperature. Most modern lightbulbs use tungsten filaments, the resistance of which increases with increasing temperature.** *(Courtesy of Central Scientific Co.)*

CHAPTER OUTLINE

21.1 Electric Current
21.2 Resistance and Ohm's Law
21.3 Superconductors
21.4 A Model for Electrical Conduction
21.5 Electrical Energy and Power
21.6 Sources of emf
21.7 Resistors in Series and in Parallel
21.8 Kirchhoff's Rules and Simple DC Circuits
21.9 *RC* Circuits

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materials depend on the materials' compositions and on temperature. A classical model is used to describe electrical conduction in metals; we shall point out some of the limitations of this model.

This chapter is also concerned with the analysis of some simple circuits the elements of which include batteries, resistors, and capacitors in varied combinations. The analysis of these circuits is simplified by the use of two rules known as *Kirchhoff's rules*, which follow from the laws of conservation of energy and conservation of charge. Most of the circuits analyzed are assumed to be in *steady state*, which means that the currents are constant in magnitude and direction. We shall close with a discussion of circuits containing resistors and capacitors, in which the current varies with time.

21.1 • ELECTRIC CURRENT

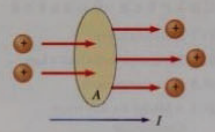


Figure 21.1 Charges in motion through an area A. The time rate of flow of charge through the area is defined as the current I. The direction of the current is the direction in which positive charge would flow if free to do so.

Electric current •

Whenever there is a net flow of charge, a **current** is said to exist. To define current more precisely, suppose the charges are moving perpendicular to a surface of area A, as in Figure 21.1. (This area could be the cross-sectional area of a wire, for example.) **The current is the rate at which charge flows through this surface.** If ΔQ is the amount of charge that passes through this area in a time interval Δt, the average current, I_{av}, is the ratio of the charge to the time interval:

$$I_{av} = \frac{\Delta Q}{\Delta t} \qquad [21.1]$$

If the rate at which charge flows varies in time, the current also varies in time. We define the **instantaneous current** I as the differential limit of the preceding expression:

$$I \equiv \frac{dQ}{dt} \qquad [21.2]$$

The SI unit of current is the **ampere** (A):

$$1\ \text{A} = 1\ \text{C/s} \qquad [21.3]$$

That is, 1 A of current is equivalent to 1 C of charge passing through a surface in 1 s.

When charges flow through a surface as in Figure 21.1, they can be positive, negative, or both. **It is conventional to give the current the same direction as the flow of positive charge.** In a common conductor such as copper, the current is due to the motion of the negatively charged electrons. Therefore, when we speak

The direction of the current •

of current in such a conductor, **the direction of the current is opposite the direction of flow of electrons.** However, if one considers a beam of positively charged protons in an accelerator, the current is in the direction of motion of the protons. In some cases—gases and electrolytes, for example—the current is the result of the flow of both positive and negative charges. It is common to refer to a moving charge (whether it is positive or negative) as a mobile **charge carrier.** For example, the charge carriers in a metal are electrons.

It is instructive to relate current to the motions of the charged particles. To illustrate this point, consider the current in a conductor of cross-sectional area A

(Fig. 21.2). The volume of an element of the conductor of length Δx is $A\,\Delta x$. If n represents the number of mobile charge carriers per unit volume, then the number of carriers in the volume element is $nA\,\Delta x$. Therefore, the charge ΔQ in this element is

$$\Delta Q = \text{number of carriers} \times \text{charge per carrier} = (nA\,\Delta x)q$$

where q is the charge on each carrier. If the carriers move with a speed of v_d, the distance they move in the time Δt is $\Delta x = v_d\,\Delta t$. Therefore, we can write ΔQ in the form

$$\Delta Q = (nAv_d\,\Delta t)q$$

If we divide both sides of this equation by Δt, we see that the current in the conductor is

$$I = \frac{\Delta Q}{\Delta t} = nqv_dA \qquad [21.4]$$

The speed of the charge carriers, v_d, is an average speed called the **drift speed.** To understand its meaning, consider a conductor in which the charge carriers are free electrons. If the conductor is isolated, these electrons undergo random motion similar to that of gas molecules. When a potential difference is applied across the conductor (say, by means of a battery), an electric field is set up in the conductor, which creates an electric force on the electrons, accelerating them, and hence producing a current. In reality, the electrons do not simply move in straight lines along the conductor. Instead, they undergo repeated collisions with the metal atoms, and the result is a complicated zigzag motion (Fig. 21.3). The energy transferred from the electrons to the metal atoms during collision causes an increase in the vibrational energy of the atoms and a corresponding increase in the temperature of the conductor. However, despite the collisions, the electrons move slowly along the conductor (in a direction opposite $\mathbf{E}$) with the drift velocity, $\mathbf{v}_d$. One can think of the collisions within a conductor as being an effective internal friction (or drag force) similar to that experienced by the molecules of a liquid flowing through a pipe stuffed with steel wool.

The **current density** J in the conductor is defined to be the current per unit area. Because $I = nqv_dA$, the current density is

$$J \equiv \frac{I}{A} = nqv_d \qquad [21.5]$$

where J has the SI units amperes per square meter. In general, the current density is a *vector quantity*. That is,

$$\mathbf{J} \equiv nq\mathbf{v}_d \qquad [21.6]$$

From this definition, we see that the current density vector is in the direction of motion of positive charge carriers and opposite the direction of motion of negative charge carriers. Because the drift velocity is proportional to the electric field $\mathbf{E}$ in the conductor, we conclude that the current density is also proportional to $\mathbf{E}$.

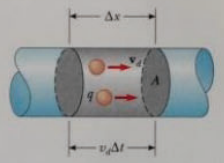


Figure 21.2 A section of a uniform conductor of cross-sectional area A. The charge carriers move with a speed v_d, and the distance they travel in a time Δt is given by $\Delta x = v_d\,\Delta t$. The number of mobile charge carriers in the section of length Δx is given by $nAv_d\,\Delta t$, where n is the number of mobile carriers per unit volume.

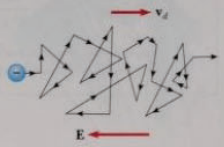


Figure 21.3 A schematic representation of the zigzag motion of a charge carrier in a conductor. The changes in direction are due to collisions with atoms in the conductor. Note that the net motion of electrons is opposite the direction of the electric field. The zigzag paths are actually parabolic segments.

• *Current density*

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Thinking Physics 1

Suppose a current-carrying wire has a cross-sectional area that gradually becomes smaller along the wire, so that the wire has the shape of a very long cone. How does the drift velocity of electrons vary along the wire?

Reasoning Every portion of the wire is carrying the same amount of current. Thus, as the cross-sectional area decreases, the drift velocity must increase to maintain the constant value of the current. This increased drift velocity is a result of the electric field lines in the wire being compressed into a smaller area, thus increasing the strength of the field.

Example 21.1 Drift Speed in a Copper Wire

A copper wire of cross-sectional area $3.00 \times 10^{-6}\ \text{m}^2$ carries a current of 10.0 A. Find the drift speed of the electrons in this wire. The density of copper is $8.95\ \text{g/cm}^3$.

Solution From the periodic table of the elements in Appendix C, we find that the atomic mass of copper is 63.5 g/mol. Recall that one atomic mass of any substance contains Avogadro's number of atoms, 6.02×10^{23} atoms. Knowing the density of copper enables us to calculate the volume occupied by 63.5 g of copper:

$$V = \frac{m}{\rho} = \frac{63.5\ \text{g}}{8.95\ \text{g/cm}^3} = 7.09\ \text{cm}^3$$

If we now assume that each copper atom contributes one free electron to the body of the material, we have

$$n = \frac{6.02 \times 10^{23}\ \text{electrons}}{7.09\ \text{cm}^3} = 8.48 \times 10^{22}\ \text{electrons/cm}^3 = \left(8.48 \times 10^{22}\ \frac{\text{electrons}}{\text{cm}^3}\right)\left(10^6\ \frac{\text{cm}^3}{\text{m}^3}\right) = 8.48 \times 10^{28}\ \text{electrons/m}^3$$

From Equation 21.4, we find that the drift speed is

$$v_d = \frac{I}{nqA} = \frac{10.0\ \text{C/s}}{(8.48 \times 10^{28}\ \text{m}^{-3})(1.60 \times 10^{-19}\ \text{C})(3.00 \times 10^{-6}\ \text{m}^2)} = 2.46 \times 10^{-4}\ \text{m/s}$$

Example 21.1 shows that typical drift speeds are very small. In fact, the drift speed is much smaller than the average speed between collisions. For instance, electrons traveling with this speed would take about 68 min to travel 1 m! In view of this low speed, you might wonder why a light turns on almost instantaneously when a switch is thrown. In a conductor, the electric field that drives the free electrons travels through the conductor with a speed close to that of light. Thus, when you flip a light switch, the message for the electrons to start moving through the wire (the electric field) reaches them at a speed on the order of $10^7\ \text{m/s}$.

EXERCISE 1 If a current of 80.0 mA exists in a metal wire, how many electrons flow past a given cross section of the wire in 10.0 min? Answer 3.0×10^{20} electrons

21.2 • RESISTANCE AND OHM'S LAW

When a voltage (potential difference) ΔV is applied across the ends of a metallic conductor, as in Figure 21.4, the current in the conductor is found to be proportional to the applied voltage; that is, $I \propto \Delta V$. If the proportionality is exact, we can write $\Delta V = IR$, where the proportionality constant R is called the resistance of the

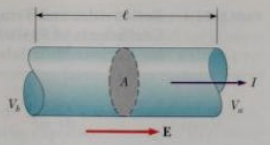


Figure 21.4 A uniform conductor of length ℓ and cross-sectional area A. A potential difference $V_b - V_a$ maintained across the conductor sets up an electric field **E** in the conductor, and this field produces a current I that is proportional to the potential difference.

conductor. In fact, we define this **resistance** as the ratio of the voltage across the conductor to the current it carries:

$$R = \frac{\Delta V}{I} \qquad [21.7]$$

• *Resistance*

Resistance has the SI units volts per ampere, called **ohms (Ω).** Thus, if a potential difference of 1 V across a conductor produces a current of 1 A, the resistance of the conductor is 1 Ω. For example, if an electrical appliance connected to a 120-V source carries a current of 6 A, its resistance is 20 Ω.

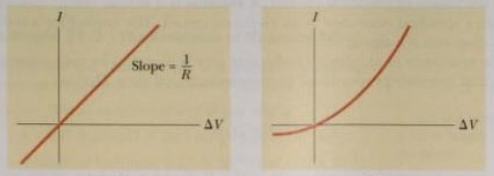
It is useful to compare the concepts of electric current, voltage, and resistance with the flow of water in a river. As water flows downhill in a river of constant width and depth, the flow rate (water current) depends on the angle of flow and the effects of rocks, the river bank, and other obstructions. Likewise, electric current in a uniform conductor depends on the applied voltage and the resistance of the conductor caused by collisions of the electrons with atoms in the conductor.

For many materials, including most metals, experiments show that **the resistance is constant over a wide range of applied voltages.** This statement is known as **Ohm's law** after Georg Simon Ohm (1787–1854), who was the first to conduct a systematic study of electrical resistance.

Ohm's law is *not* a fundamental law of nature, but an empirical relationship that is valid only for certain materials. Materials that obey Ohm's law, and hence have a constant resistance over a wide range of voltages, are said to be **ohmic.** Materials that do not obey Ohm's law are **nonohmic.** Ohmic materials have a linear current-voltage relationship over a large range of applied voltages (Fig. 21.5a). Nonohmic materials have a nonlinear current-voltage relationship (Fig. 21.5b). One



Georg Simon Ohm (1787–1854). *(Courtesy of North Wind Picture Archives)*



(a) (b)

Figure 21.5 (a) The current-voltage curve for an ohmic material. The curve is linear, and the slope gives the resistance of the conductor. (b) A nonlinear current-voltage curve for a semiconducting diode. This device does not obey Ohm's law.

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TABLE 21.1 Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient $\alpha[(°C)^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[b]	1.50×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	$10^{10} - 10^{14}$	—
Hard rubber	$\sim 10^{13}$	—
Sulfur	10^{15}	—
Quartz (fused)	75×10^{16}	—

[a] All values at 20°C.

[b] A nickel–chromium alloy commonly used in heating elements.

common semiconducting device that is nonohmic is the diode. Its resistance is small for currents in one direction (positive ΔV) and large for currents in the reverse direction (negative ΔV). Most modern electronic devices, such as transistors, have nonlinear current-voltage relationships; their operation depends on the particular ways in which they violate Ohm's law.

We can express Equation 21.7 in the form

$$\Delta V = IR \qquad [21.8]$$

where R is understood to be independent of ΔV. We shall use this expression later in the discussion of electrical circuits. A **resistor** is a simple circuit element that provides a specified resistance in an electrical circuit. The symbol for a resistor in circuit diagrams is a zigzag line (–WW–).

The resistance of an ohmic conducting wire is found to be proportional to its length and inversely proportional to its cross-sectional area. That is,

$$R = \rho \frac{\ell}{A} \qquad [21.9]$$

where the constant of proportionality ρ is called[1] the **resistivity** of the material, which has the unit ohm-meter ($\Omega \cdot m$). To understand this relationship between

[1] The symbol ρ used for resistivity should not be confused with the same symbol used earlier in the text for mass density and charge density.

resistance and resistivity, note that every ohmic material has a characteristic resistivity, a parameter that depends on the properties of the material and on temperature. However, as you can see from Equation 21.7, the resistance of a conductor depends on size and shape as well as on resistivity. Table 21.1 provides a list of resistivities for various materials measured at 20°C.

The inverse of the resistivity is defined[2] as the **conductivity, σ.** Hence, the resistance of an ohmic conductor can also be expressed in terms of its conductivity as

$$R = \frac{\ell}{\sigma A} \qquad [21.10]$$

where $\sigma (= 1/\rho)$ has the unit $(\Omega \cdot m)^{-1}$.

Equation 21.10 shows that the resistance of a cylindrical conductor is proportional to its length and inversely proportional to its cross-sectional area. This is analogous to the flow of liquid through a pipe. As the length of the pipe is increased, the resistance to liquid flow increases because of a gain in friction between the fluid and the walls of the pipe. As its cross-sectional area is increased, the pipe can transport more fluid in a given time interval, so its resistance drops.

Thinking Physics 2

We have seen that an electric field must exist inside a conductor that carries a current. How is this possible in view of the fact that in electrostatics we concluded that the electric field is zero inside a conductor?

Reasoning In the electrostatic case in which charges are stationary, the internal electric field must be zero because a nonzero field would produce a current (by interacting with the free electrons in the conductor), which would violate the condition of static equilibrium. In this chapter we deal with conductors that carry current, a nonelectrostatic situation. The current arises because of a potential difference applied between the ends of the conductor, which produces an internal electric field. So there is no paradox.

CONCEPTUAL PROBLEM 1

Newspaper articles often have statements such as, "10 000 volts of electricity surged through the victim's body." What is wrong with this statement?

Example 21.2 The Resistance of Nichrome Wire

(a) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

Solution The cross-sectional area of this wire is

$$A = \pi r^2 = \pi (0.321 \times 10^{-3}\ \text{m})^2 = 3.24 \times 10^{-7}\ \text{m}^2$$

The resistivity of Nichrome is $1.5 \times 10^{-6}\ \Omega \cdot m$ (Table 21.1). Thus, we can use Equation 21.9 to find the resistance per unit length:

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}\ \Omega \cdot \text{m}}{3.24 \times 10^{-7}\ \text{m}^2} = 4.6\ \Omega/\text{m}$$

(b) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

Solution Because a 1.0-m length of this wire has a resistance of 4.6 Ω, Equation 21.7 gives

[2] Again, do not confuse the symbol σ for conductivity with the same symbol used for surface charge density.

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$$I = \frac{\Delta V}{R} = \frac{10\ \text{V}}{4.6\ \Omega} = 2.2\ \text{A}$$

Note from Table 21.1 that the resistivity of Nichrome wire is about 100 times that of copper. Therefore, a copper wire of the same radius would have a resistance per unit length of only 0.052 Ω/m. A 1.0-m length of copper wire of the same radius would carry the same current (2.2 A) with an applied voltage of only 0.11 V.

Because of its high resistivity and its resistance to oxidation, Nichrome is often used for heating elements in toasters, irons, and electric heaters.

EXERCISE 2 What is the resistance of a 6.0-m length of 22-gauge Nichrome wire? How much current does it carry when connected to a 120-V source? Answer 28 Ω; 4.3 A

EXERCISE 3 Calculate the current density and electric field in the wire assuming that it carries a current of 2.2 A. Answer $6.7 \times 10^6\ \text{A/m}^2$; 10 N/C

Change in Resistivity with Temperature

Resistivity depends on a number of factors, one of which is temperature. For most metals, resistivity increases approximately linearly with increasing temperature over a limited temperature range, according to the expression

Variation of ρ with temperature

$$\rho = \rho_0[1 + \alpha(T - T_0)] \qquad [21.11]$$

where ρ is the resistivity at some temperature T (in degrees Celsius), ρ_0 is the resistivity at some reference temperature T_0 (usually 20°C), and α is called the **temperature coefficient of resistivity.** From Equation 21.11, we see that α can also be expressed as

Temperature coefficient of resistivity

$$\alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T} \qquad [21.12]$$

where $\Delta\rho = \rho - \rho_0$ is the change in resistivity in the temperature interval $\Delta T = T - T_0$.

The resistivities and temperature coefficients of certain materials are listed in Table 21.1. Note the enormous range in resistivities, from very low values for good conductors, such as copper and silver, to very high values for good insulators, such as glass and rubber. An ideal, or "perfect," conductor would have zero resistivity, and an ideal insulator would have infinite resistivity.

Because resistance is proportional to resistivity according to Equation 21.9, the temperature variation of the resistance can be written

$$R = R_0[1 + \alpha(T - T_0)] \qquad [21.13]$$

Precise temperature measurements are often made using this property, as shown in Example 21.3.

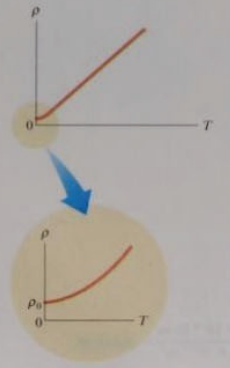


Figure 21.6 Resistivity versus temperature for a normal metal, such as copper. The curve is linear over a wide range of temperatures, and ρ increases with increasing temperature. As T approaches absolute zero (insert), the resistivity approaches a finite value ρ_0.

CONCEPTUAL PROBLEM 2

Aliens with strange powers visit Earth and double every linear dimension of every object on the surface of the earth. Does the electrical cord from the wall socket to your floor lamp now have more resistance than before, less resistance, or the same resistance? Does the lightbulb filament glow more brightly than before, less brightly, or the same? (Assume the resistivities of materials remain the same.)

CONCEPTUAL PROBLEM 3

When incandescent bulbs burn out, they usually do so just after they are switched on. Why?

Example 21.3 A Platinum Resistance Thermometer

A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made from platinum and has a resistance of 50.0 Ω at 20.0°C. When immersed in a vessel containing melting indium, its resistance increases to 76.8 Ω. What is the melting point of indium?

Solution Solving Equation 21.13 for ΔT and obtaining α from Table 21.1, we get

$$\Delta T = \frac{R - R_0}{\alpha R_0} = \frac{76.8\ \Omega - 50.0\ \Omega}{[3.92 \times 10^{-3}\ (°\text{C})^{-1}](50.0\ \Omega)} = 137°\text{C}$$

Because $T_0 = 20.0°\text{C}$, we find that $T = 157°\text{C}$.

For several metals, resistivity is nearly proportional to absolute temperature, as shown in Figure 21.6. In reality, however, there is always a nonlinear region at very low temperatures, and the resistivity usually approaches some finite value near absolute zero (see the magnified inset in Fig. 21.6). This residual resistivity near absolute zero is due primarily to collisions of electrons with impurities and to imperfections in the metal. In contrast, the high-temperature resistivity (the linear region) is dominated by collisions of electrons with the metal atoms. We shall describe this process in more detail in Section 21.4.

Semiconductors, such as silicon and germanium, have intermediate resistivity values. Their resistivity generally decreases with increasing temperature, corresponding to a negative temperature coefficient of resistivity (Fig. 21.7). This is due to the increase in the density of charge carriers at the higher temperatures. Because the charge carriers in a semiconductor are often associated with impurity atoms, the resistivity is very sensitive to the type and concentration of such impurities. A **thermistor** is a semiconducting thermometer that makes use of the large changes in its resistivity with temperature.

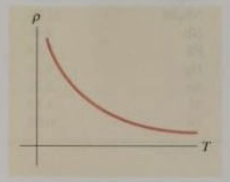


Figure 21.7 Resistivity versus temperature for a pure semiconductor, such as silicon or germanium.

EXERCISE 4 If a silver wire has a resistance of 10 Ω at 20°C, what resistance does it have at 40°C? Neglect any change in length or cross-sectional area due to the change in temperature. Answer 10.8 Ω

21.3 · SUPERCONDUCTORS

There is a class of metals and compounds the resistances of which go to zero below certain *critical temperatures*, T_c. These materials are known as **superconductors.** The resistance–temperature graph for a superconductor follows that of a normal metal at temperatures greater than T_c (Fig. 21.8). When the temperature is equal to or less than T_c, the resistivity drops suddenly to zero. This phenomenon was discovered by the Dutch physicist Heike Kamerlingh Onnes in 1911 as he worked with mercury, which is a superconductor below 4.2 K. Recent measurements have shown that the resistivities of superconductors below T_c are less than $4 \times 10^{-25}\ \Omega \cdot \text{m}$, which is around 10^{17} times smaller than the resistivity of copper and considered to be zero in practice.

Today thousands of superconductors are known. Such common metals as aluminum, tin, lead, zinc, and indium are superconductors. Table 21.2 lists the critical temperatures of several superconductors. The value of T_c is sensitive to chemical composition, pressure, and crystalline structure. It is interesting to note that copper, silver, and gold, which are excellent conductors, do not exhibit superconductivity.

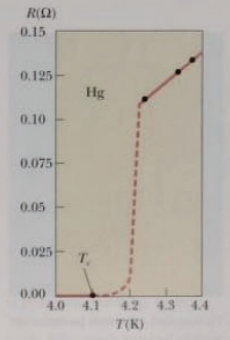


Figure 21.8 Resistance versus temperature for mercury. The graph follows that of a normal metal above the critical temperature, T_c. The resistance drops to zero at the critical temperature, which is 4.2 K for mercury.

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TABLE 21.2
Critical Temperatures for Various Superconductors

Material	T_C (K)
$YBa_2Cu_3O_7$	92
Bi-Sr-Ca-Cu-O	105
Tl-Ba-Ca-Cu-O	125
$HgBa_2Ca_2Cu_3O_8$	134
Nb_3Ge	23.2
Nb_3Sn	21.05
Nb	9.46
Pb	7.18
Hg	4.15
Sn	3.72
Al	1.19
Zn	0.88



Photograph of a small permanent magnet levitated above a disk of the superconductor $YBa_2Cu_3O_7$, which is at 77 K. This superconductor has zero electric resistance at temperatures below 92 K and expels any applied magnetic field. *(Courtesy of IBM Research Laboratory)*

One of the truly remarkable features of superconductors is the fact that once a current is set up in them, it persists *without any applied voltage* (because $R = 0$). In fact, steady currents have been observed to persist in superconducting loops for several years with no apparent decay!

An important recent development in physics that has created much excitement in the scientific community has been the discovery of high-temperature copper-oxide-based superconductors. The excitement began with a 1986 publication by Georg Bednorz and K. Alex Müller, scientists at the IBM Zurich Research Laboratory in Switzerland, in which they reported evidence for superconductivity at a temperature near 30 K in an oxide of barium, lanthanum, and copper. Bednorz and Müller were awarded the Nobel Prize in 1987 for their remarkable discovery. Shortly thereafter, a new family of compounds was open for investigation, and research activity in the field of superconductivity proceeded vigorously. In early 1987, groups at the University of Alabama at Huntsville and the University of Houston announced the discovery of superconductivity at about 92 K in an oxide of yttrium, barium, and copper ($YBa_2Cu_3O_7$). Late in 1987, teams of scientists from Japan and the United States reported superconductivity at 105 K in an oxide of bismuth, strontium, calcium, and copper. More recently, scientists have reported superconductivity at temperatures as high as 125 K in an oxide containing thallium. At this point one cannot rule out the possibility of room-temperature superconductivity, and the search for novel superconducting materials continues. It is an important search both for scientific reasons and because practical applications become more probable and widespread as the critical temperature is raised.

An important and useful application is superconducting magnets in which the magnetic field strengths are about ten times greater than those of the best normal electromagnets. Such superconducting magnets are being considered as a means of storing energy. The idea of using superconducting power lines for transmitting power efficiently is also receiving some consideration. Modern superconducting electronic devices consisting of two thin-film superconductors separated by a thin insulator have been constructed. They include magnetometers (a magnetic-field measuring device) and various microwave devices.

21.4 · A MODEL FOR ELECTRICAL CONDUCTION

The classical model of electrical conduction in metals leads to Ohm's law and shows that resistivity can be related to the motion of electrons in metals.

Consider a conductor as a regular array of atoms containing free electrons (sometimes called *conduction* electrons). Such electrons are free to move through the conductor (as we learned in our discussion of drift speed in Section 21.1) and are approximately equal in number to the atoms. In the absence of an electric field, the free electrons move in random directions with average speeds on the order of 10^6 m/s. The situation is similar to the motion of gas molecules confined in a vessel. In fact, some scientists refer to conduction electrons in a metal as an *electron gas*.

The conduction electrons are not totally free, because they are confined to the interior of the conductor and undergo frequent collisions with the array of atoms. The collisions are the predominant mechanism for the resistivity of a metal at normal temperatures. Note that there is no current through a conductor in the absence of an electric field, because the average velocity of the free electrons is zero. In other words, just as many electrons move in one direction as in the opposite direction, on the average, and so there is no net flow of charge.

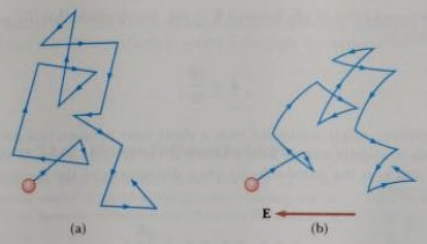


Figure 21.9 (a) A schematic diagram of the random motion of a charge carrier in a conductor in the absence of an electric field. The drift velocity is zero. (b) The motion of a charge carrier in a conductor in the presence of an electric field. Note that the random motion is modified by the field, and the charge carrier has a drift velocity.

The situation is modified when an electric field is applied to the metal. In addition to random thermal motion, the free electrons drift slowly in a direction opposite that of the electric field, with an average drift speed of v_d, which is much less (typically 10^{-4} m/s; see Example 21.1) than the average speed between collisions (typically 10^6 m/s). Figure 21.9 provides a crude depiction of the motion of free electrons in a conductor. In the absence of an electric field, there is no net displacement after many collisions (Fig. 21.9a). An electric field **E** modifies the random motion and causes the electrons to drift in a direction opposite that of **E** (Fig. 21.9b). The slight curvature in the paths in Figure 21.9b results from the acceleration of the electrons between collisions, caused by the applied field. One mechanical system somewhat analogous to this situation is a ball rolling down a slightly inclined plane through an array of closely spaced, fixed pegs (Fig. 21.10). The ball represents a conduction electron, the pegs represent defects in the crystal lattice, and the component of the gravitational force along the incline represents the electric force, $e\mathbf{E}$.

In our model, we shall assume that the excess energy acquired by the electrons in the electric field is lost to the conductor in the collision process. The energy given up to the atoms in the collisions increases the vibrational energy of the atoms, causing the conductor to warm up. The model also assumes that an electron's motion after a collision is independent of its motion *before* the collision.[3]

We are now in a position to obtain an expression for the drift velocity. When a mobile, charged particle of mass m and charge q is subjected to an electric field

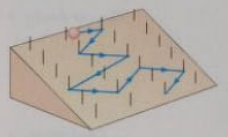


Figure 21.10 A mechanical system somewhat analogous to the motion of charge carriers in the presence of an electric field. The collisions of the ball with the pegs represent the resistance to the ball's motion down the incline.

[3]Because the collision process is random, each collision event is *independent* of what happened earlier. This is analogous to the random process of throwing a die. The probability of rolling a particular number on one throw is independent of the result of the previous throw. On the average, it would take six throws to come up with that number, starting at any arbitrary time.

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E, it experiences a force of $q\mathbf{E}$. Because $\mathbf{F} = m\mathbf{a}$, we conclude that the acceleration of the particle is

$$\mathbf{a} = \frac{q\mathbf{E}}{m} \qquad [21.14]$$

This acceleration, which occurs for only a short time between collisions, enables the electrons to acquire a small drift velocity. If t is the time since the last collision (at $t = 0$), and $\mathbf{v}_0$ is the initial velocity, then the velocity of the electron after the time t is

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t = \mathbf{v}_0 + \frac{q\mathbf{E}}{m}t \qquad [21.15]$$

We now take the average value of $\mathbf{v}$ over all possible times t and all possible values of $\mathbf{v}_0$. If the initial velocities are assumed to be randomly distributed in space, we see that the average value of $\mathbf{v}_0$ is zero. The term $(qE/m)t$ is the velocity added by the field at the end of one trip between atoms. If the electron starts with zero velocity, the average value of the second term of Equation 21.15 is $(qE/m)\tau$, where τ is the *average time between collisions*. Because the average of $\mathbf{v}$ is equal to the drift velocity, we have

Drift velocity •
$$\mathbf{v}_d = \frac{q\mathbf{E}}{m}\tau \qquad [21.16]$$

Substituting this result into Equation 21.6, we find that the magnitude of the current density is

Current density •
$$J = nqv_d = \frac{nq^2E}{m}\tau \qquad [21.17]$$

Comparing this expression with an alternative form of Equation 21.7,[4] $J = \sigma E$, we obtain the following relationships for the conductivity and resistivity:

Conductivity •
$$\sigma = \frac{nq^2\tau}{m} \qquad [21.18]$$

Resistivity •
$$\rho = \frac{1}{\sigma} = \frac{m}{nq^2\tau} \qquad [21.19]$$

According to this classical model, conductivity and resistivity do not depend on the electric field. This feature is characteristic of a conductor obeying Ohm's law. The model shows that the conductivity can be calculated from a knowledge of the density of the charge carriers, their charge and mass, and the average time between

[4]The relation $J = \sigma E$ can be derived as follows: The potential difference across a conductor of length ℓ is $\Delta V = E\ell$, and, from the definition of resistance, $\Delta V = IR$. Using these relations, together with Equations 21.5 and 21.10, we find that the magnitude of the current density is $J = I/A = \Delta V/RA = E\ell/RA = \sigma E$.

collisions, which is related to the average distance between collisions ℓ (the mean free path) and the average thermal speed $\bar{v}$ through the expression[5]

$$\tau = \frac{\ell}{\bar{v}} \qquad [21.20]$$

Example 21.4 Electron Collisions in Copper

(a) Using the data and results from Example 21.1 and the classical model of electron conduction, estimate the average time between collisions for electrons in copper at 20°C.

Solution From Equation 21.19 we see that

$$\tau = \frac{m}{nq^2\rho}$$

where $\rho = 1.7 \times 10^{-8}\ \Omega\cdot\text{m}$ for copper and the carrier density $n = 8.48 \times 10^{28}$ electrons/m^3 for the wire described in Example 21.1. Substitution of these values into the previous expression gives

$$\tau = \frac{(9.11 \times 10^{-31}\ \text{kg})}{(8.48 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})^2(1.7 \times 10^{-8}\ \Omega\cdot\text{m})} = 2.5 \times 10^{-14}\ \text{s}$$

(b) Assuming the average speed for free electrons in copper to be 1.6×10^6 m/s and using the result from part (a), calculate the mean free path for electrons in copper.

Solution

$$\ell = \bar{v}\tau = (1.6 \times 10^6\ \text{m/s})(2.5 \times 10^{-14}\ \text{s}) = 4.0 \times 10^{-8}\ \text{m}$$

which is equivalent to 40 nm (compared with atomic spacings of about 0.2 nm). Thus, although the time between collisions is very short, the electrons travel about 200 atomic distances before colliding with an atom.

Although this classical model of conduction is consistent with Ohm's law, it is not satisfactory for explaining some important phenomena. For example, classical calculations for $\bar{v}$ using the ideal-gas model are about a factor of 10 smaller than the true values. Furthermore, according to Equations 21.19 and 21.20, the temperature variation of the resistivity is predicted to vary as $\bar{v}$, which, according to an ideal-gas model (Chap. 16, Eq. 16.15), is proportional to $\sqrt{T}$. This is in disagreement with the linear dependence of resistivity with temperature for pure metals (Fig. 21.5a). It is possible to account for such observations only by using a quantum mechanical model, which we shall describe briefly.

According to quantum mechanics, electrons have wave-like properties. If the array of atoms in a conductor is regularly spaced (that is, periodic), the wave-like character of the electrons makes it possible for them to move freely through the conductor, and a collision with an atom is unlikely. For an idealized conductor there would be no collisions, the mean free path would be infinite, and the resistivity would be zero. Electron waves are scattered only if the atomic arrangement is irregular (not periodic)—for example, as a result of structural defects or impurities. At low temperatures, the resistivity of metals is dominated by scattering caused by collisions between the electrons and impurities. At high temperatures, the resistivity is dominated by scattering caused by collisions between the electrons and the atoms of the conductor, which are continuously displaced as a result of thermal agitation. The thermal motion of the atoms makes the structure irregular (compared with an atomic array at rest), thereby reducing the electron's mean free path.

[5]Recall that the average speed is the average of the speeds that particles have as a consequence of the temperature of the system of particles (Chap. 16).

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CONCEPTUAL PROBLEM 4

Why don't the free electrons in a metal fall to the bottom of the metal due to gravity? And charges in a conductor are supposed to reside on the surface—why don't the free electrons all go to the surface?

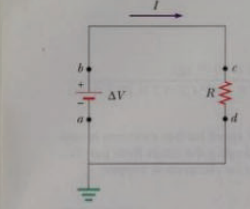


Figure 21.11 A circuit consisting of a battery of emf $\mathcal{E}$ and resistance R. Positive charge flows in the clockwise direction, from the negative to the positive terminal of the battery. Points a and d are grounded.



This versatile circuit enables the experimenter to examine the properties of circuit elements such as capacitors and resistors and their effects on circuit behavior. *(Courtesy of Central Scientific Company)*

21.5 • ELECTRICAL ENERGY AND POWER

If a battery is used to establish an electric current in a conductor, there occurs a continuous transformation of chemical energy stored in the battery to kinetic energy of the charge carriers. This kinetic energy is quickly lost as a result of collisions between the charge carriers and the lattice ions, resulting in an increase in the temperature of the conductor. Thus, the chemical energy stored in the battery is continuously transformed into thermal energy.

In order to understand the process of energy transfer in a simple circuit, consider a battery the terminals of which are connected to a resistor R, as shown in Figure 21.11. (Remember that the positive terminal of the battery is always at the higher potential.) Now imagine following a positive quantity of charge ΔQ around the circuit from point a through the battery and resistor and back to a. Point a is a reference point that is grounded (the ground symbol is ⏚), and its potential is taken to be zero. As the charge moves from a to b through the battery the potential difference of which is ΔV, its electrical potential energy increases by the amount $\Delta Q\,\Delta V$, and the chemical potential energy in the battery decreases by the same amount. (Recall from Chap. 20 that $\Delta U = q\,\Delta V$.) However, as the charge moves from c to d through the resistor, it loses this electrical potential energy during collisions with atoms in the resistor, thereby producing thermal energy. Note that, if we neglect the resistance of the interconnecting wires, no loss in energy occurs for paths bc and da. When the charge returns to point a, it must have the same potential energy (zero) as it had at the start.[6]

The rate at which the charge ΔQ loses potential energy in going through the resistor is

$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t}\,\Delta V = I\,\Delta V$$

where I is the current in the circuit. Of course, the charge regains this energy when it passes through the battery. Because the rate at which the charge loses energy equals the power P dissipated in the resistor, we have

$$P = I\,\Delta V \qquad [21.21]$$

In this case, the power is supplied to a resistor by a battery. However, Equation 21.21 can be used to determine the power transferred from a battery to *any* device carrying a current I and having a potential difference ΔV between its terminals.

Using Equation 21.21 and the fact that $\Delta V = IR$ for a resistor, we can express the power dissipated by the resistor in the alternative forms

[6]Note that when the current reaches its steady-state value, there is *no* change with time in the kinetic energy associated with the current.

$$P = I^2R = \frac{(\Delta V)^2}{R} \qquad [21.22]$$

• *Power dissipated by a resistor*

The SI unit of power is the watt, introduced in Chapter 6. The power dissipated in a conductor of resistance R is also often referred to as an I^2R *loss*.

As we learned in Chapter 6, Section 6.5, the unit of energy the electric company uses to calculate energy consumption, the kilowatt-hour, is the energy consumed in 1 h at the constant rate of 1 kW. Because 1 W = 1 J/s, we have

$$1\ \text{kWh} = (10^3\ \text{W})(3600\ \text{s}) = 3.6 \times 10^6\ \text{J} \qquad [21.23]$$

Thinking Physics 3

When is more power being delivered to a lightbulb—just after it is turned on and the glow of the filament is increasing or after it has been on for a few seconds and the glow is steady?

Reasoning Once the switch is closed, the line voltage is applied across the lightbulb. As the voltage is applied across the cold filament when first turned on, the resistance of the filament is low. Thus, the current is high, and a relatively large amount of power is delivered to the bulb. As the filament warms up, its resistance rises, and the current falls. As a result, the power delivered to the bulb falls. The large current spike at the beginning of operation is the reason that lightbulbs often fail just as they are turned on as noted in Conceptual Problem 3.

Thinking Physics 4

Two lightbulbs A and B are connected across the same potential difference, as in Figure 21.12. The resistance of A is twice that of B. Which lightbulb dissipates more power? Which carries the greater current?

Reasoning Because the voltage across each lightbulb is the same, and the power dissipated by a conductor is $P = (\Delta V)^2/R$, the conductor with the lower resistance will dissipate more power. In this case, the power dissipated by B is twice that of A and provides twice as much illumination. Furthermore, because $P = I\,\Delta V$, we see that the current carried by B is twice that of A.

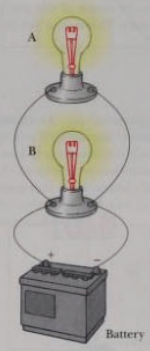


Figure 21.12 (Thinking Physics 4)

Example 21.5 Electrical Rating of a Lightbulb

A lightbulb is rated at 120 V/75 W, which means its operating voltage is 120 V and it has a power rating of 75.0 W. The bulb is powered by a 120-V direct-current power supply. Find the current in the bulb and its resistance.

Solution Because the power rating of the bulb is 75.0 W and the operating voltage is 120 V, we can use $P = I\,\Delta V$ to find the current:

$$I = \frac{P}{\Delta V} = \frac{75.0\ \text{W}}{120\ \text{V}} = 0.625\ \text{A}$$

Using $\Delta V = IR$, the resistance is calculated to be

$$R = \frac{\Delta V}{I} = \frac{120\ \text{V}}{0.625\ \text{A}} = 192\ \Omega$$

EXERCISE 5 What would the resistance be in a lamp rated at 120 V and 100 W? Answer 144 Ω

Principles of physics

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Example 21.6 The Cost of Operating a Lightbulb

How much does it cost to burn a 100-W lightbulb for 24 h if electricity costs eight cents per kilowatt hour?

Solution Because the energy consumed equals power × time, the amount of energy you must pay for, expressed in kWh, is

$$\text{Energy} = (0.10\ \text{kW})(24\ \text{h}) = 2.4\ \text{kWh}$$

If energy is purchased at 8¢ per kWh, the cost is

$$\text{Cost} = (2.4\ \text{kWh})(\$0.080/\text{kWh}) = \$0.19$$

That is, it costs 19¢ to operate the lightbulb for one day. This is a small amount, but when larger and more complex devices are being used, the costs go up rapidly.

Demands on our energy supplies have made it necessary to be aware of the energy requirements of our electric devices. This is true not only because they are becoming more expensive to operate but also because, with the dwindling of the coal and oil resources that ultimately supply us with electrical energy, increased awareness of conservation becomes necessary. On every electric appliance is a label that contains the information you need to calculate the power requirements of the appliance. The power consumption in watts is often stated directly, as on a lightbulb. In other cases, the amount of current used by the device and the voltage at which it operates are given. This information and Equation 21.21 are sufficient to calculate the operating cost of any electric device.

EXERCISE 6 If electricity costs 8¢ per kilowatt hour, what does it cost to operate an electric oven, which operates at 20.0 A and 220 V, for 5.00 h? Answer $1.76

EXERCISE 7 A 12-V battery is connected to a 60-Ω resistor. Neglecting the internal resistance of the battery, calculate the power dissipated in the resistor. Answer 2.4 W

21.6 · SOURCES OF emf

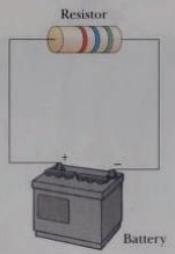


Figure 21.13 A circuit consisting of a resistor connected to the terminals of a battery.

The source that maintains the constant voltage in Figure 21.13 is called an "emf."[7] Sources of emf are any devices (such as batteries and generators) that increase the potential energy of charges circulating in circuits. One can think of a source of emf as a charge pump that forces electrons to move in a direction opposite the electrostatic field inside the source. The emf, $\mathcal{E}$, of a source describes the work done per unit charge, and hence the SI unit of emf is the volt.

Consider the circuit shown in Figure 21.13, consisting of a battery connected to a resistor. We shall assume that the connecting wires have no resistance. If we neglect the internal resistance of the battery, the potential difference across the battery (the terminal voltage) equals the emf of the battery. However, because a real battery always has some internal resistance, r, the terminal voltage is not equal to the emf. The circuit shown in Figure 21.13 can be described by the circuit diagram in Figure 21.14a. The battery within the dashed rectangle is represented by a source of emf, $\mathcal{E}$, in series with the internal resistance r. Now imagine a positive charge moving from a to b in Figure 21.14a. As the charge passes from the negative to the positive terminal within the battery, the potential of the charge increases by $\mathcal{E}$. However, as it moves through the resistance r, its potential decreases by an amount Ir, where I is the current in the circuit. Thus, the terminal voltage of the battery, $\Delta V = V_b - V_a$, is[8]

[7] The term was originally an abbreviation for *electromotive force*, but it is not a force, so the long form is discouraged.

[8] The terminal voltage in this case is less than the emf by the amount Ir. In some situations, the terminal voltage may *exceed* the emf by the amount Ir. This happens when the direction of the current is *opposite* that of the emf, as when a battery is charged with another source of emf.

$$\Delta V = \mathcal{E} - Ir \qquad [21.24]$$

Note from this expression that $\mathcal{E}$ is equivalent to the **open-circuit voltage—that is, the terminal voltage when the current is zero.** Figure 21.14b is a graphical representation of the changes in potential as the circuit is traversed clockwise. By inspecting Figure 21.14a, we see that the terminal voltage ΔV must also equal the potential difference across the external resistance R, often called the **load resistance;** that is, $\Delta V = IR$. Combining this with Equation 21.24, we see that

$$\mathcal{E} = IR + Ir \qquad [21.25]$$

Solving for the current gives

$$I = \frac{\mathcal{E}}{R + r}$$

This shows that the current in this simple circuit depends on both the resistance external to the battery and the internal resistance. If R is much greater than r, we can neglect r in our analysis. In many circuits we shall ignore this internal resistance.

If we multiply Equation 21.25 by the current I, we get

$$I\mathcal{E} = I^2R + I^2r$$

This equation tells us that the total power output of the source of emf, $I\mathcal{E}$, is equal to the power that is dissipated in the load resistance, I^2R, *plus* power that is dissipated in the internal resistance, I^2r. Again, if $r \ll R$, most of the power delivered by the battery is dissipated in the load resistance.

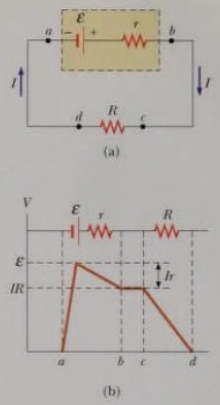


Figure 21.14 (a) Circuit diagram of an emf $\mathcal{E}$ of internal resistance r connected to an external resistor R. (b) Graphical representation showing how the potential changes as the series circuit in part (a) is traversed clockwise.

CONCEPTUAL PROBLEM 5

If the energy transferred to a dead battery during charging is E, is the total energy transferred out of the battery to an electrical load during use in which it completely discharges also E?

CONCEPTUAL PROBLEM 6

If you have your headlights on while you start your car, why do they dim while the car is starting?

CONCEPTUAL PROBLEM 7

Electrical devices are often rated with a voltage and a current—for example, 120 volts, 5 amperes. Batteries, however, are only rated with a voltage—for example, 1.5 volts. Why?

EXERCISE 8 A battery with an emf of 12 V and an internal resistance of 0.90 Ω is connected across a load resistor R. If the current in the circuit is 1.4 A, what is the value of R? Answer 7.7 Ω

21.7 · RESISTORS IN SERIES AND IN PARALLEL

When two or more resistors are connected together so that they have only one common point per pair, they are said to be in *series*. Figure 21.15 shows two resistors connected in series. Note that the current is the same through the two resistors,

- ***For a series connection of resistors, the current is the same in all the resistors.***

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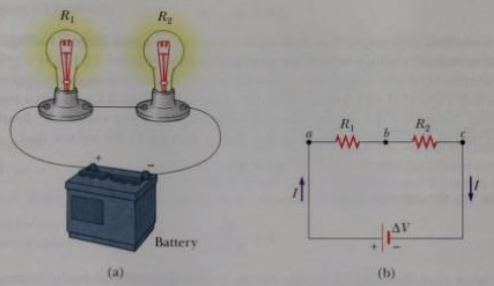


Figure 21.15 Series connection of two resistors, R_1 and R_2. The current is the same in each resistor.

because any charge that flows through R_1 must also flow through R_2. Because the potential difference between a and b in Figure 21.15b equals IR_1 and the potential difference between b and c equals IR_2, the potential difference between a and c is

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

Therefore, we can replace the two resistors in series with a single **equivalent resistance,** R_{eq}, the value of which is the *sum* of the individual resistances:

$$R_{eq} = R_1 + R_2 \qquad [21.26]$$

The resistance R_{eq} is equivalent to the series combination $R_1 + R_2$ in the sense that the circuit current is unchanged when R_{eq} replaces $R_1 + R_2$. The equivalent resistance of three or more resistors connected in series is simply

Equivalent resistance of several resistors in series

$$R_{eq} = R_1 + R_2 + R_3 + \cdots \qquad [21.27]$$

Therefore, **the equivalent resistance of a series connection of resistors is always greater than any individual resistance and is the algebraic sum of the individual resistances.**

Note that if the filament of one lightbulb in Figure 21.15 were to break, or "burn out," the circuit would no longer be complete (an open-circuit condition would exist) and the second bulb would also go out. Some Christmas-tree-light sets (especially older ones) are connected in this way, and the tedious task of determining which bulb is burned out is familiar to many people.

In many circuits, fuses are used in series with other circuit elements for safety purposes. The conductor in the fuse is designed to melt and open the circuit at some maximum current, the value of which depends on the nature of the circuit. If a fuse were not used, excessive currents could damage circuit elements, overheat wires, and perhaps cause a fire. In modern home construction, circuit breakers are used in place of fuses. When the current in a circuit exceeds some value (typically 15 A), the circuit breaker acts as a switch and opens the circuit.

Now consider two resistors connected in *parallel*, as shown in Figure 21.16. In this case, **the potential differences across the resistors are equal.** However, the currents are generally not the same. When the current I reaches point a (called a *junction*) in Figure 21.16b, it splits into two parts, I_1 going through R_1 and I_2 going through R_2. If R_1 is greater than R_2, then I_1 is less than I_2. Clearly, because charge must be conserved, the current I that enters point a must equal the total current leaving point b:

$$I = I_1 + I_2$$

The potential drops across the resistors must be the *same*, and applying $I = \Delta V/R$ gives

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{eq}}$$

From this result, we see that the equivalent resistance of two resistors in parallel is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad [21.28]$$

This can be rearranged to become

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

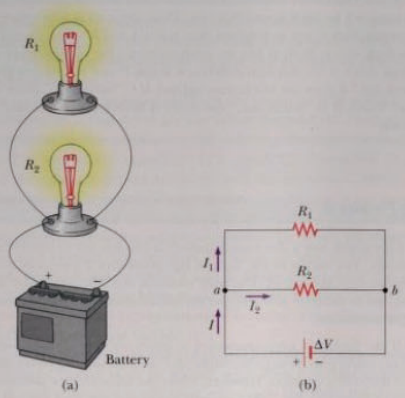


Figure 21.16 Parallel connection of two resistors, R_1 and R_2. The potential difference across each resistor is the same, and the equivalent resistance of the combination is given by $R_{eq} = R_1R_2/(R_1 + R_2)$.

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An extension of this analysis to three or more resistors in parallel yields the following general expression:

Equivalent resistance of several resistors in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \qquad [21.29]$$

From this expression it can be seen that **the equivalent resistance of two or more resistors connected in parallel is always less than the smallest resistance in the group and the inverse of the equivalent resistance is the algebraic sum of the inverses of the individual resistances.**

Household circuits are always wired so that the lightbulbs (or appliances, or whatever) are connected in parallel, as in Figure 21.16a. In this manner, each device operates independently of the others, so that if one is switched off, the others remain on. Equally important, each device operates on the same voltage.

Finally, it is interesting to note that parallel resistors combine in the same way that series capacitors combine, and vice versa.

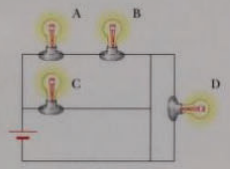


Figure 21.17 (Thinking Physics 5)

Thinking Physics 5

Predict the relative brightnesses of the four identical bulbs in Figure 21.17. What happens if bulb A "burns out," so that it cannot conduct current? What if C "burns out"? What if D "burns out"?

Reasoning Bulbs A and B are connected in series across the emf of the battery, whereas bulb C is connected by itself across this emf. Thus, the emf is split between bulbs A and B. As a result, bulb C will be brighter than bulbs A and B, which should be equally as bright as each other. Bulb D has an equipotential (the vertical wire) connected across it. Thus, there is no potential difference across D and it does not glow at all. If bulb A "burns out," B goes out but C stays lighted. If C "burns out," there is no effect on the other bulbs. If D "burns out," the event is undetectable, because D was not glowing anyway.

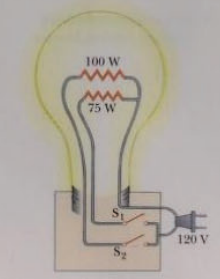


Figure 21.18 (Thinking Physics 6)

Thinking Physics 6

Figure 21.18 illustrates how a three-way lightbulb is constructed to provide three levels of light intensity. The socket of the lamp is equipped with a three-way switch for selecting different light intensities. The bulb contains two filaments. Why are the filaments connected in parallel? Explain how the two filaments are used to provide three different light intensities.

Reasoning If the filaments were connected in series and one of them were to burn out, no current could pass through the bulb, and the bulb would give no illumination, regardless of the switch position. However, when the filaments are connected in parallel and one of them (say the 75-W filament) burns out, the bulb will still operate in one of the switch positions as current passes through the other (100 W) filament. The three light intensities are made possible by selecting one of three values of filament resistance, using a single value of 120 V for the applied voltage. The 75-W filament offers one value of resistance, the 100-W filament offers a second value, and the third resistance is obtained by combining the two filaments in parallel. When switch 1 is closed and switch 2 is opened, current passes only through the 75-W filament. When switch 1 is open and switch 2 is closed, current passes only through the 100-W filament. When both switches are closed, current passes through both filaments, and a total illumination of 175 W is obtained.

CONCEPTUAL PROBLEM 8

Connecting batteries in series increases the emf. What advantage might there be in connecting them in parallel?

CONCEPTUAL PROBLEM 9

You have a large supply of lightbulbs and a battery. You start with one lightbulb connected to the battery and notice its brightness. You then add one lightbulb at a time, each new bulb being added in series to the previous bulbs. As you add the lightbulbs, what happens to the brightness of the bulbs? To the current through the bulbs? To the power transferred from the battery? To the lifetime of the battery? To the terminal voltage of the battery? Answer the same questions if the lightbulbs are added one by one in parallel with the first.

PROBLEM-SOLVING STRATEGY • Resistors

1. When two or more unequal resistors are connected in *series*, they carry the same current, but the potential differences across them are not the same. The resistors add directly to give the equivalent resistance of the series combination.
2. When two or more unequal resistors are connected in *parallel*, the potential differences across them are the same. Because the current is inversely proportional to the resistance, the currents through them are not the same. The equivalent resistance of a parallel combination of resistors is found through reciprocal addition, and the equivalent resistor is always *less* than the smallest individual resistor.
3. A complicated circuit consisting of resistors can often be reduced to a simple circuit containing only one resistor. To do so, examine the initial circuit and replace any resistors in series or any in parallel using the procedures outlined in Steps 1 and 2. Draw a sketch of the new circuit after these changes have been made. Examine the new circuit and replace any series or parallel combinations. Continue this process until a single equivalent resistance is found.
4. If the current through or the potential difference across a resistor in the complicated circuit is to be found, start with the final circuit found in Step 3 and gradually work your way back through the circuits, using $\Delta V = IR$ and the rules of Steps 1 and 2.

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Example 21.7 Find the Equivalent Resistance

Four resistors are connected as shown in Figure 21.19a. (a) Find the equivalent resistance between a and c.

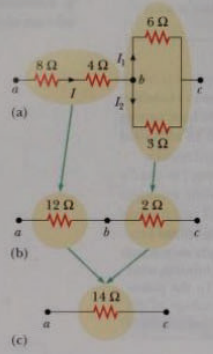


Figure 21.19 (Example 21.17) The resistances of the four resistors shown in (a) can be reduced in steps to an equivalent 14-Ω resistor.

Solution The circuit can be reduced in steps as shown in Figure 21.19. The 8.0-Ω and 4.0-Ω resistors are in series, and so the equivalent resistance between a and b is 12 Ω (Eq. 21.26). The 6.0-Ω and 3.0-Ω resistors are in parallel, and so from Equation 21.28 we find that the equivalent resistance from b to c is 2.0 Ω. Hence, the equivalent resistance from a to c is 14 Ω.

(b) What is the current in each resistor if a potential difference of 42 V is maintained between a and c?

Solution The current I in the 8.0-Ω and 4.0-Ω resistors is the same because they are in series. Using Equation 21.7 and the results from part (a), we get

$$I = \frac{\Delta V_{ac}}{R_{eq}} = \frac{42\text{ V}}{14\ \Omega} = 3.0\text{ A}$$

When this current enters the junction at b, it splits. Part of it passes through the 6.0-Ω resistor (I_1) and part goes through the 3.0-Ω resistor (I_2). Because the potential difference across these resistors, ΔV_{bc}, is the same (they are in parallel), we see that $6I_1 = 3I_2$, or $I_2 = 2I_1$. Using this result and the fact that $I_1 + I_2 = 3.0$ A, we find that $I_1 = 1.0$ A and $I_2 = 2.0$ A. We could have guessed this from the start by noting that the current through the 3.0-Ω resistor has to be twice the current through the 6.0-Ω resistor in view of their relative resistances and the fact that the same voltage is applied to each of them.

As a final check, note that $\Delta V_{bc} = 6I_1 = 3I_2 = 6.0$ V and $\Delta V_{ab} = 12I = 36$ V; therefore, $\Delta V_{ac} = \Delta V_{ab} + \Delta V_{bc} = 42$ V, as it must.

Example 21.8 Three Resistors in Parallel

Three resistors are connected in parallel, as in Figure 21.20. A potential difference of 18 V is maintained between points a and b. (a) Find the current in each resistor.

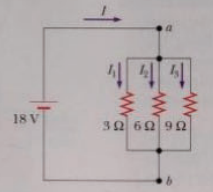


Figure 21.20 (Example 21.8) Three resistors connected in parallel. The voltage across each resistor is 18 V.

Solution The resistors are in parallel, and the potential difference across each is 18 V. Applying $\Delta V = IR$ to each resistor gives

$$I_1 = \frac{\Delta V}{R_1} = \frac{18\text{ V}}{3.0\ \Omega} = 6.0\text{ A}$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18\text{ V}}{6.0\ \Omega} = 3.0\text{ A}$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18\text{ V}}{9.0\ \Omega} = 2.0\text{ A}$$

(b) Calculate the power dissipated by each resistor and the total power dissipated by the three resistors.

Solution Applying $P = I^2R$ to each resistor gives

3.0-Ω: $P_1 = I_1^2R_1 = (6.0\text{ A})^2(3.0\ \Omega) = 110$ W

6.0-Ω: $P_2 = I_2^2R_2 = (3.0\text{ A})^2(6.0\ \Omega) = 54$ W

9.0-Ω: $P_3 = I_3^2R_3 = (2.0\text{ A})^2(9.0\ \Omega) = 36$ W

This shows that the smallest resistor dissipates the most power because it carries the most current. (Note that you can also use $P = (\Delta V)^2/R$ to find the power dissipated by each resistor.) Summing the three quantities gives a total power of 200 W.

(c) Calculate the equivalent resistance of the three resistors. We can use Equation 21.28 to find R_{eq}:

Solution

$$\frac{1}{R_{eq}} = \frac{1}{3.0} + \frac{1}{6.0} + \frac{1}{9.0}$$

$$R_{eq} = \frac{18}{11}\ \Omega = 1.6\ \Omega$$

EXERCISE 9 Use R_{eq} to calculate the total power dissipated in the circuit. **Answer** 200 W

21.8 · KIRCHHOFF'S RULES AND SIMPLE DC CIRCUITS

As indicated in the preceding section, we can analyze simple circuits using $\Delta V = IR$ and the rules for series and parallel combinations of resistors. However, there are many ways in which resistors can be connected so that the circuits formed cannot be reduced to a single equivalent resistor. The procedure for analyzing such complex circuits is greatly simplified by the use of two simple rules called **Kirchhoff's rules:**

1. The sum of the currents entering any junction must equal the sum of the currents leaving that junction. (This rule is often referred to as the **junction rule.**)
2. The sum of the potential differences across each element around any closed circuit loop must be zero. (This rule is usually called the **loop rule.**)

The junction rule is a statement of **conservation of charge.** Whatever current enters a given point in a circuit must leave that point, because charge cannot build up or disappear at a point. If we apply this rule to the junction in Figure 21.21a, we get

$$I_1 = I_2 + I_3$$

Figure 21.21b represents a mechanical analog to this situation in which water flows through a branched pipe with no leaks. The flow rate into the pipe equals the total flow rate out of the two branches.

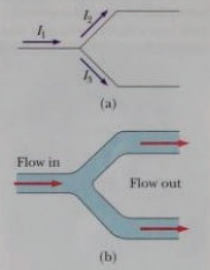


Figure 21.21 (a) A schematic diagram illustrating Kirchhoff's junction rule. Conservation of charge requires that whatever current enters a junction must leave that junction. Therefore, in this case, $I_1 = I_2 + I_3$. (b) A mechanical analog of the junction rule: The flow out must equal the flow in.

The second rule is equivalent to the law of **conservation of energy.** A charge that moves around any closed loop in a circuit (the charge starts and ends at the same point) must gain as much energy as it loses if a potential is defined for each point in the circuit. Its energy may decrease in the form of a potential drop, $-IR$, across a resistor or as a result of having the charge move in the reverse direction through an emf. In the latter case, electric potential energy is converted to chemical energy as the battery is charged. In a similar way, electrical energy may be converted to mechanical energy for operating a motor.

As an aid in applying the loop rule, the following points should be noted. They are summarized in Figure 21.22, where it is assumed that movement is from point a toward point b:

- If a resistor is traversed in the direction of the current, the change in potential across the resistor is $-IR$ (Fig. 21.22a).

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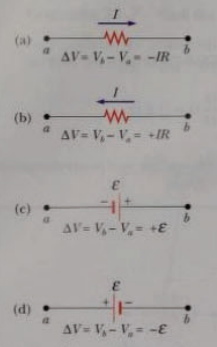


Figure 21.22 Rules for determining the potential changes across a resistor and a battery, assuming the battery has no internal resistance.



Gustav Robert Kirchhoff (1824–1887). *(Courtesy of North Wind Picture Archives)*

- If a resistor is traversed in the direction *opposite* the current, the change in potential across the resistor is $+IR$ (Figure 21.22b).
- If a source of emf is traversed in the direction of the emf (from $-$ to $+$ on the terminals), the change in potential is $+\mathcal{E}$ (Fig. 21.22c).
- If a source of emf is traversed in the direction opposite the emf (from $+$ to $-$ on the terminals), the change in potential is $-\mathcal{E}$ (Fig. 21.22d).

There are limitations on the use of the junction rule and the loop rule. You may use the junction rule as often as needed so long as each time you write an equation, you include in it a current that has not been used in a previous junction rule equation. In general, the number of times the junction rule must be used is one fewer than the number of junction points in the circuit. The loop rule can be used as often as needed so long as a new circuit element (a resistor or battery) or a new current appears in each new equation. In general, **the number of independent equations you need must equal the number of unknowns in order to solve a particular circuit problem.**

The following examples illustrate the use of Kirchhoff's rules in analyzing circuits. In all cases, it is assumed that the circuits have reached steady-state conditions—that is, the currents in the various branches are constant. If a capacitor is included as an element in one of the branches, **it acts as an open circuit: The current in the branch containing the capacitor is zero under steady-state conditions.**

PROBLEM-SOLVING STRATEGY AND HINTS • Kirchhoff's Rules

1. First, draw the circuit diagram and assign labels to all the known quantities and symbols to all the unknown quantities. You must assign *directions* to the currents in each part of the circuit. Do not be alarmed if you guess the direction of a current incorrectly; the result will have a negative value, but *its magnitude will be correct.* Although the assignment of current directions is arbitrary, you must adhere *rigorously* to the directions you assigned when you apply Kirchhoff's rules.
2. Apply the junction rule (Kirchhoff's first rule) to all but one of the junctions in the circuit; doing so provides independent equations relating the currents. (This step is easy!)
3. Now apply the loop rule (Kirchhoff's second rule) to as many loops in the circuit as are needed to solve for the unknowns. In order to apply this rule, you must correctly identify the change in potential as you cross each element in traversing the closed loop (either clockwise or counterclockwise). Watch out for signs!
4. Solve the equations simultaneously for the unknown quantities. Be careful in your algebraic steps, and check your numerical answers for consistency.

Example 21.9 Applying Kirchhoff's Rules

Find the currents I_1, I_2, and I_3 in the circuit shown in Figure 21.23.

Reasoning We choose the directions of the currents as in Figure 21.23. Applying Kirchhoff's first rule to junction c gives

(1) $I_1 + I_2 = I_3$

There are three loops in the circuit, *abcda*, *befcb*, and *aefda* (the outer loop). Therefore, we need only two loop equations to determine the unknown currents. The third loop equation

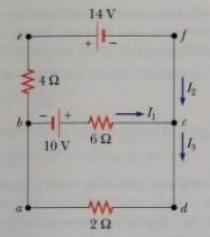


Figure 21.23 (Example 21.9) A circuit containing three loops.

would give no new information. Applying Kirchhoff's second rule to loops *abcda* and *befcb* and traversing these loops in the clockwise direction, we obtain the expressions

(2) Loop *abcda*: $10\text{ V} - (6\ \Omega)I_1 - (2\ \Omega)I_3 = 0$

(3) Loop *befcb*: $-(4\ \Omega)I_2 - 14\text{ V} + (6\ \Omega)I_1 - 10\text{ V} = 0$

Note that in loop *befcb*, a positive sign is obtained when traversing the 6-Ω resistor, because the direction of the path is opposite the direction of I_1. A third loop equation for *aefda* gives $-14 = 2I_3 + 4I_2$, which is just the sum of (2) and (3).

Solution Expressions (1), (2), and (3) represent three independent equations with three unknowns. We can solve the problem as follows: Substituting (1) into (2) gives

$$10 - 6I_1 - 2(I_1 + I_2) = 0$$

(4) $10 = 8I_1 + 2I_2$

Dividing each term in (3) by 2 and rearranging the equation gives

(5) $-12 = -3I_1 + 2I_2$

Subtracting (5) from (4) eliminates I_2, giving

$$22 = 11I_1$$

$$I_1 = 2\text{ A}$$

Using this value of I_1 in (5) gives a value for I_2:

$$2I_2 = 3I_1 - 12 = 3(2) - 12 = -6$$

$$I_2 = -3\text{ A}$$

Finally, $I_3 = I_1 + I_2 = -1$ A. Hence, the currents have the values

$$I_1 = 2\text{ A} \qquad I_2 = -3\text{ A} \qquad I_3 = -1\text{ A}$$

The fact that I_2 and I_3 are both negative indicates only that we chose the wrong direction for these currents. However, the numerical values are correct.

EXERCISE 10 Find the potential difference between points b and c. Answer $V_b - V_c = 2$ V

Example 21.10 A Multiloop Circuit

(a) Under steady-state conditions, find the unknown currents in the multiloop circuit shown in Figure 21.24.

Reasoning First note that the capacitor represents an open circuit, and hence there is no current along path *ghab* under steady-state conditions. Therefore, $I_{fg} = I_{gh} = I_{hc} = I_1$. Labeling the currents as shown in Figure 21.24 and applying Kirchhoff's first rule to junction c, we get

(1) $I_1 + I_2 = I_3$

Kirchhoff's second rule applied to loops *defcd* and *cfgbc* gives

(2) Loop *defcd*: $4.00\text{ V} - (3.00\ \Omega)I_2 - (5.00\ \Omega)I_3 = 0$

(3) Loop *cfgbc*: $(3.00\ \Omega)I_2 - (5.00\ \Omega)I_1 + 8.00\text{ V} = 0$

Solution From (1) we see that $I_1 = I_3 - I_2$, which when substituted into (3) gives

(4) $8.00\text{ V} - (5.00\ \Omega)I_3 + (8.00\ \Omega)I_2 = 0$

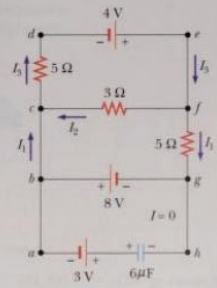


Figure 21.24 (Example 21.10) A multiloop circuit. Note that Kirchhoff's loop equation can be applied to *any* closed loop, including one containing the capacitor.

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Subtracting (4) from (2), we eliminate I_3 and find

$$I_2 = -\frac{4.00}{11.0}\,\text{A} = -0.364\,\text{A}$$

Because I_2 is negative, we conclude that the direction of I_2 is from c to f through the 3.00-Ω resistor. Using this value of I_2 in (3) and (1) gives the following values for I_1 and I_3:

$$I_1 = 1.38\,\text{A} \qquad I_3 = 1.02\,\text{A}$$

Under steady-state conditions, the capacitor represents an *open* circuit, and so there is no current in the branch *ghab*.

(b) What is the charge on the capacitor?

Solution We can apply Kirchhoff's second rule to loop *abgha* (or any other loop that contains the capacitor) to find the potential difference ΔV_c across the capacitor:

$$-8.00\,\text{V} + \Delta V_c - 3.00\,\text{V} = 0$$

$$\Delta V_c = 11.0\,\text{V}$$

Because $Q = C\,\Delta V_c$, the charge on the capacitor is

$$Q = (6.00\ \mu\text{F})(11.0\,\text{V}) = 66.0\ \mu\text{C}$$

Why is the left side of the capacitor positively charged?

EXERCISE 11 Find the voltage across the capacitor by traversing any other loop. Answer 11.0 V

21.9 • *RC* CIRCUITS

So far we have discussed circuits with constant currents, or so-called *steady-state circuits*. We shall now consider circuits containing capacitors, in which the currents may vary in time.

Charging a Capacitor

Consider the series circuit shown in Figure 21.25. Let us assume that the capacitor is initially uncharged. There is no current when switch S is open (Fig. 21.25b). If the switch is closed at $t = 0$, charges begin to flow, setting up a current in the circuit, and the capacitor begins to charge (Fig. 21.25c). Note that during the charging, charges do not jump across the plates of the capacitor, because the gap between the plates represents an open circuit. Instead, electrons move from the top plate to the bottom plate only by moving through the resistor, switch, and battery until the capacitor is fully charged. The value of the maximum charge depends on the

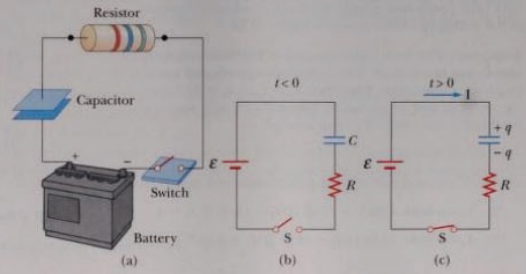


Figure 21.25 (a) A capacitor in series with a resistor, battery, and switch. (b) Circuit diagram representing this system before the switch is closed, $t < 0$. (c) Circuit diagram after the switch is closed, $t > 0$.

emf of the battery. Once the maximum charge is reached, the current in the circuit is zero.

To put this discussion on a quantitative basis, let us apply Kirchhoff's second rule to the circuit after the switch is closed. Choosing clockwise as our direction around the circuit, we get

$$\mathcal{E} - \frac{q}{C} - IR = 0 \qquad [21.30]$$

where q/C is the potential difference across the capacitor and IR is the potential difference across the resistor. Note that q and I are *instantaneous* values of the charge and current, respectively, as the capacitor is charged.

We can use Equation 21.30 to find the initial current in the circuit and the maximum charge on the capacitor. At $t = 0$, when the switch is closed, the charge on the capacitor is zero, and from Equation 21.30 we find that the initial current in the circuit I_0 is a maximum and equal to

$$I_0 = \frac{\mathcal{E}}{R} \quad \text{(current at } t = 0\text{)} \qquad [21.31]$$

• *Maximum current*

At this time, **the potential difference is entirely across the resistor.** Later, when the capacitor is charged to its maximum value Q, charges cease to flow, the current in the circuit is zero, and **the potential difference is entirely across the capacitor.** Substituting $I = 0$ into Equation 21.30 yields the following expression for Q:

$$Q = C\mathcal{E} \quad \text{(maximum charge)} \qquad [21.32]$$

• *Maximum charge on the capacitor*

To determine analytical expressions for the time dependence of the charge and current, we must solve Equation 21.30, a single equation containing two variables, q and I. In order to do this, let us substitute $I = dq/dt$ and rearrange the equation:

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC}$$

An expression for q may be found in the following way. Rearrange the equation by placing terms involving q on the left side and those involving t on the right side. Then integrate both sides:

$$\frac{dq}{(q - C\mathcal{E})} = -\frac{1}{RC}\,dt$$

$$\int_0^q \frac{dq}{(q - C\mathcal{E})} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$

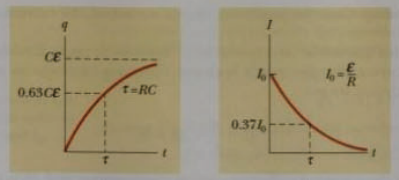
From the definition of the natural logarithm, we can write this expression as

$$q(t) = C\mathcal{E}\,[1 - e^{-t/RC}] = Q[1 - e^{-t/RC}] \qquad [21.33]$$

• *Charge versus time for a capacitor being charged through a resistor*

where e is the base of the natural logarithm and $Q = C\mathcal{E}$ is the *maximum* charge on the capacitor.

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(a) (b)

Figure 21.26 (a) Plot of capacitor charge versus time for the circuit shown in Figure 21.25. After one time constant, τ, the charge is 63.2% of the maximum value, $C\mathcal{E}$. The charge approaches its maximum value as t approaches infinity. (b) Plot of current versus time for the RC circuit shown in Figure 21.25. The current has its maximum value, $I_0 = \mathcal{E}/R$, at $t = 0$ and decays to zero exponentially as t approaches infinity. After one time constant, τ, the current decreases to 36.8% of its initial value.

An expression for the charging current may be found by differentiating Equation 21.33 with respect to time. Using $I = dq/dt$, we obtain

Current versus time •

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC} \qquad [21.34]$$

where $\mathcal{E}/R$ is the initial current in the circuit.

Plots of charge and current versus time are shown in Figure 21.26. Note that the charge is zero at $t = 0$ and approaches the maximum value of $C\mathcal{E}$ as $t \to \infty$ (Fig. 21.26a). Furthermore, the current has its maximum value of $I_0 = \mathcal{E}/R$ at $t = 0$ and decays exponentially to zero as $t \to \infty$ (Fig. 21.26b). The quantity RC, which appears in the exponential of Equations 21.33 and 21.34, is called the **time constant, τ,** of the circuit. It represents the time it takes the current to decrease to $1/e$ of its initial value; that is, in the time τ, $I = e^{-1} I_0 = 0.37 I_0$. In a time of 2τ, $I = e^{-2} I_0 = 0.135 I_0$, and so forth. Likewise, in a time τ the charge increases from zero to $C\mathcal{E}[1 - e^{-1}] = 0.63 C\mathcal{E}$.

The following dimensional analysis shows that τ has units of time:

$$[\tau] = [RC] = \left[\frac{\Delta V}{I} \times \frac{Q}{\Delta V}\right] = \left[\frac{Q}{Q/t}\right] = \mathrm{T}$$

The energy decrease of the battery during the charging process is $Q\mathcal{E} = C\mathcal{E}^2$. After the capacitor is fully charged, the energy stored in it is $\frac{1}{2}Q\mathcal{E} = \frac{1}{2}C\mathcal{E}^2$, which is just half the energy decrease of the battery. It is left to an end-of-chapter problem to show that the remaining half of the energy supplied by the battery goes into thermal energy dissipated in the resistor (Problem 52).

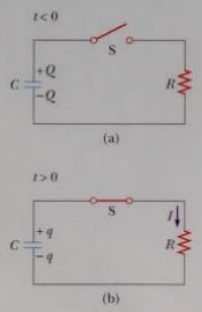


Figure 21.27 (a) A charged capacitor connected to a resistor and a switch, which is open at $t < 0$. (b) After the switch is closed, a nonsteady current is set up in the direction shown and the charge on the capacitor decreases exponentially with time.

Discharging a Capacitor

Now consider the circuit in Figure 21.27, consisting of a capacitor with an initial charge of Q, a resistor, and a switch. When the switch is open (Fig. 21.27a), there is a potential difference of Q/C across the capacitor and zero potential difference across the resistor, because $I = 0$. If the switch is closed at $t = 0$, the capacitor begins

to discharge through the resistor. At some time during the discharge, the current in the circuit is I and the charge on the capacitor is q (Fig. 21.27b). From Kirchhoff's second rule, we see that the potential difference across the resistor, IR, must equal the potential difference across the capacitor, q/C:

$$IR = \frac{q}{C} \qquad [21.35]$$

However, **the current in the circuit must equal the rate of decrease of charge on the capacitor.** That is, $I = -dq/dt$, and so Equation 21.35 becomes

$$-R\frac{dq}{dt} = \frac{q}{C}$$

$$\frac{dq}{q} = -\frac{1}{RC}\,dt$$

Integrating this expression, using the fact that $q = Q$ at $t = 0$, gives

$$\int_Q^q \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q}{Q}\right) = -\frac{t}{RC}$$

$$q(t) = Qe^{-t/RC} \qquad [21.36]$$

• *Charge versus time for a discharging capacitor*

Differentiating Equation 21.36 with respect to time gives the current as a function of time:

$$I(t) = -\frac{dq}{dt} = I_0 e^{-t/RC} \qquad [21.37]$$

• *Current versus time for a discharging capacitor*

where the initial current is $I_0 = Q/RC$. Thus we see that both the charge on the capacitor and the current decay exponentially at a rate characterized by the time constant $\tau = RC$.

Thinking Physics 7

Many automobiles are equipped with windshield wipers that can be used intermittently during a light rainfall. How does the operation of this feature depend on the charging and discharging of a capacitor?

Reasoning The wipers are part of an RC circuit the time constant of which can be varied by selecting different values of R through a multipositioned switch. The brief time that the wipers remain on and the time they are off are determined by the value of the time constant of the circuit.

Example 21.11 Charging a Capacitor in an *RC* Circuit

An uncharged capacitor and a resistor are connected in series to a battery as in Figure 21.25. If $\mathcal{E} = 12.0$ V, $C = 5.00\ \mu$F, and $R = 8.00 \times 10^5\ \Omega$, find the time constant of the circuit, the maximum charge on the capacitor, the maximum current in the circuit, and the charge and current as a function of time.

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Solution The time constant of the circuit is $\tau = RC = (8.00 \times 10^5\ \Omega)(5.00 \times 10^{-6}\ \text{F}) = 4.00\ \text{s}$. The maximum charge on the capacitor is $Q = C\mathcal{E} = (5.00 \times 10^{-6}\ \text{F})(12.0\ \text{V}) = 60.0\ \mu\text{C}$. The maximum current in the circuit is $I_0 = \mathcal{E}/R = (12.0\ \text{V})/(8.00 \times 10^5\ \Omega) = 15.0\ \mu\text{A}$. Using these values and Equations 21.33 and 21.34, we find that

$$q(t) = 60.0[1 - e^{-t/4}]\ \mu\text{C}$$

$$I(t) = 15.0e^{-t/4}\ \mu\text{A}$$

EXERCISE 12 Calculate the charge on the capacitor and the current in the circuit after one time constant has elapsed.
Answer 37.9 μC, 5.52 μA

Example 21.12 Discharging a Capacitor in an *RC* Circuit

Consider a capacitor C being discharged through a resistor R as in Figure 21.27. (a) After how many time constants is the charge on the capacitor one fourth of its initial value?

Solution The charge on the capacitor varies with time according to Equation 21.36, $q(t) = Qe^{-t/RC}$. To find the time it takes the charge q to drop to one fourth of its initial value, we substitute $q(t) = Q/4$ into this expression and solve for t:

$$\tfrac{1}{4}Q = Qe^{-t/RC}$$

$$\tfrac{1}{4} = e^{-t/RC}$$

Taking logarithms of both sides, we find

$$-\ln 4 = -\frac{t}{RC}$$

$$t = RC \ln 4 = 1.39\,RC$$

(b) The energy stored in the capacitor decreases with time as it discharges. After how many time constants is this stored energy one fourth of its initial value?

Solution Using Equations 20.29 and 21.36, we can express the energy stored in the capacitor at any time t as

$$U = \frac{q^2}{2C} = \frac{Q^2}{2C}e^{-2t/RC} = U_0 e^{-2t/RC}$$

where U_0 is the initial energy stored in the capacitor. As in part (a), we now set $U = U_0/4$ and solve for t:

$$\tfrac{1}{4}U_0 = U_0 e^{-2t/RC}$$

$$\tfrac{1}{4} = e^{-2t/RC}$$

Again, taking logarithms of both sides and solving for t gives

$$t = \tfrac{1}{2}RC \ln 4 = 0.693\,RC$$

EXERCISE 13 After how many time constants is the current in the RC circuit one half of its initial value?
Answer 0.693 RC

EXERCISE 14 An uncharged capacitor and a resistor are connected in series to a source of emf. If $\mathcal{E} = 9.0$ V, $C = 20\ \mu$F, and $R = 100\ \Omega$, find (a) the time constant of the circuit, (b) the maximum charge on the capacitor, and (c) the maximum current in the circuit.
Answer (a) 2.0 ms (b) 180 μC (c) 90 mA

SUMMARY

The **electric current** I in a conductor is defined as

$$I \equiv \frac{dQ}{dt} \qquad [21.2]$$

where dQ is the charge that passes through a cross-section of the conductor in the time dt. The SI unit of current is the ampere (A); 1 A = 1 C/s.

The current in a conductor is related to the motion of the charge carriers through the relationship

$$I = nqv_d A \qquad [21.4]$$

where n is the density of charge carriers, q is their charge, v_d is the drift speed, and A is the cross-sectional area of the conductor.

The **current density** $\boldsymbol{J}$ in a conductor is defined as the current per unit area:

$$J \equiv \frac{I}{A} = nqv_d \qquad [21.5]$$

The **resistance** $\boldsymbol{R}$ of a conductor is defined as the ratio of the potential difference across the conductor to the current:

$$R \equiv \frac{\Delta V}{I} \qquad [21.7]$$

The SI units of resistance are volts per ampere, defined as ohms (Ω). That is, $1\ \Omega = 1\ \text{V/A}$.

If the resistance is independent of the applied voltage, the conductor obeys Ohm's law, and conductors that have a constant resistance over a wide range of voltages are said to be *ohmic*.

If a conductor has a uniform cross-sectional area of A and a length of ℓ, its resistance is

$$R = \rho\frac{\ell}{A} \qquad [21.9]$$

where ρ is called the **resistivity** of the conductor. The inverse of the resistivity is defined as the **conductivity, σ.** That is, $\sigma = 1/\rho$.

The resistivity of a conductor varies with temperature in an approximately linear fashion; that is,

$$\rho = \rho_0[1 + \alpha(T - T_0)] \qquad [21.11]$$

where α is the temperature coefficient of resistivity and ρ_0 is the resistivity at some reference temperature T_0.

In a classical model of electronic conduction in a metal, the electrons are treated as molecules of a gas. In the absence of an electric field, the average velocity of the electrons is zero. When an electric field is applied, the electrons move (on the average) with a **drift velocity** $\mathbf{v}_d$, which is opposite the electric field:

$$\mathbf{v}_d = \frac{q\mathbf{E}}{m}\tau \qquad [21.16]$$

where τ is the average time between collisions with the atoms of the metal. The resistivity of the material according to this model is

$$\rho = \frac{m}{nq^2\tau} \qquad [21.19]$$

where n is the number of free electrons per unit volume.

If a potential difference ΔV is maintained across a resistor, the **power,** or rate at which energy is supplied to the resistor, is

$$P = I\,\Delta V \qquad [21.21]$$

Because the potential difference across a resistor is $\Delta V = IR$, we can express the power dissipated in a resistor in the form

$$P = I^2R = \frac{(\Delta V)^2}{R} \qquad [21.22]$$

The electrical energy supplied to a resistor appears in the form of thermal energy in the resistor.

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The **emf** of a battery is the voltage across its terminals when the current is zero. That is, the emf is equivalent to the open-circuit voltage of the battery.

The **equivalent resistance** of a set of resistors connected in **series** is

$$R_{eq} = R_1 + R_2 + R_3 + \cdots \quad [21.27]$$

The **equivalent resistance** of a set of resistors connected in *parallel* is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \quad [21.29]$$

Complex circuits involving more than one loop are conveniently analyzed using two simple rules called **Kirchhoff's rules:**

1. The sum of the currents entering any junction must equal the sum of the currents leaving that junction.
2. The sum of the potential differences across the elements around any closed-circuit loop must be *zero.*

The first rule is a statement of **conservation of charge;** the second rule is equivalent to a statement of **conservation of energy.**

When a resistor is traversed in the direction of the current, the change in potential, ΔV, across the resistor is $-IR$. If a resistor is traversed in the direction opposite the current, $\Delta V = +IR$.

If a source of emf is traversed in the direction of the emf (negative to positive) the change in potential is $+\mathcal{E}$. If it is traversed opposite the emf (positive to negative), the change in potential is $-\mathcal{E}$.

If a capacitor is charged with a battery of emf $\mathcal{E}$ through a resistance R, the charge on the capacitor and the current in the circuit vary in time according to the expressions

$$q(t) = Q[1 - e^{-t/RC}] \quad [21.33]$$

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC} \quad [21.34]$$

where $Q = C\mathcal{E}$ is the *maximum* charge on the capacitor. The product RC is called the **time constant** of the circuit.

If a charged capacitor is discharged through a resistance R, the charge and current decrease exponentially in time according to the expressions

$$q(t) = Qe^{-t/RC} \quad [21.36]$$

$$I(t) = I_0 e^{-t/RC} \quad [21.37]$$

where $I_0 = Q/RC$ is the initial current in the circuit and Q is the initial charge on the capacitor.

CONCEPTUAL QUESTIONS

1. In an analogy between automobile traffic flow and electrical current, what would correspond to the charge Q? What would correspond to the current I?
2. What factors affect the resistance of a conductor?
3. Two wires A and B of circular cross section are made of the same metal and have equal lengths, but the resistance of wire A is three times greater than that of wire B. What is the ratio of their cross-sectional areas? How do their radii compare?
4. Use the atomic theory of matter to explain why the resistance of a material should increase as its temperature increases.
5. Explain how a current can persist in a superconductor without any applied voltage.
6. What would happen to the drift velocity of the electrons in a wire and to the current in the wire if the electrons could move freely without resistance through the wire?
7. If charges flow very slowly through a metal, why does it not require several hours for a light to come on when you throw a switch?
8. If you were to design an electric heater using Nichrome wire as the heating element, what parameters of the wire could you vary to meet a specific power output, such as 1000 W?
9. Car batteries are often rated in ampere-hours. Does this designate the amount of current, power, energy, or charge that can be drawn from the battery?
10. How would you connect resistors so that the equivalent resistance is larger than the individual resistances? Give an example involving two or three resistors.
11. How would you connect resistors so that the equivalent resistance is smaller than the individual resistances? Give an example involving two or three resistors.
12. Why is it possible for a bird to sit on a high-voltage wire without being electrocuted?
13. A "short circuit" is a circuit containing a path of very low resistance in parallel with some other part of the circuit. Discuss the effect of a short circuit on the portion of the circuit it parallels. Use a lamp with a frayed line cord as an example.
14. A series circuit consists of three identical lamps connected to a battery, as in Figure Q21.14. When the switch S is closed, what happens (a) to the intensities of lamps A and B; (b) to the intensity of lamp C; (c) to the current in the circuit; and (d) to the voltage drop across the three lamps? (e) Does the power dissipated in the circuit increase, decrease, or remain the same?

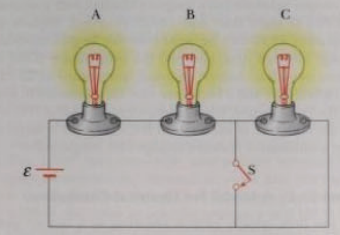


Figure Q21.14

15. Two lightbulbs both operate from 110 V, but one has a power rating of 25 W and the other of 100 W. Which bulb has the higher resistance? Which bulb carries the greater current?
16. If electrical power is transmitted over long distances, the resistance of the wires becomes significant. Why? Which mode of transmission would result in less energy loss—high current and low voltage or low current and high voltage? Discuss.
17. Two sets of Christmas tree lights are available. For set A, when one bulb is removed, the remaining bulbs remain illuminated. For set B, when one bulb is removed, the remaining bulbs do not operate. Explain the difference in wiring for the two sets.
18. Are the two headlights on a car wired in series or in parallel? How can you tell?
19. A ski resort consists of a few chair lifts and several interconnected downhill runs on the side of a mountain, with a lodge at the bottom. The lifts are analogous to batteries and the runs are analogous to resistors. Sketch how two runs can be in series. Sketch how three runs can be in parallel. Sketch a junction of one lift and two runs. One of the skiers is carrying an altimeter. State Kirchhoff's junction rule and Kirchhoff's loop rule for ski resorts.
20. In Figure Q21.20, describe what happens to the lightbulb after the switch is closed. Assume the capacitor has a large capacitance and is initially uncharged, and assume that the light illuminates when connected directly across the battery terminals.

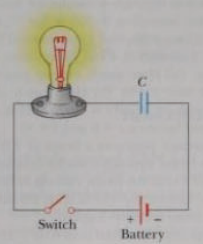


Figure Q21.20

21. Figure Q21.21 shows a series connection of three lamps, all rated at 120 V, with power ratings of 60 W, 75 W, and



Figure Q21.21 *(Henry Leap and Jim Lehman)*

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200 W. Why do the intensities of the lamps differ? Which lamp has the greatest resistance? How would their intensities differ if they were connected in parallel?

22. A student claims that a second lightbulb in series is less bright than the first, because the first bulb uses up some of the current. How would you respond to this statement?

PROBLEMS

Section 21.1 Electric Current

1. In a particular cathode ray tube, the measured beam current is 30.0 μA. How many electrons strike the tube screen every 40.0 s?
2. A teapot with a surface area of 700 cm^2 is to be silver plated. It is attached to the negative electrode of an electrolytic cell containing silver nitrate ($Ag^+NO_3^-$). If the cell is powered by a 12.0-V battery and has a resistance of 1.80 Ω, how long does it take to build up a 0.133-mm layer of silver on the teapot? (Density of silver = 10.5×10^3 kg/m^3.)
3. Suppose that the current through a conductor decreases exponentially with time according to $I(t) = I_0 e^{-t/\tau}$, where I_0 is the initial current (at $t = 0$), and τ is a constant having dimensions of time. Consider a fixed observation point within the conductor. (a) How much charge passes this point between $t = 0$ and $t = \tau$? (b) How much charge passes this point between $t = 0$ and $t = 10\tau$? (c) How much charge passes this point between $t = 0$ and $t = \infty$?
4. A Van de Graaff generator produces a beam of 2.00-MeV deuterons, which are heavy hydrogen nuclei containing a proton and a neutron. (a) If the beam current is 10.0 μA, how far apart are the deuterons? (b) Is their electrostatic repulsion a factor of beam stability? Explain.
5. An aluminum wire has cross-sectional area 4.00×10^{-6} m^2 and carries a current of 5.00 A. Find the drift speed of the electrons in the wire. The density of aluminum is 2.70 g/cm^3. (Assume one electron is supplied by each atom.)

Section 21.2 Resistance and Ohm's Law

6. A lightbulb has a resistance of 240 Ω when operating at a voltage of 120 V. What is the current through the lightbulb?
7. A 0.900-V potential difference is maintained across a 1.50-m length of tungsten wire that has a cross-sectional area of 0.600 mm^2. What is the current in the wire?
8. Suppose that you wish to fabricate a uniform wire out of 1.00 g of copper. If the wire is to have a resistance of $R = 0.500$ Ω, and all of the copper is to be used, what will be (a) the length and (b) the diameter of this wire?
9. A 12.0-Ω metal wire is cut into three equal pieces that are then connected side by side to form a new wire the length of which is equal to one third the original length. What is the resistance of this new wire?
10. (a) Make an order-of-magnitude estimate of the resistance between the ends of a rubber band. (b) Make an order-of-magnitude estimate of the resistance between the "heads" and "tails" sides of a penny. In each case state what quantities you take as data and the values you measure or estimate for them. (c) *Don't* try this at home, but each would carry current of what order of magnitude if it were connected across a 120-V power supply?
11. While traveling through Death Valley on a day when the temperature is 58.0°C, Bill Hiker finds that a certain voltage applied to a copper wire produces a current of 1.00 A. Bill then travels to Antarctica and applies the same voltage to the same wire. What current does he register there if the temperature is −88.0°C? Assume no change in the wire's shape and size.
12. An aluminum rod has a resistance of 1.234 Ω at 20.0°C. Calculate the resistance of the rod at 120°C by accounting for the changes in both the resistivity and the dimensions of the rod.
13. A certain lightbulb has a tungsten filament with a resistance of 19.0 Ω when cold and 140 Ω when hot. Assume that Equation 21.13 can be used over the large temperature range involved here, and find the temperature of the filament when hot. Assume an initial temperature of 20.0°C.
14. A carbon wire and a Nichrome wire are connected in series. If the combination has a resistance of 10.0 kΩ at 0°C, what is the resistance of each wire at 0°C so that the resistance of the combination does not change with temperature?

Section 21.4 A Model for Electrical Conduction

15. If the drift velocity of free electrons in a copper wire is 7.84×10^{-4} m/s, calculate the electric field in the conductor.
16. If the current carried by a conductor is doubled, what happens to the (a) charge carrier density? (b) current density? (c) electron drift velocity? (d) average time between collisions?

Section 21.5 Electrical Energy and Power

17. A toaster is rated at 600 W when connected to a 120-V source. What current does the toaster carry, and what is its resistance?

18. In a hydroelectric installation, a turbine delivers 1500 hp to a generator, which in turn converts 80.0% of the mechanical energy into electrical energy. Under these conditions, what current will the generator deliver at a terminal potential difference of 2000 V?
19. What is the required resistance of an immersion heater that will increase the temperature of 1.50 kg of water from 10.0°C to 50.0°C in 10.0 min while operating at 110 V?
20. What is the required resistance of an immersion heater that will increase the temperature of a mass m of water from T_1 to T_2 in a time interval Δt while operating at a voltage ΔV?
21. Suppose that a voltage surge produces 140 V for a moment. By what percentage will the power output of a 120-V, 100-W lightbulb increase assuming its resistance does not change?
22. A coil of Nichrome wire is 25.0 m long. The wire has a diameter of 0.400 mm and is at 20.0°C. If it carries a current of 0.500 A, what are (a) the electric field intensity in the wire, and (b) the power dissipated in it? (c) If the temperature is increased to 340°C and the voltage across the wire remains constant, what is the power dissipated?
23. Batteries are rated in terms of ampere hours (A·h), where a battery that can produce a current of 2.00 A for 3.00 h is rated at 6.00 A·h. (a) What is the total energy, in kilowatt hours, stored in a 12.0-V battery rated at 55.0 A·h? (b) At $0.0600 per kilowatt hour, what is the value of the electricity produced by this battery?

Section 21.6 Sources of emf

24. (a) What is the current in a 5.60-Ω resistor connected to a battery that has a 0.200-Ω internal resistance if the terminal voltage of the battery is 10.0 V? (b) What is the emf of the battery?
25. A battery has an emf of 15.0 V. The terminal voltage of the battery is 11.6 V when it is delivering 20.0 W of power to an external load resistor R. (a) What is the value of R? (b) What is the internal resistance of the battery?
26. Two 1.50-V batteries—with their positive terminals in the same direction—are inserted in series into the barrel of a flashlight. One battery has an internal resistance of 0.255 Ω, the other an internal resistance of 0.153 Ω. When the switch is closed, a current of 600 mA occurs in the lamp. (a) What is the lamp's resistance? (b) What fraction of the power dissipated is dissipated in the batteries?

Section 21.7 Resistors in Series and in Parallel

27. A television repairperson needs a 100-Ω resistor to repair a malfunctioning set. She is temporarily out of resistors of this value. All she has in her toolbox are a 500-Ω resistor and two 250-Ω resistors. How can the desired resistance be obtained from the resistors on hand?
28. (a) Find the equivalent resistance between points a and b in Figure P21.28. (b) If a potential difference of 34.0 V is applied between points a and b, calculate the current in each resistor.

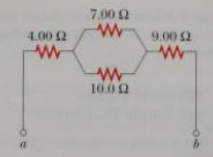


Figure P21.28

29. Consider the circuit shown in Figure P21.29. Find (a) the current in the 20.0-Ω resistor and (b) the potential difference between points a and b.

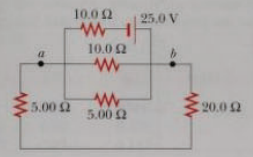


Figure P21.29

30. A lightbulb marked "75 W [at] 120 V" is screwed into a socket at one end of a long extension cord in which each of the two conductors has resistance 0.800 Ω. The other end of the extension cord is plugged into a 120-V outlet. Draw a circuit diagram and find the actual power of the bulb in this circuit.
31. Three 100-Ω resistors are connected, as shown in Figure P21.31. The maximum power that can safely be dissipated in any one resistor is 25.0 W. (a) What is the maximum voltage that can be applied to the terminals a and b? (b) For the voltage determined in part (a), what is the power dissipation in each resistor? What is the total power dissipation?

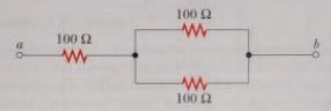


Figure P21.31

32. Four copper wires of equal length are connected in series. Their cross-sectional areas are 1.00 cm^2, 2.00 cm^2,

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3.00 cm², and 5.00 cm². If a voltage of 120 V is applied to the arrangement, determine the voltage across the 2.00-cm² wire.

Section 21.8 Kirchhoff's Rules and Simple DC Circuits

(*Note:* The currents are not necessarily in the directions shown for some circuits.)

33. Determine the current in each branch of the circuit in Figure P21.33.

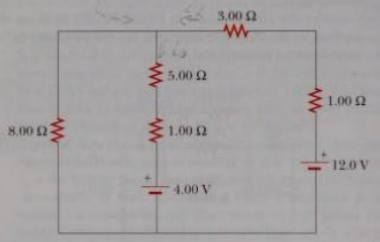


Figure P21.33

34. In Figure P21.33, show how to add just enough ammeters to measure every different current that is flowing. Show how to add just enough voltmeters to measure the potential difference across each resistor and across each battery.
35. The circuit considered in problem 33 and drawn in Figure P21.33 is connected for two minutes. (a) Find the energy converted by each battery. (b) Find the energy converted by each resistor. (c) Find the total energy converted by the circuit.
36. The ammeter in Figure 21.36 reads 2.00 A. Find I_1, I_2, and $\mathcal{E}$.

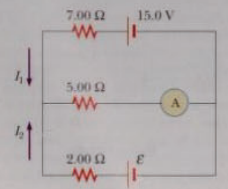


Figure P21.36

37. Using Kirchhoff's rules, (a) find the current in each resistor in Figure P21.37. (b) Find the potential difference between points *c* and *f*. Which point is at the higher potential?

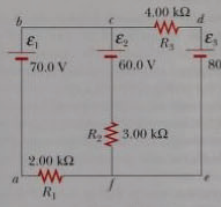


Figure P21.37

Section 21.9 *RC* Circuits

38. A 2.00×10^{-3} μF capacitor with an initial charge of 5.10 μC is discharged through a 1.30-kΩ resistor. (a) Calculate the current through the resistor 9.00 μs after the resistor is connected across the terminals of the capacitor. (b) What charge remains on the capacitor after 8.00 μs? (c) What is the maximum current in the resistor?
39. Consider a series *RC* circuit (Fig. 21.25) for which R = 1.00 MΩ, C = 5.00 μF, and $\mathcal{E}$ = 30.0 V. Find (a) the time constant of the circuit and (b) the maximum charge on the capacitor after the switch is closed. (c) If the switch is closed at $t = 0$, find the current in the resistor 10.0 s later.
40. In the circuit of Figure P21.40, the switch S has been open for a long time. It is then suddenly closed. Determine the time constant (a) before the switch is closed and (b) after the switch is closed. (c) If the switch is closed at $t = 0$ s, determine the current through it as a function of time.

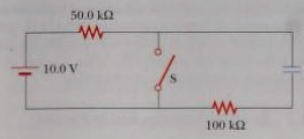


Figure P21.40

41. The circuit in Figure P21.41 has been connected for a long time. (a) What is the voltage across the capacitor? (b) If the battery is disconnected, how long does it take the capacitor to discharge to one tenth of its initial voltage?

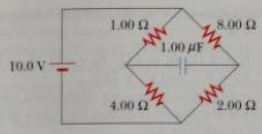


Figure P21.41

Additional Problems

42. One lightbulb is marked "25 W 120 V" and another "100 W 120 V" to mean that each converts that respective power when plugged into a constant 120-V potential difference. (a) Find the resistance of each. (b) In what time will 1.00 C pass through the dim bulb? How is this charge different on its exit versus its entry? (c) In what time will 1.00 J pass through the dim bulb? How is this energy different on its exit versus its entry? (d) Find the cost of running the dim bulb continuously for 30.0 days if the electric company sells its product at $0.0700 per kWh. What *physical quantity* does the electric company sell? What is its price for one SI unit of this quantity?
43. A high-voltage transmission line of diameter 2.00 cm and length 200 km carries a steady current of 1000 A. If the conductor is a wire made of copper with a free charge density of 8.00×10^{28} electrons/m³, how long does it take one electron to travel the full length of the cable?
44. A high-voltage transmission line carries 1000 A starting at 700 kV for a distance of 100 miles. If the resistance in the wire is 0.500 Ω/mi, what is the power loss due to resistive losses?
45. A copper cable is to be designed to carry a current of 300 A with a power loss of only 2.00 W/m. What is the required radius of the copper cable?
46. Four 1.50-V AA batteries in series are used to power a transistor radio. If the batteries can move a charge of 240 C before being depleted, how long will they last if the radio has a resistance of 200 Ω?
47. A battery has emf 9.20 V and internal resistance 1.20 Ω. (a) What resistance across the battery will dissipate heat energy from it at a rate of 12.8 W? (b) 21.2 W?
48. A 10.0-μF capacitor is charged by a 10.0-V battery through a resistance R. The capacitor reaches a potential difference of 4.00 V in a time 3.00 s after charging begins. Find R.
49. An electric heater is rated at 1500 W, a toaster at 750 W, and an electric grill at 1000 W. The three appliances are connected to a common 120-V circuit. (a) How much current does each draw? (b) Is a circuit fused at 25.0 A sufficient in this situation? Explain.
50. A more general definition of the temperature coefficient of resistivity is

$$\alpha = \frac{1}{\rho}\frac{d\rho}{dT}$$

where ρ is the resistivity at temperature T. (a) Assuming that α is constant, show that

$$\rho = \rho_0 e^{\alpha(T - T_0)}$$

where ρ_0 is the resistivity at temperature T_0. (b) Using the series expansion ($e^x \approx 1 + x$; $x \ll 1$), show that the resistivity is given approximately by the expression $\rho = \rho_0[1 + \alpha(T - T_0)]$ for $\alpha(T - T_0) \ll 1$.

51. An experiment is conducted to measure the electrical resistivity of Nichrome in the form of wires with different lengths and cross-sectional areas. For one set of measurements, a student uses #30-gauge wire, which has a cross-sectional area of 7.3×10^{-8} m². The voltage across the wire and the current in the wire are measured with a voltmeter and ammeter, respectively. For each of the measurements given in the table below taken on wires of three different lengths, calculate the resistance of the wires and the corresponding values of the resistivity. What is the average value of the resistivity, and how does it compare with the value given in Table 21.1?

ℓ(m)	ΔV(V)	I(A)	R(Ω)	ρ(Ω·m)
0.54	5.22	0.500		
1.028	5.82	0.276		
1.543	5.94	0.187		

52. A battery is used to charge a capacitor through a resistor, as in Figure 21.25. Show that in the process of charging the capacitor, half of the energy supplied by the battery is dissipated as heat in the resistor and half is stored in the capacitor.
53. A straight cylindrical wire lying along the x axis has length ℓ and diameter d. It is made of a material described by Ohm's law with resistivity ρ. Assume that potential V_0 is maintained at $x = 0$, and $V = 0$ at $x = \ell$. In terms of ℓ, d, V_0, ρ, and physical constants, derive expressions for: (a) the electric field in the wire; (b) the resistance of the wire; (c) the electric current in the wire; and (d) the current density in the wire. Express vectors in vector notation. (e) Prove that $\mathbf{E} = \rho\mathbf{J}$.

Spreadsheet Problems

S1. Spreadsheet 21.1 calculates the average annual lighting cost per bulb for fluorescent and incandescent bulbs and the average yearly savings realized with fluorescent bulbs. It also graphs the average annual lighting cost per bulb versus the cost of electrical energy. (a) Suppose that a fluorescent bulb costs $5, lasts for 5000 h, consumes 40 W of power, but provides the light intensity of a 100-W incandescent bulb. Assume that a 100-W incandescent bulb is on at all times and that energy costs 8.3 cents per kWh. How much does a con-

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sumer save each year by switching to fluorescent bulbs? (b) Check with your local electric company for their current rates, and find the cost of bulbs in your area. Would it pay you to switch to fluorescent bulbs? (c) Vary the parameters for bulbs of different wattages and reexamine the annual savings.

S2. The current-voltage characteristic curve for a semiconductor diode as a function of temperature T is given by

$$I = I_0(e^{e\Delta V/k_B T} - 1)$$

where e is the charge on the electron, k_B is Boltzmann's constant, ΔV is the applied voltage, and T is the absolute temperature. Set up a spreadsheet to calculate I and $R = \Delta V/I$ for $\Delta V = 0.40$ V to $\Delta V = 0.60$ V in increments of 0.01 V. Assume $I_0 = 1.0$ nA. Plot R versus ΔV for $T = 280$ K, 300 K, and 320 K.

S3. The application of Kirchhoff's rules to a dc circuit leads to a set of n linear equations in n unknowns. It is very tedious to solve these algebraically if $n > 3$. The purpose of this problem is to solve for the currents in a moderately complex circuit using matrix operations on a spreadsheet. You can solve equations very easily this way, and you can also readily explore the consequences of changing the values of the circuit parameters. (a) Consider the circuit in Figure S21.3. Assume the four unknown currents are in the directions shown.

- Apply Kirchhoff's rules to get four independent equations for the four unknown currents I_i, $i = 1, 2, 3,$ and 4.
- Write these equations in matrix form $\mathbf{AI} = \mathbf{B}$, that is,

$$\sum_{j=1}^{4} A_{ij} I_j = B_i \qquad i = 1, 2, 3, 4$$

The solution is $\mathbf{I} = \mathbf{A}^{-1}\mathbf{B}$, where $\mathbf{A}^{-1}$ is the inverse matrix of $\mathbf{A}$.

- Set $R_1 = 2\ \Omega$, $R_2 = 4\ \Omega$, $R_3 = 6\ \Omega$, $R_4 = 8\ \Omega$, $\mathcal{E}_1 = 3$ V, $\mathcal{E}_2 = 9$ V, and $\mathcal{E}_3 = 12$ V.
- Enter the matrix $\mathbf{A}$ into your spreadsheet, one value per cell. Use the matrix inversion operation of the spreadsheet to calculate $\mathbf{A}^{-1}$.
- Find the currents by using the matrix multiplication operation of the spreadsheet to calculate $\mathbf{I} = \mathbf{A}^{-1}\mathbf{B}$.

(b) Change the sign of $\mathcal{E}_3$, and repeat the calculations in part (a). This is equivalent to changing the polarity of $\mathcal{E}_3$. (c) Set $\mathcal{E}_1 = \mathcal{E}_2 = 0$ and repeat the calculations in part (a). For these values, the circuit can be solved using simple series-parallel rules. Compare your results using both methods. (d) Investigate any other cases of interest. For example, see how the currents change if you vary R_4.

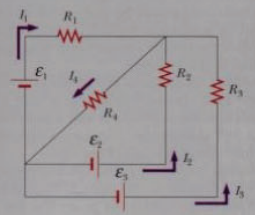


Figure S21.3

ANSWERS TO CONCEPTUAL PROBLEMS

1. A voltage is not something that "surges" through a completed circuit. A voltage is a potential difference that is applied *across* a device or a circuit. What goes *through* the circuit is *current*. Thus, it would be more correct to say, "1 ampere of electricity surges through the victim's body." Although this current would have disastrous results on the human body, a value of 1 (ampere) doesn't sound as exciting for a newspaper article as 10 000 (volts). Another possibility is to write, "10 000 volts of electricity were applied across the victim's body," which still doesn't sound quite as exciting!
2. The length of the line cord will double in this event. This would tend to increase the resistance of the line cord. But the doubling of the radius of the line cord results in the cross-sectional area increasing by a factor of 4. This would reduce the resistance more than the doubling of length increases it. The net result is a decrease in resistance. The same effect will occur for the lightbulb filament. The lowered resistance will result in more current flowing through the filament, causing it to glow more brightly.
3. The bulb filaments are cold when the lamp is first switched on, hence they have a lower resistance and draw more current than when they are hot. The increased current can overheat the filament and destroy it.
4. The gravitational force pulling the electrons to the bottom of a piece of metal is much smaller than the electrical repulsion pushing the electrons apart. Thus, they stay distributed throughout the metal. The concept of charges residing on the surface of a metal is true for a metal with an excess charge. The number of free electrons in a piece of metal is the same as the number of positive crystal lattice ions—the metal has zero net charge.
5. The total amount of energy delivered by the battery will be less than E. Recall that a battery can be considered to be an ideal, resistanceless battery in series with the internal resistance. When charging, the energy delivered to the battery includes the energy necessary to charge the ideal battery, plus the energy that goes into raising the temperature of the battery due to "joule heating" in the internal resistance. This latter energy is not available during the discharge of the battery. During discharge, part of the reduced available energy again transforms into internal energy in the internal resistance, further reducing the available energy below E.
6. The starter in the automobile draws a relatively large current from the battery. This large current causes a significant voltage drop across the internal resistance of the battery. As a result, the terminal voltage of the battery is reduced, and the headlights dim accordingly.
7. An electrical device has a given resistance. Thus, when it is attached to a power source with a known potential difference, a definite current will be drawn. The device can be labeled with both the voltage and the current. Batteries, however, can be applied to a number of devices. Each device will have a different resistance, so the current from the battery will vary with the device. As a result, only the voltage of the battery can be specified.
8. Connecting batteries in parallel does not increase the emf. A high-current device connected to batteries in parallel can draw current from both batteries. Thus, connecting the batteries in parallel does increase the possible current output, and, therefore, the possible power output.
9. As you add more lightbulbs in *series*, the overall resistance of the circuit is increasing. Thus, the current through the bulbs will decrease. This decrease in current will result in a decrease in power transferred from the battery. As a result, the battery lifetime will increase. The whole string will be dimmer than the original lightbulb. As the current drops, the terminal voltage across the battery will become closer and closer to the battery emf.

   As you add more lightbulbs in *parallel*, the overall resistance of the circuit is decreasing. The current through each bulb remains nearly the same (until the battery starts to get hot). Each new bulb will be nearly as bright as the original lightbulb. The current leaving the battery will increase with the addition of each bulb. This increase in current will result in an increase in power transferred from the battery. As a result, the battery lifetime will decrease. As the current rises, the terminal voltage across the battery will drop further below the battery emf.

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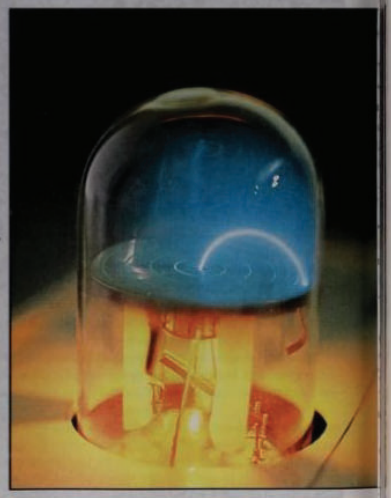
Magnetism

CHAPTER OUTLINE

22.1 Historical Overview: Magnets
22.2 The Magnetic Field
22.3 Magnetic Force on a Current-Carrying Conductor
22.4 Torque on a Current Loop in a Uniform Magnetic Field
22.5 The Biot–Savart Law
22.6 The Magnetic Force between Two Parallel Conductors
22.7 Ampère's Law
22.8 The Magnetic Field of a Solenoid
22.9 Magnetism in Matter (Optional)

The list of important technological applications of magnetism is very long. For instance, large electromagnets are used to pick up heavy loads. Magnets are also used in such devices as meters, motors, and loudspeakers. Magnetic tapes are routinely used in sound- and video-recording equipment and for computer memory, and magnetic recording material is used on computer disks. Intense magnetic fields generated by superconducting magnets are currently being used as a means of containing the plasmas (heated to temperatures on the order of 10^8 K) used in controlled nuclear fusion research.

As we investigate magnetism in this chapter, you will find that the subject cannot be divorced from electricity. For example, magnetic fields affect moving charges, and moving charges produce magnetic fields. Ultimately we shall find that the source of all magnetic fields is electric current, whether it be the current in a wire or the current produced by the motion of charges within atoms or molecules.



The white arc in this photograph indicates the circular path followed by an electron beam moving in a magnetic field. The vessel contains gas at very low pressure, and the beam is made visible as the electrons collide with the gas atoms, which in turn emit visible light. The magnetic field is produced by two coils (not shown). The apparatus can be used to measure the charge:mass ratio for the electron. *(Courtesy of CENCO)*

22.1 · HISTORICAL OVERVIEW: MAGNETS

Many historians of science believe that the compass, which uses a magnetic needle, was used in China as early as the thirteenth century B.C., its invention being of Arab or Indian origin. The phenomenon of magnetism was known to the Greeks as early as about 800 B.C. They discovered that certain stones, now called magnetite (Fe_3O_4), attracted pieces of iron. Legend ascribes the name *magnetite* to the shepherd Magnes, "the nails of whose shoes and the tip of whose staff stuck fast in a magnetic field while he pastured his flocks." In 1269 Pierre de Maricourt mapped out the directions taken by a needle when it was placed at a variety of points on the surface of a spherical natural magnet. He found that the directions formed lines that encircled the sphere and passed through two points diametrically opposite each other, which he called the *poles* of the magnet. Subsequent experiments have shown that every magnet, regardless of its shape, has two poles, called *north poles* and *south poles*, which exhibit forces on each other in a manner analogous to electrical charges. That is, like poles repel each other and unlike poles attract each other. The poles received their names because of the behavior of a magnet in the presence of the Earth's magnetic field. If a bar magnet is suspended from its midpoint by a piece of string so that it can swing freely in a horizontal plane, it will rotate until its "north" pole points to the north of the Earth and its "south" pole points to the south. (The same idea is used to construct a simple compass.)

In 1600 William Gilbert extended these experiments to a variety of materials. Using the fact that a compass needle orients in preferred directions, he suggested that magnets are attracted to land masses. In 1750 John Michell (1724–1793) used a torsion balance to show that magnetic poles exert attractive or repulsive forces on each other and that these forces vary as the inverse square of their separation. Although the force between two magnetic poles is similar to the force between two electric charges, an important difference exists. Electric charges can be isolated (witness the electron and proton), whereas **magnetic poles cannot be isolated.** That is, **magnetic poles are always found in pairs.** No matter how many times a permanent magnet is cut, each piece always has a north pole and a south pole. (There is some theoretical basis for speculating that magnetic monopoles—isolated north or south poles—may exist in nature, and attempts to detect them currently make up an active experimental field of investigation. However, none of these attempts has proved successful.)

The relationship between magnetism and electricity was discovered in 1819 when, while preparing for a lecture demonstration, the Danish scientist Hans Oersted found that an electric current in a wire deflected a nearby compass needle. Shortly thereafter, André Ampère (1775–1836) deduced quantitative laws of magnetic force between current-carrying conductors. He also suggested that electric current loops of molecular size are responsible for *all* magnetic phenomena.

In the 1820s, further connections between electricity and magnetism were identified by Faraday and, independently, Joseph Henry (1797–1878). They showed that an electric current could be produced in a circuit either by moving a magnet near the circuit or by changing the current in another, nearby circuit. Their observations demonstrated that a changing magnetic field produces an electric field.



Hans Christian Oersted (1777–1851), Danish physicist. *(The Bettmann Archive)*

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An assortment of commercially available magnets. Some of the magnets are made of metallic alloys and others are ceramic compounds. *(Courtesy of Central Scientific Co.)*

Years later, theoretical work by Maxwell showed that the reverse is also true: A changing electric field gives rise to a magnetic field.

There is a similarity between electric and magnetic effects that has given rise to methods of making permanent magnets. In Chapter 16 we learned that when rubber and wool are rubbed together, both become charged, one positively and the other negatively. In a somewhat analogous fashion, an unmagnetized piece of iron can be magnetized by stroking it with a magnet. Magnetism can also be induced in iron (and other materials) by other means. For example, if a piece of unmagnetized iron is placed near a strong permanent magnet, the piece of iron eventually becomes magnetized. The process of magnetizing the piece of iron in the presence of a strong external field can be accelerated either by heating and cooling the iron or by hammering.

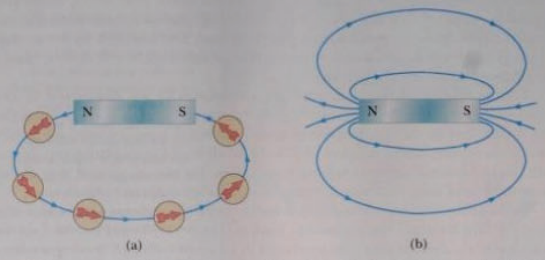
Thinking Physics 1

On a business trip to Australia, you take along your American-made compass that you used in your Boy Scout days. Does this compass work correctly in Australia?

Reasoning There is no problem with using the compass in Australia. The north pole of the magnet in the compass will be attracted to the south magnetic pole near the north geographic pole, just as it was in the United States. The only difference in the magnetic field lines is that they have an upward component in Australia, whereas they have a downward component in the United States. Your compass cannot detect this, however—it only displays the direction of the *horizontal* component of the magnetic field.

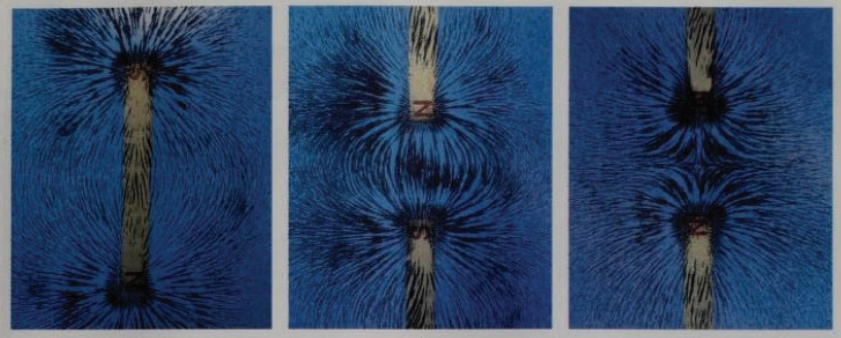
22.2 • THE MAGNETIC FIELD

In earlier chapters we found it convenient to describe the interaction between charged objects in terms of electric fields. Recall that an electric field surrounds any electric charge. The region of space surrounding a *moving* charge includes a magnetic field in addition to the electric field. A magnetic field also surrounds any material with permanent magnetism.



(a) (b)

Figure 22.1 (a) Tracing the magnetic field lines of a bar magnet. (b) Several magnetic field lines of a bar magnet.



(a) (b) (c)

Figure 22.2 (a) Magnetic field patterns surrounding a bar magnet as displayed with iron filings. (b) Magnetic field patterns between *unlike* poles of two bar magnets. (c) Magnetic field pattern between *like* poles of two bar magnets. *(Courtesy of Henry Leap and Jim Lehman)*

In order to describe any type of field, we must define its magnitude, or strength, and its direction. The direction of the magnetic field, **B**, at any location is the direction in which the north pole of a compass needle points at that location. Figure 22.1a shows how the magnetic field of a bar magnet can be traced with the aid of a compass. Several magnetic field lines of a bar magnet traced out in this manner are shown in Figure 22.1b. Magnetic field patterns can be displayed by small iron filings, as shown in Figure 22.2.

We can define a magnetic field vector, **B** (sometimes called the *magnetic induction*), at some point in space in terms of the magnetic force exerted on an appropriate test object. Our test object is taken to be a charged particle moving with a velocity of **v**. For the time being, let us assume that no electric or gravitational fields are present in the region of the charge. Experiments on the motions of various charged particles in a magnetic field give the following results:

- The magnetic force is proportional to the charge, q, and speed, v, of the particle.
- The magnitude and direction of the magnetic force depend on the velocity of the particle and on the magnitude and direction of the magnetic field.
- When a charged particle moves parallel to the magnetic field vector, the magnetic force, **F**, on the particle is zero.
- When the velocity vector makes an angle of θ with the magnetic field, the magnetic force acts in a direction perpendicular to both **v** and **B**; that is, **F** is perpendicular to the plane formed by **v** and **B** (Fig. 22.3a).
- The magnetic force on a positive charge is directed opposite the force on a negative charge moving in the same direction (Fig. 22.3b).

• ***Properties of the magnetic force on a charge moving in a* B *field***

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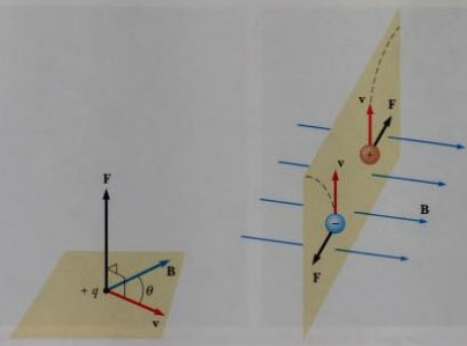


Figure 22.3 The direction of the magnetic force on a charged particle moving with a velocity **v** in the presence of a magnetic field **B**. (a) When **v** is at an angle θ to **B**, the magnetic force is perpendicular to both **v** and **B**. (b) In the presence of a magnetic field, the moving charged particles are deflected as indicated by the dotted lines.

- If the velocity vector makes an angle of θ with the magnetic field, the magnitude of the magnetic force is proportional to sin θ.

These observations can be summarized by writing the magnetic force in the form

Magnetic force on a charged particle moving in a magnetic field

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \qquad [22.1]$$

where the direction of the magnetic force is that of $\mathbf{v} \times \mathbf{B}$, which, by definition of the cross-product, is perpendicular to both **v** and **B**. We can regard this equation as an operational definition of the magnetic field at a point in space. That is, the magnetic field is defined in terms of a sideways force acting on a moving charged particle. The SI unit of magnetic field is the **tesla** (T), where

$$1\ \text{T} = 1\ \text{N}\cdot\text{s/C}\cdot\text{m}$$

Figure 22.4 reviews the right-hand rule for determining the direction of the cross product $\mathbf{v} \times \mathbf{B}$. Point the four fingers of your right hand along the direction of **v**, then turn them until they point along the direction of **B**. The thumb then points in the direction of $\mathbf{v} \times \mathbf{B}$. Because $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, **F** is in the direction of $\mathbf{v} \times \mathbf{B}$ if q is positive (Fig. 22.4a) and *opposite* the direction of $\mathbf{v} \times \mathbf{B}$ if q is negative (Fig. 22.4b). The magnitude of the magnetic force is

$$F = qvB \sin\theta \qquad [22.2]$$

where θ is the angle between **v** and **B**. From this expression, we see that F is zero when **v** is either parallel or antiparallel to **B** ($\theta = 0$ or $180°$). Furthermore, the force has its maximum value, $F = qvB$, when **v** is perpendicular to **B** ($\theta = 90°$).

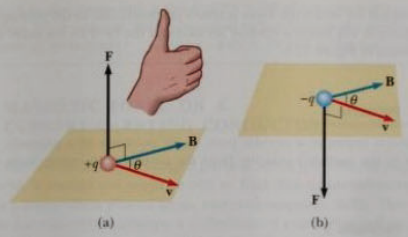


Figure 22.4 The right-hand rule for determining the direction of the magnetic force **F** acting on a charge q moving with a velocity **v** in a magnetic field **B**. If q is positive, **F** is upward in the direction of the thumb. If q is negative, **F** is downward.

There are several important differences between electric and magnetic forces on charged particles:

- The electric force is always parallel to the direction of the electric field, whereas the magnetic force is perpendicular to the magnetic field.
- The electric force acts on a charged particle independent of the particle's velocity, whereas the magnetic force acts on a charged particle only when the particle is in motion.
- The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does *no* work when a charged particle is displaced.

Differences between electric and magnetic fields

This last statement is a consequence of the fact that when a charge moves in a steady magnetic field, the magnetic force is always *perpendicular* to the displacement. That is, $\mathbf{F} \cdot d\mathbf{s} = (\mathbf{F} \cdot \mathbf{v})\,dt = 0$, because the magnetic force is a vector perpendicular to **v**. From this property and the work-kinetic energy theorem, we conclude that the kinetic energy of a charged particle *cannot* be altered by a magnetic field alone. In other words, when a charge moves with a velocity of **v**, an applied magnetic field can alter the direction of the velocity vector, but it cannot change the speed of the particle.

A magnetic field cannot change the speed of a particle.

Thinking Physics 2

Suppose a uniform magnetic field exists in a finite region of space. Can you inject a charged particle into this region and have it stay trapped in the region by the magnetic force?

Reasoning Let us consider separately the components of the particle velocity parallel and perpendicular to the field lines in the region. For the component parallel to the field lines, there will be no force on the particle—it will continue to move with the parallel component until it leaves the region of the magnetic field. Now consider the component perpendicular to the field lines. This component will result in a magnetic force that is perpendicular to both the field lines and the velocity component. The

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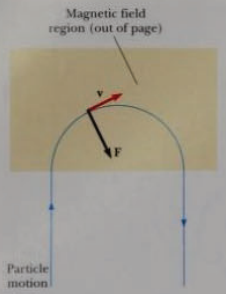


Figure 22.5 (Thinking Physics 2)

path of a particle for which the force is always perpendicular to the velocity is a circle. Thus, the particle will follow a circular arc and exit the field on the other side of the circle, as shown in Figure 22.5.

CONCEPTUAL PROBLEM 1

A charged particle moves in a circular path in the presence of a magnetic field applied perpendicular to the particle's velocity. Does the particle gain energy from the magnetic field?

CONCEPTUAL PROBLEM 2

If a charged particle moves in a straight line through some region of space, can you say that the magnetic field in that region is zero?

CONCEPTUAL PROBLEM 3

How can the motion of a charged particle be used to distinguish between a magnetic field and an electric field in a certain region?

CONCEPTUAL PROBLEM 4

Why does the picture on a television screen become distorted when a magnet is brought near the screen? *Caution:* You should not do this at home, because it may permanently affect the television set.

Example 22.1 A Proton Moving in a Magnetic Field

A proton moves with a speed of 8.0×10^6 m/s along the x axis. It enters a region where there is a magnetic field of magnitude 2.5 T, directed at an angle of 60° to the x axis and lying in the xy plane (Fig. 22.6). Calculate the initial magnetic force on and acceleration of the proton.

Solution From Equation 22.2, we get

$$\begin{aligned} F &= qvB \sin\theta \\ &= (1.60 \times 10^{-19}\ \text{C})(8.0 \times 10^6\ \text{m/s})(2.5\ \text{T})(\sin 60°) \\ &= 2.8 \times 10^{-12}\ \text{N} \end{aligned}$$

Because $\mathbf{v} \times \mathbf{B}$ is in the positive z direction (the right-hand rule), and the charge is positive, $\mathbf{F}$ is in the positive z direction.

The mass of the proton is 1.67×10^{-27} kg, and so its initial acceleration is

$$a = \frac{F}{m} = \frac{2.8 \times 10^{-12}\ \text{N}}{1.67 \times 10^{-27}\ \text{kg}} = 1.7 \times 10^{15}\ \text{m/s}^2$$

in the positive z direction.

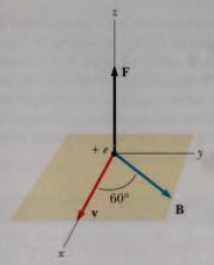


Figure 22.6 (Example 22.1) The magnetic force $\mathbf{F}$ on a proton is in the positive z direction when $\mathbf{v}$ and $\mathbf{B}$ lie in the xy plane.

EXERCISE 1 Calculate the acceleration of an electron that moves through the same magnetic field at the same speed as the proton. Answer 3.0×10^{18} m/s^2

EXERCISE 2 A proton moves with a speed of 2.5×10^6 m/s horizontally at right angles to a magnetic field. What magnetic field is required to just balance the weight of the proton and keep it moving horizontally? Answer 4.09×10^{-14} T in the horizontal plane

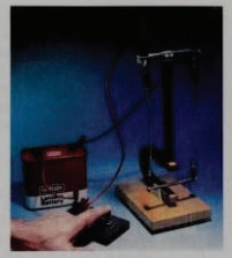
22.3 • MAGNETIC FORCE ON A CURRENT-CARRYING CONDUCTOR

If a force is exerted on a single charged particle when it moves through an external magnetic field, it should not surprise you to find that a current-carrying wire also experiences a force when placed in an external magnetic field. This follows from the fact that the current represents a collection of many charged particles in motion; hence, the resultant force on the wire is due to the sum of the individual forces on the charged particles. The force on the particles is transmitted to the "bulk" of the wire through collisions with the atoms making up the wire.

Before we continue our discussion, some explanation is in order concerning notation in many of our figures. To indicate the direction of $\mathbf{B}$, we use the following convention. If $\mathbf{B}$ is directed into the page, as in Figure 22.7, we use a series of blue crosses, which represent the tails of arrows. If $\mathbf{B}$ is directed out of the page, we use a series of blue dots, which represent the tips of arrows. If $\mathbf{B}$ lies in the plane of the page, we use a series of blue field lines with arrowheads.

The force on a current-carrying conductor can be demonstrated by hanging a wire between the faces of a magnet as in Figure 22.7, where the magnetic field is directed into the page. The wire deflects to the left or right when a current is passed through it.

Let us quantify this discussion by considering a straight segment of wire of length ℓ and cross-sectional area A, carrying a current, I, in a uniform external magnetic field, $\mathbf{B}$, as in Figure 22.8. The magnetic force on the charge q moving with the drift velocity $\mathbf{v}_d$ is $q\mathbf{v}_d \times \mathbf{B}$. To find the total force on the wire segment,



This apparatus demonstrates the force on a current-carrying conductor in an external magnetic field. Why does the bar swing *into* the magnet after the switch is closed? *(Courtesy of Henry Leap and Jim Lehman)*

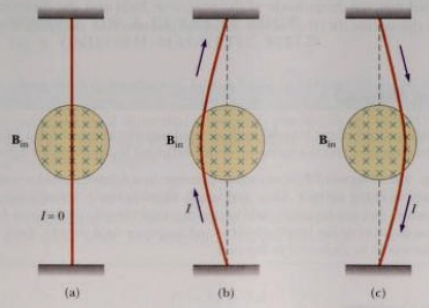


Figure 22.7 A segment of a flexible vertical wire is stretched between the poles of a magnet with the magnetic field (blue crosses) directed into the paper. (a) When there is no current in the wire, it remains vertical. (b) When the current is upward, the wire deflects to the left. (c) When the current is downward, the wire deflects to the right.

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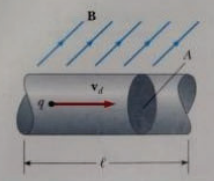


Figure 22.8 A section of a wire containing moving charges in a magnetic field **B**. The magnetic force on each charge is $q\mathbf{v}_d \times \mathbf{B}$, and the net force on a straight wire of length ℓ is $I\boldsymbol{\ell} \times \mathbf{B}$.

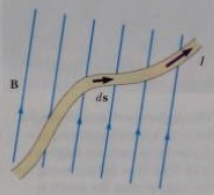


Figure 22.9 A wire of arbitrary shape carrying a current I in a magnetic field **B** experiences a magnetic force. The force on any segment $d\mathbf{s}$ is $I\,d\mathbf{s} \times \mathbf{B}$ and is directed out of the page.

we multiply the force on one charge by the number of charges in the segment. Because the volume of the segment is $A\ell$, the number of charges in the segment is $nA\ell$, where n is the number of charges per unit volume. Hence, the total magnetic force on the wire of length ℓ is

$$\mathbf{F} = (q\mathbf{v}_d \times \mathbf{B})nA\ell$$

This can be written in a more convenient form by noting that, from Equation 21.4, the current in the wire is $I = nqv_dA$. Therefore, **F** can be expressed as

$$\mathbf{F} = I\boldsymbol{\ell} \times \mathbf{B} \qquad [22.3]$$

where $\boldsymbol{\ell}$ is a vector in the direction of the current I; the magnitude of $\boldsymbol{\ell}$ equals the length of the segment. Note that this expression applies only to a straight segment of wire in a uniform external magnetic field. Furthermore, we have neglected the internal magnetic field produced by the current.

Now consider an arbitrarily shaped wire of uniform cross-section in an external magnetic field, as in Figure 22.9. It follows from Equation 22.3 that the magnetic force on a very small segment, $d\mathbf{s}$, in the presence of an external field, **B**, is

$$d\mathbf{F} = I\,d\mathbf{s} \times \mathbf{B} \qquad [22.4]$$

where $d\mathbf{F}$ is directed out of the page for the directions assumed in Figure 22.9. We can consider Equation 22.4 as an alternative definition of **B**. That is, the field **B** can be defined in terms of a measurable force on a current element, where the force is a maximum when **B** is perpendicular to the element and zero when **B** is parallel to the element.

To get the total force **F** on the wire, we integrate Equation 22.4 over the length of the wire:

$$\mathbf{F} = I\int_a^b d\mathbf{s} \times \mathbf{B} \qquad [22.5]$$

In this expression, a and b represent the end points of the wire. When this integration is carried out, the magnitude of the magnetic field and the direction the field makes with the vector $d\mathbf{s}$ (that is, the element orientation) may vary from point to point.

Thinking Physics 3

In a lightning stroke, there is rapid movement of negative charge from a cloud to the ground. In what direction is a lightning stroke deflected by the Earth's magnetic field?

Reasoning The downward flow of negative charge in a lightning stroke is equivalent to an upward-moving current. Thus, the length element vector representing the current is upward, and the magnetic field vector has a northward component. According to the cross product of the length element and magnetic field vectors, then, the lightning stroke would be deflected to the *west*.

Example 22.2 Force on a Semicircular Conductor

A wire bent into the shape of a semicircle of radius R forms a closed circuit and carries a current I. The circuit lies in the xy plane, and a uniform magnetic field is present along the positive y axis, as in Figure 22.10. Find the magnetic force on the straight portion of the wire and on the curved portion.

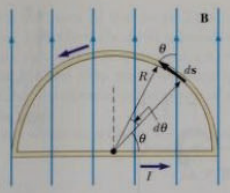


Figure 22.10 (Example 22.2) The net force on a closed current loop in a uniform magnetic field is zero. In this case, the force on the straight portion is $2IRB$ and outward, whereas the force on the curved portion is also $2IRB$ and inward.

Reasoning and Solution The force on the straight portion of the wire has a magnitude $F_1 = I\ell B = 2IRB$, because $\ell = 2R$ and the wire is perpendicular to **B**. The direction of $\mathbf{F}_1$ is out of the paper because $\boldsymbol{\ell} \times \mathbf{B}$ is outward. (That is, $\boldsymbol{\ell}$ is to the right, in the direction of the current, and so by the rule of cross products, $\boldsymbol{\ell} \times \mathbf{B}$ is outward.)

To find the force on the curved part, we must first write an expression for the force $d\mathbf{F}_2$ on the element ds. If θ is the angle between **B** and $d\mathbf{s}$ in Figure 22.10, then the magnitude of $d\mathbf{F}_2$ is

$$dF_2 = I|d\mathbf{s} \times \mathbf{B}| = IB\sin\theta\,ds$$

where ds is the length of the small element measured along the circular arc. In order to integrate this expression, we must express ds in terms of θ. Because $s = R\theta$, $ds = R\,d\theta$, and the expression for dF_2 can be written

$$dF_2 = IRB\sin\theta\,d\theta$$

To get the total force F_2 on the curved portion, we can integrate this expression to account for contributions from all elements. Note that the direction of the force on every element is the same: into the paper (because $d\mathbf{s} \times \mathbf{B}$ is inward). Therefore, the resultant force $\mathbf{F}_2$ on the curved wire must also be into the paper. Integrating dF_2 over the limits $\theta = 0$ to $\theta = \pi$ (that is, the entire semicircle) gives

$$F_2 = IRB\int_0^{\pi}\sin\theta\,d\theta = IRB\Big[-\cos\theta\Big]_0^{\pi}$$

$$= -IRB(\cos\pi - \cos 0) = -IRB(-1 - 1) = 2IRB$$

Because $F_2 = 2IRB$ and is directed into the paper and the force on the straight wire $F_1 = 2IRB$ is out of the paper, we see that the net force on the closed loop is zero.

EXERCISE 3 Calculate the magnitude of the force per unit length exerted on a conductor carrying a current of 22.0 A in a region in which a uniform magnetic field has a magnitude of 0.770 T and is directed perpendicular to the conductor. Answer 16.9 N/m

22.4 · TORQUE ON A CURRENT LOOP IN A UNIFORM MAGNETIC FIELD

In the preceding section we showed how a force is exerted on a current-carrying conductor when the conductor is placed in an external magnetic field. With this as a starting point, we shall show that a torque is exerted on a current loop placed in a magnetic field. The results of this analysis will be of great practical value when we discuss generators in Chapter 23.

Consider a rectangular loop carrying a current, I, in the presence of a uniform external magnetic field *in the plane of the loop*, as in Figure 22.11a. The forces on the sides of length a are zero, because these wires are parallel to the field; hence, $d\mathbf{s} \times \mathbf{B} = 0$ for these sides. The magnitude of the forces on the sides of length b, however, is

$$F_1 = F_2 = IbB$$

The direction of $\mathbf{F}_1$, the force on the left side of the loop, is out of the paper, and that of $\mathbf{F}_2$, the force on the right side of the loop, is into the paper. If we view the loop from an end, as in Figure 22.11b, we see the forces directed as shown. If we

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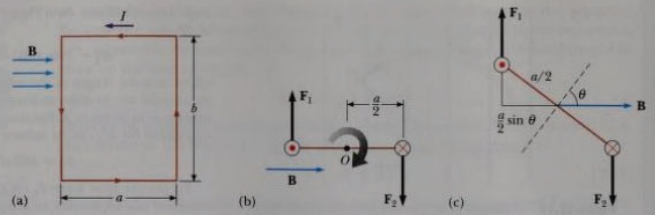


Figure 22.11 (a) Front view of a rectangular loop in a uniform magnetic field. There are no forces acting on the sides of width a parallel to **B**, but there are forces acting on the sides of length b. (b) Bottom view of the rectangular loop shows that the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ on the sides of length b create a torque that tends to twist the loop clockwise as shown. (c) If **B** is at an angle θ with respect to a line perpendicular to the plane of the loop, the torque is $IAB \sin\theta$.

assume that the loop is pivoted so that it can rotate about point O, we see that these two forces produce a torque about O that rotates the loop clockwise. The magnitude of this torque, τ_{max}, is

$$\tau_{max} = F_1 \frac{a}{2} + F_2 \frac{a}{2} = (IbB)\frac{a}{2} + (IbB)\frac{a}{2} = IabB$$

where the moment arm about O is $a/2$ for each force. Because the area of the loop is $A = ab$, the magnitude of the torque can be expressed as

$$\tau_{max} = IAB \qquad [22.6]$$

Remember that this result is valid only when the field **B** is parallel to the plane of the loop. The sense of the rotation is clockwise when the loop is viewed from the bottom end, as indicated in Figure 22.11b. If the current were reversed, the forces would reverse their directions and the rotational tendency would be counterclockwise.

Now suppose the uniform magnetic field makes an angle of θ with a line perpendicular to the plane of the loop, as in Figure 22.11c. For convenience, we shall assume that **B** is perpendicular to the sides of length b. (The end view of these sides is shown in Fig. 22.11c.) In this case, the forces on the sides of length a cancel each other and produce no torque, because they pass through a common origin. However, the forces acting on the sides of length b, $\mathbf{F}_1$ and $\mathbf{F}_2$, form a couple and hence produce a torque about *any point*. Referring to the end view in Figure 22.11c, we note that the moment arm of $\mathbf{F}_1$ about O is $(a/2)\sin\theta$. Likewise, the moment arm of $\mathbf{F}_2$ about O is also $(a/2)\sin\theta$. Because $F_1 = F_2 = IbB$, the net torque about O has the magnitude

$$\begin{aligned}\tau &= F_1 \frac{a}{2}\sin\theta + F_2 \frac{a}{2}\sin\theta \\ &= IbB\left(\frac{a}{2}\sin\theta\right) + IbB\left(\frac{a}{2}\sin\theta\right) = IabB\sin\theta \\ &= IAB\sin\theta\end{aligned}$$

where $A = ab$ is the area of the loop. This result shows that the torque has its maximum value, IAB, when the field is parallel to the plane of the loop ($\theta = 90°$) and is zero when the field is perpendicular to the plane of the loop ($\theta = 0$). As we see in Figure 22.11c, the loop tends to rotate in the direction of decreasing values of θ (that is, so that the normal to the plane of the loop rotates toward the direction of the magnetic field).

A convenient vector expression for the torque is

$$\boldsymbol{\tau} = I\mathbf{A} \times \mathbf{B} \qquad [22.7]$$

where **A**, a vector perpendicular to the plane of the loop, has a magnitude equal to the area of the loop. The sense of **A** is determined by the right-hand rule illustrated in Figure 22.12. When the four fingers of the right hand are rotated in the direction of the current in the loop, the thumb points in the direction of **A**. The product $I\mathbf{A}$ is defined to be the **magnetic moment, $\boldsymbol{\mu}$,** of the loop:

$$\boldsymbol{\mu} = I\mathbf{A} \qquad [22.8]$$

The SI unit of magnetic moment is the ampere-meter^2 (A·m^2). Using this definition, the torque can be expressed as

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} \qquad [22.9]$$



Figure 22.12 A right-hand rule for determining the direction of the vector **A**. The magnetic moment μ is also in the direction of **A**.

• *Torque on a current loop*

Although the torque was obtained for a particular orientation of **B** with respect to the loop, Equation 22.9 is valid for any orientation. Furthermore, although the torque expression was derived for a rectangular loop, the result is valid for a loop of any shape.

If a coil consists of N turns of wire, each carrying the same current and having the same area, the total magnetic moment of the coil is the product of the number of turns and the magnetic moment for one turn. The torque on an N-turn coil is N times greater than that on a one-turn coil.

Example 22.3 The Magnetic Moment of a Coil

A rectangular coil of dimensions 5.40 cm × 8.50 cm consists of 25 turns of wire. The coil carries a current of 15.0 mA. (a) Calculate the magnitude of its magnetic moment.

Solution The magnitude of the magnetic moment of a current loop is $\mu = IA$ (see Eq. 22.8). In this case, $A = (0.0540\ \text{m})(0.0850\ \text{m}) = 4.59 \times 10^{-3}\ \text{m}^2$. Because the coil has 25 turns, and assuming that each turn has the same area A, we have

$$\mu_{coil} = NIA = (25)(15.0 \times 10^{-3}\ \text{A})(4.59 \times 10^{-3}\ \text{m}^2) = 1.72 \times 10^{-3}\ \text{A·m}^2$$

(b) Suppose a magnetic field of magnitude 0.350 T is applied parallel to the plane of the loop. What is the magnitude of the torque acting on the loop?

Solution The torque is given by Equation 22.9, $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$. In this case, **B** is *perpendicular* to $\boldsymbol{\mu}_{coil}$, so that

$$\tau = \mu_{coil} B = (1.72 \times 10^{-3}\ \text{A·m}^2)(0.350\ \text{T}) = 6.02 \times 10^{-4}\ \text{N·m}$$

EXERCISE 4 Show that the units of τ, A·m^2·T, reduce to N·m.

EXERCISE 5 Calculate the magnitude of the torque on the coil when the 0.350-T magnetic field makes angles of (a) 60° and (b) 0° with $\boldsymbol{\mu}$.

Answer (a) 5.21×10^{-4} N·m (b) 0

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22.5 • THE BIOT–SAVART LAW

From their investigations on the force between a current-carrying conductor and a magnet, Jean-Baptiste Biot and Félix Savart arrived at an expression for the magnetic field at some point in space in terms of the current that produces the field. The **Biot–Savart law** says that if a wire carries a steady current, I, then at point P the magnetic field $d\mathbf{B}$ associated with a wire of length ds (Fig. 22.13) has the following properties:

Properties of the magnetic field due to a current element •

- The vector $d\mathbf{B}$ is perpendicular both to $d\mathbf{s}$ (which is in the direction of the current) and to the unit vector, $\hat{\mathbf{r}}$, directed from the element to P.
- The magnitude of $d\mathbf{B}$ is inversely proportional to r^2, where r is the distance from the element to P.
- The magnitude of $d\mathbf{B}$ is proportional to the current and to the length, ds, of the element.
- The magnitude of $d\mathbf{B}$ is proportional to $\sin\theta$, where θ is the angle between $d\mathbf{s}$ and $\hat{\mathbf{r}}$.

The **Biot–Savart law** can be summarized in the following convenient form:

Biot–Savart law •

$$d\mathbf{B} = k_m \frac{I\, d\mathbf{s} \times \hat{\mathbf{r}}}{r^2} \qquad [22.10]$$

where k_m is a constant that in SI units is exactly $10^{-7}\ \mathrm{T \cdot m/A}$. The constant k_m is usually written $\mu_0/4\pi$, where μ_0 is another constant, called the **permeability of free space:**

$$\frac{\mu_0}{4\pi} = k_m = 10^{-7}\ \mathrm{T \cdot m/A} \qquad [22.11]$$

Permeability of free space •

$$\mu_0 = 4\pi k_m = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A} \qquad [22.12]$$

Hence, the Biot–Savart law, Equation 22.10, can also be written

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\, d\mathbf{s} \times \hat{\mathbf{r}}}{r^2} \qquad [22.13]$$

It is important to note that the Biot–Savart law gives the magnetic field at a point only for a small element of the conductor. To find the total magnetic field $\mathbf{B}$ at some point due to a conductor of finite size, we must sum up contributions from all current elements making up the conductor. That is, we must evaluate $\mathbf{B}$ by integrating Equation 22.13.

There are two similarities between the Biot–Savart law of magnetism and Coulomb's law of electrostatics, and one important difference. The current element $I\,d\mathbf{s}$ produces a magnetic field, whereas the point charge q produces an electric field. Furthermore, the magnitude of the magnetic field varies as the inverse square of the distance from the current element, as does the electric field due to a point charge. However, the directions of the two fields are quite different. The electric field due to a point charge is radial; in the case of a positive point charge, $\mathbf{E}$ is directed from the charge to the field point. The magnetic field due to a current element is perpendicular to both the current element and the radius vector. Hence, if the conductor lies in the plane of the page, as in Figure 22.13, $d\mathbf{B}$ points out of the page at the point P and into the page at P'.

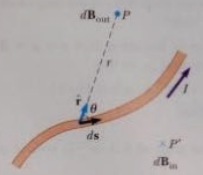


Figure 22.13 The magnetic field $d\mathbf{B}$ at a point P due to a current element $d\mathbf{s}$ is given by the Biot–Savart law. The field is out of the page at P and into the page at P'.

Figure 22.14 gives a convenient rule for determining the direction of the magnetic field due to a wire. Note that the field lines generally encircle the current. In the case of a long, straight wire, the field lines form circles that are concentric with the wire and are in a plane perpendicular to the wire. Although the magnetic field due to a long, current-carrying wire can be calculated using the Biot–Savart law (Problem 51), in Section 22.7 we use a different method to show that the magnitude of this field at a distance r from the wire is

$$B = \frac{\mu_0 I}{2\pi r} \qquad [22.14]$$

Magnetic field due to a long, straight wire •

When applying the Biot–Savart law, it is important that you recognize that **the magnetic field described in these calculations is the field due to a given current-carrying conductor.** This is not to be confused with any *external* field that may be applied to the conductor.

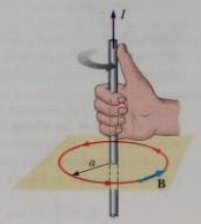


Figure 22.14 The right-hand rule for determining the direction of the magnetic field surrounding a long, straight wire carrying a current. Note that the magnetic field lines form circles around the wire.

Thinking Physics 4

In electrical circuits, it is often the case that wires carrying currents in opposite directions are twisted together. What is the advantage of this decision?

Reasoning If the wires are not twisted together, the combination of the two wires forms a current loop. Although it is not a circle, it is a loop of current surrounding an open area. Thus, there will be a magnetic field generated by the loop that might affect adjacent circuits or components.

CONCEPTUAL PROBLEM 5

Suppose you move along a wire at the same speed as the drift electrons in the current. Do you now measure a magnetic field of zero?

CONCEPTUAL PROBLEM 6

Can you use a compass to detect the currents in wires in the walls near light switches in your home?

Example 22.4 Magnetic Field on the Axis of a Circular Current Loop

Consider a circular loop of wire of radius R located in the yz plane and carrying a steady current I, as in Figure 22.15. Calculate the magnetic field at an axial point P a distance x from the center of the loop.

Reasoning In this situation, note that any element $d\mathbf{s}$ is perpendicular to $\hat{\mathbf{r}}$. Furthermore, all elements around the loop are at the same distance r from P, where $r^2 = x^2 + R^2$. Hence, the magnitude of $d\mathbf{B}$ due to the element $d\mathbf{s}$ is

$$dB = \frac{\mu_0 I}{4\pi} \frac{|d\mathbf{s} \times \hat{\mathbf{r}}|}{r^2} = \frac{\mu_0 I}{4\pi} \frac{ds}{(x^2 + R^2)} \qquad [22.15]$$

The direction of the magnetic field $d\mathbf{B}$ due to the element $d\mathbf{s}$ is perpendicular to the plane formed by $\hat{\mathbf{r}}$ and $d\mathbf{s}$, as in Figure 22.15. The vector $d\mathbf{B}$ can be resolved into a component dB_x, along the x axis, and a component $dB_\perp$, which is perpendicular to the x axis. When the components perpendicular to the x axis are summed over the whole loop, the result is zero. That is, by symmetry any element on one side of the loop sets up a perpendicular component that cancels the component set up by an element diametrically opposite it.

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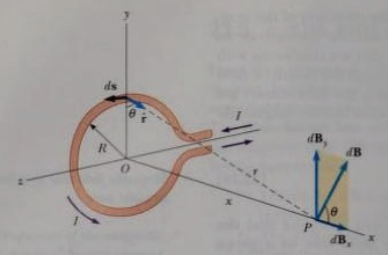


Figure 22.15 (Example 22.4) The geometry for calculating the magnetic field at an axial point P for a current loop. Note that by symmetry the total field $\mathbf{B}$ is along the x axis.

Solution For the reasons given previously, **the resultant field at *P* must be along the *x* axis** and can be found by integrating the components $dB_x = dB\cos\theta$, where this expression is obtained from resolving the vector $d\mathbf{B}$ into its components as shown in Figure 22.15. That is, $\mathbf{B} = B_x\mathbf{i}$, where

$$B_x = \oint dB\cos\theta = \frac{\mu_0 I}{4\pi}\oint \frac{ds\cos\theta}{x^2 + R^2}$$

and the integral must be taken over the entire loop. Because θ, x, and R are constants for all elements of the loop and because $\cos\theta = R/(x^2 + R^2)^{1/2}$, we get

$$B_x = \frac{\mu_0 IR}{4\pi(x^2 + R^2)^{3/2}}\oint ds = \frac{\mu_0 IR^2}{2(x^2 + R^2)^{3/2}} \qquad [22.16]$$

where we have used the fact that $\oint ds = 2\pi R$ (the circumference of the loop).

To find the magnetic field at the center of the loop, we set $x = 0$ in Equation 22.16. At this special point, this gives

$$B = \frac{\mu_0 I}{2R} \quad (\text{at } x = 0) \qquad [22.17]$$

It is also interesting to determine the behavior of the magnetic field far from the loop—that is, when x is large compared with R. In this case, we can neglect the term R^2 in the denominator of Equation 22.16 and get

$$B \approx \frac{\mu_0 IR^2}{2x^3} \quad (\text{for } x \gg R) \qquad [22.18]$$

Because the magnitude of the magnetic dipole moment μ of the loop is defined as the product of the current and the area (Eq. 22.8), $\mu = I(\pi R^2)$ and we can express Equation 22.18 in the form

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{x^3} \qquad [22.19]$$

This result is similar in form to the expression for the electric field due to an electric dipole, $E = k_e p/y^3$ (Example 19.3), where p is the electric dipole moment. The pattern of the magnetic field lines for a circular loop is shown in Figure 22.16. For clarity, the lines are drawn only for one plane that contains the axis of the loop. The field pattern is axially symmetric.

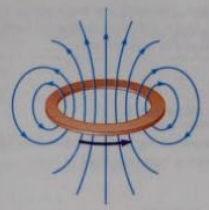


Figure 22.16 Magnetic field lines for a current loop. Far from the loop, the field lines are identical in form to those of an electric dipole.

EXERCISE 6 A wire in which there is a current of 5.00 A is to be formed into a circular loop of one turn. If the required value of the magnetic field at the center of the loop is 10.0 μT, what is the required radius? Answer 31.4 cm

22.6 • THE MAGNETIC FORCE BETWEEN TWO PARALLEL CONDUCTORS

In Section 22.3 we described the magnetic force that acts on a current-carrying conductor when the conductor is placed in an external magnetic field. Because a current in a conductor sets up its own magnetic field, it is easy to understand that two current-carrying conductors exert magnetic forces on each other. As we shall see, such forces can be used as the basis for defining the ampere and the coulomb.

Consider two long, straight, parallel wires separated by the distance a and carrying currents I_1 and I_2 in the same direction, as in Figure 22.17. We can easily determine the force on one wire due to a magnetic field set up by the other wire. Wire 2, which carries the current I_2, sets up a magnetic field, $\mathbf{B}_2$, at the position of wire 1. The direction of $\mathbf{B}_2$ is perpendicular to the wire, as shown in Figure 22.17. According to Equation 22.3, the magnetic force on a length, ℓ, of wire 1 is $\mathbf{F}_1 = I_1\boldsymbol{\ell} \times \mathbf{B}_2$. Because $\boldsymbol{\ell}$ is perpendicular to $\mathbf{B}_2$, the magnitude of $\mathbf{F}_1$ is $F_1 = I_1\ell B_2$. Because the field due to wire 2 is given by Equation 22.14,

$$B_2 = \frac{\mu_0 I_2}{2\pi a}$$

we see that

$$F_1 = I_1\ell B_2 = I_1\ell\left(\frac{\mu_0 I_2}{2\pi a}\right) = \frac{\ell\mu_0 I_1 I_2}{2\pi a}$$

We can rewrite this in terms of the force per unit length, as

$$\frac{F_1}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a} \qquad [22.20]$$

The direction of $\mathbf{F}_1$ is downward, toward wire 2, because $\boldsymbol{\ell} \times \mathbf{B}_2$ is downward. If one considers the field set up at wire 2 due to wire 1, the force $\mathbf{F}_2$ on wire 2 is found to be equal to and opposite $\mathbf{F}_1$. That is what one would expect, because Newton's third law of action–reaction must be obeyed.

When the currents are in opposite directions, the forces are reversed and the wires repel each other. Hence, we find that **parallel conductors carrying currents in the same direction attract each other,** whereas **parallel conductors carrying currents in opposite directions repel each other.**

The force between two parallel wires, each carrying a current, is used to define the **ampere:** If two long, parallel wires 1 m apart carry the same current and the force per unit length on each wire is 2×10^{-7} N/m, then the current is defined to be 1 A. The numerical value of 2×10^{-7} N/m is obtained from Equation 22.20, with $I_1 = I_2 = 1$ A and $a = 1$ m.

The SI unit of charge, the **coulomb,** can now be defined in terms of the ampere: If a conductor carries a steady current of 1 A, then the quantity of charge that flows through a cross-section of the conductor in 1 s is 1 C.

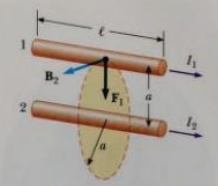


Figure 22.17 Two parallel wires each carrying a steady current exert a force on each other. The field $\mathbf{B}_2$ at wire 1 due to the current in wire 2 produces a force on wire 1 given by $F_1 = I_1\ell B_2$. The force is attractive if the currents are parallel as shown and repulsive if the currents are antiparallel.

CONCEPTUAL PROBLEM 7

Two wires carry current in opposite directions and are oriented parallel, with one above the other. The wires repel each other. Is the upper wire in a stable levitation over the lower wire? Suppose the current in one wire is reversed, so that the wires now attract. Is the lower wire hanging in a stable attraction to the upper wire?

EXERCISE 7 At what distance from a long, straight wire carrying a current of 5 A is the magnetic field due to the wire equal to the strength of the Earth's field—approximately 5×10^{-5} T? Answer 2 cm

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22.7 • AMPÈRE'S LAW



André-Marie Ampère (1775–1836), a French mathematician, chemist, and philosopher. *(Photo courtesy of AIP Niels Bohr Library)*

A simple experiment first carried out by Oersted in 1820 clearly demonstrates that a current-carrying conductor produces a magnetic field. In this experiment, several compass needles are placed in a horizontal plane near a long vertical wire, as in Figure 22.18a. When there is no current in the wire, all needles point in the same direction (that of the Earth's field), as one would expect. However, when the wire carries a strong, steady current, the needles all deflect in a direction tangent to the circle, as in Figure 22.18b. These observations show that the direction of **B** is consistent with the right-hand rule described in Section 22.6.

> If the wire is grasped in the right hand with the thumb in the direction of the current, the fingers will curl in the direction of **B**.

When the current is reversed, the needles in Figure 22.18b also reverse.

Because the needles point in the direction of **B**, we conclude that the lines of **B** form circles about the wire, as discussed in Section 22.5. By symmetry, the magnitude of **B** is the same everywhere on a circular path that is centered on the wire and lies in a plane perpendicular to the wire. By varying the current and distance from the wire, one finds that **B** is proportional to the current and inversely proportional to the distance from the wire.

Now let us evaluate the product $\mathbf{B} \cdot d\mathbf{s}$ and sum the products $B\,ds$ over the closed circular path centered on the wire. Along this path, the vectors $d\mathbf{s}$ and **B** are parallel at each point (Fig. 22.18b), so that $\mathbf{B} \cdot d\mathbf{s} = B\,ds$. Furthermore, **B** is constant in magnitude on this circle and is given by Equation 22.14. Therefore, the sum of the products $B\,ds$ over the closed path, which is equivalent to the line integral of $\mathbf{B} \cdot d\mathbf{s}$, is

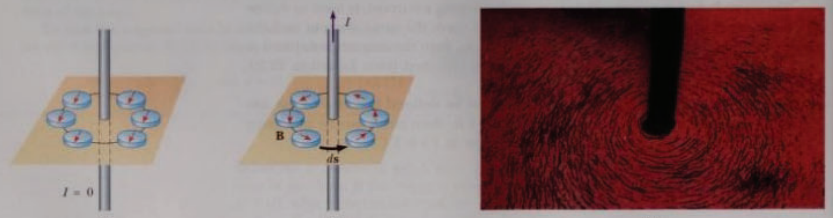


Figure 22.18 (a) When there is no current in the vertical wire, all compass needles point in the same direction. (b) When the wire carries a strong current, the compass needles deflect in a direction tangent to the circle, which is the direction of **B** due to the current. (c) Circular magnetic field lines surrounding a current-carrying conductor as displayed with iron filings. The photograph was taken using 30 parallel wires each carrying a current of 0.50 A. *(Henry Leap and Jim Lehman)*

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I \qquad [22.21]$$

where $\oint ds = 2\pi r$ is the circumference of the circle.

This result, known as **Ampère's law,** was calculated for the special case of a circular path surrounding a wire. However, it can also be applied in the general case in which a steady current passes through the area surrounded by an arbitrary closed path. That is, Ampère's law says that the line integral of $\mathbf{B} \cdot d\mathbf{s}$ around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path.

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \qquad [22.22] \quad \bullet \textit{ Ampère's law}$$

Ampère's law is valid only for steady currents. Furthermore, even though Ampère's law is true for all current configurations, it is useful only for calculating the magnetic fields of configurations with high degrees of symmetry. (Recall that Gauss's law is useful only for calculating the electric fields of highly symmetric charge distributions.) The following examples illustrate some symmetric current configurations for which Ampère's law is useful.

Example 22.5 The Magnetic Field Created by a Long Current-Carrying Wire

A long, straight wire of radius R carries a steady current I_0 that is uniformly distributed through the cross-section of the wire (Fig. 22.19). Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.

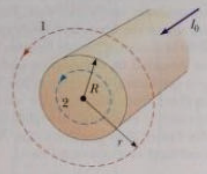


Figure 22.19 (Example 22.5) A long, straight wire of radius R carrying a steady current I_0 uniformly distributed across the wire. The magnetic field at any point can be calculated from Ampère's law using a circular path of radius r, concentric with the wire.

Solution In Region 1, where $r \geq R$, let us choose for our path of integration a circle of radius r centered at the wire. From symmetry, we see that **B** must be constant in magnitude and parallel to $d\mathbf{s}$ at every point on this circle. Because the total current passing through the plane of the circle is I_0, Ampère's law applied to the circle gives

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = B(2\pi r) = \mu_0 I_0$$

$$B = \frac{\mu_0 I_0}{2\pi r} \qquad (\text{for } r \geq R)$$

Now consider the interior of the wire—that is, Region 2—where $r < R$. Here the current I passing through the plane of the circle of radius $r < R$ is less than the total current I_0. Because the current is uniform over the cross-section of the wire, the fraction of the current enclosed by the circle of radius $r < R$ must equal the ratio of the area πr^2 enclosed by Circle 2 and the cross-sectional area πR^2 of the wire.

$$\frac{I}{I_0} = \frac{\pi r^2}{\pi R^2}$$

$$I = \frac{r^2}{R^2} I_0$$

Following the same procedure as for Circle 1, we apply Ampère's law to Circle 2:

$$\oint \mathbf{B} \cdot d\mathbf{s} = B(2\pi r) = \mu_0 I = \mu_0 \left(\frac{r^2}{R^2} I_0\right)$$

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$$B = \left(\frac{\mu_0 I_0}{2\pi R^2}\right) r \quad \text{(for } r < R\text{)} \qquad [22.23]$$

The magnetic field strength versus r for this configuration is sketched in Figure 22.20. Note that inside the wire, $B \to 0$ as $r \to 0$. This result is similar in form to that of the electric field inside a uniformly charged rod.

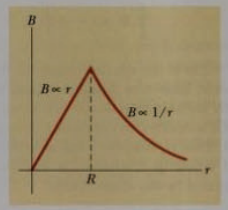


Figure 22.20 A sketch of the magnetic field versus r for the wire described in Example 22.5. The field is proportional to r inside the wire and varies as $1/r$ outside the wire.

Example 22.6 The Magnetic Field Created by a Toroid

A toroid consists of N turns of wire wrapped around a ring-shaped structure, as in Figure 22.21. You can think of a toroid as a solenoid bent into the shape of a doughnut. Assuming that the turns are closely spaced, calculate the magnetic field inside the toroid, a distance r from the center.

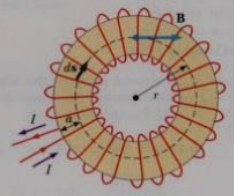


Figure 22.21 (Example 22.6) A toroid consists of many turns of wire wrapped around a doughnut-shaped structure (called a torus). If the coils are closely spaced, the field in the interior of the toroid is tangent to the dashed circle and varies as $1/r$, and the exterior field is zero.

Reasoning To calculate the field inside the toroid, we evaluate the line integral of $\mathbf{B} \cdot d\mathbf{s}$ over a circle of radius r. By symmetry, we see that the magnetic field is constant in magnitude on this circle and tangent to it, so that $\mathbf{B} \cdot d\mathbf{s} = B\, ds$.

Furthermore, note that the closed path threads N loops of wire, each of which carries a current I. Therefore, the right side of Equation 22.22 is $\mu_0 NI$ in this case.

Solution Ampère's law applied to the circle gives

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r} \qquad [22.24]$$

This result shows that B varies as $1/r$ and hence is nonuniform within the coil. However, if r is large compared with the cross-sectional radius of the toroid, then the field is approximately uniform inside the coil. Furthermore, for an ideal toroid, where the turns are closely spaced, the external magnitude field is zero. This can be seen by noting that the net current threaded by any circular path lying outside the toroid is zero (including the region of the "hole in the doughnut"). Therefore, from Ampère's law we find that $B = 0$ in the regions exterior to the coil. In reality, the turns of a toroid form a helix rather than circular loops. As a result, there is always a small field external to the coil.

Thinking Physics 5

Consider a plastic annular ring encircling a long, straight wire, which is coming out of the page in Figure 22.22. On the plastic ring are fastened two bar magnets, as shown. If a current flows in the wire in a direction out of the page, is there a net torque on the ring–magnet combination? If so, in which direction?

Reasoning The small magnetic dipoles that form the bar magnets will experience a magnetic torque due to the magnetic field set up by the wire. The torque on each magnet can be represented by a force at the south pole acting perpendicular to its body, and an oppositely directed force at the north pole. Since the circular magnetic field generated by the wire will fall off inversely with distance, the force on the north poles will be smaller than the force on the south poles. However, because the moment arms of these forces increase linearly with distance from the wire, the effects of decreases in field strength with distance is balanced by increasing moment arms. Consequently, there is no net torque on the ring–magnet combination.

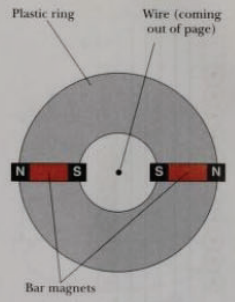


Figure 22.22 (Thinking Physics 5)

22.8 · THE MAGNETIC FIELD OF A SOLENOID

A solenoid is a long wire wound in the form of a helix. If the turns are closely spaced, this configuration can produce a reasonably uniform magnetic field within a small volume of the solenoid's interior region. Each of the turns can be regarded as a circular loop, and the net magnetic field is the vector sum of the fields due to all the turns.

Figure 22.23a shows the magnetic field lines of a loosely wound solenoid. Note that the interior field lines are nearly parallel, uniformly distributed, and close together. This indicates that the field in the solenoid's interior is uniform. The field lines between turns tend to cancel each other. The field outside the solenoid is both nonuniform and weak. To see why, consider the point P in Figure 22.23a. The field is nonuniform because of the finite length of the solenoid and a resultant field distribution that resembles that of a bar magnet. The field is weak because the field due to current elements on the upper portions tends to cancel the field due to current elements on the lower portions.

If the turns are closely spaced and the solenoid is of finite length, the field lines are as shown in Figure 22.23b. In this case, the field lines diverge from one end

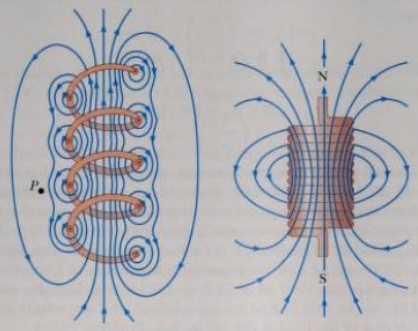


Figure 22.23 (a) The magnetic field lines for a loosely wound solenoid. (b) Magnetic field lines for a tightly wound solenoid of finite length carrying a steady current. The field in the space enclosed by the solenoid is nearly uniform and strong. Note that the field lines resemble those of a bar magnet, so that the solenoid effectively has north and south poles.

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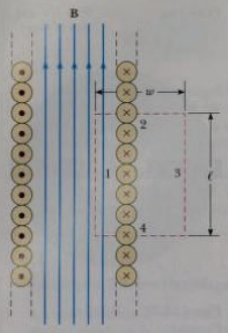


Figure 22.24 A cross-sectional view of a tightly wound solenoid. If the solenoid is long relative to its radius, we can assume that the magnetic field inside is uniform and the field outside is zero. Ampère's law applied to the red dashed rectangular path can then be used to calculate the field inside the solenoid.

and converge at the opposite end. An inspection of this field distribution exterior to the solenoid shows a similarity to the field of a bar magnet (see Fig. 22.1b). Hence, one end of the solenoid behaves like the north pole of a magnet and the opposite end behaves like the south pole. As the length of the solenoid increases, the field within it becomes more and more uniform. When the solenoid's turns are closely spaced and its length is great compared with its radius, it approaches the case of an *ideal solenoid*. The field outside the solenoid is weak compared with the field inside, and the field inside is uniform over a large volume.

We can use Ampère's law to obtain an expression for the magnetic field inside an ideal solenoid. A longitudinal cross-section of part of our ideal solenoid (Fig. 22.24) carries current I. $\mathbf{B}$ inside the ideal solenoid is uniform and parallel to the axis, and $\mathbf{B}$ outside is zero. Consider a rectangular path of length ℓ and width w, as shown in Figure 22.24. We can apply Ampère's law to this path by evaluating the integral of $\mathbf{B}\cdot d\mathbf{s}$ over each of the four sides of the rectangle. The contribution along side 3 is clearly zero, because $\mathbf{B} = 0$ in this region. The contributions from sides 2 and 4 are both zero, because $\mathbf{B}$ is perpendicular to $d\mathbf{s}$ along these paths. Side 1, the length of which is ℓ, gives a contribution of $B\ell$ to the integral, because $\mathbf{B}$ along this path is uniform and parallel to $d\mathbf{s}$. Therefore, the integral over the closed rectangular path has the value

$$\oint \mathbf{B}\cdot d\mathbf{s} = \int_{\text{path 1}} \mathbf{B}\cdot d\mathbf{s} = B\int_{\text{path 1}} ds = B\ell$$

The right side of Ampère's law involves the *total* current that passes through the area bound by the path of integration. In our case, the total current through the rectangular path equals the current through each turn of the solenoid multiplied by the number of turns. If N is the number of turns in the length ℓ, then the total current through the rectangle equals NI. Therefore, Ampère's law applied to this path gives

$$\oint \mathbf{B}\cdot d\mathbf{s} = B\ell = \mu_0 NI$$

Magnetic field inside a long solenoid •

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 nI \qquad [22.25]$$

where $n = N/\ell$ is the number of turns *per unit length* (not to be confused with N).

We also could obtain this result in a simpler manner by reconsidering the magnetic field of a toroidal coil (Ex. 22.6). If the radius, r, of the toroidal coil containing N turns is large compared with its cross-sectional radius, a, then a short section of the toroidal coil approximates a solenoid, with $n = N/2\pi r$. In this limit, we see that Equation 22.24 derived for the toroidal coil agrees with Equation 22.25.

Equation 22.25 is valid only for points near the center of a very long solenoid. As you might expect, the field near each end is smaller than the value given by Equation 22.25. At the very end of a long solenoid, the magnitude of the field is about one half that of the field at the center (see Problem 22.32).

CONCEPTUAL PROBLEM 8

A hanging Slinky toy is attached to a powerful battery and a switch. When the switch is closed so that current suddenly flows through the Slinky, does the Slinky compress or expand?

EXERCISE 8 A closely wound, long solenoid of overall length 30.0 cm has a magnetic field of magnitude 5.00×10^{-4} T at its center produced by a current of 1.00 A through its windings. How many turns of wire are on the solenoid? Answer 120 turns

22.9 • MAGNETISM IN MATTER

OPTIONAL

The magnetic field produced by a current in a coil of wire gives us a hint regarding what causes certain materials to exhibit strong magnetic properties. In order to understand why some materials are magnetic, it is instructive to begin our discussion with the classical model of the atom, in which electrons are assumed to move in circular orbits around the much more massive nucleus. In this model, each electron, with its charge of 1.6×10^{-19} C, circles the atom once in about 10^{-16} s. If we divide the electronic charge by this time interval, we find that the orbiting electron is equivalent to a current of 1.6×10^{-3} A. Therefore, each orbiting electron is viewed as a tiny current loop with a corresponding magnetic moment. Such a current loop produces a magnetic field of about 12.5 T at the center of the circular path (see Problem 16).

Consider an electron moving with a constant speed of v in a circular orbit of radius r about the nucleus, as in Figure 22.25. Because the electron travels a distance of $2\pi r$ (the circumference of the circle) in the time T, which is the time required for one revolution, the electron's orbital speed is $v = 2\pi r/T$. The current associated with this orbiting electron equals its charge divided by the time for one revolution:

$$I = \frac{e}{T} = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r}$$

The magnitude of the magnetic moment associated with this current loop is given by $\mu = IA$, where $A = \pi r^2$ is the area of the orbit. Therefore,

$$\mu = IA = \left(\frac{ev}{2\pi r}\right)\pi r^2 = \tfrac{1}{2}evr$$

Because the magnitude of the orbital angular momentum of the electron is $L = m_e vr$ (Eq. 11.24), the magnitude of the magnetic moment can be written as

$$\mu = \left(\frac{e}{2m_e}\right) L \qquad [22.26]$$

• *Orbital magnetic moment*

This result says that **the magnetic moment of an orbiting electron is proportional to its orbital angular momentum.** Note that, because the electron is negatively charged, the vectors $\boldsymbol{\mu}$ and $\mathbf{L}$ point in opposite directions. Both vectors are perpendicular to the plane of the orbit, as indicated in Figure 22.25.

As we learned in Chapter 11, Section 11.9, the Bohr theory of the atom includes the notion that the orbital angular momentum must be quantized and is always some integer multiple of $\hbar = h/2\pi = 1.06 \times 10^{-34}$ J·s, where h is Planck's constant. That is,

• *Angular momentum is quantized.*

$$L = 0, \hbar, 2\hbar, 3\hbar, \ldots$$

Hence, the smallest nonzero value of the magnetic moment of an orbiting electron is

$$\mu = \frac{e}{2m_e}\hbar \qquad [22.27]$$

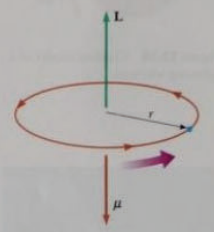


Figure 22.25 An electron moving in a circular orbit of radius r has an angular momentum $\mathbf{L}$ in one direction and a magnetic moment $\boldsymbol{\mu}$ in the opposite direction.

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Figure 22.26 Classical model of a spinning electron.

In most substances, the magnetic moment of one electron in an atom is canceled by that of another electron in the atom, orbiting in the opposite direction. The net result is that **the magnetic effect produced by the orbital motion of the electrons is either zero or very small for most materials.**

As mentioned earlier, an electron has another intrinsic property called **spin,** which also contributes to its magnetic moment. The spin of an electron is a separate angular momentum from its orbital angular momentum, just as the spin of the Earth is separate from its orbital motion around the Sun. One can model the electron as a sphere of charge spinning around its axis as it orbits the nucleus, as in Figure 22.26. (This classical description of a spinning electron should not be taken literally. The property of spin can be understood only through a relativistic model.) This spinning motion produces an effective current loop and hence a magnetic moment of the same order of magnitude as that due to the orbital motion. The magnitude of the spin angular momentum predicted by quantum theory is

$$S = \frac{\hbar}{2} = 5.2729 \times 10^{-35}\ \text{J}\cdot\text{s}$$

Every electron has an intrinsic magnetic moment associated with its spin with the value

Bohr magneton •

$$\mu_B = \frac{e}{2m_e}\hbar = 9.27 \times 10^{-24}\ \text{J/T} \qquad [22.28]$$

which is called the **Bohr magneton.**

In atoms or ions containing multiple electrons, the electrons usually pair up with their spins opposite each other, an arrangement that results in a cancellation of the spin magnetic moments. However, an atom with an odd number of electrons must have at least one "unpaired" electron and a corresponding spin magnetic moment. The magnetic moments of several atoms and ions are listed in Table 22.1.

TABLE 22.1
Magnetic Moments of Some Atoms and Ions (in Units of Bohr Magnetons)

Atom (or ion)	Magnet Moment per Atom or Ion (μ/μ_B)
H	1
He	0
Ne	0
Fe	2.22
Co	1.72
Ni	0.606
Gd	7.1
Dy	10.0
Co^{2+}	4.8
Ni^{2+}	3.2
Fe^{2+}	5.4
Ce^{3+}	2.14
Yb^{3+}	4.00

Ferromagnetic Materials

Iron, cobalt, nickel, gadolinium, and dysprosium are strongly magnetic materials and are said to be **ferromagnetic.** Ferromagnetic substances, used to fabricate permanent magnets, contain atoms with spin magnetic moments that tend to align parallel to each other even in a weak external magnetic field. Once the moments are aligned, the substance remains magnetized after the external field is removed. This permanent alignment is due to strong coupling between neighboring atoms, which can only be understood using quantum physics.

All ferromagnetic materials contain microscopic regions called **domains,** within which all magnetic moments are aligned. The domains range from about 10^{-12} to $10^{-8}\ \text{m}^3$ in volume and contain 10^{17} to 10^{21} atoms. The boundaries between domains having different orientations are called **domain walls.** In an unmagnetized sample, the domains are randomly oriented so that the net magnetic moment is zero, as in Figure 22.27a. When the sample is placed in an external magnetic field, the domains tend to align with the field, which results in a magnetized sample, as in Figure 22.27b. Observations show that domains initially oriented along the external field grow in size at the expense of the less favorably oriented domains. When the external field is removed, the sample may retain most of its magnetism.

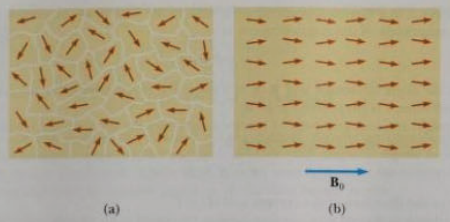


Figure 22.27 (a) Random orientation of domains in an unmagnetized substance. (b) When an external magnetic field, $\mathbf{B}_0$, is applied, the domains tend to align with the magnetic field.

The extent to which a ferromagnetic substance retains its magnetism depends on whether it is classified as being magnetically **hard** or **soft.** *Soft magnetic materials,* such as iron, are easily magnetized but also tend to lose their magnetism easily. When a soft magnetic material is magnetized and the external magnetic field is removed, thermal agitation produces domain motion and the material quickly returns to an unmagnetized state. In contrast, *hard magnetic materials,* such as cobalt and nickel, are difficult to magnetize but tend to retain their magnetism, and domain alignment persists in them after the external magnetic field is removed. Such hard magnetic materials are referred to as **permanent magnets.** Rare-earth permanent magnets, such as samarium-cobalt, are now regularly used in industry.

Thinking Physics 6

Suppose an aluminum tube is filled with very fine iron filings. Each filing is in the form of a sliver—long and thin. If the tube of filings is shaken, what is the likelihood that all of them will wind up aligned in the same direction? Suppose the shaking is repeated while the tube is in a strong magnetic field. What is the likelihood now that the filings will be aligned? Is this a macroscopic analog to anything discussed in this section?

Reasoning When the tube is shaken outside of a magnetic field, the orientation of the filings will be random—it is extremely unlikely that they will all be aligned in the same direction. If the shaking occurs in a magnetic field, the filings, which become magnetized, can align with the field while they are momentarily airborne and free to rotate. Thus, it is likely that a large number of the filings will be aligned with the field after the shaking. The filings are an analog to magnetic domains in a ferromagnetic material. When the material is placed in a magnetic field, more domains end up aligned with the field than misaligned, similar to the iron filings.



Oxygen, a paramagnetic substance, is attracted to a magnetic field. The liquid oxygen in this photograph is suspended between the poles of a permanent magnet. Paramagnetic substances contain atoms (or ions) that have permanent magnetic dipole moments. These dipoles interact weakly with each other and are randomly oriented in the absence of an external magnetic field. When the substance is placed in an external magnetic field, its atomic dipoles tend to align with the field. *(Courtesy of Leon Lewandowski)*

SUMMARY

The **magnetic force** that acts on a charge, q, moving with a velocity of $\mathbf{v}$ in an external magnetic field, $\mathbf{B}$, is

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \qquad [22.1]$$

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This force is in a direction perpendicular both to the velocity of the particle and to the field. The magnitude of the magnetic force is

$$F = qvB \sin \theta \qquad [22.2]$$

where θ is the angle between $\mathbf{v}$ and $\mathbf{B}$. $\mathbf{F} = 0$ when $\mathbf{v}$ is either parallel or antiparallel to $\mathbf{B}$, and $F = qvB$ when $\mathbf{v}$ is perpendicular to $\mathbf{B}$.

If a straight conductor of length ℓ carries a current, I, the force on that conductor when placed in a uniform *external* magnetic field, $\mathbf{B}$, is

$$\mathbf{F} = I\boldsymbol{\ell} \times \mathbf{B} \qquad [22.3]$$

where $\boldsymbol{\ell}$ is in the direction of the current and $|\boldsymbol{\ell}| = \ell$.

If an arbitrarily shaped wire carrying current I is placed in an *external* magnetic field, the force on a very small segment, $d\mathbf{s}$, is

$$d\mathbf{F} = I\, d\mathbf{s} \times \mathbf{B} \qquad [22.4]$$

To determine the total force on the wire, one has to integrate Equation 22.4.

The **magnetic moment, $\boldsymbol{\mu}$,** of a current loop carrying current I is

$$\boldsymbol{\mu} = I\mathbf{A} \qquad [22.8]$$

where $\mathbf{A}$ is perpendicular to the plane of the loop and $|\mathbf{A}|$ is equal to the area of the loop. The SI unit of $\boldsymbol{\mu}$ is the ampere-meter^2.

The torque, $\boldsymbol{\tau}$, on a current loop when the loop is placed in a uniform *external* magnetic field, $\mathbf{B}$, is

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} \qquad [22.9]$$

The **Biot–Savart law** says that the magnetic field $d\mathbf{B}$ at the point P, due to the wire element $d\mathbf{s}$ carrying a steady current, I, is

$$d\mathbf{B} = k_m \frac{I\, d\mathbf{s} \times \hat{\mathbf{r}}}{r^2} \qquad [22.10]$$

where $k_m = 10^{-7}\ \mathrm{T \cdot m/A}$ and r is the distance from the element to the point P. To find the total field at P due to a current-carrying conductor, one must integrate this vector expression over the entire conductor.

The magnitude of the **magnetic field** at a distance of r from a long, straight wire carrying current I is

$$B = \frac{\mu_0 I}{2\pi r} \qquad [22.15]$$

where $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$ is the **permeability of free space.** The field lines are circles concentric with the wire.

The force per unit length between two parallel wires separated by a distance of a and carrying currents I_1 and I_2 has the magnitude

$$\frac{F_1}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a} \qquad [22.20]$$

The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

Ampère's law says that the line integral of $\mathbf{B} \cdot d\mathbf{s}$ around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \qquad [22.22]$$

Using Ampère's law, one finds that the fields inside a toroidal coil and solenoid are

$$B = \frac{\mu_0 NI}{2\pi r} \quad \text{(toroid)} \qquad [22.24]$$

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 nI \quad \text{(solenoid)} \qquad [22.25]$$

where N is the total number of turns and n is the number of turns per unit length.

The fundamental sources of all magnetic fields are the magnetic dipole moments associated with atoms. The atomic dipole moments can arise both from the orbital motions of the electrons and from an intrinsic property of electrons known as *spin*.

CONCEPTUAL QUESTIONS

1. Two charged particles are projected into a region in which there is a magnetic field perpendicular to their velocities. If the charges are deflected in opposite directions, what can you say about them?
2. A current-carrying conductor experiences no magnetic force when placed in a certain manner in a uniform magnetic field. Explain.
3. Is it possible to orient a current loop in a uniform magnetic field so that the loop does not tend to rotate? Explain.
4. The north-seeking pole of a magnet is attracted toward the geographic North Pole of the Earth. Yet, like poles repel. What is the way out of this dilemma?
5. Which way would a compass point if you were at the north magnetic pole?
6. A magnet attracts a piece of iron. The iron can then attract another piece of iron. On the basis of domain alignment, explain what happens in each piece of iron.
7. Will a nail be attracted to either pole of a magnet? Explain what is happening inside the nail when placed near the magnet.
8. Why does hitting a magnet with a hammer cause the magnetism to be reduced?
9. Parallel wires exert magnetic forces on each other. What about perpendicular wires? Imagine two wires oriented perpendicular to each other and almost touching. Each wire carries a current. Is there a force between the wires?
10. Explain why it is not possible to determine the charge and mass of a charged particle separately via electric and magnetic forces.
11. How can a current loop be used to determine the presence of a magnetic field in a given region of space?
12. It is found that charged particles from outer space, called cosmic rays, strike the Earth more frequently at the poles than at the Equator. Why?
13. Explain why two parallel wires carrying currents in opposite directions repel each other.
14. A hollow copper tube carries a current. Why is $\mathbf{B} = 0$ inside the tube? Is $\mathbf{B}$ nonzero outside the tube?
15. Describe the change in the magnetic field in the space enclosed by a solenoid carrying a steady current I if (a) the length of the solenoid is doubled but the number of turns remains the same and (b) the number of turns is doubled but the length remains the same.
16. Consider an electron near the magnetic equator. In which direction will it tend to be deflected if its velocity is directed (a) downward? (b) northward? (c) westward? (d) southeastward?
17. Figure Q22.17 shows two permanent magnets, each having a hole through its center. Note that the upper magnet is levitated above the lower one. (a) How does this occur? (b) What purpose does the pencil serve? (c) What can you say about the poles of the magnets from this observation? (d) If the upper magnet were inverted, what do you suppose would happen?



Figure Q22.17 Magnetic levitation using two ceramic magnets. *(Courtesy of Central Scientific Company)*

18. You are an astronaut stranded on a planet with no test equipment for minerals. The planet does not have a mag-

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netic field. You have two bars of iron in your possession: One is magnetized, one is not. How could you determine which is which?

19. A Hindu ruler once suggested that he be entombed in a magnetic coffin with the polarity arranged so that he would be forever suspended between heaven and Earth. Is such magnetic levitation possible? Discuss.
20. Should the surface of a computer disk be made from a hard or a soft ferromagnetic substance?
21. The *bubble chamber* is a device used for observing tracks of particles that pass through the chamber, which is immersed in a magnetic field. If some tracks are spirals and others are straight lines, what can you say about the particles?
22. The electron beam in Figure Q22.22 is projected to the right. The beam deflects downward in the presence of a magnetic field produced by a pair of current-carrying coils. (a) What is the direction of the magnetic field? (b) What would happen to the beam if the current in the coils were reversed?

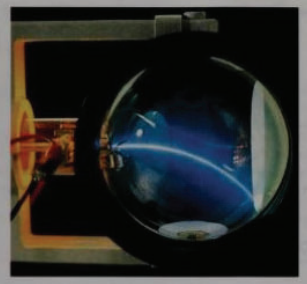


Figure Q22.22 Bending of a beam of electrons in a magnetic field. *(Courtesy of Central Scientific Company)*

PROBLEMS

Section 22.2 The Magnetic Field

1. Determine the initial direction of the deflection of charged particles as they enter the magnetic fields, as shown in Figure P22.1.

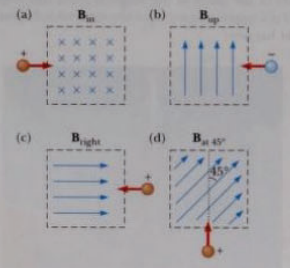


Figure P22.1

2. A proton travels with a speed of 3.00×10^6 m/s at an angle of 37.0° with the direction of a magnetic field of 0.300 T in the positive y direction. What are (a) the magnitude of the magnetic force on the proton and (b) its acceleration?
3. A proton moves perpendicular to a uniform magnetic field $\mathbf{B}$ at 1.00×10^7 m/s and experiences an acceleration of 2.00×10^{13} m/s^2 in the positive x direction when its velocity is in the positive z direction. Determine the magnitude and direction of the field.
4. An electron is accelerated through 2400 V from rest and then enters a region in which there is a uniform 1.70-T magnetic field. What are (a) the maximum and (b) the minimum values of the magnetic force this charge can experience?
5. At the Equator, near the surface of the Earth, the magnetic field is approximately 50.0 μT northward, and the electric field is about 100 N/C downward in fair weather. Find the gravitational, electric, and magnetic forces on an electron moving eastward in a straight line at 6.00×10^6 m/s in this environment.
6. A metal ball having net charge $Q = 5.00\ \mu$C is thrown out of a window horizontally at a speed $v = 20.0$ m/s. The window is at a height $h = 20.0$ m above the ground. A uniform horizontal magnetic field of magnitude $B = 0.0100$ T is perpendicular to the plane of the ball's trajectory. Find the magnetic force acting on the ball just before it hits the ground.
7. At the Fermilab accelerator in Batavia, Illinois, protons having momentum 4.80×10^{-16} kg·m/s are held in a circular orbit of radius 1.00 km by an upward magnetic field. What is the magnitude of this field?

Section 22.3 Magnetic Force on a Current-Carrying Conductor

8. A wire carries a steady current of 2.40 A. A straight section of the wire is 0.750 m long and lies along the x axis within a uniform magnetic field, $\mathbf{B} = (1.60\mathbf{k})$ T. If the current is in the positive x direction, what is the magnetic force on the section of wire?
9. A wire having a mass per unit length of 0.500 g/cm carries a 2.00-A current horizontally to the south. What are the direction and magnitude of the minimum magnetic field needed to lift this wire vertically upward?
10. In Figure P22.10, the cube is 40.0 cm on each edge. Four straight segments of wire—*ab*, *bc*, *cd*, and *da*—form a closed loop that carries a current $I = 5.00$ A, in the direction shown. A uniform magnetic field, $B = 0.0200$ T, is in the positive y direction. Determine the magnitude and direction of the magnetic force on each segment.

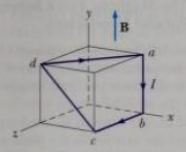


Figure P22.10

11. A strong magnet is placed under a horizontal conducting ring of radius r that carries current I, as in Figure P22.11. If, at the ring's location, the magnetic field $\mathbf{B}$ makes an angle θ with the vertical, what are the magnitude and direction of the resultant force on the ring?

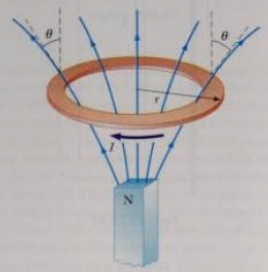


Figure P22.11

12. An arbitrarily shaped, closed loop carrying current I is placed in a uniform external magnetic field, $\mathbf{B}$, as in Figure P22.12. Show that the net magnetic force on the loop is zero.

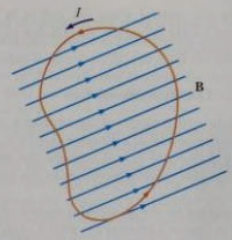


Figure P22.12

Section 22.4 Torque on a Current Loop in a Uniform Magnetic Field

13. A rectangular loop consists of $N = 100$ closely wrapped turns and has dimensions $a = 0.400$ m and $b = 0.300$ m. The loop is hinged along the y axis, and its plane makes an angle $\theta = 30.0°$ with the x axis (Fig. P22.13). What is the magnitude of the torque exerted on the loop by a uniform magnetic field $B = 0.800$ T directed along the x axis when the current is $I = 1.20$ A in the direction shown? What is the expected direction of rotation of the loop?

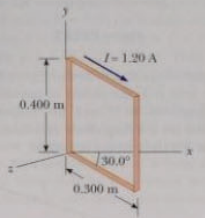


Figure P22.13

14. A small bar magnet is suspended in a uniform 0.250-T magnetic field. The maximum torque experienced by the bar magnet is 4.60×10^{-3} N·m. Calculate the magnetic moment of the bar magnet.
15. A long piece of wire of mass 0.100 kg and total length of 4.00 m is used to make a square coil with side 0.100 m. The coil is hinged along a horizontal side, carries a 3.40-A current, and is placed in a vertical magnetic field of magnitude 0.0100 T. (a) Determine the angle that the plane of the coil

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