makes with the vertical when the coil is in equilibrium. (b) Find the torque acting on the coil due to the magnetic force at equilibrium.

Section 22.5 The Biot–Savart Law

16. In Niels Bohr's 1913 model of the hydrogen atom, an electron circles the proton at a distance of 5.29×10^{-11} m with a speed of 2.19×10^6 m/s. Compute the magnetic field strength that this motion produces at the location of the proton.

17. (a) A conductor in the shape of a square of edge length $\ell = 0.400$ m carries a current $I = 10.0$ A (Fig. P22.17). Calculate the magnitude and direction of the magnetic field at the center of the square. (*Hint:* Use the result to Problem 22.51a to find the field due to each straight section.) (b) If this conductor is formed into a single circular turn and carries the same current, what is the value of the magnetic field at the center?

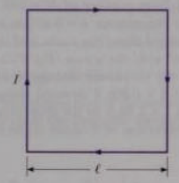


Figure P22.17

18. A current path shaped as shown in Figure P22.18 produces a magnetic field at P, the center of the arc. If the arc subtends an angle of 30.0° and the radius of the arc is 0.600 m, what are the magnitude and direction of the field produced at P if the current is 3.00 A?

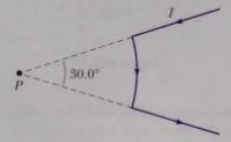


Figure P22.18

19. Determine the magnetic field at a point P located a distance x from the corner of an infinitely long wire bent at a right angle, as in Figure P22.19. The wire carries a steady current I.

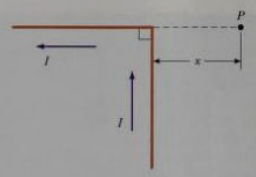


Figure P22.19

Section 22.6 The Magnetic Force between Two Parallel Conductors

20. Two long, parallel conductors, separated by 10.0 cm, carry currents in the same direction. If $I_1 = 5.00$ A and $I_2 = 8.00$ A, what is the force per unit length exerted on each conductor by the other?

21. In Figure P22.21, the current in the long, straight wire is $I_1 = 5.00$ A and the wire lies in the plane of the rectangular loop, which carries 10.0 A. The dimensions are $c = 0.100$ m, $a = 0.150$ m, and $\ell = 0.450$ m. Find the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire.

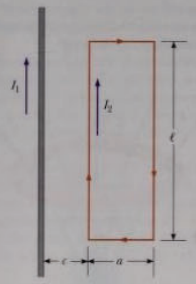


Figure P22.21

22. Two long, parallel wires, each having a mass per unit length of 40.0 g/m, are supported in a horizontal plane by strings 6.00 cm long, as in Figure P22.22. Each wire carries the same current I, causing the wires to repel each other so that the angle θ between the supporting strings is 16.0°. (a) Are the currents in the same or opposite directions? (b) Find the magnitude of each current.

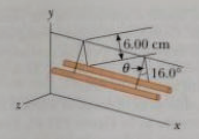


Figure P22.22

Section 22.7 Ampère's Law

23. Four long, parallel conductors carry equal currents of $I = 5.00$ A. Figure P22.23 is an end view of the conductors. The current direction is into the page at points A and B (indicated by the crosses) and out of the page at C and D (indicated by the dots). Calculate the magnitude and direction of the magnetic field at point P, located at the center of the square of edge length 0.200 m.

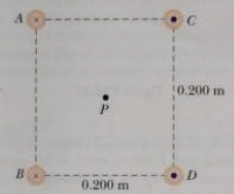


Figure P22.23

24. The magnetic field 40.0 cm away from a long, straight wire carrying current 2.00 A is 1.00 μT. (a) At what distance is it 0.100 μT? (b) At one instant, the two conductors in a long household extension cord carry equal 2.00-A currents in opposite directions. The two wires are 3.00 mm apart. Find the magnetic field 40.0 cm away from the middle of the cord, in the plane of the two wires. (c) At what distance is it one tenth as large? (d) The center wire in a coaxial cable carries current 2.00 A in one direction and the sheath around it carries current 2.00 A in the opposite direction. What magnetic field does the cable create at points outside?

25. The magnetic coils of a tokamak fusion reactor are in the shape of a toroid having an inner radius of 0.700 m and an outer radius of 1.30 m. If the toroid has 900 turns of large-diameter wire, each of which carries a current of 14.0 kA, find the magnetic field strength along (a) the inner radius and (b) the outer radius.

26. A cylindrical conductor of radius $R = 2.50$ cm carries a current of $I = 2.50$ A along its length; this current is uniformly distributed throughout the cross section of the conductor. (a) Calculate the magnetic field midway along the radius of the wire (that is, at $r = R/2$). (b) Find the distance beyond the surface of the conductor at which the magnitude of the magnetic field has the same value as the magnitude of the field at $r = R/2$.

27. A packed bundle of 100 long, straight, insulated wires forms a cylinder of radius $R = 0.500$ cm. (a) If each wire carries 2.00 A, what are the magnitude and direction of the magnetic force per unit length acting on a wire located 0.200 cm from the center of the bundle? (b) Would a wire on the outer edge of the bundle experience a force greater or smaller than the value calculated in part (a)?

28. Niobium metal becomes a superconductor when cooled below 9 K. If superconductivity is destroyed when the surface magnetic field exceeds 0.100 T, determine the maximum current a 2.00-mm-diameter niobium wire can carry and remain superconducting.

Section 22.8 The Magnetic Field of a Solenoid

29. What current is required in the windings of a long solenoid that has 1000 turns uniformly distributed over a length of 0.400 m in order to produce at the center of the solenoid a magnetic field of magnitude 1.00×10^{-4} T?

30. A superconducting solenoid is to generate a magnetic field of 10.0 T. (a) If the solenoid winding has 2000 turns/m, what is the required current? (b) What force per unit length is exerted on the windings by this magnetic field?

31. A solenoid of radius $R = 5.00$ cm is made of a long piece of wire of radius $r = 2.00$ mm, length $\ell = 10.0$ m ($\ell \gg R$), and resistivity $\rho = 1.70 \times 10^{-8}\ \Omega\cdot$m. Find the magnetic field at the center of the solenoid if the wire is connected to a battery having an emf $\mathcal{E} = 20.0$ V.

32. Consider a solenoid of length ℓ and radius R, containing N closely spaced turns and carrying a steady current I. (a) Using these parameters, find the magnetic field along the axis as a function of distance a from the end of the solenoid. (b) Show that as ℓ becomes very long, B approaches $\mu_0 NI/2\ell$ at each end of the solenoid.

Section 22.9 Magnetism in Matter (Optional)

33. The magnetic moment of the Earth is approximately 8.00×10^{22} A·m^2. (a) If this were caused by the complete magnetization of a huge iron deposit, how many unpaired electrons would this correspond to? (b) At two unpaired electrons per iron atom, how many kilograms of iron would this correspond to? (The density of iron is 7900 kg/m^3, and there are approximately 8.50×10^{28} iron atoms/m^3.)

34. In Bohr's 1913 model of the hydrogen atom, the electron is in a circular orbit of radius 5.29×10^{-11} m, and its speed is 2.19×10^6 m/s. (a) What is the magnitude of the mag-

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netic moment due to the electron's motion? (b) If the electron orbits counterclockwise in a horizontal circle, what is the direction of this magnetic moment vector?

Additional Problems

35. A lightning bolt may carry a current of 1.00×10^4 A for a short period of time. What is the resulting magnetic field 100 m from the bolt? Assume that the bolt extends far above and below the point of observation.
36. A long, straight wire lies on a horizontal table and carries a current of 1.20 μA. At a distance d above the wire, a proton moves parallel to the wire (opposite the current) with a constant velocity of 2.30×10^4 m/s. Determine the value of d. You may ignore the magnetic field due to the Earth.
37. The magnitude of the Earth's magnetic field at either pole is approximately 7.00×10^{-5} T. Using a model in which you assume that this field is produced by a current loop around the Equator, determine the current that would generate such a field. (Note that the radius of the Earth is $R_E = 6.37 \times 10^6$ m.)
38. An electron enters a region of magnetic field of magnitude 0.100 T, traveling perpendicular to the linear boundary of the region. The direction of the field is perpendicular to the velocity of the electron. (a) Determine the time it takes for the electron to leave the "field-filled" region, noting that its path is a semicircle. (b) Find the kinetic energy of the electron if the maximum depth of penetration in the field is 2.00 cm.
39. Two parallel conductors carry current in opposite directions, as shown in Figure P22.39. One conductor carries a current of 10.0 A. Point A is at the midpoint between the wires and point C is a distance $d/2$ to the right of the 10.0-A current. If $d = 18.0$ cm and I is adjusted so that the magnetic field at C is zero, find (a) the value of the current I and (b) the value of the magnetic field at A.

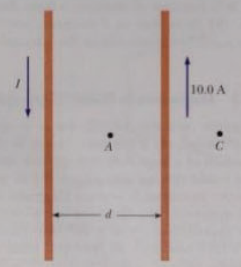


Figure P22.39

40. Suppose you install a compass on the center of the dashboard of a car. Compute an order-of-magnitude estimate for the magnetic field at this location produced by the current when you switch on the headlights. How does it compare with the Earth's magnetic field?
41. A very long, thin strip of metal of width w carries a current I along its length, as in Figure P22.41. Find the magnetic field in the plane of the strip (at an external point P) a distance b from one edge.

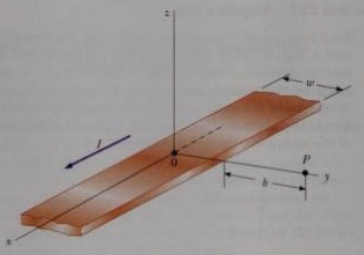


Figure P22.41

42. A 0.200-kg metal rod carrying a current of 10.0 A glides on two horizontal rails 0.500 m apart. What vertical magnetic field is required to keep the rod moving at a constant speed if the coefficient of kinetic friction between the rod and rails is 0.100?
43. A nonconducting ring of radius 10.0 cm is uniformly charged with a total positive charge 10.0 μC. The ring rotates at a constant angular speed 20.0 rad/s about an axis through its center, perpendicular to the plane of the ring. What is the magnitude of the magnetic field on the axis of the ring 5.00 cm from its center?
44. A nonconducting ring of radius R is uniformly charged with a total positive charge q. The ring rotates at a constant angular speed ω about an axis through its center, perpendicular to the plane of the ring. What is the magnitude of the magnetic field on the axis of the ring a distance $R/2$ from its center?
45. Sodium melts at 99°C. Liquid sodium, an excellent thermal conductor, is used in some nuclear reactors to remove thermal energy from the reactor core. The liquid sodium is moved through pipes by pumps that exploit the force on a moving charge in a magnetic field. The principle is as follows: Imagine the liquid metal to be in an electrically insulating pipe having a rectangular cross-section of width w and height h. A uniform magnetic field perpendicular to the pipe affects a section of length L (Fig. P22.45). An electric current directed perpendicular to the pipe and to the magnetic field produces a current density of magnitude J in the liquid sodium. (a) Explain why this arrangement produces on the liquid a force that is directed along the length of the pipe. (b) Show that the section of liquid in the magnetic field experiences a pressure increase JLB.

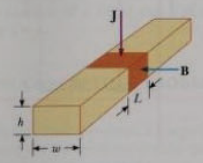


Figure P22.45

46. Two circular coils of radius R are each perpendicular to a common axis. The coil centers are a distance R apart and a steady current I flows in the same direction around each coil, as shown in Figure P22.46. (a) Show that the magnetic field on the axis at a distance x from the center of one coil is

$$B = \frac{\mu_0 I R^2}{2}\left[\frac{1}{(R^2 + x^2)^{3/2}} + \frac{1}{(2R^2 + x^2 - 2Rx)^{3/2}}\right]$$

(b) Show that $\frac{dB}{dx}$ and $\frac{d^2B}{dx^2}$ are both zero at a point midway between the coils. This means the magnetic field in the region midway between the coils is uniform. Coils in this configuration are called **Helmholtz coils.**

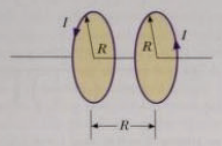


Figure P22.46

47. For a research project, a student needs a solenoid that produces an interior magnetic field of 0.0300 T. She decides to use a current of 1.00 A and a wire 0.500 mm in diameter. She winds the solenoid as layers on an insulating form 1.00 cm in diameter and 10.0 cm long. Determine the number of layers of wire needed and the total length of the wire.
48. Two circular loops are parallel, coaxial, and almost in contact, 1.00 mm apart (Fig. P22.48). Each loop is 10.0 cm in radius. The top loop carries a clockwise current of 140 A. The bottom loop carries a counterclockwise current of 140 A. (a) Calculate the magnetic force that the bottom loop exerts on the top loop. (b) The upper loop has a mass of 0.0210 kg. Calculate its acceleration, assuming that the only forces acting on it are the force in part (a) and its weight. (*Hint:* Think about how one loop looks to a bug perched on the other loop.)

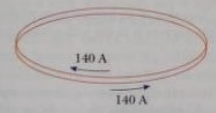


Figure P22.48

49. Protons having a kinetic energy of 5.00 MeV are moving in the positive x direction and enter a magnetic field $\mathbf{B} = (0.0500\ \text{T})\mathbf{k}$ directed out of the plane of the page and extending from $x = 0$ to $x = 1.00$ m, as in Figure P22.49. (a) Calculate the y component of the protons' momentum as they leave the magnetic field. (b) Find the angle α between the initial velocity vector of the proton beam and the velocity vector after the beam emerges from the field. (*Hint:* Neglect relativistic effects and note that $1\ \text{eV} = 1.60 \times 10^{-19}$ J.)

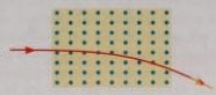


Figure P22.49

50. An infinite sheet of current lying in the yz plane carries a surface current of density $\mathbf{J}_s$. The current is in the y direction, and $\mathbf{J}_s$ represents the current per unit length measured along the z axis. Figure P22.50 is an edge view of the sheet. Find the magnetic field near the sheet. (*Hint:* Use Ampère's law and evaluate the line integral that takes a rectangular path around the sheet, represented by the dashed line in Fig. P22.50.)

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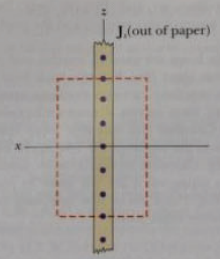


Figure P22.50 Top view of an infinite current sheet lying in the yz plane, where the current is in the y direction (out of the paper).

51. Consider a thin, straight wire segment carrying a constant current, I, and placed along the x axis, as in Figure P22.51. (a) Use the Biot–Savart law to show that the total magnetic field at the point P, located a distance a from the wire, is

$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$$

(b) If the wire is infinitely long, show that the result in (a) gives a magnetic field that agrees with that obtained using Ampère's law (see Ex. 22.5).

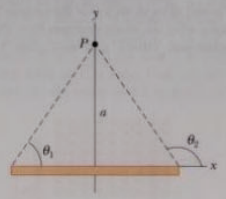


Figure P22.51

52. A long, straight wire—wire 1—oriented along the y axis carries a steady current I_1, as in Figure P22.52. A rectangular circuit located to the right of the wire carries current I_2. Consider wire 2, which is the upper horizontal segment of the circuit that runs from $x = a$ to $x = a + b$. Show that the magnetic force exerted on wire 2 is upward and has a magnitude given by

$$F = \frac{\mu_0 I_1 I_2}{2\pi}\ln\left(1 + \frac{b}{a}\right)$$

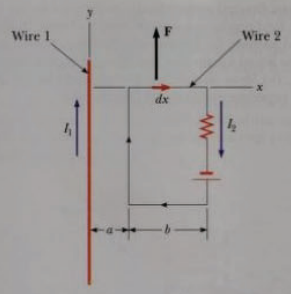


Figure P22.52

Spreadsheet Problems

S1. Spreadsheet 22.1 calculates the magnetic field along the x axis of a solenoid of length ℓ, radius R, and n turns per unit length carrying a current I. The solenoid is centered at the origin with its axis along the x axis. The field at any point x is

$$B = \frac{1}{2}\mu_0 nI[f_1(x) - f_2(x)]$$

where

$$f_1(x) = \frac{x + \frac{1}{2}\ell}{\left[R^2 + \left(x + \frac{1}{2}\ell\right)^2\right]^{1/2}}$$

$$f_2(x) = \frac{x - \frac{1}{2}\ell}{\left[R^2 + \left(x - \frac{1}{2}\ell\right)^2\right]^{1/2}}$$

(a) Let $\ell = 10$ cm, $R = 0.50$ cm, $I = 4.0$ A, and $n = 1500$ turns/m. Plot B versus x for positive values of x ranging from $x = 0$ to $x = 20$ cm. (b) Change R to the following values and observe the effects on the graph in each case: $R = 0.10$ cm, 1.0 cm, 2.0 cm, 5.0 cm, and 10.0 cm. For small values of R (small compared to ℓ), the field inside the solenoid should look like that of an ideal solenoid; for large values of R, the field should look like that of a loop. Does it?

S2. If $x \gg \ell$ (where x is measured from the center of the solenoid), the magnetic field of a finite solenoid along the x axis approaches

$$\frac{\mu_0}{4\pi}\frac{2\mu}{x^3}$$

where $\mu = NI(\pi R^2)$ is the magnetic dipole moment of the solenoid. Modify Spreadsheet 22.1 to calculate and plot $x^3B(x)$ as a function of x. Verify that $x^3B(x)$ approaches a constant for $x \gg \ell$.

S3. An electron has an initial velocity $\mathbf{v}_0$ at the origin of a coordinate system at $t = 0$. A uniform electric field $\mathbf{E} = (1.00 \times 10^3\ \text{V/m})\mathbf{i}$ and a uniform magnetic field $\mathbf{B} = (0.500 \times 10^{-4}\ \text{T})\mathbf{j}$ are turned on at $t = 0$. Write a spreadsheet or computer program to integrate the equations of motion. Because v_0 can be in any direction, you must consider the motion along x, y, and z directions. Describe the general motion of the electron for various values of $\mathbf{v}_0$.

ANSWERS TO CONCEPTUAL PROBLEMS

1. No. The magnetic field produces a magnetic force that is directed toward the center of the circular path of the particle, and this causes a change in direction of the particle's velocity vector. However, because the magnetic force is always perpendicular to the displacement of the particle, no work is done on the particle, and its speed remains constant. Hence, its kinetic energy also remains constant.
2. No. There may be another field such as an electric field or gravitational field that produces another force on the charged particle that has the same magnitude but is opposite the magnetic force, resulting in a net force of zero. Also, a charged particle moving parallel to a magnetic field would experience no magnetic force.
3. The magnetic force on a moving charged particle is always perpendicular to the direction of motion. There is no magnetic force on the charge when it moves parallel to the direction of the magnetic field. However, the force on a charged particle moving in an electric field is never zero and is always parallel to the direction of the electric field. Therefore, by projecting the charged particle in different directions, it is possible to determine the nature of the field. Note that the constant force of the electric field produces a parabolic trajectory, while the magnetic field produces a circular path.
4. The magnetic field of the magnet produces a magnetic force on the electrons moving toward the screen that produce the image. This magnetic force deflects the electrons to regions on the screen other than the ones to which they are supposed to go. The result is a distorted image.
5. If you are moving along with the electrons, you will measure zero current for the electrons, so the electrons would not produce a magnetic field according to your observations. However, the positive crystal lattice ions in the metal are now moving backward relative to you and creating a current equivalent to the forward movement of electrons when you were stationary. Thus, you will measure the same magnetic field as when you were stationary, but it will be due to the moving positive crystal lattice ions in your reference frame.
6. A compass would not detect currents in wires near light switches for two reasons. Because the cable to the light switch contains two wires, with one carrying current to the switch and the other away from the switch, the net magnetic field would be very small and fall off rapidly. The second reason is that the current is alternating at 60 Hz. As a result, the magnetic field is oscillating at 60 Hz. This frequency would be too fast for the compass to follow, so the effect on the compass reading would average to zero.
7. The levitating wire is stable with respect to vertical motion—if it is displaced upward, the repulsive force weakens, and the wire drops back down. Conversely, if it drops lower, the repulsive force increases, and it moves back up. The wire is not stable, however, with respect to lateral movement. If it moves away from the vertical position directly over the lower wire, the repulsive force will have a sideways component, which will push the wire away.

   In the case of the attracting wires, the hanging wire is not stable for vertical movement. If it rises, the attractive force increases, and the wire moves even closer to the upper wire. If the hanging wire falls, the attractive force weakens, and the wire falls further. The hanging wire is also unstable to lateral movement. If the wire moves to the right, it moves farther from the upper wire and the attractive force decreases. Although there is a restoring-force component pulling it back to the left, the vertical-force component is not strong enough to hold it up and it falls.
8. Each coil of the Slinky will generate a small magnetic dipole at its center, because a coil will act as a current loop. Because the sense of rotation of the current is the same in all coils, each coil becomes a magnet with the same orientation of poles. Thus, all of the coils attract, and the Slinky will compress.

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23

Faraday's Law and Inductance

CHAPTER OUTLINE

23.1 Faraday's Law of Induction
23.2 Motional emf
23.3 Lenz's Law
23.4 Induced emfs and Electric Fields
23.5 Self-Inductance
23.6 *RL* Circuits
23.7 Energy Stored in a Magnetic Field

Our studies so far have been concerned with the electric fields due to stationary charges and the magnetic fields produced by moving charges. This chapter deals with electric fields that originate in changing magnetic fields.

As we learned in Chapter 19, Section 19.1, experiments conducted by Michael Faraday in England in the early 1800s and—at the same time, independently—by Joseph Henry in the United States showed that an electric current could be induced in a circuit by a changing magnetic field. The results of those experiments led to a very basic and important law of electromagnetism known as *Faraday's law of induction*. This law holds that the magnitude of the emf induced in a circuit is proportional to the time rate of change of the magnetic field through the circuit.

With the exploration of Faraday's law, we complete our introduction to the fundamental laws of electromagnetism. These laws can be summarized in a set of four equations called *Maxwell's equations*. Together with the Lorentz force law, which we shall discuss briefly, they represent a complete theory for describing the interaction of charged objects. Maxwell's equations relate electric and magnetic fields to each other and to their ultimate source: electric charges.



When a strong magnet is moved toward or away from the coil attached to a galvanometer, an electric current is induced, indicated by the momentary deflection of the galvanometer during the movement of the magnet. *(Richard Megna, Fundamental Photographs)*

The phenomenon of electromagnetic induction has some practical consequences. One of the most important practical applications of electromagnetic induction is the field of power generation. We shall describe an effect known as *self-induction*, in which a time-varying current in a conductor induces an emf that opposes the external emf that set up the current. This phenomenon is the basis of a circuit element known as an *inductor*, which plays an important role in circuits with time-varying currents. Finally, we shall discuss the energy stored in the magnetic field of an inductor and the energy density associated with a magnetic field.

23.1 · FARADAY'S LAW OF INDUCTION

We begin by describing two simple experiments that demonstrate that an electric current can be produced by a changing magnetic field. First, consider a loop of wire connected to a galvanometer, a device that measures current, as in Figure 23.1. If a magnet is moved toward the loop, the galvanometer needle deflects in one direction, as in Figure 23.1a. If the magnet is moved away from the loop, the galvanometer needle deflects in the opposite direction, as in Figure 23.1b. If the magnet is held stationary relative to the loop, no deflection is observed. Finally, if the magnet is held stationary and the coil is moved either toward or away from it, the

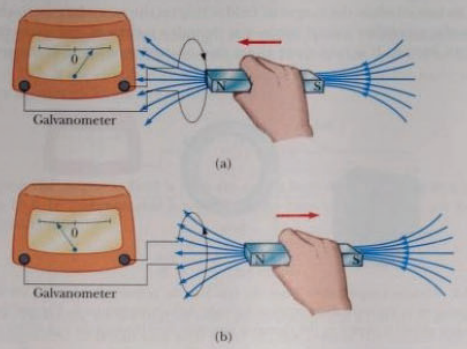


Figure 23.1 (a) When a magnet is moved toward a loop of wire connected to a galvanometer, the galvanometer deflects as shown. This shows that a current is induced in the loop. (b) When the magnet is moved away from the loop, the galvanometer deflects in the opposite direction, indicating that the induced current is opposite that shown in part (a).

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Michael Faraday (1791–1867)

Faraday was a British physicist and chemist who is often regarded as the greatest experimental scientist of the 1800s. His many contributions to the study of electricity include the invention of the electric motor, electric generator, and transformer, as well as the discovery of electromagnetic induction and the laws of electrolysis. *(By kind permission of the President and Council of the Royal Society)*

needle deflects. From these observations comes the conclusion that **an electric current is set up in the galvanometer circuit as long as there is relative motion between the magnet and the coil.**[1]

These results are quite remarkable in view of the fact that **a current is set up in the circuit even though there are no batteries in the circuit!** We call such a current an **induced current,** and it is produced by an **induced emf** or generated emf.

Now let us describe an experiment, first conducted by Faraday, that is illustrated in Figure 23.2. Part of the apparatus consists of a coil connected to a switch and a battery. We shall refer to this coil as the *primary coil* of wire and to the corresponding circuit as the primary circuit. The primary coil is wrapped around an iron ring to intensify the magnetic field produced by the current through the coil. A second coil, at the right, is also wrapped around the iron ring and is connected to a galvanometer. We shall refer to this as the *secondary coil* and to the corresponding circuit as the secondary circuit. There is no battery in the secondary circuit, and the secondary coil is not physically connected to the primary coil. The only purpose of this circuit is to detect any current that might be generated by a change in the magnetic field produced by the primary circuit.

At first you might guess that no current would ever be detected in the secondary circuit. However, something quite amazing happens when the switch in the primary circuit is suddenly closed or opened. At the instant the switch is closed, the galvanometer deflects in one direction and then returns to zero. When the switch is opened, the galvanometer deflects in the opposite direction and again returns to zero. Finally, the galvanometer reads zero when a steady current exists in the primary circuit.

As a result of these observations, Faraday concluded that **an electric current can be produced by a changing magnetic field.** A current cannot be produced by a steady magnetic field. The current produced in the secondary circuit occurs for only an instant while the magnetic field acting on the secondary coil is changing. In effect, the secondary circuit behaves as though a source of emf were connected to it for an instant. It is customary to say that an emf is induced in the secondary

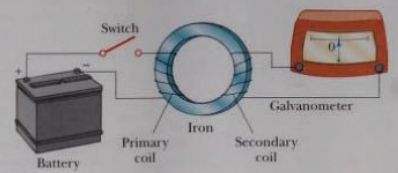


Figure 23.2 Faraday's experiment. When the switch in the primary circuit at the left is closed, the galvanometer in the secondary circuit at the right deflects momentarily. The emf induced in the secondary circuit is caused by the changing magnetic field through the coil in this circuit.

[1]The magnitude of the current depends on the particular resistance of the circuit, but the existence of the current (or its algebraic sign) does *not*.

circuit by the changing magnetic field produced by the current in the primary circuit.

In order to quantify such observations, it is necessary to define a quantity called **magnetic flux.** The flux associated with a magnetic field is defined in a similar manner to the electric flux (Chapter 19, Section 19.8). Consider an element of area dA on an arbitrarily shaped surface that is not closed, as in Figure 23.3. If the magnetic field at this element is $\mathbf{B}$, then the magnetic flux through the element is $\mathbf{B} \cdot d\mathbf{A}$, where $d\mathbf{A}$ is a vector perpendicular to the surface the magnitude of which equals the area dA. Hence, the total magnetic flux, Φ_B, through the surface is

$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A} \qquad [23.1]$$

The SI unit of magnetic flux is the tesla-meter squared, which is named the *weber* (Wb): $1\ \text{Wb} = 1\ \text{T} \cdot \text{m}^2$.

The two experiments illustrated in Figures 23.1 and 23.2 have one thing in common. In both cases, **an emf is induced in a circuit when the magnetic flux through the circuit changes with time.** In fact, a general statement summarizes such experiments involving induced emfs:

> The emf induced in a circuit is directly proportional to the time rate of change of magnetic flux through the circuit.

This statement, known as **Faraday's law of induction,** can be written

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad [23.2]$$

• *Faraday's law*

where Φ_B is the magnetic flux through the circuit, given by Equation 23.1. The negative sign in Equation 23.2 is a consequence of Lenz's law and will be discussed in Section 23.3. If the circuit is a coil consisting of N loops all of the same area and if the flux threads all loops, the induced emf is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \qquad [23.3]$$

Suppose the magnetic field is uniform over a loop of area A lying in a plane as in Figure 23.4. In this case, the flux through the loop is equal to $BA\cos\theta$; hence, the induced emf is

$$\mathcal{E} = -\frac{d}{dt}(BA\cos\theta) \qquad [23.4]$$

From this expression, we see that an emf can be induced in a circuit in several ways: (1) the magnitude of $\mathbf{B}$ can vary with time; (2) the area of the circuit can change with time; (3) the angle θ between $\mathbf{B}$ and the normal to the plane can change with time; and (4) any combination of these can occur.

Examples 23.1 and 23.2 illustrate cases in which an emf is induced in a circuit as a result of a time variation of the magnetic field.

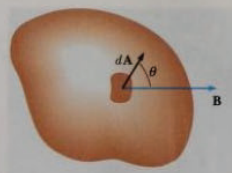


Figure 23.3 The magnetic flux through an area element $d\mathbf{A}$ is given by $\mathbf{B} \cdot d\mathbf{A} = BdA\cos\theta$. Note that $d\mathbf{A}$ is perpendicular to the surface.

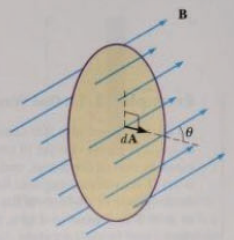


Figure 23.4 A conducting loop of area A in the presence of a uniform magnetic field $\mathbf{B}$, which is at an angle θ with the normal to the loop.

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A demonstration of electromagnetic induction. An ac voltage is applied to the lower coil. A voltage is induced in the upper coil, as indicated by the illuminated lamp connected to the upper coil. What do you think happens to the lamp's intensity as the upper coil is moved over the vertical tube? *(Courtesy of Central Scientific Company)*

Thinking Physics 1

Argentina has more land area (2.8×10^6 km^2) than Greenland (2.2×10^6 km^2). Yet the magnetic flux of the Earth's magnetic field is larger through Greenland than through Argentina. Why?

Reasoning There are two reasons for the larger magnetic flux through Greenland. Greenland (latitude 60° north to 80° north) is closer to a magnetic pole than is Argentina (latitude 20° south to 50° south). As a result, the angle that the magnetic field lines make with the vertical is smaller in Greenland than in Argentina. The dot product in the definition of the magnetic flux through an area incorporates this angle. The second reason is also a result of Greenland's proximity to a magnetic pole. The field strength is larger in the vicinity of a magnetic pole than farther from the pole. Thus, $|\mathbf{B}|$ in the definition of flux is larger for Greenland than for Argentina. These two influences will dominate over the slightly larger area of Argentina.

CONCEPTUAL PROBLEM 1

A circular loop is located in a uniform and constant magnetic field. Describe how an emf can be induced in the loop in this situation.

CONCEPTUAL PROBLEM 2

A spacecraft orbiting the Earth has a coil of wire in it. An astronaut measures a small current in the coil, although there is no battery connected to it and there are no magnets on the spacecraft. What is causing the current?

CONCEPTUAL PROBLEM 3

Suppose you would like to steal power for your home from the electric company by placing a loop of wire near a transmission cable in order to induce an emf in the loop (an illegal procedure). Should you orient your loop so that the transmission cable passes through your loop or simply place your loop near the transmission cable?

Example 23.1 One Way to Induce an emf in a Coil

A coil is wrapped with 200 turns of wire around the perimeter of a square frame of sides 18 cm. Each turn has the same area, equal to that of the frame, and the total resistance of the coil is 2.0 Ω. A uniform magnetic field is turned on perpendicular to the plane of the coil. If the field changes linearly from 0 to 0.50 T in a time of 0.80 s, find the magnitude of the induced emf in the coil while the field is changing.

Solution The area of the loop is $(0.18\ \text{m})^2 = 0.0324\ \text{m}^2$. The magnetic flux through the loop at $t = 0$ is zero, because $B = 0$. At $t = 0.80$ s, the magnetic flux through the loop is $\Phi_B = BA = (0.50\ \text{T})(0.0324\ \text{m}^2) = 0.0162$ Wb. Therefore, the magnitude of the induced emf is

$$|\mathcal{E}| = \frac{N\,\Delta\Phi_B}{\Delta t} = \frac{200(0.0162\ \text{Wb} - 0\ \text{Wb})}{0.80\ \text{s}} = 4.1\ \text{V}$$

(Note that 1 Wb/s = 1 V.)

EXERCISE 1 What is the magnitude of the induced current in the coil while the field is changing? Answer 2.0 A

Example 23.2 An Exponentially Decaying B Field

A plane loop of wire of area A is placed in a region in which the magnetic field is perpendicular to the plane. The magnitude of $\mathbf{B}$ varies in time according to the expression $B = B_0 e^{-at}$. That is, at $t = 0$ the field is B_0, and for $t > 0$, the field decreases exponentially in time (Fig. 23.5). Find the induced emf in the loop as a function of time.

Solution Because $\mathbf{B}$ is perpendicular to the plane of the loop, the magnetic flux through the loop at time $t > 0$ is

$$\Phi_B = BA = AB_0 e^{-at}$$

The induced emf can be calculated from Faraday's Law, noting that AB_0 and the parameter a are constants.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -AB_0 \frac{d}{dt} e^{-at} = aAB_0 e^{-at}$$

That is, the induced emf decays exponentially in time. Note that the maximum emf occurs at $t = 0$, where $\mathcal{E}_{\max} = aAB_0$. Why is this true? The plot of $\mathcal{E}$ versus t is similar to the B versus t curve shown in Figure 23.5.

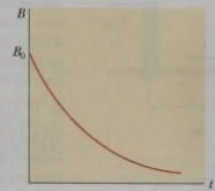


Figure 23.5 (Example 23.2) Exponential decrease of the magnetic field with time. The induced emf and induced current have similar time variations.

EXERCISE 2 A flat loop of wire consisting of a single turn of cross-sectional area 8.0 cm^2 is perpendicular to a magnetic field that increases uniformly in magnitude from 0.50 T to 2.50 T in 1.0 s. What is the resulting induced current if the loop has a resistance of 2.0 Ω? Answer 800 μA

23.2 • MOTIONAL emf

In Examples 23.1 and 23.2, we considered cases in which an emf is produced in a circuit when the magnetic field changes with time. In this section we describe **motional emf,** which is the emf induced in a conductor moving through a magnetic field.

First, consider a straight conductor of length ℓ moving with constant velocity through a uniform magnetic field directed into the page, as in Figure 23.6. For simplicity, we shall assume that the conductor is moving perpendicularly to the field. The electrons in the conductor experience a force along the conductor of $\mathbf{F}_m = q\mathbf{v} \times \mathbf{B}$. Under the influence of this force, the electrons move to the lower end and accumulate there, leaving a net positive charge at the upper end. As a result of this charge separation, an electric field is produced within the conductor. The charge at the ends builds up until the magnetic force qvB is balanced by the electric force qE. At this point, charge stops flowing, and the condition for equilibrium requires that the electric force must balance the magnetic force:

$$qE = qvB \quad \text{or} \quad E = vB$$

Because the electric field produced in the conductor is constant, it is related to the potential difference across the ends according to the relation $\Delta V = E\ell$. Thus,

$$\Delta V = E\ell = B\ell v$$

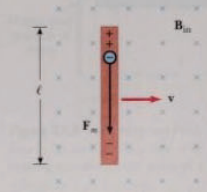


Figure 23.6 A straight conducting bar of length ℓ moving with a velocity $\mathbf{v}$ through a uniform magnetic field $\mathbf{B}$ directed perpendicular to $\mathbf{v}$. An emf equal to $B\ell v$ is induced between the ends of the bar.

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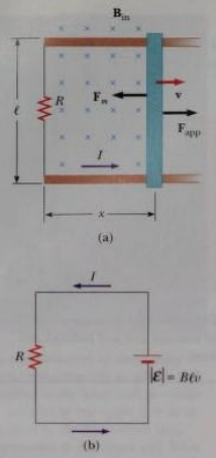


Figure 23.7 (a) A conducting bar sliding with a velocity **v** along two conducting rails under the action of an applied force $\mathbf{F}_{app}$. The magnetic force $\mathbf{F}_m$ opposes the motion, and a counterclockwise current is induced in the loop. (b) The equivalent circuit of (a).

where the upper end is at a higher potential than the lower end. Thus, **a potential difference is maintained as long as the conductor is moving through the magnetic field. If the motion is reversed, the polarity of ΔV is also reversed.**

Another interesting situation occurs if we now consider what happens when the moving conductor is part of a closed circuit. This circumstance is particularly useful for illustrating how a changing magnetic flux can cause an induced current in a closed circuit. Consider a circuit consisting of a conducting bar of length ℓ sliding along two fixed parallel conducting rails, as in Figure 23.7a. For simplicity, we assume that the moving bar has zero electrical resistance and that the stationary part of the circuit has a resistance of R. A uniform and constant magnetic field, **B**, is applied perpendicular to the plane of the circuit.

As the bar is pulled to the right with a velocity of **v**, under the influence of an applied force, $\mathbf{F}_{app}$, free charges in the bar experience a magnetic force along the length of the bar. This magnetic force sets up an induced current, because the charges on which the force is acting are free to move in the circuit. In this case, the rate of change of magnetic flux through the loop and the accompanying induced emf across the moving bar are proportional to the change in loop area as the bar moves through the magnetic field. As we shall see, if the bar is pulled to the right with a constant velocity, the work done by the applied force is dissipated in the form of thermal energy in the circuit's resistive element.

Because the area of the circuit at any instant is ℓx, the external magnetic flux through the circuit is

$$\Phi_B = B\ell x$$

where x is the width of the circuit, which changes with time. Using Faraday's law, we find that the induced emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\ell x) = -B\ell\frac{dx}{dt}$$

$$\mathcal{E} = -B\ell v \qquad [23.5]$$

Because the resistance of the circuit is R, the magnitude of the induced current is

$$I = \frac{|\mathcal{E}|}{R} = \frac{B\ell v}{R} \qquad [23.6]$$

The equivalent circuit diagram for this example is shown in Figure 23.7b.

Let us examine the system using energy considerations. Because there is no battery in the circuit, one might wonder about the origin of the induced current and the electrical energy in the system. We can understand this by noting that the external force does work on the conductor, thereby moving charges through a magnetic field. This causes the charges to move along the conductor with some average drift velocity, and hence a current is established. From the viewpoint of energy conservation, the total work done by the applied force during some time interval should equal the electrical energy that the induced emf supplies in that same period. Furthermore, if the bar moves with constant speed, the work done must equal the energy delivered to the resistor during this time interval.

As the conductor of length ℓ moves through the uniform magnetic field **B**, it experiences a magnetic force, $\mathbf{F}_m$, of magnitude $I\ell B$ (Eq. 22.3 in Chapter 22). The direction of this force is opposite the motion of the bar, or to the left in Figure 23.7a.

If the bar is to move with a *constant* velocity, the applied force must be equal to and opposite the magnetic force, or to the right in Figure 23.7a. (If the magnetic force acted in the direction of motion, it would cause the bar to accelerate once it was in motion, thereby increasing its velocity. This state of affairs would represent a violation of the principle of energy conservation.) Using Equation 23.6 and the fact that $F_{app} = I\ell B$, we find that the power delivered by the applied force is

$$P = F_{app}v = (I\ell B)v = \frac{B^2\ell^2v^2}{R} \qquad [23.7]$$

This power is equal to the rate at which energy is delivered to the resistor, I^2R, as we would expect. It is also equal to the power $I\mathcal{E}$ supplied by the induced emf. This example is a clear demonstration of the conversion of mechanical energy to electrical energy and finally to thermal energy (joule heating).

Thinking Physics 2

As an airplane flies from Los Angeles to Seattle, it cuts through the lines of the Earth's magnetic field. As a result, an emf is developed between the wingtips of the airplane. Which wingtip is positively charged and which is negatively charged? Would the wingtips be charged in the same way on an airplane traveling from Antarctica to Australia? Could this emf be a source of energy to operate the lighting system of the airplane?

Reasoning The magnetic field lines of the Earth have a downward component in the northern hemisphere. As the airplane flies northward, the right-hand rule indicates that positive charges experience a force to the left side of the airplane. Thus, the left wingtip becomes positively charged and the right wingtip negatively charged. In the southern hemisphere, the field lines have an upward component, so the charging of the wingtips would be in the opposite sense. Based on the relative weakness of the Earth's magnetic field, it is not likely that the emf induced could operate a lighting system. An order of magnitude calculation shows that the emf between the wingtips is only a fraction of a volt.

CONCEPTUAL PROBLEM 4

Is it possible to induce a constant emf for a long continuous time?

CONCEPTUAL PROBLEM 5

An electric guitar pickup consists of a permanent magnet around which is wrapped a coil of wire (Fig. 23.8). How does this detect the movement of a steel guitar string?

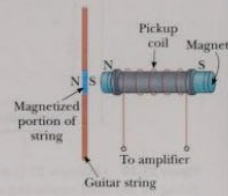


Figure 23.8 (Conceptual Problem 5) In an electric guitar, a vibrating metallic string induces a voltage in a pickup coil.

Example 23.3 emf Induced in a Rotating Bar

A conducting bar of length ℓ rotates with a constant angular speed ω about a pivot at one end. A uniform magnetic field **B** is directed perpendicular to the plane of rotation, as in Figure 23.9. Find the emf induced between the ends of the bar.

Solution Consider a segment of the bar of length dr, the velocity of which is **v**. According to Equation 23.5, the emf induced in a conductor of this length moving perpendicular to a field **B** is

(1) $d\mathcal{E} = Bv\,dr$

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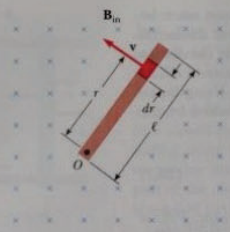


Figure 23.9 (Example 23.3) A conducting bar rotating around a pivot at one end in a uniform magnetic field that is perpendicular to the plane of rotation. An emf is induced across the ends of the bar.

Each segment of the bar is moving perpendicular to **B**, so there is an emf generated across each segment; the value of this emf is given by (1). Summing up the emfs induced across all elements, which are in series, gives the total emf between the ends of the bar. That is,

$$\mathcal{E} = \int B v \, dr$$

In order to integrate this expression, note that the linear speed of an element is related to the angular speed ω through the relationship $v = r\omega$. Therefore, because B and ω are constants, we find that

$$\mathcal{E} = B \int v \, dr = B\omega \int_0^\ell r \, dr = \tfrac{1}{2} B\omega\ell^2$$

Example 23.4 Magnetic Force on a Sliding Bar

A bar of mass m and length ℓ moves on two frictionless parallel rails in the presence of a uniform magnetic field directed into the paper (Fig. 23.10). The bar is given an initial velocity $\mathbf{v}_0$ to the right and is released. Find the velocity of the bar as a function of time.

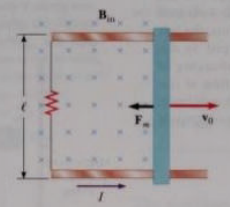


Figure 23.10 (Example 23.4) A conducting bar of length ℓ sliding on two fixed conducting rails is given an initial velocity $\mathbf{v}_0$ to the right.

Solution First note that the induced current is counterclockwise and the magnetic force on the bar is $F_x = -I\ell B$, where the negative sign denotes that the force is to the left and retards the motion. This is the only horizontal force acting on the bar, and hence Newton's second law applied to motion in the horizontal direction gives

$$F_x = ma = m\frac{dv}{dt} = -I\ell B$$

Because the induced current is given by Equation 23.6, $I = B\ell v/R$, we can write this expression as

$$m\frac{dv}{dt} = -\frac{B^2\ell^2}{R} v$$

$$\frac{dv}{v} = -\left(\frac{B^2\ell^2}{mR}\right) dt$$

Integrating this last equation using the initial condition that $v = v_0$ at $t = 0$, we find that

$$\int_{v_0}^{v} \frac{dv}{v} = -\left(\frac{B^2\ell^2}{mR}\right) \int_0^t dt$$

$$\ln\left(\frac{v}{v_0}\right) = -\left(\frac{B^2\ell^2}{mR}\right) t = -\frac{t}{\tau}$$

where the constant $\tau = mR/B^2\ell^2$. From this, we see that the velocity can be expressed in the exponential form

$$v = v_0 e^{-t/\tau}$$

Therefore, the velocity of the bar decreases exponentially with time under the action of the magnetic retarding force. Furthermore, if we substitute this result into Equation 23.6, we find that the induced emf and induced current also decrease exponentially with time. That is,

$$I = \frac{B\ell v}{R} = \frac{B\ell v_0}{R} e^{-t/\tau}$$

$$\mathcal{E} = IR = B\ell v_0 e^{-t/\tau}$$

The Alternating-Current Generator

The alternating current generator (or ac generator), a device that converts mechanical energy to electrical energy, consists of a coil of wire rotated by some external means in an external magnetic field. In commercial power plants, the energy required to rotate the loop can be derived from a variety of sources. For example, in a hydroelectric plant, falling water directed against the blades of a turbine produces the rotary motion; in a coil-fired plant, the energy from burning coal is used to convert water to steam, and this steam is directed against the turbine blades. As the loop rotates, the magnetic flux through it changes with time, inducing an emf and a current in an external circuit.

Suppose that the coil has N turns, all of the same area A, and suppose that the coil rotates with a constant angular speed ω about an axis perpendicular to the magnetic field. If θ is the angle between the magnetic field and the direction perpendicular to the plane of the coil, then the magnetic flux through the loop at any time t is given by

$$\Phi_B = BA\cos\theta = BA\cos\omega t$$

where we have used the relationship between angular displacement and angular speed, $\theta = \omega t$. (We have set the clock so that $t = 0$ when $\theta = 0$.) Hence, the induced emf in the coil is given by

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NAB\frac{d}{dt}(\cos\omega t) = NAB\omega \sin\omega t \qquad [23.8]$$

This result shows that the emf varies sinusoidally with time. From Equation 23.8 we see that the maximum emf has the value $\mathcal{E}_{max} = NAB\omega$, which occurs when $\omega t = 90°$ or 270°. In other words, $\mathcal{E} = \mathcal{E}_{max}$ when the plane of the coil is parallel to the magnetic field and the time rate of change of flux is a maximum. Furthermore, the emf is *zero* when $\omega t = 0$ or 180°—that is, when the plane of the coil is perpendicular to **B** and the time rate of change of flux is zero.

23.3 · LENZ'S LAW

The direction of an induced emf and induced current can be found from Lenz's law:

> The polarity of an induced emf is such that it produces a current the magnetic field of which opposes the change in magnetic flux through the loop. That is, the induced current is in a direction such that the induced magnetic field attempts to maintain the original flux through the circuit.

The induced current tends to keep the original flux through the circuit from changing. Interpretation of this statement depends on the circumstances. As we shall see, this law is a consequence of the law of conservation of energy.

In order to attain a better understanding of Lenz's law, let us return to the example of a bar moving to the right on two parallel rails in the presence of a uniform magnetic field directed into the page (Fig. 23.11a). As the bar moves to the right, the magnetic flux through the circuit increases with time, because the

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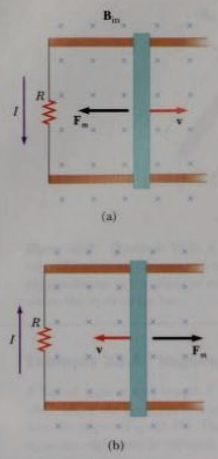


Figure 23.11 (a) As the conducting bar slides on the two fixed conducting rails, the magnetic flux through the loop increases in time. By Lenz's law, the induced current must be *counterclockwise* in order to produce a counteracting flux *out* of the paper. (b) When the bar moves to the left, the induced current must be *clockwise*. Why?

area of the loop increases. Lenz's law says that the induced current must be in such a direction that the magnetic flux *it* produces opposes the change in the magnetic flux caused by the external magnetic field. Because the magnetic flux due to the external field is increasing into the page, the induced current, if it is to oppose the change, must produce a flux out of the page. Hence, the induced current must be counterclockwise when the bar moves to the right to give a counteracting flux out of the page in the region inside the loop. (Use the right-hand rule to verify this direction.) If the bar is moving to the left, as in Figure 23.11b, the magnetic flux through the loop decreases with time. Because the flux is into the page, the induced current has to be clockwise to produce a flux into the page inside the loop. In either case, the induced current tends to maintain the original flux through the circuit.

Let us examine this situation from the viewpoint of energy considerations. Suppose that the bar is given a slight push to the right. In the preceding analysis, we found that this motion leads to a counterclockwise current in the loop. Let us see what happens if we assume that the current is clockwise. For a clockwise current, I, the direction of the magnetic force, $B\ell$, on the sliding bar would be to the right. This force would accelerate the rod and increase its speed, which in turn would cause the area of the loop to increase more rapidly. This would increase the induced current, which would increase the force, which would increase the current, which would. . . . In effect, the system would acquire energy with no additional energy input. This result is clearly inconsistent with all experience and with the law of conservation of energy. Thus, we are forced to conclude that the current must be counterclockwise.

Consider another situation, one in which a bar magnet is moved to the right toward a stationary loop of wire, as in Figure 23.12a. As the magnet moves toward the loop, the magnetic flux through the loop increases with time. To counteract this increase in flux to the right, the induced current produces a flux to the left, as in Figure 23.12b; hence, the induced current is in the direction shown. Note that the magnetic field lines associated with the induced current oppose the motion of the magnet. Therefore, the left face of the current loop is a north pole and the right face is a south pole.

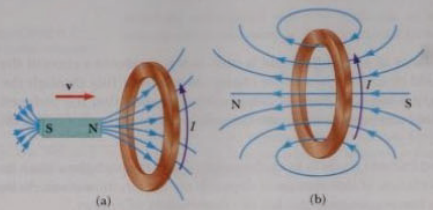


Figure 23.12 (a) When the magnet is moved toward the stationary conducting loop, a current is induced in the direction shown. (b) This induced current produces its own flux to the left to counteract the increasing external flux to the right.

If the magnet moved to the left, its flux through the loop, which is toward the right, would decrease in time. Under these circumstances, the induced current in the loop would be in a direction to set up a field through the loop from left to right, in an effort to maintain a constant number of flux lines. Hence, the direction of the induced current in the loop would be opposite that shown in Figure 23.12b. In this case, the left face of the loop would be a south pole and the right face would be a north pole.

Thinking Physics 3

In some circuits, a spark occurs between the poles of a switch when the switch is opened. Yet, when the switch for this circuit is closed, there is no spark. Why is there this difference?

Reasoning According to Lenz's law, induced emfs are in a direction such as to attempt to maintain the original magnetic flux when a change occurs. When the switch is opened, the sudden drop in the magnetic field in the circuit induces an emf in a direction which attempts to keep the original current flowing. This can cause a spark as the current bridges the air gap between the poles of the switch. The spark does not occur when the switch is closed, because the original current is zero, and the induced emf attempts to maintain it at zero.

CONCEPTUAL PROBLEM 6

A transformer is a pair of coils wrapped around an iron form, as in Figure 23.2. When ac voltage is applied to one coil, the *primary*, the magnetic field lines cutting through the other coil, the *secondary*, induce an emf. By varying the number of turns of wire on each coil, the applied ac voltage can be stepped up or stepped down. It is clear that this device will not work with dc voltage. What's more, if dc voltage of the same magnitude is applied, the primary coil sometimes will overheat and burn. Why?

CONCEPTUAL PROBLEM 7

In equal arm balances from the early twentieth century, it is sometimes observed that an aluminum sheet hangs from one of the arms and passes between the poles of a magnet. Why?

Example 23.5 Application of Lenz's Law

A coil of wire is placed near an electromagnet, as shown in Figure 23.13a. Find the direction of the induced current in the coil (a) at the instant the switch is closed, (b) after the switch has been closed for several seconds, and (c) when the switch is opened.

Reasoning (a) When the switch is closed, the situation changes from a condition in which no lines of flux pass through the coil to one in which lines of flux pass through in the direction shown in Figure 23.13b. To counteract this change in the number of lines, the coil must set up a field from left to right in the figure. This requires a current directed as shown in Figure 23.13b.

(b) After the switch has been closed for several seconds, there is no change in the number of lines through the loop; hence the induced current is zero.

(c) Opening the switch causes the magnetic field to change from a condition in which flux lines thread through the coil from right to left to a condition of zero flux. The induced current must then be as shown in Figure 23.13c, so as to set up its own field from right to left.

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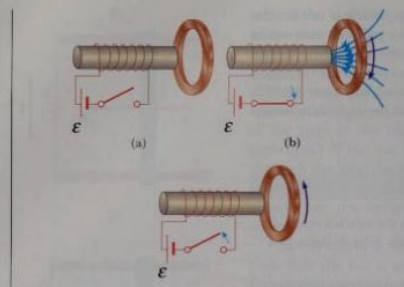


Figure 23.13 (Example 23.5)

EXERCISE 3 A small airplane with a wing span of 14 m flies due north at a speed of 70 m/s over a region in which the vertical component of the Earth's magnetic field is 1.2 μT. (a) What potential difference is developed between the wingtips? (b) How would the answer to (a) change if the plane were flying due east? Answer (a) 1.2 mV (b) same emf with the north end positive

EXERCISE 4 Coils rotating in a magnetic field are often used to measure unknown magnetic fields. As an example, consider a coil of radius 1.0 cm with 50 turns that is rotated about an axis perpendicular to the field at a rate of 20 Hz. If the maximum induced emf in the coil is 3.0 V, find the strength of the magnetic field. Answer 1.5 T

23.4 · INDUCED emfs AND ELECTRIC FIELDS

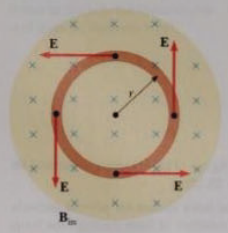


Figure 23.14 A loop of radius r in a uniform magnetic field perpendicular to the plane of the loop. If **B** changes in time, an electric field is induced in a direction tangent to the loop.

We have seen that a changing magnetic flux induces an emf and a current in a conducting loop. We therefore must conclude that **an electric field is created in the conductor as a result of changing magnetic flux.** In fact, the law of electromagnetic induction demonstrates that **an electric field is always generated by a changing magnetic flux,** even in free space in which no charges are present. However, this induced electric field has properties that are quite different from those of the electrostatic field produced by stationary charges.

We can illustrate this point by considering a conducting loop of radius r, situated in a uniform magnetic field that is perpendicular to the plane of the loop, as in Figure 23.14. If the magnetic field changes with time, then Faraday's law tells us that an emf of $\mathcal{E} = -d\Phi_B/dt$ is induced in the loop. The induced current that is produced implies the presence of an induced electric field, **E**, which must be tangent to the loop, because all points on the loop are equivalent. The work done in moving a test charge, q, once around the loop is equal to $q\mathcal{E}$. Because the electric force on the charge is qE, the work done by that force in moving the charge once around the loop is $qE(2\pi r)$, where $2\pi r$ is the circumference of the loop. These two expressions for the work must be equal; therefore, we see that

$$q\mathcal{E} = qE(2\pi r)$$

$$E = \frac{\mathcal{E}}{2\pi r}$$

Using this result along with Faraday's law and the fact that $\Phi_B = BA = \pi r^2 B$ for a circular loop, we find that the induced electric field can be expressed as

$$E = -\frac{1}{2\pi r}\frac{d\Phi_B}{dt} = -\frac{r}{2}\frac{dB}{dt}$$

This expression can be used to calculate the induced electric field if the time variation of the magnetic field is specified. The negative sign indicates that the induced electric field, **E**, opposes the change in the magnetic field. It is important to understand that **this result is also valid in the absence of a conductor.** That is, a free charge placed in a changing magnetic field experiences the same electric field.

In general, the emf for any closed path can be expressed as the line integral of $\mathbf{E}\cdot d\mathbf{s}$ over that path. Hence, the general form of Faraday's law of induction is

$$\mathcal{E} = \oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \qquad [23.9]$$

· *Faraday's law in general form*

It is important to recognize that **the induced electric field, E, that appears in Equation 23.9 is a nonconservative field that is generated by a changing magnetic field.** The field **E** that satisfies Equation 23.9 could not possibly be an electrostatic field, for the following reason. If the field were electrostatic, and hence conservative, the line integral of $\mathbf{E}\cdot d\mathbf{s}$ over a closed loop would be zero, contrary to Equation 23.9.

Thinking Physics 4

In studying electric fields, the point is made that electric field lines begin on positive charges and end on negative charges. Do *all* electric field lines begin and end on charges?

Reasoning The statement that electric field lines begin and end on charges is true only for *electrostatic* fields—electric fields due to stationary charges. Electric field lines due to changing magnetic fields form closed loops, with no beginning and no end, and are independent of the presence of charges.

EXERCISE 5 The current in a solenoid is increasing at a rate of 10 A/s. The cross-sectional area of the solenoid is π cm^2, and there are 300 turns on its 15-cm length. What is the induced emf opposing the increasing current? Answer 2.4 mV

23.5 · SELF-INDUCTANCE

Consider an isolated circuit consisting of a switch, a resistor, and a source of emf, as in Figure 23.15. When the switch is closed, the current doesn't immediately jump from zero to its maximum value, $\mathcal{E}/R$; the law of electromagnetic induction (Faraday's law) describes this behavior. What happens is that, as the current increases

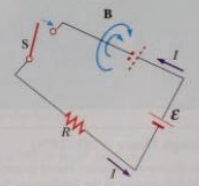


Figure 23.15 After the switch is closed, the current produces a magnetic flux through the loop. As the current increases toward its equilibrium value, the flux changes in time and induces an emf in the loop. The battery drawn with dashed lines is a symbol for the self-induced emf.

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with time, the magnetic flux through the loop due to the current also increases with time. This increasing flux induces an emf in the circuit that opposes the change in the net magnetic flux through the loop. By Lenz's law, the induced electric field in the wires must therefore be opposite the direction of the current, and this opposing emf results in a *gradual* increase in the current. This effect is called *self-induction*, because the changing flux through the circuit arises from the circuit itself. The emf that is set up in this case is called a **self-induced emf.**

To obtain a quantitative description of self-induction, we recall from Faraday's law that the induced emf is the negative time rate of change of the magnetic flux. The magnetic flux is proportional to the magnetic field, which in turn is proportional to the current in the circuit. Therefore, **the self-induced emf is always proportional to the time rate of change of the current.** For a closely spaced coil of N turns of fixed geometry (a toroidal coil or the ideal solenoid), we find that

Self-induced emf •

$$\mathcal{E}_L = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt} \qquad [23.10]$$

where L is a proportionality constant, called the **inductance** of the device, that depends on the geometric features of the circuit and other physical characteristics. From this expression, we see that the inductance of a coil containing N turns is

Inductance of an N-turn coil •

$$L = \frac{N\Phi_B}{I} \qquad [23.11]$$

where it is assumed that the same flux passes through each turn. Later we shall use this equation to calculate the inductance of some special current geometries.

From Equation 23.10 we can also write the inductance as the ratio

$$L = -\frac{\mathcal{E}_L}{dI/dt} \qquad [23.12]$$

This is usually taken to be the defining equation for the inductance of any coil, regardless of its shape, size, or material characteristics. Just as resistance is a measure of opposition to current, inductance is a measure of opposition to the *change* in current.

The SI unit of inductance is the **henry (H),** which, from Equation 23.12, is seen to be equal to 1 volt-second per ampere:

$$1\ \text{H} = 1\frac{\text{V}\cdot\text{s}}{\text{A}} = 1\frac{\text{Wb}}{\text{A}} = 1\frac{\text{T}\cdot\text{m}^2}{\text{A}}$$

As we shall see, **the inductance of a device depends on its geometry.** Because inductance calculations can be quite difficult for complicated geometries, Examples 23.6 and 23.7 involve simple situations for which inductances are easily evaluated.



Joseph Henry (1797–1878)

An American physicist, Henry became the first director of the Smithsonian Institution and first president of the Academy of Natural Science. He improved the design of the electromagnet and constructed one of the first motors. He also discovered the phenomenon of self-induction but failed to publish his findings. The unit of inductance, the henry, is named in his honor. *(North Wind Picture Archives)*

Thinking Physics 5

If inductors are connected in series and/or parallel, one expects the inductances combine like resistors—they add in series, and the inverses add in parallel. The previous statement holds true for toroidal inductors in tight quarters in an electrical circuit but not for inductors formed from solenoids. Why?

Reasoning The inductance of a coil is *self*-inductance—an effect due to the magnetic field of a toroid or solenoid cutting through its own coils. In an ideal toroidal inductor, the magnetic field is contained within the interior of the toroid. Thus, if toroidal inductors are placed close to each other, there is no magnetic interaction between the inductors. Finite solenoids, however, have magnetic fields that exist outside the solenoid. If solenoids are placed in close proximity, the field of one solenoid can interact with the coils of the other solenoid. Thus, there is *mutual* inductance in addition to self-inductance, and the simple inductance combination rules do not apply.

Thinking Physics 6

The ground fault interrupter (GFI) is a safety device that protects users of electrical power against electric shock when they touch appliances. Its essential parts are shown in Figure 23.16. How does its operation make use of Faraday's law?

Reasoning Wire 1 leads from the wall outlet to the appliance being protected, and wire 2 leads from the appliance back to the wall outlet. An iron ring surrounding the two wires confines the magnetic field set up by each wire. A sensing coil, that can activate a circuit breaker when changes in magnetic flux occur, is wrapped around part of the iron ring. (This can happen, for example, if one of the wires loses its insulation and accidentally touches the metal case of the appliance, providing a direct path to ground.) Because the currents in the two wires are in opposite directions, the net magnetic field through the sensing coil due to the currents is zero. If a short to ground occurs, the current in one of the wires will suddenly increase, producing a rapid change in flux through the coil and generating an induced voltage in it. This induced voltage is used to trigger a circuit breaker, stopping the current before it reaches a level that might be harmful to the person using the appliance.

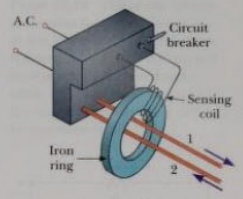


Figure 23.16 (Thinking Physics 6) Essential components of a ground fault interrupter.

Example 23.6 Inductance of a Solenoid

Find the inductance of a uniformly wound solenoid having N turns and length ℓ. Assume that ℓ is long compared with the radius and that the core of the solenoid is air.

Solution In this case, we can take the interior magnetic field to be uniform and given by Equation 22.25:

$$B = \mu_0 nI = \mu_0\frac{N}{\ell}I$$

where n is the number of turns per unit length, N/ℓ. The magnetic flux through each turn is

$$\Phi_B = BA = \mu_0\frac{NA}{\ell}I$$

where A is the cross-sectional area of the solenoid. Using this expression and Equation 23.11 we find that

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 A}{\ell}$$

This shows that L depends on geometry and is proportional to the square of the number of turns. Because $N = n\ell$, we can also express the result in the form

$$L = \mu_0\frac{(n\ell)^2}{\ell}A = \mu_0 n^2 A\ell = \mu_0 n^2\ (\text{volume})$$

where $A\ell$ is the volume of the solenoid.

Example 23.7 Calculating Inductance and emf

(a) Calculate the inductance of a solenoid containing 300 turns if the length of the solenoid is 25.0 cm and its cross-sectional area is 4.00 $\text{cm}^2 = 4.00 \times 10^{-4}\ \text{m}^2$.

Solution Using the first expression for L in Example 23.6 we get

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$$L = \frac{\mu_0 N^2 A}{\ell} = (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})\frac{(300)^2(4.00 \times 10^{-4}\ \text{m}^2)}{25.0 \times 10^{-2}\ \text{m}}$$

$$= 1.81 \times 10^{-4}\ \text{T}\cdot\text{m}^2/\text{A} = 0.181\ \text{mH}$$

(b) Calculate the self-induced emf in the solenoid described in (a) if the current through it is decreasing at the rate of 50.0 A/s.

Solution Using Equation 23.10 and given that $dI/dt = -50.0$ A/s, we get

$$\mathcal{E}_L = -L\frac{dI}{dt} = -(1.81 \times 10^{-4}\ \text{H})(-50.0\ \text{A/s}) = 9.05\ \text{mV}$$

EXERCISE 6 A 0.388-mH inductor has a length that is four times its diameter. If it is wound with 22 turns per centimeter, what is its length? Answer 0.109 m

23.6 • *RL* CIRCUITS

A circuit that contains a coil, such as a solenoid, has a self-inductance that prevents the current from increasing or decreasing instantaneously. A circuit element that has a large inductance is called an **inductor.** The circuit symbol for an inductor is . We shall always assume that the self-inductance of the remainder of the circuit is negligible compared with that of the inductor.

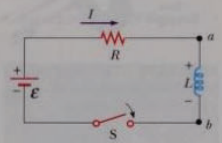


Figure 23.17 A series *RL* circuit. As the current increases toward its maximum value, the inductor produces an emf that opposes the increasing current.

Consider the circuit—consisting of a resistor, inductor, and battery—shown in Figure 23.17. The internal resistance of the battery will be neglected. Suppose the switch S is closed at $t = 0$. The current will begin to increase, and due to the increasing current, the inductor will produce an emf (sometimes referred to as a *back emf*) that opposes the increasing current. In other words, the inductor acts like a battery the polarity of which is opposite that of the real battery in the circuit. The back emf produced by the inductor is

$$\mathcal{E}_L = -L\frac{dI}{dt}$$

Because the current is increasing, dI/dt is positive; therefore $\mathcal{E}_L$ is negative. This corresponds to the fact that there is a potential drop from a to b across the inductor. For this reason, point a is at a higher potential than point b, as illustrated in Figure 23.17.

With this in mind, we can apply Kirchhoff 's loop rule to this circuit. If we begin at the battery and travel clockwise, we have

$$\mathcal{E} - IR - L\frac{dI}{dt} = 0 \qquad [23.13]$$

where IR is the voltage drop across the resistor. We must now look for a solution to this differential equation, which is similar to Equation 21.30 (Chapter 21) for the *RC* circuit.

To obtain a mathematical solution of Equation 23.13, it is convenient to change variables by letting $x = (\mathcal{E}/R) - I$, so that $dx = -dI$. With these substitutions, Equation 23.13 can be written

$$x + \frac{L}{R}\frac{dx}{dt} = 0$$

$$\frac{dx}{x} = -\frac{R}{L}dt$$

Integrating this last expression gives

$$\ln\frac{x}{x_0} = -\frac{R}{L}t$$

where the integrating constant is taken to be $-\ln x_0$. Taking the antilog of this result gives

$$x = x_0 e^{-Rt/L}$$

where the current is zero at $t = 0$ and $x_0 = \mathcal{E}/R$. Hence, the last expression is equivalent to

$$\frac{\mathcal{E}}{R} - I = \frac{\mathcal{E}}{R}e^{-Rt/L}$$

$$I = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L})$$

which represents the solution of Equation 23.13.

This mathematical solution of Equation 23.13, which represents the current as a function of time, can also be written

$$I(t) = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L}) = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) \qquad [23.14]$$

where the constant τ is the **time constant** of the *RL* circuit:

$$\tau = L/R \qquad [23.15]$$

It is left to an exercise to show that the dimension of τ is time. Physically, τ is the time it takes the current to reach $(1 - e^{-1}) = 0.63$ of its final value, $\mathcal{E}/R$.

Figure 23.18 plots the current versus time, where $I = 0$ at $t = 0$. Note that the final equilibrium value of the current, which occurs at $t = \infty$, is $\mathcal{E}/R$. This can be seen by setting dI/dt equal to zero in Equation 23.13 (at equilibrium, the change in the current is zero) and solving for the current. Thus, we see that the current rises very fast initially and then gradually approaches the equilibrium value $\mathcal{E}/R$ as $t \to \infty$.

Taking the first time derivative of Equation 23.14, we get

$$\frac{dI}{dt} = \frac{\mathcal{E}}{L}e^{-t/\tau} \qquad [23.16]$$

From Equation 23.16 we see that the rate of increase of current, dI/dt, is a *maximum* (equal to $\mathcal{E}/L$) at $t = 0$ and falls off exponentially to zero as $t \to \infty$ (Fig. 23.19).

Now consider the *RL* circuit arranged as shown in Figure 23.20. The circuit contains two switches that operate so that when one is closed, the other is opened. Now suppose that S_1 is closed for a long enough time to allow the current to reach its equilibrium value, $\mathcal{E}/R$. If S_1 is now opened and S_2 is closed at $t = 0$, we

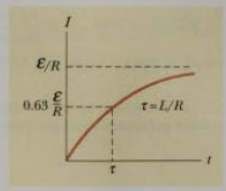


Figure 23.18 Plot of the current versus time for the *RL* circuit shown in Figure 23.17. The switch is closed at $t = 0$, and the current increases toward its maximum value, $\mathcal{E}/R$. The time constant τ is the time it takes I to reach 63% of its maximum value.

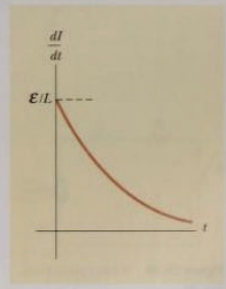


Figure 23.19 Plot of dI/dt versus time for the *RL* circuit shown in Figure 23.17. The time rate of change of current is a maximum at $t = 0$ when the switch is closed. The rate decreases exponentially with time as I increases toward its maximum value.

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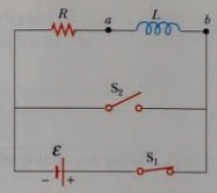


Figure 23.20 An *RL* circuit containing two switches. When S_1 is closed and S_2 is open as shown, the battery is in the circuit. At the instant S_2 is closed, S_1 is opened, and the battery is removed from the circuit.

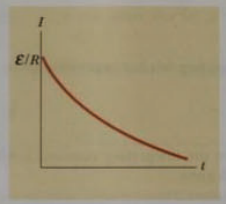


Figure 23.21 Current versus time for the circuit shown in Figure 23.20. At $t < 0$, S_1 is closed and S_2 is open. At $t = 0$, S_2 is closed, S_1 is open, and the current has its maximum value $\mathcal{E}/R$.

have a circuit with no battery ($\mathcal{E} = 0$). If we apply Kirchhoff's loop rule to this circuit, we obtain

$$IR + L\frac{dI}{dt} = 0 \qquad [23.17]$$

It is left to Problem 27 to show that the solution of this differential equation is

$$I(t) = \frac{\mathcal{E}}{R}e^{-t/\tau} = I_0 e^{-t/\tau} \qquad [23.18]$$

where the current at $t = 0$ is $I_0 = \mathcal{E}/R$ and $\tau = L/R$.

The graph of current versus time for the circuit of Figure 23.20 (Fig. 23.21) shows that the current is continuously decreasing with time, as one would expect. Furthermore, note that the slope, dI/dt, is always negative and has its maximum value at $t = 0$. The negative slope signifies that $\mathcal{E}_L = -L(dI/dt)$ is not *positive*—that is, point *a* in Figure 23.20 is at a lower potential than point *b*.

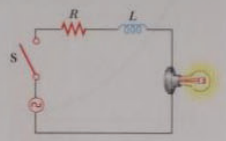


Figure 23.22 (Conceptual Problem 8)

CONCEPTUAL PROBLEM 8

The switch in the circuit in Figure 23.22 is closed and the lightbulb glows steadily. The inductor is a simple air-core solenoid. What happens to the brightness of the lightbulb as an iron rod is being inserted into the interior of the solenoid?

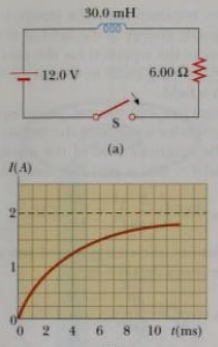
Example 23.8 Time Constant of an *RL* Circuit

The switch in Figure 23.23a is closed at $t = 0$. (a) Find the time constant of the circuit.

Solution The time constant is given by Equation 23.15

$$\tau = \frac{L}{R} = \frac{30.0 \times 10^{-3}\ \text{H}}{6.00\ \Omega} = 5.00\ \text{ms}$$

(b) Calculate the current in the circuit at $t = 2.00$ ms.



Solution Using Equation 23.14 for the current as a function of time (with t and τ in milliseconds), we find that at $t = 2.00$ ms

$$I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) = \frac{12.0\ \text{V}}{6.00\ \Omega}(1 - e^{-0.400}) = 0.659\ \text{A}$$

A plot of Equation 23.14 for this circuit is given in Figure 23.23b.

EXERCISE 7 Calculate the current in the circuit and the voltage across the resistor after one time constant has elapsed.
Answer 1.26 A, 7.59 V

Figure 23.23 (Example 23.8) (a) The switch in this *RL* circuit is closed at $t = 0$. (b) A graph of the current versus time for the circuit in part (a).

23.7 • ENERGY STORED IN A MAGNETIC FIELD

In the preceding section we found that the induced emf set up by an inductor prevents a battery from establishing an instantaneous current. Hence, a battery has to do work against an inductor to create a current. Part of the energy supplied by the battery goes into thermal energy dissipated in the resistor, and the remaining energy is stored in the inductor. If we multiply each term in Equation 23.13 by the current I and rearrange the expression, we get

$$I\mathcal{E} = I^2R + LI\frac{dI}{dt} \qquad [23.19]$$

This expression tells us that the rate at which energy is supplied by the battery, $I\mathcal{E}$, equals the sum of the rate at which thermal energy is dissipated in the resistor, I^2R, and the rate at which energy is stored in the inductor, $LI(dI/dt)$. Thus, Equation 23.19 is simply an expression of energy conservation. If we let U_B denote the energy stored in the inductor at any time, then the rate dU_B/dt at which energy is stored in the inductor can be written

$$\frac{dU_B}{dt} = LI\frac{dI}{dt}$$

To find the total energy stored in the inductor, we can rewrite this expression as $dU_B = LI\,dI$ and integrate:

$$U_B = \int_0^{U_B} dU_B = \int_0^I LI\,dI$$

$$U_B = \tfrac{1}{2}LI^2 \qquad [23.20] \quad \bullet \textit{ Energy stored in an inductor}$$

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where L is constant and so has been removed from the integral. Equation 23.20 represents the energy stored as magnetic energy in the field of the inductor when the current is I. Note that it is similar to the equation for the energy stored in the electric field of a capacitor, $U_E = Q^2/2C$ (Eq. 20.29 in Chapter 20). In either case, we see that it takes work to establish a field.

We can also determine the energy per unit volume, or energy density, stored in a magnetic field. For simplicity, consider a solenoid the inductance of which is $L = \mu_0 n^2 A\ell$ (see Example 23.6). The magnetic field of the solenoid is $B = \mu_0 nI$. Substituting the expression for L and $I = B/\mu_0 n$ into Equation 23.20 gives

$$U_B = \tfrac{1}{2}LI^2 = \tfrac{1}{2}\mu_0 n^2 A\ell \left(\frac{B}{\mu_0 n}\right)^2 = \frac{B^2}{2\mu_0}(A\ell) \qquad [23.21]$$

Because $A\ell$ is the volume of the solenoid, the energy stored per unit volume in a magnetic field—in other words, the *energy density*—is

Magnetic energy density

$$u_B = \frac{U_B}{A\ell} = \frac{B^2}{2\mu_0} \qquad [23.22]$$

Although Equation 23.22 was derived for the special case of a solenoid, **it is valid for any region of space in which a magnetic field exists.** Note that it is similar to the equation for the energy per unit volume stored in an electric field, given by $\frac{1}{2}\epsilon_0 E^2$ (Eq. 20.31). In both cases, the energy density is proportional to the square of the field strength.

Example 23.9 What Happens to the Energy in the Inductor?

Consider once again the RL circuit shown in Figure 23.20, in which switch S_2 is closed at the instant S_1 is opened (at $t = 0$). Recall that the current in the upper loop decays exponentially with time according to the expression $I = I_0 e^{-t/\tau}$, where $I_0 = \mathcal{E}/R$ is the initial current in the circuit and $\tau = L/R$ is the time constant. Let us show explicitly that all the energy stored in the magnetic field of the inductor is delivered to the resistor.

Solution The rate at which energy is dissipated in the resistor, dU/dt, (or the power) is equal to I^2R, where I is the instantaneous current:

$$\frac{dU}{dt} = I^2R = (I_0 e^{-Rt/L})^2 R = I_0^2 R e^{-2Rt/L}$$

To find the total energy dissipated in the resistor, we integrate this expression over the limits $t = 0$ to $t = \infty$ (∞ because it takes an infinite time for the current to reach zero):

$$(1) \qquad U = \int_0^\infty I_0^2 R e^{-2Rt/L}\, dt = I_0^2 R \int_0^\infty e^{-2Rt/L}\, dt$$

The value of the definite integral is $L/2R$, and so U becomes

$$U = I_0^2 R\left(\frac{L}{2R}\right) = \tfrac{1}{2}LI_0^2$$

Note that this is equal to the initial energy stored in the magnetic field of the inductor, given by Equation 23.20, as we set out to prove.

EXERCISE 8 Show that the integral on the right-hand side of Equation (1) has the value $L/2R$.

Example 23.10 The Coaxial Cable

A long coaxial cable consists of two concentric cylindrical conductors of radii a and b and length ℓ, as in Figure 23.24. The inner conductor is assumed to be a thin cylindrical shell. Each conductor carries a current I (the outer one being a return path). (a) Calculate the self-inductance, L, of this cable.

Solution To obtain L, we must know the magnetic flux through any cross-section between the two conductors. From Ampère's law (Section 22.7), it is easy to see that the magnetic field between the conductors is $B = \mu_0 I/2\pi r$. The field is zero outside the conductors ($r > b$) because the net current through a circular path surrounding both wires is zero, and hence from Ampère's law, $\oint \mathbf{B}\cdot d\mathbf{s} = 0$. The field is zero inside the inner conductor because it is hollow and there is no current within a radius $r < a$.

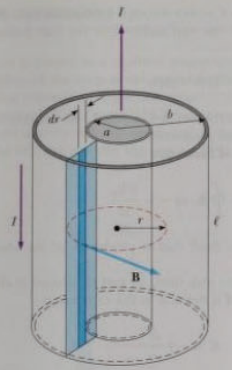


Figure 23.24 (Example 23.10) Section of a long coaxial cable. The inner and outer conductors carry equal and opposite currents.

The magnetic field is perpendicular to the outlined rectangular strip of length ℓ and width $(b - a)$, the cross section of interest because it is the surface one would choose for applying Lenz's law. Dividing this rectangle into strips of width dr, we see that the area of each strip is $\ell\, dr$ and the flux through the shaded strip is $B\, dA = B\ell\, dr$. Hence, the total flux through the region between the conductors is

$$\Phi_B = \int B\, dA = \int_a^b \frac{\mu_0 I}{2\pi r}\ell\, dr = \frac{\mu_0 I\ell}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I\ell}{2\pi}\ln\left(\frac{b}{a}\right)$$

Using this result, we find that the self-inductance of the cable is

$$L = \frac{\Phi_B}{I} = \frac{\mu_0 \ell}{2\pi}\ln\left(\frac{b}{a}\right)$$

(b) Calculate the total energy stored in the magnetic field of the cable.

Solution Using Equation 23.20 and the results to part (a) gives

$$U_B = \tfrac{1}{2}LI^2 = \frac{\mu_0 \ell I^2}{4\pi}\ln\left(\frac{b}{a}\right)$$

EXERCISE 9 A 10.0-V battery, a 5.00-Ω resistor, and a 10.0-H inductor are connected in series. After the current in the circuit has reached its maximum value, calculate (a) the power supplied by the battery, (b) the power dissipated in the resistor, (c) the power dissipated in the inductor, and (d) the energy stored in the magnetic field of the inductor.
Answer (a) 20 W (b) 20 W (c) 0 (d) 20 J

SUMMARY

Faraday's law of induction states that the emf induced in a circuit is directly proportional to the time rate of change of magnetic flux through the circuit:

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \qquad [23.3]$$

where N is the number of turns and Φ_B is the magnetic flux through each, given by

$$\Phi_B = \int \mathbf{B}\cdot d\mathbf{A} \qquad [23.1]$$

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When a conducting bar of length ℓ moves through a magnetic field, **B**, with a velocity of **v** so that **v** is perpendicular to **B**, the emf induced in the bar (called the **motional emf**) is

$$\mathcal{E} = -B\ell v \qquad [23.5]$$

Lenz's law states that the induced current and induced emf in a conductor are in such a direction as to oppose the change that produced them.

A general form of **Faraday's law of induction** is

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \qquad [23.9]$$

where **E** is a nonconservative electric field that is produced by the changing magnetic flux.

When the current in a coil changes with time, an emf is induced in the coil according to Faraday's law. The **self-induced emf** is defined by the expression

$$\mathcal{E} = -L\frac{dI}{dt} \qquad [23.10]$$

where L is the *inductance* of the coil. Inductance is a measure of the opposition of a device to a change in current. Inductance has the SI unit the **henry** (H), where 1 H = 1 V-s/A.

The **inductance** of a coil is

$$L = \frac{N\Phi_B}{I} \qquad [23.11]$$

where Φ_B is the magnetic flux through the coil and N is the total number of turns.

If a resistor and inductor are connected in series to a battery of emf $\mathcal{E}$, as shown in Figure 23.17, and a switch in the circuit is closed at $t = 0$, the current in the circuit varies in time according to the expression

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) \qquad [23.14]$$

where $\tau = L/R$ is the **time constant** of the RL circuit. That is, the current rises to an equilibrium value of $\mathcal{E}/R$ after a time interval that is long compared with τ.

If the battery is removed from an RL circuit, as in Figure 23.20 with S_1 open and S_2 closed, the current decays exponentially with time according to the expression

$$I(t) = \frac{\mathcal{E}}{R}e^{-t/\tau} \qquad [23.18]$$

where $\mathcal{E}/R$ is the initial current in the circuit.

The **energy** stored in the magnetic field of an inductor carrying a current I is

$$U_B = \tfrac{1}{2}LI^2 \qquad [23.20]$$

The **energy per unit volume** (or energy density) at a point where the magnetic field is B is

$$u_B = \frac{B^2}{2\mu_0} \qquad [23.22]$$

CONCEPTUAL QUESTIONS

1. A loop of wire is placed in a uniform magnetic field. For what orientation of the loop is the magnetic flux a maximum? For what orientation is the flux zero?
2. As the conducting bar in Figure Q23.2 moves to the right, an electric field is set up directed downward. If the bar were moving to the left, explain why the electric field would be upward.

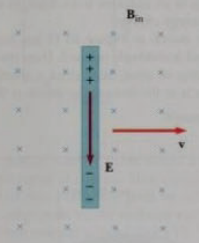


Figure Q23.2

3. As the bar in Figure Q23.2 moves perpendicular to the field, is an external force required to keep it moving with constant speed?
4. Magnetic storms on the Sun can cause difficulties with communications here on Earth. Why do these sunspots affect us in this way?
5. Wearing a metal bracelet in a region of strong magnetic field could be hazardous. Discuss.
6. How is electrical energy produced in dams (that is, how is the energy of motion of the water converted to ac electricity)?
7. A piece of aluminum is dropped vertically downward between the poles of an electromagnet. Does the magnetic field affect the velocity of the aluminum?
8. The bar in Figure Q23.8 moves on rails to the right with a velocity **v**, and the uniform, constant magnetic field is directed out of the page. Why is the induced current clockwise? If the bar were moving to the left, what would be the direction of the induced current?
9. In a beam-balance scale, an aluminum plate is sometimes used to slow the oscillations of the beam near equilibrium. The plate is mounted at the end of the beam and moves between the poles of a small horseshoe magnet attached to the frame. Why are the oscillations of the beam strongly damped near equilibrium?

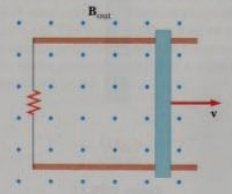


Figure Q23.8

10. When the switch in Figure Q23.10a is closed, a current is set up in the coil and the metal ring springs upward (Fig. Q23.10b). Explain this behavior.

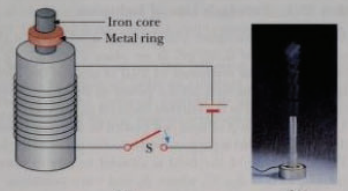


Figure Q23.10 (Questions 10 and 11) *(Photo Courtesy of Central Scientific Company)*

11. Assume that the battery in Figure Q23.10a is replaced by an ac source and the switch is held closed. If held down, the metal ring on top of the solenoid becomes hot. Why?
12. A bar magnet is held above a loop of wire in a horizontal plane, as in Figure Q23.12. The south end of the magnet is toward the loop of wire. The magnet is dropped toward the loop. Find the direction of the current through the resistor (a) while the magnet is falling toward the loop and (b) after the magnet has passed through the loop and moves away from it.
13. Find the direction of the current through the resistor in Figure Q23.13 (a) at the instant the switch is closed, (b) after the switch has been closed for several minutes, and (c) at the instant the switch is opened.
14. Discuss the similarities between the energy stored in the electric field of a charged capacitor and the energy stored in the magnetic field of a current-carrying coil.

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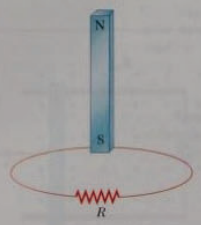


Figure Q23.12

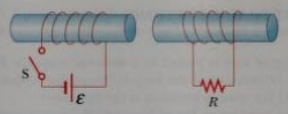


Figure Q23.13

15. What is the inductance of two inductors connected in series?

16. If the current in an inductor is doubled, by what factor does the stored energy change?

17. Suppose the switch in Figure 23.17 has been closed for a long time and is suddenly opened. Does the current instantaneously drop to zero? Why does a spark appear at the switch contacts at the instant the switch is thrown open?

PROBLEMS

Section 23.1 Faraday's Law of Induction

1. A strong electromagnet has a field of 1.60 T and a cross-sectional area of 0.200 m^2. If we place a coil having 200 turns and a total resistance of 20.0 Ω around the electromagnet and then turn off the power to the electromagnet in 20.0 ms, what is the current induced in the coil?

2. A rectangular loop of area A is placed in a region in which the magnetic field is perpendicular to the plane of the loop. The magnitude of the field is allowed to vary in time according to $B = B_0 e^{-t/\tau}$, where B_0 and τ are constants. The field has a value of B_0 at $t \leq 0$. (a) Use Faraday's law to show that the emf induced in the loop is

$$\mathcal{E} = \frac{AB_0}{\tau} e^{-t/\tau}$$

(b) Obtain a numerical value for $\mathcal{E}$ at $t = 4.00$ s when $A = 0.160\ m^2$, $B_0 = 0.350$ T, and $\tau = 2.00$ s. (c) For the values of A, B_0, and τ given in part (b), what is the maximum value of $\mathcal{E}$?

3. An aluminum ring of radius 5.00 cm and resistance 3.00×10^{-4} Ω is placed on top of a long air-core solenoid with 1000 turns per meter and radius 3.00 cm, as in Figure P23.3. Suppose that the magnetic field at the end of the solenoid due to the current in the solenoid is one half that at the center of the solenoid, and nearly parallel to its axis. Assume that the magnetic field is negligible outside of the solenoid. If the current in the solenoid is increasing at a rate of 270 A/s, (a) what is the induced current in the ring? (b) At the center of the ring, what is the magnetic field produced by the induced current in the ring? (c) What is the direction of this field?

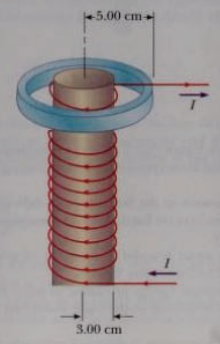


Figure P23.3

4. An aluminum ring of radius r_1 and resistance R is placed on top of a long air-core solenoid with n turns per meter and radius r_2, as in Figure P23.3. Suppose that the magnetic field at the end of the solenoid due to the current in the solenoid is one half that at the center of the solenoid, and nearly parallel to its axis. Assume that the magnetic field is negligible outside of the solenoid. If the current in the solenoid is increasing at a rate $\Delta I/\Delta t$, (a) what is the induced current in the ring? (b) At the center of the ring, what is the magnetic field produced by the induced current in the ring? (c) What is the direction of this field?

5. A long, straight wire carries a current $I = I_0 \sin(\omega t + \phi)$ and lies in the plane of a rectangular coil of N turns of wire, as shown in Figure P23.5. The quantities I_0, ω, and ϕ are all constants. Determine the emf induced in the coil by the magnetic field created by the current in the straight wire. Assume $I_0 = 50.0$ A, $\omega = 200\pi\ s^{-1}$, $N = 100$, $a = b = 5.00$ cm, and $\ell = 20.0$ cm.

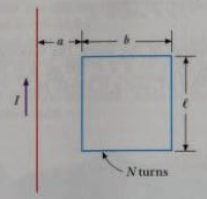


Figure P23.5

6. A magnetic field of 0.200 T exists within a solenoid of 500 turns and a diameter of 10.0 cm. How rapidly (that is, within what period of time) must the field be reduced to zero if the average magnitude of the induced emf within the coil during this time interval is to be 10.0 kV?

Section 23.3 Lenz's Law

7. Consider the arrangement shown in Figure P23.7. Assume that $R = 6.00\ \Omega$, $\ell = 1.20$ m, and a uniform 2.50-T magnetic field is directed into the page. At what speed should the bar be moved to produce a current of 0.500 A in the resistor?

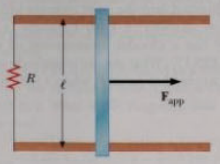


Figure P23.7

8. A conducting rod of length ℓ moves on two horizontal, frictionless rails, as in Figure P23.7. If a constant force of magnitude 1.00 N moves the bar at 2.00 m/s through a magnetic field **B** that is into the page, (a) what is the current through an 8.00-Ω resistor R? (b) What is the rate of energy dissipation in the resistor? (c) What is the mechanical power delivered by the 1.00-N force?

9. A Boeing 747 jet with a wingspan of 60.0 m is flying horizontally at 300 m/s over Phoenix, where the direction of the Earth's magnetic field is 58.0° below the horizontal. If the magnitude of the field is 50.0 μT, what is the voltage generated between the wing tips?

10. The 10.0-Ω square loop in Figure P23.10 is placed in a uniform 0.100-T magnetic field directed perpendicular to the plane of the loop. The loop, which is hinged at each vertex, is pulled as shown until the separation between points A and B is 3.00 m. If the process takes 0.100 s, what is the average current generated in the loop?

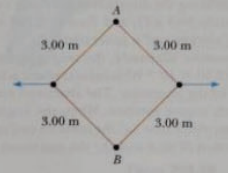


Figure P23.10

11. A helicopter has blades of length 3.00 m, extending out from a central hub and rotating at 2.00 rev/s. If the vertical component of the Earth's magnetic field is 0.500×10^{-4} T, what is the emf induced between the blade tip and the center hub?

12. Use Lenz's law to answer the following questions concerning the direction of induced currents. (a) What is the direction of the induced current in resistor R in Figure P23.12a when the bar magnet is moved to the left? (b) What

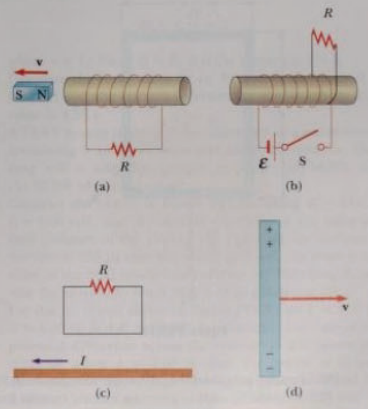


Figure P23.12

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is the direction of the current induced in the resistor R right after the switch S in Figure P23.12b is closed? (c) What is the direction of the induced current in R when the current I in Figure P23.12c decreases rapidly to zero? (d) A copper bar is moved to the right, as in Figure P23.12d, while its axis is maintained perpendicularly to a magnetic field. If the top of the bar becomes positive relative to the bottom, what is the direction of the magnetic field?

13. A 25-turn circular coil of wire has a diameter of 1.00 m. It is placed with its axis along the direction of the Earth's magnetic field (magnitude 50.0 μT), and then in 0.200 s it is flipped 180°. What average emf is generated?

14. In a 250-turn automotive alternator, the magnetic flux in each turn is $\Phi_B = (2.50 \times 10^{-4}\ \text{Wb})\cos(\omega t)$, where ω is the angular frequency of the alternator. The alternator rotates three times for each engine rotation. When the engine is running at 1000 rpm, determine (a) the induced emf in the alternator as a function of time and (b) the maximum emf in the alternator.

15. A conducting rectangular loop of mass M, resistance R, and dimensions w by ℓ falls from rest into a magnetic field $\mathbf{B}$, as in Figure P23.15. The loop accelerates until it reaches a terminal speed v_t. (a) Show that

$$v_t = \frac{MgR}{B^2 w^2}$$

(b) Why is v_t proportional to R? (c) Why is it inversely proportional to B^2?

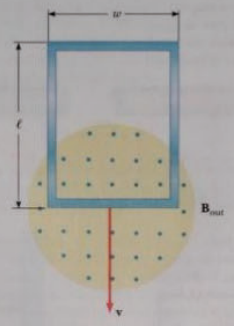


Figure P23.15

16. In 1832 Faraday proposed that the apparatus shown in Figure P23.16 could be used to generate electric current from the flowing water in the Thames River. Two conducting plates of lengths a and widths b are placed facing one another on opposite sides of the river, a distance w apart, and immersed entirely. The flow velocity of the river is $\mathbf{v}$, and the vertical component of the Earth's magnetic field is B. (a) Show that the current in the load resistor R is

$$I = \frac{abvB}{\rho + abR/w}$$

where ρ is the resistivity of the water. (b) Calculate the short-circuit current ($R = 0$) if $a = 100$ m, $b = 5.00$ m, $v = 3.00$ m/s, $B = 50.0\ \mu$T, and $\rho = 100\ \Omega\cdot$m.

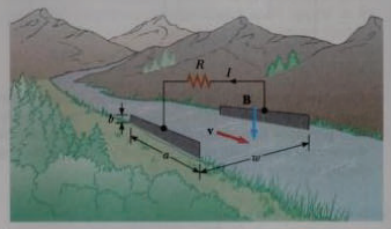


Figure P23.16

Section 23.4 Induced emfs and Electric Fields

17. A magnetic field directed into the page changes with time according to $B = (0.0300t^2 + 1.40)$ T, where t is in seconds. The field has a circular cross-section of radius $R = 2.50$ cm (Fig. P23.17). What are the magnitude and direction of the electric field at point P_1 when $t = 3.00$ s and $r_1 = 0.0200$ m?

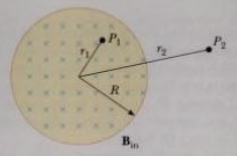


Figure P23.17

18. For the situation described in Figure P23.17, the magnetic field varies as $B = (2.00t^3 - 4.00t^2 + 0.800)$ T, and $r_2 = 2R = 5.00$ cm. (a) Calculate the magnitude and direction of the force exerted on an electron located at point P_2 when $t = 2.00$ s. (b) At what time is this force equal to zero?

19. A solenoid has a radius of 2.00 cm and 1000 turns/m. The current varies with time according to the expression $I = 3e^{0.2t}$, where I is in amperes and t is in seconds. Calculate the electric field 5.00 cm from the axis of the solenoid at $t = 10.0$ s.

20. A coil of 15 turns and radius 10.0 cm surrounds a long solenoid of radius 2.00 cm and 1.00×10^3 turns/meter (Fig. P23.20). If the current in the solenoid changes as $I = (5.00\ \text{A})\ \sin(120t)$, what is the induced emf in the 15-turn coil?

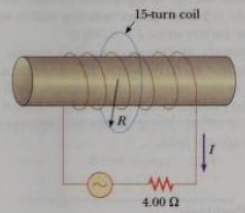


Figure P23.20

Section 23.5 Self-Inductance

21. A coil has an inductance of 3.00 mH, and a current through it changes from 0.200 A to 1.50 A in a time of 0.200 s. Find the magnitude of the average induced emf in the coil during this time.

22. An emf of 24.0 mV is induced in a 500-turn coil at an instant when the current is 4.00 A and is changing at the rate of 10.0 A/s. What is the magnetic flux through each turn of the coil?

23. A 10.0-mH inductor carries a current $I = I_{\max} \sin \omega t$, with $I_{\max} = 5.00$ A and $\omega/2\pi = 60.0$ Hz. What is the back emf as a function of time?

24. The current in a 90.0-mH inductor changes with time as $I = t^2 - 6t$ (in SI units). Find the magnitude of the induced emf at (a) $t = 1.00$ s and (b) $t = 4.00$ s. (c) At what time is the emf zero?

25. A toroid has a major radius R and a minor radius r and is tightly wound with N turns of wire, as shown in Figure P23.25. If $R \gg r$, the magnetic field within the region of the toroid of cross-sectional area $A = \pi r^2$ is essentially that of a long solenoid that has been bent into a large circle of radius R. Using the uniform field of a long solenoid, show that the self-inductance of such a toroid is given (approximately) by

$$L \cong \frac{\mu_0 N^2 A}{2\pi R}$$

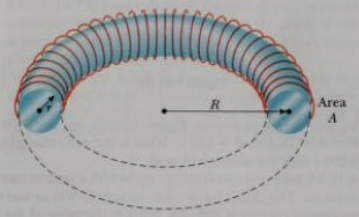


Figure P23.25

26. An inductor in the form of a solenoid contains 420 turns, is 16.0 cm in length, and has a cross-sectional area of 3.00 cm^2. What uniform rate of decrease of current through the inductor induces an emf of 175 μV?

Section 23.6 *RL* Circuits

27. Show that $I = I_0 e^{-t/\tau}$ is a solution of the differential equation

$$IR + L\frac{dI}{dt} = 0$$

where $\tau = L/R$ and $I_0 = \mathcal{E}/R$ is the current at $t = 0$.

28. Calculate the inductance in an *RL* circuit in which $R = 0.500\ \Omega$ and the current increases to one fourth its final value in 1.50 s.

29. A 12.0-V battery is about to be connected to a series circuit containing a 10.0-Ω resistor and a 2.00-H inductor. How long will it take the current to reach (a) 50.0% and (b) 90.0% of its final value?

30. Consider the circuit in Figure P23.30, taking $\mathcal{E} = 6.00$ V, $L = 8.00$ mH, and $R = 4.00\ \Omega$. (a) What is the inductive time constant of the circuit? (b) Calculate the current in the circuit 250 μs after the switch is closed. (c) What is the value of the final steady-state current? (d) How long does it take the current to reach 80.0% of its maximum value?

31. For the *RL* circuit shown in Figure P23.30, let $L = 3.00$ H, $R = 8.00\ \Omega$, and $\mathcal{E} = 36.0$ V. (a) Calculate the ratio of the potential difference across the resistor to that across the inductor when $I = 2.00$ A after the switch is closed. (b) Calculate the voltage across the inductor when $I = 4.50$ A.

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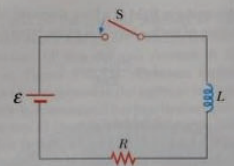


Figure P23.30

32. In the circuit shown in Figure P23.30, let $L = 7.00$ H, $R = 9.00\ \Omega$, and $\mathcal{E} = 120$ V. What is the self-induced emf 0.200 s after the switch is closed?
33. A 12.0-V battery is connected in series with a resistor and an inductor. The circuit has a time constant of 500 μs and the maximum current is 200 mA. What is the value of the inductance?
34. An inductor that has an inductance of 15.0 H and a resistance of 30.0 Ω is connected across a 100-V battery. What is the rate of increase of the current (a) at $t = 0$ and (b) at $t = 1.50$ s?
35. One application of an *RL* circuit is the generation of time-varying high-voltage from a low-voltage source, as shown in Figure P23.35. (a) What is the current in the circuit a long time after the switch has been in position *A*? (b) Now the switch is thrown quickly from *A* to *B*. Compute the initial voltage across each resistor and the inductor. (c) How much time elapses before the voltage across the inductor drops to 12.0 V?

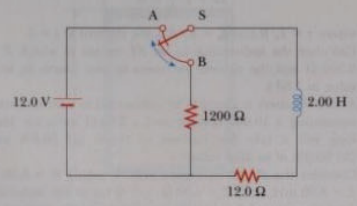


Figure P23.35

Section 23.7 Energy Stored in a Magnetic Field

36. Calculate the energy associated with the magnetic field of a 200-turn solenoid in which a current of 1.75 A produces a flux of 3.70×10^{-4} Wb in each turn.
37. The magnetic field inside a superconducting solenoid is 4.50 T. The solenoid has an inner diameter of 6.20 cm and a length of 26.0 cm. Determine (a) the magnetic energy density in the field and (b) the energy stored in the magnetic field within the solenoid.
38. At $t = 0$, an emf of 500 V is applied to a coil that has an inductance of 0.800 H and a resistance of 30.0 Ω. (a) Find the energy stored in the magnetic field when the current reaches half its maximum value. (b) After the emf is connected, how long does it take the current to reach this value?
39. On a clear day there is a 100-V/m vertical electric field near the Earth's surface. At the same time, the Earth's magnetic field has a magnitude of 0.500×10^{-4} T. Compute the energy densities of the two fields.
40. An *RL* circuit in which $L = 4.00$ H and $R = 5.00\ \Omega$ is connected to a 22.0-V battery at $t = 0$. (a) What energy is stored in the inductor when the current is 0.500 A? (b) At what rate is energy being stored in the inductor when $I = 1.00$ A? (c) What power is being delivered to the circuit by the battery when $I = 0.500$ A?
41. The magnitude of the magnetic field outside a sphere of radius R is $B = B_0(R/r)^2$, where B_0 is a constant. Determine the total energy stored in the magnetic field outside the sphere and evaluate your result for $B_0 = 5.00 \times 10^{-5}$ T and $R = 6.00 \times 10^6$ m. These values are appropriate for the Earth's magnetic field.

Additional Problems

42. A solenoid wound with 2 000 turns/m is supplied with current that varies in time according to $I = 4 \sin(120\pi t)$, where I is in A and t is in s. A small coaxial circular coil of 40 turns and radius $r = 5.00$ cm is located inside the solenoid near its center. (a) Derive an expression that describes the manner in which the emf in the small coil varies in time. (b) At what average rate is energy dissipated in the small coil if the windings have a total resistance of 8.00 Ω?
43. A flat compact coil of area 0.100 m^2 is rotating at 60.0 rev/s with the axis of rotation perpendicular to a 0.200-T magnetic field. (a) If there are 1 000 turns on the coil, what is the maximum voltage induced in it? (b) When the maximum induced voltage occurs, what is the orientation of the coil with respect to the magnetic field?
44. Figure P23.44 is a graph of the induced emf versus time for a coil of N turns rotating with angular velocity ω in a uniform magnetic field directed perpendicular to the axis of rotation of the coil. Copy this sketch (on a larger scale), and on the same set of axes show the graph of emf versus t when (a) the number of turns in the coil is doubled, (b) the angular velocity is doubled, (c) the angular velocity is doubled and the number of turns in the coil is halved.
45. A bar of mass m, length d, and resistance R slides without friction on parallel rails, as shown in Figure P23.45. A battery

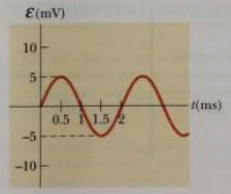


Figure P23.44

that maintains a constant emf $\mathcal{E}$ is connected between the rails, and a constant magnetic field **B** is directed perpendicular to the plane of the page. If the bar starts from rest, show that at time t it moves with a speed

$$v = \frac{\mathcal{E}}{Bd}\left(1 - e^{-B^2d^2t/mR}\right)$$

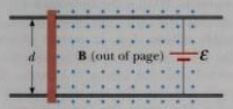


Figure P23.45

46. An automobile has a vertical radio antenna 1.20 m long. The automobile travels at 65.0 km/h on a horizontal road where the Earth's magnetic field is 50.0 μT directed downward (toward the north) at an angle of 65.0° below the horizontal. (a) Specify the direction that the automobile should move in order to generate the maximum motional emf in the antenna, with the top of the antenna positive relative to the bottom. (b) Calculate the magnitude of this induced emf.
47. The magnetic flux threading a metal ring varies with time t according to $\Phi_B = 3(at^3 - bt^2)$ T·m^2, with $a = 2.00$ s^{-3}, and $b = 6.00$ s^{-2}. The resistance of the ring is 3.00 Ω. Determine the maximum current induced in the ring during the interval from $t = 0$ to $t = 2.00$ s.
48. Magnetic field values are often determined by using a device known as a *search coil*. This technique depends on the measurement of the total charge passing through a coil in a time interval during which the magnetic flux linking the windings changes either because of the motion of the coil or because of a change in the value of B. (a) Show that if the flux through the coil changes from Φ_1 to Φ_2, the charge transferred through the coil between t_1 and t_2 will be given by $Q = N(\Phi_2 - \Phi_1)/R$, where R is the resistance of the coil and associated circuitry (galvanometer) and N is the number of turns. (b) As a specific example, calculate B when a 100-turn coil of resistance 200 Ω and cross-sectional area 40.0 cm^2 produces the following results. A total charge of 5.00×10^{-4} C passes through the coil when it is rotated in a uniform field from a position where the plane of the coil is perpendicular to the field to a position where the coil's plane is parallel to the field.
49. Assume that the switch in Figure P23.49 is initially in position 1. Show that if the switch is thrown from position 1 to position 2, all the energy stored in the magnetic field of the inductor is dissipated as thermal energy in the resistor.

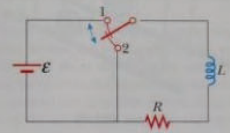


Figure P23.49

50. The inductor in Figure P23.50 has negligible resistance. When the switch is thrown open after having been closed for a long time, the current in the inductor drops to 0.250 A in 0.150 s. What is the inductance of the inductor?

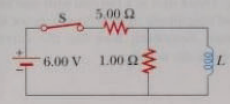


Figure P23.50

51. A novel method of storing electrical energy has been proposed. A huge underground superconducting coil, 1.00 km in diameter, would be fabricated. It would carry a maximum current of 50.0 kA through each winding of a 150-turn Nb_3Sn solenoid. (a) If the inductance of this huge coil were 50.0 H, what would be the total energy stored? (b) What would be the compressive force per meter length acting between two adjacent windings 0.250 m apart?
52. In Figure P23.52, the rolling axle, 1.50 m long, is pushed along horizontal rails at a constant speed $v = 3.00$ m/s. A resistor $R = 0.400\ \Omega$ is connected to the rails at points *a*

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and *b*, directly opposite each other. (The wheels make good electrical contact with the rails, and so the axle, rails, and *R* form a closed-loop circuit. The only significant resistance in the circuit is *R*.) There is a uniform magnetic field, $B = 0.0800$ T, vertically downward. (a) Find the induced current *I* in the resistor. (b) What horizontal force **F** is required to keep the axle rolling at constant speed? (c) Which end of the resistor, *a* or *b*, is at the higher electric potential? (d) After the axle rolls past the resistor, does the current in *R* reverse direction?

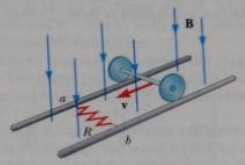


Figure P23.52

53. (a) A flat circular coil does not really produce a uniform magnetic field in the area it encloses, but estimate the self-inductance of a flat circular coil, with radius *R* and *N* turns, by supposing the field at its center is uniform over its area. (b) A circuit on a laboratory table consists of a 1.5-V battery, a 270-Ω resistor, a knife switch, and three 30-cm-long patch cords connecting them. Suppose the circuit is arranged to be circular. Think of it as a flat coil with one turn. Compute the order of magnitude of its self-inductance and (c) of the time constant describing how fast the current increases when you close the switch.

Spreadsheet Problems

S1. The size and orientation of a flat surface of area *A* can be described by a vector $\mathbf{A} = A\hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is a unit vector perpendicular to the surface. Suppose that a magnetic field **B** exists in the region of this surface. If the field is constant over the area *A*, then the magnetic flux, Φ_B, through the surface is $\Phi_B = \mathbf{B}\cdot\mathbf{A} = BA\cos\theta = B_xA_x + B_yA_y + B_zA_z$, where $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$ and $\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$ (Fig. S23.1). Spreadsheet 23.1 will calculate the flux as a function of time. It will also numerically differentiate the flux to find the induced emf, $\mathcal{E} = -\Delta\Phi_B/\Delta t$. (a) Set $A_x = 0$, $A_y = 0$, $A_z = 0.2$ m^2, $B_x = 0$, $B_y = 0.5$ T, and $B_z = 0.6$ T at $t = 0$. Copy these values down through $t = 4.5$ s.

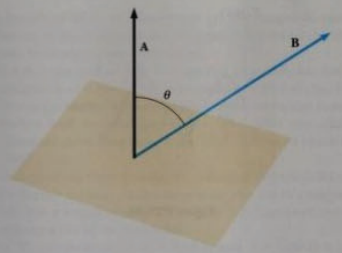


Figure S23.1

What is the induced emf? (b) Modify the spreadsheet so that B_z increases with time, $B_z = 0.2t$, where *t* is in seconds and B_z is in teslas. What is the induced emf?

S2. Modify Spreadsheet 23.1 so that $\mathbf{A} = 0.02\sin(\omega t)\mathbf{i} + 0.02\cos(\omega t)\mathbf{k}$, where *A* is in square meters. Set $\mathbf{B} = 0.5\text{ T}\,\mathbf{k}$. (a) What physical situation does this variation in **A** correspond to? (b) Let $\omega = 1$ rad/s. View the included graph and describe the induced emf. (c) Increase ω to 2 rad/s. How does the induced emf change from part (b)?

S3. (a) With **A** as given in Problem S2, modify Spreadsheet 23.1 so that $B_z = 0.2t$, where *t* is in seconds and B_z is in teslas. View the included graph, and explain your numerical results. (b) Choose any other time dependence for **B**. Explore the consequences of your choice.

S4. A coil of self-inductance *L* carries a current given by $I = I_{max}\sin 2\pi ft$. The self-induced emf in the coil is $\mathcal{E}_L = -L\,dI/dt$. Develop a spreadsheet to calculate *I* as a function of time. Numerically differentiate the current and calculate $\mathcal{E}_L$. Choose $I_{max} = 2.00$ A, $f = 60.0$ Hz, and $L = 10.0$ mH. Plot $\mathcal{E}_L$ versus *t*.

S5. Spreadsheet 23.2 calculates the current and the energy stored in the magnetic field of an *RL* circuit when the circuit is charging and when it is discharging. Use $R = 1000\ \Omega$, $L = 0.35$ H, and $\mathcal{E} = 10$ V. (a) What is the time constant of the circuit? From the graph of the current versus time when the circuit is charging, how much time has elapsed when the current is 50% of its maximum value? 90%? 99%? Give your answers both in seconds and in multiples of the time constant. (b) Repeat for the case when the circuit is discharging. (c) How much time has elapsed when the energy is 50% of its maximum value for the two cases? 90%? 99%?

ANSWERS TO CONCEPTUAL PROBLEMS

1. According to Faraday's law, an emf is induced in a wire loop if the magnetic flux through the loop changes with time. In this situation, an emf can be induced by either rotating the loop around an arbitrary axis or by changing the shape of the loop.
2. As the spacecraft moves through space, it is apparently moving from a region of one magnetic field strength to a region of a different magnetic field strength. The changing magnetic field through the coil induces an emf and a corresponding induced current in the coil.
3. The magnetic field lines around the transmission cable will be circular. If you place your loop around the cable, there will be no field lines passing through the loop, so no emf will be induced. The loop needs to be placed next to the cable, with the plane of the loop parallel to the cable, to maximize the changing flux through the area as the alternating current produces an oscillating magnetic field.
4. A constant induced emf requires a magnetic field that is changing at a constant rate in one direction—for example, always increasing or always decreasing. It is impossible for a magnetic field to increase forever, both in terms of energy considerations and technological concerns. In the case of a decreasing field, once it reaches zero and then reverses direction, we again face the problem with the field increasing without bound in the opposite direction.
5. The pickup coil is placed near the vibrating guitar string, which is made of a metal that can be magnetized. The permanent magnet inside the coil magnetizes the portion of the string nearest the coil. When the guitar string vibrates at some frequency, its magnetized segment produces a changing magnetic flux through the pickup coil. The changing flux induces a voltage in the coil, and this voltage is fed to an amplifier. The output of the amplifier is sent to the loudspeakers, producing the sound waves that we hear.
6. The primary coil of the transformer is an inductor. When an ac voltage is applied, the back emf due to the inductance will limit the current flow through the coil. If dc voltage is applied, there is no back emf, and the current can rise to a higher value. It is possible that this increased current will deliver so much energy to the resistance in the coil that its temperature rises to the point at which the insulation on the wire can burn.
7. The aluminum sheet and magnet supply *magnetic damping* to the system, so that the oscillations of the arms damp out rapidly. When the sheet moves in the magnetic field, eddy currents are induced in the aluminum. According to Lenz's law, these currents are in a direction such as to oppose the movement of the aluminum sheet, providing magnetic friction.
8. As the iron rod is being inserted into the solenoid, the inductance of the inductor increases. As a result, more potential difference appears across the inductor than before. As a consequence, less potential difference appears across the bulb, and its brightness decreases.

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24

Electromagnetic Waves

CHAPTER OUTLINE

24.1 Displacement Current and the Generalized Ampère's Law
24.2 Maxwell's Wonderful Equations
24.3 Electromagnetic Waves
24.4 Hertz's Discoveries
24.5 Production of Electromagnetic Waves by an Antenna
24.6 Energy Carried by Electromagnetic Waves
24.7 Momentum and Radiation Pressure
24.8 The Spectrum of Electromagnetic Waves
24.9 Polarization

Although we are not always aware of their presence, electromagnetic waves permeate our environment. In the form of visible light, they enable us to view the world around us; infrared waves warm our environment; radio-frequency waves carry our favorite television and radio programs; microwaves cook our food and are used in radar communication systems. The list goes on and on. The waves we described in Chapters 13 and 14 were mechanical waves, which, by definition, can exist only if a medium is present. Electromagnetic waves, in contrast, can propagate through a vacuum.

The purpose of this chapter is to explore the properties of electromagnetic waves. The fundamental laws of electricity and magnetism—Maxwell's equations—form the basis of all electromagnetic phenomena. One of these equations predicts that a time-varying electric field produces a magnetic field just as a time-varying magnetic field produces an electric field. From this generalization, Maxwell provided the final important link between electric and magnetic fields. The most dramatic prediction of his equations is the existence of electromagnetic waves that propagate through empty space with the speed of light. This discovery led to many practical applications, such as radio and television, and to the realization that light is one form of electromagnetic radiation.



An old-style rancher in St. Augustine Plains, New Mexico, rides past one of the 27 radio telescopes that comprise the Very Large Array (VLA). Arranged in a Y-shaped configuration on a system of railroad tracks, the radio telescopes of the VLA capture and focus electromagnetic waves from space. *(© Danny Lehman)*

24.1 • DISPLACEMENT CURRENT AND THE GENERALIZED AMPÈRE'S LAW

We have seen that charges in motion, or currents, produce magnetic fields. When a current-carrying conductor has high symmetry, we can calculate the magnetic field using Ampère's law, given by Equation 22.22:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \qquad [22.22]$$

where **the line integral is over any closed path through which the conduction current passes,** and the conduction current is defined by $I = dQ/dt$. (In this section, we use the term *conduction current* to refer to the current carried by charged particles such as electrons in a wire or ions in an electrolytic solution.) We now show that **Ampère's law in this form is valid only if the conduction current is continuous in space.** Maxwell recognized this limitation and modified Ampère's law to include all possible situations.

We can understand this problem by considering a capacitor being charged as in Figure 24.1. When the conduction current flows, the charge on the plate changes, but **no conduction current passes between the plates.** Now consider the two surfaces S_1 and S_2 in Figure 24.1 bounded by the same path P. Ampère's law says that the line integral of $\mathbf{B} \cdot d\mathbf{s}$ around this path must equal $\mu_0 I$, where I is the total current through any surface bounded by the path P.

When the path P is considered as bounding S_1, the result of the integral is $\mu_0 I$ because the conduction current passes through S_1. When the path bounds S_2, however, the result is zero, because no conduction current passes through S_2. Thus, we have a contradictory situation that arises from the discontinuity of the conduction current! Maxwell solved this problem by postulating an additional term on the right side of Equation 22.22, called the **displacement current,** I_d, defined as

$$I_d \equiv \epsilon_0 \frac{d\Phi_E}{dt} \qquad [24.1]$$

• *Displacement current*

Recall that Φ_E is the flux of the electric field, defined as $\Phi_E = \int \mathbf{E} \cdot d\mathbf{A}$.

As the capacitor is being charged (or discharged), the changing electric field between the plates may be modeled as a current that bridges the discontinuity in the conduction current in the wire. When the expression for the displacement current given by Equation 24.1 is added to the right side of Ampère's law, the difficulty represented by Figure 24.1 is resolved. No matter what surface bounded by the path P is chosen, some combination of conduction and displacement current will pass through it. With this new term I_d, we can express the generalized form of Ampère's law (sometimes called the **Ampère–Maxwell law**) as[1]

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 (I + I_d) = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \qquad [24.2]$$

• *Ampère-Maxwell law*

[1]Strictly speaking, this expression is valid only in a vacuum. If a magnetic material is present, a magnetizing current must also be included on the right side of Equation 24.2 to make Ampère's law fully general.

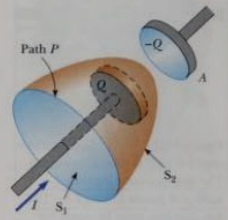


Figure 24.1 The surfaces S_1 and S_2 are bounded by the same path P. The conduction current in the wire passes only through S_1. This leads to a contradiction in Ampère's law that is resolved only if one postulates a displacement current through S_2.

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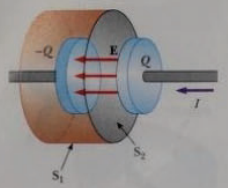


Figure 24.2 The conduction current $I = dQ/dt$ passes through S_1. The displacement current $I_d = \epsilon_0 d\Phi_E/dt$ passes through S_2. The two currents must be equal for continuity. In general, the total current through any surface bounded by some path is $I + I_d$.

The meaning of this expression can be understood by referring to Figure 24.2. The electric flux through S_2 is $\Phi_E = \int \mathbf{E} \cdot d\mathbf{A} = EA$, where A is the area of the capacitor plates and E is the uniform electric field strength between the plates. If Q is the charge on the plates at any instant, then $E = Q/\epsilon_0 A$ (Section 20.7). Therefore, the electric flux through S_2 is simply

$$\Phi_E = EA = \frac{Q}{\epsilon_0}$$

Hence, the displacement current I_d through S_2 is

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt} \qquad [24.3]$$

That is, the displacement current is precisely equal to the conduction current I through S_1!

The central point of this formalism is the fact that **magnetic fields are produced both by conduction currents and by changing electric fields.**

24.2 • MAXWELL'S WONDERFUL EQUATIONS

In this section we present four equations that can be regarded as the basis of all electrical and magnetic phenomena. These relationships, known as Maxwell's equations after James Clerk Maxwell, are as fundamental to electromagnetic phenomena as Newton's laws are to mechanical phenomena. In fact, the theory developed by Maxwell was more far-reaching than even he imagined, because it was shown by Einstein in 1905 to be in agreement with the special theory of relativity. As we shall see, Maxwell's equations represent laws of electricity and magnetism that have already been discussed. However, the equations have additional important consequences, in that they predict the existence of electromagnetic waves (traveling patterns of electric and magnetic fields), which travel with a speed of $c = 1/\sqrt{\mu_0 \epsilon_0} \approx 3 \times 10^8$ m/s, the speed of light. Furthermore, Maxwell's equations show that such waves are radiated by accelerating charges.

For simplicity, we present **Maxwell's equations** as applied to free space—that is, in the absence of any dielectric or magnetic material. The four equations are:

Maxwell's equations •

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0} \qquad [24.4]$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0 \qquad [24.5]$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \qquad [24.6]$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt} \qquad [24.7]$$

Equation 24.4 is **Gauss's law,** which states that **the total electric flux through any closed surface equals the net charge inside that surface divided by ϵ_0.** This law describes how charges create electric fields, where electric field lines originate on positive charges and terminate on negative charges.

Equation 24.5, which can be considered **Gauss's law in magnetism, says that the net magnetic flux through a closed surface is zero.** That is, the number of magnetic field lines that enter a closed volume must equal the number that leave that volume. This implies that magnetic field lines cannot begin or end at any point. If they did, this would mean that isolated magnetic monopoles existed at those points. The fact that isolated magnetic monopoles have not been observed in nature can be taken as a basis of Equation 24.5.

Equation 24.6 is **Faraday's law of induction,** which describes how a changing magnetic field creates an electric field. This law states that **the line integral of the electric field around any closed path (which equals the emf) equals the rate of change of magnetic flux through any surface area bounded by that path.** One consequence of Faraday's law is the current induced in a conducting loop placed in a time-varying magnetic field.

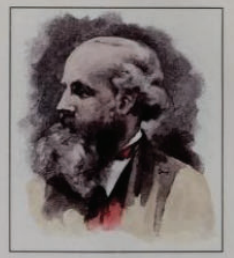
Equation 24.7, which is the **Ampère–Maxwell law,** describes how both an electric current and a changing electric field will create a magnetic field. That is, **the line integral of the magnetic field around any closed path is determined by the sum of the net current through that path and the rate of change of electric flux through any surface bounded by that path.**

Once the electric and magnetic fields are known at some point in space, the force those fields exert on a particle of charge q can be calculated from the expression

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \qquad [24.8]$$

This is called the **Lorentz force.** Maxwell's equations, together with this force law, give a complete description of all classical electromagnetic interactions.

It is interesting to note the symmetry of Maxwell's equations. Equations 24.4 and 24.5 are symmetric, apart from the absence of a magnetic monopole term in Equation 24.5. Furthermore, Equations 24.6 and 24.7 are symmetric in that the line integrals of **E** and **B** around a closed path are related to the rate of change of magnetic flux and electric flux, respectively.



James Clerk Maxwell (1839–1879)

James Clerk Maxwell is generally regarded as the greatest theoretical physicist of the 19th century. Born in Edinburgh to a well-known Scottish family, he entered the University of Edinburgh at age 15. He was appointed to his first professorship, at Aberdeen, in 1856. This was the beginning of a career during which Maxwell would develop the electromagnetic theory of light, the kinetic theory of gases, and explanations of the nature of Saturn's rings and of color vision. *(North Wind Picture Archives)*

24.3 • ELECTROMAGNETIC WAVES

The fundamental laws governing the behavior of electric and magnetic fields are Maxwell's equations, discussed in Section 24.2. In his unified theory of electromagnetism, Maxwell showed that time-dependent electric and magnetic fields satisfy a wave equation. The most significant outcome of this theory is the prediction of the existence of electromagnetic waves.

First let us assume that the electromagnetic wave is a **plane wave**—that is, it travels in one direction only. The plane wave we are describing has the following properties. It travels in the x direction (the direction of propagation), the electric field **E** is in the y direction, and the magnetic field **B** is in the z direction, as in Figure 24.3. Waves in which the electric and magnetic fields are restricted to being parallel to certain lines in the yz plane are said to be **linearly polarized waves.** Furthermore, we assume that at any point, P, E and B depend on x and t only and not on the y or z coordinate.

The properties of electromagnetic waves can be deduced from Maxwell's equations. We can relate E and B to each other by Equations 24.6 and 24.7. In empty

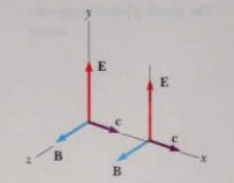


Figure 24.3 A plane-polarized electromagnetic wave traveling in the positive x direction. The electric field is along the y direction, and the magnetic field is along the z direction. These fields depend only on x and t.

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space, where $Q = 0$ and $I = 0$, Equation 24.6 remains unchanged and Equation 24.7 becomes

$$\oint \mathbf{B} \cdot d\mathbf{s} = \epsilon_0 \mu_0 \frac{d\Phi_E}{dt} \qquad [24.9]$$

Using Equations 24.6 and 24.9 and the plane wave assumption, the following differential equations relating E and B are obtained. For simplicity of notation, we drop the subscripts on the components E_y and B_z:

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \qquad [24.10]$$

$$\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t} \qquad [24.11]$$

Note that the derivatives here are partial derivatives. For example, when $\partial E/\partial x$ is evaluated, we assume that t is constant. Likewise, when evaluating $\partial B/\partial t$, x is held constant. Taking the derivative of Equation 24.10 with respect to x and combining this with Equation 24.11 we get

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial x}\left(\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right) = -\frac{\partial}{\partial t}\left(-\mu_0 \epsilon_0 \frac{\partial E}{\partial t}\right)$$

Electric field wave equation for electromagnetic waves in free space

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \qquad [24.12]$$

In the same manner, taking a derivative of Equation 24.11 and combining it with Equation 24.12, we get

Magnetic field wave equation for electromagnetic waves in free space

$$\frac{\partial^2 B}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} \qquad [24.13]$$

Equations 24.12 and 24.13 both have the form of the general wave equation,[2] with a speed, c, of

The speed of electromagnetic waves

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \qquad [24.14]$$

Taking $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$ and $\epsilon_0 = 8.854\,18 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$ in Equation 24.14, we find that $c = 2.997\,92 \times 10^8$ m/s. Because this speed is precisely the same as the speed of light in empty space,[3] one is led to believe (correctly) that light is an electromagnetic wave.

The simplest plane wave solution to Equations 24.12 and 24.13 are those for which the field amplitudes E and B vary with x and t according to the expressions

Sinusoidal electric and magnetic fields

$$E = E_{\max} \cos(kx - \omega t) \qquad [24.15]$$

$$B = B_{\max} \cos(kx - \omega t) \qquad [24.16]$$

[2]The general wave equation is of the form $(\partial^2 g/\partial x^2) = (1/v^2)(\partial^2 g/\partial t^2)$, where v is the speed of the wave and g is the wave variable.

[3]Because of the redefinition of the meter in 1983, the speed of light is now a *defined* quantity with an *exact* value of $c = 2.997\,924\,58 \times 10^8$ m/s.

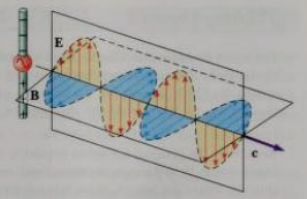


Figure 24.4 An electromagnetic wave sent out by oscillating charges in an antenna. This represents the wave at one instant of time. Note that the electric field is perpendicular to the magnetic field, and both are perpendicular to the direction of wave propagation.

where $E_{\max}$ and $B_{\max}$ are the maximum values of the fields. The angular wave number $k = 2\pi/\lambda$, where λ is the wavelength, and the angular frequency $\omega = 2\pi f$, where f is the number of cycles per second. Figure 24.4 represents one instant of a sinusoidal, linearly polarized plane wave moving in the positive x direction. Note that the electric and magnetic fields of a plane electromagnetic wave are perpendicular to each other and to the direction of propagation. Thus, **electromagnetic waves are transverse waves.**

Taking partial derivatives of Equations 24.15 and 24.16, and substituting into Equations 24.10 and 24.11, we find that

$$\frac{E}{B} = c \qquad [24.17]$$

That is, **at every instant the ratio of the electric field to the magnetic field of an electromagnetic wave equals the speed of light.**

Finally, electromagnetic waves obey the **superposition principle,** because the differential equations involving E and B are *linear* equations. For example, two waves traveling in opposite directions with the same frequency could be added by simply adding the wave fields algebraically.

Let us summarize the properties of electromagnetic waves as we have described them:

Properties of electromagnetic waves

- The solutions of Maxwell's third and fourth equations are wavelike.
- Electromagnetic waves travel through empty space with the speed of light, $c = 1/\sqrt{\epsilon_0 \mu_0}$.
- The electric and magnetic field components of plane electromagnetic waves are perpendicular to each other and also perpendicular to the direction of wave propagation. The latter property can be summarized by saying that electromagnetic waves are transverse waves.
- The relative magnitudes of **E** and **B** in empty space are related by $E/B = c$.
- Electromagnetic waves obey the superposition principle.

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Thinking Physics 1

There is a Doppler effect for light, which is demonstrated in astronomical observations by the shift of spectral lines from receding galaxies toward the red end of the visible spectrum. The equation for the Doppler effect for light is not the same equation as that for sound. Why would the equation be different?

Reasoning In general, for waves requiring a medium, the velocities of the source and observer can be separately measured with respect to a third entity, the medium. In the Doppler effect for sound, these two velocities are the velocities of the source and of the observer relative to the air. Light as an electromagnetic wave does not require a medium—there is no third entity. Thus, we cannot identify separate velocities for the source and observer—only their relative velocity can be identified. As a result, a different equation must be used, one that contains only this single velocity. This equation can be generated from the laws of relativity.

Example 24.1 An Electromagnetic Wave

A plane electromagnetic sinusoidal wave of frequency 40.0 MHz travels in free space in the x direction, as in Figure 24.5. At some point and at some instant, the electric field has its maximum value of 750 N/C and is along the y axis.

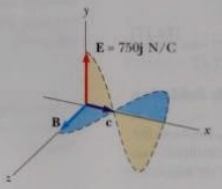


Figure 24.5 (Example 24.1) At some instant, a plane electromagnetic wave moving in the x direction has a maximum electric field of 750 N/C in the positive y direction. The corresponding magnetic field at that point has a magnitude E/c and is in the z direction.

(a) Determine the wavelength and period of the wave.

Solution Because $c = \lambda f$ and the wave has a frequency $f = 40.0\ \text{MHz} = 4.00 \times 10^7\ \text{s}^{-1}$, we get

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8\ \text{m/s}}{4.00 \times 10^7\ \text{s}^{-1}} = 7.50\ \text{m}$$

The period T of the wave equals the inverse of the frequency, and so

$$T = \frac{1}{f} = \frac{1}{4.00 \times 10^7\ \text{s}^{-1}} = 2.50 \times 10^{-8}\ \text{s}$$

(b) Calculate the magnitude and direction of the magnetic field when $\mathbf{E} = 750\mathbf{j}$ N/C.

Solution From Equation 24.17 we see that

$$B_{\max} = \frac{E_{\max}}{c} = \frac{750\ \text{N/C}}{3.00 \times 10^8\ \text{m/s}} = 2.50 \times 10^{-6}\ \text{T}$$

Because $\mathbf{E}$ and $\mathbf{B}$ must be perpendicular to each other and both must be perpendicular to the direction of wave propagation (x in this case), we conclude that $\mathbf{B}$ is in the z direction.

(c) Write expressions for the space–time variation of the electric and magnetic field components for this wave.

Solution We can apply Equations 24.15 and 24.16 directly:

$$E = E_{\max} \cos(kx - \omega t) = (750\ \text{N/C}) \cos(kx - \omega t)$$

$$B = B_{\max} \cos(kx - \omega t) = (2.50 \times 10^{-6}\ \text{T}) \cos(kx - \omega t)$$

where

$$\omega = 2\pi f = 2\pi(4.00 \times 10^7\ \text{s}^{-1}) = 8\pi \times 10^7\ \text{rad/s}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{7.50\ \text{m}} = 0.838\ \text{rad/m}$$

24.4 · HERTZ'S DISCOVERIES

In 1888, Heinrich Hertz (1857–1894) was the first to generate and detect electromagnetic waves in a laboratory setting. In order to appreciate the details of his experiment, let us first examine the properties of an LC circuit. In such a circuit, a charged capacitor is connected to an inductor, as in Figure 24.6. When the switch is closed, both the current in the circuit and the charge on the capacitor oscillate.

Let us assume that the capacitor has an initial charge of $Q_{\max}$ and that the switch is closed at $t = 0$. It is convenient to describe what happens from an energy viewpoint. When the capacitor is fully charged, the total energy in the circuit is stored in the electric field of the capacitor and is equal to $Q_{\max}^2/2C$. At this time, the current is zero, and so no energy is stored in the inductor. As the capacitor begins to discharge, the energy stored in its electric field decreases. At the same time, the current increases and an amount of energy equal to $LI^2/2$ is now stored in the magnetic field of the inductor. Thus, we see that energy is transferred from the electric field of the capacitor to the magnetic field of the inductor. When the capacitor is fully discharged, it stores no energy. At this time, the current reaches its maximum value and all of the energy is stored in the inductor. The process then repeats in the reverse direction. The energy continues to transfer between the inductor and the capacitor, corresponding to oscillations of both current and charge.

A representation of this energy transfer is shown in Figure 24.7. The behavior of the circuit is analogous to that of the oscillating mass–spring system studied in Chapter 12. The potential energy stored in a stretched spring, $kx^2/2$, is analogous to the potential energy stored in the capacitor, $Q_{\max}^2/2C$. The kinetic energy of the moving mass, $mv^2/2$, is analogous to the energy stored in the inductor, $LI^2/2$, which requires the presence of moving charges. In Figure 24.7a, all of the energy is stored as electric potential energy in the capacitor at $t = 0$ (because $I = 0$). In Figure 24.7b, all of the energy is stored as magnetic potential energy in the inductor, $LI_{\max}^2/2$, where $I_{\max}$ is the maximum current. At intermediate points, part of the energy is electric and part is magnetic.

The frequency of oscillation of an LC circuit, called the *resonance frequency*, is

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad [24.18]$$

The circuit Hertz used in his investigations of electromagnetic waves is similar to that already discussed and is shown schematically in Figure 24.8. An induction coil (a large coil of wire) is connected to two metal spheres with a narrow gap between them to form a capacitor. Oscillations are initiated in the circuit by sending short voltage pulses via the coil to the spheres, charging one positive, the other negative. Because L and C are quite small in this circuit, the frequency of oscillation is quite high, $f \approx 100$ MHz. This circuit is called a *transmitter* because it produces electromagnetic waves.

Hertz placed a second circuit, the receiver, several meters from the transmitter circuit. This receiver circuit, which consisted of a single loop of wire connected to two spheres, had its own effective inductance, capacitance, and natural frequency of oscillation. Hertz found that energy was being sent from the transmitter to the



Heinrich Rudolf Hertz
(1857–1894)

Hertz was born in Hamburg, Germany. He studied physics under Helmholtz and Kirchhoff at the University of Berlin. In 1885, Hertz accepted the position of professor of physics at Karlsruhe. Discovering radio waves (in 1887), demonstrating their generation, and determining their speed are among Hertz's many achievements. After finding that the speed of a radio wave was the same as that of light, Hertz showed that radio waves, like light waves, could be reflected, refracted, and diffracted. Hertz died of blood poisoning at the age of 36. During his short life, he made many contributions to science. The hertz, equal to one complete vibration or cycle per second, is named after him. *(The Bettman Archive)*

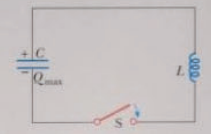


Figure 24.6 A simple LC circuit. The capacitor has an initial charge $Q_{\max}$ and the switch is closed at $t = 0$.

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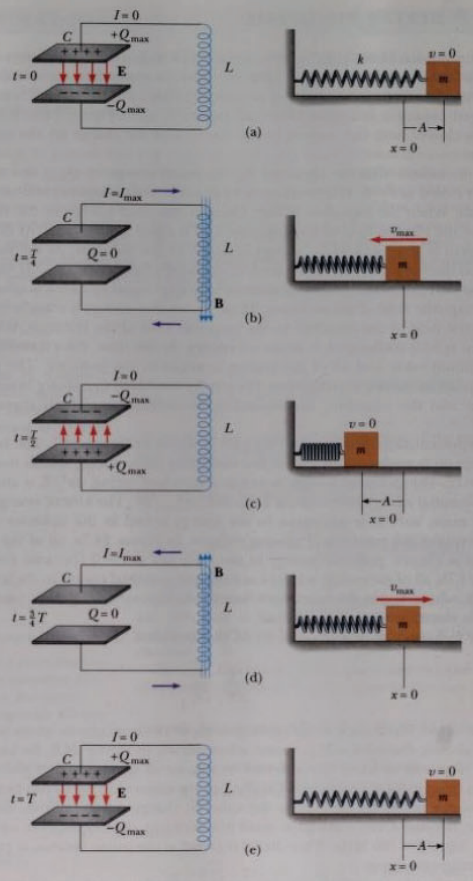


Figure 24.7 Energy transfer in a resistanceless *LC* circuit. The capacitor has a charge Q_{max} at $t = 0$ when the switch is closed. The mechanical analog of this circuit, the mass–spring system, is shown at the right.

receiver when the resonance frequency of the receiver was adjusted to match that of the transmitter.[4] The energy transfer was detected when the voltage across the spheres in the receiver circuit became high enough to ionize air molecules, which caused sparks to appear in the air gap separating the spheres. Hertz's experiment is analogous to the mechanical phenomenon in which one tuning fork picks up the vibrations from an identical vibrating fork.

Hertz assumed that the energy transferred from the transmitter to the receiver was carried in the form of waves, which are now known to have been electromagnetic waves. In a series of experiments, he also showed that the radiation generated by the transmitter exhibited the wave properties of interference, diffraction, reflection, refraction, and polarization. As we shall see shortly, all of these properties are exhibited by light. Thus, it became evident that the waves observed by Hertz had properties similar to those of light waves and differed only in frequency and wavelength.

Perhaps Hertz's most convincing experiment was his measurement of the speed of the waves from the transmitter. Waves of known frequency from the transmitter were reflected from a metal sheet so that an interference pattern was set up, much like the standing wave pattern on a stretched string. As we saw in our discussion of standing waves, the distance between nodes is $\lambda/2$, and so Hertz was able to determine the wavelength, λ. Using the relationship $v = f\lambda$, Hertz found that v was close to 3×10^8 m/s, the known speed of visible light. Thus, Hertz's experiments provided the first evidence in support of Maxwell's theory.

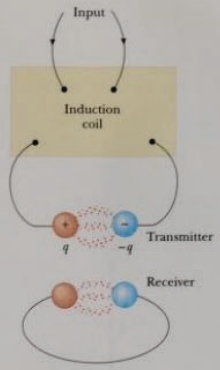


Figure 24.8 Schematic diagram of Hertz's apparatus for generating and detecting electromagnetic waves. The transmitter consists of two spherical electrodes connected to an induction coil, which provides short voltage surges to the spheres, setting up oscillations in the discharge. The receiver is a nearby loop containing a second spark gap.

24.5 · PRODUCTION OF ELECTROMAGNETIC WAVES BY AN ANTENNA

Electromagnetic waves are radiated by *any* circuit carrying an alternating current. The fundamental mechanism responsible for this radiation is the acceleration of a charged particle. **Whenever a charged particle undergoes an acceleration, it must radiate energy by electromagnetic radiation.**

An alternating voltage applied to the wires of an antenna forces an electric charge in the antenna to oscillate. This is a common technique for accelerating charged particles and is the source of the radio waves emitted by the antenna of a radio station.

Figure 24.9 illustrates how an electromagnetic wave is produced by oscillating electric charges in an antenna. Two metal rods are connected to an ac generator, which causes charges to oscillate between the two rods. The output voltage of the generator is sinusoidal. At $t = 0$, the upper rod is given a maximum positive charge and the bottom rod an equal negative charge, as in Figure 24.9a. The electric field near the antenna at this instant is also shown in Figure 24.9a. As the charges oscillate, the rods become less charged, the field near the rods decreases in strength, and the downward-directed maximum electric field produced at $t = 0$ moves away from the rod. When the charges are neutralized, as in Figure 24.9b, the electric field has dropped to zero. This occurs at a time equal to one quarter of the period of oscillation. Continuing in this fashion, the upper rod soon obtains a maximum negative charge and the lower rod becomes positive, as in Figure 24.9c, resulting

[4]Following Hertz's discoveries, Guglielmo Marconi succeeded in developing a practical, long-range radio communication system.

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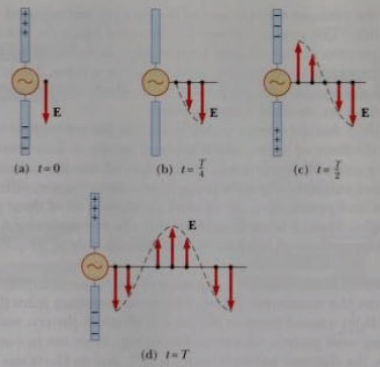


Figure 24.9 The electric field set up by charges oscillating in an antenna. The field moves away from the antenna with the speed of light.

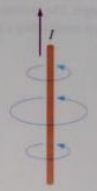


Figure 24.10 Magnetic field lines around an antenna carrying a changing current. Why do the circles have different radii?

in an electric field directed upward. This occurs after a time equal to one half the period of oscillation. The oscillations continue as indicated in Figure 24.9d.

The electric field near the antenna oscillates in phase with the charge distribution. That is, the field points down when the upper rod is positive and up when the upper rod is negative. Furthermore, the magnitude of the field at any instant depends on the amount of charge on the rods at that instant. As the charges continue to oscillate (and accelerate) between the rods, the electric field set up by the charges moves away from the antenna at the speed of light. One cycle of charge oscillation produces one full wavelength in the electric field pattern.

Because the oscillating charges create a current in the rods, a magnetic field is also generated when the current in the rods is upward, as shown in Figure 24.10. The magnetic field lines circle the antenna and are *perpendicular to the electric field at all points*. As the current changes with time, the magnetic field lines spread out from the antenna. At great distances from the antenna, the electric and magnetic fields become very weak. However, at these great distances, it is necessary to take into account the facts that (1) a changing magnetic field produces a changing electric field and (2) a changing electric field produces a changing magnetic field, as predicted by Maxwell. These induced electric and magnetic fields are in phase: At any point, the two fields reach their maximum values at the same instant. This is illustrated for one instant of time in Figure 24.11.

CONCEPTUAL PROBLEM 1

Radio antennas can be in the form of conducting lines or loops. What should the orientation of each of these types of antennas be, relative to a linear broadcasting antenna?

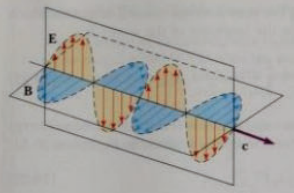


Figure 24.11 An electromagnetic wave sent out by oscillating charges in an antenna. This drawing represents the wave at one instant of time.

CONCEPTUAL PROBLEM 2

In radio transmission, a radio wave serves as a carrier wave, and the sound wave is superimposed on the carrier wave. In amplitude modulation (AM radio), the amplitude of the carrier wave varies according to the sound wave. In frequency modulation (FM radio), the frequency of the carrier wave varies according to the sound wave. The navy sometimes uses flashing lights to send Morse code to neighboring ships, a process that has similarities to radio broadcasting. Is this AM or FM? What is the carrier frequency? What is the signal frequency? What is the broadcasting antenna? What is the receiving antenna?

24.6 • ENERGY CARRIED BY ELECTROMAGNETIC WAVES

Electromagnetic waves carry energy, and as they propagate through space they can transfer energy to objects placed in their path. The intensity of an electromagnetic wave is described by a vector **S**, called the **Poynting vector,** defined by the expression

$$\mathbf{S} \equiv \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B} \qquad [24.19]$$

• *Poynting vector*

The magnitude of the Poynting vector represents the rate at which energy flows through a unit surface area perpendicular to the flow and its direction is along the direction of wave propagation (Fig. 24.12). The SI units of the Poynting vector are $J/s \cdot m^2 = W/m^2$. (These are the units **S** must have because it represents the power per unit area, where the unit area is oriented at right angles to the direction of wave propagation.)

As an example, let us evaluate the magnitude of **S** for a plane electromagnetic wave where $|\mathbf{E} \times \mathbf{B}| = EB$. In this case

$$S = \frac{EB}{\mu_0} \qquad [24.20]$$

• *Poynting vector for a plane wave*

Because $B = E/c$, we can also express this as

$$S = \frac{E^2}{\mu_0 c} = \frac{c}{\mu_0}B^2$$

These equations for S apply at any instant of time.

What is of more interest for a sinusoidal plane electromagnetic wave is the time average of S over one or more cycles, called the wave intensity, I. When this average

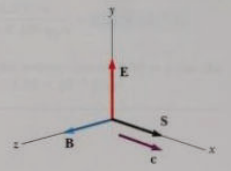


Figure 24.12 The Poynting vector **S** for a plane electromagnetic wave is along the direction of propagation.

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is taken, we obtain an expression involving the time average of $\cos^2(kx - \omega t)$, which equals $\frac{1}{2}$. Hence, the average value of S (or the intensity of the wave) is

Average power per unit area

$$I = S_{av} = \frac{E_{max} B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c} = \frac{c}{2\mu_0} B_{max}^2 \qquad [24.21]$$

Recall that the energy per unit volume, u_E, which is the instantaneous energy density associated with an electric field (Section 20.9), is given by Equation 20.31:

$$u_E = \tfrac{1}{2}\epsilon_0 E^2 \qquad [24.22]$$

and that the instantaneous energy density u_B associated with a magnetic field (Section 23.7) is given by Equation 23.22:

$$u_B = \frac{B^2}{2\mu_0} \qquad [24.23]$$

Because E and B vary with time for an electromagnetic wave, the energy densities also vary with time. Using the relationships $B = E/c$ and $c = 1/\sqrt{\epsilon_0\mu_0}$, Equation 24.23 becomes

$$u_B = \frac{(E/c)^2}{2\mu_0} = \frac{\epsilon_0\mu_0}{2\mu_0} E^2 = \tfrac{1}{2}\epsilon_0 E^2$$

Comparing this result with the expression for u_E, we see that

$$u_B = u_E = \tfrac{1}{2}\epsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

That is, **for an electromagnetic wave, the instantaneous energy density associated with the magnetic field equals the instantaneous energy density associated with the electric field.** Hence, in a given volume the energy is equally shared by the two fields.

The **total instantaneous energy density,** u, is equal to the sum of the energy densities associated with the electric and magnetic fields:

Total energy density of an electromagnetic wave

$$u = u_E + u_B = \epsilon_0 E^2 = \frac{B^2}{\mu_0}$$

When this is averaged over one or more cycles of an electromagnetic wave, we again get a factor of $\frac{1}{2}$. Hence, the total average energy per unit volume of an electromagnetic wave is

Average energy density of an electromagnetic wave

$$u_{av} = \epsilon_0 (E^2)_{av} = \tfrac{1}{2}\epsilon_0 E_{max}^2 = \frac{B_{max}^2}{2\mu_0} \qquad [24.24]$$

Comparing this result with Equation 24.21 for the average value of S, we see that

$$I = S_{av} = c u_{av} \qquad [24.25]$$

In other words, the intensity of an electromagnetic wave equals the average energy density multiplied by the speed of light.

Example 24.2 Fields Due to a Point Source

A point source of electromagnetic radiation has an average power output of 800 W. Calculate the maximum values of the electric and magnetic fields at a point 3.50 m from the source.

Solution Recall from Chapter 13 that the wave intensity, I, a distance r from a point source is

$$I = \frac{P_{av}}{4\pi r^2}$$

where P_{av} is the average power output of the source and $4\pi r^2$ is the area of a sphere of radius r centered on the source. Because the intensity of an electromagnetic wave is also given by Equation 24.21, we have

$$I = \frac{P_{av}}{4\pi r^2} = \frac{E_{max}^2}{2\mu_0 c}$$

Solving for E_{max} gives

$$E_{max} = \sqrt{\frac{\mu_0 c P_{av}}{2\pi r^2}} = \sqrt{\frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(3.00 \times 10^8\ \text{m/s})(800\ \text{W})}{2\pi(3.50\ \text{m})^2}} = 62.6\ \text{V/m}$$

We calculate the maximum value of the magnetic field using this result and the relationship $B_{max} = E_{max}/c$ (Eq. 24.17):

$$B_{max} = \frac{E_{max}}{c} = \frac{62.6\ \text{V/m}}{3.00 \times 10^8\ \text{m/s}} = 2.09 \times 10^{-7}\ \text{T}$$

EXERCISE 1 Calculate the energy density 3.50 m from the point source. Answer $1.73 \times 10^{-8}\ \text{J/m}^3$

24.7 · MOMENTUM AND RADIATION PRESSURE

Electromagnetic waves transport linear momentum as well as energy. Hence, it follows that pressure is exerted on a surface when an electromagnetic wave impinges on it. In what follows, we assume that the electromagnetic wave transports a total energy U to a surface in a time t. If the surface absorbs all the incident energy U in this time, Maxwell showed that the total momentum $\mathbf{p}$ delivered to this surface has a magnitude

$$p = \frac{U}{c} \quad \text{(complete absorption)} \qquad [24.26]$$

Momentum delivered to an absorbing surface

Furthermore, if the Poynting vector of the wave is $\mathbf{S}$, the radiation pressure P (force per unit area) exerted on the perfect absorbing surface is

$$P = \frac{S}{c} \qquad [24.27]$$

Radiation pressure exerted on a perfect absorbing surface

A black body is such a perfectly absorbing surface, for which all of the incident energy is abosrbed (none is reflected). (A more detailed discussion of a black body will be presented in Chapter 28.)

If the surface is a perfect reflector, then the momentum delivered in a time t for normal incidence is twice that given by Equation 24.26, or $p = 2U/c$. That is, a momentum U/c is delivered first by the incident wave and then again by the reflected wave, in analogy with a ball colliding elastically with a wall.[5] Finally, the

Momentum delivered to a perfectly reflecting surface

[5]For *oblique* incidence, the momentum transferred is $2U\cos\theta/c$ and the pressure is given by $P = 2S\cos\theta/c$, where θ is the angle between the normal to the surface and the direction of propagation.

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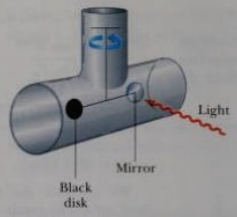


Figure 24.13 An apparatus for measuring the pressure exerted by light. In practice, the system is contained in a high vacuum.

radiation pressure exerted on a perfect reflecting surface for normal incidence of the wave is twice that given by Equation 24.27, or $P = 2S/c$.

Although radiation pressures are very small (about 5×10^{-6} N/m^2 for direct sunlight), they have been measured using torsion balances such as the one shown in Figure 24.13. Light is allowed to strike either a mirror or a black disk, both of which are suspended from a fine fiber. Light striking the black disk is completely absorbed, and so all of its momentum is transferred to the disk. Light striking the mirror (normal incidence) is totally reflected, hence the momentum transfer is twice as great as that transferred to the disk. The radiation pressure is determined by measuring the angle through which the horizontal connecting rod rotates. The apparatus must be placed in a high vacuum to eliminate the effects of air currents.

Thinking Physics 2

In the interplanetary space in the solar system, there is a large amount of dust. Although interplanetary dust can in theory have a variety of sizes, from molecular size upward, there is very little dust smaller than about 0.2 μm in our solar system. Why? (*Hint:* The solar system originally contained dust particles of all sizes.)

Reasoning Dust particles in the solar system are subject to two forces—the gravitational force toward the Sun, and the force from radiation pressure, which is away from the Sun. The gravitational force is proportional to the cube of the radius of a spherical dust particle, because it is proportional to the mass of the particle. The radiation pressure is proportional to the square of the radius, because it depends on the planar cross-section of the particle. For large particles, the gravitational force is larger than the force from radiation pressure. For small particles, less than about 0.2 μm, the larger force from radiation pressure sweeps these particles out of the solar system.

CONCEPTUAL PROBLEM 3

In *space sailing*, which is a proposed alternative method to reach the planets, a spacecraft carries a very large sail that experiences a force due to radiation pressure from the Sun. Should the sail be absorptive or reflective to be most effective?

Example 24.3 Solar Energy

The Sun delivers about 1000 W/m^2 of electromagnetic flux to the Earth's surface. (a) Calculate the total power incident on a roof of dimensions 8.00 m × 20.0 m.

Solution The magnitude of the Poynting vector is $S = 1000$ W/m^2, which represents the power per unit area, or the light intensity. Assuming the radiation is incident normal to the roof (Sun directly overhead), we get

$$\text{Power} = SA = (1000 \text{ W/m}^2)(8.00 \times 20.0 \text{ m}^2)$$
$$= 1.60 \times 10^5 \text{ W}$$

If this power could all be converted to electrical energy, it would provide more than enough power for the average home. However, solar energy is not easily harnessed, and the prospects for large-scale conversion are not as bright as they may appear from this simple calculation. For example, the conversion efficiency from solar to electrical energy is typically 10% for photovoltaic cells. Roof systems for converting solar energy to thermal energy are approximately 50% efficient; however, there are other practical problems with solar energy that must be considered, such as overcast days, geographic location, and energy storage.

(b) Determine the radiation pressure and radiation force on the roof assuming the roof covering is a perfect absorber.

Solution Using Equation 24.27 with $S = 1000$ W/m^2, we find that the radiation pressure is

$$P = \frac{S}{c} = \frac{1000 \text{ W/m}^2}{3.00 \times 10^8 \text{ m/s}} = 3.33 \times 10^{-6} \text{ N/m}^2$$

Because pressure equals force per unit area, this corresponds to a radiation force of

$$F = PA = (3.33 \times 10^{-6} \text{ N/m}^2)(160 \text{ m}^2) = 5.33 \times 10^{-4} \text{ N}$$

EXERCISE 2 How much solar energy (in joules) is incident on the roof in 1 hour? Answer 5.76×10^8 J

Example 24.4 Poynting Vector for a Wire

A long, straight wire of resistance R, radius a, and length ℓ carries a constant current I, as in Figure 24.14. Calculate the Poynting vector for this wire.

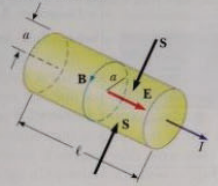


Figure 24.14 (Example 24.4) A wire of length ℓ, resistance R, and radius a carrying a current I. The Poynting vector **S** is directed radially *inward*.

Solution First, let us find the electric field **E** along the wire. If ΔV is the potential difference across its ends, then $\Delta V = IR$ and

$$E = \frac{\Delta V}{\ell} = \frac{IR}{\ell}$$

Recall that the magnetic field at the surface of the wire (Example 22.5) is

$$B = \frac{\mu_0 I}{2\pi a}$$

The vectors **E** and **B** are mutually perpendicular, as shown in Figure 24.14, and therefore $|\mathbf{E} \times \mathbf{B}| = EB$. Hence, the Poynting vector **S** is directed radially inward and has a magnitude

$$S = \frac{EB}{\mu_0} = \frac{1}{\mu_0}\frac{IR}{\ell}\frac{\mu_0 I}{2\pi a} = \frac{I^2 R}{2\pi a \ell} = \frac{I^2 R}{A}$$

where $A = 2\pi a\ell$ is the surface area of the wire and the total area through which S passes. From this result, we see that

$$SA = I^2 R$$

where SA has units of power (J/s = W). That is, **the rate at which electromagnetic energy flows into the wire, SA, equals the rate of energy (or power) dissipated as joule heat, I^2R.**

EXERCISE 3 A heater wire of radius 0.30 mm, length 1.0 m, and resistance 5.0 Ω carries a current of 2.0 A. Determine the magnitude and direction of the Poynting vector for this wire. Answer 1.1×10^4 W/m^2 directed radially inward

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24.8 · THE SPECTRUM OF ELECTROMAGNETIC WAVES

We have seen that all electromagnetic (EM) waves travel in a vacuum with the speed of light, c. These waves transport energy and momentum from some source to a receiver. In 1888, Hertz successfully generated and detected the radio-frequency electromagnetic waves predicted by Maxwell. Maxwell himself had recognized as EM waves both visible light and the near infrared radiation discovered in 1800 by William Herschel. It is now known that other forms of electromagnetic waves exist; they are distinguished by their frequencies and wavelengths.

Because all electromagnetic waves travel through vacuum with the speed c, their frequency, f, and wavelength, λ, are related by the important expression

$$c = f\lambda \qquad [24.28]$$

The various types of electromagnetic waves, all produced by accelerating charges, are listed in Figure 24.15. Note the wide range of frequencies and wavelengths. For instance, a radio wave of frequency 5 MHz (a typical value) has the wavelength

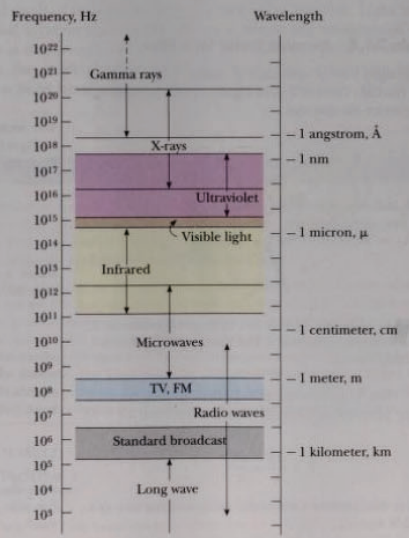


Figure 24.15 The electromagnetic spectrum. Note the overlap between adjacent wave types.

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/s}}{5 \times 10^6 \text{ s}^{-1}} = 60 \text{ m}$$

Let us briefly describe the wave types shown in Figure 24.15.

• *Radio waves*

Radio waves, which were discussed in the preceding section, are the result of charges accelerating through conducting wires. They are generated by such electronic devices as LC oscillators and are used in radio and television communication systems.

• *Microwaves*

Microwaves (short-wavelength radio waves) have wavelengths ranging between about 1 mm and 30 cm and are also generated by electronic devices. Because of their short wavelengths, they are well suited for radar systems used in aircraft navigation and for studying the atomic and molecular properties of matter. Microwave ovens are an interesting domestic application of these waves.

• *Infrared waves*

Infrared waves have wavelengths ranging from about 1 mm to the longest wavelength of visible light, 7×10^{-7} m. These waves, produced by hot bodies and molecules, are readily absorbed by most materials. The infrared energy absorbed by a substance appears as thermal energy because the energy agitates the atoms of the body, increasing their vibrational or translational motion, which results in a temperature rise. Infrared radiation has many practical and scientific applications, including physical therapy, infrared photography, and vibrational spectroscopy.

• *Visible light*

Visible light, the most familiar form of electromagnetic waves, is that part of the spectrum the human eye can detect. Light is produced by the rearrangement of electrons in atoms and molecules and hot objects like lightbulb filaments. The wavelengths of visible light are classified by color, ranging from violet ($\lambda \approx 4 \times 10^{-7}$ m) to red ($\lambda \approx 7 \times 10^{-7}$ m). The eye's sensitivity is a function of wavelength and is a maximum at a wavelength of about 5.6×10^{-7} m (yellow–green). Light is the basis of the science of optics and optical instruments, which will be discussed later.

• *Ultraviolet waves*

Ultraviolet light covers wavelengths ranging from about 3.8×10^{-7} m (380 nm) down to 6×10^{-10} m (0.6 nm). The Sun is an important source of ultraviolet waves, which is the main cause of suntans and sunburns. Most of the ultraviolet waves from the Sun are absorbed by atoms in the stratosphere. (This is fortunate, because ultraviolet waves in large quantities have harmful effects on humans.) One important constituent of the stratosphere is ozone (O_3), which results from reactions of oxygen with ultraviolet radiation. This ozone shield converts lethal high-energy ultraviolet radiation to thermal energy, which raises the temperature of the stratosphere. A great deal of controversy has arisen concerning the depletion of the protective ozone layer by the use of Freon in aerosol spray cans and as refrigerants.

• *X-rays*

X-rays are electromagnetic waves with wavelengths in the range of about 10^{-8} m (10 nm) down to 10^{-13} m (10^{-4} nm). The most common source of x-rays is the acceleration of high-energy electrons bombarding a metal target. X-rays are used as a diagnostic tool in medicine and as a treatment for certain forms of cancer. Because x-rays damage or destroy living tissues and organisms, care must be taken to avoid unnecessary exposure and overexposure. X-rays are also used in the study of crystal structure; x-ray wavelengths are comparable to the atomic separation distances (≈ 0.1 nm) in solids.

• *Gamma rays*

Gamma rays are electromagnetic waves emitted by radioactive nuclei (such as ^{60}Co and ^{137}Cs) and during certain nuclear reactions. They have wavelengths ranging from about 10^{-10} m to less than 10^{-14} m. Gamma rays are highly penetrating

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and produce serious damage when absorbed by living tissues. As a consequence, those working near such dangerous radiation must be protected with heavily absorbing materials, such as layers of lead.

Thinking Physics 3

The center of sensitivity of our eyes coincides with the center of the wavelength distribution of the Sun. Is this an amazing coincidence?

Reasoning This is not a coincidence—it is the result of biological evolution. Humans have evolved with vision most sensitive to the wavelengths that are strongest from the Sun. It is an interesting conjecture to imagine aliens from another planet, with a Sun with a different temperature, arriving at Earth. Their eyes would have the center of sensitivity at different wavelengths than ours. How would their vision of the Earth compare to ours?

CONCEPTUAL PROBLEM 4

The color of an object is said to depend on wavelength. So if you view colored objects under water, in which the wavelength of the light will be different, does the color change?

24.9 · POLARIZATION

As we learned in Section 24.3, the electric and magnetic vectors associated with an electromagnetic wave are at right angles to each other and also to the direction of wave propagation, as shown in Figure 24.16. The phenomenon of polarization described in this section is a property that specifies the directions of the electric and magnetic fields associated with an electromagnetic wave.

An ordinary beam of light consists of a large number of waves emitted by the atoms or molecules of the light source. Each atom produces a wave with its own orientation, **E**, corresponding to the direction of atomic vibration. The direction of polarization of the electromagnetic wave is defined to be the direction in which **E** is vibrating. However, because all directions of vibration are possible, the resultant electromagnetic wave is a superposition of waves produced by the individual atomic sources. The result is an **unpolarized** light wave, represented schematically in Figure 24.17a. The direction of wave propagation in this figure is perpendicular to the page. Note that *all* directions of the electric field vector, lying in a plane perpendicular to the direction of propagation, are equally probable. At any given point and at some instant of time, there is only one resultant electric field; hence, you should not be misled by the meaning of Figure 24.17a.

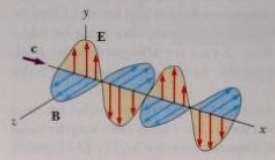


Figure 24.16 Schematic diagram of an electromagnetic wave propagating in the x direction. The electric field vector, **E**, vibrates in the xy plane, and the magnetic field vector, **B**, vibrates in the xz plane. This is an example of a linearly polarized wave.

A wave is said to be **linearly polarized** if **E** vibrates in the same direction *at all times* at a particular point, as in Figure 24.17b. (Sometimes such a wave is described as **plane-polarized.**) The wave described in Figure 24.16 is an example of a wave linearly polarized in the y direction. As the field propagates in the x direction, **E** is always in the y direction. The plane formed by **E** and the direction of propagation is called the **plane of polarization** of the wave. In Figure 24.16, the plane of polarization is the xy plane. It is possible to obtain a linearly polarized wave from an unpolarized wave by removing from the unpolarized wave all components of electric field vectors except those that lie in a single plane.

The most common technique for polarizing light is to send it through a material that passes only components of electric field vectors that are parallel to a characteristic direction of the material called the **polarizing direction.**

In 1938 E. H. Land discovered a material, which he called **polaroid,** that polarizes light through selective absorption by oriented molecules. This material is fabricated in thin sheets of long-chain hydrocarbons, which are stretched during manufacture so that the molecules align. After a sheet is dipped into a solution containing iodine, the molecules become good electrical conductors. However, the conduction takes place primarily along the hydrocarbon chains, because the valence electrons of the molecules can move easily only along the chains (recall that valence electrons are "free" electrons that can readily move through the conductor). As a result, the molecules readily *absorb* light the electric field vector of which is parallel to their length and *transmit* light the electric field vector of which is perpendicular to their length. It is common to refer to the direction perpendicular to the molecular chains as the **transmission axis.** An ideal polarizer passes the components of electric vectors that are parallel to the transmission axis. Components that are perpendicular to the transmission axis are absorbed. If light passes through several polarizers, whatever is transmitted has the plane of polarization parallel to the polarizing direction of the last polarizer it passed.

Let us now obtain an expression for the intensity of light that passes through a polarizing material. In Figure 24.18, an unpolarized light beam is incident on the

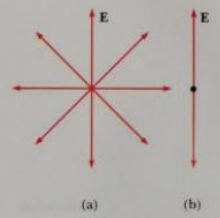


Figure 24.17 (a) An unpolarized light beam viewed along the direction of propagation (perpendicular to the page). The electric field vector can vibrate in any direction with equal probability. (b) A linearly polarized light beam with the electric field vector vibrating in the vertical direction.

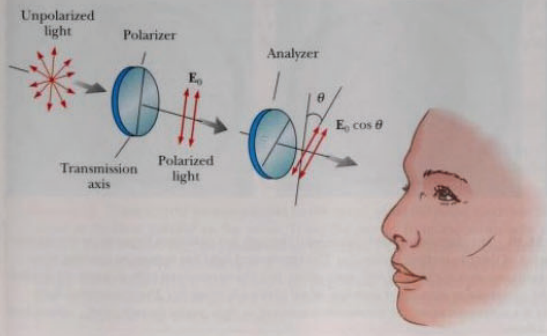


Figure 24.18 Two polarizing sheets the transmission axes of which make an angle θ with each other. Only a fraction of the polarized light incident on the analyzer is transmitted.

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first polarizing sheet, called the **polarizer,** where the transmission axis is as indicated. The light that is passing through this sheet is polarized vertically, and the transmitted electric field vector is $\mathbf{E}_0$. A second polarizing sheet, called the **analyzer,** intercepts this beam with its transmission axis at an angle of θ to the axis of the polarizer. The component of $\mathbf{E}_0$ that is perpendicular to the axis of the analyzer is completely absorbed, and the component parallel to that axis is $E_0 \cos \theta$. We know from Equation 24.18 that the transmitted intensity varies as the *square* of the transmitted amplitude, and so we conclude that the intensity of the transmitted (polarized) light varies as

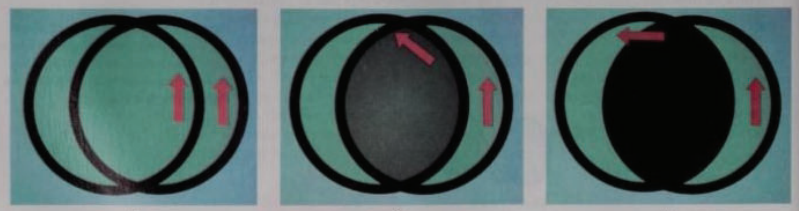
Malus's law

$$I = I_0 \cos^2 \theta \qquad [24.29]$$

where I_0 is the intensity of the polarized wave incident on the analyzer. This expression, known as **Malus's law,** applies to any two polarizing materials the transmission axes of which are at an angle of θ to each other. From this expression, note that the transmitted intensity is a maximum when the transmission axes are parallel ($\theta = 0$ or $180°$), and zero (complete absorption by the analyzer) when the transmission axes are perpendicular to each other. This variation in transmitted intensity through a pair of polarizing sheets is illustrated in Figure 24.19.

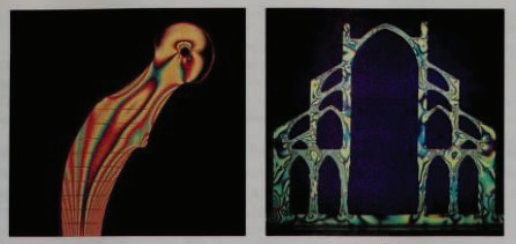
Optical Activity

Many important applications of polarized light involve materials that display **optical activity.** A substance is said to be optically active if it rotates the plane of polarization of transmitted light. The angle through which the light is rotated by a specific material depends on the length of the sample and on the concentration if the substance is in solution. One optically active material is a solution of the common sugar dextrose. A standard method for determining the concentration of sugar solutions is to measure the rotation produced by a fixed length of the solution.



(a) (b) (c)

Figure 24.19 The intensity of light transmitted through two polarizers depends on the relative orientation of their transmission axes. (a) The transmitted light has maximum intensity when the transmission axes are aligned with each other. (b) The transmitted light intensity diminishes when the transmission axes are at an angle of 45° with each other. (c) The transmitted light intensity is a minimum when the transmission axes are at right angles to each other. *(Henry Leap and Jim Lehman)*



(a) (b)

Figure 24.20 (a) Photograph showing strain distribution in a plastic model of a hip replacement used in a medical research laboratory. The pattern is produced when the plastic model is viewed between a polarizer and an analyzer oriented perpendicular to the polarizer *(Sepp Seitz 1981)* (b) A plastic model of an arch structure under load conditions observed between two crossed polarizers. Such patterns are useful in the optimum design of architectural components. *(Peter Aprahamian/Science Photo Library)*

A material is optically active because of an asymmetry in the shape of its constituent molecules. For example, some proteins are optically active because of their spiral shape. Other materials, such as glass and plastic, become optically active when stressed. Suppose an unstressed piece of plastic is placed between a polarizer and an analyzer so that light passes from polarizer to plastic to analyzer. When the analyzer axis is perpendicular to the polarizer axis, none of the polarized light passes through the analyzer. In other words, the unstressed plastic has no effect on the light passing through it. If the plastic is stressed, however, the regions of greatest stress rotate the polarized light through the largest angles. Hence, a series of bright and dark bands is observed in the transmitted light, with the bright bands corresponding to regions of greatest stress.

Engineers often use this technique, called **optical stress analysis,** to assist in designing structures ranging from bridges to small tools. A plastic model is built and analyzed under different load conditions to determine regions of potential weakness and failure under stress. Some examples of a plastic model under stress are shown in Figure 24.20.

Thinking Physics 4

A polarizer for microwaves can be made as a grid of parallel metal wires about a centimeter apart. Is the electric field vector for microwaves transmitted through this polarizer parallel to, or perpendicular to, the metal wires?

Reasoning Electric field vectors parallel to the metal wires will cause electrons in the metal to oscillate parallel to the wires. Thus, the energy from the waves with these electric field vectors will be transferred to the metal by accelerating these electrons and will eventually be transformed to internal energy through the resistance of the metal. Waves with electric field vectors perpendicular to the metal wires will not be able to accelerate electrons and will pass through. Thus, the electric field polarization will be perpendicular to the metal wires.

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CONCEPTUAL PROBLEM 5

In everyday experience, radio waves are polarized, but light is not—why?

SUMMARY

Electromagnetic waves, which are predicted by Maxwell's equations, have the following properties:

- The electric and magnetic fields satisfy the following wave equations, which can be obtained from Maxwell's third and fourth equations:

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \qquad [24.12]$$

$$\frac{\partial^2 B}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} \qquad [24.13]$$

- Electromagnetic waves travel through a vacuum with the speed of light, c, where

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3.00 \times 10^8 \text{ m/s} \qquad [24.14]$$

- The electric and magnetic fields of an electromagnetic wave are perpendicular to each other and perpendicular to the direction of wave propagation. (Hence electromagnetic waves are transverse waves.)
- The instantaneous magnitudes of **E** and **B** in an electromagnetic wave are related by the expression

$$\frac{E}{B} = c \qquad [24.17]$$

- Electromagnetic waves carry energy. The rate of flow of energy crossing a unit area is described by the **Poynting vector S,** where

$$\mathbf{S} \equiv \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \qquad [24.19]$$

- Electromagnetic waves carry momentum and hence can exert pressure on surfaces. If an electromagnetic wave the Poynting vector of which is **S** is completely absorbed by a surface on which it is normally incident, the radiation pressure on that surface is

$$P = \frac{S}{c} \quad \text{(complete absorption)} \qquad [24.27]$$

If the surface totally reflects a normally incident wave, the pressure is doubled.

The electric and magnetic fields of a sinusoidal plane electromagnetic wave propagating in the positive x direction can be written

$$E = E_{\max} \cos(kx - \omega t) \qquad [24.15]$$

$$B = B_{\max} \cos(kx - \omega t) \qquad [24.16]$$

where ω is the angular frequency of the wave and k is the angular wave number. These equations represent special solutions to the wave equations for **E** and **B**.

The average value of the Poynting vector for a plane electromagnetic wave has the magnitude

$$S_{av} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{c}{2\mu_0} B_{\max}^2 \qquad [24.21]$$

The average power per unit area (intensity) of a sinusoidal plane electromagnetic wave equals the average value of the Poynting vector taken over one or more cycles.

The electromagnetic spectrum includes waves covering a broad range of frequencies and wavelengths. The frequency, f, and wavelength, λ, of a given wave are related by

$$c = f\lambda \qquad [24.28]$$

When polarized light of intensity I_0 is incident on a polarizing film, the light transmitted through the film has an intensity equal to $I_0 \cos^2 \theta$, where θ is the angle between the transmission axis of the polarizer and the electric field vector of the incident light.

CONCEPTUAL QUESTIONS

1. What is the fundamental source of electromagnetic radiation?
2. Electrical engineers often speak of the radiation resistance of an antenna. What do you suppose they mean by this phrase?
3. Describe the physical significance of the Poynting vector.
4. If a high-frequency current is passed through a solenoid containing a metallic core, the core heats up by induction. Explain why the materials heat up in these situations.
5. Certain orientations of the receiving antenna on a television set give better reception than others. Furthermore, the best orientation varies from station to station. Explain.
6. If you charge a comb by running it through your hair and then hold the comb next to a bar magnet, do the electric and magnetic fields produced constitute an electromagnetic wave?
7. An empty plastic or glass dish removed from a microwave oven is cool to the touch right after it is removed from the oven. How can this be possible? (You can assume that your electric bill has been paid.)
8. What does a radio wave do to the charges in the receiving antenna to provide a signal for your car radio?
9. When light (or other electromagnetic radiation) travels across a given region, what is it that moves?
10. Suppose a creature from another planet had eyes that were sensitive to infrared radiation. Describe what he would see if he looked around the room you are now in. That is, what would be bright and what would be dim?
11. Why should an infrared photograph of a person look different from a photograph taken with visible light?
12. A welder must wear protective glasses and clothing to prevent eye damage and sunburn. What does this imply about the light produced by the welding?
13. Radio stations often advertise "instant news." If what they mean is that you hear the news at the instant they speak it, is their claim true? About how long would it take for a message to travel across this country by radio waves, assuming that these waves could travel this great distance and still be detected?
14. Light from the Sun takes approximately $8\frac{1}{3}$ minutes to reach the Earth. During this time the Earth has continued to move in its orbit around the Sun. How far is the actual location of the Sun from its image in the sky?
15. Do Maxwell's equations allow for the existence of magnetic monopoles?

PROBLEMS

Section 24.1 Displacement Current and the Generalized Ampère's Law

Section 24.2 Maxwell's Wonderful Equations

1. Consider the situation shown in Figure P24.1. An electric field of 300 V/m is confined to a circular area 10.0 cm in diameter and directed outward from the plane of the figure. If the field is increasing at a rate of 20.0 V/m·s, what are the direction and magnitude of the magnetic field at the point P, 15.0 cm from the center of the circle?

Section 24.3 Electromagnetic Waves

2. The speed of an electromagnetic wave traveling in a nonmagnetic transparent substance is $v = 1/\sqrt{\kappa \mu_0 \epsilon_0}$, where κ is the dielectric constant of the substance. Determine the speed of light in water, which has a dielectric constant of 1.78.
3. An electromagnetic wave in vacuum has an electric field amplitude of 220 V/m. Calculate the amplitude of the corresponding magnetic field.

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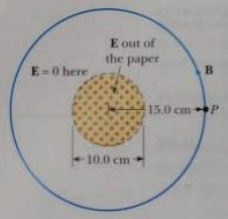


Figure P24.1

4. Calculate the maximum value of the magnetic field in a medium in which the speed of light is two thirds the speed of light in vacuum and the amplitude of the electric field is 7.60 mV/m.
5. Figure 24.4 shows a plane electromagnetic sinusoidal wave propagating in what is assumed to be the x direction. The wavelength is 50.0 m, and the electric field vibrates in the xy plane with an amplitude of 22.0 V/m. Calculate (a) the sinusoidal frequency and (b) the magnitude and direction of **B** when the electric field has its maximum value in the negative y direction. (c) Write an expression for B in the form

$$B = B_{max} \cos(kx - \omega t)$$

with numerical values for B_{max}, k, and ω.

6. Write down expressions for the electric and magnetic fields of a sinusoidal plane electromagnetic wave having a frequency of 3.00 GHz and traveling in the positive x direction. The amplitude of the electric field is 300 V/m.
7. Verify that the following equations are solutions to Equations 24.12 and 24.13, respectively:

$$E = E_{max} \cos(kx - \omega t) \qquad B = B_{max} \cos(kx - \omega t)$$

8. In SI units, the electric field in an electromagnetic wave is described by

$$E_y = 100 \sin(1.00 \times 10^7 x - \omega t)$$

Find (a) the amplitude of the corresponding magnetic wave, (b) the wavelength λ, and (c) the frequency f.

Section 24.4 Hertz's Discoveries

9. If the coil in the resonant circuit of a radio has an inductance of 2.00 μH, what range of values must the tuning capacitor have in order to cover the complete range of FM frequencies? (The FM range of frequencies is 88.0 MHz to 108 MHz.)
10. A standing wave interference pattern is set up by radio waves between two metal sheets 2.00 m apart. This is the shortest distance between the plates that will produce a standing wave pattern. What is the fundamental frequency?

Section 24.5 Production of Electromagnetic Waves by an Antenna

11. A television set uses a dipole receiving antenna for VHF channels and a loop antenna for UHF channels. The UHF antenna produces a voltage from the changing magnetic flux through the loop. (a) Using Faraday's law, derive an expression for the amplitude of the voltage that appears in a single-turn circular loop antenna with a radius r, small compared to the wavelength of the wave. The TV station broadcasts a signal with a frequency f, and the signal has an electric field amplitude E_{max} and magnetic field amplitude B_{max} at the receiving antenna's location. (b) If the electric field in the signal points vertically, what should be the orientation of the loop for best reception?
12. Two hand-held radio transceivers with dipole antennas are separated by a large fixed distance. Assuming a vertical transmitting antenna, what fraction of the maximum received power will occur in the receiving antenna when it is inclined from the vertical by (a) 15.0°? (b) 45.0°? (c) 90.0°?
13. Two radio-transmitting antennas are separated by half the broadcast wavelength and are driven in phase with each other. In which directions are (a) the strongest and (b) the weakest signals radiated?

Section 24.6 Energy Carried by Electromagnetic Waves

14. How much electromagnetic energy per cubic meter is contained in sunlight if the intensity of sunlight at the Earth's surface under a fairly clear sky is 1000 W/m^2?
15. What is the average magnitude of the Poynting vector 5.00 miles from a radio transmitter broadcasting isotropically with an average power of 250 kW?
16. An AM radio station broadcasts isotropically with an average power of 4.00 kW. A dipole receiving antenna 65.0 cm long is located 4.00 miles from the transmitter. Compute the amplitude of the emf induced by this signal between the ends of the receiving antenna.
17. A community plans to build a facility to convert solar radiation to electrical power. They require 1.00 MW of power, and the system to be installed has an efficiency of 30.0% (that is, 30.0% of the solar energy incident on the surface is converted to electrical energy). What must be the effective area of a perfectly absorbing surface used in such an installation, assuming a constant energy flux of 1000 W/m^2?
18. A monochromatic light source emits 100 W of electromagnetic power uniformly in all directions. (a) Calculate the average electric-field energy density 1.00 m from the source. (b) Calculate the average magnetic-field energy density at the same distance from the source. (c) Find the wave intensity at this location.
19. The filament of an incandescent lamp has a 150-Ω resistance and carries a direct current of 1.00 A. The filament is 8.00 cm long and 0.900 mm in radius. (a) Calculate the magnitude of the Poynting vector at the surface of the filament. (b) Find the magnitude of the electric and magnetic fields at the surface of the filament.
20. Assuming that the antenna of a 10.0-kW radio station radiates spherical electromagnetic waves, compute the maximum value of the magnetic field 5.00 km from the antenna, and compare this value with the magnetic field of the Earth.
21. In a certain region of empty space, the electric field intensity at a given instant is $\mathbf{E} = (80.0\mathbf{i} + 32.0\mathbf{j} - 64.0\mathbf{k})$ N/C and the magnetic field intensity is $\mathbf{B} = (0.200\mathbf{i} + 0.0800\mathbf{j} + 0.290\mathbf{k})$ μT. (a) Show that the two fields are perpendicular to each other. (b) Determine their Poynting vector.
22. A lightbulb's filament has a resistance of 110 Ω. The bulb is plugged into a standard 120-V (rms) outlet and emits 1.00% of the input energy as electromagnetic radiation of frequency f. Assuming that the bulb is covered with a filter, absorbing all other frequencies, find the amplitude of the magnetic field 1.00 m from the bulb.
23. High-power lasers in factories are used to cut through cloth and metal. One such laser has a beam diameter of 1.00 mm and generates an electric field at the target having an amplitude 0.700 MV/m. Find (a) the amplitude of the magnetic field produced, (b) the intensity of the laser, and (c) the power dissipated.

Section 24.7 Momentum and Radiation Pressure

24. A plane electromagnetic wave of intensity 6.00 W/m^2 strikes a small pocket mirror, of area 40.0 cm^2, held perpendicular to the approaching wave. (a) What momentum does the wave transfer to the mirror each second? (b) Find the force that the wave exerts on the mirror.
25. A radio wave transmits 25.0 W/m^2 of power per unit area. A flat surface of area A is perpendicular to the direction of propagation of the wave. Calculate the radiation pressure on it if the surface is a perfect absorber.
26. A possible means of space flight is to place a perfectly reflecting aluminized sheet into Earth's orbit and use the light from the Sun to push this "solar sail." Suppose a sail of area 6.00×10^5 m^2 and mass 6000 kg is placed in orbit facing the Sun. (a) What force is exerted on the sail? (b) What is the sail's acceleration? (c) How long does it take the sail to reach the Moon, 3.84×10^8 m away? Ignore all gravitational effects, assume that the acceleration calculated in part (b) remains constant, and assume a solar intensity of 1340 W/m^2. (*Hint:* The radiation pressure exerted by a reflected wave is given as twice the average power per area divided by the speed of light.)
27. A 15.0-mW helium-neon laser (λ = 632.8 nm) emits a beam of circular cross section the diameter of which is 2.00 mm. (a) Find the maximum electric field in the beam. (b) What total energy is contained in a 1.00-m length of the beam? (c) Find the momentum carried by a 1.00-m length of the beam.

Section 24.8 The Spectrum of Electromagnetic Waves

28. Compute an order-of-magnitude estimate for the frequency of an electromagnetic wave with wavelength equal to (a) your height, (b) the thickness of this sheet of paper. How is each wave classified on the electromagnetic spectrum?
29. The eye is most sensitive to light having a wavelength of 5.50×10^{-7} m, which is in the green–yellow region of the electromagnetic spectrum. What is the frequency of this light?
30. A "rabbit ears" television antenna consists of a pair of adjustable rods. If we set each rod to a quarter-wavelength for channel 3, which has a central frequency of 63.0 MHz, how long are the rods?
31. Suppose you are located 180 m from a radio transmitter. (a) How many wavelengths are you from the transmitter if the station calls itself 1150 AM? (The AM band frequencies are in kHz.) (b) What if this station were 98.1 FM? (The FM band frequencies are in MHz.)
32. A radar pulse returns to the receiver after a total travel time of 4.00×10^{-4} s. How far away is the object that reflected the wave?

Section 24.9 Polarization

33. Unpolarized light passes through two polaroid sheets. The axis of the first is vertical and that of the second is at 30.0° to the vertical. What fraction of the initial light is transmitted?
34. Three polarizing disks, the planes of which are parallel, are centered on a common axis. The direction of the transmission axis in each case is shown in Figure P24.34 relative to the common vertical direction. A plane-polarized beam of light with E_0 parallel to the vertical reference direction is

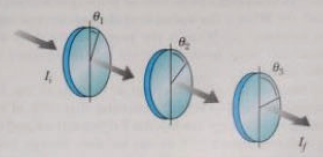


Figure P24.34

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incident from the left on the first disk with intensity I_i = 10.0 units (arbitrary). Calculate the transmitted intensity I_f when (a) $\theta_1 = 20.0°$, $\theta_2 = 40.0°$, and $\theta_3 = 60.0°$; (b) $\theta_1 = 0°$, $\theta_2 = 30.0°$, and $\theta_3 = 60.0°$.

35. Plane-polarized light is incident on a single polarizing disk with the direction of $\mathbf{E}_0$ parallel to the direction of the transmission axis. Through what angle should the disk be rotated so that the intensity in the transmitted beam is reduced by a factor of (a) 3.00, (b) 5.00, (c) 10.0?

36. In Figure P24.34, suppose that the left and right polarizing disks have their transmission axes perpendicular to each other. Also, let the center disk be rotated on the common axis with an angular speed ω. Show that if unpolarized light is incident on the left disk with an intensity I_0, the intensity of the beam emerging from the right disk is

$$I = \frac{1}{16} I_0(1 - \cos 4\omega t)$$

This means that the intensity of the emerging beam is modulated at a rate that is four times the rate of rotation of the center disk. [*Hint:* Use the trigonometric identities $\cos^2\theta = (1 + \cos 2\theta)/2$ and $\sin^2\theta = (1 - \cos 2\theta)/2$, and recall that $\theta = \omega t$.]

Additional Problems

37. Assume that the intensity of solar radiation incident on the cloudtops of Earth is 1340 W/m^2. (a) Calculate the total power radiated by the Sun, taking the average Earth–Sun separation to be 1.49×10^{11} m. (b) Determine the maximum values of the electric and magnetic fields at the Earth's location due to solar radiation.

38. Show that the instantaneous value of the Poynting vector has the magnitude

$$S = \frac{c}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right)$$

39. A dish antenna having a diameter of 20.0 m receives (at normal incidence) a radio signal from a distant source, as shown in Figure P24.39. The radio signal is a continuous sinusoidal wave with amplitude $E_{max} = 0.200\ \mu$V/m. Assume the antenna absorbs all the radiation that falls on the dish. (a) What is the amplitude of the magnetic field in this wave? (b) What is the intensity of the radiation received by this antenna? (c) What is the power received by the antenna? (d) What force is exerted on the antenna by the radio waves?

40. The intensity of solar radiation at the top of the Earth's atmosphere is 1340 W/m^2. Assuming that 60% of the incoming solar energy reaches the Earth's surface and assuming that you absorb 50% of the incident energy, make an order-of-magnitude estimate of the amount of solar energy you absorb in a 60-minute sunbath.



Figure P24.39

41. In 1965, Penzias and Wilson discovered the cosmic microwave radiation left over from the Big Bang explosion of the Universe. Suppose the energy density of this radiation is 4.00×10^{-14} J/m^3. Determine the corresponding electric field amplitude.

42. Consider a small, spherical particle of radius r located in space a distance R from the Sun. (a) Show that the ratio $F_{rad}/F_{grav} \propto 1/r$, where F_{rad} = the force exerted by solar radiation and F_{grav} = the force of gravitational attraction. (b) The result of part (a) means that, for a sufficiently small value of r, the force exerted on the particle by solar radiation exceeds the force of gravitational attraction. Calculate the value of r for which the particle is in equilibrium under the two forces. (Assume that the particle has a perfectly absorbing surface and a mass density of 1.50 g/cm^3. Let the particle be located 3.75×10^{11} m from the Sun and use 214 W/m^2 as the value of the solar flux at that point.)

43. A linearly polarized microwave of wavelength 1.50 cm is directed along the positive x axis. The electric field vector has a maximum value of 175 V/m and vibrates in the xy plane. (a) Assume that the magnetic field component of the wave can be written in the form $B = B_{max}\sin(kx - \omega t)$ and give values for B_{max}, k, and ω. Also, determine in which plane the magnetic field vector vibrates. (b) Calculate the magnitude of the Poynting vector for this wave. (c) What maximum radiation pressure would this wave exert if directed at normal incidence onto a perfectly reflecting sheet? (d) What maximum acceleration would be imparted to a 500-g sheet (perfectly reflecting and at normal incidence) of dimensions 1.00 m × 0.750 m?

44. An astronaut, stranded in space 10.0 m from her spacecraft and at rest relative to it, has a mass (including equipment) of 110 kg. Having a 100-W light source that forms a directed beam, she decides to use the beam as a photon rocket to propel herself continuously toward the spacecraft. (a) Calculate how long it takes her to reach the spacecraft by this method. (b) Suppose she instead decides to throw the light source away in a direction opposite the spacecraft. If the mass of the light source is 3.00 kg and, after being thrown, moves at 12.0 m/s relative to the recoiling astronaut, how long does she take to reach the spacecraft?

45. You want to rotate the plane of polarization of a polarized light beam by 45.0° with a maximum intensity reduction of 10.0%. (a) How many sheets of perfect polarizers do you need to achieve your goal? (b) What is the angle between adjacent polarizers?

46. Lasers have been used to suspend spherical glass beads in the Earth's gravitational field. (a) If a bead has a mass of 1.00 μg and a density of 0.200 g/cm^3, determine the radiation intensity needed to support the bead. (b) If the laser beam has a radius of 0.200 cm, what is the power required for this laser?

47. Lasers have been used to suspend spherical glass beads in the Earth's gravitational field. (a) If a bead has mass m and density ρ, determine the radiation intensity needed to support the bead. (b) If the beam has a radius r, what is the power required for this laser?

48. A singer's voice is transmitted by a radio wave to a person 100 km away. (a) How much time does it take for the radio wave to travel through the air to the person? (b) By the time the radio message reaches a listener, how far from the singer has the sound wave moved in the auditorium? Assume that the speed of sound is 345 m/s.

49. The Earth reflects approximately 38.0% of the incident sunlight by reflection from its clouds and oceans. (a) Given that the intensity of solar radiation is 1340 W/m^2, what is the radiation pressure on the Earth in Pa when the Sun is straight overhead? (b) Compare this to normal atmospheric pressure, which is 1.01×10^5 Pa, at the Earth's surface.

50. A 1.00 m diameter mirror focuses the Sun's rays onto an absorbing plate 2.00 cm in radius, which holds a can containing 1.00 liter of water at 20.0°C. (a) If the solar intensity is 1.00 kW/m^2, what is the intensity on the absorbing plate? (b) What are the **E** and **B** field amplitudes? (c) If 40.0% of the energy is absorbed, how long would it take to bring the water to its boiling point?

ANSWERS TO CONCEPTUAL PROBLEMS

1. An antenna that is a conducting line responds to the electric field of the electromagnetic wave—the oscillating electric field causes an electric force on electrons in the wire along its length. The movement of electrons along the wire is detected as a current by the radio, demodulated, and amplified. Thus, a line antenna must have the same orientation as the broadcasting antenna. A loop antenna responds to the magnetic field in the radio wave. The varying magnetic field induces a varying current in the loop, and this signal is demodulated and amplified. The axis of the loop antenna must be perpendicular to the broadcasting antenna, so that the magnetic field lines cut through the area of the loop.
2. The flashing of the light according to Morse code is a drastic amplitude modulation—the amplitude is changing from a maximum to zero. In this sense, it is similar to the on-and-off binary code used in computers and compact disks. The carrier frequency is that of the light, on the order of 10^{14} Hz. The signal frequency depends on the skill of the signal operator, but it is on the order of single hertz, as the light is flashed on and off. The broadcasting antenna for this modulated signal is the filament of the lightbulb in the signal source. The receiving antenna is the eye.
3. The sail should be as reflective as possible, so that the maximum momentum is transferred to the sail from the reflection of sunlight.
4. Assuming water does not preferentially absorb one wavelength over another, the color will not change for two reasons. First, despite the popular statement that color depends on wavelength, it actually depends on the *frequency* of the light, which does not change under water. Second, when the light enters the eye, it travels through the fluid within the eye. Thus, even if color did depend on wavelength, the important wavelength is that of the light in the ocular fluid, which does not depend on the medium through which the light traveled to reach the eye.
5. Radio waves originate from antennas, which tend to be long vertical towers. This results in a natural polarization of the radio waves in the vertical direction. Light originates from the vibrations of atoms, or electronic transitions within atoms, which represent oscillations in all possible directions. Thus, light is not generally polarized.

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25

Reflection and Refraction of Light

CHAPTER OUTLINE

25.1 The Nature of Light
25.2 The Ray Approximation in Geometric Optics
25.3 Reflection and Refraction
25.4 Dispersion and Prisms
25.5 Huygens's Principle
25.6 Total Internal Reflection

25.1 • THE NATURE OF LIGHT

Until the beginning of the 19th century, light was thought to be a stream of particles that was emitted by a light source and stimulated the sense of sight on entering the eye. The chief architect of this particle theory of light was Isaac Newton. The theory provided a simple explanation of some known experimental facts concerning the nature of light—namely, the laws of reflection and refraction.

Most scientists accepted the particle theory of light. However, during Newton's lifetime, another theory was proposed—that light might be some sort of wave motion. In 1678, a Dutch physicist and astronomer, Christian Huygens (1629–1695), showed that a wave theory of light could also explain the laws of reflection and refraction. The wave theory did not receive immediate acceptance, for several reasons. All the waves known at the time (sound, water, and so on) traveled through some sort of medium, but light from the Sun could travel to us through empty space. Furthermore, it was argued that if light were some form of wave motion, the waves would be able to bend around obstacles; hence, we should be able to see around corners. It is now known that light does indeed bend around the edges of objects. This phenomenon, known as **diffraction,** is not easy to observe, because



Photograph of a rainbow over Niagara Falls in Ontario, Canada. Rainbows are often seen by an observer located between the Sun and a rain shower. The colors of the visible spectrum that we see in a rainbow are due to the dispersion and refraction of light by raindrops. *(John Edwards/Tony Stone Images)*

light waves have such short wavelengths. Even though experimental evidence for the diffraction of light was discovered by Francesco Grimaldi (1618–1663) around 1660, most scientists for more than a century rejected the wave theory and adhered to Newton's particle theory. This was, for the most part, due to Newton's great reputation as a scientist.

The first clear demonstration of the wave nature of light was provided in 1801 by Thomas Young (1773–1829), who showed that, under appropriate conditions, light exhibits interference behavior. That is, light waves coming from a single source arriving at a point by two different paths can combine and cancel each other by destructive interference, with the energy appearing at some other point. Such behavior could not be explained at that time by a particle theory, because scientists thought that there was no conceivable way in which two or more particles could come together and cancel one another. Several years later, a French physicist, Augustin Fresnel (1788–1829), performed a number of experiments on interference and diffraction. In 1850 Jean Foucault (1791–1868) provided further evidence of the inadequacy of the particle theory by showing that the speed of light in liquids is less than that in air; according to the particle model, the speed of light would be *higher* in glasses and liquids than in air. Additional developments during the 19th century led to the general acceptance of the wave theory of light.

The most important development concerning the theory of light was the work of James Clerk Maxwell, who in 1865 predicted that light was a form of high-frequency electromagnetic wave. As discussed in Chapter 24, Hertz in 1887 provided experimental confirmation of Maxwell's theory by producing and detecting electromagnetic waves. Furthermore, Hertz and other investigators showed that **these waves exhibited reflection, refraction, and all the other characteristic properties of waves.**

Although the classical theory of electricity and magnetism could explain most known properties of light, some experiments could not be accounted for by the assumption that light was a wave. The most striking of these was the photoelectric effect, discovered by Hertz, in which electrons are ejected from a metal when the surface of the metal is exposed to light. As one example of the difficulties that arose, experiments showed that the kinetic energy of an ejected electron is independent of the light intensity. This finding contradicted the wave theory, which held that a more intense beam of light should add more energy to the electron. An explanation proposed by Einstein in 1905 used the concept of quantization developed in 1900 by Max Planck (1858–1947). Einstein's prediction that the kinetic energy of the ejected electrons would depend on the frequency of the radiation, but not on the intensity, was initially rejected. However, experiments performed between 1905 and 1912 eventually led to the acceptance of Einstein's theory. The quantization model assumes that the energy of a light wave is present in bundles of energy called *photons;* hence, the energy is said to be *quantized.* More specifically, Einstein showed that the energy of a photon is proportional to the frequency of the electromagnetic wave. That is,

$$E = hf \qquad [25.1]$$

• *Energy of a photon*

where $h = 6.63 \times 10^{-34}$ J·s is Planck's constant. This theory retains some features of both the wave theory and the particle theory of light. As we shall discuss later, the photoelectric effect is the result of energy transfer from a single photon to an electron in the metal. That is, the electron interacts with one photon of light as if

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it (the electron) had been struck by a particle. Yet this photon has wave-like characteristics, because its energy is determined by the frequency (wave-like quantity).

In view of these developments, light must be regarded as having a dual nature: **In some cases light acts like a wave and in others it acts like a particle.** Classical electromagnetic wave theory provides an adequate explanation of light propagation and interference, whereas the photoelectric effect and other experiments involving the interaction of light with matter are best explained by assuming that light is a particle. Light is light, to be sure. The question, "Is light a wave or a particle?" is inappropriate; in some experiments, we measure its wave properties; in other experiments we measure its particle properties.

25.2 · THE RAY APPROXIMATION IN GEOMETRIC OPTICS

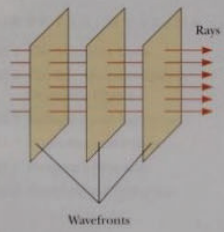


Figure 25.1 A plane wave propagating to the right. Note that the rays, which always point in the direction of wave motion, are straight lines perpendicular to the wave fronts.

A wavefront is a surface over which the wave displacement has the same value. For example, a point source of light waves generates spherical waves moving in all directions. The wavefronts in this case are concentric spherical surfaces. One wavefront is the surface where the wave displacement has its maximum value, corresponding to the crest of the wave. In studying geometric optics here and in Chapter 26, we shall use what is called the **ray approximation.** To understand it, first note that the rays of a given wave are straight lines that are perpendicular to the wavefronts, as illustrated in Figure 25.1 for a plane wave. In the ray approximation, we assume that a wave moving through a medium travels in a straight line in the direction of its rays.

If the wave meets a barrier containing a circular opening the diameter of which is large relative to the wavelength, as in Figure 25.2a, the wave emerging from the opening continues to move in a straight line (apart from some small edge effects); hence, the ray approximation continues to be valid. If the diameter of the opening is on the order of the wavelength, as in Figure 25.2b, the waves (and, as a consequence, the rays we draw) spread out from the opening in all directions. We say that the outgoing wave is noticeably diffracted. If the opening is small relative to the wavelength, the opening can be approximated as a point source of waves (Fig. 25.2c). Thus, diffraction is more pronounced as the ratio d/λ approaches zero.

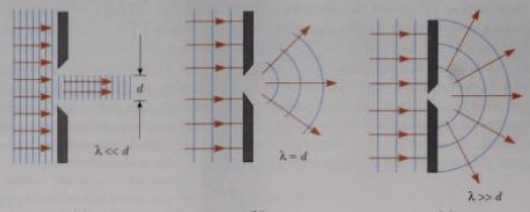


Figure 25.2 A plane wave of wavelength λ is incident on a barrier in which there is an opening of diameter d. (a) When $\lambda \ll d$, there is almost no observable diffraction and the ray approximation remains valid. (b) When $\lambda \approx d$, diffraction becomes significant. (c) When $\lambda \gg d$, the opening behaves as a point source emitting spherical waves.

Similar effects are seen when waves encounter an opaque circular object. In this case, when $\lambda \ll d$, the object casts a sharp shadow.

The ray approximation and the assumption that $\lambda \ll d$ are used here and in Chapter 26, which also deals with geometric optics. This approximation is very good for the study of mirrors, lenses, prisms, and associated optical instruments, such as telescopes, cameras, and eyeglasses. We return to the subject of diffraction (where $\lambda \approx d$) in Chapter 27.

25.3 · REFLECTION AND REFRACTION

Reflection of Light

Figure 25.3a shows several rays of a beam of light incident on a smooth, reflecting surface. The reflected rays are parallel to each other, as indicated in the figure. Reflection of light from such a smooth surface is called **specular reflection.** If the reflecting surface is rough, as in Figure 25.3b, it will reflect the rays in various directions. Reflection from any rough surface is known as **diffuse reflection.** A surface behaves as a smooth surface as long as the surface variations are small compared with the wavelength of the incident light. Figure 25.3c and 25.3d are photographs of specular reflection and diffuse reflection using laser light.

Consider the two types of reflection from a road surface that you observe when you drive at night. On a dry night, light from oncoming vehicles is scattered off the road in different directions (diffuse reflection) and the road is quite visible. On a

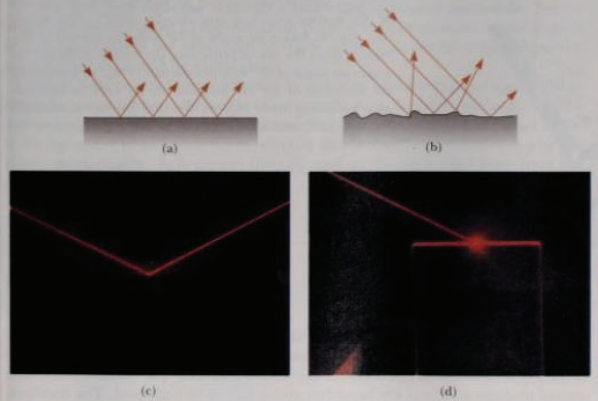


Figure 25.3 Schematic representation of (a) specular reflection, where the reflected rays are all parallel to each other, and (b) diffuse reflection, where the reflected rays travel in random directions. (c) and (d) Photographs of specular and diffuse reflection using laser light. *(Photographs courtesy of Henry Leap and Jim Lehman)*

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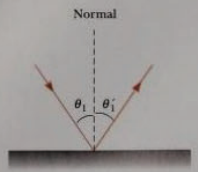


Figure 25.4 According to the law of reflection, $\theta_1 = \theta_1'$. The incident ray, the reflected ray, and the normal all lie in the same plane.

rainy night, all the little irregularities in the road surface are filled with water. Because the water surface is quite smooth, the light undergoes specular reflection and the glare from reflected light makes the road less visible. We shall concern ourselves only with specular reflection, and we shall use the term *reflection* to mean specular reflection.

Consider a light ray that is traveling in air and incident at an angle on a flat, smooth surface, as in Figure 25.4. The incident and reflected rays make angles of θ_1 and θ_1', respectively, with a line drawn normal to the surface at the point where the incident ray strikes the surface. Experiments show that **the angle of reflection equals the angle of incidence:**

$$\theta_1' = \theta_1 \qquad [25.2]$$

By convention, the angles of incidence and reflection are measured from the normal to the surface rather than from the surface itself.

Example 25.1 The Double Reflected Light Ray

Two mirrors make an angle of 120° with each other, as in Figure 25.5. A ray is incident on mirror M_1 at an angle of 65° to the normal. Find the direction of the ray after it is reflected from mirror M_2.

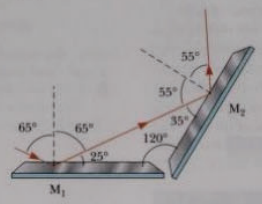


Figure 25.5 (Example 25.1) Mirrors M_1 and M_2 make an angle of 120° with each other.

Reasoning and Solution From the law of reflection, we know that the first reflected ray also makes an angle of 65° with the normal. Thus, this ray makes an angle of 90° − 65°, or 25°, with the horizontal. From the triangle made by the first reflected ray and the two mirrors, we see that the first reflected ray makes an angle of 35° with M_2 (because the sum of the interior angles of any triangle is 180°). This means that this ray makes an angle of 55° with the normal to M_2. Hence, from the law of reflection, the second reflected ray makes an angle of 55° with the normal to M_2. By comparing the direction of the ray incident on M_1 with its direction after reflecting from M_2, we see that the ray is reflected through 120°, which corresponds to the angle between the mirrors.

Refraction of Light

When a ray of light traveling through a transparent medium is obliquely incident on a boundary leading into another transparent medium, as in Figure 25.6a, part of the ray is reflected but part enters the second medium. The ray that enters the second medium is bent at the boundary and is said to be **refracted. The incident ray, the reflected ray, and the refracted ray all lie in the same plane.** The **angle of refraction,** θ_2 in Figure 25.6, depends on the properties of the two media and on the angle of incidence, through the relationship

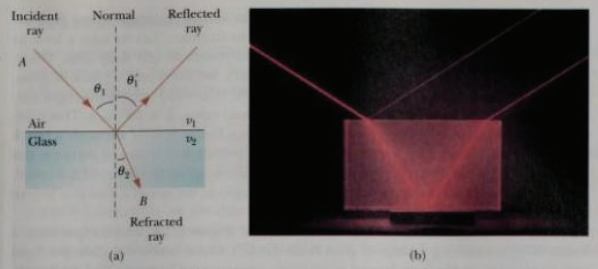


Figure 25.6 (a) A ray obliquely incident on an air–glass interface. The refracted ray is bent toward the normal because $v_2 < v_1$. All rays and the normal lie in the same plane. (b) Light incident on the Lucite block bends both when it enters the block and when it leaves the block. *(Henry Leap and Jim Lehman)*

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} = \text{constant} \qquad [25.3]$$

where v_1 is the speed of light in medium 1 and v_2 is the speed of light in medium 2. The experimental discovery of this relationship is usually credited to Willebrord Snell (1591–1627) and is therefore known as **Snell's law.**

The path of a light ray through a refracting surface is reversible. For example, the ray in Figure 25.6 travels from point *A* to point *B*. If the ray originated at *B*, it would follow the same path to reach point *A*. In the latter case, however, the reflected ray would be in the glass.

When light moves from a material in which its speed is high to a material in which its speed is lower, the angle of refraction, θ_2, is less than the angle of incidence, and so the refracted ray is bent toward the normal, as shown in Figure 25.7a. If the ray moves from a material in which it travels slowly to a material in which it travels more rapidly, θ_2 is greater than θ_1, and so the ray is bent away from the normal, as shown in Figure 25.7b.

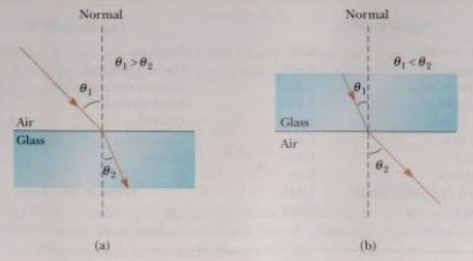


Figure 25.7 (a) When the light beam moves from air into glass, its path is bent toward the normal. (b) When the beam moves from glass into air, its path is bent away from the normal.

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The pencil partially immersed in water appears bent because light is refracted as it travels across the boundary between water and air. *(Jim Lehman)*

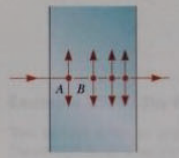


Figure 25.8 Light passing from one atom to another in a medium. The dots are electrons, and the vertical arrows represent their oscillations.

The behavior of light as it passes from air into another substance and then re-emerges into air is often a source of confusion to students. Let us take a look at why this behavior is so different from other occurrences in our daily lives. When light travels in air, its speed is equal to 3×10^8 m/s; on entry into a block of glass, its speed is reduced to approximately 2×10^8 m/s. When the light re-emerges into air, its speed instantaneously increases to its original value, 3×10^8 m/s. This is far different from what happens, for example, when a bullet is fired through a block of wood. In this case, the speed of the bullet is reduced as it moves through the wood because some of its original energy is used to tear apart the fibers of the wood. When the bullet enters the air once again, it emerges at the speed it had just before leaving the block of wood.

To see why light behaves as it does, consider Figure 25.8, which represents a beam of light entering a piece of glass from the left. Once inside the glass, the light may encounter an electron bound to an atom, represented by point *A* in the figure. Let us assume that light is absorbed by the atom, which causes the electron to oscillate (a detail represented by the double-headed arrow in the drawing). The oscillating electron then acts as an antenna and radiates (emits) the beam of light toward an atom at *B*, at which point the light is again absorbed by an atom. The details of these absorptions and emissions are best explained in terms of quantum mechanics, a subject we shall study in Chapter 28. For now, it is sufficient to think of the process as one in which the light passes from one atom to another through the glass. (The situation is somewhat analogous to a relay race in which a baton is passed between runners on the same team.) Although light travels from one atom to another with a speed of 3×10^8 m/s, the absorptions and emissions of light by the atoms cause the overall speed of light in the glass to be lowered. Once the light emerges into the air, the absorptions and emissions cease and the light's speed returns to its original value.

TABLE 25.1 Index of Refraction for Various Substances Measured with Light of Vacuum Wavelength $\lambda_0 = 589$ nm

Substance	Index of Refraction	Substance	Index of Refraction
Solids at 20°C		**Liquids at 20° C**	
Cubic zirconia	2.21	Benzene	1.501
Diamond (C)	2.419	Carbon disulfide	1.628
Fluorite (CaF_2)	1.434	Carbon tetrachloride	1.461
Silica (SiO_2)	1.458	Ethyl alcohol	1.361
Glass, crown	1.52	Glycerine	1.473
Glass, flint	1.66	Corn syrup	2.21
Ice (H_2O)	1.309	Water	1.333
Polystyrene	1.49		
Sodium chloride (NaCl)	1.544	**Gases at 0°C, 1 atm**	
Zircon	1.923	Air	1.000293
		Carbon dioxide	1.00045

The Law of Refraction

Light passing from one medium to another is refracted because the speed of light is different in the two media. In fact, light travels at its maximum speed in vacuum. It is convenient to define the **index of refraction, *n*,** of a medium to be the ratio

$$n \equiv \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v} \qquad [25.4]$$

• *Index of refraction*

From this definition we see that the index of refraction is a dimensionless number greater than or equal to unity, because v in a medium is less than c. Furthermore, n is equal to unity for vacuum. The indices of refraction for various substances are listed in Table 25.1.

As light travels from one medium to another, its frequency does not change. To see why this is so, consider Figure 25.9. Wavefronts pass an observer at point *A* in medium 1 with a certain frequency and are incident on the boundary between medium 1 and medium 2. The frequency with which the wavefronts pass an observer at point *B* in medium 2 must equal the frequency at which they arrive at point *A*. If this were not the case, either wavefronts would pile up at the boundary or they would be destroyed or created at the boundary. Because there is no mechanism for this to occur, the frequency must be a constant as a light ray passes from one medium into another.

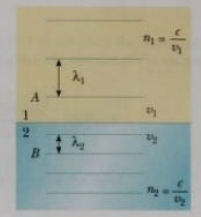


Figure 25.9 As a wave front moves from medium 1 to medium 2, its wavelength changes but its frequency remains constant.

Therefore, because the relation $v = f\lambda$ must be valid in both media and because $f_1 = f_2 = f$, we see that

$$v_1 = f\lambda_1 \quad \text{and} \quad v_2 = f\lambda_2$$

Because $v_1 \neq v_2$, it follows that $\lambda_1 \neq \lambda_2$. A relationship between index of refraction and wavelength can be obtained by dividing these two equations and making use of the definition of the index of refraction given by Equation 25.4:

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1} \qquad [25.5]$$

which gives

$$\lambda_1 n_1 = \lambda_2 n_2 \qquad [25.6]$$

If medium 1 is vacuum or, for all practical purposes, air, then $n_1 = 1$. Hence, it follows from Equation 25.5 that the index of refraction of any medium can be expressed as the ratio

$$n = \frac{\lambda_0}{\lambda_n} \qquad [25.7]$$

where λ_0 is the wavelength of light in vacuum and λ_n is the wavelength in the medium the index of refraction of which is n. A schematic representation of this reduction in wavelength is shown in Figure 25.10.

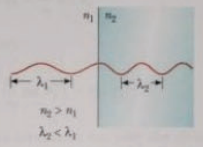


Figure 25.10 Schematic diagram of the *reduction* in wavelength when light travels from a medium of low index of refraction to one of higher index of refraction.

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We are now in a position to express Snell's law (Eq. 25.3) in an alternative form. If we combine Equations 25.5 and 25.3, we get

Snell's law of refraction •

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \qquad [25.8]$$

This form of Snell's law is practical because it is expressed in terms of n values.

Thinking Physics 1

An observer on the west-facing beach of a large lake is watching the beginning of a sunset. The water is very smooth except for some areas with small ripples. The observer notices that some areas of the water are blue and some are pink. Why does the water appear to be different colors in different areas?

Reasoning The different colors arise from specular and diffuse reflection. The smooth areas of the water will specularly reflect the light from the west, which is the pink light from the sunset. The areas with small ripples will reflect the light diffusely. Thus, light from all parts of the sky will be reflected into the observers' eyes. Because most of the sky is still blue at the beginning of the sunset, these areas will appear to be blue.

Thinking Physics 2

When looking through a glass window to the outdoors at night, you sometimes see a *double* image of yourself. Why?

Reasoning Reflection occurs whenever there is an interface between two optical media. For the glass in the window, there are two such interfaces. The first is the inner surface of the glass and the second is the outer surface. Each of these interfaces results in an image.

CONCEPTUAL PROBLEM 1

Why do face masks make observing clearer under water? A face mask includes a flat piece of glass—there are no lenses in the mask.

CONCEPTUAL PROBLEM 2

As light from the Sun passes through the atmosphere, it refracts, due to the small difference between the speeds of light in air and in vacuum. The *optical* length of the day is defined as the time interval between the instant when the top of the Sun is just visibly observed above the horizon to the instant at which the top of the Sun just disappears below the horizon. The *geometric* length of the day is defined as the time interval between the instant when a geometric straight line drawn from the observer to the actual top of the Sun just clears the horizon to the instant at which this line just dips below the horizon. Which is longer, the optical day or the geometric day?

CONCEPTUAL PROBLEM 3

If a beam of light with a given cross section enters a new medium, the cross section of the refracted beam is different from the incident beam. Is it larger or smaller, or is there no definite direction to the change?

Example 25.2 An Index of Refraction Measurement

A beam of light of wavelength 550 nm traveling in air is incident on a slab of transparent material. The incident beam makes an angle of 40.0° with the normal, and the refracted beam makes an angle of 26.0° with the normal. Find the index of refraction of the material.

Solution Snell's law of refraction (Eq. 25.8) with these data gives

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$n_2 = \frac{n_1 \sin \theta_1}{\sin \theta_2} = (1.00)\frac{\sin 40.0^\circ}{\sin 26.0^\circ} = \frac{0.643}{0.438} = 1.47$$

If we compare this value with the data in Table 25.1, we see that the material may be silica.

EXERCISE 1 What is the wavelength of light in the material? Answer 374 nm

Example 25.3 The Speed of Light in Silica

Light of wavelength 589 nm in vacuum passes through a piece of silica ($n = 1.458$). (a) Find the speed of light in silica.

Solution The speed of light in silica can be easily obtained from Equation 25.4:

$$v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.458} = 2.06 \times 10^8 \text{ m/s}$$

(b) What is the wavelength of this light in silica?

Solution We use Equation 25.7 to calculate the wavelength in silica, noting that we are given the wavelength in vacuum to be $\lambda_0 = 589$ nm:

$$\lambda_n = \frac{\lambda_0}{n} = \frac{589 \text{ nm}}{1.458} = 404 \text{ nm}$$

EXERCISE 2 Find the frequency of the light. Answer 5.09×10^{14} Hz

Example 25.4 Light Passing Through a Slab

A light beam passes from medium 1 to medium 2 with the latter being a thick slab of material the index of refraction of which is n_2 (Fig. 25.11). Show that the emerging beam is parallel to the incident beam.

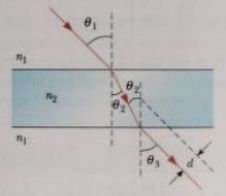


Figure 25.11 (Example 25.4) When light passes through a flat slab of material, the emerging beam is parallel to the incident beam, and therefore $\theta_1 = \theta_3$. The dashed line represents the path the light would take if the slab were not there.

Solution First, let us apply Snell's law to the upper surface:

$$(1) \quad \sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1$$

Applying Snell's law to the lower surface gives

$$(2) \quad \sin \theta_3 = \frac{n_2}{n_1} \sin \theta_2$$

Substituting (1) into (2) gives

$$\sin \theta_3 = \frac{n_2}{n_1}\left(\frac{n_1}{n_2} \sin \theta_1\right) = \sin \theta_1$$

That is, $\theta_3 = \theta_1$, and so the layer does not alter the direction of the beam as shown by the dashed line. It does, however, produce a displacement of the beam. The same result is obtained when light passes through multiple layers of materials.

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25.4 · DISPERSION AND PRISMS

An important property of the index of refraction is that its value in anything but vacuum depends on the wavelength of light. This phenomenon is called **dispersion** (Fig. 25.12). Because n is a function of wavelength, Snell's law indicates that **light rays of different wavelengths are bent at different angles when incident on a refracting material.** As we see from Figure 25.12, the index of refraction for a material decreases with increasing wavelength. This means that blue light ($\lambda \cong 470$ nm) bends more than red light ($\lambda \cong 650$ nm) when passing into a refracting material.

To understand how dispersion can affect light, let us consider what happens when light strikes a prism, as in Figure 25.13a. A single ray of light that is incident on the prism from the left emerges bent away from its original direction of travel by an angle, δ, called the **angle of deviation.** Now suppose a beam of white light (a combination of all visible wavelengths) is incident on a prism, as in Figure 25.13b. Because of dispersion, the blue component of the incident beam is bent more than the red component, and the rays that emerge from the second face of the prism spread out in a series of colors known as a **visible spectrum,** as shown in Figure 25.14. These colors, in order of decreasing wavelength, are red, orange, yellow, green, blue, indigo, and violet. Clearly, the angle of deviation, δ, depends on wavelength. Violet light ($\lambda \cong 400$ nm) deviates the most, red light ($\lambda \cong 650$ nm) deviates the least, and the remaining colors in the visible spectrum fall between these extremes.

The dispersion of light into a spectrum is demonstrated most vividly in nature through the formation of a rainbow, often seen by an observer positioned between the Sun and a rain shower (see page 730). To understand how a rainbow is formed, consider Figure 25.15. A ray of light passing overhead strikes a drop of water in the atmosphere and is refracted and reflected as follows. It is first refracted at the front surface of the drop, with the violet light deviating the most and the red light the least. At the back surface of the drop, the light is reflected and returns to the front surface, where it again undergoes refraction as it moves from water into air. The rays leave the drop so that the angle between the incident white light and the most intense returning violet light is 40°, and the angle between the white light and the most intense returning red light is 42°. This small angular difference between the returning rays causes us to see the rainbow.

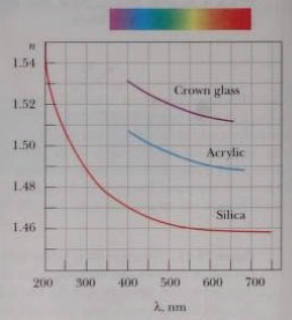


Figure 25.12 Variation of index of refraction with vacuum wavelength for three materials.

Thinking Physics 3

When a beam of light enters a glass prism, which has nonparallel sides, the rainbow of color exiting the prism is a testimonial to the dispersion occurring in the glass. Suppose a beam of light enters a slab of material with *parallel* sides. When the beam exits the other side, traveling in the same direction as the original beam, is there any evidence of dispersion?

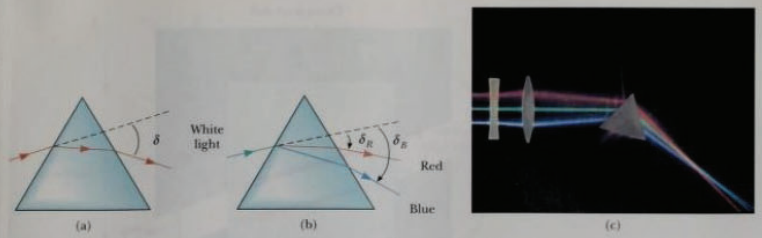


Figure 25.13 (a) A prism refracts single-wavelength light and deviates the light through an angle δ. (b) When light is incident on a prism, the blue light is bent more than the red. (c) Light of different colors passes through a prism and two lenses. As the light passes through the prism, different wavelengths are refracted at different angles. *(David Parker, SPL/Photo Researchers)*

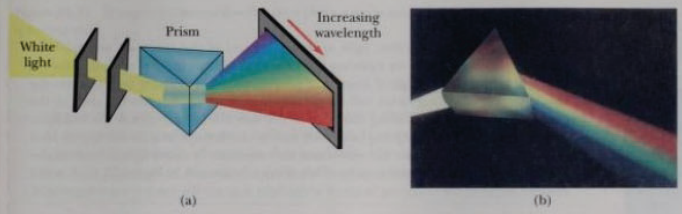


Figure 25.14 (a) Dispersion of white light by a prism. Because n varies with wavelength, the prism disperses the white light into its various spectral components. (b) Different colors are refracted at different angles because the index of refraction of the glass depends on wavelength. Violet light deviates the most; red light deviates the least. *(Photograph courtesy of Bausch and Lomb)*

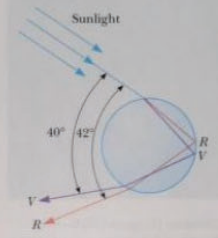


Figure 25.15 Refraction of sunlight by a spherical raindrop.

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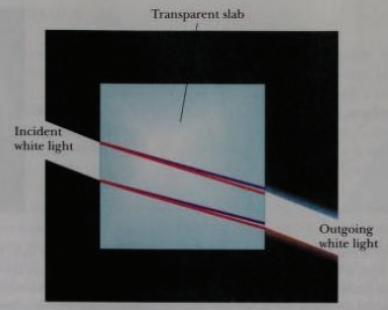


Figure 25.16 (Thinking Physics 3)

Reasoning Due to dispersion, light at the violet end of the spectrum will exhibit a larger angle of refraction on entering the glass than light at the red end. All colors of light will return to the original *direction* of propagation on refracting back out into the air. Thus, the outgoing beam will be predominantly white. But the net shift in the *position* of the violet light along the edge of the slab will be larger than the red light. Thus, one edge of the outgoing beam will have a bluish tinge to it (it will appear blue rather than violet, because the eye is not very sensitive to violet light), whereas the other edge will have a reddish tinge. This effect is indicated in Figure 25.16. It is the colored edges of the outgoing beam of white light that are the evidence for dispersion.



Christian Huygens (1629–1695). *(Courtesy of Rijksmuseum voor de Geschiedenis der Natuurwetenschappen. Courtesy AIP Niels Bohr Library)*

CONCEPTUAL PROBLEM 4

In dispersive materials, the angle of refraction for a light ray depends on the wavelength of the light. Does the angle of reflection from the surface of the material depend on the wavelength?

25.5 · HUYGENS'S PRINCIPLE

In this section, we shall discuss a geometric method proposed by Huygens in 1678, which is useful for describing the wave properties of light. Huygens assumed that light was some form of wave motion rather than a stream of particles. He had no knowledge of the nature of light or of its electromagnetic character. Nevertheless, his simplified wave model is adequate for understanding many practical aspects of the propagation of light.

Huygens's principle is a geometric construction for determining the position of a wavefront from a knowledge of an earlier wavefront. In Huygens's construction,

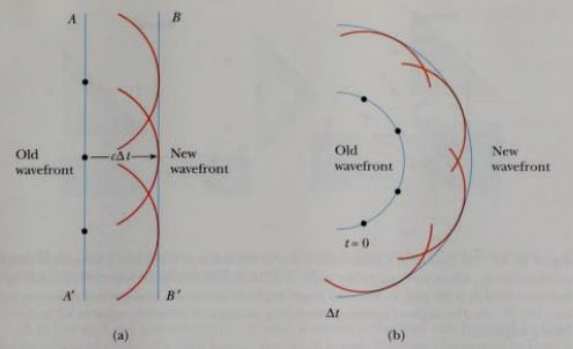


Figure 25.17 Huygens's construction for (a) a plane wave propagating to the right and (b) a spherical wave.

all points on a given wavefront are taken as point sources for the production of spherical secondary waves, called *wavelets*, that propagate outward with speeds characteristic of waves in that medium. After some time has elapsed, the new position of the wavefront is the surface tangent to the wavelets.

Figure 25.17 illustrates two simple examples of Huygens's construction. First, consider a plane wave moving through free space, as in Figure 25.17a. At $t = 0$, the wavefront is indicated by the plane labeled AA'. Each point on this wavefront is a point source for a wavelet. Showing three of these points, we draw circles, each of radius $c\,\Delta t$, where c is the speed of light in free space and Δt is the time of propagation from one wavefront to the next. The surface drawn tangent to the wavelets is the plane BB', which is parallel to AA'. In a similar manner, Figure 25.17b shows Huygens's construction for an outgoing spherical wave.

A convincing demonstration of the existence of Huygens's wavelets is obtained with water waves in a shallow tank (called a *ripple tank*), as in Figure 25.18. Plane waves produced below the slits emerge above the slits as two-dimensional circular waves propagating outward.

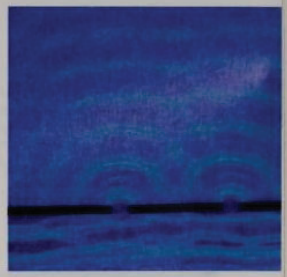


Figure 25.18 Water waves in a ripple tank, which demonstrates Huygens's wavelets. A plane wave is incident on a barrier with two small openings. Each opening acts as a source of circular wavelets. *(Erich Schrempp/Photo Researchers)*

25.6 · TOTAL INTERNAL REFLECTION

An interesting effect called **total internal reflection** can occur when light travels from a medium with a high index of refraction to one with a lower index of refraction. Consider a light beam traveling in medium 1 and meeting the boundary

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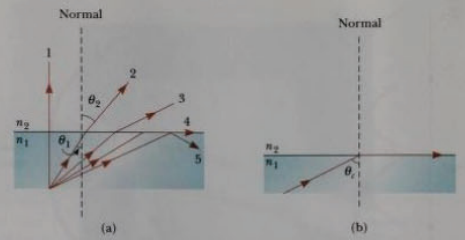
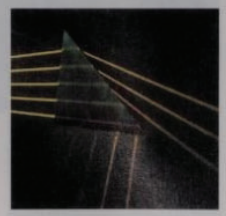


Figure 25.19 (a) Rays from a medium of index of refraction n_1 travel into a medium of index of refraction n_2, where $n_2 < n_1$. As the angle of incidence increases, the angle of refraction θ_2 increases until θ_2 is 90° (ray 4). For even larger angles of incidence, total internal reflection occurs (ray 5). (b) The angle of incidence producing an angle of refraction equal to 90° is the critical angle, θ_c.

Critical angle

between mediums 1 and 2, where $n_1 > n_2$ (Fig. 25.19). Various possible directions of the beam are indicated by rays 1 through 5. The refracted rays are bent away from the normal because $n_1 > n_2$. (Remember that when light refracts at the interface between the two media, it is also partially reflected. For simplicity, we ignore these reflected rays.) At some particular angle of incidence, θ_c, called the **critical angle,** the refracted light ray will move parallel to the boundary so that $\theta_2 = 90°$ (Fig. 25.19b). For angles of incidence greater than or equal to θ_c, the beam is entirely reflected at the boundary, as is ray 5 in Figure 25.19a. This ray is reflected at the boundary as though it had struck a perfectly reflecting surface. It and all rays like it obey the law of reflection—that is, the angle of incidence equals the angle of reflection.



This photograph shows nonparallel light rays entering a glass prism. The bottom two rays undergo total internal reflection at the longest side of the prism. The top three rays are refracted at the longest side as they leave the prism. *(Henry Leap and Jim Lehman)*

We can use Snell's law to find the critical angle. When $\theta_1 = \theta_c$, $\theta_2 = 90°$ and Snell's law (Eq. 25.8) gives

$$n_1 \sin \theta_c = n_2 \sin 90° = n_2$$

$$\sin \theta_c = \frac{n_2}{n_1} \qquad (\text{for } n_1 > n_2) \qquad [25.9]$$

This equation can be used only when n_1 is greater than n_2. That is, **total internal reflection occurs only when light travels from a medium of high index of refraction to a medium of lower index of refraction.** If n_1 were less than n_2, Equation 25.9 would give $\sin \theta_c > 1$, which is meaningless because the sine of an angle can never be greater than unity.

The critical angle for total internal reflection is small when n_1 is considerably larger than n_2. Examples of this combination are diamond ($n = 2.42$ and $\theta_c = 24°$) and crown glass ($n = 1.52$ and $\theta_c = 41°$). The angles given are only valid when the material is in air. This property combined with proper faceting causes diamonds and crystal glass to sparkle.

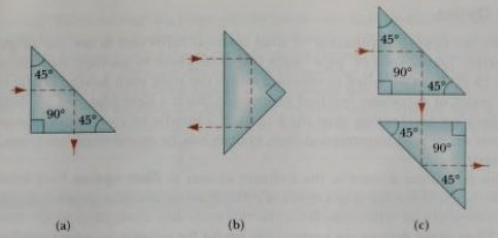


Figure 25.20 Internal reflection in a prism. (a) The ray is deviated by 90°. (b) The direction of the ray is reversed. (c) Two prisms used as a periscope.

A prism and the phenomenon of total internal reflection can be used to alter the direction of travel of a light beam. Two such possibilities are illustrated in Figure 25.20. In one case the light beam is deflected by 90° (Fig. 25.20a), and in the second case the path of the beam is reversed (Fig. 25.20b). A common application of total internal reflection is in a submarine periscope. In this device, two prisms are arranged as in Figure 25.20c so that an incident beam of light follows the path shown and the periscope user is able to "see around corners."

Example 25.5 A View from the Fish's Eye

(a) Find the critical angle for a water–air boundary if the index of refraction of water is 1.33.

Solution Applying Equation 25.9, we find the critical angle to be

$$\sin \theta_c = \frac{n_2}{n_1} = \frac{1}{1.33} = 0.752$$

$$\theta_c = 48.8°$$

(b) Use the results of part (a) to predict what a fish sees when it looks upward toward the water surface at an angle of 40°, 49°, and 60° (Fig. 25.21).

Reasoning Examine Figure 25.19a. Note that because the path of a light ray is reversible, light traveling from medium 2 into medium 1 follows the paths shown in Figure 25.19a, but in the *opposite* direction. This situation is illustrated in Figure 25.21 for the case of a fish looking upward toward the water surface. A fish can see out of the water if it looks toward the surface at an angle less than the critical angle. Thus, at 40°, the fish can see into the air above the water. At an angle of 49°, the critical angle for water, the light that reaches the fish has to skim along the water surface before being refracted to the fish's eye. At angles greater than the critical angle, the light reaching the fish comes via internal reflection at the surface. Thus, at 60°, the fish sees a reflection of some object on the bottom of the pool.

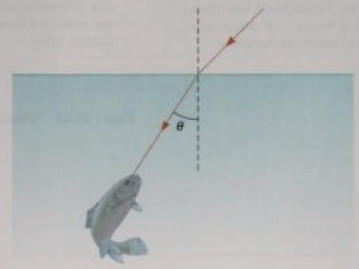


Figure 25.21 (Example 25.5)

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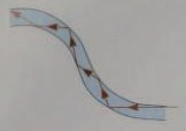


Figure 25.22 Light travels in a curved transparent rod by multiple internal reflections.



Strands of glass optical fibers are used to carry voice, video, and data signals in telecommunication networks. Typical fibers have diameters of 60 μm. *(© Richard Megna 1983, Fundamental Photographs)*

Fiber Optics

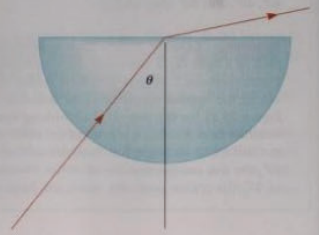
Another interesting application of total internal reflection is the use of glass or transparent plastic rods to "pipe" light from one place to another. As indicated in Figure 25.22, light is limited to traveling within the rods, even around gentle curves, as the result of successive internal reflections. Such a light pipe can be flexible if thin fibers are used rather than thick rods. If a bundle of parallel fibers is used to construct an optical transmission line, images can be transferred from one point to another.

This technique is used in the industry known as **fiber optics.** Very little light intensity is lost in the fibers as a result of reflections on the sides. Any loss in intensity is due essentially to reflections from the two ends and absorption by the fiber material. Fiber optic devices are particularly useful for viewing an image produced at an inaccessible location. Physicians often use fiber optic cables to aid in the diagnosis and repair of certain medical conditions, without the intrusive effects of major surgery. For example, a fiber optic cable can be threaded through the esophagus and into the stomach to enable the physician to look for ulcers. In this application, the cable actually consists of two fiber optic lines, one to transmit a beam of light into the stomach for illumination and the other to allow this light to be transmitted out of the stomach. The resulting image can be viewed directly by the physician in some cases but most often is displayed on a television monitor or captured on film. The field of fiber optics is also finding increasing use in telecommunications, because the fibers can carry a much higher volume of telephone calls, or other forms of communication, than electrical wires.

Thinking Physics 4

A beam of white light is incident on the curved edge of a semicircular piece of glass, as shown in Figure 25.23. The light enters the curved surface along the normal, so it shows no refraction. It encounters the straight side at the center of curvature of the curved side and refracts into the air. The incoming beam is moved clockwise (so that the angle θ increases) such that the beam always enters along the normal to the curved side and encounters the straight side at the center of curvature of the curved side. As the refracted beam approaches a direction parallel to the straight side, it becomes redder. Why?

Figure 25.23 (Thinking Physics 4)



Reasoning When the outgoing beam approaches the direction parallel to the straight side, the incident angle is approaching the critical angle for total internal reflection. But there is dispersion occurring as the light passes through the glass. The index of refraction for light at the violet end of the visible spectrum is larger than at the red end. Thus, as the outgoing beam approaches the straight side, the violet light experiences total internal reflection first, followed by the other colors. The red light is the last to experience total internal reflection, so, just before the outgoing beam disappears, it is composed of light from the red end of the visible spectrum.

Thinking Physics 5

An optical fiber consists of a transparent core, surrounded by a *cladding*, which is a material with a lower index of refraction than the core. There is a cone of angles, called the *acceptance cone*, at the entrance to the fiber. Incoming light at angles within this cone will be transmitted through the fiber, whereas light entering the core from angles outside the cone will not be transmitted. Figure 25.24 shows a light ray entering the fiber just within the acceptance cone and experiencing total internal reflection at the interface between the core and the cladding. If it is technologically difficult to produce light entering the fiber from a small range of angles, how could you adjust the indices of refraction of the core and cladding to increase the size of the acceptance cone—would you design them to be farther apart or closer together?

Reasoning The acceptance cone would become larger if the critical angle (θ_c in the diagram) could be made smaller. This can be done by making the index of refraction of the cladding smaller, so that the indices of refraction of the core and cladding would be farther apart.

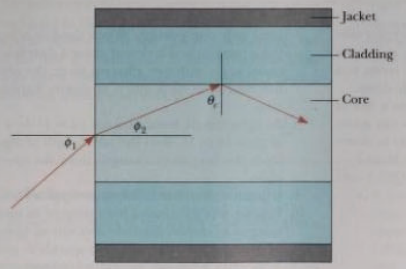


Figure 25.24 (Thinking Physics 5)

SUMMARY

In geometric optics, we use the **ray approximation,** in which we assume that a wave travels through a medium in straight lines in the direction of the rays of that wave. We neglect diffraction effects, which is a good approximation as long as the wavelength is short compared with any aperture dimensions.

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The **index of refraction** of a material, n, is defined as

$$n \equiv \frac{c}{v} \qquad [25.4]$$

where c is the speed of light in a vacuum and v is the speed of light in the material. In general, n is related to wavelength as

$$n = \frac{\lambda_0}{\lambda_n} \qquad [25.7]$$

where λ_0 is the wavelength of the light in vacuum and λ_n is its wavelength in the material.

The **law of reflection** states that a wave reflects from a surface so that the angle of reflection, θ_1', equals the angle of incidence, θ_1.

The **law of refraction,** or **Snell's law,** states that

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \qquad [25.8]$$

Huygens's principle states that all points on a wavefront can be taken as point sources for the production of secondary wavelets. At some later time, the new position of the wavefront is the surface tangent to these secondary wavelets.

Total internal reflection can occur when light travels from a medium of high index of refraction to one of lower index of refraction. The minimum angle of incidence, θ_c, for which total reflection occurs at an interface is

$$\sin \theta_c = \frac{n_2}{n_1} \quad (\text{where } n_1 > n_2) \qquad [25.9]$$

CONCEPTUAL QUESTIONS

1. You can make a corner reflector by placing three flat mirrors in the corner of a room where the ceiling meets the walls. Show that no matter where you are in the room, you can see yourself reflected in the mirrors—upside down.
2. Several corner reflectors were left on the Moon's Sea of Tranquility by the astronauts of Apollo 11. How can scientists use a laser beam sent from Earth even today to determine the precise distance from the Earth to the Moon?
3. The rectangular aquarium sketched in Figure Q25.3 contains only one goldfish. When the fish is near a corner of the aquarium and is viewed along a direction that makes an equal angle with two adjacent faces, the observer sees two fish mirroring each other, as shown. Explain this observation.

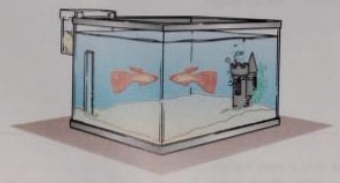


Figure Q25.3

4. As light travels from vacuum ($n = 1$) to a medium such as glass ($n > 1$), does the wavelength of the light change? Does the frequency change? Does the speed change? Explain.
5. Explain why an oar in the water appears bent.
6. Explain why a diamond loses most of its sparkle when submerged in carbon disulfide and why an imitation diamond of cubic zirconia loses all of its sparkle in corn syrup.
7. A laser beam passing through a nonhomogeneous sugar solution follows a curved path. Explain.
8. Light of wavelength λ is incident on a slit of width d. Under what conditions is the ray approximation valid? Under what circumstances does the slit produce enough diffraction to make the ray approximation invalid?
9. A laser beam ($\lambda = 632.8$ nm) is incident on a piece of Lucite, as in Figure Q25.9. Part of the beam is reflected and part is refracted. What information can you get from this photograph?



Figure Q25.9 Light from a helium–neon laser beam ($\lambda = 632.8$ nm, red light) is incident on a block of Lucite. The photograph shows both reflected and refracted rays. Can you identify the incident, reflected, and refracted rays? *(Henry Leap and Jim Lehman)*



Figure Q25.18 *(Ron Chapple/FPG)*

10. The level of water in a clear, colorless glass is easily observed with the naked eye. The level of liquid helium in a clear glass vessel is extremely difficult to see with the naked eye. Explain.
11. Describe an experiment in which internal reflection is used to determine the index of refraction of a medium.
12. Why does a diamond show flashes of color when observed under white light?
13. Explain why a diamond sparkles more than a glass crystal of the same shape and size.
14. Why do astronomers looking at distant galaxies talk about looking backward in time?
15. A solar eclipse occurs when the Moon gets between the Earth and the Sun. Use a diagram to show why some areas of the Earth see a total eclipse, other areas see a partial eclipse, and most areas see no eclipse.
16. Figure 25.15 represents sunlight striking a drop of water in the atmosphere. Use the laws of refraction and reflection and the fact that sunlight consists of a wide range of wavelengths to discuss the formation of rainbows.
17. Why does the arc of a rainbow appear with red on top and violet on the bottom?
18. How is it possible that a complete circle of rainbow can sometimes be seen from an airplane (Fig. Q25.18)?
19. Under what conditions is a mirage formed? On a hot day, what are we seeing when we observe "water on the road"?
20. Suppose you are told that only two colors of light (X and Y) are sent through a glass prism and that X is bent more than Y. Which color travels more slowly in the prism?

PROBLEMS

Section 25.3 Reflection and Refraction

(*Note:* In this section if an index of refraction value is not given, refer to Table 25.1.)

1. A narrow beam of sodium yellow light with wavelength 589 nm in vacuum is incident from air onto a smooth water surface at an angle $\theta_1 = 35.0°$. Determine the angle of refraction, θ_2, and the wavelength of the light in water.
2. The wavelength of red helium-neon laser light in air is 632.8 nm. (a) What is its frequency? (b) What is its wavelength in glass that has an index of refraction of 1.50? (c) What is its speed in the glass?
3. An underwater scuba diver sees the Sun at an apparent angle of 45.0° from the vertical. Where is the Sun actually?
4. A laser beam is incident at an angle of 30.0° to the vertical onto a solution of corn syrup in water. If the beam is refracted to 19.24° to the vertical, (a) what is the index of refraction of the syrup solution? Suppose the light is red, with vacuum wavelength 632.8 nm. Find its (b) wavelength, (c) frequency, and (d) speed in the solution.
5. A ray of light strikes a flat block of glass ($n = 1.50$) of thickness 2.00 cm at an angle of 30.0° with the normal. Trace the light beam through the glass and find the angles of incidence and refraction at each surface.
6. Light of wavelength 436 nm in air enters a fishbowl filled with water, then exits through the crown glass wall of the container. What is the wavelength of the light (a) in the water and (b) in the glass?

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7. A cylindrical tank with an open top has a diameter of 3.00 m and is completely filled with water. When the afternoon Sun reaches an angle of 28.0° above the horizon, sunlight ceases to illuminate the bottom of the tank. How deep is the tank?
8. The angle between the two mirrors in Figure P25.8 is a right angle. The beam of light in the vertical plane *P* strikes mirror 1 as shown. (a) Determine the distance the reflected light beam travels before striking mirror 2. (b) In what direction does the light beam travel after being reflected from mirror 2?

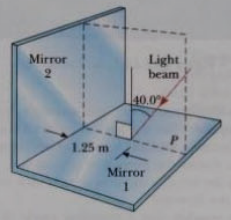


Figure P25.8

9. How many times will the incident beam shown in Figure P25.9 be reflected by each of the parallel mirrors?

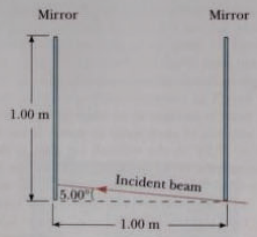


Figure P25.9

10. A beam of light strikes the surface of mineral oil at an angle of 23.1° with the normal to the surface. If the light travels with a speed of 2.17×10^8 m/s through the oil, what is the angle of refraction?

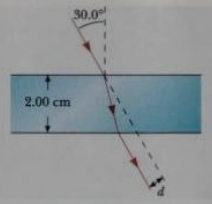


Figure P25.11

11. When the light in Figure P25.11 passes through the glass block, it is shifted laterally by the distance *d*. If $n = 1.50$, find the value of *d*.
12. Find the time required for the light to pass through the glass block described in Problem 11.
13. The light beam shown in Figure P25.13 makes an angle of 20.0° with the normal line *NN'* in the linseed oil. Determine the angles θ and θ'. (The index of refraction for linseed oil is 1.48.)

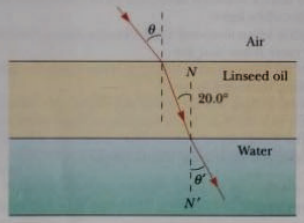


Figure P25.13

14. Two light pulses are emitted simultaneously from a source. Both pulses travel to a detector, but one first passes through 6.20 m of ice. Determine the difference in the pulses' times of arrival at the detector.
15. A piece of wire is bent through an angle θ. The bent wire is partially submerged in benzene (index of refraction = 1.50) so that looking along the dry part, the wire appears to be straight and makes an angle of 30.0° with the horizontal. Determine the value of θ.
16. When you look through a window, by how much time is the light you see delayed by having to go through glass instead of air? Make an order-of-magnitude estimate on the basis of data you specify. By how many wavelengths is it delayed?

Section 25.4 Dispersion and Prisms

17. A narrow white light beam is incident on a block of silica at an angle of 30.0°. Find the angular width of the light beam inside the silica.
18. A ray of light strikes the midpoint of one face of an equiangular glass prism ($n = 1.50$) at an angle of incidence of 30.0°. Trace the path of the light ray through the glass and find the angles of incidence and refraction at each surface.
19. The index of refraction for violet light in silica flint glass is 1.66, and that for red light is 1.62. What is the angular dispersion of visible light passing through a prism of apex angle 60.0° if the angle of incidence is 50.0°? (See Fig. P25.19.)

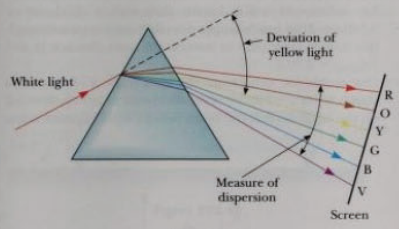


Figure P25.19

20. Light of wavelength 700 nm is incident on the face of a silica prism at an angle of 75.0° (with respect to the normal to the surface). The apex angle of the prism is 60.0°. Use the value of *n* from Figure 25.12 and calculate the angle (a) of refraction at this first surface, (b) of incidence at the second surface, (c) of refraction at the second surface, and (d) between the incident and emerging rays.
21. A light ray is incident on a prism and refracted at its first surface, as shown in Figure P25.21. Suppose the apex angle of the prism is $\Phi = 60.0°$ and its index of refraction is 1.50. (a) Find the smallest value of the angle of incidence at the first surface for which the refracted ray does not undergo total internal reflection at the second surface. (b) For what angle of incidence θ_1 does the light ray pass symmetrically through the prism and leave at the same angle θ_1?
22. A light ray is incident on a prism and refracted at the first surface, as shown in Figure P25.21. Let Φ represent the apex angle of the prism and *n* its index of refraction. Find in terms of *n* and Φ the smallest allowed value of the angle of incidence at the first surface for which the refracted ray does not undergo total internal reflection at the second surface.
23. A 1.00-cm high by 4.00-cm long glass plate is made up of two cemented prisms. The top prism has an index of refraction of 1.486 for blue light and 1.472 for red light. The bottom prism has an index of refraction of 1.878 for blue light and 1.862 for red light. A ray consisting of red and blue light is incident at 50.0° on the top face, as shown in Figure P25.23. Determine the exit angles for both rays that pass through the plate and the angle between them.

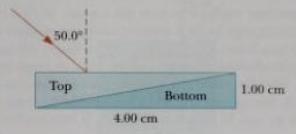


Figure P25.23

Section 25.6 Total Internal Reflection

24. For 589 nm light, calculate the critical angle for the following materials surrounded by air: (a) diamond, (b) flint glass, and (c) ice.
25. Consider a common mirage formed by superheated air just above the roadway. If an observer viewing from 2.00 m above the road (where $n = 1.0003$) has the illusion of seeing water up the road at $\theta_1 = 88.8°$, find the index of refraction of the air just above the road surface. (*Hint:* Treat this as a problem in total internal reflection.)
26. An optical fiber is made of a clear plastic for which the index of refraction is 1.50. For what angles with the surface does light remain contained within the fiber?
27. Determine the maximum angle θ for which the light rays incident on the end of the pipe in Figure P25.27 are subject

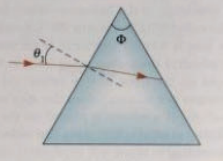


Figure P25.21

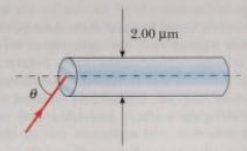


Figure P25.27

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to total internal reflection along the walls of the pipe. Assume that the pipe has an index of refraction of 1.36 and the outside medium is air.

28. A glass cube is placed on a newspaper, which rests on a table. A person reads the news through the vertical side of the cube. Determine the maximum index of refraction of the cube.

Additional Problems

29. A glass block having $n = 1.52$ and surrounded by air measures 10.0 cm × 10.0 cm. For an angle of incidence of 45.0°, what is the maximum distance x shown in Figure P25.29 so that the ray emerges from the opposite side?

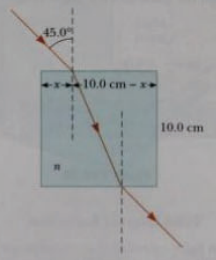


Figure P25.29

30. A light ray enters the atmosphere of a planet and propagates vertically down to the surface 20.0 km below. The index of refraction where the light enters the atmosphere is 1.000 and it increases uniformly to the surface where it has a value of 1.005. (a) How long does it take the ray to traverse this path? (b) Compare this to the time with no atmosphere.
31. A layer of ice floats on water. If light is incident on the upper surface of the ice at an angle of 30.0°, what is the angle of refraction in the water?
32. (a) Consider a horizontal interface between air above and glass of index 1.55 below. Draw a light ray incident from the air at angle of incidence 30.0°. Determine the angles of the reflected and refracted rays and show them on the diagram. (b) Suppose instead that the light ray is incident from the glass at angle of incidence 30.0°. Determine the angles of the reflected and refracted rays and show all three rays on a new diagram. (c) For rays incident from the air onto the air–glass surface, determine and tabulate the angles of reflection and refraction for all the angles of incidence at 10.0° intervals from 0° and 90.0°. (d) Do the same for light rays coming up to the interface through the glass.
33. A small light source is on the bottom of a swimming pool 1.00 m deep. The light escaping from the water forms a circle on the water surface. What is the diameter of this circle?
34. A light beam is incident on a water surface from air. What is the maximum possible value for the angle of refraction?
35. A narrow beam of light is incident from air onto a glass surface of index of refraction 1.56. Find the angle of incidence for which the corresponding angle of refraction is one half the angle of incidence. (*Hint:* You might want to use the trigonometric identity $\sin 2\theta = 2 \sin\theta \cos\theta$.)
36. Light is incident normally on a 1.00-cm layer of water that lies on top of a flat Lucite plate with a thickness of 0.500 cm. How much longer does light take to pass through this double layer than to traverse the same distance in air? ($n_{\text{Lucite}} = 1.59$.)
37. One technique to measure the apex angle A of a prism is shown in Figure P25.37. A parallel beam of light is directed on the angle so that the beam reflects from opposite sides.

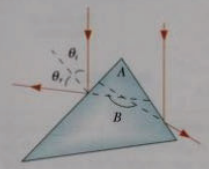


Figure P25.37

Show that the angular separation of the two beams is given by $B = 2A$.

38. The laws of refraction and reflection are the same for sound as for light. The speed of sound in air is 340 m/s and that in water is 1510 m/s. If a sound wave approaches a plane water surface at an angle of incidence of 12.0°, what is the angle of refraction?
39. A layer of kerosene ($n = 1.45$) is floating on water. For what angles of incidence at the kerosene–water interface is light totally internally reflected within the kerosene?
40. When the Sun is directly overhead, a narrow shaft of light enters a cathedral through a small hole in the ceiling and forms a spot on the floor 10.0 m below. (a) At what speed (in centimeters per minute) does the spot move across the (flat) floor? (b) If a mirror is placed on the floor to intercept the light, at what speed does the reflected spot move across the ceiling?

41. A hiker stands on a mountain peak near sunset and observes a rainbow caused by water droplets in the air about 8.00 km away. The valley is 2.00 km below the mountain peak and entirely flat. What fraction of the complete circular arc of the rainbow is visible to the hiker?
42. A fish is at a depth d under water. Show that when viewed from an angle of incidence θ_1, the apparent depth z of the fish is

$$z = \frac{3d \cos\theta_1}{\sqrt{7 + 9\cos^2\theta_1}}$$

43. A laser beam strikes one end of a slab of material, as shown in Figure P25.43. The index of refraction of the slab is 1.48. Determine the number of internal reflections of the beam before it emerges from the opposite end of the slab.

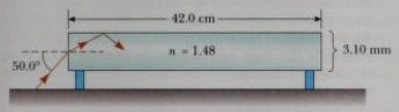


Figure P25.43

44. The prism shown in Figure P25.44 has an index of refraction of 1.55. Light is incident at an angle of 20.0°. Determine the angle θ at which the light emerges.

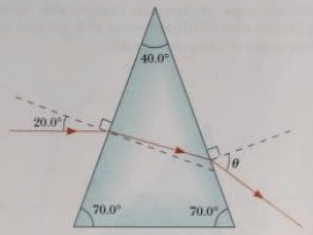


Figure P25.44

45. The light beam in Figure P25.45 strikes surface 2 at the critical angle. Determine the angle of incidence, θ_1.

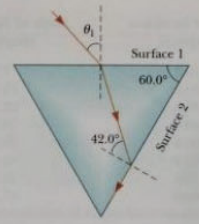


Figure P25.45

46. Figure P25.46 shows a top view of a square enclosure. The inner surfaces are plane mirrors. A ray of light enters a small hole in the center of one mirror. (a) At what angle θ must the ray enter in order to exit through the hole after being reflected once by each of the other three mirrors? (b) Are there other values of θ for which the ray can exit after multiple reflections? If so, make a sketch of one of the ray's paths.

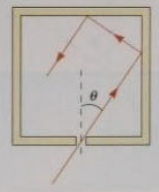


Figure P25.46

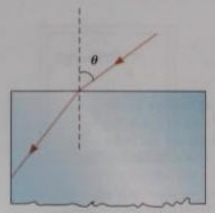
47. Students allow a narrow beam of laser light to strike a water surface. They arrange to measure the angle of refraction for selected angles of incidence and record the data shown in the accompanying table. Use the data to verify Snell's law of refraction by plotting the sine of the angle of incidence versus the sine of the angle of refraction. Use the resulting plot to deduce the index of refraction of water.

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Angle of Incidence (degrees)	Angle of Refraction (degrees)
10.0	7.5
20.0	15.1
30.0	22.3
40.0	28.7
50.0	35.2
60.0	40.3
70.0	45.3
80.0	47.7

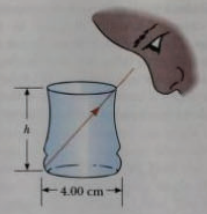
48. A 4.00-m-long pole stands vertically in a lake having a depth of 2.00 m. When the Sun is 40.0° above the horizontal, determine the length of the pole's shadow on the bottom of the lake. Take the index of refraction for water to be 1.33.

49. A light ray of wavelength 589 nm is incident at an angle θ on the top surface of a block of polystyrene, as shown in Figure P25.49. (a) Find the maximum value of θ for which the refracted ray undergoes total internal reflection at the left vertical face of the block. Repeat the calculation for the case in which the polystyrene block is immersed in (b) water and (c) carbon disulfide.



50. A ray of light passes from air into water. In order for its deviation angle $\delta = |\theta_1 - \theta_2|$ to be 10.0°, what must be its angle of incidence? You may need to use a calculator.

51. A drinking glass is 4.00 cm wide, as shown in Figure P25.51. When an observer's eye is placed as shown, the observer sees the edge of the bottom of the glass. When this glass is filled with water, an observer at the same position relative to the glass sees the center of the bottom of the glass. Find the height of the glass.

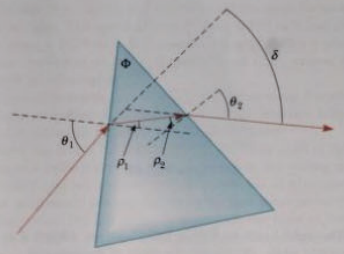


Spreadsheet Problem

S1. Spreadsheet 25.1 calculates the angle of refraction and angle of deviation as light beams of different incidence angles travel through a prism of apex angle Φ. Input parameters are the index of refraction n and the apex angle Φ (Fig. S25.1). As the angle of incidence θ_1 increases from zero, the angle of deviation δ decreases, reaches a minimum value, and then increases. At minimum deviation

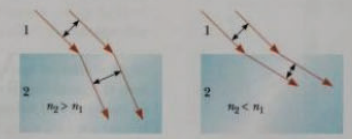
$$n \sin \frac{\Phi}{2} = \sin \frac{\Phi + \delta_{min}}{2} = \sin \theta_1$$

This equation is often used to experimentally find n of any transparent prism. (a) For crown glass ($n = 1.52$ and $\Phi = 60°$), find the minimum angle of deviation from the graph. Using the spreadsheet, verify these equations. At the angle of minimum deviation, how is angle ρ_1 related to ρ_2? (b) Because the index of refraction depends slightly on wavelength, vary n between 1.50 and 1.55, and note how the minimum angle of deviation changes with wavelength. (c) Choose several different values of Φ and note the minimum angles of deviation for each.



ANSWERS TO CONCEPTUAL PROBLEMS

1. The refraction necessary for focused viewing in the eye occurs at the air–cornea interface. The lens of the eye only performs some fine tuning of this image, allowing for accommodation for objects at various distances. When the eye is opened underwater, the interface is water–cornea, rather than the normal air–cornea for which we are designed. Thus, the image of the scene is not focused on the retina, and the scene is blurry. The face mask simply provides a layer of air in front of the eyes, so that air–cornea interface is reestablished, and the refraction is correct to form the image on the retina.
2. The optical day is longer than the geometric day. Due to the refraction of light by air, light rays from the Sun deviate slightly downward toward the surface of the Earth as the light enters the atmosphere. Thus, in the morning, light rays from the upper edge of the Sun will arrive at your eyes before the geometric line from your eyes to the top of the Sun clears the horizon. In the evening, light rays from the top of the Sun will continue to arrive at your eyes even after the geometric line from your eyes to the top of the Sun dips below the horizon.
3. The cross section can be visualized by considering just the two rays of light on the edges of the beam. If the beam of light enters a new medium with a higher index of refraction, the rays bend toward the normal, and the cross-section of the refracted beam will be larger than that of the incident beam, as suggested by the diagram on the left. If the new index of refraction is lower, the rays bend away from the normal, and the cross section of the beam is reduced, as shown on the diagram on the right.



Change in cross section of a light beam on entering a new medium.

4. There is no dependence of the angle of reflection on wavelength, because the light does not enter deeply into the material during reflection—it reflects from the surface.

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26

Mirrors and Lenses

CHAPTER OUTLINE

26.1 Images Formed by Flat Mirrors
26.2 Images Formed by Spherical Mirrors
26.3 Images Formed by Refraction
26.4 Thin Lenses
26.5 Lens Aberrations (Optional)

This chapter is concerned with the images formed when spherical waves fall on flat and spherical surfaces. We find that images can be formed by reflection or by refraction and that mirrors and lenses work because of this reflection and refraction. Such devices, commonly used in optical instruments and systems, are discussed in detail. In this chapter, we continue to use the ray approximation and to assume that light travels in straight lines, both steps valid because we are studying the field called **geometric optics.** The field of **wave optics** is discussed in Chapter 27.

26.1 · IMAGES FORMED BY FLAT MIRRORS

The main objective of this chapter is to discuss the manner in which optical elements such as lenses and mirrors form images. We begin our investigation by considering the simplest possible mirror, the flat mirror.

Consider a point source of light placed at O in Figure 26.1, a distance p in front of a flat mirror. The distance p is called the **object distance.** Light rays leave the source and are reflected from the mirror. After reflection, the rays diverge (spread apart), but they appear to the viewer to come from a point I located behind the mirror. Point I is called the **image** of the object at O. Regardless of the system under study, images are always formed in the same way. **Images are formed either at the point where rays of light intersect or at the point at which they appear to originate.** Because the rays in Figure 26.1 appear to originate at I, which is a distance of q behind the mirror, this is the location of the image. The distance q is called the **image distance.**



Some common navigational instruments. *(left)* A magnifying glass is used to view fine print and small objects. *(middle)* A compass is used to determine directions. *(right)* Binoculars are used to view distant objects. *(Murray Alcosser/The Image Bank)*

Images are classified as real or virtual. **A real image is one in which light actually intersects, or passes through, the image point; a virtual image is one in which the light does not really pass through the image point but appears to diverge from that point.** The image formed by the mirror in Figure 26.1 is a virtual image. The images seen in flat mirrors are *always virtual.* Real images can usually be displayed on a screen (as at a movie), but virtual images cannot be displayed on a screen.

We examine the properties of the images formed by flat mirrors by using the simple geometric techniques shown in Figure 26.2. To find out where an image is formed, it is necessary to follow at least two rays of light as they reflect from the mirror. One of those rays starts at P, follows the horizontal path PQ to the mirror, and reflects back on itself. The second ray follows the oblique path PR and reflects as shown. An observer to the left of the mirror would trace the two reflected rays back to the point at which they appear to have originated, point P'. A continuation of this process for points other than P on the object would result in a virtual image (drawn as a yellow arrow) to the right of the mirror. Because triangles PQR and $P'QR$ are identical, $PQ = P'Q$. Hence, we conclude that **the image formed by an object placed in front of a flat mirror is as far behind the mirror as the object is in front of the mirror.**

Geometry also shows that the object height, h, equals the image height, h'. Let us define **magnification, *M*,** as follows:

$$M \equiv \frac{\text{image height}}{\text{object height}} = \frac{h'}{h} \qquad [26.1]$$

This is a general definition of the magnification of any type of mirror. $M = 1$ for a flat mirror because $h' = h$ in this case.

Thus, we conclude that the image formed by a flat mirror has the following properties:

- The image is as far behind the mirror as the object is in front.
- The image is unmagnified, virtual, and upright. (By *upright* we mean that if the object arrow points upward, as in Figure 26.2, so does the image arrow. Later we shall discuss situations in which inverted images are formed.)

Finally, it is interesting to note that a flat mirror produces an image having an apparent right–left reversal. The apparent right–left reversal is actually a front–back reversal. This reversal can be seen by standing in front of a mirror and raising your right hand. The image you see raises its left hand. Likewise, your hair appears to be parted on the opposite side and a mole on your right cheek appears to be on your left cheek.

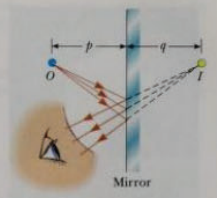


Figure 26.1 An image formed by reflection from a flat mirror. The image point, I, is located behind the mirror at a distance q, which is equal in magnitude to the object distance, p.

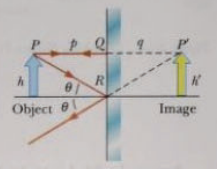


Figure 26.2 Geometric construction used to locate the image of an object placed in front of a flat mirror. Because the triangles PQR and $P'QR$ are congruent, $p = |q|$ and $h = h'$.

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Figure 26.3 (Thinking Physics 1) *(Photograph courtesy of Henry Leap and Jim Lehman)*

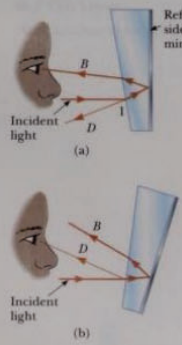


Figure 26.4 (Thinking Physics 2)

Thinking Physics 1

The professor in the box shown in Figure 26.3 appears to be balancing himself on a few fingers with both of his feet elevated from the floor. The professor can maintain this position for a long time, and he appears to defy gravity. How do you suppose this illusion was created?

Reasoning This is one example of an optical illusion, used by magicians, that makes use of a mirror. The box that the professor is standing in is a cubical frame that contains a flat vertical mirror through a diagonal plane. The professor straddles the mirror so that the foot that you see is in front of the mirror, and the other foot is behind the mirror where you cannot see it. When he raises the foot that you see in front of the mirror, the reflection of this foot also rises, so he appears to float in air.

Thinking Physics 2

Most rearview mirrors on cars have a day setting and a night setting. The night setting greatly diminishes the intensity of the image so that lights from trailing vehicles do not blind the driver. How does such a mirror work?

Reasoning Consider Figure 26.4, which represents a cross-sectional view of the mirror for the two settings. The mirror is a wedge of glass with a reflecting mirror on the back side. When the mirror is in the day setting, as in Figure 26.4a, the light from an object behind the car strikes the mirror at point 1. Most of the light enters the wedge, is refracted, and reflects from the back of the mirror to return to the front surface, where it is refracted again as it reenters the air as ray *B* (for *bright*). In addition, a small portion of the light is reflected at the front surface, as indicated by ray *D* (for *dim*). This dim reflected light is responsible for the image observed when the mirror is in the night setting, as in Figure 26.4b. In this case, the wedge is rotated so that the path followed by the bright light (ray *B*) does not lead to the eye. Instead, the dim light reflected from the front surface travels to the eye, and the brightness of trailing headlights does not become a hazard.

CONCEPTUAL PROBLEM 1

Tape a picture of yourself on a bathroom mirror. Stand several feet away from the mirror. Can you focus your eyes on *both* the picture taped to the mirror *and* your image in the mirror *at the same time*?

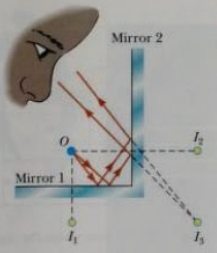
CONCEPTUAL PROBLEM 2

In the movies, you sometimes see an actor looking in a mirror. You are able to see the face of the actor. During the filming of this scene, what does the actor see in the mirror?

Example 26.1 **Multiple Images Formed by Two Mirrors**

Two flat mirrors are at right angles to each other, as in Figure 26.5, and an object is placed at point *O*. In this situation, multiple images are formed. Locate the positions of these images.

Reasoning The image of the object is at I_1 in mirror 1 and at I_2 in mirror 2. In addition, a third image is formed at I_3, which is the image of I_1 in mirror 2 or, equivalently, the image of I_2 in mirror 1. That is, the image at I_1 (or I_2) serves as the object for I_3. Note that in order to form this image at I_3, the rays reflect twice after leaving the object at *O*.



EXERCISE 1 Sketch the rays corresponding to viewing the images at I_1 and I_2 and show that the light is reflected only once in these cases.

Figure 26.5 (Example 26.1) When an object is placed in front of two mutually perpendicular mirrors as shown, three images are formed.

26.2 · IMAGES FORMED BY SPHERICAL MIRRORS

Concave Mirrors

A **spherical mirror,** as its name implies, has the shape of a segment of a sphere. Figure 26.6 shows the cross-section of a spherical mirror with light reflecting from its surface, represented by the solid curved line. Such a mirror, in which light is reflected from the inner, concave surface, is called a **concave mirror.** The mirror's radius of curvature is *R*, and its center of curvature is at point *C*. Point *V* is the center of the spherical segment, and a line drawn from *C* to *V* is called the **principal axis** of the mirror.

Now consider a point source of light placed at point *O* in Figure 26.6, on the principal axis and outside point *C*. Several diverging rays that originate at *O* are shown. After reflecting from the mirror, these rays converge and meet at *I*, called the **image point.** They then continue to diverge from *I* as if there were an object there. As a result, a real image is formed. **Whenever reflected light actually passes through a point, a real image is formed there.**

In what follows, we shall assume that all rays that diverge from an object make a small angle with the principal axis. Such rays, called **paraxial rays,** always reflect through the image point, as in Figure 26.6. Rays that are far from the principal axis, as in Figure 26.7, converge at other points on the principal axis, producing a blurred image. This effect, called **spherical aberration,** is produced to some extent by any spherical mirror and will be discussed in Section 26.5.

We can use the geometry shown in Figure 26.8 to calculate the image distance, *q*, from the object distance, *p*, and radius of curvature, *R*. By convention, these distances are measured from point *V*. Figure 26.8 shows two rays of light leaving the tip of the object. One passes through the center of curvature, *C*, of the mirror, hitting the mirror (perpendicular to the mirror surface), and reflecting back on itself. The second ray strikes the mirror at the center, point *V*, and reflects as shown, obeying the law of reflection. The image of the tip of the arrow is at the point at which these two rays intersect. From the largest right triangle in Figure 26.8 (colored gold), we see that $\tan\theta = h/p$, whereas the light blue triangle gives $\tan\theta = -h'/q$. The negative sign signifies that the image is inverted, so h' is negative. Thus,

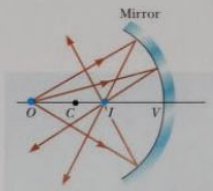


Figure 26.6 A point object at *O*, outside the center of curvature of a concave spherical mirror, forms a real image at *I*. If the rays diverge from *O* at small angles, they all reflect through the same image point.

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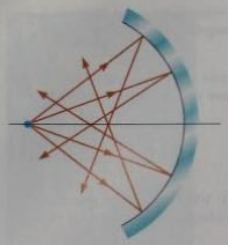


Figure **26.7** Rays at large angles from the principal axis reflect from a spherical concave mirror to intersect the principal axis at different points, resulting in a blurred image. This is called *spherical aberration.*



Red, blue, and green light rays are reflected by a parabolic mirror. Note that the focal point at which the three colors meet is white light. *(Ken Kay/Fundamental Photographs)*

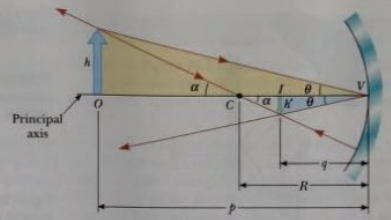


Figure 26.8 The image formed by a spherical concave mirror where the object *O* lies outside the center of curvature *C*.

from Equation 26.1 and these results, we find that the magnification of the mirror is

$$M = \frac{h'}{h} = -\frac{q}{p} \qquad [26.2]$$

We also note, from two other right triangles in the figure, that

$$\tan\alpha = \frac{h}{p - R} \quad \text{and} \quad \tan\alpha = -\frac{h'}{R - q}$$

from which we find that

$$\frac{h'}{h} = -\frac{R - q}{p - R} \qquad [26.3]$$

If we compare Equations 26.2 and 26.3, we see that

$$\frac{R - q}{p - R} = \frac{q}{p}$$

Simple algebra reduces this to

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} \qquad [26.4]$$

This expression is called the **mirror equation.** It is applicable only to paraxial rays. If the object is very far from the mirror—that is, if the object distance, p, is great enough compared with R that p can be said to approach infinity, then $1/p \approx 0$, and we see from Equation 26.4 that $q \approx R/2$. In other words, **when the object is very far from the mirror, the image point is halfway between the center of curvature and the center of the mirror**, as in Figure 26.9a. The rays are essentially parallel in this figure because their source, the object, is assumed to be very far from the mirror. In this special case, the rays focus at a point called the **focal point** of the mirror, which is a distance f from the mirror, which we call the **focal length. The image forms at the focal point only if the incoming rays**

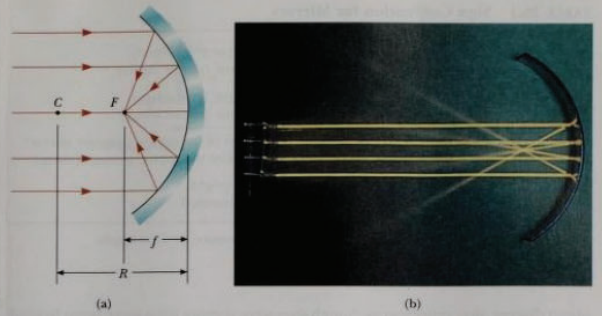


Figure 26.9 (a) Light rays from a distant object ($p \approx \infty$) reflect from a concave mirror through the focal point, *F*. In this case, the image distance $q = R/2 = f$, where f is the focal length of the mirror. (b) Photograph of the reflection of parallel rays from a concave mirror. *(Henry Leap and Jim Lehman)*

are parallel. The focal length is a parameter associated with the mirror and is given by

$$f = \frac{R}{2} \qquad [26.5]$$

• *Focal length*

The mirror equation can therefore be expressed in terms of the focal length:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad [26.6]$$

• *Mirror equation*

Convex Mirrors

Figure 26.10 shows the formation of an image by a **convex mirror**—a mirror that is silvered so that light is reflected from the outer, convex surface. Convex mirrors are sometimes called **diverging mirrors** because the rays from any point on a real

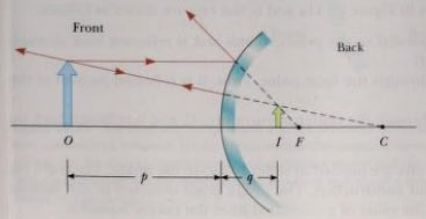


Figure 26.10 Formation of an image by a spherical convex mirror. The image formed by the real object is virtual and upright.

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TABLE 26.1 Sign Convention for Mirrors

p is + if the object is in front of the mirror (real object). p is − if the object is in back of the mirror (virtual object).
q is + if the image is in front of the mirror (real image). q is − if the image is in back of the mirror (virtual image).
Both f and R are + if the center of curvature is in front of the mirror (concave mirror). Both f and R are − if the center of curvature is in back of the mirror (convex mirror).
If M is positive, the image is upright. If M is negative, the image is inverted.

Note: p = object distance; q = image distance; R = radius of curvature, f = focal length; M = magnification.



Convex cylindrical mirror: reflection of parallel lines. The image of any object in front of the mirror is virtual, upright, and diminished in size. *(© Richard Megna 1990, Fundamental Photographs)*

object diverge after reflection as though they were coming from some point behind the mirror. The image in Figure 26.10 is virtual rather than real because it lies behind the mirror at the point at which the reflected rays appear to originate. In general, as shown in the figure, the image formed by a convex mirror is always upright, virtual, and smaller than the object.

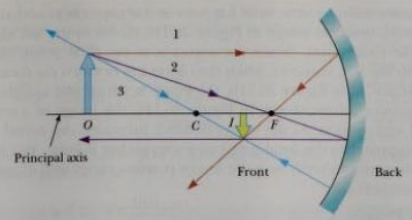
We shall not derive any equations for convex spherical mirrors. If we did, we would find that the equations developed for concave mirrors can be used with convex mirrors if we adhere to a particular sign convention. Let us refer to the region in which light rays move as the *front side* of the mirror, and the other side, where virtual images are formed, as the *back side*. For example, in Figures 26.8 and 26.10, the side to the left of the mirror is the front side, and that to the right of the mirror is the back side. Table 26.1 summarizes the sign conventions for all the necessary quantities.

Ray Diagrams for Mirrors

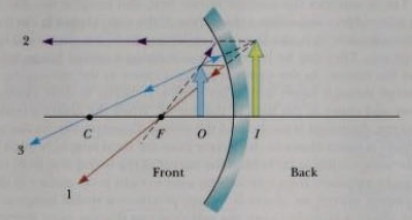
The positions and sizes of images formed by mirrors can be conveniently determined by constructing **ray diagrams.** These graphical constructions tell us the overall nature of the image and can be used to check parameters calculated from the mirror and magnification equations. To construct a ray diagram, we need to know the position of the object and the location of the center of curvature. Three rays are constructed as shown by the examples in Figure 26.11. All three rays start from the same object point; in these examples, the top of the arrow was chosen. For the concave mirrors in Figure 26.11a and b, the rays are drawn as follows:

- Ray 1 is drawn parallel to the principal axis and is reflected back through the focal point, F.
- Ray 2 is drawn through the focal point. Thus, it is reflected parallel to the principal axis.
- Ray 3 is drawn through the center of curvature, C, and is reflected back on itself.

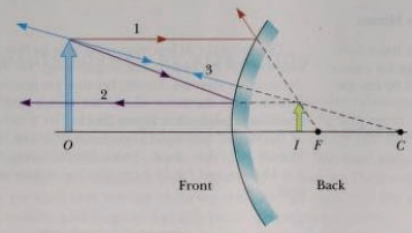
Note that only two rays are needed in order to locate the image. The third ray serves as a check on your construction. The image point obtained in this fashion must always agree with the value of q calculated from the mirror equation.



(a)



(b)



(c)

Figure 26.11 Ray diagrams for spherical mirrors. Different colors are used for the three rays so they can easily be followed. (a) An object outside the center of curvature of a spherical concave mirror. (b) An object between the spherical concave mirror and the focal point, F, gives a virtual, upright image. (c) An object located anywhere in front of a spherical convex mirror gives a virtual, upright image.

INTEL-1013
U.S. Patent 7,151,430