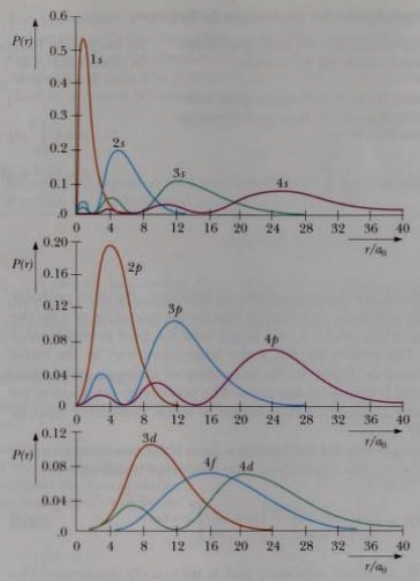


Figure 29.8 The radial probability density function versus r/a_0 for several states of the hydrogen atom. *(From E. U. Condon and G. H. Shortley, The Theory of Atomic Spectra, Cambridge, Cambridge University Press, 1953; used with permission)*

Wave functions for the 2p state •

$$\psi_{2p_x} = xF(r)$$
$$\psi_{2p_y} = yF(r)$$
$$\psi_{2p_z} = zF(r) \qquad [29.9]$$

where $F(r)$ is some exponential function of r. Wave functions with a highly directional character, such as these, are convenient for bookkeeping descriptions of chemical bonding, the formation of molecules, and chemical properties.

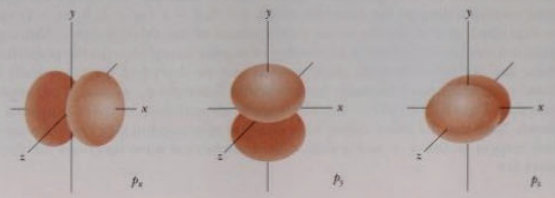


Figure 29.9 Angular dependence of the electron charge distribution for an electron in a p state. The three charge distributions p_x, p_y, and p_z have the same structure and differ only in their orientation in space.

29.5 • THE "OTHER" QUANTUM NUMBERS

The energy of a particular state depends primarily on the principal quantum number. Now let us see what the other three quantum numbers contribute to our atomic model.

The Orbital Quantum Number

If a particle moves in a circle of radius r, the magnitude of its angular momentum relative to the center of the circle is $L = mvr$. The direction of $\mathbf{L}$ is perpendicular to the plane of the circle, and the sense of $\mathbf{L}$ is given by a right-hand rule.[1] According to classical physics, L can have any value. However, the Bohr model of hydrogen postulates that the angular momentum is restricted to multiples of $\hbar$; that is, $mvr = n\hbar$. This model must be modified because it predicts (incorrectly) that the ground state of hydrogen ($n = 1$) has one unit of angular momentum. Furthermore, if L is taken to be zero in the Bohr model, one is forced to accept the electron as a particle oscillating along a straight line through the nucleus. This is a physically unacceptable situation.

These difficulties are resolved with the quantum mechanical model of the atom. According to quantum mechanics, an atom in a state the principal quantum number of which is n can take on the following *discrete* values of orbital angular momentum:

$$L = \sqrt{\ell(\ell + 1)}\,\hbar \qquad \ell = 0, 1, 2, \ldots, n - 1 \qquad [29.10]$$

• ***Allowed values of L***

Because ℓ is restricted to these values, $L = 0$ (corresponding to $\ell = 0$) is an acceptable value of the angular momentum. The fact that L can be zero in this model points out the difficulties inherent in any attempt to describe results based on quantum mechanics in terms of a purely particle-like model. In the quantum mechanical interpretation, the electron cloud for the $L = 0$ state is spherically symmetric and has no fundamental axis of revolution.

Example 29.5 Calculating L for a p State

Calculate the orbital angular momentum of an electron in a p state of hydrogen.

Solution Because we know that $\hbar = 1.054 \times 10^{-34}$ J·s, we can use Equation 29.10 to calculate L. With $\ell = 1$ for a p state, we have

$$L = \sqrt{1(1 + 1)}\,\hbar = \sqrt{2}\hbar = 1.49 \times 10^{-34}\ \text{J·s}$$

This number is extremely small relative to the orbital angular momentum of the Earth orbiting the Sun, which is about 2.7×10^{40} J·s. The quantum number that describes L for macroscopic objects, such as the Earth, is so large that the separation between adjacent states cannot be measured. Once again, the correspondence principle is upheld.

The Magnetic Orbital Quantum Number

Because angular momentum is a vector, its direction must also be specified. Recall from Chapter 22 that an orbiting electron can be considered an effective current loop with a corresponding magnetic moment. Such a moment placed in a magnetic field, $\mathbf{B}$, will interact with the field. Suppose a weak magnetic field is applied along

[1] See Sections 10.8 and 10.9 to review details on angular momentum.

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the z axis so that it defines a direction in space. According to quantum mechanics, L^2 and L_z, the projection of $\mathbf{L}$ along the z axis, can have discrete values. The magnetic orbital quantum number m_ℓ specifies the allowed values of L_z according to the expression

Allowed values of L_z

$$L_z = m_\ell \hbar \qquad [29.11]$$

Space quantization

The fact that the direction of $\mathbf{L}$ is quantized with respect to an external magnetic field is often referred to as **space quantization.**

Let us look at the possible orientations of $\mathbf{L}$ for a given value of ℓ. Recall that m_ℓ can have values ranging from $-\ell$ to ℓ. If $\ell = 0$, then $m_\ell = 0$ and $L_z = 0$. In order for L_z to be zero, $\mathbf{L}$ must be perpendicular to $\mathbf{B}$. If $\ell = 1$, then the possible values of m_ℓ are -1, 0, and 1, so that L_z may be $-\hbar$, 0, or $\hbar$. If $\ell = 2$, m_ℓ can be -2, -1, 0, 1, or 2, corresponding to L_z values of $-2\hbar$, $-\hbar$, 0, $\hbar$, or $2\hbar$, and so on.

A vector model describing space quantization for $\ell = 2$ is shown in Figure 29.10a. Note that $\mathbf{L}$ can never be aligned parallel or antiparallel to $\mathbf{B}$ because L_z must be smaller than the total angular momentum, L. From a three-dimensional viewpoint, $\mathbf{L}$ must lie on the surface of a cone that makes an angle of θ with the z axis, as shown in Figure 29.10b. From the figure, we see that θ is also quantized and that its values are specified through the relation

θ is quantized

$$\cos\theta = \frac{L_z}{|\mathbf{L}|} = \frac{m_\ell}{\sqrt{\ell(\ell+1)}} \qquad [29.12]$$

Note that m_ℓ is never greater than ℓ, and therefore θ can never be zero. (Classically, θ can have any value.)

Because of the uncertainty principle, $\mathbf{L}$ does not point in a specific direction but rather traces out a cone in space. If $\mathbf{L}$ had a definite value, then all three components L_x, L_y, and L_z would be exactly specified. For the moment, let us assume that this is the case, and let us suppose that the electron moves in the xy plane, so that $\mathbf{L}$ is in the z direction and $p_z = 0$. This means that p_z is precisely known, which is

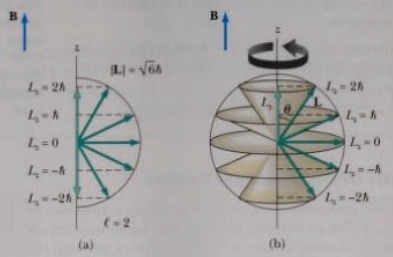


Figure 29.10 (a) The allowed projections of the orbital angular momentum for the case $\ell = 2$. (b) The orbital angular momentum vector lies on the surface of a cone and precesses about the z axis when a magnetic field $\mathbf{B}$ is applied in this direction.

in violation of the uncertainty principle, $\Delta p_z \, \Delta z \geq \hbar/2$. In reality, only the magnitude of $\mathbf{L}$ and one component (say, L_z) can have definite values. In other words, quantum mechanics allows us to specify L and L_z but not L_x and L_y. Because the direction of $\mathbf{L}$ is constantly changing as it precesses about the z axis, the average values of L_x and L_y are zero and L_z maintains a fixed value of $m_\ell \hbar$.

Example 29.6 Space Quantization for Hydrogen

For the hydrogen atom in the $\ell = 3$ state, calculate the magnitude of $\mathbf{L}$ and the allowed values of L_z and θ.

Solution We use Equation 29.10 with $\ell = 3$:

$$L = \sqrt{\ell(\ell+1)}\,\hbar = \sqrt{3(3+1)}\,\hbar = 2\sqrt{3}\hbar$$

The allowed values of L_z are $L_z = m_\ell \hbar$ with $m_\ell = -3$, -2, -1, 0, 1, 2, and 3:

$$L_z = -3\hbar,\ -2\hbar,\ -\hbar,\ 0,\ \hbar,\ 2\hbar,\ 3\hbar$$

Finally, we use Equation 29.12 to calculate the allowed values of θ. Because $\ell = 3$, $\sqrt{\ell(\ell+1)} = 2\sqrt{3}$, and we have

$$\cos\theta = \frac{m_\ell}{2\sqrt{3}}$$

Substitution of the allowed values of m_ℓ gives

$$\cos\theta = \pm 0.866,\ \pm 0.577,\ \pm 0.289,\ 0$$

$$\theta = 30.0^\circ,\ 54.8^\circ,\ 73.2^\circ,\ 90.0^\circ,\ 107^\circ,\ 125^\circ,\ 150^\circ$$

Electron Spin

In 1921 Stern and Gerlach performed an experiment that demonstrated space quantization. However, their results were not in quantitative agreement with the theory that existed at that time. In their experiment, a beam of neutral silver atoms sent through a nonuniform magnetic field was split into two components (Fig. 29.11). The experiment was repeated using other atoms, and in each case the beam split into two or more components. The classical argument is as follows: If the z direction is chosen to be the direction of the maximum inhomogeneity of $\mathbf{B}$, the net magnetic force on the atom is along the z axis and is proportional to the mag-

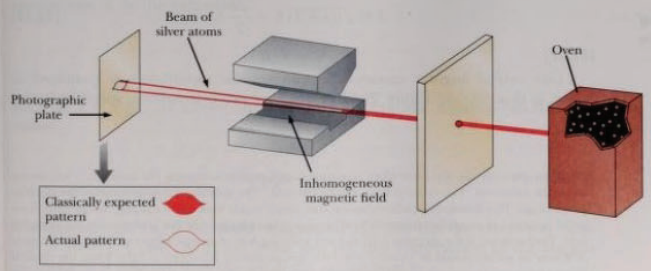


Figure 29.11 The apparatus used by Stern and Gerlach to verify space quantization. A beam of neutral silver atoms is split into two components by a nonuniform magnetic field, as shown by the actual pattern in the box.

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netic moment μ_z in the z direction. Classically, $\boldsymbol{\mu}$ can have any orientation, and so the deflected beam should be spread out continuously. According to quantum mechanics, however, the deflected beam has several components, and the number of components determines the possible values of μ_z. Hence, because the Stern–Gerlach experiment showed split beams, space quantization was at least qualitatively verified.

For the moment, let us assume that μ_z is due to the orbital angular momentum. Because μ_z is proportional to m_ℓ, the number of possible values of μ_z is $2\ell + 1$. Furthermore, because ℓ is an integer, the number of values of μ_z is always odd. This prediction is clearly not consistent with the observations of Stern and Gerlach, who observed only two components in the deflected beam of silver atoms. Hence, one is forced to conclude that either quantum mechanics is incorrect or the model is in need of refinement.

In 1927 Phipps and Taylor repeated the Stern–Gerlach experiment using a beam of hydrogen atoms. This experiment is important because it deals with an atom with a single electron in its ground state, for which the theory makes reliable predictions. Recall that $\ell = 0$ for hydrogen in its ground state, and so $m_\ell = 0$. Hence, one would not expect the beam to be deflected by the field, because μ_z would be zero. However, the beam in the Phipps–Taylor experiment is again split into two components. On the basis of this result, one can conclude only one thing: There is some contribution to the magnetic moment other than the orbital motion.

In 1925 Goudsmit and Uhlenbeck proposed that the electron has an intrinsic angular momentum apart from its orbital angular momentum. From a classical viewpoint, this intrinsic angular momentum is attributed to the charged electron spinning about its own axis, and hence is called **electron spin.**[2] In other words, the total angular momentum of the electron in a particular electronic state contains both an orbital contribution, **L**, and a spin contribution, **S**. There is a quantum number s for spin, which is analogous to ℓ for orbital angular momentum. The value for s, however, is always $s = \frac{1}{2}$.

The magnitude of the **spin angular momentum, S,** for the electron is

Spin angular momentum of an electron

$$S = \sqrt{s(s+1)}\,\hbar = \frac{\sqrt{3}}{2}\hbar \qquad [29.13]$$

Like orbital angular momentum, spin angular momentum is quantized in space, as described in Figure 29.12. It can have two orientations, specified by the spin magnetic quantum number m_s, where m_s has two possible values, $\pm\frac{1}{2}$. The z component of spin angular momentum is

$$S_z = m_s\hbar = \pm\tfrac{1}{2}\hbar \qquad [29.14]$$

The two values $\pm\hbar/2$ for S_z correspond to the two possible orientations for **S** shown in Figure 29.12.

The spin magnetic moment of the electron, $\boldsymbol{\mu}_s$, is related to its spin angular momentum, **S**, by the expression

$$\boldsymbol{\mu}_s = -\frac{e}{m_e}\mathbf{S} \qquad [29.15]$$

Because $S_z = \pm\frac{1}{2}\hbar$, the z component of the spin magnetic moment can have the values

$$\mu_{sz} = \pm\frac{e\hbar}{2m_e} \qquad [29.16]$$

The quantity $e\hbar/2m_e$ is the **Bohr magneton,** μ_B, and has the numerical value 9.27×10^{-24} J/T (see Eq. 22.28). Note that the spin contribution to the angular momentum is *twice* the contribution of the orbital motion.

Today physicists explain the outcome of the Stern–Gerlach experiment as follows. The observed moments for both silver and hydrogen are due to spin angular

[2]Physicists often use the word *spin* when referring to *spin angular momentum*. For example, it is common to use the statement "The electron has a spin of one half." The spin angular momentum of the electron *never changes*. This notion contradicts classical laws, which would hold that a rotating charge slows down in the presence of an applied magnetic field because of the Faraday emf that accompanies the changing field. Furthermore, if the electron is viewed as a spinning ball of charge subject to classical laws, parts of it near its surface would be rotating with velocities exceeding the speed of light. Thus, the classical picture must not be pressed too far; ultimately, the spinning electron is a quantum entity defying any simple classical description. It is more accurate to describe spin as a relativistic effect (which was treated classically by Sommerfeld and quantum mechanically by Dirac).

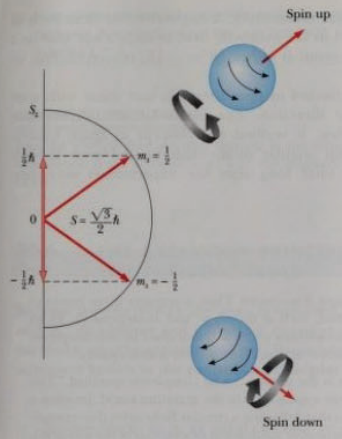


Figure 29.12 The spin angular momentum also exhibits space quantization. This figure shows the two allowed orientations of the spin vector S for a spin $\frac{1}{2}$ particle, such as the electron.

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momentum and not to orbital angular momentum. A single-electron atom such as hydrogen has its electron quantized in the magnetic field in such a way that its z component of spin angular momentum is either $\frac{1}{2}\hbar$ or $-\frac{1}{2}\hbar$, corresponding to $m_s = \pm\frac{1}{2}$.

Electrons with spin $+\frac{1}{2}$ are deflected in one direction, and those with spin $-\frac{1}{2}$ are deflected in the opposite direction. The Stern–Gerlach experiment provided two important results. First, it verified the concept of space quantization. Second, it showed that spin angular momentum existed even though this property was not recognized until long after the experiments were performed.

Thinking Physics 1

Angular momentum is a vector in three dimensions. Thus, it requires three pieces of information to be completely specified, such as the x-, y-, and z-components. In an atom, however, the orbital angular momentum vector $\mathbf{L}$ is described by only *two* pieces of information—the quantum numbers ℓ and m_ℓ. Why is there a discrepancy?

Reasoning The clue in the question is the phrase, "to be completely specified." The notion of complete specification is not appropriate in the quantum world. Imagine a completely specified vector for an electron following a circular Bohr orbit (but remember that this is inconsistent with the quantum mechanical description of the electron). The plane of the orbit would be fixed in space. Thus, in the z-direction *perpendicular* to the plane of the orbit (and parallel to $\mathbf{L}$), the position of the electron is completely specified—it is in the plane, so there is no uncertainty in position and $\Delta z = 0$. The velocity of the electron perpendicular to the plane is also completely specified—it is zero, again with no uncertainty. Thus, the uncertainty principle is violated. The uncertainty principle is satisfied by stating only two pieces of information. The length of the angular momentum vector and its z-component are known, but there is no information about the x- or y-components.

Thinking Physics 2

Does the Stern–Gerlach experiment differentiate between orbital angular momentum and spin angular momentum?

Reasoning There is a magnetic force on the magnetic moment arising from both orbital angular momentum and spin angular momentum. In this sense, the experiment does not differentiate between the two. The number of lines on the screen will tell us something, however, due to the fact that orbital angular momenta are described by an integral quantum number, whereas spin angular momentum depends on a half-integral quantum number. If there are an odd number of lines on the screen, then there are three possibilities: the atom has orbital angular momentum only, an even number of electrons with spin angular momentum, or a combination of orbital angular momentum and an even number of electrons with spin angular momentum. If there is an even number of lines on the screen, then there is at least one unpaired spin angular momentum, possibly in combination with orbital angular momentum. The only numbers of lines for which we can specify the type of angular momentum is one line (no orbital, no spin) and two lines (spin one half). Once we see more than two lines, there are multiple possibilities. For example, three lines could result from two spin one half electrons or one electron with orbital angular momentum quantum number $\ell = 1$.

CONCEPTUAL PROBLEM 2

Imagine a single free electron floating in the atmosphere. Does the spin angular momentum vector of the electron align with the magnetic field lines due to the Earth's magnetic field?

29.6 • THE EXCLUSION PRINCIPLE AND THE PERIODIC TABLE

Because any electron in any atom is specified by four quantum numbers, n, ℓ, m_ℓ, and m_s, an obvious and important question is "How many electrons can have a particular set of quantum numbers?" Pauli provided an answer in 1925 in a powerful statement known as the **exclusion principle:**

> No two electrons in an atom can ever be in the same quantum state; that is, no two electrons in the same atom can have the same set of quantum numbers.



Wolfgang Pauli (1900–1958)
An extremely talented Austrian theoretical physicist who made important contributions in many areas of modern physics, Pauli gained public recognition at the age of 21 with a masterful review article on relativity, which is still considered one of the finest and most comprehensive introductions to the subject. Other major contributions were the discovery of the exclusion principle, the explanation of the connection between particle spin and statistics, and theories of relativistic quantum electrodynamics, the neutrino hypothesis, and the hypothesis of nuclear spin. *(Photo taken by S. A. Goudsmit, AIP Niels Bohr Library)*

It is interesting to note that if this principle were not valid, every electron would end up in the lowest energy state of the atom and the chemical behavior of the elements would be grossly modified. Nature as we know it would not exist! In reality, we can view the electronic structure of complex atoms as a succession of filled levels increasing in energy, where the outermost electrons are primarily responsible for the chemical properties of the element.

As a general rule, the order of filling of an atom's subshells with electrons is as follows. Once one subshell is filled, the next electron goes into the vacant subshell that is lowest in energy. One can understand this principle by recognizing that if the atom were not in the lowest energy state available to it, it would radiate energy until it reached this state.

Before we discuss the electronic configurations of some elements, it is convenient to define an **orbital** as the state of an electron characterized by the quantum numbers n, ℓ, and m_ℓ. From the exclusion principle, we see that **there can be only two electrons in any orbital.** One of these electrons has $m_s = +\frac{1}{2}$, and the other has $m_s = -\frac{1}{2}$. Because each orbital is limited to two electrons, the numbers of electrons that can occupy the levels are also limited.

Table 29.3 shows the numbers of allowed quantum states for an atom up to $n = 3$. Each square in the bottom row of the table represents one orbital, with the $\uparrow$ arrows representing $m_s = \frac{1}{2}$ and the $\downarrow$ arrows representing $m_s = -\frac{1}{2}$. The $n = 1$ shell can accommodate only two electrons, because only one orbital is allowed with $m_\ell = 0$. The $n = 2$ shell has two subshells, with $\ell = 0$ and $\ell = 1$. The $\ell = 0$ subshell is limited to only two electrons, because $m_\ell = 0$. The $\ell = 1$ subshell

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TABLE 29.3 Allowed Quantum Numbers for an Atom up to $n = 3$

n	1	2				3								
ℓ	0	0	1			0	1			2				
m_ℓ	0	0	1	0	−1	0	1	0	−1	2	1	0	−1	−2
m_s	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓

has three allowed orbitals, corresponding to $m_\ell = 1, 0$, and -1. Because each orbital can accommodate two electrons, the $\ell = 1$ subshell can hold six electrons (and the $n = 2$ shell can hold eight). The $n = 3$ shell has three subshells and nine orbitals and can accommodate up to 18 electrons. Each shell can accommodate up to $2n^2$ electrons.

The exclusion principle can be illustrated by an examination of the electronic arrangement in a few of the lighter atoms. **Hydrogen** has only one electron, which, in its ground state, can be described by either of two sets of quantum numbers: $1, 0, 0, \frac{1}{2}$, or $1, 0, 0, -\frac{1}{2}$. The electronic configuration of this atom is often designated as $1s^1$. The notation $1s$ refers to a state for which $n = 1$ and $\ell = 0$, and the superscript indicates that one electron is present in the s subshell.

Neutral **helium** has two electrons. In the ground state, the quantum numbers of these two electrons are $1, 0, 0, \frac{1}{2}$, and $1, 0, 0, -\frac{1}{2}$. There are no other possible combinations of quantum numbers for this level, and we say that the K shell is filled. Helium is designated as $1s^2$.

The electronic configurations of some successive elements are given in Figure 29.13. Neutral **lithium** has three electrons. In the ground state, two of these are in the $1s$ subshell and the third is in the $2s$ subshell, because this subshell is lower in energy than the $2p$ subshell. Hence, the electronic configuration for lithium is $1s^2 2s^1$.

Note that the electronic configuration of **beryllium,** with its four electrons, is $1s^2 2s^2$, and **boron** has a configuration of $1s^2 2s^2 2p^1$. The $2p$ electron in boron may be described by one of six sets of quantum numbers, corresponding to six states of equal energy.

Carbon has six electrons, and a question arises concerning how to assign the two $2p$ electrons. Do they go into the same orbital with paired spins (↑ ↓), or do they occupy different orbitals with unpaired spins (↑ ↑)? Experimental data show that the most stable configuration (that is, the one that is energetically preferred) is the latter, in which the spins are unpaired. Hence, the two $2p$ electrons in carbon and the three $2p$ electrons in nitrogen have unpaired spins (Fig. 29.13). The general rule that governs such situations, called **Hund's rule,** states that **when an atom has orbitals of equal energy, the order in which they are filled by electrons is such that a maximum number of electrons will have unpaired spins.** Some exceptions to this rule occur in elements having sublevels close to being filled or half filled.

Hund's rule

A complete list of electronic configurations is provided in Table 29.4. An early attempt to find some order among the elements was made by a Russian chemist, Dmitri Mendeleev, in 1871. He arranged the atoms in a table (similar to that in Appendix C) according to their atomic masses and chemical similarities. The first table Mendeleev proposed contained many blank spaces, and he boldly stated that the gaps were there only because the elements had not yet been discovered. By noting the columns in which these missing elements should be located, he was able to make rough predictions about their chemical properties. Within 20 years of Mendeleev's announcement, the missing elements were indeed discovered.

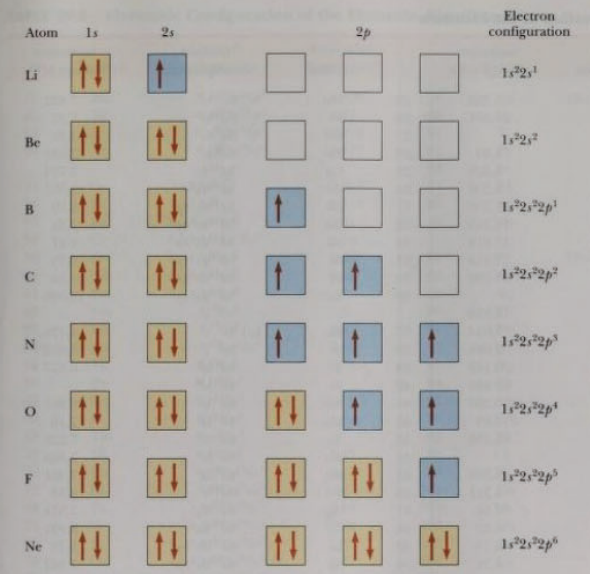


Figure 29.13 The filling of electronic states must obey the Pauli exclusion principle and Hund's rule.

The elements in the periodic table are arranged so that all those in a vertical column have similar chemical properties. For example, consider the elements in the last column: He (helium), Ne (neon), Ar (argon), Kr (krypton), Xe (xenon), and Rn (radon). The outstanding characteristic of these elements is that they do not normally take part in chemical reactions—that is, they do not join with other atoms to form molecules—and they are therefore classified as being *inert.* Because of this aloofness, they are referred to as the **noble gases.**

We can partially understand this behavior by looking at the electronic configurations in Table 29.4. This table also lists the ionization energies for certain ele-

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TABLE 29.4 Electronic Configuration of the Elements

Z	Symbol	Ground Configuration	Ionization Energy (eV)	Z	Symbol	Ground Configuration	Ionization Energy (eV)
1	H	$1s^1$	13.595	25	Mn	$3d^5 4s^2$	7.432
2	He	$1s^2$	24.581	26	Fe	$3d^6 4s^2$	7.87
				27	Co	$3d^7 4s^2$	7.86
3	Li	[He] $2s^1$	5.39	28	Ni	$3d^8 4s^2$	7.633
4	Be	$2s^2$	9.320	29	Cu	$3d^{10} 4s^1$	7.724
5	B	$2s^2 2p^1$	8.296	30	Zn	$3d^{10} 4s^2$	9.391
6	C	$2s^2 2p^2$	11.256	31	Ga	$3d^{10} 4s^2 4p^1$	6.00
7	N	$2s^2 2p^3$	14.545	32	Ge	$3d^{10} 4s^2 4p^2$	7.88
8	O	$2s^2 2p^4$	13.614	33	As	$3d^{10} 4s^2 4p^3$	9.81
9	F	$2s^2 2p^5$	17.418	34	Se	$3d^{10} 4s^2 4p^4$	9.75
10	Ne	$2s^2 2p^6$	21.559	35	Br	$3d^{10} 4s^2 4p^5$	11.84
				36	Kr	$3d^{10} 4s^2 4p^6$	13.996
11	Na	[Ne] $3s^1$	5.138				
12	Mg	$3s^2$	7.644	37	Rb	[Kr] $5s^1$	4.176
13	Al	$3s^2 3p^1$	5.984	38	Sr	$5s^2$	5.692
14	Si	$3s^2 3p^2$	8.149	39	Y	$4d^1 5s^2$	6.377
15	P	$3s^2 3p^3$	10.484	40	Zr	$4d^2 5s^2$	
16	S	$3s^2 3p^4$	10.357	41	Nb	$4d^4 5s^1$	6.881
17	Cl	$3s^2 3p^5$	13.01	42	Mo	$4d^5 5s^1$	7.10
18	Ar	$3s^2 3p^6$	15.755	43	Tc	$4d^5 5s^2$	7.228
				44	Ru	$4d^7 5s^1$	7.365
19	K	[Ar] $4s^1$	4.339	45	Rh	$4d^8 5s^1$	7.461
20	Ca	$4s^2$	6.111	46	Pd	$4d^{10}$	8.33
21	Sc	$3d^1 4s^2$	6.54	47	Ag	$4d^{10} 5s^1$	7.574
22	Ti	$3d^2 4s^2$	6.83	48	Cd	$4d^{10} 5s^2$	8.991
23	V	$3d^3 4s^2$	6.74	49	In	$4d^{10} 5s^2 5p^1$	5.78
24	Cr	$3d^5 4s^1$	6.76	50	Sn	$4d^{10} 5s^2 5p^2$	7.342

Note: The bracket notation is used as a shorthand method to avoid repetition in indicating inner-shell electrons. Thus, [He] represents $1s^2$, [Ne] represents $1s^2 2s^2 2p^6$, [Ar] represents $1s^2 2s^2 2p^6 3s^2 3p^6$, and so on.

ments. The element helium is one in which the electronic configuration is $1s^2$—in other words, one shell is filled. In addition, it is found that the energy of the electrons in this filled shell is considerably lower than the energy of the next available level, the $2s$ level. Next, look at the electronic configuration for neon, $1s^2 2s^2 2p^6$. Again, the outermost shell is filled and there is a gap in energy between the $2p$ level and the $3s$ level. Argon has the configuration $1s^2 2s^2 2p^6 3s^2 3p^6$. Here, the $3p$ subshell is filled and there is a gap in energy between the $3p$ subshell and the $3d$ subshell. We could continue this procedure through all the noble gases; the pattern remains the same. A noble gas is formed when a shell is filled and there is a gap in energy before the next possible level is encountered.

It is interesting to plot ionization energy versus the atomic number Z, as in Figure 29.14. Note the pattern 2, 8, 8, 18, 18, 32 for the ionization energies. This pattern follows from the Pauli exclusion principle and helps explain why the elements repeat their chemical properties in groups. For example, the peaks at $Z = 2$, 10, 18, and 36 correspond to the elements He, Ne, Ar, and Kr, which have filled shells. These elements have similar energies and chemical behavior.

TABLE 29.4 Electronic Configuration of the Elements *(Continued)*

Z	Symbol	Ground Configuration	Ionization Energy (eV)	Z	Symbol	Ground Configuration	Ionization Energy (eV)
51	Sb	$4d^{10} 5s^2 5p^3$	8.639	79	Au	[Xe] $4f^{14} 5d^{10} 6s^1$	9.22
52	Te	$4d^{10} 5s^2 5p^4$	9.01	80	Hg	$6s^2$	10.434
53	I	$4d^{10} 5s^2 5p^5$	10.454	81	Tl	$6s^2 6p^1$	6.106
54	Xe	$4d^{10} 5s^2 5p^6$	12.127	82	Pb	$6s^2 6p^2$	7.415
				83	Bi	$6s^2 6p^3$	7.287
55	Cs	[Xe] $6s^1$	3.893	84	Po	$6s^2 6p^4$	8.43
56	Ba	$6s^2$	5.210	85	At	$6s^2 6p^5$	
57	La	$5d^1 6s^2$	5.61	86	Rn	$6s^2 6p^6$	10.745
58	Ce	$4f^1 5d^1 6s^2$	6.54				
59	Pr	$4f^3 6s^2$	5.48	87	Fr	[Rn] $7s^1$	
60	Nd	$4f^4 6s^2$	5.51	88	Ra	$7s^2$	5.277
61	Pm	$4f^5 6s^2$		89	Ac	$6d^1 7s^2$	6.9
62	Fm	$4f^6 6s^2$	5.6	90	Th	$6d^2 7s^2$	
63	Eu	$4f^7 6s^2$	5.67	91	Pa	$5f^2 6d^1 7s^2$	
64	Gd	$4f^7 5d^1 6s^2$	6.16	92	U	$5f^3 6d^1 7s^2$	4.0
65	Tb	$4f^9 6s^2$	6.74	93	Np	$5f^4 6d^1 7s^2$	
66	Dy	$4f^{10} 6s^2$		94	Pu	$5f^6 7s^2$	
67	Ho	$4f^{11} 6s^2$		95	Am	$5f^7 7s^2$	
68	Er	$4f^{12} 6s^2$		96	Cm	$5f^7 6d^1 7s^2$	
69	Tm	$4f^{13} 6s^2$		97	Bk	$5f^8 6d^1 7s^2$	
70	Yb	$4f^{14} 6s^2$	6.22	98	Cf	$5f^{10} 7s^2$	
71	Lu	$4f^{14} 5d^1 6s^2$	6.15	99	Es	$5f^{11} 7s^2$	
72	Hf	$4f^{14} 5d^2 6s^2$	7.0	100	Fm	$5f^{12} 7s^2$	
73	Ta	$4f^{14} 5d^3 6s^2$	7.88	101	Mv	$5f^{13} 7s^2$	
74	W	$4f^{14} 5d^4 6s^2$	7.98	102	No	$5f^{14} 7s^2$	
75	Re	$4f^{14} 5d^5 6s^2$	7.87	103	Lw	$5f^{14} 6d^1 7s^2$	
76	Os	$4f^{14} 5d^6 6s^2$	8.7	104	Ku	$5f^{14} 6d^2 7s^2$	
77	Ir	$4f^{14} 5d^7 6s^2$	9.2				
78	Pt	$4f^{14} 5d^8 6s^2$	8.88				

Thinking Physics 3

As one moves from left to right across one row of the periodic table, the effective size of the atoms first decreases and then increases. What would cause this behavior?

Reasoning As one begins at the left side of the periodic table and moves toward the middle, the nuclear charge is increasing. As a result, there is an increasing Coulomb attraction between the nucleus and the electrons, and the electrons are pulled into an average position that is closer to the nucleus. Although the additional electrons repel one another, that repulsion is diffused throughout their orbital volume. From the middle of a row to the right side, the increasing number of electrons being placed in proximity to each other results in an accumulated stronger repulsion which increases their average distance from the nucleus and causes the atomic size to grow.

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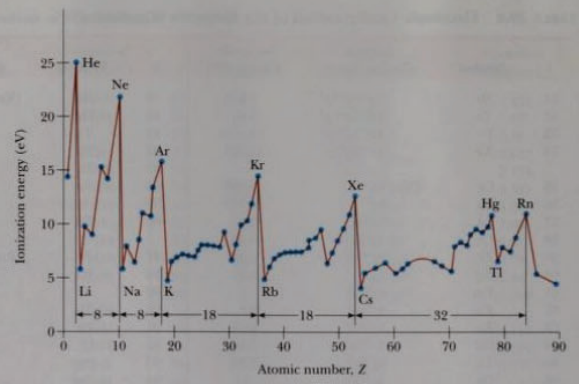


Figure 29.14 Ionization energy of the elements versus atomic number Z. *(Adapted from J. Omar, Physics, New York, Macmillan, 1979)*

29.7 · ATOMIC SPECTRA: VISIBLE AND X-RAY

In Chapter 11 we briefly discussed the origin of the spectral lines for hydrogen and hydrogen-like ions. Recall that an atom will emit electromagnetic radiation if an electron in an excited state makes a transition to a lower energy state.

The energy level diagram for hydrogen is shown in Figure 29.15. The diagonal lines represent allowed transitions between stationary states. Whenever an electron makes a transition from a higher energy state to a lower one, a photon of light is emitted. The frequency of this photon is $f = \Delta E/h$, where ΔE is the energy difference between the two levels and h is Planck's constant. The **selection rules for the allowed transitions are**

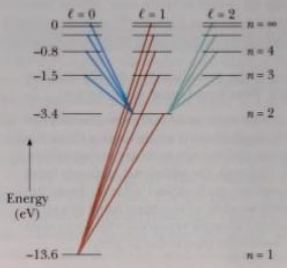


Figure 29.15 Some allowed electronic transitions for hydrogen, represented by the colored lines. These transitions must obey the selection rule $\Delta\ell = \pm 1$.

$$\Delta\ell = \pm 1 \quad \text{and} \quad \Delta m_\ell = 0 \text{ or } \pm 1 \qquad [29.17]$$

• *Selection rule for allowed atomic transitions*

Transitions that do not obey these selection rules are said to be **forbidden.** (Such transitions can occur, but their probability is negligible relative to the probability of the allowed transitions.)

Because the orbital angular momentum of an atom changes when a photon is emitted or absorbed (that is, as a result of a transition) and because angular momentum must be conserved, we conclude that **the photon involved in the process must carry angular momentum.** In fact, the photon has an angular momentum equivalent to that of a particle with a spin of 1. Also, the angular momentum of the photon is consistent with the classical description of electromagnetic radiation. Hence, **a photon has energy, linear momentum, and angular momentum.**

• *The photon carries angular momentum.*

As a generalization from Chapter 11, Equation 11.23, the allowed energies for one-electron atoms, such as hydrogen and He^+, are

$$E_n = -\frac{13.6Z^2}{n^2}\ \text{eV} \qquad [29.18]$$

• *Allowed energies for one-electron atoms*

For multi-electron atoms, the nuclear charge Ze is largely canceled or shielded by the negative charge of the inner-core electrons. Hence, the outer electrons interact with a net charge of the order of the electronic charge. The expression for the allowed energies for multi-electron atoms has the same form as Equation 29.18, with Z replaced by an effective atomic number, Z_{eff}. That is,

$$E_n = -\frac{13.6Z_{\text{eff}}^2}{n^2}\ \text{eV} \qquad [29.19]$$

• *Allowed energies for multi-electron atoms*

where Z_{eff} depends on n and ℓ. For the higher energy states, this reduction in charge increases and $Z_{\text{eff}} \rightarrow 1$.

X-Ray Spectra

X-rays are emitted from a metal target when it is bombarded by high-energy electrons. The x-ray spectrum typically consists of a broad continuous band and a series of sharp lines that depend on the type of material used for the target, as shown in Figure 29.16. These lines, called **characteristic x-rays,** were discovered in 1908, but their origin remained unexplained until the details of atomic structure were brought to light.

The continuous x-ray spectrum arises due to the change in velocity of the electrons as they strike the target. In Section 24.5, we stated that electromagnetic radiation is emitted whenever a charged particle accelerates. As the electrons in an x-ray tube are slowed down by interacting with the target atoms, they emit x-rays. The accelerations of the electrons vary over a wide range, since they interact with the atoms over a variety of separation distances. As a result, the frequencies of the emitted x-rays vary continuously.

The first step in the production of characteristic x-rays occurs when a bombarding electron collides with an electron in an inner shell of a target atom with sufficient energy to remove the electron from the atom. The vacancy created in the

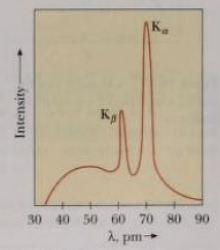


Figure 29.16 The x-ray spectrum of a metal target consists of a broad continuous spectrum plus a number of sharp lines, which are due to *characteristic x-rays.* The data shown were obtained when 35-keV electrons bombarded a molybdenum target. Note that 1 pm = 10^{-12} m = 10^{-3} nm.

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shell is filled when an electron in a higher level drops down into the level containing the vacancy. The time it takes for this to happen is very short, less than 10^{-9} s. This transition is accompanied by the emission of a photon the energy of which equals the difference in energy between the two levels. Typically, the energy of such transitions is greater than 1000 eV, and the emitted x-ray photons have wavelengths in the range of 0.01 nm to 1 nm.

Let us assume that the incoming electron has dislodged an atomic electron from the innermost shell, the K shell. If the vacancy is filled by an electron dropping from the next higher shell, the L shell, the photon emitted in the process has an energy corresponding to the K_α line on the curve of Figure 29.16. If the vacancy is filled by an electron dropping from the M shell, the line produced is called the K_β line.

Other characteristic x-ray lines are formed when electrons drop from upper levels to vacancies other than those in the K shell. For example, L lines are produced when vacancies in the L shell are filled by electrons dropping from higher shells. An L_α line is produced as an electron drops from the M shell to the L shell, and an L_β line is produced by a transition from the N shell to the L shell.

We can estimate the energy of the emitted x-rays as follows. Consider two electrons in the K shell of an atom the atomic number of which is Z. Each electron partially shields the other from the charge of the nucleus, Ze, and so each electron is subject to an effective nuclear charge of $Z_{\text{eff}} = (Z - 1)e$. We can now use Equation 29.18 to estimate the energy of either electron:

$$E_K \approx -(Z - 1)^2(13.6 \text{ eV}) \qquad [29.20]$$

As we shall show in the following example, one can estimate the energy of an electron in an L or M shell in a similar fashion. Taking the energy difference between these two levels, one can then calculate the energy and wavelength of the emitted photon.

In 1914, Henry G. J. Moseley plotted the Z values for a number of elements versus $\sqrt{1/\lambda}$, where λ is the wavelength of the K_α line of each element. He found that the plot is a straight line, as in Figure 29.17. This is consistent with rough calculations of the energy levels given by Equation 29.20. From this plot, Moseley was able to determine the Z values of some missing elements, which provided a periodic chart in excellent agreement with the known chemical properties of the elements.

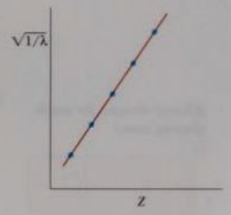


Figure 29.17 A Moseley plot for the K_α x-ray lines of a number of elements.

Thinking Physics 4

The Bohr theory gives good theoretical agreement with experimentally measured frequencies of spectral lines for the hydrogen atom, but it fails completely to predict the correct optical frequencies for heavier atoms. Moseley, however, used the Bohr theory to predict the frequencies of characteristic x-rays from heavy atoms. Why would the Bohr theory give good predictions for x-rays?

Reasoning Characteristic x-rays have their origin in transitions between the lowest energy levels in the atom. An electron in a lower energy shell is ejected from the atom by the accelerated electron in the x-ray tube, leaving a vacancy in the shell. Let us consider an electron ejected from the K shell and the resulting electrical interactions of an L shell electron with the other electrons. If we look first at the electrons in higher shells than L, these electrons spend most of their time farther from the nucleus than the L electrons. What's more, when averaged over time, the charge of these outer electrons is isotropic—there is no preferred direction. Thus, the L electrons find themselves *inside* a set of almost perfectly symmetric shells of charge. According to Gauss's law, then, **the L electron is largely unaffected by the outer electrons**—the time averaged electric field from these electrons is zero. What remains for the L electrons is a nuclear charge shielded by the one remaining K electron and the other electrons in the L shell. If we ignore the other L electrons, we can model the L electron as belonging to a hydrogen atom, with a nuclear charge of $(Z - 1)$, where the nuclear charge Z is reduced by 1 due to the shielding effect of the remaining K electron. As a result, the Bohr theory works relatively well in describing the energy of the L electron, although it is not perfect, because we have ignored the other L electrons.

CONCEPTUAL PROBLEM 3

In an x-ray tube, if the energy with which the electrons strike the metal target is increased, the wavelengths of the characteristic x-rays do not change. Why not?

CONCEPTUAL PROBLEM 4

Is it possible for a spectrum from an x-ray tube to show the continuous spectrum of x-rays without the presence of the characteristic x-rays?

CONCEPTUAL PROBLEM 5

The transition from the 3*d* state in hydrogen to the 1*s* state is not seen. Why?

Example 29.7 Estimating the Energy of an X-Ray

Estimate the energy of the characteristic x-ray emitted from a tungsten target when an electron drops from an M shell ($n = 3$ state) to a vacancy in the K shell ($n = 1$ state).

Solution The atomic number for tungsten is $Z = 74$. Using Equation 29.20, we get

$$E_K \approx -(74 - 1)^2(13.6 \text{ eV}) = -72\,500 \text{ eV}$$

The electron in the M shell is subject to an effective nuclear charge that depends on the number of electrons in the $n = 1$ and $n = 2$ states, which shield the nucleus. Because there are eight electrons in the $n = 2$ state and one electron in the $n = 1$ state, roughly nine electrons shield the nucleus, and so $Z_{\text{eff}} = Z - 9$. Hence, the energy of an electron in the M shell, following Equation 29.19, is

$$E_M \approx -Z_{\text{eff}}^2 E_3 = -(Z - 9)^2 \frac{E_0}{3^2}$$

$$= -(74 - 9)^2 \frac{(13.6 \text{ eV})}{9} = -6380 \text{ eV}$$

where E_3 is the energy of an electron in the M shell of the hydrogen atom and E_0 is the ground-state energy. Therefore, the emitted x-ray has an energy equal to $E_M - E_K \approx -6380 \text{ eV} - (-72\,500 \text{ eV}) = 66\,100 \text{ eV}$. Note that this energy difference is also equal to $hf = hc/\lambda$, where λ is the wavelength of the emitted x-ray.

EXERCISE 4 Calculate the wavelength of the emitted x-ray for this transition. Answer 0.0188 nm

29.8 • ATOMIC TRANSITIONS

We have seen that an atom emits radiation only at certain frequencies that correspond to the energy separations between the allowed states. Consider an atom with many allowed energy states, labeled $E_1, E_2, E_3, \ldots$ in Figure 29.18. When light is incident on the atom, only those photons the energy hf of which matches the energy

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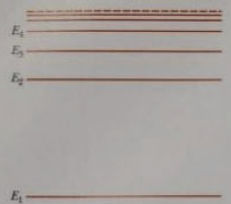


Figure 29.18 Energy level diagram of an atom with various allowed states. The lowest energy state, E_1, is the ground state. All others are excited states.

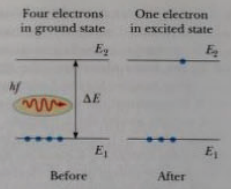


Figure 29.19 Diagram representing the *stimulated absorption* of a photon by an atom. The dots represent electrons. One electron is transferred from the ground state to the excited state when the atom absorbs a photon of energy $hf = E_2 - E_1$.

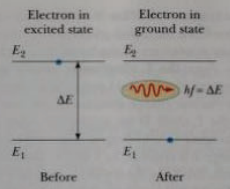


Figure 29.20 Diagram representing the *spontaneous emission of a* photon by an atom that is initially in the excited state E_2. When the electron falls to the ground state, the atom emits a photon of energy $hf = E_2 - E_1$.

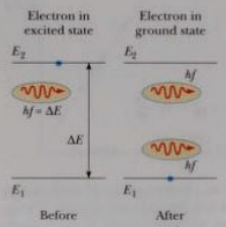


Figure 29.21 Diagram representing the *stimulated emission* of a photon by an incoming photon of energy hf. Initially, the atom is in the excited state. The incoming photon stimulates the atom to emit a second photon of energy $hf = E_2 - E_1$.

separation, ΔE, between two levels can be absorbed by the atom. Figure 29.19 is a schematic diagram representing this **stimulated absorption.** At ordinary temperatures, most of the atoms are in the ground state. If a vessel containing many atoms of a gaseous element is illuminated with a light beam containing all possible photon frequencies (that is, a continuous spectrum), only those photons of energies $E_2 - E_1$, $E_3 - E_1$, $E_4 - E_1$, and so on, can be absorbed. As a result of this absorption, some atoms are raised to allowed higher energy levels, called **excited states.**

Once an atom is in an excited state, there is a certain probability that the excited electron will jump back to a lower level by emitting a photon, as shown in Figure 29.20. This process is known as **spontaneous emission.** Typically, an atom remains in an excited state for only about 10^{-8} s.

Finally, there is a third process, **stimulated emission,** that is of importance in lasers. Suppose an atom is in the excited state E_2, as in Figure 29.21, and a photon with energy $hf = E_2 - E_1$ is incident on it. The excited state must be a metastable state having a lifetime much longer than 10^{-8} s. The incoming photon increases the probability that the excited electron will return to the ground state and thereby emit a second photon having the same energy hf. Note that the incident photon is not absorbed, so after the stimulated emission, there are two nearly identical photons—the incident photon and the emitted photon. The emitted photon is in phase with the incident photon. These photons can stimulate other atoms to emit photons in a chain of similar processes. The many photons produced in this fashion are the source of the intense, coherent light in a laser.

Thinking Physics 5

A physics student is watching a meteor shower in the early morning hours. She notices that the streaks of light from the meteoroids entering the very high regions of the atmosphere last for as long as 2 or 3 seconds before fading. She also notices a lightning storm off in the distance. The streaks of light from the lightning fade away almost immediately after the flash, certainly in much less than 1 second. Both lightning and meteors cause the air to turn into a plasma because of the very high temperatures generated. The light is given off when the stripped electrons in the plasma recombine with the ionized atoms. Why would this light last longer for meteors than for lightning?

Reasoning The answer lies in the subtle phrase in the description of the meteoroids—". . . entering the very high regions of the atmosphere." In the very high regions of the atmosphere, the pressure is very low. In addition, the *density* is very low, so that atoms of the gas are relatively far apart. Thus, after the air is ionized by the passing meteoroid, the probability of freed electrons finding an ionized atom with which to recombine is relatively low. As a result, the recombination process occurs over a relatively long time, measured in seconds.

However, lightning occurs in the lower regions of the atmosphere (the troposphere) where the pressure and density are relatively high. After the ionization by the lightning flash, the electrons and ionized atoms are much closer together than in the upper atmosphere. The probability of a recombination is much higher, and the time for the recombination to occur is much shorter.

EXERCISE 5 The wavelength of coherent ruby laser light is 694.3 nm. What is the energy difference (in electron volts) between the upper, excited state and the lower, unexcited state?
Answer 1.789 eV

29.9 • LASERS AND HOLOGRAPHY

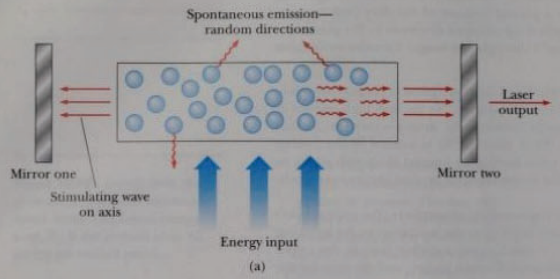
OPTIONAL

We have described how an incident photon can cause atomic transitions either upward (stimulated absorption) or downward (stimulated emission). The two processes are equally probable. When light is incident on a system of atoms, there is usually a net absorption of energy, because when the system is in thermal equilibrium, there are many more atoms in the ground state than in excited states. However, if one can invert the situation so that there are more atoms in an excited state than in the ground state, a net emission of photons can result. Such a condition is called **population inversion.**

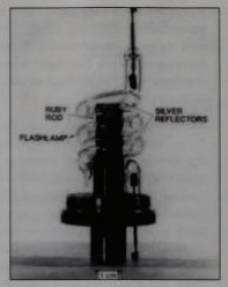
This is the fundamental principle involved in the operation of a **laser,** an acronym for **l**ight **a**mplification by **s**timulated **e**mission of **r**adiation. The amplification corresponds to a buildup of photons in the system as the result of a chain reaction of events. The following three conditions must be satisfied in order to achieve laser action:

1. The system must be in a state of population inversion (more atoms in an excited state than in the ground state).
2. The excited state of the system must be a *metastable state,* which means its lifetime must be long compared with the usually short lifetimes of excited states. When such is the case, stimulated emission will occur before spontaneous emission.
3. The emitted photons must be confined in the system long enough to enable them to stimulate further emission from other excited atoms. This is

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(a)



(b)

Figure 29.22 (a) A schematic diagram of a laser design. The tube contains atoms, which represent the active medium. An external source of energy (optical, electrical, etc.) is needed to "pump" the atoms to excited energy states. The parallel end mirrors provide the feedback of the stimulating wave. (b) Photograph of the first ruby laser showing the flash lamp surrounding the ruby rod. *(Courtesy of Hughes Aircraft Company)*

achieved by the use of reflecting mirrors at the ends of the system. One end is made totally reflecting and the other is slightly transparent to allow the laser beam to escape (Fig. 29.22a).

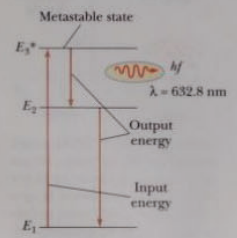


Figure 29.23 Energy level diagram for the neon atom, which emits photons at a wavelength of 632.8 nm through stimulated emission. The photon at this wavelength arises from the transition $E_3^* \rightarrow E_2$. This is the source of coherent light in the helium–neon gas laser.

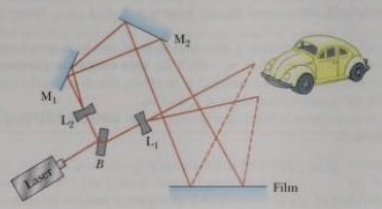
One device that exhibits stimulated emission of radiation is the helium–neon gas laser. Figure 29.23 is an energy level diagram for the neon atom in this system. The mixture of helium and neon is confined to a glass tube that is sealed at the ends by mirrors. An oscillator connected to the tube causes electrons to sweep through the tube, colliding with the atoms of the gases and raising them into excited states. Neon atoms are excited to state E_3 through this process and also as a result of collisions with excited helium atoms. Stimulated emission occurs as the neon atoms make a transition to state E_2 and neighboring excited atoms are stimulated. This results in the production of coherent light at a wavelength of 632.8 nm.

Since the development of the first laser in 1960, laser technology has experienced tremendous growth. Lasers that cover wavelengths in the infrared, visible, and ultraviolet regions are now available. Applications include surgical "welding" of detached retinas, precision surveying and length measurement, a potential source for inducing nuclear fusion reactions, precision cutting of metals and other materials, and telephone communication along optical fibers. These and other applications are possible because of the unique characteristics of laser light. In addition to its being highly monochromatic, laser light is also highly directional and can therefore be sharply focused to produce regions of extremely intense light energy (with energy densities approaching those inside the laser tube).

Holography

One interesting application of the laser is **holography,** the production of three-dimensional images of objects. Figure 29.24a shows how a hologram is made. Light from the laser is split into two parts by a half-silvered mirror at *B*. One part of the beam reflects off the object to be photographed and strikes an ordinary photographic film. The other half of the beam is diverged by lens L_2, reflects from mirrors M_1 and M_2, and finally strikes the film. The two beams overlap to form an extremely complicated interference pattern on the film. Such an interference pattern can be produced only if the phase relationship of the two waves is constant throughout the exposure of the film. This condition is met by illuminating the scene with light coming through a pinhole or with coherent laser radiation. The hologram records not only the intensity of the light scattered from the object (as in a conventional photograph) but also the phase difference between the reference beam and the beam scattered from the object. Because of this phase difference, an interference pattern is formed that produces an image with full three-dimensional perspective.

A hologram is best viewed by allowing coherent light to pass through the developed film as one looks back along the direction from which the beam comes. Figure 29.24b is a photograph of a hologram made using a cylindrical film.



(a)



(b)

Figure 29.24 (a) Experimental arrangement for producing a hologram. (b) Photograph of a hologram that uses a cylindrical film. Note the detail of the Volkswagen image. *(Courtesy of Central Scientific Company)*

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SUMMARY

The methods of quantum mechanics can be applied to the hydrogen atom using the appropriate potential energy function, $U(r) = -k_e e^2/r$, in the Schrödinger equation. The solution to this equation yields the wave functions for the allowed states and the allowed energies, given by

$$E_n = -\left(\frac{k_e e^2}{2a_0}\right)\frac{1}{n^2} = -\frac{13.6}{n^2}\ \text{eV} \qquad n = 1, 2, 3, \ldots \qquad [29.2]$$

This is precisely the result obtained in the Bohr theory. The allowed energy depends only on the **principal quantum number, *n*.** The allowed wave functions depend on three quantum numbers, n, ℓ, and m_ℓ, where ℓ is the **orbital quantum number** and m_ℓ is the **orbital magnetic quantum number.** The restrictions on the quantum numbers are as follows:

$$n = 1, 2, 3, \ldots$$
$$\ell = 0, 1, 2, \ldots, (n-1)$$
$$m_\ell = -\ell, -\ell+1, \ldots, \ell-1, \ell$$

All states with the same principal quantum number n form a **shell,** identified by the letters K, L, M, . . . (corresponding to $n = 1, 2, 3, \ldots$). All states with given values of both n and ℓ form a **subshell,** designated by the letters, *s, p, d, f,* . . . (corresponding to $\ell = 0, 1, 2, 3, \ldots$).

In order to completely describe a quantum state of the hydrogen atom, it is necessary to include a fourth quantum number, m_s, called the **spin magnetic quantum number.** This quantum number can have only two values, $\pm\frac{1}{2}$. In effect, this doubles the number of allowed states specified by the quantum numbers n, ℓ, and m_ℓ.

An atom in a state characterized by a specific n can have the following values of **orbital angular momentum, *L*:**

$$L = \sqrt{\ell(\ell+1)}\,\hbar \qquad \ell = 0, 1, 2, \ldots, n-1 \qquad [29.10]$$

The allowed values of the projection of **L** along the z axis are given by

$$L_z = m_\ell \hbar \qquad [29.11]$$

where m_ℓ is restricted to integer values lying between $-\ell$ and ℓ. Only discrete values of L_z are allowed, and these are determined by the restrictions on m_ℓ. This quantization of L_z is referred to as **space quantization.**

The electron has an intrinsic angular momentum called **spin angular momentum.** That is, the total angular momentum of an electron in an atom can have two contributions, one arising from the spin of the electron (**S**) and one arising from the orbital motion of the electron (**L**).

Electronic spin can be described by a single quantum number, $s = \frac{1}{2}$. The **magnitude of the spin angular momentum** is

$$S = \frac{\sqrt{3}}{2}\hbar \qquad [29.13]$$

and the z component of **S** is

$$S_z = m_s \hbar = \pm\tfrac{1}{2}\hbar \qquad [29.14]$$

That is, the spin angular momentum is also quantized in space, as specified by the **spin magnetic quantum number,** $m_s = \pm\frac{1}{2}$.

The magnetic moment $\boldsymbol{\mu}_s$ associated with the spin angular momentum of an electron is

$$\boldsymbol{\mu}_s = -\frac{e}{m_e}\mathbf{S} \qquad [29.15]$$

which is *twice* as large as the orbital magnetic moment. The z component of μ_s can have the values

$$\mu_{sz} = \pm\frac{e\hbar}{2m_e} \qquad [29.16]$$

The **exclusion principle** states that **no two electrons in an atom can ever have the same set of quantum numbers** n, ℓ, m_ℓ, and m_s. Using this principle and the principle of minimum energy, one can determine the electronic configuration of the elements. This serves as a basis for understanding atomic structure and the chemical properties of the elements.

The allowed electronic transitions between any two levels in an atom are governed by the selection rules

$$\Delta\ell = \pm 1 \qquad \text{and} \qquad \Delta m_\ell = 0 \text{ or } \pm 1 \qquad [29.17]$$

X-rays are emitted by atoms when an electron undergoes a transition from an outer shell into an electron vacancy in one of the inner shells. Transitions into a vacant state in the K shell give rise to the K series of spectral lines; transitions into a vacant state in the L shell create the L series of lines; and so on. The x-ray spectrum of a metal target consists of a set of sharp characteristic lines superimposed on a broad, continuous spectrum.

CONCEPTUAL QUESTIONS

1. Does the light emitted by a neon sign constitute a continuous spectrum or only a few colors? Defend your answer.
2. Must an atom first be ionized before it can emit light?
3. Discuss why the term *electron clouds* is used to describe the electronic arrangement in the quantum mechanical view of the atom.
4. It was stated in the chapter that, if the exclusion principle were not valid, every electron would end up in the lowest energy state of the atom and the chemical behavior of the elements would be grossly modified. Explain this statement.
5. When a hologram is produced, the system (including light source, object, beam splitter, and so on) must be held motionless within a quarter of a wavelength. Why?
6. Why is an inhomogeneous magnetic field used in the Stern–Gerlach experiment?
7. Could the Stern–Gerlach experiment be performed with ions rather than neutral atoms? Explain.
8. Describe some experiments that support the conclusion that the spin quantum number for electrons can only have the values $\pm\frac{1}{2}$.
9. Discuss some of the consequences of the exclusion principle.
10. Why do lithium, potassium, and sodium exhibit similar chemical properties?
11. From Table 29.4, we find that the ionization energies for Li, Na, K, Rb, and Cs are 5.390, 5.138, 4.339, 4.176, and 3.893 eV, respectively. Explain why these values are to be expected in terms of the atomic structures.
12. Does the intensity of light from a laser fall off as $1/r^2$?
13. How is it possible that electrons, which have a probability distribution around a nucleus, can exist in states of definite energy (e.g., 1*s*, 2*p*, 3*d*, . . .)?
14. It is easy to understand how two electrons (one spin up, one spin down) can fill the 1*s* shell for a helium atom. How is it possible that eight more electrons can fit into the 2*s*, 2*p* level to complete the $1s^2 2s^2 2p^6$ shell for a neon atom?
15. In 1914 Henry Moseley was able to define the atomic number of an element from its characteristic x-ray spectrum. How was this possible? (*Hint:* See Figs. 29.16 and 29.17.)
16. Why is stimulated emission so important in the operation of a laser?
17. The efficiencies of most solid-state lasers are on the order of 1% to 2%. Although the laser output is monochromatic and highly directional, can you use Figures 29.22 and 29.23 to determine why the energy input must exceed laser energy output by a factor of 50 to 100? Explain your answer.

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PROBLEMS

Section 29.2 The Hydrogen Atom Revisited

1. A photon with energy 2.28 eV is barely capable of causing a photoelectric effect when it strikes a sodium plate. Suppose that the photon is instead absorbed by hydrogen. Find (a) the minimum n for a hydrogen atom that can be ionized by such a photon and (b) the speed of the released electron far from the nucleus.
2. The Balmer series for the hydrogen atom corresponds to electronic transitions that terminate in the state with quantum number $n = 2$, as shown in Figure P29.2. (a) Consider the photon of longest wavelength; determine its energy and wavelength. (b) Consider the spectral line of shortest wavelength; find its photon energy and wavelength.

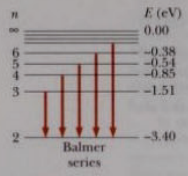


Figure P29.2 An energy-level diagram for hydrogen showing the Balmer series (not to scale).

3. A general expression for the energy levels of one-electron atoms and ions is

$$E_n = -\mu k_e^2 q_1^2 q_2^2 / 2\hbar^2 n^2$$

where k_e is the Coulomb constant, q_1 and q_2 are the charges of the two particles, and μ is the reduced mass, given by $\mu = m_1 m_2/(m_1 + m_2)$. In Problem 2 we found that the wavelength for the $n = 3$ to $n = 2$ transition of the hydrogen atom is 656.3 nm (visible red light). What are the wavelengths for this same transition in (a) positronium, which consists of an electron and a positron, and (b) singly ionized helium? (*Note:* A positron is a positively charged electron.)
4. (a) Determine the possible values of the quantum numbers ℓ and m_ℓ for the He^+ ion in the state corresponding to $n = 3$. (b) What is the energy of this state?

Section 29.3 The Spin Magnetic Quantum Number

Section 29.4 The Wave Functions for Hydrogen

5. List the possible sets of quantum numbers for electrons in (a) the 3*d* subshell and (b) the 3*p* subshell.
6. Plot the wave function $\psi_{1,s}(r)$ (Eq. 29.3) and the radial probability density function $P_{1,s}(r)$ (Eq. 29.7) for hydrogen. Let r range from 0 to $1.5a_0$, where a_0 is the Bohr radius.
7. The wave function for an electron in the 2*p* state of hydrogen is

$$\psi_{2p} = \frac{1}{\sqrt{3}(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$$

What is the most likely distance from the nucleus to find an electron in the 2*p* state? (See Fig. 29.8.)
8. During a particular period of time, an electron in the ground state of a hydrogen atom is "observed" 1000 times at a distance $a_0/2$ from the nucleus. How many times is this electron observed at a distance $2a_0$ from the nucleus during this period of "observation"?
9. Show that the 1*s* wave function for an electron in hydrogen,

$$\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

satisfies the radially symmetric Schrödinger equation,

$$-\frac{\hbar^2}{2m}\left(\frac{d^2\psi}{dr^2} + \frac{2}{r}\frac{d\psi}{dr}\right) - \frac{k_e e^2}{r}\psi = E\psi$$

Section 29.5 The "Other" Quantum Numbers

10. Calculate the angular momentum for an electron in (a) the 4*d* state and (b) the 6*f* state.
11. If an electron has orbital angular momentum 4.714×10^{-34} J·s, what is the orbital quantum number for the state of the electron?
12. A hydrogen atom is in its fifth excited state. The atom emits a 1090-nm wavelength photon. Determine the maximum possible orbital angular momentum of the electron after emission.
13. Find all possible values of L, L_z, and θ for an electron in a 3*d* state of hydrogen.
14. Two electrons in the same atom both have $n = 3$ and $\ell = 1$. List the possible states. (b) How many states would be possible if the exclusion principle were inoperative?
15. How many sets of quantum numbers are possible for an electron for which (a) $n = 1$, (b) $n = 2$, (c) $n = 3$, (d) $n = 4$, and (e) $n = 5$? Check your results to show that they agree with the general rule that the number of sets of quantum numbers is equal to $2n^2$.
16. The z component of the electron's spin magnetic moment is given by the Bohr magneton, $\mu_B = e\hbar/2m_e$. Show that μ_B has the numerical value 9.27×10^{-24} J/T = 5.79×10^{-5} eV/T.
17. (a) Find the mass density of a proton, picturing it as a solid sphere of radius 1.00×10^{-15} m. (b) Think of a classical model of an electron as a solid sphere with the same density as the proton. Find its radius. (c) If this electron possesses spin angular momentum $I\omega = \hbar/2$ because of classical rotation about the z axis, determine the speed of a point on the equator of the electron, and (d) compare this speed to the speed of light.
18. All objects, large and small, behave quantum mechanically. (a) Estimate the quantum number ℓ for the Earth in its orbit about the Sun. (b) What energy change (in joules) would occur if the Earth made a transition to an adjacent allowed state?
19. Like the electron, the nucleus of an atom has spin angular momentum and a corresponding magnetic moment. The z-component of the spin magnetic moment for a nucleus is characterized by the *nuclear magneton* $\mu_n = e\hbar/2m_p$, where m_p is the proton mass. (a) Calculate the value of μ_n in J/T and in eV/T. (b) Determine the ratio μ_n/μ_B and comment on your result.

Section 29.6 The Exclusion Principle and the Periodic Table

20. (a) Write out the electronic configuration for the ground state of oxygen ($Z = 8$). (b) Write out the values for the set of quantum numbers n, ℓ, m_ℓ, and m_s for each electron in oxygen.
21. What would you predict for the electronic configuration of element 110?
22. Devise a table similar to that shown in Figure 29.13 for atoms containing 11 through 19 electrons. Use Hund's rule and educated guesswork.
23. (a) Scanning through Table 29.4 in order of increasing atomic number, note that the electrons fill the subshells in such a way that those subshells with the lowest values of $n + \ell$ are filled first. If two subshells have the same value of $n + \ell$, the one with the lower value of n is filled first. Using these two rules, write the order in which the subshells are filled through $n + \ell = 7$. (b) Predict the chemical valence for the elements that have atomic numbers 15, 47, and 86 and compare your predictions with the actual valences.
24. Which electronic configuration has the lesser energy and the greater number of unpaired spins, $[Kr]4d^5 5s^1$ or $[Kr]4d^{10}$? Identify the element and discuss Hund's rule in this case. (*Note:* The notation [Kr] represents the filled configuration for krypton.)

Section 29.7 Atomic Spectra: Visible and X-Ray

25. If you wish to produce 10.0-nm x-rays in the laboratory, what is the minimum voltage you must use in accelerating the electrons?
26. A tungsten target is struck by electrons that have been accelerated from rest through a 40.0-kV potential difference. Find the shortest wavelength of the radiation emitted.
27. Use the method illustrated in Example 29.7 to calculate the wavelength of the x-ray emitted from a molybdenum target ($Z = 42$) when an electron moves from the L shell ($n = 2$) to the K shell ($n = 1$).
28. The wavelength of characteristic x-rays corresponding to the K_β line is 0.152 nm. Determine the material in the target.
29. The K series of the discrete spectrum of tungsten contains wavelengths of 0.0185 nm, 0.0209 nm, and 0.0215 nm. The K-shell ionization energy is 69.5 keV. Determine the ionization energies of the L, M, and N shells. Sketch the transitions.

Section 29.8 Atomic Transitions

30. The familiar yellow light from a sodium-vapor street lamp results from the $3p \rightarrow 3s$ transition in ^{11}Na. Evaluate the wavelength of this light given that the energy difference $E_{3p} - E_{3s} = 2.10$ eV.
31. A ruby laser delivers a 10.0-ns pulse of 1.00 MW average power. If the photons have a wavelength of 694.3 nm, how many are contained in the pulse?
32. A Nd:YAG laser used in eye surgery emits a 3.00-mJ pulse in 1.00 ns, focused to a spot 30.0 μm in diameter on the retina. (a) Find (in SI units) the power per unit area at the retina. (This quantity is called the irradiance.) (b) What energy is delivered to an area of molecular size—say, a circular area 0.600 nm in diameter?

Additional Problems

33. Show that the average value of r for the 1*s* state of hydrogen has the value $3a_0/2$. (*Hint:* Use Eq. 29.7.)
34. (a) How much energy is required to cause an electron in hydrogen to move from the $n = 1$ state to the $n = 2$ state? (b) Suppose the electrons gain this energy by collisions among hydrogen atoms at a high temperature. At what temperature would the thermal energy $3k_BT/2$, where k_B is the Boltzmann constant, be large enough to excite the electrons?
35. If a muon (a negatively charged particle having a mass 206 times the electron's mass) is captured by a lead nucleus, $Z = 82$, the resulting system behaves like a one-electron atom. (a) What is the "Bohr radius" for a muon captured by a lead nucleus? (*Hint:* Follow the treatment in Section 11.5.) (b) Using Equation 29.2 with one factor of e replaced by Ze, calculate the ground state energy of a muon captured by a lead nucleus. (c) What is the transition energy for a muon descending from the $n = 2$ to the $n = 1$ level in a muonic lead atom?
36. Show that the wave function for an electron in the 2*s* state in hydrogen

$$\psi(r) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0}$$

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satisfies the radially symmetric Schrödinger equation given in Problem 9.

37. A pulsed ruby laser emits light at 694.3 nm. For a 14.0-ps pulse containing 3.00 J of energy, find (a) the physical length of the pulse as it travels through space and (b) the number of photons in it. (c) If the beam has a circular cross section of 0.600 cm diameter, find the number of photons per cubic millimeter.

38. A pulsed laser emits light having wavelength λ. For a pulse of duration Δt having energy E, find (a) the physical length of the pulse as it travels through space and (b) the number of photons in it. (c) If the beam has a circular cross section having diameter d, find the number of photons per unit volume.

39. The number N of atoms in a particular state is called the population of that state. This number depends on the energy of that state and the temperature. In thermal equilibrium, the population of atoms in a state of energy E_n is given by a Boltzmann distribution expression:

$$N = N_g e^{-(E_n - E_g)/k_B T}$$

where T is the absolute temperature and N_g is the population of the ground state of energy E_g. (a) Find the equilibrium ratio of populations of the states E_2^s to E_2 for the laser in Figure 29.23, assuming $T = 27.0°C$. (b) Find the equilibrium ratio of the populations of the two states in a ruby laser that produces a light beam of wavelength 694.3 nm at 4.00 K.

40. The force on a magnetic moment μ_z in a nonuniform magnetic field B_z is given by $F_z = \mu_z(dB_z/dz)$. If a beam of silver atoms travels a horizontal distance of 1.00 m through such a field and each atom has a speed of 100 m/s, how strong must be the field gradient dB_z/dz in order to deflect the beam 1.00 mm?

41. (a) Find the most probable position for an electron in the 2*s* state of hydrogen. (*Hint:* Let $x = r/a_0$, find an equation for x, and show that $x = 5.236$ is a solution to this equation.) (b) Show that the wave function given by Equation 29.8 is normalized.

42. An electron in chromium moves from the $n = 2$ state to the $n = 1$ state without emitting a photon. Instead, the excess energy is transferred to an outer electron (one in the $n = 4$ state), which is then ejected by the atom. (This is called an Auger [pronounced "ohjay"] process, and the ejected electron is referred to as an Auger electron.) Use the Bohr theory to find the kinetic energy of the Auger electron.

43. Suppose the ionization energy of an atom is 4.10 eV. In the spectrum of this same atom, we observe emission lines with wavelengths 310 nm, 400 nm, and 1377.8 nm. Use this information to construct the energy-level diagram with the least number of levels. Assume the higher levels are closer together.

44. All atoms have the same size, to an order of magnitude. (a) To show this, estimate the diameters for aluminum, molar atomic mass = 27.0 g/mol and density 2.70 g/cm^3, and uranium, molar atomic mass = 238 g/mol and density 18.9 g/cm^3. (b) What do the results imply about the wave functions for inner-shell electrons as we progress to higher and higher atomic mass atoms? (*Hint:* The molar volume is approximately $D^3 N_A$, where D is the atomic diameter and N_A is Avogadro's number.)

45. For hydrogen in the 1*s* state, what is the probability of finding the electron farther than $2.50a_0$ from the nucleus?

46. In interstellar space, atomic hydrogen produces the sharp spectral line called the 21-cm radiation, which astronomers find most helpful in detecting clouds of hydrogen between stars. This radiation is useful because interstellar dust that obscures visible wavelengths is transparent to these radio wavelengths. The radiation is not generated by an electron transition between energy states characterized by n. Instead, in the ground state ($n = 1$), the electron and proton spins may be parallel or antiparallel, with a resultant slight difference in these energy states. (a) Which condition has the higher energy? (b) The line is actually at 21.11 cm. What is the energy difference between the states? (c) The average lifetime in the excited state is about 10^7y. Calculate the associated uncertainty in energy of this excited energy level.

47. According to classical physics, an accelerated charge e radiates at a rate

$$\frac{dE}{dt} = -\frac{1}{6\pi\epsilon_0}\frac{e^2a^2}{c^3}$$

(a) Show that an electron in a classical hydrogen atom (see Fig. 29.3) spirals into the nucleus at a rate

$$\frac{dr}{dt} = -\frac{e^4}{12\pi^2\epsilon_0{}^2 r^2 m_e{}^2 c^5}$$

(b) Find the time it takes the electron to reach $r = 0$, starting from $r_0 = 2.00 \times 10^{-10}$ m.

48. When electron clouds overlap, a detailed calculation of the effective charge exerting electrical force on another electron may be made using quantum mechanics. For the case of the lithium atom, the effective charge on each inner electron is $-0.85e$. Use this to find (a) the effective charge on the nucleus as seen by the outer valence electron and (b) the ionization energy (compare this with 5.4 eV).

49. Light from a certain He–Ne laser has a power output of 1.00 mW and a cross-sectional area of 10.0 mm^2. The entire beam is incident on a metal target that requires 1.50 eV to remove an electron from its surface. (a) Perform a classical calculation to determine how long it takes one atom in the metal to absorb 1.50 eV from the incident beam. (*Hint:* Assume the face area of an atom is 1.00×10^{-20} m^2, and first calculate the energy incident on each atom per second.) (b) Compare the (wrong) answer obtained in (a) to the actual response time for photoelectric emission ($\sim 10^{-9}$ s), and discuss the reasons for the large discrepancy.

50. Use the energy levels for hydrogen to show that when the electron falls from state n to state $n - 1$, the frequency of the emitted light is

$$f = \left(\frac{2\pi^2 m_e k_e{}^2 e^4}{h^3 n^2}\right)\frac{(2n-1)}{(n-1)^2}$$

51. Calculate the frequency of the light that a hydrogen atom should emit according to classical theory. To do so, note that the frequency of revolution of the electron is $v/2\pi r$, where r is the radius of its Bohr orbit. Show that as n approaches infinity in the equation in Problem 50, the expression varies as $1/n^3$ and reduces to the classical frequency the atom is expected to emit. This is one example of the correspondence principle, which requires that the classical and quantum models agree for large values of n.

ANSWERS TO CONCEPTUAL PROBLEMS

1. If the energy of the hydrogen atom were proportional to n (or any power of n), then the energy would become infinite as n grew to infinity. But the energy of the atom is *inversely* proportional to n^2. Thus, as n grows to infinity, the energy of the atom approaches zero from the negative side. As a result, the maximum frequency of emitted radiation approaches a value determined by the difference in energy between zero and the energy of the ground state.
2. A magnetic field exerts a torque on a macroscopic magnetic moment, in the direction such that the magnetic moment tends to align with the field. This torque will also exist on the microscopic magnetic moment associated with the spin of an electron. According to quantum physics, however, the spin angular momentum cannot line up with the magnetic field. It will point in a direction such that the component of the angular momentum along the magnetic field is $\pm\hbar/2$.
3. The characteristic x-rays originate from transitions within the atoms of the target, such as an L shell electron making a transition to a vacancy in the K shell. This vacancy in the K shell is caused when an accelerated electron in the x-ray tube supplies energy to the K electron to eject it from the atom. If the energy of the bombarding electrons were to be increased, the K electron will be ejected from the atom with more remaining kinetic energy. But the energy difference between the K and L shell has not changed, so the emitted x-ray has exactly the same wavelength.
4. A continuous spectrum without characteristic x-rays is possible. At a low accelerating potential difference for the electron, the electron may not have enough energy to eject an electron from a target atom. As a result, there will be no characteristic x-rays. The change in speed of the electron as it enters the target will result in the continuous spectrum.
5. This transition is forbidden by the selection rules, because the orbital angular momentum quantum number changes by two units.



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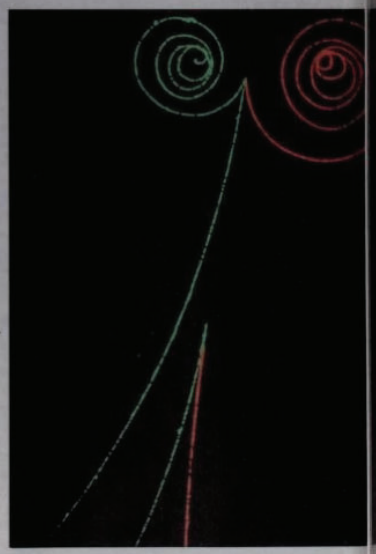
Nuclear Physics

CHAPTER OUTLINE

30.1 Some Properties of Nuclei
30.2 Binding Energy
30.3 Radioactivity
30.4 The Decay Processes
30.5 Natural Radioactivity
30.6 Nuclear Reactions

In 1896, the year that marked the birth of nuclear physics, Antoine Henri Becquerel (1852–1908) discovered radioactivity in uranium compounds. A great deal of research followed as scientists attempted to understand the radiation emitted by radioactive nuclei. Pioneering work by Rutherford showed that the radiation was of three types, which he called alpha, beta, and gamma rays. These types are classified according to the natures of their electric charges and according to their ability to penetrate matter and ionize air. Later experiments showed that alpha rays are helium nuclei, beta rays are electrons, and gamma rays are high-energy photons.

As we saw in Chapter 29, Section 29.1, the 1911 experiments of Rutherford and his students Geiger and Marsden established that the nucleus of an atom could be regarded as essentially a point mass and point charge and that most of the atomic mass is contained in the nucleus. Furthermore,



Bubble chamber photograph of electron (green) and positron (red) tracks produced by energetic gamma rays. The highly curved tracks at the top are due to an electron–positron pair that bend in opposite direction in the magnetic field. The lower tracks are produced by more energetic electrons and a positron. *(Lawrence Berkeley Laboratory/Science Photo Library, Photo Researchers, Inc.)*

such studies demonstrated a new type of force, the **nuclear force,** which is predominant at distances of less than about 10^{-14} m and zero at great distances. That is, the nuclear force is a short-range force.

Other milestones in the development of nuclear physics include

- The observation of nuclear reactions by Cockroft and Walton in 1930,
- The discovery of the neutron by Chadwick in 1932,
- The discovery of artificial radioactivity by Joliot and Irene Curie in 1933,
- The discovery of nuclear fission by Hahn and Strassman in 1938,
- The development of the first controlled fission reactor by Fermi and his collaborators in 1942.

In this chapter we discuss the properties and structure of the atomic nucleus. We start by describing the basic properties of nuclei, then discuss nuclear forces and binding energy, nuclear models, and the phenomenon of radioactivity. We also discuss nuclear reactions and the processes by which nuclei decay.

30.1 · SOME PROPERTIES OF NUCLEI

All nuclei are composed of two types of particles: protons and neutrons. The only exception is the ordinary hydrogen nucleus, which is a single proton. In describing the atomic nucleus, we must talk about the following quantities:

- The **atomic number, Z,** which equals the number of protons in the nucleus (the atomic number is sometimes called the charge number),
- The **neutron number, *N*,** which equals the number of neutrons in the nucleus,
- The **mass number, *A*,** which equals the number of nucleons (neutrons plus protons) in the nucleus.

In representing nuclei, it is convenient to have a symbolic way that shows how many protons and neutrons are present. The symbol used is ${}^{A}_{Z}\mathrm{X}$, where X represents the chemical symbol for the element. For example, ${}^{56}_{26}\mathrm{Fe}$ (iron) has a mass number of 56 and an atomic number of 26; therefore, it contains 26 protons and 30 neutrons. When no confusion is likely to arise, we omit the subscript Z because the chemical symbol can always be used to determine Z.

The nuclei of all atoms of a particular element contain the same number of protons but often contain different numbers of neutrons. Nuclei that are related in this way are called **isotopes. The isotopes of an element have the same Z value but different *N* and *A* values.** The natural abundances of isotopes can differ substantially. For example, ${}^{11}_{6}\mathrm{C}$, ${}^{12}_{6}\mathrm{C}$, ${}^{13}_{6}\mathrm{C}$, and ${}^{14}_{6}\mathrm{C}$ are four isotopes of carbon. The natural abundance of the ${}^{12}_{6}\mathrm{C}$ isotope is about 98.9%, whereas that of the ${}^{13}_{6}\mathrm{C}$ isotope is only about 1.1%. Even the simplest element, hydrogen, has isotopes: ${}^{1}_{1}\mathrm{H}$, the ordinary hydrogen nucleus; ${}^{2}_{1}\mathrm{H}$, deuterium; and ${}^{3}_{1}\mathrm{H}$, tritium. Some isotopes do not occur naturally but can be produced in the laboratory through nuclear reactions.

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Ernest Rutherford (1871–1937)

Rutherford, a New Zealander, was awarded the Nobel prize in 1908 for his studies of radioactivity. When he discovered that atoms can be broken apart by alpha rays, he remarked, "On consideration, I realized that this scattering backward must be the result of a single collision, and when I made calculations I saw that it was impossible to get anything of that order of magnitude unless you took a system in which the greater part of the mass of the atom was concentrated in a minute nucleus. It was then that I had the idea of an atom with a minute massive center carrying a charge." *(Photo courtesy of AIP Niels Bohr Library)*

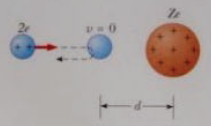


Figure 30.1 An alpha particle on a head-on collision course with a nucleus of charge Ze. Because of the Coulomb repulsion between the like charges, the alpha particle stops instantaneously at a distance d from the target nucleus, called the distance of closest approach.

TABLE 30.1 Masses of the Proton, Neutron, Hydrogen Atom, and Electron in Various Units

Particle	Mass		
	kg	u	MeV/c^2
Proton	1.6726×10^{-27}	1.007 276	938.28
Neutron	1.6750×10^{-27}	1.008 665	939.57
Hydrogen atom	1.6735×10^{-27}	1.007 825	938.78
Electron	9.109×10^{-31}	5.486×10^{-4}	0.511

Charge and Mass

The proton carries a single positive charge, $+e$, the electron carries a single negative charge, $-e$, where $e = 1.6 \times 10^{-19}$ C, and the neutron is electrically neutral. Because the neutron has no charge, it is difficult to detect.

In Chapter 1, we defined the atomic mass unit, u, in such a way that the mass of the isotope ^{12}C is exactly 12 u, where $1\ u = 1.660\ 540 \times 10^{-27}$ kg. The proton and neutron each have a mass of approximately 1 u, and the electron has a mass that is only a small fraction of an atomic mass unit:

$$\text{Mass of proton} = 1.007\ 276\ u$$

$$\text{Mass of neutron} = 1.008\ 665\ u$$

$$\text{Mass of electron} = 0.000\ 5486\ u$$

Because the rest energy of a particle is given by $E_R = mc^2$ (Chapter 9), it is often convenient to express the atomic mass unit in terms of its rest energy equivalent. For one atomic mass unit, we have

$$E_R = mc^2 = (1.660\ 540 \times 10^{-27}\ \text{kg})(2.997\ 92 \times 10^8\ \text{m/s})^2$$
$$= 931.494\ \text{MeV}$$

Nuclear physicists often express mass in terms of the unit MeV/c^2, where

$$1\ u = 931.494\ \text{MeV}/c^2$$

The masses of several simple particles are given in Table 30.1. The masses and some other properties of selected isotopes are provided in Table 30.4 and Appendix A.3.

The Size of Nuclei

The size and structure of nuclei were first investigated in the scattering experiments of Rutherford, discussed in Section 29.1. Using the principle of conservation of energy, Rutherford found an expression for how close an alpha particle moving directly toward the nucleus can come to the nucleus before being turned around by Coulomb repulsion.

In such a head-on collision, the kinetic energy of the incoming alpha particle must be converted completely to electrical potential energy when the particle stops at the point of closest approach and turns around (Fig. 30.1). If we equate the initial kinetic energy of the alpha particle to the maximum electrical potential energy of the system (alpha particle plus target nucleus), we have

$$\tfrac{1}{2}mv^2 = k_e\frac{q_1q_2}{r} = k_e\frac{(2e)(Ze)}{d}$$

where d is the distance of closest approach. Solving for d, we get

$$d = \frac{4k_eZe^2}{mv^2}$$

From this expression, Rutherford found that alpha particles approached to within 3.2×10^{-14} m of a nucleus when the foil was made of gold. Thus, the radius of the gold nucleus must be less than this value. For silver atoms, the distance of closest approach was found to be 2×10^{-14} m. From these results, Rutherford concluded that the positive charge in an atom is concentrated in a small sphere, which he called the nucleus, with a radius of no greater than about 10^{-14} m. Because such small lengths are common in nuclear physics, a convenient unit of length is the *femtometer* (fm), sometimes called the **fermi,** defined as

$$1\ \text{fm} \equiv 10^{-15}\ \text{m}$$

Since the time of Rutherford's scattering experiments, a multitude of other experiments have shown that most nuclei are approximately spherical and have an average radius of

$$r = r_0A^{1/3} \qquad [30.1]$$

• ***Radius of a nucleus***

where A is the mass number and r_0 is a constant equal to 1.2×10^{-15} m. Because the volume of a sphere is proportional to the cube of the radius, it follows from Equation 30.1 that the volume of a nucleus (assumed to be spherical) is directly proportional to A, the total number of nucleons. This suggests that **all nuclei have nearly the same density.** Nucleons combine to form a nucleus as though they were tightly packed spheres (Fig. 30.2).



Figure 30.2 A nucleus can be visualized as a cluster of tightly packed spheres, where each sphere is a nucleon.

Example 30.1 Nuclear Volume and Density

Find (a) an approximate expression for the mass of a nucleus of mass number A, (b) an expression for the volume of this nucleus in terms of the mass number, and (c) a numerical value for its density.

Solution (a) The mass of the proton is approximately equal to that of the neutron. Thus, if the mass of one of these particles is m, the mass of the nucleus is approximately Am.

(b) Assuming the nucleus is spherical and using Equation 30.1, we find that the volume is

$$V = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}\pi r_0^3A$$

(c) The nuclear density is

$$\rho_n = \frac{\text{mass}}{\text{volume}} = \frac{Am}{\frac{4}{3}\pi r_0^3A} = \frac{3m}{4\pi r_0^3}$$

$$= \frac{3(1.67 \times 10^{-27}\ \text{kg})}{4\pi(1.2 \times 10^{-15}\ \text{m})^3} = 2.3 \times 10^{17}\ \text{kg/m}^3$$

Note that the nuclear density is about 2.3×10^{14} times greater than the density of water (10^3 kg/m^3)!

Nuclear Stability

Because the nucleus consists of a closely packed collection of protons and neutrons, you might be surprised that it can exist. The very large repulsive electrostatic forces between protons in close proximity should cause the nucleus to fly apart. However,

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nuclei are stable because of the presence of another force, the **nuclear force.** This force, which is short-range (about 2 fm), is an attractive force that acts between all nuclear particles. The protons attract each other via the nuclear force, and at the same time they repel each other through the Coulomb force. The nuclear force also acts between pairs of neutrons and between neutrons and protons.

The nuclear force dominates the Coulomb repulsive force within the nucleus (at short ranges). If this were not the case, stable nuclei would not exist. Moreover, the strong nuclear force is independent of charge. In other words, the forces associated with the proton–proton, proton–neutron, and neutron–neutron interactions are the same, apart from the additional repulsive Coulomb force for the proton–proton interaction.

There are about 260 stable nuclei; hundreds of others have been observed but are unstable. A plot of N versus Z for a number of stable nuclei is given in Figure 30.3. Note that light nuclei are most stable if they contain equal numbers of protons and neutrons—that is, if $N = Z$—but heavy nuclei are more stable if $N > Z$. This can be partially understood by recognizing that as the number of protons increases, the strength of the Coulomb force increases, which tends to break the nucleus apart. As a result, more neutrons are needed to keep the nucleus stable, because neutrons experience only the attractive nuclear forces. Eventually, when $Z = 83$, the repulsive forces between protons cannot be compensated by the addition of more neutrons. In effect, the additional neutrons "dilute" the nuclear charge density. Elements that contain more than 83 protons do not have stable nuclei.

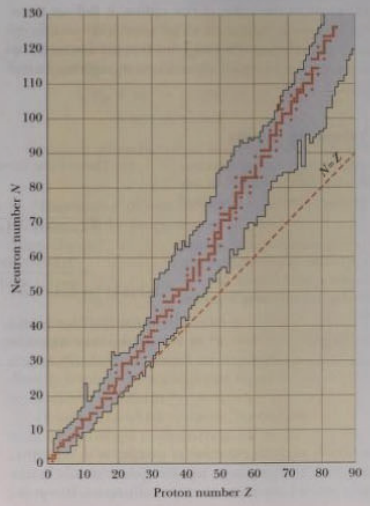


Figure 30.3 A plot of neutron number, N, versus atomic number, Z, for the stable nuclei (solid points). The dashed line corresponds to the condition $N = Z$. The shaded area shows radioactive nuclei.

It is interesting to note that most stable nuclei have even values of A. In fact, certain values of Z and N correspond to nuclei with unusually high stability. These values of N and Z, called **magic numbers,** are

$$Z \text{ or } N = 2, 8, 20, 28, 50, 82, 126 \qquad [30.2]$$

For example, the helium nucleus (two protons and two neutrons), which has $Z = 2$ and $N = 2$, is very stable.

Nuclear Spin and Magnetic Moment

In Chapter 29 we discussed the fact that an electron has an intrinsic angular momentum, which is called its spin. Nuclei, like electrons, also have an intrinsic angular momentum that arises from relativistic properties. The magnitude of the **nuclear angular momentum** is $\sqrt{I(I+1)}\hbar$, where I is a quantum number called the **nuclear spin** and may be an integer or a half-integer. The maximum component of the nuclear angular momentum projected along any direction is $I\hbar$. Figure 30.4 illustrates the possible orientations of the nuclear spin and its projections along the z axis for the case where $I = \frac{3}{2}$.

The nuclear angular momentum has a nuclear magnetic moment associated with it. The magnetic moment of a nucleus is measured in terms of the **nuclear magneton,** μ_n, a unit of magnetic moment defined as

$$\mu_n \equiv \frac{e\hbar}{2m_p} = 5.05 \times 10^{-27}\ \text{J/T} \qquad [30.3]$$

This definition is analogous to Equation 22.28 for the Bohr magneton, μ_B. Note that μ_n is smaller than μ_B by a factor of about 2000, due to the large difference in masses of the proton and electron.

The magnetic moment of a free proton is not μ_n but 2.7928 μ_n. Unfortunately, there is no general theory of nuclear magnetism that explains this value. Another surprising point is the fact that a neutron also has a magnetic moment, which has a value of $-1.9135\ \mu_n$. The minus sign indicates that the neutron's magnetic moment is opposite its spin angular momentum.

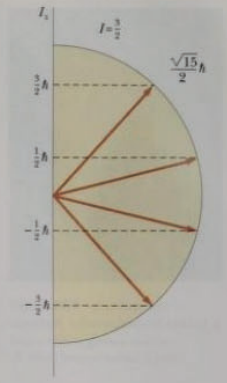


Figure 30.4 The possible orientations of the nuclear spin and its projections along the z axis for the case $I = \frac{3}{2}$.

Nuclear Magnetic Resonance and MRI

It is interesting that nuclear magnetic moments (as well as electronic magnetic moments) precess in an external magnetic field. The frequency at which they precess, called the **Larmor precessional frequency,** ω_p, is directly proportional to the magnetic field. This is described schematically in Figure 30.5a, where the magnetic field is along the z axis. For example, the Larmor frequency of a proton in a magnetic field of 1 T is equal to 42.577 MHz. The potential energy of a magnetic dipole moment in an external magnetic field is $-\boldsymbol{\mu} \cdot \mathbf{B}$. When the projection of $\boldsymbol{\mu}$ is along the field, the potential energy of the dipole moment is $-\mu B$; that is, it has its minimum value. When the projection of $\boldsymbol{\mu}$ is against the field, the potential energy is μB and it has its maximum value. These two energy states for a nucleus with a spin of $\frac{1}{2}$ are shown in Figure 30.5b.

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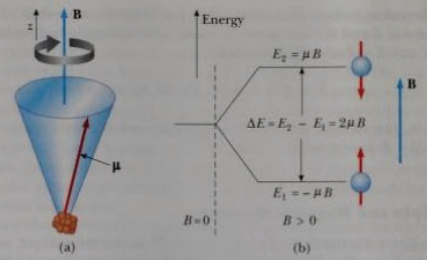


Figure 30.5 (a) When a nucleus is placed in an external magnetic field, the nuclear magnetic moment precesses about the magnetic field with a frequency that is proportional to the field. (b) A proton, the spin of which is $\frac{1}{2}$, can occupy one of two energy states when placed in an external magnetic field. The lower energy state, E_1, corresponds to the case in which the spin is aligned with the field, and the higher energy state, E_2, corresponds to the case in which the spin is opposite the field. The reverse is true for electrons.

Nuclear magnetic resonance •

It is possible to observe transitions between these two spin states by using a technique known as **nuclear magnetic resonance.** A dc magnetic field is introduced to align the magnetic moments (Fig. 30.5a), along with a second, weak, oscillating magnetic field oriented perpendicular to **B**. When the frequency of the oscillating field is adjusted to match the Larmor precessional frequency, a torque acting on the precessing moments causes them to "flip" between the two spin states. These transitions result in a net absorption of energy by the spin system. A diagram of the apparatus used to detect a nuclear magnetic resonance signal is illustrated in Figure 30.6. The absorbed energy is supplied by the generator producing the oscillating magnetic field. Nuclear magnetic resonance and a related technique called electron spin resonance are extremely important methods for studying nuclear and atomic systems and how these systems interact with their surroundings. A typical NMR spectrum is shown in Figure 30.7.

A widely used diagnostic technique called **MRI,** for **magnetic resonance imaging,** is based on nuclear magnetic resonance. In MRI, the patient is placed inside a large solenoid that supplies a spatially varying magnetic field. Because of the gradient in the magnetic field, protons in different parts of the body precess at different frequencies, so that the resonance signal can be used to provide information on the positions of the protons. A computer is used to analyze the position information to provide data for constructing a final image. An MRI scan taken on a human head is shown in Figure 30.8. The main advantage of MRI over other imaging techniques in medical diagnostics is that it causes minimal damage to cellular structures. Photons associated with the radio-frequency signals used in MRI have energies of only about 10^{-7} eV. Because molecular bond strengths are much larger (approximately 1 eV), the radio-frequency radiation causes little cellular damage. In comparison, x-rays or γ-rays have energies ranging from 10^4 to 10^6 eV and can cause considerable cellular damage.

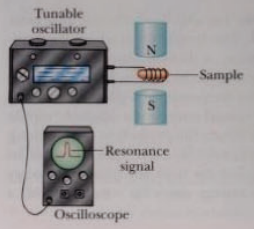


Figure 30.6 An experimental arrangement for nuclear magnetic resonance. The radio-frequency magnetic field of the coil, provided by the variable-frequency oscillator, must be perpendicular to the dc magnetic field. When the nuclei in the sample meet the resonance condition, the spins absorb energy from the field of the coil, which changes the response of the circuit in which the coil is included. Most modern NMR spectrometers use superconducting magnets at fixed field strengths and operate at frequencies of approximately 200 MHz.

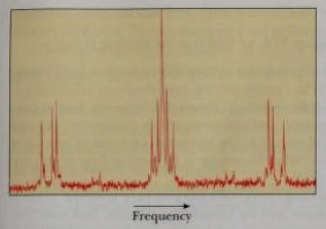


Figure 30.7 An NMR spectrum of ^{31}P in a bridged metallic complex containing platinum. The lines that flank the central strong peak are due to the interaction between ^{31}P and other distant ^{31}P nuclei. The outermost set of lines is due to the interaction between ^{31}P and neighboring platinum nuclei. The spectrum was recorded at a fixed field of about 4 T, and the mean frequency was 200 MHz.

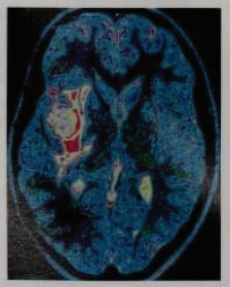


Figure 30.8 A computer-digitized, color-enhanced MRI of a brain with a glioma tumor. *(© Scott Camazine/Science Source)*

30.2 • BINDING ENERGY

The total mass of a nucleus is always less than the sum of the masses of its nucleons. Because mass is another manifestation of energy, **the total energy of the bound system (the nucleus) is less than the combined energy of the separated nucleons.** This difference in energy is called the **binding energy** of the nucleus and can be thought of as the energy that must be added to a nucleus to break it apart into its components. Therefore, in order to separate a nucleus into protons and neutrons, energy must be put into the system.

Conservation of energy and the Einstein mass–energy equivalence relationship show that the binding energy of any nucleus ${}^{A}_{Z}X$ is

$$E_b(\text{MeV}) = [ZM(\text{H}) + Nm_n - M({}^{A}_{Z}\text{X})] \times 931.494 \text{ MeV/u} \qquad [30.4]$$

• *Binding energy of a nucleus*

where $M(\text{H})$ is the atomic mass of the neutral hydrogen atom, $M({}^{A}_{Z}\text{X})$ represents the atomic mass of the element (${}^{A}_{Z}X$), m_n is the mass of the neutron, and the masses are all in atomic mass units. Note that the mass of the Z electrons included in the first term in Equation 30.4 cancels with the mass of the Z electrons included in the term $M({}^{A}_{Z}\text{X})$ within a small difference associated with the atomic binding energy of the electrons. Because atomic binding energies are typically several eV and nuclear binding energies are several MeV, this difference is negligible.

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Example 30.2 The Binding Energy of the Deuteron

Calculate the binding energy of the deuteron (the nucleus of a deuterium atom), which consists of a proton and a neutron, given that the atomic mass of deuterium is 2.014 102 u.

Solution From Table 30.1, we see that masses of a hydrogen atom and a neutron are $M(\mathrm{H}) = 1.007\,825$ u, and $m_n = 1.008\,665$ u. Therefore,

$$M(\mathrm{H}) + m_n = 2.016\,490 \text{ u}$$

To calculate the mass difference, we subtract the deuterium mass from this value:

$$\begin{aligned}\Delta m &= [M(\mathrm{H}) + m_n] - M(\mathrm{D}) \\ &= 2.016\,490 \text{ u} - 2.014\,102 \text{ u} \\ &= 0.002\,388 \text{ u}\end{aligned}$$

Using Equation 30.4, we find that the binding energy is

$$E_b = \Delta m c^2 = (0.002\,388 \text{ u})(931.494 \text{ MeV/u}) = 2.224 \text{ MeV}$$

This result tells us that, to separate a deuteron into its constituent proton and neutron, it is necessary to add 2.224 MeV of energy to the deuteron. One way of supplying the deuteron with this energy is by bombarding it with energetic particles.

If the binding energy of a nucleus were zero, the nucleus would separate into its constituent protons and neutrons without the addition of any energy; that is, it would spontaneously break apart.

A plot of binding energy per nucleon, E_b/A, as a function of mass number for various stable nuclei is shown in Figure 30.9. Except for the lighter nuclei, the average binding energy per nucleon is about 8 MeV. For the deuteron, the average binding energy per nucleon is $E_b/A = (2.224 \text{ MeV})/2 = 1.112$ MeV. Note that the curve in Figure 30.9 peaks in the vicinity of $A = 60$. That is, nuclei with mass numbers greater or less than 60 are not as strongly bound as those near the middle of the periodic table. The higher values of binding energy near $A = 60$ imply that energy is released when a heavy nucleus with $A \approx 200$ splits or fissions into several lighter nuclei which lie near $A = 60$. Energy is released in fission because the final state consisting of two lighter fragments is more tightly bound or lower in energy

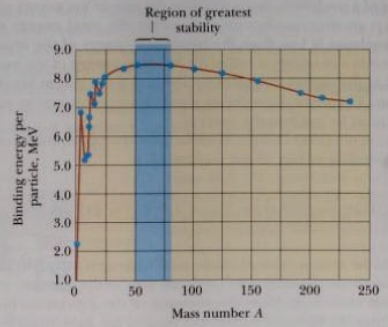


Figure 30.9 A plot of binding energy per nucleon versus mass number for the stable nuclei in Figure 30.3.

than the original nucleus. In a similar way, energy can be released when two light nuclei with $A \leq 20$ combine or fuse to form one heavier nucleus.

Another important feature of Figure 30.9 is that the binding energy per nucleon is approximately constant for $A > 20$. In this case the nuclear forces between a particular nucleon and all the other nucleons in the nucleus are said to be saturated; that is, a particular nucleon forms attractive bonds with only a limited number of other nucleons. Because of the short-range character of the nuclear force, these other nucleons can be viewed as being the nearest neighbors in the close packed structure shown in Figure 30.2.

The general features of the nuclear force responsible for the binding energy of nuclei have been revealed in a wide variety of experiments and are as follows.

- The attractive nuclear force is the strongest force in nature.
- The nuclear force is a short-range force that rapidly falls to zero when the separation between nucleons exceeds several fermis. Evidence for the limited range of nuclear forces comes from scattering experiments and from the saturation of nuclear forces previously mentioned. The short range of the nuclear force is shown in the neutron–proton (n–p) potential energy plot of Figure 30.10a obtained by scattering neutrons from a target containing hydrogen. The depth of the n–p potential energy curve is 40 MeV to 50 MeV and contains a strong repulsive component that prevents the nucleons from approaching much closer than 0.4 fm. Another interesting feature of the nuclear force is that its size depends on the relative spin orientations of the nucleons, as shown by scattering experiments using spin polarized beams and targets.

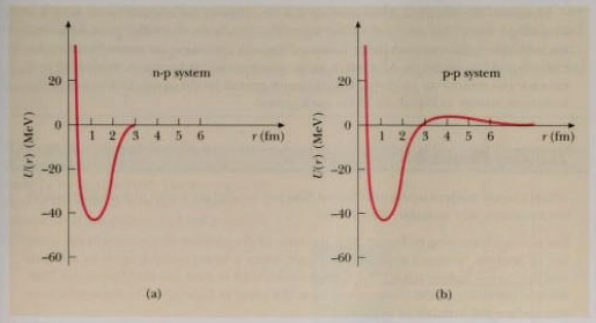


Figure 30.10 (a) Potential energy versus separation for the neutron–proton system. (b) Potential energy versus separation for the proton–proton system. The difference in the two curves is due mainly to the large Coulomb repulsion in the case of the proton–proton interaction.

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- From n–n, n–p, and p–p scattering experiments and other indirect evidence, it is found that the nuclear force is independent of the electric charge of the interacting nucleons. As might be expected from this "charge-blind" character of the nuclear force, the nuclear force does not affect electrons, enabling energetic electrons to serve as point-like probes of the charge density of nuclei. The charge independence of the nuclear force also means that the main difference between the n–p and p–p interactions is that the p–p potential energy consists of a *superposition* of nuclear and Coulomb interactions as shown in Figure 30.10b. At distances less than 2 fm, both p–p and n–p potential energies are nearly identical, but for distances greater than this, the p–p potential has a positive energy barrier with a maximum of about 1 MeV at 4 fm.

Thinking Physics 1

Figure 30.9 shows a graph of the amount of energy necessary to remove a nucleon from the nucleus. Figure 29.15 in the previous chapter shows the energy necessary to remove an electron from an atom. Why does Figure 30.9 show an *approximately constant* amount of energy necessary to remove a nucleon (above about $A = 56$), whereas Figure 29.15 shows *widely varying* amounts of energy necessary to remove an electron from the atom?

Reasoning In the case of Figure 30.9, the approximately constant value of the nuclear binding energy is a result of the short-range nature of the nuclear strong force. A given nucleon interacts only with its few nearest neighbors, rather than with all of the nucleons in the nucleus. Thus, no matter how many nucleons are present in the nucleus, pulling one nucleon out involves separating it only from its nearest neighbors. The energy to do this, therefore, is approximately independent of how many nucleons are present.

However, the electrical force holding the electrons to the nucleus in an atom is a long-range force. An electron in the atom interacts with all of the protons in the nucleus. When the nuclear charge increases, there is a stronger attraction between the nucleus and the electrons. As a result, as the nuclear charge increases, more energy is necessary to remove an electron. This is demonstrated by the upward tendency of the ionization energy in Figure 29.15 for each period.

Thinking Physics 2

When a heavy nucleus undergoes fission into just two lighter nuclei, the product nuclei are invariably very unstable. Why is this?

Reasoning According to Figure 30.3, the ratio of the number of neutrons to the number of protons increases with Z. As a result, when a heavy nucleus splits in a fission reaction to two lighter nuclei, the lighter nuclei tend to have too many neutrons. This leads to instability, as the nucleus returns to the curve in Figure 30.3 by decay processes that reduce the number of neutrons.

CONCEPTUAL PROBLEM 1

Isotopes of a given element can have different masses, different magnetic moments, but the same chemical behavior. Why is this?

30.3 • RADIOACTIVITY

In 1896 Henri Becquerel accidentally discovered that uranyl potassium sulfate crystals emitted an invisible radiation that could darken a photographic plate when the plate was covered to exclude light. After a series of experiments, he concluded that the radiation emitted by the crystals was of a new type, one that required no external stimulation and was so penetrating that it could darken protected photographic plates and ionize gases. This spontaneous emission of radiation was soon to be called **radioactivity.** Subsequent experiments by other scientists showed that other substances were also radioactive.

The most significant investigations of this type were conducted by Marie Curie (1867–1934) and Pierre Curie (1859–1906). After several years of careful and laborious chemical separation processes on tons of pitchblende, a radioactive ore, the Curies reported the discovery of two previously unknown radioactive elements. These were named polonium and radium. Subsequent experiments, including Rutherford's famous work on alpha-particle scattering, suggested that radioactivity was the result of the decay, or disintegration, of unstable nuclei.

Three types of radiation can be emitted by a radioactive substance: alpha (α) rays, where the emitted particles are ^{4}He nuclei; beta (β) rays, in which the emitted particles are either electrons or positrons; and gamma (γ) rays, in which the emitted rays are high-energy photons. A **positron** is a particle similar to the electron in all respects except that it has a charge of $+e$. (The positron is said to be the **antiparticle** of the electron.) The symbol e^- is used to designate an electron and e^+ designates a positron.

It is possible to distinguish these three forms of radiation using the scheme illustrated in Figure 30.11. The radiation from a radioactive sample is directed into a region in which there is a magnetic field. The beam splits into three components, two bending in opposite directions and the third experiencing no change in direction. From this simple observation, one can conclude that the radiation of the undeflected beam carries no charge (the gamma ray), the component deflected upward corresponds to positively charged particles (alpha particles), and the component deflected downward corresponds to negatively charged particles (e^-). If the beam includes a positron (e^+), it is deflected upward.

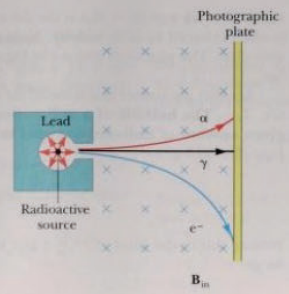


Figure 30.11 The radiation from a radioactive source can be separated into three components by a magnetic field. The photographic plate at the right records the events. The gamma ray is not deflected by the magnetic field.



Marie Curie (1867–1934)

A Polish scientist, Marie Curie shared the Nobel prize in 1903 with her husband, Pierre, and with Becquerel for their work on spontaneous radioactivity and the radiation emitted by radioactive substances. "I persist in believing that the ideas that then guided us are the only ones which can lead to the true social progress. We cannot hope to build a better world without improving the individual. Toward this end, each of us must work toward his own highest development, accepting at the same time his share of responsibility in the general life of humanity."

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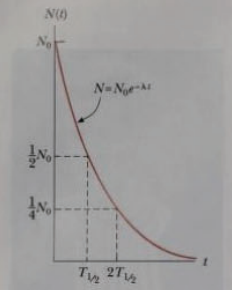


Figure 30.12 A plot of the exponential decay law for radioactive nuclei. The vertical axis represents the number of radioactive nuclei present at any time t, and the horizontal axis is time. The time $T_{1/2}$ is the half-life of the sample.

The three types of radiation have quite different penetrating powers. Alpha particles barely penetrate a sheet of paper, beta particles can penetrate a few millimeters of aluminum, and gamma rays can penetrate several centimeters of lead.

The rate at which a particular decay process occurs in a radioactive sample is proportional to the number of radioactive nuclei present (that is, those nuclei that have not yet decayed). If N is the number of radioactive nuclei present at some instant, the rate of change of N is

$$\frac{dN}{dt} = -\lambda N \qquad [30.5]$$

where λ is called either the **decay constant** or the **disintegration constant.** The minus sign indicates that dN/dt is negative; that is, N is decreasing in time.

If we write Equation 30.5 in the form

$$\frac{dN}{N} = -\lambda\, dt$$

we can integrate

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

Exponential decay •

$$N = N_0 e^{-\lambda t} \qquad [30.6]$$

The constant N_0 represents the number of radioactive nuclei at $t = 0$ and e is the base of the natural logarithm.

The **decay rate,** $R(= |dN/dt|)$, can be obtained by differentiating Equation 30.6 with respect to time:

Decay rate •

$$R = \left|\frac{dN}{dt}\right| = N_0\lambda e^{-\lambda t} = R_0 e^{-\lambda t} \qquad [30.7]$$

where $R = N\lambda$ and $R_0 = N_0\lambda$ is the decay rate at $t = 0$. The decay rate of a sample is often referred to as its **activity.** Note that both N and R decrease exponentially with time. The plot of N versus t in Figure 30.12 illustrates the exponential decay law.

Another parameter that is useful for characterizing radioactive decay is the half-life, $T_{1/2}$. **The half-life of a radioactive substance is the time it takes half of a given number of radioactive nuclei to decay.** Setting $N = N_0/2$ and $t = T_{1/2}$ in Equation 30.6 gives

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

Writing this in the form $e^{\lambda T_{1/2}} = 2$ and taking the natural logarithm of both sides, we get

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \qquad [30.8]$$

Half-life equation •

This is a convenient expression relating the half-life to the decay constant. Note that after an elapsed time of one half-life, $N_0/2$ radioactive nuclei remain (by definition); after two half-lives, half of these will have decayed and $N_0/4$ radioactive nuclei will be left; after three half-lives, $N_0/8$ will be left; and so on. In general, after n half-lives, the number of radioactive nuclei remaining is $N_0/2^n$. Thus, we see that nuclear decay is independent of the past history of the sample.

Activity can be measured with a unit called the **curie (Ci),** defined as

$$1\ \text{Ci} \equiv 3.7 \times 10^{10}\ \text{decays/s}$$

The curie •

This unit was selected as the original activity unit because it is the approximate activity of 1 g of radium. The SI unit of activity is called the **becquerel (Bq):**

$$1\ \text{Bq} \equiv 1\ \text{decay/s}$$

The becquerel •

Therefore, $1\ \text{Ci} = 3.7 \times 10^{10}\ \text{Bq}$. The most commonly used units of activity are millicuries and microcuries.

Thinking Physics 3

The isotope $^{14}_{6}$C is radioactive and has a half-life of 5730 years. If you start with a sample of 1000 carbon-14 nuclei, how many will still be around in 17 190 years?

Reasoning In 5730 years, half the sample will have decayed, leaving 500 radioactive $^{14}_{6}$C nuclei. In another 5730 years (for a total elapsed time of 11 460 years), the number will be reduced to 250 nuclei. After another 5730 years (total time 17 190 years), 125 remain.

These numbers represent ideal circumstances. Radioactive decay is an averaging process over a very large number of atoms, and the actual outcome depends on statistics. Our original sample in this example contained only 1000 nuclei, certainly not a very large number. Thus, if we were actually to count the number remaining after one half-life for this small sample, it probably would not be exactly 500.

Example 30.3 The Activity of Radium

The half-life of the radioactive nucleus $^{226}_{88}$Ra is 1.6×10^3 years. If a sample contains 3×10^{16} such nuclei, determine the activity at this initial time $t = 0$.

Solution First, let us convert the half-life to seconds:

$$T_{1/2} = (1.6 \times 10^3\ \text{years})(3.16 \times 10^7\ \text{s/year}) = 5.0 \times 10^{10}\ \text{s}$$

Now we can use this value in Equation 30.8 to get the decay constant:

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5.0 \times 10^{10}\ \text{s}} = 1.4 \times 10^{-11}\ \text{s}^{-1}$$

We can calculate the activity of the sample at $t = 0$ using $R_0 = \lambda N_0$, where R_0 is the decay rate at $t = 0$ and N_0 is the number of radioactive nuclei present at $t = 0$:

$$R_0 = \lambda N_0 = (1.4 \times 10^{-11}\ \text{s}^{-1})(3 \times 10^{16})$$
$$= 4.1 \times 10^5\ \text{decays/s}$$

Because $1\ \text{Ci} = 3.7 \times 10^{10}$ decays/s, the activity, or decay rate, at $t = 0$ is

$$R_0 = 11.1\ \mu\text{Ci}$$

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The hands and numbers of this luminous watch contain minute amounts of radium salt. The radioactive decay of radium causes the watch to glow in the dark. *(© Richard Megna 1990, Fundamental Photographs)*

Example 30.4 The Activity of Carbon

A radioactive sample contains 3.50 μg of pure $^{11}_{6}$C, which has a half-life of 20.4 min. (a) Determine the number of nuclei present initially.

Solution The atomic mass of $^{11}_{6}$C is approximately 11.0, and therefore 11.0 g contains Avogadro's number of nuclei. Hence, 3.50 μg contains N nuclei, where

$$\frac{N}{6.02 \times 10^{23}\ \text{nuclei/mol}} = \frac{3.50 \times 10^{-6}\ \text{g}}{11.0\ \text{g/mol}}$$

$$N = 1.92 \times 10^{17}\ \text{nuclei}$$

(b) What is the activity of the sample initially and after 8.00 h?

Solution Because $T_{1/2} = 20.4\ \text{min} = 1224\ \text{s}$, the decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1224\ \text{s}} = 5.66 \times 10^{-4}\ \text{s}^{-1}$$

Therefore, the initial activity of the sample is

$$R_0 = \lambda N_0 = (5.66 \times 10^{-4}\ \text{s}^{-1})(1.92 \times 10^{17})$$
$$= 1.08 \times 10^{14}\ \text{decays/s}$$

We can use Equation 30.7 to find the activity at any time t. For $t = 8.00\ \text{h} = 2.88 \times 10^{4}$ s, we see that $\lambda t = 16.3$, and so

$$R = R_0 e^{-\lambda t} = (1.09 \times 10^{14}\ \text{decays/s})e^{-16.3}$$
$$= 8.96 \times 10^{6}\ \text{decays/s}$$

EXERCISE 1 Calculate the number of radioactive nuclei remaining after 8.00 h. Answer $N = 1.58 \times 10^{10}$ nuclei

30.4 • THE DECAY PROCESSES

As we stated in the preceding section, a radioactive nucleus spontaneously decays via alpha, beta, or gamma decay. Let us discuss these three processes in more detail.

Alpha Decay

If a nucleus emits an alpha particle ($^{4}_{2}$He), it loses two protons and two neutrons. Therefore, N decreases by 2, Z decreases by 2, and A decreases by 4. The decay can be written symbolically as

Alpha decay •

$$^{A}_{Z}\text{X} \longrightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\text{He} \qquad [30.9]$$

where X is called the **parent nucleus** and Y the **daughter nucleus.** As examples, ^{238}U and ^{226}Ra are both alpha emitters and decay according to the schemes

$$^{238}_{92}\text{U} \longrightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He} \qquad [30.10]$$

$$^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He} \qquad [30.11]$$

The half-life for ^{238}U decay is 4.47×10^{9} years, and the half-life for ^{226}Ra decay is 1.60×10^{3} years. In both cases, note that the mass number A of the daughter nucleus is 4 less than that of the parent nucleus. Likewise, Z is reduced by 2. The differences are accounted for in the emitted alpha particle (the ^{4}He nucleus).

The decay of ^{226}Ra is shown in Figure 30.13. When one element changes into another, as happens in alpha decay, the process is called **spontaneous decay.** As a general rule, (1) the sum of the mass numbers A must be the same on both sides of the equation, and (2) the sum of the atomic numbers Z must be the same on both sides of the equation. In addition, the total energy must be conserved. If we call M_X the mass of the parent nucleus, M_Y the mass of the daughter nucleus, and M_α the mass of the alpha particle, we can define the **disintegration energy, Q:**

$$Q \equiv (M_X - M_Y - M_\alpha)c^2 \qquad [30.12]$$

The quantity given by Equation 30.12 is sometimes referred to as the Q value of the nuclear reaction. Note that Q will be in joules if the masses are in kilograms, and c is the usual 3.00×10^{8} m/s. We make this expression more convenient for calculations by including in all terms the masses of enough electrons to make neutral atoms. Also, we convert atomic mass units for mass to MeV units for energy:

$$Q = [M(\text{X}) - M(\text{Y}) - M(^{4}\text{He})] \times 931.494\ \text{MeV/u} \qquad [30.13]$$

The disintegration energy Q appears in the form of kinetic energy of the daughter nucleus and the alpha particle. In the case of the ^{226}Ra decay described in Figure 30.13, if the parent nucleus decays at rest, the residual kinetic energy of the products is 4.87 MeV. Most of the kinetic energy is associated with the alpha particle because this particle is much less massive than the recoiling daughter nucleus. That is, because momentum must be conserved, the lighter alpha particle recoils with a much higher speed than the daughter nucleus. Generally, light particles carry off most of the energy in nuclear decays.

Finally, it is interesting to note that if one assumed that ^{238}U (or other alpha emitters) decayed by emitting protons and neutrons, the mass of the decay products would exceed that of the parent nucleus, corresponding to negative Q values. Therefore, such spontaneous decays do not occur.

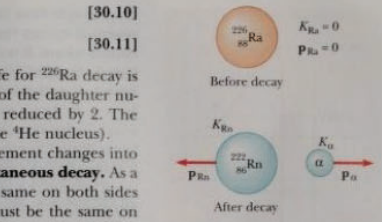


Figure 30.13 Alpha decay of radium. The radium nucleus is initially at rest. After the decay, the radon nucleus has kinetic energy K_{Rn} and momentum $\mathbf{p}_{Rn}$, and the alpha particle has kinetic energy K_α and momentum $\mathbf{p}_\alpha$.

Example 30.5 The Energy Liberated When Radium Decays

The ^{226}Ra nucleus undergoes alpha decay according to Equation 30.11. Calculate the Q value for this process. Take the mass of ^{226}Ra to be 226.025 406 u, the mass of ^{222}Rn to be 222.017 574 u, and the mass of $^{4}_{2}$He to be 4.002 603 u, as found in Table A.3.

Solution Using Equation 30.13, we see that

$$Q = [M(^{226}\text{Ra}) - M(^{222}\text{Rn}) - M(^{4}\text{He})] \times 931.494\ \text{MeV/u}$$
$$= (226.025\ 406\ \text{u} - 222.017\ 574\ \text{u} - 4.002\ 603\ \text{u}) \times 931.494\ \text{MeV/u}$$
$$= (0.005\ 229\ \text{u}) \times \left(931.494\ \frac{\text{MeV}}{\text{u}}\right) = 4.87\ \text{MeV}$$

It is left to end-of-chapter Problem 49 to show that the kinetic energy of the alpha particle is about 4.8 MeV, whereas that of the recoiling daughter nucleus is only about 0.1 MeV.

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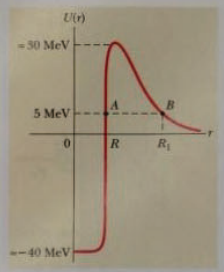


Figure 30.14 Potential energy versus separation for the alpha particle–nucleus system. Classically, the energy of the alpha particle is not sufficient to overcome the barrier, and so the particle should not be able to escape the nucleus.

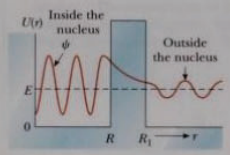


Figure 30.15 The nuclear potential energy is modeled as a square barrier. The energy of the alpha particle is E, which is less than the height of the barrier. According to quantum mechanics, the alpha particle has some chance of tunneling through the barrier, as indicated by the finite size of the wave function for $r > R_1$.

We now turn to the mechanism of alpha decay. Figure 30.14 is a plot of the potential energy versus distance r from the nucleus for the alpha particle–nucleus system, where R is the range of the nuclear force. The curve represents the combined effects of (1) the Coulomb repulsive energy, which gives the positive peak for $r > R$, and (2) the nuclear attractive force, which causes the curve to be negative for $r < R$. As we saw in Example 30.5, the disintegration energy is about 5 MeV, which is the approximate kinetic energy of the alpha particle, represented by the lower dotted line in Figure 30.14. According to classical physics, the alpha particle is trapped in the potential well. How, then, does it ever escape from the nucleus?

The answer to this question was provided by Gamow and, independently, Gurney and Condon in 1928, using quantum mechanics. In brief, the view of quantum mechanics is that there is always some probability that the particle can penetrate (or tunnel) through the barrier (Section 28.11). Recall that the probability of locating the particle depends on its wave function, ψ, and that the probability of tunneling is measured by $|\psi|^2$. Figure 30.15 is a sketch of the wave function for a particle of energy E, meeting a square barrier of finite height, which approximates the nuclear barrier. Note that the wave function is oscillating both inside and outside the barrier but is greatly reduced in amplitude because of the barrier. As the energy E increases, the probability of escaping also increases. Furthermore, the probability increases as the width of the barrier is decreased.

Beta Decay

When a radioactive nucleus undergoes beta decay, the daughter nucleus has the same number of nucleons as the parent nucleus, but the atomic number is changed by 1:

$$ {}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z+1}\mathrm{Y} + \mathrm{e}^- \qquad [30.14] $$

$$ {}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z-1}\mathrm{Y} + \mathrm{e}^+ \qquad [30.15] $$

Again, note that the nucleon number and total charge are both conserved in these decays. However, as we shall see later, these processes are not described completely by these expressions. We shall give reasons for this shortly.

It is important to note that the electron or positron involved in these decays is created within the nucleus as an initial step in the decay process. This is equivalent to saying that, during beta decay, a neutron in the nucleus is transformed into a proton. Indeed, the process $\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^-$ does occur, although it is extremely improbable outside the nucleus.

Now consider the energy of the system before and after the decay. As with alpha decay, energy must be conserved. One finds experimentally that the beta particles are emitted over a continuous range of energies (Fig. 30.16). The kinetic energy of the electrons must be balanced by the decrease in mass of the system—that is, the Q value. However, because all decaying nuclei have the same initial mass, **the Q value must be the same for each decay.** In view of this, why do the emitted electrons have different kinetic energies? The principle of conservation of energy seems to be violated! Further analysis shows that, according to the decay processes given by Equations 30.14 and 30.15, the principles of conservation of both angular momentum (spin) and linear momentum are also violated!

After a great deal of experimental and theoretical study, Pauli in 1930 proposed that a third particle must be present to carry away the "missing" energy and momentum. Fermi later named this particle the **neutrino** (little neutral one), because it had to be electrically neutral and have little or no rest mass. Although it eluded detection for many years, the neutrino (symbol ν) was finally detected experimentally in 1956. It has the following properties:

- Zero electric charge;
- A mass smaller than that of the electron, and it is possibly zero (recent experiments suggest that mass of the neutrino is less than 7 eV/c^2);
- A spin of $\frac{1}{2}$, which allows the law of conservation of angular momentum to be satisfied in beta decay;
- Very weak interaction with matter, which, therefore, makes it quite difficult to detect.

We can now write the beta decay processes in their correct form for carbon-14, nitrogen-12, and an isolated neutron:

$$ {}^{14}_{6}\mathrm{C} \longrightarrow {}^{14}_{7}\mathrm{N} + \mathrm{e}^- + \bar{\nu} \qquad [30.16] $$

$$ {}^{12}_{7}\mathrm{N} \longrightarrow {}^{12}_{6}\mathrm{C} + \mathrm{e}^+ + \nu \qquad [30.17] $$

$$ \mathrm{n} \longrightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu} \qquad [30.18] $$

where $\bar{\nu}$ represents the **antineutrino,** the antiparticle to the neutrino. We shall discuss antiparticles further in Chapter 31. For now, it suffices to say that **a neutrino is emitted in positron decay, and an antineutrino is emitted in electron decay.** Tables list isotopic masses of neutral atoms. To refer to neutral atoms in the first two processes above, we must add six electrons to each side of Equation 30.16 and seven electrons to each side of Equation 30.17.

A process that competes with e^+ decay is called **electron capture.** This occurs when a parent nucleus captures one of its own orbital electrons and emits a neutrino. The final product after decay is a nucleus the charge of which is $Z - 1$:

$$ {}^{A}_{Z}\mathrm{X} + {}^{0}_{-1}\mathrm{e} \longrightarrow {}^{A}_{Z-1}\mathrm{Y} + \nu \qquad [30.19] $$

• *Electron capture process*

In most cases, it is an inner K-shell electron that is captured, and this is referred to as **K capture.**

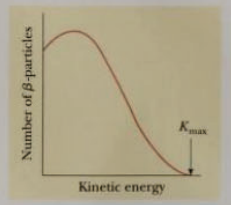


Figure 30.16 A typical beta-decay curve. The maximum kinetic energy observed for the beta particles corresponds to the value of Q for the reaction.

Carbon Dating

The beta decay of $^{14}\mathrm{C}$ given by Equation 30.16 is used to date organic samples. Cosmic rays (high-energy particles from outer space) in the upper atmosphere cause nuclear reactions that create $^{14}\mathrm{C}$. The ratio of $^{14}\mathrm{C}$ to $^{12}\mathrm{C}$ in the carbon dioxide molecules of our atmosphere has a constant value of about 1.3×10^{-12}. All living organisms have the same ratio of $^{14}\mathrm{C}$ to $^{12}\mathrm{C}$ because they continuously exchange carbon dioxide with their surroundings. When an organism dies, however, it no longer absorbs $^{14}\mathrm{C}$ from the atmosphere, and so the ratio of $^{14}\mathrm{C}$ to $^{12}\mathrm{C}$ decreases as the result of the beta decay of $^{14}\mathrm{C}$, which has a half-life of 5730 years. It is therefore possible to determine the age of a material by measuring its activity per unit mass due to the decay of $^{14}\mathrm{C}$. Using carbon dating, samples of wood, charcoal, bone, and shell have been identified as having lived 1000 to 25 000 years ago.

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A particularly interesting example is the dating of the Dead Sea Scrolls, a group of manuscripts discovered by a shepherd in 1947. Translation showed them to be religious documents, including most of the books of the Old Testament. Carbon dating applied to the material in which they were wrapped established their age at approximately 1950 years at the time of discovery.

Thinking Physics 4

In 1991, a German tourist discovered the well-preserved remains of the Ice Man trapped in a glacier in the Italian Alps. Radioactive dating of a sample of Ice Man revealed an age of 5300 years. Why did scientists date the sample using the isotope ^{14}C rather than ^{11}C, a beta emitter with a half-life of 20.4 min?

Reasoning ^{14}C has a long half-life of 5730 years, so the fraction of ^{14}C nuclei remaining after one half-life is high enough to measure accurate changes in the sample's activity. The ^{11}C isotope, which has a very short half-life, is not useful because its activity decreases to a vanishingly small value over the age of the sample, making it impossible to detect.



The Ice Man was discovered in 1991 when an Italian glacier melted enough to expose his remains. His possessions, particularly his tools, have shed light on the way people lived in the Bronze Age. Archaeologists used ^{14}C dating to determine the time frame of this discovery. *(Paul Hanny/Gamma Liaison)*

Thinking Physics 5

In comparing alpha decay energies from a number of radioactive nuclides, it is found that the half life of the decay goes down as the energy of the decay goes up. Why is this?

Reasoning As discussed in the text, the alpha particle in the nucleus is trapped in a potential well and escapes by tunneling through the barrier. In Figure 30.14, it is clear that the higher the energy of the alpha particle, the thinner is the potential barrier. The thinner barrier translates to a higher probability of escape. The higher probability of escape translates to a faster rate of decay, which appears as a shorter half-life.

Thinking Physics 6

A wooden coffin is found that contains a skeleton holding a gold statue. Which of the three objects (coffin, skeleton, and gold statue) can be carbon dated to find out how old it is?

Reasoning Only the coffin and the skeleton can be carbon dated. These two items exchanged air with the environment during their lifetimes, resulting in a fixed ratio of carbon-14 to carbon-12. Once the human and the tree died, the amount of carbon-14 began to decrease due to radioactive decay. The gold in the statue was never alive and did not breathe air; therefore, there is no carbon-14 to detect.

CONCEPTUAL PROBLEM 2

Can carbon-14 dating be used to measure the age of a stone?

Example 30.6 Radioactive Dating

A piece of charcoal of mass 25 g is found in the ruins of an ancient city. The sample shows a ^{14}C ($T_{1/2} = 5730$ years) activity of 250 decays/min. How long has the tree from which this charcoal came been dead?

Solution First, let us calculate the decay constant for ^{14}C:

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(5730\text{ y})(3.16 \times 10^7\text{ s/y})} = 3.83 \times 10^{-12}\text{ s}^{-1}$$

The number of ^{14}C nuclei can be calculated in two steps. First, the number of ^{12}C nuclei in 25 g of carbon is

$$N(^{12}C) = \frac{6.02 \times 10^{23}\text{ nuclei/mol}}{12\text{ g/mol}}(25\text{ g}) = 1.25 \times 10^{24}\text{ nuclei}$$

Assuming that the ratio of ^{14}C to ^{12}C is 1.3×10^{-12}, we see that the number of ^{14}C nuclei in 25 g *before* decay is

$$N_0(^{14}C) = (1.3 \times 10^{-12})(1.26 \times 10^{24}) = 1.63 \times 10^{12}\text{ nuclei}$$

Hence, the initial activity of the sample is

$$R_0 = N_0\lambda = (1.63 \times 10^{12}\text{ nuclei})(3.83 \times 10^{-12}\text{ s}^{-1}) = 6.25\text{ decays/s} = 375\text{ decays/min}$$

We can now calculate the age of the charcoal using Equation 30.7, which relates the activity R at any time t to the initial activity, R_0:

$$R = R_0e^{-\lambda t} \quad \text{or} \quad e^{-\lambda t} = \frac{R}{R_0}$$

Because it is given that $R = 250$ decays/min, and because we found that $R_0 = 375$ decays/min, we can calculate t by taking the natural logarithm of both sides of the last equation:

$$-\lambda t = \ln\left(\frac{R}{R_0}\right) = \ln\left(\frac{250}{375}\right) = -0.405$$

$$t = \frac{0.405}{\lambda} = \frac{0.405}{3.84 \times 10^{-12}\text{ s}^{-1}} = 1.06 \times 10^{11}\text{ s} = 3350\text{ years}$$

As discussed previously, this technique of radioactive-carbon dating has been successfully used to measure the ages of many organic relics up to 25 000 years old.

Gamma Decay

Very often, a nucleus that undergoes radioactive decay is left in an excited energy state. It can then undergo a second decay to a lower energy state, perhaps to the ground state, by emitting a photon. Photons emitted in such a de-excitation process are called *gamma rays*. Such photons have very high energy (in the range of 1 MeV to 1 GeV) relative to the energy of visible light (about 1 eV). Recall from Chapter 29 that the energy of photons emitted (or absorbed) by an atom equals

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the difference in energy between the two electronic states involved in the transition. In a similar way, a gamma ray photon has an energy, hf, that equals the energy difference, ΔE, between two nuclear energy levels. When a nucleus decays by emitting a gamma ray, the nucleus does not change, apart from the fact that it ends up in a lower energy state. We can represent gamma decay as

Gamma decay •

$$ {}^{A}_{Z}\mathrm{X}^{*} \longrightarrow {}^{A}_{Z}\mathrm{X} + \gamma \qquad [30.20] $$

where X* indicates a nucleus in an excited state.

A nucleus may reach an excited state as the result of a violent collision with another particle. However, it is more common for a nucleus to be in an excited state after it has undergone an alpha or beta decay. The following sequence of events represents a typical situation in which gamma decay occurs:

$$ {}^{12}_{5}\mathrm{B} \longrightarrow {}^{12}_{6}\mathrm{C}^{*} + {}^{0}_{-1}\mathrm{e} + \bar{\nu} \qquad [30.21] $$

$$ {}^{12}_{6}\mathrm{C}^{*} \longrightarrow {}^{12}_{6}\mathrm{C} + \gamma \qquad [30.22] $$

Figure 30.17 shows the decay scheme for ^{12}B, which undergoes beta decay to either of two levels of ^{12}C. It can either (1) decay directly to the ground state of ^{12}C by emitting an electron with energy up to 13.4 MeV or (2) undergo beta decay to an excited state of ^{12}C*, followed by gamma decay to the ground state. The latter process results in the emission of an electron with energy up to 9.0 MeV and a 4.4-MeV photon.

The pathways by which a radioactive nucleus can undergo decay are summarized in Table 30.2.

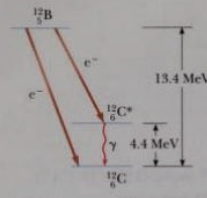


Figure 30.17 The ^{12}B nucleus undergoes beta decay to two levels of ^{12}C. The decay to the excited level, ^{12}C*, is followed by gamma decay to the ground state.

TABLE 30.2 Decay Pathways

Alpha decay	${}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A-4}_{Z-2}\mathrm{X} + {}^{4}_{2}\mathrm{He}$
Beta decay (e^-)	${}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z+1}\mathrm{X} + e^- + \bar{\nu}$
Beta decay (e^+)	${}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z-1}\mathrm{X} + e^+ + \nu$
Electron capture	${}^{A}_{Z}\mathrm{X} + {}^{0}_{-1}\mathrm{e} \longrightarrow {}^{A}_{Z-1}\mathrm{X} + \nu$
Gamma decay	${}^{A}_{Z}\mathrm{X}^{*} \longrightarrow {}^{A}_{Z}\mathrm{X} + \gamma$

CONCEPTUAL PROBLEM 3

In beta decay, the energy of the electron or positron emitted from the nucleus lies somewhere in a relatively large range of possibilities. In alpha decay, however, the alpha particle energy can only have discrete values. Why is there this difference?

CONCEPTUAL PROBLEM 4

In positron decay, a proton in the nucleus becomes a neutron, and the positive charge is carried away by the positron. But a neutron has a larger rest energy than a proton. How is this possible?

CONCEPTUAL PROBLEM 5

A student claims that a heavy form of hydrogen decays by alpha emission. How do you respond?

TABLE 30.3 The Four Radioactive Series

Series	Starting Isotope	Half-Life (years)	Stable End Product
Uranium (natural)	$^{238}_{92}$U	4.47×10^{9}	$^{206}_{82}$Pb
Actinium (natural)	$^{235}_{92}$U	7.04×10^{8}	$^{207}_{82}$Pb
Thorium (natural)	$^{232}_{90}$Th	1.41×10^{10}	$^{208}_{82}$Pb
Neptunium	$^{237}_{93}$Np	2.14×10^{6}	$^{209}_{83}$Bi

30.5 • NATURAL RADIOACTIVITY

Radioactive nuclei are generally classified into two groups: (1) unstable nuclei that are found in nature, which give rise to what is called **natural radioactivity,** and (2) nuclei produced in the laboratory through nuclear reactions, which exhibit **artificial radioactivity.**

Three series of radioactive nuclei occur naturally (Table 30.3). Each series starts with a specific long-lived radioactive isotope the half-life of which exceeds that of any of its descendents. The fourth series in Table 30.3 begins with ^{237}Np, a transuranic element (one having an atomic number greater than that of uranium) not found in nature. This element has a half-life of "only" 2.14×10^{6} years.

The two uranium series are somewhat more complex than the ^{232}Th series (Fig. 30.18). Also, there are several naturally occurring radioactive isotopes, such as ^{14}C and ^{40}K, that are not part of either decay series.

Natural radioactivity constantly resupplies our environment with radioactive elements that would otherwise have disappeared long ago. For example, because the Solar System is about 5×10^{9} years old, the supply of ^{226}Ra (the half-life of which is only 1600 years) would have been depleted by radioactive decay long ago if it were not for the decay series that starts with ^{238}U, with a half-life of 4.47×10^{9} years.

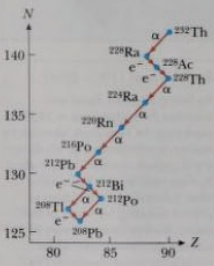


Figure 30.18 Successive decays for the ^{232}Th series.

30.6 • NUCLEAR REACTIONS

It is possible to change the structures and properties of nuclei by bombarding them with energetic particles. Such changes are called **nuclear reactions.** In 1919 Rutherford was the first to observe nuclear reactions, using naturally occurring radioactive sources for the bombarding particles. Since then, thousands of nuclear reactions have been observed following the development of charged-particle accelerators in the 1930s. With today's advanced technology in particle accelerators and particle detectors, it is possible to achieve particle energies of at least 1000 GeV = 1 TeV. These high-energy particles are used to create new particles the properties of which are helping to solve the mysteries of the nucleus.

Consider a reaction in which a target nucleus, X, is bombarded by a particle, a, resulting in a nucleus, Y, and a particle, b:

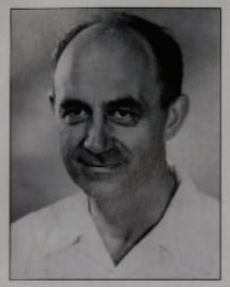
$$ \mathrm{a} + \mathrm{X} \longrightarrow \mathrm{Y} + \mathrm{b} \qquad [30.23] $$

• *Nuclear reaction*

Sometimes this reaction is written in the more compact form

$$ \mathrm{X(a, b)Y} $$

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Enrico Fermi (1901–1954)

An Italian physicist, Fermi was awarded the Nobel prize in 1938 for producing the transuranic elements by neutron irradiation and for his discovery of nuclear reactions bought about by slow neutrons. He made many other outstanding contributions to physics including his theory of beta decay, the free electron theory of metals, and the development of the world's first fission reactor in 1942. Fermi was truly a gifted theoretical and experimental physicist. He was also well known for his ability to present physics in a clear and exciting manner. "Whatever Nature has in store for mankind, unpleasant as it may be, men must accept, for ignorance is never better than knowledge."

In Section 30.4, the Q value, or disintegration energy, associated with radioactive decay was defined as the energy released during the decay process. Likewise, we define the **reaction energy, Q_r,** associated with a nuclear reaction as **the total energy released as the result of the reaction:**

$$Q_r = (M_a + M_X - M_Y - M_b)c^2 \qquad [30.24]$$

A reaction such as this, for which Q_r is positive, is called **exothermic.** After the reaction, this energy appears as an increase in kinetic energy of (Y, b). The loss of energy (and mass) of the system is balanced by an increase in the kinetic energy (and mass) of the escaping particles. A reaction for which Q_r is negative is called **endothermic.** An endothermic reaction will not occur unless the bombarding particle has a kinetic energy greater than Q_r. The minimum energy necessary for such a reaction to occur is called the **threshold energy.**

Nuclear reactions must obey the principle of conservation of linear momentum. That is, the total linear momentum of the system of interacting particles must be the same before and after the reaction. This assumes that the only force acting on the interacting particles is their mutual force of interaction; that is, no external accelerating electric fields are present near the colliding particles.

If a nuclear reaction occurs in which particles a and b are identical, so that X and Y are also necessarily identical, the reaction is called a **scattering event.** If kinetic energy is constant as a result of the reaction (that is, if $Q_r = 0$), the event is classified as **elastic scattering.** However, if $Q_r \neq 0$, kinetic energy is not constant and the reaction is called **inelastic scattering.** This terminology is identical to that used in dealing with collisions between macroscopic objects (Chapter 8, Section 8.4).

In addition to energy and momentum, the total charge and total number of nucleons must be conserved in any nuclear reaction. For example, consider the reaction $^{19}F(p, \alpha)^{16}O$, which has a Q_r value of 8.124 MeV. We can show this reaction more completely as

$$^1_1H + ^{19}_9F \longrightarrow ^{16}_8O + ^4_2He$$

We see that the total number of nucleons before the reaction ($1 + 19 = 20$) is equal to the total number after the reaction ($16 + 4 = 20$). Furthermore, the total charge ($Z = 10$) is the same before and after the reaction.

SUMMARY

A nuclear species can be represented by $^A_Z X$ where A is the **mass number,** the total number of nucleons, and Z is the **atomic number,** the total number of protons. The total number of neutrons in a nucleus is the **neutron number, N,** where $A = N + Z$. Elements with the same Z but different A and N values are called **isotopes.**

Assuming that a nucleus is spherical, its radius is

$$r = r_0 A^{1/3} \qquad [30.1]$$

where $r_0 = 1.2$ fm.

Nuclei are stable because of the **nuclear force** between nucleons. This short-range force dominates the Coulomb repulsive force at distances of less than about 2 fm and is independent of charge.

Light nuclei are most stable when the number of protons equals the number of neutrons. Heavy nuclei are most stable when the number of neutrons exceeds the number of protons. In addition, many stable nuclei have Z and N values that are both even. Nuclei with unusually high stability have Z or N values of 2, 8, 20, 28, 50, 82, and 126, called **magic numbers.**

Nuclei have an intrinsic spin angular momentum of magnitude $\sqrt{I(I+1)}\hbar$, where I is the **nuclear spin.** The magnetic moment of a nucleus is measured in terms of the **nuclear magneton,** μ_n, where

$$\mu_n = 5.05 \times 10^{-27}\ \text{J/T} \qquad [30.3]$$

The difference in mass between the separate nucleons and the nucleus containing these nucleons, when multiplied by c^2, gives the **binding energy, E_b,** of the nucleus: $E_b = \Delta m c^2$. We can calculate the binding energy of any nucleus $^A_Z X$ using the expression

$$E_b(\text{MeV}) = [ZM(\text{H}) + Nm_n - M(^A_Z X)] \times 931.494\ \text{MeV/u} \qquad [30.4]$$

A radioactive substance can undergo alpha decay, beta decay, or gamma decay. An alpha particle is the ^{4}He nucleus; a beta particle is either an electron (e^-) or a positron (e^+); a gamma particle is a high-energy photon.

If a radioactive material contains N_0 radioactive nuclei at $t = 0$, the number, N, of nuclei remaining after a time t has elapsed is

$$N = N_0 e^{-\lambda t} \qquad [30.6]$$

where λ is the **decay constant,** or **disintegration constant.** The *decay rate,* or *activity,* of a radioactive substance is given by

$$R = \left|\frac{dN}{dt}\right| = R_0 e^{-\lambda t} \qquad [30.7]$$

where $R_0 = N_0\lambda$ is the activity at $t = 0$. The **half-life,** $T_{1/2}$, is defined as the time it takes half of a given number of radioactive nuclei to decay, where

$$T_{1/2} = \frac{0.693}{\lambda} \qquad [30.8]$$

Alpha decay can occur because, according to quantum mechanics, some nuclei have barriers that can be penetrated by the alpha particles (the tunneling process). This process is energetically more favorable for those nuclei having large excesses of neutrons. A nucleus can undergo beta decay in two ways. It can emit either an electron (e^-) and an antineutrino ($\bar{\nu}$) or a positron (e^+) and a neutrino (ν). In the electron capture process, the nucleus of an atom absorbs one of its own electrons (usually from the K shell) and emits a neutrino. In gamma decay, a nucleus in an excited state decays to its ground state and emits a gamma ray.

Nuclear reactions can occur when a target nucleus, X, is bombarded by a particle, a, resulting in a nucleus, Y, and a particle, b:

$$a + X \longrightarrow Y + b \quad \text{or} \quad X(a, b)Y \qquad [30.23]$$

The energy released in such a reaction, called the **reaction energy, Q_r,** is

$$Q_r = (M_a + M_X - M_Y - M_b)c^2 \qquad [30.24]$$

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CONCEPTUAL QUESTIONS

1. In Rutherford's experiment, assume that an alpha particle is headed directly toward the nucleus of an atom. Why doesn't the alpha particle make physical contact with the nucleus?
2. Explain the main differences between alpha, beta, and gamma rays.
3. Why do heavier elements require more neutrons in order to maintain stability?
4. A proton precesses with a frequency ω_p in the presence of a magnetic field. If the magnetic field intensity is doubled, what happens to the precessional frequency?
5. Why do nearly all the naturally occurring isotopes lie above the $N = Z$ line in Figure 30.3?
6. If a nucleus has a half-life of 1 year, does this mean it will be completely decayed after 2 years? Explain.
7. What fraction of a radioactive sample has decayed after two half-lives have elapsed?
8. Two samples of the same radioactive nuclide are prepared. Sample A has twice the initial activity of sample B. How does the half-life of A compare with the half-life of B? After each has passed through five half-lives, what is the ratio of their activities?
9. If no more people were to be born, the world's population would strongly resemble radioactive decay behavior. Discuss this statement.
10. If a nucleus captures a slow-moving neutron, the product is left in a highly excited state with an energy approximately 8.0 MeV above the ground state. Explain the source of the excitation energy.
11. Use the analogy of a bullet and a rifle to explain why the recoiling nucleus carries off only a very small fraction of the disintegration energy in the α decay of a nucleus.
12. If a nucleus such as ^{226}Ra initially at rest undergoes alpha decay, which has more kinetic energy after the decay, the alpha particle or the daughter nucleus?
13. Explain why many heavy nuclei undergo alpha decay but do not spontaneously emit neutrons or protons.
14. If an alpha particle and an electron have the same kinetic energy, which undergoes the greater deflection when passed through a magnetic field?
15. If film is kept in a wooden box, alpha particles from a radioactive source outside the box cannot expose the film, but beta particles can. Explain.
16. Suppose it could be shown that the cosmic ray intensity at the Earth's surface was much greater 10 000 years ago. How would this difference affect what we accept as valid carbon-dated values of the age of ancient samples of once-living matter?
17. The radioactive nucleus $^{226}_{88}$Ra has a half-life of approximately 1.6×10^3 years. Being that the solar system is about 5 billion years old, why do we still find this nucleus in nature?
18. From Table A.3, identify the four stable nuclei that have magic numbers in both Z and N.

PROBLEMS

(Table 30.4 will be useful for many of these problems. A more complete list of atomic masses is given in Appendix A.3.)

Section 30.1 Some Properties of Nuclei

1. What is the order of magnitude of the number of protons in your body? Of the number of neutrons? Of the number of electrons?
2. Find the radius of (a) a nucleus of $^4_2$He and (b) a nucleus of $^{238}_{92}$U.
3. Determine the atomic mass number of isotopes that have a radius approximately equal to one-half the radius of uranium, $^{238}_{92}$U.
4. What would be the gravitational force between two golf balls (each with a 4.30-cm diameter), 1.00 meter apart, if they were made of nuclear matter?
5. (a) Find the speed an alpha particle must have in order to come within 3.20×10^{-14} m of a gold nucleus. (b) Find the energy of the alpha particle in MeV units.

TABLE 30.4 Some Atomic Masses

Element	Atomic Mass (u)	Element	Atomic Mass (u)
$^1_1$H	1.007 825	$^{27}_{13}$Al	26.981 541
$^4_2$He	4.002 603	$^{30}_{15}$P	29.978 310
$^7_3$Li	7.016 004	$^{40}_{20}$Ca	39.962 591
$^9_4$Be	9.012 182	$^{42}_{20}$Ca	41.958 63
$^{10}_5$B	10.012 938	$^{43}_{20}$Ca	42.958 770
$^{12}_6$C	12.000 000	$^{56}_{26}$Fe	55.934 939
$^{13}_6$C	13.003 355	$^{64}_{30}$Zn	63.929 145
$^{14}_7$N	14.003 074	$^{64}_{29}$Cu	63.929 599
$^{15}_7$N	15.000 109	$^{93}_{41}$Nb	92.906 378
$^{15}_8$O	15.003 065	$^{197}_{79}$Au	196.966 560
$^{17}_8$O	16.999 131	$^{202}_{80}$Hg	201.970 632
$^{18}_8$O	17.999 159	$^{210}_{84}$Po	216.001 790
$^{18}_9$F	18.000 937	$^{220}_{86}$Rn	220.011 401
$^{20}_{10}$Ne	19.992 439	$^{234}_{90}$Th	234.043 583
$^{23}_{11}$Na	22.989 770	$^{238}_{92}$U	238.050 786
$^{24}_{12}$Mg	22.994 127		

6. An α particle ($Z = 2$, mass 6.64×10^{-27} kg) approaches to within 1.00×10^{-14} m of a carbon nucleus ($Z = 6$). What are (a) the maximum Coulomb force on the α particle, (b) the acceleration of the α particle at this point, and (c) the potential energy of the α particle at this point?
7. Certain stars at the end of their lives are thought to collapse, combining their protons and electrons to form a neutron star. Such a star could be thought of as a gigantic atomic nucleus. If a star of mass equal to that of the Sun ($M = 1.99 \times 10^{30}$ kg) collapsed into neutrons ($m_n = 1.67 \times 10^{-27}$ kg), what would the radius be? (*Hint:* Use $r = r_0 A^{1/3}$.)
8. The Larmor precessional frequency is

$$f_p = \frac{\Delta E}{h} = \frac{2\mu B}{h}$$

Calculate the radio-wave frequency at which resonance absorption will occur for (a) free neutrons in a magnetic field of 1.00 T, (b) free protons in a magnetic field of 1.00 T, and (c) free protons in the Earth's magnetic field at a location where the field strength is 50.0 μT.

9. How much energy (in MeV units) must an α particle have to reach the surface of a gold nucleus ($Z = 79$, $A = 197$)?
10. In a Rutherford scattering experiment, alpha particles having kinetic energy of 7.70 MeV are fired toward a gold foil. (a) Use energy conservation to determine the distance of closest approach between the alpha particle and a gold nucleus. Imagine a head-on collision with a nucleus that remains at rest. (b) Calculate the de Broglie wavelength for the 7.70-MeV alpha particle and compare it to the distance obtained in part (a). (c) Based on this comparison, why is it proper to treat the alpha particle as a particle and not as a wave in the Rutherford scattering experiment?

Section 30.2 Binding Energy

11. Calculate the binding energy per nucleon for (a) ^{2}H, (b) ^{4}He, (c) ^{56}Fe, and (d) ^{238}U.
12. The peak of the stability curve occurs at ^{56}Fe. This is why iron is prominent in the spectrum of the Sun and stars. Show that ^{56}Fe has a higher binding energy per nucleon than its neighbors ^{55}Mn and ^{59}Co. Compare your results with Figure 30.9.
13. Calculate the minimum energy required to remove a neutron from the $^{40}_{20}$Ca nucleus.
14. Two nuclei having the same mass number are known as *isobars*. Calculate the difference in binding energy per nucleon for the isobars $^{23}_{11}$Na and $^{23}_{12}$Mg. How do you account for the difference?
15. The isotope $^{139}_{57}$La is stable. A radioactive isobar (see Problem 14) of this lanthanum isotope, $^{139}_{59}$Pr, is located below the line of stable nuclei in Figure 30.3 and decays by e^+ emission. Another radioactive isobar of $^{139}_{57}$La, $^{139}_{55}$Cs, decays by e^- emission and is located above the line of stable nuclei in Figure 30.3. (a) Which of these three isobars has the highest neutron-to-proton ratio? (b) Which has the greatest binding energy per nucleon? (c) Which do you expect to be heavier, ^{139}Pr or ^{139}Cs?

Section 30.3 Radioactivity

16. Ordinary potassium from milk contains 0.0117% of the radioactive isotope ^{40}K having a half-life of 1.227×10^9 y. If you drink 1 glass of milk (0.250 liter) a day, what is its activity just after you drink the milk? Assume that 1 liter of milk contains 2.00 g of potassium.
17. A sample of radioactive material contains 1.00×10^{15} atoms and has an activity of 6.00×10^{11} Bq. What is its half-life?
18. How much time elapses before 90.0% of the radioactivity of a sample of $^{73}_{33}$As disappears as measured by its activity? The half-life of $^{73}_{33}$As is 26 h.
19. The half-life of ^{131}I is 8.0 days. On a certain day, the activity of an iodine-131 sample is 6.40 mCi. What is its activity 40.0 days later?
20. Determine the activity of 1.00 g of ^{60}Co. The half-life of ^{60}Co is 5.24 y.
21. A freshly prepared pure sample of a certain radioactive isotope has an activity of 10.0 mCi. After 4.00 h, its activity is 8.00 mCi. (a) Find the decay constant and half-life. (b) How many atoms of the isotope were contained in the freshly prepared sample? (c) What is the sample's activity 30.0 h after it is prepared?
22. A building has become accidentally contaminated with radioactivity. The longest-lived material in the building is strontium-90 ($^{90}_{38}$Sr has an atomic mass 89.9077, and its half-life is 28.8 y). If the building initially contained 5.00 kg of this substance uniformly distributed throughout the building (a very unlikely situation) and the safe level is less than 10.0 decays/min, how long will the building be unsafe?
23. The most common isotope of radon is ^{222}Rn, which has a half-life of 3.8 days. (a) What fraction of the nuclei that were on Earth 1 week ago are now undecayed? (b) What fraction of those that existed 1 year ago? (c) In view of these results, explain why radon remains a problem in modern society. (*Hint:* Radon-222 is the decay product of radium-226.)
24. Consider a radioactive sample. Determine the ratio of the number of atoms decayed during the first half of its half-life to the number of atoms decayed during the second half of its half-life.
25. The radioactive isotope ^{198}Au has a half-life of 64.8 h. A sample containing this isotope has an initial activity (at $t = 0$) of 40.0 μCi. Calculate the number of nuclei that decay in the time interval between $t_1 = 10.0$ h and $t_2 = 12.0$ h.
26. A radioactive nucleus has a half-life of $T_{1/2}$. A sample containing these nuclei has an initial activity R_0. Calculate the number of nuclei that decay in the time interval between the times t_1 and t_2.

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Section 30.4 The Decay Processes

27. Enter the correct isotope symbol in each open square in Figure P30.27, which shows the sequences of decays starting with uranium-235 and ending with the stable isotope lead-207.

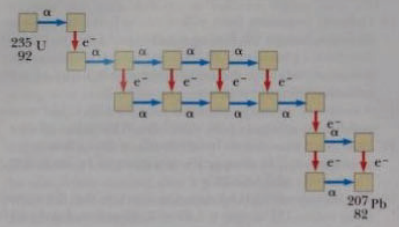


Figure P30.27

28. Identify the missing nuclide or particle (X):
(a) $X \rightarrow {}^{55}_{28}Ni + \gamma$
(b) ${}^{213}_{84}Po \rightarrow X + \alpha$
(c) $X \rightarrow {}^{55}_{26}Fe + e^+ + \nu$
(d) ${}^{109}_{48}Cd + X \rightarrow {}^{109}_{47}Ag + \nu$
(e) ${}^{14}N(\alpha, X){}^{17}O$

29. Find the energy released in the alpha decay

$$^{238}_{92}U \longrightarrow {}^{234}_{90}Th + {}^{4}_{2}He$$

You will find the following mass values useful:

$$M({}^{238}_{92}U) = 238.050\ 786\ u$$

$$M({}^{234}_{90}Th) = 234.043\ 583\ u$$

$$M({}^{4}_{2}He) = 4.002\ 603\ u$$

30. A living specimen in equilibrium with the atmosphere contains 1 atom of ^{14}C (half-life = 5730 y) for every 7.7×10^{11} stable carbon atoms. An archeological sample of wood (cellulose, $C_{12}H_{22}O_{11}$) contains 21.0 mg of carbon. When the sample is placed inside a beta counter the counting efficiency of which is 88.0%, 837 counts are accumulated in 1 week. Assuming that the cosmic-ray flux and the Earth's atmosphere have not changed appreciably since the sample was formed, find the age of the sample.

31. The nucleus ${}^{15}_{8}O$ decays by electron capture. Write (a) the basic nuclear process and (b) the decay process referring to neutral atoms. (c) Determine the energy of the neutrino. Disregard the daughter's recoil.

32. Is it energetically possible for a ${}^{8}_{4}Be$ nucleus to spontaneously decay into two alpha particles? Explain. (${}^{8}_{4}Be$ has an atomic mass of 8.005 305 u.)

Section 30.6 Nuclear Reactions

33. The reaction ${}^{27}_{13}Al(\alpha, n){}^{30}_{15}P$, achieved in 1934, was the first known in which the product nucleus is radioactive. Calculate the Q value of this reaction.

34. Find the energy released in the fission reaction

$$n + {}^{235}_{92}U \longrightarrow {}^{98}_{40}Zr + {}^{135}_{52}Te + 3n$$

The atomic masses of the fission products are: ${}^{98}_{40}Zr$, 97.912 0 u; ${}^{135}_{52}Te$, 134.908 7 u.

35. Natural gold has only one isotope, ${}^{197}_{79}Au$. If natural gold is irradiated by a flux of slow neutrons, electrons are emitted. (a) Write the appropriate reaction equation. (b) Calculate the maximum energy of the emitted electrons. The mass of ${}^{198}_{80}Hg$ is 197.966 75 u.

36. A beam of 6.61 MeV protons is incident on a target of ${}^{27}_{13}Al$. Those that collide produce the reaction

$$p + {}^{27}_{13}Al \longrightarrow {}^{27}_{14}Si + n$$

(${}^{27}_{14}Si$ has a mass of 26.986 721 u.) Neglecting any recoil of the product nucleus, determine the kinetic energy of the emerging neutrons.

37. (a) Suppose ${}^{10}_{5}B$ is struck by an alpha particle, releasing a proton and a product nucleus in the reaction. What is the product nucleus? (b) An alpha particle and a product nucleus are produced when ${}^{13}_{6}C$ is struck by a proton. What is the product nucleus?

Additional Problems

38. One method of producing neutrons for experimental use is to bombard ${}^{7}_{3}Li$ with protons. The neutrons are emitted according to the following reaction:

$${}^{1}_{1}H + {}^{7}_{3}Li \longrightarrow {}^{7}_{4}Be + {}^{1}_{0}n$$

What is the minimum kinetic energy the incident proton must have if this reaction is to occur?

39. A by-product of some fission reactors is the isotope ${}^{239}_{94}Pu$, an alpha emitter having a half-life of 24 000 years:

$$^{239}_{94}Pu \longrightarrow {}^{235}_{92}U + \alpha$$

Consider a sample of 1.00 kg of pure ${}^{239}_{94}Pu$ at $t = 0$. Calculate (a) the number of ${}^{239}_{94}Pu$ nuclei present at $t = 0$ and (b) the initial activity in the sample. (c) How long does the sample have to be stored if a "safe" activity level is 0.100 Bq?

40. (a) Can ^{57}Co decay by positron (e^+) emission? Explain. (b) Can ^{14}C decay by electron (e^-) emission? Explain. (c) If either answer is yes, what is the range of kinetic energies available for the positron or electron?

41. During the manufacture of a steel engine component, radioactive iron (^{59}Fe) is included in the total mass of 0.200 kg. The component is placed in a test engine when the activity due to this isotope is 20.0 μCi. After a 1000-h test period, oil is removed from the engine and found to contain enough ^{59}Fe to produce 800 disintegrations/min per liter of oil. The total volume of oil in the engine is 6.50 liters. Calculate the total mass worn from the engine component per hour of operation. (The half-life of ^{59}Fe is 45.1 days.)

42. The activity of a radioactive sample was measured over 12 h, with the following background-corrected count rates:

Time (h)	Counting Rate (counts/min)
1.00	3100
2.00	2450
4.00	1480
6.00	910
8.00	545
10.0	330
12.0	200

(a) Plot the activity curve on semilog paper. (b) Determine the disintegration constant and half-life of the radioactive nuclei in the sample. (c) What counting rate would you expect for the sample at $t = 0$? (d) Assuming the efficiency of the counting instrument to be 10.0%, calculate the number of radioactive atoms in the sample at $t = 0$.

43. A large nuclear power reactor produces about 3000 MW of thermal power in its core. Three months after a reactor is shut down, the core power from radioactive by-products is 10.0 MW. Assuming that each emission delivers 1.00 MeV of energy to the thermal power, estimate the activity in becquerels 3 months after the reactor is shut down.

44. In a piece of rock from the Moon, the ^{87}Rb content is assayed to be 1.82×10^{10} atoms per gram of material, and the ^{87}Sr content is found to be 1.07×10^{9} atoms per gram. (The relevant decay is $^{87}Rb \rightarrow {}^{87}Sr + e^-$. The half-life of the decay is 4.8×10^{10} years.) (a) Determine the age of the rock. (b) Could the material in the rock actually be much older? What assumption is implicit in using the radioactive dating method?

45. (a) Find the radius of the ${}^{12}_{6}C$ nucleus. (b) Find the force of repulsion between a proton at the surface of a ${}^{12}_{6}C$ nucleus and the remaining five protons. (c) How much work (in MeV) has to be done to overcome this electrostatic repulsion in order to put the last proton into the nucleus? (d) Repeat parts (a), (b), and (c) for ${}^{238}_{92}U$.

46. (a) Why is the inverse beta decay $p \rightarrow n + e^+ + \nu$ forbidden for a free proton? (b) Why is the same reaction possible if the proton is bound in a nucleus? For example, the following reaction occurs:

$${}^{13}_{7}N \longrightarrow {}^{13}_{6}C + e^+ + \nu$$

(c) How much energy is released in the reaction given in part (b)? [$m(e^+) = m(e^-) = 0.000\ 549$ u, $M({}^{13}C\ \text{atom}) = 13.003\ 355$ u, $M({}^{13}N\ \text{atom}) = 13.005\ 739$ u]

47. Carbon detonations are powerful nuclear reactions that temporarily tear apart the cores of massive stars late in their lives. These blasts are produced by carbon fusion, which requires a temperature of about 6×10^8 K to overcome the strong Coulomb repulsion between carbon nuclei. (a) Estimate the repulsive energy barrier to fusion, using the required ignition temperature for carbon fusion. (In other words, what is the average kinetic energy for a carbon nucleus at 6×10^8 K?) (b) Calculate the energy (in MeV) released in each of these "carbon-burning" reactions:

$${}^{12}C + {}^{12}C \longrightarrow {}^{20}Ne + {}^{4}He$$

$${}^{12}C + {}^{12}C \longrightarrow {}^{24}Mg + \gamma$$

(c) Calculate the energy (in kWh) given off when 2.00 kg of carbon completely fuses according to the first reaction.

48. When a material of interest is irradiated by neutrons, radioactive atoms are produced continually and some decay according to their characteristic half-lives. (a) If radioactive atoms are produced at a constant rate R and their decay is governed by the conventional radioactive decay law, show that the number of radioactive atoms accumulated after an irradiation time t is

$$N = \frac{R}{\lambda}(1 - e^{-\lambda t})$$

(b) What is the maximum number of radioactive atoms that can be produced?

49. The decay of an unstable nucleus by alpha emission is represented by Equation 30.9. The disintegration energy Q given by Equation 30.12 must be shared by the alpha particle and the daughter nucleus in order to conserve both energy and momentum in the decay process. (a) Show that Q and K_α, the kinetic energy of the alpha particle, are related by the expression

$$Q = K_\alpha\left(1 + \frac{M_\alpha}{M}\right)$$

where M is the mass of the daughter nucleus. (b) Use the result of part (a) to find the energy of the alpha particle emitted in the decay of ^{226}Ra. (See Example 30.5 for the calculation of Q.)

50. The ^{145}Pm nucleus decays by alpha emission. (a) Determine the daughter nucleus. (b) Using the values given in Table A.3, determine the energy released in this decay. (c) What fraction of this energy is carried away by the alpha particle when the recoil of the daughter is taken into account?

51. A very slow neutron (speed approximately equal to zero) can initiate the reaction

$$n + {}^{10}_{5}B \longrightarrow {}^{7}_{3}Li + {}^{4}_{2}He$$

If the alpha particle moves away with a speed of 9.30×10^6 m/s, calculate the kinetic energy of the lithium nucleus. Use nonrelativistic formulas.

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52. When, after a reaction or disturbance of any kind, a nucleus is left in an excited state, it can return to its normal (ground) state by emission of a gamma-ray photon (or several photons). This process is illustrated by Equation 30.20. The emitting nucleus must recoil in order to conserve both energy and momentum. (a) Show that the recoil energy of the nucleus is

$$E_r = \frac{(\Delta E)^2}{2Mc^2}$$

where ΔE is the difference in energy between the excited and ground states of a nucleus of mass M. (b) Calculate the recoil energy of the ^{57}Fe nucleus when it decays by gamma emission from the 14.4-keV excited state. For this calculation, take the mass to be 57 u. (*Hint:* When writing the equation for conservation of energy, use $(Mv)^2/2M$ for the kinetic energy of the recoiling nucleus. Also, assume that $hf \ll Mc^2$ and use the binomial expansion.)

53. The theory of nuclear astrophysics proposes that all the heavy elements, such as uranium, are formed in the cores of massive stars or in supernova explosions ending the lives of these stars. If we assume that at the time of the explosion there were equal amounts of ^{235}U and ^{238}U, how long ago did the star(s) explode that released the elements that formed our Earth? The present ^{235}U/^{238}U ratio is 0.007. (The half-lives of ^{235}U and ^{238}U are 0.7×10^9 years and 4.47×10^9 years.)

Spreadsheet Problems

S1. Soon after the discovery of radioactivity, investigators realized that some radioactive nuclides decayed into nuclides that were themselves radioactive. Entire chains of radioactive nuclides would form. For example, with a half-life of 3.83 days, ^{222}Rn decays to ^{218}Po, which has a half-life of 3.05 min and decays to ^{214}Pb. For the case in which the parent and daughter both decay, the rate of decay of the parent is

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

where N_1 is the number of parent nuclei present and λ_1 is the decay constant for the parent. The rate of formation of the daughter is just

$$\frac{dN_2}{dt} = \lambda_1 N_1 \qquad (\textit{formation})$$

where N_2 is the number of daughter nuclei present. The rate of decay of the daughter is

$$\frac{dN_2}{dt} = -\lambda_2 N_2 \qquad (\textit{decay})$$

where λ_2 is the decay constant for the daughter. The net production of the daughter is then

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

Spreadsheet 30.1 calculates these rates of decay and formation of any mother–daughter pair of radioactive nuclides by integrating these coupled differential equations. (a) Let's assume that we start with 5000 atoms of the parent nuclei, which have a half-life of 5 min. The daughter nuclei further decay with a half-life of 1 min. Enter these data in the spreadsheet. How many nuclei of the parent and daughter are present after 1 min? 2 min? 30 min? (b) If the half-life of the parent is changed to 2 min and the daughter's half-life is changed to 5 min, how many nuclei of each are present at the times listed in part (a)? (c) Change the parent's half-life to 1000 min and the daughter's half-life to 2 min. Discuss the number of daughter nuclei present as a function of time.

S2. Modify Spreadsheet 30.1 to take into account the decay of the granddaughter nuclei, which have a half-life of 10 min. Set the parent half-life to 5 min and the daughter half-life to 2 min. The net rate of production of the granddaughter is

$$\frac{dN_3}{dt} = \lambda_2 N_2 - \lambda_3 N_3$$

where N_3 is the number of granddaughter nuclei present and λ_3 is the decay constant for the granddaughter. How many of each species of nuclei are present at 1 min, 2 min, 5 min, 10 min, 30 min, and 60 min?

ANSWERS TO CONCEPTUAL PROBLEMS

1. Isotopes of a given element correspond to nuclei with different numbers of neutrons. This will result in different masses of the atom, and different magnetic moments, because the neutron, despite being uncharged, does have a magnetic moment. The chemical behavior, however, is governed by the electrons. All isotopes of a given element have the same number of electrons and, therefore, the same chemical behavior.
2. Carbon dating cannot generally be used to estimate the age of a stone. The stone was not alive and breathing air containing carbon-14. Only the ages of artifacts that were once alive can be estimated with carbon dating.

3. In alpha decay, there are only two final particles—the alpha particle and the daughter nucleus. There are also two conservation principles—energy and momentum. As a result, the alpha particle must be ejected with a discrete energy to satisfy both conservation principles. There are a small number of discrete energies of the alpha particle, as the daughter nucleus can be left in various excited states, but the allowed energies of the alpha particle are not continuous.

   However, beta decay is a three-particle decay—the beta particle, the neutrino (or antineutrino), and the daughter nucleus. As a result, the energy and momentum can be shared in a variety of ways among the three particles while still satisfying the two conservation principles. This allows a continuous range of energies for the beta particle.
4. The larger rest energy of the neutron means that a free proton in space will not spontaneously decay into a neutron and a positron. When the proton is in the nucleus, however, the important question is that of the total rest energy *of the nucleus*. If it is energetically lower for the nucleus to have one less proton and one more neutron, then the beta-positive decay process will occur to achieve this lower energy.
5. An alpha particle contains two protons and two neutrons. Because a hydrogen nucleus only contains one proton, it cannot emit an alpha particle.

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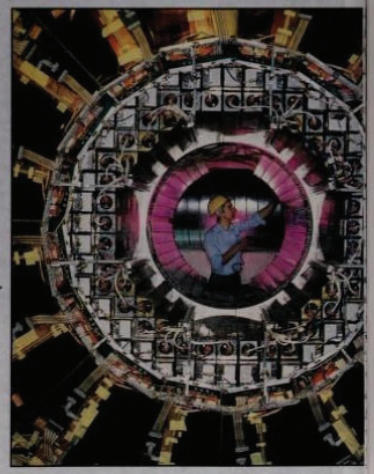
Particle Physics and Cosmology

CHAPTER OUTLINE

31.1 The Fundamental Forces in Nature
31.2 Positrons and Other Antiparticles
31.3 Mesons and the Beginning of Particle Physics
31.4 Classification of Particles
31.5 Conservation Laws
31.6 Strange Particles and Strangeness
31.7 How Are Elementary Particles Produced and Particle Properties Measured?
31.8 The Eightfold Way
31.9 Quarks
31.10 Colored Quarks
31.11 Electroweak Theory and the Standard Model
31.12 The Cosmic Connection
31.13 Problems and Perspectives

In this concluding chapter, we examine the properties and classifications of the various known subatomic particles and the fundamental interactions that govern their behavior. We also discuss the current theory of elementary particles, in which all matter is believed to be constructed from only two families of particles, quarks and leptons. Finally, we discuss how clarifications of such models might help scientists understand cosmology, which deals with the evolution of the Universe.

The word "atom" is from the Greek *atomos*, which means "indivisible." At one time, atoms were thought to be the indivisible constituents of matter; that is, they were regarded to be elementary particles. After 1932, physicists viewed all matter as consisting of only three constituent particles: electrons, protons, and neutrons. With the exception of the free neutron, these particles are very stable. Beginning in 1945, many new particles were discovered in experiments involving high-energy collisions between known particles. These new particles are characteristically very unstable and have very short half-lives, ranging between 10^{-6} and 10^{-23} s. So far more than 300 of these unstable, temporary particles have been catalogued.



A technician works on one of the particle detectors at CERN, the European center for particle physics near Geneva, Switzerland. Electrons and positrons accelerated to an energy of 50 GeV collide in a circular tunnel 2 km in circumference, located 100 m underground. *(David Parker/Science Photo Library, Photo Researchers, Inc.)*

During the past 35 years, many powerful particle accelerators have been constructed throughout the world, making it possible to observe collisions of highly energetic particles under controlled laboratory conditions in an effort to reveal the subatomic world in finer detail. Until the 1960s, physicists were bewildered by the large number and variety of subatomic particles being discovered. They wondered if the particles were like animals in a zoo, having no systematic relationship connecting them or whether a pattern was emerging that would provide a better understanding of the elaborate structure in the subnuclear world. In the past 35 years, physicists have advanced our knowledge of the structure of matter tremendously by recognizing that all particles except electrons, photons, and a few related particles are made of smaller particles called *quarks*. Thus, protons and neutrons, for example, are not truly elementary but are systems of tightly bound quarks.

31.1 • THE FUNDAMENTAL FORCES IN NATURE

As we learned in Section 5.6, all natural phenomena can be described by four fundamental forces between elemental particles. In order of decreasing strength, these are the strong force, the electromagnetic force, the weak force, and the gravitational force.

The strong force, as we mentioned in Chapter 30, represents the glue that holds nucleons together. It is very short-range and is negligible for separations greater than about 10^{-15} m (about the size of the nucleus). The electromagnetic force, which binds atoms and molecules together to form ordinary matter, has about 10^{-2} times the strength of the strong force. It is a long-range force that decreases in strength as the inverse square of the separation between interacting particles. The weak force is a short-range force that tends to produce instability in certain nuclei. It is responsible for most radioactive decay processes such as beta decay, and its strength is only about 10^{-5} times that of the strong force. Finally, the gravitational force is a long-range force that has a strength of only about 10^{-39} times that of the strong force (in SI units). Although this familiar interaction is the force that holds the planets, stars, and galaxies together, its effect on elementary particles is negligible. In modern physics, interactions between particles are often described in terms of the exchange of field particles, or quanta. In the case of the familiar electromagnetic interaction, for instance, the field particles are photons. In the language of modern physics, it is said that the electromagnetic force is *mediated* by photons and that photons are the quanta of the electromagnetic field. Likewise, the strong force is mediated by field particles called **gluons,** the weak force is mediated by particles called the W and Z **bosons,** and the gravitational force is mediated by quanta of the gravitational field called **gravitons.** These interactions, their ranges, and their relative strengths are summarized in Table 31.1.

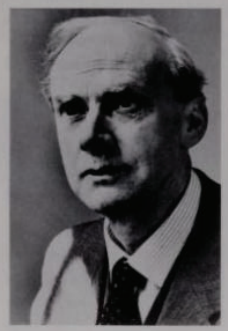
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TABLE 31.1 Particle Interactions

Interaction (Force)	Relative Strength*	Typical Lifetimes for Decays via a Given Interaction	Range of Force	Mediating Field Particle
Strong	1	$\leq 10^{-20}$ s	Short ($\sim$1 fm)	Gluon
Electromagnetic	$\sim 10^{-2}$	$\sim 10^{-16}$ s	Long ($\propto 1/r^2$)	Photon
Weak	$\sim 10^{-6}$	$\geq 10^{-10}$ s	Short ($\sim 10^{-3}$ fm)	$W^{\pm}$, Z^0 bosons
Gravitational	$\sim 10^{-43}$	?	Long ($\propto 1/r^2$)	Graviton

* For two u quarks at 3×10^{-17} m.

31.2 · POSITRONS AND OTHER ANTIPARTICLES



Paul Adrien Maurice Dirac (1902–1984), winner of the Nobel prize for physics in 1933. *(Courtesy AIP Emilio Segrè Visual Archives)*

In the 1920s, the theoretical physicist Paul Adrien Maurice Dirac (1902–1984) developed a version of quantum mechanics that incorporated special relativity. Dirac's theory automatically explained the origin of the electron's spin and its magnetic moment. However, it also presented a major difficulty. Dirac's relativistic wave equation required solutions corresponding to negative energy states even for free particles. But if negative energy states existed, one would expect an electron in a state of positive energy to make a rapid transition to one of these states, emitting a photon in the process. Dirac avoided this difficulty by postulating that all negative energy states are filled. The multitude of electrons that occupy these negative energy states is called the *Dirac sea.* Electrons in the Dirac sea are not directly observable because the Pauli exclusion principle (see Section 29.6) does not allow them to react to external forces. However, if one of these negative energy states is vacant, leaving a hole in the sea of filled states, the hole can react to external forces and would be observable.

The second, equivalent interpretation of this theory is that **for every particle, there is an antiparticle.** The antiparticle would have the same mass as the particle, but their charges would be opposite. For example, the electron's antiparticle (called a *positron*) would have a mass of 0.511 MeV/c^2 and a positive charge of 1.60×10^{-19} C. Usually we denote an antiparticle with a bar over the symbol for the particle. For example, $\bar{e}$ denotes the antielectron or positron (although in this book the notation e^+ is preferred), and $\bar{p}$ denotes the antiproton.

The positron was discovered by Carl Anderson in 1932, and in 1936 he was awarded the Nobel prize for that achievement. Anderson discovered the positron while examining tracks created by electron-like particles of positive charge in a cloud chamber. (These early experiments used cosmic rays—mostly energetic protons passing through interstellar space—to initiate high-energy reactions in the upper atmosphere, which resulted in the production of positrons at ground level.) To discriminate between positive and negative charges, Anderson placed the cloud chamber in a magnetic field, causing moving charges to follow curved paths. He noted that some of the electron-like tracks deflected in a direction corresponding to a positively charged particle.

Since Anderson's initial discovery, the positron has been observed in a number of experiments. Perhaps the most common process for producing positrons is **pair production.** In this process, a gamma ray photon with sufficiently high energy collides with a nucleus and an electron-positron pair is created. Because the total rest energy of the electron–positron pair is $2m_ec^2 = 1.02$ MeV (where m_e is the mass of the electron), the photon must have at least this much energy to create an electron–positron pair. Thus, electromagnetic energy in the form of a gamma ray is converted to mass in accordance with Einstein's famous relationship $E = mc^2$. Figure 31.1 shows tracks of electron–positron pairs created by 300-MeV gamma rays striking a lead sheet.

The reverse process can also occur. Under the proper conditions, an electron and positron can annihilate each other to produce two gamma-ray photons that have a combined energy of at least 1.02 MeV:

$$e^- + e^+ \longrightarrow 2\gamma$$

Prior to 1955, on the basis of the Dirac theory, it was expected that every particle had a corresponding antiparticle, but antiparticles such as the antiproton and antineutron had not been detected experimentally. In 1955 a team led by Emilio Segrè and Owen Chamberlain used the Bevatron particle accelerator at the University of California, Berkeley, to produce antiprotons and antineutrons, and thus established with certainty the existence of antiparticles. For this work Segrè and Chamberlain received the Nobel prize in 1959.

It is now established that every particle has a corresponding antiparticle of equal mass and spin and of equal and opposite charge, magnetic moment, and strangeness. (The property of *strangeness* is explained in Section 31.6.) The only exceptions to these rules for particles and antiparticles are the neutral photon, pion, and eta, each of which is its own antiparticle.

An intriguing aspect of antiparticle theory is that if we replace every proton, neutron, and electron in an atom with its antiparticle, we should create a stable

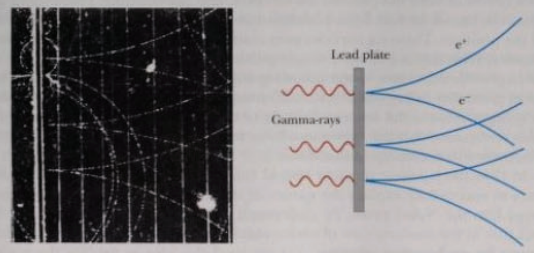


Figure 31.1 (a) Bubble-chamber tracks of electron–positron pairs produced by 300-MeV gamma rays striking a lead sheet. *(Courtesy Lawrence Berkeley Laboratory, University of California)* (b) Sketch of the pertinent pair-production events. Note that the positrons deflect upward and the electrons deflect downward in an applied magnetic field directed into the page.

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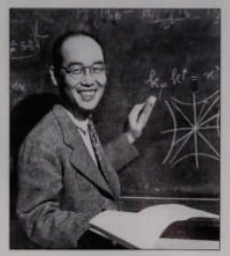
antiatom; combinations of antiatoms should form antimolecules and eventually antiworlds. As far as we know, everything would behave in the same way in an antiworld as in our world. In principle, it is possible that some distant antimatter galaxies exist, separated from normal-matter galaxies by millions of lightyears. Unfortunately, because the photon is its own antiparticle, there is no difference in the light emitted from matter and antimatter galaxies, and so astronomical observations cannot tell the two apart. Although no evidence of antimatter galaxies exists at present, it is awe-inspiring to imagine the cosmic spectacle that would result if matter and antimatter worlds were to collide: a gigantic eruption of jets of annihilation radiation, transforming the entire galactic mass into neutrinos, antineutrinos, and gamma rays, all fleeing the collision point at the speed of light.

Thinking Physics 1

When an electron and a positron meet at low speeds in free space, why are two 0.511-MeV gamma rays produced, rather than one gamma ray with an energy of 1.022 MeV?

Reasoning Gamma rays are photons, and photons carry momentum. If only one photon were produced, momentum would not be conserved, because the total momentum of the electron–positron system is approximately zero, whereas a single photon of energy 1.0022 MeV would have a very large momentum. However, the two gamma-ray photons that are produced travel off in opposite directions, so their total momentum is zero.

31.3 • MESONS AND THE BEGINNING OF PARTICLE PHYSICS



Hideki Yukawa (1907–1981), a Japanese physicist, was awarded the Nobel prize in 1949 for predicting the existence of mesons. This photograph of Yukawa at work was taken in 1950 in his office at Columbia University. *(UPI/Bettmann)*

In the mid-1930s, physicists had a fairly simple view of the structure of matter. The building blocks were the proton, the electron, and the neutron. Three other particles were known or had been postulated at the time: the photon, the neutrino, and the positron. These six particles were considered the fundamental constituents of matter. With this marvelously simple picture of the world, however, no one was able to provide an answer to the following important question: Because many protons in proximity in any nucleus should strongly repel each other because of their like charges, what is the nature of the force that holds the nucleus together? Scientists recognized that this mysterious force must be much stronger than anything encountered in nature up to that time.

In 1935 the Japanese physicist Hideki Yukawa (1907–1981) proposed the first theory to successfully explain the nature of the nuclear force, an effort that later earned him the Nobel prize. To understand Yukawa's theory, it is useful to first recall that in the modern view of electromagnetic interactions, **charged particles interact by exchanging photons.** Yukawa used this idea to explain the nuclear force by proposing a new particle whose exchange between nucleons in the nucleus produces the nuclear force. Furthermore, he established that the range of the force is inversely proportional to the mass of this particle and predicted that the mass would be about 200 times the mass of the electron. Because the new particle would have a mass between that of the electron and that of the proton, it was called a **meson** (from the Greek *meso*, "middle").

In an effort to substantiate Yukawa's predictions, physicists began an experimental search for the meson by studying cosmic rays entering the Earth's atmosphere. In 1937, Carl Anderson and his collaborators discovered a particle of mass 106 MeV/c^2, about 207 times the mass of the electron. However, subsequent experiments showed that the particle interacted very weakly with matter and hence could not be the carrier of the nuclear force. The puzzling situation inspired several theoreticians to propose that there are two mesons with slightly different masses. This idea was confirmed by the discovery in 1947 of the pi (π) meson, or simply **pion,** by Cecil Frank Powell (1903–1969) and Guiseppe P. S. Occhialini (b. 1907). The particle discovered by Anderson in 1937, the one thought to be a meson, is not really a meson. Instead, it takes part in the weak and electromagnetic interactions only, and is now called the **muon.**

The pion, Yukawa's carrier of nuclear force, comes in three varieties, corresponding to three charge states: π^+, π^-, and π^0. The π^+ and π^- particles have masses of 139.6 MeV/c^2, whereas the π^0 has a mass of 135.0 MeV/c^2. Pions and muons are very unstable particles. For example, the π^-, which has a mean lifetime of 2.6×10^{-8} s, first decays to a muon and an antineutrino. The muon, which has a mean lifetime of 2.2 μs, then decays into an electron, a neutrino, and an antineutrino:

$$\pi^- \longrightarrow \mu^- + \bar{\nu} \qquad [31.1]$$

$$\mu^- \longrightarrow e^- + \nu + \bar{\nu}$$

The interaction between two particles can be represented in a simple diagram called a *Feynman diagram,* developed by the American physicist Richard P. Feynman (1918–1988). Figure 31.2 is such a diagram for the electromagnetic interaction between two electrons. In this simple case, a photon is the field particle that mediates the electromagnetic force between the electrons. The photon transfers energy and momentum from one electron to the other in this interaction. The photon is called a *virtual photon* because it is reabsorbed by the emitting particle, without having been detected. Virtual photons can violate the law of conservation of energy by ΔE for a very short time, Δt, provided that $\Delta E\,\Delta t > \hbar/2$, and E is greater than the photon energy.

Now consider the pion exchange between a proton and a neutron via the nuclear force (Fig. 31.3). We can reason that the energy ΔE needed to create a pion of mass m_π is given by Einstein's equation $\Delta E = m_\pi c^2$. Again, the very existence of the pion would violate conservation of energy if it lasted for a time greater than $\Delta t \approx \hbar/2\,\Delta E$, (from the minimum form of the uncertainty principle), where ΔE is the energy of the pion and Δt is the time it takes the pion to transfer from one nucleon to the other. Therefore,

$$\Delta t \approx \frac{\hbar}{2\,\Delta E} = \frac{\hbar}{2m_\pi c^2} \qquad [31.2]$$

Because the pion cannot travel faster than the speed of light, the maximum distance d it can travel in a time Δt is $c\,\Delta t$. Using Equation 31.2 and $d = c\,\Delta t$, we find this maximum distance to be

$$d \approx \frac{\hbar}{2m_\pi c} \qquad [31.3]$$

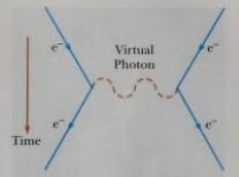


Figure 31.2 Feynman diagram showing how a photon mediates the electromagnetic force between two interacting electrons. The *t*-arrow shows the direction of increasing time.

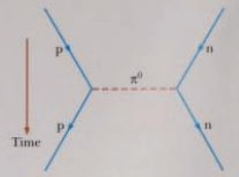


Figure 31.3 Feynman diagram representing a proton interacting with a neutron via the nuclear force. In this case, the pion mediates the nuclear force. The *t*-arrow shows the direction of increasing time.

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Richard Feynman (1918–1988)
with his son, Carl, in 1965. Feynman, together with Julian S. Schwinger and Shinichiro Tomonaga, won the 1965 Nobel prize for physics for fundamental work in the principles of quantum electrodynamics. His many important contributions to physics include the invention of simple diagrams to represent particle interactions graphically, the theory of the weak interaction of subatomic particles, a reformulation of quantum mechanics, the theory of superfluid helium, and his contribution to physics education through the magnificent three-volume text *The Feynman Lectures on Physics*. *(UPI Telephotos)*

From Chapter 30, we know that the range of the strong force is approximately 1.5×10^{-15} m. Using this value for d in Equation 31.3, we calculate the rest energy of the pion to be

$$m_\pi c^2 \approx \frac{\hbar c}{2d} = \frac{(1.05 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{2(1.5 \times 10^{-15}\ \text{m})}$$
$$\sim 100\ \text{MeV}$$

This corresponds to a mass of 100 MeV/c^2 (about 250 times the mass of the electron), a value in reasonable agreement with the observed pion mass.

The concept we have just described is quite revolutionary. In effect, it says that a proton can change into a proton plus a pion, as long as it returns to its original state in a very short time. Physicists often say that a nucleon undergoes "fluctuations" as it emits and absorbs pions. As we have seen, these fluctuations are a consequence of a combination of quantum mechanics (through the uncertainty principle) and special relativity (through Einstein's energy–mass relationship $E = mc^2$).

This section has dealt with the particles that mediate the strong force, namely the pions, and the mediators of the electromagnetic force, photons. Current ideas indicate that the nuclear force is more accurately described as an average or residual effect of the strong color force between quarks, as will be explained in Section 31.10. The graviton, which is the mediator of the gravitational force, has yet to be observed. The $W^{\pm}$ and Z^0 particles that mediate the weak force were discovered in 1983 by the Italian physicist Carlo Rubbia (b. 1934) and his associates using a proton–antiproton collider. Rubbia and Simon van der Meer, both at CERN, shared the 1984 Nobel prize for the discovery of the $W^{\pm}$ and Z^0 particles and the development of the proton–antiproton collider. In this accelerator, protons and antiprotons that have a momentum of 270 GeV/c undergo head-on collisions with each other. In some of the collisions $W^{\pm}$ and Z^0 particles are produced, which in turn are identified by their decay products.

31.4 · CLASSIFICATION OF PARTICLES

All particles other than photons can be classified into two broad categories, hadrons and leptons, according to their interactions.

Hadrons

Particles that interact through the strong force are called **hadrons.** There are two classes of hadrons, *mesons* and *baryons*, distinguished by their masses and spins.

Mesons all have zero or integer spin (0 or 1), with masses that lie between the mass of the electron and the mass of the proton. All mesons are known to decay finally into electrons, positrons, neutrinos, and photons. The pion is the lightest of known mesons; it has a mass of about 140 MeV/c^2 and a spin of 0. Another is the K meson, having a mass of about 500 MeV/c^2 and a spin of 0.

Baryons, the second class of hadrons, have masses equal to or greater than the proton mass (the name *baryon* means "heavy" in Greek), and their spin is always a noninteger value (1/2 or 3/2). Protons and neutrons are baryons, as are many other particles. With the exception of the proton, all baryons decay in such a way that the end products include a proton. For example, the baryon called the Ξ^- hyperon first decays to the Λ^0 baryon in about 10^{-10} s. The Λ^0 then decays to a proton and a π^- in approximately 3×10^{-10} s.

Today it is believed that hadrons are composed of more elemental units called quarks. Some of the important properties of hadrons are listed in Table 31.2. The symbols B, L_e, L_μ, L_τ, and S stand for baryon, electron, muon, and tau numbers, and strangeness, respectfully, and are explained in Sections 31.5 and 31.6.

TABLE 31.2 Some Particles and Their Properties

Category	Particle Name	Symbol	Anti-particle	Mass (MeV/c^2)	B	L_e	L_μ	L_τ	S	Lifetime(s)	Principal Decay Modes[a]
Leptons	Electron	e^-	e^+	0.511	0	+1	0	0	0	Stable	
	Electron–Neutrino	ν_e	$\bar{\nu}_e$	<7 eV/c^2	0	+1	0	0	0	Stable	
	Muon	μ^-	μ^+	105.7	0	0	+1	0	0	2.20×10^{-6}	$e^- \bar{\nu}_e \nu_\mu$
	Muon–Neutrino	ν_μ	$\bar{\nu}_\mu$	<0.3	0	0	+1	0	0	Stable	
	Tau	τ^-	τ^+	1784	0	0	0	+1	0	$<4 \times 10^{-13}$	$\mu^- \bar{\nu}_\mu \nu_\tau$, $e^- \bar{\nu}_e \nu_\tau$
	Tau–Neutrino	ν_τ	$\bar{\nu}_\tau$	<30	0	0	0	+1	0	Stable	
Hadrons											
Mesons	Pion	π^+	π^-	139.6	0	0	0	0	0	2.60×10^{-8}	$\mu^+ \nu_\mu$
		π^0	Self	135.0	0	0	0	0	0	0.83×10^{-16}	2γ
	Kaon	K^+	K^-	493.7	0	0	0	0	+1	1.24×10^{-8}	$\mu^+ \nu_\mu$, $\pi^+ \pi^0$
		K_S^0	$\bar{K}_S^0$	497.7	0	0	0	0	+1	0.89×10^{-10}	$\pi^+ \pi^-$, $2\pi^0$
		K_L^0	$\bar{K}_L^0$	497.7	0	0	0	0	+1	5.2×10^{-8}	$\pi^- e^+ \bar{\nu}_e$, $3\pi^0$
											$\pi^- \mu^+ \bar{\nu}_\mu$
	Eta	η	Self	548.8	0	0	0	0	0	$<10^{-18}$	2γ, 3π
		η'	Self	958	0	0	0	0	0	2.2×10^{-21}	$\eta\pi^+\pi^-$
Baryons	Proton	p	$\bar{p}$	938.3	+1	0	0	0	0	Stable	
	Neutron	n	$\bar{n}$	939.6	+1	0	0	0	0	920	$pe^-\bar{\nu}_e$
	Lambda	Λ^0	$\bar{\Lambda}^0$	1115.6	+1	0	0	0	−1	2.6×10^{-10}	$p\pi^-$, $n\pi^0$
	Sigma	Σ^+	$\bar{\Sigma}^-$	1189.4	+1	0	0	0	−1	0.80×10^{-10}	$p\pi^0$, $n\pi^+$
		Σ^0	$\bar{\Sigma}^0$	1192.5	+1	0	0	0	−1	6×10^{-20}	$\Lambda^0\gamma$
		Σ^-	$\bar{\Sigma}^+$	1197.3	+1	0	0	0	−1	1.5×10^{-10}	$n\pi^-$
	Xi	Ξ^0	$\bar{\Xi}^0$	1315	+1	0	0	0	−2	2.9×10^{-10}	$\Lambda^0\pi^0$
		Ξ^-	Ξ^+	1321	+1	0	0	0	−2	1.64×10^{-10}	$\Lambda^0\pi^-$
	Omega	Ω^-	Ω^+	1672	+1	0	0	0	−3	0.82×10^{-10}	$\Xi^0\pi^0$, $\Lambda^0 K^-$

[a] Notations in this column such as $p\pi^-$, $n\pi^0$ mean two possible decay modes. In this case, the two possible decays are $\Lambda^0 \rightarrow p + \pi^-$ and $\Lambda^0 \rightarrow n + \pi^0$.

Leptons

Leptons (from the Greek *leptos* meaning "small" or "light") are a group of particles that participate in the electromagnetic (if charged) and weak interactions. All leptons have spins of 1/2. Included in this group are electrons, muons, and neutrinos, which are less massive than the lightest hadron. Although hadrons have size and structure, **leptons appear to be truly elementary, point-like particles with no structure.**

Quite unlike hadrons, the number of known leptons is small. Currently, scientists believe there are only six leptons: the electron, the muon, and the tau and a neutrino associated with each. We now classify the six known leptons into three pairs called families:

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$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

Current evidence suggests that neutrinos travel with the speed of light and, therefore have zero mass. As we shall see later, a firm knowledge of the neutrino's mass (it may be small, not exactly zero as in Table 31.2) could have great significance in cosmological models and predictions of the future of the Universe. Finally, note that each of the six leptons has a corresponding antiparticle.

31.5 • CONSERVATION LAWS

Conservation laws are important in understanding why certain decays and reactions occur and others do not. In general, the laws of conservation of energy, linear momentum, angular momentum, and electric charge provide us with a set of rules that all processes must follow.

Certain new conservation laws are important in the study of elementary particles. Two of these laws, concerning baryon number and lepton number, are described in this section, and others will be discussed later in this chapter. Although the two described here have no theoretical foundation, they are supported by abundant empirical evidence.

Baryon Number

Conservation of baryon number •

Conservation of baryon number tells us that whenever a baryon is created in a reaction or decay, an antibaryon is also created. This scheme can be quantified by assigning a baryon number $B = +1$ for all baryons, $B = -1$ for all antibaryons, and $B = 0$ for all other particles. Thus, the **law of conservation of baryon number** states that **whenever a nuclear reaction or decay occurs, the sum of the baryon numbers before the process must equal the sum of the baryon numbers after the process.** An equivalent statement is that the net number of baryons remains constant in any process.

If baryon number is absolutely conserved, the proton must be absolutely stable. If it were not for the law of conservation of baryon number, the proton could decay to a positron and a neutral pion. However, such a decay has never been observed. At the present, we can say only that the proton has a half-life of at least 10^{31} years (the estimated age of the Universe is only 10^{10} years). In one recent version of a grand unified theory, physicists predicted that the proton is unstable. According to this theory, the baryon number is not absolutely conserved.

Example 31.1 Checking Baryon Numbers

Determine whether or not the following reactions can occur based on the law of conservation of baryon number.

(1) $p + n \longrightarrow p + p + n + \bar{p}$

(2) $p + n \longrightarrow p + p + \bar{p}$

Solution For reaction 1, recall that $B = +1$ for baryons and $B = -1$ for antibaryons. Hence the left side of reaction 1 gives a total baryon number of $1 + 1 = 2$. The right side of reaction 1 gives a total baryon number of $1 + 1 + 1 + (-1) = 2$. Thus the reaction can occur provided the incoming proton has sufficient energy.

The left side of reaction 2 gives a total baryon number of $1 + 1 = 2$. However, the right side gives $1 + 1 + (-1) = 1$. Because the baryon number is not conserved, the reaction cannot occur.

Lepton Number

There are three conservation laws involving lepton numbers, one for each variety of lepton. The **law of conservation of electron–lepton number** states that **the sum of the electron–lepton numbers before a reaction or decay must equal the sum of the electron–lepton numbers after the reaction or decay.**

Conservation of lepton number •

The electron and the electron neutrino are assigned a positive lepton number $L_e = +1$, the antileptons e^+ and $\bar{\nu}_e$ are assigned a negative lepton number $L_e = -1$, and all others have $L_e = 0$. For example, consider the decay of the neutron

$$n \longrightarrow p + e^- + \bar{\nu}_e$$

Before the decay, the electron–lepton number is $L_e = 0$; after the decay it is $0 + 1 + (-1) = 0$. Thus, the electron–lepton number is conserved. It is important to recognize that the baryon number must also be conserved. This can easily be checked by noting that before the decay $B = +1$, and after the decay B is $+1 + 0 + 0 = +1$.

In a similar way, when a decay involves muons, the muon–lepton number, L_μ, is conserved. The μ^- and the ν_μ are assigned positive numbers, $L_\mu = +1$; the antimuons μ^+ and $\bar{\nu}_\mu$ are assigned negative numbers, $L_\mu = -1$; and all others have $L_\mu = 0$. Finally, the tau–lepton number, L_τ, is conserved, and similar assignments can be made for the tau–lepton and its neutrino.

Thinking Physics 2

A student claims to have observed a decay of an electron into two neutrinos, traveling in opposite directions. What conservation laws would be violated by this decay?

Reasoning Several conservation laws are violated. Conservation of electric charge is violated, because the negative charge of the electron has disappeared. Conservation of angular momentum is violated, because the original angular momentum is that of the electron, with spin 1/2, and there are two spin-1/2 particles after the decay. Conservation of electron lepton number is also violated, because there is one lepton before the decay and two afterward. If both neutrinos were electron–neutrinos, electron–lepton number conservation would be violated in the final state. However, if one of the product neutrinos were other than an electron–neutrino, then another lepton conservation law would be violated, since there were no other leptons in the initial state.

Other conservation laws are obeyed by this decay. Energy can be conserved—the rest energy of the electron appears as the kinetic energy (and possibly some small rest energy) of the neutrinos. The opposite directions of the velocities of the two neutrinos allows for conservation of momentum. Conservation of baryon number and conservation of other lepton numbers are also upheld in the decay.

Example 31.2 Checking Lepton Numbers

Determine which of the following decay schemes can occur on the basis of conservation of electron–lepton number.

(1) $\mu^- \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$

(2) $\pi^- \longrightarrow \mu^+ + \nu_\mu + \nu_e$

Solution Because decay 1 involves both a muon and an electron, L_μ and L_e must both be conserved. Before the decay, $L_\mu = +1$ and $L_e = 0$. After the decay, $L_\mu = 0 + 0 + 1 = +1$, and $L_e = +1 - 1 + 0 = 0$. Thus, both numbers are conserved, and on this basis the decay mode is possible.

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Before decay 2 occurs, $L_\mu = 0$ and $L_e = 0$. After the decay, $L_\mu = -1 + 1 + 0 = 0$, but $L_e = +1$. Thus, the decay is not possible because the electron–lepton number is not conserved.

EXERCISE 1 Determine whether the decay $\mu^- \rightarrow e^- + \bar{\nu}_e$ can occur. Answer No. The muon–lepton number is +1 before the decay and 0 after.

31.6 • STRANGE PARTICLES AND STRANGENESS

Many particles discovered in the 1950s were produced by the nuclear interaction of pions with protons and neutrons in the atmosphere. A group of these particles—namely the kaon (K), lambda (Λ), and sigma (Σ) particles—exhibited unusual properties in production and decay and hence were called *strange particles.*

One unusual property is that these particles were always produced in pairs. For example, when a pion collides with a proton, two neutral strange particles were produced with high probability (Fig. 31.4):

$$\pi^- + p \longrightarrow K^0 + \Lambda^0$$

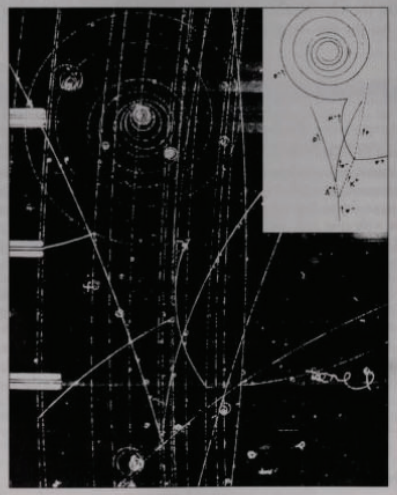


Figure 31.4 This bubble-chamber photograph shows many events, and the inset is a drawing of identified tracks. The strange particles Λ^0 and K^0 are formed at the bottom as the π^- interacts with a proton according to $\pi^- + p \rightarrow \Lambda^0 + K^0$. (Note that the neutral particles leave no tracks, as indicated by the dashed lines.) The Λ^0 and K^0 may then decay according to $\Lambda^0 \rightarrow \pi^- + p$ and $K^0 \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu$. *(Courtesy Lawrence Berkeley Laboratory, University of California, Photographic Services.)*

However, the reaction $\pi^- + p \rightarrow K^0 + n$ never occurred, even though no known conservation laws would have been violated and the energy of the pion was sufficient to initiate the reaction.

The second peculiar feature of strange particles is that, although they are produced by the strong interaction at a high rate, they do not decay into particles that interact via the strong force at a very high rate. Instead, they decay very slowly, which is characteristic of the weak interaction, as shown in Table 31.1. Their half-lives are in the range 10^{-10} s to 10^{-8} s; most other particles that interact via the strong force have lifetimes on the order of 10^{-20} s or less.

To explain these unusual properties of strange particles, a law called conservation of strangeness was introduced, together with a new quantum number S, called **strangeness.** The strangeness numbers for some particles are given in Table 31.2. The production of strange particles in pairs is explained by assigning $S = +1$ to one of the particles and $S = -1$ to the other. All nonstrange particles are assigned strangeness $S = 0$. The **law of conservation of strangeness** states that **whenever a nuclear reaction or decay occurs, the sum of the strangeness numbers before the process must equal the sum of the strangeness numbers after the process.**

• *Conservation of strangeness number*

The slow decay of strange particles can be explained by assuming that the strong and electromagnetic interactions obey the law of conservation of strangeness, but the weak interaction does not. Because the decay reaction involves the loss of one strange particle, it violates strangeness conservation and, hence, proceeds slowly via the weak interaction.

Example 31.3 Is Strangeness Conserved?

(a) Determine whether the following reaction occurs on the basis of conservation of strangeness.

$$\pi^0 + n \longrightarrow K^+ + \Sigma^-$$

Solution The initial state has strangeness $S = 0 + 0 = 0$. Because the strangeness of the K^+ is $S = +1$ and the strangeness of the Σ^- is $S = -1$, the strangeness of the final state is $+1 - 1 = 0$. Thus, strangeness is conserved and the reaction is allowed.

(b) Show that the following reaction does not conserve strangeness.

$$\pi^- + p \longrightarrow \pi^- + \Sigma^+$$

Solution The initial state has strangeness $S = 0 + 0 = 0$, and the final state has strangeness $S = 0 + (-1) = -1$. Thus, strangeness is not conserved.

EXERCISE 2 Show that the reaction $p + \pi^- \rightarrow K^0 + \Lambda^0$ obeys the law of conservation of strangeness.

31.7 • HOW ARE ELEMENTARY PARTICLES PRODUCED AND PARTICLE PROPERTIES MEASURED?

Examination of the bewildering array of entries in Table 31.2 leaves one yearning for firm ground. In fact it is natural to wonder about an entry that shows a particle that exists for 10^{-20} s and has a mass of 1192.5 MeV/c^2. How is it possible to detect a particle that exists for only 10^{-20} s, and furthermore how can its mass be measured? If a standard attribute of a particle is some type of permanence or stability, in what sense is a fleeting entity that exists for 10^{-20} s a particle? In this section we attempt to answer such questions and explain how elementary particles are produced and how their properties are measured.

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Elementary particles, most of which are unstable and created naturally only rarely in cosmic ray showers, are abundantly created in human-made collisions of high energy particles with a suitable target. Because very high energy beams of incident particles are desirable, stable charged particles such as electrons or protons generally make up the incident beam. It takes a considerable time to accelerate particles to high energies by the use of electromagnetic fields so we want to accelerate particles with long lifetimes. In a similar way, targets must be simple and stable and the simplest target, hydrogen, serves nicely as both a target and a detector. In a liquid hydrogen bubble chamber, which is basically a large container filled with hydrogen near its boiling point, a charged particle traversing the chamber ionizes the atoms along its path, and the ionization causes a visible track of tiny bubbles. The liquid hydrogen also serves as an efficient source of target protons, with a proton density sufficient to ensure many incident particle–target collisions within a reasonable time. Figure 31.4 shows a typical event in which a bubble chamber has served as both target source and detector. In this figure many parallel tracks of negative pions are visible entering the photograph from the bottom. One of the pions has hit a stationary proton in the hydrogen and produced two strange particles, the Λ^0 and K^0 according to the reaction

$$\pi^- + p \longrightarrow \Lambda^0 + K^0$$

Neither neutral strange particle leaves a track, but their subsequent decays into charged particles can be clearly seen as indicated in Figure 31.4. A magnetic field directed into the plane of the photograph causes the track of each charged particle to curve, and from the measured curvature one can determine the particle's charge and linear momentum. If the mass and momentum of the incident particle are known, we can then usually calculate the product particle mass, kinetic energy, and speed from conservation of momentum and energy. Finally, combining a product particle's speed with a measurable decay track length, we can calculate the product particle's lifetime. Figure 31.4 shows that sometimes one can use this lifetime technique even for a neutral particle that leaves no track. As long as the start and finish of the missing track are known as well as the particle speed, one can infer the missing track length and find the lifetime of the neutral particle.

Resonance Particles

With clever experimental technique and much effort, decay track lengths as short as 1 micron (10^{-6} m) can be measured. This means that lifetimes as short as 10^{-16} s can be measured with this technique for the case of high energy particles traveling at about the speed of light. We arrive at this result by assuming that a decaying particle travels 1 micron in the lab at a speed of $0.99c$ yielding a lab lifetime of $\Delta t_{lab} = 10^{-6}\ \text{m}/0.99c \approx 0.33 \times 10^{-14}$ s. Now, however, relativity helps us. Because the proper lifetime as measured in the decaying particle's restframe is shorter than Δt_{lab} by a factor of $\sqrt{1 - (v^2/c^2)}$, we can actually measure lifetimes of duration

$$\Delta t_p = \Delta t_{lab}\sqrt{1 - \frac{v^2}{c^2}} = (0.33 \times 10^{-14}\ \text{s})\sqrt{1 - \frac{(0.99\ c)^2}{c^2}} = 4 \times 10^{-16}\ \text{s}$$

Unfortunately, even with Einstein's help, we are several orders of magnitude away from 10^{-20} to 10^{-23} s lifetimes with the best efforts of the track-length method.

How, then, can we detect the presence of particles that exist for as short a time as 10^{-23} s? As we shall see, these very short-lived particles, known as **resonance particles,** can only have their mass, lifetime, and very existence inferred indirectly from peaks (resonances) in the cross section versus energy plots describing their decay products.

Let's consider this in more detail by looking at the case of the resonance particle called the delta plus (Δ^+) which has a mass of 1232 MeV/c^2 and a lifetime of about 10^{-23} s. The Δ^+ is produced in the reaction

$$e^- + p \longrightarrow e^- + \Delta^+ \qquad [31.4]$$

followed in 10^{-23} s by the decay of the delta plus according to

$$\Delta^+ \longrightarrow \pi^+ + n \qquad [31.5]$$

Because the Δ^+ lifetime is so short it leaves no measurable track and it might seem impossible to distinguish the reactions given in Equations 31.4 and 31.5 from the net direct reaction in which no Δ^+ is produced:

$$e^- + p \longrightarrow e^- + \pi^+ + n \qquad [31.6]$$

In fact we can tell whether a Δ^+ was formed by measuring the momentum and energy of the suspected decay products (pion and neutron) and using the conservation of momentum and energy to see if these values combine to give a Δ^+ mass of 1232 MeV/c^2.

The typical method for showing the existence of a resonance particle involves analyzing a large number of events in which a π^+ and a neutron are produced and then plotting a histogram of these events. (A *histogram* in this case is a plot of the number of events in a given energy range versus energy.) Following this procedure, we obtain a slowly varying curve with sharp peaks superimposed showing the existence of resonance particles. Figure 31.5 is an experimental histogram for the Δ^+

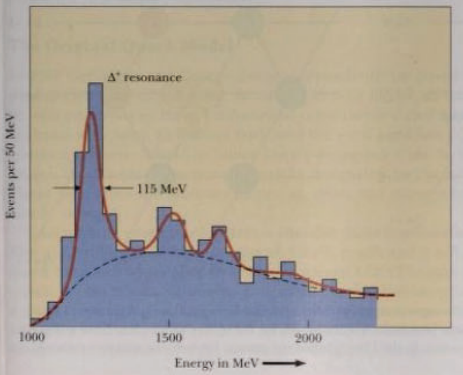


Figure 31.5 Experimental evidence for the existence of the Δ^+ particle. The sharp peak at 1232 MeV was produced by the events in which a Δ^+ formed and promptly decayed into a π^+ and neutron.

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particle. The broad background (dashed curve) is produced by direct events in which no Δ^+ was created; the sharp peak at 1232 MeV containing many events was produced by all those events in which a Δ^+ was formed and decayed into a pion and neutron with just the right energies and momenta to make up a delta particle. Peaks corresponding to two other resonance particles of larger mass can also be seen in Figure 31.5.

Histograms can tell us not only the mass of a short-lived particle but also the lifetime of the particle. The width of the resonance peak and the uncertainty relation $\Delta E\,\Delta t \cong \hbar/2$ are used to infer the lifetime Δt of the particle. The measured width of 115 MeV from Figure 31.5 leads to a lifetime of 5.7×10^{-24} s for the delta particle. In this incredibly short lifetime, a delta particle moving at the highest possible speed of c only travels 10^{-15} m, or about one nuclear diameter.

31.8 · THE EIGHTFOLD WAY

As we have seen, we associate conserved quantities such as spin, baryon number, lepton number, and strangeness with particles. Many classification schemes have been proposed that group particles into families. Consider the first eight baryons listed in Table 31.2, all having a spin of 1/2. If we plot their strangeness versus their charge using a sloping coordinate system, as in Figure 31.6a, a fascinating pattern is observed. Six of the baryons form a hexagon with the remaining two at its center.

Now consider the family of mesons listed in Table 31.2, which have spins of zero. If we count both particles and antiparticles, there are nine such mesons. Figure 31.6b is a plot of strangeness versus charge for this family. Again, a fascinating hexagonal pattern emerges. In this case, the particles on the perimeter of the hexagon lie opposite their antiparticles, and the remaining three (which form their own antiparticles) are at its center. These and related symmetric patterns, called the **eightfold way,** were proposed independently in 1961 by Murray Gell-Mann and Yuval Ne'eman.

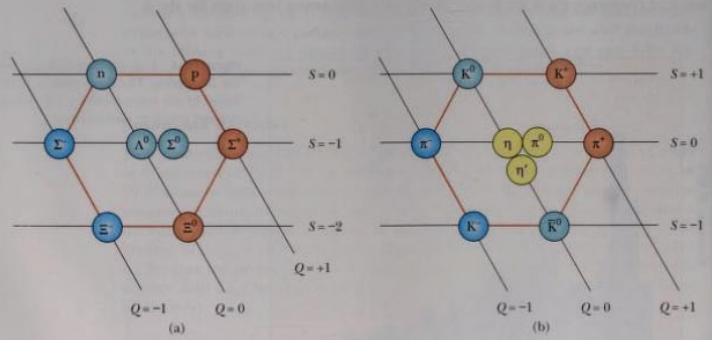
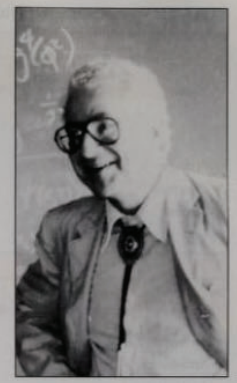


Figure 31.6 (a) The hexagonal eightfold-way pattern for the eight spin-1/2 baryons. This strangeness-versus-charge plot uses a horizontal axis for the strangeness values, S, but a sloping axis for the charge number, Q. (b) The eightfold-way pattern for the nine spin-zero mesons.

The groups of baryons and mesons can be displayed in many other symmetrical patterns within the framework of the eightfold way. For example, the family of spin-3/2 baryons contains ten particles arranged in a pattern like the tenpins in a bowling alley. When the pattern was proposed in 1961, one of the particles was missing—it had yet to be discovered. Gell-Mann predicted that the missing particle, which he called the omega minus (Ω^-), should have a spin of 3/2, a charge of -1, a strangeness of -3, and a rest energy of about 1680 MeV. Shortly thereafter, in 1964, scientists at the Brookhaven National Laboratory found the missing particle through careful analyses of bubble chamber photographs and confirmed all its predicted properties.

The patterns of the eightfold way in particle physics have much in common with the periodic table. Whenever a vacancy (a missing particle or element) occurs in the organized patterns, experimentalists have a guide for their investigations.



American physicist Murray Gell-Mann was awarded the Nobel prize in 1969 for his theoretical studies dealing with subatomic particles. *(Photo courtesy of Michael R. Dressler)*

31.9 · QUARKS

As we have noted, leptons appear to be truly elementary particles because they have no measurable size or internal structure, are limited in number, and do not seem to break down into smaller units. Hadrons, however, are complex particles having size and structure. The existence of the eightfold-way patterns suggests that baryons and mesons—in other words, hadrons—have a more elemental substructure. Furthermore, we know that hadrons decay into other hadrons and are many in number. Table 31.2 lists only those hadrons that are stable against hadronic decay; hundreds of others have been discovered. These facts strongly suggest that hadrons cannot be truly elementary. In this section we show that the complexity of hadrons can be explained by a simpler substructure.

The Original Quark Model

In 1963 Gell-Mann and George Zweig independently proposed that hadrons have a more elemental substructure. According to their model, all hadrons are composite systems of two or three fundamental constituents called **quarks** (pronounced to rhyme with *forks*). Gell-Mann borrowed the word *quark* from the passage "Three quarks for Muster Mark" in James Joyce's *Finnegan's Wake.* In the original quark model, there were three types of quarks designated by the symbols u, d, and s. These were given the arbitrary names *up, down,* and *sideways* (or now more commonly, *strange*).

An unusual property of quarks is that they have fractional electronic charges. The u, d, and s quarks have charges of $+2e/3$, $-e/3$, and $-e/3$, respectively. Each quark has a baryon number of 1/3 and a spin of 1/2. (The spin 1/2 means that all quarks are **fermions,** defined as any particle having half-integral spin.) The u and d quarks have strangeness 0, and the s quark has strangeness -1. Other properties of quarks and antiquarks are given in Table 31.3. Associated with each quark is an antiquark of opposite charge, baryon number, and strangeness.

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TABLE 31.3 Properties of Quarks and Antiquarks

Quarks								
Name	**Symbol**	**Spin**	**Charge**	**Baryon Number**	**Strangeness**	**Charm**	**Bottomness**	**Topness**
Up	u	$\frac{1}{2}$	$+\frac{2}{3}e$	$\frac{1}{3}$	0	0	0	0
Down	d	$\frac{1}{2}$	$-\frac{1}{3}e$	$\frac{1}{3}$	0	0	0	0
Strange	s	$\frac{1}{2}$	$-\frac{1}{3}e$	$\frac{1}{3}$	−1	0	0	0
Charmed	c	$\frac{1}{2}$	$+\frac{2}{3}e$	$\frac{1}{3}$	0	+1	0	0
Bottom	b	$\frac{1}{2}$	$-\frac{1}{3}e$	$\frac{1}{3}$	0	0	+1	0
Top	t	$\frac{1}{2}$	$+\frac{2}{3}e$	$\frac{1}{3}$	0	0	0	+1
Antiquarks								
Name	**Symbol**	**Spin**	**Charge**	**Baryon Number**	**Strangeness**	**Charm**	**Bottomness**	**Topness**
Anti-up	$\bar{u}$	$\frac{1}{2}$	$-\frac{2}{3}e$	$-\frac{1}{3}$	0	0	0	0
Anti-down	$\bar{d}$	$\frac{1}{2}$	$+\frac{1}{3}e$	$-\frac{1}{3}$	0	0	0	0
Anti-strange	$\bar{s}$	$\frac{1}{2}$	$+\frac{1}{3}e$	$-\frac{1}{3}$	+1	0	0	0
Anti-charmed	$\bar{c}$	$\frac{1}{2}$	$-\frac{2}{3}e$	$-\frac{1}{3}$	0	−1	0	0
Anti-bottom	$\bar{b}$	$\frac{1}{2}$	$+\frac{1}{3}e$	$-\frac{1}{3}$	0	0	−1	0
Anti-top	$\bar{t}$	$\frac{1}{2}$	$-\frac{2}{3}e$	$-\frac{1}{3}$	0	0	0	−1

TABLE 31.4 Quark Composition of Several Hadrons

Particle	Quark Composition
Mesons	
π^+	$u\bar{d}$
π^-	$\bar{u}d$
K^+	$u\bar{s}$
K^-	$\bar{u}s$
K^0	$d\bar{s}$
Baryons	
p	uud
n	udd
Λ^0	uds
Σ^+	uus
Σ^0	uds
Σ^-	dds
Ξ^0	uss
Ξ^-	dss
Ω^-	sss

The composition of all hadrons known when Gell-Mann and Zweig presented their models could be completely specified by three simple rules:

1. Mesons consist of one quark and one antiquark, giving them a baryon number of 0 as required.
2. Baryons consist of three quarks.
3. Antibaryons consist of three antiquarks.

Table 31.4 lists the quark compositions of several mesons and baryons. Note that just two of the quarks, u and d, are contained in all hadrons encountered in ordinary matter (protons and neutrons). The third quark, s, is needed only to construct strange particles with a strangeness number of either +1 or −1. Figure 31.7 is a pictorial representation of the quark composition of several particles.

Charm and Other Recent Developments

Although the original quark model was highly successful in classifying particles into families, there were some discrepancies between predictions of the model and certain experimental decay rates. Consequently, a fourth quark was proposed by several physicists in 1967. They argued that if there are four leptons (as was thought at the time), then there should also be four quarks because of an underlying symmetry in nature. The fourth quark, designated by c, was given a property called **charm.** A *charmed* quark would have charge $+2e/3$, but its charm would distinguish it from the other three quarks. The new quark would have a charm of $C = +1$, its antiquark would have a charm of $C = -1$, and all other quarks would have $C = 0$, as indicated in Table 31.3. Charm, like strangeness, would be conserved in strong and electromagnetic interactions but not in weak interactions.

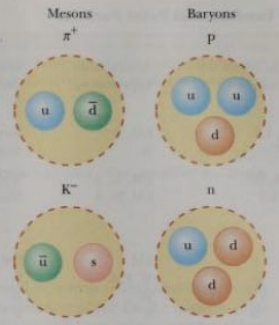


Figure 31.7 Quark compositions of two mesons and two baryons. Note that the mesons on the left each contain two quarks, whereas the baryons on the right each contain three quarks.

In 1974, a new heavy meson called the J/Ψ particle (or simply Ψ) was discovered independently by two groups: one led by Burton Richter at the Stanford Linear Accelerator (SLAC) and the other led by Samuel Ting at the Brookhaven National Laboratory. Richter and Ting were awarded the Nobel prize in 1976 for this work. The J/Ψ particle did not fit into the three-quark model but had the properties of a combination of a charmed quark and its antiquark ($c\bar{c}$). It was much more massive than the other known mesons ($\sim$3100 MeV/c^2), and its lifetime was much longer than those that decay via the strong force. Soon, related charmed mesons were discovered corresponding to such quark combinations as $\bar{c}d$ and $c\bar{d}$, all of which have large masses and long lifetimes. In 1975 researchers at Stanford University reported strong evidence for the tau (τ) lepton, mass 1784 MeV/c^2. Such discoveries led to more elaborate quark models, and the proposal of two new quarks, *top* (t) and *bottom* (b). (Some physicists prefer the designators *truth* and *beauty*.) To distinguish these quarks from the old ones, quantum numbers called *topness* and *bottomness* were assigned to these new particles and are included in Table 31.3. In 1977, researchers at the Fermi National Laboratory, under the direction of Leon Lederman, reported the discovery of a very massive new meson, Y (the Greek letter upsilon), the composition of which is considered to be $b\bar{b}$. In March of 1995, researchers at Fermilab announced the discovery of the top quark (supposedly the last of the quarks to be found) having mass 173 GeV/c^2.

You are probably wondering whether or not such discoveries will ever end. How many "building blocks" of matter really exist? At the present, physicists believe that the fundamental particles in nature are six quarks and six leptons (together with their antiparticles). Table 31.5 lists some of the properties of these particles as well as the properties of the fundamental field particles.

Despite many extensive experimental efforts, no isolated quark has ever been observed. Physicists now believe that quarks are permanently confined inside ordinary particles because of an exceptionally strong force that prevents them from

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TABLE 31.5 Properties of the Fundamental Point Particles

Particle		Mass in Terms of Proton Mass	Interactions Experienced[a]	Interaction Mediated	Electric Charge in Units of |e|	Color Charge
Matter Particles						
Leptons	e^-, μ, τ	$\frac{1}{1850}$, $\frac{1}{9}$, 1.9	EM, W	None	−1	No
	ν_e, ν_μ, ν_τ	$<10^{-8}$, $<\frac{1}{3500}$, $<\frac{1}{50}$	W	None	0	No
Quarks	u, c, t	$\frac{1}{235}$, 1.6, 165	EM, W, S	None	$+\frac{2}{3}$	Yes
	d, s, b	$\frac{1}{135}$, $\frac{1}{6}$, 5.2	EM, W, S	None	$-\frac{1}{3}$	Yes
All are fermions with spin $\frac{1}{2}$.						
Field Particles						
Photon	γ	0	EM	EM	0	No
Intermediate bosons	$W^\pm$	85	W, EM	W	±1	No
	Z^0	97	W	W	0	No
Gluons	g	0	S	S	0	Yes
All are bosons with spin 1.						

[a] EM = electromagnetic; W = weak; S = strong.

escaping. This force, called the "color" force (discussed in Section 31.10), increases with separation distance (similar to the force of a spring). The great strength of the force between quarks has been described by one author as follows:[1]

> Quarks are slaves of their own color charge . . . bound like prisoners of a chain gang . . . Any locksmith can break the chain between two prisoners, but no locksmith is expert enough to break the gluon chains between quarks. Quarks remain slaves forever.

Thinking Physics 3

We have seen a law of conservation of *lepton number* and a law of conservation of *baryon number*. Why isn't there a law of conservation of *meson number*?

Reasoning We can argue this from the point of view of creating particle–antiparticle pairs from available energy. If energy is converted to rest energy of a lepton–antilepton pair, then there is no net change in lepton number, because the lepton has a lepton number of +1 and the antilepton −1. Energy could also be transformed into rest energy of a baryon–antibaryon pair. The baryon has baryon number +1, the antibaryon −1, and there is no net change in baryon number.

But now, suppose energy is transformed into rest energy of a quark–antiquark pair. By definition in quark theory, a quark–antiquark pair *is a meson*. Thus, we have created a meson from energy—there was no meson before; now there is. Thus, there is no conservation of meson number. With more energy, we can create more mesons, with no restriction from a conservation law other than that of energy.

[1] H. Fritzsch, *Quarks, The Stuff of Matter*, London, Allen Lane, 1983.

CONCEPTUAL PROBLEM 1

If high energy electrons, with de Broglie wavelengths smaller than the size of the nucleus, are scattered from nuclei, the behavior of the electrons is consistent with scattering from very massive structures much smaller in size than the nucleus—quarks. How is this similar to another classic experiment that detected small structures in an atom?

CONCEPTUAL PROBLEM 2

Doubly charged baryons, such as the Δ^{++}, are known to exist. Why are there no doubly charged mesons?

31.10 · COLORED QUARKS

Shortly after the concept of quarks was proposed, scientists recognized that certain particles had quark compositions that were in violation of the Pauli exclusion principle. Because all quarks are fermions with spins of 1/2, they are expected to follow the exclusion principle. One example of a particle that violates the exclusion principle is the Ω^- (sss) baryon that contains three s quarks having parallel spins, giving it a total spin of 3/2. Other examples of baryons that have identical quarks with parallel spins are the Δ^{++} (uuu) and the Δ^- (ddd). To resolve this problem, Moo-Young Han and Yoichiro Nambu suggested in 1965 that quarks possess a new property called **color.** This property is similar in many respects to electric charge except that it occurs in three varieties called red, green, and blue. (The antiquarks have the colors antired, antigreen, and antiblue.) To satisfy the exclusion principle, all three quarks in a baryon must have different colors. Just as a combination of actual colors of light can produce the neutral color white, a combination of three quarks with different colors is also white, or colorless. A meson consists of a quark of one color and an antiquark of the corresponding anticolor. The result is that baryons and mesons are always colorless (or white).

Although the concept of color in the quark model was originally conceived to satisfy the exclusion principle, it also provided a better theory for explaining certain experimental results. For example, the modified theory correctly predicts the lifetime of the π^0 meson. The theory of how quarks interact with each other is called **quantum chromodynamics,** or QCD, to parallel quantum electrodynamics (the theory of interaction between electric charges). In QCD, the quark is said to carry a **color charge,** in analogy to electric charge. The strong force between quarks is often called the **color force.**

As stated earlier, the strong interaction between hadrons is mediated by massless particles called **gluons** (g), which are analogous to photons for the electromagnetic force. According to QCD, there are eight gluons, all with color charge. When a quark emits or absorbs a gluon, its color changes. For example, a blue quark that emits a gluon may become a red quark, and the red quark that absorbs this gluon becomes a blue quark. The color force between quarks is analogous to the electric force between charges; like colors repel and opposite colors attract. Therefore, two red quarks repel each other, but a red quark will be attracted to an antired quark. The attraction between quarks of opposite color to form a meson ($q\bar{q}$) is indicated in Figure 31.8a. Differently colored quarks also attract each other, but with less intensity than opposite colors of quark and antiquark. For example, a cluster of red, blue, and green quarks all attract each other to form baryons, as

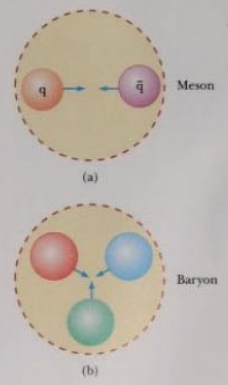


Figure 31.8 (a) A red quark is attracted to an antired quark. This forms a meson the quark structure of which is ($q\bar{q}$). (b) Three different colored quarks attract each other to form a baryon.

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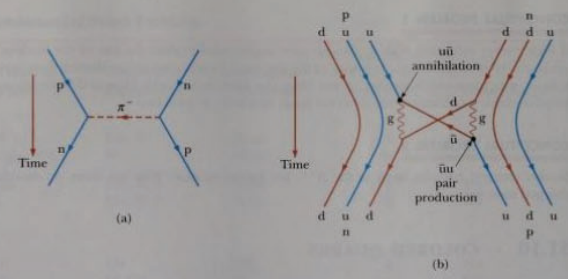


Figure 31.9 (a) A nuclear interaction between a proton and a neutron explained in terms of Yukawa's pion exchange model. (b) The same interaction as in (a), explained in terms of quarks and gluons. Note that the exchanged $\bar{u}d$ quark pair makes up a π^- meson.

indicated in Figure 31.8b. Thus, every baryon contains three quarks of three different colors.

Although the color force between two color-neutral hadrons is negligible at large separations, the strong color force between their constituent quarks does not exactly cancel at small separations. This residual strong force is in fact the nuclear force that binds protons and neutrons to form nuclei. In Section 31.3 we explained the nuclear interaction of a proton with a neutron using Yukawa's theory of pion exchange. According to QCD, a more basic explanation of nuclear force can be given in terms of quarks and gluons as shown by contrasting Feynman diagrams of the same process in Figure 31.9. Each quark within the neutron and proton is continually emitting and absorbing virtual gluons and creating and annihilating virtual ($q\bar{q}$) pairs. When the neutron and proton approach within 1 fm of each other, these virtual gluons and quarks can be exchanged between the two nucleons, and such exchanges produce the nuclear force. Figure 31.9b depicts one likely possibility or contribution to the process shown in Figure 31.9a. A down quark emits a virtual gluon (represented by a wavy line, g), which creates a $u\bar{u}$ pair. Both the recoiling d quark and the $\bar{u}$ are transmitted to the proton where the $\bar{u}$ annihilates a proton u quark (with the creation of a gluon) and the d is captured.

31.11 • ELECTROWEAK THEORY AND THE STANDARD MODEL

Recall that the weak interaction is an extremely short range force with an interaction distance of approximately 10^{-18} m. Such a short range interaction implies that the quantized particles that carry the weak field (the spin one W^+, W^-, and Z^0 bosons) are quite massive as is indeed the case (see Table 31.5). These bosons are especially amazing when we think of them as structureless, pointlike particles as massive as krypton atoms. As mentioned earlier, the weak interaction is responsible for neutron decay and the beta decay of other heavier baryons. More important, the weak interaction is responsible for the decay of the c, s, b, t quarks

into lighter, more stable u and d quarks as well as the decay of the massive μ and τ leptons into (lighter) electrons. Thus, **the weak interaction is very important because it governs the stability of the basic matter particles.**

A mysterious feature of the weak interaction is its lack of symmetry, especially compared to the high degree of symmetry shown by the strong, electromagnetic, and gravitational interactions. For example, the weak interaction, unlike the strong interaction, is not symmetric under mirror reflection or charge exchange. (Mirror reflection means that all the quantities in a given particle reaction are exchanged as in a mirror reflection—left for right, an inward motion toward the mirror for an outward motion. Charge exchange means that all the electric charges in a particle reaction are converted to their opposites—all positives to negatives and vice versa.) When we say that the weak interaction is not symmetric, we mean that the reaction with all quantities changed occurs less frequently than the direct reaction. For example, the decay of the K^0, which is governed by the weak interaction, is not symmetric under charge exchange, because $K^0 \rightarrow \pi^- + e^+ + \nu_e$ occurs much more frequently than $K^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e$.

In 1979, Sheldon Glashow, Abdus Salam, and Steven Weinberg won a Nobel prize for developing a theory that unified the electromagnetic and weak interactions. This **electroweak theory** postulates that the weak and electromagnetic interactions have the same strength at very high particle energies. Thus, the two interactions are viewed as two different manifestations of a single unifying electroweak interaction. The photon and the three massive bosons ($W^{\pm}$ and Z^0) play a key role in the electroweak theory. The theory makes many concrete predictions, but perhaps the most spectacular is the prediction of the masses of the W and Z particles at about 82 GeV/c^2 and 93 GeV/c^2, respectively. The 1984 Nobel prize was awarded to Carlo Rubbia and Simon van der Meer for their work leading to the discovery of these particles with just those masses at the CERN Laboratory in Geneva, Switzerland.

The combination of the electroweak theory and QCD for the strong interaction form what is called the **Standard Model** in high-energy physics. Although the details of the Standard Model are complex, its essential ingredients can be summarized with the help of Figure 31.10. The strong force, mediated by gluons, holds

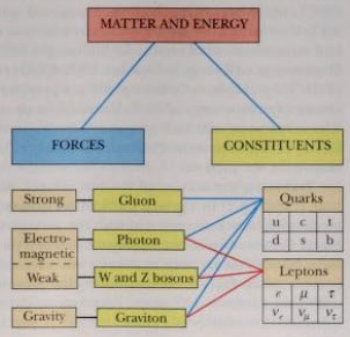


Figure 31.10 The Standard Model of particle physics.

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quarks together to form composite particles such as protons, neutrons, and mesons. Leptons participate in the electromagnetic, weak, and gravitational interactions. The electromagnetic force is mediated by photons, and the weak force is mediated by W and Z bosons. Note that all fundamental forces are mediated by spin 1 bosons with properties given, to a large extent, by symmetries involved in the theories.

The Standard Model does not answer all questions, however. A major question is why the photon has no mass, whereas the W and Z bosons do have mass. Because of this mass difference, the electromagnetic and weak forces are quite distinct at low energies but become similar at very high energies, where the rest energies of the W and Z bosons are insignificant fractions of their total energies. This behavior in the transition from high to low energies, called **symmetry breaking,** leaves open the question of the origin of particle masses. To resolve this problem, a hypothetical particle called the **Higgs boson** has been proposed. The Higgs boson provides a mechanism for breaking the electroweak symmetry. The Standard Model, including the Higgs mechanism, provides a logically consistent explanation of the massive nature of the W and Z bosons. Unfortunately, the Higgs boson has not yet been found, but physicists know that its mass should be less than 1 TeV/c^2 (10^{12} eV/c^2).

In order to determine whether the Higgs boson exists, two quarks of at least 1 TeV of energy must collide, but calculations show that this requires injecting 40 TeV of energy within the volume of a proton. Scientists are convinced that, because of the limited energy available in conventional accelerators using fixed targets, it is necessary to build colliding-beam accelerators called **colliders.** The concept of colliders is straightforward. Particles with equal masses and kinetic energies, traveling in opposite directions in an accelerator ring, collide head-on to produce the required reaction and the formation of new particles. Because the total momentum of the interacting particles is zero, all of their kinetic energy is available for the reaction. The Large Electron–Positron collider (LEP) at CERN near Geneva, Switzerland, and the Stanford Linear Collider in California collide both electrons and protons. The Super Proton Synchrotron at CERN accelerates protons and antiprotons to energies of 270 GeV, and the world's highest-energy proton accelerator, the Tevatron, at the Fermi National Laboratory in Illinois, produces protons at almost 1000 GeV (or 1 TeV). The Superconducting Super Collider (SSC), which was being built in Texas, was an accelerator designed to produce 20-TeV protons in a ring 52 mi in circumference. After much debate in Congress, and an investment of almost $2 billion, the SSC project was canceled by the U.S. Department of Energy in October 1993. CERN recently approved the development of the Large Hadron Collider (LHC), a proton–proton collider that will provide a center of mass energy of 14 TeV and allow an exploration of Higgs-boson physics. The accelerator will be constructed in the same 27-km circumference tunnel as CERN's Large Electron–Positron collider, and many countries are expected to participate in the project.

Following the success of the electroweak theory, scientists attempted to combine it with QCD in a **grand unification theory** known as GUT. In this model, the electroweak force was merged with the strong color force to form a grand unified force. One version of the theory considers leptons and quarks as members of the same family that are able to change into each other by exchanging an appropriate particle. Many GUT theories predict that protons are unstable and will decay with a lifetime of about 10^{31} years. Attempts to detect such proton decays have so far been unsuccessful.

Thinking Physics 4

Consider a car making a head-on collision with an identical car moving in the opposite direction at the same speed. Compare that collision to one of the cars making a collision with the second car at rest. In which collision is there the larger transformation of kinetic energy to other forms during the collision? How does this relate to particle accelerators?

Reasoning In the head-on collision with both cars moving, conservation of momentum causes most if not all of the kinetic energy to be transformed to other forms. In the collision between a moving car and a stationary car, the cars are still moving after the collision, in the direction of the moving car, but with reduced speed. Thus, *only part* of the kinetic energy is transformed to other forms.

This suggests the advantage of using colliding beams in a particle accelerator, as opposed to firing a beam into a stationary target. When particles moving in opposite directions collide, all of the kinetic energy is available for transformation into other forms—in this case, the creation of new particles. When a beam is fired into a stationary target, only part of the energy is available for transformation, so higher mass particles cannot be created.

31.12 · THE COSMIC CONNECTION

In this section we shall describe one of the most fascinating theories in all of science—the Big Bang theory of the creation of the Universe—and the experimental evidence that supports it. This theory of cosmology states that the Universe had a beginning and, further, that the beginning was so cataclysmic that it is impossible to look back beyond it. According to this theory, the Universe erupted from a point-like singularity about 10 billion to 20 billion years ago. Such extremes of energy occurred in the first few minutes after the Big Bang that it is believed that all four interactions of physics were unified and that all matter melted down into an undifferentiated "quark soup."

The evolution of the four fundamental forces from the Big Bang to the present is shown in Figure 31.11. During the first 10^{-43} s (the ultra-hot epoch during which $T \approx 10^{32}$ K), it is presumed that the strong, electroweak, and gravitational forces were joined to form a completely unified force. In the first 10^{-32} s following the Big Bang (the hot epoch, $T \approx 10^{29}$ K), gravity broke free of this unification and the strong and electroweak forces remained joined, as described by a grand unification theory. This was a period when particle energies were so great ($> 10^{16}$ GeV) that very massive particles as well as quarks, leptons, and their antiparticles existed. Then the Universe rapidly expanded and cooled during the warm epoch when the temperatures ranged from 10^{29} to 10^{15} K; the strong and electroweak forces parted company, and the grand unification scheme was broken. As the Universe continued to cool, the electroweak force split into the weak force and the electromagnetic force about 10^{-10} s after the Big Bang.

After the first half-hour of creation, the temperature was probably about 10^{10} K and the Universe could be viewed as a cosmic thermonuclear bomb fusing protons and neutrons into deuterium and then helium. Until about 700 000 years after the Big Bang, the Universe was dominated by radiation: Ions absorbed and re-emitted photons, thereby ensuring thermal equilibrium of radiation and matter. Energetic radiation also prevented matter from forming clumps or even single hy-



George Gamow (1904–1968)

Gamow and two of his students, Ralph Alpher and Robert Herman, were the first to take the first half hour of the Universe seriously. In a mostly overlooked paper published in 1948, they made truly remarkable cosmological predictions. They correctly calculated the abundances of hydrogen and helium after the first half hour (75% H and 25% He) and predicted that radiation from the Big Bang should still be present and have an apparent temperature of about 5 K.

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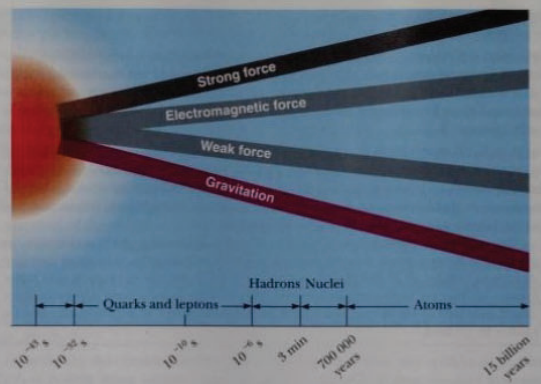


Figure 31.11 A brief history of the Universe from the Big Bang to the present. The four forces became distinguishable during the first microsecond. Following this, all the quarks combined to form particles that interact via the strong force. However, the leptons remained separate and exist as individually observable particles to this day.



Figure 31.12 Robert W. Wilson (left) and Arno A. Penzias (right) with Bell Telephone Laboratories horn-reflector antenna. *(AT&T Bell Laboratories)*

drogen atoms. When the Universe was about 700 000 years old, it had expanded and cooled to about 3000 K, and protons could bind to electrons to form neutral hydrogen atoms. Because neutral atoms do not appreciably scatter photons, the Universe suddenly became transparent to photons. Radiation no longer dominated the Universe and clumps of neutral matter steadily grew—first atoms, followed by molecules, gas clouds, stars, and finally galaxies.

Observation of Radiation from the Primordial Fireball

In 1965, Arno A. Penzias and Robert W. Wilson of Bell Labs were testing a sensitive microwave receiver and made an amazing discovery. A pesky signal producing a faint background hiss was interfering with their satellite communications experiments. Despite their valiant efforts, the signal remained. Ultimately, it became clear that they were perceiving microwave background radiation (at a wavelength of 7.35 cm) representing the leftover glow from the Big Bang.

The microwave horn that served as their receiving antenna is shown in Figure 31.12. The intensity of the detected signal remained unchanged as the antenna was pointed in different directions. The fact that the radiation had equal strengths in all directions suggested that the entire Universe was the source of this radiation. Eviction of a flock of pigeons from the 20-foot horn and cooling the microwave detector both failed to remove the "spurious" signal. Through a casual conversation, Penzias and Wilson discovered that a group at Princeton had predicted the residual radiation from the Big Bang and were planning an experiment seeking to confirm the theory. The excitement in the scientific community was high when Penzias and Wilson announced that they had already observed an excess microwave background compatible with a 3-K black-body source.

Because the measurements of Penzias and Wilson were taken at a single wavelength, they did not completely confirm the radiation as 3-K blackbody radiation. Subsequent experiments by other groups added intensity data at different wavelengths, as shown in Figure 31.13. The results confirm that the radiation is that of a black body at 2.9 K. This figure is, perhaps, the most clearcut evidence for the Big Bang theory. It also presents the earliest view of the Universe, a much earlier view than that available with the largest optical and radio telescopes. The 1978 Nobel prize was awarded to Penzias and Wilson for their most important discovery.

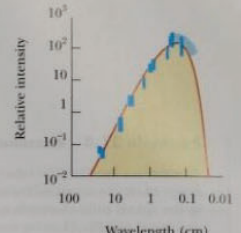


Figure 31.13 Radiation spectrum of the Big Bang. The blue areas are experimental results. The red line is the spectrum calculated for a black body at 2.9 K.

Other Evidence for the Expanding Universe

Most of the key discoveries supporting the theory of an expanding Universe, and indirectly the Big Bang theory of cosmology, were made in the 20th century. In 1913 Vesto Melvin Slipher, an American astronomer, reported that most nebulae are receding from the Earth at speeds up to several million miles per hour. Slipher was one of the first to use the methods of Doppler shifts in spectral lines to measure velocities.

In the late 1920s, Edwin P. Hubble made the bold assertion that the whole Universe is expanding. From 1928 to 1936, he and Milton Humason toiled at Mount Wilson to prove this assertion until they reached the limits of the 100-inch telescope. The results of this work and its continuation on a 200-inch telescope in the 1940s showed that the speeds of galaxies increase in direct proportion to their distance R from us (Fig. 31.14). This linear relationship, known as **Hubble's law,** may be written

$$v = HR \qquad [31.7]$$

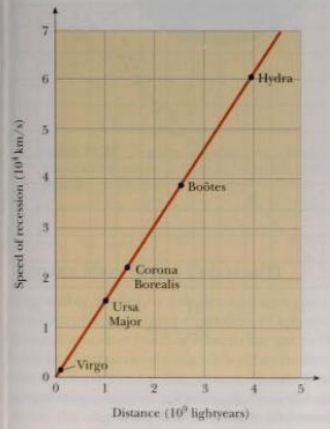


Figure 31.14 Hubble's law: The speed of recession is directly proportional to distance. The speeds of galaxies in five clusters are plotted versus distance.

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where H, called the **Hubble parameter,** has the approximate value

$$H = 17 \times 10^{-3}\ \text{m/(s·lightyear)}$$

with an uncertainty of about 50%. Note that a lightyear is the distance light travels in one Earth year (9.46×10^{15} m).

Example 31.4 Recession of a Quasar

A quasar is a star-like object that is very distant from the Earth. Its speed can be measured from Doppler shift measurements in the light it emits. A certain quasar recedes from the Earth at a speed of $0.55c$. How far away is it?

Solution We can find the distance from Hubble's law:

$$R = \frac{v}{H} = \frac{(0.55)(3.00 \times 10^8\ \text{m/s})}{17 \times 10^{-3}\ \text{m/(s·lightyear)}}$$

$$= 9.7 \times 10^9\ \text{lightyears}$$

$$= 9.2 \times 10^{25}\ \text{m}$$

EXERCISE 3 Assuming that the quasar has moved with the speed $0.55c$ ever since the Big Bang, estimate the age of the Universe. **Answer** $t = R/v = 1/H \approx 18$ billion years, which is in reasonable agreement with other estimates of the age of the Universe.

Will the Universe Expand Forever?

During the 1950s and 1960s Allan R. Sandage used the 200-inch telescope at Mount Palomar in California to measure the speeds of galaxies at distances of up to 6 billion lightyears. His measurements showed that these very distant galaxies were moving about 10 000 km/s faster than the Hubble law predicted. According to this result, the Universe must have been expanding more rapidly 1 billion years ago and, consequently, the expansion is slowing[2] (Fig. 31.15). Today astronomers and

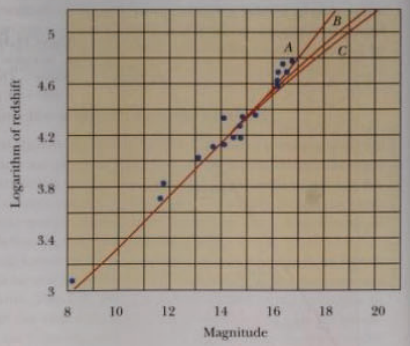


Figure 31.15 Red shift, or speed of recession, versus apparent magnitude of 18 faint clusters. Curve A is the trend suggested by the six faintest clusters of galaxies. Curve C corresponds to a Universe having a constant rate of expansion. If the data fall between B and C, the expansion slows but never stops. If the data fall to the left of B, expansion stops and contraction occurs.

[2]The data at large distances have large observational uncertainties and may be systematically in error from selection effects such as abnormal brightness in the most distant visible clusters.

physicists are trying to determine the rate of slowing. If the average mass density of the Universe is less than some critical density ($\rho_c \approx 4$ hydrogen atoms/m^3), the galaxies will slow in their outward rush but still escape to infinity. If the average density exceeds the critical value, the expansion will eventually stop and contraction will begin, possibly leading to a superdense state and another expansion or an oscillating Universe.

Example 31.5 The Critical Density of the Universe

Estimate the critical mass density of the Universe, ρ_c, using energy considerations.

Reasoning and Solution Figure 31.16 shows a large section of the Universe with radius R, containing galaxies with a total mass M. A galaxy of mass m and speed v at R will just escape to infinity with zero speed if the sum of its kinetic energy and gravitational potential energy is zero. Thus,

$$E_{\text{total}} = 0 = K + U = \tfrac{1}{2}mv^2 - \frac{GmM}{R}$$

$$\tfrac{1}{2}mv^2 = \frac{Gm\frac{4}{3}\pi R^3\rho_c}{R}$$

$$(1) \qquad v^2 = \frac{8\pi G}{3}R^2\rho_c$$

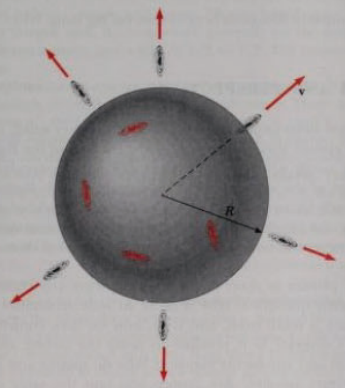


Figure 31.16 A galaxy escaping from a large cluster contained within radius R. Only the mass within R slows the mass m.

Because the galaxy of mass m obeys the Hubble law, $v = HR$, (1) becomes

$$H^2 = \frac{8\pi G}{3}\rho_c$$

$$(2) \qquad \rho_c = \frac{3H^2}{8\pi G}$$

Using $H = 17 \times 10^{-3}$ m/(s·lightyear), where 1 lightyear = 9.46×10^{15} m, and $G = 6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2$ yields $\rho_c = 5.8 \times 10^{-27}$ kg/m^3. As the mass of a hydrogen atom is 1.67×10^{-27} kg, ρ_c corresponds to about 4 hydrogen atoms per cubic meter.

Missing Mass in the Universe(?)

The visible matter in galaxies averages out to 5×10^{-30} kg/m^3. The radiation in the Universe has a mass equivalent of approximately 2% of the visible matter. Nonluminous matter (such as interstellar gas or black holes) can be estimated from the speeds of galaxies orbiting each other in the cluster. The higher the galaxy speeds, the more mass in a galaxy cluster. Results from measurements on the Coma cluster of galaxies indicate, surprisingly, that the amount of invisible matter is 20 to 30 times the amount present in stars and luminous gas clouds. Yet even this large

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invisible component, if applied to the Universe as a whole, leaves the observed mass density a factor of 10 less than ρ_c. This so-called *missing mass* (or *dark matter*) has been the subject of intense theoretical and experimental work. Exotic massive particles such as axions, photinos, and superstring particles have been suggested as candidates for the missing mass. More mundane proposals have been that the missing mass is present in certain galaxies as neutrinos. In fact, neutrinos are so abundant that a tiny neutrino mass on the order of 20 eV/c^2 could furnish the missing mass and "close" the Universe.

Although we are a bit more sure about the beginning of the Universe, we are uncertain about its end. Will the Universe expand forever? Will it collapse and repeat its expansion in an endless series of oscillations? Results and answers to these questions remain inconclusive, and the exciting controversy continues.

CONCEPTUAL PROBLEM 3

Atoms did not exist until hundreds of thousands of years after the Big Bang. Why?

31.13 · PROBLEMS AND PERSPECTIVES

While particle physicists have been exploring the realm of the very small, cosmologists have been exploring cosmic history back to the first instants of the Big Bang. Observation of events that occur when two particles collide in an accelerator is essential to a reconstruction of the early moments in cosmic history. Perhaps the key to understanding the early Universe is first to understand the world of elementary particles. Cosmologists and physicists now find they have many common goals and are joining hands to attempt to understand the physical world at its most fundamental level.

Our understanding of physics at short distances is far from complete. Particle physics is faced with many questions. Why is there so little antimatter in the Universe? Do neutrinos have a small mass, and if so, how do they contribute to the "dark matter" of the Universe? Is it possible to unify the strong and electroweak theories in a logical and consistent manner? Why do quarks and leptons form three similar but distinct families? Are muons and taus the same as electrons (apart from their different masses), or do they have other subtle differences that have not been detected? Why are some particles charged and others neutral? Why do quarks carry a fractional charge? What determines the masses of the fundamental leptons and quarks? Can isolated quarks exist? The questions go on and on. Given the rapid advances and new discoveries in the field of particle physics, some of these questions will likely be resolved by the time you read this book.

An important and obvious question that remains is whether leptons and quarks have a substructure. If they have a substructure, we can envision an infinite number of deeper structure levels. However, if leptons and quarks are indeed the ultimate constituents of matter, as physicists today tend to believe, we should be able to construct a final theory of the structure of matter as Einstein dreamed. In the view of many physicists, the end of the road is in sight, but how long it will take to reach that goal is anyone's guess.

SUMMARY

There are four fundamental forces in nature: strong, electromagnetic, weak, and gravitational. The strong, or color force, is the force between quarks that binds them into neutrons, protons, and other hadrons. The weak force is responsible for beta decay. The electromagnetic and weak forces are now considered to be manifestations of a single force called the electroweak force. Every fundamental interaction is said to be mediated by the exchange of field particles. The electromagnetic interaction is mediated by the photon; the weak interaction is mediated by the $W^{\pm}$ and Z^0 bosons; the gravitational interaction is mediated by gravitons; the strong interaction is mediated by gluons.

An antiparticle and a particle have the same mass but opposite charge; other properties may also have opposite values such as lepton number and baryon number. It is possible to produce particle–antiparticle pairs in nuclear reactions if the available energy is greater than $2mc^2$, where m is the mass of the particle (or antiparticle).

Many particles are classified as hadrons or leptons. **Hadrons** interact through the strong force. They have size and structure and are not elementary particles. There are two types of hadrons, *baryons* and *mesons*. Mesons have baryon number zero and have either zero or integral spin. Baryons, which generally are the most massive particles, have nonzero baryon number and a spin of 1/2 or 3/2. The neutron and proton are examples of baryons.

Leptons have no structure or size and are considered truly elementary. They interact only through the weak and electromagnetic forces. There are six leptons, the electron e^-, the muon μ^-, the tau τ^-; and their neutrinos ν_e, ν_μ, and ν_τ.

In all reactions and decays, quantities such as energy, linear momentum, angular momentum, electric charge, baryon number, and lepton number are strictly conserved. Certain particles have properties called **strangeness** and **charm.** These unusual properties are conserved only in those reactions and decays that occur via the strong force.

Theories in elementary particle physics have postulated that all hadrons are composed of smaller units known as **quarks.** Quarks have fractional electric charge, baryon numbers of 1/3, and come in six "flavors": up (u), down (d), strange (s), charmed (c), top (t), and bottom (b). Each baryon contains three quarks, and each meson contains one quark and one antiquark.

According to the theory of **quantum chromodynamics,** quarks have a property called **color,** and the strong force between quarks is referred to as the **color force.**

The background microwave radiation discovered by Penzias and Wilson strongly suggests that the Universe started with a Big Bang about 15 billion years ago. The background radiation is equivalent to that of a black body at approximately 3 K.

Astronomical measurements show that the Universe is expanding. According to **Hubble's law,** distant galaxies are receding from the Earth at a speed $v = HR$, where R is the distance from Earth to the galaxy and H is **Hubble's parameter,** $H \approx 17 \times 10^{-3}$ m/ (s·lightyear).

CONCEPTUAL QUESTIONS

1. Name the four fundamental interactions and the field particle that mediates each.
2. Describe the quark model of hadrons, including the properties of quarks.
3. What are the differences between hadrons and leptons?
4. Describe the properties of baryons and mesons and the important differences between them.
5. Particles known as resonances have very short lifetimes, of the order of 10^{-23} s. From this information, would you guess they are hadrons or leptons? Explain.

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6. Kaons all decay into final states that contain no protons or neutrons. What is the baryon number of kaons?
7. The Ξ^0 particle decays by the weak interaction according to the decay mode $\Xi^0 \rightarrow \Lambda^0 + \pi^0$. Would you expect this decay to be fast or slow? Explain.
8. Identify the particle decays listed in Table 31.2 that occur by the weak interaction. Justify your answers.
9. Identify the particle decays listed in Table 31.2 that occur by the electromagnetic interaction. Justify your answers.
10. Two protons in a nucleus interact via the strong interaction. Are they also subject to the weak interaction?
11. Discuss the following conservation laws: energy, linear momentum, angular momentum, electric charge, baryon number, lepton number, and strangeness. Are all of these laws based on fundamental properties of nature? Explain.
12. An antibaryon interacts with a meson. Can a baryon be produced in such an interaction? Explain.
13. Describe the essential features of the Standard Model of particle physics.
14. How many quarks are there in (a) a baryon, (b) an antibaryon, (c) a meson, (d) an antimeson? How do you account for the fact that baryons have half-integral spins and mesons have spins of 0 or 1? (*Hint:* Quarks have spin 1/2.)
15. In the theory of quantum chromodynamics, quarks come in three colors. How would you justify the statement that "all baryons and mesons are colorless"?
16. Which baryon did Murray Gell-Mann predict in 1961? What is the quark composition of this particle?
17. What is the quark composition of the Ξ^- particle? (See Tables 31.2 and 31.3.)
18. The W and Z bosons were first produced at CERN in 1983 (by having a beam of protons and a beam of antiprotons meet at high energy). Why was this an important discovery?
19. How did Edwin Hubble in 1928 determine that the Universe is expanding?

PROBLEMS

Section 31.2 Positrons and Other Antiparticles

1. A photon produces a proton–antiproton pair according to the reaction $\gamma \rightarrow p + \bar{p}$. What is the minimum possible frequency of the photon? What is its wavelength?
2. Two photons are produced when a proton and antiproton annihilate each other. What is the minimum frequency and corresponding wavelength of each photon?
3. A photon with an energy of $E_\gamma = 2.09$ GeV creates a proton–antiproton pair in which the proton has a kinetic energy of 95.0 MeV. What is the kinetic energy of the antiproton? ($m_p c^2 = 938.3$ MeV.)

Section 31.3 Mesons and the Beginning of Particle Physics

4. Occasionally, high-energy muons collide with electrons and produce two neutrinos according to the reaction $\mu^+ + e^- \rightarrow 2\nu$. What kind of neutrinos are these?
5. One of the mediators of the weak interaction is the Z^0 boson, with a mass of 96 GeV/c^2. Use this information to find an approximate value for the range of the weak interaction.
6. A free neutron beta decays by creating a proton, electron, and antineutrino according to the reaction $n \rightarrow p + e^- + \bar{\nu}$. Pretend that a free neutron beta decays by creating a proton and electron according to the reaction $n \rightarrow p + e^-$ and assume that the neutron is initially at rest in the laboratory. (a) Determine the energy released in this reaction. (b) Determine the speed of the proton and electron after the reaction. (Energy and momentum are conserved in the reaction.) (c) Are any of these particles moving at relativistic speeds? Explain.
7. When a high-energy proton or pion traveling near the speed of light collides with a nucleus, it travels an average distance of 3×10^{-15} m before interacting. From this information, estimate the duration of the strong interaction.
8. Calculate the range of the force that might be produced by the virtual exchange of a proton.
9. A neutral pion at rest decays into two photons according to

$$\pi^0 \longrightarrow \gamma + \gamma$$

Find the energy, momentum, and frequency of each photon.

Section 31.4 Classification of Particles

10. Identify the unknown particle on the left side of the following reaction:

$$? + p \longrightarrow n + \mu^+$$

11. Name one possible decay mode (see Table 31.2) for Ω^+, $\bar{K}_S^0$, $\bar{\Lambda}^0$, and $\bar{n}$.

Section 31.5 Conservation Laws

12. Each of the following reactions is forbidden. Determine a conservation law that is violated for each reaction.
    (a) $p + \bar{p} \rightarrow \mu^+ + e^-$
    (b) $\pi^- + p \rightarrow p + \pi^+$
    (c) $p + p \rightarrow p + \pi^+$
    (d) $p + p \rightarrow p + p + n$
    (e) $\gamma + p \rightarrow n + \pi^0$
13. (a) Show that baryon number and charge are conserved in the following reactions of a pion with a proton.

$$\pi^- + p \longrightarrow K^- + \Sigma^+ \qquad [1]$$

$$\pi^- + p \longrightarrow \pi^- + \Sigma^+ \qquad [2]$$

(b) The first reaction is observed, but the second never occurs. Explain.

14. For the following two reactions, the first may occur but the second cannot. Explain.

$$K_S^0 \longrightarrow \pi^+ + \pi^- \qquad \text{(can occur)}$$

$$\Lambda^0 \longrightarrow \pi^+ + \pi^- \qquad \text{(cannot occur)}$$

15. The following reactions or decays involve one or more neutrinos. Supply the missing neutrinos (ν_e, ν_μ, or ν_τ).
    (a) $\pi^- \rightarrow \mu^- + ?$
    (b) $K^+ \rightarrow \mu^+ + ?$
    (c) $? + p \rightarrow n + e^+$
    (d) $? + n \rightarrow p + e^-$
    (e) $? + n \rightarrow p + \mu^-$
    (f) $\mu^- \rightarrow e^- + ? + ?$
16. A K_S^0 particle at rest decays into a π^+ and a π^-. What will be the speed of each of the pions? The mass of the K_S^0 is 497.7 MeV/c^2, and the mass of each π is 139.6 MeV/c^2.
17. Determine which of the reactions below can occur. For those that cannot occur, determine the conservation law (or laws) violated:
    (a) $p \rightarrow \pi^+ + \pi^0$
    (b) $p + p \rightarrow p + p + \pi^0$
    (c) $p + p \rightarrow p + \pi^+$
    (d) $\pi^- \rightarrow \mu^- + \nu_\mu$
    (e) $n \rightarrow p + e^- + \bar{\nu}_e$
    (f) $\pi^+ \rightarrow \mu^+ + n$

Section 31.6 Strange Particles and Strangeness

18. The neutral ρ meson decays by the strong interaction into two pions: $\rho^0 \rightarrow \pi^+ + \pi^-$, half-life 10^{-23} s. The neutral kaon also decays into two pions: $K_S^0 \rightarrow \pi^+ + \pi^-$, half-life 10^{-10} s. How do you explain the large difference in half-lives?
19. Determine whether or not strangeness is conserved in the following decays and reactions.
    (a) $\Lambda^0 \rightarrow p + \pi^-$
    (b) $\pi^- + p \rightarrow \Lambda^0 + K^0$
    (c) $\bar{p} + p \rightarrow \bar{\Lambda}^0 + \Lambda^0$
    (d) $\pi^- + p \rightarrow \pi^- + \Sigma^+$
    (e) $\Xi^- \rightarrow \Lambda^0 + \pi^-$
    (f) $\Xi^0 \rightarrow p + \pi^-$
20. The following decays are forbidden. Determine a conservation law that each violates.
    (a) $\mu^- \rightarrow e^- + \gamma$
    (b) $n \rightarrow p + e^- + \nu_e$
    (c) $\Lambda^0 \rightarrow p + \pi^0$
    (d) $p \rightarrow e^+ + \pi^0$
    (e) $\Xi^0 \rightarrow n + \pi^0$

Section 31.9 Quarks

21. The quark composition of the proton is uud and that of the neutron is udd. Show that the charge, baryon number, and strangeness of the particle equal the sum of these numbers for their quark constituents.
22. (a) Find the number of electrons and the number of each species of quarks, in one liter of water. (b) Make an order-of-magnitude estimate of the number of each kind of fundamental matter particle in your body. State your assumptions and the quantities you take as data.
23. The quark compositions of the K^0 and Λ^0 particles are $d\bar{s}$ and uds, respectively. Show that the charge, baryon number, and strangeness of these particles equal the sums of these numbers for the quark constituents.
24. Neglect binding energies and estimate the masses of the u and d quarks from the masses of the proton and neutron.
25. Analyze each reaction in terms of constituent quarks:
    (a) $\pi^- + p \rightarrow K^0 + \Lambda^0$
    (b) $\pi^+ + p \rightarrow K^+ + \Sigma^+$
    (c) $K^- + p \rightarrow K^+ + K^0 + \Omega^-$
    (d) $p + p \rightarrow K^0 + p + \pi^+ + ?$
    In the last reaction, identify the mystery particle.
26. A Σ^0 particle traveling through matter strikes a proton and a Σ^+ and a gamma ray emerges, as well as a third particle. Use the quark model of the Σ^+ and the gamma ray to determine the identity of the third particle.

Section 31.12 The Cosmic Connection

27. Using Hubble's law (Eq. 31.7), estimate the wavelength of the 590-nm sodium line emitted from galaxies (a) 2×10^6

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lightyears away from Earth, (b) 2×10^8 lightyears away, and (c) 2×10^9 lightyears away. *Hint:* Use the relativistic Doppler formula for wavelength λ' of light emitted from a moving source:

$$\lambda' = \lambda \sqrt{\frac{1 + v/c}{1 - v/c}}$$

28. A distant quasar is moving away from Earth at such high speed that the violet 434-nm hydrogen line is observed at 650 nm in the red portion of the spectrum. (a) How fast is the quasar receding? (See the hint in the preceding problem.) (b) Using Hubble's law, determine the distance from Earth to this quasar.
29. The various spectral lines observed from a distant receding galaxy or quasar have longer wavelengths λ'_n than the wavelengths λ_n measured in a stationary laboratory. It is found that the fractional change of wavelengths toward the red are the same for all spectral lines. That is,

$$\frac{\lambda'_n - \lambda_n}{\lambda_n} = Z$$

where Z is the redshift parameter common to all spectral lines for a given object. In terms of Z, determine (a) the speed of recession of the quasar and (b) the distance from Earth to this quasar. Use the hint in Problem 27 and Hubble's law (Eq. 31.7).

Additional Problems

30. Take the range of the strong interaction as approximately 1.4×10^{-15} m. It is thought that an elementary particle is exchanged between the protons and neutrons in the nucleus, leading to an attractive force. (a) Utilize the uncertainty principle $\Delta E\, \Delta t \geq \hbar/2$ to estimate the mass of the elementary particle if it moves at nearly the speed of light. (b) Using Table 31.2, identify the particle.
31. Name at least one conservation law that prevents each of the following reactions:
    (a) $\pi^- + p \rightarrow \Sigma^+ + \pi^0$
    (b) $\mu^- \rightarrow \pi^- + \nu_e$
    (c) $p \rightarrow \pi^+ + \pi^+ + \pi^-$
32. Two protons approach each other with 70.4 MeV of kinetic energy and engage in a reaction in which a proton and positive pion emerge at rest. What third particle, obviously uncharged and therefore difficult to detect, must have been created?
33. The energy flux carried by neutrinos from the Sun is estimated to be on the order of 0.4 W/m^2 at Earth's surface. Estimate the fractional mass loss of the Sun over 10^9 years due to the radiation of neutrinos. (The mass of the Sun is 2×10^{30} kg. The Earth–Sun distance is 1.5×10^{11} m.)
34. Supernova 1987A, located about 170 000 lightyears from the Earth, is estimated to have emitted a burst of $\sim 10^{46}$ J of neutrinos. Suppose the average neutrino energy was 6 MeV and your body presented a target area of 5000 cm^2. To an order of magnitude, how many of these neutrinos passed through you?
35. A gamma-ray photon strikes a stationary electron. Determine the minimum gamma-ray energy needed to make this reaction occur:

$$\gamma + e^- \longrightarrow e^- + e^- + e^+$$

36. An unstable particle, initially at rest, decays into a proton (rest energy 938.3 MeV) and a negative pion (rest energy 139.5 MeV). A uniform magnetic field of 0.250 T exists with the field perpendicular to the velocities of the created particles. The radius of curvature of each track is found to be 1.33 m. What is the mass of the original unstable particle?
37. Calculate the kinetic energies of the proton and pion resulting from the decay of a Λ^0 at rest:

$$\Lambda^0 \longrightarrow p + \pi^-$$

38. A Σ^0 particle at rest decays according to

$$\Sigma^0 \longrightarrow \Lambda^0 + \gamma$$

Find the gamma-ray energy.
39. If a K_S^0 meson at rest decays in 0.900×10^{-10} s, how far will a K_S^0 meson travel if it is moving at $0.960c$ through a bubble chamber?
40. Two protons approach each other with equal and opposite velocities. What is the minimum kinetic energy of each of the protons if they are to produce a π^+ meson at rest in the following reaction?

$$p + p \longrightarrow p + n + \pi^+$$

41. A π-meson at rest decays according to $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$. What is the energy carried off by the neutrino? (Assume that the neutrino moves off with the speed of light.) $m_\pi c^2 = 139.5$ MeV, $m_\mu c^2 = 105.7$ MeV, $m_\nu = 0$.
42. What processes are described by the Feynman diagrams in Figure P31.42? What is the exchanged particle in each process?

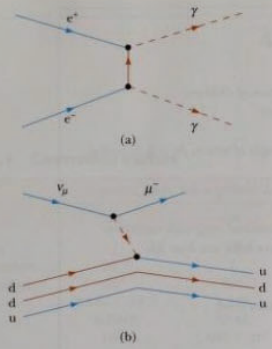


Figure P31.42



"Particles, particles, particles."

43. Identify the mediators for the two interactions described in the Feynman diagrams shown in Figure P31.43.

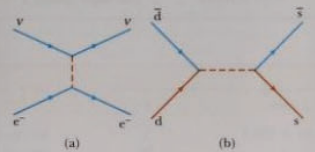


Figure P31.43

ANSWERS TO CONCEPTUAL PROBLEMS

1. The experiment described is a nice analogy to the Rutherford scattering experiment described in Section 29.1. In the Rutherford experiment, alpha particles were scattered from atoms, and the scattering was consistent with a small structure in the atom containing the positive charge.
2. The largest quark charge is $2e/3$, so a combination of only a quark and an antiquark, forming a meson, could not possibly have electric charge adding up to $+2e$. The Δ^{++} particle has three up quarks, each with charge $2e/3$, for a total charge of $+2e$.
3. Until about 700 000 years after the Big Bang, the temperature of the Universe was high enough for any atoms that formed to be ionized by ambient radiation. Once the average radiation energy dropped below the hydrogen ionization energy of 13.6 eV, hydrogen atoms could form and remain as neutral atoms for a relatively long period of time.

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The Meaning of Success

To earn the respect of intelligent people and to win the affection of children;
To appreciate the beauty in nature and all that surrounds us;
To seek out and nurture the best in others;
To give the gift of yourself to others without the slightest thought of return, for it is in giving that we receive;
To have accomplished a task, whether it be saving a lost soul, healing a sick child, writing a book, or risking your life for a friend;
To have celebrated and laughed with great joy and enthusiasm and sung with exaltation;
To have hope even in times of despair, for as long as you have hope, you have life;
To love and be loved;
To be understood and to understand;
To know that even one life has breathed easier because you have lived;
This is the meaning of success.

RALPH WALDO EMERSON
and modified by Ray Serway
December 1989

APPENDIX A

TABLE A.1 Conversion Factors

	Length					
	m	**cm**	**km**	**in.**	**ft**	**mi**
1 meter	1	10^2	10^{-3}	39.37	3.281	6.214×10^{-4}
1 centimeter	10^{-2}	1	10^{-5}	0.3937	3.281×10^{-2}	6.214×10^{-6}
1 kilometer	10^3	10^5	1	3.937×10^4	3.281×10^3	0.6214
1 inch	2.540×10^{-2}	2.540	2.540×10^{-5}	1	8.333×10^{-2}	1.578×10^{-5}
1 foot	0.3048	30.48	3.048×10^{-4}	12	1	1.894×10^{-4}
1 mile	1609	1.609×10^5	1.609	6.336×10^4	5280	1

	Mass			
	kg	**g**	**slug**	**u**
1 kilogram	1	10^3	6.852×10^{-2}	6.024×10^{26}
1 gram	10^{-3}	1	6.852×10^{-5}	6.024×10^{23}
1 slug	14.59	1.459×10^4	1	8.789×10^{27}
1 atomic mass unit	1.660×10^{-27}	1.660×10^{-24}	1.137×10^{-28}	1

	Time				
	s	**min**	**h**	**day**	**year**
1 second	1	1.667×10^{-2}	2.778×10^{-4}	1.157×10^{-5}	3.169×10^{-8}
1 minute	60	1	1.667×10^{-2}	6.994×10^{-4}	1.901×10^{-6}
1 hour	3600	60	1	4.167×10^{-2}	1.141×10^{-4}
1 day	8.640×10^4	1440	24	1	2.738×10^{-3}
1 year	3.156×10^7	5.259×10^5	8.766×10^3	365.2	1

	Speed			
	m/s	**cm/s**	**ft/s**	**mi/h**
1 meter/second	1	10^2	3.281	2.237
1 centimeter/second	10^{-2}	1	3.281×10^{-2}	2.237×10^{-2}
1 foot/second	0.3048	30.48	1	0.6818
1 mile/hour	0.4470	44.70	1.467	1

Note: 1 mi/min = 60 mi/h = 88 ft/s

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	Force		
	N	dyn	lb
1 newton	1	10^5	0.2248
1 dyne	10^{-5}	1	2.248×10^{-6}
1 pound	4.448	4.448×10^5	1

	Work, Energy, Heat		
	J	erg	ft·lb
1 joule	1	10^7	0.7376
1 erg	10^{-7}	1	7.376×10^{-8}
1 ft·lb	1.356	1.356×10^7	1
1 eV	1.602×10^{-19}	1.602×10^{-12}	1.182×10^{-19}
1 cal	4.186	4.186×10^7	3.087
1 Btu	1.055×10^3	1.055×10^{10}	7.779×10^2
1 kWh	3.600×10^6	3.600×10^{13}	2.655×10^6

	eV	cal	Btu	kWh
1 joule	6.242×10^{18}	0.2389	9.481×10^{-4}	2.778×10^{-7}
1 erg	6.242×10^{11}	2.389×10^{-8}	9.481×10^{-11}	2.778×10^{-14}
1 ft·lb	8.464×10^{18}	0.3239	1.285×10^{-3}	3.766×10^{-7}
1 eV	1	3.827×10^{-20}	1.519×10^{-22}	4.450×10^{-26}
1 cal	2.613×10^{19}	1	3.968×10^{-3}	1.163×10^{-6}
1 Btu	6.585×10^{21}	2.520×10^2	1	2.930×10^{-4}
1 kWh	2.247×10^{25}	8.601×10^5	3.413×10^2	1

	Pressure		
	Pa	dyn/cm²	atm
1 pascal = 1 newton/meter²	1	10	9.869×10^{-6}
1 dyne/centimeter²	10^{-1}	1	9.869×10^{-7}
1 atmosphere	1.013×10^5	1.013×10^6	1
1 centimeter mercury*	1.333×10^3	1.333×10^4	1.316×10^{-2}
1 pound/inch²	6.895×10^3	6.895×10^4	6.805×10^{-2}
1 pound/foot²	47.88	4.788×10^2	4.725×10^{-4}

	cm Hg	lb/in.²	lb/ft²
1 pascal = newton/meter²	7.501×10^{-4}	1.450×10^{-4}	2.089×10^{-2}
1 dyne/centimeter²	7.501×10^{-5}	1.450×10^{-5}	2.089×10^{-3}
1 atmosphere	76	14.70	2.116×10^3
1 centimeter mercury*	1	0.1943	27.85
1 pound/inch²	5.171	1	144
1 pound/foot²	3.591×10^{-2}	6.944×10^{-3}	1

* At 0°C and at a location at which the free-fall acceleration has its "standard" value, 9.80665 m/s².

TABLE A.2 Symbols, Dimensions, and Units of Physical Quantities

Quantity	Common Symbol	Unit*	Dimensions†	Unit in Terms of Base SI Units
Acceleration	**a**	m/s²	L/T²	m/s²
Amount of substance	*n*	mole		mol
Angle	θ, ϕ	radian (rad)	1	
Angular acceleration	$\boldsymbol{\alpha}$	rad/s²	T⁻²	s⁻²
Angular frequency	ω	rad/s	T⁻¹	s⁻¹
Angular momentum	**L**	kg·m²/s	ML²/T	kg·m²/s
Angular velocity	$\boldsymbol{\omega}$	rad/s	T⁻¹	s⁻¹
Area	*A*	m²	L²	m²
Atomic number	*Z*			
Capacitance	*C*	farad (F) (= C/V)	Q²T²/ML²	A²·s⁴/kg·m²
Charge	*q, Q, e*	coulomb (C)	Q	A·s
Charge density				
Line	λ	C/m	Q/L	A·s/m
Surface	σ	C/m²	Q/L²	A·s/m²
Volume	ρ	C/m³	Q/L³	A·s/m³
Conductivity	σ	1/Ω·m	Q²T/ML³	A²·s³/kg·m³
Current	*I*	AMPERE	Q/T	A
Current density	**J**	A/m²	Q/T²	A/m²
Density	ρ	kg/m³	M/L³	kg/m³
Dielectric constant	κ			
Displacement	**s**	METER	L	m
Distance	*d, h*			
Length	ℓ, L			
Electric dipole moment	**p**	C·m	QL	A·s·m
Electric field	**E**	V/m (= N/C)	ML/QT²	kg·m/A·s³
Electric flux	Φ	V·m	ML³/QT²	kg·m³/A·s³
Electromotive force	$\mathcal{E}$	volt (V)	ML²/QT²	kg·m²/A·s³
Energy	*E, U, K*	joule (J)	ML²/T²	kg·m²/s²
Entropy	*S*	J/K	ML²/T²·K	kg·m²/s²·K
Force	**F**	newton (N)	ML/T²	kg·m/s²
Frequency	*f*	hertz (Hz)	T⁻¹	s⁻¹
Heat	*Q*	joule (J)	ML²/T²	kg·m²/s²
Inductance	*L*	henry (H)	ML²/Q²	kg·m²/A²·s²
Magnetic dipole moment	$\boldsymbol{\mu}$	N·m/T	QL²/T	A·m²
Magnetic field	**B**	tesla (T) (= Wb/m²)	M/QT	kg/A·s²
Magnetic flux	Φ_B	weber (Wb)	ML²/QT	kg·m²/A·s²
Mass	*m, M*	KILOGRAM	M	kg
Molar specific heat	*C*	J/Mol·K		kg·m²/s²·mol·K
Moment of inertia	*I*	kg·m²	ML²	kg·m²
Momentum	**p**	kg·m/s	ML/T	kg·m/s
Period	*T*	s	T	s
Permeability of space	μ_0	N/A² (= H/m)	ML/Q²T	kg·m/A²·s²
Permittivity of space	ε_0	C²/N·m² (= F/m)	Q²T²/ML³	A²·s⁴/kg·m³
Potential (voltage)	*V*	volt (V) (= J/C)	ML²/QT²	kg·m²/A·s³
Power	*P*	watt (W) (= J/s)	ML²/T³	kg·m²/s³
Pressure	*P, p*	pascal (Pa) = (N/m²)	M/LT²	kg/m·s²

continued

* The base SI units are given in uppercase letters.

† The symbols M, L, T, and Q denote mass, length, time, and charge, respectively.

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TABLE A.2 *(Continued)*

Quantity	Common Symbol	Unit*	Dimensions†	Unit in Terms of Base SI Units
Resistance	R	ohm (Ω) (=V/A)	ML^2/Q^2T	$kg \cdot m^2/A^2 \cdot s^3$
Specific heat	c	J/kg·K	$L^2/T^2 \cdot K$	$m^2/s^2 \cdot K$
Temperature	T	KELVIN	K	K
Time	t	SECOND	T	s
Torque	τ	N·m	ML^2/T^2	$kg \cdot m^2/s^2$
Speed	v	m/s	L/T	m/s
Volume	V	m^3	L^3	m^3
Wavelength	λ	m	L	m
Work	W	joule (J) (=N·m)	ML^2/T^2	$kg \cdot m^2/s^2$

* The base SI units are given in uppercase letters.

† The symbols M, L, T, and Q denote mass, length, time, and charge, respectively.

TABLE A.3 **Table of Selected Atomic Masses**[a]

Z	Element	Symbol	Chemical Atomic Mass (u)	Mass Number (* Indicates Radioactive) A	Atomic Mass (u)	Percentage Abundance	Half-Life (if Radioactive) $T_{1/2}$
0	(Neutron)	n		1*	1.008 665		10.4 m
1	Hydrogen	H	1.0079	1	1.007 825	99.985	
	Deuterium	D		2	2.014 102	0.015	
	Tritium	T		3*	3.016 049		12.33 y
2	Helium	He	4.00260	3	3.016 029	0.00014	
				4	4.002 602	99.99986	
3	Lithium	Li	6.941	6	6.015 121	7.5	
				7	7.016 003	92.5	
4	Beryllium	Be	9.0122	7*	7.016 928		53.3 d
				9	9.012 174	100	
5	Boron	B	10.81	10	10.012 936	19.9	
				11	11.009 305	80.1	
6	Carbon	C	12.011	11*	11.011 433		20.4 m
				12	12.000 000	98.90	
				13	13.003 355	1.10	
				14*	14.003 242		5730 y
7	Nitrogen	N	14.0067	13*	13.005 738		9.96 m
				14	14.003 074	99.63	
				15	15.000 108	0.37	
8	Oxygen	O	15.9994	15*	15.003 065		122 s
				16	15.994 915	99.761	
				18	17.999 160	0.20	
9	Fluorine	F	18.99840	19	18.998 404	100	
10	Neon	Ne	20.180	20	19.992 435	90.48	
				22	21.991 383	9.25	
11	Sodium	Na	22.98987	22*	21.994 434		2.61 y
				23	22.989 770	100	
				24*	23.990 961		14.96 h

TABLE A.3 *(Continued)*

Z	Element	Symbol	Chemical Atomic Mass (u)	Mass Number (* Indicates Radioactive) A	Atomic Mass (u)	Percentage Abundance	Half-Life (if Radioactive) $T_{1/2}$
12	Magnesium	Mg	24.305	24	23.985 042	78.99	
				25	24.985 838	10.00	
				26	25.982 594	11.01	
13	Aluminum	Al	26.98154	27	26.981 538	100	
14	Silicon	Si	28.086	28	27.976 927	92.23	
				29	28.976 495	4.67	
				30	29.973 770	3.10	
15	Phosphorus	P	30.97376	31	30.973 762	100	
				32*	31.973 908		14.26 d
16	Sulfur	S	32.066	32	31.972 071	95.02	
				33	32.971 459	0.75	
				34	33.967 867	4.21	
				35*	34.969 033		87.5 d
17	Chlorine	Cl	35.453	35	34.968 853	75.77	
				37	36.965 903	24.23	
18	Argon	Ar	39.948	36	35.967 547	0.337	
				40	39.962 384	99.600	
19	Potassium	K	39.0983	39	38.963 708	93.2581	
				40*	39.964 000	0.0117	1.28×10^9 y
				41	40.961 827	6.7302	
20	Calcium	Ca	40.08	40	39.962 591	96.941	
				44	43.955 481	2.086	
21	Scandium	Sc	44.9559	45	44.955 911	100	
22	Titanium	Ti	47.88	46	45.952 630	8.0	
				47	46.951 765	7.3	
				48	47.947 947	73.8	
				49	48.947 871	5.5	
				50	49.944 792	5.4	
23	Vanadium	V	50.9415	50*	49.947 161	0.25	1.5×10^{17} y
				51	50.943 962	99.75	
24	Chromium	Cr	51.996	50	49.946 047	4.345	
				52	51.940 511	83.79	
				53	52.940 652	9.50	
				54	53.938 883	2.365	
25	Manganese	Mn	54.93805	55	54.938 048	100	
26	Iron	Fe	55.847	54	53.939 613	5.9	
				56	55.934 940	91.72	
				57	56.935 396	2.1	
				58	57.933 278	0.28	

continued

[a] The masses in the sixth column are atomic masses, which include the mass of Z electrons. Data are from the National Nuclear Data Center, Brookhaven National Laboratory, prepared by Jagdish K. Tuli, July 1990. The data are based on experimental results reported in *Nuclear Data Sheets* and *Nuclear Physics* and also from *Chart of the Nuclides*, 14th ed. Atomic masses are based on those by A. H. Wapstra, G. Audi, and R. Hoekstra. Isotopic abundances are based on those by N. E. Holden.

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TABLE A.3 (Continued)

Z	Element	Symbol	Chemical Atomic Mass (u)	Mass Number (* Indicates Radioactive) A	Atomic Mass (u)	Percentage Abundance	Half-Life (if Radioactive) $T_{1/2}$
27	Cobalt	Co	58.93320	59	58.933 198	100	
				60*	59.933 820		5.27 y
28	Nickel	Ni	58.693	58	57.935 346	68.077	
				60	59.930 789	26.223	
				62	61.928 346	3.634	
29	Copper	Cu	63.54	63	62.929 599	69.17	
				65	64.927 791	30.83	
30	Zinc	Zn	65.39	64	63.929 144	48.6	
				66	65.926 035	27.9	
				67	66.927 129	4.1	
				68	67.924 845	18.8	
31	Gallium	Ga	69.723	69	68.925 580	60.108	
				71	70.924 703	39.892	
32	Germanium	Ge	72.61	70	69.924 250	21.23	
				72	71.922 079	27.66	
				73	72.923 462	7.73	
				74	73.921 177	35.94	
				76	75.921 402	7.44	
33	Arsenic	As	74.9216	75	74.921 594	100	
34	Selenium	Se	78.96	74	73.922 474	0.89	
				76	75.919 212	9.36	
				77	76.919 913	7.63	
				78	77.917 307	23.78	
				80	79.916 519	49.61	
				82*	81.916 697	8.73	1.4×10^{20} y
35	Bromine	Br	79.904	79	78.918 336	50.69	
				81	80.916 287	49.31	
36	Krypton	Kr	83.80	78	77.920 400	0.35	
				80	79.916 377	2.25	
				82	81.913 481	11.6	
				83	82.914 136	11.5	
				84	83.911 508	57.0	
				86	85.910 615	17.3	
37	Rubidium	Rb	85.468	85	84.911 793	72.17	
				87*	86.909 186	27.83	4.75×10^{10} y
38	Strontium	Sr	87.62	86	85.909 266	9.86	
				87	86.908 883	7.00	
				88	87.905 618	82.58	
				90*	89.907 737		29.1 y
39	Yttrium	Y	88.9058	89	88.905 847	100	
40	Zirconium	Zr	91.224	90	89.904 702	51.45	
				91	90.905 643	11.22	
				92	91.905 038	17.15	
				94	93.906 314	17.38	
				96	95.908 274	2.80	
41	Niobium	Nb	92.9064	93	92.906 376	100	

TABLE A.3 (Continued)

Z	Element	Symbol	Chemical Atomic Mass (u)	Mass Number (* Indicates Radioactive) A	Atomic Mass (u)	Percentage Abundance	Half-Life (if Radioactive) $T_{1/2}$
42	Molybdenum	Mo	95.94	92	91.906 807	14.84	
				94	93.905 085	9.25	
				95	94.905 841	15.92	
				96	95.904 678	16.68	
				97	96.906 020	9.55	
				98	97.905 407	24.13	
				100	99.907 476	9.63	
43	Technetium	Tc		98*	97.907 215		4.2×10^6 y
				99*	98.906 254		2.1×10^5 y
44	Ruthenium	Ru	101.07	96	95.907 597	5.54	
				99	98.905 939	12.7	
				100	99.904 219	12.6	
				101	100.905 558	17.1	
				102	101.904 348	31.6	
				104	103.905 428	18.6	
45	Rhodium	Rh	102.9055	103	102.905 502	100	
46	Palladium	Pd	106.42	104	103.904 033	11.14	
				105	104.905 082	22.33	
				106	105.903 481	27.33	
				108	107.903 893	26.46	
				110	109.905 158	11.72	
47	Silver	Ag	107.868	107	106.905 091	51.84	
				109	108.904 754	48.16	
48	Cadmium	Cd	112.41	110	109.903 004	12.49	
				111	110.904 182	12.80	
				112	111.902 760	24.13	
				113*	112.904 401	12.22	9.3×10^{15} y
				114	113.903 359	28.73	
				116	115.904 755	7.49	
49	Indium	In	114.82	113	112.904 060	4.3	
				115*	114.903 876	95.7	4.4×10^{14} y
50	Tin	Sn	118.71	116	115.901 743	14.53	
				117	116.902 953	7.68	
				118	117.901 605	24.22	
				119	118.903 308	8.58	
				120	119.902 197	32.59	
				122	121.903 439	4.63	
				124	123.905 274	5.79	
51	Antimony	Sb	121.76	121	120.903 820	57.36	
				123	122.904 215	42.64	
52	Tellurium	Te	127.60	122	121.903 052	2.59	
				124	123.902 817	4.79	
				125	124.904 429	7.12	
				126	125.903 309	18.93	
				128*	127.904 463	31.70	$>8 \times 10^{24}$ y
				130*	129.906 228	33.87	$\approx 1.25 \times 10^{21}$ y

continued

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TABLE A.3 *(Continued)*

Z	Element	Symbol	Chemical Atomic Mass (u)	Mass Number (* Indicates Radioactive) A	Atomic Mass (u)	Percentage Abundance	Half-Life (if Radioactive) $T_{1/2}$
53	Iodine	I	126.9045	127	126.904 474	100	
				129*	128.904 984		1.6×10^7 y
54	Xenon	Xe	131.29	129	128.904 779	26.4	
				130	129.903 509	4.1	
				131	130.905 069	21.2	
				132	131.904 141	26.9	
				134	133.905 394	10.4	
				136*	135.907 215	8.9	$\geq 2.36 \times 10^{21}$ y
55	Cesium	Cs	132.9054	133	132.905 436	100	
56	Barium	Ba	137.33	134	133.904 492	2.42	
				135	134.905 671	6.593	
				136	135.904 559	7.85	
				137	136.905 816	11.23	
				138	137.905 236	71.70	
57	Lanthanum	La	138.905	139	138.906 346	99.9098	
58	Cerium	Ce	140.12	140	139.905 434	88.43	
				142*	141.909 241	11.13	$>5 \times 10^{16}$ y
59	Praseodymium	Pr	140.9076	141	140.907 647	100	
60	Neodymium	Nd	144.24	142	141.907 718	27.13	
				143	142.909 809	12.18	
				144*	143.910 082	23.80	2.3×10^{15} y
				145	144.912 568	8.30	
				146	145.913 113	17.19	
				148	147.916 888	5.76	
				150*	149.920 887	5.64	$>1 \times 10^{18}$ y
61	Promethium	Pm		145*	144.912 745		17.7 y
62	Samarium	Sm	150.36	144	143.911 996	3.1	
				147*	146.914 894	15.0	1.06×10^{11} y
				148*	147.914 819	11.3	7×10^{15} y
				149*	148.917 180	13.8	$>2 \times 10^{15}$ y
				150	149.917 273	7.4	
				152	151.919 728	26.7	
				154	153.922 206	22.7	
63	Europium	Eu	151.96	151	150.919 846	47.8	
				153	152.921 226	52.2	
64	Gadolinium	Gd	157.25	155	154.922 618	14.80	
				156	155.922 119	20.47	
				157	156.923 957	15.65	
				158	157.924 099	24.84	
				160	159.927 050	21.86	
65	Terbium	Tb	158.9253	159	158.925 345	100	
66	Dysprosium	Dy	162.50	161	160.926 930	18.9	
				162	161.926 796	25.5	
				163	162.928 729	24.9	
				164	163.929 172	28.2	
67	Holmium	Ho	164.9303	165	164.930 316	100	

TABLE A.3 *(Continued)*

Z	Element	Symbol	Chemical Atomic Mass (u)	Mass Number (* Indicates Radioactive) A	Atomic Mass (u)	Percentage Abundance	Half-Life (if Radioactive) $T_{1/2}$
68	Erbium	Er	167.26	166	165.930 292	33.6	
				167	166.932 047	22.95	
				168	167.932 369	27.8	
				170	169.935 462	14.9	
69	Thulium	Tm	168.9342	169	168.934 213	100	
70	Ytterbium	Yb	173.04	170	169.934 761	3.05	
				171	170.936 324	14.3	
				172	171.936 380	21.9	
				173	172.938 209	16.12	
				174	173.938 861	31.8	
				176	175.942 564	12.7	
71	Lutecium	Lu	174.967	175	174.940 772	97.41	
				176*	175.942 679	2.59	3.78×10^{10} y
72	Hafnium	Hf	178.49	176	175.941 404	5.206	
				177	176.943 218	18.606	
				178	177.943 697	27.297	
				179	178.945 813	13.629	
				180	179.946 547	35.100	
73	Tantalum	Ta	180.9479	181	180.947 993	99.988	
74	Tungsten (Wolfram)	W	183.85	182	181.948 202	26.3	
				183	182.950 221	14.28	
				184	183.950 929	30.7	
				186	185.954 358	28.6	
75	Rhenium	Re	186.207	185	184.952 951	37.40	
				187*	186.955 746	62.60	4.4×10^{10} y
76	Osmium	Os	190.2	188	187.955 832	13.3	
				189	188.958 139	16.1	
				190	189.958 439	26.4	
				192	191.961 468	41.0	
77	Iridium	Ir	192.2	191	190.960 585	37.3	
				193	192.962 916	62.7	
78	Platinum	Pt	195.08	194	193.962 655	32.9	
				195	194.964 765	33.8	
				196	195.964 926	25.3	
				198	197.967 867	7.2	
79	Gold	Au	196.9665	197	196.966 543	100	
80	Mercury	Hg	200.59	198	197.966 743	9.97	
				199	198.968 253	16.87	
				200	199.968 299	23.10	
				201	200.970 276	13.10	
				202	201.970 617	29.86	
				204	203.973 466	6.87	
81	Thallium	Tl	204.383	203	202.972 320	29.524	
				205	204.974 400	70.476	
		(Th C″)		208*	207.981 992		3.053 m

continued

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