

Introduction to Electronic Devices

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SCHOTTKY DIODES

4-8-1. Metal-semiconductor contact at zero bias.

Electrons in the conduction band of a crystal can be viewed as sitting in a potential box formed by the crystal boundaries (see Fig. 4.8.1). This potential box for electrons is usually deeper in a metal than in a semiconductor. If a metal and a semiconductor are brought together into a close proximity, some electrons from the metal will move into the semiconductor and some electrons from the semiconductor will move into the metal. However, since the barrier for the electron escape from the metal is higher, more electrons will transfer from the semiconductor into the metal than in the opposite direction. At thermal equilibrium, the metal will be charged negatively, and the semiconductor will be charged positively, forming a dipole layer that is very similar to that in a p^+-n junction (see Section 4.2). The Fermi level will be constant throughout the entire metal-semiconductor system, and the energy band diagram in the semiconductor will be similar to that for an n -type semiconductor in a p^+-n junction (see Fig. 4.8.2).

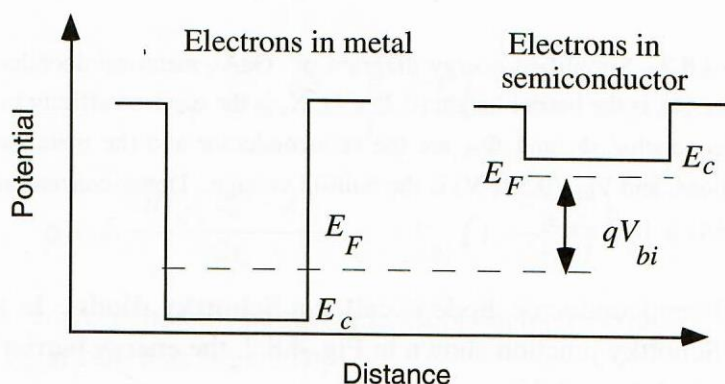


Fig. 4.8.1. Schematic energy diagram for electrons in conduction bands of a metal and of a semiconductor.

Energies Φ_m and Φ_s shown in Fig. 4.8.2 are called the metal and the semiconductor **work functions**. The **work function** is equal to the difference between the **vacuum level** (which is defined as a free electron energy in vacuum) and the Fermi level. The **electron affinity** of the semiconductor, X_s (also shown in Fig. 4.8.2), corresponds to the energy separation between the vacuum level and the conduction band edge of the semiconductor.

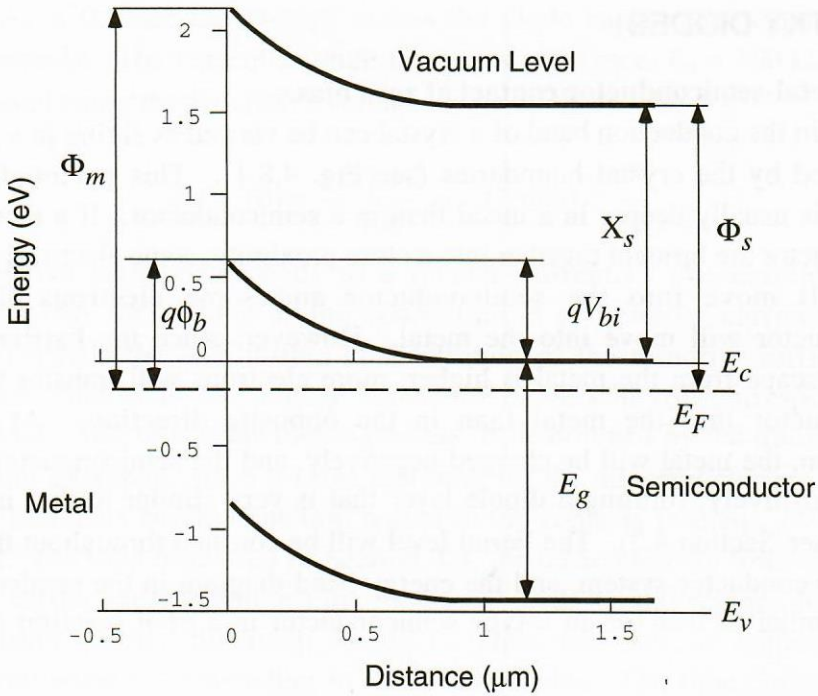


Fig. 4.8.2. Simplified energy diagram of GaAs metal-semiconductor barrier $q\phi_b$ is the barrier height (0.75 eV), X_s is the electron affinity in the semiconductor, Φ_s and Φ_m are the semiconductor and the metal work functions, and V_{bi} (0.591 V) is the built-in voltage. Donor concentration in GaAs is 10^{15} cm^{-3} .

A metal-semiconductor diode is called a **Schottky diode**. In the idealized picture of the Schottky junction shown in Fig. 4.8.2, the energy barrier between the semiconductor and the metal is

$$q\phi_b = \Phi_m - X_s \quad (4-8-1)$$

Since $\Phi_m > \Phi_s$ the metal is charged negatively. The positive net space charge in the semiconductor leads to a band bending

$$qV_{bi} = \Phi_m - \Phi_s \quad (4-8-2)$$

where V_{bi} is called the **built-in voltage**, in analogy with the corresponding quantity in a p - n junction. Note that qV_{bi} is also identical to the difference between the Fermi levels in the metal and the semiconductor when separated by a large distance

(no exchange of charge); see Fig. 4.8.1.

However, eq. (4-8-1) and Fig. 4.8.2 are not quite correct. In reality, a change in the metal work function, Φ_m , is not equal to the corresponding change in the barrier height, ϕ_b , as predicted by eq. (4-8-1). In actual Schottky diodes, ϕ_b increases with an increase in Φ_m but only by 0.1 to 0.3 eV when Φ_m increases by 1 to 2 eV. Even though a detailed and accurate understanding of Schottky barrier formation remains a challenge, many properties of Schottky barriers may be understood independently of the exact mechanism determining the barrier height. In other words, we can simply determine the effective barrier height from experimental data. Usually, as a crude and empirical rule of thumb, we can assume that the Schottky barrier height for an n -type semiconductor is close to 1/2 and 2/3 of the energy gap.

In a Schottky diode, the semiconductor band diagram looks very similar to that of an n -type semiconductor in a p^+-n diode (compare Fig. 4.3.1a and 4.8.2). Hence, the variation of the space charge density, ρ , the electric field, F , and the potential, ϕ , in the semiconductor near the metal-semiconductor interface can be found using the depletion approximation:

$$\rho = qN_d \quad (4-8-3)$$

$$F = -\frac{qN_d(x_n - x)}{\epsilon_s} \quad (4-8-4)$$

$$\phi = -\frac{qN_d(x_n - x)^2}{2\epsilon_s} = -V_{bi} \left(1 - \frac{x}{x_n}\right)^2 \quad (4-8-5)$$

(Here $x = 0$ corresponds to the metal-semiconductor interface.) The depletion layer width, x_n , at zero bias is given by

$$x_n = \sqrt{\frac{2\epsilon_s V_{bi}}{qN_d}} \quad (4-8-6)$$

4-8-2. Schottky diode under bias.

Forward bias corresponds to a positive voltage applied to the metal with respect to the semiconductor. Just as for a p^+-n junction, the depletion width under small forward bias and reverse bias may be obtained by substituting V_{bi} with $V_{bi} - V$, where V is the applied voltage. As illustrated in Fig. 4.8.3, the application of a

forward bias decreases the potential barrier for electrons moving from the semiconductor into the metal. Hence, the current-voltage characteristic of a Schottky diode can be described by a **diode equation**, similar to that for a p - n junction diode [see eq. (4-3-34)]:

$$I = I_s \left[\exp \left(\frac{V - IR_s}{\eta V_{th}} \right) - 1 \right] \quad (4-8-7)$$

where I_s is the saturation current, R_s is the series resistance, $V_{th} = k_B T/q$ is the thermal voltage, and η is the ideality factor (η typically varies from 1.02 to 1.6).

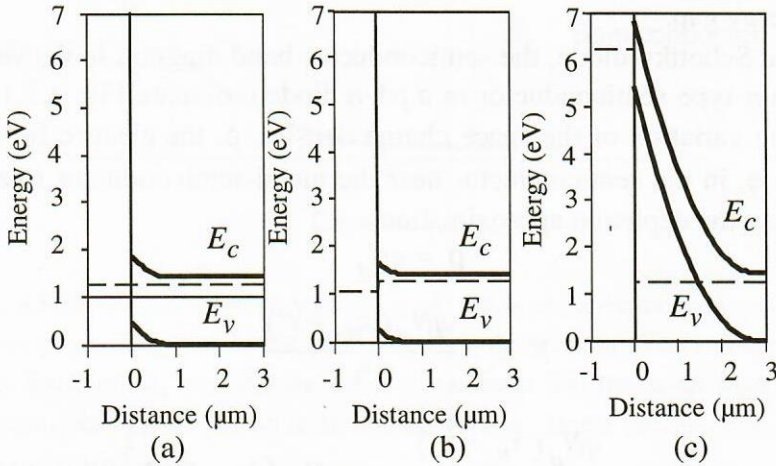


Fig. 4.8.3. Band diagrams for a GaAs Schottky barrier diode at (a) zero bias, (b) 0.2 V forward bias, and (c) 5 V reverse bias. Dashed line shows the position of the Fermi level in the metal ($x < 0$) and in the semiconductor ($x > 0$).

Example 4-8-1.

Estimate the doping density in the GaAs layer from the data in Fig. 4.8.3.

Solution:

From Fig. 4.8.3, the depletion width, x_n , at zero bias is approximately 0.7 μm . The depletion width in Fig. 4.8.2 for $N_d = 10^{15} \text{ cm}^{-3}$ is approximately 1 μm . Since the depletion width varies approximately as $1/N_d^{1/2}$, we conclude that $N_d \approx 2 \times 10^{15} \text{ cm}^{-3}$. Somewhat more accurately, we can estimate x_n from the data for

reverse bias using eq. (4-8-6) where we substitute V_{bi} with $V_{bi} - V$ where V is negative for reverse bias. Check that this estimate gives approximately the same result.

4-8-3. Thermionic emission.

The diode saturation current, I_s , is typically much larger for Schottky barrier diodes than in $p-n$ junction diodes since the Schottky barrier height is smaller than the barrier height in $p-n$ junction diodes. In a $p-n$ junction, the height of the barrier separating electrons in the conduction band of the n -type region from the bottom of the conduction band in the p -region is on the order of the energy gap. A typical Schottky barrier height is only about two thirds of the energy gap or less, as mentioned above. Also, the mechanism of the electron conduction is different. One can show that the saturation current density in a Schottky diode with a relatively low doped semiconductor is given by

$$j_{ss} = A^* T^2 \exp\left(-\frac{\phi_b}{k_B T}\right) \quad (4-8-8)$$

where A^* is called the Richardson constant. For a conduction band minimum with spherical surfaces of equal energy (such as the Γ minimum in GaAs),

$$A^* = \alpha \frac{m_n q k_B^2}{2\pi^2 \hbar^3} \approx 120 \alpha \frac{m_n}{m_e} \left(\frac{\text{A}}{\text{cm}^2 \text{K}^2} \right) \quad (4-8-9)$$

where m_n is the effective mass and α is an empirical factor on the order of unity. The Schottky diode model described by eqs. (4-8-8) and (4-8-9) is called the **thermionic emission model**. For Schottky barrier diodes fabricated on the {111} surfaces of Si, $A^* = 96 \text{ A}/(\text{cm}^2 \text{K}^2)$. For GaAs, $A^* = 4.4 \text{ A}/(\text{cm}^2 \text{K}^2)$.

The basic assumption of the thermionic model is that electrons have to pass over the barrier in order to cross the boundary between the metal and the semiconductor. Hence, to find the saturation current, we have to estimate the number of electrons passing over the barrier and their velocities. The number of electrons, $N(E)dE$, having energies between E and $E + dE$ is proportional to the product of the Fermi-Dirac distribution function, $f(E)$, and the number of states in this energy interval, $g(E)dE$, where $g(E)$ is the density of states:

$$N(E)dE = g(E)f(E)dE \quad (4-8-10)$$

$[N(E) = dn(E)dE$ where $n(E)$ is the number of electrons in the conduction band with energies higher than E ; see Fig. 2.4.2.] At high energies, the Fermi-Dirac occupation function is very close to the Boltzmann distribution function [see eqs. (2-4-4) and (2-4-5)]:

$$f_n(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)} \approx \exp\left(\frac{E_F - E}{k_B T}\right) \quad (4-8-11)$$

The next step should be to multiply the number of the electrons, $N(E)dE$, in the energy interval from E to $E + dE$ by the velocity of these electrons. We have to account for different directions of the electron velocities and integrate over energies higher than the barrier height in order to determine the flux of the electrons coming from the semiconductor into the metal. Finally, we deduct the flux of the electrons coming from the metal into the semiconductor. The difference between these two fluxes will be proportional to the current density predicted by the thermionic model. [Such a derivation is done in many books including Shur (1990)]. However, we can take a much simpler route if we are interested in understanding the physics of the thermionic model. To this end, let us consider a Schottky diode under a strong reverse bias when V is negative and $-V \gg \eta k_B T$. Then $I = -I_s$ [see eq. (4-8-7)], and the band diagram looks like that shown in Fig. 4.8.3c. In this case the energy difference between the Fermi level in the semiconductor and the top of the barrier is so large that practically no electrons are available to come from the semiconductor into the metal. However, the Fermi level in the metal is much closer to the top of the barrier, and electrons still come from the metal into the semiconductor. The flux of these electrons constitutes the saturation current. In order to estimate this flux, we should recall that the density of states is a relatively slow function of energy [$g(E)$ is proportional to $(E - E_c)^{1/2}$; see eq. (2-4-19)] compared to the distribution function, which decreases by $\exp(1) \approx 2.718$ each time E increases by $k_B T$. Hence, the largest contribution into the electron flux will come from the electrons that are a few $k_B T$ above the barrier. The number of such electrons will be proportional to the effective density of states in the semiconductor

$$N_c = 2 \left(\frac{m_n k_B T}{2\pi\hbar^2} \right)^{3/2} \quad (4-8-12)$$

[see eq. (2-4-21)] and to $\exp(-\phi_b/k_B T)$. Their velocity in the direction perpendicular to the metal semiconductor interface is proportional to the thermal velocity

$$v_{thnx} = \sqrt{\frac{k_B T}{m_n}} \quad (4-8-13)$$

[see eq. (3-6-6)]. Hence, the saturation current density is given by

$$\begin{aligned} j_{ss} &= CqN_c \exp\left(-\frac{\phi_b}{k_B T}\right) v_{thnx} = 2C \left(\frac{m_n k_B T}{2\pi\hbar^2}\right)^{3/2} \sqrt{\frac{k_B T}{m_n}} \exp\left(-\frac{\phi_b}{k_B T}\right) \\ &= \sqrt{\frac{1}{2\pi^3}} C \frac{m_n k_B^2 T^2}{\hbar^3} \exp\left(-\frac{\phi_b}{k_B T}\right) \end{aligned} \quad (4-8-14)$$

where C is a numerical constant of the order of unity. With a proper choice of C , this equation coincides with eqs. (4-8-8) and (4-8-9).

In a relatively lightly doped semiconductor with low mobility, the current through the Schottky barrier may be limited more by diffusion and drift processes in the space charge region than by the barrier at the metal-semiconductor interface. Even in this special case, eq. (4-8-7) is still applicable but eq. (4-8-8) is no longer valid.

4-8-4. Thermionic-field emission.

In relatively highly doped semiconductors, the depletion region becomes so narrow that electrons can tunnel through the barrier near the top (see Fig. 4.8.4b). This process is called **thermionic-field emission**. In order to understand thermionic-field emission, we have to recall once again that the number of electrons with energies above a given energy E decreases exponentially with energy as $\exp[-E/(k_B T)]$. On the other hand, the barrier transparency increases exponentially with the decrease in the barrier width. Hence, as the doping increases and the barrier becomes thinner, the dominant electron tunneling path occurs at lower energies than the top of the barrier (see Fig. 4.8.4b).

In degenerate semiconductors, especially in semiconductors with a small electron effective mass such as GaAs, electrons can tunnel through the barrier near or at the Fermi level, and the tunneling current is dominant. This mechanism is called **field emission** (see Fig. 4.8.4c).

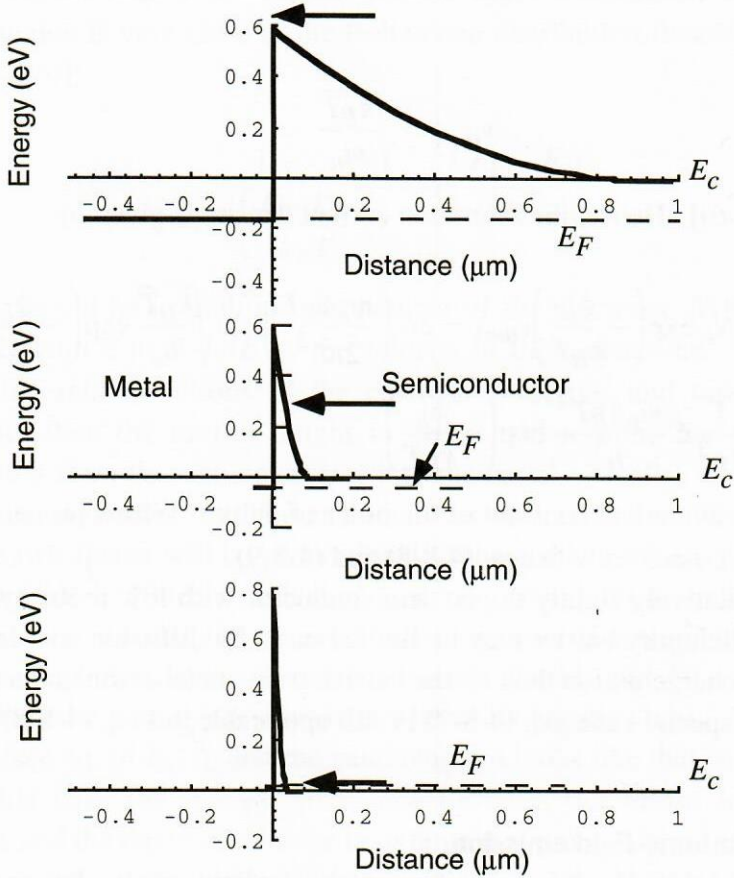


Fig. 4.8.4. Band diagrams of Schottky barrier junctions for GaAs for doping levels $N_d = 10^{15} \text{ cm}^{-3}$ (top graph), $N_d = 10^{17} \text{ cm}^{-3}$ (middle graph), and $N_d = 10^{18} \text{ cm}^{-3}$ (bottom graph). Arrows indicate electron transfer across the barrier under forward bias. At very low doping levels, electrons go over the barrier closer to the top of the barrier (this process is called thermionic emission). At moderated doping levels, electrons tunnel across the barrier closer to the top of the barrier (this process is called thermionic-field emission). In highly doped degenerate semiconductors, electrons near the Fermi level tunnel across a very thin depletion region (this process is called field emission).

The current-voltage characteristic of a Schottky diode in the case of thermionic-field emission can be calculated using the same approach as for the thermionic model, except that in thermionic-field emission case, we have to evaluate the product of the tunneling transmission coefficient and the number of electrons at a given energy as a function of energy and integrate over the states in the conduction band. Such a calculation [see Rhoderick and Williams (1988)] yields the following expression for the current density in the thermionic-field emission regime under forward bias:

$$j = j_{stf} \exp\left(\frac{qV}{E_o}\right) \quad (4-8-15)$$

where

$$E_o = E_{oo} \coth\left(\frac{E_{oo}}{k_B T}\right) \quad (4-8-16)$$

$$E_{oo} = \frac{qh}{4\pi} \sqrt{\frac{N_d}{m_n \epsilon_s}} = 1.85 \times 10^{-11} \left[\frac{N_d (\text{cm}^{-3})}{(m_n / m_o)(\epsilon_s / \epsilon_o)} \right]^{1/2} \quad (\text{eV}) \quad (4-8-17)$$

$$j_{stf} = \frac{A^* T \sqrt{\pi E_{oo} (\phi_b - qV - E_c + E_{Fn})}}{k_B \cosh(E_{oo} / k_B T)} \exp\left[-\frac{E_c - E_{Fn}}{k_B T} - \frac{(\phi_b - E_c + E_{Fn})}{E_o} \right] \quad (4-8-18)$$

In GaAs Schottky diodes, the thermionic-field emission becomes important for $N_d > 10^{17} \text{ cm}^{-3}$ at 300 K and for $N_d > 10^{16} \text{ cm}^{-3}$ at 77 K. In silicon, the corresponding values of N_d are several times larger.

Example 4-8-2.

Using eqs. (4-8-16) to (4-8-18), calculate and plot forward j - V characteristics for GaAs Schottky diodes doped at 10^{15} , 10^{17} , and 10^{18} cm^{-3} . Estimate ideality factors η . Comment on expected reverse j - V characteristics.

Solution:

The forward j - V characteristics are shown in Fig. 4.8.5.

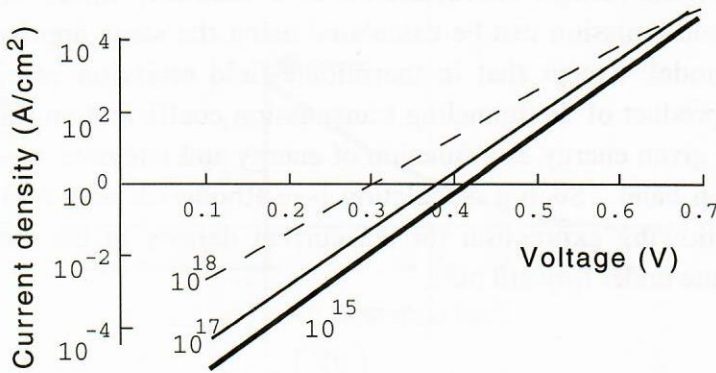


Fig. 4-8-5. Forward j - V characteristics of GaAs Schottky diodes doped at 10^{15} , 10^{17} , and 10^{18} cm^{-3} (curves are marked accordingly) at $T = 300 \text{ K}$.

We notice a larger current density (smaller differential resistance) at higher doping levels caused by tunneling. For 10^{18} cm^{-3} doping, the change in bias from 0.3 to 0.6 V changes the current density by 4 orders of magnitude. Hence, $\eta = 0.3/[4 \ln(10) 0.02584] \approx 1.26$. A similar calculation yields $\eta \approx 1.13$ for 10^{17} cm^{-3} and $\eta \approx 1$ for 10^{15} cm^{-3} . Larger values of η for higher doping levels is an indication of tunneling. Extrapolating the calculated curves to $V = 0$, we estimate the saturation current densities, j_s , of 8×10^{-5} , 9×10^{-6} , and $3 \times 10^{-7} \text{ A/cm}^2$, for 10^{18} cm^{-3} , for 10^{17} cm^{-3} , and 10^{15} cm^{-3} , respectively. At reverse bias, $j \approx j_s$, independent of bias, once $-V \gg V_{th}$.

The resistance of the Schottky barrier in the field emission regime is quite low. Therefore metal- n^+ contacts are used as ohmic contacts. The specific contact resistance, ρ_c , decreases with the increase in the doping level of the semiconductor. (This resistance may vary from $10^{-3} \text{ } \Omega\text{cm}^2$ to $10^{-7} \text{ } \Omega\text{cm}^2$ or even smaller depending on semiconductor material, doping level, contact metal, and ohmic contact fabrication technology.)

4-8-5. Small signal equivalent circuit and applications.

The small signal equivalent circuit of a Schottky barrier (see Fig. 4.8.6) is very similar to that of a p - n diode (see Fig. 4.4.4). It includes a parallel combination of the differential resistance of the Schottky barrier

$$R_d = \frac{dV}{dI} \quad (4-8-19)$$

and the differential capacitance of the space charge region:

$$C_{dep} = S \sqrt{\frac{qN_d\epsilon_s}{2(V_{bi} - V)}} \quad (4-8-20)$$

[compare with eq. (4-4-5)]. The equivalent circuit also includes the series resistance, R_s , which accounts for the contact resistance and the resistance of the neutral semiconductor region between the ohmic contact and the depletion region, the equivalent series inductance, L_s , and the device geometric capacitance:

$$C_{geom} = \epsilon_s S / L \quad (4-8-21)$$

where L is the device length and S is the device cross section

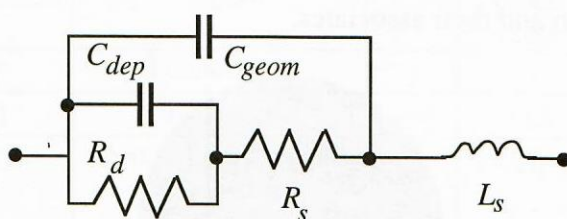


Fig. 4.8.6. The small signal equivalent circuit of a Schottky diode.

The major difference between the equivalent circuit of a Schottky diode and that of a $p-n$ junction is the absence of the diffusion capacitance in the case of the Schottky diode. The diffusion capacitance of a $p-n$ junction is associated with a time delay caused by the electron-hole recombination. A Schottky diode is a majority carrier device, where recombination is usually not important. Hence, Schottky diodes have a much faster response under forward bias conditions than $p-n$ junction diodes. Therefore, Schottky diodes are used in applications where the speed of a response is important, for example, in microwave **detectors**, **mixers**, and **varactors** [see Maas (1993)].

In most cases, $R_s \ll R_d$ and $C_{dep} \gg C_{geom}$ and the frequency response of a Schottky diode is limited by

$$f_T = \frac{1}{2\pi R_s C_{dep}} \quad (4-8-22)$$

Small Schottky diodes respond to very high frequencies (up to five terahertz – 5×10^{12} Hz or higher). Such frequencies, f , correspond to photon energies, $\hbar\omega = 2\pi\hbar f$, causing transitions between energy levels of gas molecules. Therefore, Schottky diodes are used as detectors determining the presence and concentration of certain gases in the upper atmosphere. As an example, Fig. 4.8.7 shows the distribution of chlorine monoxide in the upper atmosphere at approximately 20 km height from the sea level. [This distribution was measured by Upper Atmosphere Research Satellite (UARS) and reported by Dr. Joe Waters, Jet Propulsion Lab.] Chlorine monoxide is the gas destroying the ozone layer, and its distribution roughly coincides with the ozone “hole” in the upper atmosphere. The Schottky diodes used in this measurement were fabricated in the Semiconductor Device Laboratory at the University of Virginia by Professors Crowe and Mattauch and their associates.



Fig. 4.8.7. Distribution of chlorine monoxide in the upper atmosphere at 20 km height from sea level measured by Upper Atmosphere Research Satellite. Dark area indicates a high concentration of chlorine monoxide. (After G. Zarpette, *Sensing Climate Change*, Spectrum, p. 23, July 1993, copyright © IEEE 1993.)

A Schottky diode mixer uses a nonlinearity of the forward current-voltage characteristic of a Schottky diode. As a consequence, when two ac signals with radian frequencies ω_1 and ω_2 are applied to the Schottky diode, signals with frequencies $\omega_1 + \omega_2$ and $\omega_1 - \omega_2$ appear as well (see Problem 4-8-6).

A varactor is a device having a capacitance that varies under bias. A Schottky diode varactor uses a nonlinear capacitance of a Schottky diode [see eq.

(4-8-20)]. A Schottky diode varactor operates under reverse bias, since this allows us to obtain a larger capacitance variation with a change in the applied voltage.

Just like Schottky barrier detectors, Schottky barrier mixers and varactors operate up to very high frequencies (up to several terahertz) and find numerous applications in radioastronomy, manufacturing, and environment control, and microwave, millimeter wave, and optoelectronic circuits.

4-8-6. SPICE simulation of Schottky diodes.

The circuit simulator, SPICE, discussed in Section 4.7 does not include a separate model for a Schottky diode. However, circuits with Schottky diodes may be still simulated using a judicious choice of the parameters for the SPICE diode model. The SPICE diode parameters that must be adjusted in order to describe a Schottky diode are given in Table 4.8.1.

SPICE parameter	SPICE parameter name	Unit	Schottky diode value	Chap. 4 notation
TT	Transit time	s	0	t_{tr}
EG	Energy gap	eV	0.65	ϕ_b
XTI	Saturation current temperature exponent	–	2η	κ

Table 4.8.1. Parameters that must be adjusted to describe a Schottky diode in SPICE model (Level 1 model of AIM-Spice).

In a p - n junction, the transit time, t_{tr} , is related to the diffusion time (see Section 4.4). However, in a Schottky diode, a transit time delay is associated with the electron drift across the depletion region when the width of the depletion region changes. This time constant is given by

$$t_{tr} = \Delta d / v_T \tag{4-8-23}$$

where Δd is the change of the depletion region width during transient, and v_T is the electron thermal velocity. The value of t_{tr} is usually quite small (a fraction of a picosecond) and, in most cases, choosing $t_{tr} = 0$ as in Table 4.8.1 is appropriate.

In order to explain the suggested values of other SPICE parameters, we have to reproduce eq. (4-3-37) used by SPICE in order to describe the temperature dependence of the saturation current:

$$I_s(T) = I_s(T_o) \left(\frac{T}{T_o} \right)^{\frac{\kappa}{\eta}} \exp\left(\frac{E_g}{k_B T_o} \right) \exp\left(-\frac{E_g}{k_B T} \right) \quad (4-3-37)$$

The current-voltage characteristics of a Schottky diode and a p - n diode calculated by SPICE are compared in Fig. 4.8.8.

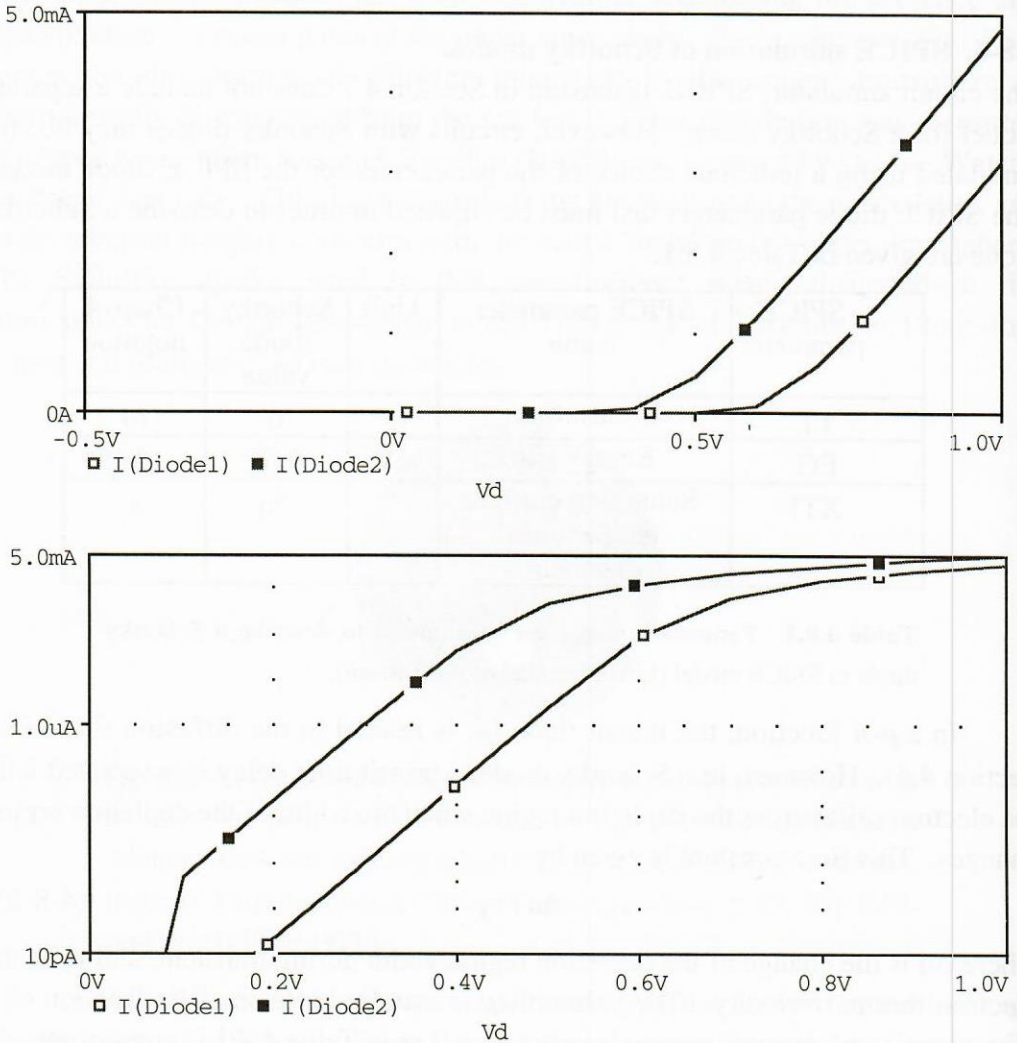


Fig. 4.8.8. Current-voltage characteristics of a Schottky diode (diode 2) and a p - n diode (diode 1) calculated by SPICE. Saturation currents are 10^{-11} A and 10^{-14} A, respectively.

As can be seen from eq. (4-8-8), the activation energy for the Schottky diode saturation current density is determined by the barrier height, ϕ_b . Hence, the value for the SPICE parameter EG (in eV) should be chosen to be equal to ϕ_b (in eV). Since the pre-exponential term in eq. (4-8-8) is proportional to T^2 , the value of XTT [the value of κ in eq. (2-3-37)] suggested in Table 4.8.1 is equal to 2η . The key differences from a p - n junction diode are a much larger saturation current density in a Schottky diode and a considerably smaller turn-on voltage. The summary of the most important results relevant to properties of Schottky junctions is given in Table 4.8.2.

Space charge density in the depletion layer	$\rho = qN_d$
Field distribution in the depletion layer	$F = -\frac{qN_d(x_n - x)}{\epsilon_s}$
Potential distribution in the depletion layer	$\phi = -\frac{qN_d(x_n - x)^2}{2\epsilon_s} = -V_{bi}\left(1 - \frac{x}{x_n}\right)^2$
Depletion layer width	$x_n = \sqrt{\frac{2\epsilon_s(V_{bi} - V)}{qN_d}}$
Empirical diode equation	$I = I_s \left[\exp\left(\frac{V}{\eta V_{th}}\right) - 1 \right]$
Parallel leakage current	$I_{leakage} = G_{min} V$
Diode equation (with series resistance)	$I = I_s \left[\exp\left(\frac{V - IR_s}{\eta V_{th}}\right) - 1 \right]$
Reverse diode current density (saturation current density)	$J_{ss} = A^* T^2 \exp\left(-\frac{\phi_b}{V_{th}}\right)$ where $A^* = \alpha \frac{m_n q k_B^2}{2\pi^2 \hbar^3} \approx 120 \alpha \frac{m_n}{m_e} \left(\frac{A}{cm^2 K^2}\right)$
Temperature dependence of the saturation current	$I_s(T) = I_s(T_o) \left(\frac{T}{T_o}\right)^2 \exp\left(\frac{\Phi_b}{k_B T_o}\right) \exp\left(-\frac{\Phi_b}{k_B T}\right)$

Table 4.8.2. Important equations describing Schottky diodes.

4-9. OHMIC CONTACTS

All electronic devices have to be connected to the outside world in order to form an electronic circuit in combination with other circuit elements. In the case of a p - n diode, for example, contacts have to be provided to both p -type and n -type regions of the device in order to connect the diode to an external circuit. These contacts have to be as unobtrusive as possible, so that the current flowing through a semiconductor device and, hence, through the contacts, leads to the smallest parasitic voltage drop possible. Whatever voltage drop does occur across the contact has to be proportional to the current so that the contacts do not introduce uncontrollable and unexpected nonlinear elements into the circuit. Since such contacts satisfy Ohm's law, they are usually called **ohmic contacts**.

As was discussed in Section 4.8, a contact between a metal and a semiconductor is typically a Schottky barrier contact. However, if the semiconductor is very highly doped, the Schottky barrier depletion region becomes very thin, as illustrated in Fig. 4.8.4. At very high doping levels, a thin depletion layer becomes quite transparent for electron tunneling (see Fig. 4.7.4). This suggests that a practical way to make a good ohmic contact is to make a very highly doped semiconductor region between the contact metal and the semiconductor.

It may have been better to use a metal with a work function, Φ_m , which is equal to or smaller than the work function of a semiconductor, Φ_s , (see Problem 4-9-1). However, for most semiconductors, it is difficult to find such a metal acceptable for practical contacts.

Current-voltage characteristics of a Schottky barrier diode (calculated using equations given in Section 4.8) and of an ohmic contact are compared in Fig. 4.9.1. As was mentioned above, a good ohmic contact should have a linear current-voltage characteristic and a very small resistance that is negligible compared to the resistance of the active region of the semiconductor device. An ohmic contact with the I - V characteristic shown in Fig. 4.9.2 does not satisfy fully these conditions since the voltage drop across this contact is not negligibly small compared with the voltage drop across the Schottky diode at moderate current densities above 0.1 kA/cm^2 .

As was discussed in Section 4.8, the barrier between a metal and a semiconductor is usually smaller for semiconductors with smaller energy gaps. Hence, another way to decrease the contact resistance is to place a layer of a

narrow gap highly doped semiconductor material between the active region of the device and the contact metal. Some of the best ohmic contacts to date have been made this way.

A quantitative measure of the contact quality is the **specific contact resistance**, ρ_c , which is the contact resistance of a unit area contact. Depending on the semiconductor material and on the contact quality, ρ_c can vary anywhere from $10^{-3} \Omega\text{cm}^2$ to $10^{-7} \Omega\text{cm}^2$ or even less.

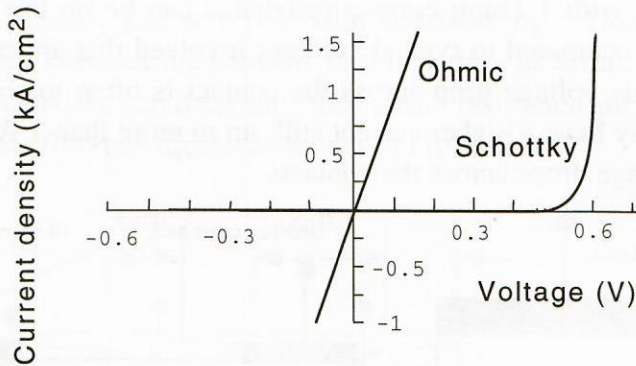


Fig. 4.9.1. Current-voltage characteristics of ohmic and Schottky barrier metal-semiconductor contacts to GaAs. (Schottky contact is to GaAs doped at 10^{15} cm^{-3} .) Ohmic contact resistance is $10^4 \Omega\text{cm}^2$.

Most semiconductor devices have either a sandwich structure or a planar structure, as illustrated in Fig. 4.9.2. The contact resistance of each contact in a sandwich structure contact is given by

$$R_c = \frac{\rho_c}{S} \tag{4-9-1}$$

A typical current density in a sandwich type device can be as high as 10^4 A/cm^2 . Hence, the specific contact resistance of $10^{-5} \Omega\text{cm}^2$ would lead to a voltage drop on the order of 0.1 V. This may be barely acceptable. A larger specific contact resistance of $10^{-4} \Omega\text{cm}^2$ or so would definitely lead to problems, as we can see from Fig. 4.9.1.

For a more typical, planar structure, the contact resistance is proportional to the contact width but no longer proportional to the contact area. In this case, the current density is larger near the contact edge. For planar devices, an important measure of the contact quality is the contact resistance related to the

unit width, R_{cm} , which can vary anywhere from 0.1 Ωmm to 5 Ωmm or more depending on a semiconductor material and the contact quality. The contact resistance is given by

$$R_c = \frac{R_{cm}}{W} \quad (4-9-2)$$

A typical current in a planar semiconductor device used in a monolithic integrated circuit can be on the order of 100 mA/mm. Hence, the voltage drop across a contact with 1 Ωmm contact resistance can be on the order of 0.1 V, quite noticeable compared to typical voltages involved that are on the order of 1 to 3 V. Still, this voltage drop across the contact is often tolerable. However, some devices may have a higher current still, up to more than 1 A/mm, leading to even higher voltage drops across the contacts.

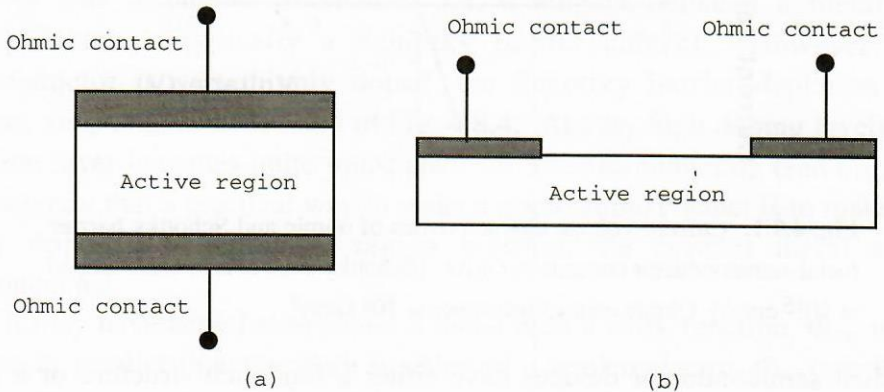


Fig. 4.9.2. (a) Sandwich and (b) planar device structures.

These estimates show that a semiconductor material can become viable for applications in electronic devices only when good ohmic contacts with low contact resistances become available. Often, poor ohmic contacts become a major stumbling block for applications of new semiconductor materials.

Figure 4.9.3 shows a characterization pattern used in integrated circuit technology for determining R_c . This pattern is called a **Transmission Line Model (TLM)** pattern because the current distribution under the planar ohmic contact is described by the same equation as a transmission line.

The resistance, $R_{n,n+1}$, between two adjacent ohmic contacts in the TLM pattern is given by

$$R_{n,n+1} = 2R_c + R_{sq} \frac{L_{n,n+1}}{W} \tag{4-9-3}$$

where $L_{n,n+1}$ is the distance between the contacts, R_{sq} is the resistance of the semiconductor film per square

$$R_{sq} = \frac{1}{\sigma t} \tag{4-9-4}$$

t is the film thickness, and σ is the film conductivity. Hence, plotting the resistances $R_{n,n+1}$ between the contacts in this pattern as a function of distances between the contacts, we can determine the contact resistance and the resistance of the semiconductor film per square as shown in Fig. 4.9.4.

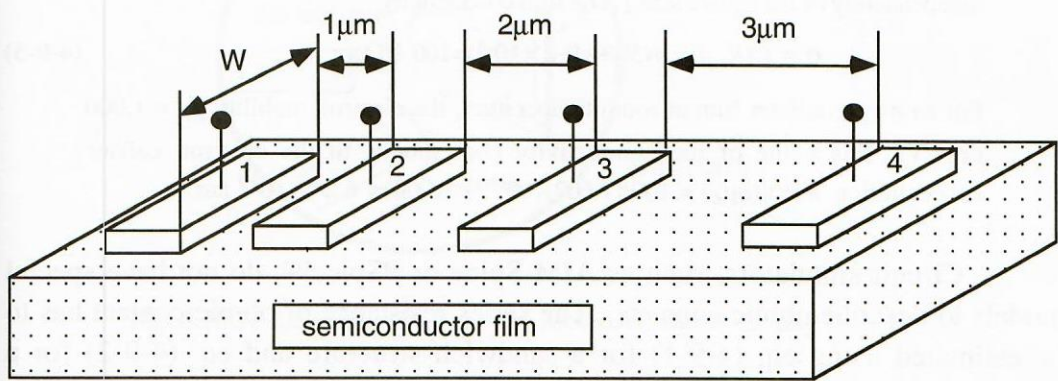


Fig. 4.9.3. TLM characterization pattern for determining contact resistances.

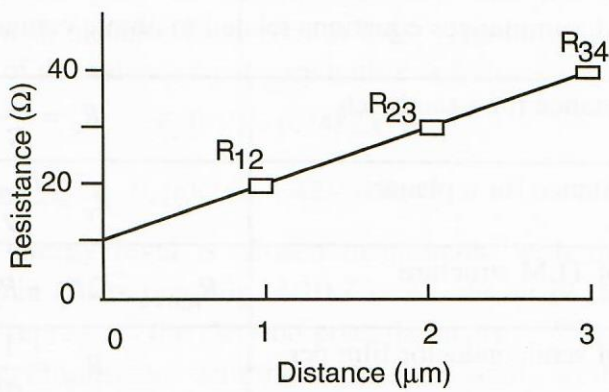


Fig. 4.9.4. TLM resistances versus distance between the contacts.

Example 4-9-1.

The plot in Fig. 4.9.4 is for a pattern with contact width $W = 50\text{ }\mu\text{m}$. The film thickness is $0.2\text{ }\mu\text{m}$. From the data shown in the figure, determine the contact resistance, the resistance of the semiconductor film per square, the film conductivity, and the carrier concentration at room temperature (assume an n -type Si film).

Solution:

The resistance intercept is $2R_c = 10\text{ }\Omega$. Hence, the contact resistance related to a 1 mm width, $R_{cm} = 0.25\text{ }\Omega\text{mm}$. The slope $R_{sq}/W = 10\text{ }\Omega/\mu\text{m}$. Hence, the film resistance per square, $R_{sq} = 10 \times 50 = 500\text{ }\Omega$ per square. [The meaning of this notation (Ω per square) is that any square of this film has the same resistance independently of the square size.] The film conductivity

$$\sigma = 1/(R_{sq}t) = 1/(500 \times 0.2 \times 10^{-4}) = 100\text{ }1/\Omega\text{cm} \tag{4-9-5}$$

For an n -type silicon film at room temperature, the electron mobility $\mu_n \approx 1,000\text{ cm}^2/\text{Vs}$, this value of the conductivity corresponds to the electron carrier concentration $n = \sigma/(q\mu_n) = 100/(1.602 \times 10^{-19} \times 1,000) = 6.24 \times 10^{17}\text{ cm}^{-3}$.

Circuit simulators, such as AIM-Spice or PSpicetm, do not have special models to describe ohmic contacts. The series resistance of ohmic contact has to be estimated using eq. (4-9-1) for a sandwich structure and eq. (4-9-2) for a planar structure and should be implemented as a part of the device series resistance (see Fig. 4.3.3, which shows the effect of a series resistance on current-voltage characteristics of a p - n diode).

Table 4.9.1 summarizes equations related to ohmic contacts.

Contact resistance for a sandwich structure	$R_c = \frac{\rho_c}{S}$
Contact resistance for a planar structure	$R_c = \frac{R_{cm}}{W}$
Resistance of TLM structure	$R_{n,n+1} = 2R_c + R_{sq} \frac{L_{n,n+1}}{W}$
Resistance of semiconductor film per square	$R_{sq} = \frac{1}{\sigma t}$

Table 4.9.1. Important equations related to ohmic contacts.