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*for Scientists and Engineers*

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Since  $I_2$  is negative, we conclude that  $I_2$  is from  $c$  to  $f$  through the  $3\text{-}\Omega$  resistor. Using this value of  $I_2$  in (3) and (1) gives the following values for  $I_1$  and  $I_3$ :

$$I_1 = 1.38 \text{ A} \quad I_3 = 1.02 \text{ A}$$

Under state-steady conditions, the capacitor represents an open circuit, and so there is no current in the branch  $ghab$ .

(b) What is the charge on the capacitor?

We can apply Kirchhoff's second rule to loop  $abgha$  (or any loop that contains the capacitor) to find the potential difference  $V_c$  across the capacitor:

$$-8 \text{ V} + V_c - 3 \text{ V} = 0$$

$$V_c = 11.0 \text{ V}$$

Since  $Q = CV_c$ , we find that the charge on the capacitor is equal to

$$Q = (6 \mu\text{F})(11.0 \text{ V}) = 66.0 \mu\text{C}$$

Why is the left side of the capacitor positively charged?

**Exercise 3** Find the voltage across the capacitor by traversing any other loop, such as the outside loop.

**Answer** 11.0 V.

## 28.4 RC CIRCUITS

So far we have been concerned with circuits with constant currents, or so-called *steady-state circuits*. We shall now consider circuits containing capacitors, in which the currents may vary in time. When a potential difference is first applied across a capacitor, the rate at which it charges depends on its capacitance and on the resistance in the circuit.

### Charging a Capacitor

Consider the series circuit shown in Figure 28.14. Let us assume that the capacitor is initially uncharged. There is no current when the switch  $S$  is open (Fig. 28.14b). If the switch is closed at  $t = 0$ , charges will begin to flow, setting up a current in the circuit, and the capacitor will begin to charge (Fig. 28.14c). Note that during the charging process, charges do not jump across the plates of the capacitor since the gap between the plates represents an open circuit. Instead, charge is transferred from one plate to the other through the resistor, switch, and battery until the capacitor is fully charged. The value of the maximum charge depends on the emf of the battery. Once the maximum charge is reached, the current in the circuit is zero.

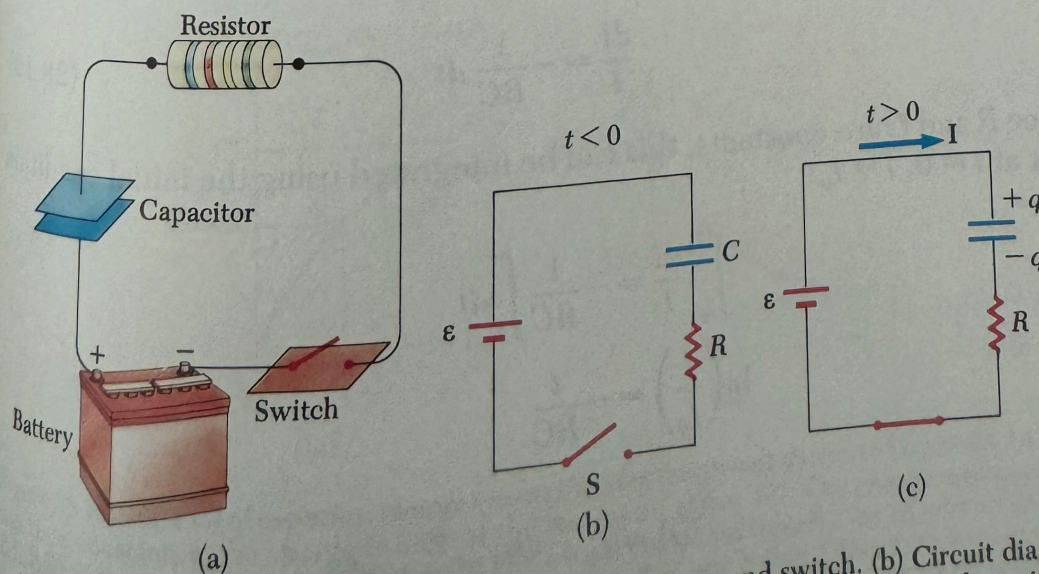


Figure 28.14 (a) A capacitor in series with a resistor, battery, and switch. (b) Circuit diagram representing this system before the switch is closed,  $t < 0$ . (c) Circuit diagram after the switch is closed,  $t > 0$ .

To put this discussion on a quantitative basis, let us apply Kirchhoff's second rule to the circuit *after* the switch is closed. This gives

$$\mathcal{E} - IR - \frac{q}{C} = 0 \quad (28.9)$$

where  $IR$  is the potential drop across the resistor and  $q/C$  is the potential drop across the capacitor. Note that  $q$  and  $I$  are *instantaneous* values of the charge and current, respectively, as the capacitor is being charged.

We can use Equation 28.9 to find the initial current in the circuit and the maximum charge on the capacitor. At  $t = 0$ , when the switch is closed, the charge on the capacitor is zero, and from Equation 28.9 we find that the initial current in the circuit,  $I_0$ , is a maximum and equal to

$$I_0 = \frac{\mathcal{E}}{R} \quad (\text{current at } t = 0) \quad (28.10)$$

Maximum current

At this time, the potential drop is entirely across the resistor. Later, when the capacitor is charged to its maximum value  $Q$ , charges cease to flow, the current in the circuit is zero, and the potential drop is entirely across the capacitor. Substituting  $I = 0$  into Equation 28.9 gives the following expression for  $Q$ :

$$Q = C\mathcal{E} \quad (\text{maximum charge}) \quad (28.11)$$

Maximum charge on the capacitor

To determine analytical expressions for the time dependence of the charge and current, we must solve Equation 28.9, a single equation containing two variables,  $q$  and  $I$ . In order to do this, let us differentiate Equation 28.9 with respect to time. Since  $\mathcal{E}$  is a constant,  $d\mathcal{E}/dt = 0$  and we get

$$\frac{d}{dt} \left( \mathcal{E} - \frac{q}{C} - IR \right) = 0 - \frac{1}{C} \frac{dq}{dt} - R \frac{dI}{dt} = 0$$

Recalling that  $I = dq/dt$ , we can express this equation in the form

$$R \frac{dI}{dt} + \frac{I}{C} = 0$$

$$\frac{dI}{I} = -\frac{1}{RC} dt \quad (28.12)$$

Since  $R$  and  $C$  are constants, this can be integrated using the initial condition that at  $t = 0$ ,  $I = I_0$ :

$$\int_{I_0}^I \frac{dI}{I} = -\frac{1}{RC} \int_0^t dt$$

$$\ln \left( \frac{I}{I_0} \right) = -\frac{t}{RC}$$

nt versus time

$$I(t) = I_0 e^{-t/RC} = \frac{\mathcal{E}}{R} e^{-t/RC} \quad (28.13)$$

where  $e$  is the base of the natural logarithm and  $I_0 = \mathcal{E}/R$  is the initial current.

In order to find the charge on the capacitor as a function of time, we can substitute  $I = dq/dt$  into Equation 28.13 and integrate once more:

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}$$

$$dq = \frac{\mathcal{E}}{R} e^{-t/RC} dt$$

We can integrate this expression using the condition that  $q = 0$  at  $t = 0$ :

$$\int_0^q dq = \frac{\mathcal{E}}{R} \int_0^t e^{-t/RC} dt$$

In order to integrate the right side of this expression, we use the fact that

$$\int e^{-ax} dx = -\frac{1}{a} e^{-ax}. \text{ The result of the integration gives}$$

$$q(t) = C\mathcal{E}[1 - e^{-t/RC}] = Q[1 - e^{-t/RC}] \quad (28.14)$$

Charge versus time for a capacitor being charged

where  $Q = C\mathcal{E}$  is the maximum charge on the capacitor.

Plots of Equations 28.13 and 28.14 are shown in Figure 28.15. Note that the charge is zero at  $t = 0$  and approaches the maximum value of  $C\mathcal{E}$  as  $t \rightarrow \infty$  (Fig. 28.15a). Furthermore, the current has its maximum value of  $I_0 = \mathcal{E}/R$  at  $t = 0$  and decays exponentially to zero as  $t \rightarrow \infty$  (Fig. 28.15b). The quantity  $RC$ , which appears in the exponential of Equations 28.13 and 28.14, is called the time constant,  $\tau$ , of the circuit. It represents the time it takes the current to decrease to  $1/e$  of its initial value; that is, in a time  $\tau$ ,  $I = e^{-1}I_0 = 0.37I_0$ . In a time  $2\tau$ ,  $I = e^{-2}I_0 = 0.135I_0$ , and so forth. Likewise, in a time  $\tau$  the charge will increase from zero to  $C\mathcal{E}[1 - e^{-1}] = 0.63C\mathcal{E}$ .

The following dimensional analysis shows that  $\tau$  has the unit of time:

$$[\tau] = [RC] = \left[ \frac{V}{I} \times \frac{Q}{V} \right] = \left[ \frac{Q}{Q/T} \right] = [T]$$

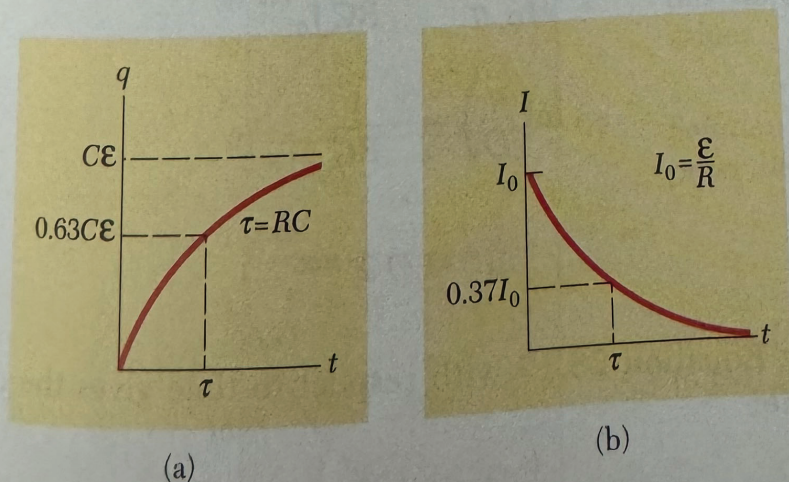


Figure 28.15 (a) Plot of capacitor charge versus time for the circuit shown in Figure 28.14. After one time constant,  $\tau$ , the charge is 63% of the maximum value,  $C\mathcal{E}$ . The charge approaches its maximum value as  $t$  approaches infinity. (b) Plot of current versus time for the RC circuit shown in Figure 28.14. The current has its maximum value,  $I_0 = \mathcal{E}/R$ , at  $t = 0$  and decays to zero exponentially as  $t$  approaches infinity. After one time constant,  $\tau$ , the current decreases to 37% of its initial value.

### 33.4 CAPACITORS IN AN AC CIRCUIT

Figure 33.6 shows an ac circuit consisting of a capacitor connected across the terminals of an ac generator. Kirchhoff's loop rule applied to this circuit gives  $v - v_C = 0$ , or

$$v = v_C = V_m \sin \omega t \quad (33.14)$$

where  $v_C$  is the *instantaneous voltage drop across the capacitor*. But from the definition of capacitance,  $v_C = Q/C$ , which when substituted into Equation 33.14 gives

$$Q = CV_m \sin \omega t \quad (33.15)$$

Since  $i = dQ/dt$ , differentiating Equation 33.15 gives the instantaneous current:

$$i_C = \frac{dQ}{dt} = \omega CV_m \cos \omega t \quad (33.16)$$

Again, we see that the current is not in phase with the voltage drop across the capacitor, given by Equation 33.14. Using the trigonometric identity  $\cos \omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$ , we can express Equation 33.16 in the alternative form

$$i_C = \omega CV_m \sin\left(\omega t + \frac{\pi}{2}\right) \quad (33.17)$$

Comparing this expression with Equation 33.14, we see that the current is  $90^\circ$  out of phase with the voltage across the capacitor. A plot of the current and voltage versus time (Fig. 33.7a) shows that the current reaches its peak value

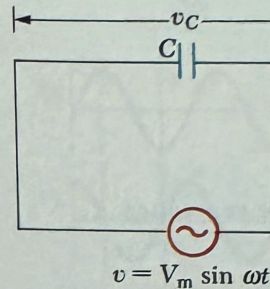


Figure 33.6 A circuit of a capacitor  $C$  connected to an ac generator.

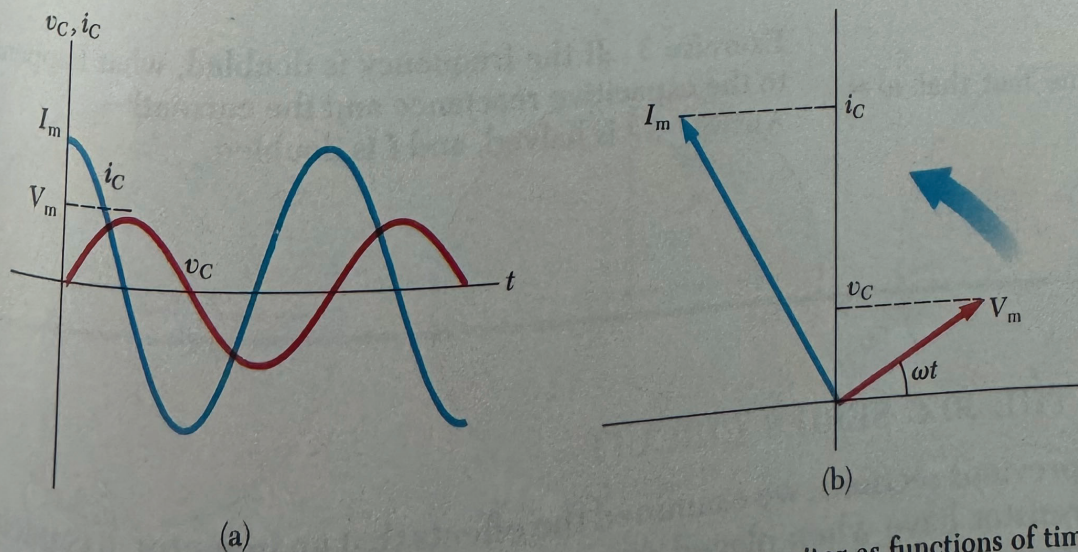


Figure 33.7 (a) Plots of the current and voltage across the capacitor as functions of time. The voltage lags behind the current by  $90^\circ$ . (b) Phasor diagram for the purely capacitive circuit. Projections of the phasors onto the vertical axis gives the instantaneous values  $v_C$  and  $i_C$ .

The current leads the voltage across the capacitor by  $90^\circ$

one quarter of a cycle sooner than the voltage reaches its peak value. The corresponding phasor diagram in Figure 33.7b also shows that for a sinusoidally applied emf, the current always leads the voltage across a capacitor by  $90^\circ$ .

From Equation 33.17, we see that the peak current in the circuit is

$$I_m = \omega C V_m = \frac{V_m}{X_C} \quad (33.18)$$

where  $X_C$  is called the **capacitive reactance**.

$$X_C = \frac{1}{\omega C} \quad (33.19)$$

Capacitive reactance

The rms current is given by an expression similar to Equation 33.18, with  $V_m$  replaced by  $V_{\text{rms}}$ .

Combining Equations 33.14 and 33.18, we can express the instantaneous voltage drop across the capacitor as

$$v_C = V_m \sin \omega t = I_m X_C \sin \omega t \quad (33.20)$$

The SI unit of  $X_C$  is also the ohm. As the frequency of the circuit increases, the current increases but the reactance decreases. For a given maximum applied voltage  $V_m$ , the current increases as the frequency increases. As the frequency approaches zero, the capacitive reactance approaches infinity. Therefore, the current approaches zero. This makes sense since the circuit approaches dc conditions as  $\omega \rightarrow 0$ . Of course, a capacitor passes no current under steady-state dc conditions.

### EXAMPLE 33.3 A Purely Capacitive AC Circuit

An  $8\text{-}\mu\text{F}$  capacitor is connected to the terminals of an ac generator whose rms voltage is  $150\text{ V}$  and whose frequency is  $60\text{ Hz}$ . Find the capacitive reactance and the rms current in the circuit.

**Solution** Using Equation 33.19 and the fact that  $\omega = 2\pi f = 377\text{ s}^{-1}$  gives

$$X_C = \frac{1}{\omega C} = \frac{1}{(377\text{ s}^{-1})(8 \times 10^{-6}\text{ F})} = 332\ \Omega$$

Hence, the rms current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{150\text{ V}}{332\ \Omega} = 0.452\text{ A}$$

**Exercise 3** If the frequency is doubled, what happens to the capacitive reactance and the current?

**Answer**  $X_C$  is halved, and  $I$  is doubled.

## 33.5 THE RLC SERIES CIRCUIT

In the previous sections, we examined the effects that an inductor, a capacitor, and a resistor have when placed separately across an ac-voltage source. We shall now consider what happens when combinations of these devices are used.