Quality Factor (Q)

The *quality factor* (Q) of a *resonator* may be defined as the *resonance frequency* divided by the *resonator bandwidth:*

$$\boxed{Q \triangleq \frac{\text{ResonanceFrequency}}{\text{Bandwidth}}}$$

where the resonance frequency and bandwidth must be given in the same units (*e.g.*, Hz). The *resonance frequency* is typically defined as the frequency at which the *peak gain* occurs. The *bandwidth* is typically defined as the *3dB-bandwidth*, *i.e.*, the bandwidth between the -3dB points (half-power-gain points) straddling the resonance frequency. A nearly equivalent definition of Q from mechanics is the *amplitude gain at resonance*.

We define Q in the context of *continuous-time* resonators, so that the transfer function $H(s)$ is the Laplace transform of the *continuous* impulse-response $h(t)$, $t \in \mathbb{R}$ (instead of the *z* transform of the discrete-time impulse-response $h(n)$, $n = 0, 1, 2, \ldots$). An introduction to Laplace-transform analysis appears in Appendix D.

The concept of Q is routinely generalized to apply to any *spectral magnitude peak* for which the peak bandwidth can meaningfully be defined. For example, the fourth-order Moog VCF lowpass filter may have a ``corner resonance'' for which Q may be defined as the Q of the two-pole resonator causing the corner boost. More generally, there may be any number of physical resonators combining to create a single magnitude-peak in the frequency response, and the Q can still be defined clearly as peak-magnitude frequency divided by 3dB-bandwidth. At the other extreme, a spectral peak may involve *no resonators at all* when the impulse response is finite in duration. Thus, the most practical definition of Q for modern usage is *peak frequency divided by peak bandwidth*. However, because the term itself refers to the quality of a simple second-order *resonator* made using a capacitor and inductor, we will keep the historical definition and generalize as needed.

Subsections

- Q of a Complex Resonator
- Q of a Real Second-Order Resonator
  - Critical Damping and Related Terms
- Damping Ratio
- Decay Time is Q Periods
- Q as Energy Stored over Energy Dissipated

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Center for Computer Research in Music and Acoustics (CCRMA), Stanford University

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