

Hyperplane, Subspace and Halfspace

Last Updated : 27 Aug, 2024

In linear algebra and geometry, **hyperplanes**, **subspaces**, and **halfspaces** are fundamental concepts used to describe and analyze the properties and behaviors of multi-dimensional spaces. These concepts have significant applications in various fields, including machine learning, optimization, and engineering.

This article explores the definitions, properties, and applications of hyperplanes, subspaces, and half-spaces.

Table of Content

- [Hyperplane](#)
 - [Hyperplane Definition](#)
 - [Hyperplane Example](#)
 - [Properties of Hyperplane](#)
- [Subspace](#)
 - [Subspace Definition](#)
 - [Subspace Example](#)
 - [Properties of Subspace](#)
- [Halfspace](#)
 - [Halfspace Definition](#)
 - [Halfspace Example](#)
 - [Properties of Halfspace](#)
- [Applications in Engineering](#)
- [Solved Examples on Hyperplane, Subspace and Halfspace](#)
- [Hyperplane, Subspace and Halfspace – Practice Problems](#)



A hyperplane is a flat affine subspace of one dimension less than its ambient space. In an n -dimensional space, a hyperplane has $n-1$ dimensions. It can be defined by a linear equation.

Hyperplane Definition

In n -dimensional space, a hyperplane can be represented as $a_1x_1 + a_2x_2 + \dots + a_nx_n = b$

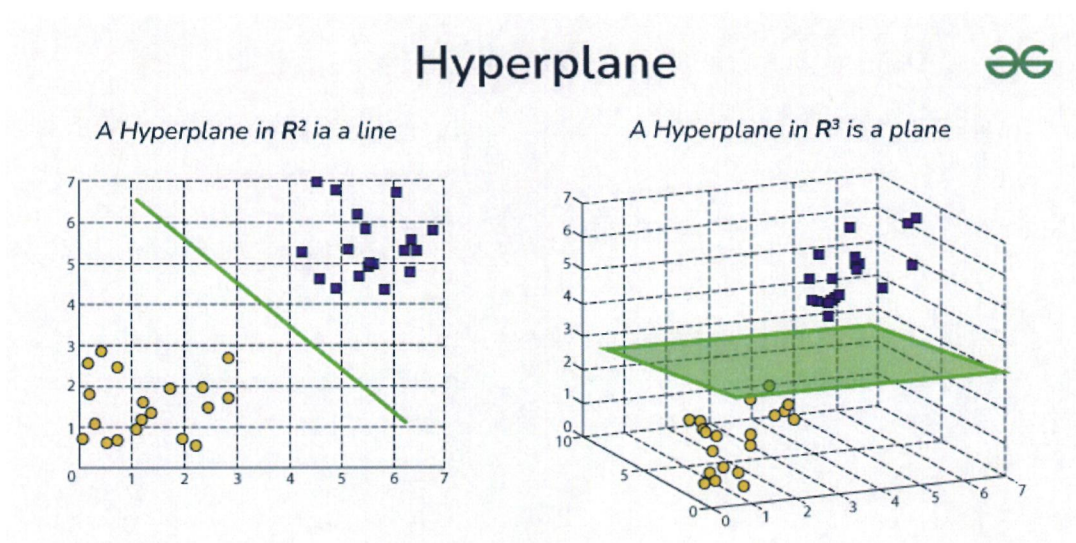
where $a_1, a_2, \dots, a_n$ are coefficients and b is a constant.

Hyperplane Example

In a 3-dimensional space ($n=3$), a hyperplane is a 2-dimensional plane defined by an equation like $2x + 3y - z = 5$.

Properties of Hyperplane

- A hyperplane divides the space into two half-spaces.
- All points on one side of the hyperplane satisfy $a_1x_1 + a_2x_2 + \dots + a_nx_n > b$, while those on the other side satisfy $a_1x_1 + a_2x_2 + \dots + a_nx_n < b$



Subspace

A subspace is a subset of a vector space that is also a vector space under the

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our [Cookie Policy](#) & [Privacy Policy](#)

Subspace Definition

A subspace W of a vector space V must satisfy three conditions:

1. The zero vector of V is in W .
2. W is closed under vector addition.
3. W is closed under scalar multiplication.

Subspace Example

In $\mathbb{R}^3$ the set of all vectors lying on the xy plane is a subspace. It includes the zero vector $(0,0,0)$, is closed under vector addition and scalar multiplication.

Properties of Subspace

- Subspaces can be of any dimension from 0 up to n , where n is the dimension of the ambient space.
- Every subspace passes through the origin.

Halfspace

A halfspace is one of the two regions into which a hyperplane divides the ambient space. It includes all points on one side of the hyperplane and can be either open or closed.

Halfspace Definition

A halfspace can be defined by a linear inequality:

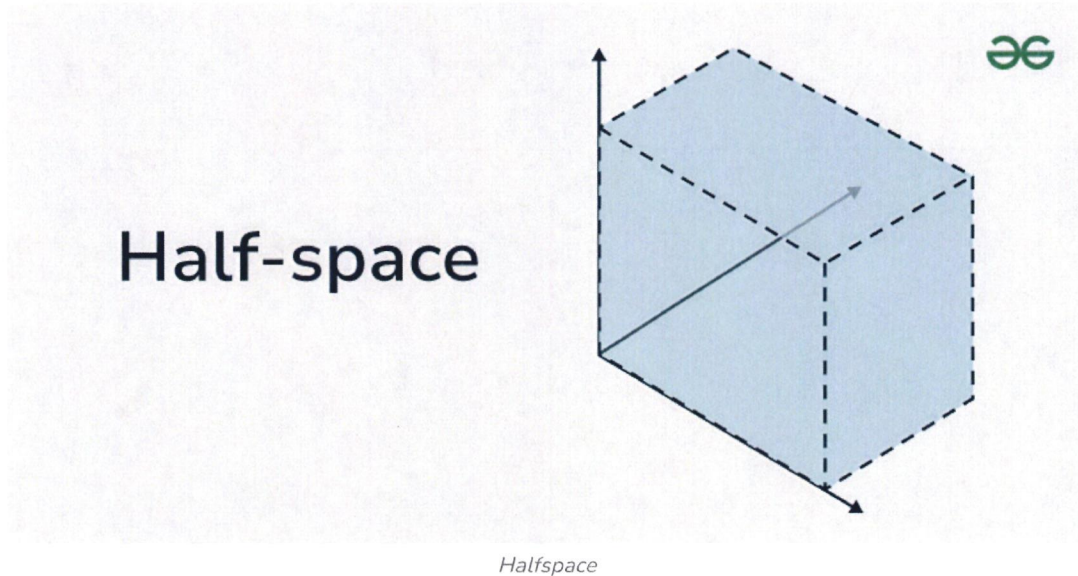
$$a_1x_1 + a_2x_2 + \dots + a_nx_n \leq b$$

$$a_1x_1 + a_2x_2 + \dots + a_nx_n < b$$

Halfspace Example

In $\mathbb{R}^3$, the region defined by $2x + 3y - z \leq 5$ is a closed halfspace.

- A halfspace includes all points satisfying the inequality defining it.
- A hyperplane is the boundary between two halfspaces.



Applications in Engineering

1. Machine Learning

In machine learning, hyperplanes are used in algorithms such as Support Vector Machines (SVMs) to classify data. The hyperplane serves as a decision boundary separating different classes of data points.

2. Optimization

In linear programming, the feasible region is often defined by a set of linear inequalities, each representing a halfspace. The optimal solution lies within the intersection of these halfspaces.

3. Signal Processing

In signal processing, subspaces are used in techniques like Principal Component Analysis (PCA) to reduce the dimensionality of data while preserving important information.

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our [Cookie Policy](#) & [Privacy Policy](#).

In control systems, state space representations use subspaces to describe the set of all possible states of the system, helping in the analysis and design of controllers.

5. Computer Graphics

In computer graphics, hyperplanes are used for clipping algorithms, which determine which parts of objects are visible within a viewing volume.

Conclusion

Understanding hyperplanes, subspaces, and halfspaces is crucial for analyzing and solving problems in multi-dimensional spaces. These concepts are fundamental in various fields of engineering, offering powerful tools for classification, optimization, signal processing, and more.

Solved Examples on Hyperplane, Subspace and Halfspace

Hyperplane

Example: Consider the 2-dimensional space (R^2). A hyperplane (which is a line in this case) can be given by:

$$3x + 4y = 12$$

Solution:

- To find the points on this line, set $y = 0$: $3x + 4(0) = 12$, which gives $x = 4$. Thus, the point $(4, 0)$ lies on the line.
- To find another point, set $x = 0$: $3(0) + 4y = 12$, which gives $y = 3$. Thus, the point $(0, 3)$ lies on the line.
- So, the line (hyperplane in R^2) passes through the points $(4, 0)$ and $(0, 3)$.

Subspace

Example: Consider $V = R^3$ and W defined by:

$$W = \{(x, y, z) \in R^3 \mid x + y + z = 0\}$$

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our [Cookie Policy](#) & [Privacy Policy](#).

- The zero vector $(0, 0, 0)$ is in W since $0 + 0 + 0 = 0$.
- If $u = (x_1, y_1, z_1)$ and $v = (x_2, y_2, z_2)$ are in W , then: $x_1 + y_1 + z_1 = 0$ $x_2 + y_2 + z_2 = 0$ Their sum, $u + v = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$, satisfies: $(x_1 + x_2) + (y_1 + y_2) + (z_1 + z_2) = (x_1 + y_1 + z_1) + (x_2 + y_2 + z_2) = 0$ Thus, W is closed under addition.
- If $u = (x, y, z)$ is in W and c is a scalar, then $cu = (cx, cy, cz)$ satisfies: $cx + cy + cz = c(x + y + z) = c \cdot 0 = 0$ Thus, W is closed under scalar multiplication.
- Therefore, W is a subspace of $\mathbb{R}^3$.

Halfspace

Example: Consider the halfspace in $\mathbb{R}^2$ given by:

$$2x + y \leq 3$$

Solution:

- To verify if the point $(1, 1)$ lies within the halfspace, substitute $x = 1$ and $y = 1$: $2(1) + 1 \leq 3$, which gives $3 \leq 3$. The point $(1, 1)$ satisfies the inequality and is in the halfspace.
- To verify if a point $(2, 2)$ lies within the halfspace, substitute $x = 2$ and $y = 2$: $2(2) + 2 \leq 3$, which gives $6 \leq 3$. The point $(2, 2)$ does not satisfy the inequality and is not in the halfspace.

Hyperplane, Subspace and Halfspace – Practice Problems

Hyperplane

1. Determine the equation of the hyperplane in $\mathbb{R}^3$ that passes through the points $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.
2. Find the distance from the point $(3, 4)$ to the hyperplane given by $2x + 3y = 6$ in $\mathbb{R}^2$.
3. Find the points of intersection with the axes for the hyperplane given by $4x - 3y + z = 12$ in $\mathbb{R}^3$.

2. Determine if the set $W = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$ is a subspace of $\mathbb{R}^3$.
3. Determine if the set $W = \{(x, y, z) \in \mathbb{R}^3 \mid x - 2y + 3z = 0\}$ is a subspace of $\mathbb{R}^3$.

Halfspace

1. Identify if the point $(2, -1)$ lies in the halfspace defined by $3x - 2y \leq 7$ in $\mathbb{R}^2$.
2. Given the halfspace $x + y \geq 4$ in $\mathbb{R}^2$, find a point that lies in this halfspace and a point that does not.
3. Identify if the point $(-1, 2)$ lies in the halfspace defined by $x - 4y \geq -5$.

Related Articles:

- [*Separating Hyperplanes in SVM*](#)
- [*Vector Space- Definition, Axioms, Properties and Examples*](#)

Comment

More info

Advertise with us

Next Article

Connection Between Eigenvectors
and Nullspace

Similar Reads

Hyperplane, Subspace and Halfspace

In linear algebra and geometry, hyperplanes, subspaces, and halfspaces are fundamental concepts used to describe and analyze the properties and behaviors of multi-dimensional spaces. These concepts have...

7 min read

Connection Between Eigenvectors and Nullspace

In linear algebra, two important concepts that are widely studied are Eigenvectors and Nullspace (also called the kernel of a matrix). Though these two concepts may seem distinct, they are fundamentally connected in...

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our [Cookie Policy](#) & [Privacy Policy](#)

A hypergraph is a graph in which hyperedges (generalized edges) can connect to a subset of vertices/nodes rather than two vertices/nodes. The edges (also known as hyperedges) of a hypergraph are arbitrary nonemp...

7 min read

Null Space of a Matrix

Null space of a matrix is a fundamental concept in linear algebra that describes the set of all possible solutions to the equation $Ax = 0$, where A is a matrix and x is a vector. This space consists of vectors that,...

6 min read

Span in Linear Algebra

In linear algebra, the concept of "span" is fundamental and helps us understand how sets of vectors can generate entire spaces. The span of a set of vectors is defined as the collection of all possible linear...

8 min read

Linear Algebra – GATE CSE Previous Year Questions

Preparing for the Graduate Aptitude Test in Engineering (GATE) in Computer Science and Engineering (CSE) is a significant undertaking. Utilizing previous years' question papers is an effective strategy to understand the...

1 min read

Different Phases of Projected Clustering in Data Analytics

We know Projected clustering is a typical dimension reduction subspace clustering method which instead of initiating from single dimensional spaces, proceeds by identifying an initial approximation of the clusters in...

3 min read

Linear Algebra for GATE Exam

Linear Algebra is an essential topic for the GATE CSE (Graduate Aptitude Test in Engineering for Computer Science Engineering) exam, as it forms the foundation for various advanced concepts in computer science,...

2 min read

LMNs -Linear Algebra

Linear algebra is the study of linear combinations. It is the study of vector spaces, lines and planes, and some mappings that are required to perform the linear transformations. It includes vectors, matrices and linear...

11 min read

Data Science | Solving Linear Equations

Linear Algebra is a very fundamental part of Data Science. When one talks about Data Science, data representation becomes an important aspect of Data Science. Data is represented usually in a matrix form....

6 min read

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our [Cookie Policy](#) & [Privacy Policy](#)

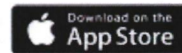


Corporate & Communications Address:

A-143, 7th Floor, Sovereign Corporate
Tower, Sector- 136, Noida, Uttar Pradesh
(201305)

Registered Address:

K 061, Tower K, Gulshan Vivante
Apartment, Sector 137, Noida, Gautam
Buddh Nagar, Uttar Pradesh, 201305



Advertise with us

Company

About Us
Legal
Privacy Policy
In Media
Contact Us
Advertise with us
GFG Corporate Solution
Placement Training Program
GeeksforGeeks Community

DSA

Data Structures
Algorithms
DSA for Beginners
Basic DSA Problems
DSA Roadmap
Top 100 DSA Interview Problems
DSA Roadmap by Sandeep Jain
All Cheat Sheets

Languages

Python
Java
C++
PHP
GoLang
SQL
R Language
Android Tutorial
Tutorials Archive

Data Science & ML

Data Science With Python
Data Science For Beginner
Machine Learning
ML Maths
Data Visualisation
Pandas
NumPy
NLP
Deep Learning

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our [Cookie Policy](#) & [Privacy Policy](#).

JavaScript
TypeScript
ReactJS
NextJS
Bootstrap
Web Design

Python Tkinter
Web Scraping
OpenCV Tutorial
Python Interview Question
Django

Computer Science

Operating Systems
Computer Network
Database Management System
Software Engineering
Digital Logic Design
Engineering Maths
Software Development
Software Testing

System Design

High Level Design
Low Level Design
UML Diagrams
Interview Guide
Design Patterns
OOAD
System Design Bootcamp
Interview Questions

School Subjects

Mathematics
Physics
Chemistry
Biology
Social Science
English Grammar
Commerce
World GK

DevOps

Git
Linux
AWS
Docker
Kubernetes
Azure
GCP
DevOps Roadmap

Interview Preparation

Competitive Programming
Top DS or Algo for CP
Company-Wise Recruitment Process
Company-Wise Preparation
Aptitude Preparation
Puzzles

GeeksforGeeks Videos

DSA
Python
Java
C++
Web Development
Data Science
CS Subjects

@GeeksforGeeks, Sanchhaya Education Private Limited, All rights reserved

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our [Cookie Policy](#) & [Privacy Policy](#).