

POCHI YEH

CLAIRE GU

Optics of Liquid Crystal Displays

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Optics of Liquid Crystal Displays

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Preface

This book introduces the basic principles and presents a systematic and self-contained treatment of the optics of liquid crystal displays (LCDs). It also describes the practical operation of various liquid crystal display systems. The book is intended as a textbook for students in electrical engineering and applied physics, as well as a reference book for engineers and scientists in the area of research and development of display technologies. It has two primary objectives: to present a clear physical picture of the fundamental principles of LCDs and to teach the reader how to analyze and design new components and subsystems for LCDs. The choice of the subject matter and the organization of the book follow closely a course on optics of LCDs that Pochi Yeh taught at the National Chiao Tung University in Hsinchu, Taiwan and at the University of California at Santa Barbara during the past 3 years.

The subject of liquid crystal displays has now grown to become an exciting interdisciplinary field of research and development, involving optics, materials, and electronics. New materials, device configurations, and processing technologies are being developed rapidly. To meet the rapid development of this display technology, this book emphasizes the fundamental principles that can be employed for understanding the limitation of various LCD systems, and to analyze and design high performance display systems. A significant effort is made to bridge the gap between theory and practice through the use of numerical examples based on real liquid crystal materials and information display systems. The book covers a very wide range of topics, including the basic physical properties of liquid crystals, the polarization of optical waves, the propagation of plane waves in liquid crystal media, the concept of phase retardation, Jones matrix method, operations of various LCD systems, passive and active matrix addressing, colors, and birefringence compensators. Near the end of the book, an advanced analytic technique known as the *extended Jones matrix method* is introduced to treat the optical transmission of LCDs at large viewing angles. In writing this book, we have assumed that the student has been introduced to basic electromagnetic plane waves in an undergraduate course in electricity and magnetism. It is further expected that the student has some background in elementary optics and linear matrix algebra.

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Optics of Liquid Crystal Displays

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Preliminaries

In an information-dominated age, the display of information is becoming increasingly essential in many aspects of daily life. This requires display systems utilizing cathode ray tubes (CRTs), electroluminescence devices, plasma display devices, field emission devices, flat CRT, vacuum fluorescence and liquid crystal displays (LCDs). Among these systems, the LCD is one of the most important optical display systems for high performance flat panel display of information. The importance arises from many of its advantages, including flat panel, light weight, high definition, low driving voltages, and low power consumption. Generally speaking, a display panel consists of a two-dimensional array of pixels. Each pixel can be turned ON or OFF independently for the display of two-dimensional images. Figure 1.1 shows a schematic drawing of a basic element (pixel; picture element) of a LCD panel.

Referring to the figure, we note that the basic components of a LCD consist of a thin layer of liquid crystal sandwiched between a pair of polarizers. To control the optical transmission of the display element electronically, the liquid crystal layer is placed between transparent electrodes [e.g., indium tin oxide (ITO)]. The polarizers and the electrodes are cemented on the surfaces of the glass plates. A minimum thickness of a few millimeters is needed to maintain the structural integrity of the panel. The thickness of the liquid crystal layer is kept uniform by using spacers that are made of glass fibers or plastic microspheres. By applying a voltage across the electrodes, an electric field inside the liquid crystal can be obtained to control the transmission of light through the liquid crystal cell. To achieve the display of information, we need a two-dimensional array of these electrodes. These electrodes can be driven electrically for data input by using two sets (x,y) of parallel array of electrodes. In what follows, we will briefly describe each of the optical components.

1.1. BASIC LCD COMPONENTS

1.1.1. Polarizers

An optical arrangement that produces a beam of polarized light from a beam of unpolarized light is called a *polarizer*. Polarizers are key components in many LCDs. Virtually all light sources used in optical displays are unpolarized. A beam of polarized light is described by its direction of propagation, frequency,

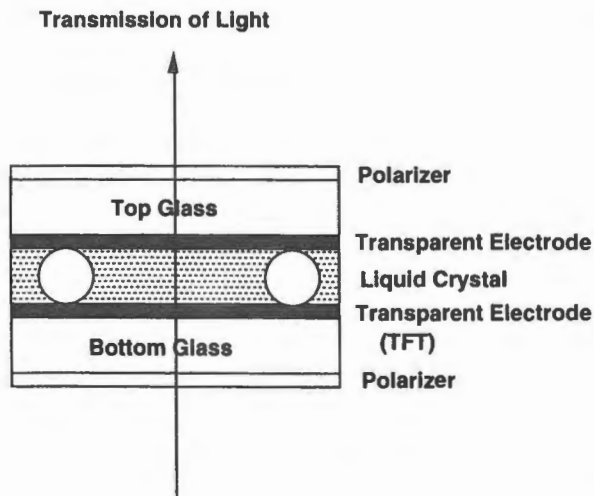


Figure 1.1. Schematic drawing of a basic element of liquid crystal display.

and a vector amplitude (electric field vector). The vector amplitude is related to the intensity of the beam and is perpendicular to the direction of propagation. Given a direction of propagation, there are two independent (orthogonal) components of the vector amplitude.

A beam of light is regarded as unpolarized provided two conditions are met: (1) the time-averaged intensity of the transmitted beam through a polarizer is independent of the orientation of the polarizer and (2) any two orthogonal components of the vector amplitude are totally uncorrelated. Thus, if we decompose the vector amplitude of a beam of unpolarized light into two mutually orthogonal transverse components (e.g., E_x, E_y for light propagating along z axis). These two components will have a random relative phase relationship between them. The random relative phase leads to a resultant amplitude vector that varies rapidly in time in a random manner.

Polarizers consist of anisotropic media that transmit one component and redirect or absorb the other component. It is known that anisotropic crystal prisms provide methods of obtaining plane polarized light by double refraction where one of the components can be removed by reflection or mechanical obstruction. Although prism polarizers can provide excellent extinction ratios, they are not suitable for flat panel applications.

In most LCDs, the polarizers consist of thin sheets of materials that transmit one polarization component and absorb the other component. This effect is known as *dichroism*. Natural dichroism of a material is, like natural birefringence, due to its anisotropic molecular structure. The ratio of the two absorption coefficients for two linear orthogonal polarizations is called the *dichroic ratio*. One of the best known materials of this kind is tourmaline—a naturally occurring mineral that exhibits uniaxial birefringence and a dichroic

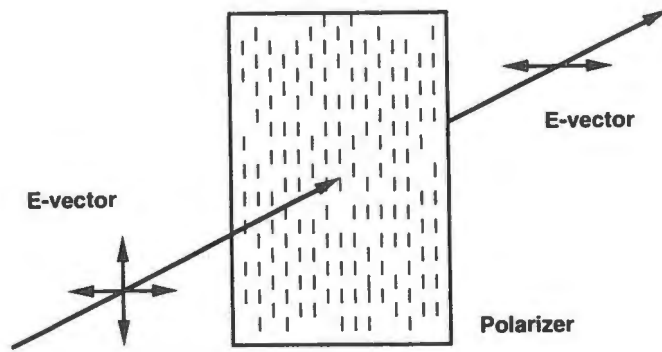


Figure 1.2. Schematic drawing of a sheet polarizer that transmits polarization component with E vector perpendicular to the direction of alignment and absorbs polarization component with E vector parallel to the direction of alignment.

ratio of about 10. In this crystal the ordinary polarization component (electric field vector perpendicular to c axis) is much more strongly absorbed (about 10 times) than the extraordinary polarization component (electric field vector parallel to c axis). A single crystal plate cut parallel to the axis can be used as a polarizer. Most sheet polarizers are, however, synthesized by stretching films that contain ultrafine rodlike or needle-like dichroic materials. The stretching provides an uniaxial alignment of the crystals or molecules (see Fig. 1.2). Dichroic ratios of greater than 100 can be easily obtained. A beam of polarized light with the vector amplitude parallel to the molecular axis may suffer more material absorption. Thus, given enough thickness, the stretched films transmit one polarization component that is perpendicular to the direction of alignment and absorb the other polarization component that is parallel to the direction of alignment. These sheet polarizers provide excellent extinction ratios while simultaneously accommodating a large angular aperture and an enormously large linear aperture. These are desirable features for large flat panel applications.

Another method of producing dichroic materials is to embed oriented ultrafine metallic needles that are not dichroic by themselves in a medium. The composite material then is said to have *form dichroism*, just as a composite medium containing oriented dielectric needles exhibits form birefringence [1].

Generally speaking, 50% of the incident energy of a beam of unpolarized light is absorbed by an ideal polarizer. This energy loss of about 3 dB accounts for the absorption of one of the polarization components resulting in a beam of plane polarized light. When two identical sheet polarizers are in series, the intensity transmission is approximately proportional to $\cos^2\theta$, where θ is the angle between the directions of alignment of the two polarizers. Thus a pair of cross-polarizers ($\theta = 90^\circ$) can block the transmission of light regardless of the polarization state of the incident beam (see Fig. 1.3).

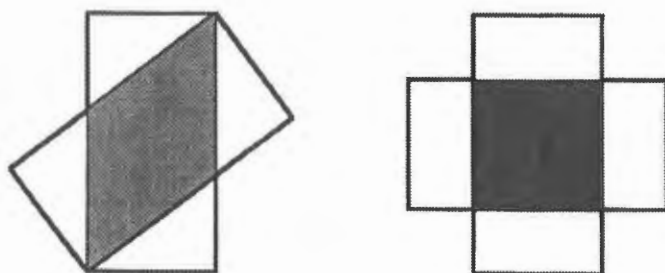


Figure 1.3. Transmission of unpolarized light through two polarizers in series.

1.1.2. Transparent Electrodes

To allow the transmission of light, the electrodes must be transparent in the spectral regime of interest. This requires transparent materials with a good electrical conductivity. For example, tin oxide (SnO_2) is a transparent material which also exhibits a good electrical conductivity. Generally speaking, all transparent semiconductors exhibit a finite electrical conductivity. The electrical conductivity in semiconductors can also be increased by an appropriate doping of impurity atoms. In recent LCD applications, ITO (indium tin oxide) is usually used because of its higher electrical conductivity. All transparent conductors exhibit a small absorption of light. Thus, the electrodes must be thin enough to allow adequate light transmission, yet thick enough to provide adequate electrical conductivity. As a result of the index mismatch between the glass substrate and the electrode materials, a Fabry–Perot cavity is formed in each of the electrodes which are sandwiched between glass and the liquid crystal. A proper choice of the thickness can ensure a constructive interference to maximize the transmission of light. In typical electrodes, the ITO thickness is in the range of 100–300 nm.

1.1.3. Liquid Crystal Cell

The space between the electrodes is filled with liquid crystal (LC) material. The thickness of the LC layer is kept uniform by using glass fibers or plastic balls as spacers. Typical thicknesses of the cell are in the range of a few micrometers. There are several different LC configurations that can be employed for display applications. For the purpose of illustrating the principle of operation of a basic element of LCD, we consider the case of a planar nematic LC cell where all the rodlike LC molecules are aligned parallel to the glass plates.

In an LC cell without any external field, the ordering of the molecules is usually determined by the anisotropy of the boundary. The surfaces of the electrodes are usually coated with a thin alignment layer (e.g., polyimide). Rubbing the surface of the alignment layer has been widely employed to align the liquid crystals in LCD panel fabrication. The LC molecules are aligned on the rubbed surface with their axes parallel to the rubbing direction, especially for

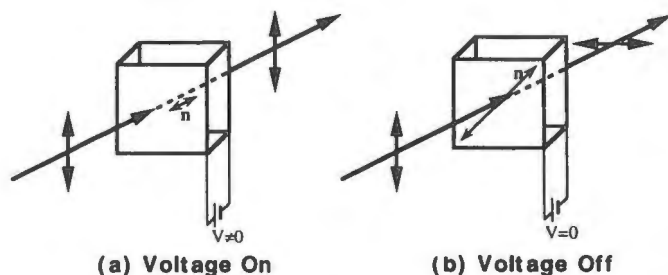


Figure 1.4. LC cell can be controlled electrically to modify the polarization state of an incoming beam (n is a unit vector indicating the direction of alignment of the liquid crystal molecules): (a) voltage ON; (b) voltage OFF.

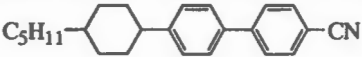
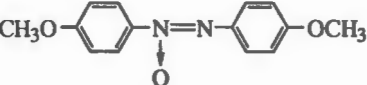
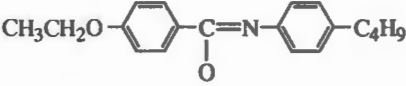
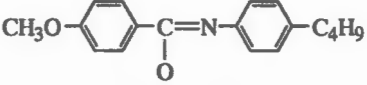
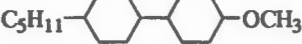
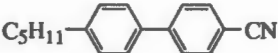
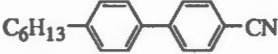
those molecules immediately next to the boundary. Depending on the boundary conditions, a long range ordering of the molecules may exist throughout the LC cell. By virtue of its electrical anisotropy, the ordering and the orientation of the molecules can be controlled electrically by applying an external field. In the presence of an applied electric field, rodlike molecules are aligned parallel to the applied electric field to minimize the electrostatic energy.

As a result of the ordering of the molecules, nematic liquid crystals exhibit a strong optical birefringence. In other words, there are two modes of optical propagation, each with a unique phase velocity. The difference in the phase velocity leads to a phase retardation between these two modes. As a consequence, the polarization state of an incoming beam of polarized light can be modified. An example is shown in Figure 1.4b, where an incoming beam of vertically polarized light is converted into a beam of horizontally polarized light as a result of a properly chosen phase retardation. If the axes of all the molecules are aligned parallel to the propagation direction of the incoming beam by applying an external electric field, the polarization state will remain unchanged. This is illustrated in Figure 1.4a. When such an LC cell is sandwiched between a pair of cross-polarizers (or parallel polarizers), the intensity of the transmitted beam can be controlled electrically.

1.2. PROPERTIES OF LIQUID CRYSTALS

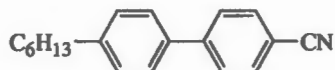
Liquid crystal is a state of matter that is intermediate between the crystalline solid and the amorphous liquid. It may also be viewed as a liquid in which an ordered arrangement of molecules exists. Liquid crystals arise under certain conditions in organic substances having sharply anisotropic molecules, that is, highly elongated (rodlike) molecules or flat (disklike) molecules. A direct consequence of the ordering of the anisotropic molecules is the anisotropy of mechanical, electric, magnetic, and optical properties. This intermediate state was first observed in 1888 in cholesteryl benzoate, a crystalline solid. It becomes

Table 1.1. Liquid Crystal Materials

Name	Formula	Nematic Range (°C)
BCH-5		96–219
PAA		118–135.5
EBBA		35–77
MBBA		22–47
CCH-501		29–36.8
5CB		24–35
6CB		15–29

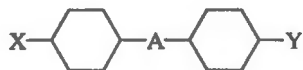
a turbid cloudy liquid, or liquid crystals, when heated to 145°C; on further heating to 179°C the liquid becomes isotropic and clear. The sequence is reversed when the substance is cooled. The cloudy intermediate phase contains domains that seem to have a crystal-like molecular structure. Color changes occur on both heating and cooling. Many organic compounds, such as hexylcyanobiphenyl (6CB) and sodium benzoate, exhibiting this behavior are known and used extensively in electric and electronic displays, electronic clocks, calculators, and similar devices dependent on temperature determination. Table 1.1 lists a number of LC materials and their nematic ranges.

Hexylcyanobiphenyl (6CB) is a nematic LC in the temperature range 15°C < T < 29°C. When 6CB is heated at room temperature, a nematic–isotropic transition occurs at $T = 29^\circ\text{C}$. The material “then becomes suddenly completely clear.” 6CB has the following rodlike molecular structure:



which consists of two benzene rings and two terminal groups. In the nematic phase, only the long axes of the molecules have a statistical preferential orientation.

Referring to Table 1.1, we note that a typical rodlike LC molecule has the following general molecular structure:



where X and Y are terminal groups and A is a linking group between two or more ring systems. We discuss the contribution of each group:

Ring Systems. The rings can be either benzene rings (unsaturated) or cyclohexanes (saturated), or a combination of both. The presence of the ring system is essential in most LCs that contain at least one ring (either phenyl or cyclohexyl). It is important to note that only σ electrons exist in cyclohexyl rings, whereas π electrons exist in phenyl rings. The presence of the rings provides the short range intermolecular forces needed to form the nematic phase. They also affect the absorption, dielectric anisotropy, birefringence, elastic constants, and viscosity. The $\sigma \rightarrow \sigma^*$ electronic transitions occur at vacuum ultraviolet regime ($\lambda < 180$ nm). Thus, the absorption of a saturated LC compound is negligible in the visible spectral regime. In unsaturated LC compound such as 5CB, the $\pi \rightarrow \pi^*$ transitions occur at $\lambda \sim 210$ nm, 280 nm with a strong dichroic ratio at 280 nm. The strong dichroic ratio is also responsible for the large dielectric anisotropy and optical birefringence in a nematic phase.

Terminal Group X (Also Called Side Chain). There are three common X-terminal groups: (1) alkyl chain C_nH_{2n+1} , (2) alkoxy chain $C_nH_{2n+1}O$, and (3) alkenyl chain that contains a double bond. The length of the chain can strongly influence the elastic constants of the nematic phase as well as the phase transition temperature. For short chains with one or two carbon atoms ($n=1,2$), the molecules are too rigid to exhibit LC phases. Terminal groups with medium chain length (e.g., $n=3-8$) are most suitable for nematic phases (which will be explained later in this section). Compounds with even longer chain length can exhibit smectic phases. Generally speaking, the melting point decreases as the chain length increases. However, some compounds are irregular and unpredictable. The nematic–isotropic transition temperature (known as the *clearing point* or N-I point) is a smooth function of n for both odd carbon number n and even carbon number n . Generally speaking, the N-I points (clearing points) for LC compounds with an even number of carbon atoms are lower than those of compounds with an odd number of carbon atoms. This is known as the “odd–even effect.” Although LC compounds with longer chain length can offer a lower melting point, they also exhibit a higher viscosity. In LCDs, it is desirable to have a low viscosity in order to have a higher frame rate.

Linking Group A. The linking group can be a single bond between the two rings. The generic name of these LCs is biphenyl. In case of an additional ring as the linking group, the generic name of these LCs is terphenyl. Some common linking groups are C_2H_4 (diphenylethane), C_2H_2

(stilbene), $-\text{C}\equiv\text{C}-$ (tolane), $-\text{N}=\text{N}-$ (azobenzene), $-\text{CH}=\text{N}-$ (Schiff's base), and COO (phenyl benzoate, ester).

Terminal Group Y. The terminal group Y plays an important role in determining the dielectric constants ϵ and its anisotropy $\Delta\epsilon$. In LCD, the operation voltage is usually a multiple of the threshold voltage, which is the minimum voltage required to cause a reorientation of the LC molecules. It is also known that the threshold voltage is inversely proportional to the square root of the dielectric anisotropy. Thus it is desirable to have a large $\Delta\epsilon$ for low voltage operations. The terminal group Y can be either polar or nonpolar. Nonpolar terminal group Y, such as the alkyl chain $\text{C}_n\text{H}_{2n+1}$, has very little effect on the dielectric anisotropy. On the other hand, a polar terminal group such as CN can contribute significantly to the dielectric anisotropy. Some common polar terminal groups Y include CN, F, and Cl. Among them, CN has the highest polarity, leading to a high dielectric anisotropy and optical birefringence. On the other hand, LC compounds containing CN terminal group Y also exhibit high viscosity, insufficient resistivity, and stability problems under UV illumination. These physical properties can affect the operation of thin film transistor (TFT) LCD. For example, LC compounds with CN terminal group Y are not suitable for high temperature operations such as those used in projection displays. In addition, LC molecules with CN terminal group Y can be dissociated under the illumination of UV. On the contrary, LC molecules with F terminal group Y exhibit low viscosity, high resistivity and relatively high stability under UV illumination. As they are relatively lower in polarity, these LC materials exhibit relatively smaller dielectric anisotropy and optical birefringence.

Eutectic. A single LC compound that can fulfill all LC specifications for most display applications is not available at the moment. An example is 5CB, which has a melting point of 24°C and a clearing point of 35.3°C . It is clear that such a small temperature window for nematic phase is not adequate for most industrial applications with operation temperature from -20 to 80°C . It is known that the melting point of a binary mixture of compounds is less than either of its constituent compounds. The melting point of the mixture depends on the mixture ratio. At the eutectic point, the melting point reaches its minimum. The clearing point of the LC mixture is usually the linear average of the composition. Thus, a mixture of two LC compounds can offer a much larger temperature range that exhibits the nematic phase for LCD applications. For example, the eutectic mixture of MBBA and EBBA can provide a nematic range from 0 to 60°C . In addition to the melting point, many other physical properties, such as dielectric constants, dielectric anisotropy, elastic constants, birefringence, and viscosity, also depend on the mixture ratio. Many of the LC mixtures are designed to provide optimum physical parameters for LCD applications.

LC Phases. Generally speaking, there are three phases of liquid crystals, known as *smectic phase*, *nematic phase*, and *cholesteric phase*. For the sake of clarity, we assume that the liquid crystals are made of rodlike molecules. Figure 1.5a illustrates the smectic phase in which one-dimensional translational order as well as orientational order exist. A unit vector $\mathbf{n}$ in the direction of the preferred orientation of the molecular axes is known as the *director*. Figure 1.5b illustrates the nematic phase in

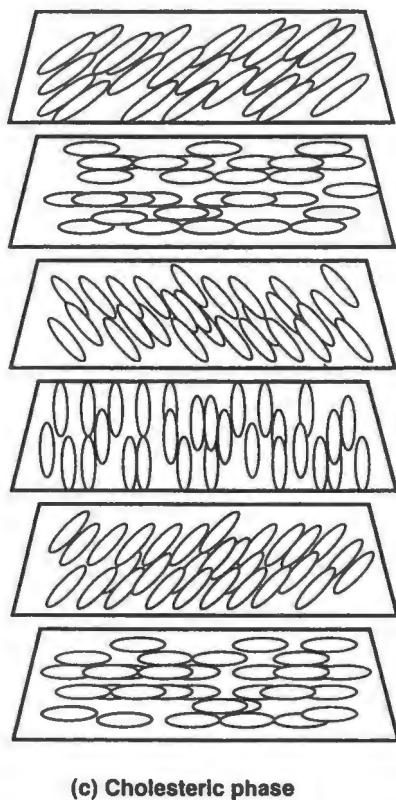
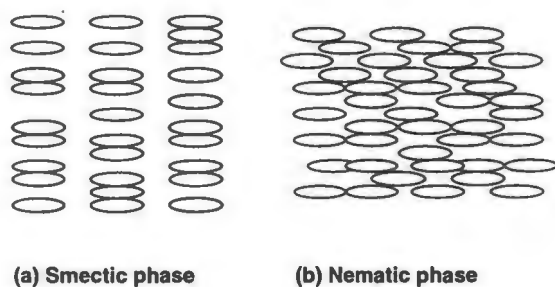


Figure 1.5. Phases of liquid crystals: (a) smectic phase; (b) nematic phase; (c) cholesteric phase.

which only a long range orientational order of the molecular axes exists. The cholesteric phase is also a nematic type of LC except that it is composed of chiral molecules. As a consequence, the structure acquires a spontaneous twist about a helical axis normal to the director. The twist may be right-handed or left-handed depending on the molecular chirality. Figure 1.5c illustrates the cholesteric phase of LC by viewing the distribution of molecules at several planes that are perpendicular to the helical axis. Note that the LC is in a nematic phase in each perpendicular plane.

A smectic LC is closest in structure to solid crystals. It is interesting to note that in substances that form both a nematic phase and a smectic phase, the sequence of phase changes on rising temperature is as follows:

Solid crystal $\rightarrow$ smectic liquid crystal $\rightarrow$ nematic liquid crystal
 $\rightarrow$ isotropic liquid

Although the smectic phase possesses the highest degree of order, it is the nematic and cholesteric phases that have the greatest number of electrooptical applications. In the nematic phase, the medium may appear milky if the orientation order exists in many different domains. The nematic liquid crystal is clear only when a long range order exists in the whole medium. At the nematic–isotropic transition temperature, the medium becomes isotropic and looks clear and transparent. Thus the temperature is also known as the *clearing point*. In the following sections, we discuss various important physical properties that are the results of the orientational order.

1.2.1. Orientational Order Parameter

In the nematic phase, the molecules are rodlike with their long axes aligned approximately parallel to one another. Thus at any point in the medium, we can define a vector $\mathbf{n}$ to represent the preferred orientation in the immediate neighborhood of the point. This vector is known as the *director*. In a homogeneous nematic LC, the director is a constant throughout the medium. In an inhomogeneous nematic liquid crystal, the director $\mathbf{n}$ can change from point to point and is, in general, a function of space (x,y,z) . If we define a unit vector to represent the long axis of each molecule, then the director $\mathbf{n}$ is the statistical average of the unit vectors over a small volume element around the point.

The order parameter S of a LC is defined as

$$S = \frac{1}{2} \langle 3 \cos^2 \theta - 1 \rangle \quad (1.2-1)$$

where θ is the angle between the long axis of an individual molecule and the director $\mathbf{n}$ and the angular brackets denote a statistical average. For perfectly

parallel alignment, $S = 1$, while for totally random orientations, $S = 0$. In the nematic phase, the order parameter S has an intermediate value that is strongly temperature dependent. It is evident that $S = 0$ at the clearing point. Typical values of the order parameter S are in the range between 0.6–0.4 at low temperatures. As the clearing point is approached, the order parameter S drops abruptly to zero. The values of the order parameter S also depend on the structure of the molecules. Experimental observation indicates that liquid crystals based on cyclohexane rings exhibit higher values of S than do aromatic systems.

1.2.2. Dielectric Constants

Because of the orientational ordering of the rodlike molecules, the smectic and nematic liquid crystals are uniaxially symmetric, with the axis of symmetry parallel to the axes of the molecules (director $\mathbf{n}$). As a result of the uniaxial symmetry, the dielectric constants differ in value along the preferred axis ($\epsilon_{\parallel}$) and perpendicular to this axis ($\epsilon_{\perp}$). The dielectric anisotropy is defined as

$$\Delta\epsilon = \epsilon_{\parallel} - \epsilon_{\perp} \quad (1.2-2)$$

The sign and magnitude of the dielectric anisotropy $\Delta\epsilon$ are of the utmost importance in the applicability of the LC material in LCDs using one of the various electrooptic effects. To illustrate this, we consider an applied electric field along the z axis in a homogeneous nematic liquid crystal. As a result of the anisotropy, the induced dipole moment of the molecules is not parallel to the applied electric field, except when the molecular axis is parallel or perpendicular to the electric field. This creates a net torque that tends to align the molecules along the direction of the electric field for most rodlike molecules. The macroscopic electrostatic energy can be written

$$U = \frac{1}{2} \mathbf{D} \cdot \mathbf{E} \quad (1.2-3)$$

where $\mathbf{E}$ is the electric field vector and $\mathbf{D}$ is the displacement field vector. In a homogeneous medium the displacement field vector $\mathbf{D}$ is independent of the orientation of the liquid crystal. Letting θ be the angle between the director and the z axis, we can express z component of the displacement field vector as

$$D_z = (\epsilon_{\parallel} \cos^2\theta + \epsilon_{\perp} \sin^2\theta)E \quad (1.2-4)$$

Thus the electrostatic energy can be written (see Appendix A)

$$U = \frac{1}{2} \frac{D_z^2}{\epsilon_{\parallel} \cos^2\theta + \epsilon_{\perp} \sin^2\theta} \quad (1.2-5)$$

Table 1.2. Properties of Liquid Crystals

	$T(^{\circ}\text{C})$	λ (nm)	n_e	n_o	$\epsilon_{\parallel}$	$\epsilon_{\perp}$	k_1	k_2	k_3		Range ($^{\circ}\text{C}$)	Reference
							(10 $^{-12}$ N)			Nematic		
MBBA ^a	22	589	1.769	1.549	4.7	5.4	6.2	3.8	8.6	20	47	4
PCH-5 ^b	30.3	589	1.604	1.4875	17.1	5	8.5	5.1	16.2	30	55	5,12
		633	1.600	1.4851								
	38.5	589	1.5956	1.4863	16.6	15.3	7.3	4.5	13.2			
		633	1.5919	1.4840								
	46.7	589	1.5849	1.4860	15.9	5.7	5.9	3.9	9.9			
		633	1.5812	1.4836								
K15 (5CB) ^c	25	515	1.736	1.5442	19.7	6.7	6.4	3	10	24	35.3	18
K21 (7CB) ^d	37	577	1.6815	1.5248	15.7	6	5.95	4	6.6	30	42.8	8,15,16,17
M15 (5OCB) ^e	50	589	1.7187	1.5259	17.9	6.7	6.1	3.74	8.4	48	68	9,11,15,16
M21 (7OCB) ^f	60	589	1.6846	1.5139	16.3	6.5	6.9		7.7	54	74	15,16
M24 (8OCB) ^g	70	589	1.6639	1.5078	14.7	6.2	7.3		9.0	67	80	15,16
E5 ^h	20	577	1.736	1.5228	19	5.9				− 8	50.5	12,15
E7 ⁱ	20	577	1.75	1.5231	19.6	5.1	12	9	19.5	− 10	60.5	15
ZLI-1646 ^j	20	589	1.558	1.478	10.6	4.6	7.7	4.0	12.2	− 20	60	12,15
ZLI-4792 ^k	20	589	1.573	1.479	8.3	3.1	13.2	6.5	18.3	− 40	92	12

^a *p*-Methoxybenzylidene-*p'*-*n*-butylaniline^c Pentylcyanobiphenyl.^e Heptyloxycyanobiphenyl.^g Octyloxycyanobiphenyl.ⁱ 47%K15 + 25%K21 + 18%M24 + 10%T15.^k SFM-TFT mixture.^b 4-(*trans*-4-Pentylcyclohexyl)benzonitrile^d Heptylcyanobiphenyl.^f Heptyloxycyanobiphenyl.^h 45%K15 + 24%K21 + 10%M15 + 9%M21 + 12%M24.^j Phenylcyclohexane and biphenylcyclohexane mixtures.

For liquid crystals with positive dielectric anisotropy ($\epsilon_{\perp} < \epsilon_{\parallel}$), lowest electrostatic energy occurs at $\theta = 0$ when the director is parallel to the applied electric field.

In classical dielectric theory, the macroscopic dielectric constant is often proportional to the molecular polarizability. For rodlike molecules, the longitudinal polarizability (parallel to molecular axis) is often greater than the transverse polarizability (perpendicular to axis). So even in nonpolar liquid crystals with rodlike molecules, the dielectric anisotropy is positive ($0 < \Delta\epsilon$). In the case of polar LC compounds, there is an additional contribution to the dielectric constant due to the permanent dipole moment. Depending on the angle between the dipole moment and the molecular axis, the dipole contribution can cause an increase or a decrease of $\Delta\epsilon$, eventually leading to a negative value of $\Delta\epsilon$ (e.g., MBBA). In practice, dielectric anisotropy varies between $-2\epsilon_0$ and $+15\epsilon_0$. Liquid crystal compounds with a large anisotropy can be synthesized by substitution of a strongly polar group (e.g., cyanide group) in specific positions. Table 1.2 lists properties of LC compounds with various dielectric anisotropy. The dielectric anisotropy also depends on the temperature and approaches zero abruptly at the clearing point. Beyond the clearing point, the dielectric constant becomes the mean dielectric constant

$$\bar{\epsilon} = \frac{\epsilon_{\parallel} + 2\epsilon_{\perp}}{3} \quad (1.2-6)$$

1.2.3. Refractive Index

In a glass flask, nematic LCs often appear as an opaque milky fluid. The scattering of light is due to the random fluctuation of the refractive index of the sample. With no proper boundaries to define the preferred orientation, the sample consists of many domains of nematic LC. The discontinuity of the refractive index at the domain boundaries is the main cause of the scattering leading to the milky appearance. Under the proper treatment (e.g., rubbing the alignment layer on the glass substrate), a slab of nematic LC can be obtained with a uniform alignment of the director. Such a sample exhibits uniaxial optical symmetry with two principal refractive indices n_o and n_e . The ordinary refractive index n_o is for light with electric field polarization perpendicular to the director and the extraordinary refractive index n_e is for light with electric field polarization parallel to the director. The birefringence (or optical anisotropy) is defined as

$$\Delta n = n_e - n_o \quad (1.2-7)$$

If $n_o < n_e$, the LC is said to be positive birefringent, whereas if $n_e < n_o$, it is said to be negative birefringent. In classical dielectric theory, the macroscopic refractive index is related to the molecular polarizability at optical frequencies.

The existence of the optical anisotropy is due mainly to the anisotropic molecular structures. Most LCs with rodlike molecules exhibit positive birefringence ranging from 0.05 to 0.45. The optical polarizability is due mainly to the presence of delocalized electrons not participating in chemical bonds and of π electrons. This is the reason that LC molecules composed of benzene rings have higher values of Δn than do the respective cyclohexane counterparts. As mentioned earlier, the substitution of a cyano terminal group (CN) can cause an increase in the optical anisotropy. Another effective way of increasing Δn or n_e is to employ triple bonds (tolane). For most LCs, the ordinary refractive index is around 1.5. The various components employed in the synthesis of LCs do not markedly affect n_o . On the other hand, the extraordinary refractive index can be higher by as much as 0.45 for some diphenyldiacetylenic compounds or only by 0.06 for bicyclohexanes. The optical anisotropy plays an essential role in changing the polarization state of light in liquid crystals.

1.2.4. Elastic Constants

Like most liquids and solids, LCs exhibit curvature elasticity. The elastic constants of LC determine the restoring torques that arise when the system is perturbed from its equilibrium configuration. These restoring torques are usually very weak compared with those of solids. In LCDs, an electric field is often applied to cause a reorientation of the molecules. It is the balance between the electric torque and the elastic restoring torque that determines the LC's static deformation pattern. Any static deformation of LCs can be divided into a combination of three basic deformations. These are splay, twist, and bend, as illustrated in Figure 1.6.

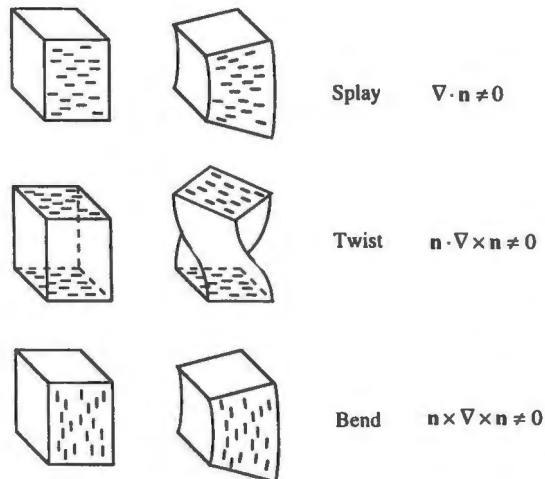


Figure 1.6. Schematic drawing of splay, twist, and bend in LC.

It can be shown that for an isothermal deformation in an incompressible fluid, the free energy (elastic energy) can be written as a quadratic function of the curvature strain tensor. Following the notation of Oseen–Frank theory, the elastic energy density of a deformed LC can be written [2,3]

$$F = \frac{1}{2}k_1(\nabla \cdot \mathbf{n})^2 + \frac{1}{2}k_2(\mathbf{n} \cdot \nabla \times \mathbf{n})^2 + \frac{1}{2}k_3(\mathbf{n} \times \nabla \times \mathbf{n})^2 \quad (1.2-8)$$

where k_1 , k_2 , and k_3 are the splay, twist, and bend elastic constants, respectively.

Like many other physical properties, the elastic constants are strongly temperature dependent. For most LC compounds, the elastic constants are in the range of 3–25 piconewtons (10^{-12} N). The ratio for the elastic constants varies from 0.7–1.8 for k_3/k_1 , and varies from 1.3 to 3.2 for k_3/k_2 . Table 1.2 lists the elastic constants for some typical LC materials.

1.2.5. Viscosity—Rotational Viscosity

The viscosity of fluid is an internal resistance to flow, defined as the ratio of shearing stress to the rate of shear. It arises from the intermolecular forces in the fluid. The viscous behavior of liquid crystals has a profound effect on the dynamical behavior of LCD systems. Like most liquids, the viscosity increases at low temperatures as a result of lower molecular kinetic energy. This can severely limit the operations of LCDs. An important parameter is the rotational viscosity coefficient γ_1 , which provides a resistance to the rotational motion of the LC molecules. In most LCDs, the directors are reoriented by the application of an electric field. The switching time is approximately proportional to $\gamma_1 d^2$, where d represents the cell spacing. For most nematic LCs used in displays, the magnitude of the rotational viscosity is in the range of 0.02–0.5 Pa·s (comparable to light machine oils). As a reference, water at 20°C has a viscosity of 1.002 mPa·s. The viscosity unit of Pa·s (pascal-second) corresponds to $1\text{N}\cdot\text{s}/\text{m}^2$ in SI (Système International) units, or 10 poise (1 poise = $1\text{dyn}\cdot\text{s}/\text{cm}^2$ in cgs (centimeter–gram–second) units). It is known experimentally that molecules with a higher number of rings or longer alkyl chains are characterized by an increasing viscosity. In addition, LCs with high values of $\Delta\epsilon$ usually exhibit higher viscosities. This is possibly due to the stronger polar interaction between the molecules.

1.2.6. Surface Alignment and Rubbing

As a result of the rodlike or disklike nature, the distribution of the orientation of LC molecules plays an essential role in the optical properties of liquid crystal in display applications. Uniform or well prescribed orientation of the LC molecules is required in most LCDs. The role of the surface is to ensure a single domain. Without special treatments of the surfaces (either physical or chemical), the LC will generally have many domains and many disinclinations, or discontinuities

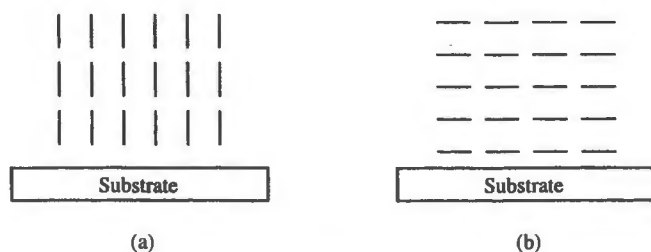


Figure 1.7. (a) Homeotropic (vertical) alignment; (b) parallel homogeneous alignment.

in orientations. These domains and discontinuities can cause a severe scattering of light, leading to a cloudy appearance.

The alignment of a LC by the surface treatment of a substrate has been one of the least understood aspects of LC behavior. It depends on both the nature of the LC and the surface. The most important factors include dipolar interactions, chemical and hydrogen bonding, van der Waals interactions, steric factors, surface topography, and the elasticity of LC molecules. For example, it is possible to deposit molecules known as *silane coupling agents* on to a glassy surface. These molecules may promote the adhesion of LC molecules, in a vertical manner, onto the surface; this is known as *homeotropic alignment* when the LC director is normal to the surface (see Fig. 1.7).

To achieve a "parallel homogeneous alignment" (see Fig. 1.7) where the LC director is uniformly parallel to the surface of the substrate, one must provide a preferred orientation to the alignment by physically or chemically treating the surface. Rubbing the surface has been a simple and effective way of achieving a preferred orientation. Typical rubbing materials include linen cloth and lens paper. Although the technique of rubbing to achieve parallel homogeneous alignment is still very much an art, it is generally believed that physically rubbing the surface produces a uniform and unidirectional tilt of the dangling bonds or side chains on the surface (see Fig. 1.8). The unidirectional tilt of these surface molecules may lead to the parallel homogeneous alignment of the LC molecules. Notice that the LC director usually tilts up in the direction of rubbing.

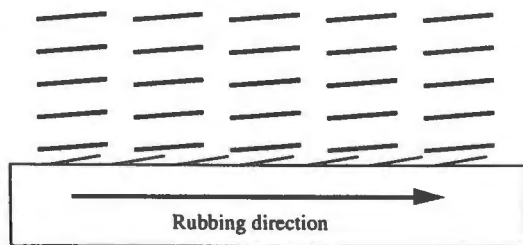


Figure 1.8. Schematic drawing of the anchoring of rodlike LC molecules near the surface of a rubbed substrate.

Pretilt Angle

As a result of the surface rubbing, a small tilt angle exists for the LC director. The pretilt angle is very important in electrooptic applications when an electric field is applied to the cell to reorient the LC director. Having no polarity, the rodlike LC molecules can turn in two ways following the application of an electric field, if the electric field is initially perpendicular to the molecular axis. A small tilt angle of the molecular axis would greatly facilitate the turning of the molecule under the application of an electric field, and ensure a single way of turning. A single way of turning for all molecules in the cell will ensure a single domain, which is essential in most display applications. Referring to Figure 1.9, we consider the distribution of director in a parallel cell of nematic LC. A uniform tilt angle can exist in a parallel cell provided the rubbing directions in the inner surfaces are antiparallel (see Fig. 1.9a). If, on the other hand, the rubbing directions are parallel as shown in Figure 1.9b,c, a nonuniform distribution of the director orientation exists with a zero tilt or 90° angle at midlayer. These are the splay or bend cells. For small pretilt angles, the splay cell has the lowest elastic energy density. This is, however, an undesirable configuration for electrooptic applications, as multiple domains may occur when an electric field is applied. The bend cell is also known as the *Pi cell*, which was originally developed by Bos et al. for an electrically controllable wave plate [13].

Twisted Nematic Liquid Crystal Cell

The pretilt angle also plays an important role in twisted nematic liquid crystal (TN-LC) cells. Referring to Figure 1.10, we consider a liquid crystal cell with a pair of orthogonal rubbing directions. The rubbing directions are important to prepare a twisted nematic liquid crystal (TN-LC) cell with a total twist angle of 90° . Having no polarity and no chirality, the rodlike molecules can twist in either direction in the absence of a small pretilt angle. In other words, both

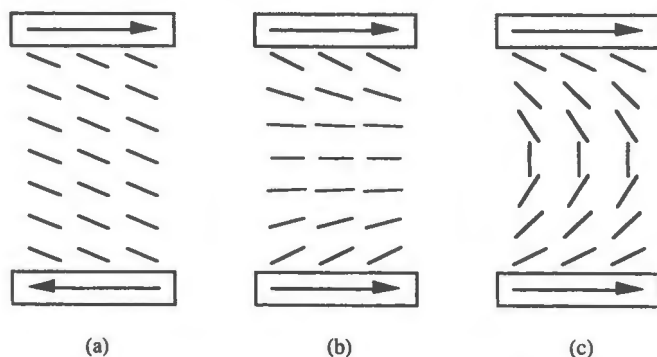


Figure 1.9. Tilt of LC molecular axes due to rubbing of inner surfaces of the LC cell. The arrows indicate the direction of rubbing. (a) Parallel alignment occurs when the rubbings are in opposite directions. Splay cell (b) or bend cell (c) can occur when the rubbings are in the same direction.

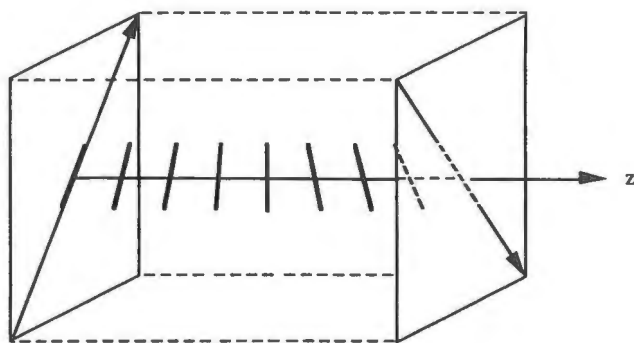


Figure 1.10. Distribution of liquid crystal molecular axes in a 90° TN-LC cell with a right-handed twist. The arrows indicate the rubbing directions at the boundaries. The rods indicate the distribution of director orientations in the cell.

right-handed and left-handed twist of 90° are possible to match the boundary conditions imposed by the rubbing. When a small pretilt angle exists, only the right-handed twist is possible in the cell described in Figure 1.10. In the cell shown in Figure 1.10, the upper parts of LC molecular axes tilt toward the right side ($+z$ direction).

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PROBLEMS

- 1.1. (a) Show that 50% of the energy of a beam of unpolarized light transmits through an ideal polarizer regardless of the orientation of the polarizer.
- (b) As discussed in this chapter, the transmitted intensity through two polarizers in series is given by $\frac{1}{2} \cos^2 \theta$. Show that the fractional error in the intensity measurement due to an error in $\Delta \theta$ is given by $-2\Delta \theta \tan \theta$.

1.2. Polarizers in series:

- (a) It is known that a pair of crossed polarizers can stop the transmission of light. Show that optical transmission occurs when a third polarizer is inserted between the crossed polarizers with a transmission axis oriented at 45° with respect to those of the crossed polarizers. Find the transmission for unpolarized light, assuming that the orientations of the crossed polarizers are at 0° and 90° .
- (b) Consider the insertion of two polarizers oriented at 30° and 60° between the two crossed polarizers. Find the transmission for unpolarized light.
- (c) By generalizing (a) and (b), we consider the insertion of N polarizers oriented at equal angular separations. Show that the transmission of unpolarized light approaches $\frac{1}{2}$ when N tends to infinity. This is a situation when the electric field vector follows the direction of the transmission axis of the polarizers as the light propagates through the whole set of polarizers in series.

1.3. Derive Eqs. (1.2-4) and (1.2-5).

- 1.4. Let $\alpha_\perp$ and $\alpha_\parallel$ be the molecular polarizability of LC molecule with $\alpha_\perp < \alpha_\parallel$. Under the application of an external electric field, the induced dipole moment of the liquid crystal molecule can be written $p_\perp = \alpha_\perp E_\perp$ and $p_\parallel = \alpha_\parallel E_\parallel$.
 - (a) Show that, in general, the dipole moment is not parallel to the applied electric field.
 - (b) Show that a net torque is exerted on the molecule that tends to turn the molecule along the direction of the electric field. Evaluate the torque in terms of the polarizability.
 - (c) Show that the torque discussed in (b) tends to minimize the electrostatic energy in Eq. (1.2-5).

1.5. Elastic energy density and electrostatic energy density:

- (a) Consider a uniformly twisted nematic LC with a director given by $\mathbf{n} = (\sin \phi, \cos \phi, 0)$, where ϕ is the twist angle given by $\phi = \alpha z$. Show that the twist elastic energy density is given by $U_t = \frac{1}{2} k_2 \alpha^2$. Evaluate the energy density in terms of J/m^3 (joules per cubic meter) for the case of a $5\text{-}\mu\text{m}$ cell of ZLI-1646 liquid crystal with a 90° twist. (Use $k_2 = 4 \times 10^{-12} \text{ N}$.)
- (b) Using Eq. (1.2-4) or (1.2-5), show that the difference between the electrostatic energy density at $\theta=0$ and $\pi/2$ can be written $\Delta U = \frac{1}{2} \Delta \epsilon E^2$. Evaluate ΔU in terms of J/m^3 for the case of a $5\text{-}\mu\text{m}$ cell of ZLI-1646 LC with an applied voltage of 1 V. Show that the electrostatic energy density is much larger than the twist elastic energy density. (Use $\Delta \epsilon = 6\epsilon_0$.) This example illustrates the advantages of LCs that can be easily reoriented by applying a small voltage.

- 1.6. Prove that the bend term in Eq. (1.2-8) can also be written as $\frac{1}{2}k_3(\mathbf{n} \cdot \nabla \mathbf{n})^2$; in other words, $(\mathbf{n} \times \nabla \times \mathbf{n})^2 = (\mathbf{n} \cdot \nabla \mathbf{n})^2$.
- 1.7. *Director distribution in a parallel cell due to pretilt:*

- (a) Show that the elastic energy density due to a tilt angle distribution $\theta(z)$ is given by

$$U_{EL} = \frac{1}{2}(k_1 \cos^2 \theta + k_3 \sin^2 \theta) \left(\frac{d\theta}{dz} \right)^2$$

where k_1, k_3 are elastic constants.

- (b) Using variational calculus, show that minimum elastic energy occurs when

$$U_{EL} = \frac{1}{2}(k_1 \cos^2 \theta + k_3 \sin^2 \theta) \left(\frac{d\theta}{dz} \right)^2 = \text{constant}$$

The absolute minimum occurs when $\theta = \text{constant}$. This is the case shown in Figure 1.9a.

- (c) Show that, in general, the tilt angle distribution $\theta(z)$ for minimum elastic energy is given by

$$\frac{d\theta}{dz} = \frac{s\sqrt{k_1}}{\sqrt{k_1 \cos^2 \theta + k_3 \sin^2 \theta}}$$

where s is a constant. In the antisymmetric case when $\theta(0) = \theta_0$, $\theta(d) = -\theta_0$, the tilt angle is zero at midlayer ($z = d/2$). In this case s is the slope $d\theta/dz$ at midlayer.

- (d) Integrate the equation in (c) and obtain an expression for s . Show that for small pretilt angle ($\theta_0 \ll 1$), the slope can be written $s = (2\theta_0/d)$ and the tilt angle distribution $\theta(z)$ can be written $\theta(z) = \theta_0 + sz$.
- (e) In a Pi cell, the distribution $\theta(z)$ for minimum elastic energy is again given by

$$\frac{d\theta}{dz} = \frac{s\sqrt{k_1}}{\sqrt{k_1 \cos^2 \theta + k_3 \sin^2 \theta}}$$

where s is a constant. In this case, $\theta(0) = \theta_0, \theta(d) = \pi - \theta_0$, the tilt angle is $\pi/2$ at midlayer ($z = d/2$). In this case s is proportional to the slope $d\theta/dz$ at midlayer. Show that

$$s = \sqrt{\frac{k_3}{k_1}} \left(\frac{d\theta}{dz} \right)_{z=d/2}$$

- (f) Explain why the splay cell has a lower elastic energy for cells with small pretilt angles.

POCHI YEH

CLAIRE GU

5

Liquid Crystal Displays

Liquid crystals can be employed for display applications using various configurations. Most of the displays produced recently involve the use of either twisted nematic (TN) or supertwisted nematic (STN) liquid crystals. In TN liquid crystal, the director of the LC is twisted by an angle of about 90° , whereas in STN the LC director is twisted by an angle larger than 90° (e.g., 180° , 240° , or 270°). As we mentioned before, each display system consists of a two-dimensional array of pixels ($M \times N$) that can be turned ON or OFF electrically. Each of pixels consists of a LC cell sandwiched between transparent electrodes. In the early days of TN displays, the liquid crystal cells are driven by two sets of electrodes ($M + N$) using multiplexing technique. The advantage of multiplexing is that $M \times N$ pixels can be addressed by using only $M + N$ electrical contacts. The simplicity of multiplexing leads to a degradation of the device performance, especially in the viewing contrast and the limited pixel resolution. The performance of TN displays was later improved by using a different driving technique that involves the use of thin film transistor (TFT) arrays. In TFT arrays, there are transistors associated with each of the pixels. This significantly increases the performance as well as the production cost. The subjects of multiplexing and active matrix (AM) addressing will be discussed in detail in the next chapter.

The STN displays was introduced recently to improve the performance of LCD without the use of TFT. It will be shown later that a larger twist angle can lead to a significantly larger electrooptical distortion. This leads to a substantial improvement in the contrast and viewing angles over TN displays.

In this chapter, we discuss the principles of operation of various displays including TN-LCD, STN-LCD, PD-LCD, reflective LCD and so on. In addition, we also discuss important properties of these displays, including the contrast and viewing angles.

5.1. TWISTED NEMATIC (TN) DISPLAYS

5.1.1. Principle of Operation

The operation of a twisted nematic display is based on the waveguiding property discussed in Chapter 4. As we recall, a beam of linearly polarized light can propagate in a TN liquid crystal cell along the direction of the twist axis. If the

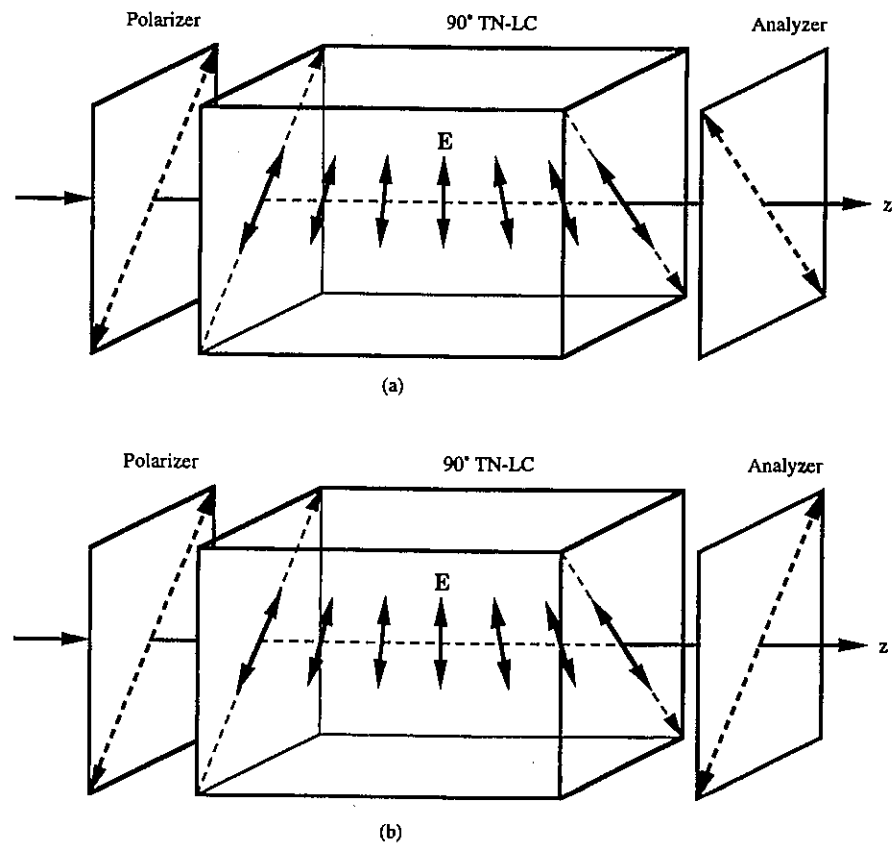


Figure 5.1. Principle of operation of TN displays (*E* mode): (a) NW operation; (b) NB operation. The dashed arrows on the surface of the TN-LC cell indicate the rubbing directions.

direction of polarization is parallel (or perpendicular) to the director at incidence, the polarization state of the light will follow the twist of the local director as the light propagates in the LC. This leads to a rotation of the output polarization state. In a 90° TN-LC cell, the direction of polarization of the beam is rotated by 90°. Figure 5.1 illustrates the principle of operation of a 90° TN-LCD.

In a typical TN display with a 90° twist, the LC is sandwiched between a pair of crossed polarizers. The polarizer is aligned so that the transmission axis is parallel to the director at the input end of the LC (*E*-mode operation). This ensures that the input beam of light is polarized along the direction of the director. The analyzer is also aligned so that the transmission axis is parallel to the director at the exit end of the liquid crystal. This ensures a total transmission of a beam of linearly polarized light by virtue of the waveguiding property. Thus, a beam of unpolarized light can transmit through the display suffering only a 3-dB loss at the front polarizer. This is known as the normally white (NW) configuration (see Fig. 5.1a).

In the TN display, each of the cells consists of a LC material sandwiched between two glass plates separated by a gap of 5–10 μm . The inner surfaces of the plates are deposited with transparent electrodes made of conductive coatings of indium tin oxide (ITO). These transparent electrodes are coated with a thin layer of polyimide with a thickness of several hundred angstroms. The polyimide films are unidirectionally rubbed to ensure that the local directors are parallel to the rubbing directions at the surfaces. In a 90° TN, the rubbing direction of the lower substrate is perpendicular to the rubbing direction of the upper surface. Thus, in the inactivated state (voltage OFF), the local director undergoes a continuous (uniform) twist of 90° in the region between the plates. Sheet polarizers are laminated on the outer surfaces of the plates. The transmission axes of the polarizers are aligned parallel to the rubbing directions of the adjacent polyimide films.

In the activated state (voltage ON) when a small voltage (3–5 V) is applied to the electrodes, a strong electric field is built up in the liquid crystal. As a result of the dielectric anisotropy, the liquid crystals are aligned parallel to the direction of the applied electric field. This leads to a homeotropically aligned configuration of the LC molecules. In other words, the directors of the liquid crystal are now perpendicular to the plates. Using the terminology of Chapter 4, this is effectively a c plate of liquid crystal. As we know, the plane of polarization remains unchanged when a beam of light propagates in the normal direction of a c plate. This leads to a zero transmission when the c plate is sandwiched between a pair of crossed polarizers (NW operation). Thus, by controlling the voltage ON or OFF, we are able to control the transmission of light through the liquid crystal cell. This is the basic principle of operation of a TN display cell.

5.1.2. Transmission Properties of Field-off State

The transmission property of a TN liquid crystal cell can be investigated by using the Jones matrix method developed in Chapter 4. To discuss the transmission property, we consider a 90° TN cell with a front local director parallel to the transmission axis of the polarizer. A beam of unpolarized light is converted into a beam of linearly polarized light by the front polarizer. We mentioned earlier that waveguiding occurs as the beam propagates in the LC cell. And the transmitted beam is linearly polarized with a rotation of the plane of polarization by 90° . Strictly speaking, the waveguiding is valid only in the limit.

$$\phi \ll \frac{2\pi}{\lambda} \Delta n d \quad (5.1-1)$$

where ϕ = total twist angle

d = thickness of LC layer

Δn = birefringence of LC material

λ = wavelength of light

This is known as the *limit of slow twist*. The inequality (5.1-1) is also known as the *Mauguin condition*. For a 90° TN cell, this condition reduces to

$$\frac{\lambda}{2} \ll \Delta nd \quad (5.1-2)$$

In general cases when the condition is not satisfied, the output beam is elliptically polarized with an ellipticity given by Eq. (4.3-20).

To discuss the polarization state, we use the eo coordinate system in which the e axis is parallel to the local director and the o axis is perpendicular to the local director. Consider an input beam of light with its electric field vector polarized parallel to the director of the liquid crystal at the input end of the LC layer. In the eo coordinate system, the input polarization state can be written as the following Jones vector:

$$\begin{pmatrix} V_e \\ V_o \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (5.1-3)$$

The output polarization state in terms of the Jones vector in the local eo coordinate at the rear end of the LC layer can be written, according to Eqs. (4.3-13) or (4.3-15), as

$$\begin{pmatrix} V'_e \\ V'_o \end{pmatrix} = \begin{pmatrix} \cos X - i \frac{\Gamma \sin X}{2X} \\ -\phi \frac{\sin X}{X} \end{pmatrix} \quad (5.1-4)$$

where

$$\Gamma = \frac{2\pi}{\lambda} \Delta nd \quad (5.1-5)$$

$$X^2 = \phi^2 + \frac{\Gamma^2}{4} \quad (5.1-6)$$

We note that the Mauguin condition, Eq. (5.1-1), is equivalent to $\phi \ll \Gamma$. We also note that for liquid crystals satisfying the Mauguin condition (with $\phi \ll \Gamma$), the second element of the column vector is near zero. Thus, Eq. (5.1-4) corresponds to a Jones vector that is approximately linearly polarized along the local director (e axis). This explains the phenomenon of waveguiding.

In most TN displays, the LC layers actually used may not satisfy the Mauguin condition. This leads to a series of performance degradation, including a

reduction in brightness and in contrast, an undesirable coloration caused by the dependence of transmission on wavelength. Using Eq. (5.1-4), we can now discuss the transmission property of the 90° TN cell. The discussion is divided into two categories:

1. *Normally Black (NB) Mode.* In this situation the 90° TN cell is sandwiched between a pair of parallel polarizers (see Fig. 5.1*b*). The director at the input end is parallel to the transmission axis of the polarizer, whereas the director at the exit end is perpendicular to the transmission axis of the analyzer. According to Eq. (5.1-4), the transmission for unpolarized monochromatic light is given by

$$T = \frac{1}{2} \phi^2 \frac{\sin^2 X}{X^2} = \frac{1}{2} \frac{\sin^2 \left[\frac{\pi}{2} \sqrt{1 + u^2} \right]}{1 + u^2} \quad (5.1-7)$$

where we used $\phi = \pi/2$ as the twist angle, u is the Mauguin parameter given by

$$u = \frac{\Gamma}{2\phi} = 2d \frac{\Delta n}{\lambda} \quad (5.1-8)$$

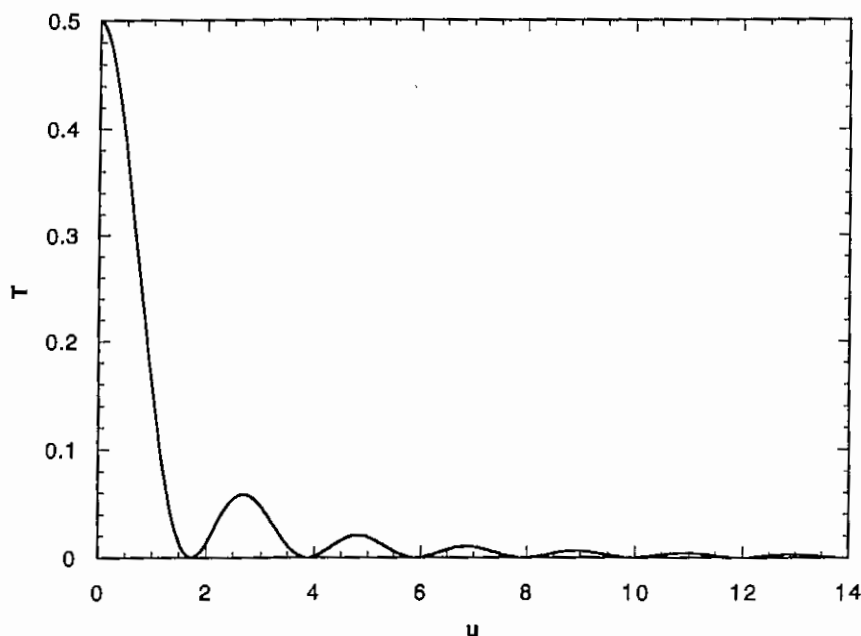


Figure 5.2. Transmission of NB TN displays (T vs. u), with $u = 2d \Delta n / \lambda$.

This equation was first derived by Gooch and Tarry [1]. From Eq. (5.1-7), we note that the transmission is near 0 for liquid crystals with $1 \ll u$, which is equivalent to the Mauguin condition. To illustrate the transmission property, we plot the transmission formula Eq. (5.1-7) as a function of the parameter u (see Fig. 5.2). The transmission is described by the square of a sinc function that consists of a main peak at $u=0$, and a series of side lobes. We note that the oscillation is caused by the $\sin^2$ term in the numerator [Eq. (5.1-7)]. The envelope of the curve decreases with increasing value of u caused by the term $(1 + u^2)$ in the denominator. We also note that the transmission is very low when u is large (i.e., when the Mauguin condition is fulfilled). Zero transmission occurs when the argument of $\sin$ is an integral number of π (i.e., $X = \pi, 2\pi, 3\pi, \dots$). This corresponds to $u = \sqrt{3}, \sqrt{15}, \sqrt{35}, \dots$. These values are known as the first, second, and third minimum conditions. In a normally black (NB) mode, it is desirable to have a zero transmission in the inactivated state (voltage OFF). According to Figure 5.2 [or Eq. (5.1-7)], zero transmission occurs at only one wavelength for a given TN cell. There is a residual transmission for other wavelengths. This is the cause of a undesirable coloration. Maximum transmission of the side lobes occurs approximately at $u = \sqrt{8}, \sqrt{24}, \sqrt{48}, \dots$. We note that the peak of the first side lobe is $\frac{1}{9}$ th of the main peak, the peak of the second side lobe is $\frac{1}{25}$ th of the main peak.

2. Normally White (NW) Mode. In this scenario the 90° TN cell is sandwiched between a pair of crossed polarizers. The director at the input end is parallel to the transmission axis of the polarizer and the director at the exit end is parallel to the transmission axis of the analyzer. According to Eq. (5.1-4), the transmission for unpolarized monochromatic light is given by

$$T = \frac{1}{2} - \frac{1}{2} \frac{\sin^2 \left[\frac{\pi}{2} \sqrt{1 + u^2} \right]}{1 + u^2} \quad (5.1-9)$$

We note that the transmission is complementary to that of the NB mode. For liquid crystals with large u , the second term in Eq. (5.1-9) is very small. This leads to a transmission of approximately $\frac{1}{2}$ for unpolarized light. Figure 5.3 shows a plot T versus u . We note that maximum transmission of $\frac{1}{2}$ occurs when the argument of $\sin$ is an integral multiple of π . This corresponds to $u = \sqrt{3}, \sqrt{15}, \sqrt{35}, \dots$.

Transmitted Luminance

In the discussion above, we note that the transmission of a TN display cell depends on the wavelength of light. For black-and-white displays in the NB mode, it is desirable to select a LC cell with the proper parameter u such that the transmitted luminance is minimized in the black state. The integrated transmitted luminance is obtained by integrating the transmission function $T(\lambda)$ with the photopic response of the human eyes $P(\lambda)$ (see Fig. 5.4) and the

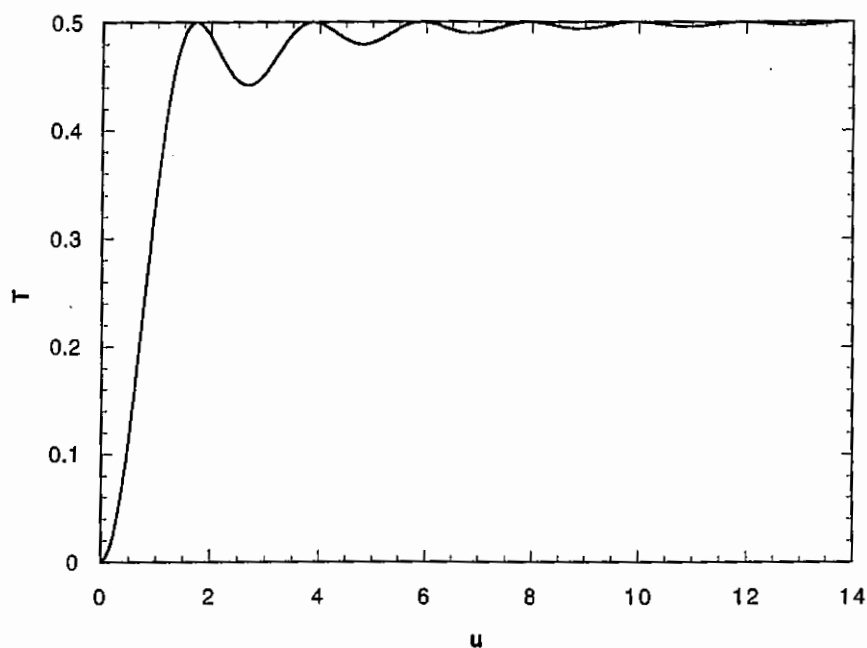


Figure 5.3. Transmission of NW TN displays (T vs. u), with $u = 2d \Delta n / \lambda$.

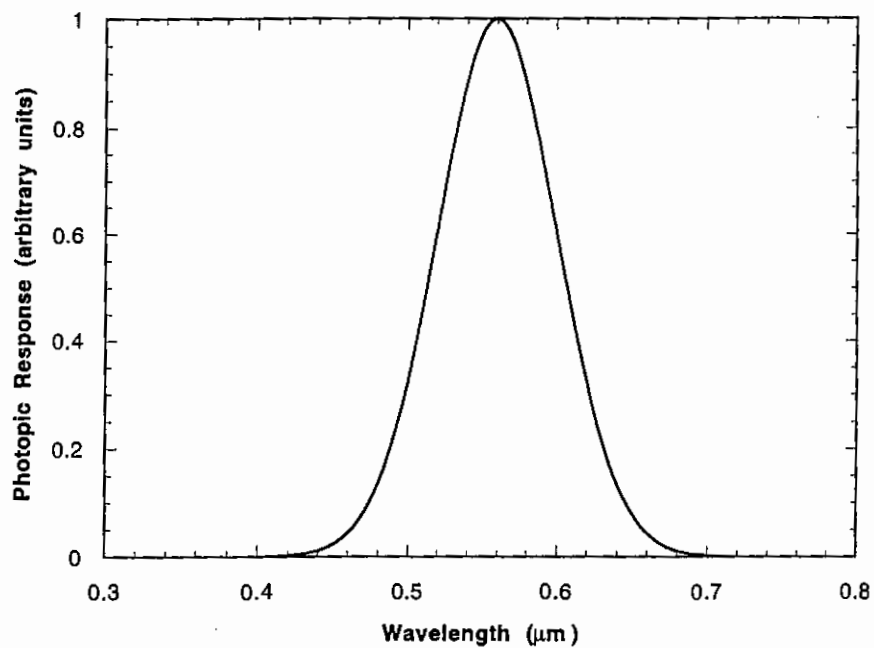


Figure 5.4. The human photopic eye response.

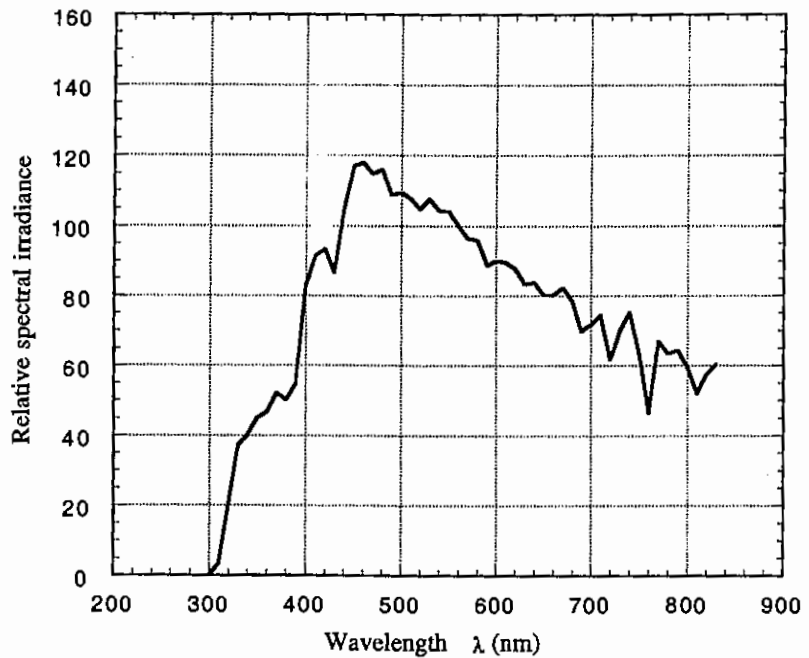


Figure 5.5. Relative spectral irradiance of CIE daylight illuminant D65.

illuminant spectral distribution $D(\lambda)$ as follows:

$$L = \frac{\int_{380}^{780} T(\lambda)D(\lambda)P(\lambda)d\lambda}{\int_{380}^{780} D(\lambda)P(\lambda)d\lambda} \quad (5.1-10)$$

where the limits of integration are from 380 nm (in the ultraviolet) and 780 nm (in the infrared). Using the D65 standard illuminating spectrum (see Fig. 5.5), a standard that approximates the emission characteristics of a blackbody held at 6500 K, we obtain a luminance that exhibits minima at $\Delta nd = 0.48$, 1.09, and 1.68 μm . These values agree very well with the minima at $u = \sqrt{3}$, $\sqrt{15}$, and $\sqrt{35}$ in Figure 5.2 by selecting $\lambda = 0.55 \mu\text{m}$, which is close to the peak of the photopic response.

5.1.3. Transmission Properties of Field-on State

When a small voltage is applied to the electrodes, a strong electric field is established in the LC cell. Recall that the typical cell thickness is in the range of

5–10 μm . So a small voltage of 1.5 V can lead to an electric field of 1.5–3 kV/cm. Under the influence of the electric field, the LC molecules are forced to align parallel to the electric field. On the other hand, the LC molecules at the boundaries must be parallel to the rubbing directions because of a strong molecular interaction with the surfaces. In addition, there is elastic inertia (restoring torque) in liquid crystals to resist a distortion of the director distribution. So the application of an electric voltage can lead to a redistribution of the director in the LC cell. In what follows, we will briefly describe the distribution of the director in a TN cell under the influence of an applied voltage. A detailed discussion can be found in Appendix D.

Field-Induced Director Reorientation

In the field-OFF state, the LC molecules are all aligned parallel to the surfaces of the glass plates. A uniform twist of the directors exists in the cell as a result of the boundary conditions. Recall that the rubbing directions of the inner surfaces of the two glass plates are mutually perpendicular in a 90° TN cell. When a sufficiently high voltage (few volts) is applied, the liquid crystal molecules are tilted toward the direction of the electric field. So in a field-ON state, both tilt angle and twist angle are needed to describe the distribution of the director.

To describe the director distribution, we choose the z axis perpendicular to the cell surfaces. According to Eq. (D-8) in Appendix D, the elastic energy density in a distorted twist cell can be written

$$U_{\text{EL}} = \frac{1}{2} (k_1 \cos^2 \theta + k_3 \sin^2 \theta) \left(\frac{d\theta}{dz} \right)^2 + \frac{1}{2} (k_2 \cos^2 \theta + k_3 \sin^2 \theta) \cos^2 \theta \left(\frac{d\phi}{dz} \right)^2 \quad (5.1-11)$$

where k_1, k_2, k_3 are the elastic constants of the LC material, ϕ is the twist angle, and θ is the tilt angle of the director. The electrostatic energy density can be written

$$U_{\text{EM}} = \frac{1}{2} \mathbf{E} \cdot \mathbf{D} = \frac{1}{2} \frac{D^2}{\epsilon_{\parallel} \sin^2 \theta + \epsilon_{\perp} \cos^2 \theta} \quad (5.1-12)$$

where $\mathbf{D}$ is the displacement field and $\epsilon_{\parallel}, \epsilon_{\perp}$ are the dielectric constants of the LC material.

The final distribution of the director $[\theta(z), \phi(z)]$ as functions of z can be obtained by minimizing the total energy integrated over the LC cell (see Appendix D). This requires the technique of variational calculus.

According to Appendix D, the redistribution of the director in a TN cell requires a *threshold voltage* given by

$$V_T = V_{c1} \left[1 + \frac{\Phi^2}{\pi^2} \left(\frac{k_3 - 2k_2}{k_1} \right) \right]^{1/2} \quad (5.1-13)$$

where Φ is the total twist angle in the field-OFF state and V_{c1} is given by

$$V_{c1} = \pi \sqrt{\frac{k_1}{\Delta\epsilon}} \quad (5.1-14)$$

where $\Delta\epsilon$ is the *dielectric anisotropy* $\Delta\epsilon = \epsilon_{\parallel} - \epsilon_{\perp}$. Physically, V_{c1} is a threshold voltage needed to tilt the LC molecules in a nematic cell with a planar orientation ($\Phi = 0$). The threshold voltage to tilt LC molecules is slightly higher in twisted nematic cells, according to Eq. (5.1-13) provided $0 < k_3 - 2k_2$. For a 90° TN cell, the threshold voltage is given by

$$V_T = V_{c1} \left[1 + \left(\frac{k_3 - 2k_2}{4k_1} \right) \right]^{1/2} \quad (5.1-15)$$

For most nematic mixtures formulated for TN displays, V_T is typically around 1 V.

Example 5.1. Using Table 1.1, we obtain $k_1 = 7.7 \times 10^{-12} \text{ N}$, $k_2 = 4 \times 10^{-12} \text{ N}$, $k_3 = 12.2 \times 10^{-12} \text{ N}$, $\epsilon_{\parallel} = 10.6\epsilon_0$, and $\epsilon_{\perp} = 4.6\epsilon_0$, for ZLI-1646. According to Eqs. (5.1-14,15), we obtain $V_{c1} = 1.196 \text{ V}$ and $V_T = 1.275 \text{ V}$. ■

Using the results described in Appendix D, we can obtain the distribution of the orientation of the director in a TN cell. This also requires numerical procedures. Figure 5.6 shows the tilt $\theta(z)$ and twist $\phi(z)$ angles of the director (optic axis) inside a TN cell filled with a LC mixture of ZLI-1646 for several applied voltages. For this LC mixture, the threshold voltage is around 1.275 V. The director distribution exhibits the following symmetry,

$$\begin{aligned} \theta(z) &= \theta(d - z) \\ \phi(z) + \phi(d - z) &= \frac{\pi}{2}. \end{aligned}$$

We note that the tilt angle is zero at both ends as a result of the boundary conditions. In practice, the inner surfaces of the glass plates are treated so that

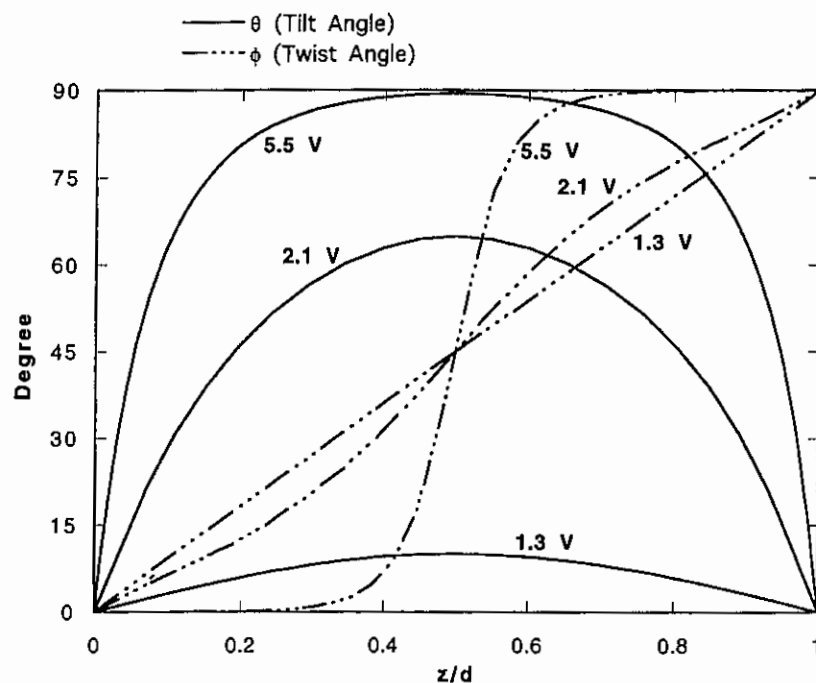


Figure 5.6. Distribution of tilt and twist angles of director, where d = cell gap.

the LC director exhibits a small pretilt angle (about 1°) at the boundaries. As we mentioned before, the pretilt is very important to maintain the distribution stability of the LC cell. According to Eq. (5.1-12), the electrostatic energy is an even function of the tilt angle. When a voltage is applied, the tilt angle for the LC molecules can be either positive or negative. In actual operations without a pretilt, this can lead to a distribution instability in the LC cell. Thus, a small pretilt can facilitate the tilt of the LC molecules when a small voltage is applied. The tilt angle distribution resembles a sinusoidal function at small voltages and becomes squaring off at higher voltages. Because the tilt angle is symmetric about the center of the cell, it is maximum at the center. The midlayer tilt angle increases monotonically as a function of the applied voltage and asymptotically approaches 90° at higher applied voltages. Figure 5.7 shows the midlayer tilt angle as a function of the applied voltage.

The twist angle remains relatively uniform at low voltages (e.g., 1.3 V). Significant changes occur at voltages that cause a midlayer tilt of greater than 45° (e.g., 2.1 V). In the case when the midlayer tilt approaches 90° , the twist angle undergoes a sharp transition at midlayer from 0° to 90° . The twist angle remains parallel to the rubbing directions of the surfaces at both ends. A very fast twisting of 90° occurs within a thin layer at the center of the cell. We also note that the twist angle remains 45° at the center of the cell regardless of the applied

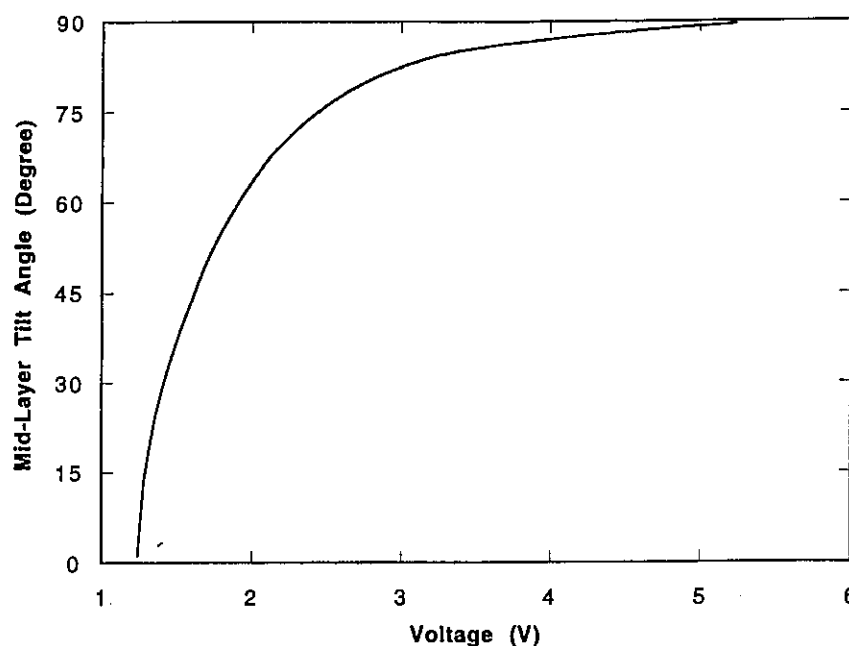


Figure 5.7. The midlayer tilt angle as a function of the applied voltage.

voltages. Table 5.1 lists the tilt and twist angles at 20 equally spaced locations in the LC layer and at the boundaries for three different applied voltages.

Transmission at Normal Incidence

Given the distribution of the tilt and twist angles, we can then use the standard Jones matrix method to calculate the transmission of the TN cell. In the numerical approach, the TN cell is divided into a large number of plates ($N=20-100$) with an identical thickness. Each of the plates is treated as a homogeneous and uniaxially anisotropic crystal plate. In the matrix method, each of the uniaxial crystal plates can be represented by a Jones matrix that transforms an input polarization state into an output polarization state. The overall Jones matrix is then obtained by multiplying all the individual Jones matrices in sequence. In what follows, we discuss the transmission at normal incidence of a 90° TN cell as a function of the applied voltage.

Normally Black Mode. Referring to Figure 5.8, we consider the transmission of a TN cell with $\Delta nd = 0.48 \mu\text{m}$ as a function of the applied voltage. From the discussion above, this thickness corresponds to the first minimum of a normally black TN display for green light at $\lambda = 550 \text{ nm}$ ($u_{\text{green}} \approx \sqrt{3}$). Thus, in the field-OFF state ($V=0$), the transmission for green light at 550 nm is zero. Residual transmission exists for both red and blue (about 0.03). This is a result of an

Table 5.1. Tilt and Twist Angles versus z where d is the LC Cell Gap

z/d	$V = 1.29 \text{ V}$		$V = 2.10 \text{ V}$		$V = 5.49 \text{ V}$	
	$\theta(^{\circ})$	$\phi(^{\circ})$	$\theta(^{\circ})$	$\phi(^{\circ})$	$\theta(^{\circ})$	$\phi(^{\circ})$
0.000	0.0	0.0	0.0	0.0	0.0	0.0
0.025	0.8	2.3	7.5	1.6	24.6	0.0
0.075	2.3	6.8	22.3	4.8	54.7	0.0
0.125	3.8	11.4	33.7	7.8	69.4	0.0
0.175	5.2	15.9	42.5	10.9	77.9	0.1
0.225	6.5	20.5	49.4	14.4	82.8	0.2
0.275	7.6	25.0	54.7	18.3	85.4	0.6
0.325	8.5	29.4	58.8	23.0	87.2	2.3
0.375	9.2	33.9	61.7	28.5	88.3	4.9
0.425	9.7	38.3	63.7	34.6	89.0	16.9
0.475	9.9	42.8	64.7	41.4	89.3	35.6
0.525	9.9	47.2	64.7	48.6	89.3	54.4
0.575	9.7	51.7	63.7	55.4	89.0	73.1
0.625	9.2	56.1	61.7	61.5	88.3	85.1
0.675	8.5	60.6	58.8	67.0	87.2	87.7
0.725	7.6	65.0	54.7	71.7	85.4	89.4
0.775	6.5	69.5	49.4	75.6	82.8	89.8
0.825	5.2	74.1	42.5	79.1	77.9	89.9
0.875	3.8	78.6	33.7	82.2	69.4	90.0
0.925	2.3	83.2	22.3	85.2	54.7	90.0
0.975	0.8	87.7	7.5	88.4	24.6	90.0
1.000	0.0	90.0	0.0	90.0	0.0	90.0

overshoot for blue light ($u_{\text{blue}} > \sqrt{3}$) and a undershoot for red light ($u_{\text{red}} < \sqrt{3}$) due to the $1/\lambda$ dependence. When the field is turned ON, the transmission remains the same for applied voltages between 0 and the threshold voltage V_T . As we know, the LC director remains unchanged for voltages under the threshold voltage. As the applied voltage increases beyond the threshold voltage V_T , the LC molecules are tilted toward the direction of the electric field. This effectively decreases the phase retardation leading to an increase in the transmission. For red light, the tilt of the directors leads to a further decrease of u_{red} below $\sqrt{3}$, leading to an increase of the transmission. For blue light, the parameter u_{blue} passes through $\sqrt{3}$ as the voltage increases, leading to a zero transmission at a voltage of around 1.5 V. Beyond this voltage, the tilt angle continues to increase as the applied voltage increases, leading to a further decrease of the effective phase retardation and the value of u_{blue} . This leads to an increase of transmission as a function of the voltage. At a very large applied voltage, all the LC molecules are practically aligned parallel to the electric field. This resembles the situation of a c plate sandwiched between parallel polarizers. Thus, all three colors of light are transmitted completely.

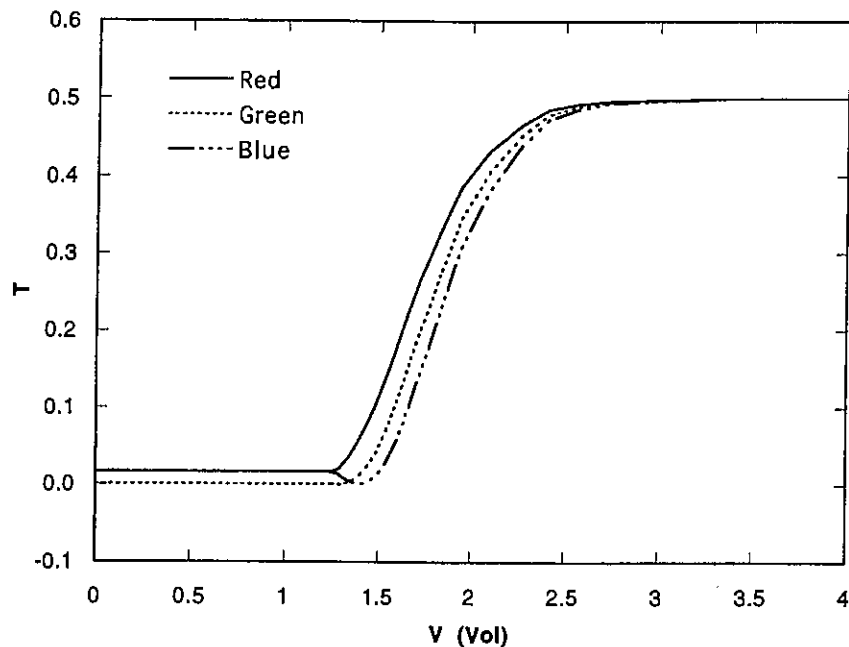


Figure 5.8. Transmission of a TN cell in the NB mode as a function of the applied voltage.

If we were to pick a TN cell with $\Delta nd = 1.09 \mu\text{m}$, the electrooptical transmission curves would be slightly different. A TN cell with $\Delta nd = 1.09 \mu\text{m}$ corresponds to the second minimum of a normally black TN display for green light at $\lambda = 550 \text{ nm}$ ($u_{\text{green}} \approx \sqrt{15}$). The transmission for the green light is again zero at 0 voltage. As the applied voltage increases, the transmission of the green light will first increase when the voltage is greater than the threshold voltage as the parameter u_{green} decreases. The transmission will then decrease and become zero when the parameter reaches $u_{\text{green}} = \sqrt{3}$. Beyond this point, the transmission will increase as the applied voltage continues to increase. For blue light, the transmission curves will dip to zero 2 times, as the parameter passes through $u_{\text{blue}} = \sqrt{3}$ and $u_{\text{blue}} = \sqrt{15}$. For red light, the transmission curves will dip to zero only once, as the parameter passes through $u_{\text{red}} = \sqrt{3}$.

Normally White (NW) Mode. In this mode, the TN cell is sandwiched between a pair of crossed polarizers. Assuming ideal polarizers, the electrooptical transmission curves are complementary to those of the NB mode. Figure 5.9 shows the electrooptical transmission curves for the NW modes of operation.

In designing TN-LC cells for display application, it is desirable to select cells with $\Delta n = 0.48 \mu\text{m}$ ($u = \sqrt{3}$) to minimize the dependence of the transmission on λ . The minimum cell gap also ensures a larger viewing angle.

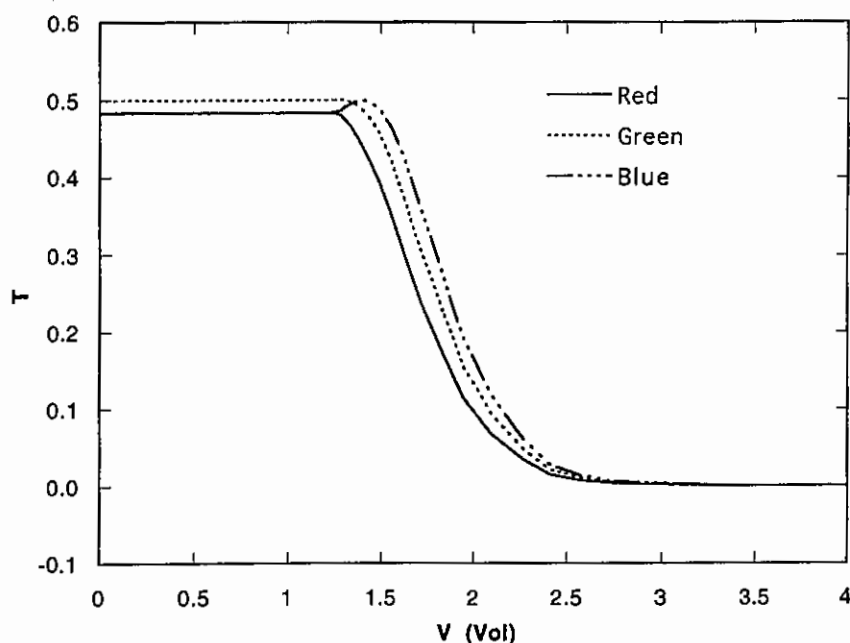


Figure 5.9. Transmission of a TN cell in the NW mode as a function of the applied voltage.

Contrast at Normal Incidence

We now discuss the contrast of the TN displays. The contrast of any display system is defined as the ratio of the transmission of the bright state (high transmission) to that of the dark state (low or zero transmission). Since the transmission depends on the applied voltage, the contrast is also a function of the applied voltage. For normally black mode, the contrast is defined as

$$C_{NB} = \frac{T_{NB}(V)}{T_{NB}(0)} \quad (5.1-16)$$

where T_{NB} is the transmission of the TN cell. Similarly, the contrast of a normally white mode is defined as

$$C_{NW} = \frac{T_{NW}(0)}{T_{NW}(V)} \quad (5.1-17)$$

where T_{NW} is the transmission of the TN cell. On the basis of the complementary electrooptical transmission of these two modes, C_{NW} can also be written as

$$C_{NW} = \frac{T_{NW}(0)}{T_{NW}(V)} = \frac{0.5 - T_{NB}(0)}{0.5 - T_{NB}(V)} \quad (5.1-18)$$

Let the residual transmission leakage be written

$$T_{NB}(0) = T_0 \quad (5.1-19)$$

where T_0 is a small number in the range of 0.0–0.05, depending on whether T_{NB} is the integrated transmission over all wavelengths, or the transmission for an individual wavelength. Also, for the purpose of discussion, let

$$T_{NB}(V) = T_1 \quad (5.1-20)$$

where T_1 can vary from T_0 to 0.5 as the applied voltage increases. Using T_1 and T_0 , the contrasts can be written

$$C_{NB} = \frac{T_{NB}(V)}{T_{NB}(0)} = \frac{T_1}{T_0} \quad (5.1-21)$$

$$C_{NW} = \frac{T_{NW}(0)}{T_{NW}(V)} = \frac{0.5 - T_0}{0.5 - T_1} \quad (5.1-22)$$

We note that both contrasts are monotonically increasing functions of the applied voltage. As $T_{NB}(V)$ reaches the maximum value of 0.5 at high applied voltages, we find that the contrast of the normally black mode is primarily limited by the leakage $T_{NB}(0)$ (residual transmission) because of the slight ellipticity of the output polarization state, according to Eq. (5.1-21). A contrast ratio of 100 : 1 can be easily achieved in this mode. In the normally white mode of operation, the contrast ratio can be much higher at high applied voltages as the denominator approaches zero, according to Eq. (5.1-22). As we know, at high applied voltages, all LC molecules are aligned parallel to the electric field, the TN cell becomes a homeotropically aligned cell. In practice, the leakage of light $T_{NW}(V)$ at this state is limited by the extinction ratio of the crossed polarizers. Contrast ratio of 1000 : 1 can be achieved with the use of sheet polarizers.

Assuming $T_0 = 0.0005$, we find that the contrasts are 1 at zero voltage. As the applied voltage increases, T_1 increases from 0.0005 to 0.5. At high applied voltages, $T_1 = 0.5$, the contrast C_{NB} reaches 1000, whereas C_{NW} reaches infinity as the denominator approaches zero asymptotically. Equal contrast of these modes occurs at $T_1 = 0.5 - T_0$, which corresponds to transmission of $T_{NB}(V) = 0.4995$. Based on the electrooptical transmission curves discussed above, this requires an applied voltage of about 3.3 V. So for operations below 3.3 V in this example, the normally black mode exhibits a higher contrast ratio. Figure 5.10 shows the calculated contrast ratios for these two modes of operation for green light. Since the cell is designed for minimum leakage at green light, the contrast is very good. The contrast ratio for the integrated luminance follows a similar pattern, with a lower value of the maximum contrast.

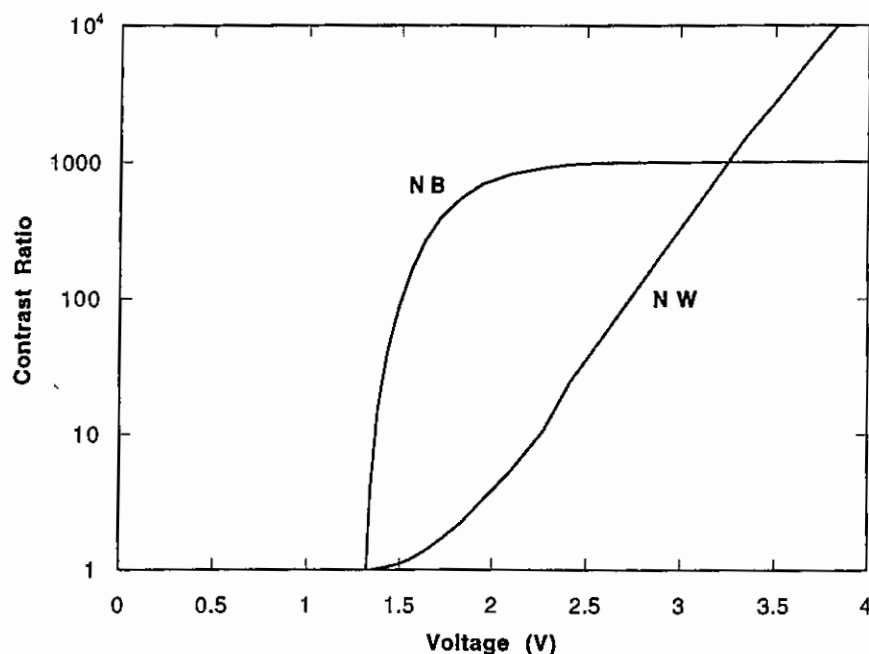


Figure 5.10. Contrast ratio as a function of the applied voltage.

Viewing Angles

In the preceding discussion, we considered only the transmission properties of TN-LCD at normal incidence. The Jones matrix method can also be employed to investigate the transmission properties at a general angle of incidence. As we know, the phase retardation depends on the angle of incidence. The transmission is now a function of several variables, including the angles of incidence θ , ϕ , wavelength λ , and the applied voltage V :

$$T = T(\theta, \phi, \lambda, V) \quad (5.1-23)$$

The angles of incidence are defined such that θ is the angle between the incidence direction and the normal vector to the surface, ϕ is the azimuth angle of the incidence direction. For the azimuthal angles we use the convention that the director at the entrance face is oriented at $\phi = 45^\circ$ and the director at the exit face is oriented at $\phi = 135^\circ$. We further assume that the transmission axes of the polarizers for the normally white mode are perpendicular to the LC alignment directions on the surface of the adjacent substrates (*O* mode; see Fig. 4.6b). In calculating the transmission for off-axis light using the Jones matrix method, it is important to use the phase retardation $\Gamma(\theta, \phi, \lambda)$ for off-axis incidence (Eq. (4.4-2)), according to the discussion in Chapter 4. More accurate results can be calculated using the extended Jones matrix method (Chapter 8). For a given

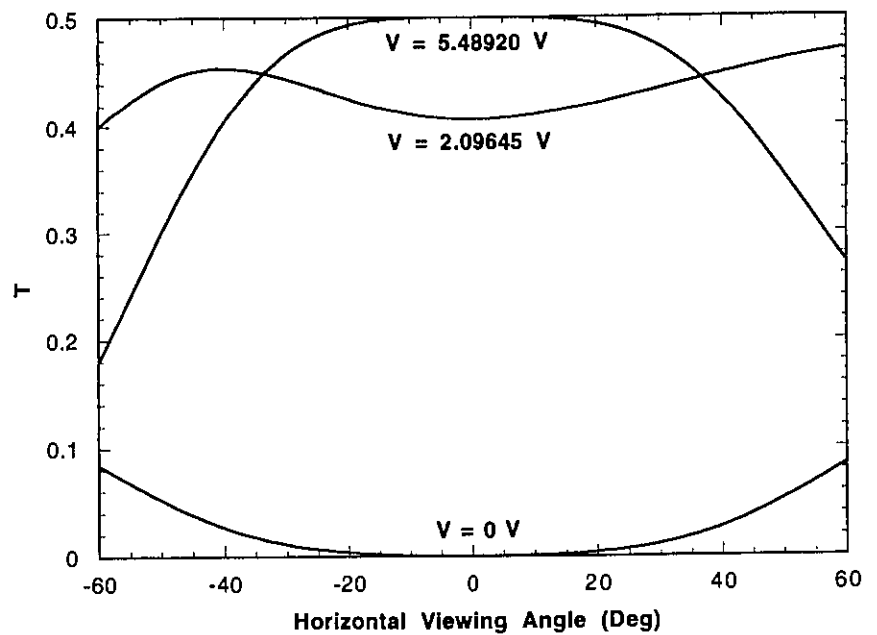
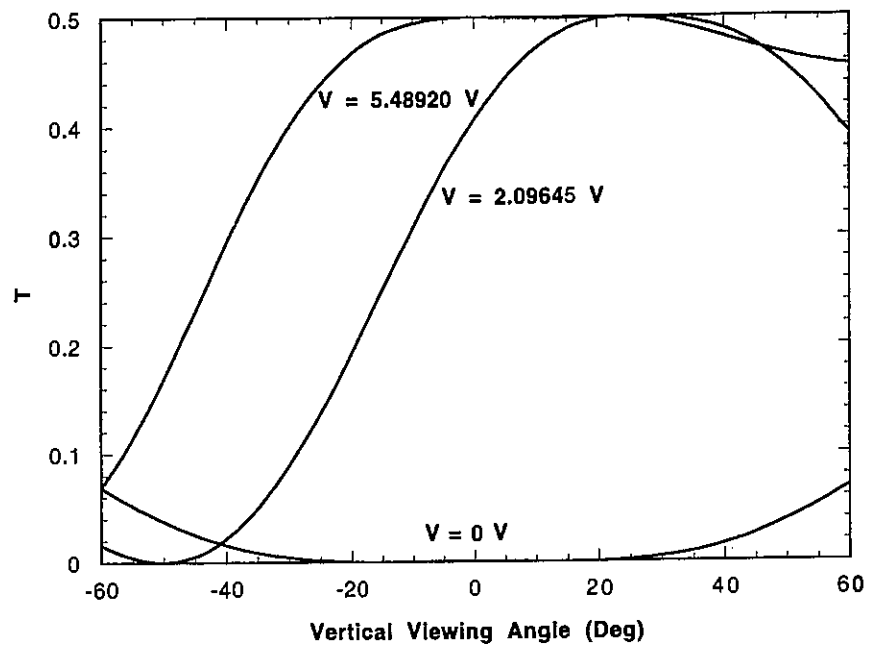


Figure 5.11. (a) Horizontal and vertical viewing characteristics of a NB TN-LCD at three different applied voltages ($V=0, 2.10, 5.49 \text{ V}$).

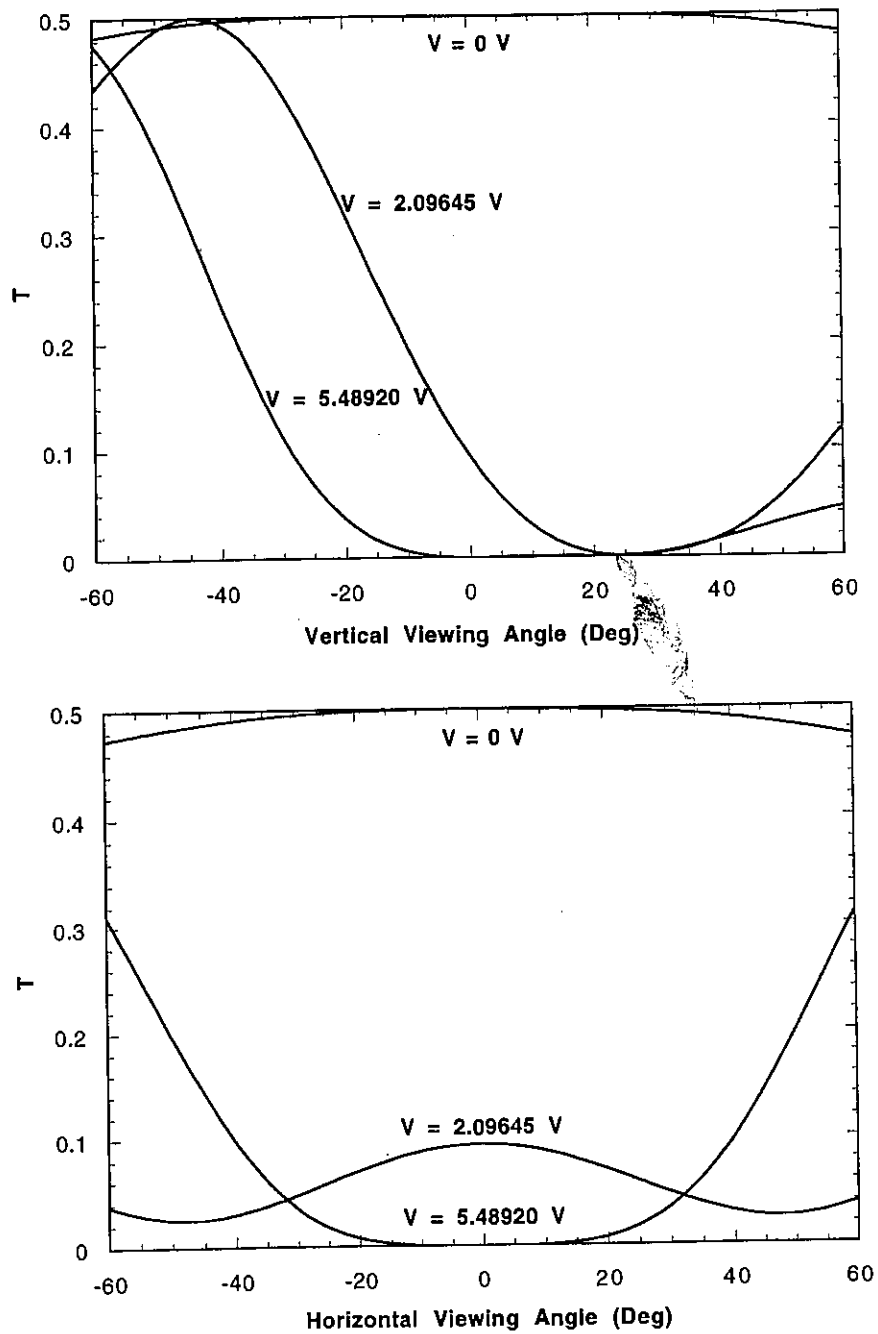


Figure 5.11. (b) Horizontal and vertical viewing characteristics of a NW TN-LCD at three different applied voltages ($V=0$, 2.10, 5.49 V).

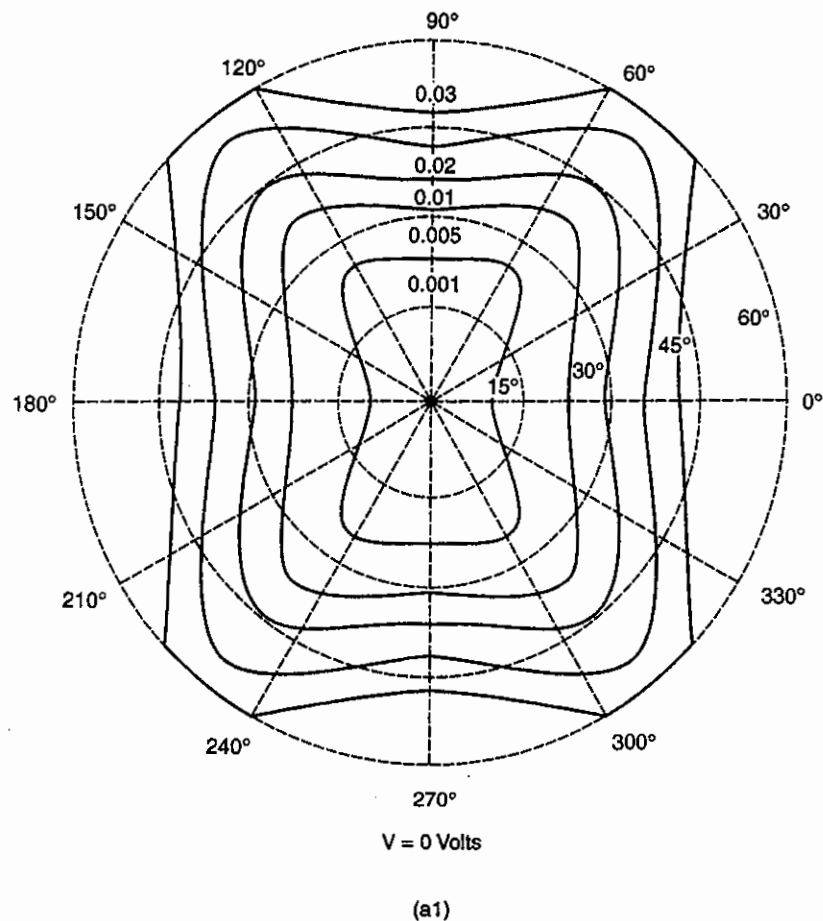


Figure 5.12. (a) Isotransmittance viewing diagrams of a NB TN-LCD at three different applied voltages ($V = 0, 2.10, 5.49$ V).

wavelength λ and an applied voltage V , it is possible to display the transmission $T(\theta, \phi)$ in polar diagrams by using either density plots or contour plots (curves of equal transmission). In the polar diagrams, each point corresponds to a certain viewing angle (θ, ϕ) . The center corresponds to normal incidence (on-axis viewing) with $\theta = 0$ and $\phi = 0$. The viewing angle characteristics can also be displayed by plotting $T(\theta, \phi)$ for vertical viewing ($\phi = \pi/2$) or horizontal viewing ($\phi = 0$) as a function of θ .

To illustrate the angular viewing properties of LCD, we consider the case of a TN cell with $\Delta nd = 0.48 \mu\text{m}$. Figure 5.11 shows the horizontal and vertical viewing characteristics of the TN-LCD at three different voltages. In the normally black (NB) mode, the LCD is in a black state when the applied voltage is zero. Referring to Figure 5.11a, we note that the transmittance is indeed zero at normal incidence. In the horizontal viewing, we note that leakage occurs for

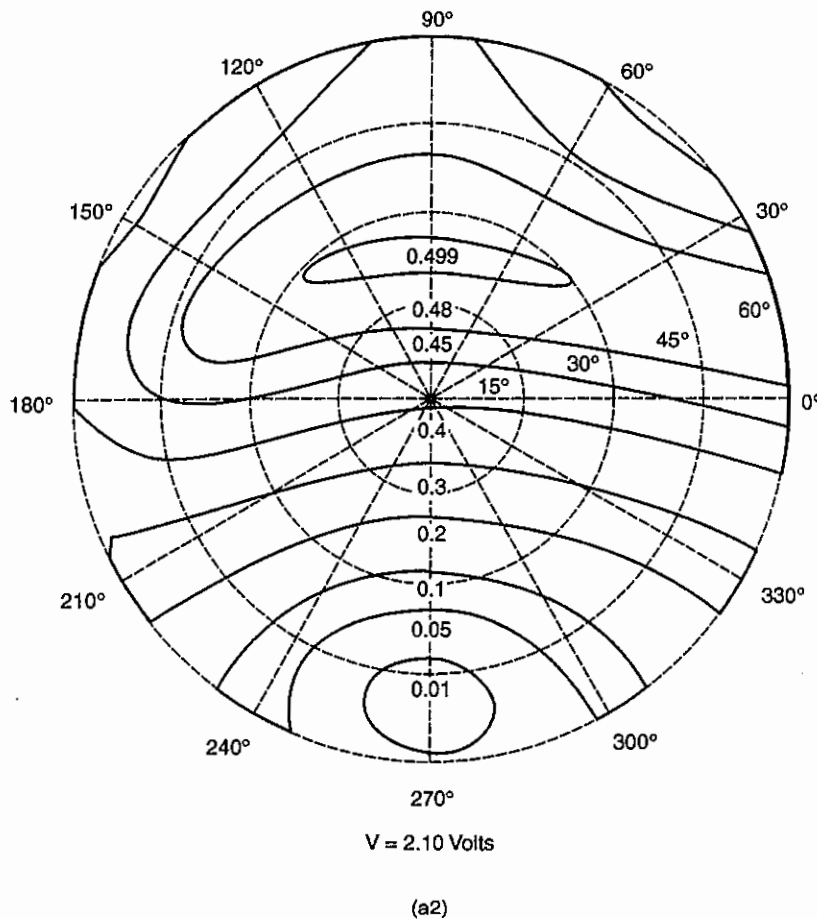


Figure 5.12. (Continued)

viewing angles greater than 20° . The leakage becomes severe at large viewing angles. In the vertical viewing, similar leakage occurs at large viewing angles. As we increase the applied voltage, the transmittance increases with the applied voltage. This is true for normal incidence as shown in Figure 5.11a. We note that the transmittance again depends on the angle of viewing. The variation of the transmittance with the viewing angle may degrade the performance of the display. An ideal LCD would have a transmittance that is insensitive to the variation of the viewing angles, and the transmittance varies monotonically with the applied voltage at all angles of viewing. Figure 5.11b shows the same viewing angle characteristics of a normally white TN-LCD. We note that the transmittance curves of NW TN-LCDs exhibit a complete left-right viewing symmetry. The symmetry is a result of the symmetry of the LCD system which exhibits a 180° rotation around the x axis (see Fig. 5.1a and Problem 5.10). It is important to note that normally black TN-LCDs, however, do not exhibit an

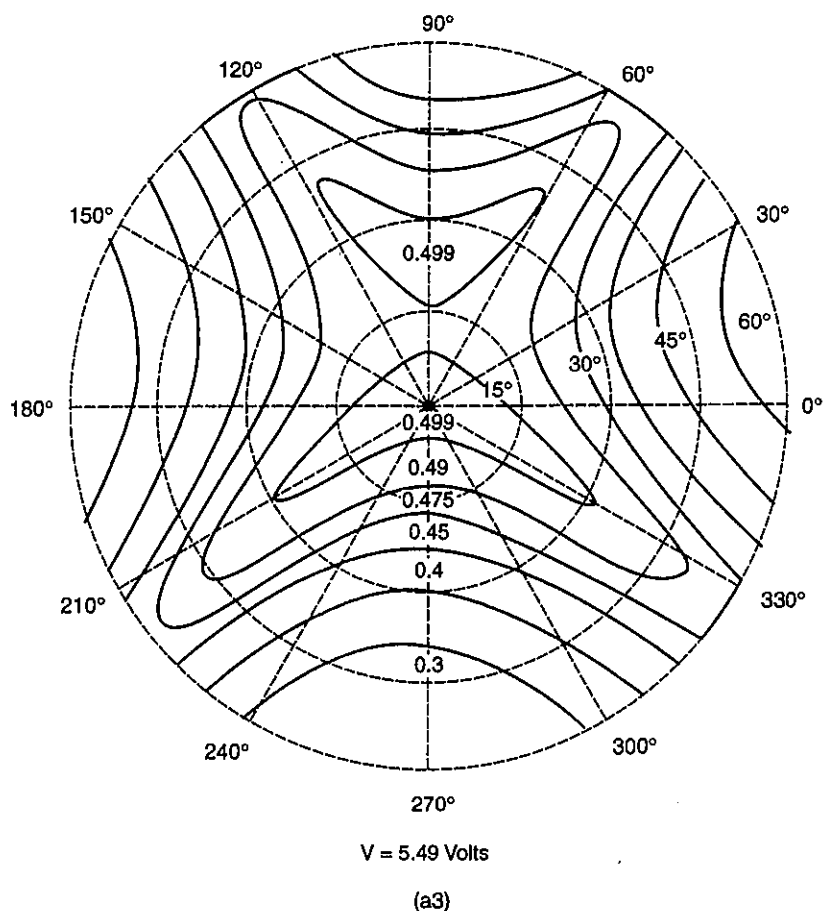


Figure 5.12. (Continued)

exact left-right viewing symmetry. The issue of viewing angle characteristics will be further discussed in Chapter 9.

As mentioned earlier, the viewing angle characteristics of TN-LCDs can also be presented by using iso-transmittance contours in polar diagrams. This is illustrated in Figure 5.12 with three different applied voltages. These curves are contours of equal transmittance in the polar coordinate. As an example, we now examine the iso transmittance curves of a NB TN-LCD as shown in Figure 5.12a. In the black state when the applied voltage is zero, we note that the transmission exhibits an approximate left-right viewing symmetry and up-down viewing symmetry. It can be shown that the approximate symmetry disappears at large viewing angles. The leakage of light in the black state is illustrated by the contours of equal transmittance at $T = 0.001, 0.005, 0.01, 0.02$, and 0.03 . We note that the angular window of viewing with a leakage of < 0.005 covers a range of -20° to $+20^\circ$ in the horizontal direction and -30° to $+30^\circ$

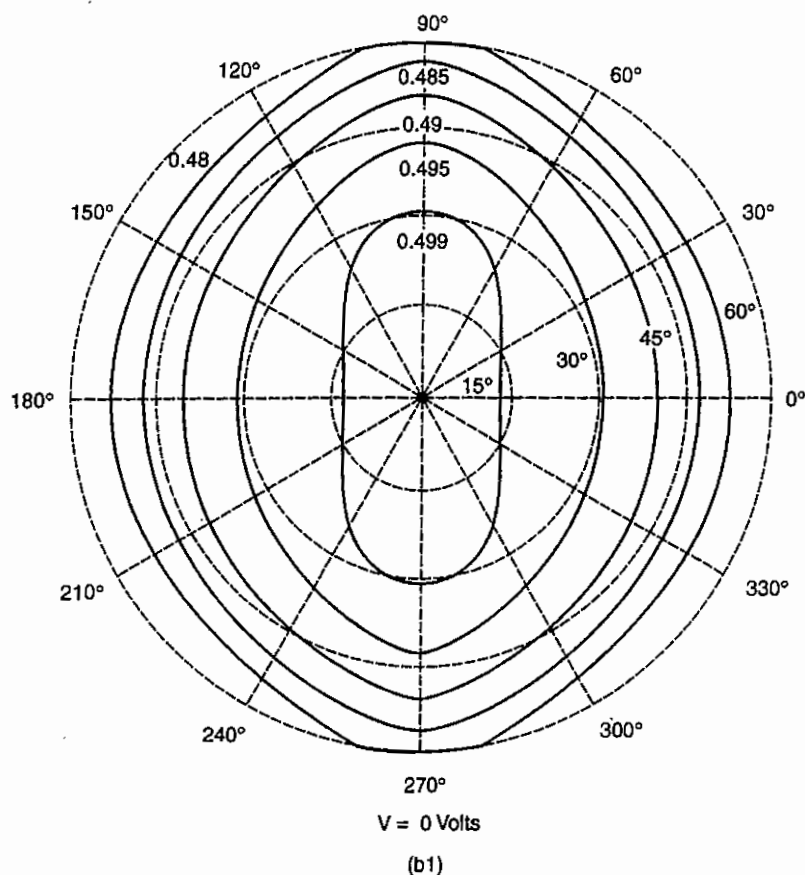


Figure 5.12. (b) Isotransmittance viewing diagrams of a NW TN-LCD at three different applied voltages ($V = 0, 2.10, 5.49$ V).

in the vertical direction. At an applied voltage of 2.1 V, the approximate viewing symmetries disappear. The TN-LCD exhibits a brighter transmission in the upper viewing directions. This is in agreement with the vertical viewing characteristics shown in Figure 5.11a. At an higher applied voltage of 5.49 V, the transmission of the LCD is higher at most angles of viewing as expected. However, the angular dependence still exists, as illustrated by the contours. It is important to note that the isotransmittance contours in the polar diagrams reveal more information about the viewing angle characteristics as compared with the simple vertical and horizontal plots. On the other hand, the simple vertical and horizontal plots are easier in recognizing the angular variation of the transmittance function. Figure 5.12b shows isotransmittance contours in polar diagrams for the normally white mode of the TN-LCD. Here, we note that the isotransmittance curves exhibit an exact left-right viewing symmetry at all voltages. The transmittance curves in Figures 5.11 and 5.12 are obtained by using the extended Jones matrix method, which will be discussed in Chapter 8.

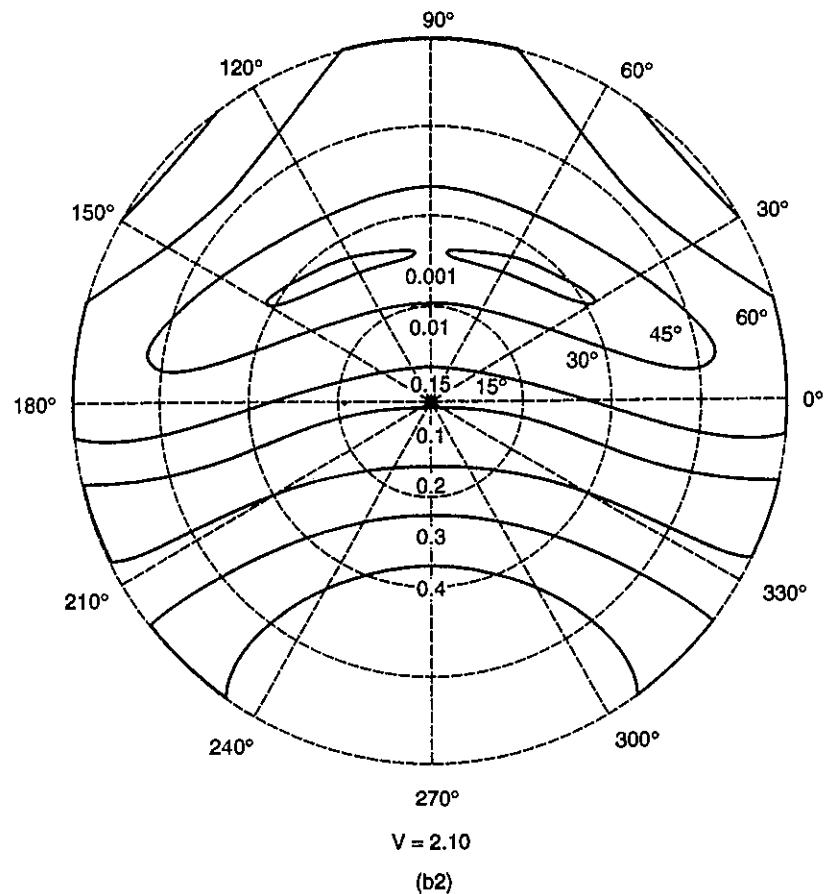


Figure 5.12. (Continued)

As we know, the standard Jones matrix method is no longer adequate for the range of viewing angles shown in the figures.

The issue of contrast was discussed in the section entitled "Contrast at Normal Incidence" for normal incidence only. Since the transmittance is a function of $(\theta, \phi, \lambda, V)$, the contrast ratio is, strictly speaking, also a function of $(\theta, \phi, \lambda, V)$. We first investigate the dependence of the contrast ratio on (θ, ϕ) for a TN-LCD illuminated with a monochromatic source (e.g., a laser beam). The contrast ratio of a TN-LCD can be defined as

$$C_{NB}(\theta, \phi, \lambda, V) = \frac{T(\theta, \phi, \lambda, V)}{T(\theta, \phi, \lambda, 0)} \quad (5.1-24)$$

$$C_{NW}(\theta, \phi, \lambda, V) = \frac{T(\theta, \phi, \lambda, 0)}{T(\theta, \phi, \lambda, V)} \quad (5.1-25)$$

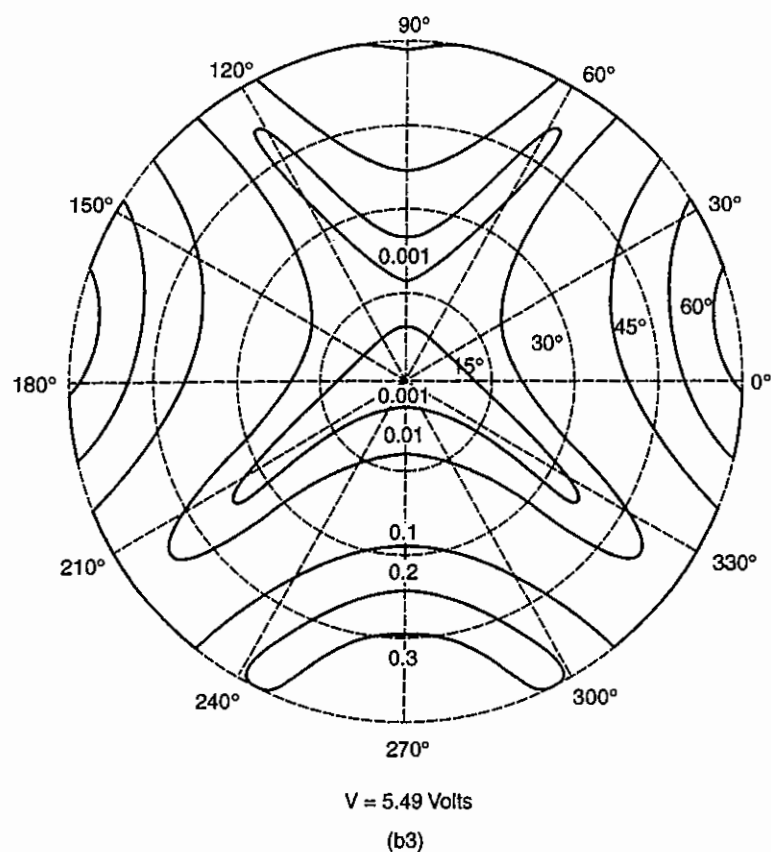


Figure 5.12. (Continued)

where NB stands for normally black and NW denotes normally white and V is the voltage of operation. The dependence on the applied voltage V was discussed earlier. We now focus our attention on the dependence on the viewing angles (θ, ϕ) at a given wavelength of illumination λ . Figure 5.13 and 5.14 show the angular dependence of the contrast ratio for a beam of monochromatic green light. As we know, the transmittance of a TN-LCD depends on the wavelength λ . It is important to note that the contrast ratio also depends on the wavelength of illumination.

Figure 5.13a shows the contrast ratio as a function of the horizontal and vertical viewing angle θ for a normally black mode of TN-LCD. In the horizontal viewing, we note that the contrast ratio is a symmetric function of θ with a peak of around 1000 at normal incidence. The contrast ratio drops monotonically as a function of θ , reaching ~ 50 for viewing angles over 30° . In the vertical viewing, the contrast ratio is no longer a symmetric function of θ . Sharp peaks of high

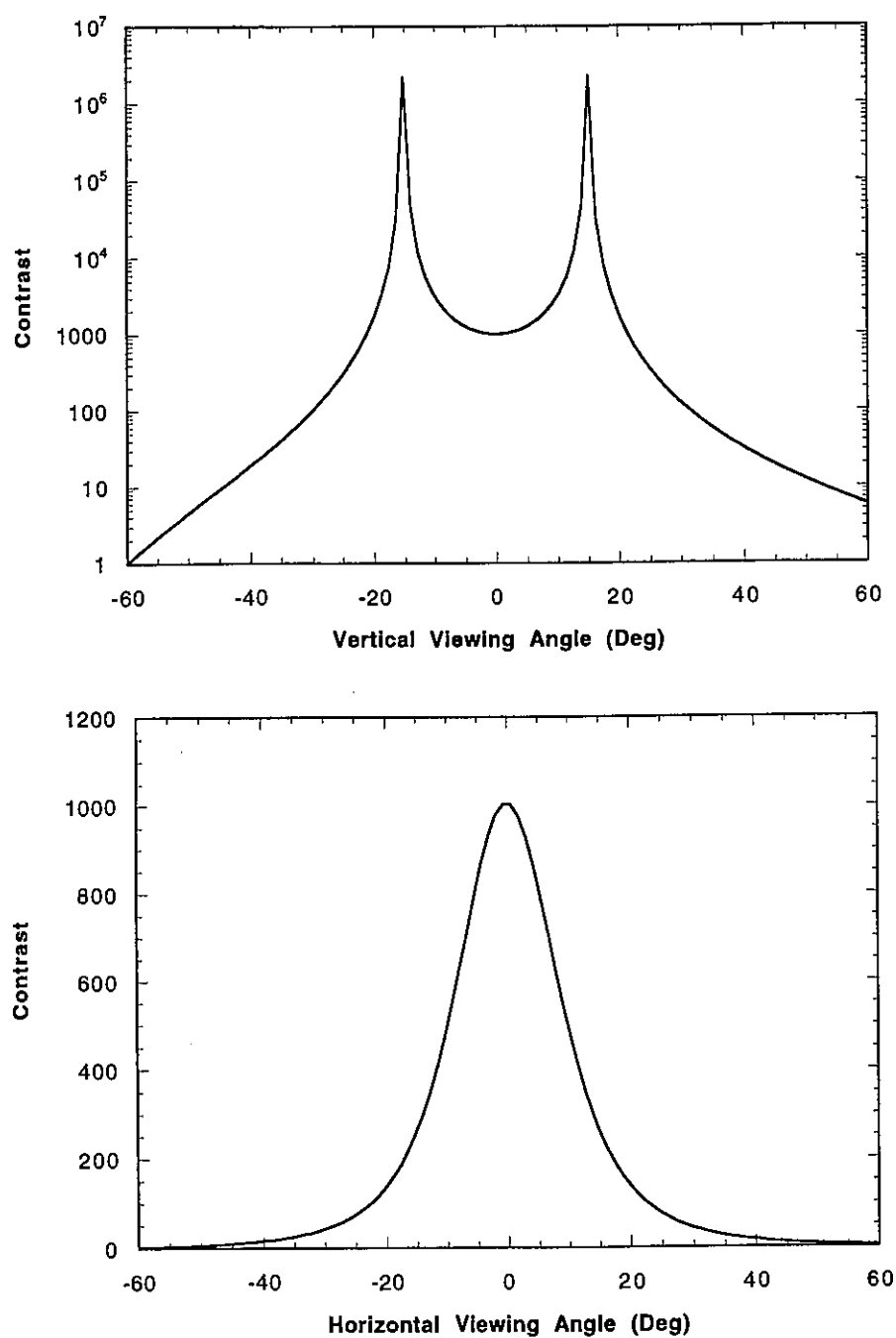


Figure 5.13. (a) Horizontal and vertical viewing contrast of a NB TN-LCD at $V = 5.49$ V and $\lambda = \text{green}$.

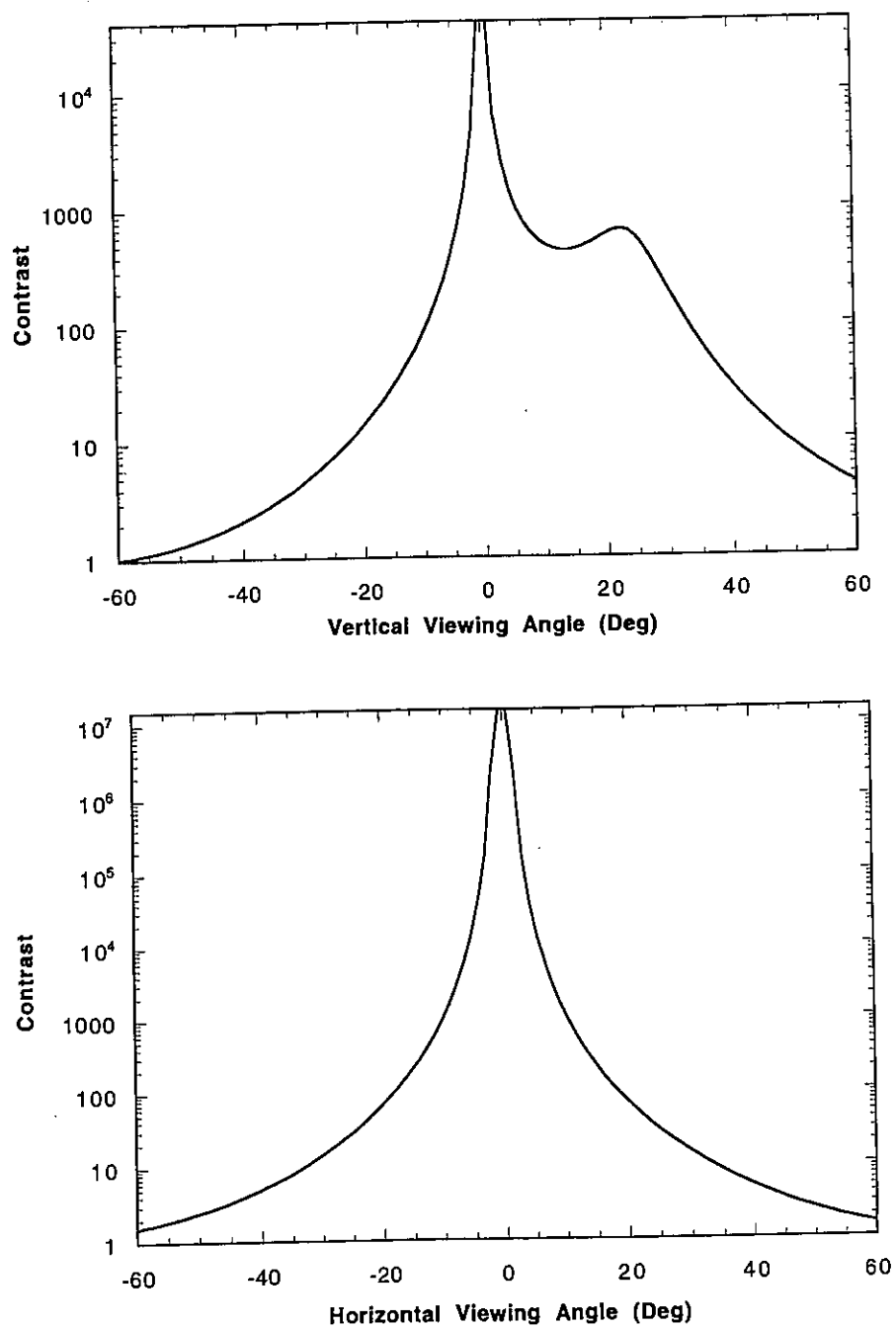


Figure 5.13. (b) Horizontal and vertical viewing contrast of a NW TN-LCD at $V=5.49$ V and $\lambda = \text{green}$.

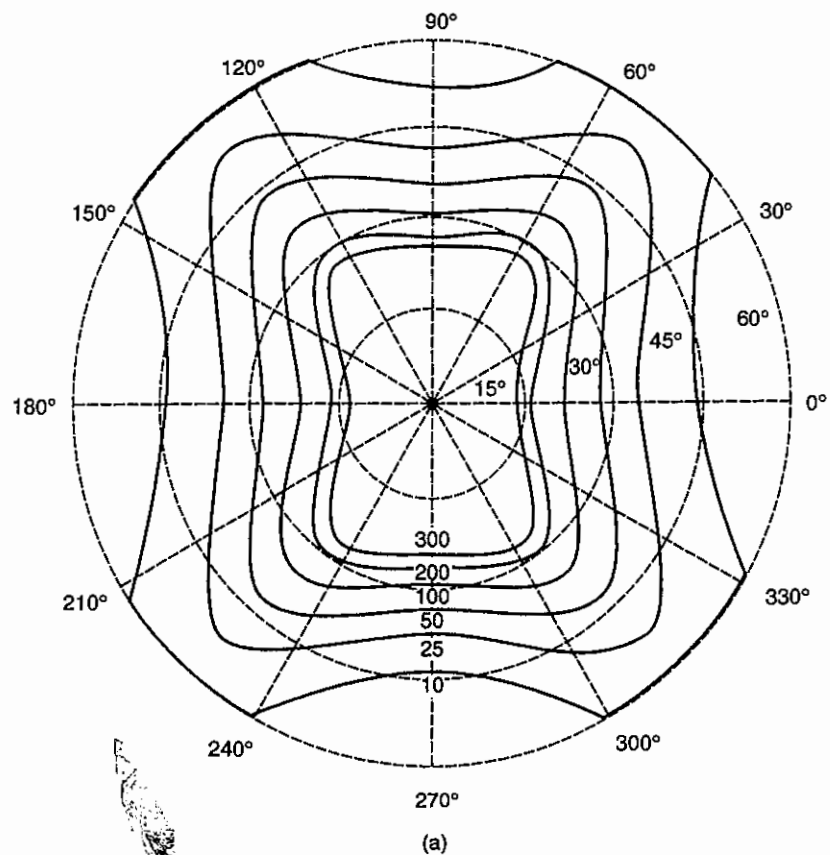


Figure 5.14. (a) Isocontrast viewing diagrams of a NB TN-LCD at $V = 5.49$ V and $\lambda = \text{green}$.

contrast ratio exist at some particular viewing angles. The contrast ratio drops to less than 100 for viewing angles over 30° . It is important to note that the presence of the peaks of high contrast is a result of the monochromatic illumination, which can lead to a near zero transmission of the black state at some particular angle of incidence. Figure 5.13b shows the contrast ratio as a function of the horizontal and vertical viewing angle θ for a normally white mode of TN-LCD. Generally speaking, these two modes of operation (NB and NW) have different contrast ratios. As most of the display systems are designed to provide the optimum viewing at normal incidence, the contrast ratio of both modes of operation drops as a function of the viewing angle θ . We note that the high contrast ratio at normal incidence is a result of the monochromatic illumination. The contrast ratio as a function of the viewing angles (θ, ϕ) can also be displayed by using isocontrast contours in the polar diagrams. Figure 5.14 shows the isocontrast viewing diagrams of the TN-LCD with $V = 5.49$ V and $\lambda = \lambda_{\text{green}}$. Each contour defines a field of viewing with a given contrast ratio.

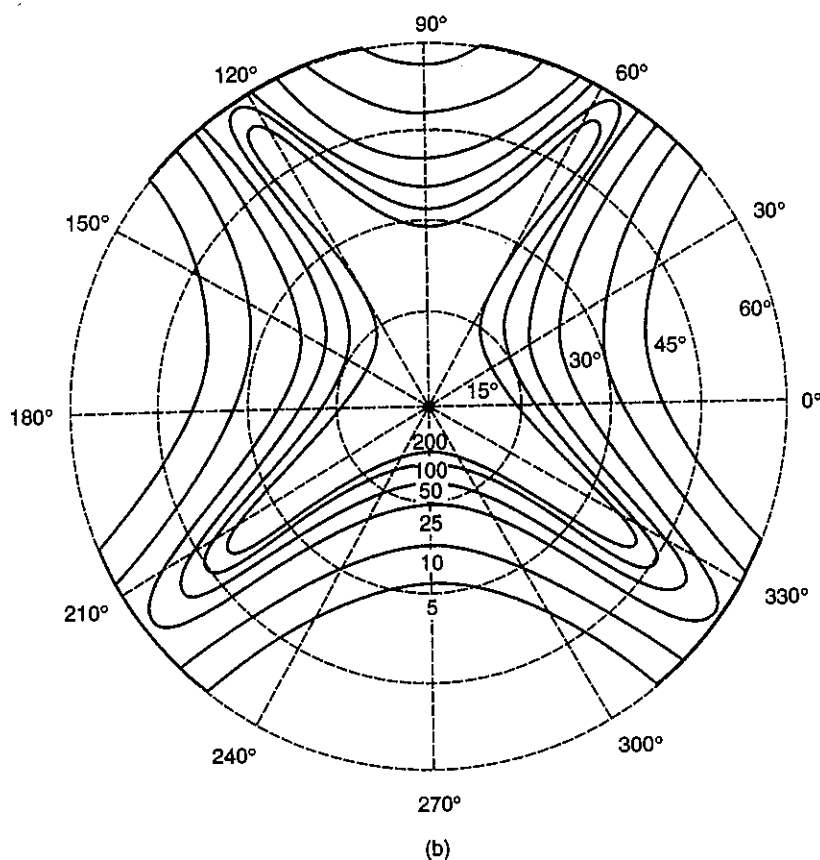


Figure 5.14. (b) Isocontrast viewing diagrams of a NW TN-LCD at $V = 5.49$ V and $\lambda = \text{green}$.

In the discussion above, we described the transmission properties at large viewing angles by using only one color (e.g., green). For displays with white light illumination, the integrated luminance given by Eq. (5.1-10) is the most important parameter. Using the integrated luminance, the contrasts are given by

$$C_{NB}(\theta, \phi, V) = \frac{L(\theta, \phi, V)}{L(\theta, \phi, 0)} \quad (5.1-26)$$

$$C_{NW}(\theta, \phi, V) = \frac{L(\theta, \phi, 0)}{L(\theta, \phi, V)} \quad (5.1-27)$$

where the luminance $L(\theta, \phi, V)$ is obtained by integrating the transmittance function $T(\theta, \phi, \lambda, V)$ over all wavelengths using Eq. (5.1-10). Figures 5.15 and 5.16 describe the isocontrast curves in the viewing plane. Each data point in Figure 5.15 is obtained by integrating over 50 wavelengths from 380 to 780 nm

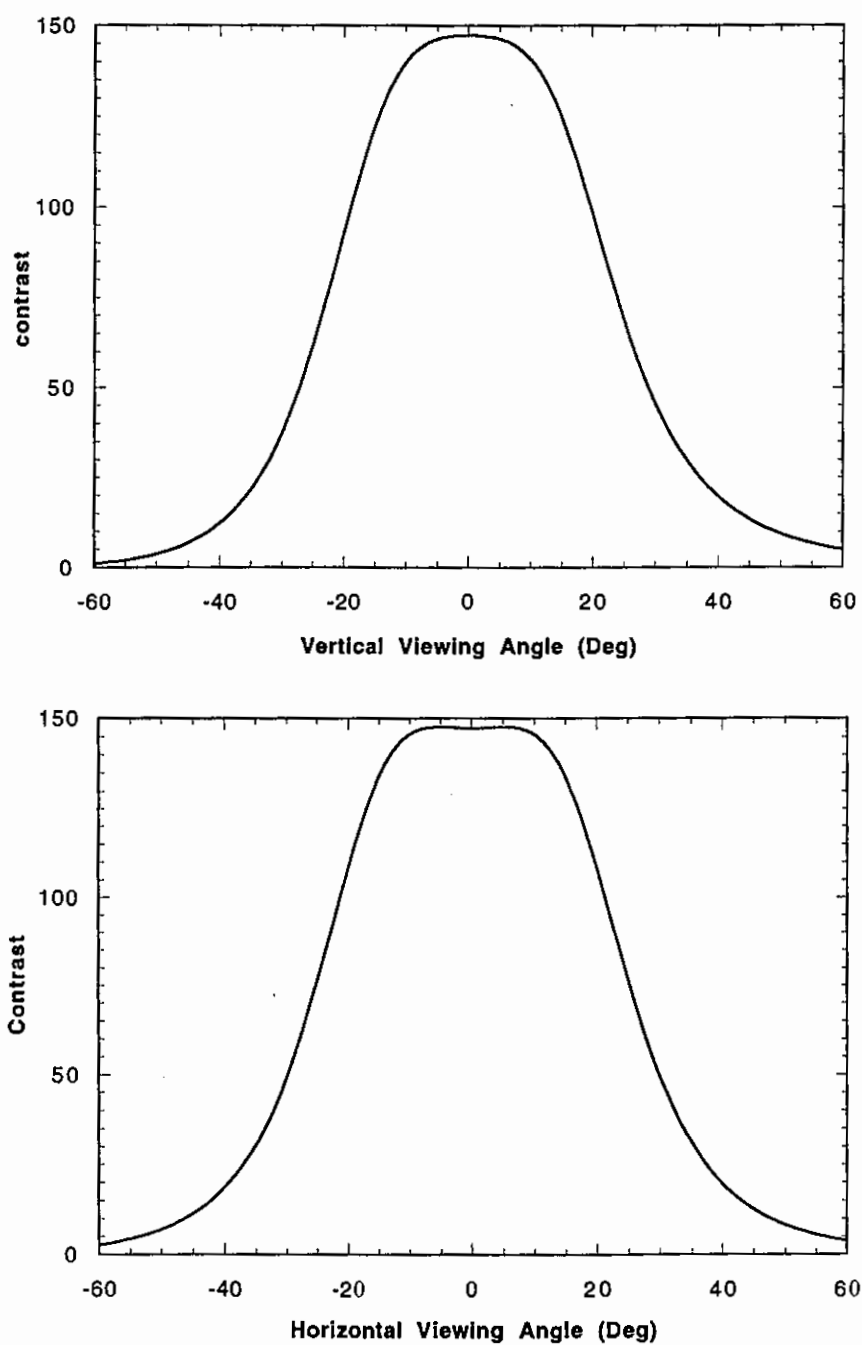


Figure 5.15. (a) Horizontal and vertical viewing contrast of a NB TN-LCD at $V = 5.49$ V.

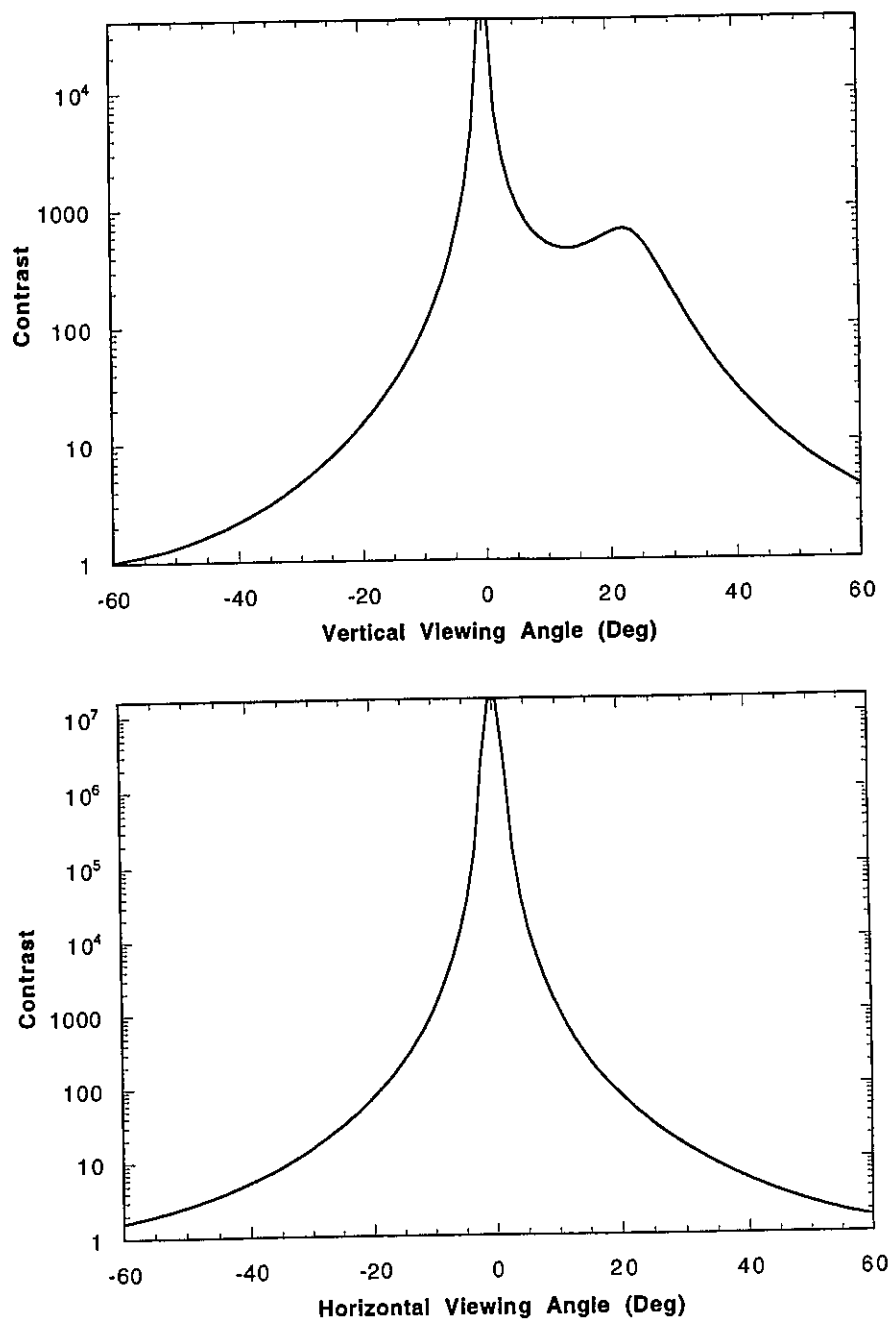


Figure 5.15. (b) Horizontal and vertical viewing contrast of a NW TN-LCD at $V=5.49$ V.

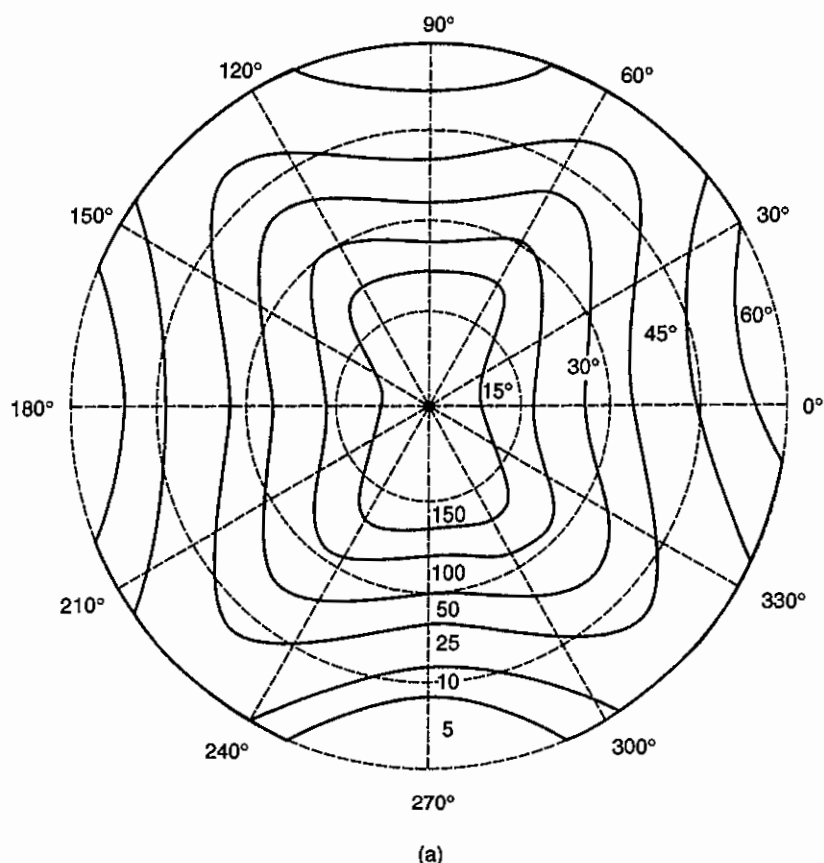


Figure 5.16. (a) Isocontrast viewing diagrams of a NB TN-LCD at $V = 5.49$ V.

using Eq. (5.1-10). Here we used the emission characteristics of a blackbody radiation at 6500 K as the illuminant spectral distribution $D(\lambda)$. For different illumination sources, the isocontrast curves will look slightly different. Figure 5.16 is obtained by averaging over the three primary colors (red, green and blue).

Figure 5.15a shows the contrast ratio of a normally black TN-LCD as a function of the vertical and horizontal viewing angles with an applied voltage of $V = 5.49$ V at the white state. We note that the contrast ratio is around 150 at normal incidence. The contrast ratio remains around 150 for viewing angles within a field of view of $\pm 10^\circ$. The contrast ratio then drops to below 100 for viewing angles greater than 20° . We also note that the contrast ratio remains above 10 for viewing angles up to $\pm 40^\circ$.

Figure 5.15b shows the contrast ratio of a normally white TN-LCD as a function of the vertical and horizontal viewing angles with an applied voltage of 5.49 V at the black state. The contrast ratio exhibits a rather asymmetric dependence on the vertical viewing angle. We note that the contrast ratio is very

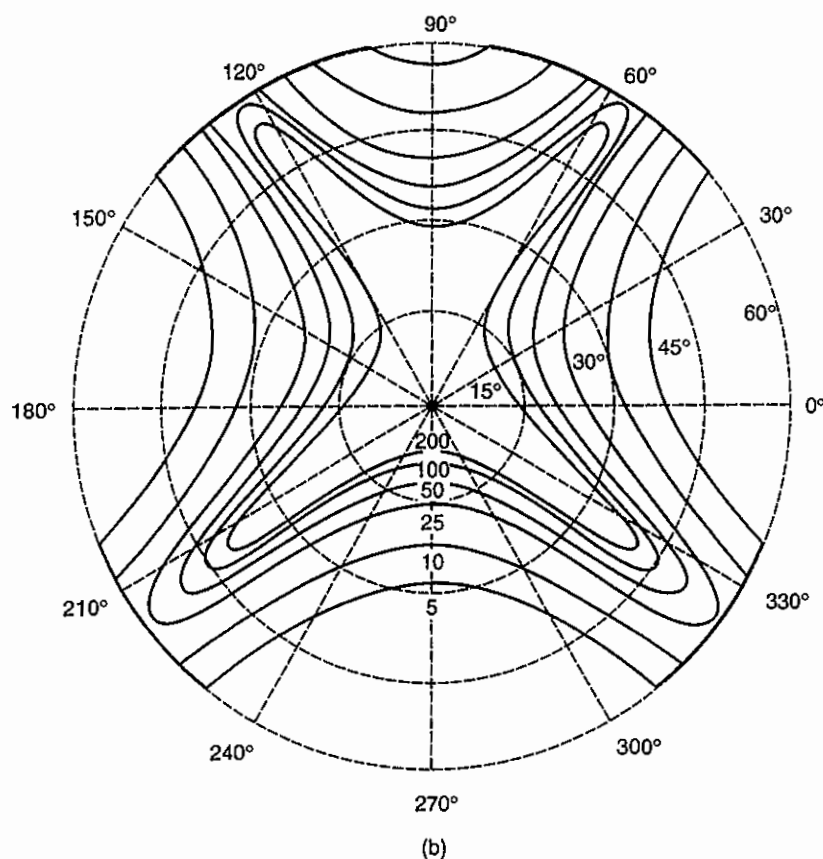


Figure 5.16. (b) Isocontrast viewing diagrams of a NW TN-LCD at $V = 5.49$ V.

high ($>10^4$) at normal incidence. This is in agreement with our earlier discussion (see Fig. 5.10). In the vertical viewing, the contrast ratio remains above 100 in the angular range of $(-10^\circ, 30^\circ)$. In the horizontal viewing, the contrast ratio remains above 100 in the angular range of $\pm 20^\circ$. We note that the normally white mode of TN-LCD exhibits an exact left-right viewing symmetry.

Figure 5.16a shows the isocontrast contours of a normally black TN-LCD at an applied voltage of 5.49 V in the white state. Such a contour plot shows the angular viewing region in which the contrast ratio is above a specified value. Figure 5.16b shows a similar polar diagram for the normally white TN-LCD. The iso-contrast contours in the polar diagram give a better description of the angular dependence of the contrast ratio.

In the preceding discussion of the transmittance of LCD at large viewing angles, we assumed a zero pretilt angle for the purpose of illustrating the effect of off-axis incidence. In practice, a small pretilt angle ($\leq 1^\circ$) is present in most LCD's for the stability of the liquid crystal when voltages are applied.

5.2. SUPERTWISTED NEMATIC (STN) DISPLAYS

As we discussed earlier, when a voltage is applied to a LC cell, the directors of the LC molecules are forced to align along the direction of the electric field. A balance between the elastic energy and the electrostatic energy leads to a distribution of the director orientation in the LC cell. The redistribution of the director leads to a change in the transmittance of the LC cell. Typical panels for information displays requires a 2D array (say $M \times N$) of LC pixels. Each of the pixels consists of a small LC cell that can be turned ON or OFF electrically. Thus, each of the individual LC cells must be addressed electrically for information displays. In principle this can be achieved by using $M \times N$ electrical connections. However, for practical purposes, the electrical addressing is achieved by using multiplexing techniques that require only $M + N$ electrodes. The subject of multiplexing and other addressing schemes will be discussed in the next chapter.

As a result of the multiplexing, the voltages applied at one cell cannot be arbitrarily changed without affecting the applied voltages at other cells. This is known as the *crosstalk*. As a result of the crosstalk, the ON and OFF voltages applied at any of the cells cannot be too different. This severely limits the contrast in display applications.

A high contrast display requires a significant distortion (or redistribution) of the LC directors in the cell at a reasonable applied voltage. Supertwisted nematic (STN) liquid crystals offer a solution to this problem. This will be explained by using the electrooptical distortion curves discussed in this section. Figure 5.17 shows examples of STN-LC cells with total twist angles of 180° and 270° . In what follows, we discuss the principle of operation of STN cells.

5.2.1. Steepness of Electrooptical Distortion Curves of STN Cells

Referring to Figure 5.17b, we considered a TN cell with orthogonal rubbing directions on the surfaces of the cell. The arrows indicate the direction of the LC directors. It is important to note that the LC molecules are usually nonpolar. The dashed arrows indicate the rubbing directions at the boundaries. It is also important to note that the liquid crystal in this cell can have either a right-handed 270° twist or a left-handed 90° twist. Both configurations satisfy the boundary conditions determined by the rubbing directions. Based on the argument in Appendix D, the left-handed 90° would have a lower elastic energy. Thus, when an ordinary nematic liquid crystal is filled in the cell, the liquid crystal would maintain a left-handed 90° twist that has a lower elastic energy.

To maintain a cell twist angle greater than 90° requires a nematic liquid crystal with an intrinsically twisted nematic structure known as a *chiral nematic*. Chiral nematics are ordinary nematic liquid crystals doped with a small percentage of other materials that exhibit intrinsic optical rotatory power. As we know, these optically active materials exhibit intrinsic "handedness" in the rotation of the plane of polarization of a beam of incident light. The handedness

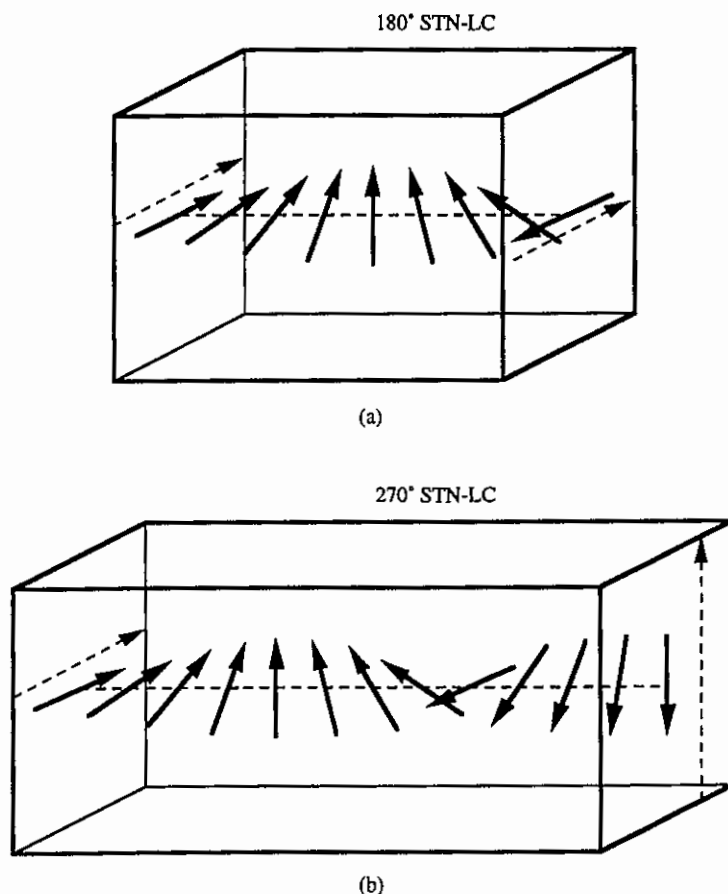


Figure 5.17. (a) STN cells with total twist angles of (a) 180° and (b) 270°. The dashed lines with arrows show the rubbing directions.

of the dopant molecules imparts a macroscopic twist to the whole nematic structure. The amount of twisting is characterized by a pitch length p , which is defined as the distance measured along the helical axis for the twist of the director to rotate by a full 360°. In an infinite medium of helically twisted nematic structure, the pitch is exactly the period of the medium along the axis of the helix. The structure is identical to that of the cholesteric phase described in Chapter 1. When the chiral nematic liquid crystal is filled into the cell, the director at the substrate planes anchor along the alignment directions, whereas the director in the cell twist according to the handedness of the chiral molecules. The pitch length is, in general, slightly different from that of the infinite medium, as a result of the boundary conditions. In an infinite medium, the pitch length is inversely proportional to the concentration of the chiral dopant. Thus, we can easily modify the pitch length by adjusting the chiral dopant concentration to suit the cell spacing d . The pitch p , the cell spacing d , and the cell twist angle Φ are

related by the following condition:

$$\frac{d}{p} = \frac{\Phi}{2\pi} \quad (5.2-1)$$

This condition provides a precise match of the intrinsic pitch length of the infinite medium to the pitch defined by the cell spacing and the alignment directions of a cell with a zero pretilt.

To account for the natural twist due to the presence of chiral molecules, the elastic energy density is written

$$U_{EL} = \frac{1}{2}k_1(\nabla \cdot \mathbf{n})^2 + \frac{1}{2}k_2(\mathbf{n} \cdot \nabla \times \mathbf{n} + q_0)^2 + \frac{1}{2}k_3(\mathbf{n} \times \nabla \times \mathbf{n})^2 \quad (5.2-2)$$

where $\mathbf{n}$ is a unit vector representing the director distribution in the cell, k_1, k_2, k_3 are the three principal elastic constants, and $q_0 = 2\pi/p$ is the natural twist rate in the absence of the boundaries. We adopt a sign convention of positive q_0 for right-handed twist and negative q_0 for left-handed twist.

Consider the special case of a supertwisted nematic LC cell with a zero tilt angle and a uniform twist rate of q . Using Eq. (5.2-2), we obtain the following expression for the elastic energy density:

$$U_{EL} = \frac{1}{2}k_2(q - q_0)^2 \quad (5.2-2a)$$

We note that, in the absence of boundaries, the lowest energy configuration occurs when the actual twist rate is exactly the same as the natural twist rate (i.e., $q = q_0$). Any deviation from the natural twist rate can lead to a higher elastic energy density. Thus, when the boundaries are present with prescribed rubbed directions, the actual twist rate is often slightly different from the natural twist rate in order to comply with the boundary conditions. The condition of d/p ratio in Eq. (5.2-1) corresponds to $q = q_0$.

The presence of the additional term q_0 in Eq. (5.2-2) for STN cells leads to more complicated electrodistortional curves (see Appendix E). The extent of the electrodistortion is often measured by examining the midlayer tilt angle as a function of the applied voltage. Figure 5.18 shows the electrodistortional curve for a TN (or STN) cell with various cell twist angles. We note that the steepness of the curve depends on the cell twist angle. By increasing the cell twist angle from 90° to 180° – 270° , we find that the steepness of the electrodistortional curve can be dramatically increased. As we mentioned earlier, a steep electrodistortional curve is a precondition to achieve a high contrast in a highly multiplexed display system. According to Figure 5.18, we find that the steepness increases as the cell twist angle increases. An infinite slope is obtained at a certain critical cell twist angle of about 270° . At higher cell twist angles, the curves become

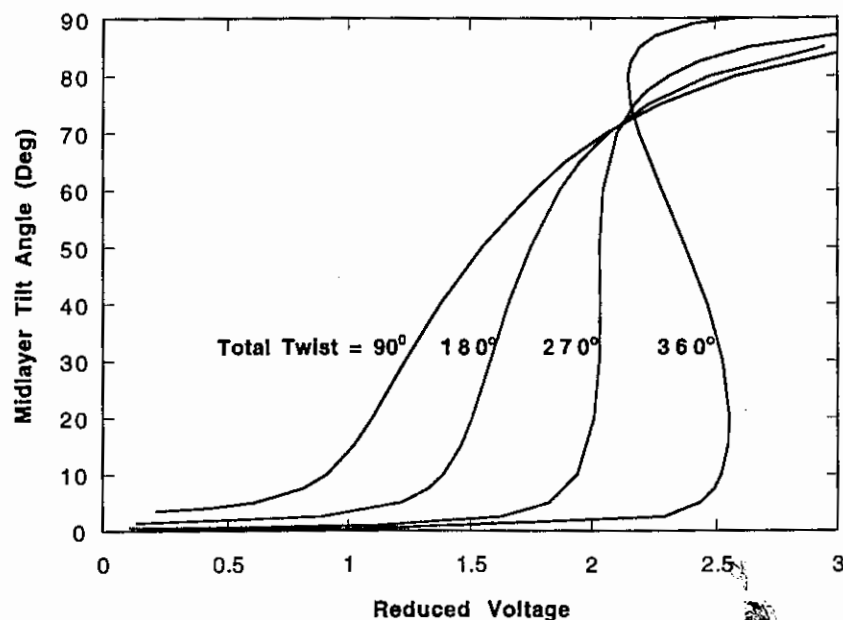


Figure 5.18. Electrodistortional curve for a TN (or STN) cell with various cell twist angles.

doubled-valued, indicating a region of instability, bistability and hysteresis. It is important to note that the onset of the bistability occurs at a specific twist angle that depends on several device and material parameters, including d/p ratio, pretilt angle dielectric and elastic constants. Depending on these parameters, the steepest slope may occur at other twist angles, such as 240° or 210° .

Effect of Other Device and Material Parameters

In addition to the cell twist angle, the pretilt angle Θ at the boundaries can also affect the shape and the steepness of the electrodistortional curve. As we know, the electrodistortional curve is a result of the balance between the elastic restoring force and the electrostatic force. Thus, it is natural that the shape of the electrodistortional curve may depend on the elastic coefficients k_1, k_2, k_3 , as well as the dielectric constants $\epsilon_{\parallel}, \epsilon_{\perp}$. The d/p ratio is both a device and material parameter. In what follows, we will discuss the dependence of the shape of the electrodistortional curve on these device and material parameters. The following discussion is based on a reference STN cell used in previous works [2,3].

Pretilt Angle

As we mentioned earlier, a small pretilt angle is important in TN cells to ensure a single domain under the influence of an applied electric field. The pretilt angle is even more important in STN cells. As a result of the larger twist angles, unwanted striped textures may occur in STN cells without a pretilt angle. In fact, the pretilt angles are often larger in STN cells, ranging from 5° to as large as 60° .

Generally speaking, increasing the pretilt angle causes the electrodistortional curve to shift toward lower voltages with a slight increase in the steepness of the curve. For the special case of a zero pretilt angle, there exists, a true threshold voltage below which there is uniform twist and zero tilt throughout the layer. When a pretilt angle exists, the tilt angle distribution $\theta(z)$ exhibits a minimum at the center of the cell (see Fig. 5.21). A uniform tilt angle $\theta(z) = \text{constant}$ may exist provided an appropriate voltage is applied to the cell (see Section 5.2.3).

Thickness Pitch Ratio d/p

As a result of the presence of the chiral molecules, a nature twist exists without the boundaries. Minimum elastic energy density occurs when the d/p ratio is exactly $\Phi/2\pi$. Any deviation from this value will cause an increase in the elastic energy density. This affects the balance between the electrostatic and the elastic energy, leading to a different distribution of the director. Thus, we expect to have a different electrodistortional curve. Many STN cells for display applications have a d/p ratio that is within the Grandjean zone defined by the following inequality [4]: $\Phi/2\pi - 0.25 < d/p < \Phi/2\pi + 0.25$. We note that the variation of the cell gap is within a quarter of a pitch ($p/4$). Generally speaking, lower d/p ratios shift the curves to lower voltages and make them steeper.

Bend/Splay Elastic Constant Ratio (k_3/k_1)

Increasing the k_3/k_1 ratio in an STN cell increases the steepness of the electrodistortional curves. Known nematic LC materials have k_3/k_1 ratios in the range 0.5–2.0, but most are in the range 1.2–1.8 (see Table 1.2).

Twist/Splay Elastic Constant Ratio (k_2/k_1)

The twist/splay elastic constant ratio lies between 0.5 and 0.6 for practically all LC materials. Decreasing the ratio shifts the upper portions of the electrodistortional curves to lower voltages, thereby increasing their steepness.

Dielectric Parameter γ

Generally speaking, increasing the dielectric parameter γ , defined as $\gamma = (\epsilon_{\parallel} - \epsilon_{\perp})/\epsilon_{\perp}$, decreases the steepness of the electrodistortion curve, as it does in the TN case.

5.2.2. Transmission Properties of STN-LCD in the Field-OFF State

We now consider the transmission properties of STN-LCDs. In a manner similar to TN-LCDs, there are various modes of operation in STN-LCDs. These include normally white (NW) and normally black (NB) operations. As a result of the relatively faster twist rate, the effect of waveguiding is no longer present. Thus, the polarizers in STN-LCDs are, in general, not parallel or perpendicular. In fact, the transmission axes of the polarizers are no longer parallel or perpendicular to

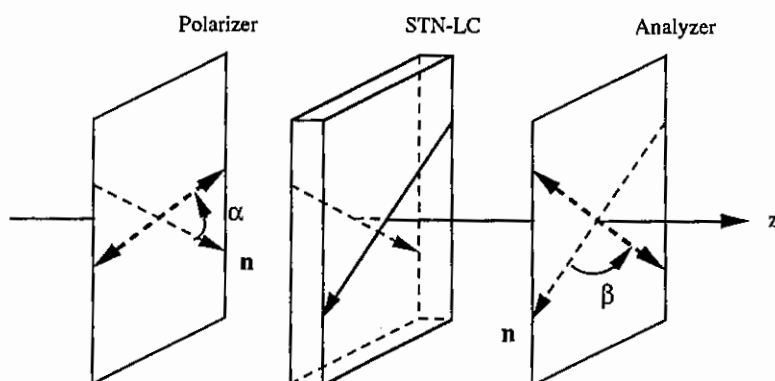


Figure 5.19. Schematic drawing of an STN-LCD, where α is the azimuth angle of the polarizer measured from the local director $\mathbf{n}(z=0)$ and β is the azimuth angle of the analyzer measured from the local director $\mathbf{n}(z=d)$, where d is the cell thickness. The arrows on the STN cell indicate the director orientations at the surfaces. The double arrows indicate the transmission axes of the polarizers.

the local directors (rubbed directions) of the STN-LC. Generally speaking, two normal modes are excited by the incidence of a beam of polarized light. These two modes interfere after exiting the LC cell to produce either a bright state or a dark state. The general results obtained in Chapter 4 can now be employed to investigate the transmission of STN-LCDs in the field-OFF state.

Referring to Figure 5.19, we consider an STN-LCD that consists of a supertwisted nematic LC cell sandwiched between a pair of polarizers. We assume that the twist is right-handed. The azimuth angles of the transmission axes of the polarizers are denoted as α and β as shown in Figure 5.19.

If we assume a uniform tilt angle (either zero or small tilt angle) throughout the cell, then the transmission of the STN-LCD in the field-OFF state can be obtained by using Eq. (4.3-29). The polarizer's azimuth angles in Eq. (4.3-29) are related to α and β of Figure 5.19 by the following relationships:

$$\Phi_{\text{ent}} = \alpha, \quad \Phi_{\text{exit}} = \phi + \beta \quad (5.2-3)$$

where ϕ is the total twist angle. Note that the azimuthal angular separation between the polarizers is $(\Phi_{\text{exit}} - \Phi_{\text{ent}}) = \phi + \beta - \alpha$.

Using the new definition for the polarizer azimuth angles, we obtain the following transmission formula, according to Eq. (4.3-29):

$$\begin{aligned} T = \frac{1}{2} \bigg\{ & \cos^2(\alpha - \beta) - \sin^2 X \sin 2\beta \sin 2\alpha \\ & + \frac{\phi}{2X} \sin 2X \sin 2(\alpha - \beta) \\ & - \phi^2 \frac{\sin^2 X}{X^2} \cos 2\beta \cos 2\alpha \bigg\} \end{aligned} \quad (5.2-4)$$

where we recall that X is given by

$$X = \sqrt{\phi^2 + \frac{\Gamma^2}{4}} = \phi \sqrt{1 + u^2} \quad (5.2-5)$$

where Γ is the phase retardation

$$\Gamma = \frac{2\pi}{\lambda} d(n_e - n_o) \quad (5.2-6)$$

and u is the Mauguin parameter

$$u = \frac{\Gamma}{2\phi} \quad (5.2-7)$$

It is important to note that if a uniform tilt angle exists ($\theta \neq 0$) in the cell, then n_e in Eq. (5.2-6) should be replaced by $n_e(\theta)$ given by

$$\frac{1}{n_e^2(\theta)} = \frac{\cos^2 \theta}{n_e^2} + \frac{\sin^2 \theta}{n_o^2} \quad (5.2-8)$$

where θ is the tilt angle. The transmission formula Eq. (5.2-4) can also be written

$$T = \frac{1}{2} \left\{ \cos^2(\alpha + \beta) + \cos^2 X \cos 2\alpha \cos 2\beta \left(\tan 2\alpha - \frac{\phi}{X} \tan X \right) \right. \\ \left. \left(\tan 2\beta + \frac{\phi}{X} \tan X \right) \right\} \quad (5.2-9)$$

We also note that the transmission formula is simpler by using the new definition for the polarizer azimuth angles. The transmission formula exhibits a 90° rotation symmetry. In other words

$$T\left(\alpha + \frac{\pi}{2}, \beta + \frac{\pi}{2}\right) = T(\alpha, \beta) \quad (5.2-10)$$

Thus, if we rotate both the polarizer and the analyzer by 90° relative to the STN-LC cell, the transmission remains unchanged.

Although Eq. (5.2-4) or Eq. (5.2-9) can be employed to design the various modes of operation of STN-LCD, the techniques described in Chapter 4 offer

some useful insight for this purpose. In Eqs. (4.3-38)–(4.3-40), we derived the input linear polarization states that can be transformed into linear polarization states (output beams) by an arbitrarily twisted LC cell with a uniform tilt. Using these formulas, we can obtain the azimuth angle of the polarizers for various modes of operation.

Before we consider the general cases, let us examine a special case of interest. According to Eq. (5.2-4), the transmission can be written

$$T = \frac{1}{2} \cos^2(\alpha - \beta) \quad (5.2-11)$$

provided

$$X = \phi \sqrt{1 + u^2} = \pi, 2\pi, 3\pi, \dots \quad (5.2-12)$$

Thus, in this special case, the transmission is 100% if $\alpha = \beta$, and zero if $\alpha - \beta = \pi/2$. For a 270° twist cell, $\phi = 1.5\pi$, solutions for $X = 2\pi, 3\pi, \dots$ are possible. These correspond to a Mauguin parameter of $u = \sqrt{7}/3, \sqrt{3}, \dots$ for the 270° STN cell. For $\lambda = 0.55 \mu\text{m}$, these correspond to $\Delta nd = 0.728 \mu\text{m}, 1.429 \mu\text{m}$, and so on. For applications in this special case, the relative azimuth angle between the polarizers is fixed by either $\alpha = \beta$, or $\alpha - \beta = \pi/2$, whereas the absolute azimuth angle can be arbitrarily chosen.

These numbers $\Delta nd = 0.728 \mu\text{m}, 1.429 \mu\text{m}$, and so on for a 270° twist cell are unfortunately too large for some LCD applications. In the following, we consider the general case when the parameter X is not an integral number of π . In the general cases, the azimuth angles (α, β) of both the polarizer and the analyzer must be properly chosen to obtain either a unity transmission or zero transmission for a specific wavelength. As we recall the transmission of a TN-LCD is somewhat insensitive to wavelength. This is a result of the waveguiding phenomenon in the Mauguin regime ($1 \ll u$). The transmission of an STN-LCD is no longer insensitive to the wavelength as a result of the higher twist rate. The following discussions are based on a single wavelength only.

Normally White Mode I (NW-I)

In the normally white operation, the transmission is 0.5 in the field-OFF state at a specific wavelength. Unlike the case in TN, the transmission of light at other wavelengths is much lower than $\frac{1}{2}$ in STN cells. So the white state is not quite white. The azimuth angle of the entrance polarizer is given by, according to Eq. (4.3-41)

$$\tan \alpha_1 = \sqrt{1 + v^2} - v \quad (5.2-13)$$

or equivalently

$$\tan 2\alpha_1 = \frac{1}{v} = \frac{\phi \tan X}{X} \quad (5.2-14)$$

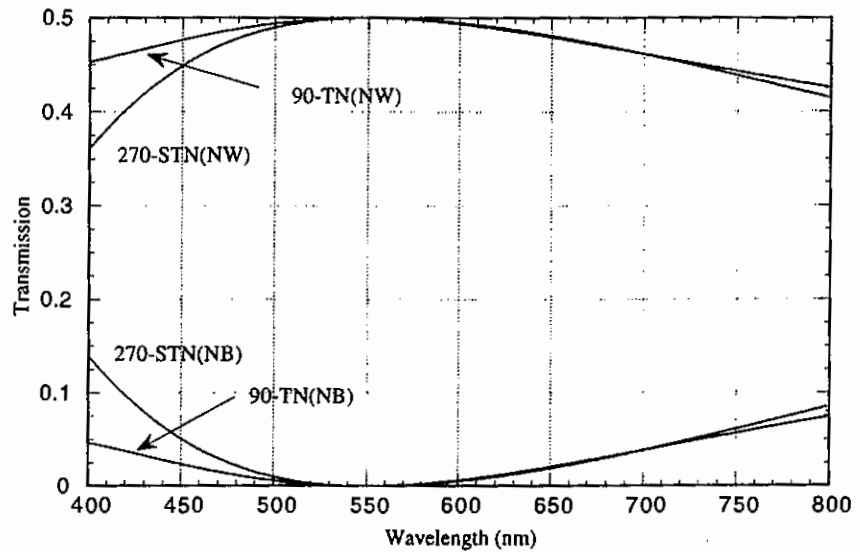


Figure 5.20. Transmission spectra of a 270° STN cell in the field-OFF state with $\alpha = -22^\circ$, $\beta = 22^\circ$, $\Delta nd = 0.48 \mu\text{m}$, and a 90° TN cell in the field-OFF state with $\alpha = 0^\circ$, $\beta = 0^\circ$, $\Delta nd = 0.48 \mu\text{m}$. The polarizer azimuth angles are chosen to have maximum (or zero) transmission in the field-OFF state for $\lambda = 550 \text{ nm}$.

It is important to remember that these angles α_1, β_2 are measured from the local director as indicated in Figure 5.19. And the azimuth angle of the analyzer (exit polarizer) is given by, according to Eq. (4.3-43)

$$\beta_1 = -\alpha_1 \quad (5.2-15)$$

We note that the transmission is $T = \frac{1}{2}$ for these polarizer azimuth angles, according to Eq. (5.2-9). It is important to note that $T = \frac{1}{2}$ is applicable only for a specific design wavelength. A lower transmission will be obtained for other wavelengths (see Fig. 5.20). By examining Eq. (5.2-9), we find that the dependence on the wavelength is due to the $1/\lambda$ factor in Γ [Eq. (5.2-6)] and X . Thus, the deviation from $T = \frac{1}{2}$ depends on Δnd of the cell. Generally speaking, lower values of Δnd are desirable. If the STN cell is designed for a maximum transmission at around $\lambda = 0.55 \mu\text{m}$, the field-OFF state (bright state) will have a yellowish green appearance, and the field-ON state (dark state) will have a black appearance. This is often referred to as the yellow mode [2].

Normally White Mode II (NW-II)

This mode is obtained by rotating both the polarizer and analyzer by 90° relative to the STN-LC cell. The transmission properties at normal incidence is identical to those of NW-I, according to Eq. (5.2-10). Specifically, the azimuth angle of

the entrance polarizer is given by, according to Eq. (4.3-41)

$$\tan \alpha_2 = -\sqrt{1 + v^2} - v \quad (5.2-16)$$

or equivalently

$$\tan 2\alpha_2 = \frac{1}{v} = \frac{\phi \tan X}{X} \quad (5.2-17)$$

Note that although the two azimuth angles (α_1, α_2) are different, they have the same value for $\tan 2\alpha$. In fact, these two values of α are related by

$$\alpha_2 = \alpha_1 + \frac{\pi}{2} \quad (5.2-18)$$

The azimuth angle of the analyzer (exit polarizer) is given by, according to Eq. (4.3-43)

$$\beta_2 = -\alpha_2 \quad (5.2-19)$$

Normally Black Mode I

This mode can be obtained by keeping the azimuth angle of the entrance polarizer unchanged, while rotating the analyzer (exit polarizer) by 90° . Zero transmission is obtained for a specific wavelength. Leakage of light at other wavelengths is relatively stronger in STN cells than in TN cells. This is a result of the lack of waveguiding in STN cells. Thus, the black state is actually not quite black. Specifically, the azimuth angle of the entrance polarizer is given by, according to Eq. (4.3-41)

$$\tan \alpha_1 = \sqrt{1 + v^2} - v \quad (5.2-20)$$

or equivalently

$$\tan 2\alpha_1 = \frac{1}{v} = \frac{\phi \tan X}{X} \quad (5.2-21)$$

where X is given by Eq. (5.2-5). And the azimuth angle of the analyzer (exit polarizer) is given by

$$\beta_1 = \frac{\pi}{2} - \alpha_1 \quad (5.2-22)$$

Note that the transmission axis of polarizers has no polarity. Thus, azimuth angles of ϕ and $\phi \pm \pi$ are the same orientation. We note that normally black mode I can be easily obtained from normally white mode I by rotating the analyzer (exit polarizer) by 90° in either direction. The zero transmission ($T = 0$) for these polarizer azimuth angles can also be obtained; according to Eq. (5.2-9). It is important to note that $T = 0$ is only for a specific design wavelength. A leakage of light occurs for other wavelengths, due to the dependence on the wavelength in Γ [Eq. (5.2-6)] and X (see Fig. 5.20). Thus, if the STN cell is designed for a zero transmission at around $\lambda = 0.55 \mu\text{m}$, the field-OFF state (dark state) will have a blue-purple appearance, and the field-ON state (bright state) will have a white appearance. This is often referred to as the *blue mode* [2].

Normally Black Mode II

This mode is obtained by rotating both the polarizer and the analyzer in the NB-I mode by 90° relative to the STN-LC cell. The transmission properties at normal incidence is identical to those of NB-I, according to Eq. (5.2-10). Specifically, the azimuth angle of the entrance polarizer is given by, according to Eq. (4.3-41)

$$\tan \alpha_2 = -\sqrt{1 + v^2} - v \quad (5.2-23)$$

or equivalently

$$\tan 2\alpha_2 = \frac{1}{v} = \frac{\phi \tan X}{X} \quad (5.2-24)$$

$$\alpha_2 = \alpha_1 + \frac{\pi}{2} \quad (5.2-25)$$

where X is given by Eq. (5.2-5). The azimuth angle of the analyzer (exit polarizer) is given by

$$\beta_2 = \frac{\pi}{2} - \alpha_2 \quad (5.2-26)$$

Example 5.2. Consider a 270° STN-LCD with a uniform tilt angle throughout the cell. Let us calculate the polarizer azimuth angles α and β for three wavelengths of operation: $\lambda = 0.45, 0.55$, and $0.65 \mu\text{m}$. The azimuth angles for the polarizer and the analyzer depend on the birefringence-thickness product $\Delta n d = [n_e(\theta) - n_o]d$, where θ is the tilt angle of the LC director. These angles (α, β) for $\Delta n d$ between 0.40 and $1.00 \mu\text{m}$ are listed in Table 5.2. ■

Table 5.2. Polarizer Azimuth Angles

λ (μm)	Δnd (μm)	NW-I		NB-I	
		α	β	α	β
0.45	0.40	-20.9°	20.9°	-20.9°	-69.1°
0.45	0.45	-15.3°	15.3°	-15.3°	-74.7°
0.45	0.50	-9.8°	9.8°	-9.8°	-80.2°
0.45	0.55	-4.6°	4.6°	-4.6°	-85.4°
0.45	0.60	0.5°	-0.5°	0.5°	89.5°
0.45	0.65	5.4°	-5.4°	5.4°	84.6°
0.45	0.70	10.5°	-10.5°	10.5°	79.5°
0.45	0.75	16.3°	-16.3°	16.3°	73.7°
0.45	0.80	23.3°	-23.3°	23.3°	66.7°
0.45	0.85	32.7°	-32.7°	32.7°	57.3°
0.45	0.90	45.0°	-45.0°	45.0°	45.0°
0.45	0.95	-31.7°	31.7°	-31.7°	-58.3°
0.45	1.00	-20.6°	20.6°	-20.6°	-69.4°
0.55	0.40	-28.6°	28.6°	-28.6°	-61.4°
0.55	0.45	-24.4°	24.4°	-24.4°	-65.6°
0.55	0.50	-19.9°	19.9°	-19.9°	-70.1°
0.55	0.55	-15.3°	15.3°	-15.3°	-74.7°
0.55	0.60	-10.8°	10.8°	-10.8°	-79.2°
0.55	0.65	-6.5°	6.5°	-6.5°	-83.5°
0.55	0.70	-2.3°	2.3°	-2.3°	-87.7°
0.55	0.75	1.8°	-1.8°	1.8°	88.2°
0.55	0.80	5.9°	-5.9°	5.9°	84.1°
0.55	0.85	10.0°	-10.0°	10.0°	80.0°
0.55	0.90	14.6°	-14.6°	14.6°	75.4°
0.55	0.95	19.9°	-19.9°	19.9°	70.1°
0.55	1.00	26.4°	-26.4°	26.4°	63.6°
0.65	0.40	-33.3°	33.3°	-33.3°	-56.7°
0.65	0.45	-30.2°	30.2°	-30.2°	-59.8°
0.65	0.50	-26.7°	26.7°	-26.7°	-63.3°
0.65	0.55	-23.0°	23.0°	-23.0°	-67.0°
0.65	0.60	-19.2°	19.2°	-19.2°	-70.8°
0.65	0.65	-15.3°	15.3°	-15.3°	-74.7°
0.65	0.70	-11.5°	11.5°	-11.5°	-78.5°
0.65	0.75	-7.8°	7.8°	-7.8°	-82.2°
0.65	0.80	-4.2°	4.2°	-4.2°	-85.8°
0.65	0.85	-0.7°	0.7°	-0.7°	-89.3°
0.65	0.90	2.7°	-2.7°	2.7°	87.3°
0.65	0.95	6.2°	-6.2°	6.2°	83.8°
0.65	1.00	9.7°	-9.7°	9.7°	80.3°

It is important to note that these polarizer azimuth angles for maximum transmission ($T = \frac{1}{2}$) or complete extinction ($T = 0$) depend on the operation wavelength λ . To illustrate the dependence on the wavelength, we plot the transmission of an STN cell as a function of wavelength in Figure 5.20. We note that in the normally black (NB) mode, the STN-LCD has a much stronger leakage in the blue spectral regime, leading to a bluish appearance in the black state. In the normally white (NW) mode, the STN-LCD has a much lower transmission in the blue spectral regime, leading to a yellowish green appearance in the white state. The transmission spectra of a TN cell are also shown in the same figure. We note that the TN cell is relatively insensitive to the wavelength variation as compared with the STN cell. Although the azimuth angles of the polarizers can be obtained for either zero or $\frac{1}{2}$ transmission for a given wavelength, the actual azimuth angles for STN-LCDs should be determined by optimizing the integrated transmission over the spectrum of the illuminating light source.

The colors in the field-OFF state of STN-LCDs are due to leakage of light, which is a result of the absence of waveguiding in the LC cell. These colors can be completely removed by using a double-layer STN cell, leading to true white or black in the field-OFF states. The double layer STN cells consist of an electrically switchable STN cell and a fixed STN cell with a reversed twist for color compensation in the field-OFF states.

5.2.3. Transmission Properties of STN-LCD in the Field-ON State

When an electric field of sufficient strength is turned ON, the LC molecules will be aligned parallel to the electric field. Similar to the situation in TN-LCD, the actual director distribution depends on the balance between the elastic energy density and the electrostatic energy density in the cell. Before we consider the general case of director distribution, let us examine a special case of approximately homeotropic alignment of the LC molecules in the cell due to the action of the electric field. The transmission for this special case is particularly simple. In the approximation of homeotropic alignment, the phase retardation Γ becomes zero. In this case, $X = \phi$, according to Eq. (5.2-5), and the transmission formula Eq. (5.2-4) becomes

$$T = \frac{1}{2} \cos^2(\beta + \phi - \alpha) \quad (5.2-27)$$

which is exactly the transmission of a pair of polarizers with an azimuth angle of $(\beta + \phi - \alpha)$ between them. Equation (5.2-27) is a good estimate of the transmission of the STN cell in the field-ON state. However, in general, the director is not exactly in homeotropic alignment with the electric field. Jones matrix method and numerical techniques must be employed to calculate the transmission of the cell. However, we must obtain the director distribution $\theta(z)$ and $\phi(z)$ first before we can calculate the transmission.

Field-Induced Director Reorientation

According to Appendix E, the total electrostatic and elastic energy in the cell can be written

$$U = \frac{1}{2} \int_0^d \left(F_1 \theta'^2 + F_2 \phi'^2 + F_3 \phi' + \frac{D_z^2}{\epsilon_{\parallel} \sin^2 \theta + \epsilon_{\perp} \cos^2 \theta} \right) dz \quad (5.2-28)$$

where F_1, F_2 , and F_3 are as follows:

$$F_1 = k_1 \cos^2 \theta + k_3 \sin^2 \theta \quad (5.2-29)$$

$$F_2 = [k_2 \cos^2 \theta + k_3 \sin^2 \theta] \cos^2 \theta \quad (5.2-30)$$

$$F_3 = -2k_2 q_0 \cos^2 \theta \quad (5.2-31)$$

where θ' and ϕ' are the derivatives of the tilt and twist angles with respect to z , as defined in Eqs. (E-9) and (E-10), and q_0 is the natural twist rate of the LC material. Using the variational technique described in Appendix E, we obtain the following expression for the threshold voltage needed to achieve a uniform tilt angle throughout the cell:

$$V_{th} = \frac{1}{\sqrt{\epsilon_{\parallel} - \epsilon_{\perp}}} \left[4\pi k_2 \frac{d}{p_0} \Phi - \Phi^2 [k_3 - 2(k_3 - k_2) \cos^2 \theta_0] \right]^{1/2} \quad (\theta_0 \neq 0) \quad (5.2-32)$$

where θ_0 is the nonzero pretilt angle, $p_0 = 2\pi/q_0$ is the natural pitch, and Φ is the total twist angle. For the special case of zero pretilt angle ($\theta_0 = 0$), the minimum voltage needed to create a small distortion in the cell is given by, according to Appendix E

$$V_{th} = V_{cl} \left[1 + \frac{k_3 - 2k_2}{k_1} \left(\frac{\Phi}{\pi} \right)^2 + 4 \frac{k_2}{k_1} \frac{d}{p_0} \frac{\Phi}{\pi} \right]^{1/2} \quad (\theta_0 = 0) \quad (5.2-33)$$

where V_{cl} is the critical voltage needed to produce a finite tilt angle in a parallel nematic cell:

$$V_{cl} = \pi \frac{\sqrt{k_1}}{\sqrt{\epsilon_{\parallel} - \epsilon_{\perp}}} \quad (5.2-34)$$

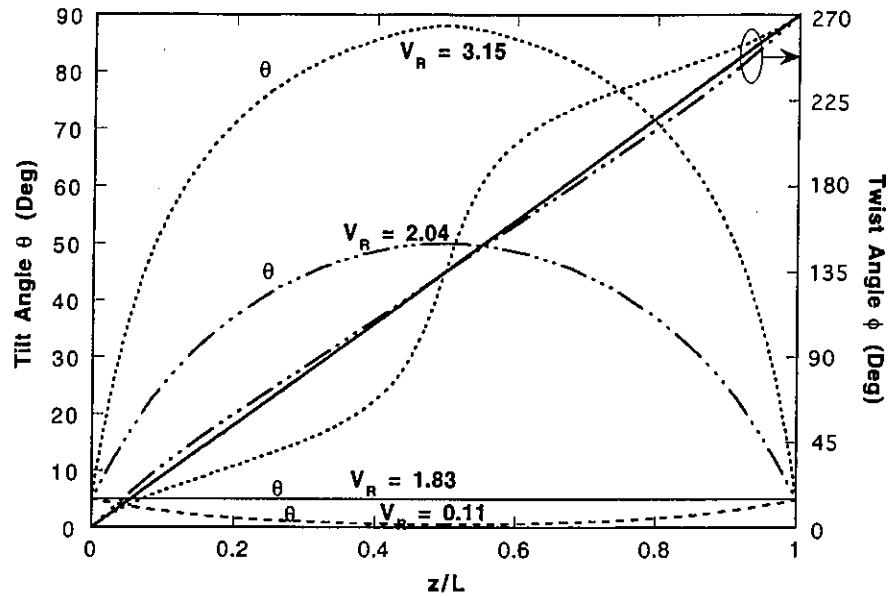


Figure 5.21. Director distribution $\theta(z)$ and $\phi(z)$ in an STN cell with 270° total twist angle at different reduced voltages. Reduced voltage $V_R = V/V_{cl}$.

It is important to remember that Eq. (5.2-33) is valid for the special case of zero pretilt angle ($\theta_0 = 0$). A discontinuity exists between Eqs. (5.2-32) and (5.2-33) at $\theta_0 = 0$. In other words, Eq. (5.2-32) does not reduce to Eq. (5.2-33) when θ_0 approaches 0. We note that Eq. (5.2-33) reduces to the result obtained previously for TN-LC by putting $p_0 = \text{infinity}$:

$$V_T = V_{cl} \left[1 + \frac{k_3 - 2k_2}{k_1} \frac{\Phi^2}{\pi^2} \right]^{1/2} \quad (5.2-35)$$

The director distribution $\theta(z)$ and $\phi(z)$ can be obtained by solving the equations in Appendix E using numerical techniques. Figure 5.21 shows a typical director distribution in an STN cell at different applied voltages. In this case, a small pretilt angle of $\theta = 5^\circ$ exists at the cell boundaries. We note that in the field-OFF ($V=0$) state, the LC tilt angle $\theta(z)$ decreases as z increases from $z=0$ to $z=d/2$ (midlayer), reaching a minimum value of the tilt angle at the midlayer point. At an applied voltage of $V_R = 1.83$ (reduced voltage $V_R = V/V_{cl}$), a uniform tilt $\theta(z) = 5^\circ$ exists throughout the cell. For applied voltages V_R greater than 1.83, the tilt angle $\theta(z)$ is an increasing function of z between $z=0$ and $z=d/2$, reaching a maximum at the midlayer point. The midlayer tilt angle $\theta(d/2)$ approaches 90° at applied voltages $1 \ll V_R$.

Transmission at Normal Incidence

Using the director distribution $\theta(z)$ and $\phi(z)$, we can now employ the Jones method to calculate the transmission spectrum of the STN cell. Here again, we can divide an STN cell into 20 or 40 sublayers of equal thickness. Each of these layers will be treated as a homogeneous uniaxially birefringent medium.

Figure 5.22 shows the transmission spectra of two different STN-LCDs at various applied voltages. The polarizers' transmission axes of these STN-LCDs are obtained from Table 5.2. These polarizers' azimuth angles are designed to yield a maximum transmission of 50%, for unpolarized light, at $\lambda = 0.55 \mu\text{m}$ in the field-OFF state. Figure 5.22b plots the transmission spectrum of a different STN-LCD which offers a better contrast ratio. This is a result of the near-orthogonal orientation of the transmission axes of the polarizers. As we recall, the transmission in the field-ON state with $1 \ll V_R$ is given by Eq. (5.2-27). Thus a cell configuration with $(\beta + \phi - \alpha)$ near 90° or 270° will provide a better contrast ratio.

Contrast at Normal Incidence

As we indicated earlier, high-information-content displays with multiplexing addressing techniques suffer from the poor contrast ratios as a result of crosstalk between the addressing rows. Although the STN cells offer a solution for this problem, by virtue of their steep electrodistortional curves, the contrast ratio is still relatively poor for displays with high information content because of the absence of waveguiding. In TN-LCD, the transmission is relatively insensitive to the wavelength. In other words, if a TN cell is designed to have a zero transmission for the green light, the leakage is relatively low for the red and blue lights as well. This is no longer true in STN-LCDs due to the absence of waveguiding. In other words, if an STN cell is designed to have a zero transmission for green light, the leakage is relatively high for the red and blue lights. This can be seen from Figure 5.20. The leakage of light is a main source of the poor contrast.

The definitions of contrast ratios for an individual STN cell are the same as those of the TN cells. These contrast ratios for individual cells are only useful for low level multiplexed displays (alphanumeric) such as those used in automobiles panels, gas stations, watches, and calculators. For high-information-content displays with multiplexed addressing, the selection ratio of the applied voltage is limited by the following law of multiplexing [5]:

$$\frac{V_{\text{ON}}}{V_{\text{OFF}}} = \sqrt{\frac{\sqrt{N} + 1}{\sqrt{N} - 1}} \quad (5.2-36)$$

where N is the number of rows in the displays panel (see Chapter 6 for more detailed discussion). For $N = 100$, the ratio is only 1.105.

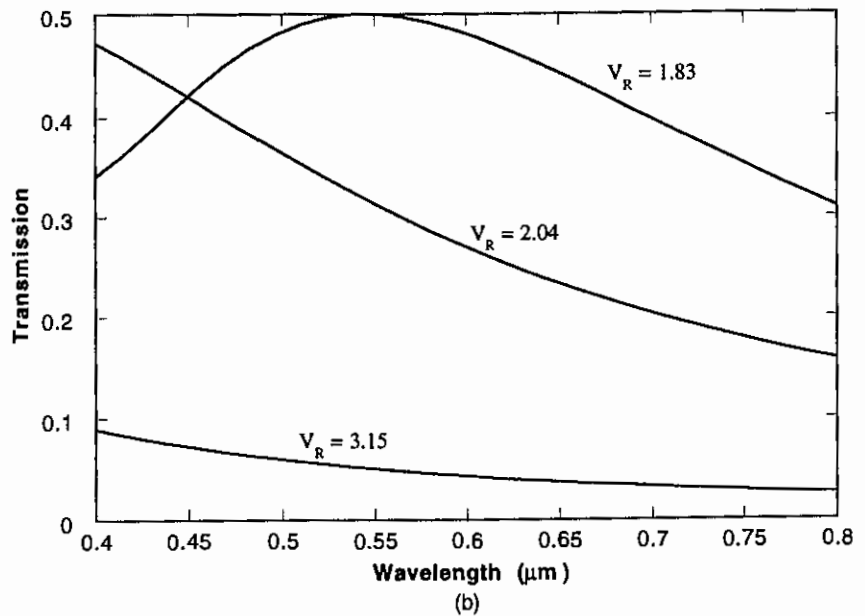
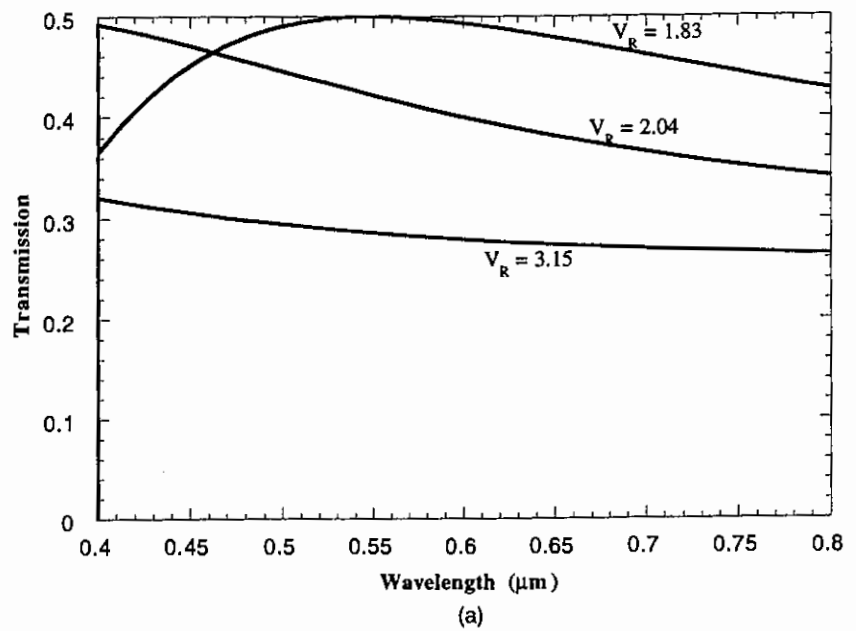


Figure 5.22. Transmission spectra (normal incidence) of a 270° STN-LCD at different reduced voltages, with (a) $\alpha = -22^\circ$, $\beta = 22^\circ$, $\Delta nd = 0.48 \mu\text{m}$, reduced voltage $V_R = V/V_{cl}$ (b) $\alpha = 1.8^\circ$, $\beta = -1.8^\circ$, $\Delta nd = 0.75 \mu\text{m}$, $V_R = V/V_{cl}$.

The contrast ratios for STN-LCDs are defined as follows:

$$C_{NW} = \frac{T(V_{OFF})}{T(V_{ON})} \quad (\text{normally white}) \quad (5.2-37)$$

$$C_{NB} = \frac{T(V_{ON})}{T(V_{OFF})} \quad (\text{normally black}) \quad (5.2-38)$$

As mentioned earlier, the leakage of light in the black state is relatively higher in STN-LCDs. This leads to a poor contrast ratio in STN display systems. In the case of TN-LCDs with multiplexed addressing, the voltages for ON and OFF states are governed by Eq. (5.2-36). For TN displays with a large number of rows N , V_{ON} is very close to V_{OFF} . This leads to $T(V_{ON}) \approx T(V_{OFF})$ and thus a very poor contrast ratio. In an STN cell with the proper twist angle, a very steep (near vertical) electrodistortional curve can be obtained (see Fig. 5.18 or 5.23). This is a very desirable property of the STN cell, allowing the possibility of high contrast ratios with a small voltage selection ratio. Thus, the applied voltages V_{ON} and V_{OFF} are often selected on both sides of the steepest slope (see Fig. 5.23). We note that the midlayer tilt angle is around 10° for a reduced voltage of

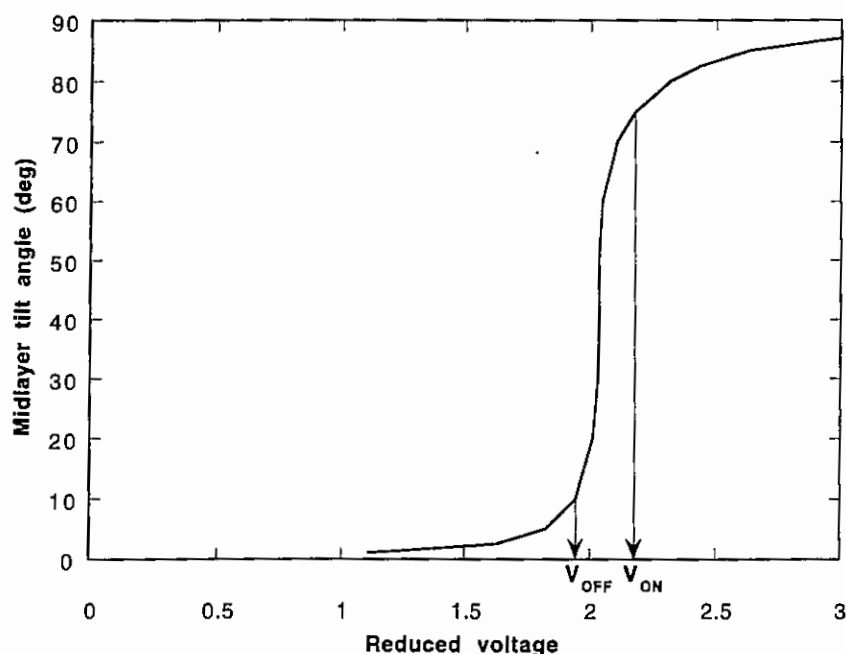


Figure 5.23. Electrodistortional curve of a 270° STN cell. The vertical lines indicate the nonselect voltage V_{OFF} and the select voltage V_{ON} .

$V_{\text{OFF}} = 1.9$. The midlayer tilt angle becomes about 75° at a reduced voltage of $V_{\text{ON}} = 2.2$.

As we know, a high contrast ratio requires a near zero denominator in Eqs. (5.2-37) and (5.2-38). As a result of the fast twist rate, the transmission of the field-OFF state $T(V_{\text{OFF}})$ varies with the wavelength (see Fig. 5.20). A near-zero integrated transmission of $T(V_{\text{OFF}})$ over the visible wavelengths is difficult to achieve. On the other hand, if the LC molecules can be homeotropically aligned because of an applied field, the transmission of the STN cell is simply $\cos^2(\beta + \phi - \alpha)$, which can be zero if the polarizers are oriented at the appropriate azimuth angles. It can be shown that $\beta + \phi - \alpha = \pm \pi/2$ corresponds to a pair of crossed polarizers. Therefore the NW mode (yellow mode) is expected to have a higher contrast ratio.

Viewing Angles

Referring to Figures 5.24 and 5.25, we consider the viewing characteristics of a 270° STN-LCD at three different applied voltages. Figure 5.24 shows the transmission as a function of the horizontal viewing angle. We note that the display exhibits a left-right viewing symmetry. A severe leakage of light in the black state occurs for viewing angles greater than $\pm 30^\circ$. The viewing exhibits no up-down symmetry. A significant leakage of light in the black state occurs for vertical viewing angles greater than $\pm 40^\circ$.

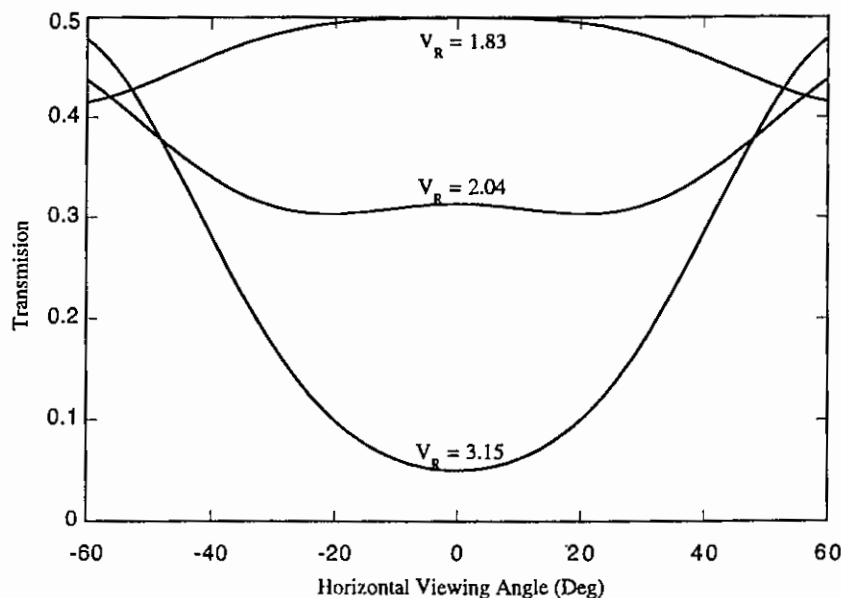


Figure 5.24. Horizontal viewing characteristics of a 270° STN-LCD at different reduced voltages, with $\alpha = 1.8^\circ$, $\beta = -1.8^\circ$, $\Delta nd = 0.75 \mu\text{m}$, $\lambda = 0.55 \mu\text{m}$. Reduced voltage $V_R = V/V_{c1}$, where V_{c1} is as given by Eq. (5.2-34).

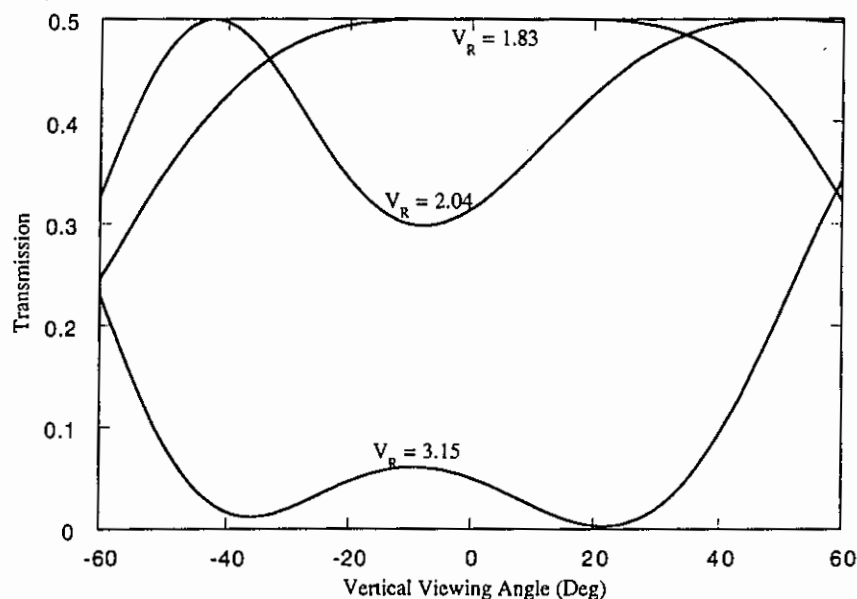


Figure 5.25. Vertical viewing characteristics of a 270° STN-LCD at different reduced voltages, with $\alpha = 1.8^\circ$, $\beta = -1.8^\circ$, $\Delta nd = 0.75 \mu\text{m}$, $\lambda = 0.55 \mu\text{m}$. Reduced voltage $V_R = V/V_{c1}$, where V_{c1} is as given by Eq. (5.2-34).

For NW-STN cells, in order to achieve a higher contrast ratio, the polarizers must be crossed. In this case, a high contrast can be achieved by using a high applied voltage in the black state. From Table 5.2, we realize that the cell gap d must be properly chosen.

5.3. NEMATIC LIQUID CRYSTAL DISPLAY (N-LCD) MODES

In both TN and STN, the LC director $\mathbf{n}$ undergoes a twist in the cell. The twisting of the director allows the phenomenon of waveguiding to occur in TN cells. As a result of the twist, the displays show asymmetric viewing characteristics (in the vertical direction). Birefringence phase compensation for improving the viewing characteristics (Chapter 9) in these cells is more difficult to realize because of the complicated director distribution. We now consider the use of nematic LC cells for display applications. In this section, we consider three important cases of interest. These are homogeneously parallel aligned cell, vertically aligned (VA) cell, and bend-aligned (BA) cell.

5.3.1. Parallel Aligned (PA) Cells

The LC cell in this case is effectively an a plate of uniaxially birefringent material in the field-OFF state. An electric field can be applied to reorient the LC

director in the cell to control the phase retardation. The discussion is divided into the two following cases depending on the direction of the applied electric field.

Vertical Switching (*E* Field Perpendicular to LC Layer)

Referring to Figure 5.26, we consider a nematic liquid crystal display (N-LCD) consisting of a homogeneously parallel aligned cell sandwiched between a pair of crossed polarizers. The *c* axis (director) of the cell is oriented at 45° relative to the transmission axes of the polarizers. For display applications, the LC cell is coated with transparent electrodes so that an electric field along the *z* axis can be applied to the cell. In the field-off state, the LC cell exhibits a phase

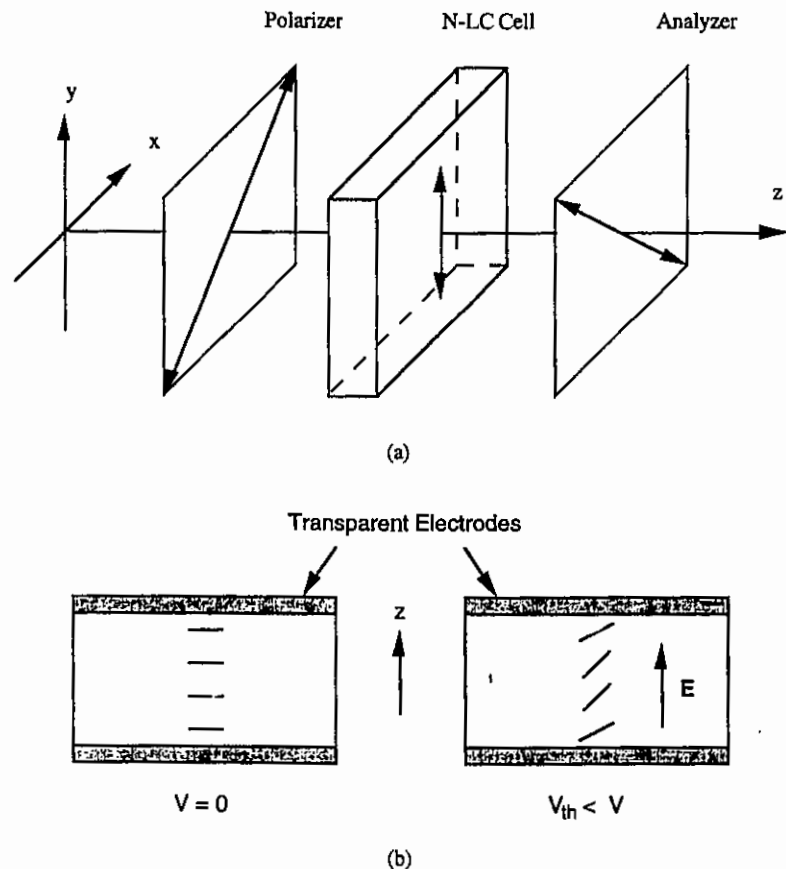


Figure 5.26. (a) Schematic drawing of a nematic liquid crystal display (N-LCD) using a parallel aligned cell; (b) director distribution in the field-off and field-on states (V =applied voltage; V_{th} = threshold voltage).

retardation of

$$\Gamma = \frac{2\pi}{\lambda} (n_e - n_o) d \quad (5.3-1)$$

where n_e, n_o are the principal refractive indices of the LC and material and d is the thickness of the LC layer. In a normally white (NW) operation, Γ is chosen to be an odd integral number of π (i.e., $\Gamma = \pi, 3\pi, 5\pi, 7\pi, \dots$). Usually, the lowest order ($\Gamma = \pi$) is chosen for the best viewing characteristics at large viewing angles. The transmission of the LCD is given by

$$T = \frac{1}{2} \sin^2 \frac{\Gamma}{2} \quad (\text{crossed polarizers}) \quad (5.3-2)$$

When an electric field is applied to the LC cell, the LC director is reoriented (tilt) toward the direction of the electric field. This leads to a decrease of the phase retardation, which is now written

$$\Gamma = \frac{2\pi}{\lambda} \int [n_e(\theta) - n_o] dz \quad (5.3-3)$$

where the integral is from $z = 0$ to $z = d$, $n_e(\theta)$ is given by

$$\frac{1}{n_e^2(\theta)} = \frac{\sin^2 \theta}{n_o^2} + \frac{\cos^2 \theta}{n_e^2} \quad (5.3-4)$$

where θ is the tilt angle of the LC director. It is important to note that $\theta(z)$ is a function of position z , and the tilt angle distribution $\theta(z)$ depends on the applied voltage. In the extreme case of homeotropical alignment ($\theta = 90^\circ$) by applying a strong electric field, the phase retardation reduces to zero. This leads to a zero transmission in a crossed polarizer configuration, according to Eq. (5.3-2). This mode of switching is sometimes referred to as the *electrically controlled birefringence mode* (ECB-LCD).

In principle, the same structure shown in Figure 5.26a can also be employed for a normally black (NB) operation. In this case Γ is chosen to be an integral number of 2π (i.e., $\Gamma = 2\pi, 4\pi, 6\pi, 8\pi, \dots$). Again, for viewing characteristics at large viewing angles, Γ is usually chosen to be 2π ($\Gamma = 2\pi$). The white state is obtained by applying a voltage so that the integrated phase retardation in Eq. (5.3-3) becomes π . This operation is, however, not practical for two reasons: (1) the LC cell, with $\Gamma = 2\pi$, must be two times thicker, which is very undesirable in LCDs; and (2) leakage of light occurs when the phase retardation becomes less than π , as the applied voltage becomes too high.

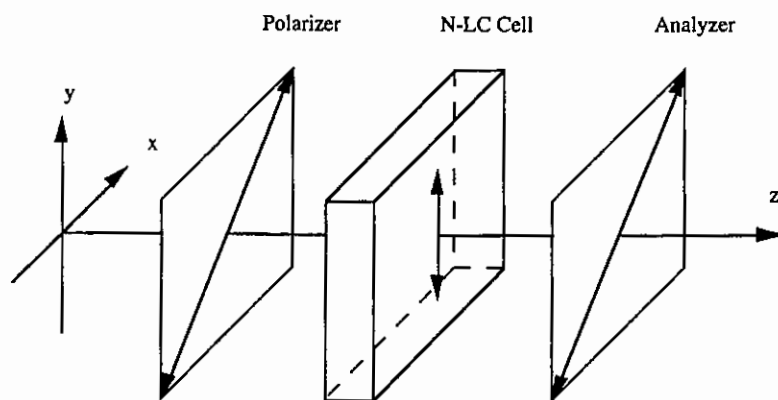


Figure 5.27. Schematic drawing of a normally black vertical switching mode of N-LCD.

Referring to Figure 5.27, we consider a more practical normally black (NB) operation that involves the use of parallel polarizers. In the field-OFF state, the LC cell behaves like a half-wave plate, where Γ is an odd integral number of π (i.e., $\Gamma = \pi, 3\pi, 5\pi, 7\pi, \dots$). Again, the lowest order ($\Gamma = \pi$) is usually chosen for the best viewing characteristics at large angles of viewing. The transmission of the LCD shown in Figure 5.27 is given by

$$T = \frac{1}{2} \cos^2 \frac{\Gamma}{2} \quad (\text{parallel polarizers}) \quad (5.3-5)$$

When an electric field is applied to the LC cell, the phase retardation Γ is as given by Eq. (5.3-3). In the extreme case of homeotropic alignment ($\theta = 90^\circ$) by applying a strong electric field, the phase retardation reduces to zero. This leads to a maximum transmission in a parallel-polarizer configuration, according to Eq. (5.3-5).

Electrodistortion (Electrooptical Distortion) in the Vertical Switching Mode. When an electric field is applied to the cell along the z axis, the LC molecules are aligned toward the z axis. The distribution of the director tilt angle $\theta(z)$ is a result of the balance between the elastic restoring force and the electric force, as well as the boundary conditions. According to Appendix B, the elastic energy density can be written

$$U_{EL} = \frac{1}{2} (k_1 \cos^2 \theta + k_3 \sin^2 \theta) \left(\frac{d\theta}{dz} \right)^2 \quad (5.3-6)$$

where θ is the tilt angle and k_1, k_3 are the elastic constants. The change of the electrostatic energy due to the reorientation of the directors can be written,

according to Eq. (B-2) in Appendix B

$$\Delta U_{EM} = \frac{1}{2} \frac{D_z^2}{(\epsilon_{\parallel} \sin^2 \theta + \epsilon_{\perp} \cos^2 \theta)} - \frac{1}{2} \frac{D_z^2}{\epsilon_{\perp}} \quad (5.3-7)$$

where D_z (= constant) is the z component of the displacement field vector. We note that the second term is a constant independent of the director orientation $\theta(z)$. According to the discussion in Appendix B, distortion occurs when the applied voltage is higher than a critical voltage:

$$V_{cl} = E_{cl}d = \pi \frac{\sqrt{k_1}}{\sqrt{\epsilon_{\parallel} - \epsilon_{\perp}}} \quad (\text{tilt mode}) \quad (5.3-8)$$

A numerical analysis can be carried out to obtain the director distribution $\theta(z)$ by using the equations in Appendix B. Figure 5.28 shows the director distribution $\theta(z)$ at various applied voltages. We note that the tilt angle $\theta(z)$ of the LC director is an increasing function of the applied voltage. In addition, the tilt angle $\theta(z)$ is a symmetric function with respect to the midlayer $z = d/2$. At an applied voltage of 5.71 V, the nematic cell is virtually homeotropically aligned with a tilt angle $\theta(z)$ of about 90° .

Figure 5.29 shows the transmission of the N-LCD at normal incidence as a function of the applied voltage. Using the green light as the design wavelength,

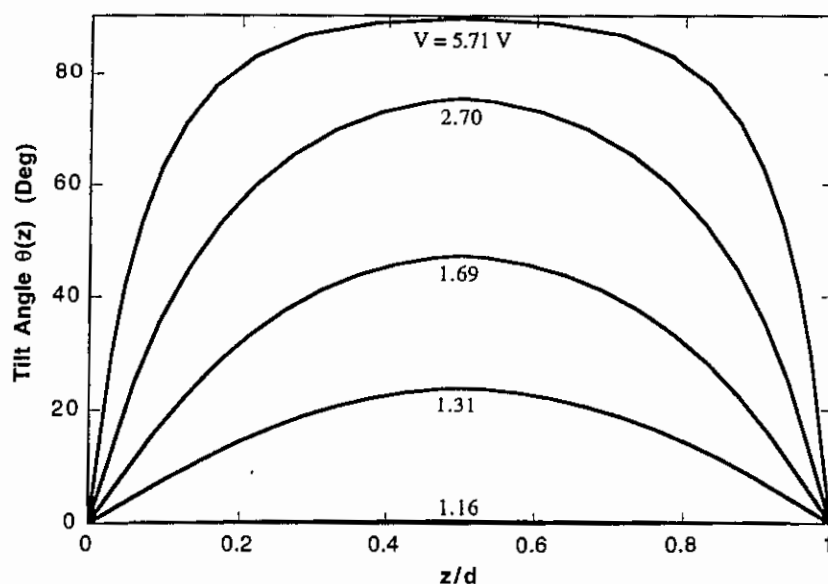


Figure 5.28. Director distribution $\theta(z)$ at various applied voltages.

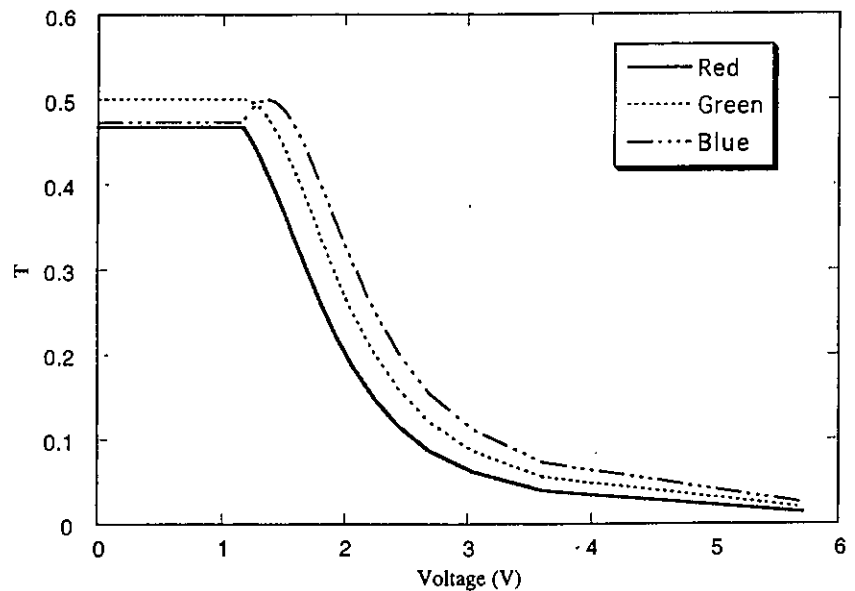


Figure 5.29. Transmission of the N-LCD at normal incidence as a function of the applied voltage.

the cell exhibits a maximum transmission of 0.5 for the green light. The transmission for the green light is a monotonically decreasing function of the applied voltage for $V_{th} < V$. The cell exhibits a lower transmission for the blue and red lights in the field-OFF ($V = 0$) state. As the applied voltage increases, the transmission for the blue light first increases to a maximum of 0.5, and then drops monotonically. As we know, the phase retardation $\Gamma(\lambda_{blue})$ is greater than π in the field-OFF state. As the LC director tilts, $\Gamma(\lambda_{blue})$ decreases. The maximum transmission of 0.5 occurs when $\Gamma(\lambda_{blue}) = \pi$. For the red light, the transmission is a monotonically decreasing function of the applied voltage. The phase retardation $\Gamma(\lambda_{red})$ is less than π and is a decreasing function of the applied voltage.

Figure 5.29 resembles Figure 5.9 for a NW TN-LCD. However, further comparison show that the transmission of TN-LCDs is less sensitive to wavelength variation than that of N-LCDs. This is a result of the waveguiding in the field-OFF state of TN-LCD's. In N-LCDs the transmission is based on polarization interference, which is often sensitive to wavelength.

In-Plane Switching (IPS)

Referring to Figure 5.30, we consider another switching mode of the homogeneously parallel aligned nematic cell for LCDs. This is known as the

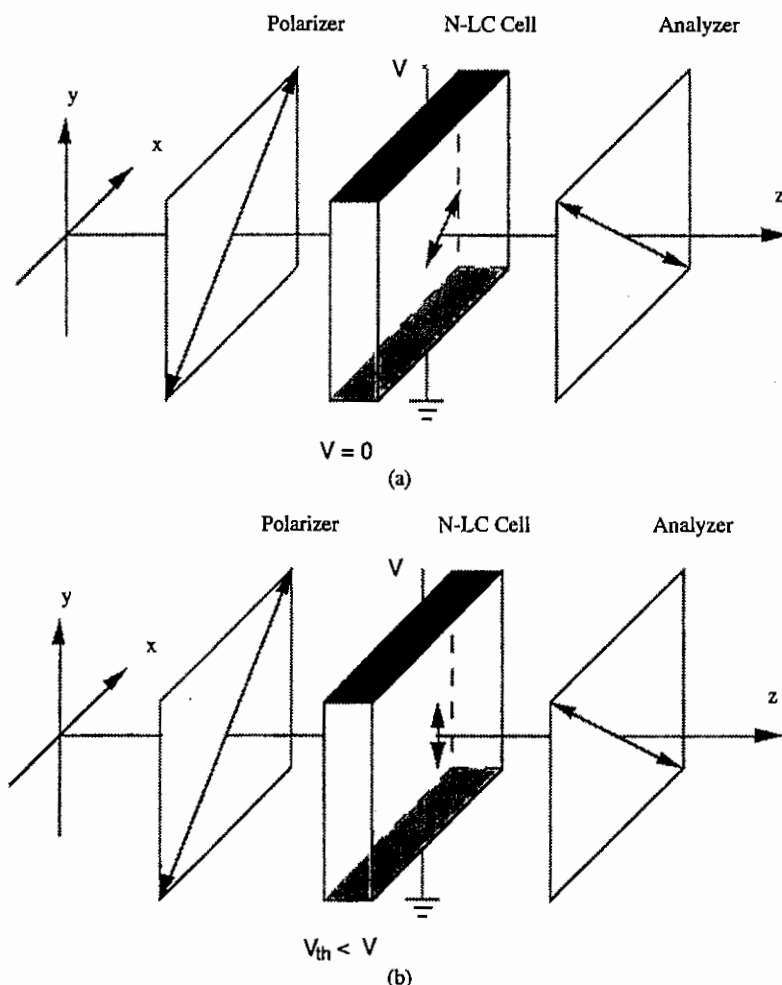


Figure 5.30. Schematic drawing of the in-plane switching mode of the N-LCD with normally black operations, where the shaded areas represent electrodes (V =applied voltage; V_{th} = threshold voltage). (a) In the field-OFF state, the LC director is parallel to the transmission axis of the polarizer. The transmission is zero, due to the crossed polarizers. (b) In the field-ON state, the director is aligned parallel to the electric field. Maximum transmission occurs with a proper choice of the cell thickness.

in-plane switching (IPS) [6,7] mode. In this case, the applied electric field is in the xy plane parallel to the LC layer surface. In the normally black (NB) operation, the transmission axis of the polarizer is parallel to the LC director at the input plane. The optical wave traversing through the LC cell is an extraordinary wave whose polarization state remains unchanged. This leads to a zero transmission due to the crossed polarizer configuration.

When an electric field is applied to the LC cell, the LC molecules are aligned toward the electric field (along the y axis). This leads to a new director

distribution with a twist $\phi(z)$ in the xy plane. As a result of the boundary condition due to the anchoring of LC molecules with the rubbed directions, the twist angle ϕ is a function of position z . Except for a small pretilt, the tilt angle θ is zero since the electric field is in plane. Unlike the TN-LC cells, the twist here is not a simple function of z . Thus, a simple transmission formula is not available for this case. The Jones matrix method must be employed by dividing the LC cell into N thin layers of equal thickness (with N in the range of 20). However, if the applied field is sufficiently strong enough to ensure alignment of most of the LC molecules along the direction of the field, the LC cell is approximately in a homogeneously parallel alignment, by neglecting the thin boundary layers at the surfaces. In this approximation, the transmission can be written

$$T = \frac{1}{2} \sin^2 2\Delta\phi \sin^2 \frac{\Gamma}{2} \quad (5.3-9)$$

where $\Delta\phi$ is the twist angle relative to the transmission axis of the polarizer due to the applied field, and Γ is the phase retardation of the LC cell. In the field-off ($V = 0$) state, $\Delta\phi = 0$, Eq. (5.3-9) yields a transmission of $T = 0$. Selection of a proper cell thickness such that $\Gamma = \pi$, and a proper reorientation of the LC director such that $\Delta\phi = 45^\circ$, Eq. (5.3-9) yields a transmission of $T = \frac{1}{2}$.

If we rotate the analyzer in Figure 5.30 by 90° , a normally white (NW) operation with parallel polarizers can be obtained. Several other modes of operation can be devised by choosing proper orientations of the LC cell relative to the transmission axes of the polarizers. Focusing on the operation shown in Figure 5.30, we now consider the electrodistortion of the LC cell under the application of an in-plane electric field.

Electrodistortion in the In-plane Switching (IPS) Mode. When an electric field is applied to the cell along the y axis, the LC molecules are aligned toward the y axis. The distribution of the director twist angle $\phi(z)$ is a result of the balance between the elastic restoring force and the electric force, as well as the boundary conditions. According to Appendix C, the elastic energy density can be written

$$U_{EL} = \frac{1}{2} k_2 \left(\frac{d\phi}{dz} \right)^2 \quad (5.3-10)$$

where ϕ is the twist angle measured from the x axis and k_2 is the twist elastic constant.

By taking into account the work done by the battery (or power supply), the net change in the electromagnetic energy density due to the reorientation of the LC director can be written, according to Eq. (C-4) in Appendix C

$$\Delta U_{EM} = \frac{1}{2} \epsilon_{\perp} E^2 - \frac{1}{2} (\epsilon_{\parallel} \sin^2 \phi + \epsilon_{\perp} \cos^2 \phi) E^2 \quad (5.3-11)$$

where ϕ is the twist angle, which depends on z . We note that the first term is a constant independent of the twist angle.

According to Appendix C, there exists a critical field E_{c1} , which is needed to create any twist of the director in the LC medium. For applied electric field $E < E_{c1}$, the midlayer twist angle $\Delta\phi$ is zero, with $\phi = 45^\circ$ throughout the cell. The critical field can be written

$$E_{c1}d = \pi \frac{\sqrt{k_2}}{\sqrt{\epsilon_{\parallel} - \epsilon_{\perp}}} \quad (5.3-12)$$

where k_2 is the twist elastic constant, and d is the cell gap. The critical voltage (threshold voltage) needed is thus given by

$$V_{th} = E_{c1}w = \pi \frac{\sqrt{k_2}}{\sqrt{\epsilon_{\parallel} - \epsilon_{\perp}}} \frac{w}{d} \quad (5.3-13)$$

where w is the distance between the electrodes of the LC cell. We note that the critical voltage is proportional to the square root of the twist elastic constant and is inversely proportional to the square root of dielectric anisotropy. Figure 5.31 shows the twist angle distribution $\phi(z)$ at various applied voltages. We note that

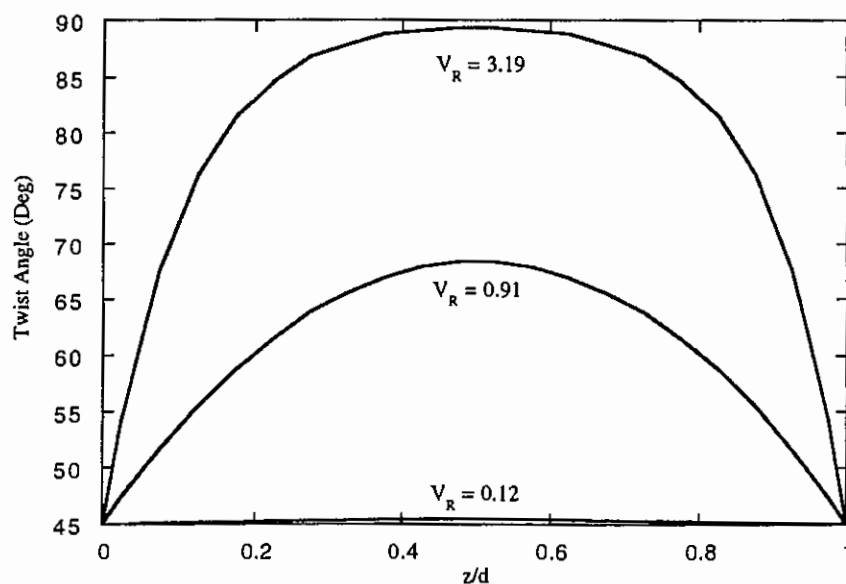


Figure 5.31. The twist angle distribution $\phi(z)$ at various reduced voltages, $V_R = V/V_{c1}$. Note that ϕ is measured from the x axis. So, $\phi = 45^\circ$ in the field-OFF state.

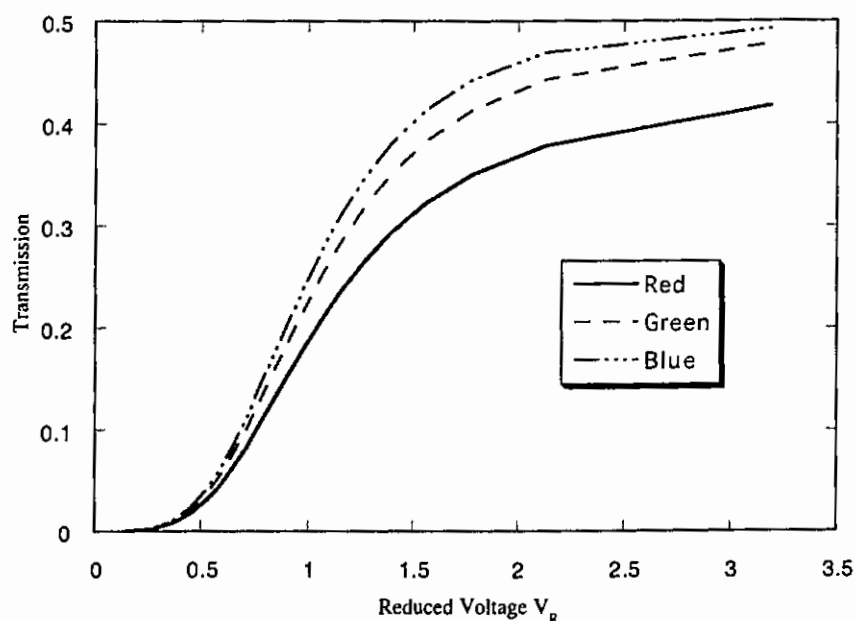


Figure 5.32. The transmission at normal incidence as a function of reduced voltage, $V_R = V/V_{cl}$.

the twist angle $\phi(z)$ is an increasing function of the applied voltage. The twist angle $\phi(z)$ is also symmetric with respect to the midlayer $z = d/2$. At a higher reduced voltage of $V_R = 3.19$, the twist angle $\Delta\phi$ due to the applied field is approximately 45° throughout the cell. Figure 5.32 shows the transmission at normal incidence as a function of applied voltage. We note that the transmission is an increasing function of the applied voltage. This is in agreement with Eq. (5.3-9), where $\Delta\phi$ is an increasing function of the applied voltage.

The cell gap used in Figure 5.32 is chosen so that the phase retardation for the green light is π when the LC director is oriented at 45° . Because of the boundary conditions, only molecules near the center of the cell are twisted by 45° (see Fig. 5.31). Therefore, the LC cell is slightly different from a half-wave plate and the transmission is not 50%. In addition, we note that the transmission of IPS-LCDs is also sensitive to wavelength variation. For IPS-LCDs to respond uniformly to all wavelengths, pixels with different colors need to have different cell gaps.

5.3.2. Vertically Aligned (VA) Cells

In a vertically aligned LC cell, the LC director is perpendicular to the surfaces of the cell (see Fig. 1.7). The LC cell in this case is effectively a c plate of uniaxially birefringent material in the field-off state ($V = 0$). An electric field can be applied to reorient the LC director in the cell to control the phase retardation.

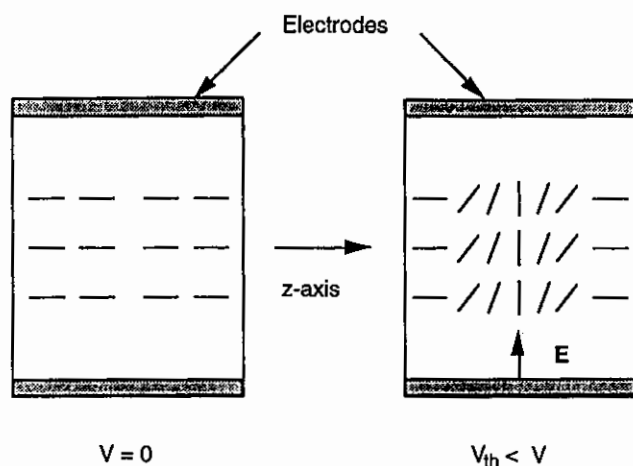


Figure 5.33. Schematic drawing and the director distribution in a VA cell under the influence of an applied electric field E that is perpendicular to the vertical alignment.

This mode of operation is also called *electrically controlled birefringence* (ECB). Figure 5.33 shows a schematic drawing of the director distribution in a VA cell under the influence of an applied electric field. The cells are enlarged for the illustration of the director distribution. In the VA-LCDs, the light is propagating along the z axis, whereas the electric field is perpendicular to the z axis.

Referring to Figure 5.34, we consider a normally black (NB) operation of the vertically aligned (VA) LCD using a pair of crossed polarizers. For normally incident light, the transmission of the LCD is, again, given by

$$T = \frac{1}{2} \sin^2 \frac{\Gamma}{2} \quad (\text{crossed polarizers}) \quad (5.3-14)$$

In the field-OFF state ($V = 0$), the LC molecules are homeotropically aligned along the z axis. For normally incident light, the propagation is along the c axis of the cell with a zero phase retardation ($\Gamma = 0$). Thus, the transmission is zero due to the crossed polarizers. When an electric field is applied to the LC cell, the LC director is reoriented (tilted) toward the direction of the electric field (see Fig. 5.34b). This leads to a distribution of the director tilt angle $\theta(z)$ and an increase of the phase retardation, which is now written

$$\Gamma = \frac{2\pi}{\lambda} \int [n_e(\theta) - n_o] dz \quad (5.3-15)$$

where the integral is from $z = 0$ to $z = d$, $n_e(\theta)$, is as given by Eq. (5.3-4). The

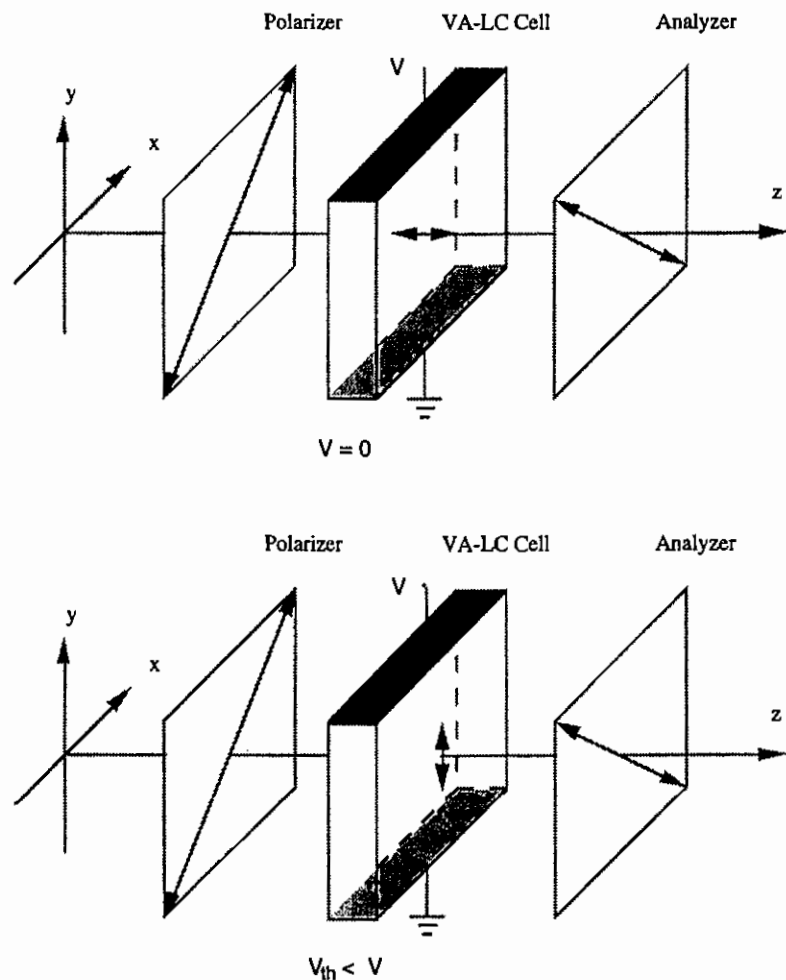


Figure 5.34. Normally black operation of a VA-LCD using a pair of crossed polarizers: (a) field-off state (black state); (b) field-on state (white state).

phase retardation is an increasing function of the applied voltage. Under the appropriate conditions, a phase retardation of π can be achieved, which leads to a maximum transmission. The director distribution $\theta(z)$ in a vertically aligned cell can be obtained by using the technique described in Appendix B.

5.3.3. Bend-Aligned (BA) Cells

In a bend-aligned LC cell (also known as the *Pi cell*), the LC director has a bend distribution with a total bend of 180° (π) from surface to surface. This cell configuration was originally proposed by Bos and Koehler/Beran for an electrically controllable wave plate [8]. An electric field can be applied to

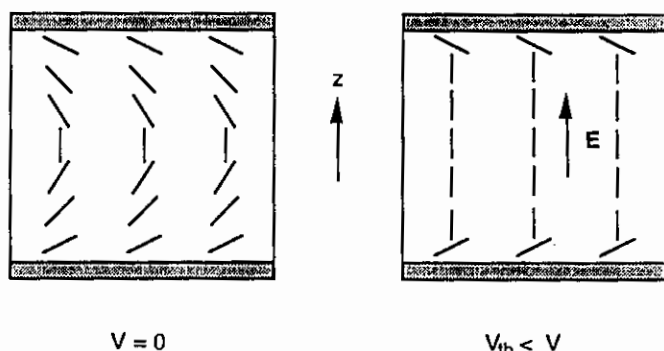


Figure 5.35. A schematic drawing of the director distribution in a bend-aligned cell under the influence of an applied electric field. The shaded areas are transparent electrodes.

reorient the LC director in the cell to control the phase retardation. Figure 5.35 shows a schematic drawing of the director distribution in a BA cell under the influence of an applied electric field. The cells are enlarged for the illustration of the director distribution. In the BA-LCDs, transparent electrodes are needed as the light is propagating along the z axis, which is also the direction of the electric field.

The BA cells, which have a phase retardation that is electrically controllable, can be employed for display applications. Figure 5.36 shows an example of BA-LCDs with a normally white (NW) operation using a pair of crossed polarizers. For normally incident light, the transmission of the LCD is, again, given by

$$T = \frac{1}{2} \sin^2 \frac{\Gamma}{2} \quad (\text{crossed polarizers}) \quad (5.3-16)$$

where Γ is the phase retardation given by

$$\Gamma = \frac{2\pi}{\lambda} \int [n_e(\theta) - n_o] dz \quad (5.3-17)$$

where the integral is from $z = 0$ to $z = d$, $n_e(\theta)$ is given by Eq. (5.3-4). In a (NW) operation, Γ is chosen to be an odd integral number of π (i.e., $\Gamma = \pi, 3\pi, 5\pi, 7\pi, \dots$) in the field-off state ($V = 0$). Usually, the lowest order ($\Gamma = \pi$) is chosen for the best viewing characteristics at large viewing angles. This can be achieved by a proper choice of the cell thickness and the LC birefringence. When an electric field is applied to the LC cell, the LC director is reoriented (tilted) toward the direction of the electric field (see Fig. 5.35). This leads to a decrease of the phase retardation. When the field is sufficiently strong, the cell becomes homeotropically aligned, except at the boundaries. For normally incident light,

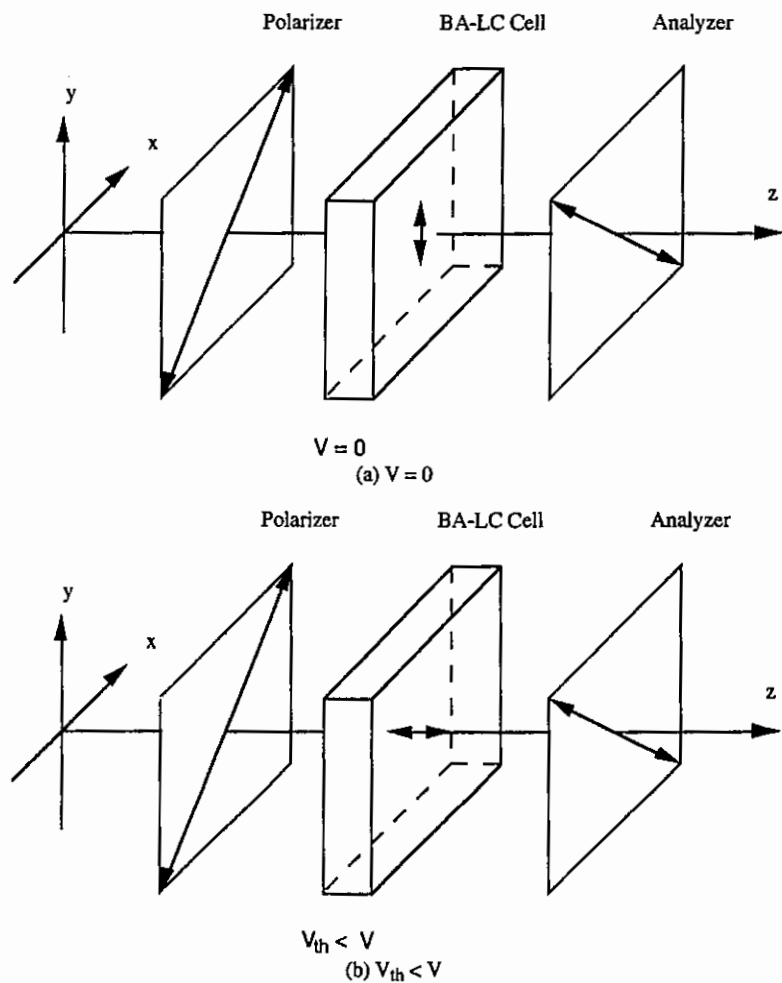


Figure 5.36. A schematic drawing of a BA-LCD in the normally white operation. (a) $V = 0$: The plane of bending (yz -plane) is oriented at 45° relative to the transmission axes of the polarizers. (b) $V_{th} < V$: In the field-on state, the liquid crystal director is homeotropically aligned along the z axis, leading to a dark state.

Eq. (5.3-17) yields a zero phase retardation ($\Gamma = 0$). Thus, the transmission is zero as a result of the crossed polarizers. The director distribution $\theta(z)$ in a BA cell (or Pi cell) can be obtained by using the technique described in Appendix B.

Symmetry Property of Pi Cell

The BA cell has a unique property of viewing characteristics. As a result of the bend alignment, the phase retardation in the field-off state is a symmetrical function of the vertical viewing angles whose bend plane is in the yz plane. This viewing symmetry can be explained as follows with the aid of Figure 5.37.

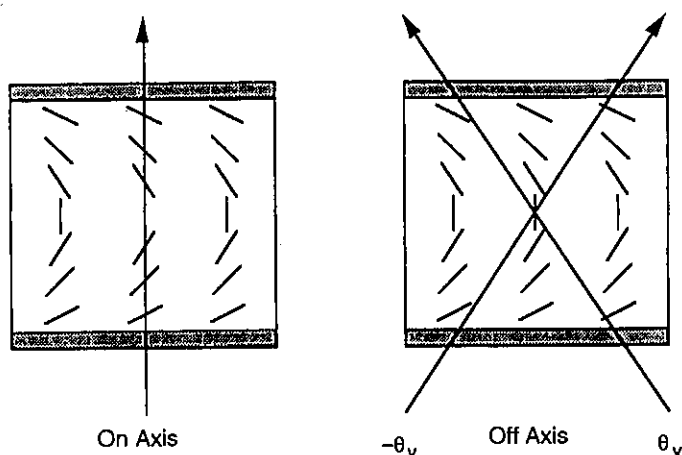


Figure 5.37. On-axis and off-axis viewing through a bend-aligned cell in the field-OFF state ($V = 0$).

As a result of the bend alignment, the phase retardation for viewing in the bend plane has a self-compensating nature. The increase in the phase retardation in the first half of the cell due to off-axis viewing is compensated by the decrease in the phase retardation in the second half of the cell. This can be seen by decomposing the phase retardation integral into two parts:

$$\Gamma = \Gamma_1 + \Gamma_2 \quad (5.3-18)$$

where

$$\Gamma_1 = \frac{2\pi}{\lambda} \int_0^{d/2} [n_e(\theta) - n_o] dz \quad (5.3-19)$$

$$\Gamma_2 = \frac{2\pi}{\lambda} \int_{d/2}^d [n_e(\theta) - n_o] dz \quad (5.3-20)$$

In other words, Γ_1 is the phase retardation due to the first half of the cell, whereas Γ_2 is that of the second half of the cell. Both Γ_1 and Γ_2 are functions of the vertical viewing angle θ_v . As a result of the symmetric bend alignment relative to the center of the cell, it can be easily shown, by examining Figure 5.37, that the partial retardations, Γ_1 , Γ_2 , satisfy the following relation

$$\Gamma_1(\theta_v) = \Gamma_2(-\theta_v) \quad (5.3-21)$$

It follows immediately that, according to Eqs. (5.3-18)

$$\Gamma(\theta_v) = \Gamma(-\theta_v) \quad (5.3-22)$$

which leads to a vertical viewing symmetry. The symmetry in the vertical viewing direction is also responsible for the wide viewing angle of LCDs based on BA cells. It is important to note that the viewing will remain symmetric in the field-ON state provided the bend alignment remains symmetric relative to the center of the cell. The same result [Eq. (5.3-22)] can also be easily obtained using the principle of reciprocity and the symmetry of the cell [See Problem 5.15].

5.4. POLYMER DISPERSED LIQUID CRYSTAL DISPLAYS (PD-LCDs)

We recall that, in Chapter 1, we discussed the milky appearance of nematic liquid crystal in a glass bottle. The milky appearance is a result of the severe scattering of light due to the discontinuities of the refractive index at the boundaries of nematic LC droplets or domains whose physical dimensions are in the micrometer range. The milky appearance disappears when the sample is (1) heated beyond the clearing point so that the liquid crystal becomes optically isotropic or (2) poled into a single domain by using physical or chemical means. The scattering phenomenon of nematic LC droplets can be employed for display applications.

Referring to Figure 5.38, we consider a suspension of nematic LC droplets in a host medium (polymer) of matched refractive index n_p . The refractive indices of the nematic LC droplets depend on the order parameter S of the LC molecules in the droplets. Let the effective refractive indices be written

$$n'_e = n'_e(S), \quad n'_o = n'_o(S) \quad (5.4-1)$$

where S is the order parameter. In the case of a homogeneously parallel alignment, the order parameter S is 1. In this case the refractive indices are written

$$n'^2_e(S=1) = n_e^2 \quad n'^2_o(S=1) = n_o^2 \quad (5.4-2)$$

where n_e, n_o are the principal refractive indices of the nematic liquid crystal. In the extreme case of disorder (or a uniform distribution of $\mathbf{n}$ over all angles) with $S=0$, the droplet becomes optically isotropic with the refractive index given by

$$n^2 = \frac{n_e^2 + 2n_o^2}{3} \quad (S=0) \quad (5.4-3)$$

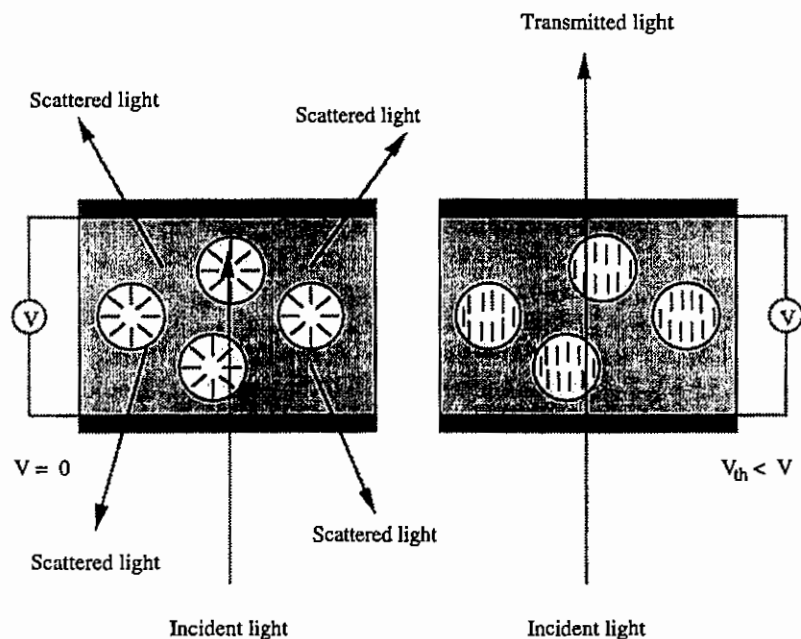


Figure 5.38. Schematic drawing of a polymer dispersed liquid crystal display (PD-LCD). The liquid crystal droplets are dispersed in a polymer host medium with a matching refractive index. Scattering occurs in the field-OFF state ($V = 0$) when the liquid crystal molecules are not fully aligned in the droplets.

In most general cases, the LC molecules in the droplets are partially aligned because of the boundary conditions and the intermolecular forces. The effective refractive indices are somewhere between n_o and n_e . In other words

$$n_o < n'_e(S) < n_e, \quad n_o < n'_o(S) < n_e \quad (5.4-4)$$

The refractive index of the host medium is chosen to be n_o (i.e., $n_p = n_o$). Thus there exists a discontinuity of the refractive index at the droplet's boundary. As a result, scattering of light occurs. Consider now the incidence of a beam of unpolarized light into the LC cell. In the field-off state, the LC molecules within the droplets are only partially aligned. In addition, the averaged director orientation $\mathbf{n}$ of the droplets (if $S \neq 0$) exhibits a random distribution of orientation within the cell. The beam is significantly attenuated as a result of the energy loss via scattering.

When a sufficiently strong field ($\approx 1 \text{ V}_{\text{rms}}/\mu\text{m}$) is applied to the cell, all the LC molecules are aligned parallel to the electric field. If the light is also propagating in the direction parallel to the electric field, then the beam of light "sees" a refractive index of n_o , which is matched with the refractive index of the surrounding medium. In this state, the whole cell appears to be optically

homogeneous (like a thin sheet of glass) for normally incident light. Thus the beam of light transmits through the cell with no scattering and no attenuation. This is the principle of operation of the PD-LC.

When the electric field is turned OFF, the droplets return to their original orientations with a partial alignment of the LC molecules within the droplets. This requires a restoring force. Polymer materials are ideal for this purpose. In most PD-LCDs, encapsulation procedures are employed to create micrometer-size capsules of LC droplets inside a polymer. By virtue of the rodlike or chainlike nature of the polymers, LC molecules within the droplets are likely to anchor in the directions parallel to the local polymer chains. Without an appropriate boundary to provide the restoring force, the switching from the ordered state to a disordered state must rely on the Brownian motion, which can take a very long time. It is important to note that there are several different configurations of director distribution in the droplets in the field-OFF state, depending on the specific boundary conditions and the shape of each droplet.

The contrast ratio of the PD-LCDs is defined as

$$C(V) = \frac{T(V)}{T(0)} \quad (5.4-5)$$

where V is the applied voltage, $T(V)$ is the transmission of the field-ON state and $T(0)$ is the transmission of the field-OFF state. A high contrast ratio requires a high transmission in field-ON state and a near-zero transmission in the field-OFF state. High transmission in the field-ON state requires a good matching of the refractive index n_p of the host medium with the ordinary refractive index n_o of the liquid crystal. In other words, $n_p(\lambda) = n_o(\lambda)$ for most wavelengths λ in the spectral regime of operation.

The residual transmission (leakage) $T(0)$ depends on the attenuation of the incident beam due to scattering loss in the field-OFF state. Assuming a spherical shape of radius a , the scattering cross section σ can be written, according to Mie scattering theory [9-11]

$$\sigma = 2\pi a^2 \left[1 - \frac{\sin 2ka \delta n}{ka \delta n} + \frac{\sin^2 ka \delta n}{(ka \delta n)^2} \right] \quad (5.4-6)$$

where k is the wavenumber of light in vacuum ($k = 2\pi/\lambda$) and δn is a measure of the index discontinuity

$$\delta n = \left(\frac{n'}{n_p} \right) - 1 \quad (5.4-7)$$

where n' is the refractive index of the LC droplet according to Eq. (5.4-1),

depending on the polarization state, and n_p is the index of refraction of the host medium. It is important to note that Eq. (5.4-6) is derived by assuming a sphere of isotropic medium with a refractive index of n' , and is valid only when the index discontinuity is very small, e.g., ($\delta n' \ll 1$).

In the limit of large spheres with $1 \ll ka \delta n$, the scattering cross section becomes, according to Eq. (5.4-6)

$$\sigma = 2\pi a^2 \quad (5.4-8)$$

which is exactly twice the geometric cross section of the sphere. This result can also be obtained by assuming that the portion of light falling in the geometric area of the sphere is refracted and deflected into other directions. In addition, diffraction of light due to the removal of the amount of light in the geometric cross section also causes a scattering loss of the same amount of energy. This leads to the factor of 2 in Eq. (5.4-8). It is important to note that Eq. (5.4-8) is valid only when the index discontinuity is large enough ($1 \ll ka \delta n$) to justify the complete removal of light falling into the geometric area via refraction. In most PD-LCDs with micrometer-size particles ($a \approx 1 \mu\text{m}$), where $ka \delta n$ is near unity, the scattering cross section must be obtained by using Eq. (5.4-6). In case when $ka \delta n \ll 1$, the scattering cross section can be written approximately as

$$\sigma = 2\pi a^2 (ka \delta n)^2 \quad (5.4-9)$$

Figure 5.39 plots the normalized scattering cross section ($\sigma/2\pi a^2$) as a function of the parameter $ka \delta n$. We note that the normalized cross section increases as a quadratic function according to Eq. (5.4-9) when $ka \delta n$ is small. The normalized cross section oscillates around 1 when $ka \delta n$ exceeds 2. This figure can be employed to determine the optimum droplet size for maximum scattering cross section. According to this figure, maximum scattering cross section occurs at around $ka \delta n = 2$.

The attenuation of a beam of light in a scattering medium is given by

$$I(z) = I(0) \exp \{-\alpha z\} \quad (5.4-10)$$

where $I(z)$ is the intensity at position z ; the attenuation coefficient α is related to the scattering cross section by

$$\alpha = N\sigma \quad (5.4-11)$$

where N is the number of droplets per unit volume. The transmission in the

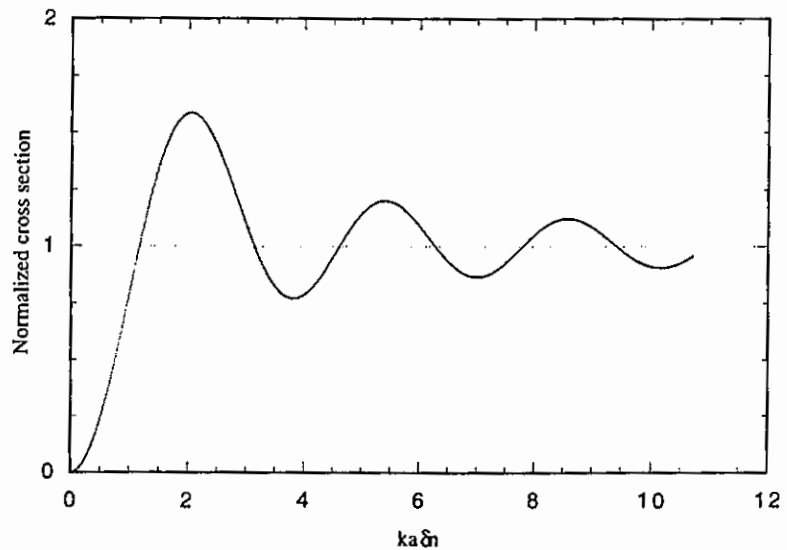


Figure 5.39. The normalized scattering cross section ($\sigma/2\pi a^2$) as a function of the parameter $ka\delta n$.

field-OFF state can thus be written

$$T(0) = \exp \{-\alpha d\} \quad (5.4-12)$$

where d is the thickness of the cell.

Example 5.3: 1- μm Droplets. Consider a droplet size of $a = 1\mu\text{m}$, with a concentration of $N = 0.1\mu\text{m}^{-3}$. This corresponds to a fill factor of about 40%. Using $\lambda = 0.55\mu\text{m}$, $\delta n = 0.1$, and Eqs. (5.4-6), we obtain a scattering cross section of $\sigma = 2\pi a^2(0.97) = 6.09\mu\text{m}^2$ and an attenuation coefficient of $\alpha = 0.609\mu\text{m}^{-1}$, which yields a transmission of $\exp(-6.09)$ in a cell $10\mu\text{m}$ thick, or $\exp(-12.2)$ in a cell $20\mu\text{m}$ thick. ■

Example 5.4: 2- μm Droplets. Consider a droplet size of $a = 2\mu\text{m}$, with a concentration of $N = 0.0125\mu\text{m}^{-3}$. This corresponds to the same fill factor of about 40%. Using $\lambda = 0.55\mu\text{m}$, $\delta n = 0.1$, and Eq. (5.4-6), we obtain a scattering cross section of $\sigma = 2\pi a^2(1.54) = 38.7\mu\text{m}^2$ and an attenuation coefficient of $\alpha = 0.484\mu\text{m}^{-1}$, which yields a transmission of $\exp(-4.84)$ in a cell $10\mu\text{m}$ thick, or $\exp(-9.68)$ in a cell $20\mu\text{m}$ thick. ■

So far, we have considered the transmission only at normal incidence (along the direction of the applied electric field). This is the situation when the beam of light is propagating along the c axis of the nematic LC droplets. For off-axis transmission, the LC droplets are optically birefringent. Thus, one polarization

component will see a refractive index of n_o , while the other polarization component will see a refractive index of $n_e(\theta)$ given by

$$\frac{1}{n_e^2(\theta)} = \frac{\sin^2 \theta}{n_e^2} + \frac{\cos^2 \theta}{n_o^2} \quad (5.4-13)$$

where θ is the angle between the c -axis and the direction of propagation in the droplet. Since the refractive index of the host medium is matched with n_o , the extraordinary polarization component of the incident beam will suffer from scattering loss at the boundary of the droplets as a result of the difference between n_p and $n_e(\theta)$ in the field-ON state. This leads to a decrease in the transmission and a degradation of the display at large viewing angles.

PD-LCDs offer several unique advantages, including high throughput due to the absence of polarizers, and simple cell construction due to the absence of alignment layers. They are particularly well suited for use in projection displays where high intensity operations are needed. The residual scattering in the bright state due to the extraordinary polarization component at large viewing angles is undesirable for direct-view displays. In the projection displays, the beam of light is often collimated. Thus, the scattering for off-axis light in the bright state is not a problem in projection displays. The PD-LCDs are also particularly useful for applications such as a variable light blocker, or a smart window whose transmission (from dark to clear) can be controlled electrically. Further discussion on the properties of PD-LCDs can be found in Doane [12].

5.5. REFLECTIVE LCDs

So far we have discussed LC displays only in the transmission configuration. In reflective LCDs, a mirror or reflector is placed at the back of the LC cell. Thus, the incident beam of light passes through the LC cell twice before exiting the device. Figure 5.40 shows a schematic drawing of reflective and transmissive modes of LCDs. We note that a rear mirror is an essential part in the reflective LCDs. Generally speaking, all transmissive LCDs discussed earlier can be employed for reflective LCDs. As a result of the double passage, only one polarizer is needed to obtain polarization interference to display the bright or dark state of reflection. On the other hand, reflective LCDs have one less degree of freedom in designing the normally white or normally black state. As a result, zero or unity reflectivity occurs only at specific values of Δnd in TN- or STN-based reflective LCDs. For example, normally black reflective LCDs based on a 45° TN cell with $R=0$ in the field-OFF state is possible only when $\Delta nd/\lambda = 0.29, 0.68, 1.23$, and higher and with a polarizer azimuth angle of $-15.3^\circ, 34.3^\circ, -39.1^\circ$, and higher respectively. This issue was discussed in Section 4.6.

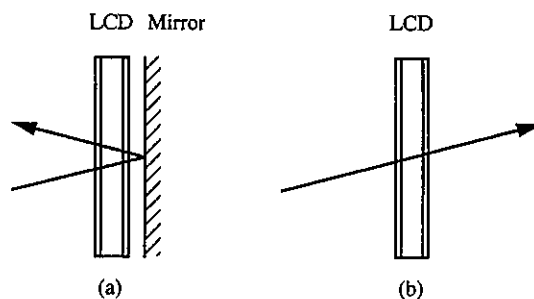


Figure 5.40. (a) A reflective LCD; (b) a transmissive LCD.

5.5.1. Reflection Properties at Normal Incidence

We now consider the reflectivity of a reflective LCD as a function of the applied voltage. Under the influence of an applied field, the LC director in the TN-LC cell undergoes a redistribution. The techniques described in Section 5.1 (or Appendix D) can be applied to obtain the director distribution functions $\theta(z)$ and $\phi(z)$. Once these functions are obtained, the Jones matrix method can then be employed to calculate the reflection properties. Again, we need to divide the LC layer into several thin sublayers (20–100). Each of these sublayers will be treated as a thin homogeneous birefringent plate with its own c axis orientation. Figure 5.41 shows the reflectivity $R(\lambda, V)$ of a reflective LCD based on a 45° TN-LC cell as a function of wavelength and applied voltage. The cell is designed to have a zero transmission at $\lambda = 0.55 \mu\text{m}$ in the field-OFF state ($V = 0$). A slight leakage of light occurs at other wavelengths. As the applied voltage increases, the zero transmission occurs at a shorter wavelength (blue shift). Other than that, the transmission in the visible spectrum increases with the applied voltage. In the limiting case of a high applied voltage, the cell becomes homeotropically aligned with a zero phase retardation. As it has only one polarizer, the cell transmits all wavelengths of light.

5.5.2. Reflection Properties at Oblique Incidence

We now consider the reflection properties of reflective LCDs at large viewing angles (θ, ϕ) . In the case when two polarizers are employed in the reflective LCDs (see Fig. 5.42b), the reflectivity can be easily obtained as follows:

$$R(\theta, \phi) = T(\theta, \phi)T(\theta, \phi + \pi) \quad (5.5-1)$$

where $T(\theta, \phi)$ is the transmission of the LCD without the mirror. The transmission $T(\theta, \phi)$ and $T(\theta, \phi + \pi)$ can be obtained by using the Jones matrix method.

The reflection properties of TN-based reflective LCDs using only one polarizer have been discussed in Section 4.6 for normal incidence. For a general

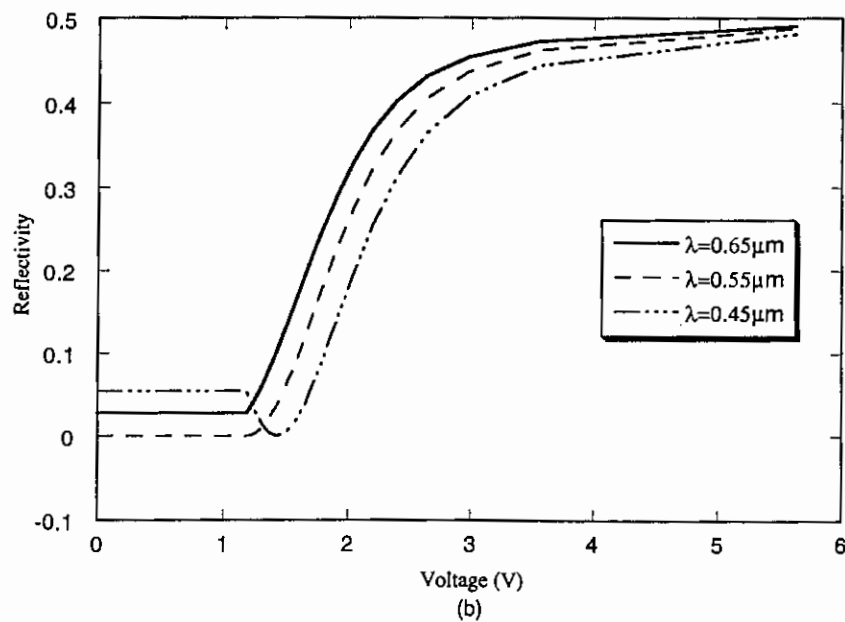
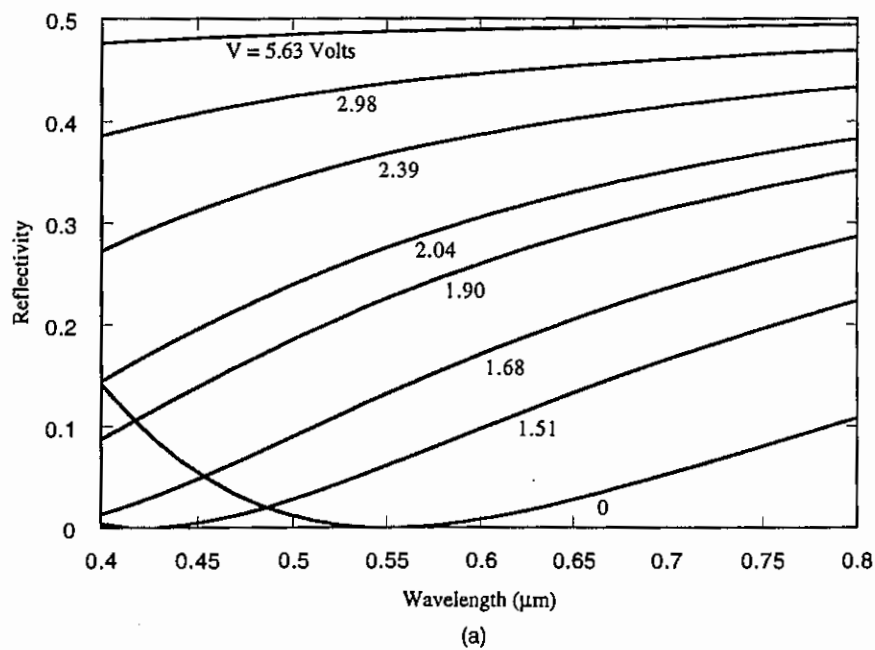


Figure 5.41. (a) The reflectivity spectrum $R(\lambda, V)$ of a reflective LCD at various applied voltages. (b) The reflectivity $R(\lambda, V)$ versus applied voltage V of a reflective LCD based on a 45° TN-LC cell, at $\lambda = 0.45, 0.55$, and $0.65 \mu\text{m}$.

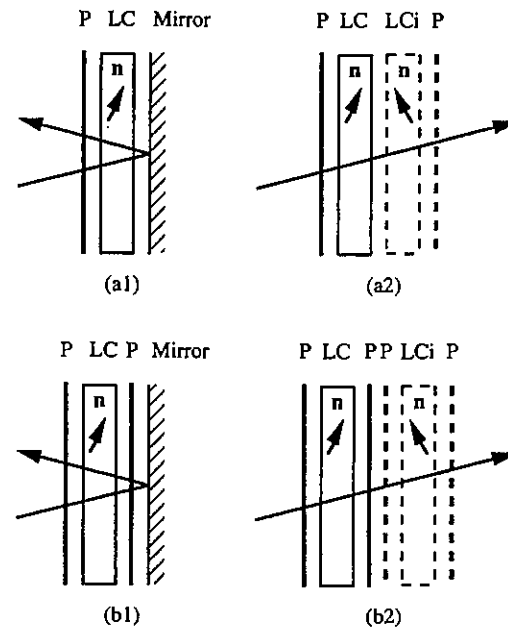


Figure 5.42. (a1) Schematic drawing of a reflective LCD with one polarizer (P=polarizer, LC=liquid crystal cell). (a2) An equivalent circuit for the calculation of the reflection properties of the display in (a1). LCi is the mirror image of the LC cell. Note that the mirror image of the director $\mathbf{n}$ is reversed in direction. In addition, the twist direction is also reversed in LCi. (b1) A reflective LCD with two polarizers. (b2) An equivalent circuit for the calculation of the reflection properties of the display in (b1).

angle of incidence (θ, ϕ), an equivalent circuit such as that shown in Figure 5.42 can be employed in conjunction with the Jones matrix method to calculate the reflectivity of the displays.

Using the general properties of the Jones matrix under mirror reflection derived in 4.1.3, we can express the reflection for the reflective LCD as

$$R(\theta, \phi) = |\mathbf{V} \cdot \tilde{M}(\theta, \phi + \pi) M(\theta, \phi) \mathbf{V}|^2 \quad (5.5-2)$$

where M is the Jones matrix of the LC cell and $\mathbf{V}$ is the Jones vector of the incident beam as defined by the polarizer.

It is known that the transmission of NW TN-LCDs exhibits a left-right symmetry in its viewing angle characteristics. In other words

$$T(\theta, \phi) = T(\theta, \pi - \phi) \quad (\text{NW TN-LCD}) \quad (5.5-3)$$

The left-right symmetry in viewing is a desirable property of the displays. This

symmetry, however, does not exist in a general reflective LCD based on TN or STN cells. In a reflective LCD an incident beam along the direction (θ, ϕ) is reflected into the direction $(\theta, \phi + \pi)$. The principle of reciprocity yields the following symmetry (twofold rotation symmetry):

$$R(\theta, \phi) = R(\theta, \phi + \pi) \quad (5.5-4)$$

This is a general property of reflective LCDs (i.e., a 180° rotation symmetry). The absence of left-right symmetry in a general reflective LCD is undesirable for direct-viewing applications. This is not a problem in projection displays. In a manner very similar to BA-LCDs, the reflective LCDs also exhibit a self-phase-compensation property. This leads to a wide viewing angle characteristics. By properly selecting the twist angle and the polarizer orientation, it is possible to design a reflective LCD which exhibits a left-right viewing symmetry. In fact, a display such as this will also exhibit a up-down viewing symmetry, according to Eq. (5.5-4).

5.6. PROJECTION DISPLAYS

In general, all LCDs described so far can be employed for projection displays. Most projection displays, especially for large screen projectors, require high brightness and high contrast ratios. Generally speaking, three LCDs are need for the three spectral regimes red, blue, and green in the projection displays. With the use of properly designed dielectric mirrors, the fundamental energy loss of $\frac{2}{3}$ due to color filters in direct-view displays can be eliminated or minimized. Referring to Figure 5.43, we consider a projection display system that involves the use of reflective mode LCDs. For projection displays, the incidence angles are usually relatively small (i.e., near normal incidence). Low loss polarizing beamsplitters (PBS) are employed to improve the optical energy throughput (brightness). The incident beam of light is split into three spectral components—R, G, and B—using spectral reflectors (dichroic mirrors). In fact, two high quality spectral filters for the blue and green spectral regimes and a mirror are adequate for the purpose. All the spectral components are polarized by the polarizing beamsplitters. Each of the polarizing beamsplitters is oriented at the proper azimuth angle as determined by the recipe described in Section 4.6. For the purpose of description, let us assume that the LC cells are 45° TN-LC and the RGB wavelengths are 0.65, 0.55, and 0.45 μm . All the TN-LC cells are located at the focal plane of the projection lens. In the normally white transmission (NWT) operation ($T=0.5$, or $R=0$) for a 45° TN-LC cell, consider the example in the first row of Table 4.3 with a polarizing beamsplitter azimuth of -15.26° and a Mauguin parameter of $u=1.16$. To achieve a normally white operation with $T=0.5$, the $\Delta n d$ for the LC cells must be 0.19 μ , 0.16 μ , and 0.13 μm for the three wavelengths $\lambda=0.65, 0.55$, and 0.45 μm , respectively.

with corresponding wavelengths. In practice, some of the spectral reflectors (R and B') can be replaced by mirrors to eliminate or minimize the energy loss due to dichroic filters.

5.7. OTHER DISPLAY SYSTEMS

Up to this point, all LCDs described in this book involve the use of nematic liquid crystals as the electrooptical medium to modulate the intensity of light. Actually, several other phases of the liquid crystals can also be employed for display applications. In the following, we discuss ferroelectric LCDs and cholesteric LCDs.

5.7.1. Ferroelectric LCD

The smectic $C(S_c)$ phase of LC materials can be employed for display applications. Figure 5.45 shows the molecular arrangement in the S_c phase. We note that the S_c phase consists of layers of LC molecules. The thicknesses of the layers are less than the molecular length. As a result, the molecules must tilt at an angle relative to the layer normal. Within each layer, the positions of the molecules are random with the long molecular axis $\mathbf{n}$ inclined at an angle θ with respect to the layer normal ($\mathbf{k}$). Because the angle θ is fixed, the molecular orientation is confined in a cone with a half-apex angle of θ . If the azimuth angle ϕ is fixed within a layer and remains the same throughout all the layers, the structure has a symmetry of the monoclinic class (see Table 3.1b), with a plane of symmetry formed by the vector $\mathbf{n}$ and the unit vector $\mathbf{k}$. The structure also exhibits a twofold rotational symmetry C_2 with the axis of symmetry parallel to $\mathbf{n} \times \mathbf{k}$. In other words, the LC medium is invariant under a rotation of 180°

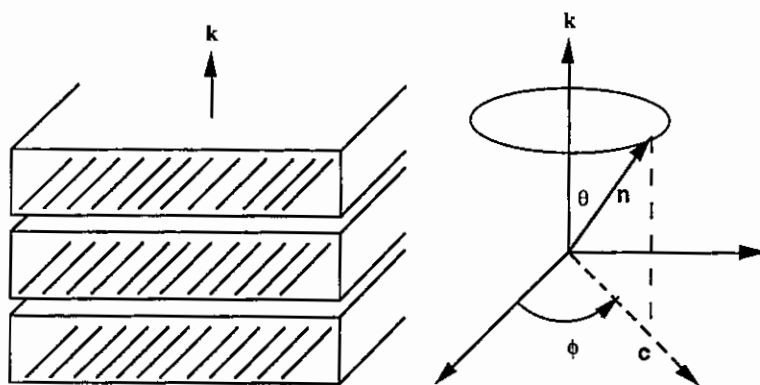


Figure 5.45. The structure of smectic C phase of liquid crystals, where $\mathbf{k}$ is a unit vector normal to the molecular layer, θ is the angle between the molecular axis $\mathbf{n}$ and the layer normal $\mathbf{k}$, and ϕ is the azimuth angle of the molecular axis in the plane of the layer.

around the axis of symmetry along the direction $\mathbf{n} \times \mathbf{k}$. this symmetry is a result of an equal population distribution of molecules with axes in $\mathbf{n}$ and $-\mathbf{n}$.

Generally speaking, compounds exhibiting the S_c phase have a transverse component of the permanent electric dipole moments. In other words, the molecular dipole moment is not parallel to the molecular axis $\mathbf{n}$. However, the net dipole moment in a layer is zero due to the inversion symmetry and C_2 symmetry. The inversion symmetry here is a result of an equal population distribution of nonchiral molecules with axes in $\mathbf{n}$ and $-\mathbf{n}$. This is no longer true in a smectic C^* phase. If the liquid crystal in the smectic phase is doped with molecules with chirality, the structure distorts when the director precesses around the $\mathbf{k}$ axis leading to a helical configuration. The inversion symmetry and the mirror reflection symmetry are destroyed by the chirality. This is known as the smectic C^* phase. The pitch of the helix is often several thousand times the layer thickness. The optical properties of this structure are similar to those of a cholesteric liquid crystal where the local dielectric tensor in each layer is biaxial. In addition, a net dipole moment exists in each layer. As a result of the C_2 symmetry, the net dipole moment is always parallel to the C_2 axis that is in the direction of $\mathbf{n} \times \mathbf{k}$. Note that this net dipole moment is perpendicular to the molecular axis $\mathbf{n}$.

When an electric field is applied normal to the helical axis, the structure gets distorted. Above a critical field given by

$$E_c = \frac{\pi^4}{4} \frac{k_2}{P p^2} \quad (5.7-1)$$

where p is the pitch of the undistorted helix, k_2 is the 'twist' elastic constant, and P is the polarization; the helix is completely unwound and the sample is poled. This leads to a homogeneously aligned LC cell with the director in a plane perpendicular to the electric field. When the field is reversed, the polarization vector is also reversed. The reversal of the polarization corresponds to a rotation of the molecular axis from (θ, ϕ) to $(\theta, \phi + \pi)$. It appears that the long molecular axis is rotated by an angle of 2θ .

Surface Stabilized Ferroelectric Liquid Crystal (SSF-LC) Cells

Clark and Lagerwall proposed the idea of using a very thin cell such that the boundary condition imposed by the alignment layers would be strong enough to suppress the helix [13]. The helical structure is indeed suppressed in a thin slab with a thickness less than the pitch of the helix, resulting in a homogeneously parallel alignment of the molecular axis. This is known as the "book-shelf" geometry, resembling a bookshelf with all the books tilting toward the same direction (see Fig. 5.46), with $\mathbf{k}$ parallel to the y axis). The addition of chirality destroys the inversion symmetry and the mirror reflection symmetry. Thus, if the molecules possess a transverse component of the permanent electric dipole moment, a net polarization may exist along the C_2 axis (z axis in

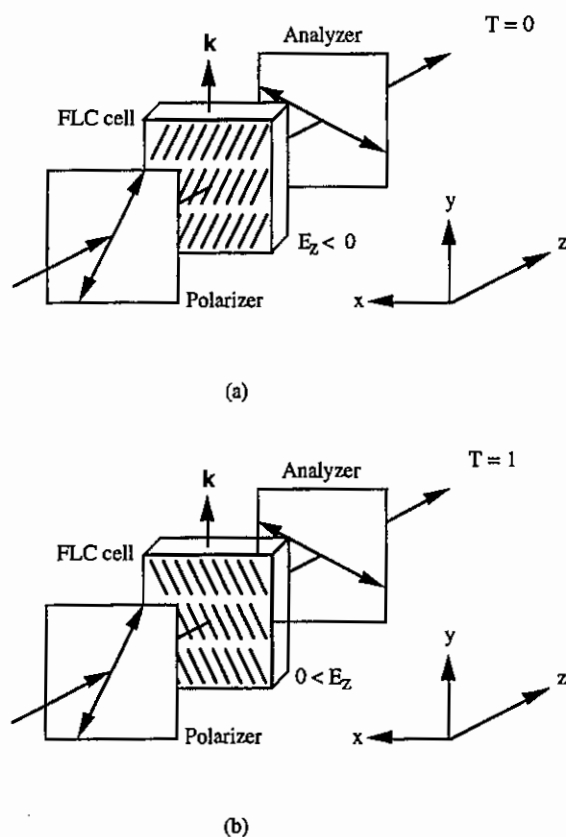


Figure 5.46. Schematic drawing of a ferroelectric LC cell for display applications. (a) When $E_z < 0$, the LC director is tilt toward upper right with an azimuth angle of π ; (b) when $0 < E_z$, the LC director is tilt toward upper left with an azimuth angle of 0.

Fig. 5.46) in a layer. As a result of the C_2 symmetry, the net polarization is zero in the plane (xy plane in Fig. 5.46) perpendicular to the C_2 axis of symmetry. This is very similar to the spontaneous polarization in ferroelectric crystals. Note that this net polarization vector (dipole moment per unit volume) is perpendicular to the long molecular axis $\mathbf{n}$ and parallel to the layer. Thus, if the director $\mathbf{n}$ is in the xy plane (forced by the boundary condition), then the net polarization vector is along the z axis (see Fig. 5.46). This net polarization can point either up or down along the z axis, depending on whether the azimuth angle (measured from the x axis) is π or 0. In other words, the structure is bistable. An electric field along the z axis can be employed to switch the polarization. The switching rotates the director orientation around $\mathbf{k}$ from (θ, π) to $(\theta, 0)$ and vice versa. Here the angle θ is measured from the vector $\mathbf{k}$ that is parallel to the y axis in Figure 5.46. Although the molecular axes are actually rotated around the y axis ($\mathbf{k}$) by an angle π , they appear like a rotation of the molecular axes around the z axis by an angle of 2θ .

For display applications, a surface stabilized F-LC cell is sandwiched between a pair of crossed polarizers. The transmission axis of the entrance polarizer is aligned parallel to the F-LC director in one of the bistable states. This corresponds to the black state (zero transmission; see Fig. 5.46a). When a proper electric field is applied, the polarization of F-LC is switched. The switching of the polarization leads to a reorientation of the molecular axis by an angle of 2θ relative to the polarizer. The transmission becomes

$$T = \frac{1}{2} \sin^2 4\theta \sin^2 \frac{\Gamma}{2} \quad (5.7-2)$$

where Γ is the phase retardation of the F-LC cell. By a proper choice of the cell thickness, it is possible to obtain a phase retardation of $\Gamma = \pi$. With a tilt angle of $\theta = 22.5^\circ$, Eq. (5.7-2) yields a maximum transmission of $T = 0.5$. Thus, the transmission can be switched from 0 to 0.5 by reversing the polarity of the applied voltage (see Fig. 5.46).

It is important to note that F-LCD described in Figure 5.46 is a bistable device. In the operation, an applied electric field is employed to switch the molecular orientations from (a) to (b). Once the switching is done, the molecular orientation can be sustained without an applied electric field. Thus, the applied electric field can be removed without affecting the transmission of the device.

5.7.2. Cholesteric LCD

Cholesteric liquid crystals can also be employed for display applications. In Chapter 7 we will discuss the optical properties of cholesteric liquid crystals in the equilibrium planar state. In the equilibrium planar state (P) the medium exhibits a uniform twist of the director that is perpendicular to the helical axis. As a result of the uniform twist, the cholesteric liquid crystal is a periodic medium. For display applications, we consider a thin layer of cholesteric liquid crystal sandwiched between a pair of glass substrates. The helical axis of the liquid crystal is perpendicular to the substrates. It is known that selective reflection (Bragg reflection) occurs for light in the following wavelength range:

$$n_o p < \lambda < n_e p \quad (5.7-3)$$

where p is the pitch of the helix, and n_e, n_o are the principal refractive indices of the liquid crystal. The strong Bragg reflection may disappear if the helical structure in the cholesteric LC cell is destroyed or severely distorted.

In addition to the equilibrium planar state (P), there are several other states of the cholesteric liquid crystal in the cell. These include the homeotropic state (H), the focal conic state (FC), and some intermediate states (see Fig. 5.47). In a homeotropic state, the twist disappears and the LC director is uniformly aligned perpendicular to the substrates, resembling a vertically aligned (VA) nematic

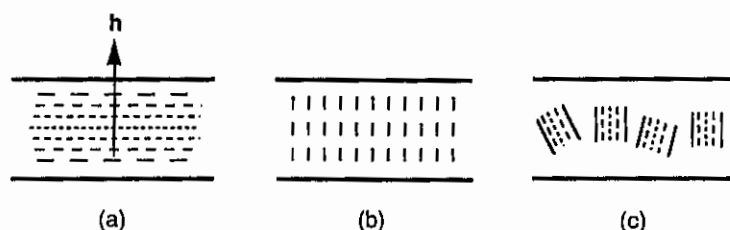


Figure 5.47. States (or textures) in cholesteric liquid crystals: (a) planar state; (b) homeotropic state; (c) focal conic state. The planar and focal conic states are stable at zero applied field. The homeotropic state can exist only with the presence of an applied electric field. Here h is a unit vector representing the helical axis.

cell. An applied electric field is needed to maintain the state. Thus, this state is often referred to as the field-induced nematic state. The Bragg reflection disappears completely in the homeotropic state. Thus, the cell appears optically like a plate of transparent medium. The focal conic state is similar to the planar state with the same intrinsic pitch of the helix. However, there are multiple domains in the focal conic state. Within each domain, the director is twisted with a helical axis approximately parallel to the substrates. At the domain boundary, the helical axis changes its azimuth angle abruptly. Forward scattering of light occurs in the focal conic state. If the rear substrate is painted black, then the focal conic state will appear dark. LCDs can be designed based on the different transmission and reflection properties of these states.

Transition from one state to another state can be achieved by applying an external electric field. For example, if an electric field is applied parallel to the substrates, a focal conic state can be transformed into a planar state. On the other hand, if an electric field is applied perpendicular to the substrates, a focal conic state can be transformed into a homeotropic state. These transitions can be employed for display applications. A cholesteric reflective display can be designed based on the transition between the focal conic state and the planar state. Since both of these states are stable, the device is bistable; in other words, the application of an external field can switch the states back and forth. In addition, the state can remain unchanged after the removal of the field. A cholesteric reflective or transmissive display can be designed based on the transition between the homeotropic state and the planar state. This would require an electric field perpendicular to the substrates. When the field is turned ON, the cholesteric liquid crystal cell is transformed into the homeotropic state, which transmits light. On removal of the field, the cholesteric LC cell returns to the planar state, which reflects light in selected spectral regimes.

5.8. SUMMARY

In summary, we have discussed the transmission properties of various liquid crystal displays, including TN-LCD, STN-LCD, N-LCD, VA-LCD, BA-LCD,

reflective LCD, PD-LCD, ferroelectric LCD, and cholesteric LCD. It is important to note that LCD technology is developing very rapidly. New modes of operation and new cell configuration are constantly being invented and developed. The fundamental principles described in this chapter and the previous chapters will be very useful for the design and development of new LCDs. It is also important to note that there are other display systems that are not covered in this chapter, including plasma addressed LCDs. Interested readers are referred to the references and the proceedings of LC display conferences.

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PROBLEMS

5.1. Birefringent dielectrics between parallel electrodes:

Consider a slab of uniform anisotropic dielectric material sandwiched between a pair of parallel electrodes. Let ϵ be the dielectric tensor and d be the thickness of the slab. A voltage V is applied on the electrodes. Let z be the axis perpendicular to the electrodes, and assuming that the slab and the electrodes are of infinite extent.

- Show that $E_x = E_y = 0$ regardless of the orientation of the principal axes of the dielectric.
- Show that the displacement field vector $\mathbf{D}$ is, in general, not parallel to the z axis.
- Let the dielectric be a uniaxial crystal with its c axis oriented at angle θ relative to the z axis. Find the displacement field vector $\mathbf{D}$ in terms of E_z .
- Show that $E_x = E_y = 0$ even if $\epsilon(z)$ is a function of z .

5.2. Electrostatic energy of capacitors with birefringent dielectrics:

Consider an empty capacitor made of a pair of electrodes. A slab of uniform anisotropic dielectric material (of dielectric tensor ϵ) is moved from infinity into the capacitor.

- If the electrodes are isolated from a power supply (or battery), the surface charge density σ on the electrodes is kept constant. Find the change in the electrostatic energy.
- Assume that the electrodes are connected to a power supply (or battery), so that the voltage drop between the electrodes is kept a constant. Find the net change in the electrostatic energy. (Note that there is an additional work done by the battery as the dielectric is moved into the capacitor.)

5.3. Polarization ellipses in TN-LC:

Consider the polarization states of transmitted wave through a TN-LC with a right-handed twist. Use $n_o = 1.5$, $n_e = 1.6$, $L = 10 \mu\text{m}$, and $\lambda = 0.45, 0.55, 0.65 \mu\text{m}$.

- Consider a linearly polarized incident light propagating along z axis. Letting θ be the angle between the $\mathbf{E}$ vector and the director (c axis) at

- $z = 0$, find the ellipticity e of the transmitted wave as a function of θ . Plot e as a function of θ . Also find the sense of rotation of the transmitted wave as a function of θ .
- (b) Consider the incidence of a right-handed circularly (RHC) polarized light along the z axis. Find the output polarization state. Discuss the possibility of converting RHC into LHC polarized light by using a TN-LC.
- 5.4. Consider a 90° TN-LC and a (-90°) TN-LC in series with a continuous c axis (director) at the interface. Find the output wave $\mathbf{E}_o$ as a function of the input wave $\mathbf{E}_i$ (propagating along the z axis). Use $n_o = 1.5$, $n_e = 1.6$, $L = 10 \mu\text{m}$, and $\lambda = 0.45, 0.55, 0.65 \mu\text{m}$. Compare the results with a 180° TN-LC.
- 5.5. Derive Eq. (A-16)—elastic energy density due to twist and tilt in a TN-LCD.
- 5.6. Derive Eqs. (B-7) and (B-22)—electrooptical distortion (tilt mode).
- 5.7. Derive Eq. (C-9)—electrooptical distortion (twist mode—IPS).
- 5.8. Derive Eqs. (D-20), and (D-25)—electrooptical distortion (TN-LCD).
- 5.9. Consider a 90° TN-LC and a (-90°) TN-LC in series with a continuous c axis (director) at the interface. The whole structure is sandwiched between a pair of parallel polarizers with the transmission axes parallel to the directors.
- (a) Using $\Delta nd = 0.48 \mu\text{m}$ for the cells, find the transmission for $\lambda = 0.45, 0.5, 0.65 \mu\text{m}$. Compare the results with a 180° TN-LC.
- (b) Find the minimum cell thickness such that green light at $\lambda = 0.5 \mu\text{m}$ has a zero transmission in the 180° TN-LCD. Find the residual transmission of blue and red light at $\lambda = 0.45, 0.65 \mu\text{m}$. Compare the result with the $90^\circ + (-90^\circ)$ TN-LCD.
- 5.10. Determine the symmetry of viewing in normally white TN-LCDs and STN-LCDs. The transmission of light through a TN-LCD as shown in Figure 5.1a is dependent on the incidence wavevector $\mathbf{k} = (k_x, k_y, k_z)$.
- (a) Show that the TN-LCD shown in Figure 5.1a is invariant under a rotation of 180° around the x axis. Thus, the transmission exhibits the following symmetry: $T(k_x, k_y, k_z) = T(k_x, -k_y, -k_z)$
- (b) Show that the principle of reciprocity can be written $T(k_x, k_y, k_z) = T(-k_x, -k_y, -k_z)$.
- (c) Using the result in (a) and (b), show that $T(k_x, k_y, k_z) = T(-k_x, k_y, k_z)$, which implies that a left-right symmetry exists in the viewing.
- (d) Show that the same symmetry in (c) can also exist in STN-LCD, provided the transmission axes of the polarizers are aligned

symmetrically with respect to the bisector of the rubbing directions.

- 5.11. Calculate the leakage of light in a 90° TN-LCD at normal incidence. Consider a 90° TN-LCD in the normally black mode of operation. The Mauguin parameter should be $u = \sqrt{3}, \sqrt{15}, \sqrt{35}, \dots$ to obtain a zero transmission (assuming ideal polarizers).

- (a) Find the values of $\Delta n d$ of the LC cell for the wavelength $\lambda = 0.55 \mu\text{m}$.
 (b) Find the transmission of the LCD for $\lambda = 0.45, 0.65 \mu\text{m}$.

- 5.12. Determine the leakage of light in a 90° TN-LCD at normal incidence. Consider a 90° TN-LCD in the normally black mode of operation with a Mauguin parameter of $u = \sqrt{3}$. Let the transmission of the sheet polarizers be $T_1 = 0.8$ and $T_2 = 0.05$, where T_1 is the polarization parallel to the transmission axis of the polarizer, and T_2 is the polarization perpendicular to the transmission axis of the polarizer. Find the transmission of the TN-LCD.

- 5.13. *Minimum elastic energy density in TN-LC cells:*

- (a) Show that in a TN cell with a uniform twist rate and zero tilt, the elastic energy density is

$$U_{\text{EL}} = \frac{1}{2} k_2 \left(\frac{2\pi}{4d} \right)^2$$

where d = cell gap.

- (b) Show that, according to Eq. (5.1-11), in a TN cell with a uniform twist rate and non-zero-pretilt angle θ_0 , the elastic energy density is minimum when $\theta(z) = \theta_0$.

- 5.14. *Difference between E and O modes of operation:*

- (a) Show that for normal incidence the transmission of these two modes are identical.
 (b) Show that, in general, the transmission of these two modes are different.

- 5.15. The BA-LCD shown in Figure 5.36 exhibits an exact rotation symmetry of 180° around the y axis C_{2y} . Using the procedure similar to those used in Problem 5.10, show that the transmission exhibits an exact vertical viewing symmetry. An approximate horizontal viewing symmetry also exists in the same BA-LCD. Show that the slight asymmetry in the horizontal viewing is due to the polarizers.

6

Matrix Addressing, Colors, and Properties of LCDs

In Chapter 5, we described the principle of operation of an individual LC cell for display applications. To control the information display, the LC cell must be electrically addressed. For the display of high content information (e.g., TV and computer monitors), we need a 2D array (say, $M \times N$) of LC pixels. Each pixel consists of a small LC cell that can be turned ON or OFF electrically. In principle, this can be achieved by using $M \times N$ electrical connections. However, for practical purposes, the electrical addressing is achieved by using multiplexing techniques, which requires only $M + N$ electrodes. The addressing of the display pixels can be achieved by two different methods. They are multiplexed addressing (passive) and the active matrix (AM) addressing. In this chapter, we will discuss these two methods of addressing, as well as some important properties of LCDs.

6.1. MULTIPLEXED DISPLAYS

As a result of the multiplexing, the voltages applied at one cell cannot be arbitrarily changed without affecting the applied voltages at other cells. This is known as *crosstalk*. As a consequence, the effective ON and OFF voltages applied at any of the cells cannot be too different. This severely limits the contrast ratio in display applications. In this section, we discuss the effect of the crosstalk.

Referring to Figure 6.1, we consider 2D array ($M \times N$) LCD pixels that are addressed by N rows and M columns of electrodes. In the multiplexing, a positive voltage V_s is applied to one row at a time, starting from the first row. Focusing on the first row at the moment, we find that the M elements in the first row can be turned ON or OFF depending on the voltages applied to each element. Let V_d or $-V_d$ be the voltages applied to the elements (assume a positive V_d). The instantaneous voltage drops at the pixel electrodes in the selected row are

$$\text{ON state:} \quad V = V_s - (-V_d)$$

$$\text{OFF state:} \quad V = V_s - V_d$$

We note that a negative voltage $-V_d$ at the m th column and a positive voltage V_s at the n th row can produce an applied voltage of $V_s + V_d$ across the LC cell. This

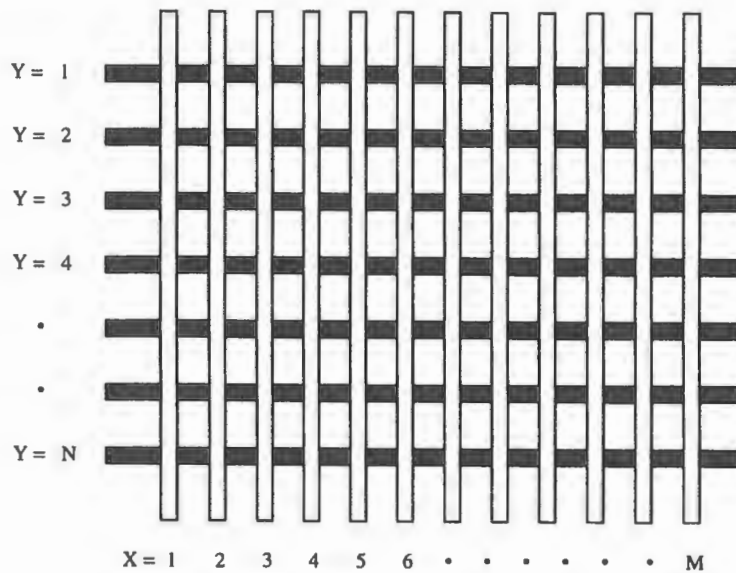


Figure 6.1. A 2D array of LC pixels that can be addressed by $M \times N$ electrical connections using N rows and M columns of electrodes.

creates an ON pixel at the (m,n) location. On the other hand, voltage drops also exist at other pixels. These include the voltage drop of $\pm V_d$ at all pixels in the nonselected row, and the voltage drop of $(V_s - V_d)$ at the OFF pixels of the selected row.

It is known that liquid crystals do not respond to the applied electric field instantaneously, because of the finite time needed in the reorientation of the directors. Thus, the actual molecular orientations depend on the root mean square (rms) average of the applied voltage. In each multiplexing cycle, each row is selected only during $1/N$ of the cycle time T . Thus, the rms voltage of the ON and OFF states are

$$V_{\text{ON}}^2 = \frac{1}{N}(V_s + V_d)^2 + \frac{N-1}{N}(V_d)^2 \quad (6.1-1)$$

$$V_{\text{OFF}}^2 = \frac{1}{N}(V_s - V_d)^2 + \frac{N-1}{N}(V_d)^2 \quad (6.1-2)$$

Subtracting Eq. (6.1-2) from Eq. (6.1-1), we obtain

$$\frac{4V_d V_s}{N} = V_{\text{ON}}^2 - V_{\text{OFF}}^2 \quad (6.1-3)$$

Adding Eqs. (6.1-1) and (6.1-2), we obtain

$$V_{\text{ON}}^2 + V_{\text{OFF}}^2 = 2V_d^2 + \frac{2}{N} V_s^2 \quad (6.1-4)$$

According to Eq. (6.1-3), we find that the difference of the rms voltages of the ON state and the OFF state is near zero when N is large. This may lead to a poor contrast in the display.

In what follows, we find the optimum value of N given the desired rms voltages V_{ON} and V_{OFF} . By eliminating V_s from Eqs. (6.1-3,4), we obtain

$$N = 8V_d^2 \frac{V_{\text{ON}}^2 + V_{\text{OFF}}^2}{(V_{\text{ON}}^2 - V_{\text{OFF}}^2)^2} - \frac{16V_d^4}{(V_{\text{ON}}^2 - V_{\text{OFF}}^2)^2} \quad (6.1-5)$$

To find the optimum value for N , we differentiate Eq. (6.1-5) with respect to V_d while keeping V_{ON} and V_{OFF} constant. This leads to

$$V_d = \frac{1}{2} \sqrt{V_{\text{ON}}^2 + V_{\text{OFF}}^2} \quad (6.1-6)$$

with

$$N_{\text{max}} = \frac{(V_{\text{ON}}^2 + V_{\text{OFF}}^2)^2}{(V_{\text{ON}}^2 - V_{\text{OFF}}^2)^2} \quad (6.1-7)$$

According to this equation, the maximum number of rows in multiplexing is limited by the difference between the desired rms voltages V_{ON} and V_{OFF} . Equation (6.1-7) can also be written

$$\frac{V_{\text{ON}}}{V_{\text{OFF}}} = \sqrt{\frac{\sqrt{N_{\text{max}}} + 1}{\sqrt{N_{\text{max}}} - 1}} \quad (6.1-8)$$

where we note that the ratio of the rms voltages (also known as the *selection ratio*) approaches unity when N_{max} is very large, leading to a poor contrast. This equation was first obtained by Alt and Pleshko [1] in 1974, and is often referred to as the “iron law of multiplexing.” For $1 \ll N_{\text{max}}$, this equation can be written approximately as

$$\frac{V_{\text{ON}}}{V_{\text{OFF}}} = 1 + \frac{1}{\sqrt{N_{\text{max}}}} \quad (6.1-9)$$

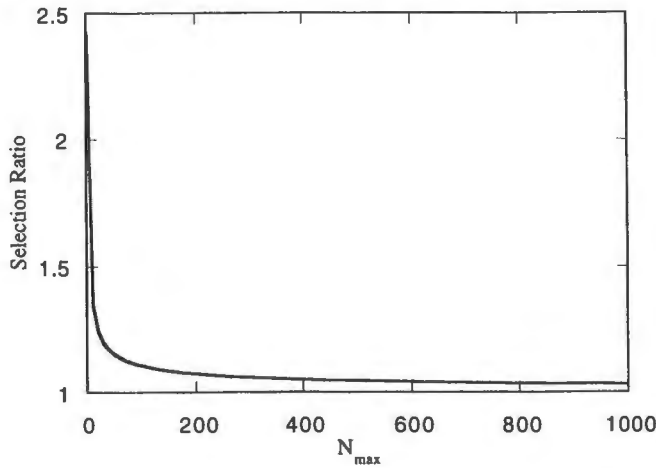


Figure 6.2. Selection ratio of multiplexed addressing versus column number [Eq. (6.1-8)].

For a column number of $N_{\max} = 400$, Eq. (6.1-9) yields a selection ratio of only 1.05, which means that the rms voltage of the ON state is only 5% more than that of the OFF state. Figure 6.2 shows the selection ratio as a function of the row number. We note that the selection ratio approaches unity at large $N_{\max}$.

The ratio V_d/V_s is often referred to as the *bias ratio*. At the optimum multiplexing, the bias ratio is given by, according to Eqs. (6.1-3), (6.1-6), and (6.1-7)

$$\frac{V_d}{V_s} = \frac{1}{\sqrt{N_{\max}}} \quad (6.1-10)$$

Example 6.1. Consider a multiplexed LC display of 600×400 pixels. Assuming $V_{\text{OFF}} = 2.0$ V, we obtain the following rms ON voltage, according to Eq. (6.1-8): $V_{\text{ON}} = 2.10$ V, and $V_s = 29.0$ V, $V_d = 1.45$ V. We note that the difference between V_{ON} and V_{OFF} is very small. ■

For practical applications in LCDs, it is important that

$$V_{\text{OFF}} \leq V_{\text{th}} \quad (6.1-11)$$

$$V_{\text{ON}} \geq V_{\text{th}} + \Delta \quad (6.1-12)$$

where V_{th} is the threshold voltage needed to induce a reorientation of the LC director and Δ is a transition voltage. From the operational point of view, it is desirable to define a device parameter

$$P \equiv \frac{\Delta}{V_{\text{th}}} \quad (6.1-13)$$

which is a measure of the nonlinearity of the electrooptical characteristics of the device. This parameter completely specifies the maximum scanned lines and the corresponding voltages. According to Eqs. (6.1-9) and (6.1-11)–(6.1-13), we obtain

$$P \equiv \frac{\Delta}{V_{th}} \leq \frac{1}{\sqrt{N_{max}}} \quad (6.1-14)$$

A technique known as “dual scan” is often employed in multiplexed displays. In a dual-scan operation, an upper column driver provides the voltages (V_d or $-V_d$) for the upper $N/2$ rows, and a lower column driver provides the voltages for the lower $N/2$ rows. The dual scan improves the bias ratios or the device parameter P by a factor of $\sqrt{2}$.

6.2. ACTIVE-MATRIX DISPLAYS

As we discussed earlier, the multiplexing technique of addressing (also known as *passive matrix*) leads to a poor contrast as the number of rows N increases. Although the use of STN liquid crystals can provide a significant improvement in the contrast when N is large, severe limitations still exist in cell performance. These include response time, viewing angles, and gray-scale stability. These problems can be eliminated by using active-matrix addressing. The performance of active-matrix (AM) LC cells is compared to that of simple multiplexed LC cells in Table 6.1. As the manufacturing techniques continue to improve, it is now recognized that the ultimate solution for high quality, large area, high-information-content, color gray-scale display applications requires the use of AM addressing. The discussion that follows is based on a classic book chapter by F. C. Luo [2], to which the reader is referred to for more details.

6.2.1. Principle of TFT Operation

An active-matrix liquid crystal display (AM-LCD) incorporates a 2D array (matrix) of circuits to provide the electrical addressing of the individual pixels.

Table 6.1. Performance of AM LC Cells and Simple Multiplexed LC Cells

	Active-Matrix LC cells	Simple Multiplexed LC cells
LC mode	TN	STN
Contrast ratio (CR)	100 : 1	15 : 1
Viewing angles—horizontal	(-60° , $+60^\circ$)	(-30° , $+30^\circ$)
Viewing angles—vertical	(-30° , $+45^\circ$)	(-25° , $+25^\circ$)
Response time	30–50 m sec	150 m sec
Multiplexed lines	>1000	400
Gray scale	>16	8

The AM circuit has an active device in each pixel element defined by the crossover of the row and the column bus lines. The active devices in typical AM-LCDs are thin film transistors (TFTs). A simple matrix circuit can incorporate a transistor and a capacitor in each of the pixel elements for addressing of the display. Electrical charges can be stored in the capacitors to maintain a steady state voltage across the electrodes. Being isolated from other pixels by the transistors, the voltages remain constant while other pixel elements are being addressed electrically. Thus, the contrast ratio in AM-LCDs is not subject to the Alt-Pleshko limitation, Eq. (6.1-8). In what follows, we describe briefly the principle of operation of the active matrix (AM) LCDs, especially the TFT-LCDs.

Referring to Figure 6.3, we consider a color TFT-LCD that consists of a TFT matrix circuit on a glass substrate and a color filter glass plate with the LC layer sandwiched in between. Each pixel element is defined by the row and column bus lines in the TFT matrix circuit. Figure 6.4a is a schematic diagram of the TFT array circuit for the active-matrix addressing of LCD. A detailed electrical circuit diagram of each pixel element is shown in Figure 6.4b. We note that each pixel element has one TFT and a LC capacitor formed between the ITO transparent output electrode on the TFT matrix circuit and the ITO backplane electrode on the top glass substrate/color filter with the LC layer as the insulator (see also Fig. 6.5a).

Figure 6.5a is a schematic drawing of a side view of the TFT-LCD module. We note that the top glass plate contains pixel color filters and a black matrix, which are then covered with a passivation layer and the ITO common electrode. A thin layer of polyimide is then coated as the alignment layer. The main purpose of the black matrix is to improve the display quality by blocking the stray light passing through the gap between the pixel electrodes. The black matrix also protects the semiconductor layer from external optical illumination,

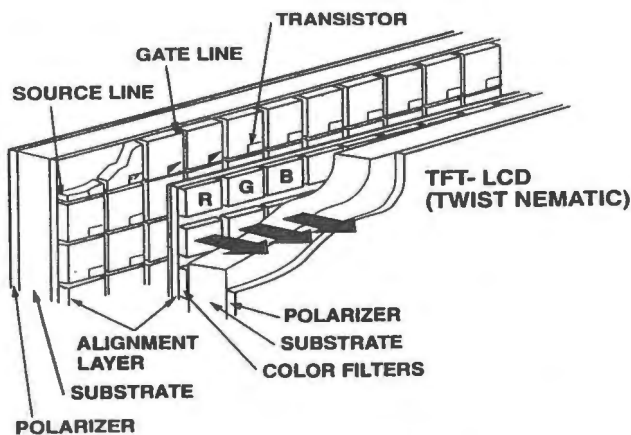


Figure 6.3. Schematic drawing of a TFT-LCD; R, G, and B stand for color filters (red, green, and blue).

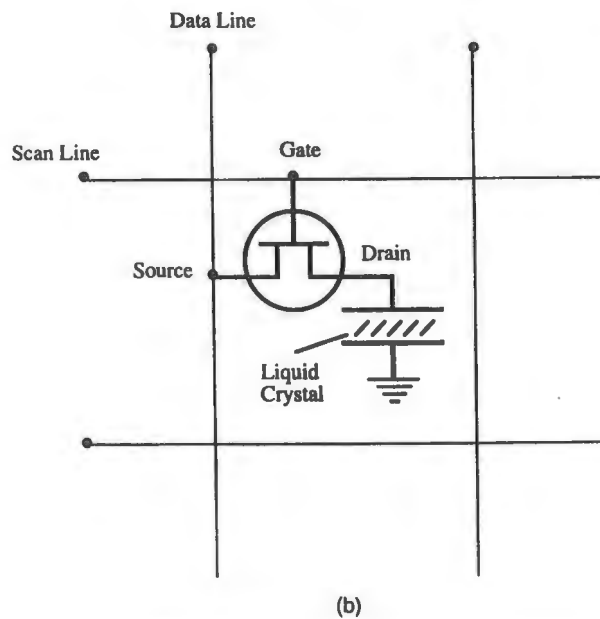
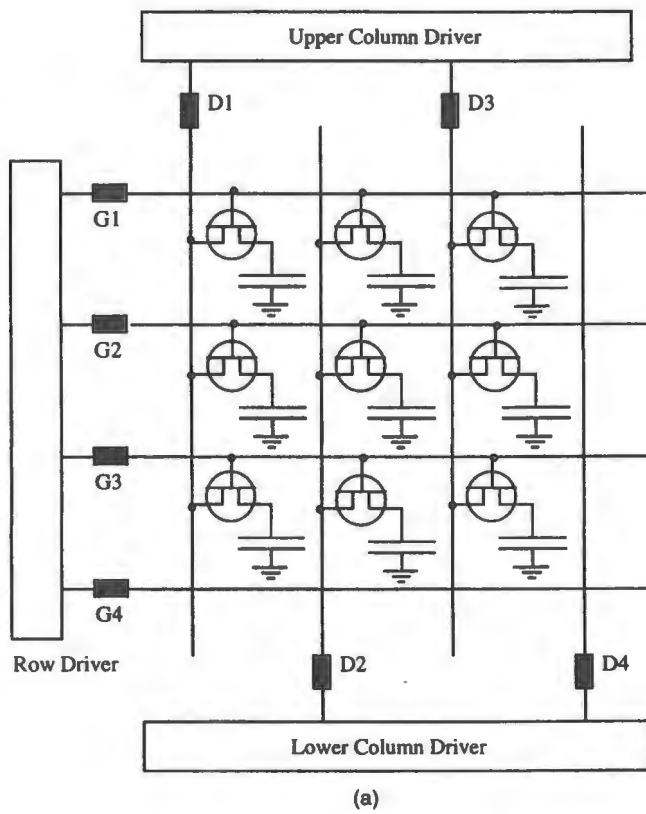


Figure 6.4. (a) Schematic drawing of TFT array circuit for LCD; (b) electrical circuit diagram of one pixel element.

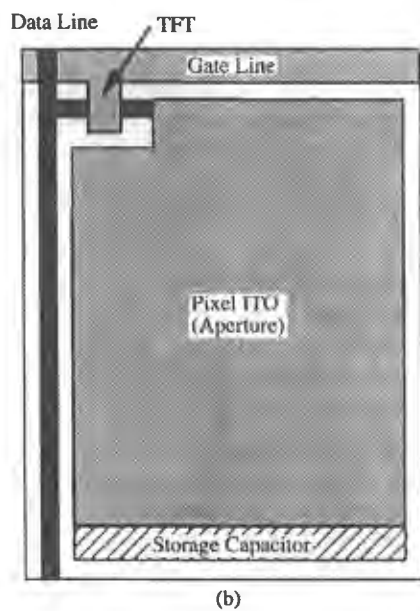
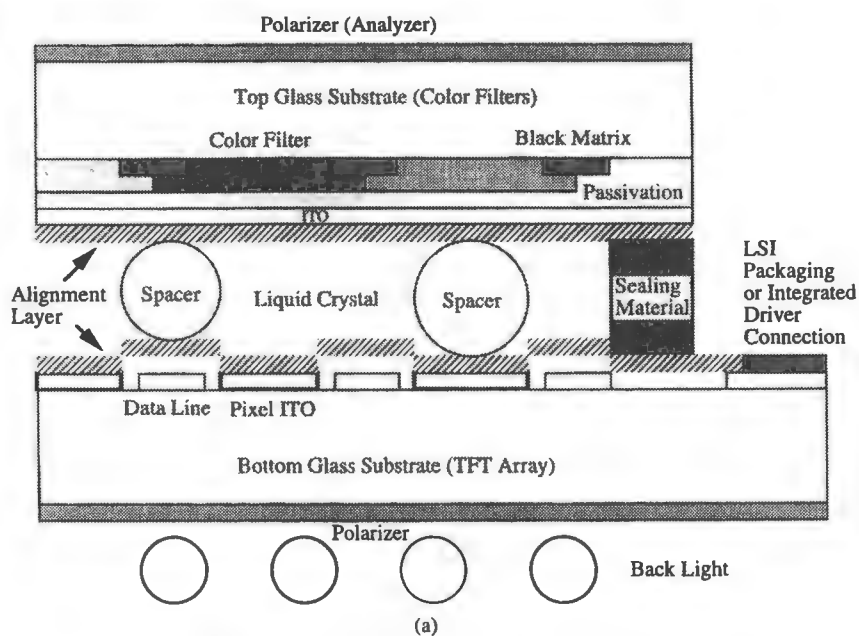


Figure 6.5. (a) Schematic drawing of a side view of AM-LCD module; (b) schematic drawing of a typical pixel layout in a-Si TFT-LCD.

which can affect the conductivity in the TFTs via photoconductivity of the semiconductor. We also note that the cell gap is kept uniform by using transparent spacer balls (or fibers). Figure 6.5*b* is a schematic drawing of a typical layout of a pixel in a-Si (amorphous silicon) TFT-LCD. We note that the useful area for light transmission (aperture) is only a fraction of the total pixel area, due to the real estate needed for the driving circuits, storage capacitor, and scan and data bus lines.

The principle of operation is described as follows. Again, the cells are addressed one line at a time. When a row (scan or gate line) is addressed, a positive voltage pulse (of duration T/N (where N is the number of rows and T is the frame time)) is applied to the line turning on all the transistors along the row. The transistors act as switches transferring electrical charges to the LC capacitors from the respective columns (data or source line). When addressing other rows, a negative voltage is applied to the gate lines turning OFF all the transistors along the line and holding the electrical charges in the LC capacitors for one frame time T until the line is addressed again. If the LC material used in the cell is twisted nematic (TN), it is desirable to use AC voltage to drive the LC element. This is often achieved by switching the polarity of the data voltage in alternate frames. An example of the driving voltage waveforms of both the gate and source lines, and the voltage on the pixel electrode is shown in Figure 6.6 [3, 4].

The operation of the TFT in LCD can be divided into four steps in time [4]:

1. At time 1 (see Fig. 6.6*a*), during the addressing time of an odd frame (at the end of the frame), a positive voltage of duration T/N is applied to the gate line V_G turning on the TFT. Thus, the LC pixel electrode (ITO) is charged from $V_p = -0.9 V_{ON}$ at time 1 to greater than $V_p = 0.9 V_{ON}$ at time 2 within the time duration of T/N , due to the positive source voltage $V_{SD} = V_{ON}$.
2. At time 2, the gate voltage becomes negative, turning OFF the TFT and simultaneously the source voltage V_{SD} changes from $+V_{ON}$ to $-V_{ON}$. During the time period between 2 and 3 with a duration of $(N-1)T/N$, the voltage on the LC pixel electrode V_p remains about $0.9 V_{ON}$ as the LC capacitor is now isolated from the data lines.
3. At time 3 (the next addressing time), the TFT is turned ON again by applying a positive gate voltage of duration T/N . The LC capacitor now sees a negative source-to-drain voltage $V_{SD} = -V_{ON}$. Thus, the pixel electrode is charged (actually discharged) from $V_p = +0.9 V_{ON}$ at time 3 to less than $V_p = -0.9 V_{ON}$ at time 4 within the time duration of T/N .
4. At time 4, the TFT is turned OFF by the negative gate voltage and simultaneously the source voltage V_{SD} changes from $-V_{ON}$ to $+V_{ON}$.

This completes the cycle of the four steps in time. We note that the voltage on the LC pixel electrode V_p does not remain constant during the duration $(N-1)T/N$ because of a slight leakage of current at the LC cell. The leakage of

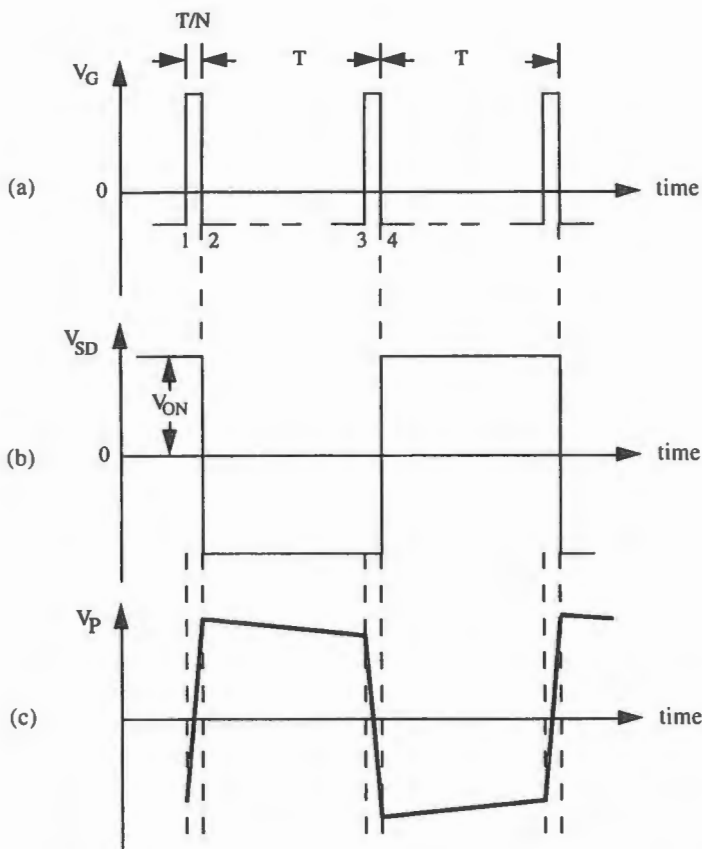


Figure 6.6. An example of the driving voltage waveforms: (a) applied to one of the gate bus lines; (b) applied to one of the source bus lines; (c) on the LC pixel electrode [3,4].

current is due to a very small residual conductivity of the LC material. We note that the pixel voltage V_p across the LC cell in the field-ON state is actually alternating in time. This is necessary to maintain the domain stability in the TN cell. The director of the liquid crystal responds to the root mean square of the pixel voltage V_p . To turn off a pixel, we simply apply a zero source-to-drain voltage ($V_{SD} = 0$) to the bus line. This removes all the charges in the LC capacitor, leading to a zero pixel voltage ($V_p = 0$).

The TFT device in AM-LCDs works like a metal oxide semiconductor field effect transistor (MOSFET). A typical MOSFET consists of a conducting channel (e.g., n-type semiconductor) between two ohmic contacts, called *source* and *drain*, respectively; the width of the conducting channel is controlled electrically by a rectifying contact called the *gate*; the gate voltage controls the current flowing between source and drain. The metal gate is separated from the conducting channel by an oxide insulating layer. This insulating oxide layer ensures an almost zero gate current. When an appropriate voltage is applied to

the gate, an n-type conducting channel is induced. If a voltage V_{SD} is applied between the source and the drain, a current I_{SD} will flow from source to drain, and the magnitude of this current is controlled by the gate voltage V_G . The conducting channel can be completely cut off by reverse-biasing the gate. This feature is particularly useful for the operation of AM-LCDs. For detail characteristics of MOSFET, the reader is referred to Reference [5].

The current and voltage characteristics of an MOSFET based TFT can be written [2,5]

$$\begin{aligned} I_{SD} &= \frac{W\mu C_G}{L} V_{SD} \left(V_G - V_T - \frac{V_{SD}}{2} \right) & \text{when } V_{SD} < (V_G - V_T) \\ I_{SD} &= \frac{W\mu C_G}{2L} (V_G - V_T)^2 & \text{when } V_{SD} > (V_G - V_T) \end{aligned} \quad (6.2-1)$$

where W = width

L = length of conducting channel of TFT

μ = effective mobility of semiconductor

C_G = gate capacitance per unit area

V_{SD} = source-drain voltage

V_G = gate voltage

I_{SD} = source-drain current

V_T = threshold voltage of the TFT.

In an MOSFET based TFT, a conducting channel is induced when the gate voltage is larger than the threshold voltage. We note that the TFT device is in the ohmic regime when $V_{SD} < (V_G - V_T)$. In this regime, the current is approximately proportional to the voltage. We also note that the current I_{SD} is practically independent of the voltage V_{SD} when $V_{SD} > (V_G - V_T)$. This is the so-called pinchoff or saturation regime, and the current is called the *saturation current*.

We now consider the requirement on the characteristics of the TFT for AM-LCD operations. The electrical current I_{ON} of the TFT to charge the LC cell, which acts like a capacitor, during a time period of T/N must satisfy the following relation

$$2K_1 C_{LC} V_{ON} < \frac{T}{N} I_{ON} \quad (6.2-2)$$

where K_1 = an engineering operation constant

V_{ON} = ON voltage of the LC element

C_{LC} = capacitance of the LC element

N = number of addressed lines (rows) in the display

T = frame time.

For bilevel operations such as an alphanumeric or graphical displays, the requirement on the pixel voltage is not very tight since all voltages above the ON voltage of the LC element will be acceptable. The constant K_1 can be a value between 1 and 10 to allow for any nonuniformity in the TFTs in the panel and the fact that the ON current of the TFT is not a constant during the charging of the pixel. For a 6.25×6.25 -in. panel with 1024×1024 pixel elements, I_{ON} should be greater than 1.09×10^{-6} A if we pick K_1 to be 5. For gray-scale operations, the RC (resistance $\times$ capacitance) time constant is required to be at least one third of the charging period to obtain less than 5% error; also K_1 should be greater than 10. In this case, I_{ON} should be greater than 2.18×10^{-6} A.

During the OFF period, the TFTs are turned off. The OFF currents (leakage) of the TFTs should be low enough so that the charge stored in the LC capacitors will not leak away to affect the appearance of the panel. The leakage current is generally attributed to thermionic field emission and residual thermal charge carriers. For bilevel display panels, there are two separate conditions. For OFF pixels, the voltage on the pixels should not increase to a level exceeding the threshold voltage of the LC material and turn the pixels partially ON. For the ON pixels, the voltage on the pixels should not drop below a voltage at which the pixels appear partially ON or OFF. For typical LC materials, this voltage change ΔV should be smaller than 1.5 V. The requirement on the OFF current (leakage current) of the TFT is thus

$$I_{OFF}T < K_2\Delta VC_{LC} \quad (6.2-3)$$

where T is the frame time, K_2 is another engineering operation constant that allows for the uniformity variations of TFT characteristics in a TFT array and the temperature variations of the OFF currents of the TFTs. For a typical a-Si TFT, the OFF current can increase by a factor of 10 when the temperature is raised from 20 to 70°C. Again using the 1024×1024 array of LC cells as an example, $\Delta V = 1.5$ V and $K_2 = 0.1$, I_{OFF} should be less than 3.28 pA. For gray-scale operations, ΔV should be less than one gray level, or 0.08 V for a 16-level display panels, I_{OFF} should be less than 0.175 pA.

The charge stored in the LC capacitors can also leak through the RC relaxation due to a small residual conductivity of the LC materials (i.e., the resistance R is not infinite). For a 5% error of the rms voltage on the LC capacitors, the RC time constant of the LC materials is required to be 10 times the frame time of addressing. For a display with a 16-ms frame time, the RC time constant should be longer than 160 ms. When considering the requirements of elevated temperature performance of the cell, the room temperature RC time constant should be over 1 s. Typical resistivities of TN-LC materials are in the range from 10^{11} to 10^{12} $\Omega \cdot \text{cm}$. The actual resistivity of TN-LC in a display cell may be affected by the alignment polyimide material, the buffing material, or the cleaning procedure after buffing and the sealing epoxy. It is important to note that great care in handling all these areas needs to be exercised to ensure optimized LCD performance.

In order to reduce the severe requirements on the TFT OFF current I_{OFF} and the LC cell's RC time constant, a storage capacitor can be added in parallel to each pixel element. This leads to a larger capacitance C_{LC} that relaxes the OFF current condition according to Eq. (6.2-3), and also increases the RC time constant. The storage capacitor can be either a general ground capacitor or a capacitor formed between the ITO output electrode and the following scan line. The additional capacitor leads to an increased fabrication complexity and a reduced yield. In addition, the storage capacitor takes up real estate of the pixel, leading to a smaller aperture ratio. For LCDs with moderate or low resolutions (e.g., ≤ 200 lines per inch), the storage capacitor is desirable but not required. For high resolution displays with 500 lines or more per inch, the storage capacitor is considered to be essential. In AM-LCD technology using polysilicon TFTs, the leakage current can also be reduced by replacing a single TFT with multiple TFTs in series connected to a common gate [6,7].

In a TFT circuit, there are many parasitic capacitances, including capacitance C_{GD} due to the physical overlap of the gate and the drain electrodes. There also exists a finite capacitance C_{GP} between the gate bus line and the ITO output electrode. Some of the more important parasitic capacitances are shown in Figure 6.7. In most TFTs for LCDs, $C_{\text{GP}} \ll C_{\text{GD}}$. During the addressing of a line, when the scan line voltage is dropped from V_{ON} to V_{OFF} as shown in Figure 6.8, a negative voltage shift of

$$\Delta V_P = \Delta V_G \frac{C_{\text{GD}}}{C_{\text{LC}} + C_{\text{GD}}} \quad (6.2-4)$$

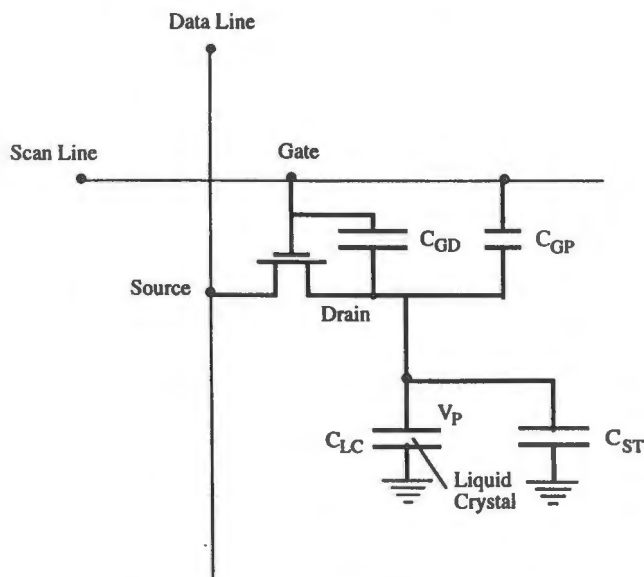


Figure 6.7. Parasitic capacitances in a TFT circuit.

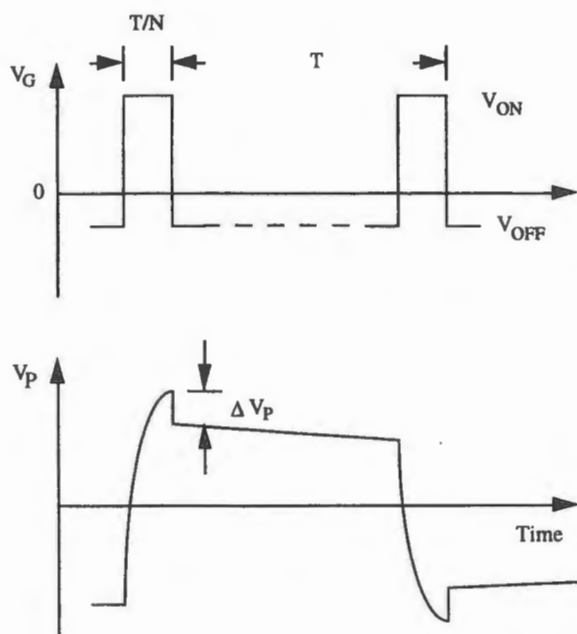


Figure 6.8. Voltage shift in the pixel voltage caused by the scan line voltage change and the presence of parasitic capacitance.

results, where $\Delta V_G = V_{ON} - V_{OFF}$. The voltage shift ΔV_P can lead to the problem of flicker and the degradation of the gray-scale performance. Unfortunately, this voltage shift cannot be compensated by using a simple voltage shift of the backplane electrode, because (1) C_{GD} is not always uniform throughout the TFT array and (2) C_{LC} is dependent on the voltage applied to the LC cell element due to the intrinsic dielectric anisotropy of the material. The voltage shift ΔV_P can, however, be reduced by the addition of a storage capacitor C_{ST} in each pixel. A voltage compensation scheme can be developed to achieve a symmetrical driving voltage across the LC cell. The discussion of the compensation scheme is beyond the scope of this book. Interested readers are referred to Reference 10. There are other parasitic capacitance (not shown in Fig. 6.7), including the source-drain capacitance C_{SD} of the TFT, the capacitance between the data line, and the ITO pixel electrode C_{SP} . The capacitance coupling from the data voltage can lead to gray scale voltage distortion. Again, electrical addressing techniques can be developed to reduce the distortion. Interested readers are referred to references 2, 8, and 9 and references cited therein.

6.2.2. Array Fabrication

Amorphous silicon (a-Si), polycrystalline silicon (poly-Si), and cadmium selenide (CdSe) are by far the most widely investigated semiconductor materials for

application in AM-LCDs. Each of the materials has its own unique fabrication processes. For instance, CdSe TFTs have the longest history of technological development. Although CdSe TFTs have all the required characteristics to drive the matrix circuits and the peripheral circuits, large scale high resolution color displays have not yet been demonstrated. On the other hand, a-Si TFTs have been the most popular and widely developed materials, possibly because of their low OFF current (I_{OFF}) and adequate ON current (I_{ON}) characteristics and the convenience of using the same processing and fabrication equipment as Si-MOS devices. Polysilicon (poly-Si) TFTs are currently being actively developed for future AM-LCDs. By using poly-Si materials, the peripheral driving circuits can be integrated with the TFT matrix circuit on the same substrate. In addition, poly-Si materials offer a higher electron (and hole) mobility, potentially up to $300 \text{ cm}^2/\text{V}\cdot\text{s}$. The higher mobility of poly-Si TFT translates into smaller transistors (higher pixel density) and storage capacitors, hence a higher aperture ratio than an a-Si TFT-LCD of comparable drive current and pixel size [11,12]. Crystalline-Si (c-Si) materials are also being investigated for future TFT-LCD applications. Higher resolution [$>2000 \text{ dpi}$ (dots/inch)] and electron (and hole) mobility are important features of c-Si. Table 6.2 summarizes a comparison of the a-Si, poly-Si, and CdSe TFT materials.

TFT Structures

Generally speaking, two types of TFT structure are commonly used in display applications. Figure 6.9a shows a bottom gate (inverted staggered) structure, as the gate is at the bottom and the source-drain electrodes are on the top side of the semiconductor. This structure has been used extensively in a-Si TFT arrays. Figure 6.9b shows a top gate (coplanar) structure, as the gate is on the top and all electrodes (gate, source, drain) are on the same side of the semiconductor. This structure is quite popular for poly-Si TFT arrays. Both structures have been used in CdSe TFT arrays.

Table 6.2. Comparison of the Three TFT Materials

	Amorphous Si	Poly-Si	CdSe
Mobility ($\text{cm}^2/\text{V}\cdot\text{s}$)	0.3–1	10–300	25–150
Substrate	Hard glass / sodalime	Hard glass / quartz	Hard glass / sodalime
Processing temperature	$<300^\circ\text{C}$	$600\text{--}1000^\circ\text{C}$	$<350^\circ\text{C}$
ON/OFF ratio	$10^5\text{--}10^7$	10^6	$10^5\text{--}10^7$
OFF current	$<1 \text{ pA}$	$1\text{--}10 \text{ pA}$	$1\text{--}10 \text{ pA}$
Integrated circuit	Difficult	Product	Lab demo
Largest size	14 in.	9.5 in.	9.5 in.
Most pixels	1100×1440	480×960	400×600

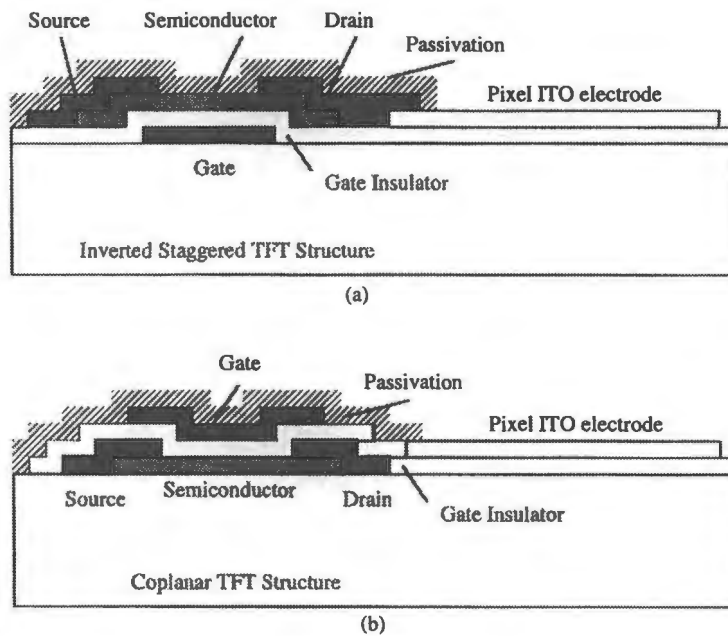


Figure 6.9. (a) An inverted staggered TFT structure; (b) a coplanar TFT structure.

TFT Types

a-Si TFT. The development of *a-Si* TFT for AM-LCDs has benefited significantly from the early advancement of silicon technology for *a-Si* solar cells in the 1970s. The extensive knowledge on the deposition techniques and electrical characteristics accumulated during these years can be readily applied to TFT-LCD applications. By virtue of its intrinsically high resistivity, *a-Si* TFT arrays have been very successful in AM-LCD applications. Although there have been rapid developments in the fabrication processes, they can be generally divided into two types: A and B. For TFT-LCDs, the substrate must be a thin and transparent flat plate with a strain point at least 100°C above the maximum processing temperature, alkaline-free and enough structure integrity, such as Corning 7059 glass. The substrate must be able to sustain the TFT-LCD processing cycles (different temperature, pressure, chemical environments) without significant chemical attack, shrinkage, or thermal mismatch to ensure the manufacturability. The gate material can be conducting metals such as Ti, Ti-Mo, Ta, and Mo-Ta, which can be deposited by DC magnetron sputtering. Both Ta and Mo-Ta gates have been widely used as the materials can be easily anodized to yield a pinhole-free gate insulator. These two fabrication processes for bottom gate structures are discussed as follows:

Type A fabrication process

1. Deposition of a pattern of gate metal for gate delineation.
2. Deposition of a sandwich of SiN_x (gate insulator), i-a-Si (semiconductor), and n^+ a-Si (source, drain) layers in one vacuum pump down by using the plasma-enhanced chemical vapor deposition (PECVD) process.
3. Etching of n^+ a-Si and i-a-Si layers into islands for individual TFTs.
4. Deposition and delineation of source, drain, and ITO electrodes.
5. Etching of n^+ a-Si layer to form the conducting channels for the TFTs using the source-drain electrodes as the mask.
6. Etching (timed or differential) of n^+ a-Si layer to reveal the intrinsic a-Si layer.
7. Deposition of a thin passivation layer.

Type B fabrication process

1. Deposition of a pattern of gate metal for gate delineation.
2. Deposition of a sandwich of SiN_x (gate insulator), i-a-Si (semiconductor), and SiN_x (passivation) layers in one vacuum pump down by using the same PECVD process.
3. Etching of top SiN_x and i-a-Si layers into islands for individual TFTs.
4. Etching of the top SiN_x layer to expose i-a-Si layer surface for source and drain regions.
5. Deposition of thin layers of n^+ a-Si and source-drain metal (Cr, Mo, Cr-Al, Ti) in sequence.
6. Etching and patterning of source-drain electrodes.
7. Deposition of a thin ITO layer by sputtering.
8. Etching to form the LC output electrodes.

We note that process A requires only four mask levels, whereas process B requires five mask levels. However, the etch stop of the n^+ a-Si layer (step 6) in process A is difficult to control. As a result, the i-a-Si layer cannot be reduced to a few hundred angstroms to minimize the photoconductivity effect. Thus, the photocurrent of the TFT obtained in this process is in general higher than that of process B. Since the passivation layer is not deposited in the same pump down as the i-a-Si layer, the OFF (leakage) current of the TFT obtained in this process is, in general, higher than those obtained in process B. We also note that process B requires a very careful cleaning of the interface before the deposition of the n^+ a-Si layer (step 5) to ensure good ohmic contact for the TFTs. An ON/OFF current ratio greater than 10^6 and mobility of $0.3\text{--}1\text{ cm}^2/\text{V}\cdot\text{s}$ have been achieved.

Poly-Si TFT

The fabrication processes of poly-Si TFTs are generally divided into two paths according to the maximum processing temperatures and the substrate size [11]. The first path leads to high temperature (HT) devices for high resolution view finders and projection displays, with a maximum processing temperature above 900°C. The second path is aimed at low temperature (LT) devices to be processed on large glass substrate of about 40 cm [12,13], with a maximum processing temperature below 600°C. Currently, the LT process has been the focus of most development efforts for reasons of cost-effectiveness. As for the device configurations, there are again, top gate and bottom gate structures. The top gate configuration with the self-aligned ion implantation architecture based on low pressure chemical vapor deposition (LPCVD)-deposited Si thin films is popular among MOS manufacturers. The bottom gate configuration using crystallized PECVD a-Si:H as the active material is preferred among a-Si TFT-LCD manufacturers for the apparent compatibility with large area a-Si TFT technology. Poly-Si layers are typically based on solid-phase crystallization (SPC) of a-Si layers deposited by the low pressure chemical vapor deposition (LPCVD) at around 600°C. The substrate can be either quartz or hard glass depending on whether a thermally grown oxide or a deposited low temperature oxide is used as the gate insulator. As mentioned earlier, top gate (coplanar) structures are often used for poly-Si TFTs. An example of the fabrication process of a top gate low temperature (LT) poly-Si TFT structure is described as follows [2]:

1. Deposition of a 150 nm-thick-phosphorous-doped poly-Si layer (a-Si with solid-phase crystallization) on a hard glass substrate.
2. Etching and patterning to form the source-drain electrodes.
3. Deposition of a very thin (25-nm) layer of undoped poly-Si layer by LPCVD at 600°C.
4. Etching to form TFT.
5. Deposition of ITO layer.
6. Etching to form the ITO data lines and pixel ITO output electrodes.
7. Deposition of the gate insulator (150 nm SiO₂) using thermal CVD.
8. Sputtering and delineation of gate electrode (Cr).

For the high temperature poly-Si TFT, the gate insulator is thermally grown at around 1000°C and the source-drain doping is accomplished by self-aligned ion implantation. Self-aligned-gate processes have the advantage of minimizing the gate-to-drain electrode overlapping. Figure 6.10 shows an example of a self-aligned-gate process. In the self-aligned process, a poly-Si layer is deposited and patterned into islands for TFT. Then a layer of SiO₂ and a layer of poly-Si are deposited in sequence. The stack is then etched to expose the two ends of each poly-Si island. After the etching, the exposed poly-Si areas are treated with ion

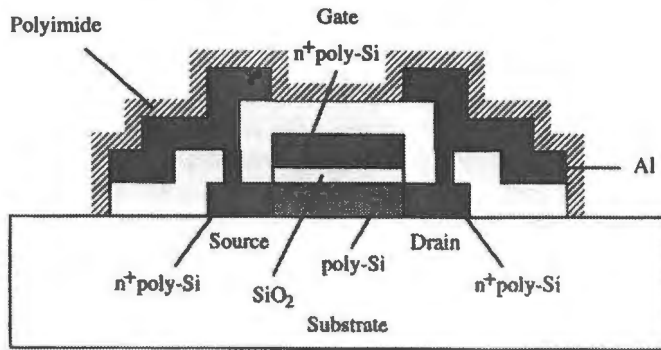


Figure 6.10. A self-aligned gate poly-Si TFT fabrication process.

implantation, converting the poly-Si into n^+ -doped poly-Si. The doping process by the ion implantation creates the gate, source, and drain, where the top poly-Si layer is the gate electrode. In addition, the top poly-Si layer (gate) along with the SiO_2 layer underneath act as a mask defining the source and drain regions of the TFT, thus achieving the process of self-alignment. After depositing another layer of SiO_2 and via hole etching, Al is deposited and delineated to serve as the ohmic contact for the source-drain electrodes. Finally, ITO is sputter deposited and etched as output electrodes.

As mentioned earlier, poly-Si materials offer the capability of integrating the TFT arrays with the driver circuits and the peripheral circuits. Another promising aspect of poly-Si TFT is the possibility of converting a-Si films into poly-Si films by using laser annealing (LA). By using this laser-assisted conversion, both poly-Si and a-Si can be processed on the same substrate where a-Si TFT is the matrix array and poly-Si is the driver circuits. The integration can significantly reduce the required display interconnections and increase the compactness and the reliability of the display. The significantly higher electron (and hole) mobility in poly-Si offers the possibility of a higher addressing speed. According to Eq. (6.2-1), the source-drain current I_{SD} is proportional to the electron mobility μ . With a higher current, it takes less time to charge the LC capacitor, leading to the capability of higher operation frequency for the column driver.

CdSe TFT. CdSe has the advantages of being a low temperature processing material and having a relatively higher electron mobility of up to $150 \text{ cm}^2/\text{V} \cdot \text{s}$. In an all-photolithography CdSe fabrication process for an inverted staggered structure, all layers are delineated by liftoff processes except the ITO layer, which was patterned by etching. The process is described as follows.

1. Deposition of a pattern of gate metal (Ni) for gate delineation by sputtering.
2. Deposition of gate insulator layer (Al_2O_3) by sputtering.

3. Deposition of a pattern of CdSe for TFT's.
4. Delineation of In-Au source and drain electrodes.
5. Deposition of a thin passivation layer.
6. Delineation of ITO electrodes.

CdSe TFTs can have an ON current of greater than $1\text{ }\mu\text{A}$ and an OFF currents of less than 1 pA . One of the potential advantages of the CdSe TFT process is the possibility of integrating the row and column driver circuits with the matrix circuits.

6.2.3. Cell Assembly

Here we discuss the assembly of the display panel of TFT-LCDs. As described earlier, the display panel consists of mainly two glass plates with LC material sandwiched in between [2,14]. One glass plate contains the TFT array, and the other contains the color filters and the common electrode. In the assembly, the inner surfaces of both glass plates are first coated with a thin layer of polyimide with a thickness ranging from a few hundreds to one thousand angstroms. The polyimide layers are then rubbed in prearranged directions for the alignment of the LC director. Next, a narrow strip of epoxy is either dispensed or printed on the borders of the color filter glass plate to form a ring with a narrow opening. To maintain a uniform spacing ($4\text{--}10\text{ }\mu\text{m}$) between the glass plates, fiber glass spacer rods or plastic spacer beads are dispensed uniformly on the TFT glass plate. The uniform spacing can also be maintained by delineating a dyed polyimide post on each TFT in the array. We note that the post not only maintains the proper spacing, but also serves a light-blocking function. The two plates are then assembled together with the pixels in the TFT array properly aligned with the pixels in the color filters. In addition, the two plates are also properly oriented so that a desired twist angle (TN-LC) is obtained. After pressing and curing, the assembly is vacuum-filled with LC and plugged by a quick-setting epoxy. After the completion of the LC cell assembly process, the final step is performed with external components. Plastic polarizers are carefully adhered to both sides of the LC cell with the transmission axes aligned in the predetermined directions depending on the director twist. Finally, the LC cell is mounted on the circuit board or rigid substrate connected with the driving and peripheral circuits of LSI (large scale integration).

In the discussion above we described several TFT structures and some of the fabrication processes for a-Si, poly-Si, and CdSe TFTs and the cell assembly process. These discussions were intended to illustrate the basic operation principles. It is important to note that the fabrication technologies, device architectures and the assembly process are advancing very rapidly. It is important to keep an eye on the latest developments in these areas, including new materials and new fabrication processes.

6.3. OPTICAL THROUGHPUT OF TFT-LCDs

The presence of the TFT array circuits, scan and data bus, polarizers, transparent electrodes, and other elements will affect the brightness or light throughput of the displays. In color displays, the presence of the color filters leads to a significant decrease of the brightness due to the absorption of light in the filters. In what follows, we consider the energy loss at each of the components.

6.3.1. Polarizers

A beam of unpolarized light suffers 50% energy loss due to a perfect polarizer. In practice, sheet polarizers are used for flat panel displays. Most commercially available polarizers consist of polarizer materials such as HN22, HN32, HN38S, and HN42HE sandwiched between two thin high quality glass substrates. These polarizer materials are usually made of stretched PVA (poly vinyl alcohol) films containing highly concentrated iodine dyes. The polarizers for LCD applications have typical transmission around 45% or less for visible wavelengths. For example, a polarizer material, HN42HE, manufactured by Polaroid corporation transmits roughly 42% of an unpolarized input beam. The deviation of 8% from perfect polarizers is due to residual absorption for the transmitted polarization component. For display applications, the polarizer materials are cemented to the outer surfaces of the glass plates of the LC cell.

Example 6.2. Consider a LCD using polarizers with a maximum transmission of 42% for unpolarized light. The energy loss due to the polarizer is 58%, whereas the energy loss due to the analyzer is about 8%. Thus, the total energy loss due to the polarizers alone is 62% ($= 1 - 0.42 \times 0.92$).

The loss of 3 dB (50%) of energy at the first polarizer is very undesirable. Using sheet polarizers, the 50% of the energy is absorbed by the polarizer. There is no possibility of recovering the energy loss. However, if polarizing beamsplitters (PBS) are used, 50% of the energy is deflected out of the viewing field. The 50% of the energy deflected may be recycled by using optical techniques. This may improve the optical energy throughput. Unfortunately, ordinary polarizing beam splitters are often bulky and unsuitable for flat panel displays. However, for projection displays, these polarizing beamsplitters can be very important in improving the optical energy throughput. Fig. 6.11 describes an example of the use of polarizing beam splitters for projection displays [15].

Referring to Figure 6.11, we consider a Sagnac ring interferometer consisting of a polarizing beamsplitter (PBS) and three mirrors (M). In practice, two mirrors are adequate. An incident beam of unpolarized light, entering at port A is split into two which are then directed toward each other via the mirrors. The beams recombine at the same PBS and then exit the interferometer at port B. A TN-LC cell without polarizers is inserted in the beam path between the mirrors. In the field-OFF state, the TN-LC cell with the proper orientation serves as a

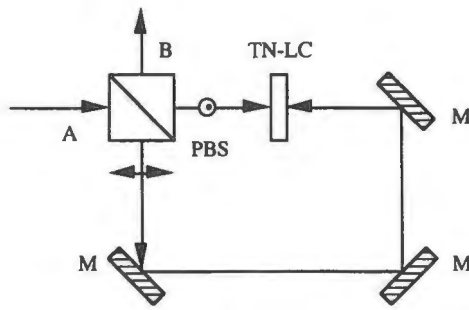


Figure 6.11. A schematic drawing of the use of polarizing beamsplitters in a Sagnac interferometer configuration for projection displays.

polarization rotator, rotating the polarization state by 90° . As a result, the beams recombine at the PBS and exit the interferometer through port A, in the direction opposite to the incident beam. In the field-ON state, the LC molecules are homeotropically aligned, and the cell exhibits no phase retardation to the beams. As a result the polarization states of the beams remain unchanged. Thus, the beams recombine at the PBS and exit the interferometer through port B. In this configuration, there is no energy loss due to the polarizer.

The use of cholesteric liquid crystal (CLC) as polarizers for LCDs has been proposed. A thin sheet of CLC can work as a polarizer that can separate right-handed circularly (RHC) polarized waves from left-handed circularly (LHC) polarized waves within a spectral regime of interest. Specifically, a CLC-based polarizer can reflect LHC polarized light and transmit RHC polarized light. The reflected light can be recycled by using a mirror behind the backlight, leading to a higher energy throughput. This concept is illustrated in Figure 6.12.

Unpolarized light emitted from the backlight is split into two parts of circularly polarized light. The RHC polarized light is transmitted through the CLC cell and then converted into linear polarized light by using a quarter-wave plate. The LHC polarized light is reflected at the CLC cell. The reflected LHC light is then reflected by the rear mirror. On reflection, the light becomes RHC polarized, and can then transmit through the CLC cell. This is the basic principle of recycling the light by using CLC polarizers. The polarizing properties of CLC will be discussed in detail in Chapter 7.

In addition to the energy loss due to the polarizers, the transmission of light through the LC cell suffers energy loss due to the presence of the black matrix and storage capacitor (aperture ratio), residual absorption of the ITO layers, and scan and data bus lines. As a result the typical light throughput is around 20% for monochrome and 5% for color LCD. Color filters are the major source of energy loss in color LCDs. Each color filter absorbs at least 67% (two-thirds) of the energy entering into that particular color pixel. The energy loss and the typical fraction of light transmission are listed in Table 6.3, where we note that the major source of energy loss in the color filters.

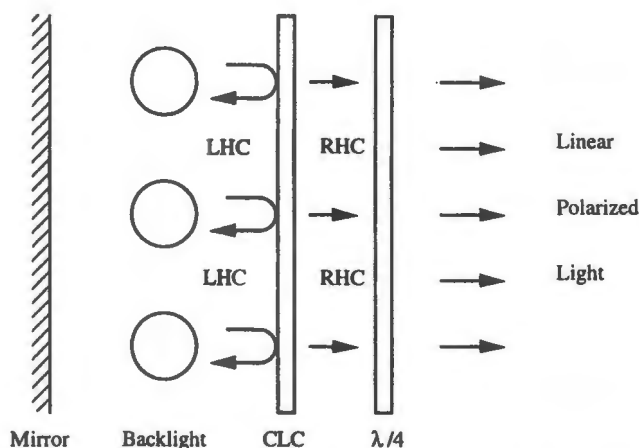


Figure 6.12. CLC polarizers for throughput improvement in LCD applications.

Table 6.3. Transmission of Various LCD Components

Energy Loss	Transmission (%)
Polarizer	43
Color filters	25
TFT aperture ratio	80
Liquid Crystal	95
Analyzer	80
Overall (color LCDs)	5

6.3.2. Color Filters

Many types of color filters have been reported for the color LCDs. These include dyed gelatin, dyed polyimide, and colored inks. Dyed gelatin is by far the most widely used material for color filters. Currently, photolithography is employed to generate small color pixels. At first, a thin layer (2–3 μm thick) of base material (e.g., glue, gelatin, PVA, polyimide) that has dye acceptance property is coated on the glass substrate. The base is then covered with a thin layer of photoresist. Selected areas formed by photoexposure with photomasks are then opened for red dyeing by dipping into the red dye solution. After photoresist is removed, this procedure is repeated for green and blue dyes. Finally, the surface of the dyed base material is coated with a passivation layer (polyimide), thus isolating the LC molecules from the color filter layer and providing a smooth surface for the deposition of the ITO transparent electrode. Precision printing with RGB inks is a cost-effective way to eliminate the photolithography technique.

Generally speaking, three color filters are needed for the display of all colors: red, green, and blue filters. If we assume that the optical energy is evenly

distributed in these three spectral regimes, then it is clear that each color filter can transmit only one-third of the total optical energy. In other words, two-thirds of the energy is absorbed by the color filters, if the filters are based on absorption. The transmission bandwidth of the color filter is usually very wide to allow a higher brightness at the expense of a lower color purity. In practice, the color filters have a bell-shape spectral transmission curve centered at their respective colors that match with the peaks of the RGB-enhanced backlighting. Thus the color filters can only transmit less than one-third of the total energy. This explains the typical transmission of only 20% as shown in Table 6.3.

The use of diffractive gratings or interference filters for color LCDs has been proposed. The basic principles of recycling the energy using either diffractive gratings or interference filters are depicted in Figure 6.13.

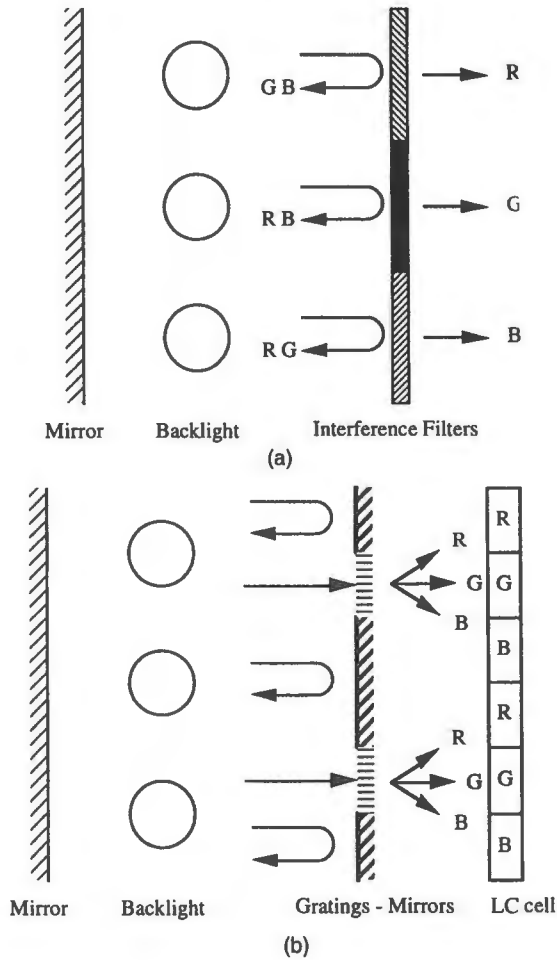


Figure 6.13. The use of (a) interference filters and (b) diffractive gratings for improving the light throughput in LCDs.

Figure 6.13a shows a schematic drawing of the use of interference filters for improving the light throughput in LCDs. Optical interference filters based on multilayer thin films can be designed to transmit the desired colors in R,G,B spectral regimes of interest. The optical energy outside the transmission band will be reflected by the interference filters. The reflected energy is then recycled by using a rear mirror.

Figure 6.13b shows a schematic drawing of the use of diffractive gratings for improving the light throughput in LCDs. By using a grating-mirror combination in which about one-third of the area is occupied by diffractive gratings. Thus, one-third of the incident light is diffracted into R,G,B beams, whereas two-thirds of the incident light is reflected. The one-third open aperture for the gratings is important to ensure the color separation in space. The reflected light is then recycled by using a rear mirror.

6.4. COLORS IN LCDs

From the fundamental point of view, any source of color light can be described by its spectral distribution. This can be easily obtained experimentally by using a spectrometer such as a diffraction grating or a prism. Human eyes, however, do not function like a spectrometer. It is generally believed that human eyes contain three different pigments (say, R,G,B) which receive the light with different absorption spectra, so that R pigments absorb strongly in the red region, G pigments absorb strongly in the green, and B pigments absorb strongly in the blue. Thus, when a beam of light is directed into the eyes, we perceive a color that is the result of the mixing of the amount of absorption received by the three different sets of pigments. As a result of the mechanism of color vision of human eyes, different spectral distribution can produce the same color. In addition, any color can be made from three different colors (e.g., red, green, and blue). Furthermore these fundamental laws of colors can be described mathematically [16].

The discussion in the previous chapters, suggests that TN-LCD can be employed to modulate the transmission of light. In black-and-white (B/W) displays, the transmission properties of the LCD must be insensitive to wavelength. This can be achieved by using TN-LCD with Mauguin parameters in the regime $1 \ll u$. For color displays, the easiest way to achieve colors is to use color filters in series with B/W displays. Using three spectral filters (red, green and blue) for each pixel, we are able to obtain different colors by controlling the intensity transmission of each of the three subpixels. As a result of the color mixing, we are able to generate almost all colors of interest.

In most color LCDs, each pixel of information consists of three subpixels with color filters. The color perceived by our eyes is a mixture (sum) of the three colors in the pixel. Letting the intensity transmission for each of the subpixels be t_1, t_2, t_3 , and the transmission spectra of the three color filters be

$f_1(\lambda), f_2(\lambda), f_3(\lambda)$, the effective transmission spectrum of the pixel can be written

$$T(\lambda) = t_1 f_1(\lambda) + t_2 f_2(\lambda) + t_3 f_3(\lambda) \quad (6.4-1)$$

Here we assume that t_1, t_2, t_3 are insensitive to the variation of the wavelength. Strictly speaking, t_1, t_2, t_3 are functions of wavelength. However, the spectral dependence of $T(\lambda)$ is dominated by the transmission of the color filters $f_1(\lambda), f_2(\lambda)$, and $f_3(\lambda)$. In color LCDs, t_1, t_2, t_3 can be independently controlled by applying different voltages to each subpixel. Once $T(\lambda)$ is obtained, it is possible to calculate the chromaticity coordinates of the color transmitted by the pixel. In what follows, we discuss the calculation of the chromaticity coordinates and the mixing of colors.

To calculate the chromaticity coordinates (x, y) , we must first obtain the tristimulus values (X, Y, Z) by using the following formulas [17,18],

$$X = k \int S(\lambda) T(\lambda) \bar{x}(\lambda) d\lambda \quad (6.4-2)$$

$$Y = k \int S(\lambda) T(\lambda) \bar{y}(\lambda) d\lambda \quad (6.4-3)$$

$$Z = k \int S(\lambda) T(\lambda) \bar{z}(\lambda) d\lambda \quad (6.4-4)$$

where $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ are the *color matching functions*, $S(\lambda)$ is the source spectrum (backlight illuminant), and $T(\lambda)$ is the transmission spectrum of the information pixel according to Eq. (6.4-1); k is the normalization factor defined as

$$k = \frac{100}{\int S(\lambda) \bar{y}(\lambda) d\lambda} \quad (6.4-5)$$

This definition of k makes the Y tristimulus value equal to 100 for a perfect transmission system with $T(\lambda) = 1$ for all λ . The color matching functions for 10° viewing angles (CIE 1964 *Standard Colorimetric Observer*) are written as $\bar{x}_{10}(\lambda), \bar{y}_{10}(\lambda), \bar{z}_{10}(\lambda)$ whose spectral tristimulus values are tabulated in Table 6.4, and plotted as functions of the wavelength in Figure 6.14. The tristimulus values X_{10}, Y_{10}, Z_{10} are obtained if the color matching functions $\bar{x}_{10}(\lambda), \bar{y}_{10}(\lambda), \bar{z}_{10}(\lambda)$ are used. The color matching functions of CIE 1931 *Standard Colorimetric Observer* are denoted $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$ for $1-4^\circ$ viewing angles.

Table 6.4. Spectral Tristimulus Values of Color Matching Functions for 10° Viewing Angles $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$ ^a

λ (nm)	$\bar{x}_{10}(\lambda)$	$\bar{y}_{10}(\lambda)$	$\bar{z}_{10}(\lambda)$	x	y
380	0.00015995	0.00001736	0.00070478	0.18133	0.019685
385	0.00066244	0.00007156	0.00292780	0.18091	0.019542
390	0.00236160	0.00025340	0.01048220	0.18031	0.019348
395	0.00724230	0.00076850	0.03234400	0.17947	0.019044
400	0.01910970	0.00200440	0.08601090	0.17839	0.018711
405	0.04340000	0.00450900	0.19712000	0.17712	0.018402
410	0.08473600	0.00875600	0.38936600	0.17549	0.018134
415	0.14063799	0.01445600	0.65675998	0.17323	0.017806
420	0.20449200	0.02139100	0.97254199	0.17063	0.017849
425	0.26473701	0.02949700	1.28250000	0.1679	0.018708
430	0.31467900	0.03867600	1.55348000	0.16503	0.020283
435	0.35771900	0.04960200	1.79849990	0.16217	0.022487
440	0.38373399	0.06207700	1.96728000	0.15902	0.025725
445	0.38672599	0.07470400	2.02729990	0.15539	0.030017
450	0.37070200	0.08945600	1.99480000	0.151	0.036439
455	0.34295699	0.10625600	1.90070000	0.14594	0.045217
460	0.30227301	0.12820099	1.74537000	0.13892	0.05892
465	0.25408500	0.15276100	1.55490010	0.12952	0.07787
470	0.19561800	0.18519001	1.31756000	0.11518	0.10904
475	0.13234900	0.21994001	1.03020000	0.095732	0.15909
480	0.08050700	0.25358900	0.77212501	0.072777	0.22924
485	0.04107200	0.29766500	0.57006001	0.045194	0.32754
490	0.01617200	0.33913299	0.41525400	0.020987	0.44011
495	0.00513200	0.39537901	0.30235600	0.007302	0.56252
500	0.00381600	0.46077701	0.21850200	0.005586	0.67454
505	0.01544400	0.53135997	0.15924899	0.021874	0.75258
510	0.03746500	0.60674101	0.11204400	0.04954	0.8023
515	0.07135800	0.68566000	0.08224800	0.085024	0.81698
520	0.11774900	0.76175702	0.06070900	0.12524	0.81019
525	0.17295299	0.82332999	0.04305000	0.16641	0.79217
530	0.23649099	0.87521100	0.03045100	0.20706	0.76628
535	0.30421299	0.92381001	0.02058400	0.24364	0.73987
540	0.37677199	0.96198797	0.01367600	0.27859	0.7113
545	0.45158401	0.98220003	0.00791800	0.31323	0.68128
550	0.52982599	0.99176103	0.00398800	0.3473	0.65009
555	0.61605299	0.99910998	0.00109100	0.38116	0.61816
560	0.70522398	0.99734002	0.00000000	0.41421	0.58579
565	0.79383200	0.98237997	0.00000000	0.44692	0.55308
570	0.87865502	0.95555198	0.00000000	0.47904	0.52096
575	0.95116198	0.91517502	0.00000000	0.50964	0.49036
580	1.01416000	0.86893398	0.00000000	0.53856	0.46144
585	1.07430010	0.82562298	0.00000000	0.56544	0.43456
590	1.11852000	0.77740502	0.00000000	0.58996	0.41004
595	1.13430000	0.72035301	0.00000000	0.6116	0.3884
600	1.12399010	0.65834099	0.00000000	0.63063	0.36937

Table 6.4. (Continued)

λ (nm)	$\bar{x}_{10}(\lambda)$	$\bar{y}_{10}(\lambda)$	$\bar{z}_{10}(\lambda)$	x	y
605	1.08910000	0.59387797	0.00000000	0.64713	0.35287
610	1.03048000	0.52796298	0.00000000	0.66122	0.33878
615	0.95073998	0.46183401	0.00000000	0.67305	0.32695
620	0.85629702	0.39805701	0.00000000	0.68266	0.31734
625	0.75493002	0.33955401	0.00000000	0.68976	0.31024
630	0.64746702	0.28349301	0.00000000	0.69548	0.30452
635	0.53511000	0.22825401	0.00000000	0.70099	0.29901
640	0.43156701	0.17982800	0.00000000	0.70587	0.29413
645	0.34369001	0.14021100	0.00000000	0.71025	0.28975
650	0.26832899	0.10763300	0.00000000	0.71371	0.28629
655	0.20430000	0.08118700	0.00000000	0.71562	0.28438
660	0.15256800	0.06028100	0.00000000	0.71679	0.28321
665	0.11221000	0.04409600	0.00000000	0.71789	0.28211
670	0.08126060	0.03180040	0.00000000	0.71873	0.28127
675	0.05793000	0.02260170	0.00000000	0.71934	0.28066
680	0.04085080	0.01590510	0.00000000	0.71976	0.28024
685	0.02862300	0.01113030	0.00000000	0.72002	0.27998
690	0.01994130	0.00774880	0.00000000	0.72016	0.27984
695	0.01384200	0.00537510	0.00000000	0.7203	0.2797
700	0.00957688	0.00371774	0.00000000	0.72036	0.27964
705	0.00660520	0.00256456	0.00000000	0.72032	0.27968
710	0.00455263	0.00176847	0.00000000	0.72023	0.27977
715	0.00314470	0.00122239	0.00000000	0.72009	0.27991
720	0.00217496	0.00086419	0.00000000	0.71565	0.28435
725	0.00150570	0.00058644	0.00000000	0.71969	0.28031
730	0.00104476	0.00040741	0.00000000	0.71945	0.28055
735	0.00072745	0.00028404	0.00000000	0.71919	0.28081
740	0.00050826	0.00019873	0.00000000	0.71891	0.28109
745	0.00035638	0.00013955	0.00000000	0.71861	0.28139
750	0.00025097	0.00009843	0.00000000	0.71829	0.28171
755	0.00017773	0.00006982	0.00000000	0.71796	0.28204
760	0.00012639	0.00004974	0.00000000	0.71761	0.28239
765	0.00009015	0.00003554	0.00000000	0.71724	0.28276
770	0.00006453	0.00002549	0.00000000	0.71686	0.28314
775	0.00004634	0.00001834	0.00000000	0.71646	0.28354
780	0.00003341	0.00001325	0.00000000	0.71606	0.28394

^a Where x , y are the chromaticity coordinates of monochromatic light.

The chromaticity coordinates of any given color are defined as follows:

$$x = \frac{X}{X + Y + Z}, \quad x_{10} = \frac{X_{10}}{X_{10} + Y_{10} + Z_{10}} \quad (6.4-6)$$

$$y = \frac{Y}{X + Y + Z}, \quad y_{10} = \frac{Y_{10}}{X_{10} + Y_{10} + Z_{10}} \quad (6.4-7)$$

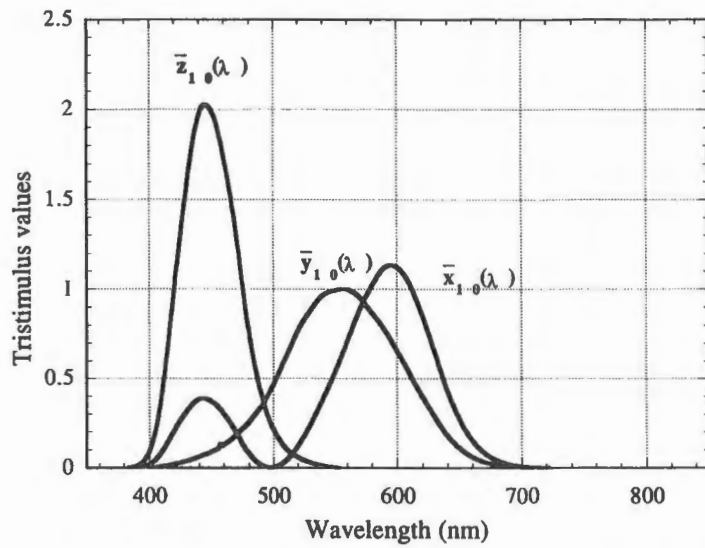


Figure 6.14. The spectral tristimulus values of the color matching functions for 10° viewing angles as functions of the wavelength.

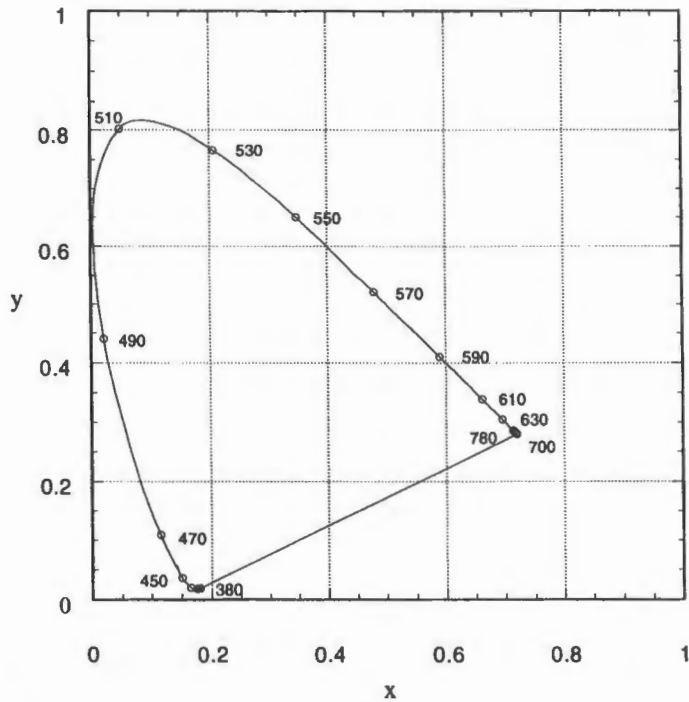


Figure 6.15. CIE 1964 (x,y) chromaticity diagram showing the spectrum locus of monochromatic stimuli from 380 to 780 nm, and the purple line.

Each color specification is represented by a point (x, y) in the chromaticity coordinate system. Figure 6.15 shows the curve by connecting all points of the chromaticity coordinate of monochromatic light, and the purple line, which is the straight line between 380 and 780 nm.

6.4.1. Addition (or Mixing) of Colors and CIE 1976 Color Space ($L^*u^*v^*$)

Let (x_1, y_1) be the chromaticity coordinates of the color due to the color filter of the first subpixel, and similarly (x_2, y_2) for the color filter of the second subpixel, and (x_3, y_3) for the color filter of the third subpixel. Let (X_1, Y_1, Z_1) , (X_2, Y_2, Z_2) , (X_3, Y_3, Z_3) be the corresponding tristimulus values for the subpixels. It can be easily shown that the tristimulus values for the whole pixel can be written

$$X = t_1X_1 + t_2X_2 + t_3X_3 \quad (6.4-8)$$

$$Y = t_1Y_1 + t_2Y_2 + t_3Y_3 \quad (6.4-9)$$

$$Z = t_1Z_1 + t_2Z_2 + t_3Z_3 \quad (6.4-10)$$

where t_1, t_2 , and t_3 are the transmittance of the subpixels. We note that the resultant tristimulus values are weighted sums of the individual tristimulus values. The chromaticity coordinates for the whole pixel can thus be calculated by using Eqs. (6.4-6) and (6.4-7). If all the intensity transmission t_1, t_2, t_3 can be varied independently between 0 and 1, then all colors within the triangle formed by the three points (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) can be obtained. If leakage of light exists at the dark state, then t_1, t_2, t_3 cannot be very close to zero. This may lead to the problem of color desaturation. When this happens, the color LCD cannot display some of the colors within the triangle formed by the three points (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) , especially the colors near the corners and the edges of the triangle.

In 1976, the CIE recommended CIE $L^*u^*v^*$ for applications dealing with additive mixtures of colored light such as the lighting and color TV industries. The same color space can be employed for LCDs. Given a chromaticity coordinate (x, y) for each gray-scale level of the pixel, we can calculate the coordinate (u', v') as follows:

$$u' = \frac{4x}{-2x + 12y + 3} \quad (6.4-11)$$

$$v' = \frac{9y}{-2x + 12y + 3} \quad (6.4-12)$$

The rectangular coordinates of CIE 1976 include L^* (metric lightness), u^*

(redness–greenness), and v^* (yellowness–blueness) scales, which are defined as follows;

$$L^* = 116\sqrt[3]{\frac{Y}{Y_0}} - 16 \quad (6.4-13)$$

$$u^* = 13L^*(u' - u'_0) \quad (6.4-14)$$

$$v^* = 13L^*(v' - v'_0) \quad (6.4-15)$$

where u', v', Y and u'_0, v'_0, Y_0 are the (u', v') chromaticity coordinates and the luminance of the pixel under examination and those of a suitable reference (e.g., a chosen white background), respectively. For investigating the angular dependence of the color viewing in LCDs, u', v', Y are the (u', v') coordinates and luminance of the pixel under examination and u'_0, v'_0, Y_0 are the (u', v') coordinates and luminance of the backlight at normal incidence.

The polar coordinates of the CIE 1976 color space include S_{uv} (saturation), C_{uv} (metric chroma), and H_{uv} (hue angle) scales, which are calculated as follows:

$$S_{uv} = 13\sqrt{(u' - u'_0)^2 + (v' - v'_0)^2} \quad (6.4-16)$$

$$C_{uv} = \sqrt{u^{*2} + v^{*2}} \quad (6.4-17)$$

$$H_{uv} = \tan^{-1} \frac{v^*}{u^*} \quad (6.4-18)$$

For the purpose of quantitatively comparing the difference between a pair of samples (L_2^*, u_2^*, v_2^*) , (L_1^*, u_1^*, v_1^*) , we define the following parameters in the CIE 1976 $(L^*u^*v^*)$ color space:

$$\text{Color difference: } \Delta E = \sqrt{(L_2^* - L_1^*)^2 + (u_2^* - u_1^*)^2 + (v_2^* - v_1^*)^2} \quad (6.4-19)$$

$$\text{Chromaticity difference: } \Delta_{uv} = \sqrt{(u_2^* - u_1^*)^2 + (v_2^* - v_1^*)^2} \quad (6.4-20)$$

$$\text{Chroma difference: } \Delta C_{uv} = \sqrt{u_2^{*2} + v_2^{*2}} - \sqrt{u_1^{*2} + v_1^{*2}} \quad (6.4-21)$$

$$\text{Hue angle difference: } \Delta H_{uv} = \tan^{-1} \frac{v_2^*}{u_2^*} - \tan^{-1} \frac{v_1^*}{u_1^*} \quad (6.4-22)$$

For investigating the angular dependence of the color viewing in LCDs, u_2^*, v_2^*, Y_2 are the (u^*, v^*) chromaticity coordinates and luminance of the pixel

under examination and u_1^*, v_1^*, Y_1 are the (u^*, v^*) chromaticity coordinates and luminance of the same pixel at normal incidence.

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PROBLEMS

6.1. *Multiplexed addressing:*

(a) Differentiate Eq. (6.1-5) with respect to V_d while keeping V_{ON} and V_{OFF} constant. Derive Eqs. (6.1-6) and (6.1-7).

(b) Derive Eq. (6.1-8):

$$\frac{V_{ON}}{V_{OFF}} = \sqrt{\frac{\sqrt{N_{\max}} + 1}{\sqrt{N_{\max}} - 1}}$$

(c) Find the selection ratio for $N = 400$.

6.2. Derive the following expressions Eq. (6.2-1) for the current in MOSFET:

$$I_{SD} = \frac{W\mu C_G}{L} V_{SD} \left(V_G - V_T - \frac{V_{SD}}{2} \right) \quad \text{when} \quad V_{SD} < (V_G - V_T)$$

$$I_{SD} = \frac{W\mu C_G}{2L} (V_G - V_T)^2 \quad \text{when} \quad V_{SD} > (V_G - V_T)$$

where W = width

L = length of conducting channel of TFT

μ = effective mobility of the semiconductor

C_G = gate capacitance per unit area

V_{SD} = source-drain voltage

V_G = is the gate voltage

I_{SD} = source-drain current

V_T = threshold voltage of the TFT

6.3. Derive Eq. (6.2-2):

$$2K_1 C_{LC} V_{ON} < \frac{T}{N} I_{ON}$$

where K_1 = an engineering operation constant

V_{ON} = ON voltage of the LC element

C_{LC} = capacitance of the LC element

N = number of addressed lines in the display

T = frame time.

6.4. Derive Eq. (6.2-3):

$$I_{OFF} T < K_2 \Delta V C_{LC}$$

where T is the frame time and K_2 is another engineering operation constant that allows for the uniformity variations of TFT characteristics in a TFT array and the temperature variations of the OFF currents of the TFTs.

6.5. Derive Eq. (6.2-4):

$$\Delta V_P = \Delta V_G \frac{C_{GD}}{C_{LC} + C_{GD}}$$