

On the Equivalence of Space-Time Block Coding with Multipath Propagation and/or Cyclic Delay Diversity in OFDM

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ABSTRACT

In this paper, we show the equivalence of Space-Time block coding to multipath propagation with respect to Cyclic Delay Diversity for OFDM systems. Hence, we can apply existing Space-Time code design criteria and analyze methods for Space-Time code construction using Cyclic Delay Diversity OFDM systems in multipath propagation environments. i.e. terms like "diversity gain" and "coding gain" can be used to design, analyze, compare and understand these OFDM systems from the Space-Time block coding point of view.

1 INTRODUCTION

Mobile communication systems mainly suffer from time-varying multipath fading with extremely different multipath intensity profiles [1]. For systems which work in multipath environments, an improvement in error performance may become very difficult. In present systems, a slight improvement in the bit error rate may require a huge amount of additional transmission power, which contradicts the goal of economical use of system resources. It is an enormous challenge to design wireless communication systems capable of dealing with these varying scenarios.

Orthogonal frequency division multiplexing (OFDM) [2] is a suitable technique for broadband transmission in multipath fading environments and is implemented in new broadcast standards like terrestrial digital video broadcasting (DVB-T) [3] as well as wireless local area network (WLAN) standards [4] such as HIPERLAN/2 or IEEE 802.11a.

Because of the poor error performance of OFDM in multipath environments, it is necessary for wireless communications systems to use techniques like interleaving and channel coding in addition to OFDM. These techniques add redundancy and diversity in time and frequency direction. For many scattering environments, spatial diversity is another effective way to improve the error performance of wireless radio systems. In [5], [6] space-time-coding is proposed in order to get the benefits of channel coding in combination with spatial (antenna) diversity. Another approach to achieve spatial diversity effects for OFDM systems are techniques like Delay Diversity, Cyclic Delay Diversity or Phase Diversity ([7], [8]).

In this paper we will describe Cyclic Delay Diversity [8] and multipath propagation as a Space-Time block code. Section 2 first introduces the application of Cyclic Delay Diversity to OFDM systems. Further more, the equivalence of Cyclic Delay Diversity with respect to multipath propagation in OFDM and Space-Time block coding is shown. In Section 3 we will analyze using an example the diversity and the coding gain for a CDD OFDM system in a Rayleigh fading environment.

2 CYCLIC DELAY DIVERSITY AND MULTIPATH PROPAGATION AS SPACE-TIME BLOCK CODE

In this section we introduce *Cyclic Delay Diversity* (CDD) and show the equivalence between multipath propagation and CDD. Then multipath propagation and CDD are described as Space-Time block codes and analyzed using techniques from [5] and [9].

2.1 Introduction to Cyclic Delay Diversity for OFDM

Below we will briefly introduce the application of CDD to OFDM systems as described in detail in [8]. Fig. 1 shows the block diagram of an N -antenna OFDM transmitter with CDD. The OFDM modulated signal is transmitted over N antennas, whereas the particular signals only differ in an antenna specific cyclic shift. After cyclic shifting, a cyclic prefix is inserted to avoid intersymbol interference and maintain subcarrier orthogonality for multipath channels. The functional block "UC" performs an upconversion of the signals from the base-band into RF-band. $\delta_{cy\ n}, n = 1, \dots, N - 1$, denote cyclic shifts of the OFDM symbol in the time domain.

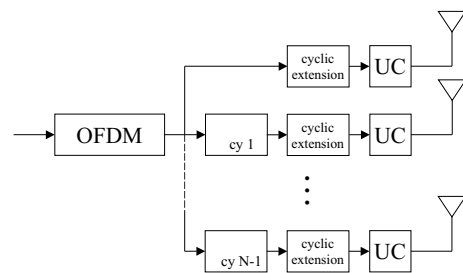


Figure 1: Application of CDD to an OFDM transmitter

2.2 Equivalence of multipath propagation and CDD

The equivalence between multipath propagation and CDD is illustrated in Fig. 2. We assume that an OFDM time signal with cyclic extension – as commonly used for OFDM systems – is transmitted from antenna 1 (TX-antenna 1). The cyclic extension allows different OFDM symbols to be decoded without intersymbol interference when

$$T_{\text{Guard}} \geq \delta_{\text{max}} \quad (\geq \delta_1).$$

T_{Guard} is the period of the cyclic extension, and δ_{max} is the maximum multi-path delay of the channel. For clarity, one carrier is drawn as a sine wave with a different phase per OFDM symbol in Fig. 2. We observe that we may receive the OFDM time signal from TX-antenna 1 superimposed with a multi-path signal from antenna 1 delayed by δ_1 .

Now we transmit a 2nd CDD OFDM time signal from TX-antenna 2. As can be seen from Fig. 2, the OFDM time domain symbols, transmitted from TX-antenna 2, are built by a cyclic shift δ_{cy} of the respective OFDM time domain symbols at TX-antenna 1.

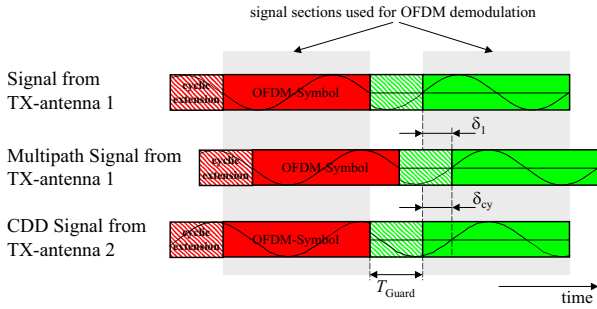


Figure 2: Equivalence of CDD and multipath propagation for OFDM

If we now compare the signal sections which are used for OFDM demodulation, one can see that a cyclic delay δ_{cy} behaves exactly like a multipath channel delay δ_1 based on the channel model. So an arbitrary OFDM receiver cannot distinguish between multipath propagation and CDD within this signal range.

2.3 Space-Time Block Code Description of CDD and Multipath Propagation

As can be seen from the length K IDFT definition (Eq. 1), a cyclic delay δ_{cy} in time domain corresponds to a phase factor of $e^{-j\frac{2\pi}{K}k\delta_{\text{cy}}}$ in frequency domain.

$$\begin{aligned} s((\ell - \delta_{\text{cy}}) \bmod K) &= \\ \text{CDD OFDM time signal} &= \\ &= \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} e^{-j\frac{2\pi}{K}\delta_{\text{cy}} \cdot k} \cdot S(k) \cdot e^{j\frac{2\pi}{K} \cdot k \cdot \ell} \\ &= \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} S_{\delta}(k) \cdot e^{j\frac{2\pi}{K} \cdot k \cdot \ell} \end{aligned} \quad (1)$$

where ℓ and k denote the discrete time and frequency indices respectively with $\ell, k = 0 \dots K-1$. $s(\ell)$ and $S(k)$

represent the complex signals in the time and frequency domain respectively. δ_{cy} stands for a cyclic time shift in samples. We assume that the $S(k)$ are elements of a signal constellation $\mathcal{Q}$, where the average power over all elements of $\mathcal{Q}$ is 1. Commonly used signal constellations are QAM or PSK. Note, that in time domain the choice of any integer value for δ_{cy} is possible, but due to the characteristics $(\delta_{\text{cy}} + K) \bmod K = \delta_{\text{cy}}$ and $e^{j\frac{2\pi}{K}(\delta_{\text{cy}}+K)} = e^{j\frac{2\pi}{K}\delta_{\text{cy}}}$, the range of values for δ_{cy} can reasonably be restricted to $\delta_{\text{cy}} = 0 \dots K-1$. To simplify notation, we define $\delta = \delta_{\text{cy}}$ in the following. If we define

$$\begin{aligned} \Phi(\delta) &= \text{diag} \left(e^{-j\frac{2\pi}{K}\delta \cdot 0}, e^{-j\frac{2\pi}{K}\delta \cdot 1}, \dots, e^{-j\frac{2\pi}{K}\delta \cdot (K-1)} \right) \\ &= \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & e^{-j\frac{2\pi}{K}\delta \cdot 1} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & e^{-j\frac{2\pi}{K}\delta \cdot (K-1)} \end{pmatrix}, \end{aligned}$$

we can express $\mathbf{S}_{\delta} = (S_{\delta}(0), S_{\delta}(1), \dots, S_{\delta}(K-1))$ as

$$\mathbf{S}_{\delta} = \mathbf{S} \cdot \Phi(\delta),$$

where $\mathbf{S} = (S(0), S(1), \dots, S(K-1))$.

Investigations in [5] and [9] were done for independent Rayleigh fading with average energy of 1 per path, i.e. the channel coefficients are complex valued Gaussian random variables with zero mean and variance 0.5 per complex dimension. We introduce real valued normalization coefficients $\lambda_{n,m}$, which allow the adjustment of transmission power for propagation path m from TX-antenna n such that the overall average received energy is 1. If we have N TX-antennas and assume M propagation paths per TX-antenna, then

$$\begin{aligned} n &= 0 \dots N-1 \quad \text{and} \\ m &= 0 \dots M-1. \end{aligned}$$

We define an index $\ell = m + n \cdot M$. If $\delta_{\text{cy}n}$ denotes the cyclic shift at TX-antenna n and $\delta_{n,m}$ is the multipath delay of propagation path m at TX-antenna n , then $\delta_{\ell} = \delta_{\text{cy}n} + \delta_{n,m}$. For simplicity, we define $\Phi(\delta_{\ell}) = \Phi_{\ell}$. Now the Space-Time codeword matrix can be written as

$$\begin{aligned} \mathbf{W} &= \begin{pmatrix} \mathbf{S} \cdot \lambda_0 \cdot \Phi(\delta_0) \\ \mathbf{S} \cdot \lambda_1 \cdot \Phi(\delta_1) \\ \vdots \\ \mathbf{S} \cdot \lambda_{MN-1} \cdot \Phi(\delta_{MN-1}) \end{pmatrix} \\ &\quad \underbrace{\hspace{10em}}_{M \cdot N \times K} \\ &= \begin{pmatrix} \mathbf{S} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{S} & & \vdots \\ \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{S} \end{pmatrix} \cdot \begin{pmatrix} \lambda_0 \cdot \Phi_0 \\ \lambda_1 \cdot \Phi_1 \\ \vdots \\ \lambda_{MN-1} \cdot \Phi_{MN-1} \end{pmatrix} \\ &\quad \underbrace{\hspace{10em}}_{M \cdot N \times K \cdot M \cdot N} \quad \underbrace{\hspace{10em}}_{K \cdot M \cdot N \times K} \end{aligned} \quad (2)$$

Eq. 2 shows that it does not make any difference whether the delays are caused by multipath propagation

$$\begin{aligned} \mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}}) &= (\mathbf{W} - \tilde{\mathbf{W}}) \cdot (\mathbf{W} - \tilde{\mathbf{W}})^H \\ &= \begin{pmatrix} \lambda_0^2 \cdot |\Delta \mathbf{S}|^2 & \lambda_0 \lambda_1 \cdot \Delta \mathbf{S} \Phi_0 \Phi_1^H \Delta \mathbf{S}^H & \cdots & \lambda_0 \lambda_{MN-1} \cdot \Delta \mathbf{S} \Phi_0 \Phi_{MN-1}^H \Delta \mathbf{S}^H \\ \lambda_1 \lambda_0 \cdot \Delta \mathbf{S} \Phi_1 \Phi_0^H \Delta \mathbf{S}^H & \lambda_1^2 \cdot |\Delta \mathbf{S}|^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \lambda_{MN-2} \lambda_{MN-1} \cdot \Delta \mathbf{S} \Phi_{MN-2} \Phi_{MN-1}^H \Delta \mathbf{S}^H \\ \lambda_{MN-1} \lambda_0 \cdot \Delta \mathbf{S} \Phi_{MN-1} \Phi_0^H \Delta \mathbf{S}^H & \cdots & \lambda_{MN-1} \lambda_{MN-2} \cdot \Delta \mathbf{S} \Phi_{MN-1} \Phi_{MN-2}^H \Delta \mathbf{S}^H & \lambda_{MN-1}^2 \cdot |\Delta \mathbf{S}|^2 \end{pmatrix} \end{aligned} \quad (3)$$

or CDD. Hence, for OFDM systems, a multipath channel can be considered as an "inherent" Space-Time block encoder.

Following analysis in [5] and [9] we define the Hermitian difference matrix as a $M \cdot N \times M \cdot N$ matrix $\mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}})$ as depicted in Eq. 3. where $\mathbf{W}^H$ denotes the Hermitian of $\mathbf{W}$ and $\Delta \mathbf{S} = \mathbf{S} - \tilde{\mathbf{S}}, \forall \mathbf{S} \neq \tilde{\mathbf{S}}$. The diagonal element of row/column i is defined as

$$\lambda_i^2 \cdot |\Delta \mathbf{S}|^2 = \lambda_i^2 \cdot \sum_{m=0}^{K-1} |\Delta S_m|^2. \quad (4)$$

For asymptotically good channels, the diversity gain (rank criterion) is defined as [9]

$$\min_{\mathbf{S} \neq \tilde{\mathbf{S}}} \text{rank}[\mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}})] \quad (5)$$

This parameter determines the asymptotic slope of the error rate curve with respect to the signal to noise ratio. We define r and γ_i as the rank and i -th nonzero eigenvalue of the matrix $\mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}})$, then

$$\min_{\mathbf{S} \neq \tilde{\mathbf{S}}} \prod_{i=1}^r \gamma_i, \quad (6)$$

over all codeword pairs $\mathbf{W}, \tilde{\mathbf{W}}$ defines the coding gain (eigen product criterion). This criterion is responsible for shifting the error rate curve of the chosen construction to the left for high signal to noise ratios.

3 ANALYSIS

As an example, we analyze a 2-antenna CDD OFDM system in a Rayleigh fading environment. In this case $N = 2, M = 1$ and $\lambda_0 = \lambda_1 = 1$. For the delays we choose $\delta_0 = 0$ and $\delta_1 = 1$. Then Φ_0 is an identity matrix of size $K \times K$.

As modulation alphabet $\mathcal{Q}$, we assume 4-QAM with $|S|^2 = 1, \forall S \in \mathcal{Q}$ and equiprobable symbols S as shown in Fig. 3. If we define $\Delta \mathcal{Q} = \{\Delta \mathbf{S} = \mathbf{S} - \tilde{\mathbf{S}} | S, \tilde{\mathbf{S}} \in \mathcal{Q}\}$, then the probabilities of the elements $\Delta \mathbf{S} \in \Delta \mathcal{Q}$ are illustrated in Fig. 4 and

$$E\{|\Delta \mathbf{S}|^2\} = E\{|\mathbf{S}|^2\} + E\{|\tilde{\mathbf{S}}|^2\} = 2 \quad (7)$$

for independent S and $\tilde{S}$.

3.1 Rank Criterion

The parameters and assumptions above lead to a Hermitian difference matrix as shown in Eq. 8.

$$\mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}}) = \begin{pmatrix} |\Delta \mathbf{S}|^2 & \Delta \mathbf{S} \Phi_1^H \Delta \mathbf{S}^H \\ \Delta \mathbf{S} \Phi_1 \Delta \mathbf{S}^H & |\Delta \mathbf{S}|^2 \end{pmatrix} \quad (8)$$

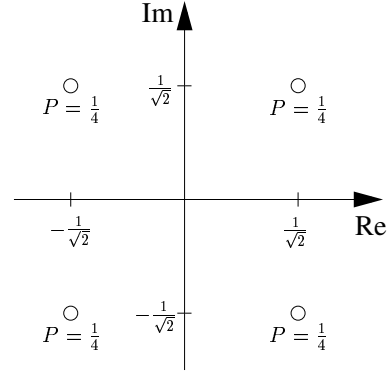


Figure 3: 4-QAM Constellation and Probabilities

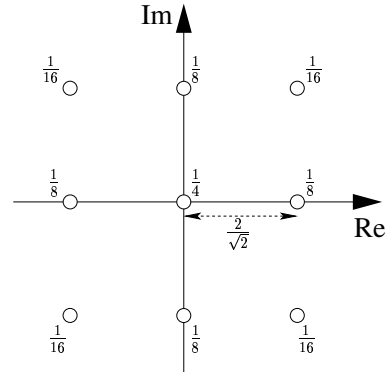


Figure 4: Differential 4-QAM Constellation and Probabilities

The rank of matrix $\mathbf{A}$ is as follows:

1. $\Delta \mathbf{S} = (0, \dots, 0) \in \Delta \mathcal{Q}^K \iff \text{rank}[\mathbf{A}] = 0$
2. $\Delta \mathbf{S} = \Delta S \cdot \mathbf{e}_j \in \Delta \mathcal{Q}^K, \Delta S \neq 0, j \in \{0, 1, \dots, K-1\} \iff \text{rank}[\mathbf{A}] = 1$
 $\mathbf{e}_j$ is a vector where the j th coordinate is 1, and all other coordinates are zero.
3. else $\iff \text{rank}[\mathbf{A}] = 2$

Proof: We will first proof the " $\implies$ " direction

1. The rank of a null-matrix is 0.
2. In this case:
 $|\Delta \mathbf{S}|^2 > 0 \implies \text{rank}[\mathbf{A}] > 0$
and
 $\det(\mathbf{A}) = 0 \implies \text{rank}[\mathbf{A}] < 2 \implies \text{rank}[\mathbf{A}] = 1.$

3. Let us arbitrarily choose 2 components $\Delta S_a \neq 0$ and $\Delta S_b \neq 0$ of the vector $\Delta \mathbf{S}$ with $a \neq b$ and $a, b \in \{0, 1, \dots, K-1\}$. Then

$$\begin{aligned} |\Delta \mathbf{S} \Phi_1 \Delta \mathbf{S}^H| &= \left| \sum_{i=0}^{K-1} |\Delta S_i|^2 \cdot e^{-j \frac{2\pi}{K} i} \right| \\ &\leq \underbrace{|\Delta S_a|^2 e^{-j \frac{2\pi}{K} a} + |\Delta S_b|^2 e^{-j \frac{2\pi}{K} b}}_{< |\Delta S_a|^2 + |\Delta S_b|^2 \ \forall a \neq b} + \\ &\quad + \sum_{\substack{i=0 \\ i \neq a \wedge i \neq b}}^{K-1} |\Delta S_i|^2 \cdot \underbrace{e^{-j \frac{2\pi}{K} i}}_{=1} \\ &< |\Delta S_a|^2 + |\Delta S_b|^2 + \\ &\quad + \sum_{\substack{i=0 \\ i \neq a \wedge i \neq b}}^{K-1} |\Delta S_i|^2 = |\Delta \mathbf{S}|^2 \end{aligned}$$

and therefore $\det(\mathbf{A}) = |\Delta \mathbf{S}|^2 \cdot |\Delta \mathbf{S}|^2 - |\Delta \mathbf{S} \Phi_1 \Delta \mathbf{S}^H|^2 > 0$, which means $\text{rank}[\mathbf{A}] = 2$.

The cases 1-3 define disjunctive sets of $\Delta \mathbf{S}$. The union of these sets yields the set of all possible vectors $\Delta \mathbf{S}$. This implies that the mapping of the sets defined in the cases 1-3 onto the set $\{0, 1, 2\}$ (rank) is a one-to-one mapping. Therefore, the " $\Leftarrow$ " direction also holds. ■

Based on the calculations in this section and the signal constellation probabilities in Fig. 4 we can derive the probabilities for $\text{rank}[\mathbf{A}] = 0, 1, 2$.

1. From this section we know: $\text{rank}[\mathbf{A}] = 0 \implies \Delta \mathbf{S} = (0, \dots, 0)$
This gives

$$\text{Prob}(\text{rank}[\mathbf{A}] = 0) = \left(\frac{1}{4}\right)^K,$$

where $\frac{1}{4}$ is the probability for $\Delta S = 0$ (see Fig.4). Note that in fact $\text{rank}[\mathbf{A}] = 0$ is not possible because of the condition $\Delta \mathbf{S} = \mathbf{S} - \tilde{\mathbf{S}}, \mathbf{S} \neq \tilde{\mathbf{S}}$ in Eq. 5 and 6.

2. $\text{rank}[\mathbf{A}] = 1 \implies \Delta \mathbf{S} = \Delta S \cdot \mathbf{e}_j \in \Delta \mathcal{Q}^K, \Delta S \neq 0, j \in \{0, 1, \dots, K-1\}$
and consequently

$$\text{Prob}(\text{rank}[\mathbf{A}] = 1) = K \cdot \frac{3}{4} \cdot \left(\frac{1}{4}\right)^{K-1},$$

where $\frac{3}{4} = 1 - \frac{1}{4}$ is the probability for $\Delta S \neq 0$.

- 3.

$$\begin{aligned} \text{Prob}(\text{rank}[\mathbf{A}] = 2) &= 1 - \text{Prob}(\text{rank}[\mathbf{A}] = 0) \\ &\quad - \text{Prob}(\text{rank}[\mathbf{A}] = 1) \\ &= 1 - \frac{3K+1}{4^K} \end{aligned}$$

Therefore, the larger the number of subcarriers, the higher the probability that we have full rank. Also, the denominator 4^K depends directly on the number of signals in the modulation constellation. Therefore by increasing the cardinality of the signal constellation we also drive the full rank probability towards 1.

3.2 Calculation of the Determinant

In the case of our example, $\det \mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}})$ is

$$\begin{aligned} \det \mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}}) &= \\ &= |\Delta \mathbf{S}|^2 \cdot |\Delta \mathbf{S}|^2 - \Delta \mathbf{S} \Phi_1 \Delta \mathbf{S}^H \cdot \Delta \mathbf{S} \Phi_1^H \Delta \mathbf{S}^H \\ &= |\Delta \mathbf{S}|^2 \cdot |\Delta \mathbf{S}|^2 - |\Delta \mathbf{S} \Phi_1 \Delta \mathbf{S}^H|^2. \end{aligned} \quad (9)$$

The determinant of $\mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}})$ is closely related to the coding gain criterion which is based on the product of non-zero eigenvalues.

The minimum product of non-zero eigenvalues will give the coding gain in asymptotically good channels. However, by calculating the average determinant value, we can see how the statistics depend on the chosen code parameters. The mean value μ of $\det[\mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}})]$ is:

$$\begin{aligned} \mu &= E \left\{ \det \mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}}) \right\} \\ &= E \left\{ |\Delta \mathbf{S}|^2 \cdot |\Delta \mathbf{S}|^2 \right. \\ &\quad \left. - \Delta \mathbf{S} \Phi_1 \Delta \mathbf{S}^H \cdot (\Delta \mathbf{S} \Phi_1 \Delta \mathbf{S}^H)^H \right\} \\ &= E \left\{ \sum_{\ell=0}^{K-1} |\Delta S_\ell|^2 \sum_{m=0}^{K-1} |\Delta S_m|^2 \right. \\ &\quad \left. - \sum_{\ell=0}^{K-1} |\Delta S_\ell|^2 \cdot \exp \left(-j \frac{2\pi}{K} \cdot \ell \right) \cdot \sum_{m=0}^{K-1} |\Delta S_m|^2 \cdot \exp \left(j \frac{2\pi}{K} \cdot m \right) \right\} \\ &= \sum_{\ell, m=0}^{K-1} E \left\{ |\Delta S_\ell|^2 |\Delta S_m|^2 \right\} \\ &\quad - \sum_{\ell, m=0}^{K-1} E \left\{ |\Delta S_\ell|^2 \cdot |\Delta S_m|^2 \right\} \cdot \exp \left(j \frac{2\pi}{K} \cdot (m - \ell) \right) \\ &= \sum_{\substack{\ell, m=0 \\ \ell \neq m}}^{K-1} E \left\{ |\Delta S_\ell|^2 |\Delta S_m|^2 \right\} + \sum_{\ell=0}^{K-1} E \left\{ |\Delta S_\ell|^4 \right\} \\ &\quad - \sum_{\substack{\ell, m=0 \\ \ell \neq m}}^{K-1} E \left\{ |\Delta S_\ell|^2 \cdot |\Delta S_m|^2 \right\} \cdot \exp \left(j \frac{2\pi}{K} \cdot (m - \ell) \right) \\ &\quad - \sum_{\ell=0}^{K-1} E \left\{ |\Delta S_\ell|^4 \right\} \\ &= E \left\{ |\Delta \mathbf{S}|^2 \right\}^2 \cdot K(K-1) - E \left\{ |\Delta \mathbf{S}|^2 \right\}^2 \cdot \\ &\quad \cdot \sum_{\substack{\ell, m=0 \\ \ell \neq m}}^{K-1} \exp \left(j \frac{2\pi}{K} \cdot (m - \ell) \right) \\ &= E \left\{ |\Delta \mathbf{S}|^2 \right\}^2 \cdot K(K-1) \end{aligned}$$

$$\begin{aligned}
& - E \{ |\Delta S|^2 \}^2 \cdot \left(\sum_{\ell, m=0}^{K-1} \exp \left(j \frac{2\pi}{K} \cdot (m - \ell) \right) \right. \\
& \quad \left. - \sum_{\ell=0}^{K-1} \exp \left(j \frac{2\pi}{K} \cdot 0 \right) \right) \\
& = E \{ |\Delta S|^2 \}^2 \cdot K(K-1) \\
& - E \{ |\Delta S|^2 \}^2 \cdot \left(\underbrace{\sum_{\ell, m=0}^{K-1} \exp \left(j \frac{2\pi}{K} \cdot (m - \ell) \right)}_{=0} - K \right) \\
\mu & = K^2 \cdot E \{ |\Delta S|^2 \}^2 \tag{10}
\end{aligned}$$

Equation 10 shows that the average value of the determinant will increase with K , the number of OFDM carriers and the average separation between signals $E \{ |\Delta S|^2 \}^2$. Therefore densely packed signal constellations would be expected to have a low coding gain advantage.

3.3 Channel Coding

From the calculations in Section 3.1 and 3.2 we can see that unfortunately $\min_{\mathbf{S} \neq \mathbf{S}} \text{rank}[\mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}})] = 1$, i.e. for the considered example, multipath propagation or antenna diversity does not provide full diversity gain, if it is used on its own. The calculations in Section 3.1 imply that we have to guarantee a minimum Hamming Weight of 2 for $\Delta \mathbf{S}$. This can be fulfilled by using a single parity check code in the real and imaginary part of the considered 4-QAM alphabet.

4 CONCLUSIONS

In this paper we have presented a system model which uses multiple antennas to transmit cyclic delayed copies of an OFDM symbol. We term this technique Cyclic Delay Diversity (CDD). Furthermore, we extend the transmission model to include multipath components, and derive a condition on the antenna delays which insure that multipath and antenna delay components have equivalent descriptions from the receiver point of view.

In Section 2.3 the Hermitian difference matrix of a CDD codeword pair with multipath components is derived. The rank and non-zero eigen product of this matrix determines the diversity and coding gain of the code as presented in [5].

Section 3 gives a method for finding a bound for non-full spacial diversity codeword pairs for a specific scenario. In general, it was shown that by increasing the number of OFDM carriers, or the cardinality of the signal space, the probability of having full rank codeword pairs approaches 1. The determinant between codeword pairs was shown to increase with the number of OFDM carriers, and the average separation among signal constellation points. The determinant is closely related to the product of non-zero eigen values which determine the space-time coding gain. Though a general proof of the diversity properties of CDD systems has not yet been completed, the similarity with traditional temporal delay di-

versity schemes which achieve full spatial diversity ([5]) motivates further theoretical research in this direction.

The requirements for full rank derived in our example can easily be met with a single parity check code. This motivates further investigations of combining cyclic delay diversity with coding.

The error rate performance improvement of CDD on practical systems has already been shown through simulations ([8]) using a standard OFDM receiver. A link to Space-Time codes may lead to better receiver designs which exploit the inherent CDD diversity, or to modifications of CDD which improve error rate performance, complexity, or information rate.

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