

Uplink Power Control in Multi-Cell Spatial Multiplexing Wireless Systems

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Abstract—This paper proposes a power control strategy for the uplink of cellular MIMO spatial multiplexing systems, with a linear MMSE receiver applied at the base station and a single active user per time instant. A fixed per-stream SINR target is employed that allows guaranteed QoS for delay-sensitive applications. A straightforward application of single antenna power control is not possible in the MIMO context due to coordination between receive antennas and nonlinear dependence between interference and eigenspaces of the channel matrices. Two schemes are proposed to solve the problem. The first equally allocates power to all transmit antennas. Deriving an SINR lower bound based on an eigenvalue approximation of the composite interference, allows application of the conventional single antenna power control framework to solve this problem. To improve the feasibility performance, a second scheme is proposed that adaptively allocates power on the transmit antennas, where an iterative algorithm based on game theory is used to sequentially update each user's power distribution. The optimal solution with full channel knowledge, and a practical near-optimal solution requiring only partial channel knowledge, are both derived. Numerical results show that power control, compared to supposedly optimal waterfilling strategies, actually achieves higher throughput at the low SINRs typical in cellular systems, with significantly lower overhead and complexity. Due to its better exploitation of spatial diversity and reduced transmit power (and hence reduced interference), adaptive power allocation increases the achievable SINR by an order of magnitude over equal power allocation, resulting in far better coverage.

Index Terms—MIMO systems, power control.

I. INTRODUCTION

IN an interference-limited multiuser system, the performance of each user depends not only on its own transmission, but also on the transmissions of the other users. As one user increases its power to improve its performance, it generates more interference and degrades the other users' performance. Power control is a well-known technique to coordinate the transmitters such that the users can achieve a balanced Quality of Service (QoS). This paper studies power control for uplink transmission of a cellular multiple-input multiple-output (MIMO) system, where antenna arrays are

used at both the transmitter and receiver. We specifically consider the spatial multiplexing case where each transmit antenna sends an independent data substream, with an MMSE detector at base stations. In particular, we consider one active mobile per cell at each time instant,¹ but the extension to other multi-access schemes with multiple mobiles per cell is also discussed. The objective is to allow *each* substream of *each* user to meet a certain signal-to-interference and noise ratio (SINR), while minimizing the sum transmit power, and therefore, mutual interference.

MIMO systems [1]–[4] have shown great potential in a single-user link, but their performance degrades seriously in a cellular network [5]–[6] which is typically interference-dominated and operates in a very low SINR regime (i.e., significantly below 10 dB, [7]). Novel techniques for dealing with interference are therefore critical to make MIMO viable in cellular networks. A significant amount of prior work on MIMO cellular systems focused on capacity analysis and receiver design for interference mitigation [8]–[11]. To the best of our knowledge, however, previous work has not considered multi-cell MIMO power control. In this paper we adopt a minimum SINR constraint, because it is a critical QoS metric for circuit-switching and even packet-switching data networks [12]–[13]. Many delay-sensitive data applications (e.g., VoIP, video streaming) need a SINR guarantee to maintain these services.

Prior power control schemes for single-input single-output (SISO) cellular systems were primarily based on a fixed SINR objective [14]–[18]. Several other researchers have studied SISO power control with a utility-based game theory approach, where utility is defined as the ratio of throughput to power [19]–[22]. It was proven that the Nash equilibrium of the power control game corresponds to the SINR-balanced point [19][22], where the output SINR of each user is equal to a fixed value that maximizes an *efficiency function*. Hence with such an utility definition, utility-based power control is a special case of the fixed-SINR power control. Pricing was introduced in [23]–[24] to further improve the sum utility. For more complicated multi-dimensional power control problems, each user needs to determine the power over a higher dimension (e.g., frequency or spatial domain). For example, Yu *et al.* studied power control for a multicarrier interference channel in [25], and proposed an iterative waterfilling algorithm for each user to reach a target rate. This approach for interference-limited channels (like a cellular network) is quite different from the well-known iterative waterfilling for

Manuscript received December 19, 2005; revised June 21, 2006 and September 18, 2006; accepted September 18, 2006. The associate editor coordinating the review of this paper and approving it for publication was K.-K. Wong.

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Digital Object Identifier 10.1109/TWC.2007.051007.

¹This assumption is valid for TDMA, FDMA and orthogonal CDMA networks, and has been used in existing cellular MIMO literature [10][11]

the broadcast and MAC channels (like a single-cell system), where the iteration is performed with individual [26] or sum power constraints [27]. In [20], utility-based power control was extended to the multi-carrier CDMA scenario, which is essentially a set of SISO power control problems.

The cellular MIMO power control problem in this paper is different from prior power control research in several aspects. First, it is a non-orthogonal problem where each antenna interferes with *all* antennas of *all* users, hence it includes previous orthogonal multi-carrier algorithms [20][25] as special cases. Secondly, unlike in the SISO system where interference on each receive antenna is solely a *linear scalar function of the power*, the effect of interference in MIMO system relies on the specific MIMO receiver structure. Since MIMO receivers are typically non-linear in terms of transmit power vector, which will be discussed in Section IV, the interference is also non-linear and is correlated via the eigenspaces of the channel matrices. On one hand, interference from one transmit antenna does not scale linearly with its power. On the other hand, if one transmit antenna alters its power, interference from other co-cell transmit antennas to the remaining cells will also change even if their power remains constant. Given these fundamental differences, the existing SISO schemes cannot be directly applied even by considering each transmit antenna as an independent user.

To address these issues, two power control techniques are proposed in this paper. In the first method, a user's power is equally distributed on its transmit antennas. We derive a lower bound on the post-processing SINR with a linear MMSE receiver, expressed in terms of an eigenvalue approximation of both the desired and interfering users' channel. Using this bound, we formulate the MIMO power control problem in a similar framework in the SISO scenario, and derive closed-form optimal (in terms of the SINR bound) and low-complexity sub-optimal solutions with either full or partial channel knowledge. Intuitively, this approach enforces the worse stream of every user to meet the SINR requirement, causing excess power on antennas which better channels. To reduce the infeasibility probability, a second algorithm based on game theory is proposed that adaptively distributes transmit power on the transmit antennas. The adaptive allocation solution can more effectively exploit the variation of antenna array response, thereby resulting in a higher power efficiency. Numerical results show that adaptive power allocation reduces the sum power by approximately 80%, and leads to 10-12 dB target SINR improvement under the same infeasibility probability.

In Section II, we present the multi-cell MIMO system model. In Section III, an overview of power control is provided. In Section IV, we systematically investigate the power control with equal power allocation, including the closed-form optimal solution, and a low complexity 1-bit iterative algorithm. A more practical suboptimal algorithm requiring only the in-cell user's channel knowledge is proposed in Section V. In Section VI, we develop the power control with adaptive power allocation based on game theory. Numerical results are provided in Section VII, with conclusions drawn in Section VIII.

II. SYSTEM MODEL

A. Signal Model

Consider the uplink transmission of a multi-cell system with K cells. For now we assume each cell consists of one base station (BS) with N_r receive antennas, and one active mobile station (MS) with N_t transmit antennas and spatial multiplexing. A narrow-band quasi-static flat-fading channel is assumed, where channel remains constant within several frames. The received signal at the k th BS is represented as

$$\mathbf{y}_k = \sqrt{\frac{P_k d_{k,k}^{-\rho} \chi_{k,k}}{N_t}} \mathbf{H}_{k,k} \mathbf{T}_k \mathbf{x}_k + \sum_{j \neq k} \sqrt{\frac{P_j d_{k,j}^{-\rho} \chi_{k,j}}{N_t}} \mathbf{H}_{k,j} \mathbf{T}_j \mathbf{x}_j + \mathbf{n}_k, \quad (1)$$

where $\mathbf{x}_k \in \mathbb{C}^{N_t \times 1}$ is the transmit symbol vector from user k , satisfying $E_x(\mathbf{x}_k \mathbf{x}_k^\dagger) = \mathbf{I}_{N_t}$.² $\chi_{k,j}$, $d_{k,j}$, and $\mathbf{H}_{k,j}$ denote the log-normal shadow fading, the distance, and the channel transfer matrix from the j th MS to the k th BS, respectively. Pathloss exponent is represented by ρ and assumed the same for $\forall k$. We have no specific assumption on the small scale fading captured in $\mathbf{H}_{k,j}$, and our work applies to a variety of MIMO channels (e.g., Rayleigh, Rician, independent or correlated). Matrix $\mathbf{T}_k = \text{diag}\{\mathbf{T}_k^{(1,1)}, \dots, \mathbf{T}_k^{(N_t, N_t)}\}$ is the $N_t \times N_t$ diagonal power loading matrix of user k , P_k is the transmit power of user k , $\mathbf{n}_k$ denotes the additive white Gaussian noise (AWGN) with zero mean and variance $E(\mathbf{n}_k \mathbf{n}_k^\dagger) = \sigma_n^2 \mathbf{I}_{N_r}$. To keep the transmit power of user k at constant, $\mathbf{T}_k$ must satisfy

$$\text{trace}(\mathbf{T}_k \mathbf{T}_k^\dagger) = \sum_{j=1}^{N_t} |\mathbf{T}_k^{(j,j)}|^2 = N_t, \quad k = 1, 2, \dots, K. \quad (2)$$

The signal model in (1) applies to a set of multi-dimensional power control problems. For example, it incorporates the multi-carrier (OFDM) power control as a special case by modeling $\mathbf{H}_{k,j}$ as a diagonal matrix containing the subcarrier gain. Introducing off-diagonal elements in $\mathbf{H}_{k,j}$ can represent an OFDM system with inter-carrier interference, which is more general.

Spatial multiplexing is used throughout this paper where each transmit antenna sends an independent stream. Dynamically adjusting the number of substreams has been considered in the *single-user* scenario [28]-[29], but the extension to the multi-cell context is not straightforward. Power control with other signaling schemes such as beamforming or linear dispersion codes [30] is an interesting topic for future work.

To avoid the complexity associated with either multiuser receiver [11] or non-linear single-user receivers (e.g., maximum likelihood, BLAST [3]), we focus our study on linear MMSE receiver, and defer the study on other receivers for future research. It was shown in [33] that the MMSE receiver maximizes the SINR for each user, while mitigating the noise enhancement problem. The power control objective is to enforce the receive SINR γ to meet the fixed target Γ , which can be adjusted according to the propagation environment.

² $(\cdot)^T$ denotes the transpose, $(\cdot)^\dagger$ denotes the complex conjugate transpose, $(\cdot)^{(s,j)}$ represents the element on the s th row and j th column of a matrix, $E_s(\cdot)$ denotes the expectation with respect to variable s , $\text{trace}(\cdot)$ denotes the summation of the diagonal element of a square matrix, $\|\cdot\|_F$ denotes the Frobenius norm.

Two levels of channel awareness are studied in this paper. In the first case, each BS knows the channel of both its in-cell user $\mathbf{H}_{k,k}$ and the out-of-cell users $\mathbf{H}_{k,j}, \forall j$. We refer to this case as the *full-CSI* situation. The channel information is sent back to a central controller to perform power control, or power distribution is calculated at each BS in a distributed way. In the second case, each BS knows only the channel information of its in-cell user, and we refer to this case as the *self-CSI* situation. In Sections IV and V, the equal power allocation is studied under both full-CSI and self-CSI environments. In Section VI we investigate the adaptive power allocation under a self-CSI assumption, which is more practical.

B. Multiple Mobiles Per Cell

In most practical deployments of MIMO, multiple MSs in each cell will likely be orthogonalized by TDMA, OFDMA or orthogonal CDMA to maintain the highest possible SINR [10][11]. Although the above model implicitly assumes TDMA, other multi-access options such as SDMA, non-orthogonal CDMA can also apply our proposed approaches.

- With SDMA, multiple mobiles in the same cell occupy the same spectrum and are detected at the base station simultaneously. Suppose there are L users in the k th cell. If $N_r \geq LN_t$, the base station antenna array provides sufficient spatial diversity to decode L users with a linear receiver, therefore we can stack $\mathbf{x}_k = [\mathbf{x}_{k,1}^T, \dots, \mathbf{x}_{k,L}^T]^T$, where $\mathbf{x}_{k,l}$ is the l th user's signal vector. If $N_r < LN_t$, the receive antenna array has insufficient spatial diversity to separate multiple users with a linear receiver. In such a case, SIC can be used to detect the users sequentially, and our proposed work can be applied by treating users decoded afterwards as interference. The MIMO power control techniques proposed in the rest of this paper can be applied to both these scenarios.
- With non-orthogonal CDMA, each user also receives interference from intra-cell users which is attenuated by a spreading factor. Our approaches can be applied by treating the system as a KL pseudo-cell system, where K is the actual cell number, L is the number of mobiles per cell.

III. OVERVIEW OF POWER CONTROL

Conventional power control objective functions include a fixed SINR target, a fixed capacity target or the maximization of utility, defined as the throughput per unit of power.

A. SISO Power Control With Fixed SINR Target

Many SISO cellular power control algorithms aim to achieve a pre-defined SINR requirement [14]-[18]. Define $\mathbf{P} = (P_1, P_2, \dots, P_K)$ as the power vector. Let $I(\mathbf{P}) = (I_1(\mathbf{P}), I_2(\mathbf{P}), \dots, I_K(\mathbf{P}))$ represent the interference vector, where $I_k(\mathbf{P})$ denotes the *effective* interference that user k must overcome. The users' SINR requirement can be described by a vector inequality of the form

$$\mathbf{P} \geq I(\mathbf{P})$$

We say that a solution is *feasible* if there exists $\mathbf{P} \geq 0$ such that $I(\mathbf{P})$ that satisfies (3).³

Power control for SISO systems is typically a non-convex optimization problem and difficult to solve. Prior work showed that if the interference function satisfies the *standard* property, then a unique optimal solution can be found by resorting to the following iterative algorithm

$$\mathbf{P}(t+1) = I(\mathbf{P}(t)), \quad (4)$$

where t denotes the time instant. An interference function $I(\mathbf{P})$ is *standard* if it satisfies [15]:

- 1) *Positivity*: $I(\mathbf{P}) > 0$
- 2) *Monotonicity*: If $\mathbf{P} > \mathbf{P}'$, then $I(\mathbf{P}) > I(\mathbf{P}')$
- 3) *Scalability*: For all $\alpha > 1$, $\alpha I(\mathbf{P}) > I(\alpha \mathbf{P})$

If $I(\mathbf{P})$ is a standard interference function, the iteration (4) is called *standard power control algorithm* and we have the following theorems [15].

Theorem 1: If the standard power control algorithm has a fixed point, then it is unique.

Theorem 2: If $I(\mathbf{P})$ is feasible, then for any initial power vector $\mathbf{P}$, the standard power control algorithm converges to a unique fixed point $\mathbf{P}^*$.

B. Power Control With Fixed Capacity Target

In [25], Yu et al. studied the power control for a multi-carrier DSL system. The interference channel was modeled as a noncooperative game where self-interested users compete with each other to reach their target capacity. A distributed iterative algorithm was proposed, where each user applied waterfilling to maximize its own capacity, and then adjusted the power iteratively until the capacity target was obtained. This algorithm can also be applied to the cellular MIMO systems, by applying the power allocation in the frequency domain to the spatial domain.

C. Power Control for Utility Maximization

Utility can be defined in different ways. In most utility-based power control research, the fundamental definition is the throughput per unit of energy, expressed as $u = \frac{LRf(\gamma)}{MP}$, where L and M are the number of information bits and total number of bits in a packet, R is the transmission rate, P is the transmit power, and $f(\gamma)$ is the *efficiency function* representing the packet success rate. It was shown in [21]-[23] that in a utility maximizing power control game, there exists a unique Nash equilibrium where no user can improve its utility given the power level of other users. This equilibrium corresponds to a point where all users are SINR-balanced with the output SINR equal to γ^* , which satisfies $f(\gamma^*) = \gamma^* f'(\gamma^*)$. Hence, when utility is defined as above, the utility-based power control corresponds to a special case of the fixed SINR target power control problem.

³If we let $I_k(\mathbf{P})$ denote the interference received by user k , the equation

IV. MIMO POWER CONTROL WITH EQUAL POWER ALLOCATION AND FULL-CSI

We start from a simple case where each user's transmit power is equally distributed to its antenna array, i.e. $\mathbf{T}_k = \mathbf{I}_{N_t}$, for $k = 1, \dots, K$.

The basic idea in this section is to enforce a group of antennas associated with the same user to transmit at the same power, and enforces the worse channel to reach the SINR target. Although this is a suboptimal approach where user with better channel is allocated with excess power, it significantly simplifies the problem formulation and enables to approximate the interference as a linear function of the power vector, therefore a similar framework as in the SISO case can be applied. We present a necessary and sufficient condition on the existence of a feasible solution, and develop the closed-form solution optimal in terms of a SINR lower bound. We also show that this problem falls into the *standard* power control category, and then propose a low-complexity 1-bit power control algorithm. Full CSI is assumed through this section.

A. Signal Model

With a linear MMSE receiver, an MMSE weighting matrix $\mathbf{G}_k$ is applied to the received signal $\mathbf{y}_k$ to obtain the estimate

$$\hat{\mathbf{x}}_k = \mathbf{G}_k \mathbf{y}_k. \quad (5)$$

According to [11][31][32], the linear MMSE receiver is given as

$$\mathbf{G}_k = \sqrt{\frac{N_t}{P_k d_{k,k}^{-\rho} \chi_{k,k}}} \mathbf{H}_{k,k}^\dagger \left(\mathbf{H}_{k,k} \mathbf{H}_{k,k}^\dagger + \sum_{j \neq k} \frac{P_j d_{k,j}^{-\rho} \chi_{k,j}}{P_k d_{k,k}^{-\rho} \chi_{k,k}} \mathbf{H}_{k,j} \mathbf{H}_{k,j}^\dagger + \frac{N_t \sigma_n^2}{P_k d_{k,k}^{-\rho} \chi_{k,k}} \mathbf{I} \right)^{-1} \quad (6)$$

Denoting the post-processing SINR of the k th user's s th stream as $\gamma_{k,s}$, we can derive a lower bound on the minimum SINR of the k th user.

Lemma 1: Consider a cellular MIMO system with K cells. With the linear MMSE receiver, the minimum post-processing SINR of the k th user is bounded as

$$\gamma_{k,\min} = \min_{s \in [1, N_t]} \gamma_{k,s} \geq \frac{P_k d_{k,k}^{-\rho} \chi_{k,k}}{\mu_{\max} \left(\sum_{j \neq k} P_j d_{k,j}^{-\rho} \chi_{k,j} \mathbf{\Omega}_{k,j,1} + N_t \sigma_n^2 \mathbf{\Omega}_{k,j,2} \right)}, \quad (7)$$

where

$$\mathbf{\Omega}_{k,j,1} = \left(\mathbf{H}_{k,k}^\dagger \mathbf{H}_{k,k} \right)^{-1} \mathbf{H}_{k,k}^\dagger \mathbf{H}_{k,j} \mathbf{H}_{k,j}^\dagger \mathbf{H}_{k,k} \left(\mathbf{H}_{k,k}^\dagger \mathbf{H}_{k,k} \right)^{-1}, \quad (8)$$

$$\mathbf{\Omega}_{k,j,2} = \left(\mathbf{H}_{k,k}^\dagger \mathbf{H}_{k,k} \right)^{-1}, \quad (9)$$

and $\mu_{\max}(\cdot)$ is the maximum eigenvalue of a Hermitian matrix.

Proof: see the Appendix. $\square$

It is easy to see that the MIMO receiver $\mathbf{G}_k$ relies on the matrix channel and is non-linear in $\mathbf{P}$, hence the interference after MMSE detection is also non-linear in $\mathbf{P}$. As a result of

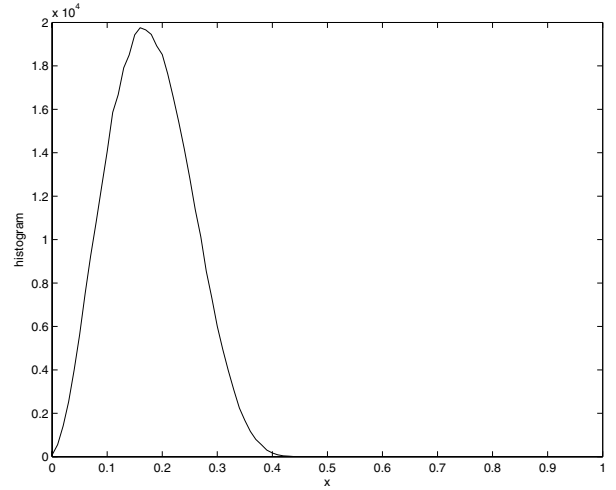


Fig. 1. Histogram of the normalized approximation error x .

the non-linearity, it is very difficult to obtain a closed-form optimal solution by solving $\mathbf{P} = \mathbf{I}(\mathbf{P})$, thus the conventional SISO power control framework cannot be directly applied.

B. Problem Formulation

The multi-cellular MIMO power control aims to minimize the sum transmit power $P_\Sigma = \sum_{k=1}^K P_k$ subject to

$$\gamma_{k,\min} \geq \Gamma_k. \quad (10)$$

The constraint functions (10) of the optimization problems are in terms of the maximum eigenvalue $\mu_{\max} \left(\sum_{j \neq k} P_j d_{k,j}^{-\rho} \chi_{k,j} \mathbf{\Omega}_{k,j,1} + N_t \sigma_n^2 \mathbf{\Omega}_{k,j,2} \right)$ in (7), which cannot be expressed as a linear function of $\mathbf{P}$ to obtain a closed-form solution as in the SISO case. To ease the problem, we wish to relax the constraint function to be a linear function of the power. The following theorem regarding the eigenvalues of a composite matrix proves to be useful.

Theorem 3: (Weyl [39]) Let $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{n \times n}$ be Hermitian and let the eigenvalues $\mu_k(\mathbf{A})$, $\mu_k(\mathbf{B})$, and $\mu_k(\mathbf{A} + \mathbf{B})$ be arranged in increasing order. For each $k = 1, 2, \dots, n$, we have

$$\mu_k(\mathbf{A}) + \mu_1(\mathbf{B}) \leq \mu_k(\mathbf{A} + \mathbf{B}) \leq \mu_k(\mathbf{A}) + \mu_n(\mathbf{B}). \quad (11)$$

Choosing $k = n$, the above theorem states that the maximum eigenvalue of a composite matrix is upper bounded by the sum of the maximum eigenvalue of each matrix. The tightness of this bound can be evaluated by the relative error

$$x = \frac{\mu_{\max}(\mathbf{H}_1 \mathbf{H}_1^\dagger) + \mu_{\max}(\mathbf{H}_2 \mathbf{H}_2^\dagger) - \mu_{\max}(\mathbf{H}_1 \mathbf{H}_1^\dagger + \mathbf{H}_2 \mathbf{H}_2^\dagger)}{\mu_{\max}(\mathbf{H}_1 \mathbf{H}_1^\dagger + \mathbf{H}_2 \mathbf{H}_2^\dagger)} \quad (12)$$

where elements of $\mathbf{H}_1, \mathbf{H}_2$ are i.i.d. complex Gaussian random variables with zero mean and variance 1. Fig. 1 plots the histogram of x obtained with 4×10^6 independent channel realizations. It is shown that for 95% of the channel realizations, the approximation error is less than 30%, and almost never exceeds 40%. Therefore we consider this upper bound reasonably tight.

Extending the above theorem to K matrices, we can easily obtain the following corollary.

Corollary 1: Let $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_K \in \mathbb{C}^{n \times n}$ be Hermitian, then for $1 \leq k \leq K, 1 \leq j \leq n$, we have

$$\sum_k \mu_{\min}(\mathbf{A}_k) \leq \mu_j \left(\sum_k \mathbf{A}_k \right) \leq \sum_k \mu_{\max}(\mathbf{A}_k). \quad (13)$$

Hence the constraint in (10) is relaxed to

$$\begin{aligned} \gamma_{k,\min} &\geq \frac{P_k d_{k,k}^{-\rho} \chi_{k,k}}{\mu_{\max} \left(\sum_{j \neq k} P_j d_{k,j}^{-\rho} \chi_{k,j} \Omega_{k,j,1} + N_t \sigma_n^2 \Omega_{k,j,2} \right)} \\ &\geq \frac{P_k d_{k,k}^{-\rho} \chi_{k,k}}{\sum_{j \neq k} P_j d_{k,j}^{-\rho} \chi_{k,j} \cdot \mu_{\max}(\Omega_{k,j,1}) + \sigma_n^2 N_t \mu_{\max}(\Omega_{k,j,2})}. \end{aligned} \quad (14)$$

Problem Formulation: The objective of the multi-cellular power optimization problem is to minimize the sum transmit power $P_\Sigma = \sum_{k=1}^K P_k$ subject to

$$\begin{aligned} P_k &\geq \frac{d_{k,k}^{-\rho} \Gamma_k}{\chi_{k,k}} \left(\sum_{j \neq k} P_j d_{k,j}^{-\rho} \chi_{k,j} \mu_{\max}(\Omega_{k,j,1}) + \sigma_n^2 N_t \mu_{\max}(\Omega_{k,j,2}) \right), \end{aligned} \quad (15)$$

Note that this formulation is based on the proposed SINR bound under the *equal allocation* assumption, and the remaining discussion in this section is based on this bound.

C. Optimal Solution

With the derivation in the previous section, the interference function $I(\mathbf{P})$ is expressed as

$$I(\mathbf{P}) = \mathbf{\Lambda}(\mathbf{F}\mathbf{P} + \mathbf{N}), \quad (16)$$

where $\mathbf{\Lambda} = \text{diag}\{\Gamma_1, \Gamma_2, \dots, \Gamma_K\}$ is a diagonal target SINR matrix for the K users. Referring to (15), $\mathbf{F}$ is given as

$$\mathbf{F}_{k,j} = \begin{cases} 0 & k = j \\ \frac{d_{k,j}^{-\rho} \chi_{k,j}}{d_{k,k}^{-\rho} \chi_{k,k}} \mu_{\max}(\Omega_{k,j,1}) & k \neq j \end{cases} \quad (17)$$

$\mathbf{N}$ is a $N_t \times 1$ noise vector where the k th element is $\frac{N_t \sigma_n^2}{d_{k,k}^{-\rho} \chi_{k,k}} \mu_{\max}(\Omega_{k,j,2})$. Recall that $\Omega_{k,j,2}$ is Hermitian and positive semi-definite, thus we have $\mu_{\max}(\Omega_{k,j,2}) > 0$, $\mathbf{N} > 0$.

To find a feasible power vector $\mathbf{P}$ that achieves the target SINR, the parameters of the system must allow for a *feasible* solution. The following lemma describes the necessary and sufficient conditions for the existence of a feasible solution.

Lemma 2: The multi-cell MIMO power control problem has a feasible power vector $\mathbf{P}$ satisfying the SINR constraint, if $\rho_G < 1$ where $\mathbf{G} \triangleq \mathbf{\Lambda}\mathbf{F}$, ρ_G is the spectral radius (magnitude of maximum singular value) of $\mathbf{G}$.

Proof: Proof for this lemma can be found in [16][38]. $\square$

If the MIMO power control is feasible, the closed-form feasible power vector is derived as

$$\mathbf{P}^* = (\mathbf{I} - \mathbf{\Lambda}\mathbf{F})^{-1} \mathbf{\Lambda}\mathbf{N}. \quad (18)$$

It is a fixed point because it satisfies $\mathbf{P} = I(\mathbf{P})$. We will prove in the next section that it is optimal in terms of minimum sum power. After the optimal solution is achieved, each BS only needs to send P_k^* to its MS, therefore the feedback overhead is small.

D. Standard Power Control Algorithm

The main complexity of the closed-form solution (18) comes from the matrix inversion and computing the maximum eigenvalues. An alternative method is to apply the iterative algorithm (4) that avoids matrix inversion. As discussed in Section III, if the interference function $I(\mathbf{P})$ is standard, the iterative algorithm (4) converges to a unique fixed point whenever a feasible solution exists. In view of this fact, we prove that $I(\mathbf{P})$ in (15) is standard.

Lemma 3: The cellular MIMO power control problem in (15) satisfies the positivity, monotonicity and scalability properties and is therefore standard. Given it is feasible, for any initial power vector $\mathbf{P}$, the standard power control algorithm converges to a unique fixed point $\mathbf{P}^*$.

Proof: The positivity, monotonicity and scalability can be proven by noting that $\Omega_{k,j,1}, \Omega_{k,j,2}$ are positive semi-definite, and hence $\mu_{\max}(\Omega_{k,j,1}) > 0, \mu_{\max}(\Omega_{k,j,2}) > 0$. For brevity, the full proof is not included in the paper. $\square$

Now we have proven that the iterative algorithm converges to a fixed point, whenever the problem is feasible. Because the fixed point for any standard power control problem is unique (see Theorem 1), the solution obtained with the iterative approach should be exactly the same as the closed form solution (18). Finally according to Lemma 1 in [15], this unique fixed point achieves the minimum sum power among all feasible power $\mathbf{P}$, hence we conclude that this fixed point is the optimal solution.

E. Low Complexity Iterative Power Control

The closed-form and the iterative solutions are based on the assumption that a centralized controller has perfect instantaneous channel information (e.g. $\mathbf{H}_{k,j}$ and $d_{k,j}, \forall k, j$). In wireless communication, the radio propagation channel varies constantly due to the changing in the environment, hence updating channel information instantaneously requires a lot of feedback overhead to the centralized controller. These facts motivate a distributed power control scheme, which allows each BS to determine the power of its MS independently.

A fixed-step, 1-bit binary control algorithm was proposed in [18]. In this algorithm, each BS measures the SINR value and commands its associated mobile to increase/decrease its power if the SINR is below/above Γ_k , expressed as

$$P_k(n+1) = \begin{cases} \delta P_k(n) & \text{if } P_k(n) \leq I_k(\mathbf{P}(n)) \\ \delta^{-1} P_k(n) & \text{if } P_k(n) > I_k(\mathbf{P}(n)) \end{cases} \quad (19)$$

where $\delta > 1$ is the power control step size. This scheme requires only 1-bit UP/DOWN command every power control cycle. In addition, it is simple to implement because each receiver independently determines the power control command based on a single comparison [16].

The convergence of this 1-bit algorithm for the standard power control problem was proven in [18] for different step sizes. Since we have proven that multi-cell MIMO power control is standard, the proof of convergence follows implicitly.

V. MIMO POWER CONTROL WITH EQUAL POWER ALLOCATION AND SELF-CSI

In the previous section, we developed the optimum power control solution with equal power allocation, given that the

system is full-CSI aware. The perfect CSI requirement, however, is usually not satisfied in a practical cellular system. In this section, we propose an enhanced multi-cell MIMO power control scheme that requires only self-CSI information. The fundamental idea is to estimate the covariance matrix of the *composite noise and interference* at each BS, and then perform power control with the estimated covariance matrix. Only the sum interference covariance is required, instead of each interfering users' true channels (e.g. $\mathbf{H}_{k,j}$ and $d_{k,j}$, $\forall k, j$).

A. Problem Formulation

The received signal model is the same as in (1). In contrast to the full-CSI aware system, the k th BS has only knowledge of its in-cell user's channel $\mathbf{H}_{k,k}$, while $\mathbf{H}_{k,j}$, $j \neq k$ are not known.

The k th BS detects the k th user's symbol $\mathbf{x}_k$ and obtains the estimate $\hat{\mathbf{x}}_k$. The k th user's contribution to the k th BS is subtracted from $\mathbf{y}_k$ to obtain the composite noise estimate

$$\begin{aligned} \tilde{\mathbf{y}}_k &= \sqrt{\frac{P_k d_{k,k}^{-\rho} \chi_{k,k}}{N_t}} \mathbf{H}_{k,k} (\mathbf{x}_k - \hat{\mathbf{x}}_k) \\ &+ \sum_{j \neq k} \sqrt{\frac{P_j d_{k,j}^{-\rho} \chi_{k,j}}{N_t}} \mathbf{H}_{k,j} \mathbf{x}_j + \mathbf{n}_k, \end{aligned} \quad (20)$$

where the first item on the right hand side is the cancellation error. With a properly chosen Γ_k and channel coding, it is reasonable to assume an acceptably low SER and accurate composite noise estimate. In addition, the noise variance σ_n^2 is normally estimated with a separate subroutine, therefore we can subtract it and obtain the estimate of the interference covariance matrix as

$$\bar{\Sigma}_k = \sum_{j \neq k} \frac{P_j d_{k,j}^{-\rho} \chi_{k,j}}{N_t} \mathbf{H}_{k,j} \mathbf{H}_{k,j}^\dagger. \quad (21)$$

Using the estimated covariance matrix, the post-processing SINR lower bound for $\gamma_{k,\min}$ is derived in (22), where $\Omega_{k,j,1}$, $\Omega_{k,j,2}$ can be found in (8)(9).

As a result, the multi-cell MIMO power control problem with self-CSI is formulated as

$$\begin{aligned} P_k &\geq \frac{d_{k,k}^\rho}{\chi_{k,k}} \left(\mu_{\max} \left(\sum_{j \neq k} P_j d_{k,j}^{-\rho} \chi_{k,j} \Omega_{k,j,1} \right) \right. \\ &\quad \left. + \sigma_n^2 N_t \mu_{\max}(\Omega_{k,j,2}) \right) \Gamma_k, \end{aligned} \quad (23)$$

Note that this formulation is similar to the case with perfect channel knowledge as in (15). The key observation is that it involves the maximum eigenvalue of the composite noise covariance matrix which can be estimated, rather than the covariance matrix of *each* user.

B. Standard Power Control Algorithm

Similar to the full-CSI scenario, we can prove that $I(\mathbf{P})$ in (23) is a standard function, therefore the self-CSI problem is a standard power control problem.

Lemma 4: The interference function in (23) satisfies the positivity, monotonicity and the scalability properties, therefore the self-CSI power control problem is *standard*.

Proof: The positivity, monotonicity and scalability can be proven by noting $\Omega_{k,j,1}$, $\Omega_{k,j,2}$ are positive semi-definite and $\mu_{\max}(\Omega_{k,j,1}) > 0$, $\mu_{\max}(\Omega_{k,j,2}) > 0$. For brevity, the full proof is not included in the paper. $\square$

Because the self-CSI power control is standard, we can use the iterative algorithm in (4) to obtain the fixed point. Following Theorem 1 and 2, if the problem is feasible, then there exists a unique fixed point and the iterative algorithm converges to this unique point regardless of the initial power.

C. Discussion on Synchronization

For easier analysis, this paper has assumed perfect synchronization among the multiple cells and users. Although this is clearly unrealizable in current systems, this assumption does not jeopardize the integrity of the results. In [15], it was proven that if $I(\mathbf{P})$ is feasible, the standard power control algorithm converges asynchronously, where some users adjust their power more frequently than other users and perform power updates with the outdated information of the other users' interference. Because we have proven that both the full-CSI and self-CSI scenarios are standard, it follows that the proposed schemes are valid for asynchronous systems.

VI. MIMO POWER CONTROL WITH ADAPTIVE POWER ALLOCATION

A common issue for any power control problem is the infeasibility. Typically, users with poor channels (e.g., located near cell edge) are commanded to use more power, and generate significant interference to the adjacent cells. Other cells then need to increase their power to meet their own SINR target, which consequently force the original cell to increase

$$\begin{aligned} \gamma_{k,\min} &\geq \frac{P_k d_{k,k}^{-\rho} \chi_{k,k}}{N_t \max_i \left\{ \left(\mathbf{H}_{k,k}^\dagger \mathbf{H}_{k,k} \right)^{-1} \mathbf{H}_{k,k}^\dagger \bar{\Sigma}_k \mathbf{H}_{k,k} \left(\mathbf{H}_{k,k}^\dagger \mathbf{H}_{k,k} \right)^{-1} + \sigma_n^2 \left(\mathbf{H}_{k,k}^\dagger \mathbf{H}_{k,k} \right)^{-1} \right\}^{(i,i)}} \\ &\geq \frac{P_k d_{k,k}^{-\rho} \chi_{k,k}}{N_t \mu_{\max} \left(\left(\mathbf{H}_{k,k}^\dagger \mathbf{H}_{k,k} \right)^{-1} \mathbf{H}_{k,k}^\dagger \bar{\Sigma}_k \mathbf{H}_{k,k} \left(\mathbf{H}_{k,k}^\dagger \mathbf{H}_{k,k} \right)^{-1} \right) + \sigma_n^2 \mu_{\max} \left(\left(\mathbf{H}_{k,k}^\dagger \mathbf{H}_{k,k} \right)^{-1} \right)} \\ &= \frac{P_k d_{k,k}^{-\rho} \chi_{k,k}}{\mu_{\max} \left(\sum_{j \neq k} P_j d_{k,j}^{-\rho} \chi_{k,j} \Omega_{k,j,1} \right) + \sigma_n^2 N_t \mu_{\max}(\Omega_{k,j,2})}. \end{aligned} \quad (22)$$

its power in response. In certain channel settings, the target SINR cannot be achieved by all users simultaneously, thus infeasibility occurs.

In Sections IV and V, antennas of the same user are allocated with the same power, which is chosen such that the worst antenna can reach the SINR target. Because antennas with better channels are allocated with more than necessary power, this approach results in excess interference and leads to a higher infeasibility rate. An intuitively better approach is to adaptively distribute a user's power to its antenna array, such that its power is minimized subject to the SINR constraint. In contrast to the equal allocation scheme whose feasible region is a set of scalars, the feasible region of the adaptive schemes is a set of both scalars and matrices (i.e., P_k and $\mathbf{T}_k$), therefore it has an exponentially increasing number of possible solutions. More importantly, due to the non-linearity of the composite interference in terms of $\mathbf{P}$, obtaining a closed-form solution is very difficult. In this section we will propose a game theory based algorithm where users sequentially update their own power until converging at a fixed point.

A. Signal Model

The signal model follows (1), where $\mathbf{T}_k \in \mathbb{C}^{N_t \times N_t}$ is the diagonal power loading matrix of user k satisfying sum power constraint $\sum_{j=1}^{N_t} (\mathbf{T}_k^{(j,j)})^2 = N_t, k = 1, 2, \dots, K$. Following similar derivation in the Appendix, the post-processing SINR of stream s for cell k is given as

$$\gamma_{k,s} = \frac{P_k (\mathbf{T}_k^{(s,s)})^2}{\zeta_{k,s}} - 1, \quad (24)$$

where

$$\zeta_{k,s} = \left(\left(d_{k,k}^{-\rho} \chi_{k,k} \mathbf{H}_{k,k}^\dagger \left(\sum_{j \neq k} P_j d_{k,j}^{-\rho} \chi_{k,j} \mathbf{H}_{k,j} \mathbf{T}_j \mathbf{T}_j^\dagger \mathbf{H}_{k,j}^\dagger + N_t \sigma_n^2 \mathbf{I} \right)^{-1} \mathbf{H}_{k,k} + \frac{1}{P_k} \mathbf{I} \right)^{-1} \right)^{(s,s)} \quad (25)$$

denotes the effective interference after MMSE processing. An estimate of $\zeta_{k,i}$ can be obtained at the k th BS by the method in Section V, therefore we assume a partial-CSI in this section. Denote $\mathbf{T} = [\mathbf{T}_1^T, \dots, \mathbf{T}_K^T]^T$, the objective is to find

$$\{\mathbf{P}_{\text{opt}}, \mathbf{T}_{\text{opt}}\} = \arg \min_{\mathbf{P} \geq \mathbf{0}, \text{trace}(\mathbf{T}_k \mathbf{T}_k^\dagger) = N_t, k \in [1, K]} \sum_{k=1}^K P_k, \quad (26)$$

such that $\gamma_{k,s} \geq \Gamma_k, \forall s \in [1, N_t], k \in [1, K]$.

B. Iterative Algorithm for Adaptive Power Allocation

Obtaining a closed-form optimal solution $\{\mathbf{P}_{\text{opt}}, \mathbf{T}_{\text{opt}}\}$ for the adaptive scheme is very difficult. In this section, we will propose a low-complexity suboptimal approach based on game theory. The basic idea is to sequentially update each user's transmission, by treating other users as fixed interference, until all users converge to a fixed point. We will first specify a necessary condition of the fixed point to minimize sum power, provided in the following Lemma. We enforce each

user to satisfy this necessary condition at each iteration of our algorithm, thus if the algorithm converges at a fixed point, the necessary condition will be satisfied and the sum power is minimized.

Lemma 5: Consider a K -user system. At the fixed point, $\{\mathbf{P}_{\text{opt}}, \mathbf{T}_{\text{opt}}\}$ enforces the effective SINR of the k th user's N_t substreams to be equal to Γ_k , for $k = 1, \dots, K$.

Proof: Without loss of generality, consider user k . The remaining $K - 1$ users are treated as fixed interference, and the problem reduces to deciding $\{P_k, \mathbf{T}_k\}$ such that the N_t streams of user k meet Γ_k . It is well known in the power control literature that for a set of parallel N_t eigenchannels and fixed power P , the optimal approach for maximizing the minimum SINR is *channel inversion*, i.e., allocate power inverse proportionally to the effective channel gain $\frac{1}{\zeta_{k,s}}$. In other words, under a given Γ_k , P_k is minimized by enforcing $\gamma_{k,s} = \Gamma_k$, for $s \in [1, N_t]$, otherwise P_k can always be decreased while meeting Γ_k . $\square$

Lemma 5 suggests that the optimum power loading follows the *channel inversion* principle. If we consider $\frac{1}{\zeta_{k,s}}$ as the *effective* channel gain, antennas with good channels are allocated less power, while antennas in deep fades are given more power to meet the SINR target. This is different from the waterfilling approach where more power is given to better channels.

The merit of Lemma 5 is that it specifies a criterion to perform power adaptation for each user. Our game-theory based algorithm is proposed as follows. At each iteration, $\{P_k, \mathbf{T}_k\}$ is optimized from user 1 to user K in numerical order. For each user k , we treat the other users as interference and assume they maintain the power in the previous iteration, and optimize $\{P_k, \mathbf{T}_k\}$. Power optimization is conducted according to channel inversion (e.g., Lemma 5) to minimize the power of *each* single user. This process repeats until a fixed point is reached.

Algorithm 1: Iterative Channel Inversion Power Control

- 1) Let $i = 0$, set the initial power vector $\mathbf{P}^0 = (1, 1, \dots, 1)$, where $P_k^0 = 1$ is the initial power level for user k . Let $\mathbf{T}^0 = [\mathbf{T}_1^0; \dots; \mathbf{T}_K^0] = [\mathbf{I}_{N_t}; \dots; \mathbf{I}_{N_t}]$ be the initial power loading matrix.

- 2) Let $i = i + 1$.

For $k = 1$ to K ,

- a) Given $\{\mathbf{P}^{i-1}, \mathbf{T}^{i-1}\}$, compute the effective interference $\zeta_{k,s} = f(\mathbf{P}^{i-1}, \mathbf{T}^{i-1})$ as in (25), for $s \in [1, N_t]$.
- b) Calculate the optimum loading matrix $\mathbf{T}_k^i$ as

$$(\mathbf{T}_k^i)^{(s,s)} = \sqrt{\frac{\zeta_{k,s} N_t}{\sum_{j=1}^{N_t} \zeta_{k,j}}}, \forall s \in [1, N_t]. \quad (27)$$

- c) Calculate the optimum transmit power P_k^i , according to

$$P_k^i = \frac{\zeta_{k,s}}{((\mathbf{T}_k^i)^{(s,s)})^2} (\Gamma_k + 1), \forall s \in [1, N_t]. \quad (28)$$

end.

- 3) If $\max_k \|P_k^i - P_k^{i-1}\|/P_k^{i-1} \geq \eta$, go to step 2), otherwise stop. η is a small value scalar to determine whether convergence is reached.

After the optimum strategy of user k is obtained, the k th BS send $N_t + 1$ numbers (i.e. $P_{k,\text{opt}}, \mathbf{T}_{k,\text{opt}}$) to the k th mobile, where $\mathbf{T}_{k,\text{opt}}$ is diagonal. The feedback can be implemented on the specific feedback channel, such as in WiMAX/IEEE802.16 [35].

In terms of complexity, waterfilling performs SVD and calculates the optimum water-level in each iteration, so is highly computationally demanding. In contrast, equal power allocation power control is much less complicated because it only requires the *largest eigenvalue*, and adaptive power allocation requires no eigenvalue computation at all. The feedback overhead of power control and waterfilling are approximately the same, as the power on each antenna is the information to be fed back.

C. Discussion on Convergence of Adaptive Power Allocation

For MIMO power control with adaptive power allocation, it is straightforward to obtain the following corollary.

Corollary 2: For the multi-cell MIMO power control with adaptive power allocation, if the power loading matrices $\{\mathbf{T}_k\}_{k=1}^K$ are arbitrarily fixed and not considered for optimization, the power control problem is standard and therefore converges, if a feasible solution exists. The equal power allocation case (e.g., $\mathbf{T}_k = \mathbf{I}_{N_t}$) is a special case of this category.

It is difficult to analyze the convergence if we also consider the loading matrix $\mathbf{T}_k$ optimization in each iteration. The feasible region of $\mathbf{T}_k$ is non-linear due to the power constraint, $\sum_{s=1}^{N_t} (\mathbf{T}_k^{(s,s)})^2 = N_t$. As a result, the problem is not standard and it is impossible to evaluate the monotonicity (i.e., $I(\mathbf{P}) \geq I(\mathbf{P}')$ if $\mathbf{P} \geq \mathbf{P}'$), and the scalability ($I(\alpha\mathbf{P}) \geq \alpha I(\mathbf{P})$, if $\alpha \geq 1, \mathbf{P} \geq I(\mathbf{P})$) without violating the power constraint. Since the power control problem is no longer standard, we are not able to theoretically prove the convergence of the iterative channel inversion power control algorithm at this point. Numerical results, however, indicate that algorithm 1 converges within less than 10 iterations if the problem is feasible.

VII. NUMERICAL RESULTS

Simulation results are provided to demonstrate the performance of our proposed schemes. All cells are hexagonal and have radius of $R = 1000$ meters. A universal frequency reuse pattern is applied, and mobiles are uniformly distributed within the cells. The pathloss exponent is $\rho = 3$ for $\forall k$, and the variance of log-normal shadow-fading is 8 dB. For brevity, a system with N_t transmit and N_r receive antennas is denoted as a $N_t \times N_r$ system.

A. MIMO Power Control With Equal Power Allocation

Fig. 2 presents the convergence of the 1-bit algorithm in a 7-cell system. The target SINR is 6 dB, and the AWGN noise variance is $\sigma_n^2 = 0.01$. The vertical axis shows the normalized

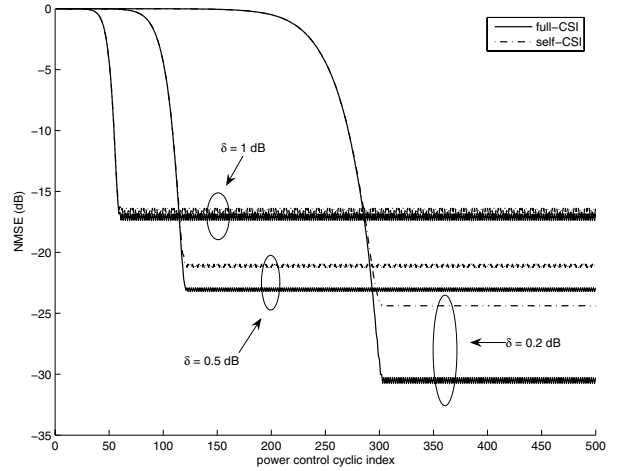


Fig. 2. Convergence of the 1-bit power control algorithm with equal power allocation and different step size δ : $K = 7$, $\sigma_n^2 = 0.01$, $\Gamma = 6$ dB.

mean square error (NMSE) for each iteration $NMSE = \|\mathbf{P} - \mathbf{P}^*\|^2 / \|\mathbf{P}^*\|^2$, averaged over 1000 independent channel realizations. Fig. 2 confirms the convergence with step size δ ranging from 0.2 dB to 1 dB, with either self-CSI and full-CSI. As expected, larger δ allows faster convergence, but smaller δ allows finer resolution and lower steady-state NMSE. Because a finer resolution can help more if the available channel information is more abundant, the full-CSI case enables more NMSE reduction and provides a higher advantage than the self-CSI case, as δ decreases.

B. MIMO Power Control With Adaptive Power Allocation and Self-CSI

Fig. 3 compares the infeasibility probability of equal and adaptive power allocation, where 19 cells are considered. For each target SINR, 1000 independent channel realizations are simulated. For each channel realization, infeasibility occurs if the iterative algorithm does not converge after 1000 loops. Simulation results suggest that the adaptive algorithm substantially reduces the infeasibility probability, and consequently increases the achievable target SINR under a certain infeasibility rate. For example, for a 2×4 system, if the infeasibility is required to be less than 10%, the maximum achievable target SINR with equal power allocation is 7 dB, while adaptive power allocation can achieve a target SINR up to 17 dB.

Fig. 4 plots the ratio of average sum transmit power obtained with these two approaches, when both algorithms obtain a feasible solution. It is shown that the adaptive allocation scheme substantially reduces the sum power, by up to 80%. Additionally, we find that the power reduction is more effective when the number of users K , the number of transmit antennas N_t , and the target SINR Γ are large. This is because adaptive power allocation efficiently exploits the multiuser MIMO channels (e.g., multiuser diversity), hence power reduction is more significant with larger K . A larger receive antenna number N_r , however, reduces the channel variation by providing more receive diversity, and diminishes the improvement of the adaptive scheme.

Convergence of the adaptive scheme is evaluated in Fig. 5 by plotting the NMSE vs. iteration number, where $\Gamma = 6$ dB

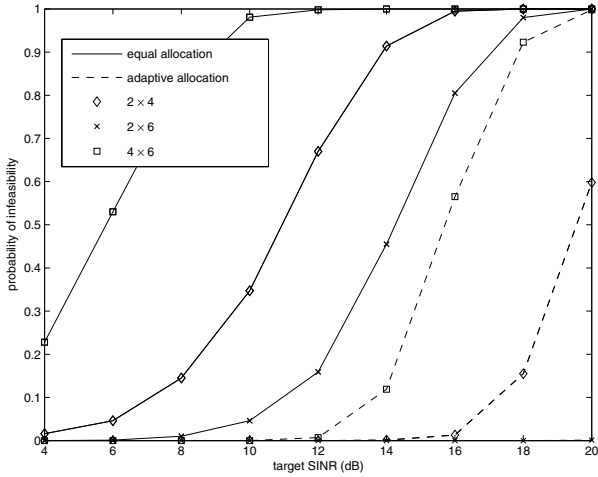


Fig. 3. Probability of infeasibility with equal power allocation and adaptive power allocation: $K = 19$, $\sigma_n^2 = 0.01$.

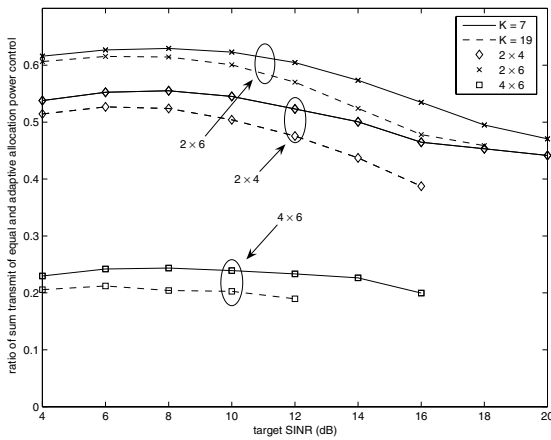


Fig. 4. Average ratio of sum transmit power between adaptive power allocation and equal power allocation power control: $\sigma_n^2 = 0.01$.

and $\sigma_n^2 = 0.01$, averaged over 1000 channel realizations. It is shown that if there exists a feasible solution, the algorithm typically requires less than 10 iterations to converge. Convergence is slightly faster with a larger receive antenna array or a smaller transmit antenna array, due to the higher receive diversity $N_r - N_t + 1$. Similarly, a smaller number of cells lead to a faster convergence.

C. Cell Coverage With Equal and Adaptive Power Control

In this section we investigate the cell coverage capability by evaluating the coverage radius $D(P_{\text{out}}, \Gamma)$, defined such that if all MSs are within D meters from their associated BSs, the infeasibility probability is less than $P_{\text{out}}(D, \Gamma)$, under a target SINR of Γ . Likewise, $P_{\text{out}}(D, \Gamma)$ is defined as the outage probability.

Fig. 6 shows the outage probability P_{out} with different coverage radius D and target Γ , for both equal and adaptive allocation schemes. The adaptive power allocation can dramatically reduce the outage probability under the same coverage and SINR constraint. Fig. 7 depicts the cell coverage rate $D^2(P_{\text{out}}, \Gamma)/R^2$ for both schemes. The adaptive power allocation can significantly increase the cell coverage rate. For

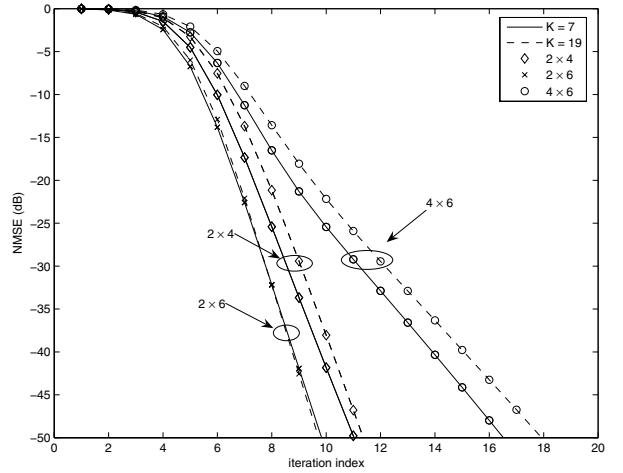


Fig. 5. Convergence of the adaptive allocation power control given that the channel provides a feasible solution: $\Gamma = 6$ dB, $\sigma_n^2 = 0.01$.

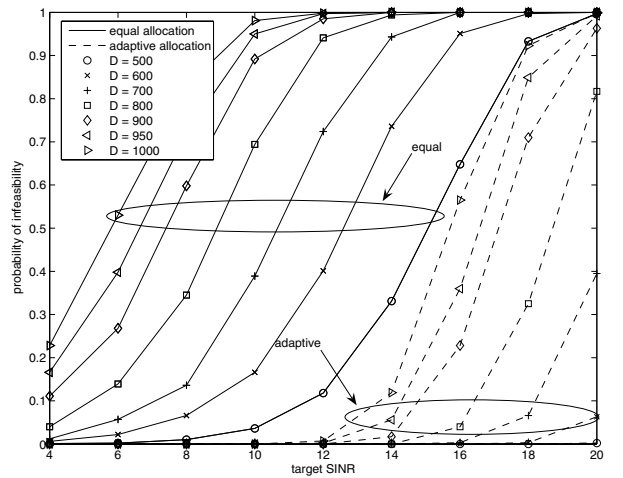


Fig. 6. Outage probability P_{out} with equal power allocation and adaptive power allocation when all MSs are within D meters to their BS: cell radius $R = 1000$, $K = 19$, $N_t = 4$, $N_r = 6$, $\sigma_n^2 = 0.01$.

example at $P_{\text{out}} = 10\%$ and $\Gamma = 8$ dB, the equal power allocation covers only 50% of the cell, while adaptive power allocation can serve the entire cell.

D. Comparison of Power Control and Iterative Waterfilling

Fig. 8 compares the throughput per user for both iterative waterfilling and our proposed power control schemes in a 7-cell system. The probability of infeasibility is compared in Fig. 9. The target SINR Γ translates into the target throughput for iterative waterfilling as $T = N_t \log_2(1 + \Gamma)$. The simulation setting is given as follows:

- The throughput is calculated at the *fixed point* obtained by each algorithm.
- For iterative waterfilling, the mobile's maximum power P_{max} is chosen to be significantly higher than $\bar{P}$, which is the average power at the fixed points of power control. Note that waterfilling can transmit at any power level from 0 to P_{max} .
- Waterfilling is infeasible if one or more users do not achieve the target throughput.

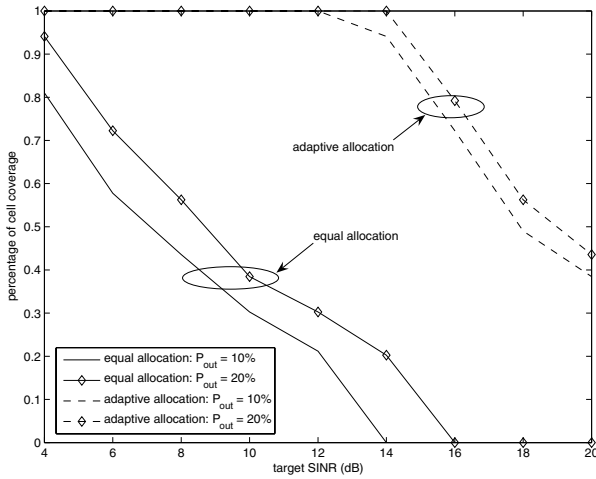


Fig. 7. Cell coverage $D^2(P_{out}, \Gamma)/R^2$ of equal and adaptive power allocation power control: $K = 19$, $N_t = 4$, $N_r = 6$, $\sigma_n^2 = 0.01$.

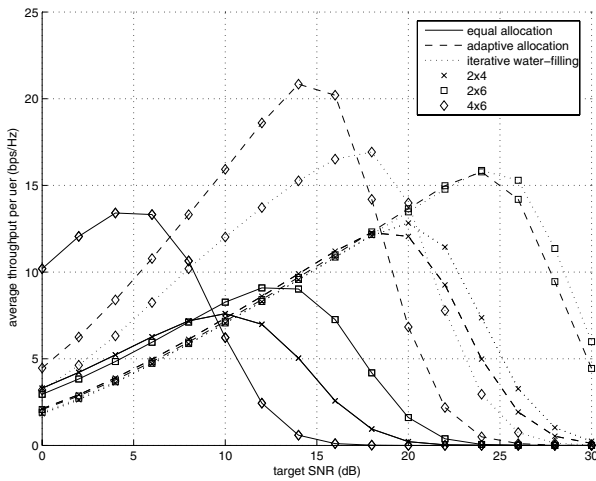


Fig. 8. Throughput per user of the proposed power control algorithms and the iterative waterfilling algorithm: $K = 7$, $\sigma_n^2 = 0.01$.

- The throughput of power control is defined to be zero in the case of infeasibility, while iterative waterfilling has a non-zero throughput even when infeasible. To make a fair comparison, the throughput is averaged over channels that are feasible for both power control and iterative waterfilling.

Fig. 8 shows that the power control schemes obtain higher average throughput than waterfilling at low-to-moderate SINRs. For 2×4 or 2×6 cases, the average throughput of power control is approximately 2 bps/Hz higher than iterative water-filing, when $\Gamma \leq 18$ dB. For 4×6 case, power control has 3-5 bps/Hz higher throughput when $\Gamma \leq 14$ dB. The throughput of power control drops quickly to zero at high SINRs due to infeasibility, while waterfilling can still maintain a link. Note that power control and waterfilling are performed with completely different objectives (e.g., target SINR vs target rate). The throughput in Fig. 8 is obtained at the *fixed point* of different algorithms, which is different from the conventional problem of *throughput maximization* with a fixed sum power. Additionally, the complexity of iterative waterfilling is significantly higher, due to the iterative

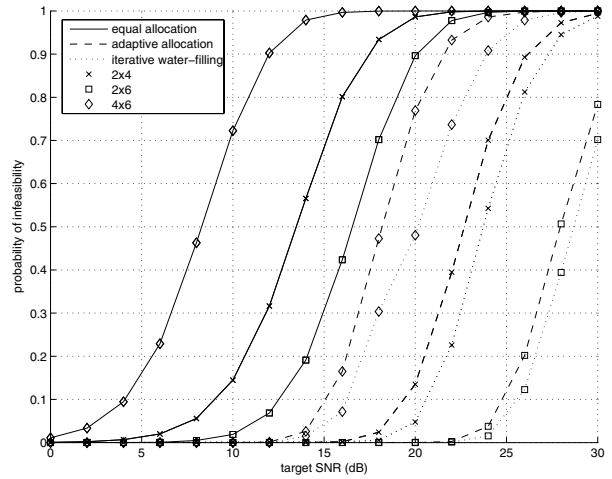


Fig. 9. Probability of infeasibility of proposed power control algorithms and the iterative waterfilling algorithm: $K = 7$, $\sigma_n^2 = 0.01$.

SVD operations. Hence we conclude that power control is more effective at low-to-moderate SINRs, which is the more common case in cellular networks. Waterfilling is well suited to high SINRs, so is probably more appropriate for wireless LANs.

The performance gain of power control over waterfilling is possibly the result from the non-convexity of the sum capacity in terms of the power vector $\mathbf{P}$. In a single-user MIMO link, it is well-known that waterfilling is optimal due to the convexity of the capacity with regards to the power. In cellular systems, however, the sum capacity is no longer convex because each user's power appears in the denominator of the other users' SINR.

Fig. 9 compares the infeasibility probability of power control and waterfilling. We find that the adaptive power allocation power control significantly outperforms the equal allocation scheme, approximately achieving a SINR improvement of 10-12 dB in most channel settings. Waterfilling outperforms power control in terms of infeasibility probability by 1-2 dB SINR improvement, at the cost of higher computational cost and reduced throughput at lower SINR. In summary, power control - suitably modified for MIMO systems - is a promising means for alleviating the interference problem in cellular MIMO systems.

VIII. CONCLUSIONS

This paper studied the multi-dimensional power control problem for an uplink cellular MIMO spatial multiplexing system. Our work include several previous OFDM power control algorithms as special cases by taking inter-carrier interference into account, and included many utility maximizing power control as special cases if the utility maximizing SINR is used as the target. Solutions were provided where each user's power is equally or adaptively allocated to its antenna array. Simulation results showed that the average throughput of adaptive power control outperforms iterative waterfilling by 2-5 bps/Hz at low-to-moderate SINRs (e.g., below 10 dB), while iterative waterfilling is more effective at high SINR. In terms of the probability of infeasibility, adaptive power allocation provides 10-12 dB target SINR improvement at

the same infeasibility probability, compared to equal power allocation, which effectively increases the cell coverage and reduces power consumption at mobile terminals. Waterfilling outperforms the adaptive power allocation power control by 1-2 dB target SINR gain at the same infeasibility probability, at the cost of much higher computational complexity.

APPENDIX

For simplicity of notation, denote

$$\mathbf{z}_k = \sum_{j \neq k} \sqrt{\frac{P_j d_{k,j}^{-\rho} \chi_{k,j}}{P_k d_{k,k}^{-\rho} \chi_{k,k}}} \mathbf{H}_{k,j} \mathbf{x}_j, \quad (29)$$

$$\tilde{\mathbf{n}}_k = \sqrt{\frac{N_t}{P_k d_{k,k}^{-\rho} \chi_{k,k}}} \mathbf{n}_k$$

$$\mathbf{R}_{\mathbf{z}_k} = E(\mathbf{z}_k \mathbf{z}_k^\dagger) = \sum_{j \neq k} \frac{P_j d_{k,j}^{-\rho} \chi_{k,j}}{P_k d_{k,k}^{-\rho} \chi_{k,k}} \mathbf{H}_{k,j} \mathbf{H}_{k,j}^\dagger, \quad (30)$$

$$\mathbf{R}_{\tilde{\mathbf{n}}_k} = E(\tilde{\mathbf{n}}_k \tilde{\mathbf{n}}_k^\dagger) = \frac{N_t \sigma_n^2}{P_k d_{k,k}^{-\rho} \chi_{k,k}}. \quad (31)$$

The estimate $\hat{\mathbf{x}}_k$ is given as $\hat{\mathbf{x}}_k = \mathbf{G}_k \mathbf{y}_k$ and the estimation error matrix with the MMSE receiver is given as [32]

$$\Sigma_{\mathbf{x}_k} = \left(\mathbf{H}_{k,k}^\dagger (\mathbf{R}_{\mathbf{z}_k} + \mathbf{R}_{\tilde{\mathbf{n}}_k})^{-1} \mathbf{H}_{k,k} + \mathbf{I} \right)^{-1}. \quad (32)$$

The minimum post-processing SINR of the k th user is bounded as (36). The factor -1 in step (33) removes the estimation bias [31], and $\mathbf{e}_i$ in (34) denotes the i th column of a $N_t \times N_t$ identity matrix. Step (35) follows the Rayleigh-Ritz theorem [39], which has widely used in deriving a SNR bound for MIMO systems [36][37].

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$$\gamma_{k,\min} = \min_{1 \leq i \leq N_t} \gamma_{k,i} = \frac{1}{\max_i \left\{ \left(\mathbf{H}_{k,k}^\dagger (\mathbf{R}_{\mathbf{z}_k} + \mathbf{R}_{\tilde{\mathbf{n}}_k})^{-1} \mathbf{H}_{k,k} + \mathbf{I} \right)^{-1} \right\}^{(i,i)}} - 1 \quad (33)$$

$$= \frac{1}{\max_{\mathbf{z}=\mathbf{e}_i, i=1, \dots, N_t} \mathbf{z}^\dagger \left(\mathbf{H}_{k,k}^\dagger (\mathbf{R}_{\mathbf{z}_k} + \mathbf{R}_{\tilde{\mathbf{n}}_k})^{-1} \mathbf{H}_{k,k} + \mathbf{I} \right)^{-1} \mathbf{z}} - 1 \quad (34)$$

$$\geq \frac{1}{\max_{\|\mathbf{z}\|_F^2=1} \mathbf{z}^\dagger \left(\mathbf{H}_{k,k}^\dagger (\mathbf{R}_{\mathbf{z}_k} + \mathbf{R}_{\tilde{\mathbf{n}}_k})^{-1} \mathbf{H}_{k,k} + \mathbf{I} \right)^{-1} \mathbf{z}} - 1 \quad (35)$$

$$\begin{aligned} &= \frac{1}{\mu_{\max} \left(\left(\mathbf{H}_{k,k}^\dagger (\mathbf{R}_{\mathbf{z}_k} + \mathbf{R}_{\tilde{\mathbf{n}}_k})^{-1} \mathbf{H}_{k,k} + \mathbf{I} \right)^{-1} \right)} - 1 \\ &= \mu_{\min} \left(\mathbf{H}_{k,k}^\dagger (\mathbf{R}_{\mathbf{z}_k} + \mathbf{R}_{\tilde{\mathbf{n}}_k})^{-1} \mathbf{H}_{k,k} + \mathbf{I} \right) - 1 \\ &= \mu_{\min} \left(\mathbf{H}_{k,k}^\dagger (\mathbf{R}_{\mathbf{z}_k} + \mathbf{R}_{\tilde{\mathbf{n}}_k})^{-1} \mathbf{H}_{k,k} \right) + \mu_{\min} (\mathbf{I}) - 1 \\ &= \frac{1}{\mu_{\max} \left(\left(\mathbf{H}_{k,k}^\dagger \mathbf{H}_{k,k} \right)^{-1} \mathbf{H}_{k,k}^\dagger (\mathbf{R}_{\mathbf{z}_k} + \mathbf{R}_{\tilde{\mathbf{n}}_k}) \mathbf{H}_{k,k} \left(\mathbf{H}_{k,k}^\dagger \mathbf{H}_{k,k} \right)^{-1} \right)} + 1 - 1 \\ &= \frac{P_k d_{k,k}^{-\rho} \chi_{k,k}}{\mu_{\max} \left(\sum_{j \neq k} P_j d_{k,j}^{-\rho} \chi_{k,j} \mathbf{\Omega}_{k,j,1} + N_t \sigma_n^2 \mathbf{\Omega}_{k,j,2} \right)}. \end{aligned} \quad (36)$$

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