

Low Complexity Multiuser MIMO Scheduling with Chordal Distance

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Abstract—In multiuser MIMO systems, block diagonalization (BD) eliminates inter-user interference with perfect CSI since the transmitter decomposes a multiuser MIMO channel into multiple parallel independent single-user MIMO channels. For a large number of users, the transmitter selects a subset of users who have good channels to maximize the system throughput. In this paper, a new low complexity scheduling algorithm with BD is proposed since an exhaustive search for the optimal user set is impractical. Simulation results show that the proposed algorithm has much lower complexity while it achieves the total throughput close to the optimal algorithm. It also has lower complexity than previous low complexity scheduling algorithms.

Index Terms—multiuser, multiple-input multiple- output (MIMO), chordal distance, sum capacity.

I. INTRODUCTION

Multiple-input multiple output (MIMO) systems has been an attractive research field in recent years. It was shown that MIMO system can increase the channel capacity of the system significantly [1], [2]. It was also shown that to allocate all the resources to the user who has the best channel is the optimal strategy in multiuser single-input single-output (SISO) systems [3]. On the other hand, in multiuser MIMO systems it was shown that it is better that the resources are allocated to multiple users [4].

However, in multiuser MIMO (MU-MIMO) system the inter-user interference is generated when the resources are allocated to multiple users. In order to avoid the inter-user interference we can use dirty paper coding (DPC) [5]. It was shown that the interference which is known a priori to the transmitter does not affect the channel capacity. DPC is an optimal precoding method that achieves the channel capacity. However, it is not practical in real systems since its complexity is prohibitive.

Zero-forcing (ZF) and block diagonalization (BD) are proposed to reduce the system complexity [6]–[8]. ZF looks for the pseudo inverse of channels of the selected users. It eliminates interference among antennas in the overall system. That is, performing ZF in a system with K users and N antennas is equivalent to performing ZF in a channel with KN single antenna receivers. On the other hand, BD looks for

the null space of interference channels. It eliminates the interference among the selected users. Since each user's precoding matrix must lie in the null space of all other users' channels, the number of users that can be simultaneously supported with BD is limited by the number of transmit antennas and the number of receive antennas.

In MU-MIMO systems, the optimal scheduling algorithm which maximizes the system throughput needs a brute-force exhaustive search for all possible user subsets, and it has large complexity. A suboptimal scheduling algorithm to reduce complexity was proposed when the number of receiver antennas is one [9]. First, it selects the user who has the maximum Frobenius norm. The algorithm successively selects the users who are near-orthogonal with previous selected users by using Gram-Schmidt orthogonalization (GSO).

GSO of [9] can be used only in vectors. In conventional MU-MIMO systems, the channels of users are represented with matrices. Therefore [9] cannot be used in conventional MU-MIMO systems. In [10], two low complexity suboptimal scheduling algorithms, the capacity based algorithm and the Frobenius norm based algorithm, that can be applied in conventional MU-MIMO systems were proposed. These algorithms achieve the total throughput similar to the optimal algorithm with much lower complexity. However, the capacity based algorithm needs heavy computation of singular value decomposition, and the Frobenius norm based algorithm also needs heavy GSO computation.

In this paper, we propose a new low complexity scheduling algorithm with BD to maximize the total throughput. We introduce chordal distance as the distance metric, and select the user whose channel has the maximum chordal distance with previously selected users' channels. It has lower computational complexity since it needs a fewer GSO than the Frobenius norm based algorithm.

The rest of the paper is organized as follows. Section II introduces the system model and the backgrounds. The proposed algorithm is described in Section III. Section IV presents the computational complexity analysis and Section V shows the simulation results. Finally, conclusions are given in Section VI.

II. SYSTEM MODEL AND BACKGROUNDS

We assume that the receivers estimate their channels perfectly and the transmitter knows the exact channel state information (CSI) of all the users. We consider an MU-MIMO downlink channel with a single base station (BS) which has M transmit antennas. The total number of users is K_T . Only K out of K_T users can be selected in BS simultaneously, and each user has N antennas. The broadcast channel is described as

$$y_i = H_i^H x + n_i, \quad i = 1, \dots, K \quad (1)$$

where H_i is the $M \times N$ channel matrix to user i whose channel elements are independent identically distributed (i.i.d.) complex Gaussian with zero mean and unit variance, and x is an $M \times 1$ transmitted signal. n_i is an $N \times 1$ noise vector, whose elements are i.i.d. complex Gaussian with zero mean and unit variance. y_i is the $N \times 1$ received signal vector at user i .

A. Block Diagonalization

If s_i is the N complex symbols intended for user i and V_i is the $M \times N$ precoding matrix, the transmitted vector is given by

$$x = \sqrt{\frac{P}{K}} \sum_{i=1}^K V_i s_i, \quad (2)$$

and the received signal at the user i is given by

$$y_i = \sqrt{\frac{P}{K}} H_i^H V_i s_i + \sqrt{\frac{P}{K}} \sum_{j=1, j \neq i}^K H_j^H V_j s_j + n_i. \quad (3)$$

Using the BD strategy, the precoding matrix V_i is selected such that $H_j^H V_i = 0, \forall i \neq j$. In order to maintain the power constraint, it is assumed that $V_i^H V_i = I_N$ and $E[\|s_i\|^2] \leq 1$. This is an orthonormal basis for the null space of the matrix formed by stacking all $H_j^H, (j \neq i)$ matrices together. This reduces the interference term in (3) to zero at each user. At most $K = \lfloor M/N \rfloor$ users are allocated simultaneously because of the constraint of null space.

B. Chordal Distance

Let us introduce a distance metric between subspaces. Grassmannian space $G(m, n)$ is the set of all the n -dimensional subspaces of Euclidean m -dimensional space. We need to define the distance between two elements of $G(m, n)$. We associate principal angles $\theta_i \in [0, \pi/2]$ and principal vectors $u_i \in P$ and $v_i \in Q$ for $i = 1, \dots, n$ with two n -planes P and Q as follows. Choose $u_i \in P$ and $v_i \in Q$

having length 1, and such that $u_i \perp v_i$ is maximal. Inductively, define $u_i \in P$ and $v_i \in Q$ having length 1 and such that $u_i \perp v_i$ is maximal, subject to the conditions $u_i \perp u_j = 0$ and $u_i \perp v_j = 0$ for all $1 \leq j \leq i$. Then set $\theta_i = \arccos u_i \perp v_i$. The chordal distance is given by

$$d_c(P, Q) = \sqrt{\sin^2 \theta_1 + \dots + \sin^2 \theta_n} \quad (4)$$

A generator matrix for an n -plane $P \in G(m, n)$ is an $m \times n$ matrix whose columns span P . The orthogonal group $O(m)$ acts on $G(m, n)$ by right multiplication of generator matrices. Applying a suitable element of $O(m)$ and choosing appropriate basis vectors for the planes, we can reduce the generator matrices of any given pair of n -planes P, Q with $n \leq m/2$ to the forms

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}, \quad \begin{bmatrix} \cos \theta_1 & 0 & \dots & 0 \\ 0 & \cos \theta_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \cos \theta_n \\ \sin \theta_1 & 0 & \dots & 0 \\ 0 & \sin \theta_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sin \theta_n \end{bmatrix} \quad (5)$$

respectively. If A is a generator matrix for P whose columns are orthogonal unit vectors, then the projection is represented by the matrix $P_p = AA^H$. Using (5), we can see that $\text{trace } P_p$ is n . For $P, Q \in G(m, n)$, with orthonormal generator matrices A and B , and principal angles $\theta_1, \dots, \theta_n$, an elementary calculation using (5) shows that

$$\begin{aligned} d_c^2(P, Q) &= n - (\cos^2 \theta_1 + \dots + \cos^2 \theta_n) \\ &= n - \text{trace } AA^H BB^H \\ &= \frac{1}{2} \|P_p - Q_p\|_F^2 \end{aligned} \quad (6)$$

where P_p, Q_p are the corresponding projection matrices.

III. LOW COMPLEXITY SCHEDULING ALGORITHM

A. Optimal Scheduling Algorithm

Since we assume that the BS knows the perfect CSI of all the users, the capacity with water-filling can be written by

$$C(H^H, P) = \sum_{i=1}^r \log_2 \left(1 + \frac{P \gamma_i}{M} \lambda_i \right) \quad (7)$$

where P is the transmit power, r is the rank of channel matrix, and λ_i ($i=1, \dots, r$) is the positive singular value of H^H . Also

γ_i satisfies $\gamma_i = \left(\mu - \frac{M}{P\lambda_i}\right)^+$ and $\sum_{i=1}^n \gamma_i = M$. Therefore the

sum capacity of the MU-MIMO system with BD and equal power among the selected users can be written by

$$R_{CD}^{EP}(S) = \sum_{i=1}^K C\left(H_i V_i, \frac{P}{K}\right) \quad (8)$$

The sum capacity of an MU-MIMO system with optimal scheduling method can then be written as

$$R_{CD}^{EP} = \max_{S \subseteq \{1, \dots, K_T\}, |S| \leq K} R_{CD}^{EP}(S) \quad (9)$$

From (8) and (9), we can see that the optimal scheduling algorithm requires brute-force exhaustive search over all possible user sets. The number of user subsets with the optimal scheduling algorithm is $\sum_{i=1}^K \binom{K_T}{i}$. However, when K_T is large, the search complexity of the optimal scheduling algorithm is too large.

B. Low Complexity Scheduling Algorithm

We propose a new low complexity scheduling algorithm. We consider the Frobenius norm of channels. With BD, (3) can be rewritten by

$$y_i = \sqrt{\frac{P}{K}} H_i^H V_i s_i + n_i \quad (10)$$

We should consider not H_i^H , but $H_i^H V_i$ since the effective channel is $H_i^H V_i$. Even if the Frobenius norm of H_i^H is large, the Frobenius norm of $H_i^H V_i$ can be small since V_i eliminates not only inter-user interference but also the common subspace of H_i^H and H_j^H ($i \neq j$). Therefore orthogonality among the selected users is important. If users are close to orthogonal, the common subspaces removed by V_i are small. We use the chordal distance to select the users to maximize the channel matrix distance between the previously selected users and the new user.

The chordal distance between P and Q is given by

$$d_c(P, Q) = \frac{1}{\sqrt{2}} \|P_o P_o^H - Q_o Q_o^H\|_F \quad (11)$$

where P_o, Q_o are orthonormal bases for subspaces P, Q respectively. The summary of the proposed algorithm is

TABLE I
LOW COMPLEXITY SCHEDULING ALGORITHM
STEP I : Initialization

$$\begin{aligned} T &= \{1, 2, \dots, K_T\}; \quad C_k = \|H_k^H\|_F; \quad s_1 = \max_k C_k; \\ S &= \{s_1\}; \quad T = T - \{s_1\}; \quad R_{last} = 0; \quad P_o = \emptyset; \end{aligned}$$

STEP II : Loop

FOR $i = 2$ to K

$$P_o = [P_o \ H_{s_{i-1}}]_o; \quad A = P_o P_o^H;$$

$$s_i = \max_{k \in T} (C_k^\alpha \times \|A - H_{k,o} H_{k,o}^H\|_F),$$

where $H_{k,o}$ is orthonormal basis of H_k ;

$$R_{temp} = R_{CD}^{EP}(S_{temp}), \text{ where } S_{temp} = S + \{s_i\};$$

$$IF (R_{temp} < R_{last})$$

the selected user set is S , end;

ELSE

$$S_{temp} = S; \quad R_{last} = R_{temp}; \quad T = T - \{s_i\};$$

END-IF

END-FOR

described in Table I. First, the user who has the maximum Frobenius norm of H_k^H ($k \in T$) is selected among all the users. Since the effective channel is $H_k^H V_k$ ($k \in T$), we must consider not only the chordal distance between P_o and $H_{k,o}$ but also the Frobenius norm of H_k^H . The Frobenius norm of the effective channel is proportional to the chordal distance between P_o and $H_{k,o}$ and the Frobenius norm of H_k^H . Therefore the next selected user is the user who has the maximum value of $C_k^\alpha \times d_c(P_o, H_{k,o})$, where α is the importance factor between the chordal distance and the Frobenius norm. That is, when α is large, the Frobenius norm is more important than the chordal distance, and vice versa. α is dependent on N, K , which will be shown in Section VI.

IV. COMPUTATIONAL COMPLEXITY ANALYSIS

The complexity is counted as the number of flops, denoted as ψ . A real addition, a multiplication, and a division are counted as one flop. Therefore a complex addition needs two flops and a complex multiplication needs six flops [10].

A. Complexity of Typical Matrix Operations

For an $M \times N$ complex valued matrix H ,

□ Frobenius norm $\|H\|_F^2$ needs $2MN$ real multiplications and $2MN$ real additions. Therefore, $4MN$ flops are needed.

□ GSO needs $4N^2M - 2NM$ real multiplications, $4N^2M - 2NM$ real additions and $2MN$ real divisions.

Therefore total $8N^2M - 2NM$ flops are needed.

1) Optimal Scheduling Algorithm

$$\psi_{cs} \geq O\left(\left(\left\lfloor \frac{K_T}{M/N} \right\rfloor \left\lfloor \frac{M}{N} \right\rfloor M^3\right)\right) \quad (12)$$

2) Suboptimal Scheduling Algorithm Capacity Based Algorithm

$$\psi_c \approx O\left(K_T \left\lfloor \frac{M}{N} \right\rfloor^2 M^3\right) \quad (13)$$

Frobenius Norm Based Algorithm

$$\psi_n \approx O\left(K_T \left\lfloor \frac{M}{N} \right\rfloor^2 M^3\right) \quad (14)$$

3) Proposed Scheduling Algorithm

$$\begin{aligned} \psi_{ch} &\approx \sum_{i=2}^{\lfloor M/N \rfloor} \{ [8(i-1)^2 N^2 M - 2(i-1)NM + 7(i-1)NM^2] \\ &\quad + [8N^2 M - 2NM + 7NM^2 + 4M^2] \\ &\quad \times (K_T - i + 1) \} + 4K_T NM \\ &\approx O\left(K_T \left\lfloor \frac{M}{N} \right\rfloor^2 NM^2\right) \approx O\left(K_T \left\lfloor \frac{M}{N} \right\rfloor M^3\right) \end{aligned} \quad (15)$$

The following is the detailed explanation of (15).

□ $i = 1$: the Frobenius norm of K_T users needs $4K_T MN$ flops.
□ $i \geq 2$: Since $P_o P_o^H$ is unrelated to the total user set T , it is calculated only once. P_o needs $8(i-1)^2 N^2 M - 2(i-1)NM$ flops by GSO. Therefore $P_o P_o^H$ needs $8(i-1)^2 N^2 M - 2(i-1)NM + 7(i-1)NM^2$ flops. Since the size of matrix $H_{k,o}$ is independent of i , $H_{k,o} H_{k,o}^H$ needs $8N^2 M - 2NM + 7NM^2$ flops. Finally, the Frobenius norm of $H_{k,o} H_{k,o}^H$ needs $4M^2$. The proposed scheduling algorithm has lower complexity than the optimal algorithm and the two algorithms in [10].

V. SIMULATION RESULTS

In this section we compare the performance of the proposed scheduling algorithm with the optimal algorithm and the two suboptimal algorithms in [10]. Fig. 1 and Fig. 2 compare the sum capacities of various scheduling algorithms with respect to K_T and SNR with fixed M, N and K . It is shown that the proposed algorithm achieves around 95% of the sum capacity of the optimal algorithm. When $M = 4, N = 2$, the proposed algorithm has slightly lower sum capacity compared to the capacity based algorithm and outperforms the Frobenius norm based algorithm. When $M = 6, N = 2$ it has the sum capacity similar to the Frobenius norm based algorithm. The capacity

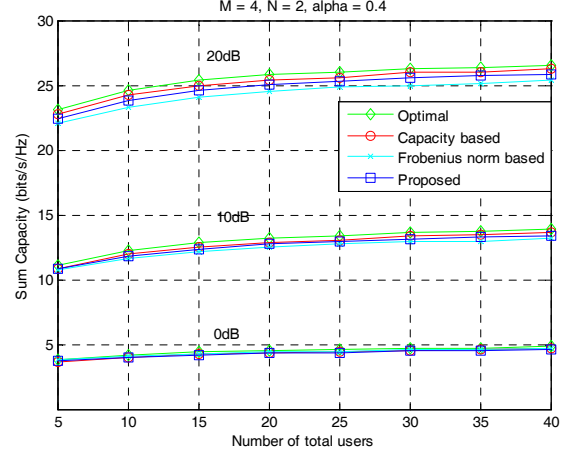


Fig. 1. Sum-capacity comparison of various scheduling algorithms with 4 transmit antennas at the BS and 2 receive antennas at each other

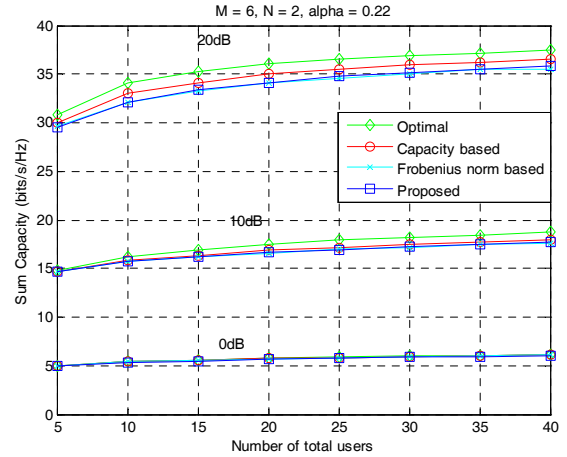


Fig. 2. Sum-capacity comparison of various scheduling algorithms with 6 transmit antennas at the BS and 2 receive antennas at each other

based algorithm performs slightly better than the proposed algorithm and the Frobenius norm based algorithm. That is because the Frobenius norm does not accurately represent the channel capacity.

Fig. 3 shows the average CPU (Pentium IV 3.20 GHz PC) run time of various scheduling algorithms with respect to total users with 6 transmit antennas at BS and 2 receive antennas at each receiver. Three low complexity scheduling algorithms have linear behavior with respect to the total number of users since the complexities of these are proportional to the total number of users. We can see that the proposed algorithm has smaller complexity than the other two algorithms because the capacity based algorithm computes SVD heavily, and the Frobenius norm based algorithm needs more GSO than the proposed algorithm.

Fig. 4 shows the average CPU run time of various scheduling algorithms with respect to the number of selected users (K) with 12 transmit antennas at BS and (6,4,3,2,1) receive

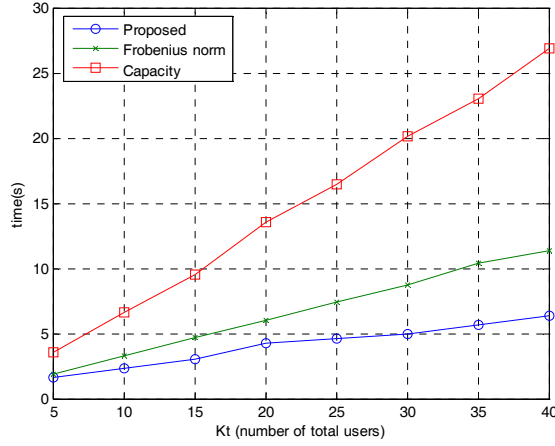


Fig. 3. Average run time comparison for various scheduling algorithms with 6 transmit antennas at BS and 2 receive antennas at each receiver

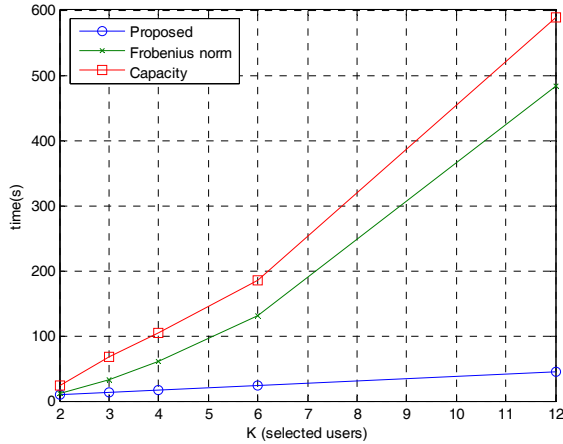


Fig. 4. Average run time comparison for various scheduling algorithm with 12 transmit antennas at BS and (6,4,3,2,1) receive antennas at each receiver

antennas at each receiver. Therefore possible K is (2,3,4,6,12). We can see that as K increases, the proposed algorithm has linear behavior while the other two algorithms have much higher complexity. That is because the complexity of the proposed algorithm is proportional to K while the complexity of the other two proposed algorithms is proportional to K^2 . Therefore the proposed algorithm has lower complexity especially in the system that can allocate resources to many users simultaneously.

Table II shows empirical α values that maximizes the sum capacity. We can see that α is dependent on the number of transmit antennas and the number of receiver antennas. As M gets larger, α gets smaller when N is 2. That is, when N is fixed α is inversely proportional to K since K is $\lfloor M/N \rfloor$. It is because, as K gets larger, more common subspaces are eliminated by that. When K is 2, α gets smaller as N gets larger. That is also because we get more common subspaces as

TABLE II
Empirically chosen α according to M, N

M \ N	2	3	4
4	0.4	•	•
6	0.22	0.28	•
12	0.07	0.1	0.15

N gets larger.

VI. CONCLUSIONS

In this paper, we proposed a new low complexity scheduling algorithms. Simulation results show that the proposed algorithm achieves around 95% of the sum capacity of the optimal search algorithm. It also has the sum capacity similar to the existing low complexity scheduling algorithms while its complexity is smaller. In particular, it has an advantage when the number of selected users is large. The importance factor α between the Frobenius norm and the chordal distance is inversely proportional to the number of receiver antennas and the number of selected users.

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