

Adaptive Mode Switching in the MIMO Broadcast Channel

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Abstract— This paper proposes an adaptive mode switching between single user and multi-user multiple input multiple output (MIMO) transmissions for Gaussian MIMO broadcast channels with partial channel state information at the transmitter. With full channel state information, it is known that multi-user transmission outperforms all other policies. With alternate channel state information available at the transmitter, it is not true that one transmission strategy always dominates the other. In particular, under some practical side information assumptions, there is instead a transition point - when the number of users is below this point, single user transmission outperforms multi-user transmission, and the reverse is true when the number of users exceeds this critical value. In this paper, we determine this mode switching point both analytically and numerically for a MIMO channel with transmit side correlation.

Index Terms— MIMO Broadcast Channels, Partial Channel State Information, Single User and Multi-user Transmissions

I. INTRODUCTION

Given the increasing importance of multiple input multiple output (MIMO) technology in emerging wireless standards, understanding the fundamental capacity limits on MIMO communication systems is invaluable. Researchers have been actively uncovering these limits, with recent work on the capacity of MIMO communication channels under perfect channel state information (CSI). These limits for both single and multi-user systems are well summarized in [1]–[5].

In the MIMO broadcast channel (BC), the methods utilizing multiple streams can be largely categorized as either single user or multi-user transmission. A single user transmission strategy implies that a transmitter opportunistically chooses a single user in the system and all data streams are allocated to that user, while a multi-user transmission strategy focuses on distributing data streams opportunistically across multiple users for its transmission.

With perfect CSI at the transmitter, multi-user transmission strategies outperform single user transmission strategies with time division multiple access (TDMA) from the point of view of sum rate [6]. Perfect CSI is, however, impractical thus single user transmission strategies can show better performance than multi-user transmission strategies under different situations. Specifically, under some partial

CSI assumptions, the throughput of single user transmission is higher than that of multi-user transmissions when the number of users is below a threshold [7] and simulation results were presented on the potential benefits of *mode switching* between single and multi-user MIMO transmissions in [8]. In [9], the authors derived switching points as a function of offered load and other parameters, and [10] showed some asymptotic analysis without spatial correlation.

In this paper, we propose an analytical framework for the mode switching point and investigate the impact of transmit side spatial correlation.

II. SYSTEM MODEL

Consider a K -user MIMO BC with partial CSI. For simplicity, we assume all users (receivers) have same number of antennas N_r and the transmitter has N_t antennas. Then the received signal at user k is expressed as ¹

$$\mathbf{y}_k = \sqrt{\frac{\rho}{N_t}} \mathbf{H}_k \mathbf{x} + \mathbf{n}_k, \quad k = 1, \dots, K, \quad (1)$$

where $\mathbf{y}_k$ is an $N_r \times 1$ receive vector, $\mathbf{x}$ is an $N_t \times 1$ transmit vector, $\mathbf{H}_k$ is an $N_r \times N_t$ channel matrix and $\mathbf{n}_k$ is an $N_r \times 1$ additive Gaussian noise vector. Without loss of generality, we assume that $\mathbf{n}_k$ has its covariance matrix as the identity matrix $\mathbf{I}$ and ρ is the received signal to noise ratio (SNR) and $\mathbb{E}[\mathbf{x}^H \mathbf{x}] = N_t$.

Our partial CSI is similar to that in [7]. The receivers know the channel state perfectly, while the transmitter requires feedback information from the receivers as partial CSI. This information available at the transmitter is detailed in Sections III-A and III-B, respectively.

We consider correlation at the transmit antennas while receive antennas are uncorrelated. This assumption is reasonable since the transmitter in a broadcast channel is usually placed in an environment with limited scattering,

¹Assume $\mathbf{A}$ denotes a complex matrix. Then $\mathbf{A}^T$, $\mathbf{A}^H$, and $\mathbf{A}^{-1}$ denote the transpose, Hermitian, and inverse of $\mathbf{A}$, respectively. We also assume that the lower case bold letter $\mathbf{a}$ and the upper case bold letter $\mathbf{A}$ mean vector and matrix, respectively. $\mathbb{E}$ and $[\mathbf{A}]_{i,j}$ denote expectation and the (i,j) th element of matrix $\mathbf{A}$. $\mathbf{I}_N$ denotes the $N \times N$ identity matrix and $\xrightarrow{P}$ indicates convergence in probability.

while the receivers are usually located in a rich scattering environment. Thus, we have the channel matrix

$$\mathbf{H}_k = \mathbf{H}_w \mathbf{R}_t^{\frac{1}{2}}, \quad (2)$$

where correlation $\mathbf{R}_t$ is identical for all users, $\mathbf{H}_w$ has its elements as i.i.d. unit variance zero-mean complex Gaussian random variables and $\mathbf{R}_t$ is the transmit correlation matrix with correlation factor $0 \leq \alpha < 1$, i.e., $[\mathbf{R}_t]_{i,j} = \alpha^{|i-j|}$. In this paper, we assume $\mathbf{R}_t$ is the same for all users for simplicity.

III. PERFORMANCE ANALYSIS

We assume that the received SNR is high enough to utilize spatial multiplexing effect. This high SNR assumption leads us to simple analytical expressions.

A. Single User MIMO Transmission

The achievable rate of user k for single user MIMO transmission with equal power allocation² is given by

$$R_k = \log_2 \left| \mathbf{I}_{N_r} + \frac{\rho}{N_t} \mathbf{H}_w \mathbf{R}_t \mathbf{H}_w^H \right|. \quad (3)$$

The transmitter obtains K achievable rates as partial CSI and serves the user with maximum achievable rate in each time slot. Thus, the maximum rate R_{SU} of single user transmission is

$$R_{SU} = \max_{k=1,\dots,K} R_k. \quad (4)$$

Since $\mathbf{R}_t$ is full rank for $0 \leq \alpha < 1$, the rate of single user transmission (3) with high SNR assumption can be rewritten as

$$R_k \approx \min(N_r, N_t) \log_2 \frac{\rho}{N_t} + \log_2 |\mathbf{H}_w \mathbf{H}_w^H| + \log_2 |\mathbf{R}_t|.$$

In [13], it was shown that according to the number of antennas, R_k converges to the following Gaussian random variable³

$$R_k \sim \mathcal{N}(\mu, \sigma^2) \quad (5)$$

where

$$\begin{aligned} \mu &= n \log_2 \frac{\rho}{N_t} + n \log_2 e \left(\sum_{i=1}^{N-n} \frac{1}{i} - 0.5772 \right) \\ &\quad + \log_2 |\mathbf{R}_t| + \log_2 e \sum_{i=1}^{n-1} \frac{i}{N-i}, \\ \sigma^2 &= (\log_2 e)^2 \left(\sum_{i=1}^{n-1} \frac{i}{(N-n+i)^2} + \frac{n\pi^2}{6} - \sum_{i=1}^{N-1} \frac{n}{i^2} \right), \end{aligned}$$

and $n = \min(N_r, N_t)$, $N = \max(N_r, N_t)$.

²This may not be always the optimal choice, but the loss in system throughput by equal power allocation compared with optimal power allocation is small if SNR (ρ) is high [11], [12].

³This convergence to a Gaussian random variable is reasonably rapid enough to make this approximation valuable at practical situations.

To acquire asymptotic analysis for R_{SU} , we introduce the following lemma.

Lemma 1: Let R_k , R_{SU} be expressed in (4) and (5). Then as $K \rightarrow \infty$,

$$\frac{R_{SU}}{\sqrt{2\sigma^2 \ln K}} \xrightarrow{p} 1.$$

Proof: See [14, p. 83] or [15, p. 71]. ■

Lemma 1 says that as K increases

$$R_{SU} \xrightarrow{p} C(\mu, \sigma^2, K) + \sqrt{2\sigma^2 \ln K}$$

and $C(\mu, \sigma^2, K)$ behaves as $o(\sqrt{\ln K})$.⁴ Therefore as K increases,

$$C_{SU} = \mathbb{E}[R_{SU}] \rightarrow C(\mu, \sigma^2, K) + \sqrt{2\sigma^2 \ln K}. \quad (6)$$

B. Multi-user MIMO Transmission

With ZF receivers, the post-processing signal to noise ratio (SNR) of i th stream to user k is given by

$$S_{i,k} = \frac{\rho}{N_t} \frac{1}{\left[(\mathbf{H}_k^H \mathbf{H}_k)^{-1} \right]_{i,i}}, \quad (7)$$

where $i = 1, \dots, N_t$ and $k = 1, \dots, K$. The transmitter obtains KN_t post-processing SNRs from all users as partial CSI. Then the maximum achievable rate of the i th stream amongst K users is

$$R_{MU,i} = \max_{k=1,\dots,K} \log_2(1 + S_{i,k}), \quad (8)$$

thus maximum sum rate of multi-user transmission is the summation of the rates for all streams, i.e.,

$$R_{MU} = \sum_{i=1}^{N_t} R_{MU,i}. \quad (9)$$

In [17], it was shown that post-processing SNR of ZF receiver behaves as a weighted chi-squared distributed random variable with degree of freedom $2(N_r - N_t + 1)$. In equation,

$$S_{i,k} = \frac{\rho}{N_t} \frac{1}{\left[(\mathbf{H}_k^H \mathbf{H}_k)^{-1} \right]_{i,i}} = \frac{\rho}{N_t} V_{i,k}, \quad (10)$$

where the p.d.f. $f_{V_{i,k}}(v)$ of $V_{i,k}$ is given by [17]

$$\begin{aligned} f_{V_{i,k}}(v) &= \frac{\lambda_i \exp(-v\lambda_i) (v\lambda_i)^{N_r - N_t}}{(N_r - N_t)!}, \\ \lambda_i &= \left[\mathbf{R}_t^{-1} \right]_{i,i}. \end{aligned} \quad (11)$$

⁴ $C(\cdot)$ is expressed as a function of $\mu - \frac{a[\ln \ln(\sigma^2 K) + b]}{\sqrt{2 \ln(\sigma^2 K)}}$ where a and b are constant [15], [16].

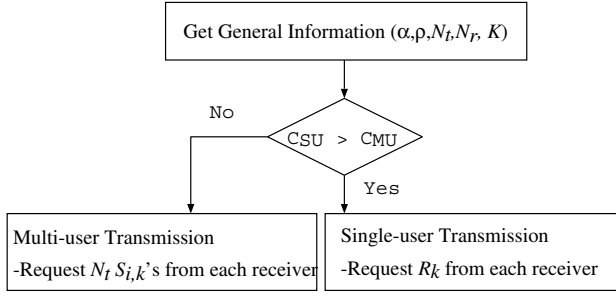


Fig. 1. Adaptive mode switching algorithm at the transmitter

By combining (8) and (10), we have

$$\begin{aligned}
 R_{MU,i} &= \max_{k=1,\dots,K} \log_2 \left(1 + \frac{\rho}{N_t} V_{i,k} \right) \\
 &= \log_2 \left(1 + \frac{\rho}{N_t} \max_{k=1,\dots,K} V_{i,k} \right) \\
 &= \log_2 \left(1 + \frac{\rho}{N_t} M_i \right), \quad (12)
 \end{aligned}$$

where $M_i = \max_k V_{i,k}$. Note that $\max$ moves into $\log_2$ since $\log_2$ is a non-decreasing function. Now let us see the relationship between M_i and $V_{i,k}$ in the following lemma.

Lemma 2: Let M_i and $V_{i,k}$ be expressed in (10) and (12). Then as $K \rightarrow \infty$,

$$\frac{M_i}{\ln K / \lambda_i} \xrightarrow{p} 1.$$

Proof: The c.d.f. of a weighted chi-squared distributed random variable is a Von Mises function [15]. Then there exist a_K and b_K satisfying that as K increases

$$\mathbb{P} \left(\frac{M_i - b_K}{a_K} \leq x \right) \rightarrow \exp(-e^{-x}).$$

Specifically, it is shown that $a_K \approx \frac{1}{\lambda_i}$ and $b_K \approx \frac{\ln K}{\lambda_i}$ [16]. Then for any fixed $\epsilon > 0$, as K increases

$$\begin{aligned}
 &\mathbb{P} \left(\left| \frac{M_i}{b_K} - 1 \right| > \epsilon \right) \\
 &= \mathbb{P}(M_i - b_K > \epsilon b_K) + \mathbb{P}(M_i - b_K < -\epsilon b_K) \\
 &= 1 - \exp(-e^{-\epsilon b_K / a_K}) + \exp(-e^{\epsilon b_K / a_K}) \\
 &\approx 1 - \exp(-e^{-\epsilon \ln K}) + \exp(-e^{\epsilon \ln K}) \\
 &\rightarrow 0.
 \end{aligned}$$

This means that M_i/b_K is with ϵ distance with probability 1 as $K \rightarrow \infty$, i.e.,

$$\frac{M_i}{b_K} \approx \frac{M_i}{\ln K / \lambda_i} \xrightarrow{p} 1.$$

Lemma 2 says that as K increases

$$M_i \xrightarrow{p} C_i(\lambda_i, N_t, N_r, K) + \frac{\ln K}{\lambda_i} \quad (13)$$

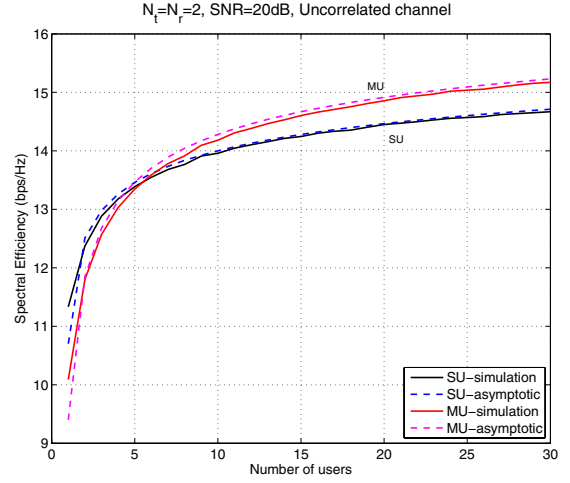


Fig. 2. Ergodic rates for single user and multi-user transmissions

and $C_i(\lambda_i, N_t, N_r, K)$ behaves as $o(\ln K)$.⁵ From (9), the ergodic sum rate of MIMO BC with multi-user transmission is written as

$$\begin{aligned}
 C_{MU} &= \mathbb{E}[R_{MU}] = \sum_{i=1}^{N_t} \mathbb{E}[R_{MU,i}] \quad (14) \\
 &= \sum_{i=1}^{N_t} \mathbb{E} \left[\log_2 \left(1 + \max_{k=1,\dots,K} \frac{\rho}{N_t} V_{i,k} \right) \right] \\
 &= \sum_{i=1}^{N_t} \mathbb{E} \left[\log_2 \left(1 + \frac{\rho}{N_t} M_i \right) \right].
 \end{aligned}$$

Therefore as K increases

$$C_{MU} \rightarrow \sum_{i=1}^{N_t} \log_2 \left(1 + \frac{\rho}{N_t} \left(C_i(\cdot) + \frac{\ln K}{\lambda_i} \right) \right). \quad (15)$$

C. Adaptive Mode Switching Point

Based on the expressions (6) and (15), we determine the mode switching point by the following inequality

$$C_{SU}(\alpha, N_t, N_r, K) \geq C_{MU}(\alpha, N_t, N_r, K),$$

where $\alpha, (N_t, N_r)$ and K are correlation factor, the number of antennas and the number of users, respectively. The detailed algorithm is illustrated in Fig. 1.

IV. SIMULATION RESULTS

In Fig. 2, we show the achievable sum rates of single and multi-user transmissions and determine the mode switching point between the two strategies. When there is only one user, single user transmission has the greater (sum) rate than multi-user transmission as the former

⁵ $C_i(\cdot)$ is expressed as a function of $\frac{c_i(N_r - N_t + 1)}{\lambda_i} \ln \ln K$ where c_i is a constant [15], [16].

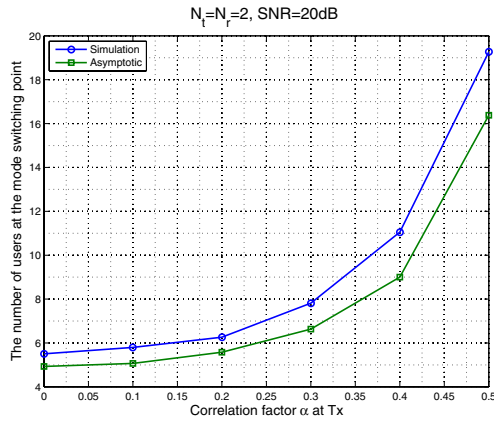


Fig. 3. The relationship between the number of users at the mode switching point and the correlation factor α

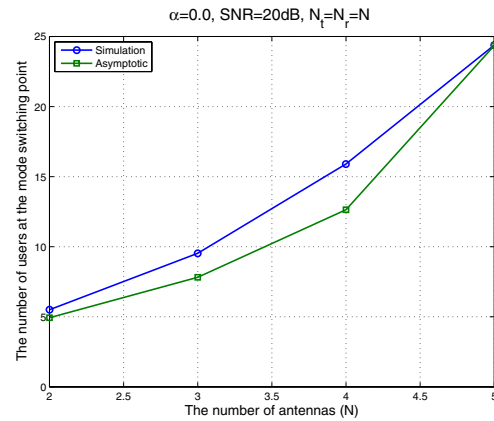


Fig. 4. The relationship between the number of users at the mode switching point and the number of antennas

uses a more sophisticated receiver. The growth rate of multi-user transmission, however, is larger than that of single user transmission owing to multi-user diversity [7]. Figs. 3 and 4 illustrate the number of users at the mode switching point according to two different parameters: the transmit side correlation factor and the number of antennas (assuming $N_t = N_r = N$). From these results, we confirm that our analytic expressions match simulation results fairly closely. In addition, we are able to know which transmission mode is better from these results. For example, multi-user transmission has higher throughput than single user transmission in upper regions over curves and vice versa.

V. CONCLUSION

In this paper, we investigated two transmission strategies termed single user transmission and multi-user transmission in the MIMO broadcast channels with partial CSI at the transmitter. In a multi-user MIMO transmission, more than one user is served at a time where ZF receiver is used at the user. The single user MIMO strategy transmits to one user, while utilizing a receiver that incorporates inter-stream interference mitigation by successive interference cancellation. Thus, there is an inherent tradeoff between interference loss suffered by multi-user MIMO with the performance gains obtained by serving multiple users at once. This translates into a mode switching making single user transmissions better in some regimes and multi-user ones better in others. The key contribution of this paper is an analytical framework for determining this mode switching point, and to study the impact of correlation and number of antennas on this switching point.

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