



On chaotic advection in a static mixer

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ABSTRACT

We present a numerical study of chaotic advection and mixing in a spatially periodic, three-dimensional flow. The flow inside a cylindrical tube, into which one or more Kenics KM static mixer elements are inserted to promote mixing, is studied by solving the momentum equations numerically and then applying tools for analysis of mixing in time-periodic two-dimensional and three-dimensional chaotic flows. It is found that for this particular static mixer geometry, the initial location of a blob of dye is important for its spreading over the cross section of the tube only in the first few mixing elements, its influence decreases afterwards. The influence of the Reynolds number on mixing is mainly a finer grain of the resulting mixture, this is visible after several elements. A symmetry breaking modification of the standard Kenics KM static mixer geometry is suggested and subject to the same analysis as the original geometry. A slight improvement in the mixing efficiency is obtained with the modified geometry: it increases the extensional efficiency and gives a more homogeneous distribution of the stretching rates.

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1. Introduction

Since the last part of the 19th century, it is well known that fluid flows in a horizontal tube either in a laminar or in a turbulent regime. This experiment, performed by Osborne Reynolds in 1883 [1], is sometimes associated with the beginning of modern fluid mechanics. A review of the papers of Osborne Reynolds has appeared recently [2]. In laminar flow, particle trajectories are parallel to the tube axis, the pressure drop per unit length is small, compared to turbulent flow, but there is no mixing on a given cross section. During turbulent flow, intense mixing is present in a given cross section perpendicular to the tube axis, but the pressure drop per unit length is much higher.

Engineers have developed tools to obtain mixing on a cross section under laminar flow conditions. One such tool is the use of static mixers, which are inserts put inside the tube to cut, fold, twist, and re-combine fluid elements. Static mixers are much used in the chemical industry to enhance mixing, to increase the heat transfer rate from the wall to the fluid, and to promote mass transfer in gas–liquid or in liquid–liquid dispersions. Many different shapes of static mixers are available; the choice of a given shape depends on the process and on the induced pressure drop increase.

For certain static mixer geometries and under laminar flow conditions, a phenomenon known as chaotic advection can occur. This

phenomenon can be simply described as chaotic trajectories, as in turbulent flow, in a flow dominated by viscous forces. Chaotic advection can occur in two-dimensional periodic flows, such as the flow between two eccentric cylinders rotating at time-periodic velocities [3–6], and in three-dimensional flows [7–10]; it is the latter type of flow that interest us here.

Over the last 15 years and with the development of Computational Fluid Dynamics (CFD) and faster computers, it has become possible to solve numerically the three-dimensional flow through a static mixer. The complex static mixer geometry can be modeled using an appropriate grid generation software and the conservation equations are solved using a control volume or finite element methods [11–16]. Many of these studies have involved the Kenics static mixer, which is widely used in the chemical industry. CFD has allowed researchers to simulate the complex flow induced by static mixers and to modify the geometry of these elements in order to obtain better performance.

In this paper we first solve numerically the continuity and momentum equations in the complex geometry of the Kenics static mixer; the numerical code is validated by comparing pressure drops with available literature results. We then use tools developed for the understanding of time-periodic, three-dimensional flows [17–20] to study both the Kenics static mixer and a symmetry breaking modified version. The influence of the initial tracer location, stretching calculations, and other tools are used to highlight the efficiency of these static mixers and to compare the performance of the modified geometry with respect to the standard one.

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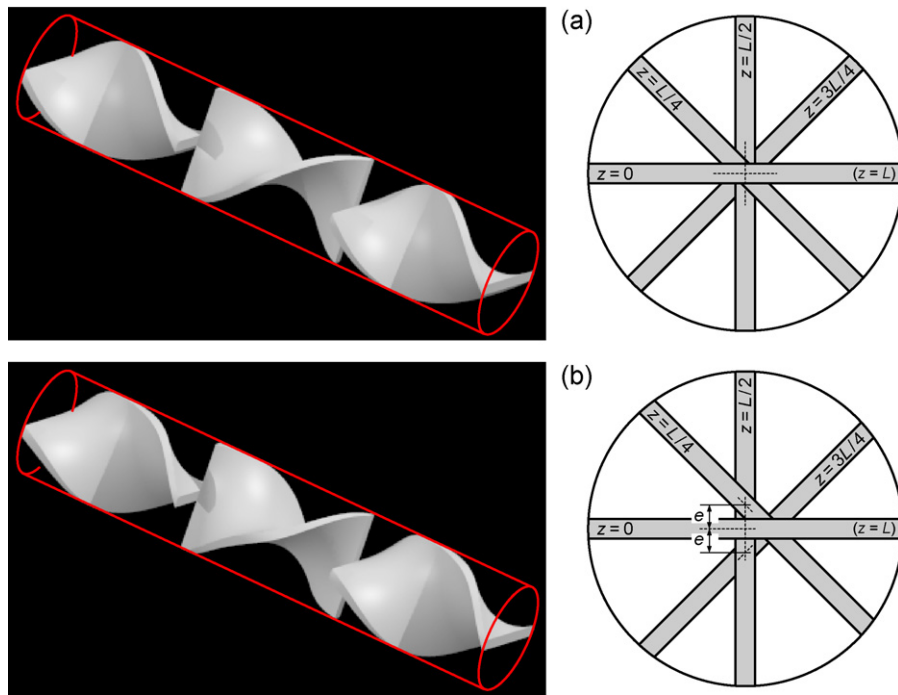


Fig. 1. (a) Standard helical twisted tape inserts of the Kenics static mixer and (b) internal elements obtained by deforming the plates with an eccentricity $\epsilon = 2e/D = 0.15$. In both cases, $L/D = 1.5$, the twist angle per element is 180° , and the elements abut at 90° . The right-hand schematics show cross-sectional cuts of the twisted plate at various axial positions along the first element.

2. Problem formulation and numerical solution

The geometry considered in the present study is based on the Kenics KM static mixer manufactured by Chemineer (Dayton, OH). The Kenics KM mixer has helical twisted tape inserts; these are illustrated in the schematics of Fig. 1a. The tapes are twisted through 180° and abut one another at a 90° angle, providing successive right- and left-hand rotation to the fluid elements. The alternating helical elements of the KM Static Mixer continually divide, stretch, and reorient the flow stream to produce complete mixing with low pressure drop.

The main characteristics of the mixer considered in the present study are given in Table 1. The length-to-diameter ratio (L/D) of each twisted tape element is 1.5, which is the value for most commercial Kenics mixers. The plate thickness is 0.3175 cm, which corresponds to a thickness-to-diameter ratio of 1/16. The mixer consists of six elements tightly inserted into a pipe with an entrance and exit zones of empty tube; the length of each of these zones is equivalent to two pipe diameters. Thus, the overall length of the pipe is $2 \times (2D) + 6 \times (1.5D) = 13D$.

The Kenics KM static mixer has a symmetry with respect to an axis defined by the helical element. To break the symmetry, we devised a hypothetical static mixer, just like the Kenics KM mixer,

but in which each plate is twisted around a virtual axis defined by the equation

$$y = e \sin \left(\frac{2\pi z}{L} \right), \quad (1)$$

where z is the axial coordinate along the pipe, y is an axial coordinate perpendicular to the leading edge of the plate, L is the length of the element, and e is the amplitude of the modulation. The eccentricity ϵ can be defined as follows:

$$\epsilon = \frac{2e}{D}, \quad (2)$$

where D is the diameter of the static mixer. For the standard KM mixer, $\epsilon = 0$. The elements of the eccentric static mixer for $\epsilon = 0.15$

Table 1
Main characteristics of the static mixer configuration.

<i>Twisted plate</i>	
Diameter, D	5.08 cm
Length, L	7.62 cm
Thickness, w	0.3175 cm
Twist angle, θ	180°
<i>Pipe</i>	
Entrance zone	$2D$
Mixing zone (6 elements)	$6L$
Exit zone	$2D$

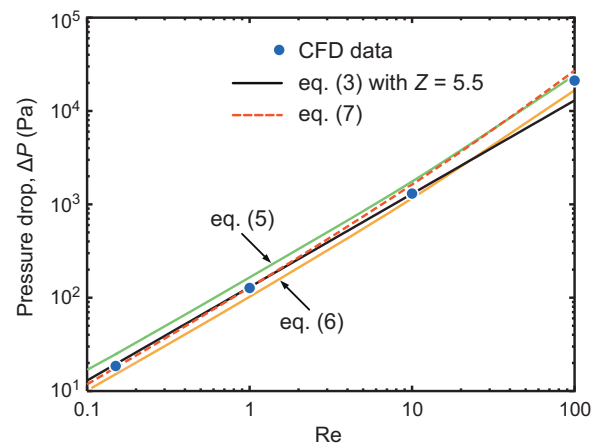


Fig. 2. Pressure drop across six elements of the standard Kenics KM mixer as a function of Reynolds number, Re . The filled circles represent our simulation values, black solid line is given by Eq. (3) with $Z = 5.5$, and the other lines represent the experimental correlations given in the text.

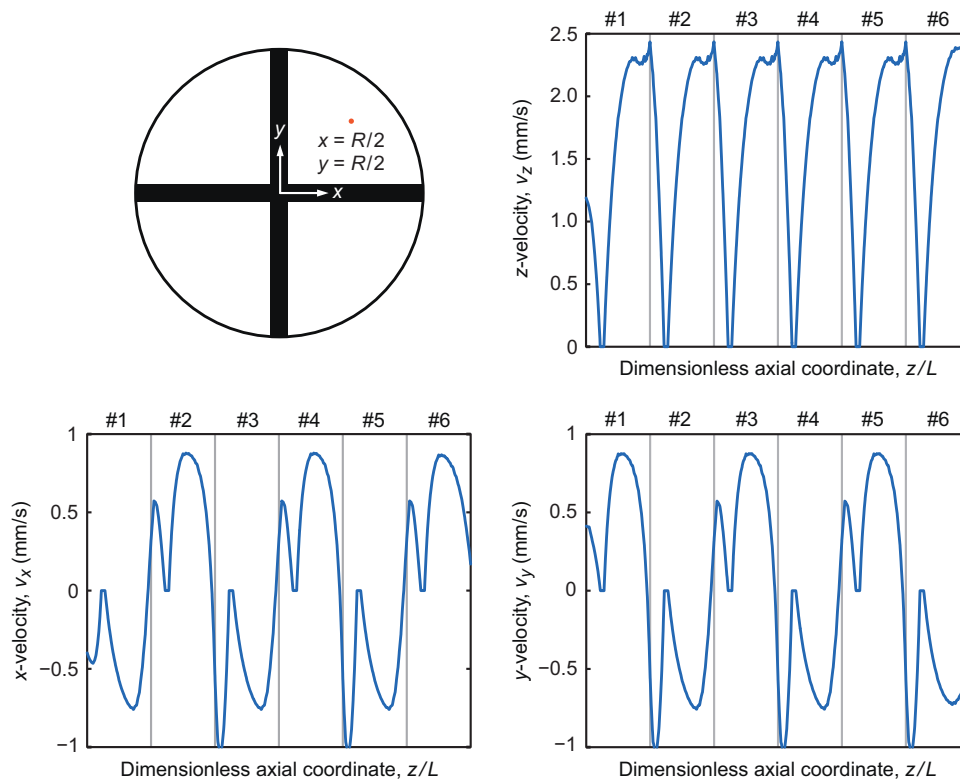


Fig. 3. Axial profiles along the mixer of the three components of the velocity vector (v_x , v_y , and v_z) for $x = y = R/2$ (see top-left schematic of mixer's cross section).

are illustrated in the schematics of Fig. 1b. The standard Kenics mixer and its modified version are analyzed in this paper.

The computational domain of the static mixer was meshed using a structured hexahedral grid. First, a 2D mesh with quadrilateral cells was generated for the cross section of the mixer at the intersection between two adjacent helical elements; this 2D mesh is symmetrical about the horizontal and vertical axes that divide the cross section into four equal quarter-circles. Then, a 3D mesh for one element of the mixer was generated by twisting and extruding the 2D mesh along the length of the element in the same way a plate is twisted to form the helix of a Kenics element. Finally, this 3D mesh was replicated n times (n being the number of mixing elements) along the axial length of the mixer with the blocks arranged alternatively in the pipe with co- and counter-clockwise rotations of 90° . The finite thickness of the twisted plates was explicitly taken into account in the meshing procedure to differentiate between the fluid and solid domains. The structured hexahedral grid thus generated has several advantages over the unstructured tetrahedral grids typically generated automatically by preprocessors for geometry modeling and mesh generation: our grid is aligned with the axial flow, which minimizes numerical diffusion in that direction, the cell faces parallel to the axial flow are less skewed, and structured hexahedral cells typically require less memory by the solver, which speeds up the convergence of the numerical simulations. At the end, the grid quality was checked for the cell aspect ratio and skewness. The aspect ratio was kept below 3 and the skewness below 0.85.

The continuity and momentum equations were solved numerically using OpenFOAM®, an open source CFD toolbox [21]. This code uses a control-volume formulation to solve the primitive Navier–Stokes equations. A second-order upwind scheme was used for the discretization of the convective terms. The pressure field was corrected at each iteration step using the SIMPLE algorithm. The inflow velocity at the upstream end of the entrance zone was defined by specifying an inlet velocity profile that was set equal to the parabolic profile for fully developed laminar flow in a

circular tube. The outflow boundary condition at the downstream end of the exit zone of empty tube was imposed by specifying a static (gauge) pressure at the outlet boundary.

To generate grid-independent results, the static mixer was meshed with different grid resolutions. These preliminary tests allowed us to determine the appropriate node distribution and cell density in terms of computational load and numerical accuracy. The grid resolution was increased until the pressure drop values reported below stabilized within a minimum deviation of less than 1% from the nearest mesh. From these results, it was concluded that the structured mesh with 8×10^5 cells was adequate for the Kenics mixer with six elements.

In order to validate the accuracy of our numerical code, the pressure drop calculated in the Kenics static mixer is compared

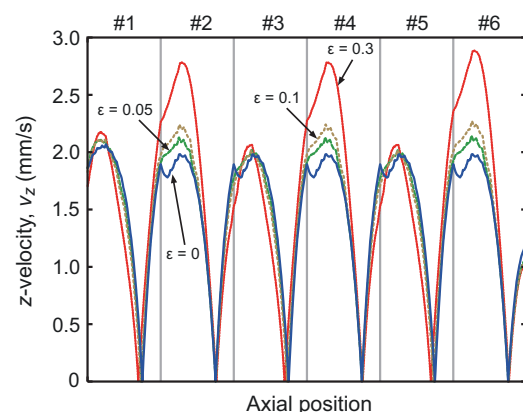


Fig. 4. Axial profile of the z -component of the velocity vector along a straight line with cross-sectional coordinates $x = y = R/2$ (see top-left schematic of mixer's cross section in Fig. 3). The velocity profile for the standard Kenics geometry ($\epsilon = 0$) is compared to those of asymmetric geometries with $\epsilon = 0.05$, 0.1 , and 0.3 .

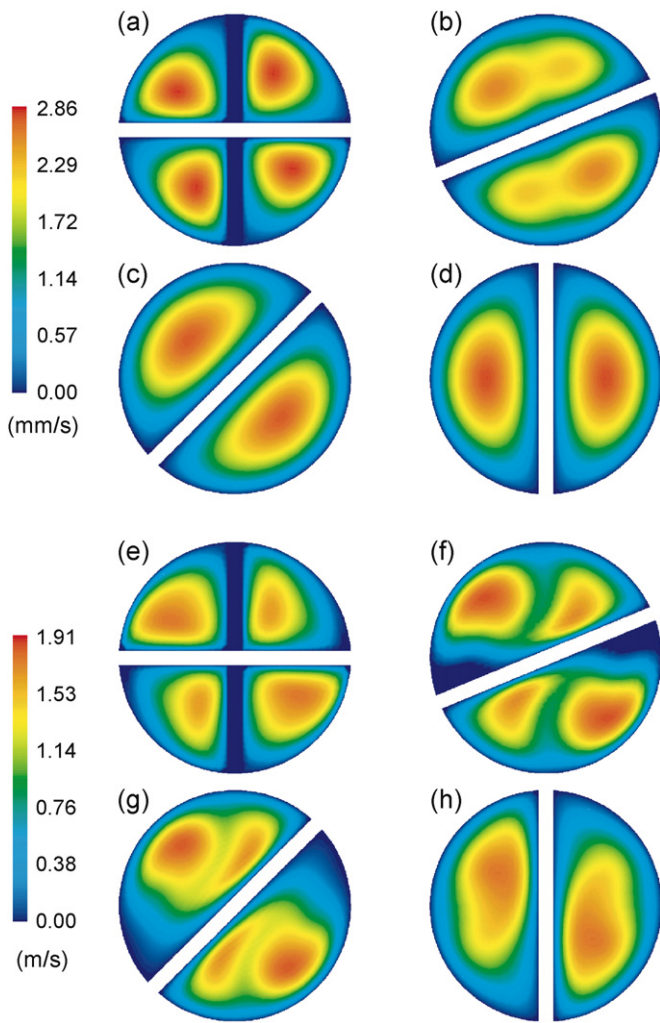


Fig. 5. Contour fields of the axial velocity component, $v_z(x, y)$, at different cross sections of the third element of the Kenics mixer for $Re = 0.15$ (a–d) and $Re = 100$ (e–h). The cross sections are located at the following positions relative to the leading edge of the element: (a, e) $z = 0$; (b, f) $z = L/8$; (c, g) $z = L/4$; (d, h) $z = L/2$.

to known experimental correlations. Several empirical correlations for the pressure drop in Kenics static mixers have appeared in the literature. Most of these empirical correlations are expressed as a function of a Z-factor, dependent on Reynolds number, defined as

$$Z = \frac{(\Delta P)_{mx}}{(\Delta P)_{ep}}, \quad (3)$$

where $(\Delta P)_{mx}$ is the pressure drop along the pipe with the static mixer and $(\Delta P)_{ep}$ is the pressure drop in the empty pipe, i.e., the same pipe but without the static mixing elements. For a circular, horizontal pipe and under laminar flow conditions, the pressure drop per unit length without a static mixer, is obtained by solving the Stokes equations; it leads to

$$\frac{(\Delta P)_{ep}}{L} = \frac{32}{Re} \frac{\rho V^2}{D}, \quad (4)$$

where L is the length of tube over which the pressure drop is measured, $Re = \rho V D / \mu$ is the Reynolds number based on the empty pipe (ρ and μ are the density and viscosity of the fluid, respectively, and V is the average axial velocity) and D is the inner diameter of the pipe.

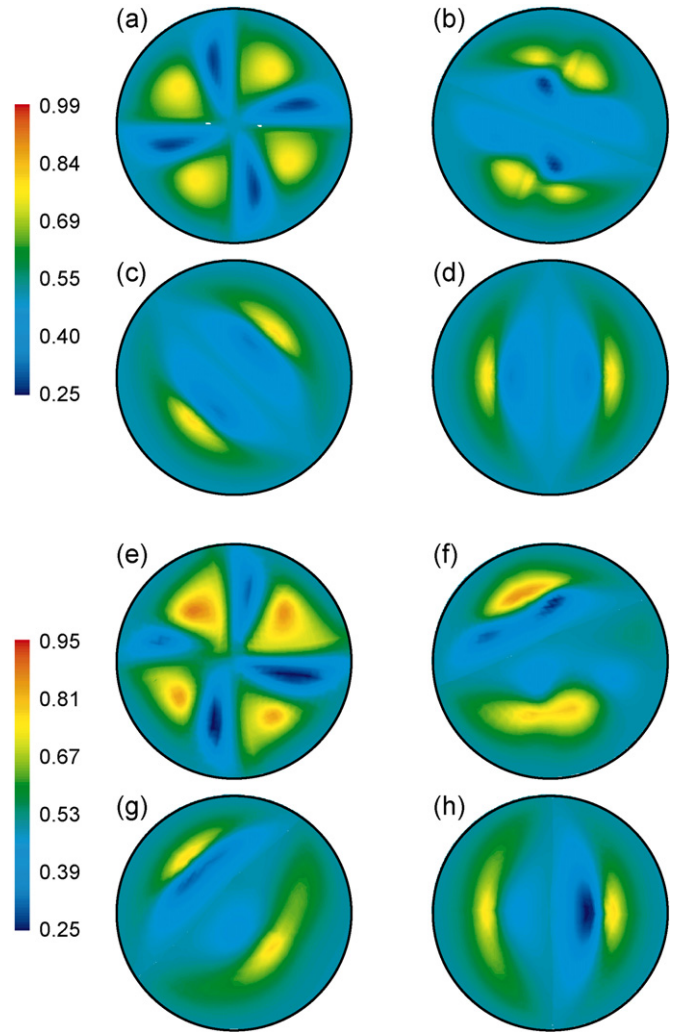


Fig. 6. Contour fields of extensional efficiency, α , for $Re = 0.15$, at different cross sections of the third element of the Kenics mixer (a–d) and of the modified geometry for $\epsilon = 0.3$ (e–h): (a, e) $z = 0$; (b, f) $z = L/8$; (c, g) $z = L/4$; (d, h) $z = L/2$.

One pressure drop correlation for the Kenics mixer was proposed by Wilkinson and Cliff [22]; it can be written as

$$Z = 7.19 + \frac{Re}{32}. \quad (5)$$

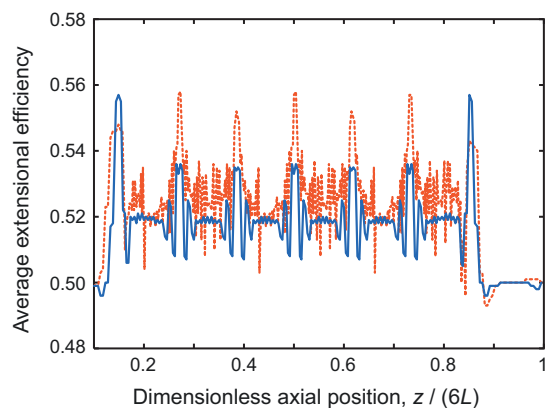


Fig. 7. Cross-sectional average extensional efficiency plotted against the axial distance, z/L , for the Kenics mixer (—) and for the modified geometry with $\epsilon = 0.3$ (·····); the Reynolds number is $Re = 0.15$.

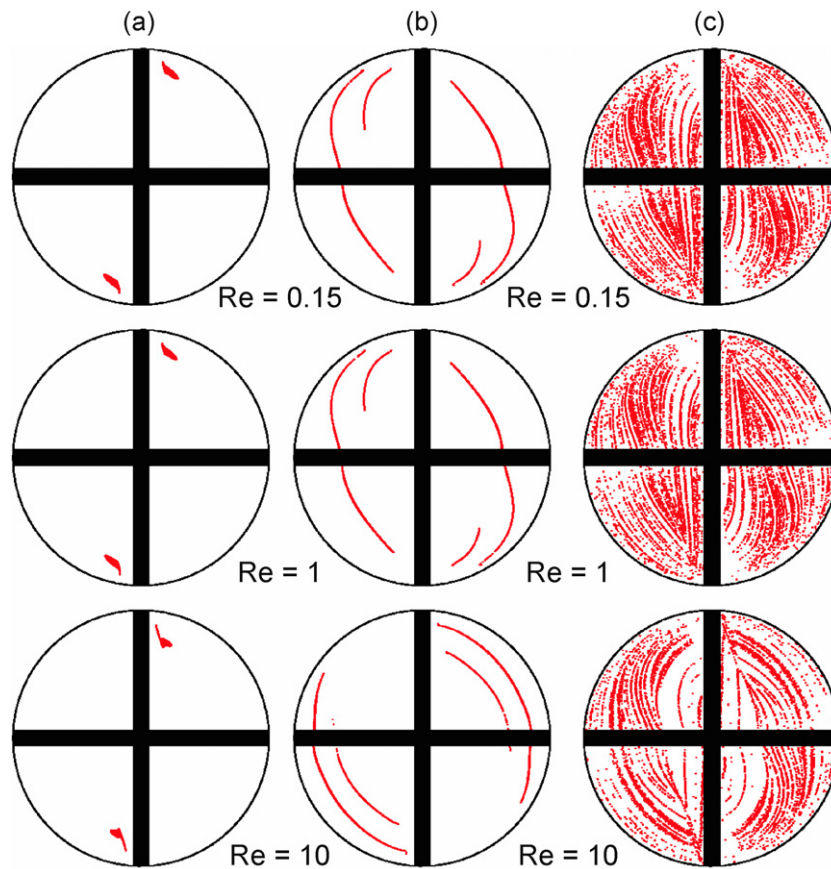


Fig. 8. Numerical experiments of passive tracer tracking for a 1 vol% injection at the central axial axis of the Kenics mixer ($x=y=0$, $z=-2D$) for different Reynolds numbers. The particle locations are recorded at (a) $z=2L$ (trailing edge of the second element), (b) $z=6L$ (trailing edge of the sixth element), and (c) $z=12L$ (trailing edge of the twelfth element). The injected blob of dye consists of over 10^4 material points.

A second correlation available in the literature is that of Shah and Kale [23], which for a Newtonian fluid can be written as

$$Z = \frac{1}{16\epsilon} \left[64.06 + 1.84 \left(\frac{Re}{\epsilon} \right)^{0.66} \right], \quad (6)$$

where ϵ is the void fraction of the static mixer; for our static mixer $\epsilon = 0.92$. Yet another well known correlation is that given by Grace [24]:

$$Z = 3.24(1.5 + 0.21\sqrt{Re}). \quad (7)$$

Fig. 2 shows the calculated pressure drop across the six Kenics KM static elements as a function of the Reynolds number; Re values up to 100 are considered. Our numerical results (blue circles) are compared to the three correlations given above and to Eq. (3) with $Z = 5.5$ (black, solid line), which was fitted to the simulation data for the smallest Reynolds number considered, $Re = 0.15$. The constant Z -factor of 5.5 applies well for $Re < 10$, but for $Re = 100$ the pressure drop given by our numerical code is better predicted by Eq. (7). Our numerical results are tightly bounded between the estimates given by Eqs. (5) and (6). Overall, the agreement between our numerical results and literature data is very good. These comparisons allow us to have confidence in the results furnished by the numerical code.

3. Results and discussion

The solution of the equations of motion in a tube composed of six standard Kenics KM static elements gives the Eulerian velocity field for the volume of fluid flowing through the mixer. Fig. 3 shows the three components of the velocity field (v_x , v_y , and v_z), as a function

of axial distance, along a straight line whose cross-sectional coordinates are $x=y=R/2$ (the cross-sectional location of the line under study is shown in the top-left schematic); the boundary between each mixing element is identified by a vertical, gray line. Note that both the entrance ($z < 0$) and exit ($z > 6L$) zones of empty tube are excluded from the horizontal axis of the graphics in Fig. 3.

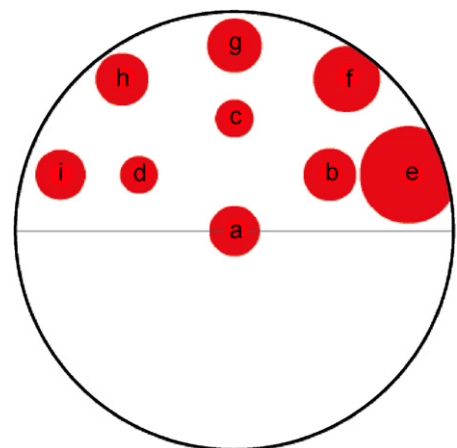


Fig. 9. Initial conditions for Lagrangian particle tracking of 1 vol% passive tracer injections in a modified Kenics mixer geometry with $\epsilon = 0.05$. In each case the injector is located 0.1 cm upstream of the leading edge of the first static mixing element. The cross-sectional coordinates, x and y , of the centers of the disks are (a) 0, 0 (b) 0.44, 0.26, (c) 0, 0.51, (d) -0.43, 0.26, (e) 0.80, 0.26, (f) 0.52, 0.69, (g) 0, 0.85, (h) -0.51, 0.69 and (i) -0.79, 0.26.

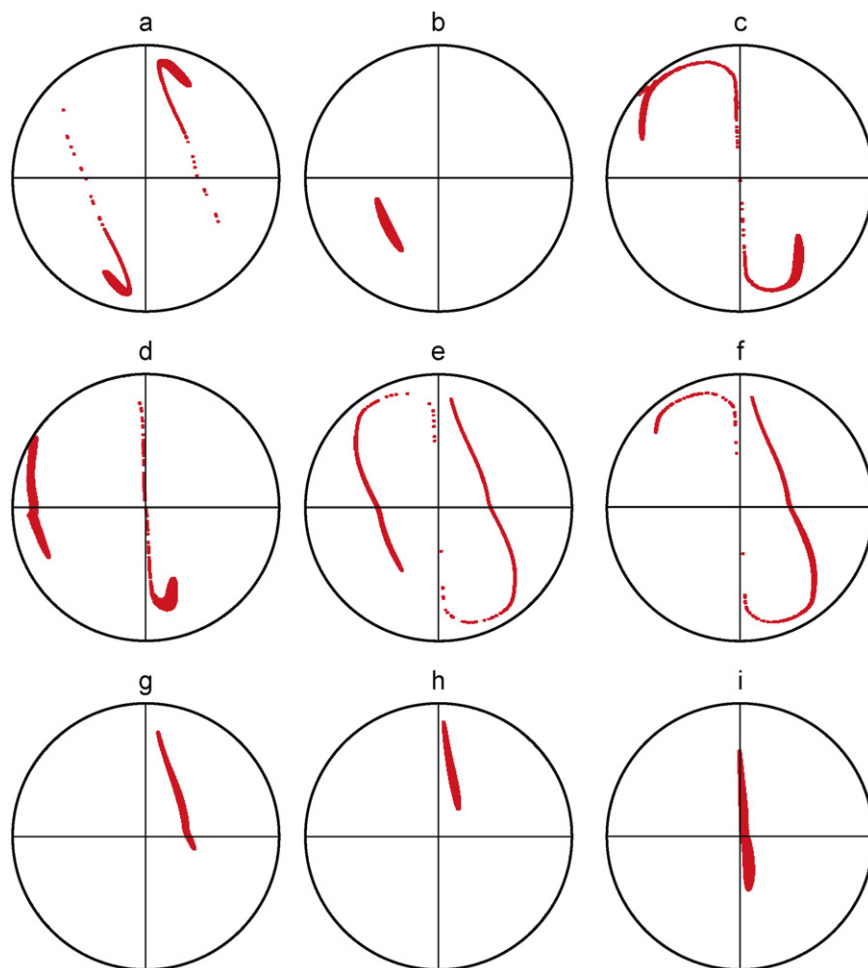


Fig. 10. Positions of blob of dye after two modified Kenics elements ($\epsilon = 0.05$) for $Re = 0.15$. The 1 vol% injection locations are given in Fig. 9.

This flow is spatially periodic, the flow field repeats itself every two Kenics static mixer elements. Notice that both of the cross sectional velocity components, v_x and v_y , can be either positive or negative. This is because the helical mixing element directs the flow of material radially toward the pipe walls and back to the center. Additional velocity reversal and flow division results from the combination of the alternating right- and left-hand elements.

We have investigated if the modification of the standard geometry discussed earlier in the paper can increase even further this ability. At this point it is worth recalling Eqs. (1) and (2), which define the distortion of the helical element and introduce the eccentricity, ϵ , as a measure of that distortion. Because of the sinusoidal distortion of the helical tape, the flow is divided into two uneven zones, one with a larger cross section than the other. This results in an acceleration of the flow in one zone and deceleration in the other zone, thus creating a local de-synchronization of the two fluid zones because of the different average residence times of a fluid particle in the two zones of the element. However, the combination of the alternating right- and left-hand elements periodically balances the volumes of fluid on opposite sides of the tape inserts.

To illustrate what has just been discussed, we have performed the same calculations as the ones given above for eccentricities of 0.05, 0.1, and 0.3. For the same Reynolds number ($Re = 0.15$) and for the same inlet cross-sectional position ($x = y = R/2$), the z -component of the velocity vector is plotted as a function of axial distance in Fig. 4 for the Kenics static mixer and for the modified geometries. In these calculations the tape inserts were rendered infinitely thin in order to remove from the results the influence of

the tape thickness (w) for a case in which the ratio w/D is non-negligible. Fig. 4 shows that, in all even numbered elements, the axial velocity for the most eccentric case, $\epsilon = 0.3$, is much higher than in the standard Kenics geometry. For small eccentricities, $\epsilon = 0.05$ and 0.1, the velocity components are practically identical to those of the unmodified Kenics static mixer.

Fig. 5 shows the influence of the Reynolds number on the axial velocity, under the form of color plots of cross-sectional contours of constant axial velocity at four different axial coordinate locations on the third mixing element, namely, $z = 0, L/8, L/4$ and $L/2$, where L is the length of the element. The top 4 contour plots (Fig. 5a–d) correspond to $Re = 0.15$, whereas the bottom ones (Fig. 5e–h) correspond to $Re = 100$; note that the color scale for Fig. 5a–d (mm/s) is not the same as for Fig. 5e–h (m/s). The contour fields of axial velocity for the modified Kenics static mixers are qualitatively similar to those plotted for the conventional geometry.

The extensional efficiency, α , is a quantity that has been used in the past to quantify mixing. It is defined as the ratio of the norm of the rate of deformation tensor, $|\gamma|$, to the sum of $|\gamma|$ plus the norm of the vorticity tensor ($|\omega|$):

$$\alpha = \frac{|\gamma|}{|\gamma| + |\omega|}. \quad (8)$$

If the extensional efficiency has a value of zero, the flow is purely rotational, and if it has a value of 1, the flow is purely extensional.

Cross-sectional contours of extensional efficiency are plotted in Fig. 6 for $Re = 0.15$ and for $z = 0, L/8, L/4$, and $L/2$ along the third element of the Kenics KM static mixer (a–d) and along the third

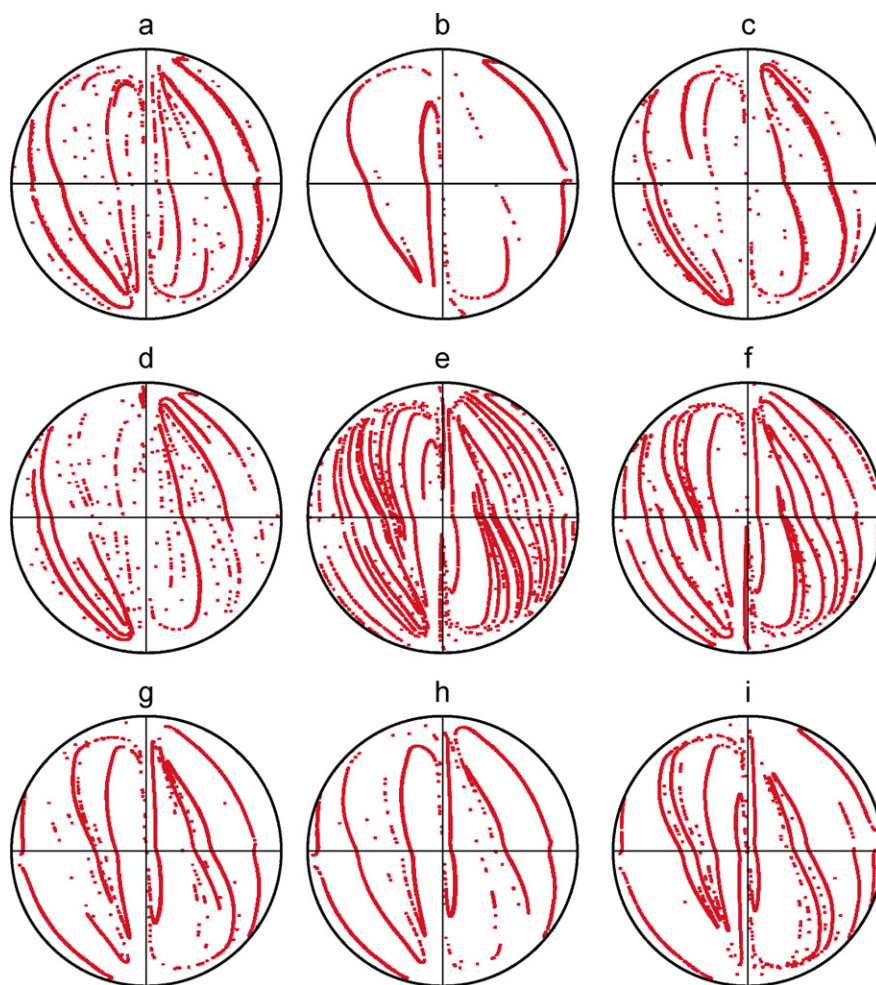


Fig. 11. Position of blob of dye after six modified Kenics elements ($\epsilon = 0.05$) for $Re = 0.15$. The 1 vol% injection locations are given in Fig. 9.

element of a modified Kenics static mixer with $\epsilon = 0.3$ (e–h). Notice how the zones of high extensional mixing turn due to the 180° twisting of the helical element of the static mixer. The values of the extensional efficiency in the transition region between elements of the modified geometry are somewhat higher than those for the conventional geometry.

The modified geometry with the highest eccentricity ($\epsilon = 0.3$) has a higher extensional efficiency than the standard Kenics static mixer geometry. This is shown in Fig. 7, where the average extensional efficiency over a cross section is plotted against the axial distance for the standard Kenics static mixer and for its modified version with $\epsilon = 0.3$. Inspection of Fig. 7 shows that over most part of the mixer the modified geometry has a higher extensional efficiency.

A higher extensional efficiency in a particular region of a mixer may not be a good indicator of good overall mixing because the region may be segregated [25,26]. For this reason we have also performed stretching calculations, which are discussed below.

Another tool that provides valuable information on static mixers is the study of the trajectories of a given material point. This is an excellent tool to highlight chaos because two particles, initially very close to each other, will follow very different trajectories if chaotic advection is present in the flow. In Fig. 8 we simulated the injection of a blob of dye (passive scalar) at the center of the inlet cross section ($x=y=0$), and recorded the position of each material point at the cross section of the tube after 2, 6, and 12 Kenics elements. The initial blob is composed of more than 10^4 particles.

These simulations were done for Reynolds numbers of 0.15, 1, and 10.

In these numerical experiments, the blob of dye was advected with the flow by solving the evolution equations,

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(\mathbf{x}), \quad \mathbf{x}_{t=0} = \mathbf{x}_0, \quad (9)$$

for the position vector $\mathbf{x}$ of each material point comprising the initial blob. The ODE solver employed is an implicit fifth-order Gear integration method with adaptive step-size control [27].

The main difference between the results of Fig. 8 at the end of 12 elements for $Re = 0.15$ and $Re = 10$ is the finer grain observed at the higher Reynolds number, otherwise the area covered is practically the same. In all cases, the stretching and folding of an element, a typical consequence of chaotic advection, can be seen.

Also of interest is the location of the initial blob of fluid and its effect on the mixing in the static mixer. Would it be different if in Fig. 8 the blob of dye had been placed elsewhere? Fig. 9 shows different blobs of fluid placed on the upper part of the inlet cross section of a modified Kenics mixer with $\epsilon = 0.05$; the circles correspond to 9 different initial conditions (or injector locations), but the area covered by each circle gives the same injected volumetric flow rate (1% of the total volumetric flow rate). For each injection location, 10^4 tracer particles were evenly distributed over the cross section of the injector tip placed 0.1 cm upstream of the leading edge of the first static mixing element, and then the particles were advected with flow as described above.

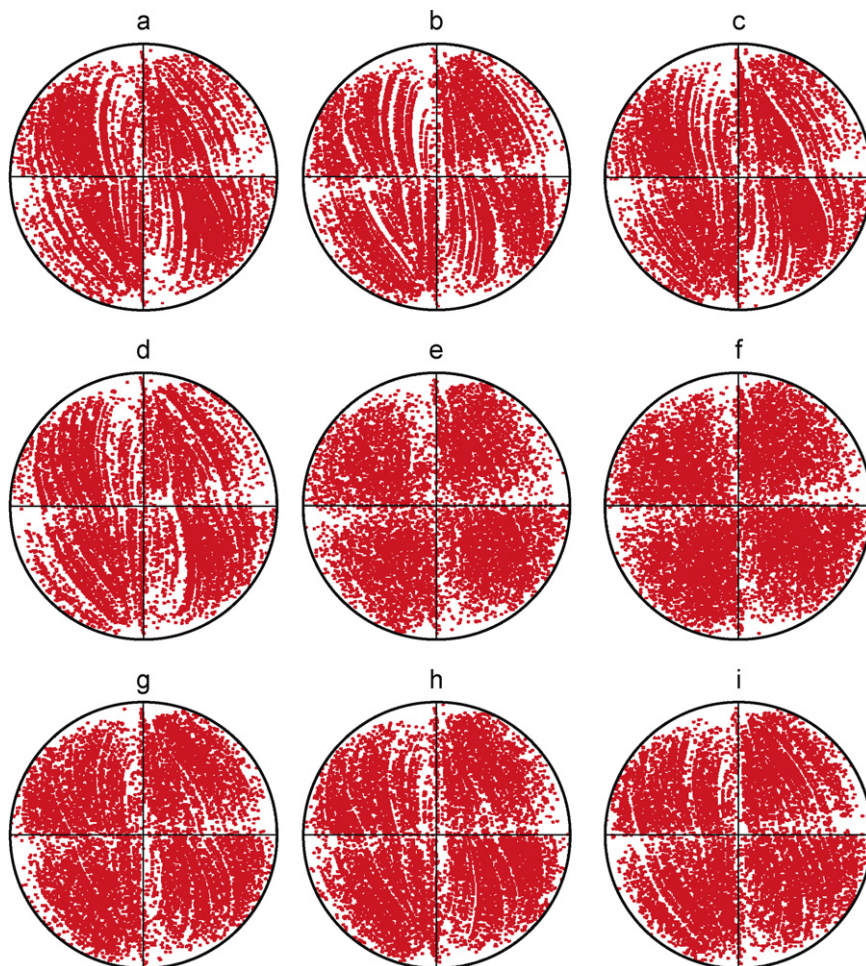


Fig. 12. Position of blob of dye after twelve modified Kenics elements ($\epsilon=0.05$) for $Re=0.15$. The 1 vol% injection locations are given in Fig. 9.

Figs. 10–12 show the positions of the tracer particles at the end of 2, 6, and 12 modified static mixer elements ($\epsilon=0.05$). Note that positions d, e, and f seem to mix better over the cross section; this is more true when the fluid has crossed just a few static mixer elements. After 12 elements, the difference is still noticeable but to a much less extent. It is worth noting that in this numerical experiment the results obtained with the modified geometry ($\epsilon=0.05$) are qualitatively similar to those obtained with the conventional geometry ($\epsilon=0$).

In Fig. 13 we show an initial condition which is half blue and half red, each color occupying one half of the entrance. The flow is turned on at $Re=0.15$ and we plot the position of each point at the end of 2, 4, 6, 8 and 10 elements of a modified Kenics mixer $\epsilon=0.1$. Again it is worth noting how a very thorough mixing is obtained at this small Reynolds number after only 10 elements.

Finally, we calculate the stretching rate in both the Kenics and in the modified geometry. The rate of stretching determines the rate of the micromixing process both by increasing the intermaterial area over which inter-diffusion of components can occur but also by decreasing the required diffusional distance. Points experiencing high and low stretching correspond to regions of good and poor micromixing, respectively. The stretching distribution provides a means of characterizing the distribution of mixing intensities within the flow [28].

The stretching vector $\mathbf{F}$ of a fluid element $\mathbf{I}_0$, associated with a given marker particle initially positioned at $\mathbf{x}_0$, can be monitored

by simultaneously integrating the advection equations, Eq. (9), and solving

$$\frac{d\mathbf{F}}{dt} = (\nabla \mathbf{v})^T \cdot \mathbf{F}, \quad \mathbf{F}_{t=0} = \mathbf{I}_0. \quad (10)$$

The total accumulated stretching per element at the end of given time span, λ , is defined as:

$$\lambda = \frac{|\mathbf{F}|}{|\mathbf{I}_0|}. \quad (11)$$

The stretching rate distribution is calculated using approximately 20,000 tracer particles uniformly distributed over the inlet cross section of the mixer. The initial stretching rate vector for each particle is $\mathbf{I}_0 = [0, 0, 1]$. At the end of each spatial period, or $2n$ mixing elements ($n = 1, 2, \dots$), the position and the accumulated stretching rate for each particle are registered. After each spatial period we also calculate the geometric average value of the stretching rate for the N marker particles:

$$\Lambda_n = \log_{10} \langle \lambda \rangle = \frac{1}{N} \sum_{i=1}^N \log_{10} \lambda_i. \quad (12)$$

In Fig. 14a the geometrical average of the stretching rate, Λ_n , is plotted against the number of helicoidal elements of the static mixer for both the Kenics and modified Kenics mixers ($\epsilon=0.05$ and $\epsilon=0.1$). After the first two elements, the increase is exponential, as expected for a chaotic flow. The Λ_n -curves for the two

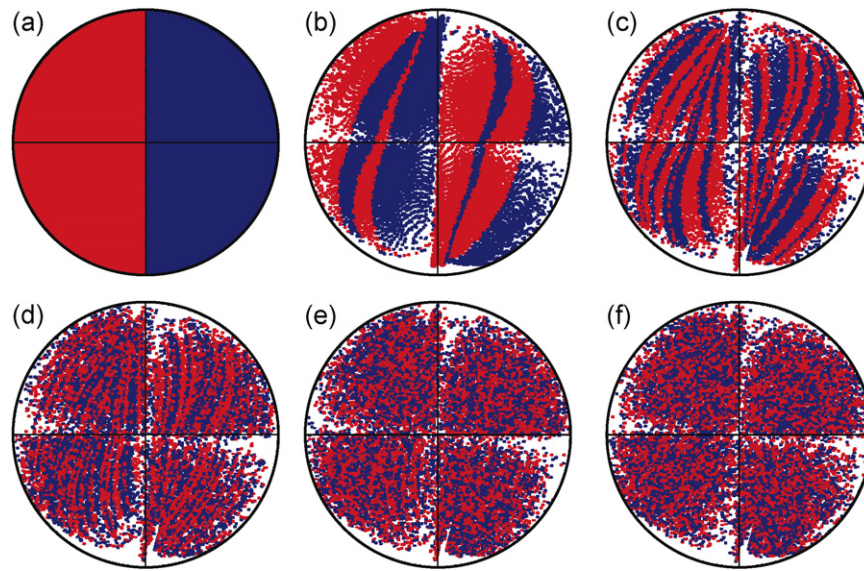


Fig. 13. Mixing of red and blue fluid of passive particles, each one occupying half of the inlet cross section of a modified Kenics mixer ($\epsilon = 0.05$). The particle distributions are plotted at (a) the inlet cross section and after (b) 2, (c) 4, (d) 6, (e) 8, and (f) 10 mixing elements.

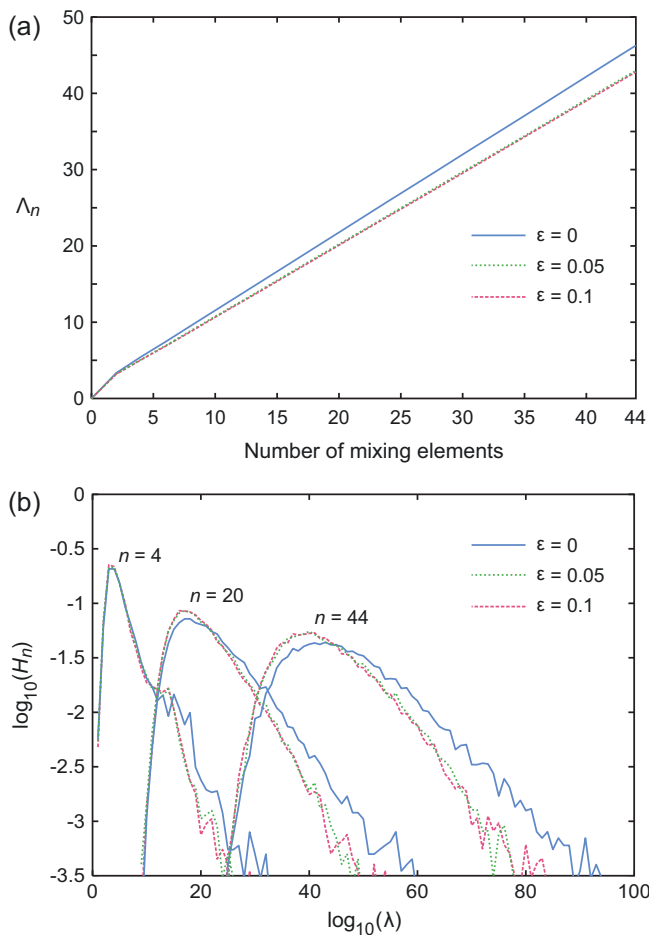


Fig. 14. (a) Geometric mean of the stretching rate, Λ_n , as a function of the number of static mixing elements, and (b) probability density distribution, H_n , after 4, 20 and 44 mixing elements. Each curve is for a different value of the eccentricity of the mixing element.

modified geometries ($\epsilon = 0.05$ and $\epsilon = 0.1$) are nearly identical and are slightly below the Λ_n -curve for the standard Kenics mixer. As will be seen later, this slight decrease is compensated by a more uniform distribution of the stretching rates.

The stretching rate distribution can also be described by the probability density function of the logarithm of stretching rate:

$$H_n(\log_{10} \lambda) = \frac{1}{N} \frac{dN(\log_{10} \lambda)}{d\log_{10} \lambda}, \quad (13)$$

where $dN(\log_{10} \lambda)$ is the number of particles having a stretching rate comprised between $\log_{10} \lambda$ and $\log_{10} \lambda + d(\log_{10} \lambda)$. The value of $H_n(\log_{10} \lambda)$ is calculated at the end of each spatial period of the mixer; its value can be interpreted as a spectral intensity measure of the micromixing process.

In Fig. 14b we plot the probability density function, $H_n(\log_{10} \lambda)$, of the stretching rate distribution calculated after 4, 20, and 44 mixing elements for the standard Kenics mixer and modified geometries with $\epsilon = 0.05$ and $\epsilon = 0.1$. The width of the probability density distribution is larger for the standard Kenics mixer than for the two modified geometries; moreover, the maximum of the H_n -curve for the standard Kenics mixer is slightly lower than those for the two modified geometries. This implies that the modified geometry has a more homogeneous stretching rate distribution. This is confirmed in Fig. 15 where the variance of the logarithm of the stretching rate, σ_n^2 , is plotted against the number of static mixer elements for both the Kenics and the modified geometry. The variance in the modified Kenics geometry is smaller than for the conventional case implying a more homogeneous distribution.

Finally, Fig. 16 shows the relative pressure drop increase with respect to the standard Kenics mixer, $\Delta(\Delta P)$, as a function of the Reynolds number for eccentricities $\epsilon = 0.1$ and $\epsilon = 0.3$. Here, $\Delta(\Delta P) = 100 \times [(\Delta P)_\epsilon - (\Delta P)_{\epsilon=0}] / (\Delta P)_{\epsilon=0}$, where $(\Delta P)_\epsilon$ is the pressure drop for a given value of ϵ and $(\Delta P)_{\epsilon=0}$ is the pressure drop in the standard Kenics mixer for the same Reynolds number. The eccentric geometries with $\epsilon = 0.05$ and $\epsilon = 0.1$ are the ones that were more thoroughly analyzed in the present work, whereas the eccentric geometry with $\epsilon = 0.3$ was taken as an extreme case of deformation of the twisting axis of the inserts and is included here for reference. The results show that the pressure drop for $\epsilon = 0.1$ increases by only 0.52% with respect to the pressure drop of the standard Kenics mixer, and this relative increase is

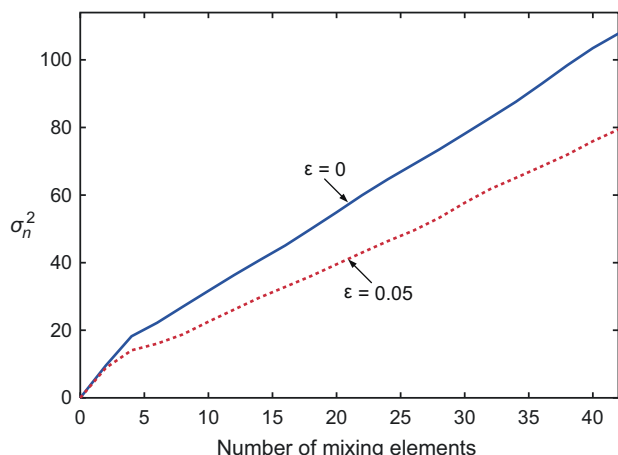


Fig. 15. Variance, σ_n^2 , of the logarithm of stretching rate, as a function of the number of static mixing elements, for the standard Kenics mixer ($\epsilon = 0$) and for the modified geometry ($\epsilon = 0.05$).

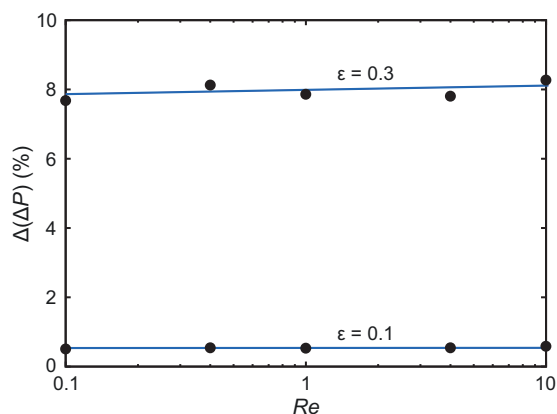


Fig. 16. Relative pressure drop increase, $\Delta(\Delta P) = 100 \times [(\Delta P)_\epsilon - (\Delta P)_{\epsilon=0}] / (\Delta P)_{\epsilon=0}$, as a function of Reynolds number for different eccentricities (ϵ) of the modified Kenics geometry. Here, $(\Delta P)_\epsilon$ is the pressure drop for a given value of ϵ and $(\Delta P)_{\epsilon=0}$ is the pressure drop in the standard Kenics mixer for the same Reynolds number. The symbols are values determined from the CFD simulations, the lines are linear fitting of the numerical data as serve as a guide to the eye.

nearly independent of the Reynolds number. For $\epsilon = 0.3$, the relative pressure drop increase is ca. 7.9% and, again, increases only slightly with the Reynolds number.

It can thus be concluded that an improvement in the mixing efficiency is obtained with the modified geometry: it increases the extensional efficiency, gives a more homogeneous distribution of the stretching rates, and induces a negligible pressure drop increase with respect to the standard geometry.

4. Conclusions

Chaotic advection in a spatially periodic flow was studied numerically and using tools developed in the analysis of mixing by chaotic advection. The flow inside a Kenics static mixer was solved numerically for Reynolds numbers up to 100 using the OpenFOAM software package. The injection location is of little importance in the mixing performance if a sufficiently high number of elements is employed. The Kenics static mixer is effective at practically all Reynolds numbers.

The geometry of the static mixer was also modified in order to break a symmetry. Stretching results show that this modification slightly improves the mixing efficiency of the static mixer. The effects of Reynolds number, injection location, and number of elements on the mixing efficiency are highlighted.

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