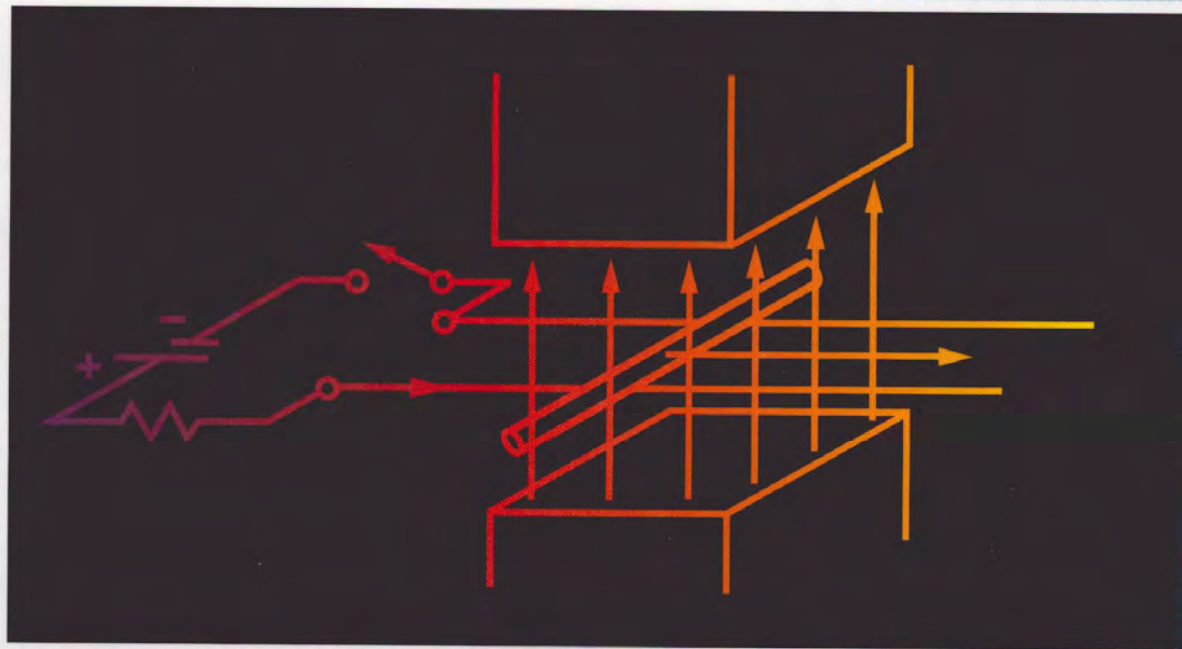


FOUNDATIONS OF
**ELECTRICAL
ENGINEERING**

S E C O N D  E D I T I O N



J. R. C O G D E L L

COGDELL



F O U N D A T I O N S O F
ELECTRICAL ENGINEERING

**SECOND
EDITION**



**PRENTICE
HALL**

FOUNDATIONS OF ELECTRICAL ENGINEERING

SECOND EDITION

J. R. COGDELL

A thorough revision of a popular text that is appropriate for introduction to electrical engineering courses for majors and non-majors. The author provides an in-depth, yet concise presentation of the important principles that form the basis of electrical engineering.

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Second Edition

J . R . C O G D E L L

**Department of Electrical and Computer Engineering
University of Texas at Austin**



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1. To explain well the fundamental ideas of electrical engineering.
2. To emphasize the unity of the subject by frequent reminders of those basic ideas.
3. To bring understanding of circuits, electronics, systems, and electrical power engineering within the reach of all engineers.

My equal emphasis on electrical power engineering meets, I predict, an emerging need of our time. In the eyes of EE students, the technical challenge, professional opportunities, and personal enjoyment of electronics have totally eclipsed interest in electric power engineering. To most students, comparing power and electronics is like comparing a draft horse with a racing thoroughbred, with the result that many electrical engineering students avoid taking a single elective course in power. Thus, electric motors, like illumination engineering, must fall to the engineer who needs the information, such as the mechanical engineer who needs to select a motor to drive a pump. This book anticipates the need of all engineers to have more than superficial knowledge of electric power, motors, in particular.

IMPROVEMENTS IN THE SECOND EDITION

While sharing in the goals of the first edition, this second edition differs in many ways. The entire text has been revised and refined, a new chapter has been added, and material has been rearranged to improve the logic and flow. The presentation has been im-

$$L_{eq} = L_1 \parallel L_2 = \frac{1}{1/L_1 + 1/L_2} \quad (\text{parallel inductors}) \quad (3.9)$$

Capacitor Basics

Physical capacitors. Electric (electrostatic) forces arise from interactions between separated charges, and the associated energy is called electric energy. To store electric energy, we must separate charges, as in Fig. 3.9.

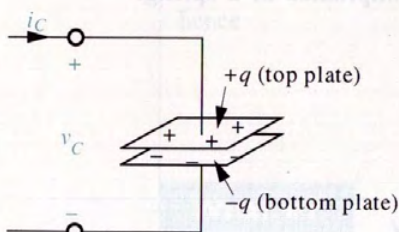


Figure 3.9 Structure having capacitance.

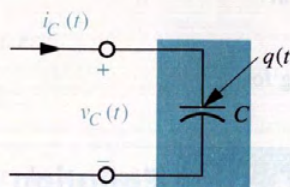


Figure 3.10 Circuit symbol for capacitance.

OBJECTIVE 2

To understand the properties of capacitors and to be able to find the capacitor voltage from the capacitor current and vice versa

Circuit-theory definition of a capacitor. Figure 3.10 shows the circuit symbol for a capacitance. Capacitance is defined as the constant relating charge and voltage in a structure that supports a charge separation. If q is the charge on the + side of the capacitor and the voltage is v_C , as shown in Fig. 3.9, the capacitance, C , is defined as

$$q = Cv_C \quad (3.10)$$

Chapter 13 discusses the physics of capacitance. For every charge arriving at the + side of the capacitor, a charge of like sign will depart from the - side and the structure as a whole will remain charge-neutral. Thus, KCL will be obeyed because charge flowing into the + terminal side is matched by charge flowing out of the - terminal, as for a resistor. If current is positive into the + terminal, positive charge will accumulate there and will be increasing in proportion to the current. From the definition of current, we can relate the charge in Eq. (3.10) and current as follows:

$$i_C = \frac{dq(t)}{dt} \quad (3.11)$$

where $q(t)$ is the charge in the + side of the capacitor. Thus, we may define the relationship between current and voltage for a capacitor as

$$i_C = \frac{d}{dt} Cv_C(t) = C \frac{dv_C(t)}{dt} \quad (3.12)$$

where the last form of Eq. (3.12) is valid if the capacitance is constant. Note that we have used a load set in Fig. 3.10 for the voltage and current variables.

Units of capacitance. The unit of capacitance is the coulomb/volt, but to honor Michael Faraday (1791–1867), we use the name *farad* (F). Realistic capacitor values come small and usually are specified in microfarads (10^{-6} F, or μF), nanofarads (10^{-9} F, or nF), or picofarads (10^{-12} F, or pF). When a capacitor is constructed from parallel

farad,
microfarad,
nanofarad,
picofarad

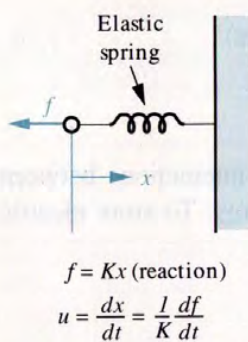


Figure 3.11
Mechanical analog for capacitance.

plates, as in Fig. 3.9, the capacitance depends on the area, separation, and material (if any) between the plates.

Mechanical analog of capacitance. The mechanical analog of a capacitor is a spring, as shown in Fig. 3.11. The analog of velocity is current and thus displacement of the spring is analogous to charge. The voltage across a capacitor is analogous to the force produced by the spring. Comparison of the second equation in Fig. 3.11 with Eq. (3.12) shows that capacitance corresponds to the inverse of the stiffness constant, K , and thus capacitance is analogous to the compliance of a spring.

EXAMPLE 3.3 Equation of RC circuit

Derive the equation for the current in Fig. 3.12(a). Assume $v_C(0) = 0$.

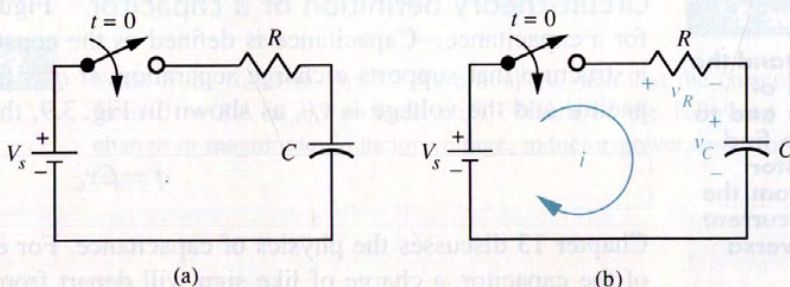


Figure 3.12 (a) Closing the switch will cause a momentary current. (b) The same circuit with variables defined.

SOLUTION:

Figure 3.12(b) defines the capacitor and resistor voltages in a load-set convention. With the switch closed, KVL is

$$-V_s + v_R(t) + v_C(t) = 0 \quad (3.13)$$

We may express both voltages in terms of the current if we differentiate Eq. (3.13):

$$0 + \frac{d}{dt} Ri(t) + \frac{dv_C(t)}{dt} = R \frac{di(t)}{dt} + \frac{i(t)}{C} = 0 \quad (3.14)$$

The last form in Eq. (3.14) is readily manipulated to the form

$$\frac{di}{i} = -\frac{1}{RC} dt \Rightarrow \ln i = -\frac{t}{RC} + \ln A \Rightarrow i(t) = Ae^{-t/RC} \quad (3.15)$$

where A is a constant. The current will charge the capacitor until the capacitor voltage is equal to the battery voltage. Equation (3.16) equates the final charge on C to the integral of the current.

$$q - q(0) = CV_s = \int_0^{\infty} i(t) dt = \frac{Ae^{-t/RC}}{-1/RC} \bigg|_0^{\infty} = A(RC) \quad (3.16)$$

Because $v_C(0) = 0$, $q(0) = 0$, and

$$A = \frac{V_s}{R} \quad (3.17)$$

hence

$$i(t) = \frac{V_s}{R} e^{-t/RC} \quad (3.18)$$

WHAT IF?

What if the switch is opened after being closed a long time? What would happen to the capacitor voltage?⁴

Integral i - v equation. Equation (3.12) is useful in determining the current through a capacitor, given the voltage as a function of time. If we know the current and wish to determine the voltage, we must integrate. Let us consider that we know the capacitor voltage at some time, say, $t = 0$, and wish to determine the voltage at a later time, t . We can integrate Eq. (3.12) from 0 to t , with the result

$$v_C(t) = v_C(0) + \frac{1}{C} \int_0^t i_C(t') dt' \quad (3.19)$$

We have used t' as the dummy variable of the integration process because t is one limit of the integral.

EXAMPLE 3.4 Capacitor discharge and charge

As an example of the use of Eq. (3.19), consider a capacitor with current, as shown in Fig. 3.13.

Here we know the voltage at the beginning time, $v_C(0) = -2$ V, and the current through the capacitor. We wish to compute the voltage for all $t > 0$. We will use Eq. (3.19), noting that we must divide the integration into several regions due to the piecewise nature of the current function. During interval I, $0 < t < 1$ ms, the current is zero; hence, the voltage does not change. During interval II, the slope of the current is 0.1 A/1 ms = 100 A/s; hence, the current is

⁴ No current could flow after the switch opened, and according to Eq. (3.12), the voltage would remain constant. A real capacitor would discharge over time due to leakage current.

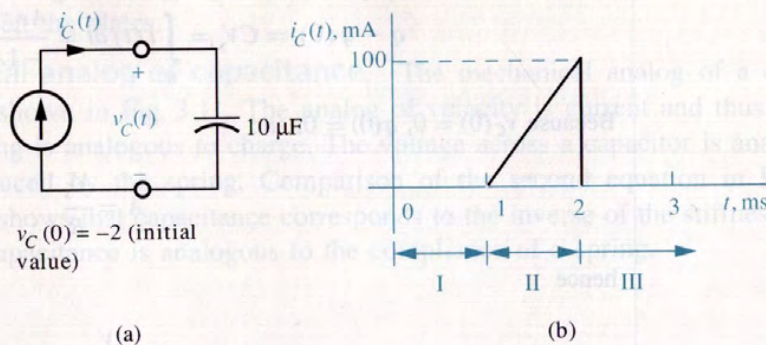


Figure 3.13 (a) The capacitor has an initial voltage. (b) Current into the capacitor for positive time.

$$i_C(t) = 100(t - 1 \text{ ms}) \text{ A}, \quad 1 \text{ ms} < t < 2 \text{ ms} \quad (3.20)$$

Thus, during interval II, the voltage will be

$$\begin{aligned} v_C(t) &= -2.0 + \frac{1}{10 \mu\text{F}} \int_{1 \text{ ms}}^t 100(t' - 1 \text{ ms}) dt' \Big|_{\text{II}: 1 < t < 2 \text{ ms}} \\ &= -2.0 + \frac{10^{-5} \times 100 (t - 1 \text{ ms})^2}{2} \text{ V} \end{aligned} \quad (3.21)$$

which indicates a parabolic increase, as shown in Fig. 3.14. The final value at $t = 2 \text{ ms}$ is $+3 \text{ V}$, which is easily checked from the area under the current curve:

$$\Delta v_C = \frac{\Delta q}{C} = \frac{\text{area under current curve}}{C} = \frac{\frac{1}{2} \times 1 \text{ ms} \times 100 \text{ mA}}{10 \mu\text{F}} = +5 \text{ V} \quad (3.22)$$

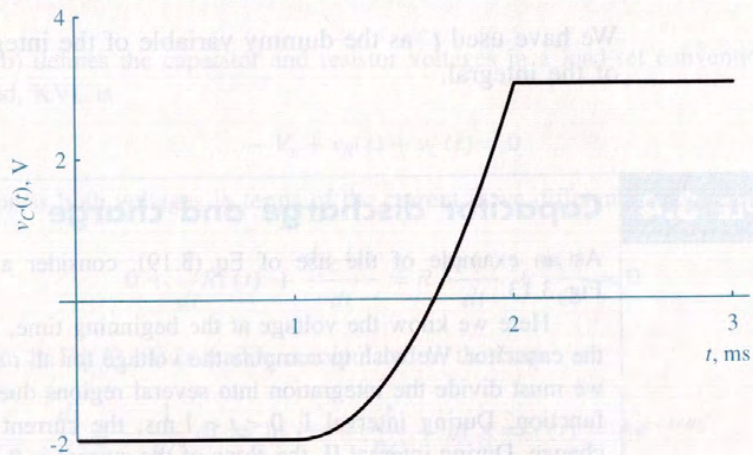


Figure 3.14 Capacitor voltage for positive time.

After the current pulse, the capacitor voltage remains constant because the current is zero. This

constant voltage would hold for an ideal capacitor; a physical capacitor would discharge eventually due to leakage current.

WHAT IF?

What if the current were constant at 100 mA for $1 \text{ ms} < t < 2 \text{ ms}$? What then would be the final value?⁵

Mechanical analogy. The process of integrating current through a capacitor to determine voltage is analogous to integrating velocity to determine change in force in a spring problem. The only difference is that the value of the capacitance scales the integral of the current, whereas the scale factor depends on K , the spring constant, in the mechanical analog.

Stored energy in a capacitor. The charge separation in a capacitor stores electric energy. This energy is analogous to potential energy stored in a stressed spring. We may derive the stored electric energy in a capacitor by integrating the power into the capacitor,

$$W_e = \int p_C dt = \int v_C \times C \frac{dv_C}{dt} dt = \int d\left(\frac{1}{2} C v_C^2\right) = \frac{1}{2} C v_C^2 = \frac{1}{2} \frac{q^2}{C} \quad (3.23)$$

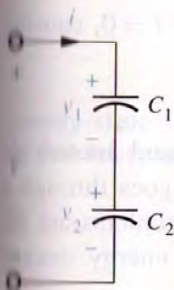
where p_C is the power into the capacitor and W_e is the stored electric energy in the capacitor. The constant of integration must be zero because the uncharged capacitor stores no energy. We see from Eq. (3.23) that the stored energy depends uniquely on the voltage, or the charge, and the capacitance. For example, if we take a $10\text{-}\mu\text{F}$ capacitor and connect it briefly to a 12.6-V battery, the capacitor will receive $0.5 \times 10^{-5} (12.6)^2 = 7.94 \times 10^{-4} \text{ J}$ from the battery. Although this is not much energy, it would take only about $1 \mu\text{s}$ to charge the capacitor. Hence, the rate of energy flow would be rather high, about 800 W .



Equivalent Circuits

Figure 3.15

Capacitors in series add like resistors in parallel: $C_{eq} = C_1 \parallel C_2$.



Capacitors in series and parallel. Figure 3.15 shows two capacitors in series. KVL yields

$$v(t) = v_1 + v_2 = \frac{1}{C_1} \int_0^t i dt + \frac{1}{C_2} \int_0^t i dt = \frac{1}{C_{eq}} \int_0^t i dt \quad (3.24)$$

where

$$C_{eq} = C_1 \parallel C_2 = \frac{1}{1/C_1 + 1/C_2} \quad (\text{series capacitors})$$

Thus, capacitors connected in series combine numerically like resistors in parallel. Similarly, we can easily show that capacitors in parallel add like resistors in series, that is,

$$C_{eq} = C_1 + C_2 \quad (\text{parallel capacitors}) \quad (3.25)$$

Mechanical analog for resistance. A mechanical analog for a resistance is frictional loss of a special type. Voltage is analogous to force, and current analogous to ve-

⁵+8 V.

TABLE 3.1 Summary of Mechanical and Electrical Analogs

Mechanical	Electrical
Force	Voltage
Velocity	Current
Displacement	Charge
Mass	Inductance
Spring compliance	Capacitance
Shock absorber	Resistance

locity. Voltage/current, or resistance, would thus imply a force that is proportional to velocity. We experience such a force when we try to move underwater. A common mechanical component having this property would be an automotive shock absorber. Table 3.1 summarizes the analogies between mechanical and electrical quantities.



Check Your Understanding

1. An inductor has a stored energy of 5 J and an inductance of 0.1 H. What is the current through the inductor?
2. If an ideal 10-V battery were connected for 1 s to an ideal 1-H inductor, how much energy would be given to the inductor?
3. A capacitor has a stored energy of 500 μJ and a capacitance of 0.15 μF . What is the voltage across this capacitor?
4. What is the mechanical analog of an inductor?

Answers. (1) 10 A; (2) 50.0 J; (3) 81.6 V; (4) mass.

3.2 FIRST-ORDER TRANSIENT RESPONSE OF RL AND RC CIRCUITS

transients, first-order transient

First-order transients. In the major part of this chapter, we show how to analyze an important class of circuit problems. Typical situations are shown in Fig. 3.16. These circuits are characterized by having the following:

- A single energy storage element, one capacitor or inductor
- Loss, represented by one or more resistors
- DC sources
- A switch⁶ that either opens or closes at a known time, usually $t = 0$, causing a sudden change in the circuit
- Only linear components.

In all cases, the circuit will have one dc state before the switch action and another dc state long after the switch action. Consequently, the state of the circuit goes through a transition. Because this transition lasts for a brief period of time, these problems are often called *transient* problems. We consider primarily circuits with one energy storage element, which lead to *first-order transients*.

⁶The pulse in Fig. 3.16(c) can be considered a voltage switched ON, then OFF.

mately 10–12 pulses/second and relays counted the pulses at the exchange to route the call. The ring signal was about 90 to 100 V ac at approximately 20 Hz.

The modern system used a tone system to transmit switching information and semiconductor switching computers have replaced the relays. The “ring” signal is commonly a tone that activates an audible device in the telephone receiver.

The local exchange switches up to 10^4 lines with the signal at the base band with a 4-kHz bandwidth. Between exchanges, the voice information is modulated with signal sideband plus carrier into groups of 12 adjacent channels between 60 and 108 kHz. These groups, 12 channels, are then placed into a supergroup of 60 channels between 312 and 552 kHz, and finally 10 supergroups, 600 channels, are placed into a master-group between 564 and 3084 kHz. For long distance, the voice channels are modulated to yet higher frequency for transmission by coaxial cable, line-of-sight microwave, or satellite link. Currently digital systems using optical fibers are replacing traditional analog systems.

Noise in Receivers



Digital Information

noise, interference

Importance of noise. When we spoke of op amps in Chapter 9, we asserted that gain is cheap. With radio receivers, gain is not as cheap as with op amps, but we can still achieve as high a gain as we wish by adding more stages of amplification. We might suppose, therefore, that we can successfully detect any input RF signal, no matter how weak, simply by having adequate gain in the receiver. This is not true, however, because the sensitivity of a radio receiver is limited by random noise. *Noise*, by definition, is any undesired signal in the system, whether natural or man-made. Man-made noise is called *interference*; Chapter 10 showed techniques to reduce its effects.

blackbody radiation, shot noise, flicker noise

Sources of natural noise. Natural noise in communication systems comes from two sources. Some noise comes into the receiver from the antenna; for example, in an AM receiver, we can hear a sporadic crackling noise from lightning during storms. Other sources of noise that enter the system through the antenna are atmospheric gases (thermal noise) and extraterrestrial objects such as the sun and our galaxy (cosmic noise).

The second source of noise is the receiver itself. Just as any body radiates *blackbody radiation* due to the thermal motion of its atomic constituents, so also resistors radiate their own type of thermal noise into the circuits of which they are a part. We investigate resistor noise in what follows. Transistors add, in addition to thermal noise, *shot noise* due to the discrete charges that carry the current across the *pn* junctions, and *flicker* or *1/f* noise, due to slow changes in device properties from aging, temperature, and other physical changes.



Equivalent Circuits



The Frequency Domain

white noise

Thermal noise. The thermal radiation of resistors plays an important role in the description of noise because resistors provide a simple calibration for noise signals. Figure 11.16(a) shows a resistor, R , and indicates the noise voltage generated by the thermal motion of the carriers in the resistor. The Thévenin equivalent circuit in Fig. 11.16(b) separates the noise voltage from the resistance, which here is noise-free. The power spectrum of the noise voltage in volts²/hertz is shown in Fig. 11.16(c),

$$S_n(f) = 4kTR \text{ V}^2/\text{Hz} \quad (11.14)$$

with k = Boltzmann’s constant of 1.38×10^{-23} J/K. We note the following:

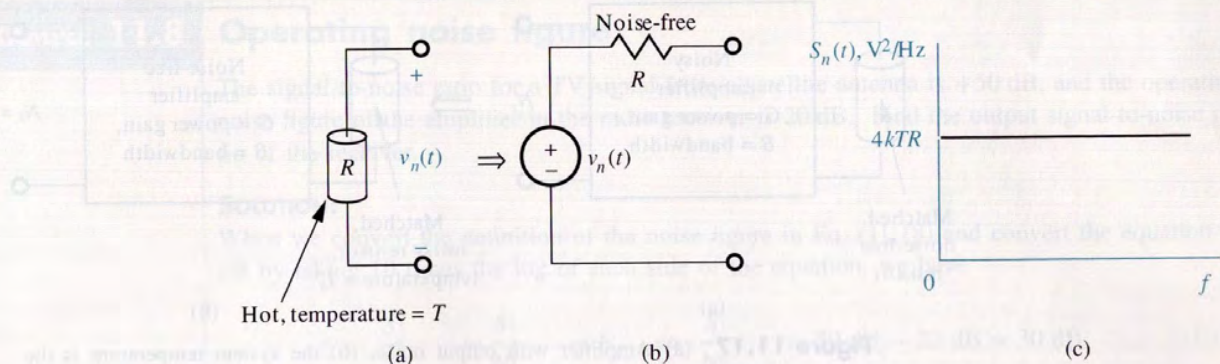


Figure 11.16 (a) A noisy resistor; (b) Thévenin equivalent circuit of a noisy resistor; (c) power spectrum of thermal noise.

- The noise spectrum is *white noise*, meaning that all frequencies are present equally, at least throughout the radio spectrum. Thus, a resistor produces a noise power that is proportional to the bandwidth of the circuit.
- The intensity of the power spectrum is proportional to the temperature of the resistor. For sensitive amplifiers, we may measure the noise of the system by comparing system noise to the noise from a resistor of known temperature.
- The intensity of the power spectrum is proportional to the resistance. For this reason, large values of resistance are avoided in sensitive electronic amplifiers. The *available power* out of a circuit is the maximum power that the circuit is capable of delivering into a matched load (see Chapter 2, page 69). The available power spectrum from the circuit in Fig. 11.16(b) is.

available power

$$S_{av}(f) = \frac{S_n(f)}{4R} = kT \text{ W/Hz} \quad (10.15)$$

where $S_{av}(f)$ is the available power spectrum in watts/hertz. Thus, the total power a noisy resistor will contribute to a matched circuit is

$$P = kTB \text{ W} \quad (11.16)$$

where B is the bandwidth in hertz. Equation (11.16) is useful in determining the signal-to-noise ratio in a communication circuit, once we have defined the noise figure and system temperature of an amplifier.

system temperature

System temperature. The noise originating in the resistors and semiconductors in a radio amplifier is generally broadband or white noise, at least over the bandwidth of the radio. Thus, we may represent the noise *as if* it originated in a resistor at the input of the radio. Figure 11.17(a) shows the true situation: An amplifier with a power gain of G and a bandwidth of B has internal noise sources that produce N_s watts of noise at its output. In Fig. 11.17(b) we show an equivalent amplifier, assumed to be noise-free, with the same noise at its output *attributed* to a noisy resistor at the input. The *system temperature* is defined as the temperature the resistor required in Fig. 11.17(b) to produce the same amount of output noise as in Fig. 11.17(a). Using Eq. (11.16), we can express

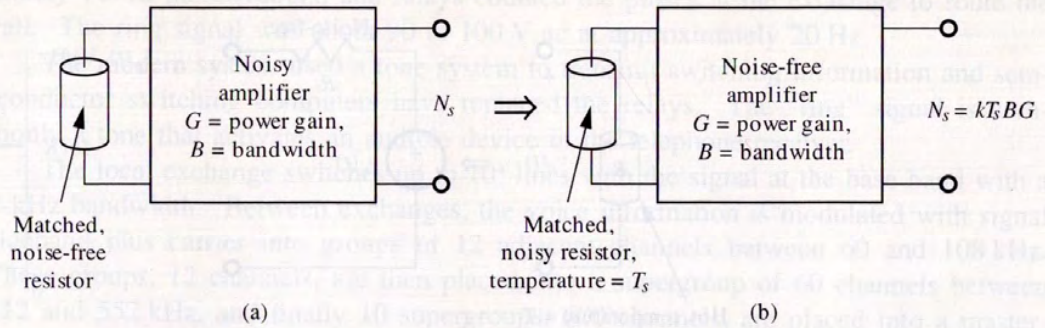


Figure 11.17 (a) Amplifier with output noise; (b) the system temperature is the temperature of the input resistor to give the same output noise.

the output noise as

$$N_s = kT_s GB \Rightarrow T_s = \frac{N_s}{kGB} \text{ K} \quad (11.17)$$

where T_s is the system temperature and N_s is the output noise produced by the amplifier. All the internally generated noise is *referred to the input* of the receiver and then expressed as a temperature. The system temperature describes the sensitivity of the amplifier; the lower the system temperature, the more sensitive the amplifier.

operating noise figure

Noise figure. The *operating noise figure* is defined as the input signal-to-noise ratio divided by the output signal-to-noise ratio. Figure 11.18 shows an amplifier with input signal and noise, S_{in} and N_{in} , and output signal and noise, S_{out} and N_{out} . The *operating noise figure* is defined to be

$$F = \frac{S_{in}/N_{in}}{S_{out}/N_{out}} = \frac{N_{out}}{N_{in}G} = \frac{N_{in}G + N_s}{N_{in}G} = 1 + \frac{N_s/G}{N_{in}} = 1 + \frac{T_s}{T_{in}} \quad (11.18)$$

where F is the operating noise figure of the amplifier and is often expressed in decibels: $F_{dB} = 10 \log F$ and T_{in} is the input noise expressed as a temperature. The first form on the right side of Eq. (11.18) is the definition of operating noise figure. The second form introduces the gain, $G = S_{out}/S_{in}$, and the third form breaks the output noise into two components, the amplified input noise and the internally generated noise. The last form shows that the noise figure depends on the internally generated noise, referred to the input of the amplifier, compared with the input noise. Thus, the operating noise figure depends in part on the input noise and not on the amplifier only. The operating noise figure is useful in performing signal-to-noise calculations in a communication system.

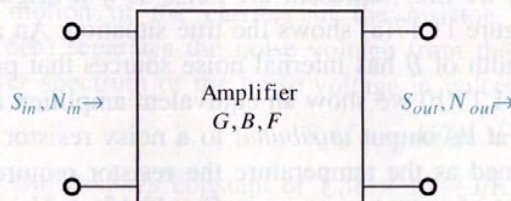


Figure 11.18 Amplifier with signal and noise at input and output.