

Technical Report on Equations and Calculations or Casing, Tubing, and Line Pipe Used as Casing or Tubing; and Performance Properties Tables for Casing and Tubing

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ISO 10400:2007 (Modified), Petroleum, petrochemical and natural gas industries—Equations and calculations for the properties of casing, tubing, drill pipe and line pipe used as casing or tubing



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Petroleum and natural gas industries — Equations and calculations for the properties of casing, tubing, drill pipe and line pipe used as casing or tubing

1 Scope

This Technical Report illustrates the equations and templates necessary to calculate the various pipe properties given in International Standards, including

- pipe performance properties, such as axial strength, internal pressure resistance and collapse resistance,
- minimum physical properties,
- product assembly force (torque),
- product test pressures,
- critical product dimensions related to testing criteria,
- critical dimensions of testing equipment, and
- critical dimensions of test samples.

For equations related to performance properties, extensive background information is also provided regarding their development and use.

Equations presented here are intended for use with pipe manufactured in accordance with ISO 11960 or API 5CT, ISO 11961 or API 5D, and ISO 3183 or API 5L, as applicable. These equations and templates may be extended to other pipe with due caution. Pipe cold-worked during production is included in the scope of this Technical Report (e.g. cold rotary straightened pipe). Pipe modified by cold working after production, such as expandable tubulars and coiled tubing, is beyond the scope of this Technical Report.

Application of performance property equations in this Technical Report to line pipe and other pipe is restricted to their use as casing/tubing in a well or laboratory test, and requires due caution to match the heat-treat process, straightening process, yield strength, etc., with the closest appropriate casing/tubing product. Similar caution should be exercised when using the performance equations for drill pipe.

This Technical Report and the equations contained herein relate the input pipe manufacturing parameters in ISO 11960 or API 5CT, ISO 11961 or API 5D, and ISO 3183 or API 5L to expected pipe performance. The design equations in this Technical Report are not to be understood as a manufacturing warrantee. Manufacturers are typically licensed to produce tubular products in accordance with manufacturing specifications which control the dimensions and physical properties of their product. Design equations, on the other hand, are a reference point for users to characterize tubular performance and begin their own well design or research of pipe input properties.

This Technical Report is not a design code. It only provides equations and templates for calculating the properties of tubulars intended for use in downhole applications. This Technical Report does not provide any guidance about loads that can be encountered by tubulars or about safety margins needed for acceptable design. Users are responsible for defining appropriate design loads and selecting adequate safety factors to develop safe and efficient designs. The design loads and safety factors will likely be selected based on historical practice, local regulatory requirements, and specific well conditions.

6.6.2 Initial yield pressure of pipe body, historical API equation

6.6.2.1 General

The Barlow Equation for pipe yield, which is the historical API equation, is based on a one-dimensional (not triaxial) approximate equation of the von Mises yield condition, combined with an approximate expression for the hoop stress in the pipe. In essence, the Barlow Equation approximates the hoop stress and then equates this approximation to the yield strength. This approximation is less accurate than the Lamé Equation of yield discussed in 6.6.1. Because the Barlow Equation neglects axial stress, there is no distinction between pipe with capped ends, pipe with open ends or pipe with tension end load.

6.6.2.2 Historical, one-dimensional yield pressure design equation

Initial yield of a thin tube is defined by the following expression:

$$p_{iYAPI} = [2f_{ymn}(k_{wall}t)/D] \quad (10)$$

where

D is the specified pipe outside diameter;

f_{ymn} is the specified minimum yield strength;

k_{wall} is the factor to account for the specified manufacturing tolerance of the pipe wall. For example, for a tolerance of $-12,5\%$, $k_{wall} = 0,875$;

p_{iYAPI} is the internal pressure at yield for a thin tube;

t is the specified pipe wall thickness.

This equation is subject to the same assumptions and limitations as the more general expressions from which it may be derived (by methods other than that used by Barlow) (see 6.2).

6.6.3 Pipe body yield strength, historical API equation

6.6.3.1 General

Pipe body yield strength is the axial load required to yield the pipe in the absence of internal and external pressure, bending and torsion. It is taken as the product of the cross-sectional area and the specified minimum yield strength for the particular grade of pipe:

$$F_{YAPI} = f_{ymn} A_p \quad (11)$$

where

A_p is the area of the pipe cross section, $A_p = \pi/4 (D^2 - d^2)$;

D is the specified pipe outside diameter;

d is the pipe inside diameter, $d = D - 2t$;

F_{YAPI} is the axial force at yield, historical API equation;

f_{ymn} is the specified minimum yield strength;

t is the specified pipe wall thickness.

Since both bending and torsion are zero in this example, there is no need to calculate σ_e at both the inner and outer radii of the tube, as yield will always occur at the inner radius. The calculations have been included here for completeness.

As the equivalent stress is less than the yield stress, the tube does not yield at its uppermost cross section.

7 Ductile rupture of the pipe body

7.1 General

The equations for ductile rupture pertain to the actual failure of the pipe body due to internal pressure. While the yield equations of Clause 6 are intended to describe the onset of permanent plastic deformation and not loss of pressure integrity, the rupture equations are intended to describe the ultimate pressure capacity of the pipe at a pressure which fails the pipe body with loss of internal pressure integrity.

The equations for ductile rupture depend on the minimum physical wall thickness, the pipe outer diameter, the maximum depth of imperfections which have a reasonable probability of passing through the inspection process undetected, the fracture toughness of the material, the work hardening of the material and the ultimate tensile strength of the pipe. Yield strength has no direct impact on the ductile rupture pressure except through the correlation of the work hardening parameter n .

The ductile rupture equations can be derived from the mechanics of pipe equilibrium combined with a model of pipe plasticity and a model of the effect of imperfections. The selection of the basic equation, the application of the basic equation to ISO/API tubular products, and the verification of the equation with actual test data are discussed in detail in Annex B. The ductile rupture limit state and design equations consist of three interlinked concepts:

- a) an equation for equilibrium-plasticity based rupture of a pipe with known physical wall thickness and diameter;
- b) reduction in performance due to wall loss in proportion to depths of imperfections which may not be detected by the manufacturing and inspection system;
- c) a criterion for minimum toughness at which ductile rupture applies.

These equations are applicable to direct pressure and axial loading, but do not describe the capacity of the pipe under fatigue loading. The subtraction to the pipe wall for the presence of imperfections and the interrelated role of pipe toughness are based on a fracture mechanics approach relating J_{Ic} toughness measurements of sample pipes to numerically calculated crack-tip intensities (J-Integrals) as a function of imperfection depth. This is explained in detail in Annex B.

7.2 Assumptions and limitations

These equations are applicable only when the pipe material in its environment has sufficient toughness to meet a minimum criterion such that the deformation of the pipe, in its environment, through to rupture is ductile and not brittle, even in the presence of small imperfections.

Bending stresses (for instance, due to buckling or bending due to curvature of the well trajectory) are not included in the equation for ductile rupture pressure. Hence, the ductile rupture equation may not apply to pipe which is buckled (nor to pipe in a dogleg).

7.3 Data requirements

7.3.1 General

The following input data are required to complete the calculation for ductile rupture of the pipe body:

- a_N imperfection depth associated with a specified inspection threshold, i.e. the maximum depth of a crack-like imperfection that could reasonably be missed by the pipe inspection system. For example, for a 5 % imperfection threshold inspection in a 12,7 mm (0.500 in) wall thickness pipe, $a_N = 0,635$ mm (0.025 in);
- D specified pipe outside diameter;
- f_{umn} specified minimum tensile strength;
- k_a burst strength factor, having the numerical value 1,0 for quenched and tempered (martensitic structure) or 13Cr products and 2,0 for as-rolled and normalized products based on available test data; and the default value set to 2,0 where the value has not been measured. The value of k_a can be established for a specific pipe material based on testing;
- k_{wall} factor to account for the specified manufacturing tolerance of the pipe wall. For example, for a tolerance of -12,5 %, $k_{wall} = 0,875$;
- n dimensionless hardening index used to obtain a curve fit (see B.2.3.3) of the true stress-strain curve derived from the uniaxial tensile test;
- t specified pipe wall thickness.

7.3.2 Determination of the hardening index

In the absence of stress-strain information, the following values of n are suggested.

Table 2 — Suggested values for hardening index in ductile rupture equation

API grade	n
H40	0,14
J55	0,12
K55	0,12
M65	0,12
N80	0,10
L80 Type 1	0,10
L80 Chrome	0,10
C90	0,10
C95	0,09
T95	0,09
P110	0,08
Q125	0,07

If the grade of the material is unknown but is not high-hardening, the hardening index can alternatively be determined from the following correlation.

$$n = 0,1693 - k_n f_y \quad (13)$$

where f_y has units of MPa (psi) and

f_y is the yield strength of a representative tensile specimen;

k_n is the stress conversion factor, equal to $1,18 \times 10^{-4}$ MPa⁻¹ for SI units and 8.12×10^{-7} psi⁻¹ for USC units;

n is the dimensionless hardening index used to obtain a curve fit (see B.2.3.3) of the true stress-strain curve derived from the uniaxial tensile test.

The effort expended to determine n should be weighed against the fact that the equation for ductile rupture is relatively insensitive to this quantity for commonly used Oil Country Tubular Goods (OCTG). However, if a high-hardening material such as duplex steel is used, it is important to determine n to avoid a non-conservative rupture strength prediction. Values of n for these materials can be as high as 0,30.

7.3.3 Determination of the burst strength factor, k_a

7.3.3.1 General

The burst strength factor k_a quantifies the impact that the material toughness has on ductile rupture when a crack of depth a_N is present. The value of k_a does not need to be determined for each pipe order; instead, it is recommended that k_a be determined for a fixed tubular product line with a fixed process control plan. For a material with high material toughness, k_a will be 1,0 or lower, and the influence the crack has on the ductile rupture pressure will be no greater than the depth of the crack. However, for a pipe material with lower material toughness, k_a can be larger, say 2,0, so that the penalty from having the crack is as though the crack had twice its actual depth.

When k_a has not been determined for the pipe material, use a k_a value of 2,0. The value of k_a has been measured to be 1,0 for quenched and tempered (martensitic structure) pipe and for 13Cr products. The value of k_a can be established for a specific pipe material based on testing. One of two methods presented below can be used to calculate k_a .

7.3.3.2 Analytical method for determining k_a

The following procedure should be used.

- Construct a finite element model of a pipe with three separate cases of crack depth: zero, 5 %, and 12,5 %. Model the crack as an infinitely long, longitudinal crack on the ID of the pipe. In the finite element model, use the specified pipe wall thickness (do not reduce the pipe wall for manufacturing eccentricity) and a typical stress-strain curve for the material being analysed.
- Use the finite element model to simulate applying internal pressure to the pipe model in order to calculate the value of the J-integral of the pipe as a function of internal pressure (see B.7.2 for examples).
- Experimentally measure the critical J_{1c} value of the pipe material in air. J_{1c} is a parameter similar to K_{1c} , but based on a different type of test. See ASTM E1152-95 [1] for the precisely defined test methodology.
- Terminate each of the finite element J-integral curves at the critical value of J_{1c} measured in the test. The internal pressure corresponding to this terminal point (where the J-integral equals J_{1c}) represents the rupture pressure in the presence of the crack, for a material with toughness reflected by J_{1c} .
- Next, divide the rupture pressure for pipe with a crack depth 5 % by the rupture pressure for pipe with no crack, using the finite element results combined with the J_{1c} measurement. Next, set this ratio equal to $(1-k_a alt)$ where alt is the ratio of the crack depth to the specified pipe wall thickness (i.e. 5 % in this case). Solve this equation for the value of k_a .

- f) Repeat the above calculation using the finite element model results for the crack with 12,5 % depth.
- g) Average the two values of k_a from steps e) and f) and use the average in the rupture equation.

7.3.3.3 Experimental method for determining k_a

An alternative way to measure k_a in lieu of the J_{1C} measurement and the finite element calculation is to conduct full-scale pipe burst tests using a pipe with no measurable crack-like imperfection, another pipe with a crack-like imperfection of 5 % depth, and another pipe with a crack-like imperfection of 12,5 % depth. Then construct the ratios of the rupture pressures as described above and calculate k_a . The limitation and difficulty with this approach is that it is not valid unless the imperfection is sharp, and crack-like. A mechanical or EDM notch is not adequate for this purpose. This means that the crack-like imperfection needs to be generated either as part of the pipe manufacturing process or else through fatigue precracking. In general, the finite element approach combined with the J_{1C} measurement is easier to pursue.

7.3.3.4 Sensitivity of k_a

It turns out that k_a is a weakly sensitive parameter in regard to the grade of the pipe; that is, k_a primarily depends on the manufacturing process and does not vary much by grade within a fixed process. This is understandable since k_a only represents a potential amplification of the pipe's response to the presence of the crack while being loaded to rupture. Because of this, it is recommended that k_a only be determined for a specific grade of product and not for specific sizes or orders of product.

7.4 Design equation for capped-end ductile rupture

Minimum ductile rupture of a tube is defined by:

$$p_{iR} = 2k_{dr}f_{umt}(k_{wall}t - k_a a_N)/[D - (k_{wall}t - k_a a_N)] \quad (14)$$

where

- a_N is the imperfection depth associated with a specified inspection threshold, i.e. the maximum depth of a crack-like imperfection that could reasonably be missed by the pipe inspection system. For example, for a 5 % imperfection threshold inspection in a 12,7 mm (0.500 in) wall thickness pipe, $a_N = 0,635$ mm (0.025 in);
- D is the specified pipe outside diameter;
- f_{umt} is the specified minimum tensile strength;
- k_a is the burst strength factor, having the numerical value 1,0 for quenched and tempered (Martensitic structure) or 13Cr products and 2,0 for as-rolled and normalized products based on available test data; and the default value set to 2,0 where the value has not been measured. The value of k_a can be established for a specific pipe material based on testing;
- k_{dr} is the correction factor based on pipe deformation and material strain hardening, having the numerical value $[(1/2)^{n+1} + (1/\sqrt{3})^{n+1}]$;
- k_{wall} is the factor to account for the specified manufacturing tolerance of the pipe wall. For example, for a tolerance of -12,5 %, $k_{wall} = 0,875$;
- n is the dimensionless hardening index used to obtain a curve fit (see B.2.3.3) of the true stress-strain curve derived from the uniaxial tensile test;
- p_{iR} internal pressure at ductile rupture of an end-capped pipe;
- t specified pipe wall thickness.

B.2.3.3 Determination of the hardening index

A specimen from material with true stress-strain relation of the form given in Equation (B.3) would reach a maximum load in a uniaxial tensile test at a logarithmic strain of n . The best method is to derive n from fitting the actual true stress-strain curve with Equation (B.3) in the relevant strain range of % to the strain at maximum load. Although less accurate, one could alternatively approximate n equal to the actual logarithmic strain at the maximum load point of the tensile test. If the engineering strain at the maximum load is given, the logarithmic strain, ϵ_{ln} , is

$$\epsilon_{ln} = \ln(1 + \epsilon_{eng}) \quad (B.5)$$

where ϵ_{eng} is the engineering strain.

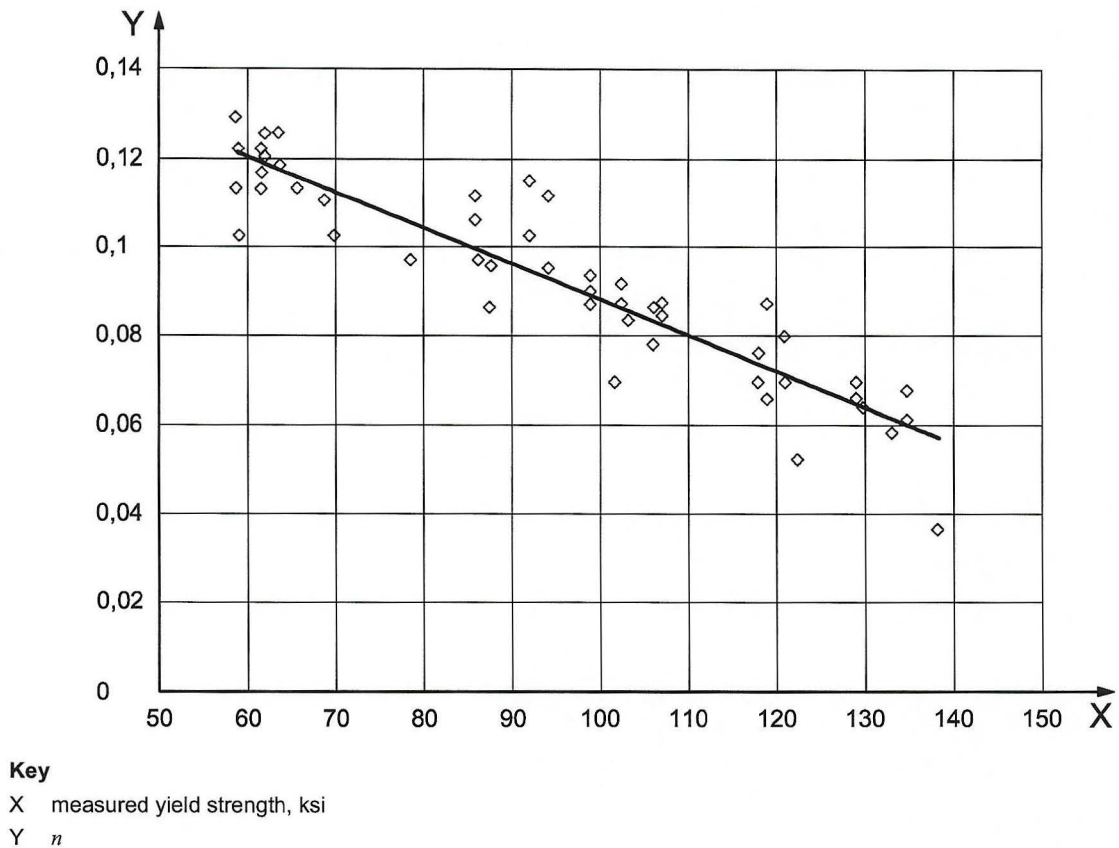
The relatively flat nature of the stress-strain curve in the plastic region of most OCTG grades makes determination of n through this last method often rather difficult. As a third alternative, in the absence of stress-strain information, the following values of n are suggested.

Table B.2 — Suggested values for hardening index in ductile rupture equation

Grade	n
H40	0,14
J55	0,12
K55	0,12
M65	0,12
N80	0,10
L80 Type 1	0,10
L80 Chrome	0,10
C90	0,10
C95	0,09
T95	0,09
P110	0,08
Q125	0,07

If the grade of the material is unknown but is not high hardening, the hardening index can alternatively be determined from the correlation given in Figure B.1. The effort expended to determine n should be weighed against the fact that the equation for ductile rupture is relatively insensitive to this quantity for commonly used OCTG. However, if a high-hardening material such as duplex steel is used, it is important to determine n to avoid a non-conservative rupture strength prediction. Values of n for these materials can be as high as 0,30.

The parameter n has relatively small effect on the value of p_{IR} , but should not be ignored. At $n = 0,12$ a 14 % change in n is necessary to produce a 1 % change in predicted rupture pressure. Two materials may have the same tensile strength, but if one material has $n = 0,12$ it will have a 4 % lower rupture strength than a material with $n = 0,06$. When selecting a value of n for OCTG grades in the absence of experimental data, estimating n using specified minimum yield strength, f_{ymn} , is conservative.



NOTE Least squares fit of data results in $n = 0,169\ 3 - 0,000\ 812 \times \text{measured yield strength}$; coefficient of variance = 0,10.

Figure B.1 — Correlations for hardening index from typical experimental data for steel grades listed in Table B.2

B.2.4 Adjustment for the effect of axial tension and external pressure

B.2.4.1 General

The ductile rupture strength Equation (B.1) was developed for the situation of an end-capped pipe, where the axial tension is determined by the internal pressure acting on the closed inner pipe surface area. This is a special case of a more general situation where a pipe can reach a maximum internal pressure load, that is a rupture load, under the simultaneous action of arbitrary external pressure and arbitrary axial tension or compression. The combined loads together determine when the pipe is going to yield and how it will plastically deform towards the point of rupture. A fundamental criterion when this rupture load is attained can still be expressed, but this will now be a more involved equation governed by the equation of the von Mises or Tresca yield surface in terms of axial stress, radial stress and hoop stress.

Moreover, rupture is only the prevailing failure mechanism if the axial tension is not too large. For large axial tension and smaller internal over-pressure, a maximum axial load (a precursor to necking and axial splitting of the pipe) is reached before the maximum pressure phenomenon occurs.

Below, equations for both rupture and necking under combined loads are described, together with a criterion to identify which phenomenon occurs first. The equation is given in terms of "effective axial tension" associated with "effective axial stress" defined in A.1.3.2.4. For effective axial tension, these approximate equations are very accurate when compared to the exact theoretical Equation (24); performance against combined loading test data is given in B.6.2.