



COANDA FLOW OF A TWO-DIMENSIONAL
WALL JET ON THE OUTSIDE OF
A CIRCULAR CYLINDER

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NOTATION

- a - radius of circular cylinder
- a' - constant in Equation (2.4)
- A - constant in Equation (A.III .1)
- b - slot width
- b' - constant in Equation (2.4)
- B - constant in Equation (A.III .1)
- C - constant in Equations (4.4) and (2.5)
- C' - constant in Equation (2.5)
- C₁ - constant in Equation (4.2)
- C₂ - constant in Equation (4.1)
- $\overline{C}_f$ - coefficient of total skin friction
- D - ratio of square profile velocity to the corresponding maximum jet velocity $D = \frac{u_1}{u_m}$
- d_w - diameter of hot wire
- E - ratio of width of square velocity profile to the half width of the jet. $E = \frac{y_1}{\frac{y_m}{2}}$
- f - function of (not a particular function)
- h - r.m.s. height of roughness elements
- H - heat loss from hot wire
- J_o - theoretical jet momentum at slot = (P - p_∞)b
- k - constant in Equation (4.4)
- k_f - thermal conductivity of fluid
- K - ratio of theoretical to actual jet momentum
- l - length of hot wire, or mixing length

- m - constant in Equation (A.III .1)
- M - Mach number
- n - constant in Equation (A.III.1)
- N_u - Nusselt number = $\frac{H}{\pi l k_f (T_w - T_a)}$
- p - static pressure
- p_s - static pressure at surface of cylinder
- p_t - total pressure
- p_∞ - static pressure of surrounding fluid
- P - stagnation pressure in plenum chamber
- Re - Reynolds number $\left[\frac{(P-p_\infty)ba}{\rho v^2} \right]^{1/2}$
- $Re_{lim.}$ - lowest value of Reynolds number at which $\theta_{1_{sep.}}$ maximum is reached
- Re_w - Reynolds number of hot wire = $\frac{u d_w}{\nu}$
- T_a - ambient absolute temperature
- T_m - arithmetic mean absolute temperature = $\frac{T_w + T_a}{2}$
- T_w - absolute temperature of hot wire
- u - mean velocity component tangential to cylinder, or fluid velocity by hot wire
- u' - tangential turbulent velocity component
- u_1 - square profile velocity
- u_{jo} - velocity of laminar jet at slot
- u_m - maximum jet velocity
- $u_{max.c}$ - maximum jet velocity at centre line of cylinder
- v - radial mean velocity component

- v' - radial turbulent velocity component
- $\bar{V}$ - mean velocity vector
- x - distance along the surface = $a\theta$
- x_o - position of hypothetical origin from slot measured along surface = $a\theta_o$
- y - distance measured from and perpendicular to cylinder
- y_1 - width of square velocity profile
- y_m - distance from surface where $u = u_m$
- $\frac{y_m}{2}$ - distance from surface where $u = \frac{1}{2} u_m$
- z - spanwise coordinate
- S - total skin friction force
- δ_f - length scale defined by $\delta_f = \left| u_m / \left[\left(\frac{du}{dy} \right)_{y_{\frac{m}{2}}} \right] \right|$
- ϵ - eddy viscosity
- θ - angular distance measured from slot
- θ_1 - angular distance measured from hypothetical origin
- θ_o - angular position of hypothetical origin from slot
- $\theta_{sep.}$ - angular position of separation from slot
- $\theta_{1sep.}$ - angular position of separation from hypothetical origin
- ν - kinematic viscosity of fluid
- ρ - density of fluid.

1. INTRODUCTION

The study of wall jets is of interest for boundary layer control to prevent separation, and basic information about curved jets is important when blowing over the knee of flaps is considered, or in pneumatic computer design where the deflection of fluid streams is desired. (1 and 2)

An experimental investigation of the relatively simple flow where a jet flows around the surface of a circular cylinder, the surroundings being stationary and the fluid in the jet being the same as that in the surroundings, is described.

The flow shown in Figure (1) is a wall jet which emanates from a slot of width b and flows around the outer surface of a circular cylinder of radius a . The surrounding fluid is at rest at pressure p_{∞} , and the jet entrains fluid from it. Hence the flow resembles a plane wall jet in still air. But, whereas the static pressure across the plane wall jet can be assumed to be equal to the pressure of the surroundings (3) there is a pressure drop across the curved flow, the surface pressure being lower than that of the surroundings. The pressure force is related to the centrifugal acceleration associated with the curved flow, and keeps the jet attached to the wall. As the jet proceeds around the cylinder, the jet velocity decreases due to air being entrained from the surroundings. In consequence, even though the jet

width increases, the surface pressure gradually rises to p_∞ , producing an adverse pressure gradient and the flow finally separates from the cylinder. Experiments show that separation does not occur before the value of θ exceeds well over 180° when the Reynolds number is high and b/a is small.⁽⁴⁾

As in the case of a free jet or wall jet, the flow consists of two distinct parts, one near the slot where the mixing region rapidly becomes turbulent and spreads into the laminar core, and a fully turbulent region which starts further downstream where the laminar jet core has disappeared. Only the fully turbulent flow will be considered in this paper and, in addition, the flow will be taken as incompressible. ($M < 0.3$).

2. DETAILS OF PREVIOUS INVESTIGATIONS

2.1 General

There were three previous investigations of note, those of Newman⁽⁴⁾, Nakaguchi⁽⁵⁾ and Sawyer⁽⁶⁾. Bradshaw and Gee⁽⁷⁾ also made some experimental measurements.*

Dimensional and theoretical analysis was carried out by Newman, and the results were compared with experiment. Newman noted that for plane uncurved wall jets the skin friction, the variation of static pressure across the flow associated with the cross component of turbulence v' , and the longitudinal normal stresses can be considered negligible compared to the jet momentum per unit area of wall at sufficiently high Reynolds numbers. Hence, the mean jet momentum is conserved. It was also noted that flows having the same jet momentum, but which emerge from slots having various widths, produce the same velocity profiles far downstream provided a suitable origin is chosen for the streamwise coordinate.

It was, therefore, assumed that for the curved flow the skin friction is also small compared to the jet momentum at high Reynolds numbers; hence the sum of the moment of momentum and the moment of the pressure forces about the centre of the circular cylinder is constant. In applying this concept, it was assumed that the streamlines are circles with centres at the centre of the cylinder, that the profiles are similar

* For other work not directly applicable here because compressible flow and small θ are considered see References 28, 29, 30 .

at all downstream positions and that they can be replaced by a uniform velocity profile having the same mass flow and the same momentum.

On the basis of these assumptions, the surface pressure and the decay of the maximum jet velocity were theoretically predicted. A law for the jet growth was suggested and its constants were determined experimentally. This was done by assuming the position of the hypothetical origin for an equivalent infinitesimally thin slot to be located at the slot for small gap widths. The assumed uniform velocity and the width of the uniform profile were determined experimentally by matching its mass flow and momentum to that of the measured similar profile.

Experimental measurements of surface pressure, maximum jet velocity, and position of separation broadly confirmed the predictions of the dimensional analysis. The maximum jet velocity coefficient predicted theoretically agreed well with experiments, but the theoretical surface pressure coefficient was considerably higher than the experimental one.

Nakaguchi made a comprehensive theoretical analysis for incompressible flows by solving the Navier-Stokes equations of motion, after making suitable assumptions to simplify them. It was assumed:

- (i) that the familiar boundary layer approximation holds, i.e. the radial velocity component is very much smaller than the tangential one;

- (ii) that the width of the flow is very much smaller than the radius of the flow curvature, i.e. $\frac{y}{a} \sim O(\delta)$ and $\frac{1}{a+y} \sim \frac{1}{a}$;
- (iii) that the pressure changes in the flow direction are negligible in so far as they affect the momentum of the flow;
- (iv) that the velocity profiles are similar and the boundary layer portion of the flow is negligible, and that the non-dimensional velocity profiles can therefore be represented by a half free jet profile;
- (v) that the rate of spread of the jet is proportional to v' , which in turn is proportional to the centrifugal pressure change across the flow, the constants of proportionality being established experimentally;
- (vi) that the turbulent sheer stress can be simply related to eddy viscosity as was done by Görtler⁽⁸⁾ for a free jet;
- (vii) that self preservation of $\overline{u'^2}$ and $\overline{v'^2}$ exists as stipulated by Townsend⁽⁹⁾;
- (viii) that all flows can be reduced to a universal flow which emanates from a hypothetical origin.

Assumptions (i) and (viii) are the same, (ii) and (iii) are simpler, and (iv), (vi) and (vii) are more sophisticated than Newman's assumptions.

Formulae were derived for the radial velocity distribution, the local maximum jet velocity and the wall pressure.

Integrating the equation of motion across the flow, the effect of skin friction forces was included in the analysis, yet the universal flow concept was retained.

In order that the local maximum jet velocity and the wall pressure could be determined, it was necessary to know from experiment the maximum jet velocity at some arbitrarily chosen point in the fully developed flow.

Experiments were carried out by measuring the velocity and pressure distributions and wall pressures in jets flowing over cylindrical surfaces of various radii. The slot width was kept constant in all cases, and measurements were made at two slot jet velocities.

Sawyer⁽⁶⁾ studied two-dimensional turbulent jets with adjacent boundaries. He investigated, among other flow configurations, a curved flow where the walls (concave and convex) were designed in such a manner that the ratio of jet width to radius of curvature was maintained constant along the jet. His treatment of the relationship between jet growth and the flow curvature is relevant to the present investigation and will be examined in some detail in Section (2.2).

2.2 The Effect of Curvature on the Rate of Growth for Curved Jets.

In order to establish the jet growth rate, Sawyer⁽⁶⁾ based his approach on Prandtl's⁽¹⁰⁾ mixing length concept and argued that, when the radius of curvature is positive, there is an increase in mixing length in the outer part of the flow due to centrifugal effects. He contended that when a parcel of fluid moves outward in the outer part of a jet with convex curvature, its velocity is greater than the velocity of the surroundings. The centrifugal force per unit mass acting on the parcel of fluid is higher than that acting on the surrounding fluid. But since the centrifugal forces and the pressures balance in the mean, the parcel of fluid is subject to an instantaneous total force, moving it further out, with a consequent increase in the mixing length. On this basis the mixing length is increased by a factor of

$$\frac{l + \delta l}{l} = 1 - \text{constant} \frac{2u}{a} / \frac{\partial u}{\partial y} \quad \dots (2.1)$$

compared with a straight jet, the constant being established experimentally.

Following Görtler⁽⁸⁾, mixing length and jet growth are related by

$$\epsilon \sim l^2 \left(\frac{\partial u}{\partial y} \right) \sim l^2 \frac{u_m}{y_m^2}$$

hence

$$\epsilon \frac{y_m^2}{u_m} \sim l^2$$

The growth of a curved jet is therefore increased or decreased, compared to the growth of a straight jet when the curvature is convex or concave respectively.

Stratford et al.⁽¹¹⁾ approached the problem slightly differently, but also based their analysis on the mixing length concept. They related the increase in mixing length to the flow curvature by examining the transverse motion of turbulent eddies from a kinematic point of view. They found that the mixing length for a wall jet flowing over a convex cylindrical surface, as compared with that for a straight jet, was increased by a factor of

$$\frac{l + \delta l}{l} = \frac{1}{1 - \frac{2u}{a} / (-\frac{\partial u}{\partial y})} \quad \dots(2.2)$$

due to flow curvature in the outer part of the jet. This is the same as Sawyer's Equation (2.1) for small $\frac{u}{u_m} \frac{\partial u}{\partial y}$, which was assumed in both analyses. Taking a mean value for $u \frac{\partial u}{\partial y}$ over the velocity profile, it was found that this factor could be expressed as

$$\frac{l + \delta l}{l} = 1 + \frac{2}{3} \frac{b}{a} + \frac{4}{9} \left(\frac{b}{a}\right)^2 + \dots \quad \dots(2.3)$$

for a jet in still air, where b is the width of the mixing region. The width of the mixing region was defined as the distance between the point of maximum jet velocity and the outer edge of the jet.

Nakaguchi⁽⁵⁾ seems to base his argument on strict similarity of the flow in all details, including turbulence, so that $(c \rho u_m^2 \frac{y_m}{2a}) \sim \text{wall pressure} \sim \text{magnitude of } \overline{v'^2}$, and

since $|v'| \sim u_m \frac{dy_m}{dx}$

$$\frac{dy_m}{dx} = \sqrt{a' + b' \frac{y_m}{a}} \quad \dots(2.4)$$

Since $x = a\theta_1$ where θ_1 is measured from the hypothetical origin

where $\frac{y_m}{a}$ vanishes,

$$\frac{y_m}{a\theta_1} = C + C' \theta_1 \quad \dots(2.5)$$

was obtained for the spread of the jet. The constant C represents the growth of a plane wall jet and Nakaguchi fixed its value as 0.086. The value of the constant C' was to be found experimentally.

Newman⁽⁴⁾ postulated that the curved jet growth will be equal to the growth of a plane wall jet plus a term due to the flow curvature and proportional to the jet half width to

radius ratio $\frac{y_m}{a}$, i.e.

$$\frac{\frac{y_m}{2}}{a\theta_1} = C + Ck \frac{\frac{y_m}{2}}{a} \quad \dots(2.6)$$

where θ_1 is measured from the hypothetical origin, k is of order unity, and both C and k are established experimentally.

Comparison of Sawyer's and Stratford's work, Equations (2.1) and (2.2), shows comparable results ($\delta l \ll l$) in spite of the somewhat different approaches taken. These relationships express the local change in mixing length due to flow curvature. Stratford obtained Equation (2.3) as an average value and it should be noted that there is no arbitrary constant in this equation. The width of the mixing region, b , is very difficult to measure with any degree of accuracy, however, and it can be expressed more conveniently as being proportional to $(\frac{y_m}{2} - y_m)$. When this is done and the growth of the jet is obtained from Equation (2.3), the relationship for the jet growth contains at least two arbitrary constants. Hence, in spite of the apparent theoretical treatment, an empirical growth law is obtained. This statement applies also to Sawyer's work.

Newman's growth law was based on a simple extension of the linear growth for a plane wall jet, but it will be shown in Sections (3) and (4) that it can be obtained by dimensional reasoning and series expansion.

Hence all flow investigators obtained growth relationships containing empirical constants when they investigated the effect of curvature on the rates of growth for curved jets. The simplest and most useful for the present investigation appear to be Newman's and Nakaguchi's growth laws, and they will be compared with experimental results in Section (6).

3. DIMENSIONAL ANALYSIS

Newman's Analysis Modified

Since the flow can be considered effectively incompressible, the independent parameters which define the flow are:

- $P - p_{\infty}$ the plenum chamber pressure relative to the surroundings
- ρ the density of the fluid
- ν the kinematic viscosity of the fluid
- b the slot width
- a the cylinder radius
- θ the angular distance from the slot
- the geometry of the contraction.

The non-dimensional criteria of similarity may be expressed as

$$\theta, \frac{b}{a}, \left[\frac{(P - p_{\infty})ba}{\rho \nu^2} \right]^{\frac{1}{2}}, \quad \text{non-dimensional numbers describing the contraction}$$

For the particular apparatus used, the geometry of the contraction is a function of b , hence the criteria of similarity reduce to

$$\theta, \frac{b}{a}, \left[\frac{(P - p_{\infty})ba}{\rho \nu^2} \right]^{\frac{1}{2}}$$

The coefficient of actual jet momentum emanating

from the slot may be written as

$$\frac{J_0}{2(P-p_\infty)b} = K = f\left(\frac{b}{a}, \left[\frac{(P-p_\infty)ba}{\rho v^2}\right]^{\frac{1}{2}}\right) \quad \dots(3.1)$$

The non-dimensional length scales for similar

velocity profiles $\frac{y_m}{2a}$ and $\frac{y_m}{a\theta}$ can be written as

$$\frac{y_m}{2a} = f\left(\theta, \frac{b}{a}, \left[\frac{(P-p_\infty)ba}{\rho v^2}\right]^{\frac{1}{2}}\right) \quad \dots(3.2)$$

and

$$\frac{y_m}{a\theta} = f\left(\theta, \frac{b}{a}, \left[\frac{(P-p_\infty)ba}{\rho v^2}\right]^{\frac{1}{2}}\right) \quad \dots(3.3)$$

It is permissible to write then

$$\frac{y_m}{a\theta} = f\left(\frac{y_m}{2a}, \frac{b}{a}, \left[\frac{(P-p_\infty)ba}{\rho v^2}\right]^{\frac{1}{2}}\right) \quad \dots(3.4)$$

The angular position of separation becomes

$$\theta_{sep.} = f\left(\frac{b}{a}, \left[\frac{(P-p_\infty)ba}{\rho v^2}\right]^{\frac{1}{2}}\right) \quad \dots(3.5)$$

Similarly, the surface pressure coefficient becomes

$$\frac{(p_{\infty} - p_s)a}{(P - p_{\infty})b} = f\left(\theta, \frac{b}{a}, \left[\frac{(P - p_{\infty})ba}{\rho v^2}\right]^{\frac{1}{2}}\right) \quad \dots(3.6)$$

and the maximum jet velocity coefficient is given by

$$\frac{\rho u_m^2 a \theta}{(P - p_{\infty})b} = f\left(\theta, \frac{b}{a}, \left[\frac{(P - p_{\infty})ba}{\rho v^2}\right]^{\frac{1}{2}}\right) \quad \dots(3.7)$$

If the surface roughness is also important, an additional non-dimensional criterion of similarity $\frac{h}{a}$ should be added, where h is a measure of the surface roughness.

If the skin friction forces are negligible compared with the jet momentum, a suitable zero can be chosen for the angular position θ , and a universal flow can be assumed to emanate from this hypothetical origin. The location of the hypothetical origin measured from the slot can be expressed as

$$\theta_0 = f\left(\frac{b}{a}, \left[\frac{(P - p_{\infty})ba}{\rho v^2}\right]^{\frac{1}{2}}\right) \quad \dots(3.8)$$

At a sufficient distance downstream from the slot the jet momentum $2(P - p_{\infty})b$ replaces the two separate independent parameters $(P - p_{\infty})$ and b ⁽⁴⁾. Hence the independent parameters for the universal flow become:

$K(P - p_{\infty})b$ one half of the jet momentum emanating
from the slot

ρ, v, a as defined before

θ_1 the angular position measured from the

hypothetical origin ($\theta_1 = \theta + \theta_0$).

The non-dimensional length scales for the similar velocity profiles $\frac{y_m}{a}$ and $\frac{y_m}{a\theta_1}$ are then written as

$$\frac{y_m}{a} = f\left(\theta_1, \left[\frac{K(P-p_\infty)ba}{\rho v^2} \right]^{\frac{1}{2}} \right) \quad \dots(3.9)$$

and

$$\frac{y_m}{a\theta_1} = f\left(\theta_1, \left[\frac{K(P-p_\infty)ba}{\rho v^2} \right]^{\frac{1}{2}} \right) \quad \dots(3.10)$$

from which

$$\frac{y_m}{a\theta_1} = f\left(\frac{y_m}{a}, \left[\frac{K(P-p_\infty)ba}{\rho v^2} \right]^{\frac{1}{2}} \right) \quad \dots(3.11)$$

is obtained.

The angular position of separation $\theta_{1\text{sep.}}$ becomes

$$\theta_{1\text{sep.}} = f\left(\left[\frac{K(P-p_\infty)ba}{\rho v^2} \right]^{\frac{1}{2}} \right) \quad \dots(3.12)$$

Furthermore, for sufficiently high Reynolds numbers

$$R_e = \left[\frac{K(P-p_\infty)ba}{\rho v^2} \right]^{\frac{1}{2}}$$

the structure of the large-scale eddies, hence the mean flow,

tends to become insensitive to viscosity⁽⁹⁾, therefore independent of changes in Reynolds number. It should be noted however, that the associated limiting Reynolds number may be a function of $\frac{b}{a}$. The rate of growth of the jet is then

$$\frac{Y_m^2}{a\theta_1} = f\left(\frac{Y_m^2}{a}\right) \quad \dots(3.13)$$

and the position of separation

$$\theta_{1\text{sep.}} = \text{constant} \quad \dots(3.14)$$

The surface pressure coefficient becomes

$$\frac{(p_\infty - p_s)a}{K(P - p_\infty)b} = f(\theta_1) \quad \dots(3.15)$$

and the maximum jet velocity coefficient is given by

$$\frac{\rho u_m^2 a \theta_1}{K(P - p_\infty)b} = f(\theta_1) \quad \dots(3.16)$$

4. THEORETICAL ANALYSIS

Newman's Analysis Modified

Newman's⁽⁴⁾ assumptions and universal flow model are retained, but whereas Newman assumed that the hypothetical origin was located at the slot for small $\frac{b}{a}$, and that the slot boundary layers are negligible, the position of the hypothetical origin and the effect of the slot boundary layers on the slot jet momentum are now taken into account.

It was shown by dimensional considerations in Section (3) that for high Reynolds numbers the position of the hypothetical origin from the slot can be written as

$$\theta_o = f\left(\frac{b}{a}\right)$$

Expressing the function as a power series with $\theta_o \rightarrow 0$ as $\frac{b}{a} \rightarrow 0$,

$$\theta_o = c_1 \frac{b}{a} + c_2 \left(\frac{b}{a}\right)^2 + \dots \quad \dots(4.1)$$

is obtained.

Experimental results of plane wall jets in still air show that the position of the hypothetical origin from the slot varies linearly with the slot width. Since similar behaviour is expected for the curved flow, Equation (4.1) is reduced to

$$\theta_o = c_1 \frac{b}{a} \quad \dots(4.2)$$

Dimensional reasoning gave for the spread of the jet width:

$$\frac{Y_m}{a\theta_1} = f\left(\frac{Y_m}{a}\right)$$

which, expressed as a power series, becomes

$$\frac{Y_m}{a\theta_1} = d_0 + d_1 \frac{Y_m}{a} + d_2 \left(\frac{Y_m}{a}\right)^2 + \dots \quad \dots(4.3)$$

Newman showed that the growth of the jet can be expressed as

$$\frac{Y_m}{a\theta_1} = c + ck \frac{Y_m}{a} \quad \dots(4.4)$$

which corresponds to the first two terms of Equation (4.3).

$$\theta_1 = \theta + \theta_0 \quad \text{by definition.} \quad \dots(4.5)$$

Taking the slot boundary layers into account, and noting that dimensional analysis showed that

$$\frac{J_0}{K(P-p_\infty)b} = K = f\left(\frac{b}{a}, R_e\right)$$

where the value of K is found experimentally, Newman's surface pressure coefficient becomes:

$$\frac{(p_{\infty} - p_s)a}{K(p - p_{\infty})b} = \frac{8 \ln \left(E \frac{y_m^2}{a} + 1 \right)}{\left(E \frac{y_m^2}{a} + 1 \right)^2 - 1 + 2 \ln \left(E \frac{y_m^2}{a} + 1 \right)} \dots(4.6)$$

and the maximum jet velocity coefficient

$$\frac{\rho u_m^2 a \theta_1}{K(p - p_{\infty})b} = \frac{8 \theta_1}{D^2 \left[\left(E \frac{y_m^2}{a} + 1 \right)^2 - 1 + 2 \ln \left(E \frac{y_m^2}{a} + 1 \right) \right]} \dots(4.7)$$

where

$$D = \frac{u_1}{u_m} \dots(4.8)$$

the ratio of the square profile velocity to the corresponding maximum jet velocity, and

$$E = \frac{y_1}{\frac{y_m}{2}} \dots(4.9)$$

the ratio of the width of the square velocity profile to the half width of the jet as postulated by Newman.

It should be noted that the moment of the pressure forces at the slot is neglected on the basis that it is small compared with the jet momentum.

5. EXPERIMENTAL INVESTIGATION

Experimental measurements were carried out on a 6 inch diameter plexiglass cylinder fitted with end plates and supplied with air from a plenum chamber located on the outside of the cylinder. This was the apparatus used by Newman at Laval University. It is shown in Figure (2) and it is described in Appendix I. The experimental arrangement is shown in Figure (3.a)

Exploratory measurements made on Newman's apparatus showed that two-dimensionality was very difficult to obtain, the apparatus having been slightly damaged during shipment to McGill. After repairs and modifications, satisfactory two-dimensionality was established, however, (Figure 4), so that a set of surface pressure, velocity and separation position measurements could be made. The velocity measurements were made with pressure probes. Details of the experimental procedure followed are given in Appendix I.

It became quite clear, however, that excessive effort would be required to maintain satisfactory flow conditions for an extensive investigation. It appeared also that the flow, in particular the position of separation, was affected by the wall of the plenum chamber. A steel cylinder having the plenum chamber located within the cylinder was therefore designed and subsequently produced to very close tolerances.

Appendix II contains a description of this improved

apparatus and details of the experimental procedure.

Figures (5) and (3.b) show the apparatus and the experimental arrangement.

This improved apparatus gave consistent two-dimensionality (typical results are shown in Figures (6) and (7)), and it was used for an extensive range of experimental measurements.

Wall static pressure measurements were made, slot velocity profiles were taken by hot wires and a flat total probe, and the actual jet momentum, $\int_0^b \rho u^2 dy$, emanating from the slot compared with the theoretical jet momentum, $2(P-p_\infty)b$, was established for various $\frac{b}{a}$ ratios for a range of Reynolds numbers. The position of separation was determined for smooth and rough cylinders, and the direction of the mean velocity was established in the fully developed turbulent flow by means of very flexible tufts. The longitudinal component of the turbulent intensity was also measured.

A comparison made between velocity profiles taken by hot wires using a DISA non-linearized constant temperature anemometer and by total-static probes showed considerable discrepancy (Figure 8). A further test indicated negligible error in the hot wire anemometer output due to the lack of linearization (Figure 9, Appendix III). It was concluded, therefore, and for reasons given in Appendix III, that velocities measured with hot wires in highly curved flows are likely to be more accurate than those measured by

total-static probes. Hot wires were used, therefore, for all velocity measurements around the improved cylinder.

Special precautions were taken and appropriate calibrating and measuring techniques were used for hot wire measurements. These are described in Appendix III together with the conditions that made them necessary.

All measurements were carried out with the stagnation to atmospheric pressure ratio being less than 1.05, hence the flow could be considered effectively incompressible.

The range of the experiments and the accuracies expected are shown in Table I for Newman's and the improved apparatus.

6. DISCUSSION OF EXPERIMENTAL RESULTS AND COMPARISON WITH THEORY

6.1 Velocity Profiles

Velocity profiles measured in wall jets flowing over the surface of the improved cylinder are shown in Figures (10) for four $\frac{b}{a}$ ratios at high Reynolds numbers ($Re = 4.0 \times 10^4$). This value was found from Newman's experiments to be sufficiently high for the flow to be independent of Reynolds number. Of particular interest are Figures (10.5) to (10.7), showing the velocity traverses taken at three different times at a $\frac{b}{a}$ ratio of 0.0379 and at a Reynolds number of 4.0×10^4 with two hot wires of different diameters and materials. These figures also show the r.m.s. longitudinal component of turbulence as a percentage of the local mean velocity, $\sqrt{u'^2}/u$. The mean velocity measurements exhibit an experimental scatter especially in the outer part of the flow; but there is also a definite systematic difference between the results obtained by hot wires of different materials. The maximum deviation at the velocity maximum is of the order of $\pm 5\%$, giving an indication of the maximum error of the velocity measurements. The difference between velocity as measured by wire N7 is of the order of $\pm 2\frac{1}{2}\%$ at the maximum velocity, and since all the traverses obtained at the other three $\frac{b}{a}$ ratios were made using wire N7, a reasonable consistency in accuracy can be expected over the whole range of measurements.

Turbulence intensity profiles obtained using the two

different wires show considerable deviation where the turbulence intensity is high and/or the velocity is low. The reason for this is that the turbulence intensity as a proportion of mean velocity is very sensitive to errors in calibration which occur at low velocities.

Comparison of turbulence intensity $\sqrt{u'^2}/u$ is made in Figure (11) between present results and the measurements of Bradshaw and Gee⁽⁷⁾ on a circular cylinder. Bradshaw and Gee did not indicate the angular position where the turbulence profile was taken, but, judging from their apparatus, it must have been obtained between $\theta = 75^\circ$ and 145° . To permit comparison, the results of the present measurements taken at $\theta = 75^\circ$ and $\theta = 145^\circ$ are shown in Figure (11). Agreement with Bradshaw and Gee is good within the accuracy of the measurements. The results of the present measurement made at the two angular positions suggest experimental inaccuracy rather than a systematic difference.

The percentage turbulence intensities measured on the circular cylinder at $\theta = 75^\circ$ and $\theta = 145^\circ$ are plotted in Figure (12) against $\frac{(Y - Y_m)}{\delta_f}$, where $\delta_f = \left| u_m / \left(\frac{\partial u}{\partial y} \right)_{y_m/2} \right|$, and they are compared with similar results obtained in plane wall jets by Bradshaw and Gee and by Patel.⁽¹²⁾ Although there is some discrepancy between the results, it seems that this is due to experimental errors, and the longitudinal turbulence intensities appear comparable for plane and curved wall jets.

6.2 Non-Dimensional Velocity Profiles

Non-dimensional velocity profiles are shown for Newman's cylinder in Figure (13), where $\frac{u}{u_m}$ is plotted vs. $\frac{y}{\frac{y_m}{2}}$ from total pressure traverses taken at $\theta = 45^\circ, 90^\circ, 135^\circ, 180^\circ$ and 210° . Also shown is the theoretical curve obtained from Glauert's⁽¹³⁾ theory, for $\frac{y_m}{\frac{y_m}{2}} = 0.14$, for plane wall jets. With the exception of those at $\theta = 210^\circ$, experimental data and theory are in very good agreement except in the outer part, where the measurements are expected to be in error (see Appendices I and II). The results fall on a single curve showing similarity of velocity profiles over most of the flow. Thus Newman's results are reproduced and his conclusions concerning similarity* are confirmed.

Figures (14) show the non-dimensional velocity profiles obtained from measurements made on the improved cylinder, the traverses being taken by hot wires at $\theta = 40^\circ, 75^\circ, 110^\circ, 145^\circ, 180^\circ$ and 200° for all four gap widths. The results collapse fairly well on a single line, confirming similarity,* and they are also in very good agreement with Glauert's theory for plane wall jets, even in the outer part of the flow. At very large θ near separation, the boundary layer part of the flow grows much more rapidly than the outer part and similarity is no longer possible. Since the velocity measurements carried out by hot wires are considered to be reasonably accurate even at the outer edge of the flow,

* Similarity of velocity profiles.

their agreement with Glauert's theory tends to confirm its accuracy over the whole width of the flow.

6.3 Hypothetical Origin and Growth Law

In order to find the position of the hypothetical origin, and the values of the growth law constants, Equation (4.4) is re-written by substituting Equations (4.2) and (4.5), to yield

$$\frac{\frac{Y_m}{2}}{a(\theta + C_1 \frac{b}{a})} = C + Ck \frac{Y_m}{a} \quad \dots(6.1)$$

A straight line representing this relationship is fitted to the experimental data by varying the value of C_1 until the sum of the root mean square deviation of the experimental data from the straight line is a minimum. This was done using an IBM 1401 computer. The results are plotted in Figure (15) for Newman's cylinder (pitot tube measurements) and in Figure (16) for the improved cylinder (hot wire measurements). Newman's results are also presented in Figure (15). For comparison, the improved cylinder data are also shown in Figure (17) without a correction for the hypothetical origin.

It should be noted that Equation (6.1) reduces to

$$\frac{\frac{Y_m}{2}}{(a\theta + C_1 b)} = \frac{\frac{Y_m}{2}}{(a\theta + a\theta_o)} = \frac{\frac{Y_m}{2}}{(X + X_o)} = C \quad \dots(6.2)$$

when the cylinder radius, a , tends to infinity. Hence the growth law is universal and should reduce in the limit to that for the plane wall jet.

Table II shows the constants obtained in the present experiment, together with the corresponding plane wall jet constants of various investigators. It can be seen that the constants obtained from the hot wire data (improved cylinder) correspond very well to the plane wall jet constants, and that the pitot data constants (Newman's cylinder) do not. Hence the growth law obtained from hot wire measurements is universal and the one obtained from pitot measurements cannot be considered universal.

It is interesting to note that although the velocity profiles measured by hot wires of different materials showed a discrepancy (Figures 10.5 to 10.7), they give rise to effectively the same growth law.

Figure (18) shows the growth laws for the Newman and improved cylinders and Nakaguchi's growth law re-plotted in terms of $\frac{Y_m}{2/a\theta_1}$ vs. $\frac{Y_m}{2/a}$, together with the experimental data of Nakaguchi and Bradshaw and Gee for comparison. It can be seen that Nakaguchi's experimental range was considerably smaller than the present one. Nakaguchi's growth law does not deviate substantially from the Newman cylinder growth law within his limited experimental range, but does so rapidly beyond this range (the constants for both were established experimentally from pressure measurements). Both growth laws are certainly within the scatter of Nakaguchi's experimental results, although his results tend to follow his growth law rather than a straight line relationship.

Bradshaw and Gee's data also show moderate scatter.

Nakaguchi's and the improved cylinder growth law and experimental data are also shown plotted in polar coordinates, for comparison, (Figure 19) as $\frac{y_m}{2/a}$ vs. θ_1 .

The positions of the hypothetical origins are compared in Figure (20), where $\frac{a\theta_o}{b}$ is plotted vs. $\frac{b}{a}$ showing Nakaguchi's results and the present ones. The range of $\frac{a\theta_o}{b}$ values obtained by various investigators for plane wall jets is also given. The agreement between the improved cylinder and plane wall jet hypothetical origin positions was mentioned before. It can be seen that Nakaguchi's results also fall within the plane wall jet range when $\frac{b}{a}$ tends to zero. Furthermore, it becomes evident that the position of the hypothetical origin from the slot in Nakaguchi's experiments can be expressed fairly well by the first two terms of Equation (4.1). It appears, then, that the present growth law is universal, and Nakaguchi's growth law is only universal if one chooses to measure the flow with a pitot static tube, a procedure which introduces error.

6.4 The Effect of Slot Boundary Layers on Jet Momentum

The effect of the slot boundary layers on the jet momentum leaving the slot is shown in Figure (21) for Newman's cylinder, where the measured actual-to-theoretical jet-

momentum ratio $\frac{\int_0^b \rho u^2 dy}{2(P-p_\infty)b} = K$ is plotted vs. Reynolds number

for various $\frac{b}{a}$ ratios.

For the improved cylinder, comparison is made in Figure (22) between the non-dimensional velocity profiles obtained by the flat pitot tube and by the two hot wires at a $\frac{b}{a}$ ratio of 0.0074 and a Reynolds number of 4.0×10^4 . The measured velocity profiles are shown and also the profiles obtained when Wills' ⁽¹⁴⁾ wall effect corrections were applied to the hot wire readings. The agreement between the results of the two hot wires and the discrepancy between hot wire and pitot results is evident. Although the discrepancy in absolute magnitude of velocities is large in the boundary layers, the effect on the accuracy of the jet momentum measurements is small, as can be seen from the table accompanying Figure (23), where $(\frac{u}{u_{jo}})^2$ is plotted vs. y . The values obtained are within 5%, and this is very satisfactory for this type of measurement. Figure (24) shows the jet momentum ratios obtained by hot wire measurements for various $\frac{b}{a}$ ratios and Reynolds numbers. The flat pitot results for the smallest $\frac{b}{a}$ ratio investigated are also shown in Figure (24) (see discussion in Appendix II). The results suggest that the accuracy is good at small $\frac{b}{a}$, and at high Reynolds numbers, but there is some doubt about the accuracy at large $\frac{b}{a}$ when the Reynolds numbers are low.

Since the jet momentum ratios are also a function of upstream conditions, they are expected to change from apparatus to apparatus. Furthermore, the actual-to-theoretical

jet-momentum ratio is expected to be little affected by the cylinder radius, a . Hence the results shown in Figures (21) and (24) are not strictly universal and apply only for the particular apparatus. It is interesting to note, however, that the jet momentum ratios obtained for comparable $\frac{b}{a}$ ratios on Newman's and the improved cylinder are within 5%. This leads to the conclusion that these results may be used for other apparatus with similar duct geometry, provided it is clearly understood that they represent only an approximation in that case.

6.5 Surface Pressure

The surface pressure coefficient $(p_{\infty} - p_s)a/(P - p_{\infty})b$, measured on the improved cylinder, is shown as a function of the Reynolds number and the angular position measured from the slot in Figures (25) for the four $\frac{b}{a}$ ratios investigated, while the high Reynolds number data are plotted in Figure (26). It can be seen that the pressure coefficients are a function of both the $\frac{b}{a}$ ratio and the Reynolds number, in addition to being a function of θ , as was predicted by dimensional analysis. Figure (27) shows the high Reynolds number data for Newman's cylinder obtained by Newman and during the present investigation for comparison. The data of the two investigations agree quite well. In order that the Newman and the improved cylinder results might be compared, mean lines were fitted through the experimental data for comparable

Reynolds numbers and $\frac{b}{a}$ ratios in Figures (26) and (27). Both experimental mean lines are reproduced in Figure (28) together with Nakaguchi's and Bradshaw and Gee's experimental results for various $\frac{b}{a}$ ratios and Reynolds numbers. The two experimental mean lines are in remarkable agreement except at large θ , where the accuracy of the measurements is very low. Bradshaw and Gee's results are rather low, and Nakaguchi's results show considerable deviation. It is surprising that Nakaguchi's highest Reynolds number data gave rise to the lowest values for the coefficients, while the low Reynolds number data gave rise to the highest values. This is discussed in section 6.8.

When the surface pressure coefficient is based on half the actual jet momentum, $(p_{\infty} - p_s)a/K(P - p_{\infty})b$, and it is plotted against θ_1 , the angular position measured from the hypothetical origin, the high Reynolds number data collapse on a single line for both Newman's (Figure 29) and the improved cylinders (Figure 30), although the $\frac{b}{a} = 0.0074$ improved cylinder data show slight but consistent deviation. The agreement with theory is reasonably good for Newman's cylinder up to $\theta_1 = 170^\circ$, although the theory is expected to be in error due to the use of an incorrect growth law. The improved cylinder results and the theory based on the correct growth law show good agreement for $50^\circ < \theta_1 < 180^\circ$. Discrepancy between experiment and theory is expected when $\theta_1 > 180^\circ$ because the assumptions made in

the derivation of the theory (in particular for the geometry of the streamlines) fail when $\theta_1 \gtrsim 160^\circ - 180^\circ$. The two experimental mean lines re-plotted in Figure (31) show good agreement, the maximum deviation not exceeding 5% except at large θ_1 . This is as expected because the accuracy of the measurements is of the same order for both cylinders. The two theoretical curves also plotted in Figure (31) show that the theory is not sensitive to small changes in the growth law constants until θ_1 exceeds approximately 130° . The experimental data of Nakaguchi and Bradshaw and Gee are also plotted in this Figure. In order to do so, assumptions had to be made concerning the actual jet momentum ratio. The improved cylinder jet momentum ratios were assumed to be applicable to Nakaguchi's data without causing excessive errors, but a speculative extrapolation of Figure (24), made to correct Bradshaw and Gee's data, can be expected to be in considerable error. Nonetheless, all the data are of the right order of magnitude, although Nakaguchi's results show rather large variation.

It can be finally concluded that, although Newman's theory incorporates rather gross assumptions, agreement between theory and experiment is nevertheless satisfactory within the accuracy of the measurements, when the growth law obtained with hot wires is used in computing the theoretical curve.

6.6 Maximum Jet Velocity

The maximum velocity coefficient $\frac{\rho u_m^2 a \theta}{(P - p_\infty) b}$ is shown in Figure (32) for Newman's and the improved cylinders. The results of Newman and of the present investigation obtained from Newman's cylinder fall on a single line and appear to show only an experimental scatter. The improved cylinder data show a larger scatter ($\frac{b}{a} = 0.0379$ data) in addition to showing dependability on $\frac{b}{a}$. The experimental mean lines are of the same shape for both cylinders. The results of Nakaguchi and Bradshaw and Gee for various $\frac{b}{a}$ ratios and Reynolds numbers are compared with the two mean lines in Figure (33). It can be seen that Nakaguchi's results are of the same order of magnitude as those obtained during the present investigation, while Bradshaw and Gee's results are much lower. As was the case for the pressures, Nakaguchi's high Reynolds number measurements result in low coefficients and low Reynolds number measurements in high coefficients.

The maximum velocity coefficient based on actual jet momentum and θ_1 , hence being representative of the postulated universal flow, is shown in Figures (34) and (35) for both cylinders, together with the theoretical curves obtained using the appropriate growth laws. The data obtained on Newman's cylinder exhibit an experimental scatter and the agreement with the theory is not very good. The improved cylinder experimental data straddle the theoretical curve and in addition to a fair experimental scatter

($\frac{b}{a} = 0.0379$ data) appear to exhibit a definite variation with the $\frac{b}{a}$ ratio. This is inconsistent with the universal flow assumption. The revised data of Nakaguchi and of Bradshaw and Gee are also shown in Figure (36). (Assumptions made concerning the actual jet momentum ratios are the same as in the case of the surface pressure coefficients.) Their results show general agreement with the old cylinder results, but they exhibit a variation with $\frac{b}{a}$ ratio as did the improved cylinder data. Examination of this maximum velocity coefficient shows that it is very sensitive at small θ_1 to errors in the position of the hypothetical origin and to errors made in the velocity measurements. It is unlikely, however, that the dependence of the velocity coefficient on $\frac{b}{a}$ can be attributed solely to such errors. For further discussion see Section (6.8).

6.7 Position of Separation

The angular position of separation θ , measured from the slot, is shown as a function of Reynolds number for various $\frac{b}{a}$ ratios in Figures (37) and (38) for Newman's cylinder and the improved cylinder respectively. The Newman cylinder data fall on a single line within some experimental scatter and become independent of Reynolds number at high Reynolds numbers. Variation with $\frac{b}{a}$ ratio is not readily apparent. There is a discrepancy between

Newman's data and the present results although the same experimental procedure was followed in both cases. It is thought that the difference in assessing the time mean of the backward and forward oscillations of the flexible tuft gave rise to this discrepancy. The present results are expected to be more accurate, judging by the consistency of data obtained from a large number of observations. The improved cylinder data show some dependence on $\frac{b}{a}$, but for each $\frac{b}{a}$, $\theta_{\text{sep.}}$ maximum becomes independent of Reynolds number.

Figure (39) shows the position of separation θ_1 , measured from the hypothetical origin for high Reynolds numbers for the improved cylinder. Within the experimental scatter, $\theta_{1\text{sep.}}$ maximum is found to be a constant for all $\frac{b}{a}$ ratios at sufficiently high Reynolds numbers. The value of the Reynolds number at which $\theta_{1\text{sep.}}$ maximum is reached, $Re_{\text{lim.}}$, is a function of the $\frac{b}{a}$ ratio.⁽⁴⁾ Close examination of Figure (39) suggests that, for small $\frac{b}{a}$, ($\frac{b}{a} = 0.0074$), the limiting Reynolds number would be somewhat higher than $Re_{\text{lim.}} = 4.0 \times 10^4$, the value assumed on the basis of Newman's results.

The position of separation θ , measured from the slot, is shown as a function of Reynolds number and surface roughness, and comparison is made with the smooth cylinder results in Figures (40) and (41) for $\frac{b}{a}$ ratios of 0.0074 and 0.0531 respectively. As expected, separation occurs earlier

on the rough cylinder and $\theta_{\text{sep. maximum}}$ is a function of $\frac{b}{a}$ and surface roughness.

6.8 General Discussion

The basic criterion for the existence of a universal flow is that the total skin friction force should be negligible compared to the jet momentum. This is equivalent to the assumption that the boundary layer part of the flow is negligible compared to the outer part of the flow. This condition is partially satisfied when the Reynolds number exceeds a certain lower limiting value.

It can also be shown⁽⁴¹⁾ that the ratio of the total skin friction force (over an angular distance θ) to the theoretical jet momentum is approximately equal to

$$\frac{s}{2K(P-p_{\infty})b} = 9.2 \bar{c}_f \ln\left(1 + \frac{1}{7.43} \frac{a\theta}{b}\right) \quad \dots(6.3)$$

for small $\frac{y_m}{a}$. The assumption that the skin friction force is negligible is valid, therefore, only if $\frac{a\theta}{b}$ is sufficiently small. This puts a lower limit on the value of $\frac{b}{a}$ for which the solution will hold. The results for $\frac{b}{a} = 0.0074$, Bradshaw's results for $\frac{b}{a} = 0.0020$ and Nakaguchi's results for $\frac{b}{a} = 0.0060$ appear to have exceeded the limit slightly. It is difficult to establish a definite value for this limit at this stage because the value of the limit also depends on $\bar{c}_f$, which in turn depends on the internal slot boundary-layer

Reynolds number. However, it is probably about 500 and is better expressed as a limiting value on $\frac{a\theta}{Kb}$. In addition, the $\frac{b}{a} = 0.0074$ data were apparently obtained at a Reynolds number which seems to be below $R_{e,lim.}$ (see Section 6.7). In spite of this, the position of separation, $\theta_{1sep.}$ maximum, for the small $\frac{b}{a} = 0.0074$ test (Figure 39) does not appear to be affected. A possible explanation for this is that the initial boundary layer at the slot being thin, the boundary layer is thin at large θ even though the skin friction is large. For larger $\frac{b}{a}$ ratios, the initial slot boundary layer is usually thicker. This compensating effect will also reduce the error in the growth law. On the other hand, the momentum loss due to friction is largest for the smallest $\frac{b}{a}$ ratio and is important in determining the decay in maximum jet velocity. This may explain why all the small $\frac{b}{a}$ experimental data result in low maximum velocity coefficients in spite of the relatively high Reynolds numbers.

It appears, then, that the inclusion of the $\frac{b}{a} = 0.0074$ data has probably introduced some error when the growth law and the relation for the position of the hypothetical origin were calculated. The error in growth law constants would only slightly affect the theoretical prediction for the surface pressure coefficient, but the prediction of the maximum velocity coefficient could be considerably affected. The inclusion of the slightly low Reynolds number and low $\frac{b}{a}$ data in the surface pressure coefficient

plot is unlikely to cause noticeable discrepancies, but its use in the maximum velocity coefficient graph, especially with erroneous hypothetical origin values, is expected to give rise to fairly large errors at low θ_1 .

On the positive side it appears that, if allowance is made for the above errors, there is adequate evidence to support the existence of a universal flow sufficiently far from the slot, as long as $\frac{a\theta}{Kb}$ does not exceed a value of about 500 and the Reynolds number is high enough ($> 4 \times 10^4$).

The relative merits of Newman's and Nakaguchi's analyses can finally be compared.

Newman's universal flow model and analysis are very attractive because of their simplicity, because only the growth law constants need to be established experimentally in order to predict theoretically the surface pressure and the maximum velocity coefficients, and because these predictions appear to be in reasonable agreement with experiment. Their draw-back is that they apply only if the Reynolds number and the $\frac{b}{a}$ ratio are sufficiently high, and even then it is necessary to know the actual-to-theoretical jet-momentum ratio in order to predict the actual surface pressures and maximum velocities. Fortunately, a reasonable estimate of the jet-momentum ratio is possible and can be based on previous experiments, and in most practical applications high, rather than low, Reynolds number flows are of interest.

Nakaguchi's analysis is more sophisticated than Newman's, in that he takes friction into account and also uses a more realistic velocity profile. However, all streamlines were assumed to have the same curvature as the surface and in order to relate theory and experiment it was necessary to match the two at an arbitrary pivotal point. Newman's analysis is therefore certainly more useful.

7. CONCLUSIONS

It is apparent that the present measurements constitute a distinct improvement over those of previous investigators. In addition to velocity and surface pressure measurements around the cylinder, slot velocity profiles were taken and the ratio of actual-to-theoretical jet momentum at the slot was established. Improved measuring techniques were used for the velocity measurements and strong evidence was found that velocity and, especially, static pressure measurements in highly curved flows using conventional total and static probes can be in considerable error.

It was confirmed that the velocity profiles are similar over the major portion of the flow, except near separation, and it was found that they show good agreement with Glauert's theoretical prediction for plane wall jets, even near the edge of the flow.

A method for finding the growth law constants and for finding a relation for the position of the hypothetical origin was developed. The constants for Equation (6.1)

$$\frac{\frac{Y_m^2}{2}}{a(\theta + C_1 \frac{b}{a})} = C + Ck \frac{Y_m^2}{a}$$

obtained by hot wire measurements, were found to be equal to

$$C_1 = 7.43, \quad C = 0.0731 \quad \text{and} \quad k = 3.19.$$

These may be in error (Section 6.8), but the error is expected to be very small.

When the surface pressure coefficient was based on the actual jet momentum that emanates from the slot, satisfactory agreement was found between the experimental data and the predictions of Newman's theory.

The results for the maximum velocity coefficient were inconclusive, in all probability because data from a flow having Reynolds number and $\frac{b}{a}$ value too low to be valid for the universal flow were included. Furthermore, the same data were used when the constants for the hypothetical origin relation and for the growth law were established. As a consequence these constants are expected to be slightly in error, resulting in additional errors in the presented values of the maximum velocity coefficient. The theoretical curve for the maximum velocity coefficient would also be affected by these errors, but not the theoretical surface pressure coefficient which is insensitive to even fairly large changes in the constants just mentioned.

The maximum position of separation measured from the hypothetical origin ($\theta_{1sep. \text{ maximum}}$) became independent of $\frac{b}{a}$ and Reynolds number at sufficiently high Re ($> 4 \times 10^4$), the value being equal to approximately 225° for smooth cylinders. The maximum position of separation measured from the slot ($\theta_{sep. \text{ maximum}}$) for rough cylinders was also obtained ($\frac{h}{a} = 0.0014, 0.0024, 0.0045$). It was found that

$\theta_{\text{sep.}}$ maximum varied with $\frac{b}{a}$ ratio and surface roughness and that it was reduced by approximately 40° for the largest $\frac{h}{a}$ ratios.

The experimental surface pressure, separation and relevant maximum velocity data tend to support the validity of Newman's universal flow model.

Comparison of Newman's and Nakaguchi's analyses showed Newman's analysis to be the more useful.

Further investigation is required, by taking measurements on cylinders having widely varying radii, to establish a more accurate value for the limiting Reynolds number and a value for the limiting $\frac{a\theta}{Kb}$ ratio, and to verify the growth law and the position of the hypothetical origin.

An extensive investigation appears to be justified to establish with certainty the relative accuracy of velocity measurements made in highly curved flows by means of total-static probes and by means of hot wires using a linearized anemometer.

Comprehensive measurements of turbulence intensities and of the structure of turbulence would be valuable for comparison with plane wall jets, especially since the non-dimensional velocity profiles and the turbulence measurements of the present investigation appear to show comparable results for plane and curved wall jets.

APPENDIX I

APPARATUS AND EXPERIMENTAL PROCEDURE -

NEWMAN'S APPARATUS

Experimental Arrangement

Newman's apparatus is shown in Figure (2). It consisted basically of a 6 in. O.D. plexiglass cylinder which was very carefully selected for straightness and roundness and had a wall thickness of one half inch, two circular end plates of 24 in. diameter, and a surface ground steel bar which formed the outer lip of the slot. The contraction was formed by the plexiglass cylinder on one side and a sheet metal wall, extending backward from the steel lip, on the other. A small plenum chamber preceded the contraction. Air was supplied to the plenum chamber through a deep cell honeycomb after the air had been decelerated through a 40° diffuser. A perforated plate was installed in the diffuser to prevent flow separation from the walls. A 45-gallon drum was used as a reservoir to damp out pressure fluctuations, and a bleed valve rather than a throttling valve was used between the reservoir and the centrifugal pump, to regulate the flow in order to reduce pressure fluctuations to a minimum. In fact, the plenum chamber stagnation pressure was maintained within $\pm 1\%$ of its mean value.

Static pressure holes of 0.015 in. diameter were accurately drilled at 5° intervals around the cylinder and they were located in two rows at one half inch to the right

and to the left of the mid-point of the cylinder.

The slot width could be adjusted by moving the ground steel lip by means of two screws, the maximum slot width obtainable being 0.120 in. Since the cylinder length was 24 in. between the end plates, the slot aspect ratio varied from 200:1 to 2400:1 as the slot width was changed. The uniformity of the slot width could be kept to within a $\frac{b}{a}$ of ± 0.001 over the middle portion of the cylinder. The contraction ratio from plenum chamber to slot ranged from 17:1 to 200:1.

A two-inch wide cut-out was made along the length of the cylinder between the end plates, starting at an angular position of 245° from the slot to vent the flow to atmosphere after separation. This reduced the probability of interference between the air supply box and the flow. Metal spacers were glued in place in the cut-out to prevent the distortion of the cylinder.

An aluminum beam was attached to the end plates by means of friction pads. This beam supported a dial indicator traversing gear. Probes were mounted on the traversing gear by suitable supports, and could be positioned with an accuracy of 0.001 in. from the wall in the radial direction. The beam could be moved along the edges of the end plates, thereby making it possible to position the attached traversing gear and probe at any angular distance from the slot around the cylinder. The probe support could be tilted, permitting the

alignment of the probes with the flow direction.

An inclined multi-tube alcohol manometer was connected to the surface static pressure holes by tygon tubing, and a single tube manometer was used to measure the total pressure. The experimental arrangement is shown in Figure (3.a).

Experimental Procedure

In order to check two-dimensionality the total pressure was measured along the slot at several positions at the mid-point of the gap width. When no deviation from two-dimensionality could be detected, a further check was made by taking velocity profiles far downstream from the slot in the fully developed turbulent wall jet. These measurements showed a marked deviation from two-dimensionality. Examination of the ground steel lip showed several small nicks and areas where the sharp edge of the lip was slightly rounded. When the end of a steel ruler was pulled along the front face of the slot lip, thereby scraping the edge slightly, the two-dimensionality changed somewhat. This showed that even a small imperfection of the slot lip will have a detrimental effect on two-dimensionality, due to Coanda effect. After the sharp edge was restored by grinding, two-dimensionality was improved but was still not satisfactory. In order to see whether this was possibly due to uneven slot width caused by waviness of the plexiglass wall, measurements

were made first with the slot width kept uniform and then with the slot width being much larger at one end of the cylinder than at the other. Figures (42) and (43) show the results of these measurements. The effect of non-uniformity of slot width was as expected, but the dominant feature was still the previously exhibited lack of two-dimensionality superimposed on that due to non-uniformity of slot width. Examination of the sheet metal contraction wall showed some, but not conspicuous, unevenness. After it was very carefully smoothed, satisfactory two-dimensionality was obtained (Figure 4).

The flow temperature was measured at the slot with a mercury thermometer. The measured temperature was assumed to be the static temperature since the difference between static and total temperatures was calculated to be only of the order of one degree Fahrenheit. The Reynolds number was calculated on the basis of these measurements, and the atmospheric pressure read on a mercury barometer.

A very flexible tuft (down from a bird's wing), attached to a thin wire probe, was used to traverse the flow to find the approximate flow direction. The accuracy obtained was not very great. The results of the survey were conclusive enough, however, to permit the positioning of a chamfered, round, 0.028 in. O.D. total pressure probe with accuracy sufficient to measure the stagnation pressure, i.e. $\pm 15\%$ ⁽¹⁵⁾.

The uncertainty of the flow direction, the high degree of turbulence, and the relatively large diameter of the available probe could give rise to fairly large errors if static pressure measurements were made in the curved flow (18, 19, 20). No attempt was made, therefore, to obtain the static pressure distribution across the flow and it was thought best to assume that the static pressure rose linearly from the accurately known wall pressure to atmospheric pressure, at a distance from the wall where the total pressure fell to zero with reference to atmospheric pressure. The assumption of a linear static pressure distribution was to some extent justified by Newman's⁽⁴⁾ measurements. Velocities were calculated from the total pressures measured and from the assumed linear static pressure distribution.

The tuft traverses also showed, in spite of their relative inaccuracy, that the direction of the mean velocity vector did not deviate substantially from the direction tangential to the cylinder until it became somewhat larger than $\frac{y_m}{2}$. The measured velocity was therefore assumed to represent the tangential component of velocity fairly accurately.

When velocity measurements were made at the slot the slot static pressure was assumed to be atmospheric, because the surface pressures registered at the slot did not give conclusive evidence that there was a deviation from atmospheric pressure, although downstream from, but close to the slot, higher than atmospheric pressures were measured.⁽¹⁶⁾

Before static pressure measurements were made the cylinder was carefully wiped and the static pressure holes were cleaned and reamed with a nylon bristle. This was necessary because Newman⁽⁴⁾ found that even a slight imperfection of a static hole (due to dust or burrs) results in erroneous pressure readings, especially close to the slot.

Down from a bird's wing, mounted at the end of a fine wire, was moved along the surface of the cylinder and its oscillations were observed in order to find the mean position of separation. The tuft showed very clearly the flow direction at the surface before the point of separation, and also the back flow well past it. As the point of separation was approached from either direction, the tuft began to oscillate back and forth. It was assumed that the mean position of separation was located where the time average of the forward and backward oscillations equalled zero. This is the same method as Newman used.

APPENDIX II

APPARATUS AND EXPERIMENTAL PROCEDURE - IMPROVED APPARATUS

Experimental Arrangement

Measurements on Newman's cylinder indicated that the flow was affected by the wall of the plenum chamber. In order to see whether the flow would separate earlier or would flow further around the cylinder when this wall constraint was removed, an improved cylinder without an external plenum chamber was designed. An 8 in. diameter, very accurately machined, steel cylinder formed the main part of this improved apparatus shown in Figures(5). The cylinder was fitted with 4 ft. diameter end plates and it was supplied with air from a plenum chamber located within the cylinder, the air reaching the plenum chamber through bulkheads located at the ends of the cylinder. This arrangement permitted the smooth cylinder surface to extend to 290° from the slot, and the outer wall of the contraction, which started there, and the slot lip protruded only slightly above the cylinder profile. The jet was thus free to flow 360° or more around the cylinder.

A centrifugal blower supplied air to the apparatus and a bleed valve was used for flow control. When hot wire measurements were made, the air was passed through a bag-type air filter mounted at the suction side of the blower. A semi-cylinder made of perforated sheet metal formed the

plenum chamber within the main cylinder; it served to reduce the supply pressure pulsations and in particular to distribute the flow uniformly along the length of the cylinder.

The cylinder surface static pressure holes were 0.015 in. in diameter and were positioned around the cylinder at its centre line at angular positions shown in Figure (5.2). Tygon tubing led from the pressure taps through the plenum chamber to the pressure bulkheads at the cylinder ends, and from there to a multitube manometer.

The inner wall of the contraction was formed by a small semi-cylinder, the surface of which was tangential to the surface of the main cylinder at the slot. The outer wall of the contraction was formed by a segment of a large diameter pipe, its surface being parallel to the tangent of the main cylinder where the outer wall formed the slot lip. The pipe segment was bolted to the main cylinder and the slot width could be varied by inserting shims of various thicknesses between the two.

Slot aspect ratio varied from 650 to 1200 with changing slot width, and the slot width uniformity was kept to within a $\frac{b}{a}$ of ± 0.001 over the whole 36 in. length of the cylinder and was considerably better than this over its middle portion.

Stagnation pressure and temperature in the cylinder were measured respectively by means of a pressure probe and the sensing element of a tele-thermometer inserted through

one of the bulkheads into the area behind the small semi-cylinder.

The cylinder ends rested in bearings mounted in the end plates and the whole cylinder assembly could be rotated to any angular position about the cylinder axis. It was, therefore, possible to mount the traversing gear support permanently at a certain position on the end plates and align it accurately with the cylinder. A dial indicator traversing gear could be moved axially and radially while the whole cylinder assembly was rotated about its axis when the angular position was changed.

A simple probe support was incorporated in the traversing gear, which permitted probe alignment with the flow while maintaining the sensing point of the probe in a particular position (Figure 3.c).

Pressures were measured by vertical and inclined manometers and hot wire measurements were made using a DISA constant temperature anemometer⁽¹⁷⁾, by means of a 0.000226 in. diameter platinum plated tungsten (N7) and a 0.000423 in. diameter 80% platinum-20% iridium (N2) wire. The experimental arrangement is shown in Figure (3.b), and details of the hot wire anemometry are given in Appendix III.

A low temperature oven, in conjunction with a potentiometer, was used to calibrate hot wires as resistance thermometers. A short length of pipe with a bellmouth entry, fed by air from a large reservoir, was used for velocity calibrations (Figure 44).

Experimental Procedure

Tests were carried out to see if rotation of the cylinder to various angular positions, or asymmetry of the air supply ducts would alter the flow. Surface static pressure and total pressure measurements and tuft surveys were made, while the cylinder was rotated to extreme angular positions and the air supply ducts were severely distorted. No change could be detected in the flow due to these manipulations.

Similar measurements were made to study the effect of cylinder and slot aspect ratio by progressively moving additional end-plates inwards towards the centre. The results are shown in Figure (45). It can be seen that the maximum variation in the pressure coefficients was only $3\frac{1}{2}\%$ when the aspect ratio was changed from 8 to 2.75. The pressure coefficients tended to constant values at high aspect ratios, showing that the cylinder and slot aspect ratio was sufficiently large.

Two-dimensionality was checked by taking velocity profiles at various spanwise positions and at an angular position of 110° from the slot. Typical results are shown in Figures (6) and (7) for a Reynolds number of 4.0×10^4 and a $\frac{b}{a}$ ratio of 0.0531. The maximum spanwise non-dimensional

velocity gradient at the centre line $\frac{d(\frac{u}{u_{max_c}})}{dz}$ was found to be 0.01/in., and this was considered sufficiently small for the flow to be effectively two-dimensional.

Calculation of the Reynolds numbers was based on plenum chamber total pressure and temperature, and the barometric pressure.

The surface static pressure holes were carefully cleaned before readings were taken and the measurements were repeated three times in order to increase accuracy.

Slot velocity profiles were measured with hot wires because probe size limitations prevented the measurement of the static pressure in the region of greatest interest, very near the walls. The hot wires were calibrated in the slot in order to eliminate temperature effects due to the temperature difference between calibrating flow and the flow in the slot. First a hot wire profile was taken, then a static pressure profile using a 0.020 in. diameter static probe. The radial position of the maximum jet velocity was established from the hot wire data, and the static pressure at that position obtained from the static pressure profile. This static pressure with the plenum chamber pressure was used to establish the maximum jet velocity. The hot wire reading corresponding to this maximum jet velocity was then used with the zero velocity reading to obtain the wire calibration⁽¹⁷⁾. The velocities obtained using this calibration were in error near the wall, however, because of additional heat loss to the wall. Wills⁽¹⁴⁾ investigated this problem and the corrections obtained by him were applied to find the boundary layer velocities. This procedure was

used in the case of all slot widths investigated except in that of the smallest ($b = 0.0295$), which was too small to permit the use of the static probe. It was found, however, when the static pressures used for the large slot width maximum jet velocity calculations were extrapolated, that the static gauge pressure for the smallest slot width would be of the order of 1.5% of the plenum chamber gauge pressure. It could be assumed, therefore, that the slot static pressure at this gap width was equal to atmospheric pressure, and it was permissible to calculate velocities from total pressure profiles obtained by the use of the 0.007 in. high flattened total pressure probe for comparison with the hot wire profiles.

The mean flow direction at various angular positions was established by mounting a very flexible tuft at the end of a thin wire probe on the traversing gear and observing its oscillations in the vertical plane with the aid of a protractor device and a mirror (Figure 3.c). The mean position of the oscillations was taken as the mean flow direction. Figure (46) shows the results. The angles so found were used to calculate the tangential velocity components from the hot wire velocity measurements.

Since it was known that the turbulence level is high in the curved flow, and that there exists a radial pressure and temperature gradient, and since the eddy structure was unknown, it was expected that the hot wire,

the total pressure, and especially the static pressure readings (mainly due to yaw effects) would be in error (18, 19, 20). Exploratory measurements were made, therefore, at $\theta = 120^\circ$ at a Reynolds number of 8.25×10^4 with all the available probes, in order to assess these errors. The probes included a 0.060 in. diameter double probe, the total pressure tube being chamfered and the static pressure tube having two horizontal holes, i.e. holes aligned with the cylinder axis, a 0.028 in. diameter chamfered total pressure tube, a 0.039 in. diameter static pressure tube having two holes in the horizontal direction and two in the vertical, a similar probe having only two holes in the horizontal direction, and the two hot wires (Figure 3.c). The velocity traverses taken with the hot wires and the double probe were repeated and one profile each of the two-hole static, four-hole static-total tube combinations was obtained. The results, plotted in Figure (8), show remarkable consistency for the hot wire data and for the total-static probe data, but show considerable discrepancy between the two. In order to find which of these is likely to be more accurate, an attempt was made to linearize the anemometer output signal and thereby eliminate errors due to turbulent fluctuations in measuring mean velocity. The linearizer could not be stabilized, however, with the available equipment. One test was made, nevertheless, to get some indication of the errors associated with the high

turbulence. This was done by taking a mean velocity profile in the cylinder flow, noting the anemometer output signal and the linearizer output signal. A calibration was made immediately after this, followed by the measurement of the same velocity profile again. It was found that the two profiles as measured by the anemometer were in agreement, that portions of the linearized readings taken just before (outer part of velocity profile) and just after the calibration (inner part of velocity profile) agreed with the anemometer readings (Figure 9). It is clear that this test cannot be considered conclusive, yet it could be assumed that the effect of turbulence on the accuracy of the non-linearized hot wire readings was very small or negligible for the flow investigated, and that the total-static probe measurements were in error. Thereafter, all the velocity traverses were taken by means of hot wires.

The position of separation was established in the same manner as in the case of Newman's cylinder, except that for the smooth wall measurements an array of tufts mounted on the surface was also used. When tests were made to find the effect of surface roughness on the position of separation, the cylinder surface was covered with garnet papers of various grit sizes for $\theta > 40^\circ$.

APPENDIX III

HOT WIRE ANEMOMETRY

General

There is a divergence of opinion about the use of hot wires for mean velocity measurements. Kováshnay⁽²¹⁾ does not recommend the use of hot wires for mean velocity measurements mainly because dust accumulation on the wires changes their calibration and gives rise to an output signal which is in considerable error.

Temperature variations affect hot wire calibration, and errors made in reading the anemometer output voltage are magnified when the corresponding velocities are calculated.

The errors associated with ambient temperature change became evident when a simple calibration was made when the flow temperature was varied. The calibration scatter was unacceptable.

In spite of these difficulties, Collis⁽²²⁾ obtained very good results by keeping the wires always clean, by very careful experimentation, and by modifying the functional form of King's⁽²³⁾ law.

It is recalled that Collis found a universal relationship

$$N_u \left(\frac{T_m}{T_a} \right)^{-m} = A + B R_{ew}^n \quad \dots (A.III. 1)$$

where N_u	Nusselt number = $\frac{H}{\pi k_f \ell (T_w - T_a)}$,
T_w	absolute temperature of wire,
T_a	ambient absolute temperature,
T_m	arithmetic mean temperature $\frac{(T_w + T_a)}{2}$
R_{ew}	Reynolds number of the wire = $\frac{u d_w}{\nu}$
H	heat loss from hot wire
k_f	thermal conductivity of fluid
ℓ	length of hot wire
u	mean velocity of fluid past the wire
d_w	diameter of wire
$A = 0.24$	for $0.02 < R_{ew} < 44$
$B = 0.56$	
$m = 0.17$	
$n = 0.45$	

provided the wire aspect ratio was greater than 10^3 . The Nusselt number and the wire Reynolds number were evaluated at the mean temperature T_m as defined above, and the data were corrected for molecular effects and for metallic conduction to the wire supports.

The wires used in the present investigation, a 0.000226 in. diameter platinum plated tungsten wire (N7) and a 0.000425 in. diameter 80% platinum-20% iridium wire (N2), had an aspect ratio of 200 and 120 respectively.

Since the wire aspect ratios were considerably lower

than 10^3 , it was expected that the constants, A, B and n in Collis' law would change due to conduction to the metallic wire supports.

Since A, B and n were to be evaluated experimentally to give only a calibration and not a universal relationship, no corrections were made for metallic conduction or molecular effects in order to simplify the calculations.

It was known from preliminary measurements that the maximum ambient temperature variation in the flow around the cylinder was about 10°F , and that the maximum temperature difference between calibrating flow and cylinder flow did not exceed 10°F . A test was therefore made by calibrating the wires by the use of Equation (A.III. 1) in two flows having a temperature difference of 10°F , in order to see if measurements using Collis' law would give satisfactory results over the expected experimental temperature range. Figures (47) and (48) show the results of these calibrations for wires N2 and N7 respectively. It can be seen that the results lie on a straight line for each temperature and that the two straight lines deviate only slightly from each other. Examination of Collis' equation shows that the maximum error in velocity due to this deviation, and hence due to temperature variation, is of the order of 3%. This was thought to be acceptable and Collis' formula was used for hot wire measurements.

Detailed Procedure for Use of the Hot Wires

The wire was placed under a 100-power microscope and its physical condition, including straightness, condition of the support welds and cleanliness, examined. The wire was then washed in chromic acid, rinsed in distilled water and alcohol, and examined again under the microscope. If found completely clean, the wire was placed into a low temperature oven and connected to the anemometer. The wire was retained in a plastic cap very close to a permanently-installed copper constantan thermocouple so that the thermocouple and the wire were always at the same temperature. The resistance of the wire could be measured by the anemometer and the wire temperature by a potentiometer. The oven temperature was varied and the wire was calibrated as a resistance thermometer. For velocity calibration the wire was placed into the laminar core of a jet emanating from a short piece of pipe (Figure 44). Air flowed from a large plenum chamber through a suitable contraction to this pipe and the total pressure in the laminar jet core was known for all jet velocities from previous calibration of the drum pressure against a total pressure tube placed in the jet core. The resistance of the wire corresponding to the flow temperature was measured first, then the output of the anemometer, the wire overheat ratio being kept constant. This was repeated for various flow velocities covering the whole range of velocities to be measured using this calibration. From these data the constants for Collis' law were

determined, using an IBM 1401 computer. When velocities were measured in the flow around the cylinder, the wire resistance corresponding to the flow temperature was measured at each point of the traverse before the corresponding anemometer output reading was taken. The wire overheat ratio was kept constant and identical to the calibration overheat ratio. The velocities were calculated from this information, using Collis' law with the constants established from the previous calibration.

One set of six velocity profiles was measured after each calibration.

The wire was then examined under the microscope, washed and the whole calibration procedure repeated before further measurements were taken, in spite of the fact that very little, if any, dust accumulation could usually be detected on it, owing to the efficiency of the upstream filter system that had been installed (see Appendix II).

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TABLE I

RANGE OF EXPERIMENTS AND EXPECTED ACCURACY

Quantity	Improved Cylinder		Newman's Cylinder		Notes
	Range	Accuracy $\pm$ %	Range	Accuracy $\pm$ %	
a	4.0 in.	0.2	3.0 in.	1	
$\frac{b}{a}$	0.0074 0.531	2 0.5	0.0200 0.0400	5 2	
θ	$2\frac{1}{2}^\circ$ 250°	10 0.2	5° 240°	10 0.2	
R_e	0.25×10^4 8.00×10^4	2 1	0.50×10^4 4.45×10^4	4 2	
$\frac{y_m}{a}$	0.073 1.59	3 4	0.1 1.26	6 7	
α	-	$7\frac{1}{2}$	-	15	
$P - p_\infty$	-	2 0.2	- -	2 0.2	at Low R_e at High R_e
$p_\infty - p_s$	-	2 0.4	- -	5 1	at Low R_e and $\theta < 180^\circ$ at High R_e and $\theta < 180^\circ$
u	-	5	-	10	
$\frac{\sqrt{u'^2}}{u}$	-	10	-	-	
K	-	3	-	5	
$\theta_{sep.}$	-	7	-	7	

TABLE II

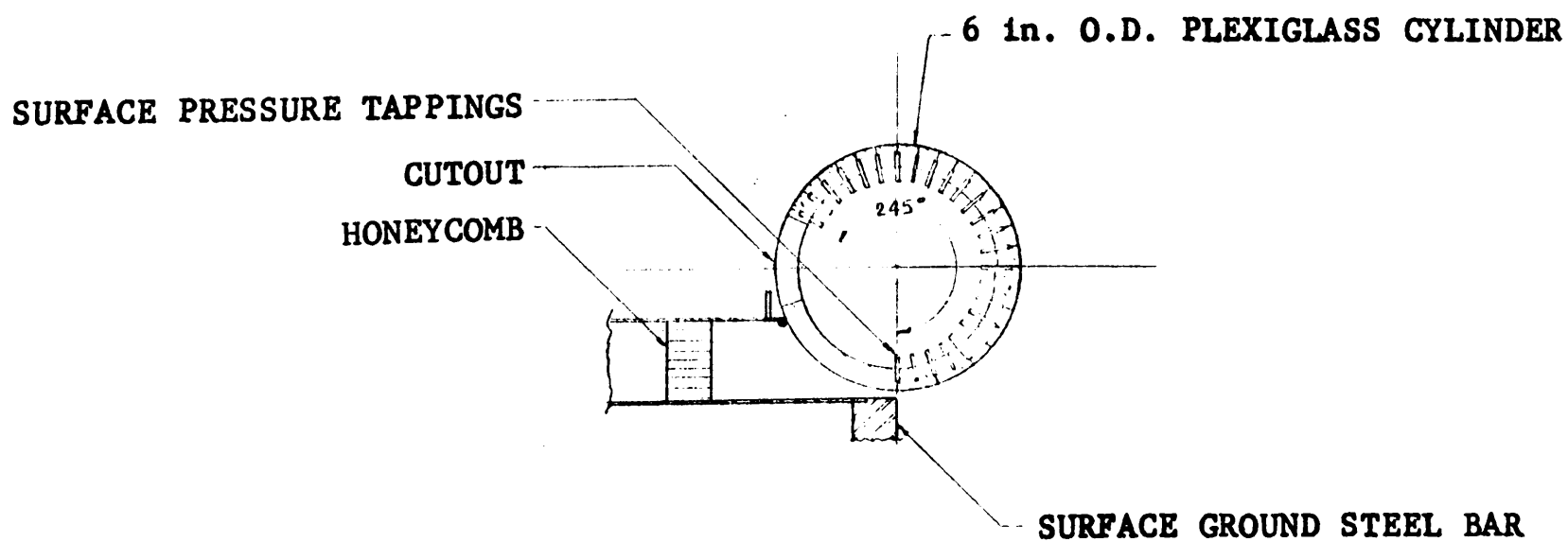
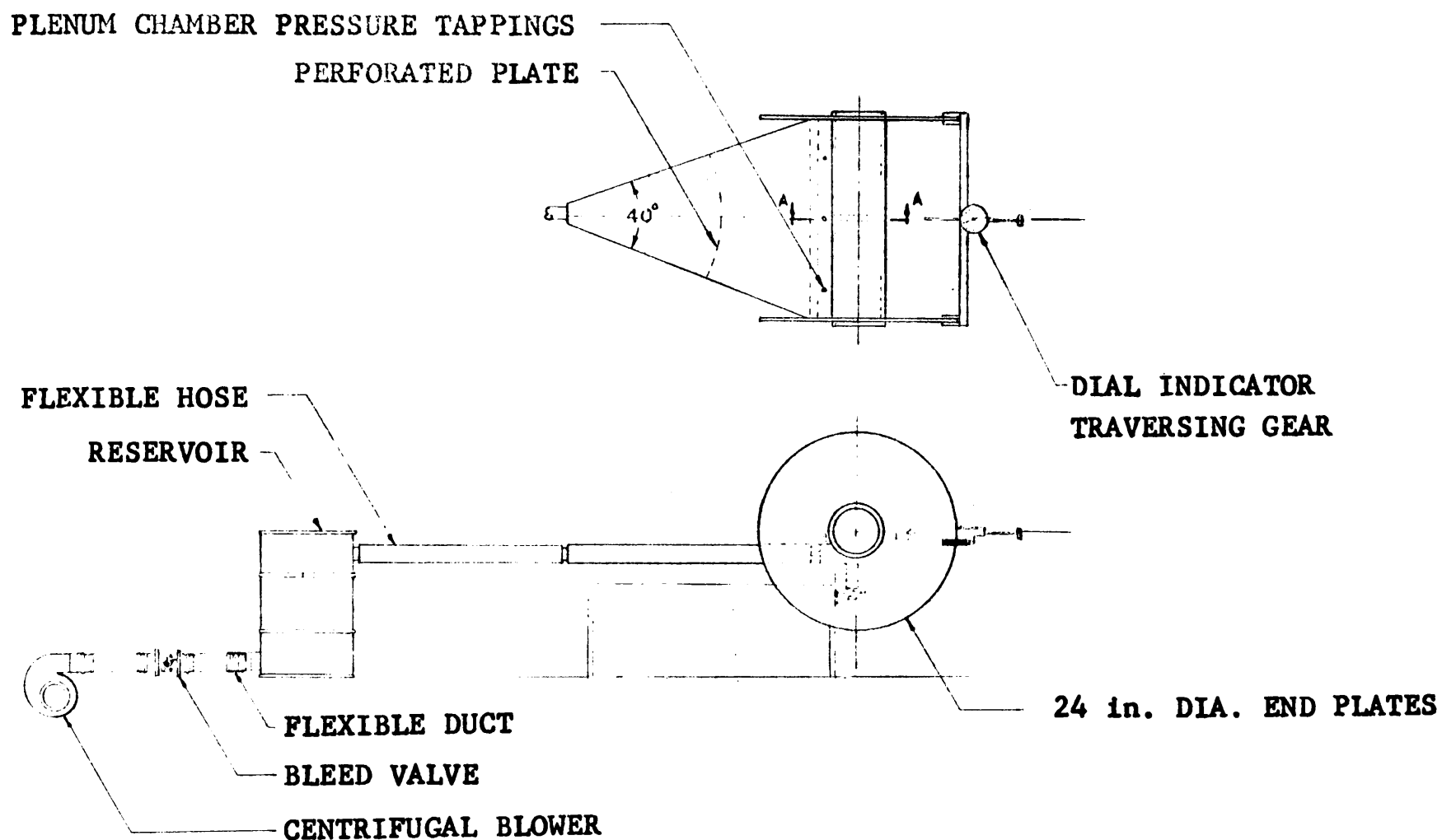
CONSTANTS IN EQUATION 6.1

	INVESTIGATOR	PROBE	C	C ₁	k	$Re = \frac{u_{jo}^b}{\nu}$
Plane Wall Jet	Sigalla ⁽¹⁶⁾	Pitot	0.0638	8.33	-	2.34×10^4 to 5.35×10^4
	Myers et al. ⁽³²⁾	Kiel	0.076	6.5	-	1.86×10^4 to 3.47×10^4
	Schwarz and Cosart ⁽³¹⁾	Hot Wire	0.0851	7.2	-	1.35×10^4
			0.0694	15.5	-	2.01×10^4
			0.0561	12.9	-	3.00×10^4
			0.0607	8.97	-	4.16×10^4
Wall Jet on Cylinder	Newman ⁽⁴⁾	Pitot	0.11	-	1.5	1.00×10^4
	Present Investigation	Pitot	0.1004	2.93	1.839	0.80×10^4
		Hot Wire	0.0731	7.43	3.185	0.80×10^4

1
88
1

FIG. 2

NEWMAN'S APPARATUS



SECTION A - A

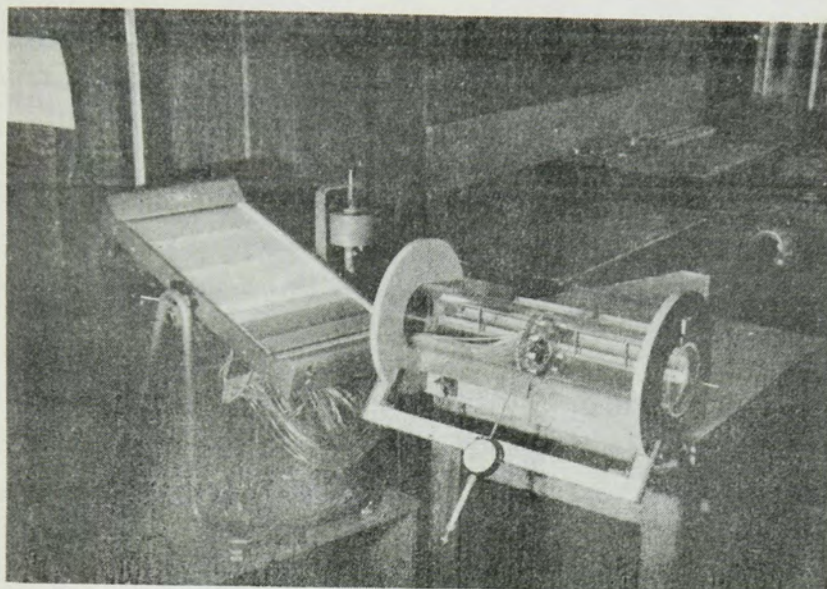


FIG. 3.a

EXPERIMENTAL ARRANGEMENT - NEWMAN'S CYLINDER

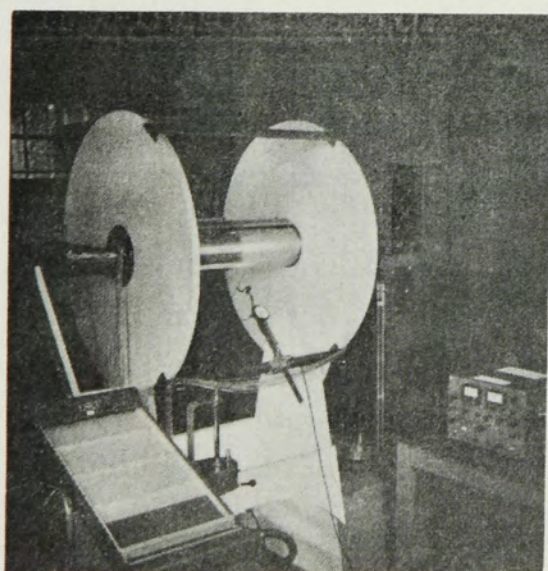


FIG. 3.b

EXPERIMENTAL ARRANGEMENT -
IMPROVED CYLINDER

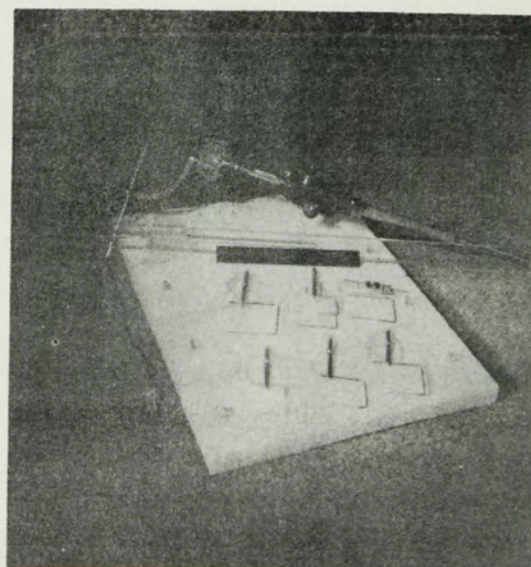


FIG. 3.c

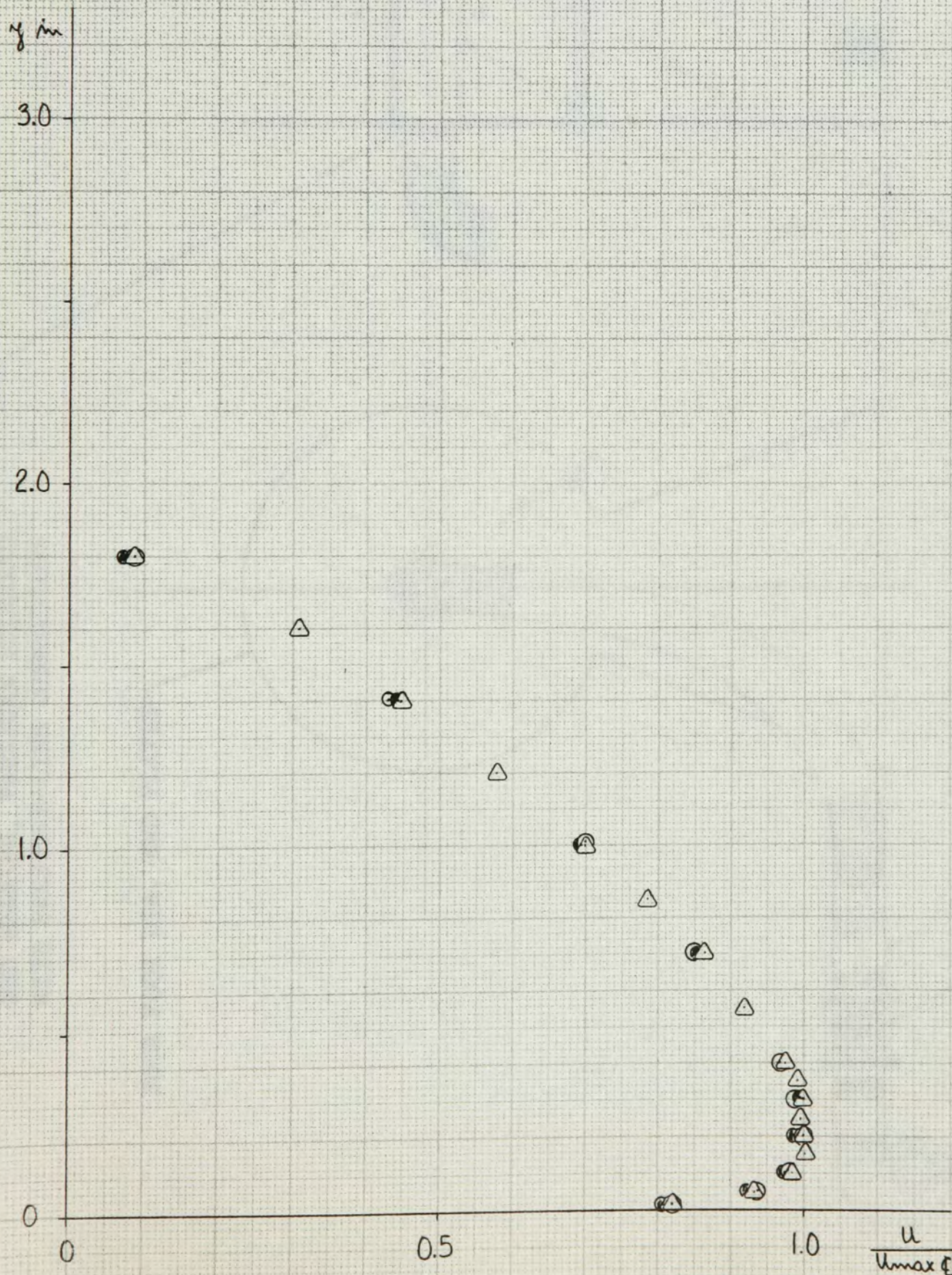
TRAVERSING GEAR, PROTRACTOR
DEVICE, AND PROBES

FIG. 4

TWO-DIMENSIONALITY CHECK- NEWMAN'S CYLINDER

$$a = 3.0 \text{ in.}, \frac{b}{a} = 0.0200, \theta = 135^\circ, Re = 4.1 \times 10^4$$

	○	△	●
z in.	-2	0	+2



IMPROVED APPARATUS

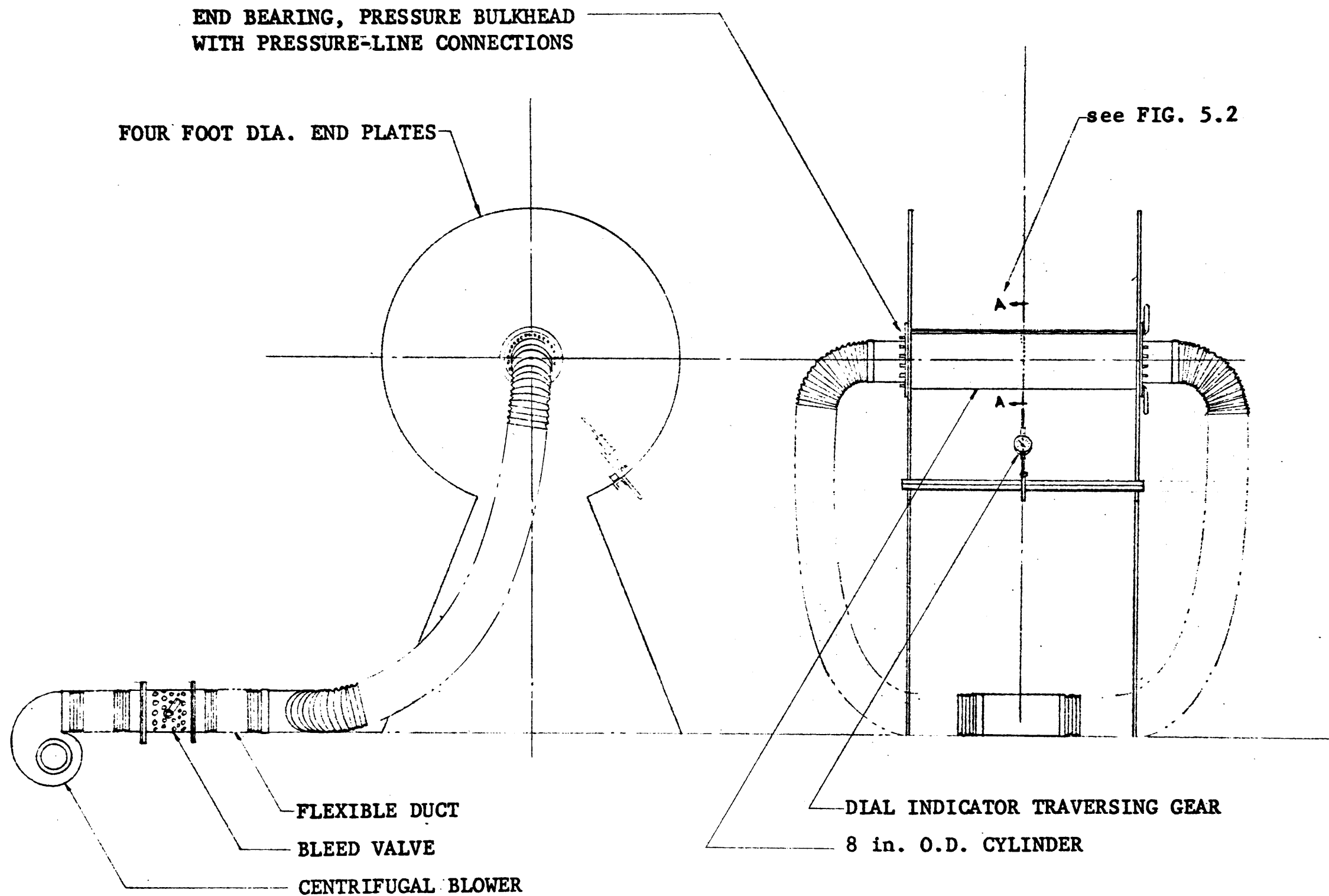


FIG. 5.2

IMPROVED APPARATUS - DETAIL

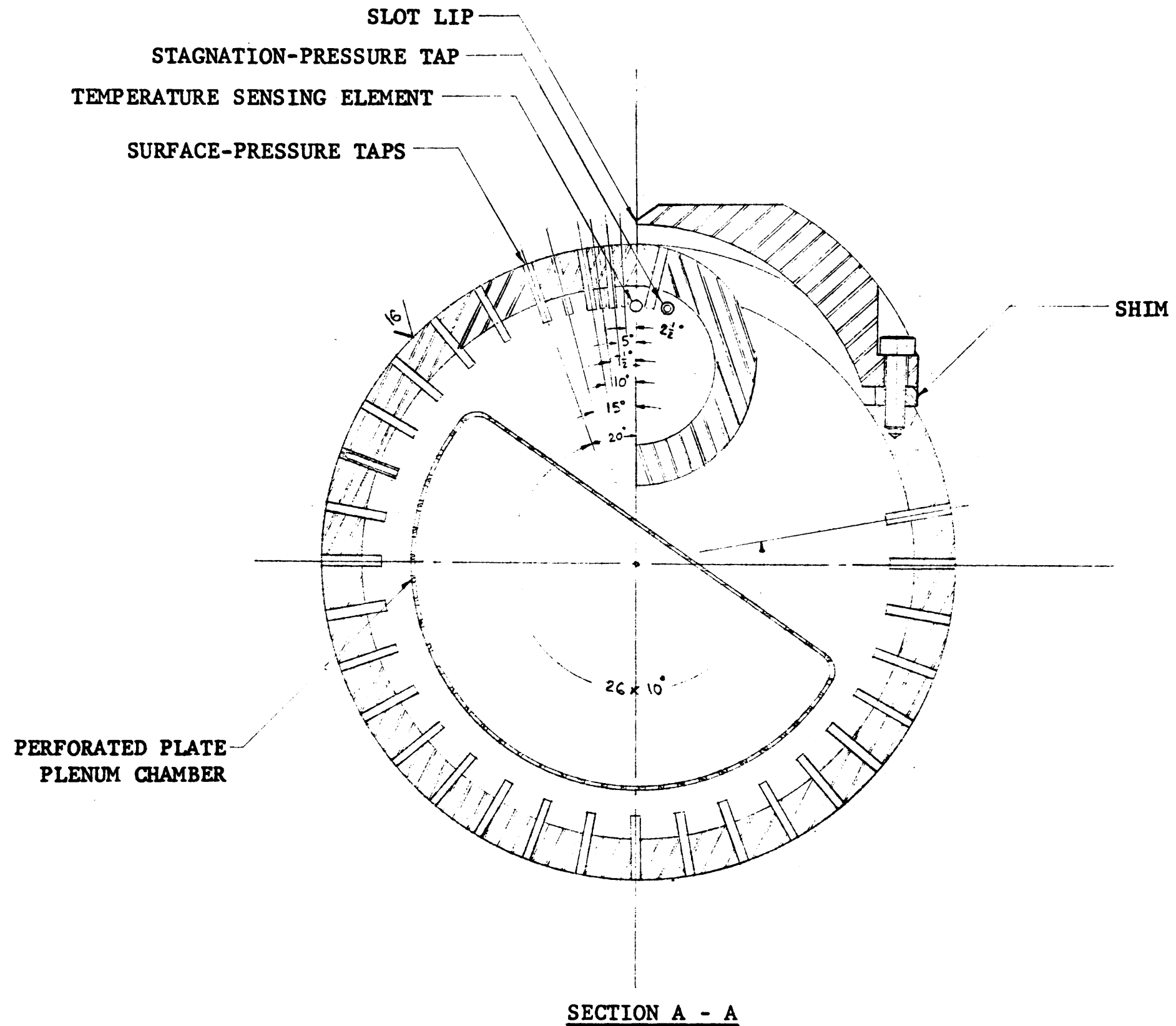


FIG. 5.2

TWO-DIMENSIONALITY CHECK-IMPROVED CYLINDER

FIG. 6

$$a = 4.0 \text{ in.}, \quad \frac{b}{a} = 0.0531, \quad \theta = 110^\circ, \quad R_e = 4.0 \times 10^4$$

	◇	○	△	●	◆
Z in.	-4	-2	0	+2	+4

(see also Fig.7)

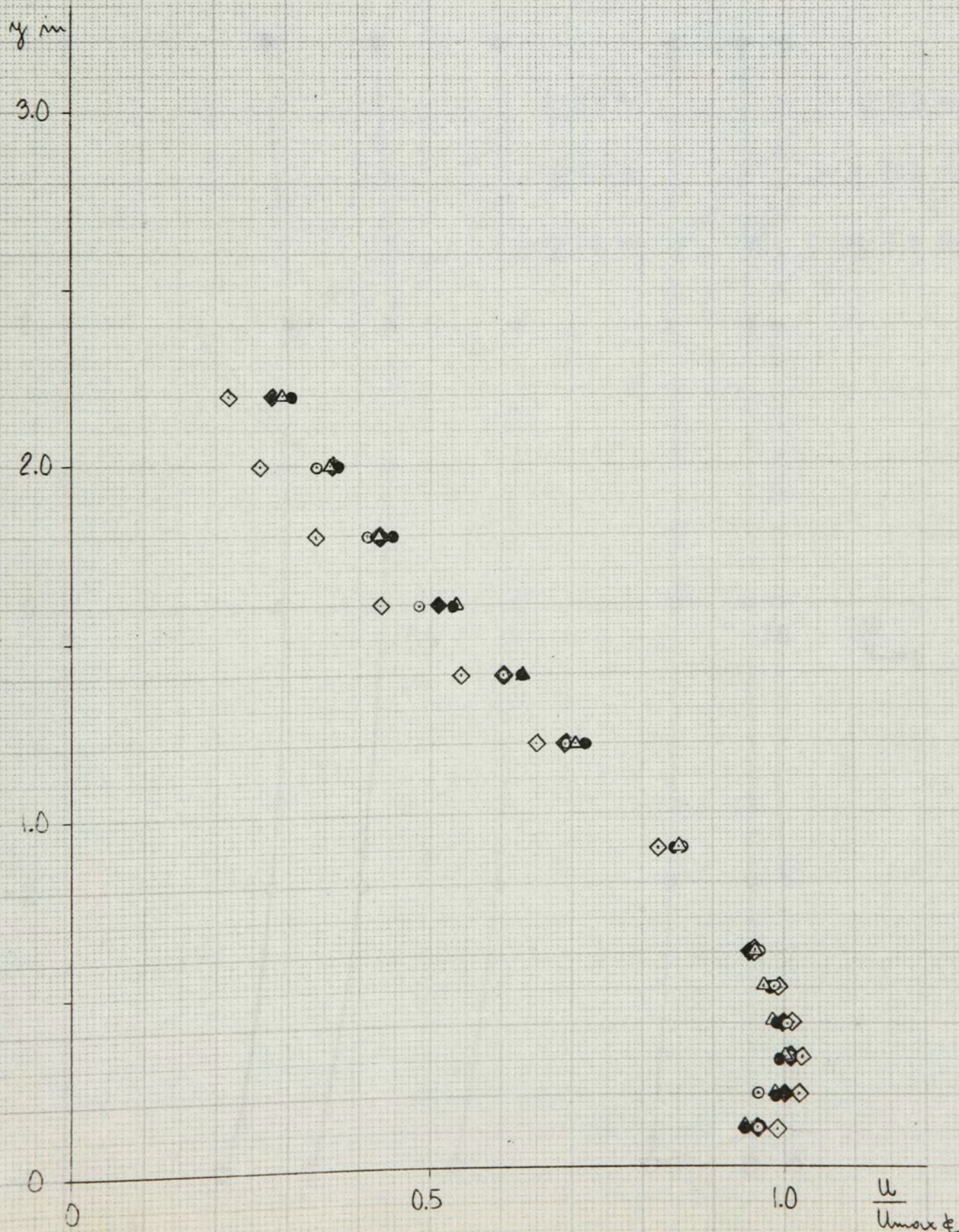


FIG. 7

TWO-DIMENSIONALITY CHECK-IMPROVED CYLINDER
 (Data of Fig. 6 re-plotted)

$$a = 4.0 \text{ in.}, \frac{b}{a} = 0.0531, \theta = 110^\circ, Re = 4.0 \times 10^4$$

	◇	○	△	●	◆
z in.	-4	-2	0	+2	+4

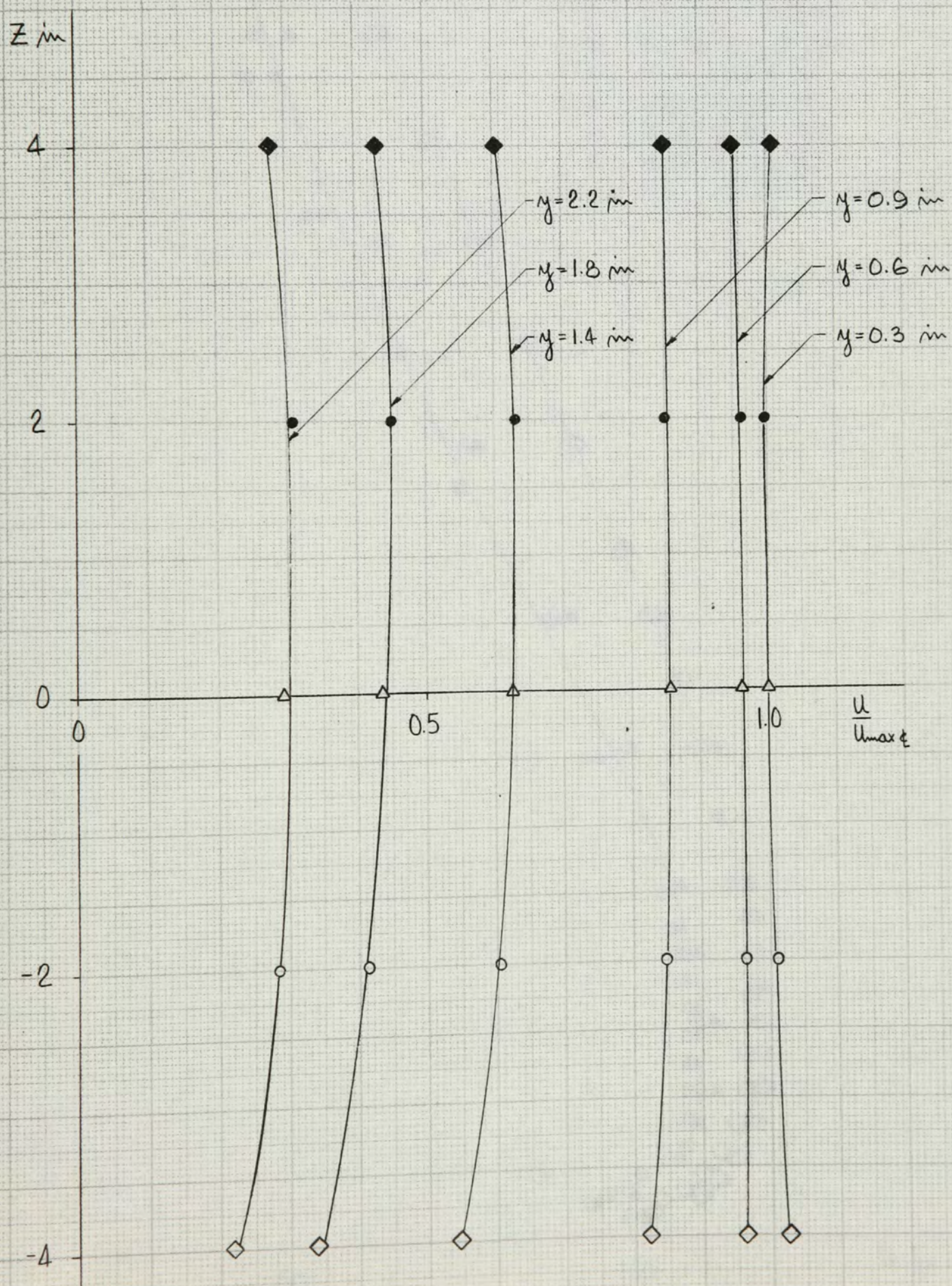
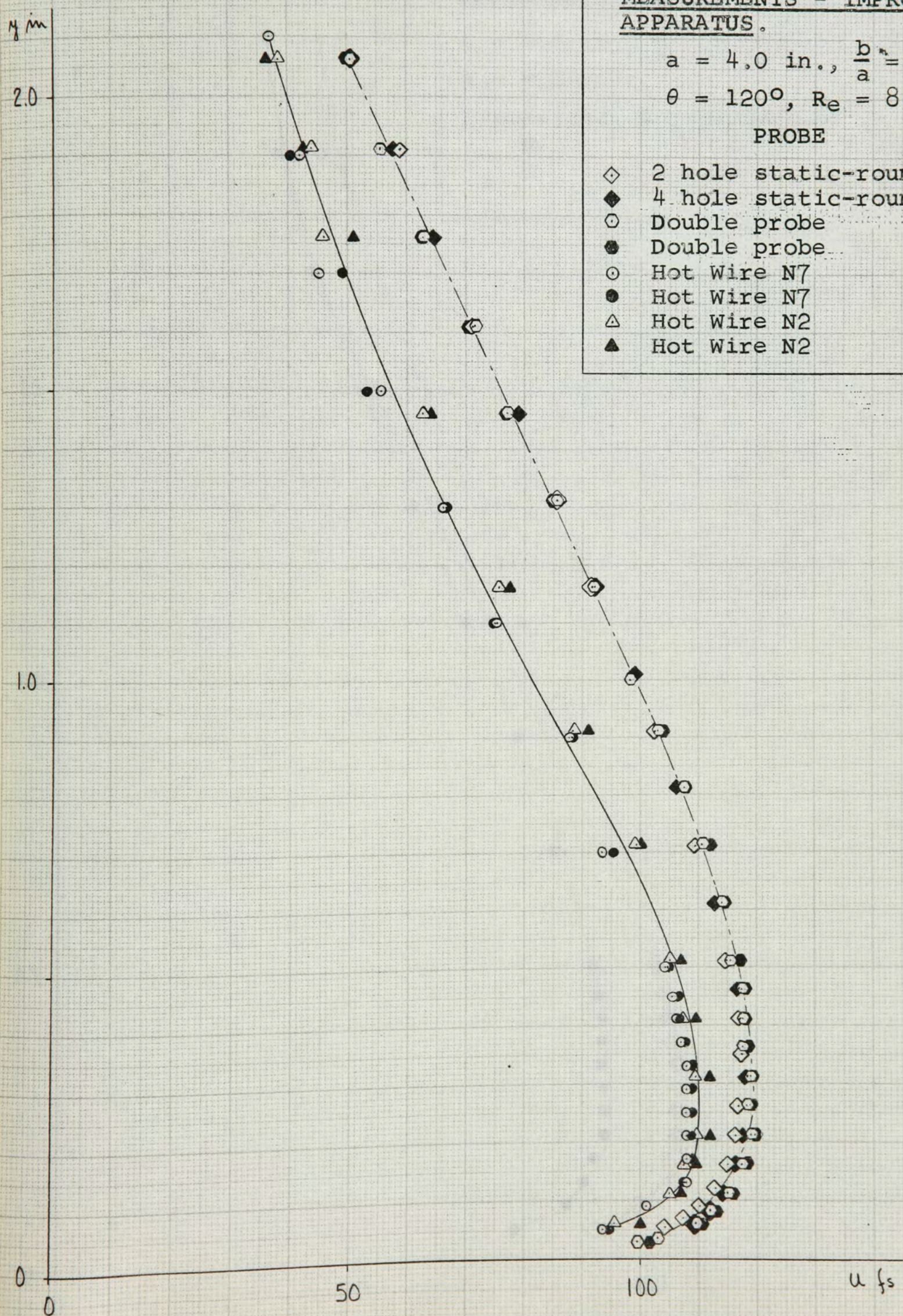


FIG. 8



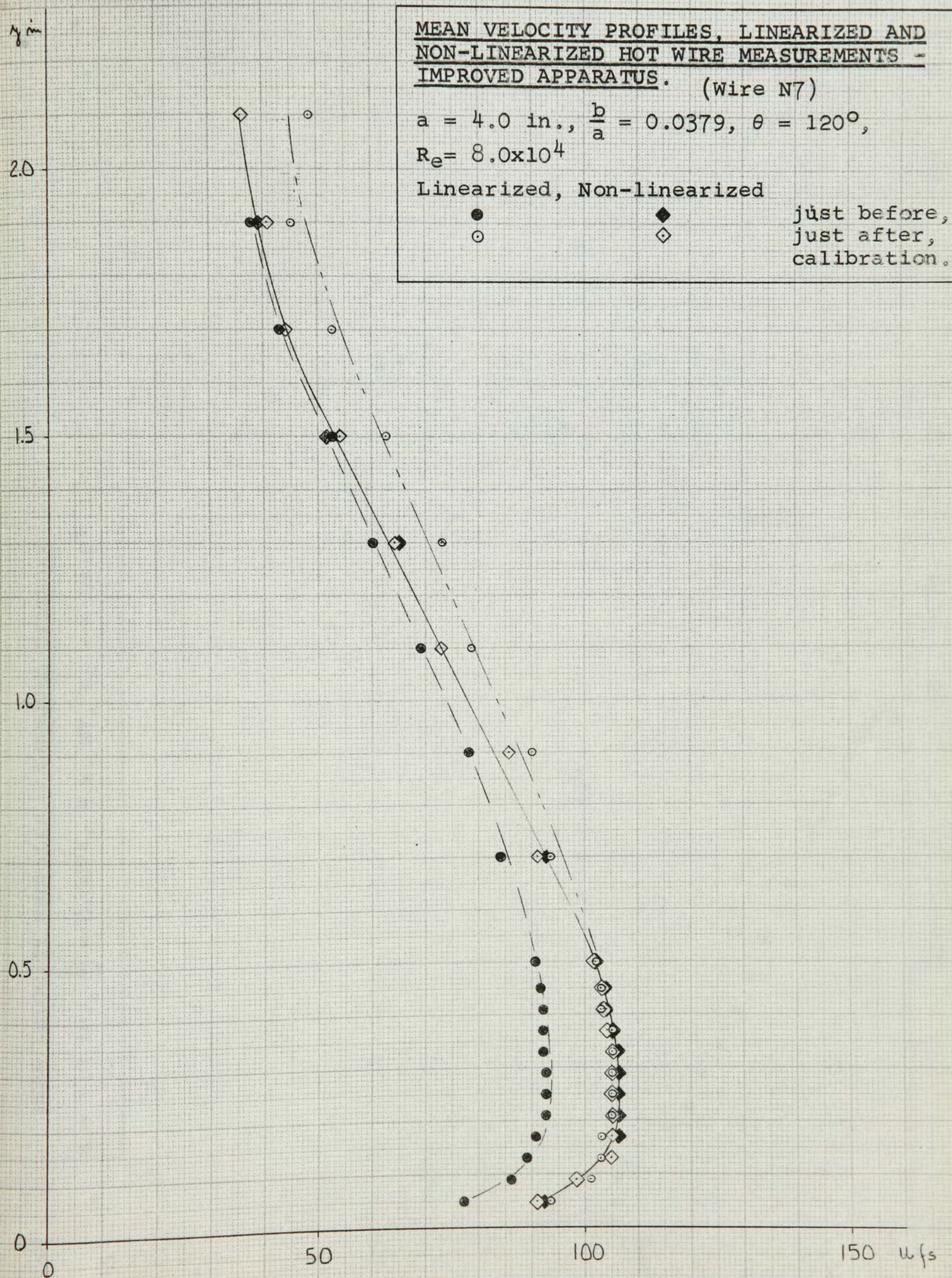
MEAN VELOCITY PROFILES,
PRESSURE PROBE AND HOT WIRE
MEASUREMENTS - IMPROVED
APPARATUS.

$a = 4.0 \text{ in.}, \frac{b}{a} = 0.0379,$
 $\theta = 120^\circ, R_e = 8.25 \times 10^4$

PROBE

- ◇ 2 hole static-round total
- ◆ 4 hole static-round total
- Double probe
- Double probe
- Hot Wire N7
- Hot Wire N7
- △ Hot Wire N2
- ▲ Hot Wire N2

FIG. 9



MEAN VELOCITY PROFILES - IMPROVED CYLINDER

$a = 4.0 \text{ in.}, \frac{b}{a} = 0.0074, \text{ H.W. N7, } Re = 4.0 \times 10^4$

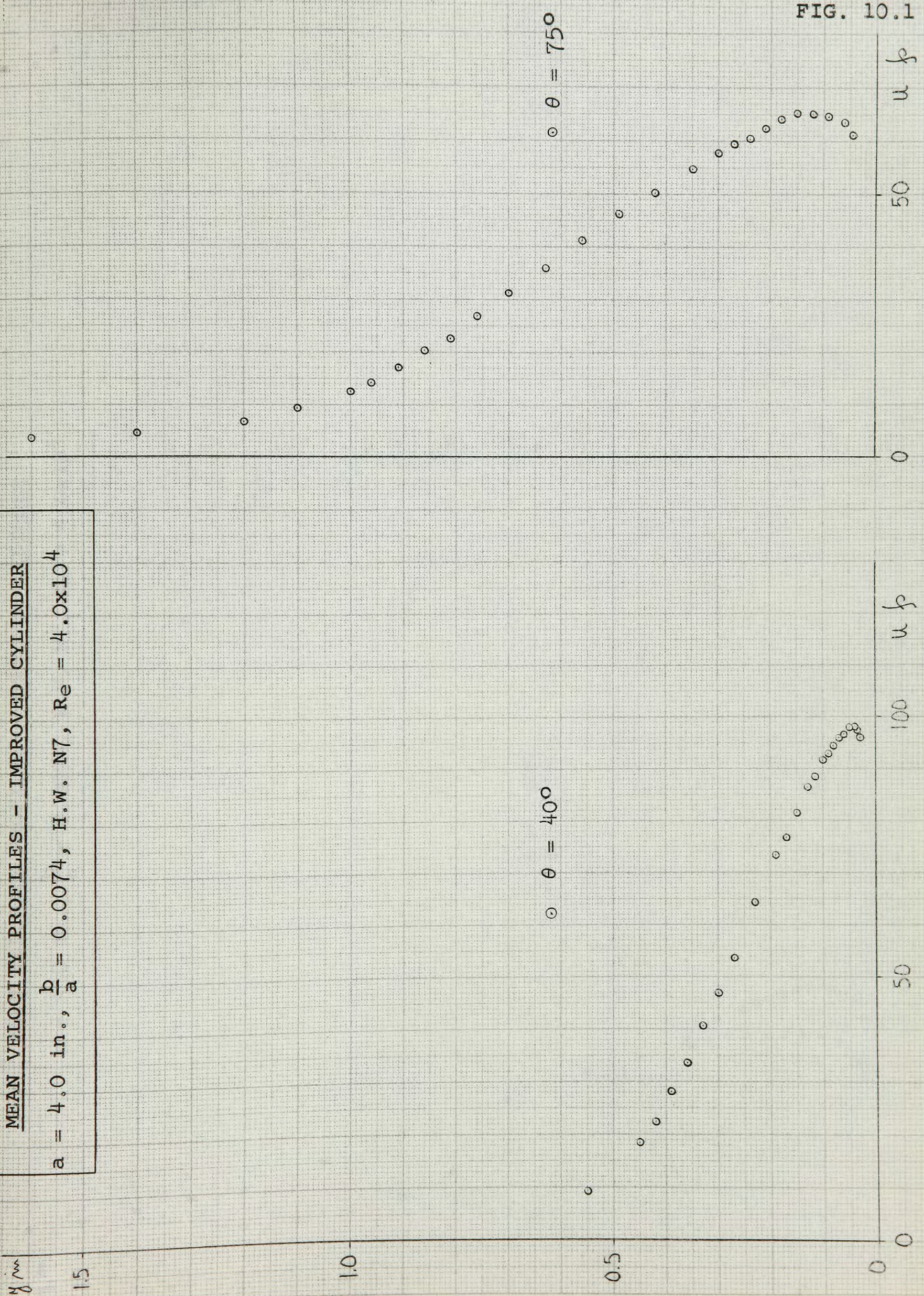


FIG. 10.1

MEAN VELOCITY PROFILES - IMPROVED CYLINDER

$a = 4.0 \text{ in.}$, $\frac{b}{a} = 0.0074$, H.W. N7, $Re = 4.0 \times 10^4$

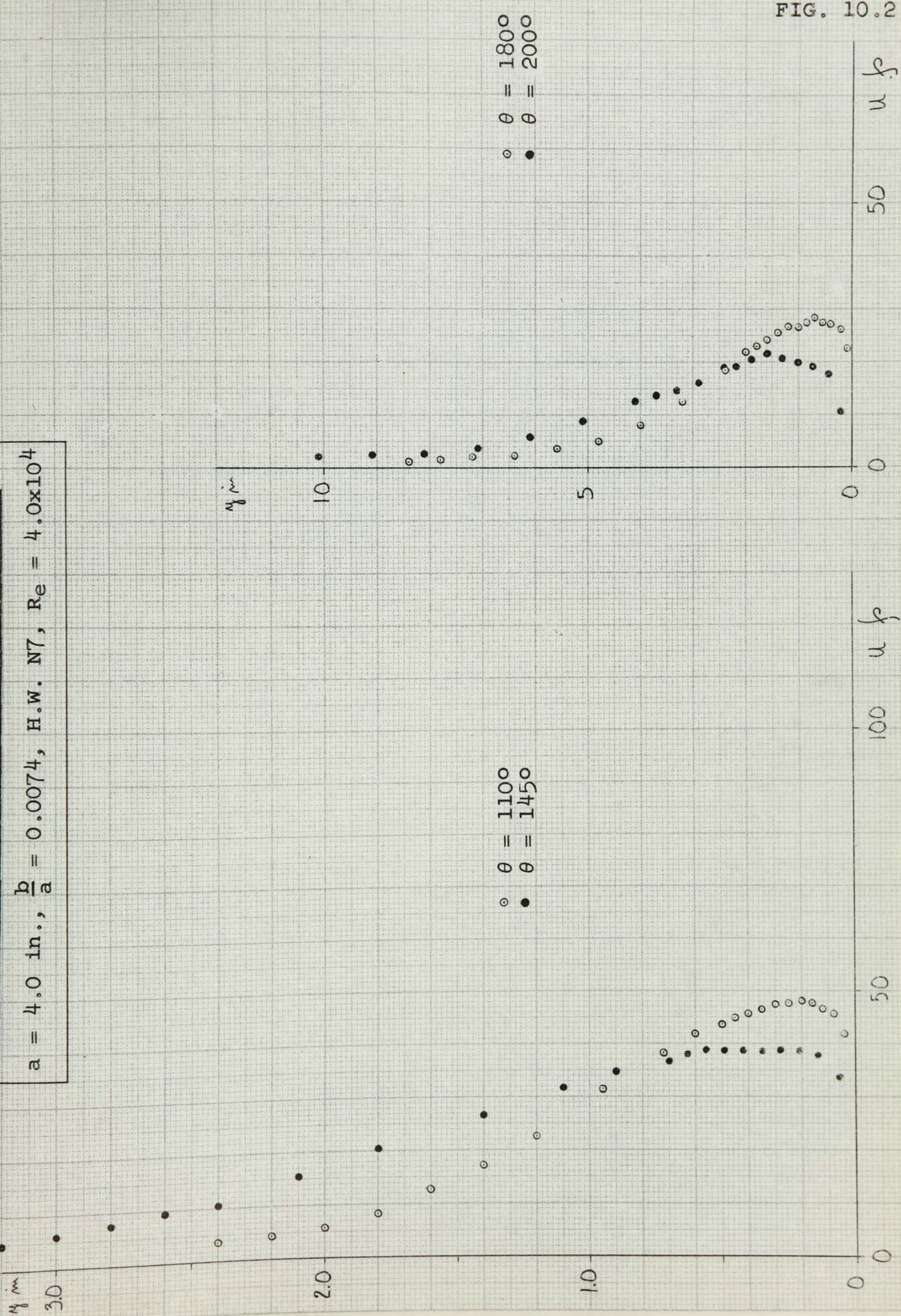


FIG. 10.2

MEAN VELOCITY PROFILES - IMPROVED CYLINDER

$a = 4.0 \text{ in.}, \frac{b}{a} = 0.0229, \text{ H.W. N7, } Re = 4.0 \times 10^4$

u_f
1.5

1.0

0.5

0

$\theta = 40^\circ$

50

u_f

100

$\theta = 75^\circ$

50

u_f

0

FIG. 10.3

MEAN VELOCITY PROFILES - IMPROVED CYLINDER

$a = 4.0 \text{ in.}$, $\frac{b}{a} = 0.0229$, H.W. N7, $Re = 4.0 \times 10^4$

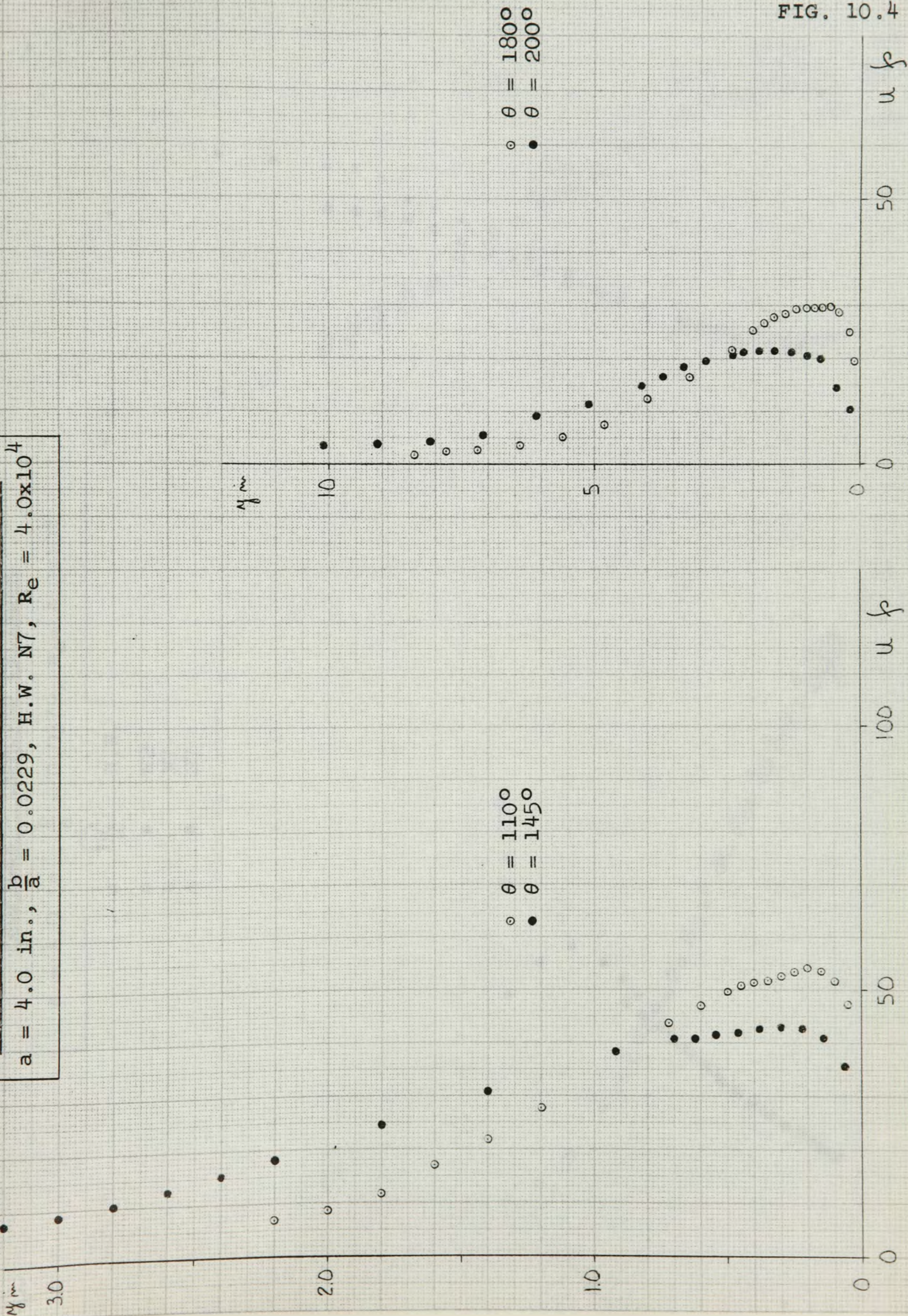


FIG. 10.4

MEAN VELOCITY AND TURBULENCE INTENSITY
PROFILES - IMPROVED CYLINDER

$a = 4.0 \text{ in.}, \frac{b}{a} = 0.0379, Re = 4.0 \times 10^4$

u	$\frac{\sqrt{u'^2}}{u}$	H.W.
○	●	N2
○	●	N7
△	▲	N7

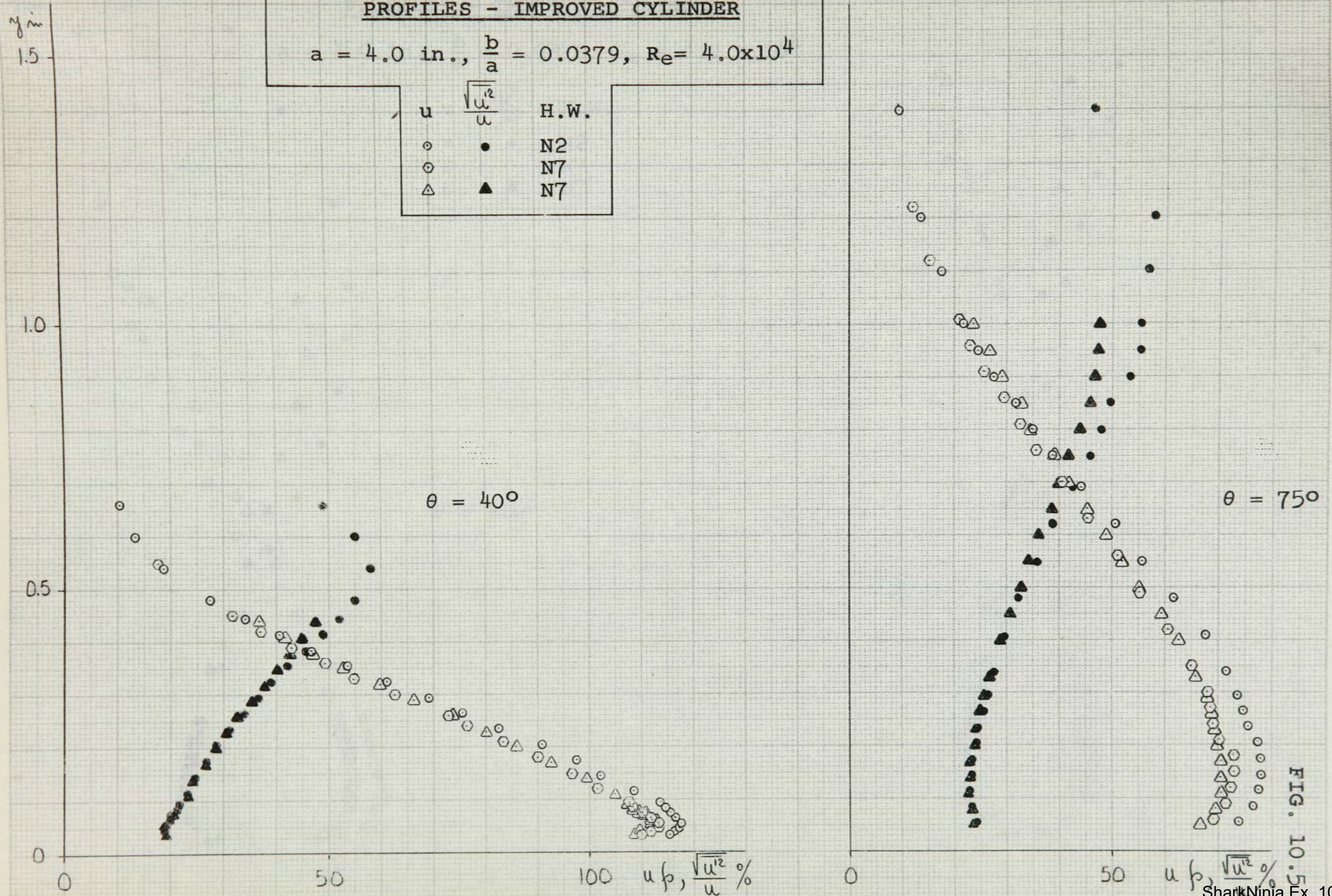


FIG. 10.5

MEAN VELOCITY AND TURBULENCE INTENSITY PROFILES - IMPROVED CYLINDER

$a = 4.0 \text{ in.}$, $\frac{b}{a} = 0.0379$, $Re = 4.0 \times 10^4$

u	$\frac{\sqrt{u'^2}}{u}$	H.W.
○	●	N2
○	●	N7
△	▲	N7

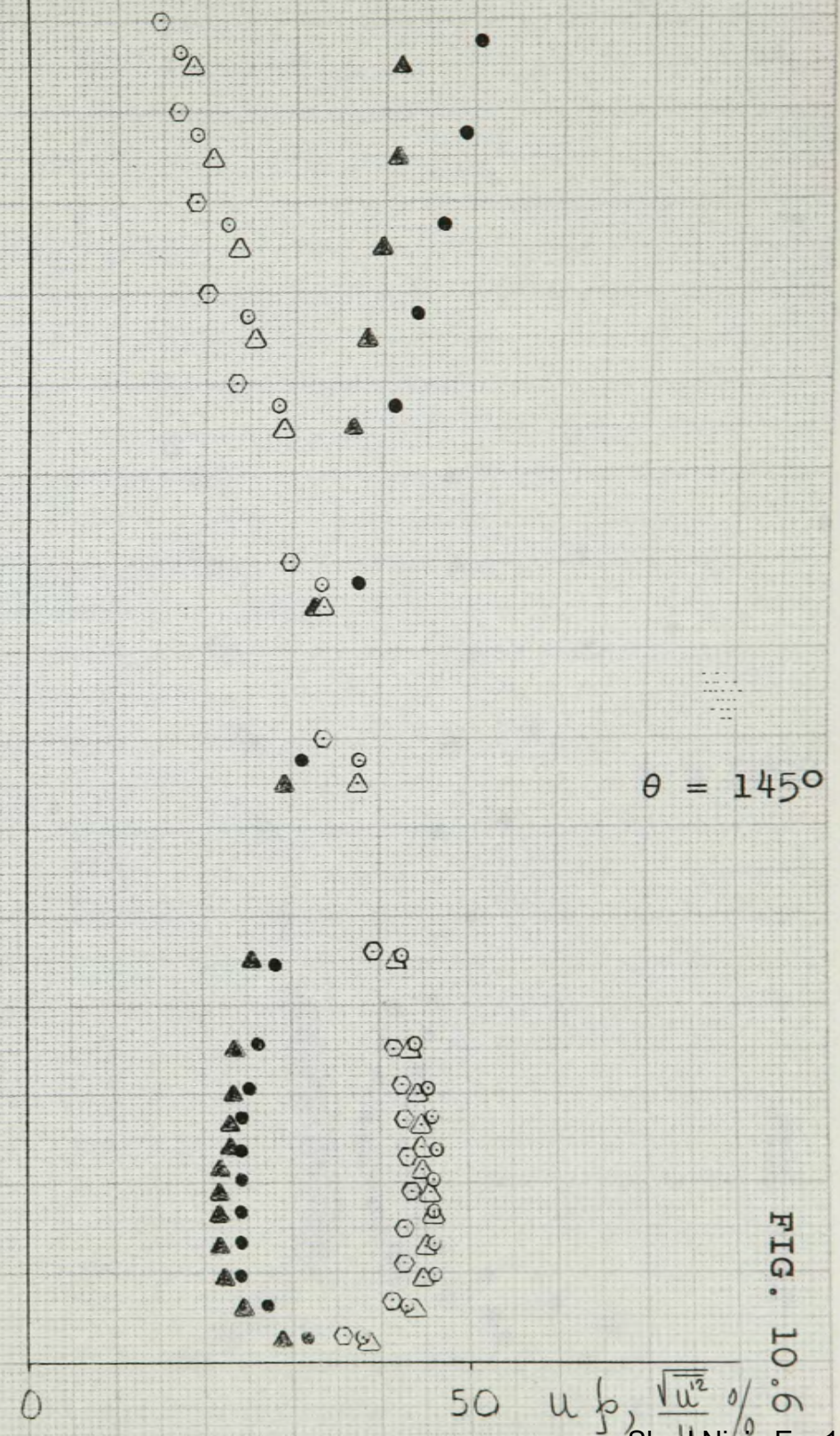
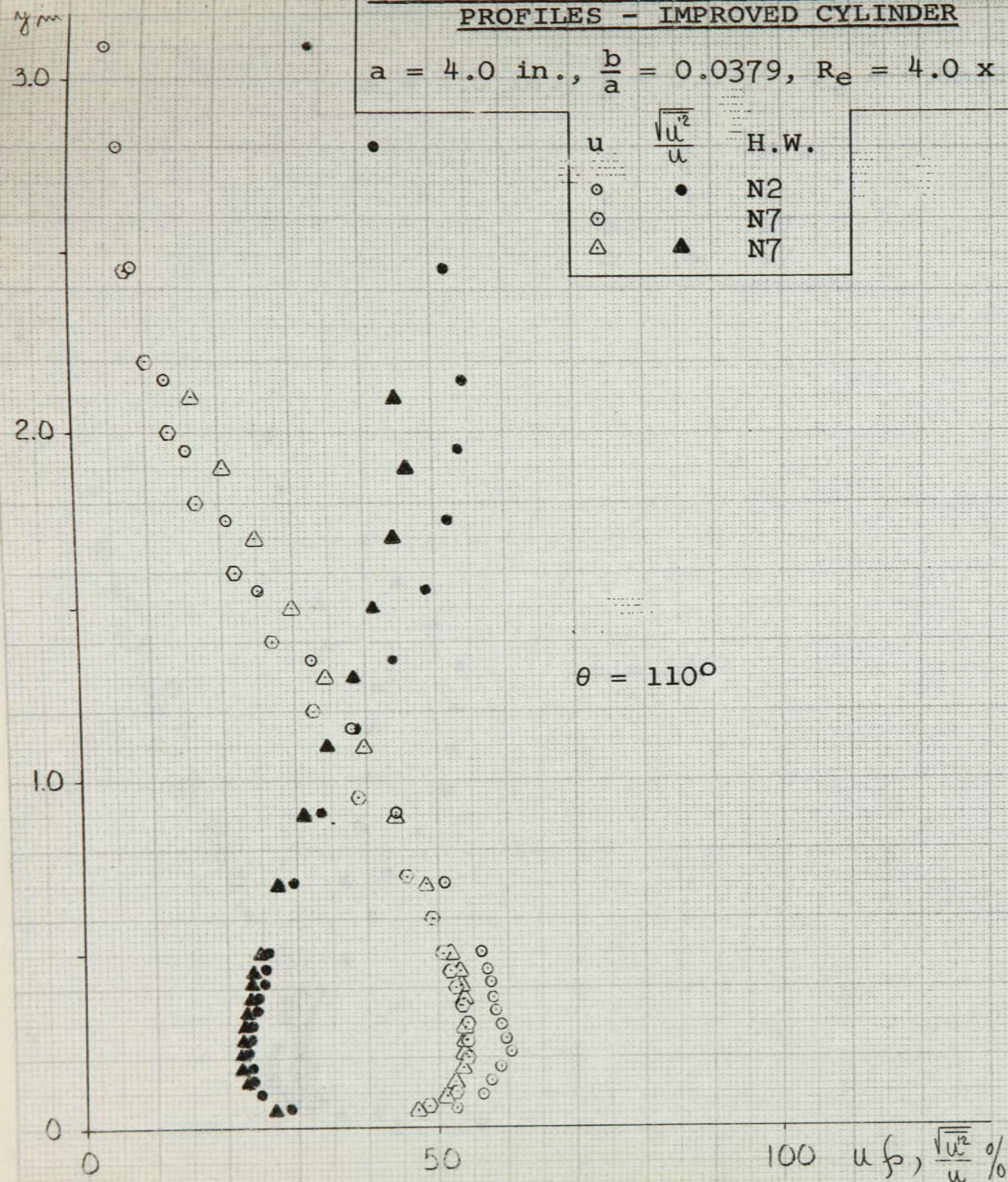


FIG. 10.6

MEAN VELOCITY AND TURBULENCE INTENSITY
PROFILE - IMPROVED CYLINDER

$a = 4.0 \text{ in.}$, $\frac{b}{a} = 0.0379$, $Re = 4.0 \times 10^4$

u	$\frac{\sqrt{u'^2}}{u}$	H.W.
$\circ$	$\bullet$	N2
$\odot$		N7
Δ	$\blacktriangle$	N7

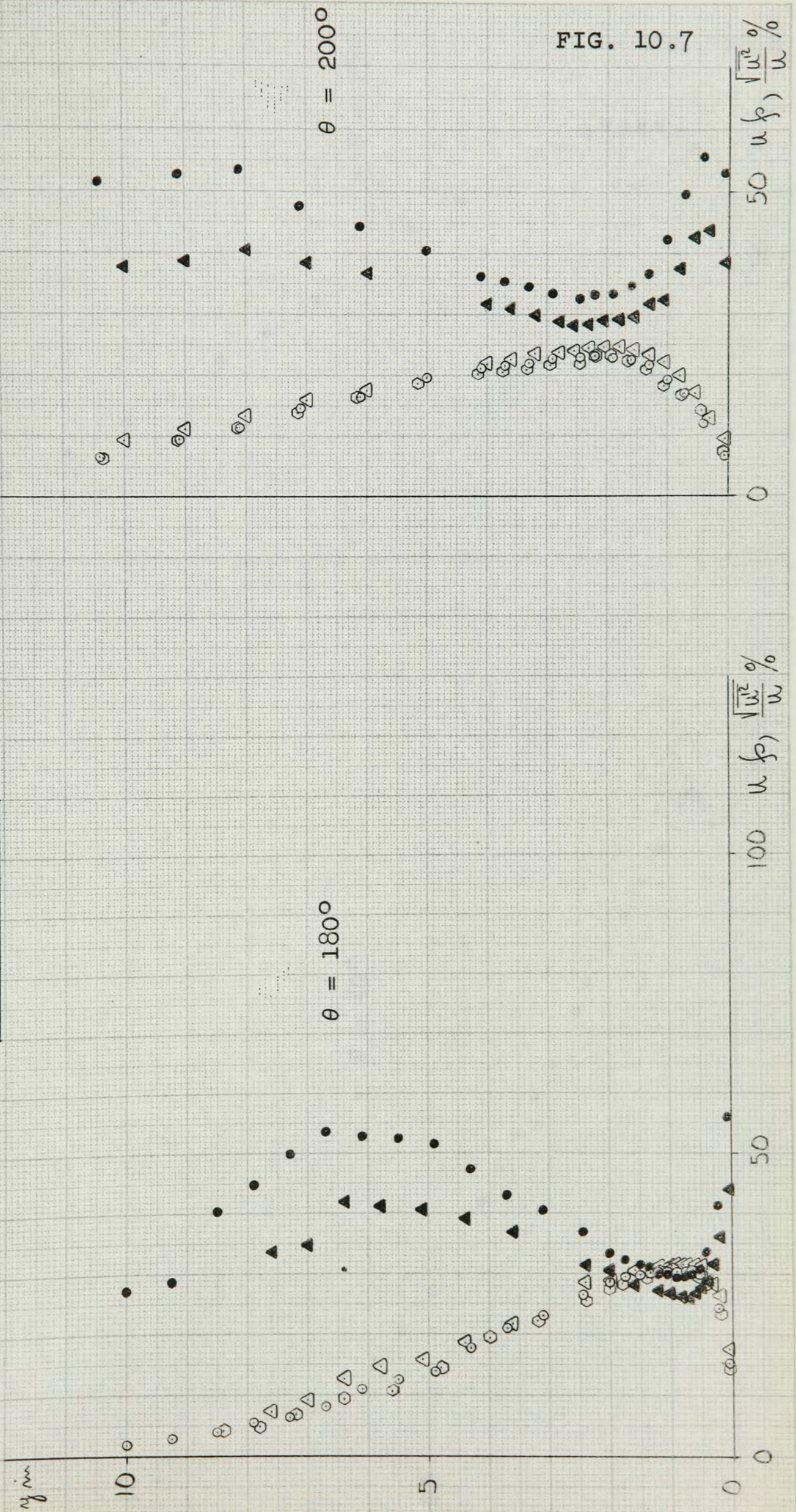


FIG. 10.7

MEAN VELOCITY PROFILES - IMPROVED CYLINDER

$a = 4.0 \text{ in.}$, $\frac{b}{a} = 0.0531$, H.W. N7, $Re = 4.0 \times 10^4$

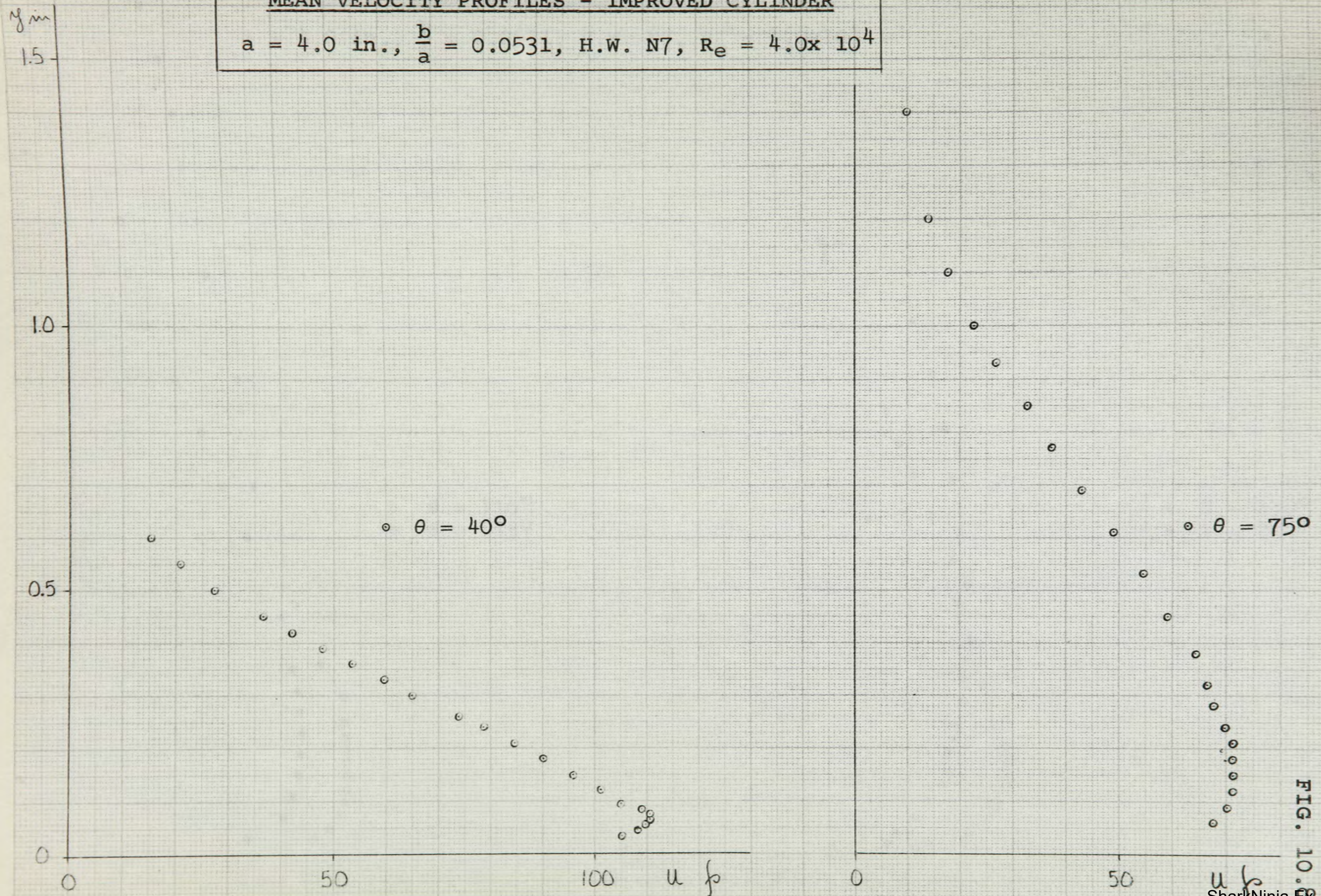


FIG. 10.8

MEAN VELOCITY PROFILES - IMPROVED CYLINDER

$a = 4.0 \text{ in.}$, $\frac{b}{a} = 0.0531$, H.W. N7, $R_e = 4.0 \times 10^4$

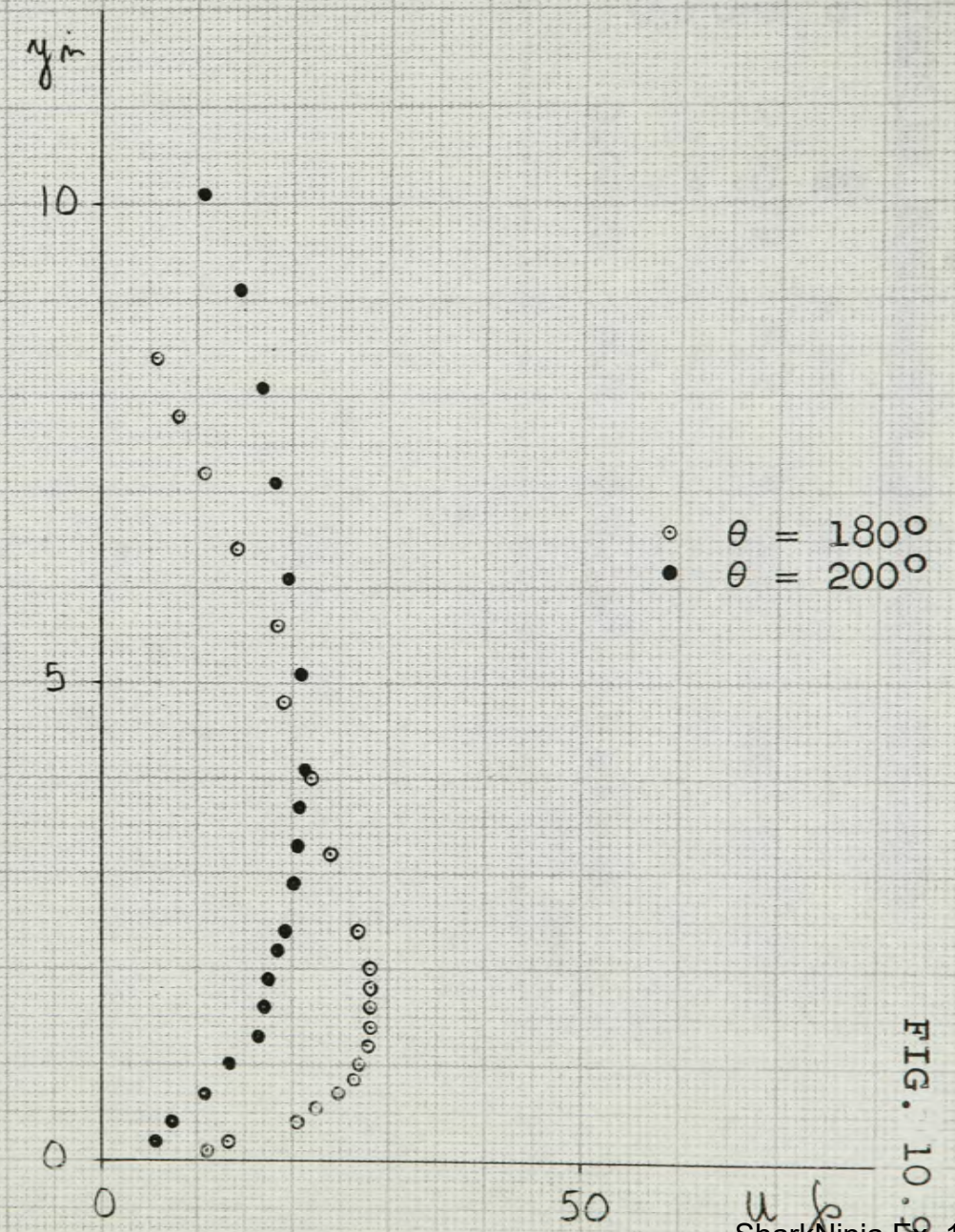
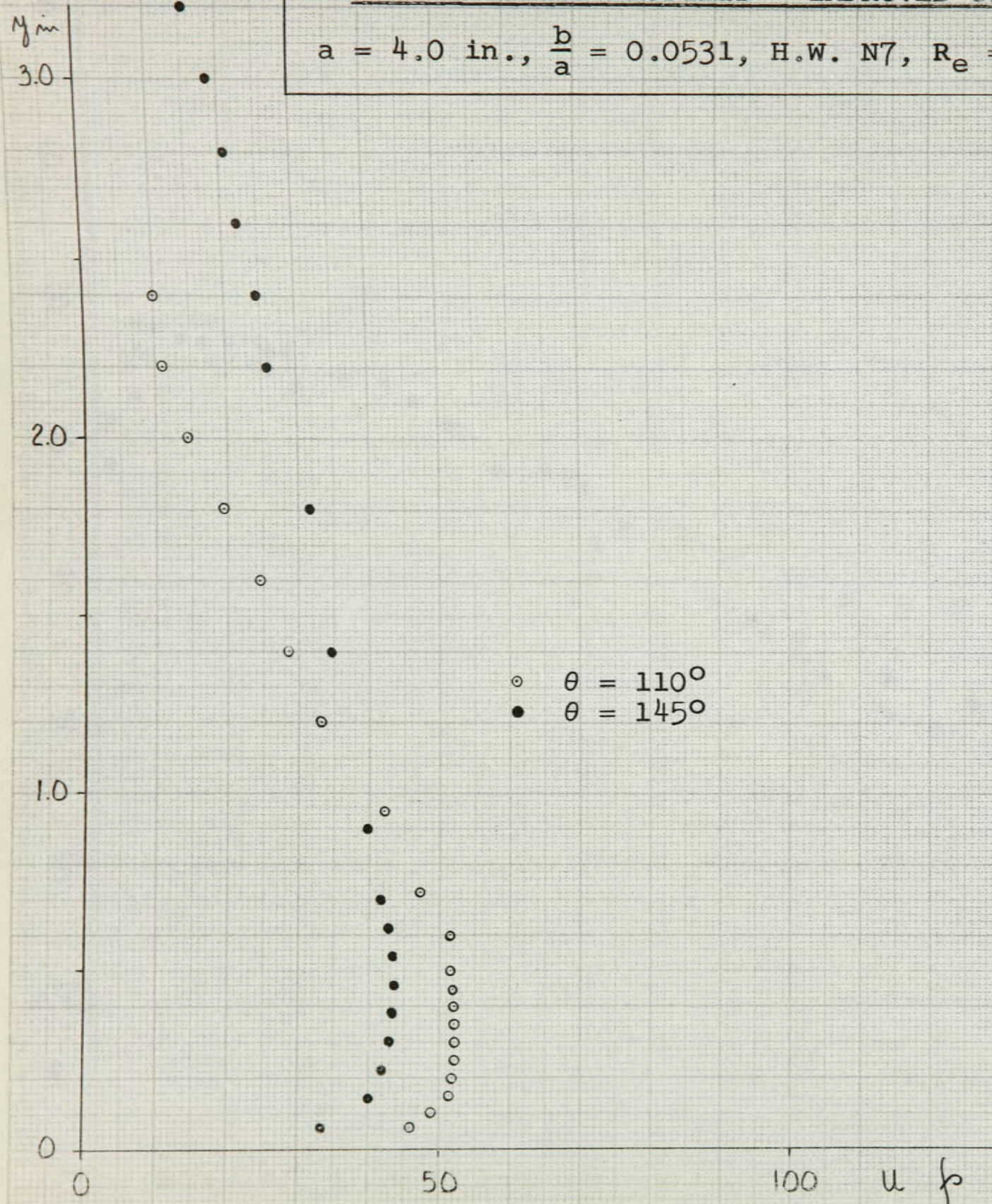


FIG. 10.8

FIG. 11

COMPARISON OF TURBULENCE INTENSITY PROFILES

	H.W.	θ	$\frac{b}{a}$	a in.	Re	
▲	N7	75	0.0379	4.0	4.0×10^4	Improved Cyl.
△	"	145	"	"	"	"
●	N2	75	"	"	"	"
○	"	145	"	"	"	"
⊗			0.0020	9.0	9.8×10^4	Bradshaw and Gee

$\frac{u}{u_{\infty}}$

2.0

1.0

0

0

10

20

30

40

50

60

$\frac{\sqrt{u'^2}}{u} \%$

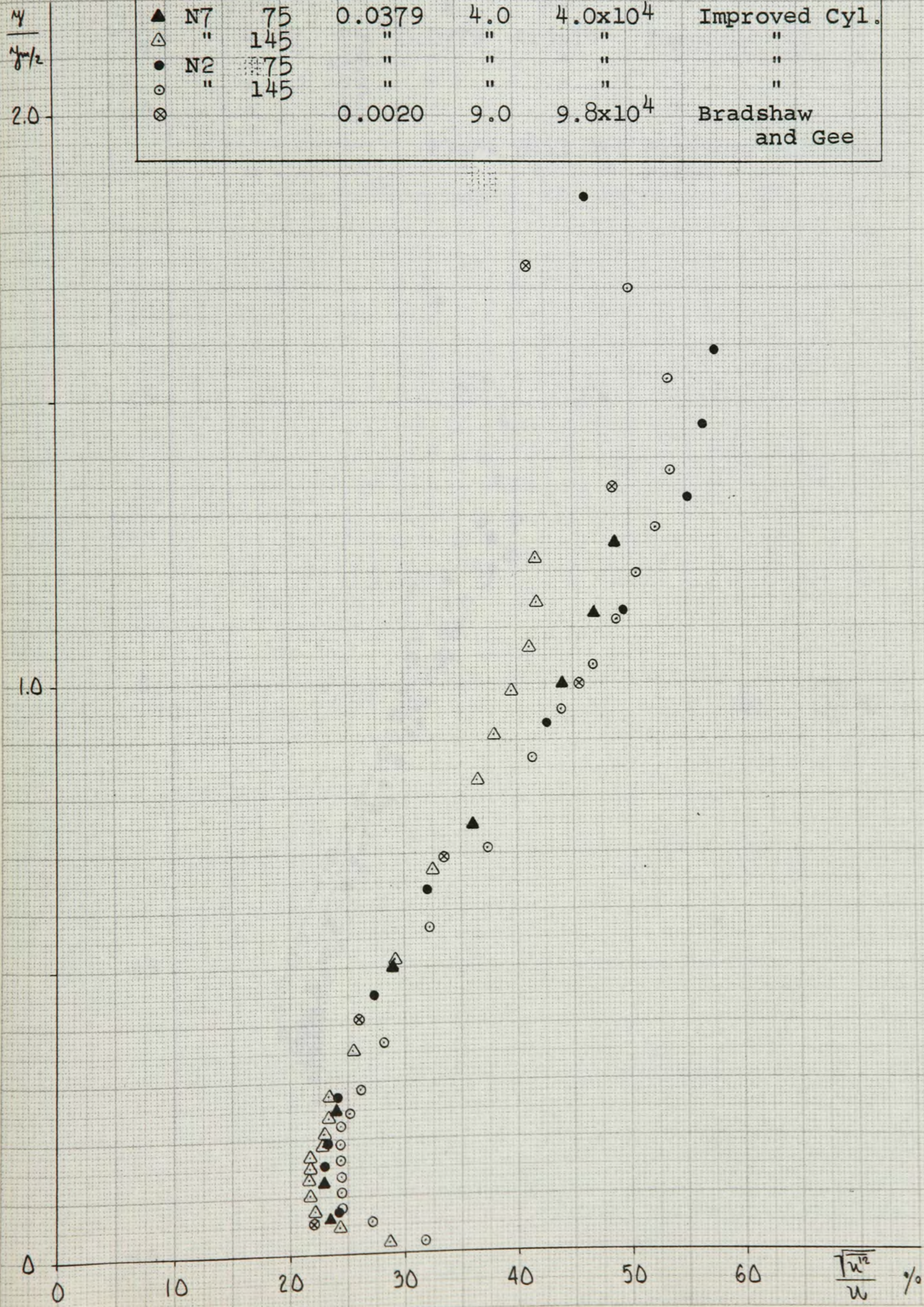
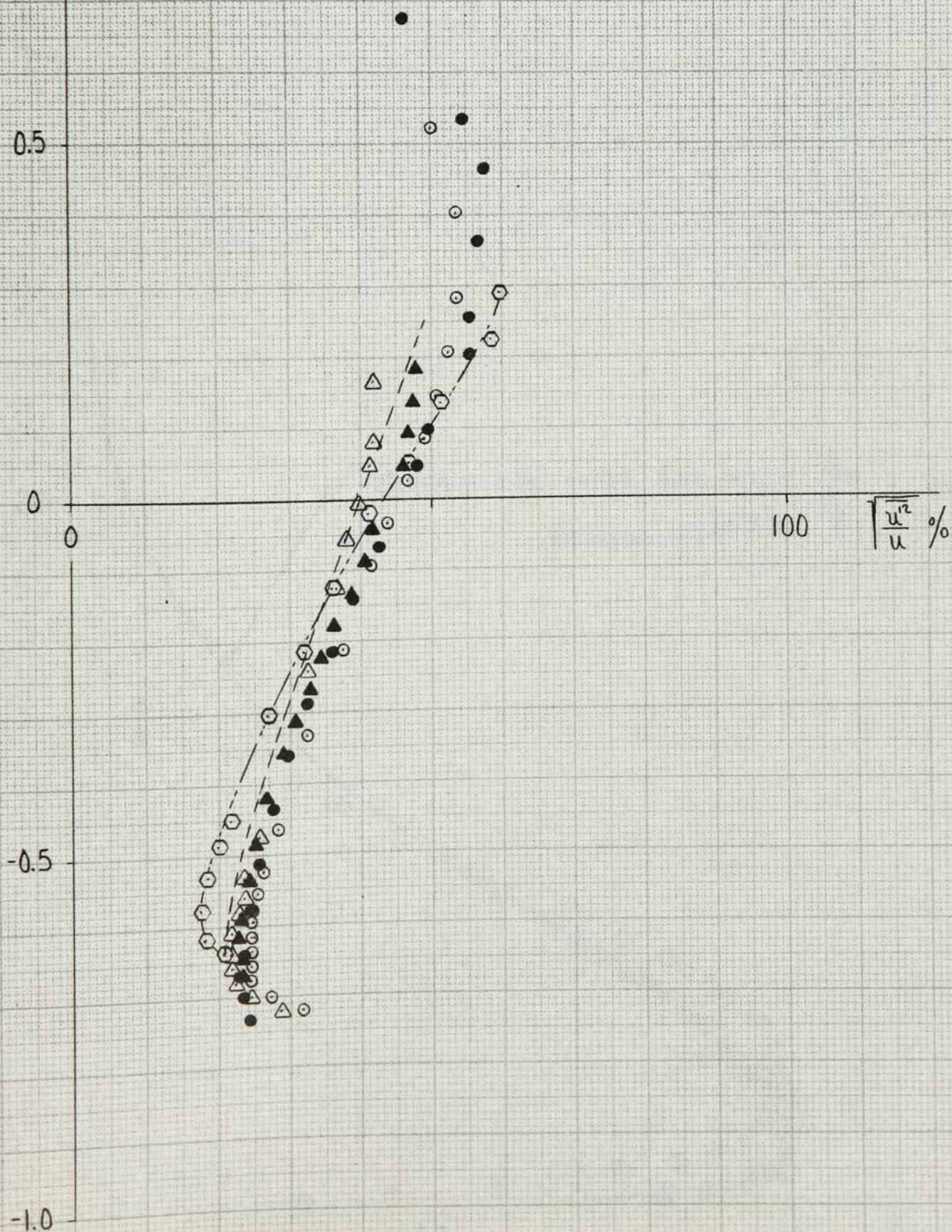


FIG.12

COMPARISON OF CYLINDER AND PLANE WALL
JET TURBULENCE INTENSITY PROFILES.

	H.W.	θ°	$\frac{b}{a}$	Re	
$\frac{u_y - u_{y/2}}{\delta_s}$	▲ N7	75	0.0379	4.0×10^4	Improved Cyl.
	△ "	145	"	"	"
	● N2	75	"	"	"
	○ "	145	"	"	"
1.0	---				Plane Wall Jet Bradshaw and Gee
	⊖				Patel

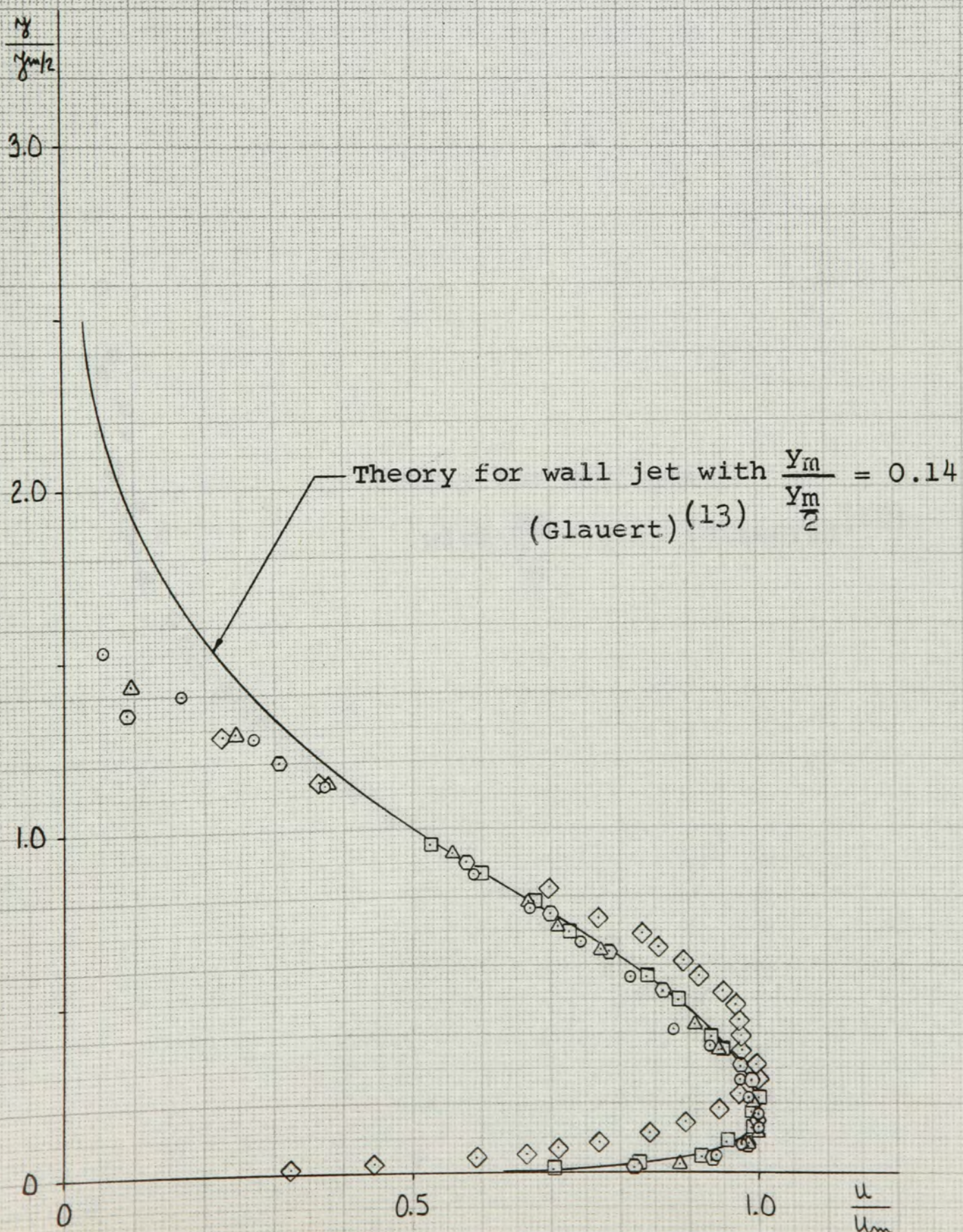


NON-DIMENSIONAL MEAN VELOCITY PROFILES
- NEWMAN'S CYLINDER.

FIG. 13

$a = 3.0 \text{ in.}, \frac{b}{a} = 0.0200, R_e = 4.1 \times 10^4$

	θ°
○	45
△	90
⊙	135
□	180
◇	210



NON-DIMENSIONAL MEAN VELOCITY PROFILES- IMPROVED CYLINDER

$$a = 4.0 \text{ in.}, \frac{b}{a} = 0.0074, Re = 4.0 \times 10^4$$

θ°	θ°
$\circ$ 40	$\diamond$ 145
$\triangle$ 75	$\circ$ 180
$\square$ 110	∇ 200

$$\frac{y_m}{y_{m/2}} \div 0.17 \text{ for } \theta < 180^\circ$$

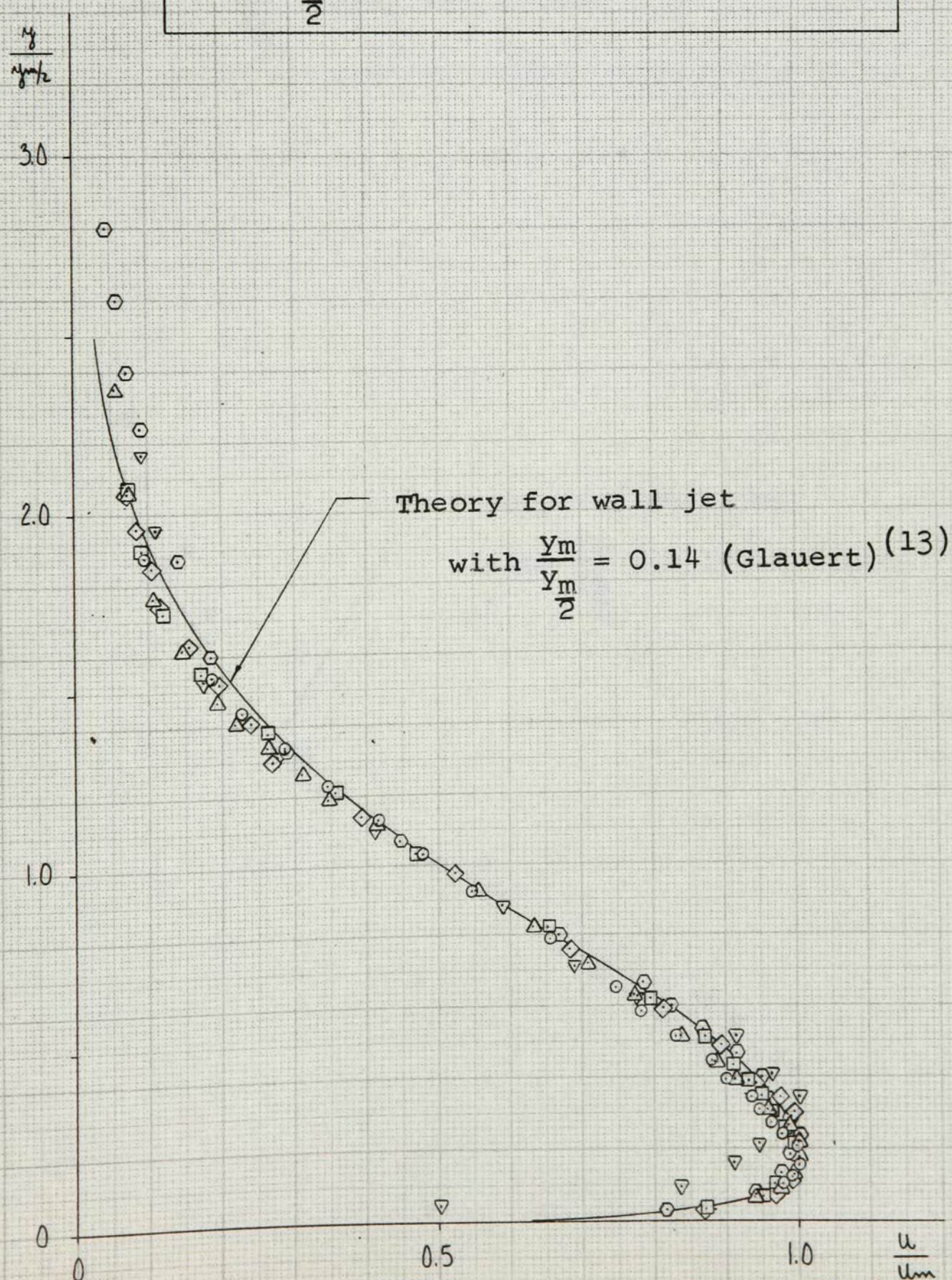


FIG. 14.2

NON-DIMENSIONAL MEAN VELOCITY PROFILES- IMPROVED CYLINDER

$$a = 4.0 \text{ in.}, \frac{b}{a} = 0.0229, R_e = 4.0 \times 10^4$$

 θ°

○ 40

△ 75

□ 110

 θ°

◇ 145

◊ 180

▽ 200

$$\frac{y_m}{y_m^2} \div 0.17 \text{ for } \theta < 180^\circ$$

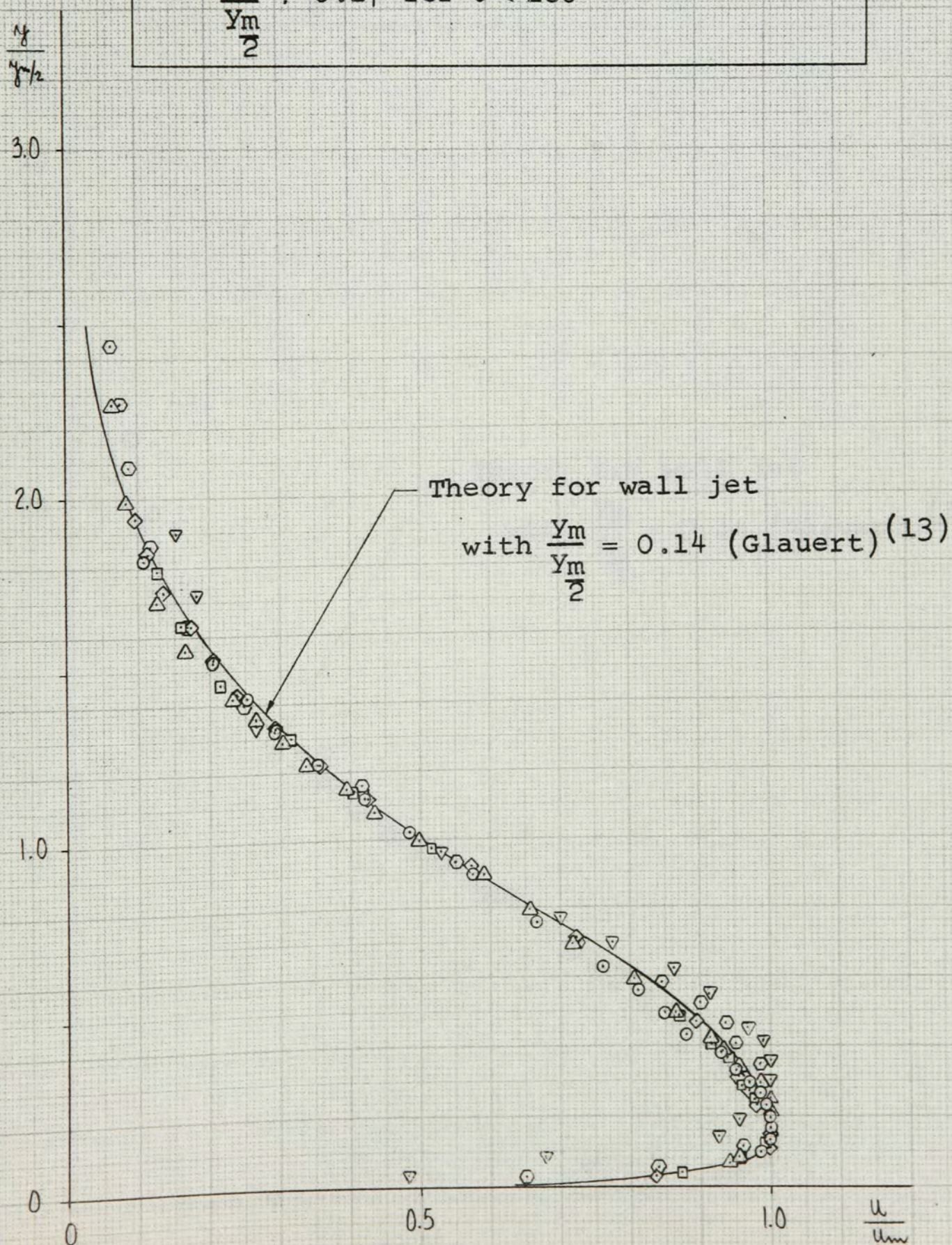


FIG. 14.3

NON-DIMENSIONAL MEAN VELOCITY PROFILES- IMPROVED CYLINDER

$$a = 4.0 \text{ in.}, \frac{b}{a} = 0.0379, R_e = 4.0 \times 10^4$$

θ°	θ°
$\circ$ 40	$\diamond$ 145
$\triangle$ 75	$\circ$ 180
$\square$ 110	∇ 200

$$\frac{y_m}{y_m/2} \div 0.17 \text{ for } \theta < 180^\circ$$

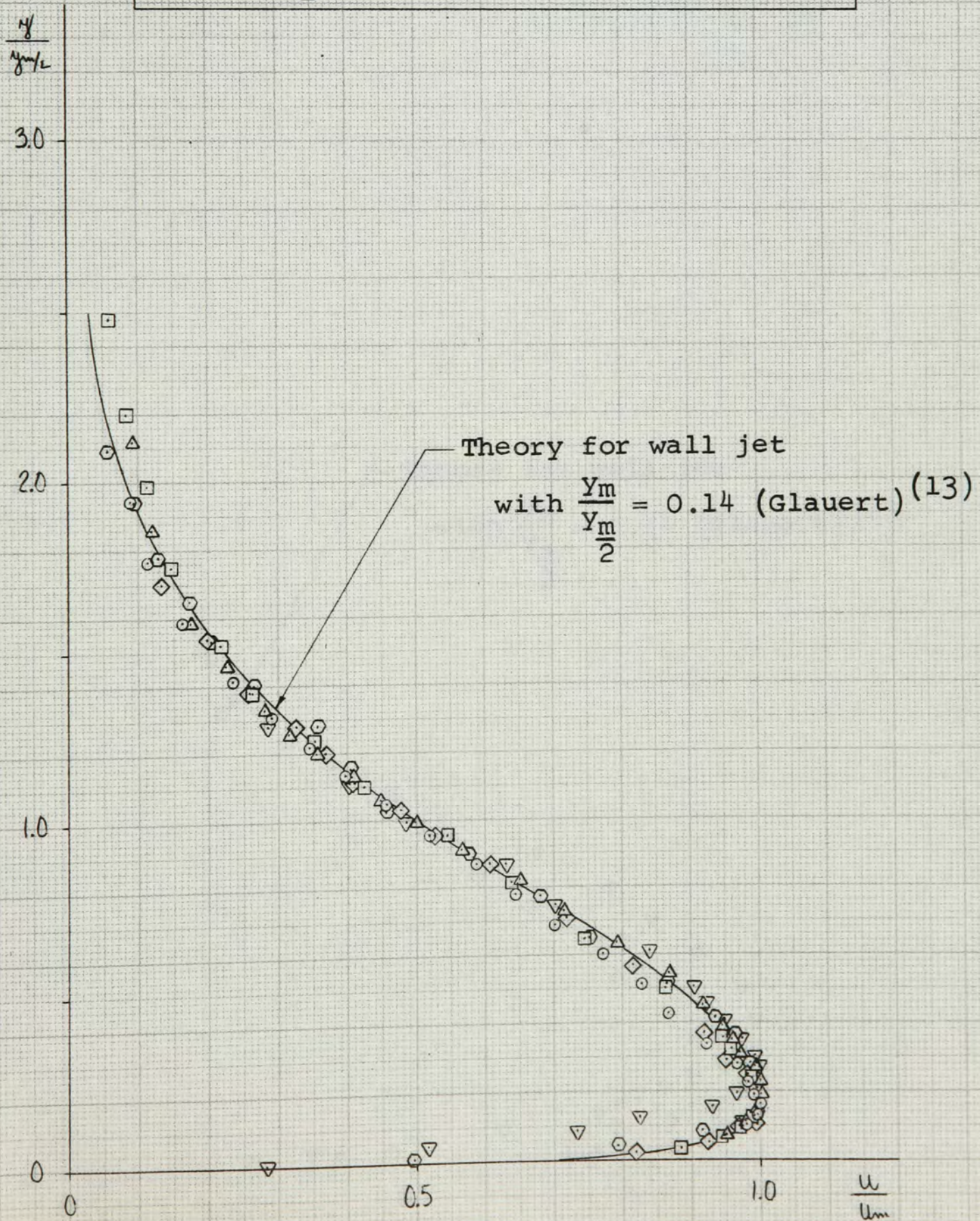


FIG. 14.4

NON-DIMENSIONAL MEAN VELOCITY PROFILES- IMPROVED CYLINDER

$$a = 4.0 \text{ in.}, \frac{b}{a} = 0.0531, R_e = 4.0 \times 10^4$$

θ°	θ°
$\circ$ 40	$\diamond$ 145
$\triangle$ 75	$\odot$ 180
$\square$ 110	∇ 200

$$\frac{y_m}{y_m/2} \doteq 0.19 \text{ for } \theta < 180^\circ$$

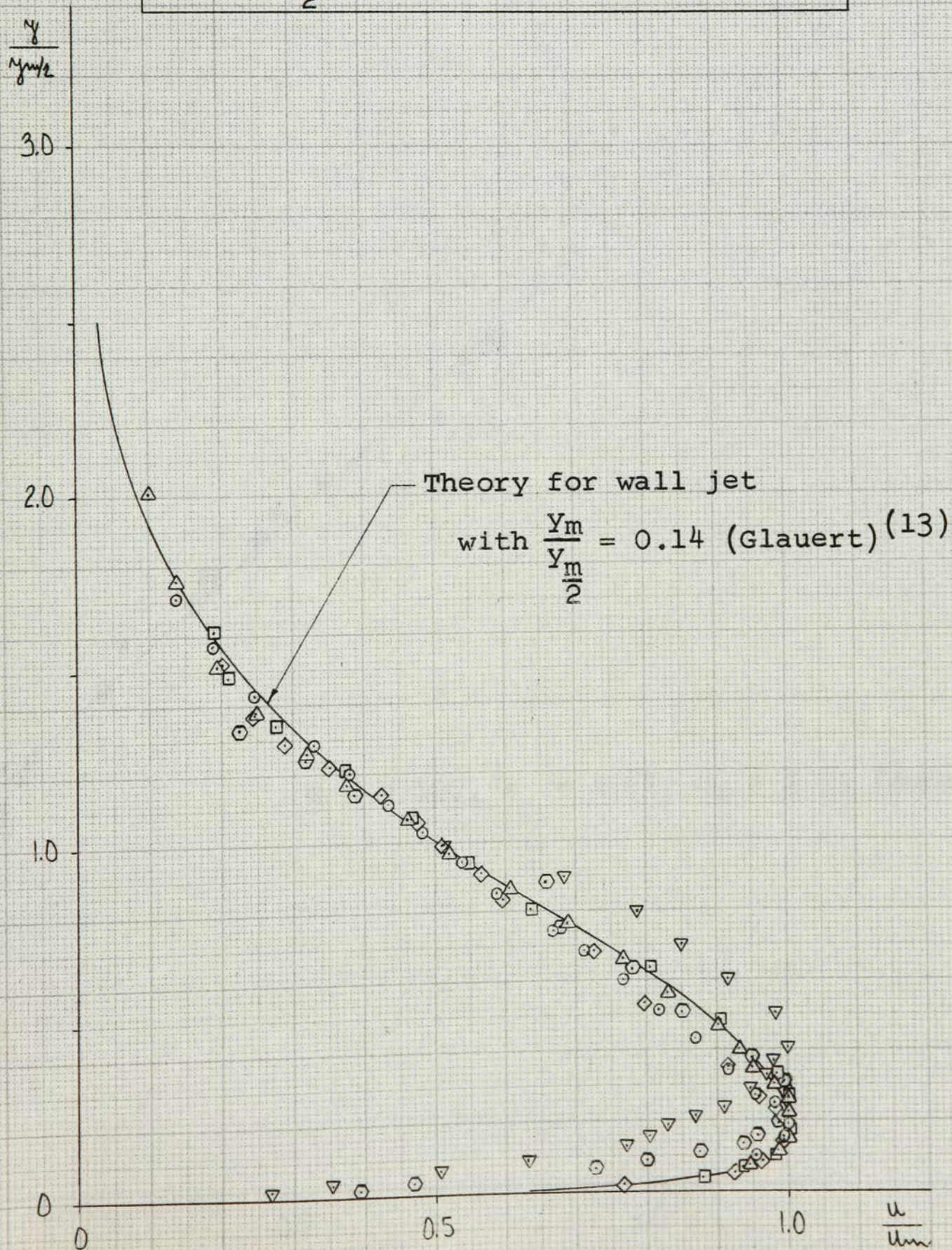


FIG. 15

STREAMWISE VARIATION OF $\frac{y_m/2}{a\theta}$ WITH $\frac{y_m/2}{a}$ -
 NEWMAN'S CYLINDER

	$\frac{b}{a}$	a	Re	
◇	0.0200	3.0	4.1×10^4	Present
◇	0.0267	"	4.0×10^4	"
▽	0.0400	"	4.2×10^4	"
⊕	0.0267	3.0	4.9×10^4	Newman
⊗	0.0282	2.55	4.6×10^4	"

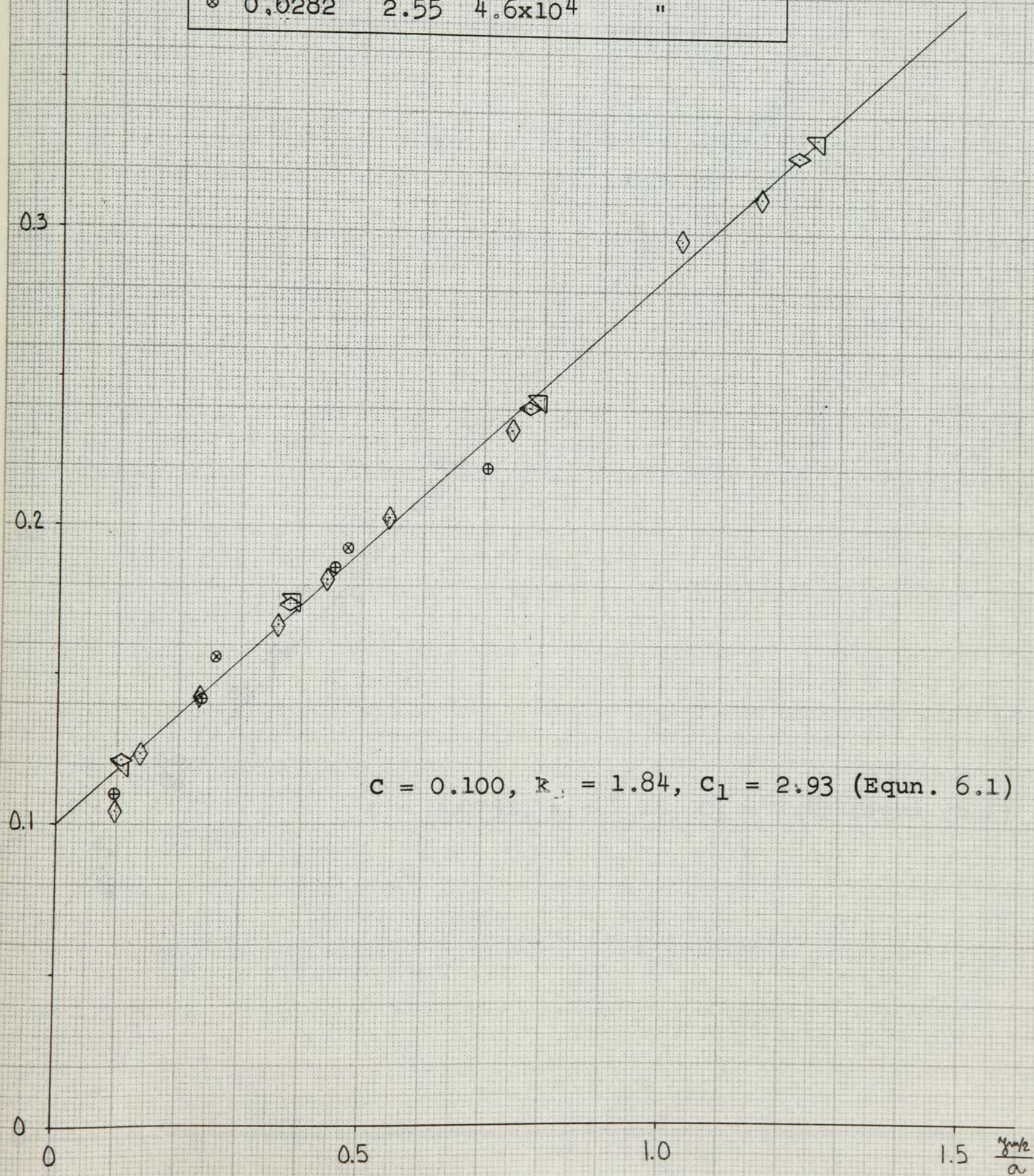


FIG. 16

STREAMWISE VARIATION OF $\frac{y_m/2}{a\theta_1}$ WITH $\frac{y_m/2}{a}$ -
IMPROVED CYLINDER

	$\frac{b}{a}$	a	Re
○	0.0074	4.0	4.0×10^4
◊	0.0229	"	"
◐	0.0379	"	"
●	0.0531	"	"

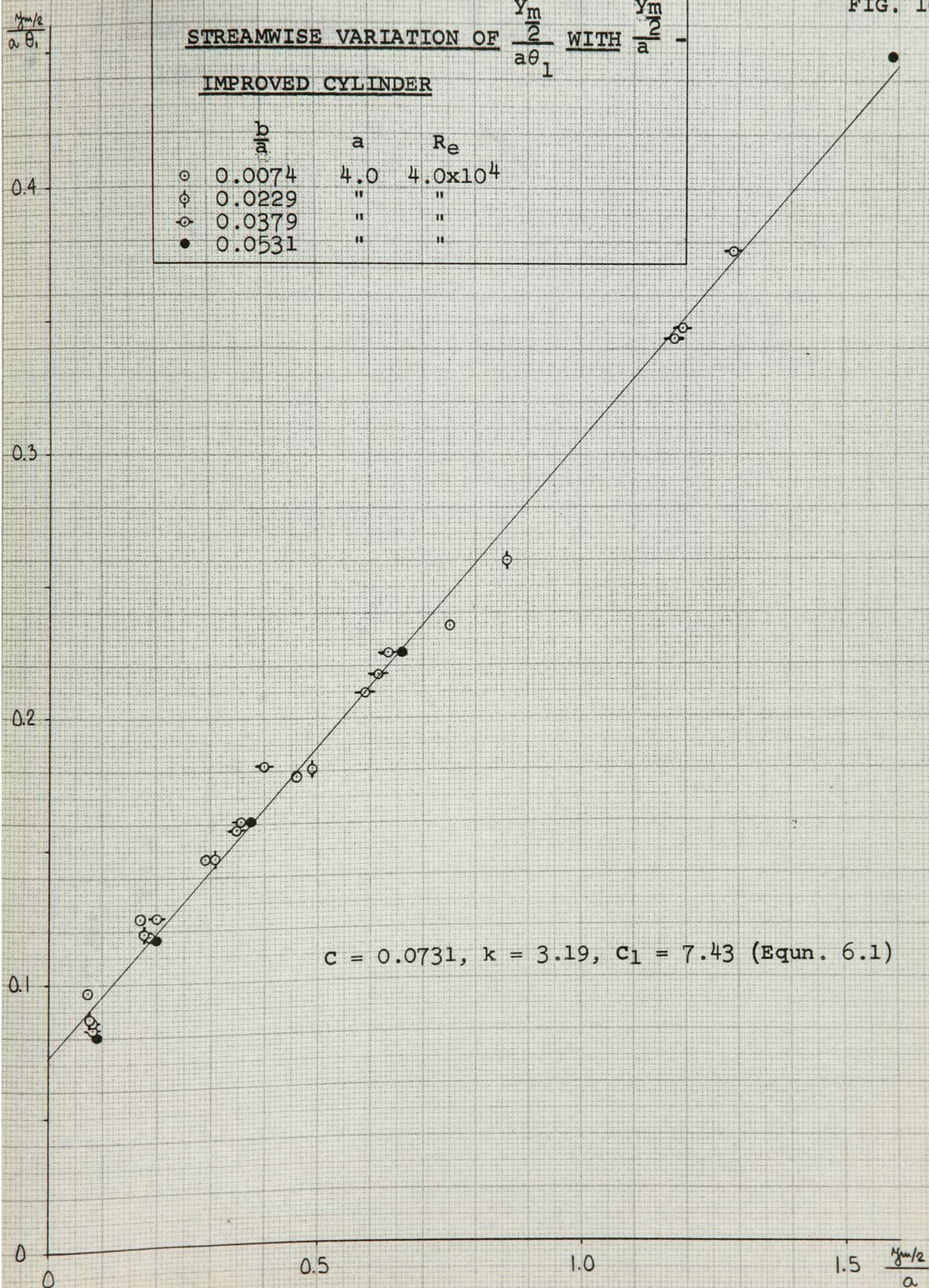


FIG. 17

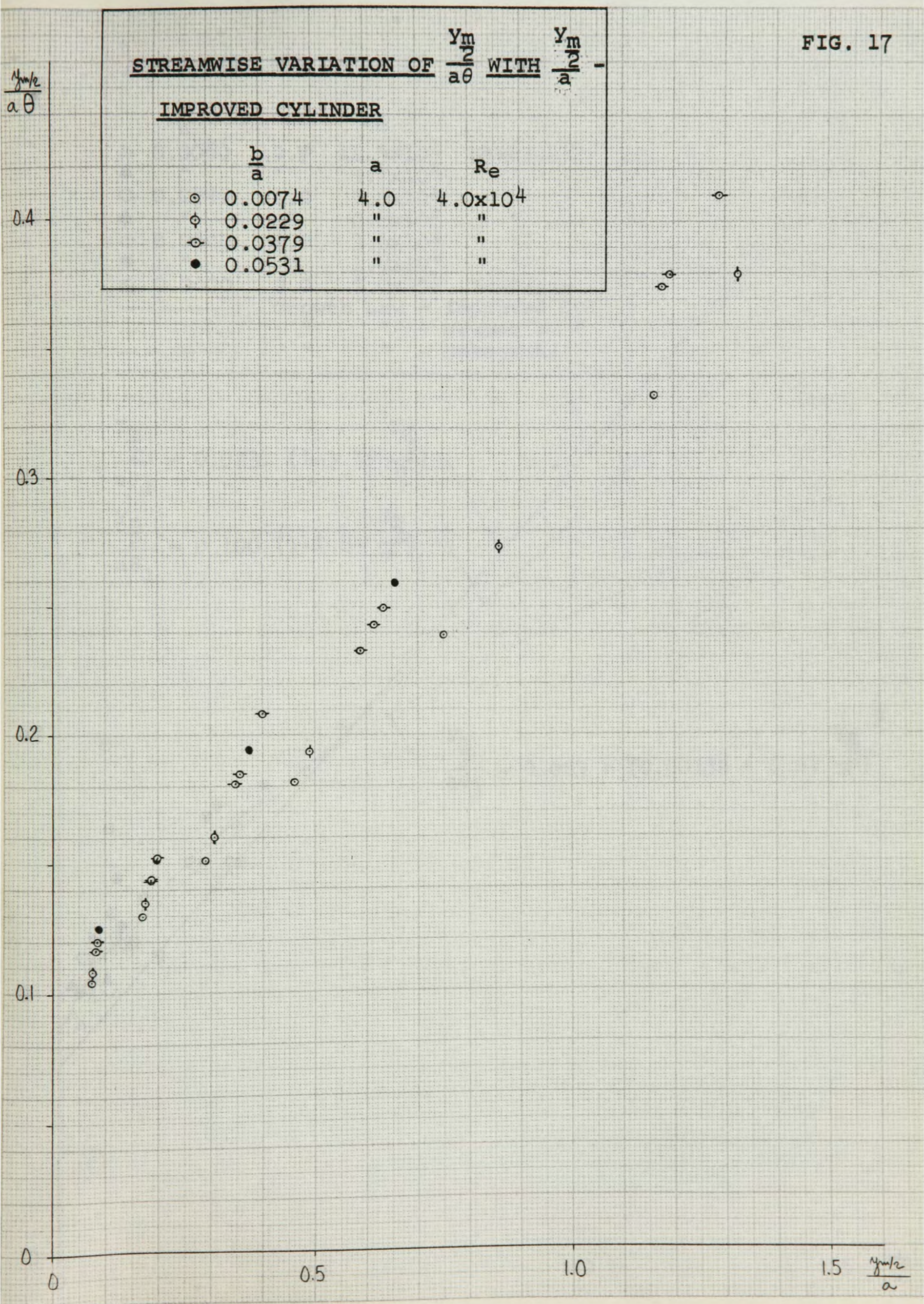
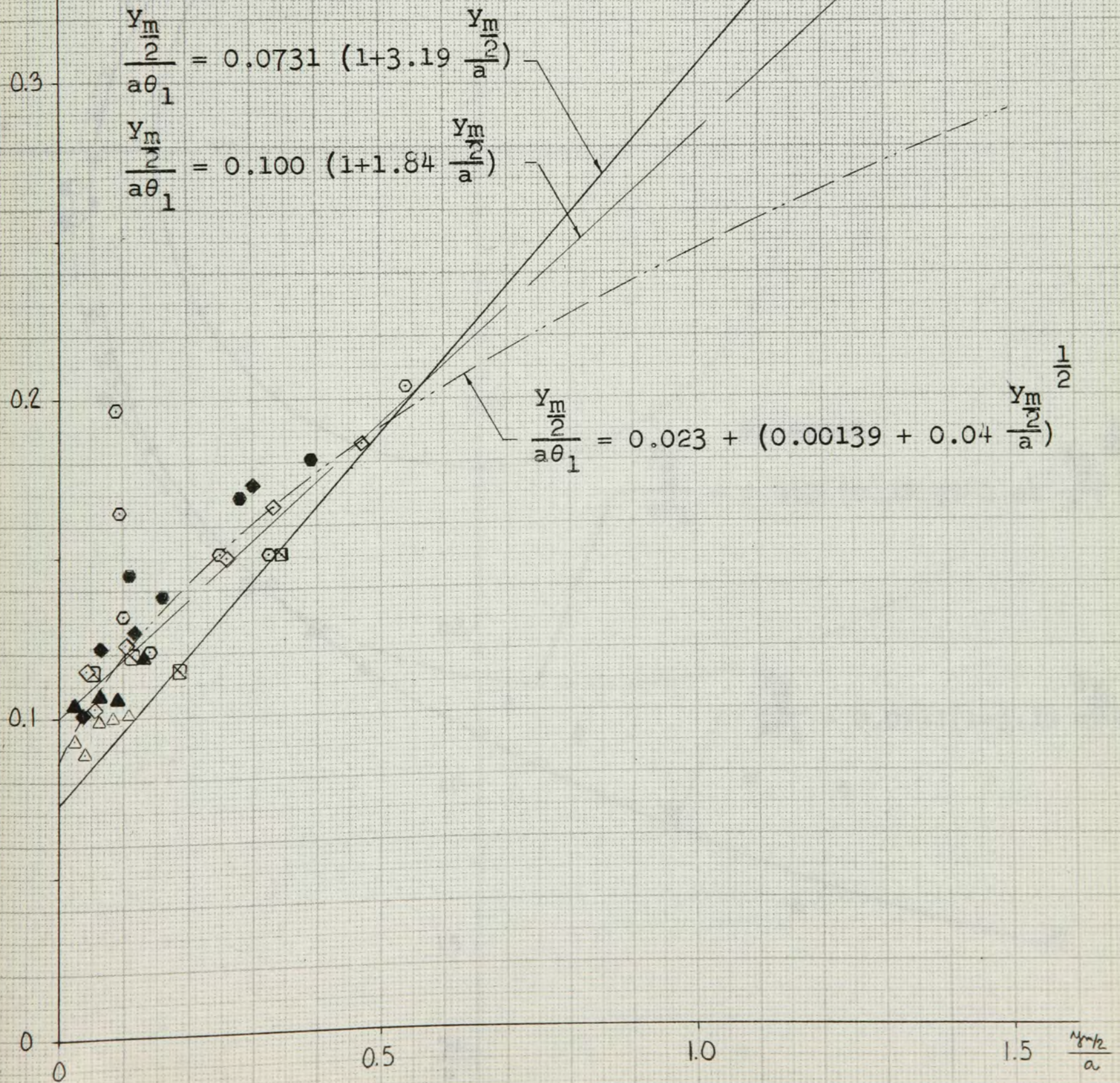


FIG. 18

STREAMWISE VARIATION OF $\frac{Y_m^2}{a\theta_1}$ WITH $\frac{Y_m^2}{a}$

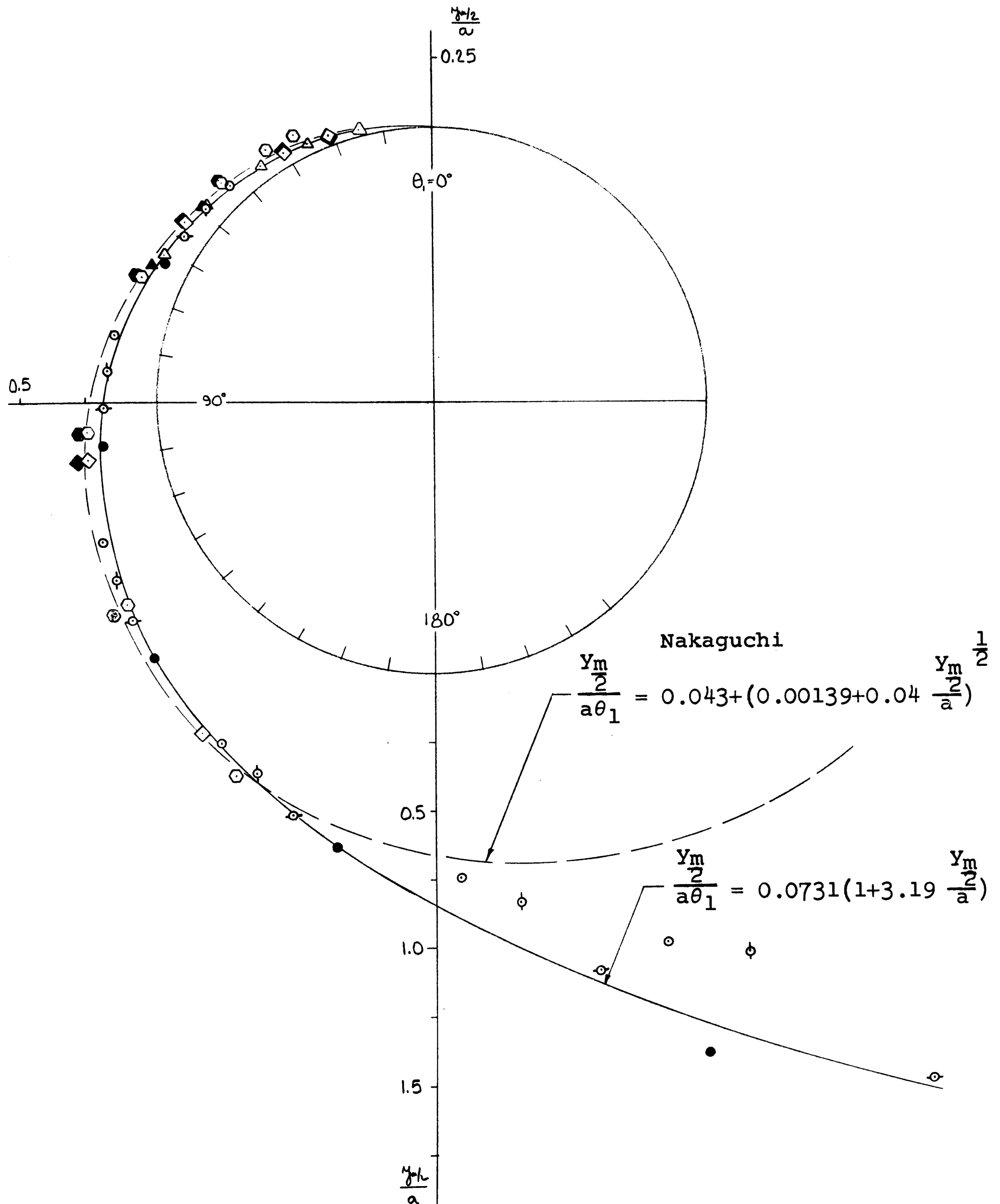
	$\frac{b}{a}$	a	Re	
$\triangle$	0.0060	19.7	11.3×10^4	Nakaguchi
$\blacktriangle$	"	"	5.7×10^4	"
$\diamond$	0.0240	4.9	5.6×10^4	"
$\blacklozenge$	"	"	2.8×10^4	"
$\circ$	0.0600	2.0	3.6×10^4	"
$\bullet$	"	"	1.8×10^4	"
$\square$	0.0020	9.0	9.0×10^4	Bradshaw
—				Growth Law - Improved Cyl.
---				Newman's "
- - -				Nakaguchi



STREAMWISE VARIATION OF $\frac{y_m}{2}$ WITH $\frac{y_m}{a}$

Data of Figures 16 and 18 Re-plotted
in Polar Coordinates.

FIG. 19



POSITION OF HYPOTHETICAL ORIGIN

FIG. 20

	Re	
$\triangle$	11.3×10^4 and 5.7×10^4	Nakaguchi
$\diamond$	5.6×10^4 " 2.8×10^4	"
$\odot$	3.6×10^4 " 1.8×10^4	"
—	4.0×10^4	Improved Cylinder
— —	"	Newman's

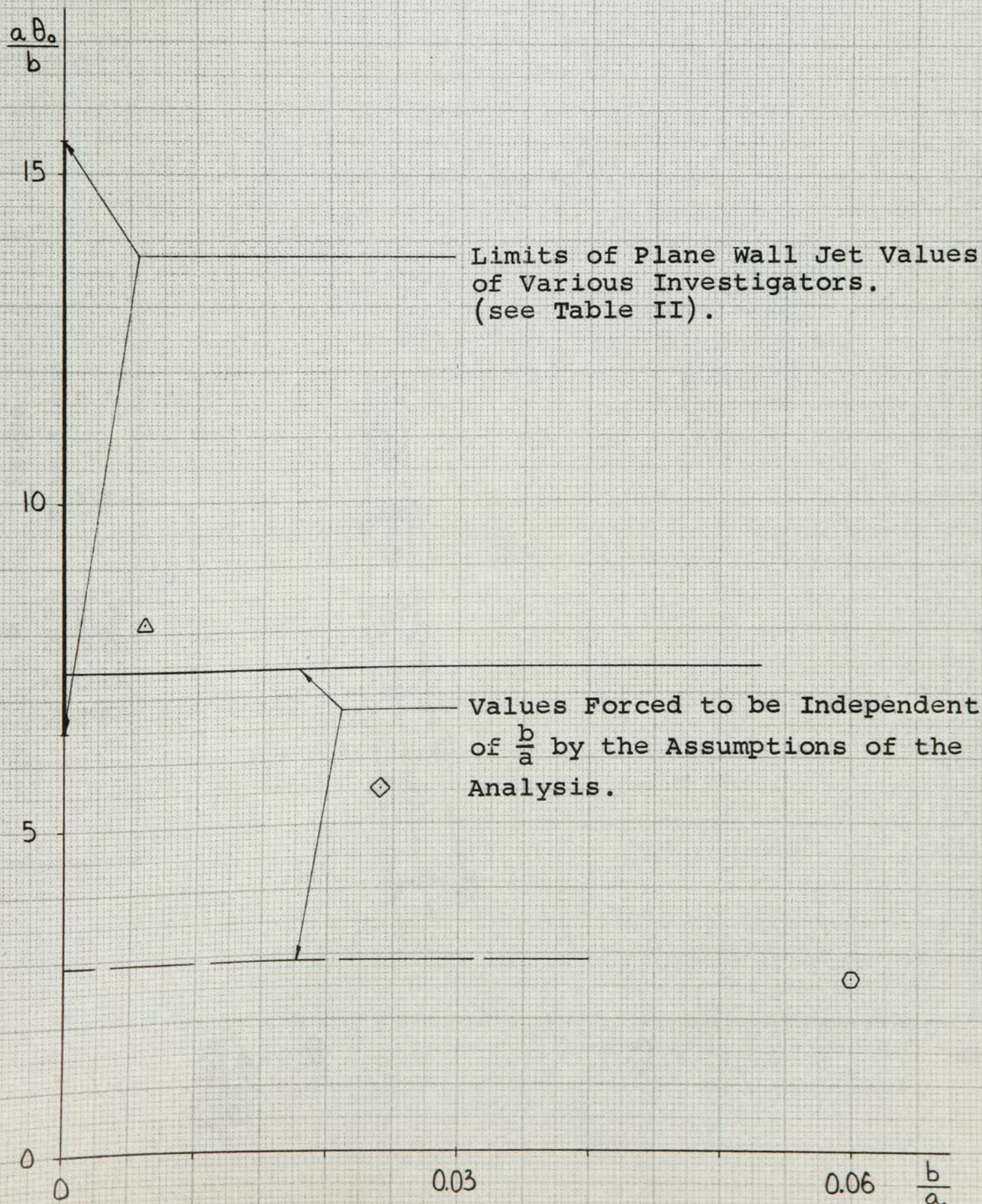


FIG. 21

RATIO OF ACTUAL-TO-THEORETICAL JET-MOMENTUM -

NEWMAN'S CYLINDER

$\frac{b}{a}$	Probe
$\triangle$ 0.0134	Flat Pitot
$\diamond$ 0.0207	"
∇ 0.0400	"

