

ANALYSIS OF M-QAM-OFDM TRANSMISSION SYSTEM PERFORMANCE IN THE PRESENCE OF PHASE NOISE AND NONLINEAR AMPLIFIERS

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ABSTRACT

The performance evaluation of M-QAM-OFDM transmission systems in the presence of phase noise and non-linear distortions due to High-Power-Amplifiers (HPA) is presented. A theoretical analysis enables us to derive the Bit Error Rate and to relate it to the HPA-distorted signal spectrum. The phase noise distortion effects are also taken into account by modeling the phase noise as a Wiener Lévy process. It is shown that with a simple way to compensate the phase distortion within an OFDM symbol, the system can still work also in the presence of relevant phase noise. An example of application is reported.

INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) systems have been developed for digital audio (DAB) and digital video (DVB) broadcasting over radio channels where selective fading and multipath cause severe degradation in single carrier wideband modulation formats Cimini (1). The major system impairment for these applications is due to the distortion generated by the High Power Amplifier (HPA) at the transmitter-end. In recent years some research programs have been looking to the possible use of the OFDM at carrier frequencies in the range of 40 GHz and up for the so called *cellular TV*. In these systems, oscillators phase noise is a new significant source of impairment. The effects induced by nonlinear and phase noise distortions have been separately investigated Tomba and Krzymien (2), Santella and Mazzenga (3) and Costa et al (4), but the joint effects have not been studied yet. Hereafter we lay the basis for a global study and we present some preliminary results.

SYSTEM MODEL

Reference Linear Model

Let us consider an OFDM transmission system Cimini (1), where the information to be transmitted is distributed over N orthogonal narrowband modulated subcarriers, spaced by $\Delta f = f_s/N$, where $f_s = 1/\Delta t$ is the symbol rate of the information signal. Each OFDM symbol has a duration $T = N\Delta t = N/f_s$. Let $a_k(i)$ be the data symbol modulating the k -th subcarrier in the OFDM symbol interval $\mathcal{I}_i \triangleq [iT, iT + T]$. Using an M-QAM modulation scheme for each subcarrier, the real and imaginary parts of $a_k(i)$ belong to the alphabet $\mathcal{A} = \{2m - 1 - \sqrt{M}; m = 1, 2, \dots, \sqrt{M}\}$. The OFDM transmitter and receiver block diagrams are shown in Fig. 1. The complex envelope of the OFDM signal in the interval $\mathcal{I}_i$ can be written as Weinstein and Ebert (5)

$$s(t) = \sum_{k=0}^{N-1} a_k(i) q_0 \exp(j2\pi f_k t) \quad t \in \mathcal{I}_i \quad (1)$$

where $f_k = k\Delta f$ is the center frequency of the k -th subchannel and q_0 is a scaling factor common to all subchannels, that will be assumed equal to 1. We remark that the k -th subchannel may be demodulated by multiplying $s(t)$ by the k -th subcarrier tone and applying the result to an Integrate & Dump (I&D) filter, i.e.

$$S_k(i) \triangleq \frac{1}{T} \int_{iT}^{iT+T} s(t) \exp(-j2\pi f_k t) dt = a_k(i) q_0, \quad (2)$$

System Impaired by HPA and Phase Noise

HPA Model

The HPA is modelled by a nonlinear memoryless transformation between the complex envelopes of the input and output signals Saleh (6). The amplified signal can be expressed as $u(t) \triangleq s(t)G[|s(t)|]$, with

$$G[|s(t)|] \triangleq \frac{A[|s(t)|] \exp \{j\Phi[|s(t)|]\}}{|s(t)|}, \quad (3)$$

where the functions $A[\cdot]$ and $\Phi[\cdot]$ represent the AM/AM and AM/PM conversion, respectively.

Phase Noise Model

We model the phase noise of the oscillators as a Wiener-Lévy process

$$\theta(t) = 2\pi \int_0^t \mu(\tau) d\tau, \quad (4)$$

where $\mu(\tau)$ is a white Gaussian frequency noise with power spectral density $R_\mu(f) = N_0$ Corvaja and Pupolin (7). The baseband equivalent of the noisy carrier turns out to be Lorentzian Kreß et al (8) with the two-sided 3-dB bandwidth $B = 2\pi N_0$. Moreover, the phase noise $\theta(t)$ has zero mean and variance $2\pi B|t|$.

Received Signal

The effect of phase noise on the amplified signal $u(t)$ can be modelled by a multiplicative error term $e(t) \triangleq \exp \{j\theta(t)\}$.

Thus, at the receiver-end, the distorted signal is

$$r(t) = z(t) + \eta(t), \quad (5)$$

with $z(t) \triangleq u(t)e(t)$, and $\eta(t)$ the baseband equivalent of the channel white Gaussian noise.

Distortion

The k -th subchannel distortion, $D_k(i)$, occurring in the i -th OFDM symbol interval is

$$D_k(i) \triangleq Z_k(i) - \alpha S_k(i), \quad (6)$$

where $S_k(i)$ is given by eq. (2) and $Z_k(i)$ is the value detected at the output of the k -th I&D

$$Z_k(i) \triangleq \frac{1}{T} \int_{iT}^{iT+T} z(t) \exp(-j2\pi f_k t) dt. \quad (7)$$

In eq. (6), α is a complex gain factor accounting for both amplitude and phase distortion generated by the HPA and the phase noise.

NONLINEAR AMPLIFIER DISTORTION

As shown in Costa et al (4), the nonlinear distortions induced by an HPA on an OFDM signal cause the following effects:

- warping of the signal constellation in each subchannel (amplitude and phase distortion);
- intermodulation effects between the subcarriers, which generate a cluster of received values around each constellation point, instead of a single one. Simulation results show that the distortion may be regarded as additive Gaussian noise. In Costa et al (4) it has been shown that the complex gain α and the HPA distortion power M_{hpa} can be obtained from spectral analysis. In particular, by observing that for a number of carriers N greater than 100 the OFDM signal $s(t)$ turns out to be a zero mean Gaussian random process, in Costa et al (4) the power spectral density of the signal after the HPA, $R_u(f)$, has been computed as the Fourier Transform of the autocorrelation function

$$r_u(\tau) = E[u(t+\tau)u^*(t)] = E[s(t+\tau)G(|s(t+\tau)|)s^*(t)G^*(|s(t)|)] \quad (8)$$

with $e(t) = 1$ because of the absence of phase noise and $u(t) = z(t)$. The coefficient α , defined in eq. (6), turns out to be $\alpha = r_{su}^*(0)/r_s(0)$. In a real system, a correcting factor $1/\alpha$ is mandatory in each subchannel. The variance of the in-phase and quadrature components of the distortion, $\sigma_{hpa}^2 = M_{hpa}/2$, is then given by

$$\sigma_{hpa}^2 = \frac{1}{2} \frac{1}{|\alpha|^2} P_{d,k}, \quad (9)$$

with

$$P_{d,k} = \frac{2}{3} (M-1) |\alpha|^2 \cdot \frac{R_d(f_k)}{R_u(f_k)}. \quad (10)$$

PHASE NOISE DISTORTION

Distortion Model

Without loss of generality we consider the symbol interval $\mathcal{I}_0$. The samples driving the FFT demodulator are

$$z_n = u_n e^{j[\phi_n + \Phi_0]}, \quad (11)$$

where Φ_0 is the initial phase value, ϕ_n is a Gaussian random variable given by

$$\phi_n = \phi_{n-1} + \psi_n \quad ; \quad \phi_0 = 0 \quad (12)$$

and ψ_n is the random phase increment in the n -th time slot of length Δt . Based on the phase noise model considered, $\{\psi_n\}$ turns out to be a sequence of mutually independent identically distributed Gaussian random variables, with variance $\sigma_\psi^2 = 2\pi B \Delta t = 2\pi BT/N$. The signal samples at the output of the FFT are

$$Z_k = \frac{1}{N} \sum_{n=0}^{N-1} z_n e^{-j \frac{2\pi}{N} kn}. \quad (13)$$

In the absence of nonlinear distortions, and assuming ϕ_n small, in Armada and Calvo (10) it has been proven that

$$Z_k = (a_k + e_k) e^{j\Phi_0}, \quad (14)$$

where e_k can be further separated into two independent error terms $e_k = e_{k1} + e_{k2}$ with

$$e_{k1} = ja_k \frac{1}{N} \sum_{n=0}^{N-1} \phi_n, \quad (15)$$

and

$$e_{k2} = j \sum_{r=0, r \neq k}^{N-1} a_r \frac{1}{N} \sum_{n=0}^{N-1} \phi_n e^{j \frac{2\pi}{N} (r-k)n}. \quad (16)$$

The quantity

$$\Phi \triangleq \frac{1}{N} \sum_{n=0}^{N-1} \phi_n + \Phi_0 \quad (17)$$

represents the common phase rotation (CPR) of the constellation, which is the same for all the subcarriers in one OFDM symbol interval, while the term e_{k1} represents the average symbol displacement which is proportional to the data symbol magnitude. The second error term e_{k2} represents the intercarrier interference and it behaves like a zero mean complex white Gaussian noise superimposed to each subcarrier useful signal. So, even in the absence of nonlinearity, phase noise produces two effects:

- a rotation of the constellation in each subchannel;
- a cluster of received values for each constellation point.

Phase Noise Distortion Second Order Statistics

From the definition (17) Φ is a Gaussian random variable with mean value Φ_0 and variance

$$\sigma_\Phi^2 = \sigma_\psi^2 \frac{(N+1)(2N+1)}{6N}. \quad (18)$$

As a consequence, if we represent the received points over many OFDM symbols, we see a series of circular clouds with radius related to the variance of e_{k2} centered, for each OFDM symbol, on a different position of

the constellation rotated by the CPR Φ . By simulation, we checked that the CPR can be estimated in each OFDM symbol interval by averaging the phase errors detected in all subcarriers. By compensating the CPR, the received clouds appear to be centered around the expected constellation points and have a small residual phase noise error Φ_r , whose variance is $\sigma_{\Phi_r}^2$. This result is shown in Fig. 2 for the first quadrant constellation point of a QPSK-OFDM system with $BT = 5 \cdot 10^{-3}$.

For a given symbol γ , let $(m_{Z_I|\gamma} + jm_{Z_Q|\gamma})$ the average detected sample. Moreover, let $\sigma_{Z_I|\gamma}^2 \triangleq E[(Z_I - m_{Z_I|\gamma})^2|\gamma]$, $\sigma_{Z_Q|\gamma}^2 \triangleq E[(Z_Q - m_{Z_Q|\gamma})^2|\gamma]$ be the variances of the in-phase and quadrature components and $c_{Z_I Z_Q|\gamma} = E[(Z_I - m_{Z_I|\gamma})(Z_Q - m_{Z_Q|\gamma})|\gamma]$ the in-phase and quadrature cross-correlation. By assuming Φ_r small, after some algebra, we get

$$\sigma_{Z_I|\gamma}^2 \simeq \sigma_{pn}^2 + m_{Z_Q|\gamma}^2 \sigma_{\Phi_r}^2 \quad ; \quad \sigma_{Z_Q|\gamma}^2 \simeq \sigma_{pn}^2 + m_{Z_I|\gamma}^2 \sigma_{\Phi_r}^2 \quad (19)$$

$$c_{Z_I Z_Q|\gamma} \simeq -m_{Z_I|\gamma} m_{Z_Q|\gamma} \sigma_{\Phi_r}^2 \quad (20)$$

where σ_{pn}^2 is the variance of the in-phase (quadrature) component of e_{k2} . From simulation we derive an estimate of $m_{Z_I|\gamma}$, $m_{Z_Q|\gamma}$, $\sigma_{Z_I|\gamma}^2$, $\sigma_{Z_Q|\gamma}^2$ and $c_{Z_I Z_Q|\gamma}$ for each of the M-QAM alphabet symbols. Then eq. (20) enables us to estimate $\sigma_{\Phi_r}^2$ and equation (19) allows us to estimate σ_{pn}^2 . Moreover, as stated in Section 3, also the nonlinear distortion induced by an HPA may be regarded as additive complex Gaussian noise superimposed to the received symbols. Thus, in the presence of both HPA nonlinearity and phase noise, the total distortion may be regarded as a zero mean Gaussian complex random process with in-phase and quadrature variance given by

$$\sigma_{tot}^2 = \sigma_{hpa}^2 + \sigma_{pn}^2. \quad (21)$$

EXAMPLE OF APPLICATION

We refer to a 16-QAM-OFDM system with $N = 128$ subcarriers transmitting over a channel with bandwidth $f_s = 9$ MHz. The bandwidth per subcarrier is $\Delta f_s = 9/128$ MHz. For the sake of simplicity we have not considered the presence of a guard interval, so the OFDM symbol duration is equal to $T = 1/\Delta f_s = 14.22 \mu s$. We have considered a TWTA (Travelling Wave Tube Amplifier) at the transmitter-end with different values of Input Back-Off (IBO). We now report the results obtained for a 16-QAM-OFDM system encompassing a TWTA with IBO = 14 dB, in the presence of phase noise characterized by a 3-dB bandwidth $B = 5 \cdot 10^{-3}/T$. In this case $\sigma_{\Phi}^2 = 0.0105$. Fig. 3a shows the superimposition of the received signal constellations over all the OFDM symbol intervals, without CPR compensation at the receiver-end. The reduction in the constellation warping achieved by means of CPR compensation is apparent in Fig. 3b.

In the presence of particularly detrimental nonlinearity characteristics (IBO ≤ 14 dB) the residual phase noise variance results $\sigma_{\Phi_r}^2 = 0.002$, while in the absence of nonlinearity $\sigma_{\Phi_r}^2$ reduces to 0.001. An accurate estimate of the system BER can be derived using standard techniques for M-QAM modulation scheme with additive Gaussian noise, which in this case is given by the superimposition of the channel Gaussian noise and the distortion due to phase noise and HPA nonlinearity. Fig. 4 shows the BER for the considered OFDM system, versus the signal-to-channel-noise power ratio, for several IBO.

CONCLUSIONS

By means of computer simulations and spectral analysis, we have investigated the joint effects of HPA nonlinear distortions and phase noise distortions on OFDM signals. It results that HPA nonlinearity enhances the detrimental effects due to phase noise, and that CPR compensation is mandatory to make the system working. Further study should be done to verify if spectral analysis is a sufficient tool to derive the estimate of the overall system performance.

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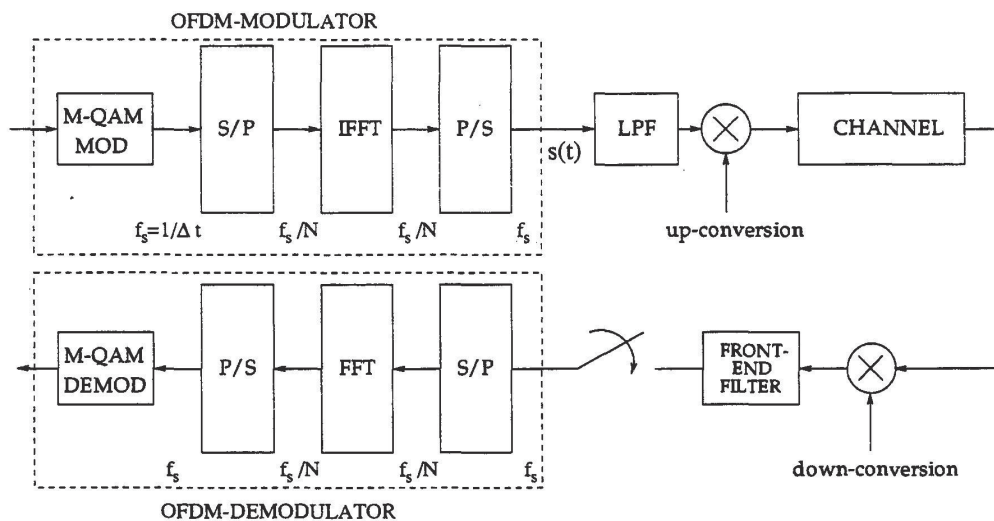


Figure 1: Transmitter and receiver block diagrams of the OFDM system.

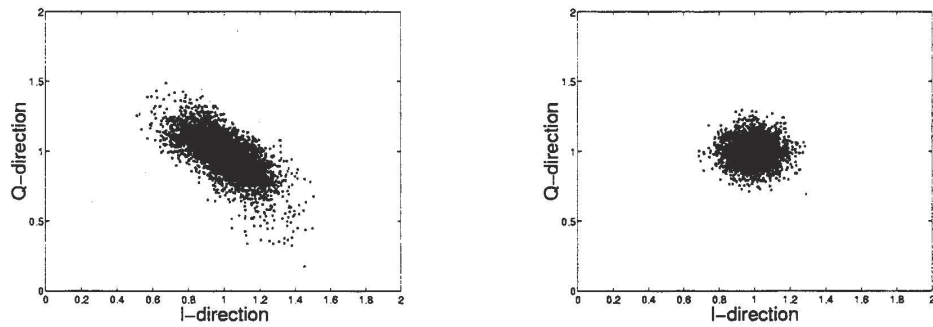


Figure 2: QPSK constellation point with phase noise distortion before and after CPR compensation.

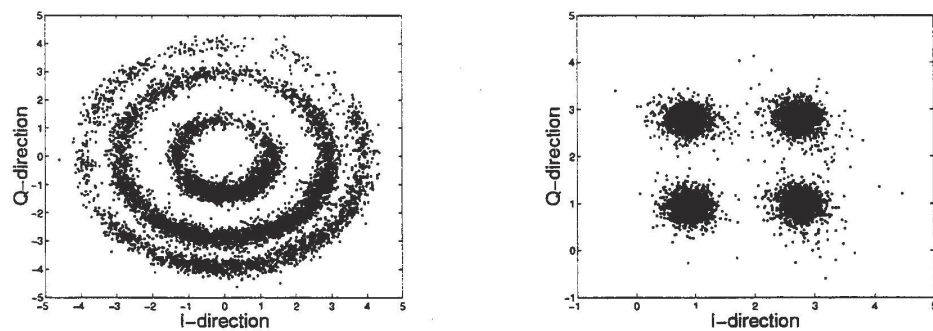


Figure 3: 16-QAM constellation with phase noise distortion and TWTA (IBO= 14 dB) before and after CPR compensation.

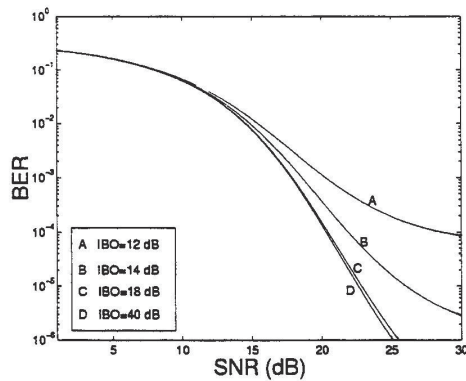


Figure 4: BER of 16-QAM-OFDM system with phase noise distortion and TWTA with CPR compensation.