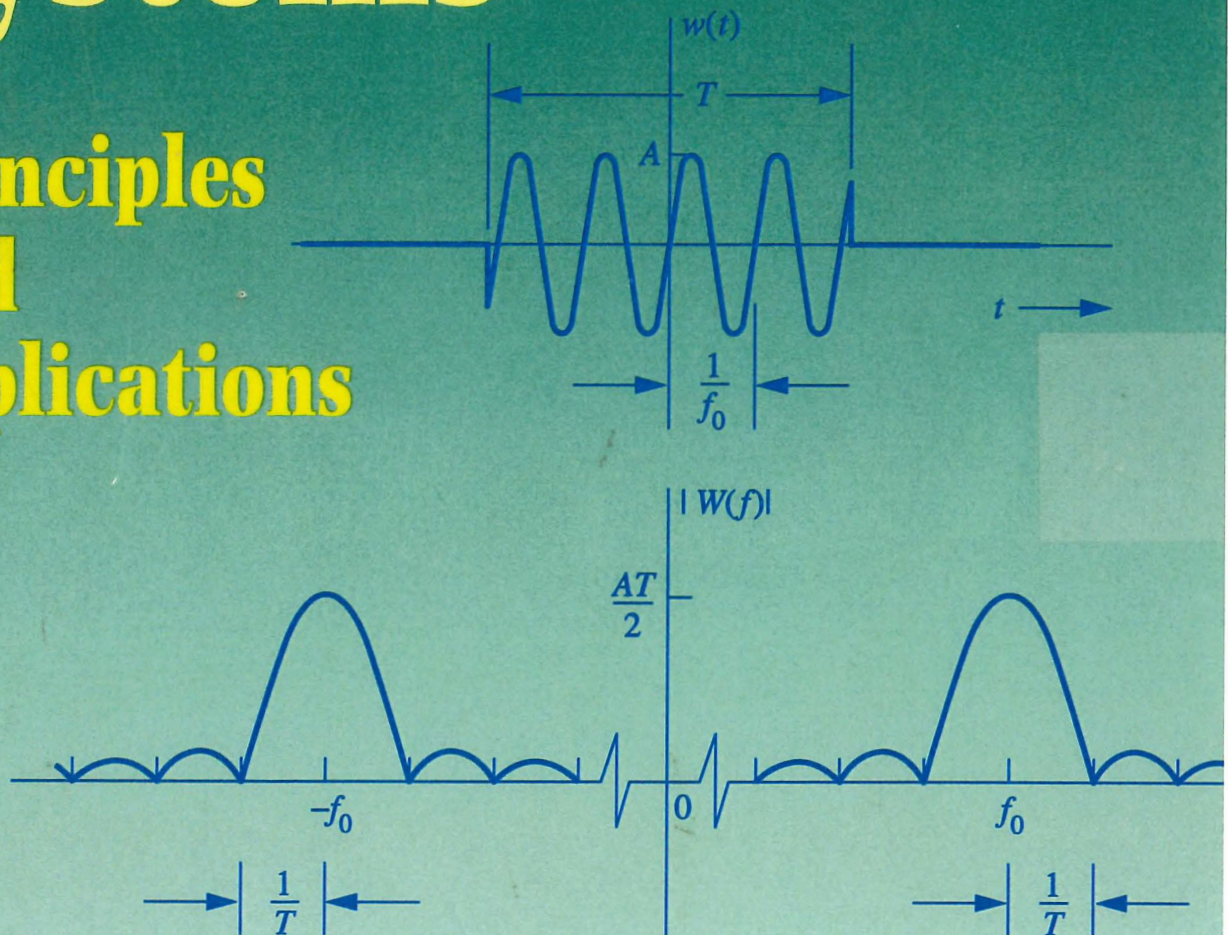


# Modern Communication Systems

Principles  
and  
Applications



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This result is illustrated in Fig. 7-8. If a raised cosine-rolloff premodulation filter is used, the transmission bandwidth of the FSK signal becomes

$$B_T = 2\Delta F + (1 + r)R \quad (7-22)$$

For wideband FSK, where  $\beta \gg 1$ ,  $\Delta F$  dominates in these equations and we have  $B_T = 2\Delta F$ . For narrowband FSK the transmission bandwidth is  $B_T = 2B$ .

The exact PSD for continuous-phase FSK signals is difficult to evaluate for the case of random data modulation. However, it can be done by using some elegant statistical techniques [Anderson and Salz, 1965; Bennett and Rice, 1963; Proakis, 1983, pp. 130–135]. The resulting PSD for the complex envelope of the FSK signal is<sup>†</sup>



$$\mathcal{P}_s(f) = \frac{A_c^2 T_b}{2} \times (A_1^2(f)[1 + B_{11}(f)] + A_2^2(f)[1 + B_{22}(f)] + 2B_{12}(f)A_1(f)A_2(f)) \quad (7-23a)$$

where

$$A_n(f) = \frac{\sin[\pi T_b(f - \Delta F(2n - 3))]}{\pi T_b(f - \Delta F(2n - 3))} \quad (7-23b)$$

and

$$B_{nm}(f) = \frac{\cos[2\pi f T_b - 2\pi \Delta F T_b(n + m - 3)] - \cos(2\pi \Delta F T_b) \cos[2\pi \Delta F T_b(n + m - 3)]}{1 + \cos^2(2\pi \Delta F T_b) - 2 \cos(2\pi \Delta F T_b) \cos(2\pi f T_b)} \quad (7-23c)$$

where  $\Delta F$  is the peak frequency deviation,  $R = 1/T_b$  is the bit rate, and the digital modulation index is  $h = 2\Delta F/R$ . The PSD is plotted in Fig. 7-9 for several values of the digital modulation index. The curves in this figure were obtained by using the MathCAD computation shown in Appendix D.

FSK can be detected by using either a frequency (noncoherent) detector or two product detectors (coherent detection), as shown in Fig. 7-10. For optimum detection—that is, to obtain the lowest probability of bit error when the received FSK signal is corrupted by AWGN—coherent detection with matched filter processing is required [Couch, 1993]. The probability of bit error for noncoherent detection and optimum coherent detection of FSK is given in Sec. 7-4.

## 7-2 Multilevel Signaling

With multilevel signaling, digital inputs with more than two modulation levels are allowed on the transmitter input. This is illustrated in Fig. 7-11, which shows how multilevel signals can be generated from a serial binary input stream using a digital-to-analog converter (DAC). For example, suppose that an  $\ell = 2$ -bit DAC is used. Then the number of

<sup>†</sup>This result assumes that the carrier oscillator phase is not synchronized to any external source and that  $h \neq 0, 1, 2, \dots$ . When  $h = 0, 1, 2, \dots$ , there are also discrete terms (delta functions) in the spectrum.

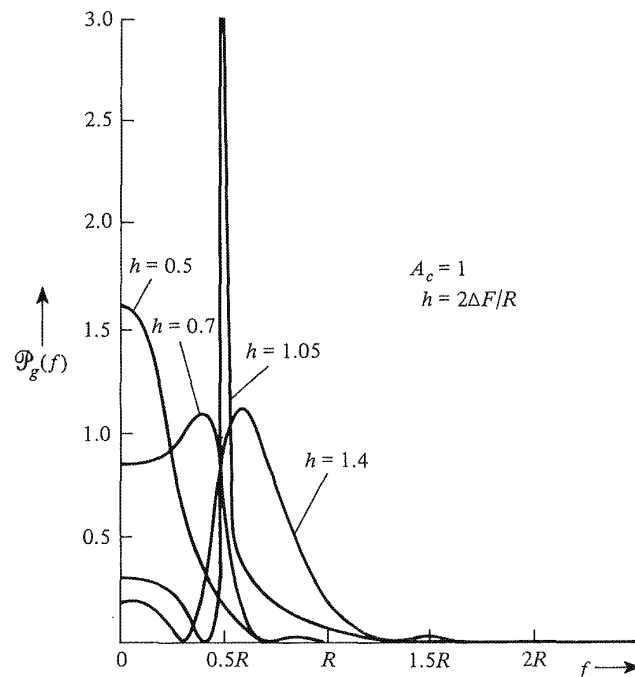


FIGURE 7-9 PSD for the complex envelope of FSK (positive frequencies shown).

levels (symbols) in the multilevel signal is  $M = 2^\ell = 2^2 = 4$ . The symbol rate (baud) of the multilevel signal is  $D = R/\ell = \frac{1}{2}R$ , where the bit rate is  $R = 1/T_b$  bits/sec.

### Quadrature Phase-Shift Keying (QPSK) and $M$ -ary Phase-Shift Keying (MPSK)

If the transmitter is a PM transmitter with an  $M = 4$  level digital modulation signal,  $M$ -ary phase-shift keying (MPSK) is generated at the transmitter output. A plot of the permitted

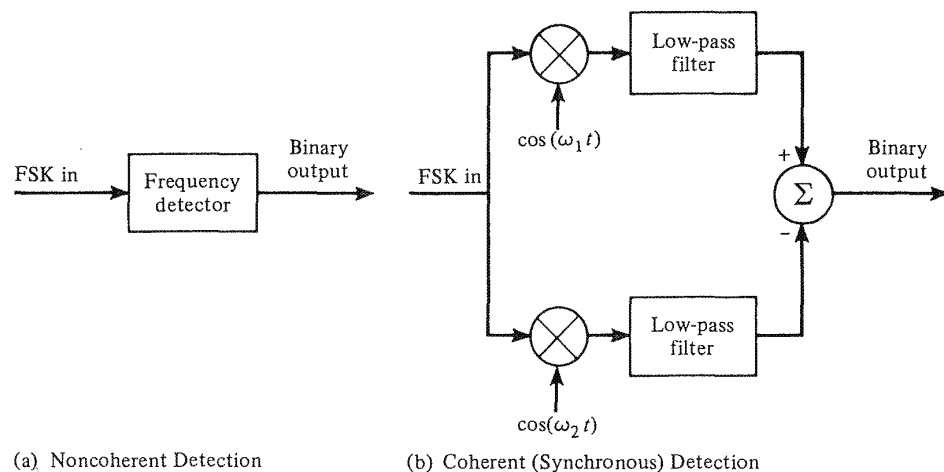


FIGURE 7-10 Detection of FSK.

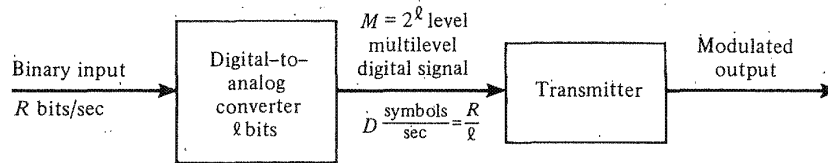


FIGURE 7-11 Multilevel digital transmission system.

values of the complex envelope,  $g(t) = A_c e^{j\theta(t)}$ , would contain four points, one value of  $g$  (a complex number in general) for each of the four multilevel values, corresponding to the four phases that  $\theta$  is permitted to have. A plot of two possible sets of  $g(t)$  is shown in Fig. 7-12. For instance, suppose that the permitted multilevel values at the DAC are  $-3$ ,  $-1$ ,  $+1$ , and  $+3$  V; then in Fig. 7-12a these multilevel values might correspond to PSK phases of  $0^\circ$ ,  $90^\circ$ ,  $180^\circ$ , and  $270^\circ$ , respectively. In Fig. 7-12b these levels would correspond to carrier phases of  $45^\circ$ ,  $135^\circ$ ,  $225^\circ$ , and  $315^\circ$ , respectively. These two signal constellations are essentially the same except for a shift in the carrier-phase reference.<sup>†</sup> This example of  $M$ -ary PSK where  $M = 4$  is called *quadrature-phase-shift-keyed* (QPSK) signaling.

MPSK can also be generated by using two quadrature carriers modulated by the  $x$  and  $y$  components of the complex envelope (instead of using a phase modulator)

$$g(t) = A_c e^{j\theta(t)} = x(t) + jy(t) \quad (7-24)$$

where the permitted values of  $x$  and  $y$  are

$$x_i = A_c \cos \theta_i \quad (7-25)$$

$$y_i = A_c \sin \theta_i \quad (7-26)$$

for the permitted phase angles  $\theta_i$ ,  $i = 1, 2, \dots, M$ , of the MPSK signal. This is illustrated by Fig. 7-13, where the signal processing circuit implements (7-25) and (7-26). Figure

<sup>†</sup> A constellation is an  $N$ -dimensional plot of the possible signal vectors corresponding to the possible digital signals (see Sec. 3-6).

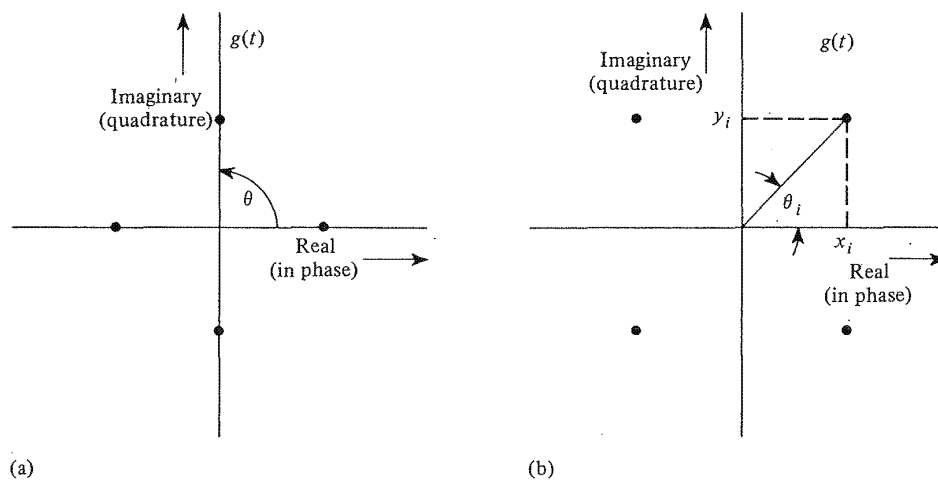
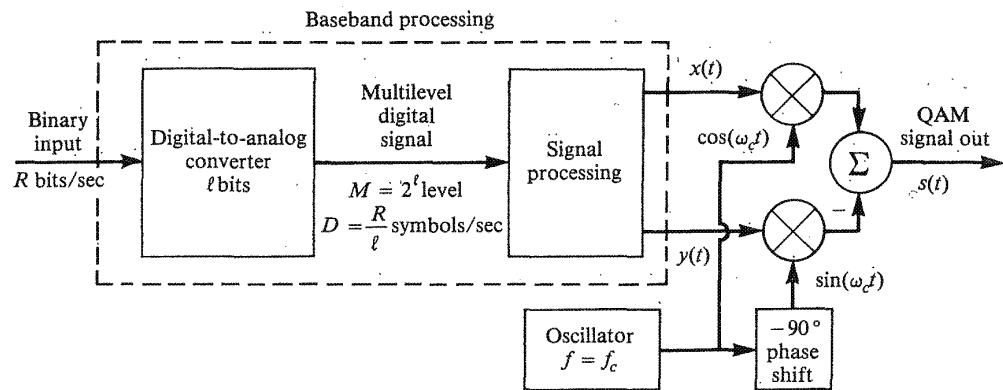
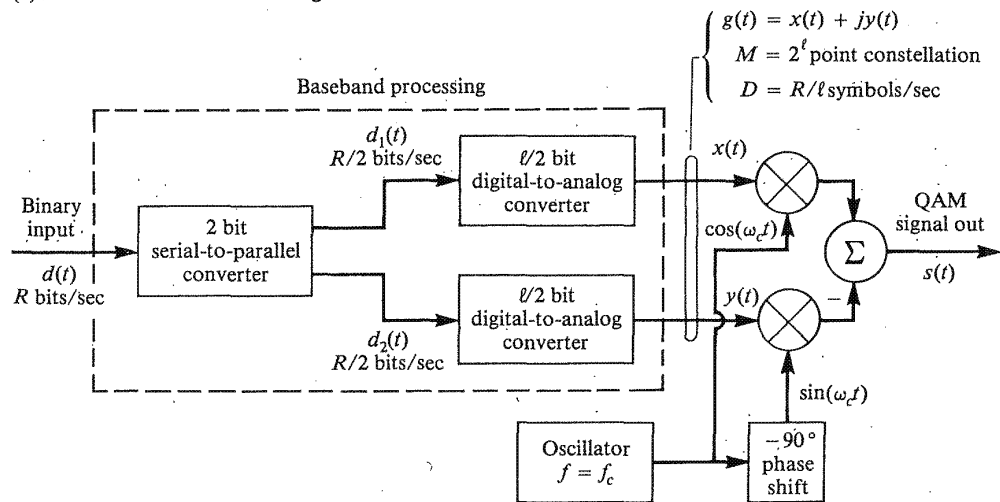


FIGURE 7-12 QPSK signal constellations (permitted values of the complex envelope).



(a) Modulator for Generalized Signal Constellation



(b) Modulator for Rectangular Signal Constellation

FIGURE 7-13 Generation of QAM signals.

7-12 gives the relationship between the permitted phase angles  $\theta_i$  and the  $(x_i, y_i)$  components for two QPSK signal constellations. Note that this is identical to the quadrature method of generating modulated signals presented in Fig. 5-27.

### Quadrature Amplitude Modulation (QAM)

Quadrature carrier signaling, as shown in Fig. 7-13, is called *quadrature amplitude modulation* (QAM). In general, QAM signal constellations are not restricted to having permitted signaling points only on a circle (of radius  $A_c$ , as was the case for MPSK). The general QAM signal is

$$s(t) = x(t) \cos \omega_c t - y(t) \sin \omega_c t \quad (7-27)$$

where

$$g(t) = x(t) + jy(t) = R(t)e^{j\theta(t)} \quad (7-28)$$



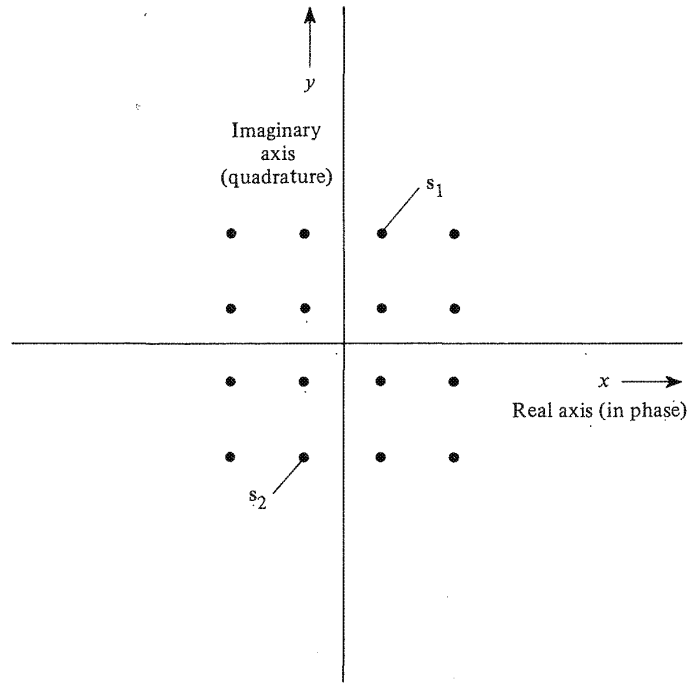


**FIGURE 7-14** 16-symbol QAM constellation (four levels per dimension).

For example, a popular 16-symbol ( $M = 16$  levels) QAM constellation is shown in Fig. 7-14, where the relationship between  $(R_i, \theta_i)$  and  $(x_i, y_i)$  can readily be evaluated for each of the 16 signal values permitted. This type of signaling is used by 2400 bit/sec V.22 bis computer modems (see Table C-13). Here  $x_i$  and  $y_i$  are each permitted to have four levels per dimension. This 16-symbol QAM signal may be generated by using two ( $\ell/2 = 2$ )-bit digital-to-analog converters and quadrature balanced modulators as shown in Fig. 7-13b. The waveforms of  $x$  and  $y$  are represented by

$$x(t) = \sum_n x_n h_1\left(t - \frac{n}{D}\right) \quad (7-29)$$

and

$$y(t) = \sum_n y_n h_1\left(t - \frac{n}{D}\right) \quad (7-30)$$

where  $D = R/\ell$  and  $(x_n, y_n)$  denotes one of the permitted  $(x_i, y_i)$  values during the symbol time that is centered on  $t = nT = n/D$  seconds. (It takes  $T$  seconds to send each symbol.)  $h_1(t)$  is the pulse shape that is used for each symbol. If the bandwidth of the QAM signal is not to be restricted, the pulse shape will be rectangular of  $T$ -second duration. In some applications the timing between the  $x(t)$  and  $y(t)$  components is offset by  $T/2 = 1/(2D)$

seconds. That is,  $x(t)$  would be described by (7-29), and the offset  $y(t)$  would be described by

$$y(t) = \sum_n y_n h_1\left(t - \frac{n}{D} - \frac{1}{2D}\right) \quad (7-31)$$

One popular type of offset signaling is *offset QPSK (OQPSK)*, which is offset QAM when  $M = 4$ . A special case of OQPSK when  $h_1(t)$  has a sinusoidal type of pulse shape is called *minimum-shift keying (MSK)*. This type will be studied immediately after the next section. Furthermore, a QPSK signal is said to be *unbalanced* if the  $x(t)$  and  $y(t)$  components have unequal powers and/or unequal data rates.

### Power Spectral Density for MPSK and QAM

The PSD for MPSK and QAM signals is relatively easy to evaluate for the case of rectangular bit shape signaling. In this case, the PSD has the same spectral shape that was obtained for BPSK, *provided* that proper frequency scaling is used. For example, take the case of  $M = 16$  QAM signaling illustrated in Fig. 7-14. The PSD can be evaluated by taking two *antipodal* signal vectors at a time and evaluating the PSD for the condition of signaling only with data modulation corresponding to those signal vectors. That is, referring to the figure, suppose that the vectors corresponding to the points  $s_1$  and  $s_2$  are used. This is equivalent to BPSK signaling. When we use (7-12), the *conditional* PSD of the complex envelope is of the form

$$\mathcal{P}_g(f|s_1 \text{ or } s_2) = A_i \left( \frac{\sin \pi f T}{\pi f T} \right)^2 \quad (7-32)$$

where the time that it takes to send one symbol is  $T$  and  $T = 1/D = \ell/R = \ell T_b$ . The resulting PSD, considering all possible antipodal signal pairs, is (using the probability theory of Appendix B)

$$\mathcal{P}_g(f) = \sum_{\substack{i=1 \\ i=\text{odd}}}^M \mathcal{P}(f|s_i \text{ or } s_{i+1}) P(s_i \text{ or } s_{i+1}) \quad (7-33)$$

where  $i = 1, 3, 5, \dots$  and  $P(s_i \text{ or } s_{i+1})$  is the probability of sending the  $s_i$  or  $s_{i+1}$  antipodal vector pair. For equally likely random data (i.e., the usual situation),  $P(s_i \text{ or } s_{i+1}) = 1/(M/2)$ , so (7-33) becomes

$$\mathcal{P}_g(f) = \frac{2}{M} \sum_{\substack{i=1 \\ i=\text{odd}}}^M A_i \left( \frac{\sin \pi f T}{\pi f T} \right)^2 = \frac{2}{M} \left( \frac{\sin \pi f T}{\pi f T} \right)^2 \sum_{\substack{i=1 \\ i=\text{odd}}}^M A_i \quad (7-34)$$

But  $\sum A_i$  is just a constant. Thus the PSD for the complex envelope of MPSK or QAM signals with data modulation of rectangular bit shape is

$$\mathcal{P}_g(f) = K \left( \frac{\sin \pi f \ell T_b}{\pi f \ell T_b} \right)^2 \quad \text{for MPSK and QAM} \quad (7-35)$$

where  $M = 2^\ell$  is the number of points in the signal constellation and the bit rate is  $R = 1/T_b$ . For a total transmitted power of  $P$  watts the value of  $K$  is  $K = 2P\ell T_b$  since

