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Electric Circuits

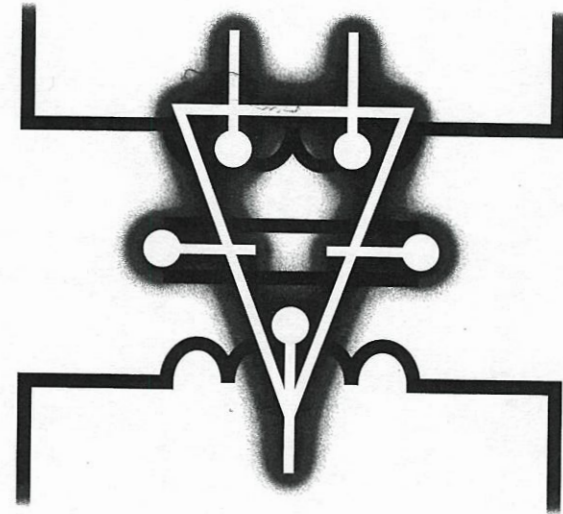
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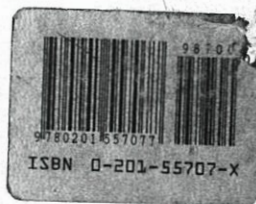
Addison
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Electric Circuits

FIFTH EDITION



James W. Nilsson
Susan A. Riedel



 Addison-Wesley Publishing Company

ELECTRIC CIRCUITS
FIFTH EDITION

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Nilsson, James William.
Electric circuits / James W. Nilsson, Susan A. Riedel. — 5th ed.

p.
cm.

Includes index.
ISBN 0-201-55707-X

I. Electric circuits. I. Riedel, Susan A. II. Title.

TK545.N54 1966

621.31972—dc20

94-17612
CIP

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ISBN 0-201-55707-X

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TO ANNA

PRACTICAL PERSPECTIVE

Electrical Safety

"Danger—High Voltage." This commonly seen warning is misleading. All forms of energy, including electrical energy, can be hazardous. But it's not only the voltage that harms. The static electricity shock you receive when you walk across a carpet and touch a doorknob is annoying but does not injure. Yet that spark is caused by a voltage hundreds or thousands of times larger than the voltages which can cause harm.

The thing that can actually cause injury is electrical current and how it flows through the body. Why, then, does the sign warn of high voltage? Because of the way electrical power is produced and distributed, it is easier to determine voltages than currents. Also, most electrical sources produce constant, specified voltages. So the signs warn about what is easy to measure. Determining whether and under what conditions a source can supply potentially dangerous currents, however, is more difficult, as this requires an understanding of electrical engineering.

Before we can examine this aspect of electrical safety, we have to learn how voltages and currents are produced and the relationship between them. The electrical behavior of objects, such as the human body, is quite complex and often beyond complete comprehension. To allow us to predict and control electrical phenomena, we use simplifying models in which simple mathematical relationships between voltage and current are used to approximate the actual relationships in real objects. Such models and analytical methods form the core of the electrical engineering techniques that will allow us to

understand all electrical phenomena, including those relating to electrical safety.

At the end of this chapter we will use a simple electric circuit model to describe how and why people are injured by electric currents. While we may never develop a complete and accurate explanation of the electrical behavior of the human body, we can obtain a close approximation using simple circuit models to assess and improve the safety of electrical systems and devices. Developing models which provide an understanding that is imperfect but adequate for solving practical problems lies at the heart of engineering. Much of the art of electrical engineering, which you will learn with experience, is in knowing when and how to solve difficult problems by using simplifying models.

2

CIRCUIT ELEMENTS

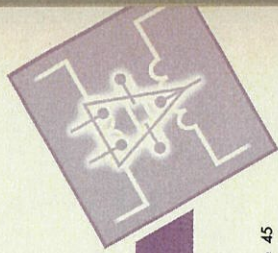
CHAPTER CONTENTS

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There are five ideal basic circuit elements:

1. voltage sources,
2. current sources,
3. resistors,
4. inductors, and
5. capacitors.

In this chapter we discuss the characteristics of voltage sources, current sources, and resistors. Although this may seem like a small number of elements with which to begin analyzing circuits, many practical systems can be modeled with just sources and resistors. They are also a useful starting point because of their relative simplicity; the mathematical relationships between voltage and current in sources and resistors are algebraic. Thus you will be able to begin learning the basic techniques of circuit analysis with only algebraic manipulations.



We will postpone introducing inductors and capacitors until Chapter 6, because their use requires that you solve integral and differential equations. However, the basic analytical techniques for solving circuits with inductors and capacitors are the same as those introduced in this chapter. So, by the time you need to begin manipulating more difficult equations, you should be very familiar with the methods of writing them.

2.1 VOLTAGE AND CURRENT SOURCES

Before discussing ideal voltage and current sources, we need to consider the general nature of electrical sources. An **electrical source** is a device that is capable of converting nonelectric energy to electric energy and vice versa. A discharging battery converts chemical energy to electric energy, whereas a battery being charged converts electric energy to chemical energy. A dynamo is a machine that converts mechanical energy to electric energy and vice versa. If operating in the mechanical-to-electric mode, it is called a generator. If transforming from electric to mechanical energy, it is referred to as a motor. The important thing to remember about these sources is that they can either deliver or absorb electric power, generally maintaining either voltage or current. This behavior is of particular interest for circuit analysis and led to the creation of the ideal voltage source and the ideal current source as basic circuit elements. The challenge is to model practical sources in terms of the ideal basic circuit elements.

An **ideal voltage source** is a circuit element which maintains a prescribed voltage across its terminals regardless of the current flowing in those terminals. Similarly, an **ideal current source** is a circuit element which maintains a prescribed current through its terminals regardless of the voltage across those terminals. These circuit elements do not exist as practical devices—they are idealized models of actual voltage and current sources.

Using an ideal model for current and voltage sources places an important restriction on how we may describe them mathematically. Since an ideal voltage source provides the same voltage even if the current in the element changes, it is impossible to specify the current in an ideal voltage source as a function of its voltage. Likewise, if the only information you have about an ideal current source is the value of current supplied, it is impossible to determine the voltage across that current source. We have sacrificed our ability to relate voltage and current in a practical source for the simplicity of using ideal sources in circuit analysis.

2.1 VOLTAGE AND CURRENT SOURCES

Ideal voltage and current sources can be further described as either independent sources or dependent sources. An **independent source** establishes a voltage or current in a circuit without relying on voltages or currents elsewhere in the circuit. The value of the voltage or current supplied is specified by the value of the independent source alone. In contrast, a **dependent source** establishes a voltage or current whose value depends on the value of a voltage or current elsewhere in the circuit. You cannot specify the value of a dependent source unless you know the value of the voltage or current on which it depends.

The circuit symbols for the ideal independent sources are shown in Fig. 2.1. Note that a circle is used to represent an independent source. To completely specify an ideal independent voltage source in a circuit, you must include the value of the supplied voltage and the reference polarity, as shown in Fig. 2.1(a). Similarly, to completely specify an ideal independent current source, you must include the value of the supplied current and its reference direction, as shown in Fig. 2.1(b).

The circuit symbols for the ideal dependent sources are shown in Fig. 2.2. A diamond is used to represent a dependent source. Both the dependent current source and the dependent voltage source may be controlled by either a voltage or a current elsewhere in the circuit, so there are a total of four variations, as indicated by the symbols in Fig. 2.2. Dependent sources are sometimes called **controlled sources**.

To completely specify an ideal dependent voltage-controlled voltage source, you must identify the controlling voltage, the equation which permits you to compute the supplied voltage from the controlling voltage, and the reference polarity for the supplied voltage. In Fig. 2.2(a), the controlling voltage is named v_x , the equation which determines the supplied voltage v_y is

$$v_y = \mu v_x,$$

and the reference polarity for v_y is as indicated. Note that μ is a multiplying constant which is dimensionless.

Similar requirements exist for completely specifying the other ideal dependent sources. In Fig. 2.2(b), the controlling current is i_x , the equation for the supplied voltage v_y is

$$v_y = \rho i_x,$$

the reference polarity is as shown, and the multiplying constant ρ has the dimension volts per ampere. In Fig. 2.2(c) the controlling voltage is v_x , the equation for the supplied current i_y is

$$i_y = \alpha v_x,$$

the reference direction is as shown, and the multiplying constant α has the dimension amperes per volt. In Fig. 2.2(d), the

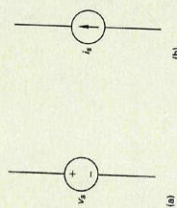


FIGURE 2.1 The circuit symbols for (a) an ideal independent voltage source and (b) an ideal independent current source.

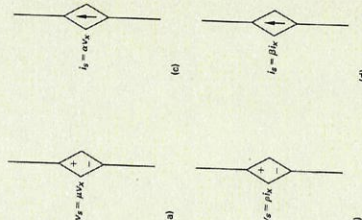


FIGURE 2.2 The circuit symbols for (a) an ideal dependent voltage-controlled voltage source, (b) an ideal dependent current-controlled voltage source, (c) an ideal dependent voltage-controlled current source, and (d) an ideal dependent current-controlled current source.

controlling current is i_s , the equation for the supplied current i_s is

$$i_s = \beta i_z,$$

the reference direction is as shown, and the multiplying constant β is dimensionless.

Finally, in our discussion of ideal sources, we note that they are examples of active circuit elements. An **active element** is one which models a device capable of generating electric energy. **Passive elements** model physical devices which cannot generate electric energy. Resistors, inductors, and capacitors are examples of passive circuit elements. Examples 2.1 and 2.2 illustrate how the characteristics of ideal independent and dependent sources limit the types of permissible interconnections of the sources.

EXAMPLE 2.1

Using the definitions of the ideal independent voltage and current sources, state which interconnections in Fig. 2.3 are permissible and which violate the constraints imposed by the ideal sources.

SOLUTION

Connection (a) is valid. Each source supplies voltage across the same pair of terminals, marked a.b. This requires that each source supply the same voltage with the same polarity, which they do.

Connection (b) is valid. Each source supplies current through the same pair of terminals, marked a.b. This requires that each source supply the same current in the same direction, which they do.

Connection (c) is not permissible. Each source supplies voltage across the same pair of terminals, marked a.b. This requires that each source supply the same voltage with the same polarity, which they do not.

Connection (d) is not permissible. Each source supplies current through the same pair of terminals, marked a.b. This requires that each source supply the same current in the same direction, which they do not.

Connection (e) is valid. The voltage source supplies voltage across the pair of terminals marked a.b. The current source supplies current through the same pair of terminals. Since an ideal voltage source supplies the same voltage regardless of the current, and an ideal current source supplies the same current regardless of the voltage, this is a permissible connection.

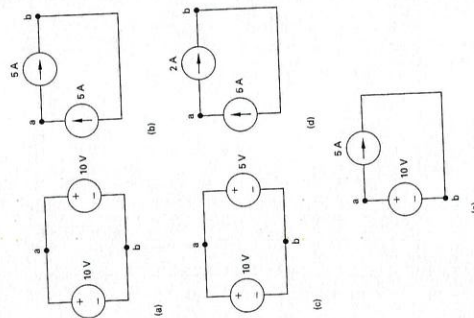


FIGURE 2.3 The circuits for Example 2.1.

EXAMPLE 2.2

Using the definitions of the ideal independent and dependent sources, state which interconnections in Fig. 2.4 are valid and which violate the constraints imposed by the ideal sources.

SOLUTION

Connection (a) is invalid. Both the independent source and the dependent source supply voltage across the same pair of terminals, labeled a.b. This requires that each source supply the same voltage with the same polarity. The independent source supplies 5 V, but the dependent source supplies 15 V.

Connection (b) is valid. The independent voltage source supplies voltage across the pair of terminals marked a.b. The dependent current source supplies current through the same pair of terminals. Since an ideal voltage source supplies the same voltage regardless of current, and an ideal current source supplies the same current regardless of voltage, this is an allowable connection.

Connection (c) is valid. The independent current source supplies current through the pair of terminals marked a.b. The dependent voltage source supplies voltage across the same pair of terminals. Since an ideal current source supplies the same current regardless of voltage, and an ideal voltage source supplies the same voltage regardless of current, this is an allowable connection.

Connection (d) is invalid. Both the independent source and the dependent source supply current through the same pair of terminals, labeled a.b. This requires that each source supply the same current in the same reference direction. The independent source supplies 2 A, but the dependent source supplies -6 A.

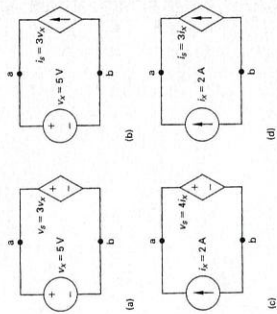


FIGURE 2.4 The circuits for Example 2.2.



FIGURE 2.5 The circuit symbol for a resistor having a resistance R .

2.2 ELECTRICAL RESISTANCE (OHM'S LAW)

Resistance is the capacity of materials to impede the flow of current or, more specifically, the flow of electric charge. The circuit element used to model this behavior is the **resistor**. Figure 2.5 shows the circuit symbol for the resistor, with R denoting the resistance value of the resistor.

Conceptually, we can understand resistance if we think about the moving electrons which make up electric current interacting with and being resisted by the atomic structure of the material through which

they are moving. In the course of these interactions, some amount of electric energy is converted to thermal energy and dissipated in the form of heat. This effect may be undesirable. However, many useful electrical devices take advantage of resistance heating, including stoves, toasters, irons, and space heaters.

Most materials exhibit measurable resistance to current. The amount of resistance depends upon the material. Metals such as copper and aluminum have small values of resistance, making them good choices for wiring used to conduct electric current. In fact, when represented in a circuit diagram, copper or aluminum wiring isn't usually modeled as a resistor; the resistance of the wire is so small compared to the resistance of other elements in the circuit that we can neglect the wiring resistance to simplify the diagram.

For purposes of circuit analysis, we must reference the current in the resistor to the terminal voltage. We can do so in two ways: either in the direction of the voltage drop across the resistor, or in the direction of the voltage rise across the resistor, as shown in Fig. 2.6. If we choose the former, the relationship between the voltage and current is

$$v = iR, \quad (2.1)$$

where

v = the voltage in volts,

i = the current in amperes, and

R = the resistance in ohms.

If we choose the second method, we must write

$$v = -iR, \quad (2.2)$$

where v , i , and R are, as before, measured in volts, amperes, and ohms, respectively. The algebraic signs used in Eqs. (2.1) and (2.2) are a direct consequence of the passive sign convention, which we introduced in Chapter 1. In both Eqs. (2.1) and (2.2), we assume that the resistance is positive.

Equations (2.1) and (2.2) are known as **Ohm's law** after Georg Simon Ohm, a German physicist who established its validity early in the nineteenth century. Ohm's law is the algebraic relationship between voltage and current for a resistor to which we referred in the introduction to this chapter. In SI units, resistance is measured in ohms. The Greek letter omega (Ω) is the standard symbol for an ohm.

The circuit diagram symbol for an $8\ \Omega$ resistor is shown in Fig. 2.7. Ohm's law expresses the voltage as a function of the current. However, expressing the current as a function of the voltage also is convenient. Thus, from Eq. (2.1),

$$i = \frac{v}{R}, \quad (2.3)$$

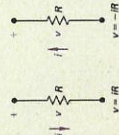


FIGURE 2.6 Two possible reference choices for the current and voltage at the terminals of a resistor, and the resulting equations.



FIGURE 2.7 The circuit symbol for an $8\ \Omega$ resistor.

or, from Eq. (2.2),

$$i = -\frac{v}{R}. \quad (2.4)$$

The reciprocal of the resistance is referred to as **conductance** is symbolized by the letter G , and is measured in siemens (S). Thus

$$G = \frac{1}{R} \text{ S}. \quad (2.5)$$

An $8\ \Omega$ resistor has a conductance value of $0.125\ \text{S}$. In much of the professional literature, the unit used for conductance is the mho (ohm spelled backward), which is symbolized by an inverted omega (Ω). Therefore we may also describe an $8\ \Omega$ resistor as having a conductance of $0.125\ \text{mho}$, or $0.125\ \text{S}$.

We use ideal resistors in circuit analysis to model the behavior of physical devices. Using the qualifier *ideal* reminds us that the resistor model makes several simplifying assumptions about the behavior of actual resistive devices. The most important of these simplifying assumptions is that the resistance of the ideal resistor is constant and its value does not vary over time. Most actual resistive devices do not have constant resistance, and their resistance does vary over time. The ideal resistor model can be used to represent a physical device whose resistance doesn't vary much from some constant value over the time period of interest in the circuit analysis. In this book we assume that the simplifying assumptions about resistance devices are valid, and we thus use ideal resistors in circuit analysis.

We may calculate the power at the terminals of a resistor in several ways. The first approach is to use the defining equation and simply calculate the product of the terminal voltage and current. For the reference systems shown in Fig. 2.6 we write

$$p = vi \quad (2.6)$$

when $v = iR$ and

$$p = -vi \quad (2.7)$$

when $v = -iR$.

A second method of expressing the power at the terminals of a resistor expresses power in terms of the current and the resistance. Substituting Eq. (2.1) into Eq. (2.6), we obtain

$$p = vi = (iR)i = i^2 R. \quad (2.8)$$

Likewise, substituting Eq. (2.2) into Eq. (2.7), we have

$$p = -vi = -(-iR)i = i^2 R. \quad (2.9)$$

Equations (2.8) and (2.9) are identical and demonstrate clearly that, regardless of voltage polarity and current direction, the power at the

terminals of a resistor is positive. Therefore a resistor absorbs power from the circuit.

A third method of expressing the power at the terminals of a resistor is in terms of the voltage and resistance. The expression is independent of the polarity references, so

$$p = \frac{v^2}{R} \quad (2.10)$$

Sometimes a resistor's value will be expressed as a conductance rather than as a resistance. Using the relationship between resistance and conductance given in Eq. (2.5), we may also write Eqs. (2.9) and (2.10) in terms of the conductance, or

$$p = \frac{i^2}{G} \quad (2.11)$$

and

$$p = v^2 G. \quad (2.12)$$

Equations (2.6)–(2.12) provide a variety of different methods for calculating the power absorbed by a resistor. Each yields the same answer. In analyzing a circuit, look at the information provided and choose the power equation which uses that information directly.

Example 2.3 illustrates the application of Ohm's law in conjunction with an ideal source and a resistor. Power calculations at the terminals of a resistor also are illustrated.

EXAMPLE 2.3

In each circuit in Fig. 2.8, either the value of v or i is not known.

- Calculate the values of v and i .
- Determine the power dissipated in each resistor.

SOLUTION

- The voltage v_a in Fig. 2.8(a) is a drop in the direction of the current in the resistor. Therefore,

$$v_a = (1)(8) = 8 \text{ V.}$$

The current i_b in the resistor with a conductance of 0.2 S in Fig. 2.8(b) is in the direction of the voltage drop across the

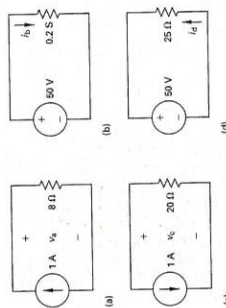


FIGURE 2.8 The circuits for Example 2.3.

resistor. Thus

$$i_b = (50)(0.2) = 10 \text{ A.}$$

The voltage v_c in Fig. 2.8(c) is a rise in the direction of the current in the resistor. Hence

$$v_c = -(1)(20) = -20 \text{ V.}$$

The current i_d in the 25Ω resistor in Fig. 2.8(d) is in the direction of the voltage rise across the resistor. Therefore

$$i_d = \frac{-50}{25} = -2 \text{ A.}$$

- The power dissipated in each of the four resistors is

$$p_{8\Omega} = \frac{(8)^2}{8} = (1)^2(8) = 8 \text{ W;}$$

$$p_{0.2\text{S}} = (50)^2(0.2) = 500 \text{ W;}$$

$$p_{20\Omega} = \frac{(-20)^2}{20} = (1)^2(20) = 20 \text{ W;}$$

$$p_{25\Omega} = \frac{(50)^2}{25} = (-2)^2(25) = 100 \text{ W.}$$

Having introduced the general characteristics of ideal sources and resistors, we next show how to use these elements to build the circuit model of a practical system.

2.3 CONSTRUCTION OF A CIRCUIT MODEL

We have already stated that one reason for an interest in the basic circuit elements is that they can be used to construct circuit models of practical systems. The skill required to develop a circuit model of a device or system is as complex as the skill required to solve the derived circuit. Although this text emphasizes the skills required to solve circuits, you also will need other skills in the practice of electrical engineering, and one of the most important is modeling.

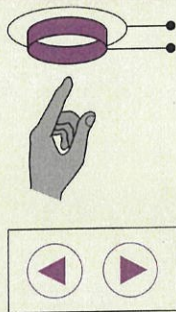
We develop circuit models in the next two examples. In Example 2.4 we construct a circuit model based on a knowledge of the behavior of the system's components and how the components are interconnected. In Example 2.5 we create a circuit model by measuring the terminal behavior of a device.

PRACTICAL PERSPECTIVE

Proximity Switches

The electrical devices we use in our daily lives contain many switches. Most switches are mechanical, such as the one used in the flashlight introduced in Chapter 2. Mechanical switches use an actuator which is pushed, pulled, slid, or rotated, causing two pieces of conducting metal to touch and create a short circuit. Sometimes designers prefer to use switches without moving parts, to increase the safety, reliability, convenience, or novelty of their products. Such switches are called proximity switches. Proximity switches can employ a variety of sensor technologies. For example, some elevator doors stay open whenever a light beam is obstructed.

Another sensor technology used in proximity switches detects people by responding to the disruption they cause in electric fields. This type of proximity switch is used in some desk lamps which turn on and off when touched, and in elevator buttons with no moving parts (as shown in the figure). The switch is based on a capacitor. As you are about to discover in this chapter, a capacitor is a circuit element whose terminal



characteristics are determined by electric fields. When you touch a capacitive proximity switch, you produce a change in the value of a capacitor, causing a voltage change which activates the switch. The design of a capacitive touch-sensitive switch is the topic of the Practical Perspective example at the end of this chapter.

INDUCTORS AND CAPACITORS

6

CHAPTER CONTENTS

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- 6.2 The Capacitor 221
- 6.3 Series-Parallel Combinations of Inductance and Capacitance 226

We now introduce the last two ideal circuit elements mentioned in Chapter 2, namely, inductors and capacitors. Be assured that the circuit analysis techniques introduced in Chapters 3 and 4 apply to circuits containing inductors and capacitors. Therefore, once you understand the terminal behavior of these elements in terms of current and voltage, you can use Kirchhoff's laws to describe any interconnections with the other basic elements. Like other components, inductors and capacitors are easier to describe in terms of circuit variables rather than electromagnetic field variables. However, before we focus on the circuit descriptions, a brief review of the field concepts underlying these basic elements is in order.

The behavior of inductors is based on phenomena associated with magnetic fields. The source of the magnetic field is charge in motion, or current. If the current is varying with time, the magnetic field is varying with time. A time-varying magnetic field induces a voltage in any conductor linked by the field. The circuit parameter of inductance

relates the induced voltage to the current. We discuss this quantitative relationship in Section 6.1.

The behavior of capacitors is based on phenomena associated with electric fields. The source of the electric field is separation of charge, or voltage. If the voltage is varying with time, the electric field is varying with time. A time-varying electric field produces a displacement current in the space occupied by the field. The circuit parameter of capacitance relates the displacement current to the voltage, where the displacement current is equal to the conduction current at the terminals of the capacitor. We discuss this quantitative relationship in Section 6.2. Section 6.3 describes techniques used to simplify circuits with series or parallel combinations of capacitors or inductors.

Energy can be stored in both magnetic and electric fields. Hence you should not be too surprised to learn that inductors and capacitors are capable of storing energy. For example, energy can be stored in an inductor and then released to fire a spark plug. Energy can be stored in a capacitor and then released to fire a flashlight. In ideal inductors and capacitors, only as much energy can be extracted as has been stored. Because inductors and capacitors cannot generate energy, they are classified as **passive elements**.

6.1 THE INDUCTOR

Inductance is the circuit parameter used to describe an inductor. Inductance is symbolized by the letter L , is measured in henrys (H), and is represented graphically as a coiled wire—a reminder that inductance is a consequence of a conductor linking a magnetic field. Figure 6.1(a) shows an inductor. Assigning the reference direction of the current in the direction of the voltage drop across the terminals of the inductor, as shown in Fig. 6.1(b), yields

$$v = L \frac{di}{dt} \quad (6.1)$$

where v is measured in volts, L in henrys, i in amperes, and t in seconds. Equation (6.1) reflects the passive sign convention shown in Fig. 6.1(b), that is, the current reference is in the direction of the voltage drop across the inductor. If the current reference is in the direction of the voltage rise, Eq. (6.1) is written with a minus sign.

Note from Eq. (6.1) that the voltage across the terminals of an inductor is proportional to the time rate of change of the current in the inductor. We can make two important observations here. First, if the current is constant, the voltage across the ideal inductor is zero. Thus

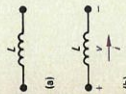


FIGURE 6.1 The graphic symbol for an inductor with an inductance of L henrys.

6.1 THE INDUCTOR

the inductor behaves as a short circuit in the presence of a constant, or dc, current. Second, current cannot change instantaneously in an inductor; that is, the current cannot change by a finite amount in zero time. Equation (6.1) tells us that this change would require an infinite voltage, and infinite voltages are not possible. For example, when someone opens the switch on an inductive circuit in an actual system, the current initially continues to flow in the air across the switch, a phenomenon called **arcing**. The arc across the switch prevents the current from dropping to zero instantaneously. Switching inductive circuits is an important engineering problem because arcing and voltage surges must be controlled to prevent equipment damage. The first step to understanding the nature of this problem is to master the inductor material presented in this and the following two chapters. Example 6.1 illustrates the application of Eq. (6.1) to a simple circuit.

EXAMPLE 6.1

The independent current source in the circuit shown in Fig. 6.2 generates zero current for $t < 0$ and a pulse $10te^{-5t}$ for $t > 0$.

- Sketch the current waveform.
- At what instant of time is the current maximum?
- Express the voltage across the terminals of the 100 mH inductor as a function of time.
- Sketch the voltage waveform.
- Is the voltage maximum when the current is maximum?
- At what instant of time does the voltage change polarity?
- Is there ever an instantaneous change in voltage across the inductor? If so, at what time?

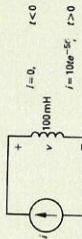


FIGURE 6.2 The circuit for Example 6.1.

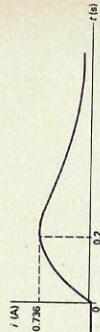


FIGURE 6.3 The current waveform for Example 6.1.

Use a computer tool to confirm your sketch.



FIGURE 6.4 The voltage waveform for Example 6.1.

SOLUTION

- Figure 6.3 shows the current waveform.
- $\frac{di}{dt} = 10(-5te^{-5t} + e^{-5t}) = 10e^{-5t}(1 - 5t)$, $di/dt = 0$ when $t = \frac{1}{5}$ s. (See Fig. 6.3.)
- $v = L \frac{di}{dt} = (0.1)10e^{-5t}(1 - 5t) = e^{-5t}(1 - 5t)$ V, $t > 0$; $v = 0$, $t < 0$.
- Figure 6.4 shows the voltage waveform.
- No; the voltage is proportional to di/dt , not i .

- f) At 0.2 s, which corresponds to the moment when di/dt is passing through zero and changing sign.
 g) Yes, at $t = 0$. Note that the voltage can change instantaneously across the terminals of an inductor.

CURRENT IN AN INDUCTOR IN TERMS OF THE VOLTAGE ACROSS THE INDUCTOR

Equation (6.1) expresses the voltage across the terminals of an inductor as a function of the current in the inductor. Also desirable is the ability to express the current as a function of the voltage. To find i as a function of v , we start by multiplying both sides of Eq. (6.1) by a differential time dt :

$$v dt = L \left(\frac{di}{dt} \right) dt. \quad (6.2)$$

Multiplying the rate at which i varies with t by a differential change in time generates a differential change in i , so we write Eq. (6.2) as

$$v dt = L di. \quad (6.3)$$

We next integrate both sides of Eq. (6.3). For convenience, we interchange the two sides of the equation and write

$$L \int_{i(t_0)}^{i(t)} dx = \int_{t_0}^t v d\tau. \quad (6.4)$$

Note that we use x and τ as the variables of integration, while i and t become limits on the integrals. Then from Eq. (6.4),

$$i(t) = \frac{1}{L} \int_{t_0}^t v d\tau + i(t_0), \quad (6.5)$$

where $i(t)$ is the current corresponding to t , and $i(t_0)$ is the value of the inductor current when we initiate the integration, namely, t_0 . In many practical applications, t_0 is zero and Eq. (6.5) becomes

$$i(t) = \frac{1}{L} \int_0^t v d\tau + i(0). \quad (6.6)$$

Equations (6.1) and (6.5) both give the relationship between the voltage and current at the terminals of an inductor. Equation (6.1) expresses the voltage as a function of current, whereas Eq. (6.5) expresses the current as a function of voltage. In both equations the reference direction for the current is in the direction of the voltage drop across the terminals. Note that $i(t_0)$ carries its own algebraic sign. If the initial current is in the same direction as the reference direction for i , it is a positive quantity. If the initial current is in the

Use a symbolic equation solver to confirm this result.

opposite direction, it is a negative quantity. Example 6.2 illustrates the application of Eq. (6.5).

EXAMPLE 6.2

The voltage pulse applied to the 100 mH inductor shown in Fig. 6.5 is 0 for $t < 0$ and is given by the expression

$$v(t) = 20te^{-10t} \text{ V}$$

for $t > 0$. Also assume $i = 0$ for $t \leq 0$.

- Sketch the voltage as a function of time.
- Find the inductor current as a function of time.
- Sketch the current as a function of time.

SOLUTION

a) The voltage as a function of time is shown in Fig. 6.6.

b) The current in the inductor is 0 at $t = 0$. Therefore the current for $t > 0$ is

$$\begin{aligned} i &= \frac{1}{0.1} \int_0^t 20te^{-10\tau} d\tau + 0 \\ &= 200 \left[\frac{-e^{-10\tau}}{100} (10\tau + 1) \right]_0^t \\ &= 2(1 - 10te^{-10t} - e^{-10t}) \text{ A}, \quad t > 0. \end{aligned}$$

c) Figure 6.7 shows the current as a function of time.

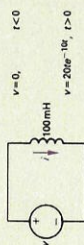


FIGURE 6.5 The circuit for Example 6.2.

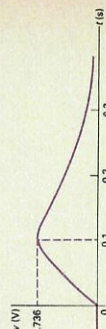


FIGURE 6.6 The voltage waveform for Example 6.2.

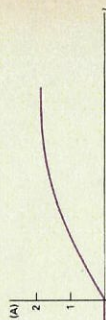


FIGURE 6.7 The current waveform for Example 6.2.

Note in Example 6.2 that i approaches a constant value of 2 A as t increases. We say more about this result after discussing the energy stored in an inductor.

POWER AND ENERGY IN THE INDUCTOR

The power and energy relationships for an inductor can be derived directly from the current and voltage relationships. If the current reference is in the direction of the voltage drop across the terminals of the inductor, the power is

$$p = vi. \quad (6.7)$$

Remember that power is in watts, voltage is in volts, and current is in amperes. If we express the inductor voltage as a function of the

inductor current, Eq. (6.7) becomes

$$p = Li \frac{di}{dt} \quad (6.8)$$

We can also express the current in terms of the voltage:

$$p = v \left[\frac{1}{L} \int_{t_0}^t v d\tau + i(t_0) \right] \quad (6.9)$$

Equation (6.8) is most useful in expressing the energy stored in the inductor. Power is the time rate of expending energy, so

$$p = \frac{dw}{dt} = Li \frac{di}{dt} \quad (6.10)$$

Multiplying both sides of Eq. (6.10) by a differential time gives the differential relationship

$$dw = Li di \quad (6.11)$$

Both sides of Eq. (6.11) are integrated with the understanding that the reference for zero energy corresponds to zero current in the inductor. Thus

$$\int_0^w dx = L \int_0^i y dy; \quad (6.12)$$

$$w = \frac{1}{2} Li^2.$$

As before, we use different symbols of integration to avoid confusion with the limits placed on the integrals. In Eq. (6.12), the energy is in joules when inductance is in henrys and current is in amperes. To illustrate the application of Eqs. (6.7) and (6.12), we return to Examples 6.1 and 6.2 by means of Example 6.3.

EXAMPLE 6.3

- For Example 6.1, plot i , v , p , and w versus time. Line up the plots vertically to allow easy assessment of each variable's behavior.
- In what time interval is energy being stored in the inductor?
- In what time interval is energy being extracted from the inductor?
- What is the maximum energy stored in the inductor?
- Evaluate the integrals

$$\int_0^{0.2} p dt \quad \text{and} \quad \int_{0.2}^{\infty} p dt$$

and comment on their significance.

- Repeat parts (a)–(c) for Example 6.2.

- In Example 6.2, why is there a sustained current in the inductor as the voltage approaches zero?

SOLUTION

- The plots of i , v , p , and w follow directly from the expressions for i and v obtained in Example 6.1 and are shown in Fig. 6.8. In particular, $p = vi$ and $w = (\frac{1}{2})Li^2$.

- An increasing energy curve indicates that energy is being stored. Thus energy is being stored in the time interval 0 to 0.2 s. Note that this corresponds to the interval when $p > 0$.

- A decreasing energy curve indicates that energy is being extracted. Thus energy is being extracted in the time interval 0.2 s to ∞ . Note that this corresponds to the interval when $p < 0$.

- From Eq. (6.12) we see that energy is at a maximum when current is at a maximum; glancing at the graphs confirms this. From Example 6.1, maximum current = 0.736 A. Therefore, $w_{\max} = 27.07 \text{ mJ}$.

- From Example 6.1,

$$i = 10te^{-5t} \text{ A} \quad \text{and} \quad v = e^{-5t}(1 - 5t) \text{ V.}$$

Therefore

$$p = vi = 10te^{-10t} - 50t^2e^{-10t} \text{ W.}$$

Thus

$$\begin{aligned} \int_0^{0.2} p dt &= 10 \left[\frac{e^{-10t}}{100} (-10t - 1) \right]_0^{0.2} \\ &\quad - 50 \left[\frac{t^2 e^{-10t}}{100} + \frac{2}{10} \left[\frac{e^{-10t}}{100} (-10t - 1) \right] \right]_0^{0.2} \\ &= 0.2e^{-2} = 27.07 \text{ mJ;} \\ \int_{0.2}^{\infty} p dt &= 10 \left[\frac{e^{-10t}}{100} (-10t - 1) \right]_{0.2}^{\infty} \\ &\quad - 50 \left[\frac{t^2 e^{-10t}}{100} + \frac{2}{10} \left[\frac{e^{-10t}}{100} (-10t - 1) \right] \right]_{0.2}^{\infty} \\ &= -0.2e^{-2} = -27.07 \text{ mJ.} \end{aligned}$$

Based on the definition of p , the area under the plot of p versus t represents the energy expended over the interval of integration. Hence the integration of the power between 0 and 0.2 s represents

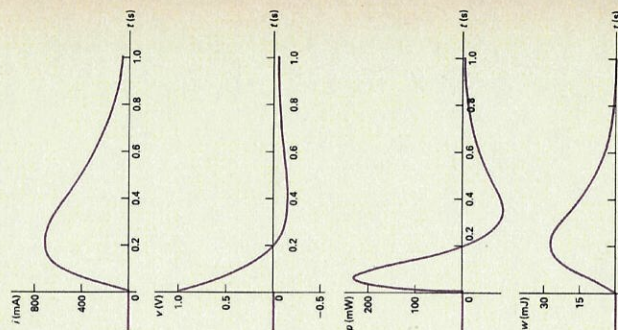


FIGURE 6.8 The variables i , v , p , and w versus t for Example 6.1.

the energy stored in the inductor during this time interval. The integral of p over the interval 0.2 s to ∞ is the energy extracted. Note that in this time interval all the energy originally stored is removed; that is, after the current peak has passed, no energy is stored in the inductor.

- f) The plots of v , i , p , and w follow directly from the expressions for v and i given in Example 6.2 and are shown in Fig. 6.9. Note that in this case the power always is positive, and hence energy always is being stored during the duration of the voltage pulse.
- g) The application of the voltage pulse stores energy in the inductor. Because the inductor is ideal, this energy cannot dissipate after the voltage subsides to zero. Therefore a sustained current circulates in the circuit. A lossless inductor obviously is an ideal circuit element. Practical inductors require a resistor in the circuit model. (More about this later.)

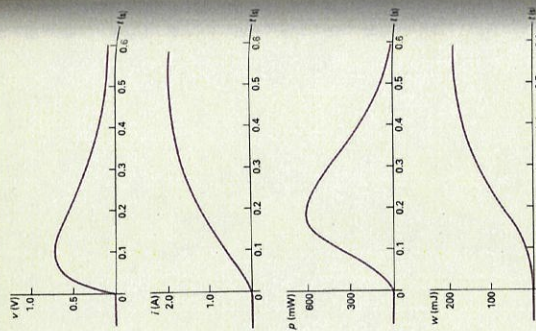


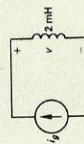
FIGURE 6.9 The variables v , i , p , and w versus t for Example 6.2.

DRILL EXERCISE

- 6.1 The current source in the circuit shown generates the current pulse

$$i_g(t) = 0, \quad t < 0;$$

$$i_g(t) = 5e^{-200t} - 5e^{-800t} \text{ A}, \quad t \geq 0.$$



ANSWER: (a) 6 V; (b) 2.31 ms; (c) $50\mu\text{C}$; $100\mu\text{J}$; $40\mu\text{W}$; (d) 616.58 μs ; (e) 4.26 W; (f) 5.58 mJ, and (g) 2.31 ms.

Find (a) $v(0)$; (b) the instant of time, greater than zero, when the voltage v passes through zero; (c) the expression for the power delivered to the inductor; (d) the instant when the power delivered to the inductor is maximum; (e) the maximum power; (f) the maximum energy stored in the inductor; and (g) the instant of time when the stored energy is maximum.

6.2 THE CAPACITOR

The circuit parameter of capacitance is represented by the letter C , is measured in farads (F), and is symbolized graphically by two short parallel conductive plates, as shown in Fig. 6.10(a). Because the farad is an extremely large quantity of capacitance, practical capacitors are based on submultiples of the farad. The most frequently encountered values lie in the picofarad (pF) to microfarad (μF) range.

The graphic symbol for a capacitor is a reminder that capacitance occurs whenever electrical conductors are separated by a dielectric, or insulating, material. This condition implies that electric charge is not transported through the capacitor. Although applying a voltage to the terminals of the capacitor cannot move a charge through the dielectric, it can displace a charge within the dielectric. As the voltage varies with time, the displacement of charge also varies with time, causing what is known as the **displacement current**.

At the terminals, the displacement current is indistinguishable from a conduction current. The current is proportional to the rate at which the voltage across the capacitor varies with time, or, mathematically,

$$i = C \frac{dv}{dt} \quad (6.13)$$

where i is measured in amperes, C in farads, v in volts, and t in seconds.

Equation (6.13) reflects the passive sign convention shown in Fig. 6.10(b); that is, the current reference is in the direction of the voltage drop across the capacitor. If the current reference is in the direction of the voltage rise, Eq. (6.13) is written with a minus sign.

Two important observations follow from Eq. (6.13). First, voltage cannot change instantaneously across the terminals of a capacitor. Equation (6.13) indicates that such a change would produce infinite current, a physical impossibility. Second, if the voltage across the

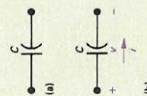


FIGURE 6.10 The circuit symbol for a capacitor.

terminals is constant, the capacitor current is zero. The reason is that a conduction current cannot be established in the dielectric material of the capacitor. Only a time-varying voltage can produce a displacement current. Thus a capacitor behaves as an open circuit in the presence of a constant voltage.

Equation (6.13) gives the capacitor current as a function of the capacitor voltage. Expressing the voltage as a function of the current is also useful. To do so we multiply both sides of Eq. (6.13) by a differential time dt and then integrate the resulting differentials:

$$i dt = C dv \quad \text{or} \quad \int_{v(t_0)}^{v(t)} dx = \frac{1}{C} \int_{t_0}^t i d\tau.$$

Carrying out the integration of the left-hand side of the second equation gives

$$v(t) = \frac{1}{C} \int_{t_0}^t i d\tau + v(t_0). \quad (6.14)$$

In many practical applications of Eq. (6.14), the initial time is zero; that is, $t_0 = 0$. Thus Eq. (6.14) becomes

$$v(t) = \frac{1}{C} \int_0^t i d\tau + v(0). \quad (6.15)$$

We can easily derive the power and energy relationships for the capacitor. From the definition of power,

$$p = vi = C v \frac{dv}{dt}, \quad (6.16)$$

or

$$p = i \left[\frac{1}{C} \int_{t_0}^t i d\tau + v(t_0) \right]. \quad (6.17)$$

Combining the definition of energy with Eq. (6.16) yields

$$dw = C v dv,$$

from which

$$\int_0^w dx = C \int_0^v y dy,$$

or

$$w = \frac{1}{2} C v^2. \quad (6.18)$$

In the derivation of Eq. (6.18), the reference for zero energy corresponds to zero voltage.

Examples 6.4 and 6.5 illustrate the application of the current, voltage, power, and energy relationships for a capacitor.

EXAMPLE 6.4

The voltage pulse described by the following equations is impressed across the terminals of a $0.5 \mu\text{F}$ capacitor:

$$\begin{aligned} v(t) &= 0, & t < 0; \\ v(t) &= 4t \text{ V}, & 0 \leq t \leq 1; \\ v(t) &= 4e^{-(t-1)} \text{ V}, & 1 \leq t < \infty. \end{aligned}$$

a) Derive the expressions for the capacitor current, power, and energy. b) Sketch the voltage, current, power, and energy as functions of time. Line up the plots vertically.

c) Specify the interval of time when energy is being stored in the capacitor.

d) Specify the interval of time when energy is being delivered by the capacitor.

e) Evaluate the integrals

$$\int_0^1 p dt \quad \text{and} \quad \int_1^\infty p dt$$

and comment on their significance.

SOLUTION

a) From Eq. (6.13),

$$\begin{aligned} i &= (0.5 \times 10^{-6})(0) = 0, & t < 0; \\ i &= (0.5 \times 10^{-6})(4) = 2 \mu\text{A}, & 0 \leq t < 1; \\ i &= (0.5 \times 10^{-6})(-4e^{-(t-1)}) = -2e^{-(t-1)} \mu\text{A}, & 1 \leq t < \infty. \end{aligned}$$

The expression for the power is derived from Eq. (6.16):

$$\begin{aligned} p &= 0, & t < 0; \\ p &= (4t)(2) = 8t \mu\text{W}, & 0 \leq t < 1; \\ p &= (4e^{-(t-1)})(-2e^{-(t-1)}) = -8e^{-2(t-1)} \mu\text{W}, & 1 \leq t < \infty. \end{aligned}$$

The energy expression follows directly from Eq. (6.18):

$$\begin{aligned} w &= 0 & t < 0; \\ w &= \frac{1}{2}(0.5)(6t^2) = 4t^2 \mu\text{J}, & 0 \leq t < 1, \\ w &= \frac{1}{2}(0.5)(16e^{-2(t-1)}) = 4e^{-2(t-1)} \mu\text{J}, & 1 \leq t < \infty. \end{aligned}$$

b) Figure 6.11 shows the voltage, current, power, and energy as functions of time.

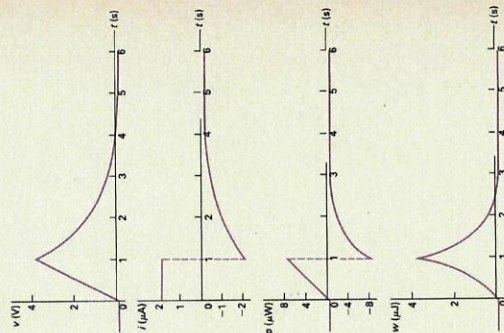


FIGURE 6.11 The variables v , i , p , and w versus t for Example 6.4.

Use a symbolic equation solver to derive these equations.

- c) Energy is being stored in the capacitor whenever the power is positive. Hence energy is being stored in the interval 0 to 1 s.
 d) Energy is being delivered by the capacitor whenever the power is negative. Thus energy is being delivered for all t greater than 1 s.
 e) The integral of $p dt$ is the energy associated with the time interval corresponding to the limits on the integral. Thus the first integral represents the energy stored in the capacitor between 0 and 1 s, whereas the second integral represents the energy returned, or delivered, by the capacitor in the interval 1 s to ∞ :

$$\int_0^1 p dt = \int_0^1 8t dt = 4t^2 \Big|_0^1 = 4 \text{ J};$$

$$\int_1^\infty p dt = \int_1^\infty (-8e^{-2(t-1)}) dt = (-8) \frac{e^{-2(t-1)}}{-2} \Big|_1^\infty = -4 \text{ J}.$$

The voltage applied to the capacitor returns to zero as time increases without limit, so the energy returned by this ideal capacitor must equal the energy stored.

EXAMPLE 6.5

An uncharged $0.2 \mu\text{F}$ capacitor is driven by a triangular current pulse. The current pulse is described by

$$i(t) = \begin{cases} 0, & t \leq 0; \\ 5000t \text{ A}, & 0 \leq t \leq 20 \mu\text{s}; \\ 0.2 - 5000t \text{ A}, & 20 \leq t \leq 40 \mu\text{s}; \\ 0, & t \geq 40 \mu\text{s}. \end{cases}$$

- a) Derive the expressions for the capacitor voltage, power, and energy for each of the four time intervals needed to describe the current.
 b) Plot i , v , p , and w versus t . Align the plots as specified in the previous examples.
 c) Why does a voltage remain on the capacitor after the current returns to zero?

SOLUTION

- a) For $t \leq 0$, v , p , and w all are zero. For $0 \leq t \leq 20 \mu\text{s}$,

$$v = 5 \times 10^6 \int_0^t 5000\tau d\tau + 0 = 12.5 \times 10^9 t^2 \text{ V};$$

$$p = vi = 62.5 \times 10^{12} t^3 \text{ W};$$

$$w = \frac{1}{2} C v^2 = 15.625 \times 10^{12} t^4 \text{ J}.$$

For $20 \mu\text{s} \leq t \leq 40 \mu\text{s}$,

$$v = 5 \times 10^6 \int_{20 \mu\text{s}}^t (0.2 - 5000\tau) d\tau + 5.$$

(Note that 5 V is the voltage on the capacitor at the end of the preceding interval.) Then,

$$v = (10^6 t - 12.5 \times 10^9 t^2 - 10) \text{ V};$$

$$p = vi$$

$$= (62.5 \times 10^{12} t^3 - 7.5 \times 10^9 t^2 + 2.5 \times 10^4 t - 2) \text{ W};$$

$$w = \frac{1}{2} C v^2$$

$$= (15.625 \times 10^{12} t^4 - 2.5 \times 10^9 t^3 + 0.125 \times 10^6 t^2 - 2t + 10^{-5}) \text{ J}.$$

For $t \geq 40 \mu\text{s}$,

$$v = 10 \text{ V};$$

$$p = vi = 0;$$

$$w = \frac{1}{2} C v^2 = 10 \mu\text{J}.$$

- b) The excitation current and the resulting voltage, power, and energy are plotted in Fig. 6.12.

- c) Note that the power is always positive for the duration of the current pulse, which means that energy is continuously being stored in the capacitor. When the current returns to zero, the stored energy is trapped because the ideal capacitor offers no means for dissipating energy. Thus a voltage remains on the capacitor after i returns to zero.

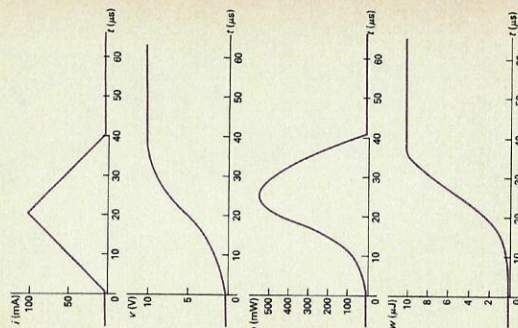


FIGURE 6.12 The variables i , v , p , and w versus t for Example 6.5.

Use a computer tool to investigate the effect of pulse shape on the current, voltage, power, and energy.

DRILL EXERCISES

- 6.2 The voltage at the terminals of the $0.5 \mu\text{F}$ capacitor shown in the figure is 0 for $t < 0$ and $100e^{-20,000t} \sin 40,000t \text{ V}$ for $t \geq 0$. Find (a) $i(0)$; (b) the power delivered to the capacitor at $t = \pi/80 \text{ ms}$; and (c) the energy stored in the capacitor at $t = \pi/80 \text{ ms}$.



ANSWER: (a) 2 A; (b) -20.79 W;
 (c) 519.70 μJ.