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EXHIBIT B





Low Complex Standard Conformable Antenna Diversity Techniques for OFDM Systems and its Application to the DVB-T System

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Abstract—In this paper, we investigate different antenna diversity concepts, which can be easily applied to orthogonal frequency division multiplexing (OFDM) systems. The focus is on standard compatibility, i.e. this schemes can be implemented to already existing OFDM systems without changing the standards. The introduced diversity techniques are applied exemplarily to the DVB-T system. Bit error performance investigations were done by simulation for different DVB-T and diversity parameter sets.

I. INTRODUCTION

FUTURE mobile wireless systems are required to provide high data rate services in a spectral efficient manner due to the high costs of bandwidth resources, e.g. $\approx 400,000,000$ €/MHz for UMTS in Germany. In terms of power efficiency - especially for mobiles - and electromagnetic pollution it is required to keep the isotropic radiated power as low as possible. Particularly the electromagnetic radiation charge becomes more and more important for the acceptance of wireless systems in society. Wireless systems have to operate in different environments. So a mobile is expected to work reliably in scenarios like rural, urban, indoor, outdoor, etc.

Mobile communication systems mainly suffer from time-varying multipath fading with extremely different multipath intensity profiles [1]. For systems, which have to work in multipath environments, an improvement in error performance may become very difficult. Already a slight improvement in the bit error rate can necessitate a huge amount of additional transmitter power, which contradicts the aforementioned item of an economically use of transmission power. It is an enormous challenge to design wireless communication systems, which are capable to deal with these varying scenarios.

Orthogonal frequency division multiplexing (OFDM) [2] is a suitable technique for broadband transmission in multipath fading environments and is implemented in new broadcast standards like digital audio broadcasting (DAB) [3] or terrestrial digital video broadcasting (DVB-T) [4] as well as wireless local area network (WLAN) standards [5] such as HIPERLAN/2 or IEEE 802.11a.

Because of the poor error performance of OFDM in multipath environments, it is necessary for wireless communications systems to use techniques like inter-

leaving and channel coding in addition to OFDM. These techniques add redundancy and diversity in time and frequency direction. For many scattering environments, spatial diversity is another effective way to improve the error performance of wireless radio systems. In [6], [7] space-time-coding is proposed in order to get the benefits of channel coding in combination with spatial (antenna) diversity. Unfortunately space-time-coding is not suitable for extending existing systems, because this would make non standard conformable modifications necessary. Therefore for standardized systems additional spatial diversity techniques can only be implemented, if this modifications keep the systems standard compatible. In [8] such techniques for the transmitter side are proposed.

In this paper we will investigate standard conformable antenna diversity techniques, which are well suited for the extension of existing standardized OFDM systems. Section II introduces diversity techniques for both transmitter and receiver. At this, the main idea is to increase the frequency selectivity of the resulting channel transfer function by specific cyclic delays at the transmitter and/or receiver antennas. The transmitter sided delay diversity is also investigated in combination with receiver sided maximum ratio combining. It is shown in Section III how the mentioned diversity techniques are applicable to the DVB-T system in order to improve the bit error performance in multipath environments. In Section IV the DVB-T system and transmission parameters as well as the used channel models are described. Finally, simulation results for the bit error rates are presented for various DVB-T system parameter sets in indoor and outdoor environments.

II. SPATIAL ANTENNA DIVERSITY WITH OFDM

In this section we will introduce Delay Diversity (DD), Cyclic Delay Diversity (CDD), Phase Diversity (PD) and Maximum Ratio Combining (MRC), which can easily be applied to existing OFDM system standards with little efforts. A combination of this diversity techniques is easily possible. Thus, we can even with hindsight improve OFDM systems and find an optimal tradeoff be-



tween complexity and performance.

A. Delay Diversity

This section will briefly introduce delay diversity (DD), which was described in [8]. Fig. 1 shows the block diagram of an N -transmitter-antenna OFDM system with DD. The OFDM modulated signal is transmitted over N antennas, whereas the particular signals only differ in an antenna specific delay. Before shifting, an additional cyclic prefix as guard interval may be inserted. The functional blocks "UC" stands for upconversion from the baseband into the RF-band. Note, that in case of DD $\delta_n, n = 1, \dots, N-1$, denote simple time shifts.

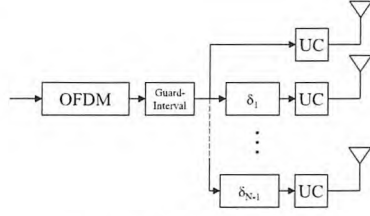


Fig. 1. OFDM transmitter with DD

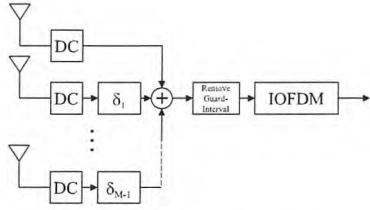


Fig. 2. OFDM receiver with DD

Because of linearity, it is also possible to implement DD at the receiver side. The appropriate block diagram of a M -antenna receiver with DD is shown in Fig. 2. The received signals are downconverted ("DC") into the baseband and shifted in time direction according to $\delta_m, m = 1, \dots, M-1$. After the superposition from these signals is built, the guard interval section is removed and the resulting signal is finally transformed into frequency domain (IOFDM).

To avoid intersymbol interference (ISI) the time delays δ_n, δ_m must hold the following condition:

$$\delta_n, \delta_m \leq \tau_g - \tau_{\max}, \quad n=1, \dots, N-1; \quad m=1, \dots, M-1. \quad (1)$$

Where τ_g is the guard interval length and $\tau_{\max}$ denotes the multipath channel delay spread. For tight dimensioned guard intervals, where τ_g is only slightly larger than $\tau_{\max}$, Eq. 1 strongly restricts the choice of the time delays δ_n, δ_m . In the next section cyclic delay diversity is introduced, which overcomes this problem.

B. Cyclic Delay Diversity

Fig. 3 illustrates the difference between DD and CDD in the time domain and shows the transmission of 2 consecutive OFDM symbols with their cyclic prefixes as

guard intervals. For clarity, the 1st subcarrier is plotted as a sine wave. The reference signal is undelayed and transmitted (resp. received) for both DD and CDD. In the case of DD it can be seen, that the DD signal is a simple copy of the reference signal, but delayed by δ . It's also observable, that OFDM symbols of the DD signal partly overlap the guard interval of the subsequent OFDM symbol in the reference signal at about δ . The result is the above mentioned restriction in the choice of δ (see Eq. 1). In the case of CDD one can see, that there is no overlapping of CDD OFDM symbols with the reference signal OFDM symbols, whereas the phase of the subcarriers are equal to that of the DD signal. This makes the performance of CDD equal to DD while there is no ISI in case of DD, i.e. Eq. 1 holds. Concurrently, no ISI can occur due to a too large dimensioned cyclic delay. The OFDM symbols of the CDD signal can be generated from the reference signal OFDM symbols just by applying a cyclic time shift of δ_{cy} to the reference signals' OFDM symbols and subsequent insertion of the cyclic prefix.

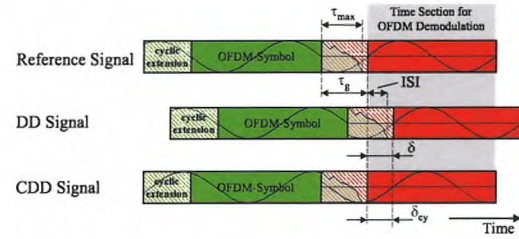


Fig. 3. Time signals for OFDM transmitters with DD and CDD

Fig. 4 shows the block diagram of an N -transmitter-antenna OFDM system with CDD. The OFDM modulated signal is transmitted over N antennas, whereas the particular signals only differ in an antenna specific cyclic shift. After cyclic shifting, the guard interval is inserted. The functional block "UC" again does upconversion of the signals from the baseband into RF-band.

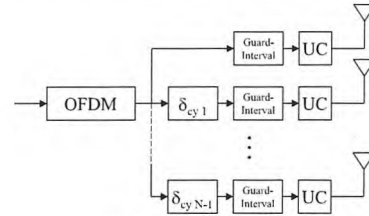


Fig. 4. OFDM transmitter with CDD

Again because of linearity CDD can be implemented at the receiver. Fig. 5 shows the principle block diagram of a CDD receiver.

Because of simple time shifts for DD resp. cyclic time shifts for CDD, these techniques are implementable with only slight additional complexity as long as the delays δ_n and δ_{cy_n} are multiples of the system sampling time.



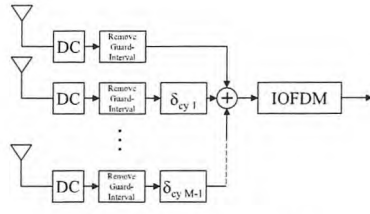


Fig. 5. OFDM receiver with CDD

The next section introduces phase diversity (PD), which is done in the frequency domain and allows implicit the choice of arbitrary delays.

C. Phase Diversity

The equivalence between PD and CDD is a property of the Discrete Fourier Transformation (DFT) and can directly be seen from the length K IDFT definition

$$\begin{aligned}
 s(\ell) &= \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} S(k) \cdot e^{j\frac{2\pi}{K}k\ell} \\
 \underbrace{s((\ell - \delta_{cy}) \bmod K)}_{\text{CDD signal}} &= \\
 &= \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} \underbrace{e^{-j\frac{2\pi}{K}k \cdot \delta_{cy}} \cdot S(k)}_{\text{PD signal}} \cdot e^{j\frac{2\pi}{K}k \cdot \ell}
 \end{aligned} \quad (2)$$

where ℓ , k , $s(\ell)$ and $S(k)$ denote the discrete time, frequency and the complex-valued signals in time- and frequency-domain respectively with $\ell, k = 1 \dots K-1$. As it can be seen from Eq. 2, a cyclic delay δ_{cy} in the time domain corresponds to a phase factor of $e^{-j\frac{2\pi}{K}k\delta_{cy}}$ in the frequency domain. PD is not only restricted to linearly incremented phases. It's also possible to choose phase factors $e^{j\phi(k)}$, where $\phi(k)$ is an arbitrary function of the discrete frequency k . Eq. 2 also shows that the operation for PD has to be done *before* OFDM modulation. So for an M -antenna PD system, M OFDM transformations have to be done and therefore the implementation of PD is more complex compared to CDD.

CDD and PD are independent of the existence of a cyclic prefix (guard interval) and are capable to increase the channel frequency selectivity without increasing the *overall* channel delay spread because these operations are done *before* guard interval insertion and are restricted to the OFDM symbol itself. The effect can be seen, if we have a look on the overall channel transfer function

$$H_m^{\text{res}}(k, \ell) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{K}k \cdot \delta_{cy,n}} \cdot H_{n,m}(k, \ell), \quad (3)$$

where $H_{n,m}(k, \ell)$ denotes the channel transfer function from the n^{th} transmitter antenna to the m^{th} receiver antenna and $\delta_{cy,n}$ stands for the transmitter antenna specific cyclic delay ($\delta_{cy,0} = 0$). Fig. 6 shows a snapshot

of $|H_m^{\text{res}}(k, \ell)|^2 = |H_{0,m}(k, \ell)|^2$ for a single antenna system over 512 adjacent subcarriers and the number of processed OFDM symbols. $|H_m^{\text{res}}(k, \ell)|^2$ denotes the squared absolute value of the resulting channel transfer function at receiver antenna m . The OFDM symbol duration is $T_U = 224 \mu\text{s}$. This yields a subcarrier spacing of $\Delta f_c = 1/T_U = 4464 \text{ Hz}$. In contrast, Fig. 7 shows a snapshot of $|H_m^{\text{res}}(k, \ell)|^2 = |H_{0,m}(k, \ell) + e^{-j\frac{2\pi}{K}k\delta_{cy,1}} H_{1,m}(k, \ell)|^2$ with $\delta_{cy,1} = 10$ (samples) and $K = 2048$ for a 2-transmit-antenna-system.

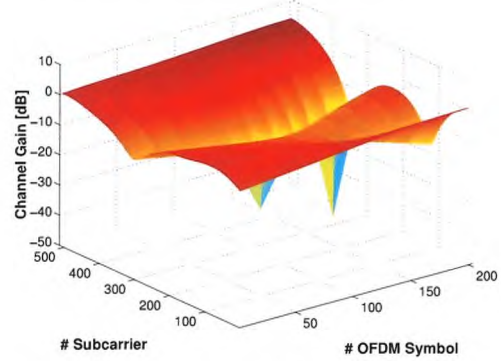


Fig. 6. Indoor channel snapshot for a single antenna system

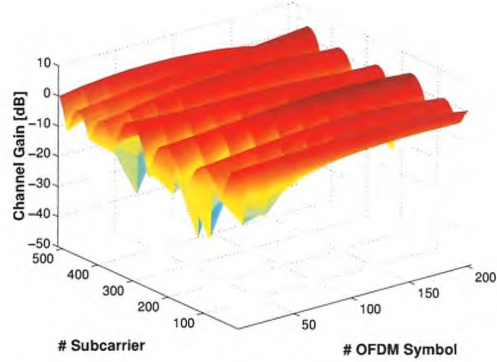


Fig. 7. Indoor channel snapshot for a CDD system

For Fig. 6 and 7, the "Indoor Commercial Channel B" channel model, which is briefly described in Section IV-A, has been used. It can obviously be seen, that an additional transmitter antenna with CDD increases the frequency selectivity, i.e. decreases the coherence bandwidth. In Section IV-C simulations will show, that a lower coherence bandwidth lead to a better error performance for the considered DVB-T system in multipath environment. In order to achieve any diversity effects, i.e. to get constructive and destructive interference within the OFDM signal bandwidth B , the inserted (cyclic) delays $\delta_{cy,n}$ have to fulfill

$$\delta_{cy} \geq \frac{1}{B}.$$



D. Maximum Ratio Combining

Another well known standard conformable diversity technique is MRC. Fig. 8 shows the block diagram of a classical M -branch MRC-OFDM receiver. After down-conversion into the baseband, the M time-signals are OFDM demodulated and combined by subcarriers, using the MRC scheme, which optimizes the signal-to-noise-ratio (SNR) for each subcarrier and is described e.g. in [1], [7].

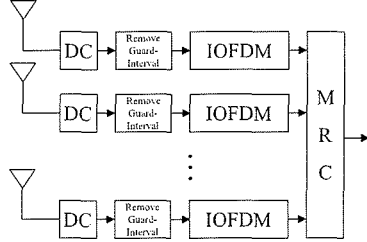


Fig. 8. OFDM receiver with MRC

III. APPLICATION TO THE DVB-T SYSTEM

In this section we will apply CDD and MRC, introduced in Section II, to the DVB-T system. DVB-T is basically a coded OFDM system containing an outer shortened Reed-Solomon code concatenated with an inner (punctured) convolutional code [4]. The investigations in this paper restrict to the "inner system" of DVB-T for non hierarchical transmission parameter sets, i.e. outer coding and decoding are not considered.

A. Transmitter

For the implementation of CDD at the DVB-T transmitter, only a second signal path after the OFDM-modulation has to be added. Fig. 9 shows the transmitter sided inner part with transmitter CDD. After chan-

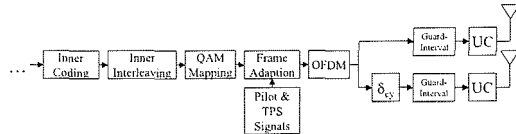


Fig. 9. Inner non hierarchical DVB-T transmitter part with CDD

nel coding and interleaving, the bit-stream is mapped to complex-valued QAM-symbols. The functional block "Frame Adaption" is responsible for QAM-symbol interleaving, pilot insertion and transmission parameter signaling (TPS). The resulting symbol-stream is OFDM-modulated. Finally, the signal is splitted, upconverted and transmitted directly on the one hand and cyclic shifted on the other hand. It is important to note that signal splitting does not increase the overall transmission power.

B. Receiver

As mentioned above, we use a MRC receiver for our simulations. Fig. 10 shows a block diagram of the DVB-T MRC receiver implementation. After downcon-

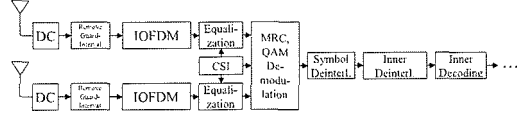


Fig. 10. Non hierarchical DVB-T MRC receiver

version and guard interval removal, the received signal is OFDM-demodulated and equalized using zero forcing. For our investigations we assume perfect knowledge of the channel state information (CSI). Both complex-valued symbol-streams are combined and QAM demodulated with soft-out values before symbol- and bit-deinterleaving is done. Finally, the bit stream is soft-decision-maximum-likelihood (SDML) decoded.

IV. SIMULATIONS

In this section we will present simulation results for DVB-T with antenna diversity in indoor and outdoor scenarios. The simulations were done for Doppler frequencies, which follow from typical pedestrian movement in indoor environment and mobile vehicle movement in outdoor parking-lot scenarios.

A. Channel Models

Table I shows the main properties of the wide sense stationary uncorrelated scattering (WSSUS) channel models, which were used for simulations. For the individual scatterers Rayleigh fading is assumed. The mobile radio channel models are described in [9]. For the indoor scenario the "Indoor Commercial Channel B" channel model is used with a maximum Doppler frequency of 10 Hz, which correspond to a mobile velocity of $1.25 \frac{m}{s}$ ($4.5 \frac{km}{h}$) for operation in the 2.4 GHz band. For the outdoor scenario the "Outdoor Residential - High Antenna" (Channel B) channel model is used. The maximum Doppler frequency for the outdoor scenario is 50 Hz, which results in a mobile velocity of $6.25 \frac{m}{s}$ ($22.5 \frac{km}{h}$). Antenna diversity simulations were done for pairwise uncorrelated WSSUS channels $H_{n,m}(k, \ell)$.

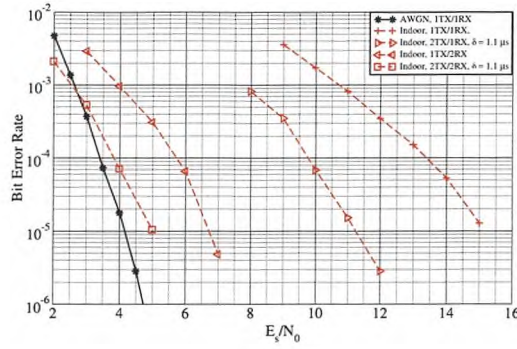
TABLE I
MAIN CHANNEL MODEL PROPERTIES

	indoor	outdoor
Number of Taps	7	10
max. Channel Delay τ_{max}	750 ns	15 μs
max. Doppler Frequency f_{Dmax}	10 Hz	50 Hz
Doppler Spectrum Form	rectangular	Jakes

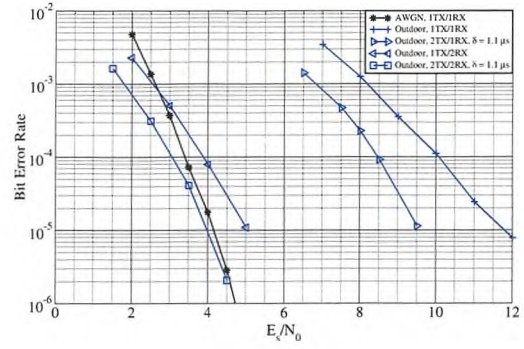
B. DVB-T System Parameters

For 8 MHz channels the DVB-T standard defines some basic parameters. In the 2k-mode the OFDM symbol





(a) Indoor



(b) Outdoor

Fig. 11. BER vs. SNR, 2k-Mode, 4-QAM, Code-Rate 1/2

duration is $T_U = 224 \mu\text{s}$. The number of used carriers is $K = 1705$. This yields a subcarrier spacing of $1/T_U = 4464 \text{ Hz}$ and a spacing between the spectrum edge carriers of $(K - 1)/T_U = 7.61 \text{ MHz}$. For the 8k-mode, the basic parameters are $T_U = 896 \mu\text{s}$ and $K = 6817$. Subcarrier spacing and edge carrier spacing will follow equivalently. Beside this basic parameters, there are some additional transmission parameters regarding modulation, guard interval length and channel coding. For 2k systems and the outdoor channel a guard interval length of $\tau_g = 1/8 \cdot 224 \mu\text{s} = 28 \mu\text{s}$ have to be chosen because of the maximum outdoor channel delay of $\tau_{\text{max}} = 15 \mu\text{s}$. Therefore, a guard interval length of "1/32" is enough for 8k systems. Table II summarizes the main DVB-T simulation parameters.

TABLE II
DVB-T SYSTEM PARAMETERS FOR SIMULATIONS

Simulation #	1	2	3	4
Code-Rate	1/2		2/3	
Signal Constellation	4-QAM		16-QAM	
DVB-T Mode	2k		8k	
Guard Interval	$1/8 \cdot T_U = 28 \mu\text{s}$		$1/32 \cdot T_U = 28 \mu\text{s}$	

C. Results

The Monte Carlo simulations are restricted to the "inner" DVB-T system as it is described in Section III. It is important to note, that the overall transmitted power is kept equal for all simulation runs, i.e., the transmitted power per transmit antenna decreases with increasing number of antennas. The signal power at the receiver antennas has not been normalized. This means that the overall received signal power increases as the number of receive antennas increases. For simulations we will restrict to the use of at most 2 antennas at the transmitter respectively the receiver side.

Fig. 11(a) shows the bit error rate vs. the SNR for the indoor channel with different diversity techniques, applied to the DVB-T system in non hierarchical 2k-mode

with 4-QAM-modulation and code-rate 1/2. Note that the SNR here equals E_s/N_0 (the signal energy divided by noise power spectral density). A single-antenna-system is given as a reference, i.e., for this system no spatial diversity is implemented. As it can be seen from Fig. 11(a), the receiver-MRC system outperforms the single-antenna system about 7.4 dB in SNR at a BER of $2 \cdot 10^{-4}$. The reason therefore is the 2nd receiver antenna, which provides the receiver with additional signal power. Furthermore the two propagation paths (channel transfer functions) are uncorrelated. So subcarriers, which are in a deep fade for receiver-antenna 1 may have good channel properties for antenna 2. With additional transmitter-CDD, a further gain of about 1.8 dB can be achieved. Note, that the transmitter-CDD gain decreases with the use of MRC. CDD increases channel selectivity and therefore decreases the occurrence of packet errors after demodulation. This packet errors may appear also before decoding in spite of interleaving due to extremely wide deep fades, which particularly occur in environments with small channel delay spreads, e.g. indoor scenarios.

Fig. 11(b) shows the bit error performance for a DVB-T system in non hierarchical 2k-mode with 4-QAM-modulation and code rate 1/2, which now operates in an outdoor environment. Again, most can be gained, if MRC is used at the receiver. In general the error performance is better for all system combinations compared to the indoor scenario. Note, that the outdoor channel provides a higher maximum channel delay and therefore a higher frequency selectivity, which explains the better error performance compared to the indoor channel. It is also interesting to see, that the gains *between* the different system combinations for the outdoor channel become smaller compared to indoor channels. So the lower the maximum channel delay spread τ_{max} , the higher is the achievable gain for additional use of CDD.

Fig. 12 shows the transmitter-CDD gain for single-



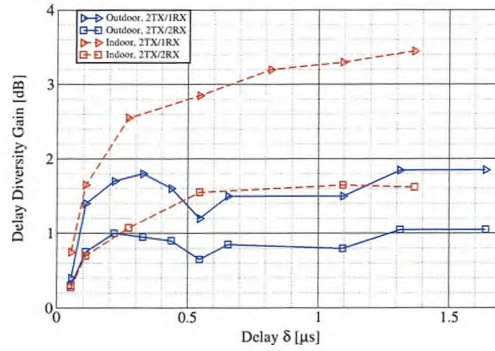


Fig. 12. Delay Diversity Gain vs. Delay at $\text{BER} = 2 \cdot 10^{-4}$, 2k-Mode, 4-QAM, Code-Rate 1/2

antenna and 2-antenna MRC-receivers in indoor and outdoor environments vs. the cyclic delay at the 2nd transmit antenna of a CDD transmitter. As the results, showed in Fig. 11(a) and 11(b), signify, there is a saturation effect in terms of the cyclic diversity delay. It can be observed, that a cyclic delay of $\delta > 1.5 \mu\text{s}$ results in no further improvement. Fig. 12 also shows, that the achievable gain is much higher for indoor scenarios compared to outdoor scenarios due to the extremely different maximum channel delays of $\tau_{\max} = 15 \mu\text{s}$ for outdoor and $\tau_{\max} = 750 \text{ ns}$ for indoor scenarios. Different from the indoor results, the outdoor delay diversity gain shows local minima at about $\delta = 0.55 \mu\text{s}$. A reason is the structure of tap delay model for the outdoor channel. For $\delta = 0.55 \mu\text{s}$, relatively powerful paths of the outdoor channel model come close to each other. This tight adjacent echos yield a reduced frequency selectivity of the overall channel transfer function $|H_m^{\text{res}}(\ell, k)|^2$, $m = 1, 2$.

The BER and delay diversity gain results for various 8k systems are illustrated in Figs. 13-15. Table II shows the main parameter sets. It can be observed that the achievable delay diversity gains for the 8k systems remain in the same dimensions as the respective 2k results, shown in Fig. 12. The influence of the choice of the DVB-T mode (FFT length) can be seen if we compare Figs. 11 and 13(a). It can be seen that there are only nonessential differences between the resp. BERs. A rule of thumb says that the intercarrier interference (ICI) due to Doppler spread can be neglected, if the Doppler spread of the channel is below 10% of the subcarrier spacing. If we have a look on Table I, we can see that this rule holds for all of the simulations. However, if we would choose higher Doppler spreads, a higher degradation of the BER performance of 8k systems compared to their 2k counterparts can be expected. Simulation results on further 2k systems can be found in [10].

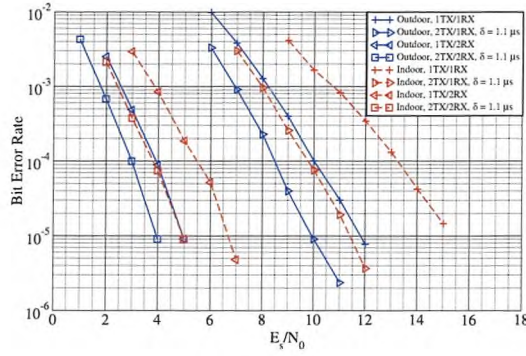
V. CONCLUSIONS

In this paper delay diversity, cyclic delay diversity, phase diversity and maximum ratio combining have been presented. The (conditional) equivalence between cyclic delay diversity and phase diversity respectively delay diversity have been shown. It is the purpose of the proposed techniques to increase the delay spread resp. the frequency selectivity of the resulting channel transfer function by specific cyclic delays at the transmitter and/or receiver antennas. The presented antenna diversity schemes can easily be implemented in existing OFDM systems without changing the standards or the receivers. The main advantage of cyclic delay diversity and phase diversity compared to delay diversity is that no additional ISI can occur at all due to the additional implementation of that techniques. It was shown exemplarily, how this techniques are applicable to the DVB-T system. Simulations showed, that the higher the delay spread of the channel, the better is the bit error performance. Therefore, the lower the channel delay spread of the channel model for single-antenna systems, the higher is the achievable gain for additional implementation of the proposed diversity techniques.

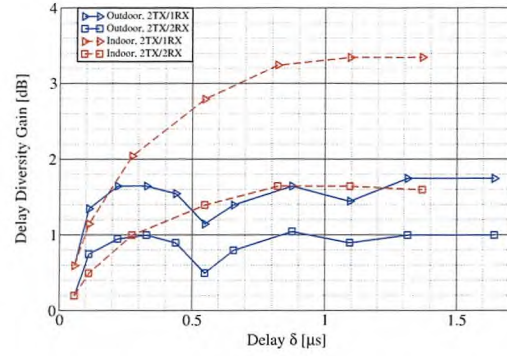
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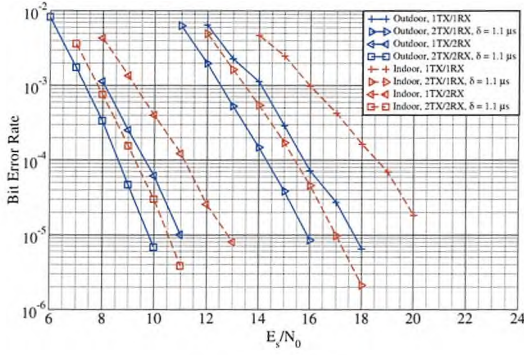


(a) BER vs. SNR

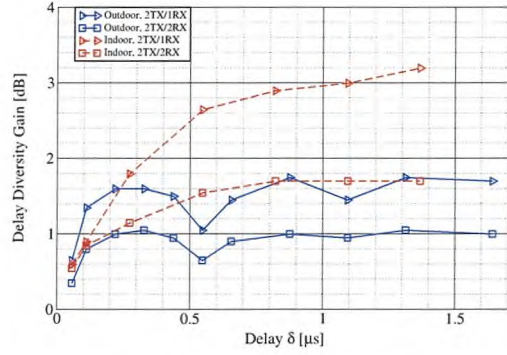


(b) Delay Diversity Gain vs. Delay at $\text{BER} = 2 \cdot 10^{-4}$

Fig. 13. Simulation results for 8k-Mode, 4-QAM, Code-Rate 1/2

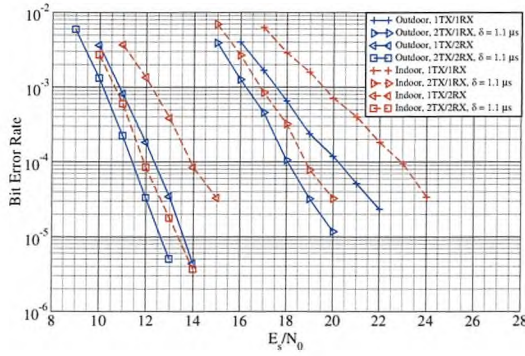


(a) BER vs. SNR

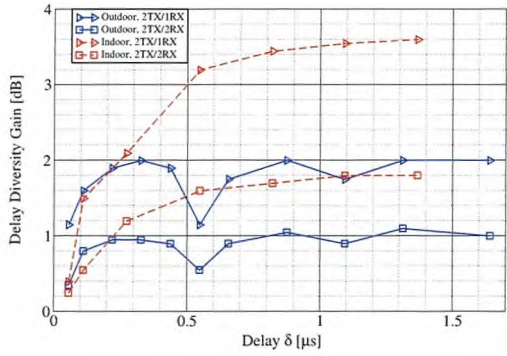


(b) Delay Diversity Gain vs. Delay at $\text{BER} = 2 \cdot 10^{-4}$

Fig. 14. Simulation results for 8k-Mode, 16-QAM, Code-Rate 1/2



(a) BER vs. SNR



(b) Delay Diversity Gain vs. Delay at $\text{BER} = 2 \cdot 10^{-4}$

Fig. 15. Simulation results for 8k-Mode, 16-QAM, Code-Rate 2/3



<https://web.archive.org/web/20030330174357/http://www.dlr.de:80/KN/KN-S/Dammann/papers/EW2002.pdf>



On the Equivalence of Space-Time Block Coding with Multipath Propagation and/or Cyclic Delay Diversity in OFDM

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ABSTRACT

In this paper, we show the equivalence of Space-Time block coding to multipath propagation with respect to Cyclic Delay Diversity for OFDM systems. Hence, we can apply existing Space-Time code design criteria and analyze methods for Space-Time code construction using Cyclic Delay Diversity OFDM systems in multipath propagation environments. i.e. terms like "diversity gain" and "coding gain" can be used to design, analyze, compare and understand these OFDM systems from the Space-Time block coding point of view.

1 INTRODUCTION

Mobile communication systems mainly suffer from time-varying multipath fading with extremely different multipath intensity profiles [1]. For systems which work in multipath environments, an improvement in error performance may become very difficult. In present systems, a slight improvement in the bit error rate may require a huge amount of additional transmission power, which contradicts the goal of economical use of system resources. It is an enormous challenge to design wireless communication systems capable of dealing with these varying scenarios.

Orthogonal frequency division multiplexing (OFDM) [2] is a suitable technique for broadband transmission in multipath fading environments and is implemented in new broadcast standards like terrestrial digital video broadcasting (DVB-T) [3] as well as wireless local area network (WLAN) standards [4] such as HIPERLAN/2 or IEEE 802.11a.

Because of the poor error performance of OFDM in multipath environments, it is necessary for wireless communications systems to use techniques like interleaving and channel coding in addition to OFDM. These techniques add redundancy and diversity in time and frequency direction. For many scattering environments, spatial diversity is another effective way to improve the error performance of wireless radio systems. In [5], [6] space-time-coding is proposed in order to get the benefits of channel coding in combination with spatial (antenna) diversity. Another approach to achieve spatial diversity effects for OFDM systems are techniques like Delay Diversity, Cyclic Delay Diversity or Phase Diversity ([7], [8]).

In this paper we will describe Cyclic Delay Diversity [8] and multipath propagation as a Space-Time block code. Section 2 first introduces the application of Cyclic Delay Diversity to OFDM systems. Further more, the equivalence of Cyclic Delay Diversity with respect to multipath propagation in OFDM and Space-Time block coding is shown. In Section 3 we will analyze using an example the diversity and the coding gain for a CDD OFDM system in a Rayleigh fading environment.

2 CYCLIC DELAY DIVERSITY AND MULTIPATH PROPAGATION AS SPACE-TIME BLOCK CODE

In this section we introduce *Cyclic Delay Diversity* (CDD) and show the equivalence between multipath propagation and CDD. Then multipath propagation and CDD are described as Space-Time block codes and analyzed using techniques from [5] and [9].

2.1 Introduction to Cyclic Delay Diversity for OFDM

Below we will briefly introduce the application of CDD to OFDM systems as described in detail in [8]. Fig. 1 shows the block diagram of an N -antenna OFDM transmitter with CDD. The OFDM modulated signal is transmitted over N antennas, whereas the particular signals only differ in an antenna specific cyclic shift. After cyclic shifting, a cyclic prefix is inserted to avoid intersymbol interference and maintain subcarrier orthogonality for multipath channels. The functional block "UC" performs an upconversion of the signals from the baseband into RF-band. $\delta_{cy\ n}, n = 1, \dots, N - 1$, denote cyclic shifts of the OFDM symbol in the time domain.

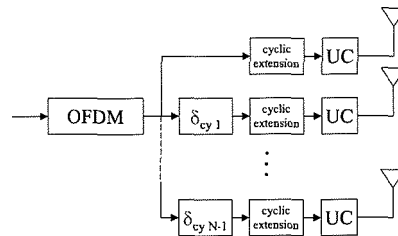


Figure 1: Application of CDD to an OFDM transmitter



2.2 Equivalence of multipath propagation and CDD

The equivalence between multipath propagation and CDD is illustrated in Fig. 2. We assume that an OFDM time signal with cyclic extension – as commonly used for OFDM systems – is transmitted from antenna 1 (TX-antenna 1). The cyclic extension allows different OFDM symbols to be decoded without intersymbol interference when

$$T_{\text{Guard}} \geq \delta_{\text{max}} \quad (\geq \delta_1).$$

T_{Guard} is the period of the cyclic extension, and δ_{max} is the maximum multi-path delay of the channel. For clarity, one carrier is drawn as a sine wave with a different phase per OFDM symbol in Fig. 2. We observe that we may receive the OFDM time signal from TX-antenna 1 superimposed with a multi-path signal from antenna 1 delayed by δ_1 .

Now we transmit a 2nd CDD OFDM time signal from TX-antenna 2. As can be seen from Fig. 2, the OFDM time domain symbols, transmitted from TX-antenna 2, are built by a cyclic shift δ_{cy} of the respective OFDM time domain symbols at TX-antenna 1.

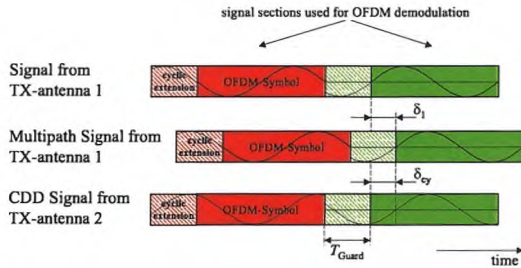


Figure 2: Equivalence of CDD and multipath propagation for OFDM

If we now compare the signal sections which are used for OFDM demodulation, one can see that a cyclic delay δ_{cy} behaves exactly like a multipath channel delay δ_1 based on the channel model. So an arbitrary OFDM receiver cannot distinguish between multipath propagation and CDD within this signal range.

2.3 Space-Time Block Code Description of CDD and Multipath Propagation

As can be seen from the length K IDFT definition (Eq. 1), a cyclic delay δ_{cy} in time domain corresponds to a phase factor of $e^{-j\frac{2\pi}{K}\delta_{\text{cy}}k}$ in frequency domain.

$$\begin{aligned} s((\ell - \delta_{\text{cy}}) \bmod K) &= \quad (1) \\ &\text{CDD OFDM time signal} \\ &= \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} e^{-j\frac{2\pi}{K}\delta_{\text{cy}}k} \cdot S(k) \cdot e^{j\frac{2\pi}{K}k\ell} \\ &= \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} S_\delta(k) \cdot e^{j\frac{2\pi}{K}k\ell} \end{aligned}$$

where ℓ and k denote the discrete time and frequency indices respectively with $\ell, k = 0 \dots K-1$. $s(\ell)$ and $S(k)$

represent the complex signals in the time and frequency domain respectively. δ_{cy} stands for a cyclic time shift in samples. We assume that the $S(k)$ are elements of a signal constellation $\mathcal{Q}$, where the average power over all elements of $\mathcal{Q}$ is 1. Commonly used signal constellations are QAM or PSK. Note, that in time domain the choice of any integer value for δ_{cy} is possible, but due to the characteristics $(\delta_{\text{cy}} + K) \bmod K = \delta_{\text{cy}}$ and $e^{j\frac{2\pi}{K}(\delta_{\text{cy}}+K)} = e^{j\frac{2\pi}{K}\delta_{\text{cy}}}$, the range of values for δ_{cy} can reasonably be restricted to $\delta_{\text{cy}} = 0 \dots K-1$. To simplify notation, we define $\delta = \delta_{\text{cy}}$ in the following. If we define

$$\begin{aligned} \Phi(\delta) &= \text{diag} \left(e^{-j\frac{2\pi}{K}\delta \cdot 0}, e^{-j\frac{2\pi}{K}\delta \cdot 1}, \dots, e^{-j\frac{2\pi}{K}\delta \cdot (K-1)} \right) \\ &= \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & e^{-j\frac{2\pi}{K}\delta \cdot 1} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & e^{-j\frac{2\pi}{K}\delta \cdot (K-1)} \end{pmatrix}, \end{aligned}$$

we can express $\mathbf{S}_\delta = (S_\delta(0), S_\delta(1), \dots, S_\delta(K-1))$ as

$$\mathbf{S}_\delta = \mathbf{S} \cdot \Phi(\delta),$$

where $\mathbf{S} = (S(0), S(1), \dots, S(K-1))$.

Investigations in [5] and [9] were done for independent Rayleigh fading with average energy of 1 per path, i.e. the channel coefficients are complex valued Gaussian random variables with zero mean and variance 0.5 per complex dimension. We introduce real valued normalization coefficients $\lambda_{n,m}$, which allow the adjustment of transmission power for propagation path m from TX-antenna n such that the overall average received energy is 1. If we have N TX-antennas and assume M propagation paths per TX-antenna, then

$$\begin{aligned} n &= 0 \dots N-1 \quad \text{and} \\ m &= 0 \dots M-1. \end{aligned}$$

We define an index $\ell = m + n \cdot M$. If $\delta_{\text{cy},n}$ denotes the cyclic shift at TX-antenna n and $\delta_{n,m}$ is the multipath delay of propagation path m at TX-antenna n , then $\delta_\ell = \delta_{\text{cy},n} + \delta_{n,m}$. For simplicity, we define $\Phi(\delta_\ell) = \Phi_\ell$. Now the Space-Time codeword matrix can be written as

$$\begin{aligned} \mathbf{W} &= \begin{pmatrix} \mathbf{S} \cdot \lambda_0 \cdot \Phi(\delta_0) \\ \mathbf{S} \cdot \lambda_1 \cdot \Phi(\delta_1) \\ \vdots \\ \mathbf{S} \cdot \lambda_{MN-1} \cdot \Phi(\delta_{MN-1}) \end{pmatrix} \quad (2) \\ &\quad \underbrace{\hspace{10em}}_{M \cdot N \times K} \\ &= \underbrace{\begin{pmatrix} \mathbf{S} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{S} & & \vdots \\ \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{S} \end{pmatrix}}_{M \cdot N \times K \cdot M \cdot N} \cdot \underbrace{\begin{pmatrix} \lambda_0 \cdot \Phi_0 \\ \lambda_1 \cdot \Phi_1 \\ \vdots \\ \lambda_{MN-1} \cdot \Phi_{MN-1} \end{pmatrix}}_{K \cdot M \cdot N \times K}. \end{aligned}$$

Eq. 2 shows that it does not make any difference whether the delays are caused by multipath propagation



$$\mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}}) = (\mathbf{W} - \tilde{\mathbf{W}}) \cdot (\mathbf{W} - \tilde{\mathbf{W}})^H \quad (3)$$

$$= \begin{pmatrix} \lambda_0^2 \cdot |\Delta \mathbf{S}|^2 & \lambda_0 \lambda_1 \cdot \Delta \mathbf{S} \Phi_0^H \Delta \mathbf{S}^H & \dots & \lambda_0 \lambda_{MN-1} \cdot \Delta \mathbf{S} \Phi_0^H \Phi_{MN-1}^H \Delta \mathbf{S}^H \\ \lambda_1 \lambda_0 \cdot \Delta \mathbf{S} \Phi_1^H \Phi_0^H \Delta \mathbf{S}^H & \lambda_1^2 \cdot |\Delta \mathbf{S}|^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \lambda_{MN-2} \lambda_{MN-1} \cdot \Delta \mathbf{S} \Phi_{MN-2}^H \Phi_{MN-1}^H \Delta \mathbf{S}^H \\ \lambda_{MN-1} \lambda_0 \cdot \Delta \mathbf{S} \Phi_{MN-1}^H \Phi_0^H \Delta \mathbf{S}^H & \dots & \lambda_{MN-1} \lambda_{MN-2} \cdot \Delta \mathbf{S} \Phi_{MN-1}^H \Phi_{MN-2}^H \Delta \mathbf{S}^H & \lambda_{MN-1}^2 \cdot |\Delta \mathbf{S}|^2 \end{pmatrix}$$

or CDD. Hence, for OFDM systems, a multipath channel can be considered as an "inherent" Space-Time block encoder.

Following analysis in [5] and [9] we define the Hermitian difference matrix as a $M \cdot N \times M \cdot N$ matrix $\mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}})$ as depicted in Eq. 3. where $\mathbf{W}^H$ denotes the Hermitian of $\mathbf{W}$ and $\Delta \mathbf{S} = \mathbf{S} - \tilde{\mathbf{S}}, \forall \mathbf{S} \neq \tilde{\mathbf{S}}$. The diagonal element of row/column i is defined as

$$\lambda_i^2 \cdot |\Delta \mathbf{S}|^2 = \lambda_i^2 \cdot \sum_{m=0}^{K-1} |\Delta S_m|^2. \quad (4)$$

For asymptotically good channels, the diversity gain (rank criterion) is defined as [9]

$$\min_{\mathbf{S} \neq \tilde{\mathbf{S}}} \text{rank}[\mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}})] \quad (5)$$

This parameter determines the asymptotic slope of the error rate curve with respect to the signal to noise ratio. We define r and γ_i as the rank and i -th nonzero eigenvalue of the matrix $\mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}})$, then

$$\min_{\mathbf{S} \neq \tilde{\mathbf{S}}} \prod_{i=1}^r \gamma_i, \quad (6)$$

over all codeword pairs $\mathbf{W}, \tilde{\mathbf{W}}$ defines the coding gain (eigen product criterion). This criterion is responsible for shifting the error rate curve of he chosen construction to the left for high signal to noise ratios.

3 ANALYSIS

As an example, we analyze a 2-antenna CDD OFDM system in a Rayleigh fading environment. In this case $N = 2, M = 1$ and $\lambda_0 = \lambda_1 = 1$. For the delays we choose $\delta_0 = 0$ and $\delta_1 = 1$. Then Φ_0 is an identity matrix of size $K \times K$.

As modulation alphabet $\mathcal{Q}$, we assume 4-QAM with $|\mathbf{S}|^2 = 1, \forall \mathbf{S} \in \mathcal{Q}$ and equiprobable symbols $\mathbf{S}$ as shown in Fig. 3. If we define $\Delta \mathcal{Q} = \{\Delta \mathbf{S} = \mathbf{S} - \tilde{\mathbf{S}} | \mathbf{S}, \tilde{\mathbf{S}} \in \mathcal{Q}\}$, then the probabilities of the elements $\Delta \mathbf{S} \in \Delta \mathcal{Q}$ are illustrated in Fig. 4 and

$$E\{|\Delta \mathbf{S}|^2\} = E\{|\mathbf{S}|^2\} + E\{|\tilde{\mathbf{S}}|^2\} = 2 \quad (7)$$

for independent $\mathbf{S}$ and $\tilde{\mathbf{S}}$.

3.1 Rank Criterion

The parameters and assumptions above lead to a Hermitian difference matrix as shown in Eq. 8.

$$\mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}}) = \begin{pmatrix} |\Delta \mathbf{S}|^2 & \Delta \mathbf{S} \Phi_1^H \Delta \mathbf{S}^H \\ \Delta \mathbf{S} \Phi_1 \Delta \mathbf{S}^H & |\Delta \mathbf{S}|^2 \end{pmatrix} \quad (8)$$

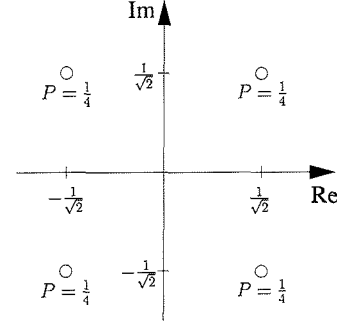


Figure 3: 4-QAM Constellation and Probabilities

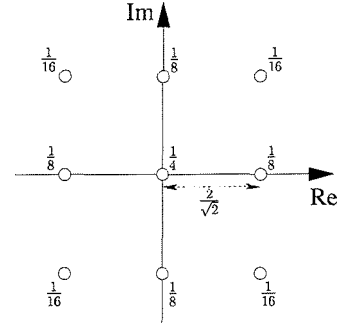


Figure 4: Differential 4-QAM Constellation and Probabilities

The rank of matrix $\mathbf{A}$ is as follows:

1. $\Delta \mathbf{S} = (0, \dots, 0) \in \Delta \mathcal{Q}^K \iff \text{rank}[\mathbf{A}] = 0$
2. $\Delta \mathbf{S} = \Delta \mathbf{S} \cdot \mathbf{e}_j \in \Delta \mathcal{Q}^K, \Delta \mathbf{S} \neq 0, j \in \{0, 1, \dots, K-1\} \iff \text{rank}[\mathbf{A}] = 1$
 $\mathbf{e}_j$ is a vector where the j th coordinate is 1, and all other coordinates are zero.
3. else $\iff \text{rank}[\mathbf{A}] = 2$

Proof: We will first proof the " $\implies$ " direction

1. The rank of a null-matrix is 0.
2. In this case:
 $|\Delta \mathbf{S}|^2 > 0 \implies \text{rank}[\mathbf{A}] > 0$
and
 $\det(\mathbf{A}) = 0 \implies \text{rank}[\mathbf{A}] < 2 \implies \text{rank}[\mathbf{A}] = 1.$



3. Let us arbitrarily choose 2 components $\Delta S_a \neq 0$ and $\Delta S_b \neq 0$ of the vector $\Delta \mathbf{S}$ with $a \neq b$ and $a, b \in \{0, 1, \dots, K-1\}$. Then

$$\begin{aligned} |\Delta \mathbf{S} \Phi_1 \Delta \mathbf{S}^H| &= \left| \sum_{i=0}^{K-1} |\Delta S_i|^2 \cdot e^{-j \frac{2\pi}{K} i} \right| \\ &\leq \underbrace{|\Delta S_a|^2 e^{-j \frac{2\pi}{K} a} + |\Delta S_b|^2 e^{-j \frac{2\pi}{K} b}}_{< |\Delta S_a|^2 + |\Delta S_b|^2 \forall a \neq b} + \\ &\quad + \sum_{\substack{i=0 \\ i \neq a \wedge i \neq b}}^{K-1} |\Delta S_i|^2 \cdot \underbrace{\left| e^{-j \frac{2\pi}{K} i} \right|}_{=1} \\ &< |\Delta S_a|^2 + |\Delta S_b|^2 + \\ &\quad + \sum_{\substack{i=0 \\ i \neq a \wedge i \neq b}}^{K-1} |\Delta S_i|^2 = |\Delta \mathbf{S}|^2 \end{aligned}$$

and therefore $\det(\mathbf{A}) = |\Delta \mathbf{S}|^2 \cdot |\Delta \mathbf{S}|^2 - |\Delta \mathbf{S} \Phi_1 \Delta \mathbf{S}^H|^2 > 0$, which means $\text{rank}[\mathbf{A}] = 2$.

The cases 1-3 define disjunctive sets of $\Delta \mathbf{S}$. The union of these sets yields the set of all possible vectors $\Delta \mathbf{S}$. This implies that the mapping of the sets defined in the cases 1-3 onto the set $\{0, 1, 2\}$ (rank) is a one-to-one mapping. Therefore, the " $\Leftarrow$ " direction also holds. ■

Based on the calculations in this section and the signal constellation probabilities in Fig. 4 we can derive the probabilities for $\text{rank}[\mathbf{A}] = 0, 1, 2$.

1. From this section we know: $\text{rank}[\mathbf{A}] = 0 \implies \Delta \mathbf{S} = (0, \dots, 0)$
This gives

$$\text{Prob}(\text{rank}[\mathbf{A}] = 0) = \left(\frac{1}{4}\right)^K,$$

where $\frac{1}{4}$ is the probability for $\Delta S = 0$ (see Fig. 4). Note that in fact $\text{rank}[\mathbf{A}] = 0$ is not possible because of the condition $\Delta \mathbf{S} = \mathbf{S} - \tilde{\mathbf{S}}$, $\mathbf{S} \neq \tilde{\mathbf{S}}$ in Eq. 5 and 6.

2. $\text{rank}[\mathbf{A}] = 1 \implies \Delta \mathbf{S} = \Delta S \cdot \mathbf{e}_j \in \Delta \mathcal{Q}^K$, $\Delta S \neq 0$, $j \in \{0, 1, \dots, K-1\}$
and consequently

$$\text{Prob}(\text{rank}[\mathbf{A}] = 1) = K \cdot \frac{3}{4} \cdot \left(\frac{1}{4}\right)^{K-1},$$

where $\frac{3}{4} = 1 - \frac{1}{4}$ is the probability for $\Delta S \neq 0$.

- 3.

$$\begin{aligned} \text{Prob}(\text{rank}[\mathbf{A}] = 2) &= 1 - \text{Prob}(\text{rank}[\mathbf{A}] = 0) \\ &\quad - \text{Prob}(\text{rank}[\mathbf{A}] = 1) \\ &= 1 - \frac{3K+1}{4^K} \end{aligned}$$

Therefore, the larger the number of subcarriers, the higher the probability that we have full rank. Also, the denominator 4^K depends directly on the number of signals in the modulation constellation. Therefore by increasing the cardinality of the signal constellation we also drive the full rank probability towards 1.

3.2 Calculation of the Determinant

In the case of our example, $\det \mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}})$ is

$$\begin{aligned} \det \mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}}) &= \\ &= |\Delta \mathbf{S}|^2 \cdot |\Delta \mathbf{S}|^2 - \Delta \mathbf{S} \Phi_1 \Delta \mathbf{S}^H \cdot \Delta \mathbf{S} \Phi_1^H \Delta \mathbf{S}^H \\ &= |\Delta \mathbf{S}|^2 \cdot |\Delta \mathbf{S}|^2 - |\Delta \mathbf{S} \Phi_1 \Delta \mathbf{S}^H|^2. \end{aligned} \quad (9)$$

The determinant of $\mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}})$ is closely related to the coding gain criterion which is based on the product of non-zero eigenvalues.

The minimum product of non-zero eigenvalues will give the coding gain in asymptotically good channels. However, by calculating the average determinant value, we can see how the statistics depend on the chosen code parameters. The mean value μ of $\det[\mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}})]$ is:

$$\begin{aligned} \mu &= E \{ \det \mathbf{A}(\mathbf{W}, \tilde{\mathbf{W}}) \} \\ &= E \{ |\Delta \mathbf{S}|^2 \cdot |\Delta \mathbf{S}|^2 \\ &\quad - \Delta \mathbf{S} \Phi_1 \Delta \mathbf{S}^H \cdot (\Delta \mathbf{S} \Phi_1 \Delta \mathbf{S}^H)^H \} \\ &= E \left\{ \sum_{\ell=0}^{K-1} |\Delta S_\ell|^2 \sum_{m=0}^{K-1} |\Delta S_m|^2 \right. \\ &\quad \left. - \sum_{\ell=0}^{K-1} |\Delta S_\ell|^2 \cdot \exp \left(-j \frac{2\pi}{K} \cdot \ell \right) \cdot \sum_{m=0}^{K-1} |\Delta S_m|^2 \cdot \exp \left(j \frac{2\pi}{K} \cdot m \right) \right\} \\ &= \sum_{\ell, m=0}^{K-1} E \{ |\Delta S_\ell|^2 |\Delta S_m|^2 \} \\ &\quad - \sum_{\ell, m=0}^{K-1} E \{ |\Delta S_\ell|^2 \cdot |\Delta S_m|^2 \} \cdot \exp \left(j \frac{2\pi}{K} \cdot (m - \ell) \right) \\ &= \sum_{\substack{\ell, m=0 \\ \ell \neq m}}^{K-1} E \{ |\Delta S_\ell|^2 |\Delta S_m|^2 \} + \sum_{\ell=0}^{K-1} E \{ |\Delta S_\ell|^4 \} \\ &\quad - \sum_{\substack{\ell, m=0 \\ \ell \neq m}}^{K-1} E \{ |\Delta S_\ell|^2 \cdot |\Delta S_m|^2 \} \cdot \exp \left(j \frac{2\pi}{K} \cdot (m - \ell) \right) \\ &\quad - \sum_{\ell=0}^{K-1} E \{ |\Delta S_\ell|^4 \} \\ &= E \{ |\Delta \mathbf{S}|^2 \}^2 \cdot K(K-1) - E \{ |\Delta \mathbf{S}|^2 \}^2 \cdot \\ &\quad \cdot \sum_{\substack{\ell, m=0 \\ \ell \neq m}}^{K-1} \exp \left(j \frac{2\pi}{K} \cdot (m - \ell) \right) \\ &= E \{ |\Delta \mathbf{S}|^2 \}^2 \cdot K(K-1) \end{aligned}$$



$$\begin{aligned}
& - E \{ |\Delta S|^2 \}^2 \cdot \left(\sum_{\ell, m=0}^{K-1} \exp \left(j \frac{2\pi}{K} \cdot (m - \ell) \right) \right. \\
& \quad \left. - \sum_{\ell=0}^{K-1} \exp \left(j \frac{2\pi}{K} \cdot 0 \right) \right) \\
& = E \{ |\Delta S|^2 \}^2 \cdot K(K-1) \\
& - E \{ |\Delta S|^2 \}^2 \cdot \underbrace{\left(\sum_{\ell, m=0}^{K-1} \exp \left(j \frac{2\pi}{K} \cdot (m - \ell) \right) \right)}_{=0} - K \\
\mu & = K^2 \cdot E \{ |\Delta S|^2 \}^2 \quad (10)
\end{aligned}$$

Equation 10 shows that the average value of the determinant will increase with K , the number of OFDM carriers and the average separation between signals $E \{ |\Delta S|^2 \}^2$. Therefore densely packed signal constellations would be expected to have a low coding gain advantage.

3.3 Channel Coding

From the calculations in Section 3.1 and 3.2 we can see that unfortunately $\min_{\mathbf{s} \neq \mathbf{t}} \text{rank}[\mathbf{A}(\mathbf{W}, \mathbf{\tilde{W}})] = 1$, i.e. for the considered example, multipath propagation or antenna diversity does not provide full diversity gain, if it is used on its own. The calculations in Section 3.1 imply that we have to guarantee a minimum Hamming Weight of 2 for $\Delta \mathbf{S}$. This can be fulfilled by using a single parity check code in the real and imaginary part of the considered 4-QAM alphabet.

4 CONCLUSIONS

In this paper we have presented a system model which uses multiple antennas to transmit cyclic delayed copies of an OFDM symbol. We term this technique Cyclic Delay Diversity (CDD). Furthermore, we extend the transmission model to include multipath components, and derive a condition on the antenna delays which insure that multipath and antenna delay components have equivalent descriptions from the receiver point of view.

In Section 2.3 the Hermitian difference matrix of a CDD codeword pair with multipath components is derived. The rank and non-zero eigen product of this matrix determines the diversity and coding gain of the code as presented in [5].

Section 3 gives a method for finding a bound for non-full spacial diversity codeword pairs for a specific scenario. In general, it was shown that by increasing the number of OFDM carriers, or the cardinality of the signal space, the probability of having full rank codeword pairs approaches 1. The determinant between codeword pairs was shown to increase with the number of OFDM carriers, and the average separation among signal constellation points. The determinant is closely related to the product of non-zero eigen values which determine the space-time coding gain. Though a general proof of the diversity properties of CDD systems has not yet been completed, the similarity with traditional temporal delay di-

versity schemes which achieve full spatial diversity ([5]) motivates further theoretical research in this direction.

The requirements for full rank derived in our example can easily be met with a single parity check code. This motivates further investigations of combining cyclic delay diversity with coding.

The error rate performance improvement of CDD on practical systems has already been shown through simulations ([8]) using a standard OFDM receiver. A link to Space-Time codes may lead to better receiver designs which exploit the inherent CDD diversity, or to modifications of CDD which improve error rate performance, complexity, or information rate.

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JURAT ATTACHMENT

A notary public or other officer completing this certificate verifies only the identity of the individual who signed the document to which this certificate is attached, and not the truthfulness, accuracy, or validity of that document.

STATE OF Texas }

COUNTY OF Tarrant }

The foregoing instrument was subscribed and sworn before me this date of
06/26/2024, by Nathaniel Frank-White

This notarial act was an online notarization.



Notary's Signature Brittany Rene Copeland

Registration No.: 133645929

Commission Expiration Date: March 15, 2026

