

Shigley's Mechanical Engineering Design

Ninth Edition



Richard G. Budynas and J. Keith Nisbett



SHIGLEY'S MECHANICAL ENGINEERING DESIGN, NINTH EDITION

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The true strain corresponding to 15 percent cold work is obtained from Eq. (2-17). Thus

$$\varepsilon_i = \ln \frac{A_0}{A_i} = \ln 1.176 = 0.1625$$

Since $\varepsilon_i < \varepsilon_u$, Eqs. (2-18) and (2-19) apply. Therefore,

Answer
$$S'_y = \sigma_0 \varepsilon_i^m = 90(0.1625)^{0.25} = 57.1 \text{ kpsi}$$

Answer
$$S'_u = \frac{S_u}{1 - W} = \frac{49.5}{1 - 0.15} = 58.2 \text{ kpsi}$$

2-4 Hardness

The resistance of a material to penetration by a pointed tool is called *hardness*. Though there are many hardness-measuring systems, we shall consider here only the two in greatest use.

Rockwell hardness tests are described by ASTM standard hardness method E-18 and measurements are quickly and easily made, they have good reproducibility, and the test machine for them is easy to use. In fact, the hardness number is read directly from a dial. Rockwell hardness scales are designated as *A*, *B*, *C*, . . . , etc. The indenters are described as a diamond, a $\frac{1}{16}$ -in-diameter ball, and a diamond for scales *A*, *B*, and *C*, respectively, where the load applied is either 60, 100, or 150 kg. Thus the Rockwell *B* scale, designated R_B , uses a 100-kg load and a No. 2 indenter, which is a $\frac{1}{16}$ -in-diameter ball. The Rockwell *C* scale R_C uses a diamond cone, which is the No. 1 indenter, and a load of 150 kg. Hardness numbers so obtained are relative. Therefore a hardness $R_C = 50$ has meaning only in relation to another hardness number using the same scale.

The *Brinell hardness* is another test in very general use. In testing, the indenting tool through which force is applied is a ball and the hardness number H_B is found as a number equal to the applied load divided by the spherical surface area of the indentation. Thus the units of H_B are the same as those of stress, though they are seldom used. Brinell hardness testing takes more time, since H_B must be computed from the test data. The primary advantage of both methods is that they are nondestructive in most cases. Both are empirically and directly related to the ultimate strength of the material tested. This means that the strength of parts could, if desired, be tested part by part during manufacture.

Hardness testing provides a convenient and nondestructive means of estimating the strength properties of materials. The Brinell hardness test is particularly well known for this estimation, since for many materials the relationship between the minimum ultimate strength and the Brinell hardness number is roughly linear. The constant of proportionality varies between classes of materials, and is also dependent on the load used to determine the hardness. There is a wide scatter in the data, but for rough approximations for *steels*, the relationship is generally accepted as

$$S_u = \begin{cases} 0.5H_B & \text{kpsi} \\ 3.4H_B & \text{MPa} \end{cases} \quad (2-21)$$

Similar relationships for *cast iron* can be derived from data supplied by Krause.⁵ The minimum strength, as defined by the ASTM, is found from these data to be

$$S_u = \begin{cases} 0.23H_B - 12.5 \text{ kpsi} \\ 1.58H_B - 86 \text{ MPa} \end{cases} \quad (2-22)$$

Walton⁶ shows a chart from which the SAE minimum strength can be obtained, which is more conservative than the values obtained from Eq. (2-22).

EXAMPLE 2-2

It is necessary to ensure that a certain part supplied by a foundry always meets or exceeds ASTM No. 20 specifications for cast iron (see Table A-24). What hardness should be specified?

Solution From Eq. (2-22), with $(S_u)_{\min} = 20 \text{ kpsi}$, we have

Answer
$$H_B = \frac{S_u + 12.5}{0.23} = \frac{20 + 12.5}{0.23} = 141$$

If the foundry can control the hardness within 20 points, routinely, then specify $145 < H_B < 165$. This imposes no hardship on the foundry and assures the designer that ASTM grade 20 will always be supplied at a predictable cost.

2-5 Impact Properties

An external force applied to a structure or part is called an *impact load* if the time of application is less than one-third the lowest natural period of vibration of the part or structure. Otherwise it is called simply a *static load*.

The *Charpy* (commonly used) and *Izod* (rarely used) *notched-bar tests* utilize bars of specified geometries to determine brittleness and impact strength. These tests are helpful in comparing several materials and in the determination of low-temperature brittleness. In both tests the specimen is struck by a pendulum released from a fixed height, and the energy absorbed by the specimen, called the *impact value*, can be computed from the height of swing after fracture, but is read from a dial that essentially “computes” the result.

The effect of temperature on impact values is shown in Fig. 2-7 for a material showing a ductile-brittle transition. Not all materials show this transition. Notice the narrow region of critical temperatures where the impact value increases very rapidly. In the low-temperature region the fracture appears as a brittle, shattering type, whereas the appearance is a tough, tearing type above the critical-temperature region. The critical temperature seems to be dependent on both the material and the geometry of the notch. For this reason designers should not rely too heavily on the results of notched-bar tests.

The average strain rate used in obtaining the stress-strain diagram is about $0.001 \text{ in}/(\text{in} \cdot \text{s})$ or less. When the strain rate is increased, as it is under impact conditions,

⁵D. E. Krause, “Gray Iron—A Unique Engineering Material,” ASTM Special Publication 455, 1969, pp. 3–29, as reported in Charles F. Walton (ed.), *Iron Castings Handbook*, Iron Founders Society, Inc., Cleveland, 1971, pp. 204, 205.

⁶Ibid.

where r is the radius to the stress element under consideration, ρ is the mass density, and ω is the angular velocity of the ring in radians per second. For a rotating disk, use $r_i = 0$ in these equations.

3-16 Press and Shrink Fits

When two cylindrical parts are assembled by shrinking or press fitting one part upon another, a contact pressure is created between the two parts. The stresses resulting from this pressure may easily be determined with the equations of the preceding sections.

Figure 3-33 shows two cylindrical members that have been assembled with a shrink fit. Prior to assembly, the outer radius of the inner member was larger than the inner radius of the outer member by the *radial interference* δ . After assembly, an interference contact pressure p develops between the members at the nominal radius R , causing radial stresses $\sigma_r = -p$ in each member at the contacting surfaces. This pressure is given by¹²

$$p = \frac{\delta}{R \left[\frac{1}{E_o} \left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o \right) + \frac{1}{E_i} \left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i \right) \right]} \quad (3-56)$$

where the subscripts o and i on the material properties correspond to the outer and inner members, respectively. If the two members are of the same material with $E_o = E_i = E$, $\nu_o = \nu_i$, the relation simplifies to

$$p = \frac{E\delta}{2R^3} \left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{r_o^2 - r_i^2} \right] \quad (3-57)$$

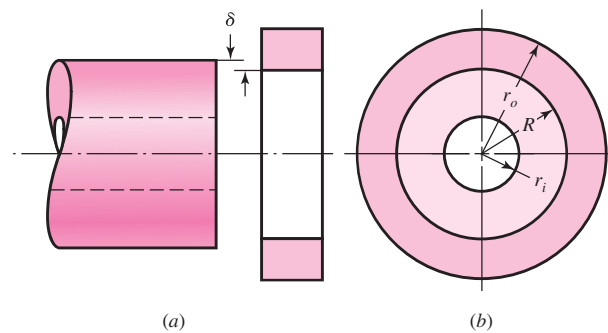
For Eqs. (3-56) or (3-57), diameters can be used in place of R , r_i , and r_o , provided δ is the diametral interference (twice the radial interference).

With p , Eq. (3-49) can be used to determine the radial and tangential stresses in each member. For the inner member, $p_o = p$ and $p_i = 0$. For the outer member, $p_o = 0$ and $p_i = p$. For example, the magnitudes of the tangential stresses at the transition radius R are maximum for both members. For the inner member

$$(\sigma_t)_i \Big|_{r=R} = -p \frac{R^2 + r_i^2}{R^2 - r_i^2} \quad (3-58)$$

Figure 3-33

Notation for press and shrink fits. (a) Unassembled parts; (b) after assembly.



¹²Ibid, pp. 348–354.

and, for the outer member

$$(\sigma_t)_o \Big|_{r=R} = p \frac{r_o^2 + R^2}{r_o^2 - R^2} \quad (3-59)$$

Assumptions

It is assumed that both members have the same length. In the case of a hub that has been press-fitted onto a shaft, this assumption would not be true, and there would be an increased pressure at each end of the hub. It is customary to allow for this condition by employing a stress-concentration factor. The value of this factor depends upon the contact pressure and the design of the female member, but its theoretical value is seldom greater than 2.

3-17 Temperature Effects

When the temperature of an unrestrained body is uniformly increased, the body expands, and the normal strain is

$$\epsilon_x = \epsilon_y = \epsilon_z = \alpha(\Delta T) \quad (3-60)$$

where α is the coefficient of thermal expansion and ΔT is the temperature change, in degrees. In this action the body experiences a simple volume increase with the components of shear strain all zero.

If a straight bar is restrained at the ends so as to prevent lengthwise expansion and then is subjected to a uniform increase in temperature, a compressive stress will develop because of the axial constraint. The stress is

$$\sigma = -\epsilon E = -\alpha(\Delta T)E \quad (3-61)$$

In a similar manner, if a uniform flat plate is restrained at the edges and also subjected to a uniform temperature rise, the compressive stress developed is given by the equation

$$\sigma = -\frac{\alpha(\Delta T)E}{1 - \nu} \quad (3-62)$$

The stresses expressed by Eqs. (3-61) and (3-62) are called *thermal stresses*. They arise because of a temperature change in a clamped or restrained member. Such stresses, for example, occur during welding, since parts to be welded must be clamped before welding. Table 3-3 lists approximate values of the coefficients of thermal expansion.

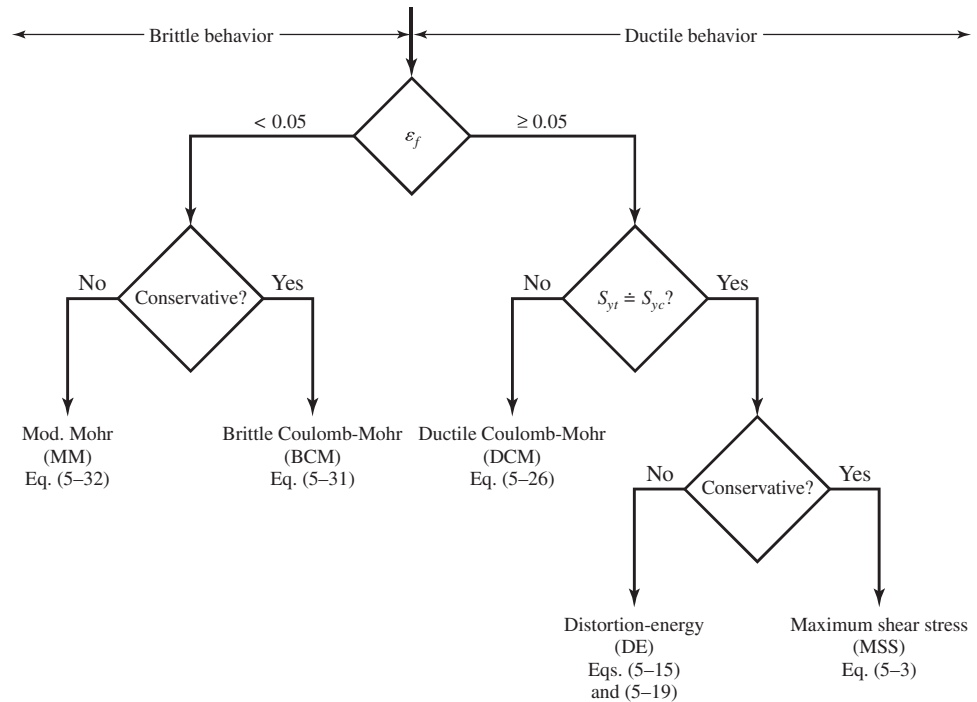
Table 3-3

Coefficients of Thermal Expansion (Linear Mean Coefficients for the Temperature Range 0–100°C)

Material	Celsius Scale (°C ⁻¹)	Fahrenheit Scale (°F ⁻¹)
Aluminum	23.9(10) ⁻⁶	13.3(10) ⁻⁶
Brass, cast	18.7(10) ⁻⁶	10.4(10) ⁻⁶
Carbon steel	10.8(10) ⁻⁶	6.0(10) ⁻⁶
Cast iron	10.6(10) ⁻⁶	5.9(10) ⁻⁶
Magnesium	25.2(10) ⁻⁶	14.0(10) ⁻⁶
Nickel steel	13.1(10) ⁻⁶	7.3(10) ⁻⁶
Stainless steel	17.3(10) ⁻⁶	9.6(10) ⁻⁶
Tungsten	4.3(10) ⁻⁶	2.4(10) ⁻⁶

Figure 5-21

Failure theory selection flowchart.



namely, Coulomb Mohr, or modified Mohr. Figure 5-21 provides a summary flowchart for the selection of an effective procedure for analyzing or predicting failures from static loading for brittle or ductile behavior. Note that the maximum-normal-stress theory is excluded from Fig. 5-21 as the other theories better represent the experimental data.

5-12 Introduction to Fracture Mechanics

The idea that cracks exist in parts even before service begins, and that cracks can grow during service, has led to the descriptive phrase “damage-tolerant design.” The focus of this philosophy is on crack growth until it becomes critical, and the part is removed from service. The analysis tool is *linear elastic fracture mechanics* (LEFM). Inspection and maintenance are essential in the decision to retire parts before cracks reach catastrophic size. Where human safety is concerned, periodic inspections for cracks are mandated by codes and government ordinance.

We shall now briefly examine some of the basic ideas and vocabulary needed for the potential of the approach to be appreciated. The intent here is to make the reader aware of the dangers associated with the sudden brittle fracture of so-called ductile materials. The topic is much too extensive to include in detail here and the reader is urged to read further on this complex subject.⁹

⁹References on brittle fracture include:

H. Tada, P. C. Paris, and G. R. Irwin, *The Stress Analysis of Cracks Handbook*, 3rd ed., ASME Press, New York, 2000.

D. Broek, *Elementary Engineering Fracture Mechanics*, 4th ed., Martinus Nijhoff, London, 1985.

D. Broek, *The Practical Use of Fracture Mechanics*, Kluwer Academic Pub., London, 1988.

David K. Felbeck and Anthony G. Atkins, *Strength and Fracture of Engineering Solids*, 2nd ed., Prentice-Hall, Englewood Cliffs, N.J., 1995.

Kåre Hellan, *Introduction to Fracture Mechanics*, McGraw-Hill, New York, 1984.

The use of elastic stress-concentration factors provides an indication of the average load required on a part for the onset of plastic deformation, or yielding; these factors are also useful for analysis of the loads on a part that will cause fatigue fracture. However, stress-concentration factors are limited to structures for which all dimensions are precisely known, particularly the radius of curvature in regions of high stress concentration. When there exists a crack, flaw, inclusion, or defect of unknown small radius in a part, the elastic stress-concentration factor approaches infinity as the root radius approaches zero, thus rendering the stress-concentration factor approach useless. Furthermore, even if the radius of curvature of the flaw tip is known, the high local stresses there will lead to local plastic deformation surrounded by a region of elastic deformation. Elastic stress-concentration factors are no longer valid for this situation, so analysis from the point of view of stress-concentration factors does not lead to criteria useful for design when very sharp cracks are present.

By combining analysis of the gross elastic changes in a structure or part that occur as a sharp brittle crack grows with measurements of the energy required to produce new fracture surfaces, it is possible to calculate the average stress (if no crack were present) that will cause crack growth in a part. Such calculation is possible only for parts with cracks for which the elastic analysis has been completed, and for materials that crack in a relatively brittle manner and for which the fracture energy has been carefully measured. The term *relatively brittle* is rigorously defined in the test procedures,¹⁰ but it means, roughly, *fracture without yielding occurring throughout the fractured cross section*.

Thus glass, hard steels, strong aluminum alloys, and even low-carbon steel below the ductile-to-brittle transition temperature can be analyzed in this way. Fortunately, ductile materials blunt sharp cracks, as we have previously discovered, so that fracture occurs at average stresses of the order of the yield strength, and the designer is prepared for this condition. The middle ground of materials that lie between “relatively brittle” and “ductile” is now being actively analyzed, but exact design criteria for these materials are not yet available.

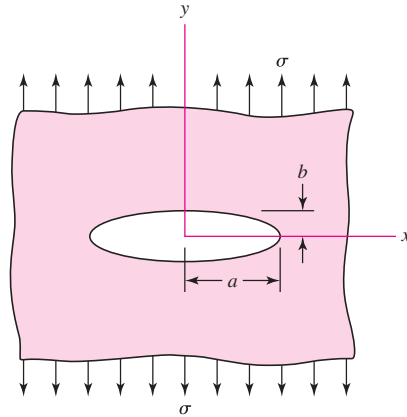
Quasi-Static Fracture

Many of us have had the experience of observing brittle fracture, whether it is the breaking of a cast-iron specimen in a tensile test or the twist fracture of a piece of blackboard chalk. It happens so rapidly that we think of it as instantaneous, that is, the cross section simply parting. Fewer of us have skated on a frozen pond in the spring, with no one near us, heard a cracking noise, and stopped to observe. The noise is due to cracking. The cracks move slowly enough for us to see them run. The phenomenon is not instantaneous, since some time is necessary to feed the crack energy from the stress field to the crack for propagation. Quantifying these things is important to understanding the phenomenon “in the small.” In the large, a static crack may be stable and will not propagate. Some level of loading can render the crack unstable, and the crack propagates to fracture.

The foundation of fracture mechanics was first established by Griffith in 1921 using the stress field calculations for an elliptical flaw in a plate developed by Inglis in 1913. For the infinite plate loaded by an applied uniaxial stress σ in Fig. 5–22, the maximum stress occurs at $(\pm a, 0)$ and is given by

$$(\sigma_y)_{\max} = \left(1 + 2\frac{a}{b}\right)\sigma \quad (5-33)$$

¹⁰BS 5447:1977 and ASTM E399-78.

| **Figure 5-22**

Note that when $a = b$, the ellipse becomes a circle and Eq. (5-33) gives a stress-concentration factor of 3. This agrees with the well-known result for an infinite plate with a circular hole (see Table A-15-1). For a fine crack, $b/a \rightarrow 0$, and Eq. (5-34) predicts that $(\sigma_y)_{\max} \rightarrow \infty$. However, on a microscopic level, an infinitely sharp crack is a hypothetical abstraction that is physically impossible, and when plastic deformation occurs, the stress will be finite at the crack tip.

Griffith showed that the crack growth occurs when the energy release rate from applied loading is greater than the rate of energy for crack growth. Crack growth can be stable or unstable. Unstable crack growth occurs when the *rate* of change of the energy release rate relative to the crack length is equal to or greater than the *rate* of change of the crack growth rate of energy. Griffith's experimental work was restricted to brittle materials, namely glass, which pretty much confirmed his surface energy hypothesis. However, for ductile materials, the energy needed to perform plastic work at the crack tip is found to be much more crucial than surface energy.

Crack Modes and the Stress Intensity Factor

Three distinct modes of crack propagation exist, as shown in Fig. 5-23. A tensile stress field gives rise to mode I, the *opening crack propagation mode*, as shown in Fig. 5-23a. This mode is the most common in practice. Mode II is the *sliding mode*, is due to in-plane shear, and can be seen in Fig. 5-23b. Mode III is the *tearing mode*, which arises from out-of-plane shear, as shown in Fig. 5-23c. Combinations of these modes can also occur. Since mode I is the most common and important mode, the remainder of this section will consider only this mode.

| **Figure 5-23**

Crack propagation modes.

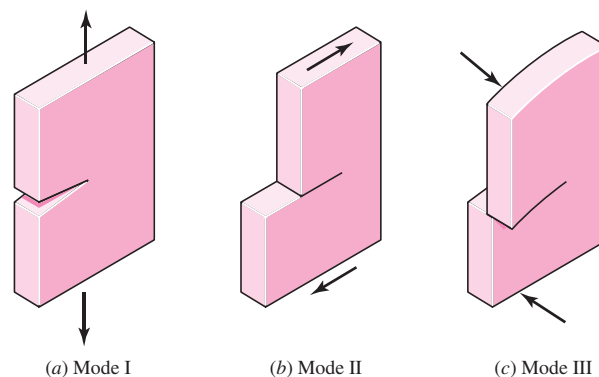
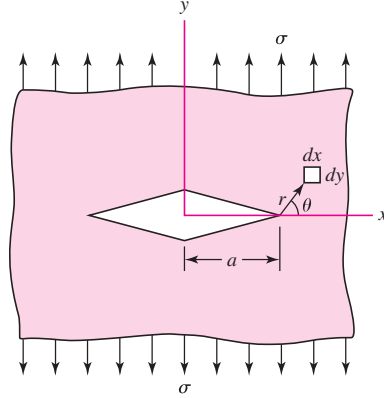


Figure 5–24

Mode I crack model.



Consider a mode I crack of length $2a$ in the infinite plate of Fig. 5–24. By using complex stress functions, it has been shown that the stress field on a $dx \, dy$ element in the vicinity of the crack tip is given by

$$\sigma_x = \sigma \sqrt{\frac{a}{2r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \quad (5-34a)$$

$$\sigma_y = \sigma \sqrt{\frac{a}{2r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \quad (5-34b)$$

$$\tau_{xy} = \sigma \sqrt{\frac{a}{2r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \quad (5-34c)$$

$$\sigma_z = \begin{cases} 0 & \text{(for plane stress)} \\ \nu(\sigma_x + \sigma_y) & \text{(for plane strain)} \end{cases} \quad (5-34d)$$

The stress σ_y near the tip, with $\theta = 0$, is

$$\sigma_y|_{\theta=0} = \sigma \sqrt{\frac{a}{2r}} \quad (a)$$

As with the elliptical crack, we see that $\sigma_y|_{\theta=0} \rightarrow \infty$ as $r \rightarrow 0$, and again the concept of an infinite stress concentration at the crack tip is inappropriate. The quantity $\sigma_y|_{\theta=0} \sqrt{2r} = \sigma \sqrt{a}$, however, does remain constant as $r \rightarrow 0$. It is common practice to define a factor K called the *stress intensity factor* given by

$$K = \sigma \sqrt{\pi a} \quad (b)$$

where the units are $\text{MPa}\sqrt{\text{m}}$ or $\text{kpsi}\sqrt{\text{in.}}$. Since we are dealing with a mode I crack, Eq. (b) is written as

$$K_I = \sigma \sqrt{\pi a} \quad (5-35)$$

The stress intensity factor is *not* to be confused with the static stress-concentration factors K_t and K_{ts} defined in Secs. 3–13 and 5–2.

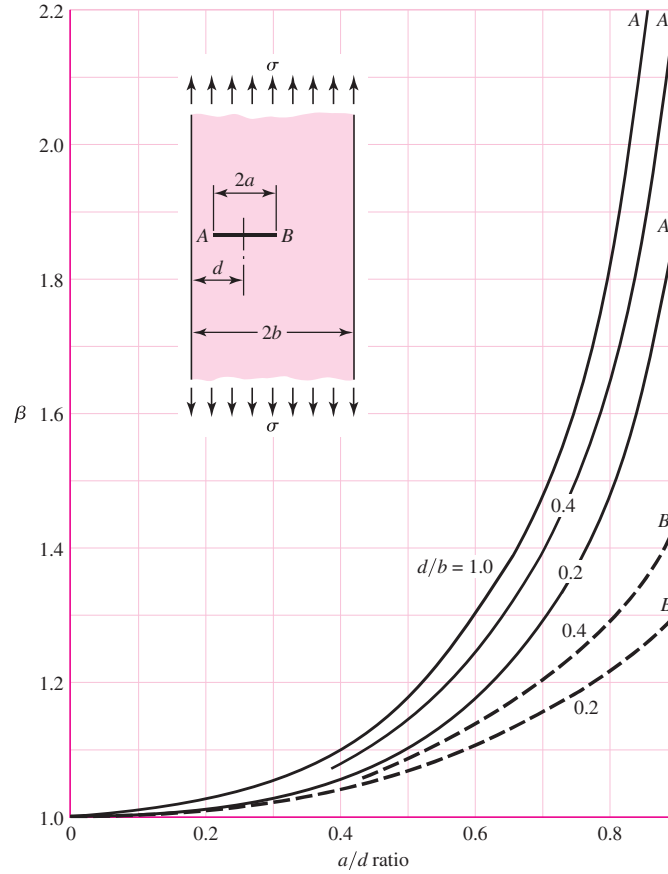
Thus Eqs. (5–34) can be rewritten as

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \quad (5-36a)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \quad (5-36b)$$

Figure 5–25

Off-center crack in a plate in longitudinal tension; solid curves are for the crack tip at A; dashed curves are for the tip at B.



$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \quad (5-36c)$$

$$\sigma_z = \begin{cases} 0 & \text{(for plane stress)} \\ \nu(\sigma_x + \sigma_y) & \text{(for plane strain)} \end{cases} \quad (5-36d)$$

The stress intensity factor is a function of geometry, size and shape of the crack, and the type of loading. For various load and geometric configurations, Eq. (5–35) can be written as

$$K_I = \beta \sigma \sqrt{\pi a} \quad (5-37)$$

where β is the *stress intensity modification factor*. Tables for β are available in the literature for basic configurations.¹¹ Figures 5–25 to 5–30 present a few examples of β for mode I crack propagation.

¹¹See, for example:

H. Tada, P. C. Paris, and G. R. Irwin, *The Stress Analysis of Cracks Handbook*, 3rd ed., ASME Press, New York, 2000.

G. C. Sib, *Handbook of Stress Intensity Factors for Researchers and Engineers*, Institute of Fracture and Solid Mechanics, Lehigh University, Bethlehem, Pa., 1973.

Y. Murakami, ed., *Stress Intensity Factors Handbook*, Pergamon Press, Oxford, U.K., 1987.

W. D. Pilkey, *Formulas for Stress, Strain, and Structural Matrices*, 2nd ed. John Wiley & Sons, New York, 2005.

Figure 5-26

Plate loaded in longitudinal tension with a crack at the edge; for the solid curve there are no constraints to bending; the dashed curve was obtained with bending constraints added.

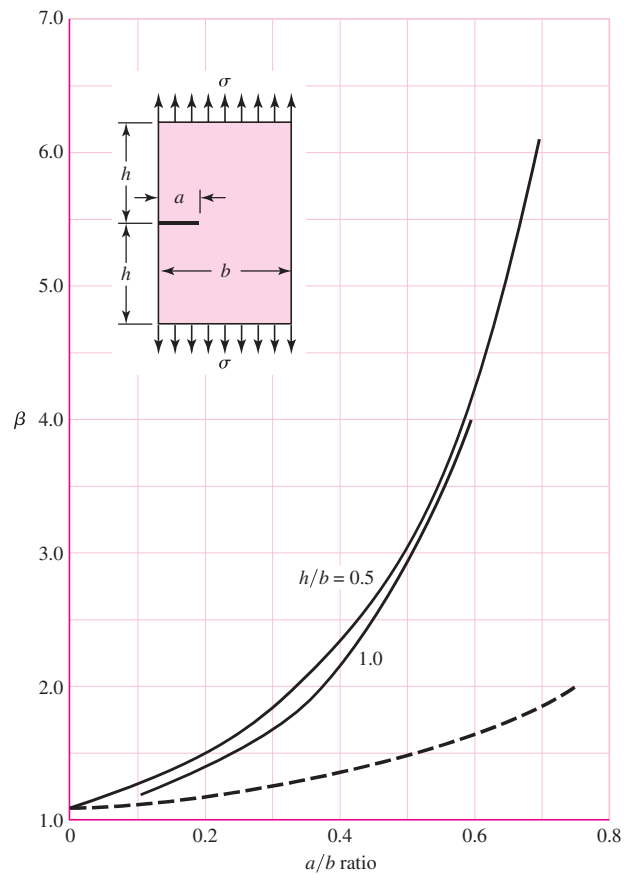


Figure 5-27

Beams of rectangular cross section having an edge crack.

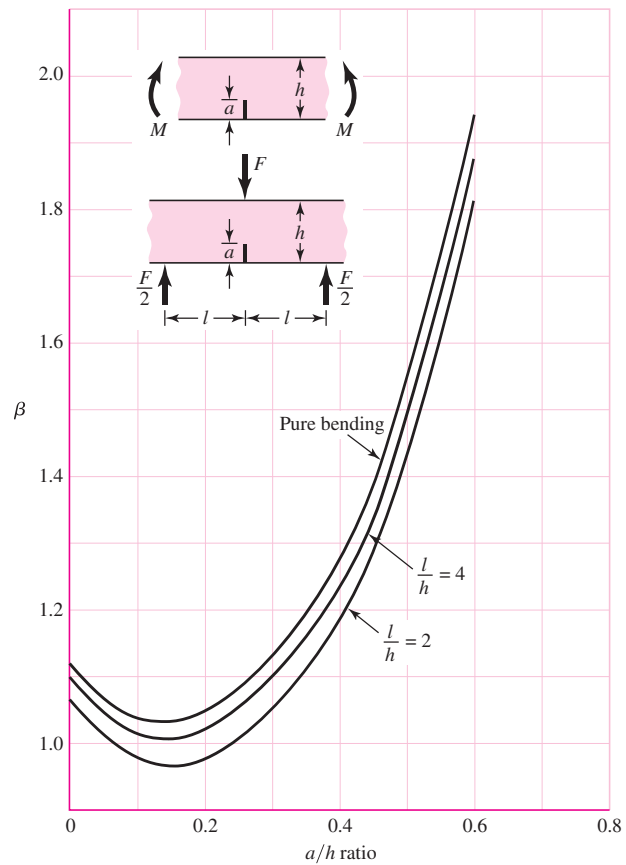
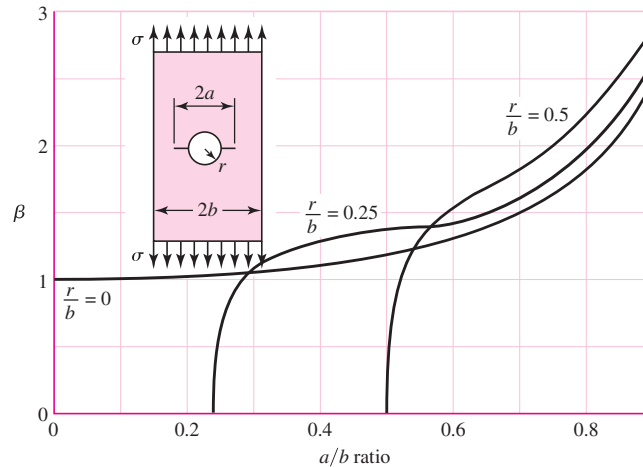
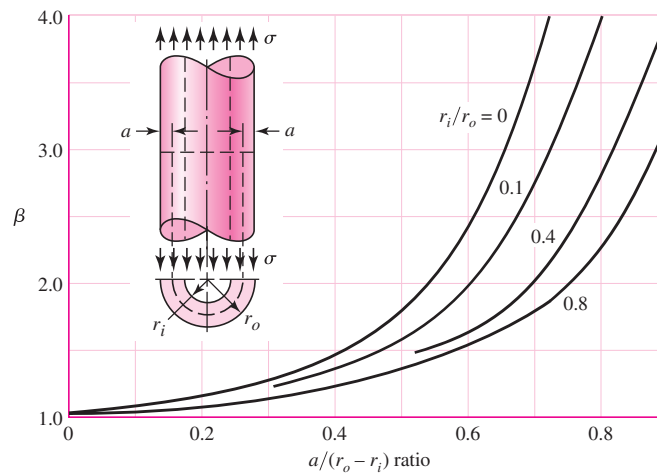


Figure 5-28

Plate in tension containing a circular hole with two cracks.

**Figure 5-29**

A cylinder loading in axial tension having a radial crack of depth a extending completely around the circumference of the cylinder.



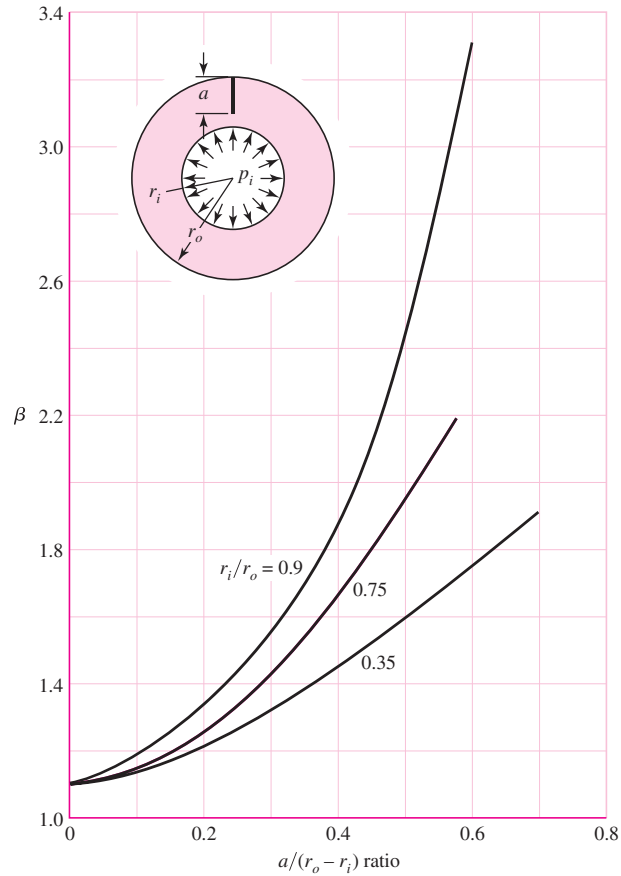
Fracture Toughness

When the magnitude of the mode I stress intensity factor reaches a critical value, K_{Ic} , crack propagation initiates. The *critical stress intensity factor* K_{Ic} is a material property that depends on the material, crack mode, processing of the material, temperature, loading rate, and the state of stress at the crack site (such as plane stress versus plane strain). The critical stress intensity factor K_{Ic} is also called the *fracture toughness* of the material. The fracture toughness for plane strain is normally lower than that for plane stress. For this reason, the term K_{Ic} is typically defined as the *mode I, plane strain fracture toughness*. Fracture toughness K_{Ic} for engineering metals lies in the range $20 \leq K_{Ic} \leq 200 \text{ MPa} \cdot \sqrt{\text{m}}$; for engineering polymers and ceramics, $1 \leq K_{Ic} \leq 5 \text{ MPa} \cdot \sqrt{\text{m}}$. For a 4340 steel, where the yield strength due to heat treatment ranges from 800 to 1600 MPa, K_{Ic} decreases from 190 to 40 $\text{MPa} \cdot \sqrt{\text{m}}$.

Table 5-1 gives some approximate typical room-temperature values of K_{Ic} for several materials. As previously noted, the fracture toughness depends on many factors and the table is meant only to convey some typical magnitudes of K_{Ic} . For an actual application, it is recommended that the material specified for the application be certified using standard test procedures [see the American Society for Testing and Materials (ASTM) standard E399].

Figure 5-30

Cylinder subjected to internal pressure p , having a radial crack in the longitudinal direction of depth a . Use Eq. (4-51) for the tangential stress at $r = r_0$.



One of the first problems facing the designer is that of deciding whether the conditions exist, or not, for a brittle fracture. Low-temperature operation, that is, operation below room temperature, is a key indicator that brittle fracture is a possible failure mode. Tables of transition temperatures for various materials have not been published, possibly because of the wide variation in values, even for a single material. Thus, in many situations, laboratory testing may give the only clue to the possibility of a brittle fracture. Another key indicator of the possibility of fracture is the ratio of the yield strength to the ultimate strength. A high ratio of S_y/S_u indicates there is only a small

Table 5-1

Values of K_{Ic} for Some Engineering Materials at Room Temperature

Material	K_{Ic} , $\text{MPa}\sqrt{\text{m}}$	S_y , MPa
Aluminum		
2024	26	455
7075	24	495
7178	33	490
Titanium		
Ti-6AL-4V	115	910
Ti-6AL-4V	55	1035
Steel		
4340	99	860
4340	60	1515
52100	14	2070

ability to absorb energy in the plastic region and hence there is a likelihood of brittle fracture.

The strength-to-stress ratio K_{Ic}/K_I can be used as a factor of safety as

$$n = \frac{K_{Ic}}{K_I} \quad (5-38)$$

EXAMPLE 5-6

A steel ship deck plate is 30 mm thick and 12 m wide. It is loaded with a nominal uniaxial tensile stress of 50 MPa. It is operated below its ductile-to-brittle transition temperature with K_{Ic} equal to 28.3 MPa. If a 65-mm-long central transverse crack is present, estimate the tensile stress at which catastrophic failure will occur. Compare this stress with the yield strength of 240 MPa for this steel.

Solution

For Fig. 5-25, with $d = b$, $2a = 65$ mm and $2b = 12$ m, so that $d/b = 1$ and $a/d = 65/12(10^3) = 0.00542$. Since a/d is so small, $\beta = 1$, so that

$$K_I = \sigma \sqrt{\pi a} = 50 \sqrt{\pi (32.5 \times 10^{-3})} = 16.0 \text{ MPa } \sqrt{\text{m}}$$

From Eq. (5-38),

$$n = \frac{K_{Ic}}{K_I} = \frac{28.3}{16.0} = 1.77$$

The stress at which catastrophic failure occurs is

Answer

$$\sigma_c = \frac{K_{Ic}}{K_I} \sigma = \frac{28.3}{16.0} (50) = 88.4 \text{ MPa}$$

The yield strength is 240 MPa, and catastrophic failure occurs at $88.4/240 = 0.37$, or at 37 percent of yield. The factor of safety in this circumstance is $K_{Ic}/K_I = 28.3/16 = 1.77$ and *not* $240/50 = 4.8$.

EXAMPLE 5-7

A plate of width 1.4 m and length 2.8 m is required to support a tensile force in the 2.8-m direction of 4.0 MN. Inspection procedures will detect only through-thickness edge cracks larger than 2.7 mm. The two Ti-6AL-4V alloys in Table 5-1 are being considered for this application, for which the safety factor must be 1.3 and minimum weight is important. Which alloy should be used?

Solution

(a) We elect first to estimate the thickness required to resist yielding. Since $\sigma = P/wt$, we have $t = P/w\sigma$. For the weaker alloy, we have, from Table 5-1, $S_y = 910$ MPa. Thus,

$$\sigma_{\text{all}} = \frac{S_y}{n} = \frac{910}{1.3} = 700 \text{ MPa}$$

Thus

$$t = \frac{P}{w\sigma_{\text{all}}} = \frac{4.0(10)^3}{1.4(700)} = 4.08 \text{ mm or greater}$$

For the stronger alloy, we have, from Table 5–1,

$$\sigma_{\text{all}} = \frac{1035}{1.3} = 796 \text{ MPa}$$

and so the thickness is

Answer
$$t = \frac{P}{w\sigma_{\text{all}}} = \frac{4.0(10)^3}{1.4(796)} = 3.59 \text{ mm or greater}$$

(b) Now let us find the thickness required to prevent crack growth. Using Fig. 5–26, we have

$$\frac{h}{b} = \frac{2.8/2}{1.4} = 1 \quad \frac{a}{b} = \frac{2.7}{1.4(10^3)} = 0.00193$$

Corresponding to these ratios we find from Fig. 5–26 that $\beta \doteq 1.1$, and $K_I = 1.1\sigma\sqrt{\pi a}$.

$$n = \frac{K_{Ic}}{K_I} = \frac{115\sqrt{10^3}}{1.1\sigma\sqrt{\pi a}}, \quad \sigma = \frac{K_{Ic}}{1.1n\sqrt{\pi a}}$$

From Table 5–1, $K_{Ic} = 115 \text{ MPa}\sqrt{\text{m}}$ for the weaker of the two alloys. Solving for σ with $n = 1$ gives the fracture stress

$$\sigma = \frac{115}{1.1\sqrt{\pi(2.7 \times 10^{-3})}} = 1135 \text{ MPa}$$

which is greater than the yield strength of 910 MPa, and so yield strength is the basis for the geometry decision. For the stronger alloy $S_y = 1035 \text{ MPa}$, with $n = 1$ the fracture stress is

$$\sigma = \frac{K_{Ic}}{nK_I} = \frac{55}{1(1.1)\sqrt{\pi(2.7 \times 10^{-3})}} = 542.9 \text{ MPa}$$

which is less than the yield strength of 1035 MPa. The thickness t is

$$t = \frac{P}{w\sigma_{\text{all}}} = \frac{4.0(10^3)}{1.4(542.9/1.3)} = 6.84 \text{ mm or greater}$$

This example shows that the fracture toughness K_{Ic} limits the geometry when the stronger alloy is used, and so a thickness of 6.84 mm or larger is required. When the weaker alloy is used the geometry is limited by the yield strength, giving a thickness of only 4.08 mm or greater. Thus the weaker alloy leads to a thinner and lighter weight choice since the failure modes differ.

5–13 Stochastic Analysis¹²

Reliability is the probability that machine systems and components will perform their intended function satisfactorily without failure. Up to this point, discussion in this chapter has been restricted to deterministic relations between static stress, strength, and the design factor. Stress and strength, however, are statistical in nature and very much tied to the reliability of the stressed component. Consider the probability density functions

¹²Review Chap. 20 before reading this section.

Because of this necessity for testing, it would really be unnecessary for us to proceed any further in the study of fatigue failure except for one important reason: the desire to know why fatigue failures occur so that the most effective method or methods can be used to improve fatigue strength. Thus our primary purpose in studying fatigue is to understand why failures occur so that we can guard against them in an optimum manner. For this reason, the analytical design approaches presented in this book, or in any other book, for that matter, do not yield absolutely precise results. The results should be taken as a guide, as something that indicates what is important and what is not important in designing against fatigue failure.

As stated earlier, the stress-life method is the least accurate approach especially for low-cycle applications. However, it is the most traditional method, with much published data available. It is the easiest to implement for a wide range of design applications and represents high-cycle applications adequately. For these reasons the stress-life method will be emphasized in subsequent sections of this chapter. However, care should be exercised when applying the method for low-cycle applications, as the method does not account for the true stress-strain behavior when localized yielding occurs.

6-5 The Strain-Life Method

The best approach yet advanced to explain the nature of fatigue failure is called by some the *strain-life* method. The approach can be used to estimate fatigue strengths, but when it is so used it is necessary to compound several idealizations, and so some uncertainties will exist in the results. For this reason, the method is presented here only because of its value in explaining the nature of fatigue.

A fatigue failure almost always begins at a local discontinuity such as a notch, crack, or other area of stress concentration. When the stress at the discontinuity exceeds the elastic limit, plastic strain occurs. If a fatigue fracture is to occur, there must exist cyclic plastic strains. Thus we shall need to investigate the behavior of materials subject to cyclic deformation.

In 1910, Bairstow verified by experiment Bauschinger's theory that the elastic limits of iron and steel can be changed, either up or down, by the cyclic variations of stress.² In general, the elastic limits of annealed steels are likely to increase when subjected to cycles of stress reversals, while cold-drawn steels exhibit a decreasing elastic limit.

R. W. Landgraf has investigated the low-cycle fatigue behavior of a large number of very high-strength steels, and during his research he made many cyclic stress-strain plots.³ Figure 6-12 has been constructed to show the general appearance of these plots for the first few cycles of controlled cyclic strain. In this case the strength decreases with stress repetitions, as evidenced by the fact that the reversals occur at ever-smaller stress levels. As previously noted, other materials may be strengthened, instead, by cyclic stress reversals.

The SAE Fatigue Design and Evaluation Steering Committee released a report in 1975 in which the life in reversals to failure is related to the strain amplitude $\Delta\epsilon/2$.⁴

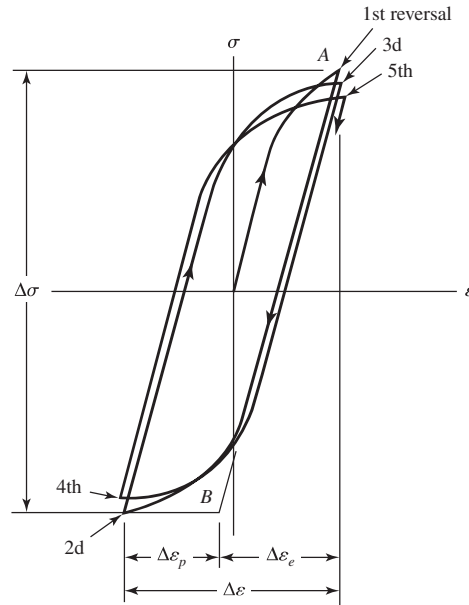
²L. Bairstow, "The Elastic Limits of Iron and Steel under Cyclic Variations of Stress," *Philosophical Transactions*, Series A, vol. 210, Royal Society of London, 1910, pp. 35-55.

³R. W. Landgraf, *Cyclic Deformation and Fatigue Behavior of Hardened Steels*, Report no. 320, Department of Theoretical and Applied Mechanics, University of Illinois, Urbana, 1968, pp. 84-90.

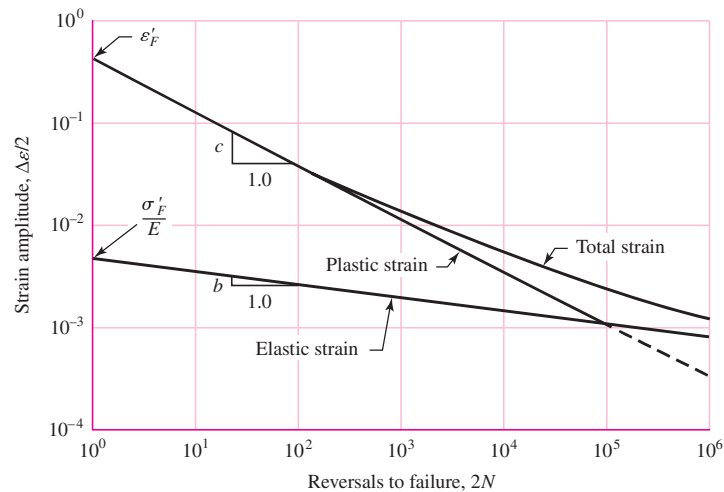
⁴*Technical Report on Fatigue Properties*, SAE J1099, 1975.

Figure 6-12

True stress–true strain hysteresis loops showing the first five stress reversals of a cyclic-softening material. The graph is slightly exaggerated for clarity. Note that the slope of the line AB is the modulus of elasticity E . The stress range is $\Delta\sigma$, $\Delta\epsilon_p$ is the plastic-strain range, and $\Delta\epsilon_e$ is the elastic strain range. The total-strain range is $\Delta\epsilon = \Delta\epsilon_p + \Delta\epsilon_e$.

**Figure 6-13**

A log-log plot showing how the fatigue life is related to the true-strain amplitude for hot-rolled SAE 1020 steel. (Reprinted with permission from SAE J1099_200208 © 2002 SAE International.)



The report contains a plot of this relationship for SAE 1020 hot-rolled steel; the graph has been reproduced as Fig. 6-13. To explain the graph, we first define the following terms:

- **Fatigue ductility coefficient** ϵ'_F is the true strain corresponding to fracture in one reversal (point A in Fig. 6-12). The plastic-strain line begins at this point in Fig. 6-13.
- **Fatigue strength coefficient** σ'_F is the true stress corresponding to fracture in one reversal (point A in Fig. 6-12). Note in Fig. 6-13 that the elastic-strain line begins at σ'_F/E .
- **Fatigue ductility exponent** c is the slope of the plastic-strain line in Fig. 6-13 and is the power to which the life $2N$ must be raised to be proportional to the true plastic-strain amplitude. If the number of stress reversals is $2N$, then N is the number of cycles.

- *Fatigue strength exponent b* is the slope of the elastic-strain line, and is the power to which the life $2N$ must be raised to be proportional to the true-stress amplitude.

Now, from Fig. 6–12, we see that the total strain is the sum of the elastic and plastic components. Therefore the total strain amplitude is half the total strain range

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} \quad (a)$$

The equation of the plastic-strain line in Fig. 6–13 is

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_F (2N)^c \quad (6-1)$$

The equation of the elastic strain line is

$$\frac{\Delta\varepsilon_e}{2} = \frac{\sigma'_F}{E} (2N)^b \quad (6-2)$$

Therefore, from Eq. (a), we have for the total-strain amplitude

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_F}{E} (2N)^b + \varepsilon'_F (2N)^c \quad (6-3)$$

which is the Manson-Coffin relationship between fatigue life and total strain.⁵ Some values of the coefficients and exponents are listed in Table A–23. Many more are included in the SAE J1099 report.⁶

Though Eq. (6–3) is a perfectly legitimate equation for obtaining the fatigue life of a part when the strain and other cyclic characteristics are given, it appears to be of little use to the designer. The question of how to determine the total strain at the bottom of a notch or discontinuity has not been answered. There are no tables or charts of strain-concentration factors in the literature. It is possible that strain-concentration factors will become available in research literature very soon because of the increase in the use of finite-element analysis. Moreover, finite element analysis can of itself approximate the strains that will occur at all points in the subject structure.⁷

6–6 The Linear-Elastic Fracture Mechanics Method

The first phase of fatigue cracking is designated as stage I fatigue. Crystal slip that extends through several contiguous grains, inclusions, and surface imperfections is presumed to play a role. Since most of this is invisible to the observer, we just say that stage I involves several grains. The second phase, that of crack extension, is called stage II fatigue. The advance of the crack (that is, new crack area is created) does produce evidence that can be observed on micrographs from an electron microscope. The growth of

⁵J. F. Tavernelli and L. F. Coffin, Jr., “Experimental Support for Generalized Equation Predicting Low Cycle Fatigue,” and S. S. Manson, discussion, *Trans. ASME, J. Basic Eng.*, vol. 84, no. 4, pp. 533–537.

⁶See also, Landgraf, *Ibid.*

⁷For further discussion of the strain-life method see N. E. Dowling, *Mechanical Behavior of Materials*, 2nd ed., Prentice-Hall, Englewood Cliffs, N.J., 1999, Chap. 14.

the crack is orderly. Final fracture occurs during stage III fatigue, although fatigue is not involved. When the crack is sufficiently long that $K_I = K_{Ic}$ for the stress amplitude involved, where K_{Ic} is the critical stress intensity for the undamaged metal, then there is sudden, catastrophic failure of the remaining cross section in tensile overload (see Sec. 5–12). Stage III fatigue is associated with rapid acceleration of crack growth then fracture.

Crack Growth

Fatigue cracks nucleate and grow when stresses vary and there is some tension in each stress cycle. Consider the stress to be fluctuating between the limits of $\sigma_{\min}$ and $\sigma_{\max}$, where the stress range is defined as $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$. From Eq. (5–37) the stress intensity is given by $K_I = \beta\sigma\sqrt{\pi a}$. Thus, for $\Delta\sigma$, the stress intensity range per cycle is

$$\Delta K_I = \beta(\sigma_{\max} - \sigma_{\min})\sqrt{\pi a} = \beta\Delta\sigma\sqrt{\pi a} \quad (6-4)$$

To develop fatigue strength data, a number of specimens of the same material are tested at various levels of $\Delta\sigma$. Cracks nucleate at or very near a free surface or large discontinuity. Assuming an initial crack length of a_i , crack growth as a function of the number of stress cycles N will depend on $\Delta\sigma$, that is, ΔK_I . For ΔK_I below some threshold value $(\Delta K_I)_{th}$ a crack will not grow. Figure 6–14 represents the crack length a as a function of N for three stress levels $(\Delta\sigma)_3 > (\Delta\sigma)_2 > (\Delta\sigma)_1$, where $(\Delta K_I)_3 > (\Delta K_I)_2 > (\Delta K_I)_1$ for a given crack size. Notice the effect of the higher stress range in Fig. 6–14 in the production of longer cracks at a particular cycle count.

When the rate of crack growth per cycle, da/dN in Fig. 6–14, is plotted as shown in Fig. 6–15, the data from all three stress range levels superpose to give a sigmoidal curve. The three stages of crack development are observable, and the stage II data are linear on log-log coordinates, within the domain of linear elastic fracture mechanics (LEFM) validity. A group of similar curves can be generated by changing the stress ratio $R = \sigma_{\min}/\sigma_{\max}$ of the experiment.

Here we present a simplified procedure for estimating the remaining life of a cyclically stressed part after discovery of a crack. This requires the assumption that plane strain

Figure 6–14

The increase in crack length a from an initial length of a_i as a function of cycle count for three stress ranges, $(\Delta\sigma)_3 > (\Delta\sigma)_2 > (\Delta\sigma)_1$.

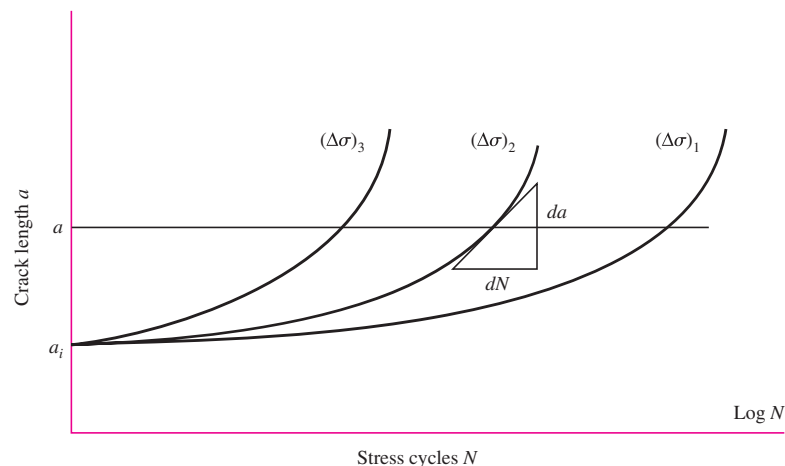
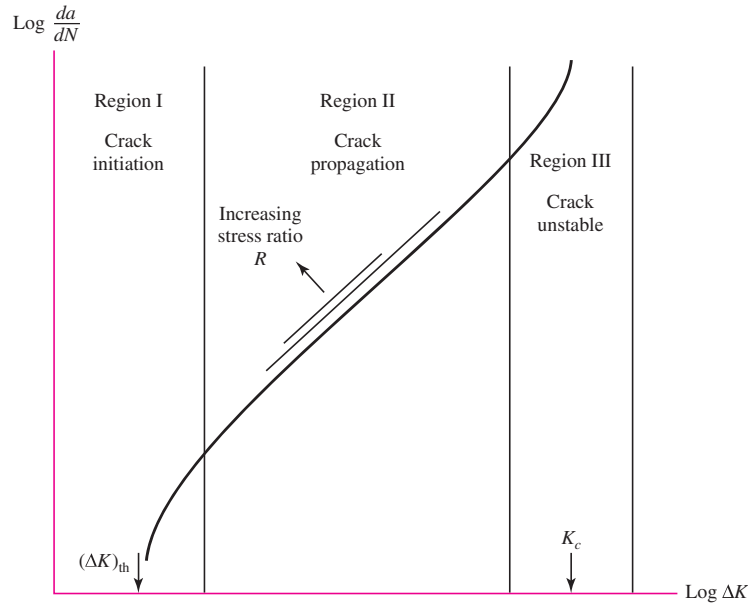


Figure 6-15

When da/dN is measured in Fig. 6-14 and plotted on log-log coordinates, the data for different stress ranges *superpose*, giving rise to a sigmoid curve as shown. $(\Delta K_I)_{th}$ is the threshold value of ΔK_I , below which a crack does not grow. From threshold to rupture an aluminum alloy will spend 85–90 percent of life in region I, 5–8 percent in region II, and 1–2 percent in region III.

**Table 6-1**

Conservative Values of Factor C and Exponent m in Eq. (6-5) for Various Forms of Steel ($R = \sigma_{\max}/\sigma_{\min} \doteq 0$)

Material	$C, \frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$	$C, \frac{\text{in/cycle}}{(\text{kpsi}\sqrt{\text{in}})^m}$	m
Ferritic-pearlitic steels	$6.89(10^{-12})$	$3.60(10^{-10})$	3.00
Martensitic steels	$1.36(10^{-10})$	$6.60(10^{-9})$	2.25
Austenitic stainless steels	$5.61(10^{-12})$	$3.00(10^{-10})$	3.25

From J. M. Barsom and S. T. Rolfe, *Fatigue and Fracture Control in Structures*, 2nd ed., Prentice Hall, Upper Saddle River, NJ, 1987, pp. 288–291, Copyright ASTM International. Reprinted with permission.

conditions prevail.⁸ Assuming a crack is discovered early in stage II, the crack growth in region II of Fig. 6-15 can be approximated by the *Paris equation*, which is of the form

$$\frac{da}{dN} = C(\Delta K_I)^m \quad (6-5)$$

where C and m are empirical material constants and ΔK_I is given by Eq. (6-4). Representative, but conservative, values of C and m for various classes of steels are listed in Table 6-1. Substituting Eq. (6-4) and integrating gives

$$\int_0^{N_f} dN = N_f = \frac{1}{C} \int_{a_i}^{a_f} \frac{da}{(\beta \Delta \sigma \sqrt{\pi a})^m} \quad (6-6)$$

Here a_i is the initial crack length, a_f is the final crack length corresponding to failure, and N_f is the estimated number of cycles to produce a failure *after* the initial crack is formed. Note that β may vary in the integration variable (e.g., see Figs. 5-25 to 5-30).

⁸Recommended references are: Dowling, op. cit.; J. A. Collins, *Failure of Materials in Mechanical Design*, John Wiley & Sons, New York, 1981; H. O. Fuchs and R. I. Stephens, *Metal Fatigue in Engineering*, John Wiley & Sons, New York, 1980; and Harold S. Reemsnyder, "Constant Amplitude Fatigue Life Assessment Models," *SAE Trans.* 820688, vol. 91, Nov. 1983.

If this should happen, then Reemsnyder⁹ suggests the use of numerical integration employing the algorithm

$$\begin{aligned}\delta a_j &= C(\Delta K_I)_j^m (\delta N)_j \\ a_{j+1} &= a_j + \delta a_j \\ N_{j+1} &= N_j + \delta N_j \\ N_f &= \sum \delta N_j\end{aligned}\tag{6-7}$$

Here δa_j and δN_j are increments of the crack length and the number of cycles. The procedure is to select a value of δN_j , using a_i determine β and compute ΔK_I , determine δa_j , and then find the next value of a . Repeat the procedure until $a = a_f$.

The following example is highly simplified with β constant in order to give some understanding of the procedure. Normally, one uses fatigue crack growth computer programs such as NASA/FLAGRO 2.0 with more comprehensive theoretical models to solve these problems.

EXAMPLE 6-1

The bar shown in Fig. 6-16 is subjected to a repeated moment $0 \leq M \leq 1200 \text{ lbf} \cdot \text{in.}$ The bar is AISI 4430 steel with $S_{ut} = 185 \text{ kpsi}$, $S_y = 170 \text{ kpsi}$, and $K_{Ic} = 73 \text{ kpsi}\sqrt{\text{in.}}$ Material tests on various specimens of this material with identical heat treatment indicate worst-case constants of $C = 3.8(10^{-11})(\text{in/cycle})/(\text{kpsi}\sqrt{\text{in.}})^m$ and $m = 3.0$. As shown, a nick of size 0.004 in has been discovered on the bottom of the bar. Estimate the number of cycles of life remaining.

Solution

The stress range $\Delta\sigma$ is always computed by using the nominal (uncracked) area. Thus

$$\frac{I}{c} = \frac{bh^2}{6} = \frac{0.25(0.5)^2}{6} = 0.01042 \text{ in}^3$$

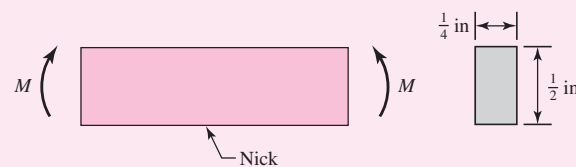
Therefore, before the crack initiates, the stress range is

$$\Delta\sigma = \frac{\Delta M}{I/c} = \frac{1200}{0.01042} = 115.2(10^3) \text{ psi} = 115.2 \text{ kpsi}$$

which is below the yield strength. As the crack grows, it will eventually become long enough such that the bar will completely yield or undergo a brittle fracture. For the ratio of S_y/S_{ut} it is highly unlikely that the bar will reach complete yield. For brittle fracture, designate the crack length as a_f . If $\beta = 1$, then from Eq. (5-37) with $K_I = K_{Ic}$, we approximate a_f as

$$a_f = \frac{1}{\pi} \left(\frac{K_{Ic}}{\beta\sigma_{\max}} \right)^2 \doteq \frac{1}{\pi} \left(\frac{73}{115.2} \right)^2 = 0.1278 \text{ in}$$

| Figure 6-16



⁹Op. cit.

6

Fatigue Failure Resulting from Variable Loading

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Collins said it well: “In spite of all the problems cited, the Palmgren linear damage rule is frequently used because of its simplicity and the experimental fact that other more complex damage theories do not always yield a significant improvement in failure prediction reliability.”²⁶

6-16 Surface Fatigue Strength

The surface fatigue mechanism is not definitively understood. The contact-affected zone, in the absence of surface shearing tractions, entertains compressive principal stresses. Rotary fatigue has its cracks grown at or near the surface in the presence of tensile stresses that are associated with crack propagation, to catastrophic failure. There are shear stresses in the zone, which are largest just below the surface. Cracks seem to grow from this stratum until small pieces of material are expelled, leaving pits on the surface. Because engineers had to design durable machinery before the surface fatigue phenomenon was understood in detail, they had taken the posture of conducting tests, observing pits on the surface, and declaring failure at an arbitrary projected area of hole, and they related this to the Hertzian contact pressure. This compressive stress did not produce the failure directly, but whatever the failure mechanism, whatever the stress type that was instrumental in the failure, the contact stress was an *index* to its magnitude.

Buckingham²⁷ conducted a number of tests relating the fatigue at 10^8 cycles to endurance strength (Hertzian contact pressure). While there is evidence of an endurance limit at about $3(10^7)$ cycles for cast materials, hardened steel rollers showed no endurance limit up to $4(10^8)$ cycles. Subsequent testing on hard steel shows no endurance limit. Hardened steel exhibits such high fatigue strengths that its use in resisting surface fatigue is widespread.

Our studies thus far have dealt with the failure of a machine element by yielding, by fracture, and by fatigue. The endurance limit obtained by the rotating-beam test is frequently called the *flexural endurance limit*, because it is a test of a rotating beam. In this section we shall study a property of *mating materials* called the *surface endurance shear*. The design engineer must frequently solve problems in which two machine elements mate with one another by rolling, sliding, or a combination of rolling and sliding contact. Obvious examples of such combinations are the mating teeth of a pair of gears, a cam and follower, a wheel and rail, and a chain and sprocket. A knowledge of the surface strength of materials is necessary if the designer is to create machines having a long and satisfactory life.

When two surfaces roll or roll and slide against one another with sufficient force, a pitting failure will occur after a certain number of cycles of operation. Authorities are not in complete agreement on the exact mechanism of the pitting; although the subject is quite complicated, they do agree that the Hertz stresses, the number of cycles, the surface finish, the hardness, the degree of lubrication, and the temperature all influence the strength. In Sec. 3-19 it was learned that, when two surfaces are pressed together, a maximum shear stress is developed slightly below the contacting surface. It is postulated by some authorities that a surface fatigue failure is initiated by this maximum shear stress and then is propagated rapidly to the surface. The lubricant then enters the crack that is formed and, under pressure, eventually wedges the chip loose.

²⁶J. A. Collins, *Failure of Materials in Mechanical Design*, John Wiley & Sons, New York, 1981, p. 243.

²⁷Earle Buckingham, *Analytical Mechanics of Gears*, McGraw-Hill, New York, 1949.

To determine the surface fatigue strength of mating materials, Buckingham designed a simple machine for testing a pair of contacting rolling surfaces in connection with his investigation of the wear of gear teeth. Buckingham and, later, Talbourdet gathered large numbers of data from many tests so that considerable design information is now available. To make the results useful for designers, Buckingham defined a *load-stress factor*, also called a *wear factor*, which is derived from the Hertz equations. Equations (3-73) and (3-74), p. 124, for contacting cylinders are found to be

$$b = \sqrt{\frac{2F(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{\pi l(1/d_1 + 1/d_2)}} \quad (6-59)$$

$$p_{\max} = \frac{2F}{\pi bl} \quad (6-60)$$

where b = half width of rectangular contact area

F = contact force

l = length of cylinders

ν = Poisson's ratio

E = modulus of elasticity

d = cylinder diameter

It is more convenient to use the cylinder radius, so let $2r = d$. If we then designate the length of the cylinders as w (for width of gear, bearing, cam, etc.) instead of l and remove the square root sign, Eq. (6-59) becomes

$$b^2 = \frac{4F}{\pi w} \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/r_1 + 1/r_2} \quad (6-61)$$

We can define a *surface endurance strength* S_C using

$$p_{\max} = \frac{2F}{\pi bw} \quad (6-62)$$

as

$$S_C = \frac{2F}{\pi bw} \quad (6-63)$$

which may also be called *contact strength*, the *contact fatigue strength*, or the *Hertzian endurance strength*. The strength is the contacting pressure which, after a specified number of cycles, will cause failure of the surface. Such failures are often called *wear* because they occur over a very long time. They should not be confused with abrasive wear, however. By squaring Eq. (6-63), substituting b^2 from Eq. (6-61), and rearranging, we obtain

$$\frac{F}{w} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) = \pi S_C^2 \left[\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right] = K_1 \quad (6-64)$$

The left expression consists of parameters a designer may seek to control independently. The central expression consists of material properties that come with the material and condition specification. The third expression is the parameter K_1 , Buckingham's load-stress factor, determined by a test fixture with values F , w , r_1 , r_2 and the number of

cycles associated with the first tangible evidence of fatigue. In gear studies a similar K factor is used:

$$K_g = \frac{K_1}{4} \sin \phi \quad (6-65)$$

where ϕ is the tooth pressure angle, and the term $[(1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2]$ is defined as $1/(\pi C_P^2)$, so that

$$S_C = C_P \sqrt{\frac{F}{w} \left(\frac{1}{r_1} + \frac{1}{r_2} \right)} \quad (6-66)$$

Buckingham and others reported K_1 for 10^8 cycles and nothing else. This gives only one point on the $S_C N$ curve. For cast metals this may be sufficient, but for wrought steels, heat-treated, some idea of the slope is useful in meeting design goals of other than 10^8 cycles.

Experiments show that K_1 versus N , K_g versus N , and S_C versus N data are rectified by log-log transformation. This suggests that

$$K_1 = \alpha_1 N^{\beta_1} \quad K_g = a N^b \quad S_C = \alpha N^\beta$$

The three exponents are given by

$$\beta_1 = \frac{\log(K_1/K_2)}{\log(N_1/N_2)} \quad b = \frac{\log(K_{g1}/K_{g2})}{\log(N_1/N_2)} \quad \beta = \frac{\log(S_{C1}/S_{C2})}{\log(N_1/N_2)} \quad (6-67)$$

Data on induction-hardened steel on steel give $(S_C)_{10^7} = 271$ kpsi and $(S_C)_{10^8} = 239$ kpsi, so β , from Eq. (6-67), is

$$\beta = \frac{\log(271/239)}{\log(10^7/10^8)} = -0.055$$

It may be of interest that the American Gear Manufacturers Association (AGMA) uses $\beta = -0.056$ between $10^4 < N < 10^{10}$ if the designer has no data to the contrary beyond 10^7 cycles.

A longstanding correlation in steels between S_C and H_B at 10^8 cycles is

$$(S_C)_{10^8} = \begin{cases} 0.4H_B - 10 \text{ kpsi} \\ 2.76H_B - 70 \text{ MPa} \end{cases} \quad (6-68)$$

AGMA uses

$$0.99(S_C)_{10^7} = 0.327H_B + 26 \text{ kpsi} \quad (6-69)$$

Equation (6-66) can be used in design to find an allowable surface stress by using a design factor. Since this equation is nonlinear in its stress-load transformation, the designer must decide if loss of function denotes inability to carry the load. If so, then to find the allowable stress, one divides the load F by the design factor n_d :

$$\sigma_C = C_P \sqrt{\frac{F}{w n_d} \left(\frac{1}{r_1} + \frac{1}{r_2} \right)} = \frac{C_P}{\sqrt{n_d}} \sqrt{\frac{F}{w} \left(\frac{1}{r_1} + \frac{1}{r_2} \right)} = \frac{S_C}{\sqrt{n_d}}$$

and $n_d = (S_C/\sigma_C)^2$. If the loss of function is focused on stress, then $n_d = S_C/\sigma_C$. It is recommended that an engineer

- Decide whether loss of function is failure to carry load or stress.
- Define the design factor and factor of safety accordingly.
- Announce what he or she is using and why.
- Be prepared to defend his or her position.

In this way everyone who is party to the communication knows what a design factor (or factor of safety) of 2 means and adjusts, if necessary, the judgmental perspective.

6-17 Stochastic Analysis²⁸

As already demonstrated in this chapter, there are a great many factors to consider in a fatigue analysis, much more so than in a static analysis. So far, each factor has been treated in a deterministic manner, and if not obvious, these factors are subject to variability and control the overall reliability of the results. When reliability is important, then fatigue testing must certainly be undertaken. There is no other way. Consequently, the methods of stochastic analysis presented here and in other sections of this book constitute guidelines that enable the designer to obtain a good understanding of the various issues involved and help in the development of a safe and reliable design.

In this section, key stochastic modifications to the deterministic features and equations described in earlier sections are provided in the same order of presentation.

Endurance Limit

To begin, a method for estimating endurance limits, the *tensile strength correlation method*, is presented. The ratio $\Phi = S'_e / \bar{S}_{ut}$ is called the *fatigue ratio*.²⁹ For ferrous metals, most of which exhibit an endurance limit, the endurance limit is used as a numerator. For materials that do not show an endurance limit, an endurance strength at a specified number of cycles to failure is used and noted. Gough³⁰ reported the stochastic nature of the fatigue ratio Φ for several classes of metals, and this is shown in Fig. 6-36. The first item to note is that the coefficient of variation is of the order 0.10 to 0.15, and the distribution varies for classes of metals. The second item to note is that Gough's data include materials of no interest to engineers. In the absence of testing, engineers use the correlation that Φ represents to estimate the endurance limit S'_e from the mean ultimate strength $\bar{S}_{ut}$.

Gough's data are for ensembles of metals, some chosen for metallurgical interest, and include materials that are not commonly selected for machine parts. Mischke³¹ analyzed data for 133 common steels and treatments in varying diameters in rotating bending,³² and the result was

$$\Phi = 0.445d^{-0.107} \text{LN}(1, 0.138)$$

where d is the specimen diameter in inches and $\text{LN}(1, 0.138)$ is a unit lognormal variate with a mean of 1 and a standard deviation (and coefficient of variation) of 0.138. For the standard R. R. Moore specimen,

$$\Phi_{0.30} = 0.445(0.30)^{-0.107} \text{LN}(1, 0.138) = 0.506 \text{LN}(1, 0.138)$$

²⁸Review Chap. 20 before reading this section.

²⁹From this point, since we will be dealing with statistical distributions in terms of means, standard deviations, etc. A key quantity, the ultimate strength, will here be presented by its mean value, $\bar{S}_{ut}$. This means that certain terms that were defined earlier in terms of the minimum value of S_{ut} will change slightly.

³⁰In J. A. Pope, *Metal Fatigue*, Chapman and Hall, London, 1959.

³¹Charles R. Mischke, "Prediction of Stochastic Endurance Strength," *Trans. ASME, Journal of Vibration, Acoustics, Stress, and Reliability in Design*, vol. 109, no. 1, January 1987, pp. 113–122.

³²Data from H. J. Grover, S. A. Gordon, and L. R. Jackson, *Fatigue of Metals and Structures*, Bureau of Naval Weapons, Document NAVWEPS 00-2500435, 1960.