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History of Finite Element Method

List of Articles

by Dominique Madier - Monday, 15 November 2021

The exact date of the emergence of the Finite Element Method is quite difficult, if not impossible, to determine precisely. Its origins can be traced to the need to solve practical problems in Aerospace, Mechanical and Civil Engineering.

The Pioneers

Usage of the finite element method began in the early 1940s, with the work of the Russian-Canadian structural engineer, ***Alexander Hrennikoff***, and German American mathematician, ***Richard Courant***.

Although very different, the work of these two men shared a common characteristic— the discretization of a continuous domain into several elements. ***Hrennikoff's*** proposals were based on lattice discretization, while ***Courant's*** approach was based on triangular elements.

The finite element method was developed in parallel in Europe and North America, by a number of independent actors:

- ***Ray Clough*** and colleagues at UC Berkeley (California, USA)
- ***John Argyris*** and colleagues at the University of Stuttgart (Germany)
- ***Olgierd Zienkiewicz*** and colleagues at the University of Swansea (UK)
- ***Bruce Irons*** and colleagues at the University of Swansea (UK)
- ***Richard Gallagher*** and colleagues at Cornell University (New York, USA)

A Very Popular Domain

The Finite Element Method (FEM) and Finite Element Analysis (FEA) have been generalized to a wide variety of engineering disciplines such as structural analysis, electromagnetism, heat transfer, and fluid dynamics, for the numerical modeling of physical systems.

The FEA domain is very popular in the universities due to its wide range of applications. Consequently, the number of papers published on the subject has increased exponentially over time.

The table below is showing the number of papers about FEM published from 1956 to present.

Year	1956	1961	1966	1971	1976	1986	Today
Papers	1	10	134	844	7,000	20,000	> 10^5

FEM Timeline

Early 1940s

Alexander Hrennikoff and ***Richard Courant*** establish the mathematical foundations of the present form of the Finite Element Method for solving elasticity and structural analysis problems in civil and aeronautical engineering.

1941 – 1943

Hrennikoff presents the discretization of a continuous domain into several elements using the lattice analogy to model membrane and plate bending parts of a structure. However, this analogy could not be applied to nonrectangular areas.

Hrennikoff, A. "Solution of Problems in Elasticity by the Framework Method", Journal of Applied Mechanics, 8, pp. 169-175, 1941.

1943

Richard Courant presents his approach based on constant strain triangular elements using the Ritz method. This approach was not pursued at that time due to the lack of high-speed computers.

Courant, C., "Variational Methods for Solution of Equilibrium and Vibration", Bull. Am. Math Soc., Vol. 49, 1943, pp. 1-43.

1952 – 1953

Ray Clough worked with ***John Turner*** at the Boeing Structural Dynamics Unit on the calculation of bending and torsional flexibility influence coefficients on low aspect wings. Static experimental results had been obtained for a swept-back box wing structure, but they did not agree with the results produced by a structural analysis model using the lattice analogy. They conceived the procedure for the development of the constant strain triangle.

In addition to the constant strain triangular membrane element, a rectangular membrane element, based on equilibrium stress patterns, was presented which avoided shear locking. The node equilibrium equations were formed by the direct stiffness method.

Their work was presented at Boeing in 1954 and a paper was published in 1956 (see reference below).

1954 – 1955

John Argyris, from the University of Stuttgart, unified many different approximate methods for the solution of both continuous and one-dimensional frame structures. By using matrix transformation methods, he clearly shown that most structural analysis methods could be categorized as either a force or a displacement method.

Argyris, J, "Energy Theorems and Structural Analysis", Aircraft Engineering, 1954 and 1955.

1956

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Stiffness and Deflection Analysis of Complex Structures

M. J. TURNER,* R. W. CLOUGH H. C. MARTIN,† and L. J. TOPP**

M. J. Turner, R. W. Clough, H. C. Martin, and ***L. J. Topp*** published *"Stiffness and Deflection Analysis of Complex Structures,"* in the *Journal of the Aeronautical Sciences,* one of the first articles concerning the application of the finite element method.

1957 – 1960

Clough initiated a new structural analysis research program at UC Berkeley and started a new graduate course entitled Matrix Analysis of Structures. An IBM 701 digital computer, with 4k of 16-bit memory, was installed in the College of Engineering. The maximum number of equations that could be solved by this computer was approximately 40. ***Clough*** developed a matrix algebra program to permit students learning programming in order to solve finite element problems. So, by using submatrix techniques and tape storage it was possible to solve larger systems.

Under the direction of ***Clough***, graduate student ***Ari Adini*** used the matrix algebra program to solve several plane stress problems using triangular elements. Even a simple structure required a significant amount of time. Hence, only coarse mesh solutions were possible. This approach was used to develop all examples for the paper *"The Finite Element Method in Plane Stress Analysis"* presented by ***Clough*** in 1960 (see reference below).

1958

Ed Wilson, a graduate student, who shared an office with ***Adini*** at UC Berkeley, was not satisfied with the large amount of work required to solve finite element problems by using the matrix algebra program. Under the direction of ***Clough***, he started the development of an automated finite element program based on the rectangular plane stress finite element developed at Boeing. ***Wilson*** created a limited capacity, semiautomated program which was based on the force method.

1959 – 1960

The IBM 704 computer is deployed at UC Berkeley (it had 32K of 32-bit memory and a floating-point arithmetic unit which was approximately 100 times faster than the IBM 701).

Under the direction of ***Clough***, ***Wilson*** wrote a two-dimensional frame analysis program with a nonlinear, moment-curvature relationship defined by the classical Ramberg-Osgood equation. The loads were applied incrementally and produced a pushover type of analysis. The incremental load approach was general and could be used for all types of finite element systems. ***Wilson*** presented his work in 1960 at the 2nd *American Society of Civil Engineers* (ASCE) *Conference on Electronic Computation.*

1960

Ray Clough coins the term "Finite Element Method" in his paper, *"The Finite Element Method in Plane Stress Analysis"* presented at the 2nd *American Society of Civil Engineers* (ASCE) *Conference on Electronic Computation*, Pittsburgh, September 1960.

1960

Clough and ***Wilson*** developed a fully automated finite element program in which the basic input was the location of the nodes and the node numbers where the triangular plane stress elements were attached.

It was then possible for structural engineers, without a strong mathematical background in continuum mechanics, to solve practical plane stress structures of arbitrary geometry built by using several different materials. The work required to prepare the computer input data was simple and could be completed in a few hours for most structures.

1960s

Many different research activities are pursued at ***UC Berkeley.*** FEA is the analysis tool that complements all such analytical and experimental research activities and contributes to the development of the finite element method:

- The US Department of Defense studies the cost and ability to reinforce buildings and underground structures to withstand nuclear blasts.
- A very significant program on earthquake engineering research, including the construction of the world's largest shaking table, is initiated by ***Profs. Jack Bouwkamp, Ray Clough, Joseph Penzien,*** and ***Harry Seed.***
- The US Federal Government and California Department of Transportation rapidly expand the state freeway system and sponsor research at Berkeley on the behavior of bridges and overpass structures, led by ***Profs. Alex Scordelis*** and ***Carl Monismith.***
- The US human spaceflight program becomes a national priority, and ***Profs. Karl Pister, Joseph Penzien, Egor Popov, Jerry Sackman, Bob Taylor,*** and ***Edward Wilson*** are very active conducting related research.
- Offshore deep-water oil drilling and construction of the Alaska pipeline require new technology for steel structures, which is developed by ***Profs. Egor Popov, Jack Bouwkamp,*** and ***Graham Powell.***
- Construction of nuclear reactors and cooling towers requires the development of new analytical methods and materials. ***Profs. Egor Popov, Alex Scordelis,*** and ***T.Y. Lin*** are consultants in the design and construction of many important long-span shell structures.

1960 – 1962

Adini continued his finite element research at UC Berkeley by using the matrix algebra program to solve plate bending problems using rectangular finite elements and demonstrated that this class of structures could be modeled accurately by the method. He demonstrated that plate bending problems could also be solved by the finite element method. ***Adini*** solved several simple shell structures using the matrix algebra approach and additional commands to form membrane and bending stiffness matrices for rectangular elements.

Adini, A. and Clough, R. W., "Analysis of Plate Bending by the Finite Element Method", NSF Report, Grant G7337, 1960.

Adini, A., "Analysis of Shell Structures By the Finite Element Method", University of California, Berkeley, Ph.D. Dissertation, 1961.

Late 1960s

James Tocher, a Ph.D. student working under the direction of ***Clough*** at UC Berkeley, started a search for a practical triangular plate-bending element. With the help of a former student of ***Clough***, ***T. K. Hsieh***, they developed the first triangular plate-bending elements. The resulting plate bending element was implemented and tested by ***Tocher*** while working at Boeing. The element produced excellent results and was named the HCT element.

Clough, R. W. and J. L. Tocher, "Finite Element Stiffness Matrices For the Analysis of Plate Bending", Proc. Matrix Methods in Structural Analysis, Wright-Patterson Air Force Base, Ohio, October 26-28, 1965.

1964 – 1965

Dr. Olgierd Zienkiewicz, who was installed as a Professor at the University of Wales in Swansea (UK), asked ***Clough*** and many other leading specialists on the development of new methods of analysis to take part in a conference on Stress Analysis at the University. These lectures were compiled in a book entitled *"Stress Analysis"*.

Stress Analysis – Recent Developments in Numerical and Experimental Methods, John Wiley & Sons Ltd. 1965.

1965

Conference on *Matrix Methods in Structural Analysis* at the *Wright-Patterson Air Force Base (USA)* in October. It brought together the major structural analysis research groups from many areas of the world. The presented works used two and three-dimensional elements to solve problems in continuum mechanics.

A session chaired by ***Professor Richard Gallagher*** was devoted to the Finite Element Method. ***John Argyris*** presented many applications on the analysis of solids, plates and shells. In addition, he presented the six-node triangular plane element formulated in a natural area coordinate system and a ten-node solid tetrahedral element formulated in a natural volume coordinate system.

1965

NASA issues a request for a proposal for the development of a structural analysis software.

The result is the **NA**sa **STR**uctural **AN**alysis application (**NASTRAN**), which implements available FEA technology to solve structural problems.

1965

The terminology ***"Finite Element Method"*** was accepted as replacement terminology for the ***"Direct Stiffness Method"***.

1967

The book *"The Finite Element Method"* is published by ***Olgierd Zienkiewicz, Robert Taylor,*** and ***Jianzhong Zhu***. It remains, to this day, the standard reference text on the theoretical basis of the method.

1968

Bruce Irons and ***Olgierd Zienkiewicz***, from the University of Wales in Swansea (UK), presented the isoparametric formulation of finite element stiffness matrices. This work had a significant impact on the finite element research.

1968

Computer Sciences Corporation (CSC) releases the FEA program **NASTRAN** (**NA**sa **STR**uctural **AN**alysis) to NASA.

1969

Kenneth Kavanagh, under the direction of ***Clough***, used the eight-node solid element for structural analysis of 3D problems.

1969

Wilson and ***Taylor*** solve the problem of shear locking which occurred with the use of the four-node plane element and the eight-node solid element. They successfully solved the problem by using reduced integration and incompatible displacement modes.

1969

MacNeal-Schwendler Corporation (MSC, now MSC Software) initiates the first commercially available version of **NASTRAN**, dubbing it MSC/NASTRAN (now **MSC NASTRAN**), which would become known as the first generation of FEA software.

MacNeal-Schwendler Corporation was formed in 1963 by Dr. Richard H. MacNeal and Robert Schwendler.

1970

John Swanson releases the first version of his **AN**alysis **SYS**tems (**ANSYS**) FEA software.

1972

"Lectures on Mathematical Foundations of the Finite Element Method," the first mathematical proofs on the properties of the finite element method, are published by ***Ivo Babuska*** and ***A. Aziz.*** Until then, the method had been implemented but not mathematically proven.

1978

Hibbitt, Karlsson, and ***Sorensen*** release the first version of the FEA software **ABAQUS**.

1980s

Graphical and computational developments accelerate quickly.

1990s

Low-cost, powerful personal computer (PC) workstations emerge, and FEA is adopted by small- and medium-sized industries.

1991

Implementation of model hierarchies in FEA software is successfully completed, meaning that for the first time in FEA history, FEA analysts are able to separate discretization and idealization errors, a process essential to verification, validation, and uncertainty quantification.

2006

The *Guide for Verification and Validation in Computational Solid Mechanics* is released by the *American Society of Mechanical Engineers* (**ASME**). This ground-breaking guide lays the foundation of requirements for controlling errors in numerical simulations.

2008

NASA releases the *"Standard for the Development of Models and Simulations"*, NASA-STD-7009. Any FEA simulation must pass strict verification requirements to be deemed compliant.

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