

Mathematical Analysis of Stability and Drift Behavior of Hydraulic Cylinders Driven by a Servovalve

Edouard Richard

Jean C. Vivalda

INRIA-Lorraine (CONGE project)-ISGMP. Bât. A,
Université de Metz-le du Saulcy,
57045 Metz Cedex 01, France
e-mail: {richard,vivalda}@loria.f

Hydraulic cylinders are commonly used and many works deal with modeling and control of such devices. This article deals with the stability properties of hydraulic cylinders drift in various situations. The study is based on a classic nonlinear model of these physical systems. The cases of system models without leakages and models with cylinder leakages or servovalve leakages are distinguished and lead to distinct behaviors. The stability properties are proven by various mathematical arguments such as first integrals, LYAPUNOV theorems, LASALLE invariance principle, BARBALAT's lemma, and the center manifold theory. [DOI: 10.1115/1.1433482]

1 Introduction

Hydraulic devices including cylinders are commonly used in industrial processes and in manufactured products, and many research works are devoted to modeling, analysis, control, and design of such hydraulic systems (for example see [1–4] as reference books and the works [5–14]).

This article deals with the stability properties of the hydraulic cylinders drift in various situations. These properties are described by nonlinear models, and the proofs are based on mathematical arguments such as first integrals, LYAPUNOV theorems, LASALLE invariance principle, BARBALAT's lemma and the center manifold theory. In the proofs, auxiliary functions derived from physical considerations are used. This study leads to a better understanding of the natural behavior of this kind of hydraulic system and can be useful in design of nonlinear control laws. From a practical point of view, these properties are roughly known but are not clearly explained by mathematical models. Moreover, to the best of our knowledge, no precise mathematical analysis of these properties is available in the literature. For example, in the case of a system model without leakages, using the tangent linearized model, which presents two eigenvalues equal to zero and two eigenvalues on the imaginary axis, it is often asserted that the solution is a periodic function of the time. Although the result is valid, the mathematical analysis is wrong.

The first section is concerned with the description of the physical hydraulic device and its corresponding model, and in the second section the existence of solutions is proved. For the flows model only the properties used in the proofs are given. In the third section the drift dynamic behavior of the hydraulic actuator without any leakages induced in the model is studied. The cases with and without viscous friction are examined. The fourth section deals with the dynamic behavior of the hydraulic actuator with internal actuator leakages. For these leakages, it should be noted that the mass flow rate is used instead of the usual volumetric flow rate which prevents from using the mass conservation property. Finally, in the fifth section the dynamic system behavior of the system with servovalve leakages is analyzed and a conclusion summarizes the obtained results.

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2 A Hydraulic Cylinder Driven by a Four-Way Servovalve

2.1 Description of the Actuator System. The system under consideration is composed of a four-way servovalve driving a linear cylinder which carries a mass M submitted to a viscous friction $-bv$ and a constant external force $-F_0$. The control input u is the servovalve voltage; the scheme of Fig. 1 represents this system. The origin of the position y is chosen such that the volumes of both cylinder chambers are the same and equal to V_0 . The useful piston areas are denoted by A_p and A_n and the chamber pressures by P_p and P_n . The position verifies $-l_p \leq y \leq l_n$ and the volume of the chambers are $V_p(y) = V_0 + S_p y$ and $V_n(y) = V_0 - S_n y$. If dead volumes are taken into account then $V_p(-l_p) > 0$ and $V_n(l_n) > 0$, moreover, at each cylinder end stroke, a stop represented by a force $F_s(y)$ is introduced. Otherwise, $V_p(-l_p) = V_n(l_n) = 0$, that is to say $l_p = V_0/S_p$, $l_n = V_0/S_n$, and the stop force $F_s(y)$ is removed.

2.2 Assumptions and Nonlinear Model of the Actuator System. In order to obtain the system models, the following main assumptions are usually made (see [1,2,4]): (i) the dynamic of the servovalve is negligible, (ii) the pressures are homogeneous in each cylinder chamber, (iii) the cylinder has no dry friction, (iv) the temperature is constant.

Under the previous assumptions, the system model is deduced from the three following laws: (i) the pressure evolution law written for each cylinder chamber, (ii) the flow rates provided by the servovalve ports to the cylinder chambers, (iii) the mechanical equation of the moving part.

2.2.1 Cylinder Chamber Model. Consider a pressure chamber of volume filled with a mass m of fluid with a density ρ (see Fig. 2). The continuity equation leads to $\dot{m} = \rho \dot{V} + V \dot{\rho} = \dot{m}_{in} - \dot{m}_{out}$, where $\dot{m}_{in}$ and $\dot{m}_{out}$ represent respectively the mass flow rates entering and leaving the chamber. Using the bulk modulus definition

$$B \triangleq \rho \frac{\partial P}{\partial \rho}, \quad (1)$$

gives the pressure evolution law ([1,3,4]):

$$\dot{P} = \frac{B}{V} \left(\frac{\dot{m}_{in}}{\rho} - \frac{\dot{m}_{out}}{\rho} - \frac{dV}{dt} \right). \quad (2)$$

Into the working range domain of pressures evolution, it can be assumed that B remains constant, and by integrating definition (1), it follows ([12]):

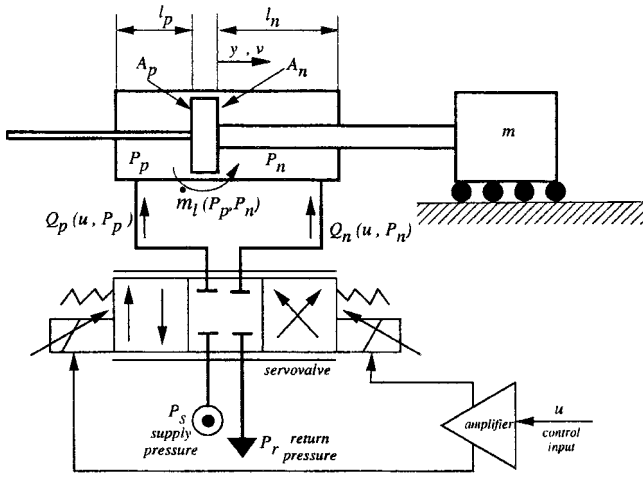


Fig. 1 Scheme of the hydraulic actuator

$$\frac{\rho}{\rho_0} = \exp\left(\frac{P - P_0}{B}\right). \quad (3)$$

where P_0 is an arbitrary reference pressure and ρ_0 the corresponding density.

2.2.2 Compressibility Energy. Now, the compressibility energy of the fluid in a chamber, which will be used later, is computed. The compressibility work by unity of mass is $df^* = (P_0 - P)dv$ where $v \triangleq 1/\rho$ represents the specific volume and P_0 is an arbitrary reference pressure. By using Eq. (3), the previous relation can be integrated and leads to the compressibility potential energy for a chamber ([12]):

$$\mathcal{E}_{ch}(V, P, P_0) = mf^* = BV \left(\exp\left(\frac{P - P_0}{B}\right) - \frac{P - P_0 + B}{B} \right). \quad (4)$$

It is interesting to note that $\mathcal{E}_{ch}(V, P_0, P_0) = 0$ and that $\mathcal{E}_{ch}(V, P, P_0) > 0$ if $V > 0$.

2.2.3 Flow Equations. In this section, the properties of the flow rates needed in the theorem proofs are listed below. Formulations of flow rates can be found in [1,4,5,7,13]. The servovalve, whose dynamics have been neglected, can be schematized by a Wheatstone bridge in which the four restrictions areas are modulated by the control input u . Usually, the volumetric flow rates,

$$Q_p(u, P_p) = \frac{\dot{m}_p(u, P_p)}{\rho_p}, \quad Q_n(u, P_n) = \frac{\dot{m}_n(u, P_n)}{\rho_n},$$

provided by the servovalve ports connected to the cylinder chambers, are used instead of the mass flow rates. These volumetric flow rates verifies:

$$\frac{\partial Q_p(u, P_p)}{\partial P_p} < 0, \quad \frac{\partial Q_n(u, P_n)}{\partial P_n} < 0 \quad \forall u \neq 0, \quad (5)$$

and, in addition, for a model of servovalve assuming null leakages:

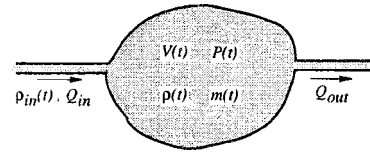


Fig. 2 A variable volume Chamber

$$Q_p(0, P_p) \equiv Q_n(0, P_n) \equiv 0. \quad (6)$$

In the case of a model of servovalve including leakages, inequalities (5) are also satisfied when $u = 0$. Moreover, the equations $Q_p(u, P_p) = 0$ and $Q_n(u, P_n) = 0$ define two smooth implicit functions, called pressure gain curves, verifying:

$$\varphi_p: [-u_n, u_n] \rightarrow [P'_r, P'_s]$$

is onto and is an increasing function (7)

$$\varphi_n: [-u_n, u_n] \rightarrow [P'_s, P'_r]$$

is onto and is a decreasing function

here P'_r and P'_s are two pressures closed to P_r and P_s , u_n denotes the maximal servovalve voltage. Regarding the internal leakage $\dot{m}_l(P_p, P_n)$ between the cylinder chambers, it is assumed:

$$\dot{m}_l(P_p, P_n)(P_p - P_n) > 0 \quad \forall P_p \neq P_n \quad \text{and} \quad \dot{m}_l(P, P) \equiv 0 \quad \forall P. \quad (8)$$

2.2.4 Piston Motion. On the subject of the mechanical point of view, at the end stroke of the piston ($y = -l_p$ or $y = l_n$), a force $F_s(y)$ corresponding to the piston stop is considered. The function $F_s(y)$ is a nondecreasing smooth function such that:

$$\begin{cases} F_s(y) \equiv 0 & \forall y \in [-l_p, l_n] \\ F_s(y) < 0 & \forall y \in [-l_p - \delta, -l_p] \text{ and } [l_n, l_n + \delta] \end{cases} \quad (9)$$

and

$$\lim_{y \rightarrow -l_p - \delta} \mathcal{E}_s(y) = \lim_{y \rightarrow l_n + \delta} \mathcal{E}_s(y) = +\infty \quad \text{with} \quad \mathcal{E}_s(y)$$

$$\triangleq \int_{y^0}^y F_s(\sigma) d\sigma \quad (y^0 \in [-l_p, l_n]) \quad (10)$$

where $\delta > 0$ is a very small distance such that $V_p(-l_p - \delta) > 0$ and $V_n(l_n + \delta) > 0$. The application of the main principle of mechanics to the motion mass M gives:

$$M\dot{v} = A_p P_p - A_n P_n - F_0 - F_s(y) - bv. \quad (11)$$

Because, the external force F_0 is assumed to be less than the maximum actuator force F_0 is within the range $[S_p P_s - S_n P_r, S_p P_r - S_n P_s]$.

2.2.5 Nonlinear State Model of the Electrohydraulic System Under Consideration. Taking into account the various flow rates, shown by Fig. 1 and applying relation (2) to each cylinder chamber in addition to Eq. (11), the hydraulic state model (12) is deduced.

$$\begin{cases} \dot{P}_p = -\frac{B}{V_0 + A_p y} A_p v + \frac{B}{V_0 + A_p y} \left(Q_p(u, P_p) - \frac{1}{\rho_0} \exp\left(\frac{P_0 - P_p}{B}\right) \dot{m}_l(P_p, P_n) \right) \\ \dot{P}_n = \frac{B}{V_0 - A_n y} A_n v + \frac{B}{V_0 - A_n y} \left(Q_n(u, P_n) + \frac{1}{\rho_0} \exp\left(\frac{P_0 - P_n}{B}\right) \dot{m}_l(P_p, P_n) \right) \\ \dot{v} = \frac{1}{M} (A_p P_p - A_n P_n - F_0 - F_s(y) - bv) \end{cases} \quad \dot{y} = v \quad (12)$$

3 Uniqueness of the Solution and Pressures Positivity

Within the open set $\mathcal{U} = \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times]-l_p - \delta, l_n + \delta[$, the vector field corresponding to model (12) is of class C^1 for a constant value of u . In the following proofs, it will be shown that $\mathcal{U}$ is a positive invariant set. By virtue of CAUCHY-LIPSCHITZ's theorem (see [15,16]), for any initial state x_0 in $\mathcal{U}$, there exist $\delta_m(x_0) > 0$ and $\delta_M(x_0) > 0$ such that the solution initialized at x_0 exists for t in the interval $]-\delta_m(x_0), \delta_M(x_0)[$ and is unique.

Regarding the pressure positivity, remarks that it is possible to choose initial states in $\mathcal{U}$ such that the pressures become negative. Without going into too much detail, it should be noticed that this behavior can be avoided by taking into account cavitation phenomena or by considering the bulk modulus B as a function of the pressure P such that $B(0) = 0$.

4 Dynamic Behavior of the System Model Drift Without Leakages

In this section, system model (12) with $u \equiv 0$ and conditions $Q_p(0, P_p) \equiv Q_n(0, P_n) \equiv 0$ and $\dot{m}_i(P_p, P_n) \equiv 0$ is considered.

First of all, let us look at the state equations corresponding to the pressures evolution. As a result of the mass conservation law in each cylinder chamber, a simple integration of the two first state equations of (12) leads to the following first integrals:

$$P_p + B \ln(V_0 + A_p y) \quad \text{and} \quad P_n + B \ln(V_0 - A_n y)$$

so, given two constants PK_p and PK_n , if the initial conditions are such that

$$\begin{cases} P_p(0) + B \ln(V_0 + S_p y(0)) = PK_p \\ P_n(0) + B \ln(V_0 - S_n y(0)) = PK_n, \end{cases} \quad (13)$$

the trajectories of the system evolve on the two-dimensional manifold $\mathcal{M}(PK_p, PK_n)$ defined by the equations:

$$\begin{cases} P_p + B \ln(V_0 + S_p y) = PK_p \\ P_n + B \ln(V_0 - S_n y) = PK_n. \end{cases} \quad (14)$$

In view of the previous first integrals, the system model (12) can be reduced to the equation:

$$\begin{cases} \dot{v} = \frac{1}{M} (F_h(y) - F_0 - F_s(y) - bv) \\ \dot{y} = v \end{cases} \quad (15)$$

where

$$F_h(y) \triangleq A_p P_p - A_n P_n = A_p PK_p - A_n PK_n - B(A_p \ln(V_0 + S_p y) - A_n \ln(V_0 - S_n y)) \quad (16)$$

represents the hydraulic force. These equations represent the evolution of the system model restricted to the manifold $\mathcal{M}(PK_p, PK_n)$ defined by Eqs. (14).

The set E of equilibrium points of system (12) is a two-dimensional manifold defined by the equations: $v = 0$ and $A_p P_p - A_n P_n - F_0 - F_s(y) = 0$.

In the following, the mechanical energy $\mathcal{E}_m$ will be considered; it is defined by:

$$\mathcal{E}_m(v, y) \triangleq \mathcal{E}_k(v) + \mathcal{E}_p(y) + \mathcal{E}_s(y) \quad (17)$$

with:

$$\begin{aligned} \mathcal{E}_k(v) &\triangleq \frac{1}{2} M v^2 & \mathcal{E}_p(y) &\triangleq \int_{y^e}^y (F_0 \\ & & - F_h(\sigma)) d\sigma & \mathcal{E}_s(y) &\triangleq \int_{y^e}^y F_s(\sigma) d\sigma \end{aligned} \quad (18)$$

where y^e represents an equilibrium position. A straightforward computation of the hydraulic potential energy gives:

$$\begin{aligned} \mathcal{E}_p(y) &= B(V_0 + S_p y) \ln \left(\frac{V_0 + S_p y}{V_0 + S_p y^e} \right) + B(V_0 - S_n y) \ln \left(\frac{V_0 - S_n y}{V_0 - S_n y^e} \right) \\ &+ B(y^e - y)(S_p - S_n). \end{aligned} \quad (19)$$

4.1 System Model Without Viscous Friction. Theorem 1. Here, system (15) with $b = 0$ is considered.

1. The constants, PK_p and PK_n being given, the reduced system (15) has a single equilibrium point $(0, y^e)$, corresponding to the intersection of E and $\mathcal{M}(PK_p, PK_n)$. Moreover $y^e \in [-l_p, l_n]$ iff $F_0 \in [F_h(l_n), F_h(-l_p)]$.

2. If the initial state is not the equilibrium point, then the solution of (15) is periodic (and also the one of (12)).

3. In the case of $y^e \in [-l_p, l_n]$ and $0 < \mathcal{E}_m(0) \leq \inf(\mathcal{E}_p(-l_p), \mathcal{E}_p(l_n))$, the piston does not reach the actuator ends and the period can be approximated by: $T_h \approx 2\pi \sqrt{M/\mathcal{K}_h(y^e)}$ with the hydraulic stiffness $\mathcal{K}_h(y^e) = BS_p^2/V_0 + S_p y^e + BS_n^2/V_0 - S_n y^e$.

4. Moreover, if $\mathcal{E}_m(0) > \sup(\mathcal{E}_p(-l_p), \mathcal{E}_p(l_n))$, at least one actuator end is reached by the piston. This result is still true when there are no dead volumes although the hydraulic force F_h tends to infinity when the piston comes near to the cylinder end.

5. Proof

1. First, an equilibrium point $(0, y^e)$ of (15) is such that $F_h(y^e) - F_s(y^e) = F_0$. The function $F_h(y) :]-V_0/A_p, V_0/A_n[\rightarrow \mathbb{R}$ being decreasing from $+\infty$ to $-\infty$, the function $F_h(y) - F_s(y)$ is also decreasing, therefore, the previous equation has a single solution and system (15) admits an unique equilibrium point. Clearly, this point is the intersection of E and $\mathcal{M}(PK_p, PK_n)$.

Now, as $F_h - F_s$ is decreasing, $y^e \in [-l_p, l_n]$ is equivalent to $(F_h - F_s)(y^e) \in [(F_h - F_s)(l_n), (F_h - F_s)(-l_p)]$ which is itself equivalent to $F_0 \in [F_h(l_n), F_h(-l_p)]$, because $F_h(y^e) - F_s(y^e) = F_0$ and $F_s(-l_p) = F_s(l_n) = 0$.

2. In order to show the periodicity of the movement, it is noticed first that the mechanical energy $\mathcal{E}_m$ is a proper positive definite function with respect to $(0, y^e)$. Indeed, $\mathcal{E}_k$ is non-negative, $\mathcal{E}_p + \mathcal{E}_s$ is equal to $\int_{y^e}^y (F_0 - (F_h - F_s)(\sigma)) d\sigma$ and is non-negative because the function $\sigma \mapsto (F_0 - (F_h - F_s)(\sigma))$ has the same sign as $y - y^e$. Also, it is clear that $\mathcal{E}_k$ vanishes only when $v = 0$ and $\mathcal{E}_p + \mathcal{E}_s$ only when $y = y^e$. The fact that $\mathcal{E}_m$ is proper on $\mathcal{U}_R \triangleq \mathbb{R} \times]-l_p - \delta, l_n + \delta[$ arises from $\lim_{(v, y) \rightarrow \partial \mathcal{U}_R} \mathcal{E}_m(v, y) = \infty$ (this is due to property (10)).

Then, a straightforward, computation shows that $\dot{\mathcal{E}}_m = 0$ which proves that the trajectories of (15) remain on the level surfaces of $\mathcal{E}_m$ which are contained in $\mathcal{U}_R$ since $\mathcal{E}_m$ is a proper function. Consider an initial condition such that $\mathcal{E}_m(0) \triangleq \mathcal{E}_m(v(0), y(0)) > 0$, the reduced system trajectory $(v(t), y(t))$ remains on the curve $\mathcal{E}_m(v, y) = \mathcal{E}_m(0)$ whose equation can be written as

$$v = \pm \sqrt{\frac{2(\mathcal{E}_m(0) - (\mathcal{E}_p + \mathcal{E}_s)(y))}{M}} \quad (20)$$

which is a closed curve $\mathcal{C}$. The periodicity of the movement results from the fact that $\mathcal{C}$ is a closed curve, which does not contain the unique equilibrium point since $\mathcal{E}_m(0) > 0$.

3. Now, point 3 is proved. Denote by $(v(t), y(t))$ the trajectory issued from initial conditions $(v(0), y(0))$. Since $\mathcal{E}_m(0) \leq \inf(\mathcal{E}_p(-l_p), \mathcal{E}_p(l_n))$, $y(t) \in [-l_p, l_n]$ for all t , so $\mathcal{E}_s(y(t)) = 0$ for all t . The function $\mathcal{E}_p$ being decreasing (resp. increasing) on the interval $[-l_p, y^e]$ (resp. on $[y^e, l_n]$) from $\mathcal{E}_p(-l_p)$ to $\mathcal{E}_p(y^e) = 0$ (resp. from $\mathcal{E}_p(y^e) = 0$ to $\mathcal{E}_p(l_n)$), there exists a unique $y_{\min} \in [-l_p, y^e]$ (resp. $y_{\max} \in [y^e, l_n]$) such that $\mathcal{E}_m(0) = \mathcal{E}_p(y_{\min})$ (resp. $\mathcal{E}_m(0) = \mathcal{E}_p(y_{\max})$).

The points on the trajectory such that $y(t) = y_{\min}$ or $y(t) = y_{\max}$ are such that $v(t) = 0$ and the time needed to go from point $A(0, y_{\min})$ to $B(0, y_{\max})$ on the trajectory can be computed as in

[17]:

$$T = \int_{y_{\min}}^{y_{\max}} \frac{1}{\sqrt{\frac{2(\mathcal{E}_m(0) - \mathcal{E}_p(y))}{M}}} dy. \quad (21)$$

Due to the symmetry in the relation (20), the period T_h is equal to $2T$.

Since the previous integral cannot be analytically computed, a second-order series development of the potential energy around y^e is used:

$$\mathcal{E}_p(y) = \frac{1}{2} \mathcal{K}_h(y^e)(y - y^e)^2 + \sigma(y - y^e)^3. \quad (22)$$

In the light of approximation (22) the extremum positions are $y_{\max, \min} = y^e \pm \sqrt{2\mathcal{E}_m(0)/\mathcal{K}_h(y^e)}$ and using an adequate change of variable, an approximation of the period is given by:

$$T_h \approx 2 \sqrt{\frac{M}{\mathcal{K}_h(y^e)}} \int_{-1}^1 \frac{d\sigma}{\sqrt{1 - \sigma^2}} = 2\pi \sqrt{\frac{M}{\mathcal{K}_h(y^e)}}. \quad (23)$$

One may point out that $d^2\mathcal{E}_p(y)/d^2y|_{y=y^e} = -dF_h(y)/dy|_{y=y^e}$ leads to the well-known (see [1,3]) hydraulic stiffness:

$$\mathcal{K}_h(y^e) = -\left. \frac{dF_h}{dy} \right|_{y=y^e} = \frac{A_p^2 B}{V_0 + S_p y^e} + \frac{A_n^2 B}{V_0 - A_n y^e} \quad (24)$$

Notice also that the pressure evolution of system (12) is also periodic with the same period since from (13) and (14) it follows:

$$\begin{cases} P_p = P_p(0) + B \ln\left(\frac{V_0 + S_p y(0)}{V_0 + S_p y}\right) \\ P_n = P_n(0) + B \ln\left(\frac{V_0 - S_n y(0)}{V_0 - S_n y}\right) \end{cases}. \quad (25)$$

The maximum pressure $P_{p \max}$ and minimum pressure $P_{p \min}$ correspond, respectively, to the minimum position $y_{\min}$ and to the maximum position $y_{\max}$ and conversely concerning P_n .

1. Let us now turn our attention to point 4. The first thing that must be pointed out is that although $\lim_{y \rightarrow -V_0/S_p} F_h(y) = \infty$ and $\lim_{y \rightarrow V_0/S_n} F_h(y) = -\infty$ the corresponding limits of $\mathcal{E}_p(y)$ are finite. It follows that if $\mathcal{E}_m(0) > \sup(\mathcal{E}_p(-l_p), \mathcal{E}_p(l_n))$, then the equation $\mathcal{E}_m(0) = \mathcal{E}_p(y)$ has at most one solution in the interval $[-l_p, l_n]$, which means that the piston reaches at least one end of the actuator. ■

4.2 System Model With Viscous Friction. Theorem 2. Here, system (15) with $b \neq 0$ is considered.

1. The constants, PK_p and PK_n being given, the reduced system (15) has a single equilibrium point $(0, y^e)$, which is globally asymptotically stable. (This equilibrium point sets the equilibrium pressures of system (12)).

2. If $y^e \in [-l_p, l_n]$ and $0 < \mathcal{E}_m(0) \leq \inf(\mathcal{E}_p(-l_p), \mathcal{E}_p(l_n))$, the piston does not reach the actuator ends.

3. Moreover, if $\mathcal{E}_m(0) > \sup(\mathcal{E}_p(-l_p), \mathcal{E}_p(l_n))$, the piston can reach the actuator end stroke. This result is still true when there are no dead volumes.

Proof

A simple computation leads to $\dot{\mathcal{E}}_m(v, y) = -bv^2 \leq 0$. Now, consider a trajectory $(v(t), y(t))$ of the reduced system such that $\dot{\mathcal{E}}_m(v(t), y(t)) = 0$. Along this trajectory, $v(t) = 0$, then $\dot{v}(t) = 0$ and using Eq. (15) it follows $F_h(y(t)) = F_0 + F_s(y(t))$ and from the uniqueness of the equilibrium point it results $y(t) = y^e$. By using LASALLE invariance principle (see [15,16]) and the fact that $\mathcal{E}_m$ is a proper function the global asymptotic stability of (15) follows. Clearly, using the first integrals (14), the equilibrium pressures are determined.

Point 2 is obvious in view of theorem 1.

Point 3 is obvious in the case of the existence of dead volumes

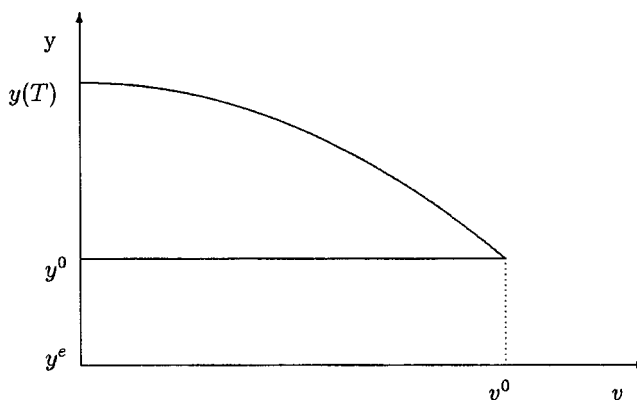


Fig. 3 Shape of reduced system trajectory

and the case without dead volumes is studied. In fact, it will be shown that there are trajectories $(v(t), y(t))$ such that $y(t) \rightarrow -V_0/S_p$ or V_0/S_n in finite time.

Consider the case $\mathcal{E}_m(0) > \mathcal{E}_p(-l_p)$ and assume that the equilibrium point $(v, y) = (0, y^e)$ is globally asymptotically stable. Then a trajectory beginning at the point $(v(0), y(0))$ with $v(0) > 0$ and $y(0) > y^e$ intersects the y axis at a time T with $y(T) < V_0/A_n$ (see Fig. 3).

Then consider the integral:

$$I = \oint_{\Gamma} \left(-\frac{b}{M} v + \frac{1}{M} F_h(y) \right) dy - v dv,$$

where Γ represents the path defined by the segment $[0, v(0)] \times \{y(0)\}$, the trajectory part going from $(v(0), y(0))$ to $(0, y(T))$ and the segment $\{0\} \times [y(T), y(0)]$. On one hand, by calculating I along the path Γ , it follows:

$$I = \int_{y(T)}^{y(0)} \frac{1}{M} F_h(y) dy - \frac{1}{2} v(0)^2.$$

On the other hand, by using GREEN's formula it is obtained:

$$I = \iint_{\mathcal{A}} -\frac{b}{M} dv dy = -\frac{b}{M} \text{Area}(\mathcal{A})$$

Equating both expressions of I the following relation is deduced:

$$\frac{1}{2} M v(0)^2 = \int_{y(T)}^{y(0)} F_h(y) dy + b \text{Area}(\mathcal{A}).$$

On Fig. 3 the obvious inequality $\text{Area}(\mathcal{A}) < v(0)(y(T) - y(0))$ is noticeable. Then:

$$\frac{1}{2} M v(0) \leq \frac{1}{v(0)} \int_{y(T)}^{y(0)} F_h(y) dy + b(y(T) - y(0)).$$

In view of the fact that $\mathcal{E}_p(y)$ is bounded, for values of $v(0)$ large enough, the previous inequality is impossible with the condition $y(T) < V_0/A_n$. ■

5 Dynamic Behavior of the System Model Drift With Actuator Leakage and Without Servovalve Leakages

In this section, Eq. (12) with $u = 0$ is considered and it is assumed that $Q_p(0, P_p) = Q_n(0, P_n) = 0$, $\dot{m}_i(P_p, P_n) \neq 0$ and $b \neq 0$. From the condition $\dot{m}_i \neq 0$, it follows that the right-hand members of (14) are no more first integrals but the conservation of the fluid mass contained in both cylinder chambers leads to the consideration of:

$$m_T(P_p, P_n, y) = \rho_0(V_0 + S_p y) \exp\left(\frac{P_p - P_0}{B}\right) + \rho_0(V_0 - S_n y) \exp\left(\frac{P_n - P_0}{B}\right) \quad (26)$$

where P_0 and ρ_0 are respectively an arbitrary reference pressure and the corresponding density of the fluid. An easy computation shows that $\dot{m}_T(P_p(t), P_n(t), y(t)) \equiv 0$ along the trajectories of system (12) and so:

$$m_T(P_p(t), P_n(t), y(t)) = m_T(0) \text{ for all } t. \quad (27)$$

Theorem 3. System (12) with the above conditions is considered.

1. In the case of a non symmetrical actuator ($S_p \neq S_n$) and F_0 such that

$$-l_p \leq \frac{1}{S_p - S_n} \left(\frac{m_T(0)}{\rho_0} \exp\left(\frac{P_0 - \frac{F_0}{S_p - S_n}}{B}\right) - 2V_0 \right) \leq l_n, \quad (28)$$

$P_p \rightarrow P^e$, $P_n \rightarrow P^e$, $v \rightarrow 0$, $y \rightarrow y^e$ with $y^e \in [-l_p, l_n]$ where P^e and y^e depends on the initial pressures and on the initial position.

2. In the case of a symmetrical cylinder ($S_p = S_n$) and without external load ($F_0 = 0$), $P_p \rightarrow P^e$, $P_n \rightarrow P^e$, $v \rightarrow 0$ and $y \rightarrow y^e \in [-l_p, l_n]$ where P^e , given in the proof, and y^e depend on the initial pressures and on the initial position.

In this proof the total energy of system (12) will be used:

$$\mathcal{E}(P_p, P_n, v, y) = \frac{1}{2} M v^2 + \mathcal{E}_{ch}(V_0 + S_p y, P_p, P_0) + \mathcal{E}_{ch}(V_0 - S_n y, P_n, P_0) + \mathcal{E}_s(y). \quad (29)$$

with P_0 any arbitrary reference pressure which will be set later. The time derivative of $\mathcal{E}(P_p, P_n, v, y)$ along the system trajectories gives:

$$\dot{\mathcal{E}} = ((S_p - S_n)P_0 - F_0)v - b v^2 - \frac{B}{\rho_0} \dot{m}_l(P_p, P_n) \left(\exp\left(\frac{P_p - P_n}{B}\right) - 1 \right) \exp\left(\frac{P_0 - P_p}{B}\right).$$

For a symmetrical actuator without external force or for a non-symmetrical actuator by choosing $P_0 = F_0 / (A_p - A_n)$ it follows that $((S_p - S_n)P_0 - F_0)v = 0$. From property (8), it is clear that $\dot{\mathcal{E}} \leq 0$, so $\mathcal{E}$ is a nonincreasing function. As, $\lim_{y \rightarrow -l_p - \delta} \mathcal{E} = \lim_{y \rightarrow l_n + \delta} \mathcal{E} = +\infty$, it follows that there exists $\varepsilon > 0$ such that the position verifies $-l_p - \delta + \varepsilon \leq y \leq l_n + \delta - \varepsilon$, then $\mathcal{E}$ remains positive. From this, it can be deduced that $\lim_{t \rightarrow \infty} \mathcal{E}(t) = l$ with $0 \leq l < +\infty$ and also:

$$\begin{aligned} 0 &\leq \frac{1}{2} M v^2 \leq \mathcal{E}(t) \leq \mathcal{E}(0) \\ 0 &\leq \mathcal{E}_{ch}(V_0 + A_p y(t), P_p(t), P_0) \leq \mathcal{E}(t) \leq \mathcal{E}(0) \\ 0 &\leq \mathcal{E}_{ch}(V_0 - A_n y(t), P_n(t), P_0) \leq \mathcal{E}(t) \leq \mathcal{E}(0) \\ 0 &\leq n \mathcal{E}_s(y(t)) \leq \mathcal{E}(t) \leq \mathcal{E}(0) \end{aligned}$$

which proves that all the state variables remain in a compact set.

Moreover, the initial conditions being fixed, $\dot{\mathcal{E}}(t)$ can be expressed as a function of the state variables, these variables being bounded, it follows that $\dot{\mathcal{E}}(t)$ is bounded too and so $\mathcal{E}$ is uniformly continuous. From BARBALAT's lemma (see [18]), it follows that $\lim_{t \rightarrow \infty} \dot{\mathcal{E}}(t) = 0$ and, since $\dot{\mathcal{E}} = 0$ iff $v = 0$ and $P_p = P_n$, $\lim_{t \rightarrow \infty} v(t) = 0$ and $\lim_{t \rightarrow \infty} (P_p - P_n)(t) = 0$. By derivating the state equation corresponding to v , it is clear that $\dot{v}$ is bounded, and using again BARBALAT's lemma $\dot{v} \rightarrow 0$ is deduced.

Now, two situations have to be distinguished: the case of a nonsymmetrical actuator with an external force and the case of a symmetrical actuator without external force.

First Case ($A_p \neq A_n$). From $\lim_{t \rightarrow \infty} (P_p - P_n) = 0$, it follows $P_p(t) = P_n(t) + \varepsilon(t)$ with $\lim_{t \rightarrow \infty} \varepsilon(t) = 0$. Substituting for P_p in the third equation of (12) and in Eq. (27) and using $\lim_{t \rightarrow \infty} \dot{v}(t) = 0$, the following relations are deduced:

$$\lim_{t \rightarrow \infty} (S_p - S_n) P_p(t) - F_s(y(t)) = F_0 \quad (30a)$$

$$\lim_{t \rightarrow \infty} \exp\left(\frac{P_p(t) - P_0}{B}\right) (2V_0 + (S_p - S_n)y(t)) = \frac{m_T(0)}{\rho_0} \quad (30b)$$

Equation (30a) can be written under the form $P_p(t) = 1/A_p - A_n(F_0 + F_s(y(t)) + \varepsilon(t)) \triangleq \varphi(y(t)) + \bar{\varepsilon}(t)$ with $\lim_{t \rightarrow \infty} \bar{\varepsilon}(t) = 0$. Substituting for $P_p(t)$ in Eq. (30b) and taking into account $\lim_{t \rightarrow \infty} \bar{\varepsilon}(t) = 0$, the following equality is deduced:

$$\lim_{t \rightarrow \infty} \exp\left(\frac{\varphi(y(t)) - P_0}{B}\right) (2V_0 + (A_p - A_n)y(t)) = \frac{m_T(0)}{\rho_0}. \quad (31)$$

Since the function of y appearing in Eq. (31) is monotonous, $y(t)$ admits a limit, y^e , as t tends to infinity, hence the same is true for $P_p(t)$. Now these two limits are easy to compute: from condition (28), it is clear that $y^e \in [-l_p, l_n]$ and Eq. (30a) gives $P_p^e = F_0 / (A_p - A_n)$; substituting for P_p in Eq. (30b), the expression of y^e follows:

$$y^e = \frac{1}{S_p - S_n} \left(\frac{m_T(0)}{\rho_0} \exp\left(\frac{P_0 - P^e}{B}\right) - 2V_0 \right).$$

Second Case ($A_p = A_n$). From $\lim_{t \rightarrow \infty} (P_p - P_n)(t) = 0$ and Eq. (27), it follows that $\lim_{t \rightarrow \infty} P_p(t) = \lim_{t \rightarrow \infty} P_n(t) = P^e$ with:

$$P^e = P_0 + B \ln\left(\frac{m_T(0)}{2\rho_0 V_0}\right). \quad (32)$$

Now, it is shown that $y(t)$ tends to some limit y^e as t tends to infinity. To this end, first it is shown that each singular point of system (12) is Lyapunov stable; this can be done by using the center manifold theory [19,20]. The equilibrium points (P_p^e, P_n^e, v^e, y^e) are such that:

$$\begin{cases} v^e = 0 \\ \dot{m}_l(P_p^e, P_n^e) = 0 \Rightarrow P_p^e = P_n^e \triangleq P^e \end{cases} \quad (33)$$

In order to compute the Jacobian matrix of the right hand member of (12), notice that, in the hydraulic case, the leakage $\dot{m}_l$ can be expressed as $\dot{m}_l(P_p, P_n) = \varphi(P_p - P_n)$ where φ is a C^1 function satisfying $\varphi(0) = 0$ and $x\varphi(x) > 0$ if $x \neq 0$. This Jacobian matrix is equal to:

$$\begin{pmatrix} -\alpha(P^e, y^e) & \alpha(P^e, y^e) & -\gamma_1(y^e) & 0 \\ \beta(P^e, y^e) & -\beta(P^e, y^e) & \gamma_2(y^e) & 0 \\ S/M & -S/M & -b/M & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

where:

$$\begin{aligned} \alpha(P^e, y^e) &= \frac{B}{\rho_0(V_0 + S y^e)} \exp\left(\frac{P_0 - P^e}{B}\right) \lambda \\ \beta(P^e, y^e) &= \frac{B}{\rho_0(V_0 - S y^e)} \exp\left(\frac{P_0 - P^e}{B}\right) \lambda \end{aligned}$$

$$\gamma_1(y^e) = \frac{BS}{V_0 Sy^e} \quad \gamma_2(y^e) = \frac{BS}{V_0 - Sy^e},$$

λ being equal to $\varphi'(0)$ (due to assumption (8), λ is positive). The characteristic polynomial of this matrix is equal to $X^2(X^2 + (\alpha + \beta + b/M)X + S/M(\gamma_1 + \gamma_2) + B/M(\alpha + \beta))$ and it can be easily seen that this polynomial has two roots with negative real parts and the root 0 of multiplicity two. Making the change of variables:

$$\begin{cases} x_1 = \beta(P_p - P^e) + \alpha(P_n - P^e) + (\beta\gamma_1 - \alpha\gamma_2)(y - y^e) \\ x_2 = S/M(P_p - P^e) + \alpha v + (S/M\gamma_1 + b/M\alpha)(y - y^e) \\ z_1 = P_p - P_n \\ z_2 = v \end{cases}$$

system (12) can be express as:

$$\begin{cases} \dot{x}_1 = f_1(x, z) \\ \dot{x}_2 = f_2(x, z) \\ \dot{z}_1 = -(\alpha + \beta)z_1 - (\gamma_1 + \gamma_2)z_2 + g(x, z) \\ \dot{z}_2 = S/Mz_1 - b/Mz_2 \end{cases}$$

the functions $f_i (i=1,2)$ and g being C^2 and such that $f_i(0,0) = g(0,0) = 0$, $f'_i(0,0) = g'(0,0) = 0$ (the ' denotes the differential). Moreover, the manifold defined by the equations $z_1 = z_2 = 0$ is the set of singular points of system (12), so it is an invariant manifold for (12).

It was shown that the conditions of application of the theorem of the center manifold (see [19,20]) are met, the restriction of system (12) to the above invariant manifold is, obviously, Lyapunov stable about an equilibrium point $(P^e, P^e, 0, y^e)$ so it is possible to conclude that system (12) is Lyapunov stable about every singular point.

It is now possible to prove that $\lim_{t \rightarrow \infty} y(t)$ exists by contradiction. Assume that there exists $\varepsilon_0 > 0$ such that for all $T > 0$, there exist t' and t'' such that $|y(t') - y(t'')| > \varepsilon_0$. All the state variables being bounded, the ω -limit set of a trajectory is non empty, let Ω be a point in this set, since $\lim_{t \rightarrow \infty} P_p(t) = \lim_{t \rightarrow \infty} P_n(t) = P^e$ and $\lim_{t \rightarrow \infty} v(t) = 0$, Ω is under the form: $\Omega = (P^e, P^e, 0, y^e)$, so Ω is a singular point. Now for all $\alpha > 0$, there exists an index k such that $(P_p(t_k), P_n(t_k), v(t_k), y(t_k))$ belongs to $B(\Omega, \alpha)$ the ball of center Ω and radius α . Since system (12) is Lyapunov stable about Ω , if α is chosen small enough, the trajectory remains in $B(\Omega, \varepsilon_0/2)$ for all $t \geq t_k$, which contradicts our assumption. ■

6 Dynamic Behavior of the System Drift With Servo-valve Leakages and Without Actuator Leakage

This section consider system model (12) with the conditions $\dot{m}_l(P_p, P_n) \equiv 0$ and $Q_p(0, P_p) \neq 0$, $Q_p(0, P_p) \neq 0$. First, the singular points are searched, these points verify the systems of equations:

$$\begin{cases} v^e = 0 \\ A_p P_p^e - A_n P_n^e - F_0 - F_s(y^e) = 0 \\ Q_p(u^e, P_p^e) = Q_n(u^e, P_n^e) = 0 \end{cases} \quad (34)$$

As only the singular points such that $y^e \in [-l_p, l_n]$ are interesting, the second equation reduces to $A_p P_p^e - A_n P_n^e - F_0 = 0$ whereas, from the last two equations, it follows $P_p^e = \varphi_p(u^e)$ and $P_n^e = \varphi_n(u^e)$ (see (7)). Now the function

$$F_{stath}: [-u_n, u_n] \rightarrow [S_p P_s' - S_n P_r', S_p P_r' - S_n P_s']$$

$$u_e \mapsto A_p \varphi_p(u^e) - A_n \varphi_n(u^e) - F_0$$

is increasing, so if F_0 belongs to the interval $I \triangleq [S_p P_s' - S_n P_r', S_p P_r' - S_n P_s']$, there exists an unique $u^e \in [-u_n, u_n]$ such that the second equation of (34) (with $F_s(y^e) = 0$) is satisfied.

In conclusion, if $F_0 \in I$, the equilibrium points of system (12) are $(\varphi_p(u^e), \varphi_n(u^e), 0, y^e)$, where u^e is the unique solution of $F_{stath}(u^e) = 0$ and $y^e \in [-l_p, l_n]$ is indeterminate. Regarding the behavior of the system, the following theorem is stated.

Theorem 4. If $F_0 \in I$ and if the control u^e , solution of the equation $F_{stath}(u^e) = 0$, is applied then, for any initial conditions and as t tends to infinity, the pressures $P_p(t)$ and $P_n(t)$ tend to $P_p^e = \varphi_p(u^e)$ and $P_n^e = \varphi_n(u^e)$ respectively, $v(t)$ tends to 0 and $y(t)$ tends to a value y^e belonging to $[-l_p, l_n]$.

Proof

As in the proof of theorem (3), the total energy of system (12) is considered:

$$\begin{aligned} \mathcal{E}(P_p, P_n, v, y) = & \frac{1}{2} M v^2 + \mathcal{E}_{ch}(V_0 + S_p y, P_p, P_p^e) \\ & + \mathcal{E}_{ch}(V_0 - S_n y, P_n, P_n^e) + \mathcal{E}_s(y). \end{aligned} \quad (35)$$

Clearly, $\mathcal{E} \geq 0$ and $\mathcal{E} = 0 \Leftrightarrow P_p = P_p^e$, $P_n = P_n^e$, $v = 0$ for $y \in]-l_p - \delta, l_n + \delta[$. The time derivative of $\mathcal{E}$ along the system trajectories is equal to:

$$\begin{aligned} \dot{\mathcal{E}} = & -b v^2 + B Q_p(u^e, P_p) \left(\exp\left(\frac{P_p - P_p^e}{B}\right) - 1 \right) + B Q_n(u^e, P_n) \\ & \times \left(\exp\left(\frac{P_n - P_n^e}{B}\right) - 1 \right) \end{aligned} \quad (36)$$

Now, using (5), the reasoning is as in the prove of theorem (3): the consideration of $\dot{\mathcal{E}}$ and the use of the BARBALAT lemma (see [18]) lead to:

$$\lim_{t \rightarrow \infty} P_p(t) = P_p^e \quad \lim_{t \rightarrow \infty} P_n(t) = P_n^e \quad \lim_{t \rightarrow \infty} v(t) = 0$$

Now, it is proved that $y(t)$ admits a limit as t tends to infinity, again, the scheme of the proof is the same as in the proof of theorem (3). First, the Jacobian matrix of the left hand of system (12) about an equilibrium point is computed, this matrix is equal to:

$$\begin{pmatrix} \alpha & 0 & -\gamma_1 & 0 \\ 0 & \beta & \gamma_2 & 0 \\ A_p/M & -A_n/M & -b/M & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

with:

$$\begin{aligned} \alpha = & \frac{B}{V_0 + A_p y^e} \frac{\partial Q_p}{\partial P_p}(u^e, P_p^e) \quad \beta = \frac{B}{V_0 - A_n y^e} \frac{\partial Q_n}{\partial P_n}(u^e, P_n^e) \\ \gamma_1 = & \frac{B A_p}{V_0 + A_p y^e} \quad \gamma_2 = \frac{B A_n}{V_0 - A_n y^e}. \end{aligned}$$

Notice that α and β are negative while γ_1 and γ_2 are positive. The characteristic polynomial of the above matrix can be written $X P_3(X)$ where P_3 is equal to:

$$\begin{aligned} P_3(X) = & X^3 + \left(\frac{b}{M} - \alpha - \beta \right) X^2 + \left(\alpha \beta + \gamma_1 \frac{A_p}{M} + \gamma_2 \frac{A_n}{M} - \alpha \frac{b}{M} \right. \\ & \left. - \beta \frac{b}{M} \right) X - \beta \gamma_1 \frac{A_p}{M} - \alpha \gamma_2 \frac{A_n}{M} + \alpha \beta \frac{b}{M} \end{aligned}$$

All the roots of P_3 have negative real parts: calling a_2 , a_1 and a_0 the coefficients of X^2 , X and X^0 , it is clear that these coefficients are all positive and an easy computation shows that $a_1 a_2 - a_0 > 0$. From the Routh criterion, the result follows.

After the change of variables:

$$\left\{ \begin{array}{l} x = -\beta \frac{A_p}{M} (P_p - P_p^e) + \alpha \frac{A_n}{M} (P_n - P_n^e) + \alpha \beta v + \left(\alpha \beta \frac{b}{M} - \beta \gamma_1 \frac{A_p}{M} - \alpha \gamma_2 \frac{A_n}{M} \right) (y - y^e) \\ z_1 = P_p - P_p^e \\ z_2 = P_n - P_n^e \\ z_3 = v \end{array} \right.$$

system (12) writes:

$$\left\{ \begin{array}{l} \dot{x} = f(x, z) \\ \dot{z}_1 = \alpha z_1 - \gamma_1 z_3 + g_1(x, z) \\ \dot{z}_2 = \beta z_2 + \gamma_2 z_3 + g_2(x, z) \\ \dot{z}_3 = A_p / M z_1 - A_n / M z_2 - b / M z_3 + g_3(x, z) \end{array} \right.$$

with $f(0,0)=g(0,0)=0$, $f'(0,0)=0$ and $g'(0,0)=0$. The manifold defined by the equations $z_1=z_2=z_3=0$ is the set of singular points of system (12), so it is an invariant manifold for (12). It is concluded by using exactly the same arguments as in the proof of theorem (3). ■

7 Conclusion

In this article, the dynamic behavior of a hydraulic actuator that consist of a cylinder driven by a four-way servovalve in several situations of leakages, has been studied. The study is based on a mathematical analysis of the nonlinear model.

In the case of a system without any leakages, there are two first integrals which enable to define the hydraulic force as a function of the position. This hydraulic force has been used in order to reduce the system model to its mechanical part. Moreover, on the one hand if there is no friction, it is rigorously shown that the solution is periodic. On the other hand, for a system with viscous friction, it is proved that the reduced system has a unique globally asymptotically stable equilibrium point which depends on the initial conditions of the whole system.

Regarding a device with internal leakages between the cylinder chambers, it has been proved that P_p , P_n , v , y converge, respectively, to P^e , P^e , O , y^e . Meanwhile, the situations are different for a symmetrical and a dissymmetrical actuator. In the case of a dissymmetrical actuator the common pressure value P^e is determined by the equilibrium conditions and the equilibrium position y^e is related to the initial mass contained in both cylinder chambers. In the case of a symmetrical actuator the common pressure value P^e depends on the mass contained in both cylinder chambers and the equilibrium position which is reached cannot be simply deduced from the initial conditions.

As far as a device with servovalve leakages is concerned, the equilibrium pressures reached are determined only by the control value and are independent of the initial conditions. In this case it is also proved that v , y converge, respectively, to O and y^e where y^e depends on the initial conditions.

The study of the nonlinear model properties leads to a better understanding of the open-loop system functioning. It can also be useful to carry out a nonlinear control design by using the open-loop dissipative properties. This study points out some shortcomings of classic nonlinear models of the literature. In particular, it emphasizes the necessity of the modelization of the actuator end stroke in order to avoid the finite escape time phenomenon. A second important point is that the compressibility modulus should vanish with the pressure in order to avoid negative pressure.

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Nomenclature

A_p , A_n	= useful piston areas
B	= bulk modulus
b	= coefficient of viscous friction
F_0	= load force
$F_h(y)$	= hydraulic force
$F_s(y)$	= stop force
l_p , l_n	= extreme values of position
M	= cylinder piston and load mass
P_p , P_n	= chamber pressures
$Q_p(u, P_p)$, $Q_n(u, P_n)$	= volumetric flow rates
T_h	= hydraulic period
u	= control input
u_n	= maximal servovalve voltage
v	= piston velocity
$V(t)$	= volume of a chamber
V_0	= chambers volume at null position
$V_p(y)$, $V_n(y)$	= volumes of the chambers with the piston at position y
y	= position of the load
y^e	= equilibrium position
$\dot{m}$	= mass flow rate
$\dot{m}_l(P_p, P_n)$	= internal leakage between the cylinder chambers
$\mathcal{E}_k$	= kinetic energy
$\mathcal{E}_m$	= mechanical energy
$\mathcal{E}_{ch}(V, P, P_0)$	= compressibility energy of the fluid
ρ	= density of the fluid
φ_p , φ_n	= pressure gain curves
$\mathcal{K}_h(y^e)$	= hydraulic stiffness at position y^e

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